Title: Moduli Stabilization and Holographic RG

Date: Dec 04, 2012 02:00 PM

URL: http://www.pirsa.org/12120002

Abstract: In this talk, I will
relate moduli stabilization in AdS or de Sitter space to basic properties of the Wilsonian action in the holographic dual theory living on dS (of one lower dimension): the single-trace terms in the action have vanishing beta functions, and higher-trace couplings are determined purely from lower-trace ones (a property we refer to as the iterative structure of RG). In the dS case, this encodes the maximal symmetry of the bulk spacetime in a quantity which is accessible within a single observer's patch.<strongr>

</strongr>

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Stanford University
December 4th, 2012

Based on arXiv: 1209.5392

MODULI STABILIZATION AND HOLOGRAPHIC RG IN (A)dS

Quantum Fields and String Seminar, Perimeter Institute

Question

If some Lorentzian field theory is holographically dual to de Sitter space, what properties should it have?

Progress in understanding quantum gravity

- Holographic principle

  Entropy / area law

$$S = \frac{\text{Area}}{4G_N}$$

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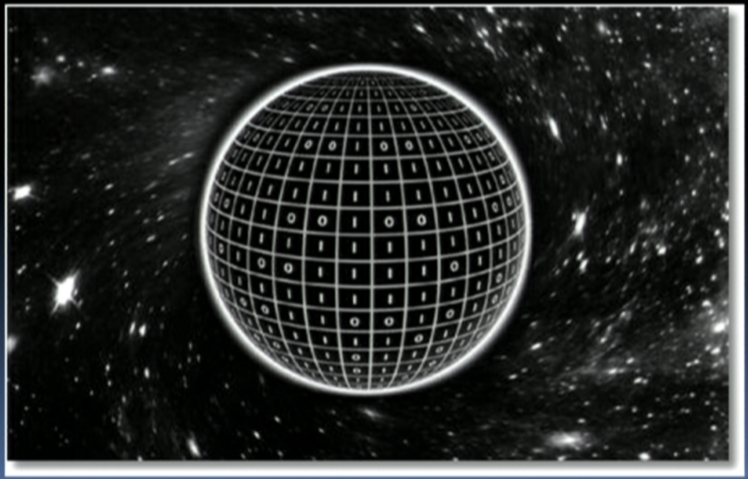

Progress in understanding quantum gravity

- Holographic principle

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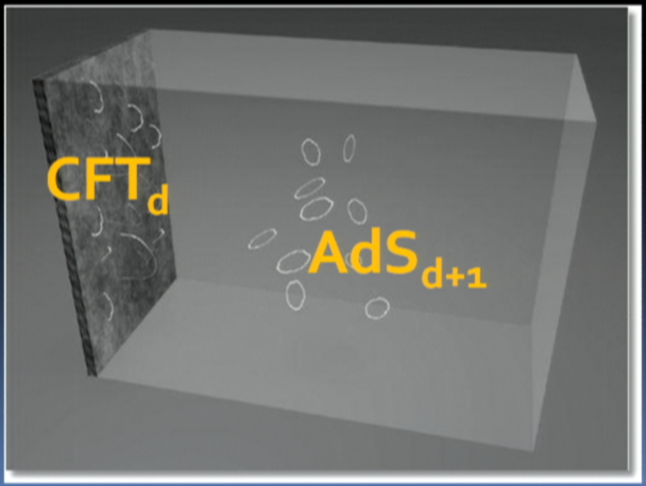
$$\mathcal{S} = \frac{\text{Area}}{4G_N}$$

- AdS/CFT: provides complete description of quantum gravity in AdS.



Yet to be understood:

- At present, we lack a theoretical framework for quantum gravity in cosmological spacetimes (e.g. de Sitter, FRW).
- Can we use what we learned from AdS/CFT?

Challenges in generalizing AdS/CFT to cosmology

- Absence of a non-fluctuating time-like boundary.
- Absence of supersymmetry.
- Dynamical gravity usually present in the dual theory (e.g. in dS/dS, FRW/CFT, and dS/CFT).
- Cosmological horizons, singularities, metastability, time-dependence, ...

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dS/dS Correspondence

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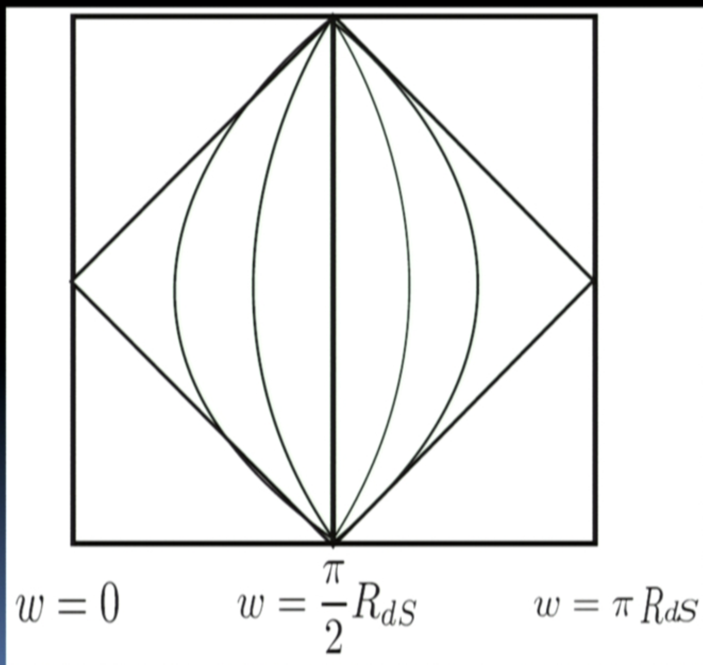
[Alishahiha, Karch, Silverstein, Tong '04]
A brane realization given in [XD, Horn, Silverstein, Torroba '10]

$$ds^2_{dS_{d+1}} = dw^2 + \sin^2\left(\frac{w}{R_{dS}}\right) ds^2_{dS_d}$$



- Two throats joined smoothly in the UV.
- d-dimensional EFT dual for each throat.
- EFT has finite UV cutoff + dynamical gravity.
- Analogous to Randall-Sundrum / warped compactification.

dS/dS correspondence

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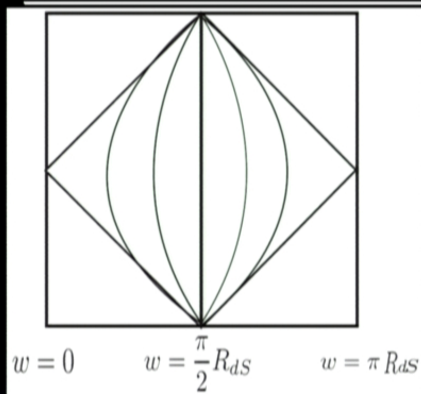

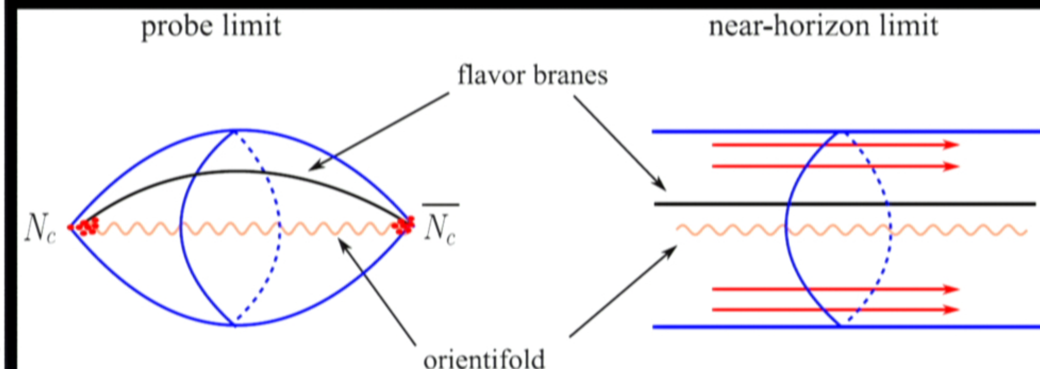

- Put color branes at the tips of a compact cone supported by flavor branes and orientifolds
- Near horizon limit gives dS as two warped throats
- Counting microscopic d.o.f gives de Sitter entropy

Contrast dS/dS with dS/CFT

dS/dS

- Lorentzian
- Manifestly unitary
- Covers static patch: accessible to one observer
- Global de Sitter symmetry not explicit

dS/CFT

- Euclidean
- Non-unitary CFT
- Contains meta-observables
- Manifestly preserves maximal symmetry of global de Sitter through conformal invariance

Can we find manifestation of de Sitter symmetry in dS/dS?

It is not conformal invariance, but should be something equally constraining.

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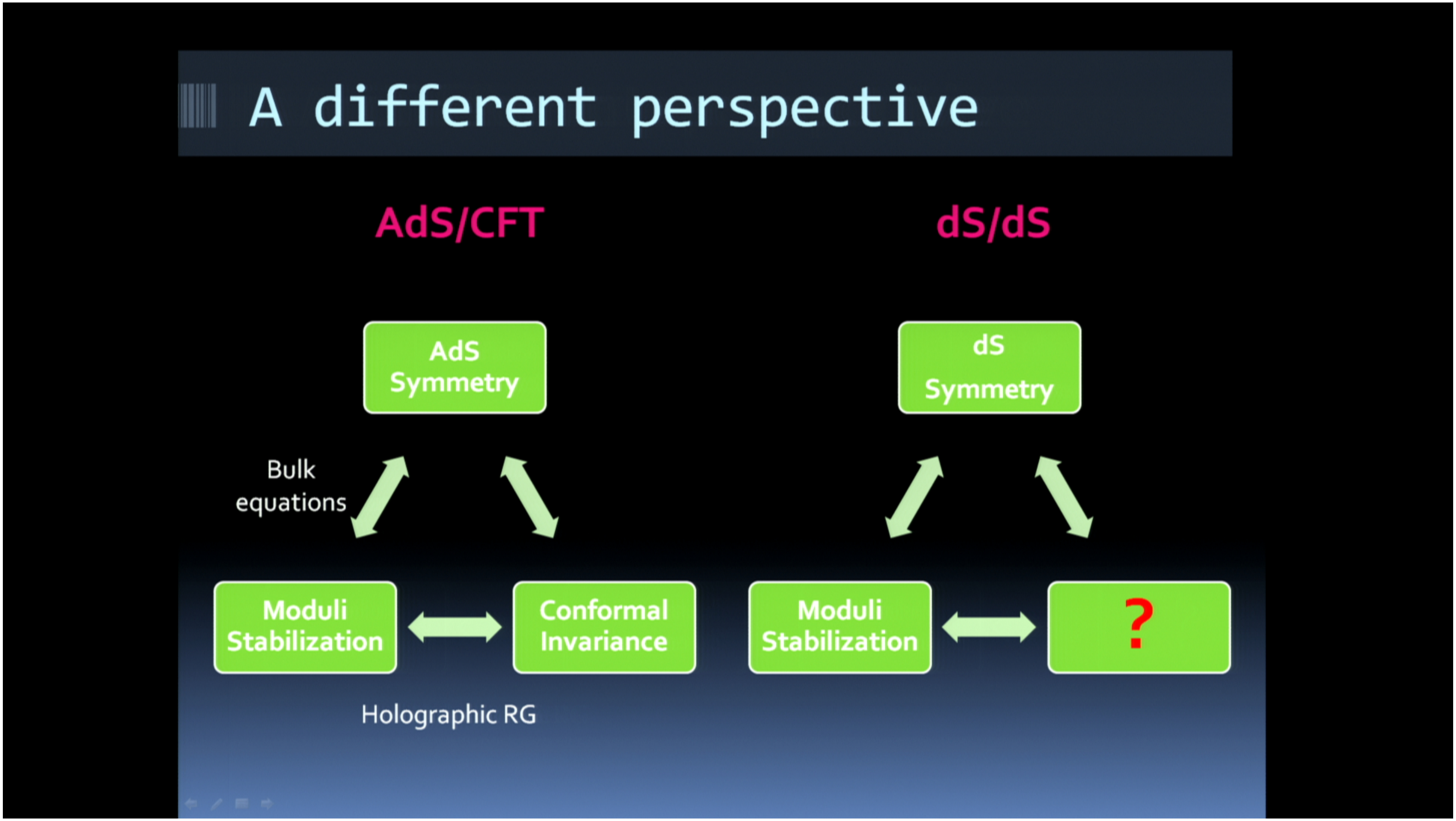
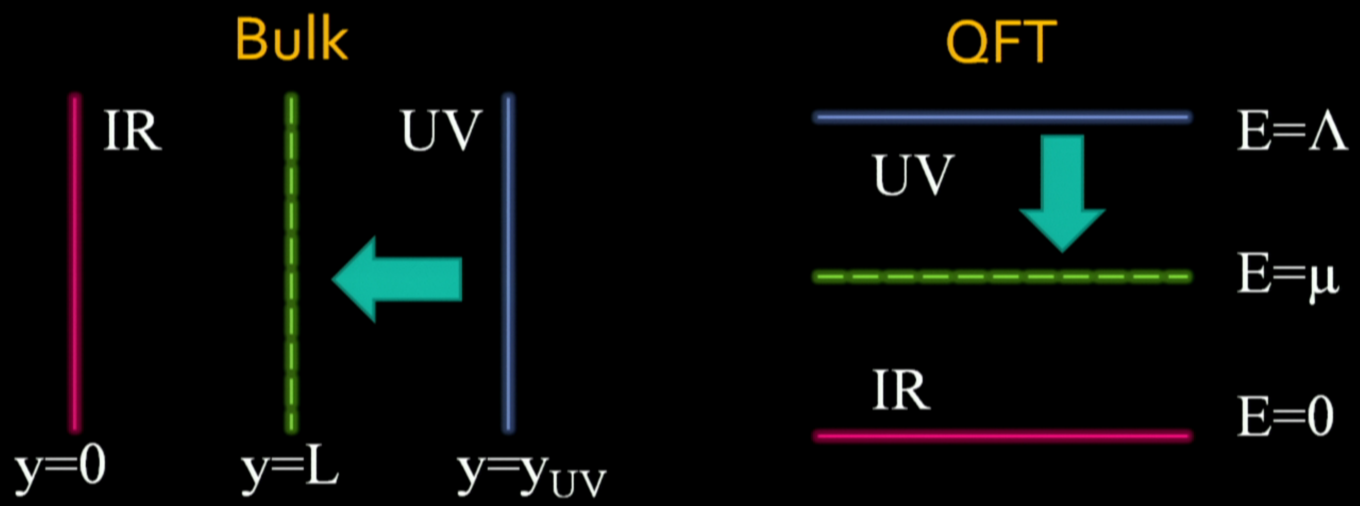

A different perspective
AdS/CFT
dS/dS
AdS Symmetry
Bulk equations
Moduli Stabilization
Conformal Invariance
Holographic RG
dS Symmetry
Moduli Stabilization
?

Holographic Wilsonian RG

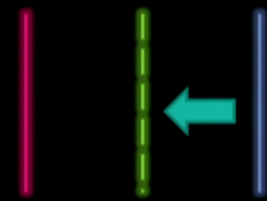
[Heemskerk, Polchinski '10]



$$ds^2 = dy^2 + a(y)^2 \hat{g}_{\mu\nu} dx^\mu dx^\nu$$

- Identify bulk radius y with QFT scale: $E_{\text{QFT}} = E_{\text{proper}} a(y)$
- Supergravity modes: $E_{\text{proper}} \sim \frac{1}{R}$
- String modes: $E_{\text{proper}} \sim \frac{1}{\sqrt{\alpha'}}$

Holographic Wilsonian RG



$$Z_{\mathrm{QFT}} = Z_{\mathrm{bulk}} = \int \mathcal{D}\phi \, e^{-\frac{1}{\kappa^2}\mathcal{S}}$$

$$= \int \mathcal{D}\tilde{\phi} \underbrace{\int \mathcal{D}\phi|_{y>L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y>L}}}_{\Psi_{\mathrm{UV}}(\tilde{\phi}, L)} \underbrace{\int \mathcal{D}\phi|_{y<L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y<L}}}_{\Psi_{\mathrm{IR}}(\tilde{\phi}, L)}$$

- In QFT, integrate out UV modes ➡ effective action
- Ψ_{UV} contains information about effective action
- Need to express it in terms of QFT variables

Holographic Wilsonian RG

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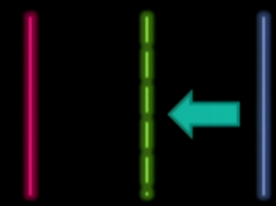


$$= \int \mathcal{D}\tilde{\phi} \int \mathcal{D}\phi|_{y>L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y>L}} \int \mathcal{D}\phi|_{y<L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y<L}}$$

$$\underbrace{\int \mathcal{D}\phi|_{y>L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y>L}}}_{\Psi_{\rm UV}(\tilde{\phi}, L)} \qquad \underbrace{\int \mathcal{D}\phi|_{y<L} e^{-\frac{1}{\kappa^2}\mathcal{S}|_{y<L}}}_{\Psi_{\rm IR}(\tilde{\phi}, L)}$$

Motivated by standard AdS/CFT dictionary:

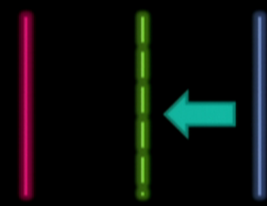
$$\boxed{\Psi_{\rm IR} = \int \mathcal{D}M|_{E<\mu_L} \, e^{-S_{\rm QFT}[M] + \frac{1}{\kappa^2}\int d^d x \, \tilde{a}^d \sqrt{\tilde{g}} \, \tilde{\phi}\mathcal{O}}}$$

$M =$ matrix field

$\mathcal{O} =$ single-trace operator $\mathrm{tr}(\partial^n M \cdots \partial^{n'} M)$

$\mu_L \sim \frac{a(L)}{R}$ (for supergravity modes)

Holographic Wilsonian RG



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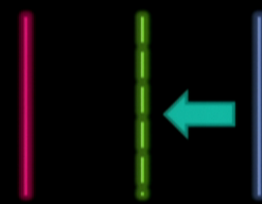
where $\boxed{e^{-\frac{1}{\kappa^2}s[\mathcal{O}]} = \int \mathcal{D}\tilde{\phi} \, \Psi_{\rm UV}[\tilde{\phi}] \, e^{\frac{1}{\kappa^2}\int d^dx \, \tilde{a}^d \sqrt{\tilde{g}} \, \tilde{\phi}\mathcal{O}}}$

Effective action $s[\mathcal{O}] =$ integral transform of $\Psi_{\rm UV}[\tilde{\phi}]$

Holographic Wilsonian RG



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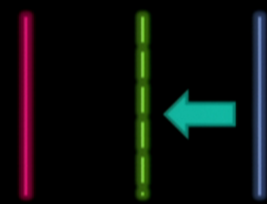

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Finally, expand in single- and multi-trace operators:

$$s = \int d^dx\, \tilde{a}^d \sqrt{\hat{g}} \left(\sigma_0 + \sigma_1 \mathcal{O} + \frac{1}{2}\sigma_2 \mathcal{O}^2 + \ldots \right)$$

- $\sigma_n(L)$ = basically the dimensionless couplings of QFT
- For CFT (on R^d), they are all independent of L.

Results [for (A)dS_{d+1}/dS_d at large N]

$$s = \int d^d x\, \tilde{a}^d \sqrt{\hat{g}} \left(\sigma_0 + \sigma_1 \mathcal{O} + \frac{1}{2}\sigma_2 \mathcal{O}^2 + \dots \right)$$

If ϕ sits at extremum ϕ_* of bulk potential V(ϕ), then

1. $\sigma_1 = -\phi_*$ does not depend on L ➡ $\beta_{\text{single-trace}} = 0$
2. σ_n ($n \neq 1$) do depend on L, but their beta function only involves lower-trace couplings (σ_m with $m \leq n$).

"Iterative structure of RG flow"

$$\partial_L \sigma_0 = -d \frac{\partial_L \tilde{a}}{\tilde{a}} \sigma_0 - V(-\sigma_1)$$

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$$\partial_L \sigma_n = (n-1) d \frac{\partial_L \tilde{a}}{\tilde{a}} \sigma_n + \sigma_{n+1} V'(-\sigma_1) + \text{terms involving } \sigma_2 \text{ through } \sigma_n$$

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AdS flat slicing:

$$\sigma_0 = -V_*/d$$

$$\sigma_1 = -\phi_*$$

$$\sigma_2 = -1/\Delta_\pm$$

$$\dots$$

Comments

- A set of strong constraints for candidate dual theories for dS static patch
- Help construct and elucidate specific duals, e.g. the brane construction in [XD, Horn, Silverstein, Torroba '10]
- Allow explicit reconstruction of the bulk potential from QFT variables

$$
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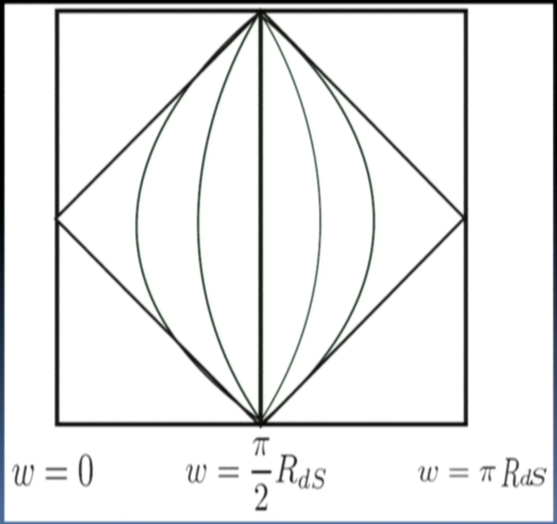
\end{aligned}
$$

Generalizations

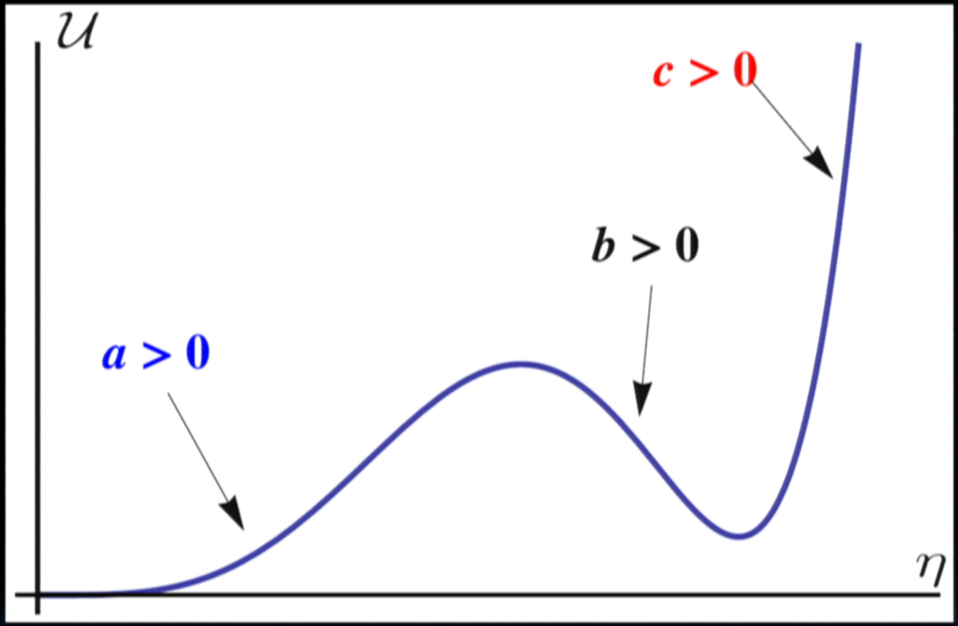
- Non-zero modes
- Multiple scalar fields
- Bulk fields with nonzero spins
- Backreaction on the metric (dual to stress tensor)
- Counterterms $e^{S_{\mathrm{CT}}[\phi_{\mathrm{UV}}]}$

(which is absent in dS/dS

by symmetry, leading to T=0)



Metastability



$$\Psi_{\mathrm{UV}}[\tilde{\phi}] = e^{-W_{\mathrm{UV,pert}}[\tilde{\phi}]} + \sum_{\mathrm{instantons}} e^{-W_{\mathrm{inst}}[\tilde{\phi}]} iK[\tilde{\phi}]$$

Conclusion & future directions

- We have identified necessary (and perhaps sufficient at large N) conditions for a dual theory of de Sitter static patch.
- $\beta_{\text{single-trace}}=0$
- Multi-trace couplings have iterative RG structure.
- Apply to previous brane constructions of de Sitter?
- These constraints hold for each stabilized scalar field, so we can study simple uplifted examples where only some moduli are stabilized.
- Lower-dimensional dynamical gravity in dS/dS not yet included. Liouville gravity in d=2?