

Title: Approaches to modeling pulsar magnetospheres

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Abstract: Pulsars are rotating magnetized neutron stars that emit broadband pulses of radiation. Our ability to model magnetospheres of pulsars has been hampered by the difficulty of solving the self-consistent behavior of strongly magnetized relativistic plasmas. I will describe
 recent progress in numerical modeling of magnetically-dominated plasmas and show applications to pulsar magnetospheres in increasing levels of realism, including ideal and resistive force-free,
 relativistic MHD and kinetic models. The knowledge of the magnetospheric shape together with the new observations of gamma-ray emission from pulsars with Fermi telescope allow to directly constrain the location and physics of the acceleration regions in the magnetosphere and the origin of high energy emission. The pulsar magnetosphere is a prototype for other strongly magnetized
 astrophysical objects, and I will discuss how the lessons from pulsar modeling can be useful in predicting EM counterparts to gravitational wave sources.

Approaches to modeling pulsar magnetospheres

Anatoly Spitkovsky (Princeton)

(with J. Arons, X. Bai, J. Li, L. Sironi, A. Tchekhovskoy)

Pulsars are rotating neutron stars, born in supernova explosions. They emit periodic pulses of radiation and slow down with time.

The Crab Nebula

In the year 1054 A.D., Chinese astronomers were startled by the appearance of a new star, so bright that it was visible in broad daylight for several weeks. Today, the Crab Nebula is visible at the site of this violent stellar explosion. In this view, NASA's Hubble Space Telescope has zoomed in on a portion of the Crab to reveal its detailed structure.

Located about 2 kpc (6,500 ly) from Earth in the direction of the constellation Taurus, the Crab Nebula is the remnant of a star that began its life with about 8-10 times the mass of our Sun. Such a massive star consumes its nuclear fuel so rapidly that it lives only about 50 million years before exploding as a supernova. For this star, the end came on July 4, 1054. The explosion was witnessed as a "mystery" "Guest Star" by Chinese astronomers, and is also depicted in rock paintings of native Americans in the southwestern United States.

This image was created by the Hubble Heritage Team from data obtained by Hubble's Wide Field and Planetary Camera 2. Images taken with five different color filters, totaling over 10 hours of exposure time, have been combined to construct this false-color picture. Resembling an abstract painting, the image shows ragged gaseous shreds of the original star that are expanding away from the explosion site at over 1,500 km/s (3.4 million mph). The colorful network of filaments is the material from the outer layers of the star that was expelled during the explosion.

The core of the star has survived the explosion as a "pulsar," visible in the Hubble image as the lower right of the two moderately bright stars near the center. The pulsar has about 1.4 times the mass of the Sun, crumpled by gravity into an object only about 10 miles in diameter. This incredible object, a "neutron star," is even more remarkable because it spins on its axis 30 thirty times a second. The spinning pulsar heats its surroundings, creating the ghostly diffuse bluish-green synchrotron cloud in its vicinity, including a blur at toward the upper right of the neutron star.

The picture is somewhat deceptive in that the filaments appear to be close to the pulsar. In reality, the yellowish green filaments toward the right side of the image are closer to us, and approaching at some 350,000 km/s. The orange and pink filaments toward the top of the picture, including the "backwards question mark," is material behind the pulsar, rushing away from us at 200,000 km/s.

The various colors in the picture arise from different chemical elements in the expanding gas, including hydrogen (orange), nitrogen (red), sulfur (pink), and oxygen (greenish-blue). The shades of color represent variations in the temperature and density of the gas, as well as changes in the elemental composition.

These chemical elements, some of them newly created during the evolution and explosion of the star and now flung back into space, will eventually be incorporated into new stars and planets. Astronomers believe that the chemical elements in the Earth and even in our own bodies, such as carbon, oxygen, and iron, were made in other exploding stars billions of years ago.

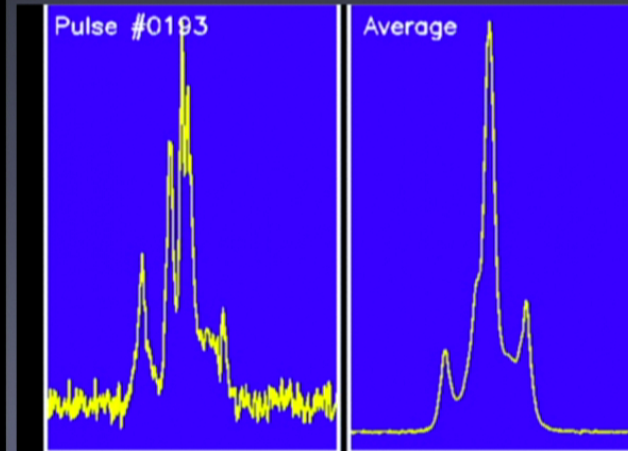
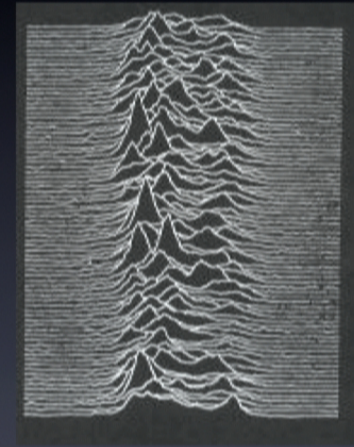
Blair, W. P., Davidson, K., Fesen, R. A., Uemoto, A., MacAlpine, G. M., & Henry, R. B. C., "HST/WFPC2 Imaging of the Crab Nebula. I. Observational Overview," 1997, *ApJS*, 109, 473

<http://heritage.stsci.edu>

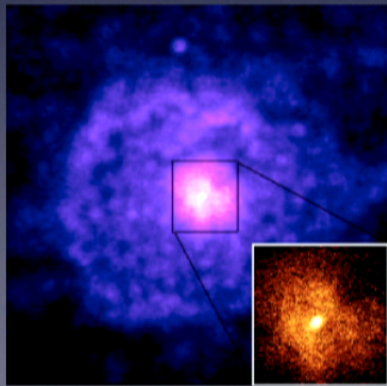
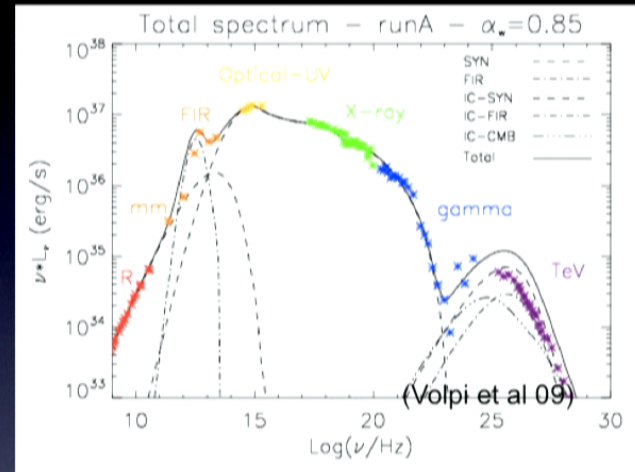
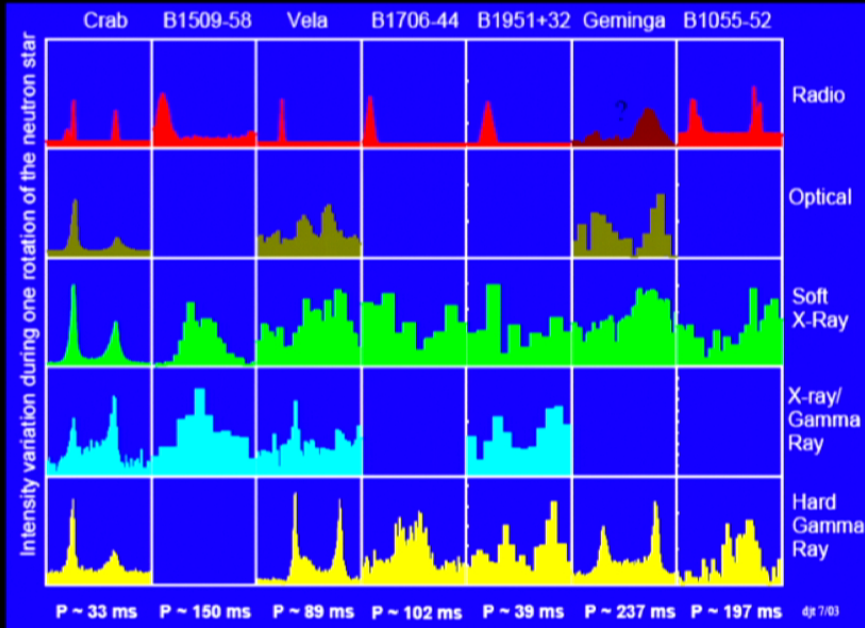
HST • WFPC2

W.P. Blair (JHU),
K. Davidson (U. Minnesota) and
The Hubble Heritage Team:
K. Noll, H. Bond,
C. Christian, J. English,
L. Frattare, F. Hamilton,
and Z. Levay (STScI)

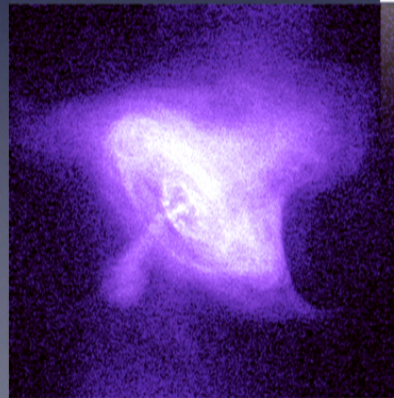
Green F502N [O III]
Blue F547M Strömgren y
Orange F656W H α
Red F658N [N II]
Pink F673W [S II]



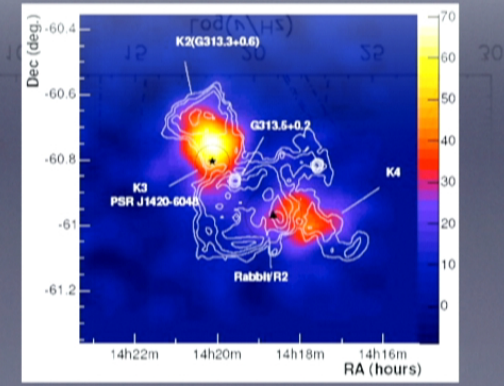
- Broadband pulsed emission, now > 100 GeV (Veritas).
- PWNe: radio-TeV. 10^{40} pairs/sec. Also, flares!



G21.9 (Safi-Harb et al 2004)

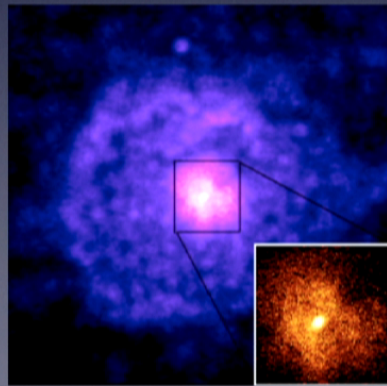
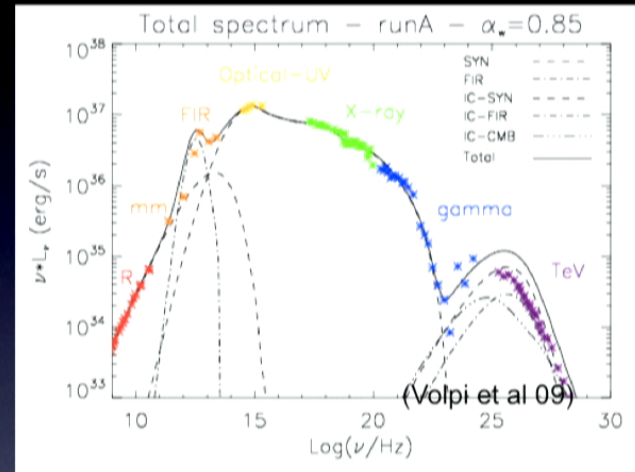
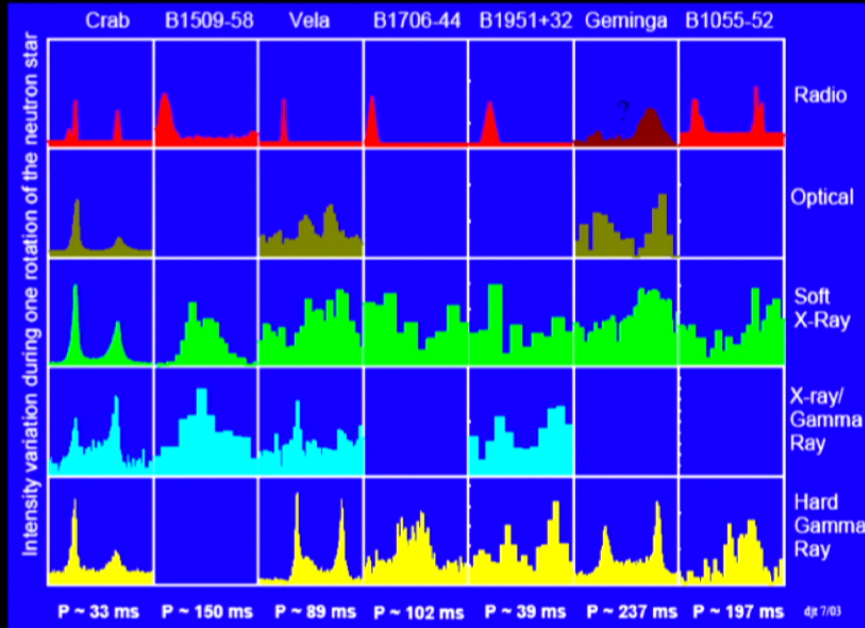


Crab (Weisskopf et al 2000)

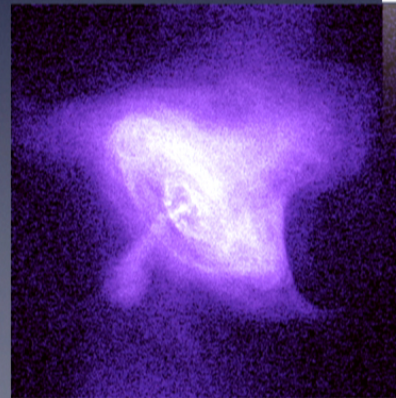


HESS J1420 (Aharonian et al 2006)

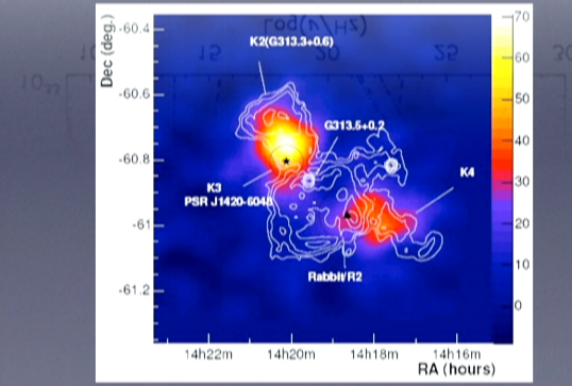
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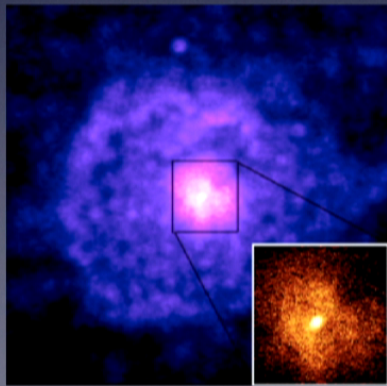
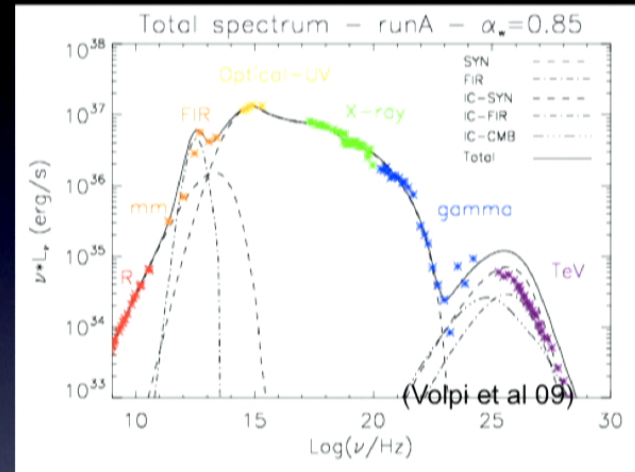
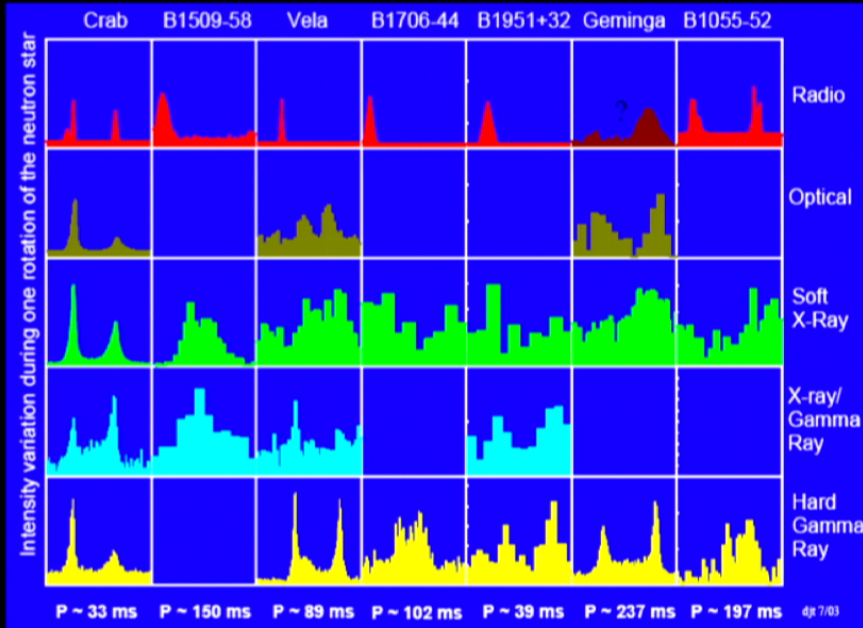


Crab (Weisskopf et al 2000)

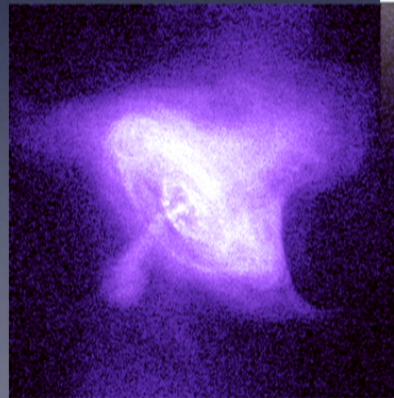


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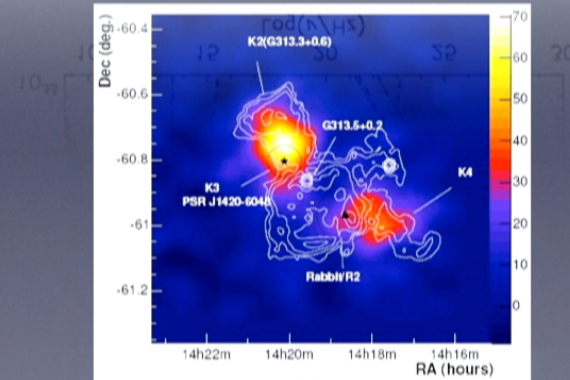
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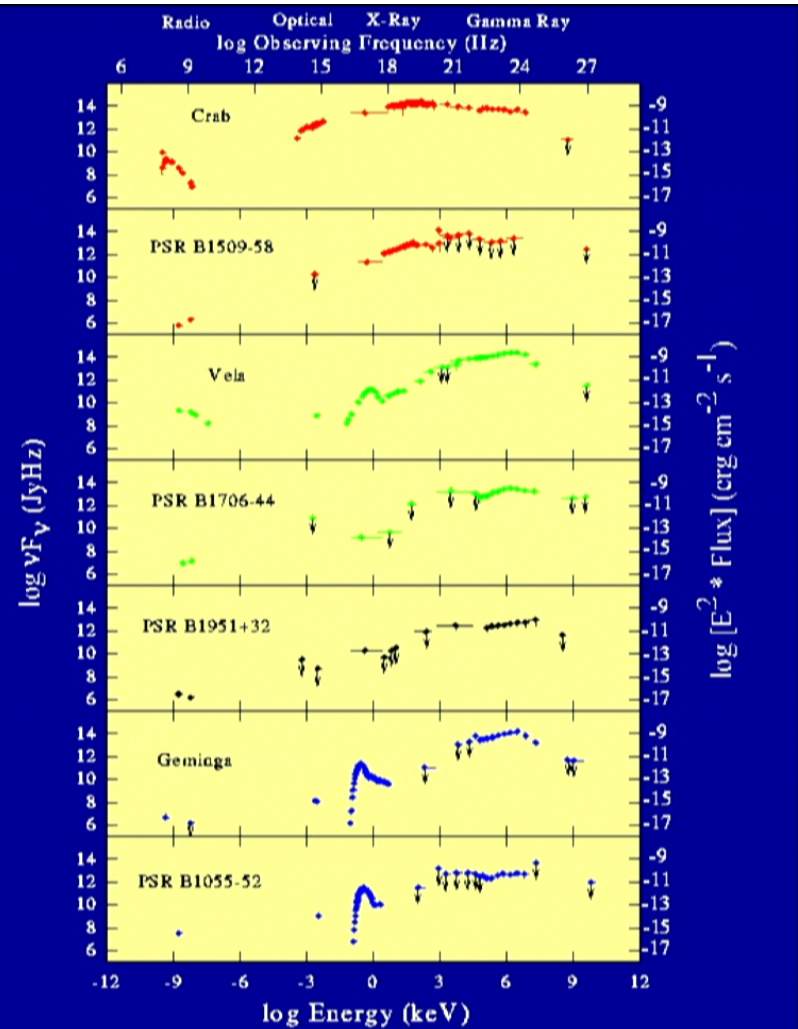
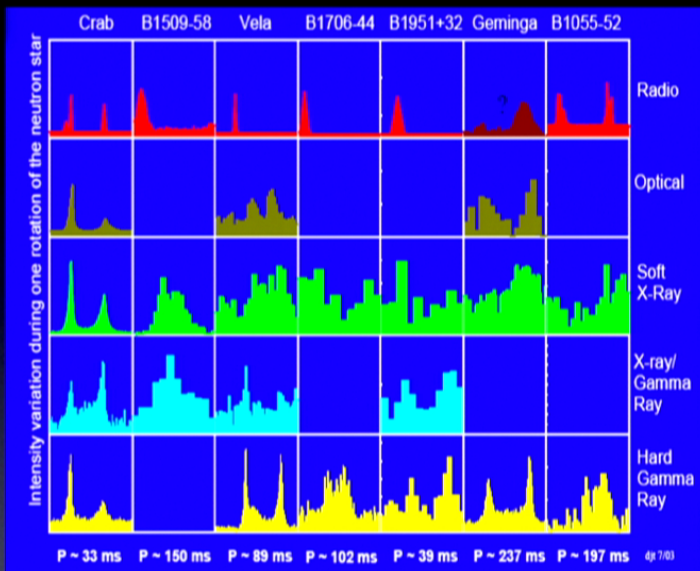
G21.9 (Safi-Harb et al 2004)



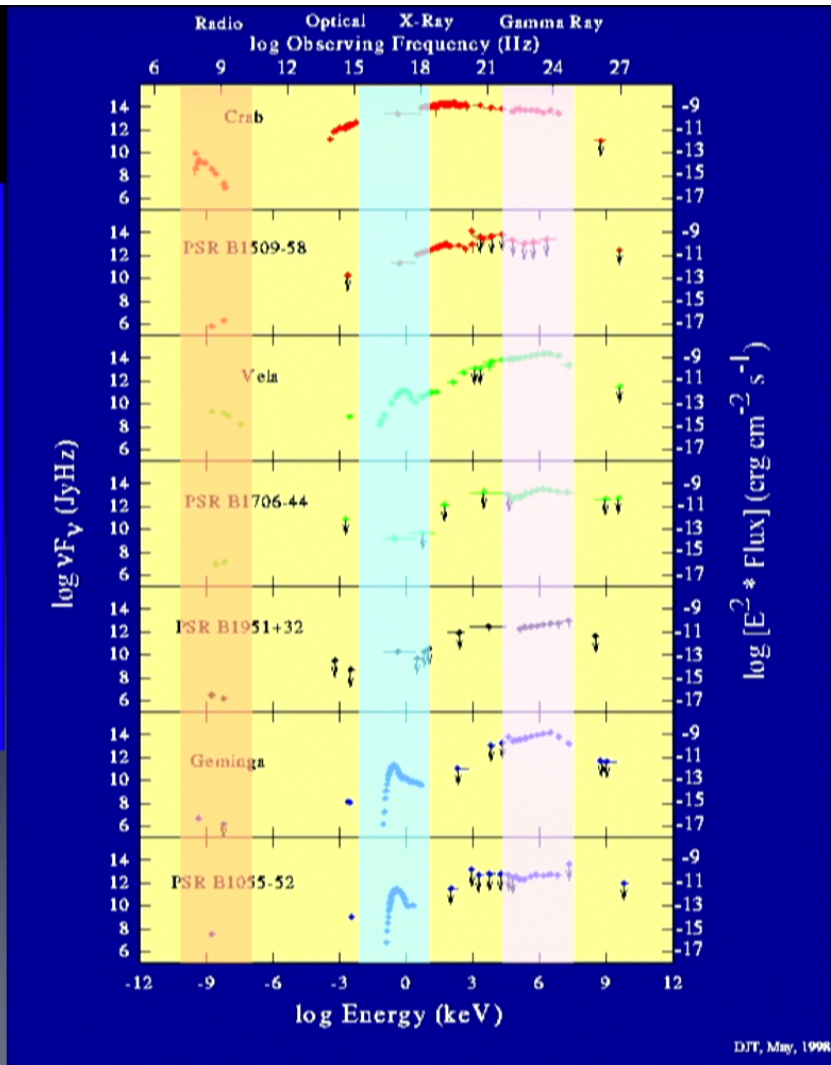
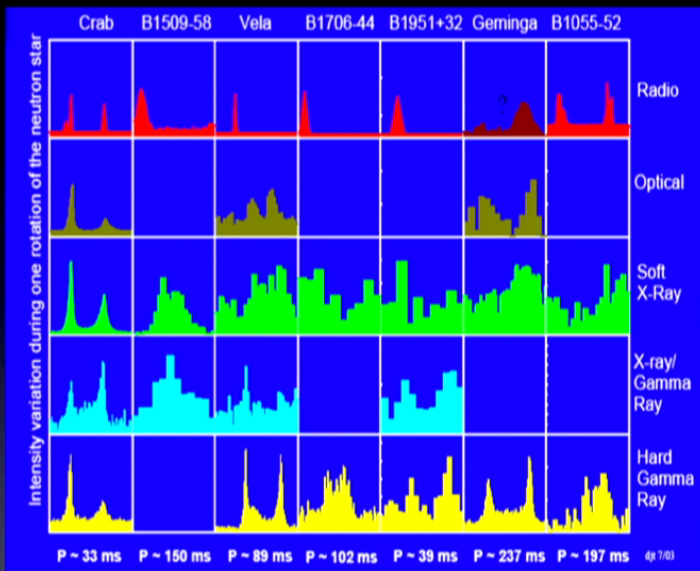
Crab (Weisskopf et al 2000)



HESS J1420 (Aharonian et al 2006)



DJT, May, 1998

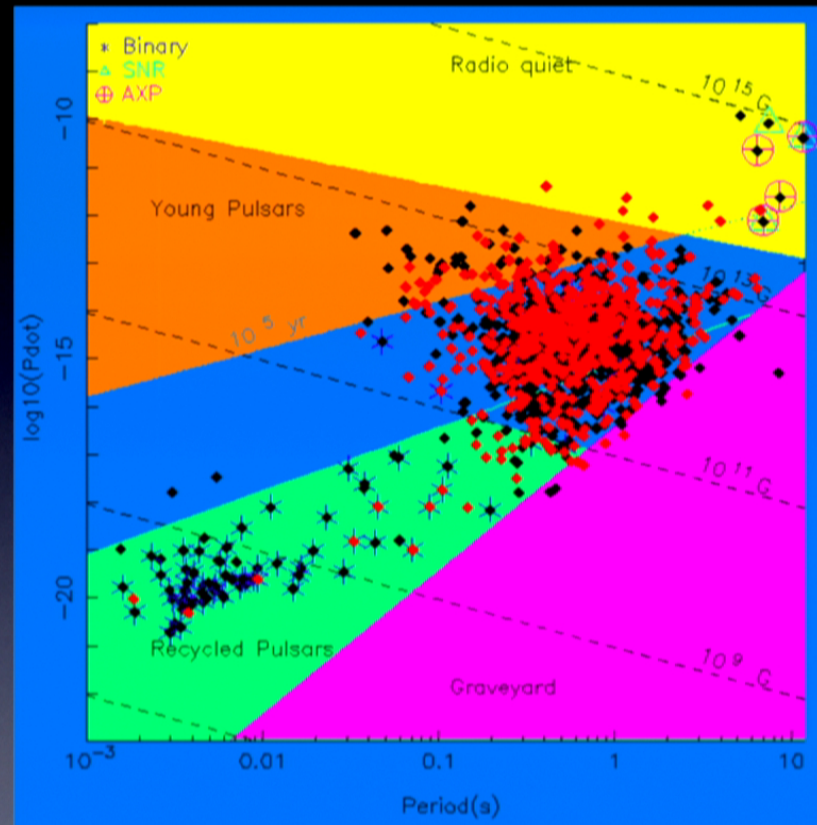


Main energy loss is invisible,
but detectable -- pulsar
spin-down

Leaves as magnetized wind
(carrying Poincaré flux)

The fact that γ -ray power
reaches 10-s of percent of
spin-down power implies
that we are tapping the
main magnetospheric
currents

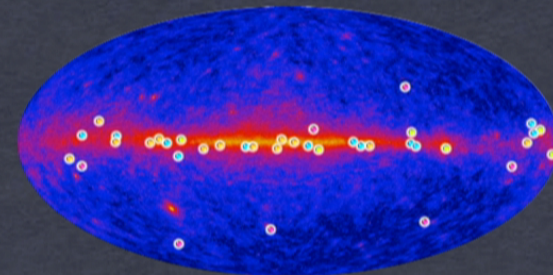
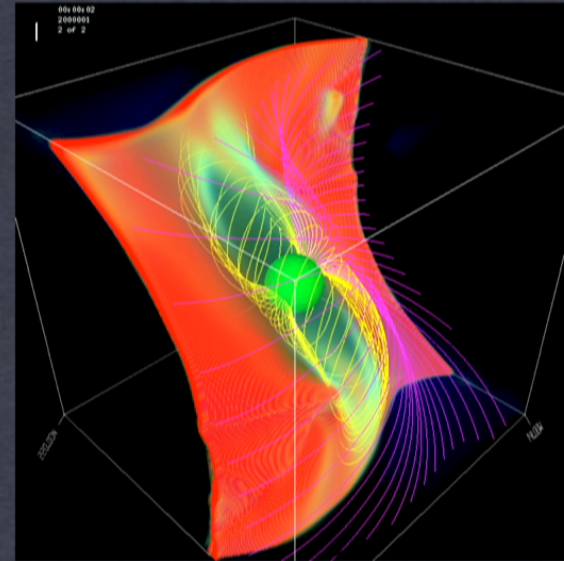
Need to understand how
magnetosphere works

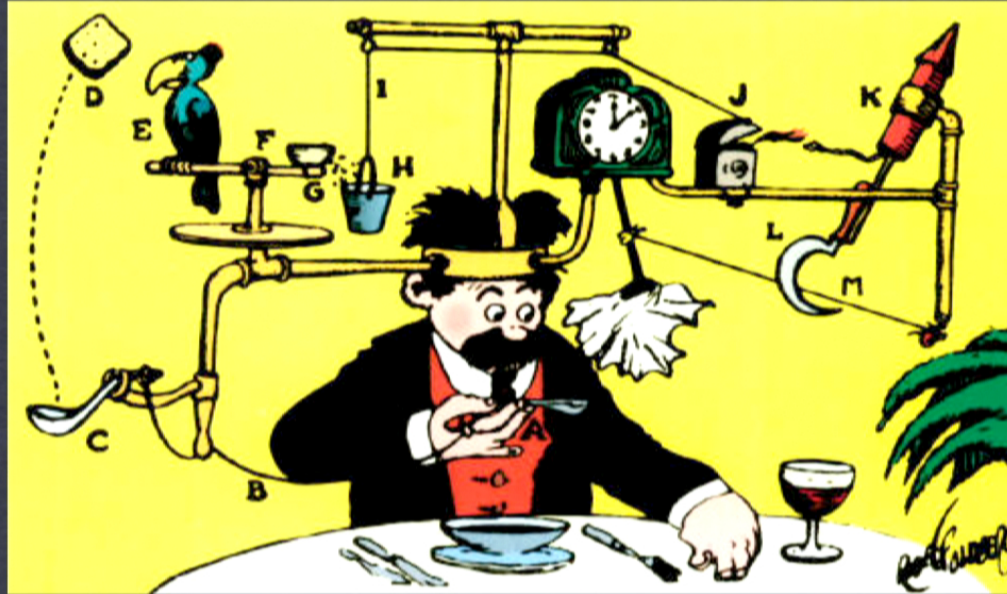


Pulsars in Fermi era

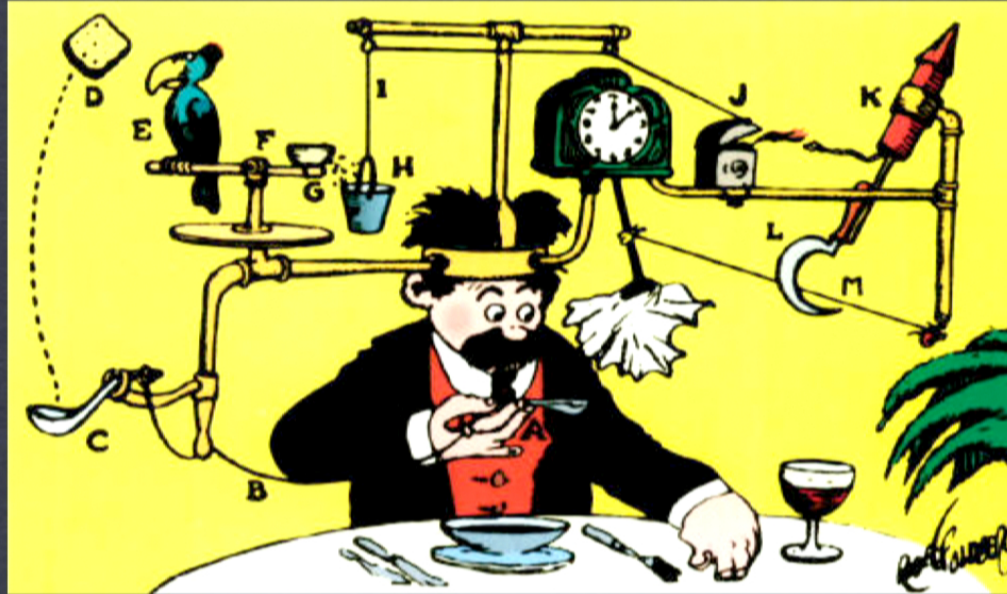
Why pulsars are interesting?

- Unique laboratory for strong B fields and relativistic plasmas
- Prototypes of other astrophysical objects: accretion disks, jets, black hole magnetospheres
- Incredible electromagnetic machines
- Not understood for > 40 yrs
- Prime sources for Fermi





self-operating napkin



self-operating napkin

Pulsars: observationally driven

Pulsars: observationally driven



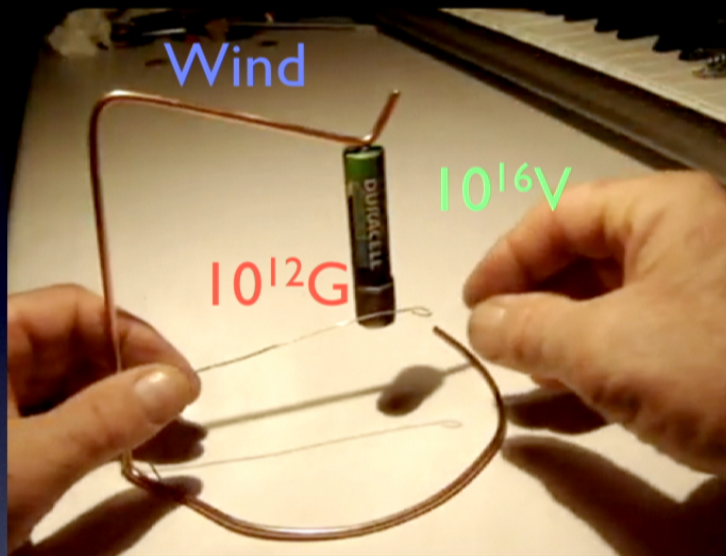
Pulsar theory:



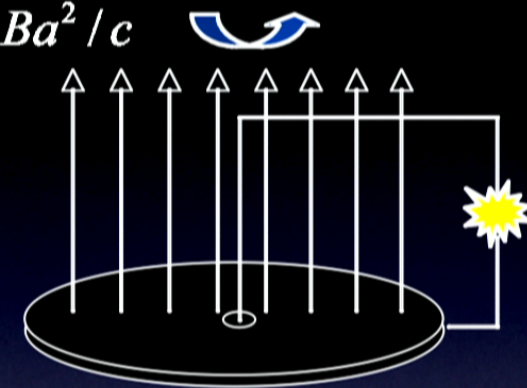
Open questions:

- * The structure of the magnetosphere is the primary question, as all emission physics must be done in the context of proper magnetospheric geometry.
- * Related question is the nature of the spin-down: most energetic, but mostly invisible, process in normal pulsars.

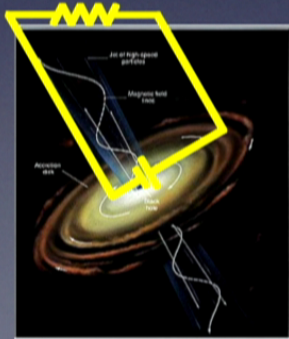
Pulsar physics in space



$$\phi_0 = \Omega B a^2 / c$$



Faraday disk
Unipolar induction



from R. Blandford

Rule of thumb: $V \sim \Omega \Phi$; $P \sim V^2 / Z_0 = I V$

Crab Pulsar

$B \sim 10^{12} \text{ G}$, $\Omega \sim 200 \text{ rad s}^{-1}$, $R \sim 10 \text{ km}$

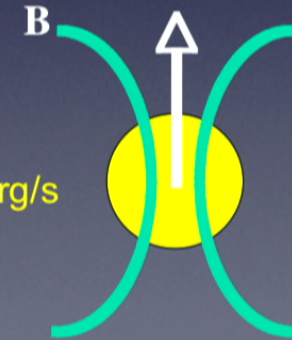
Voltage $\sim 3 \times 10^{16} \text{ V}$; $I \sim 3 \times 10^{14} \text{ A}$; $P \sim 10^{38} \text{ erg/s}$

Magnetar

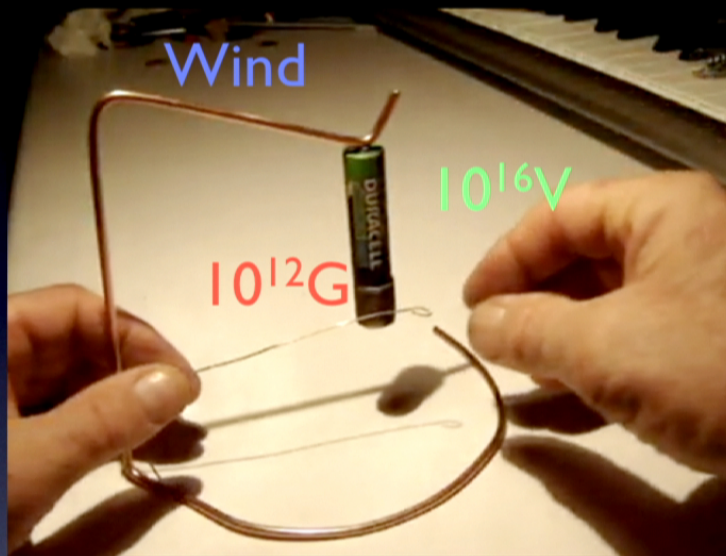
$B \sim 10^{14} \text{ G}$; $P \sim 10^{44} \text{ erg/s}$

Massive Black Hole in AGN

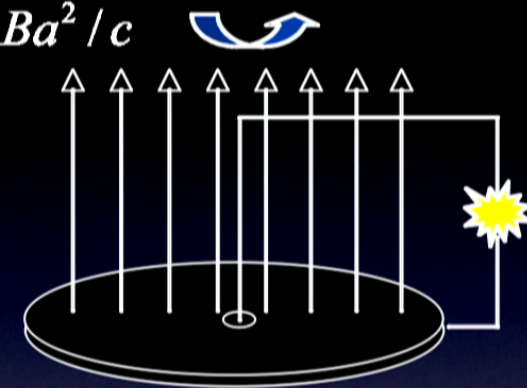
$B \sim 10^4 \text{ G}$; $P \sim 10^{46} \text{ erg/s}$



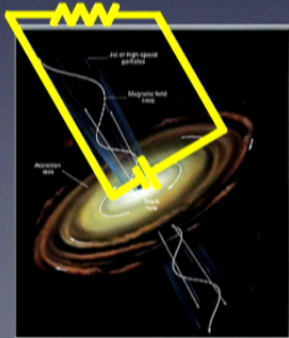
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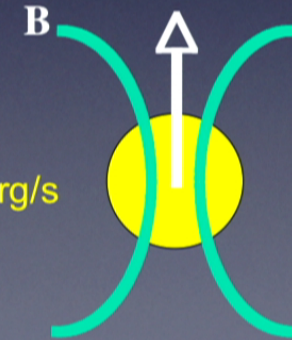
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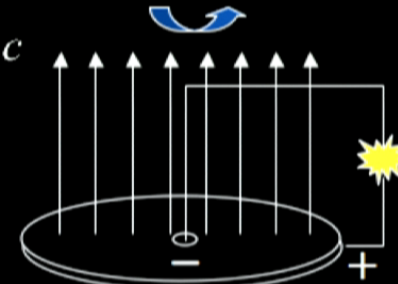
Plasma source

Where does the plasma come from?

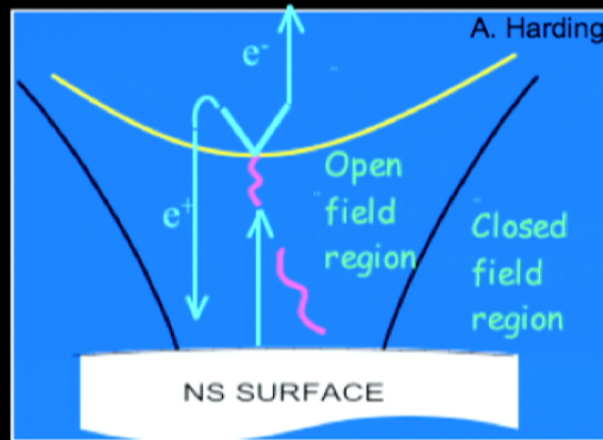
$$\phi_0 = \Omega B a^2 / c$$

Polar cap is a space-charge limited accelerator.
Accelerated primary particles radiate curvature radiation, and pair produce in the strong field.
Pair cascade shorts out $E \cdot B$.

$$\gamma_{\text{primary}} \sim 10^7 \quad \gamma_{\text{secondary}} \sim 10^{2-3} \quad \sigma_{\text{LC}} \sim 10^4$$



Faraday disk: unipolar induction



Arons & Scharleman 79, Muslimov & Harding 03

Electrostatic accelerator, non-MHD region

$$\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = -\frac{\vec{\Omega}}{c} \times \vec{R} \times \vec{B}$$

$$\frac{1}{4\pi} \nabla \cdot \vec{E} = \rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c}$$

$$\vec{j}_{GJ} = \rho_{GJ} c = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi}$$

After pair formation front -- enough plasma to use MHD.

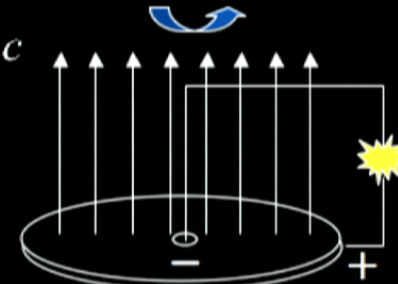
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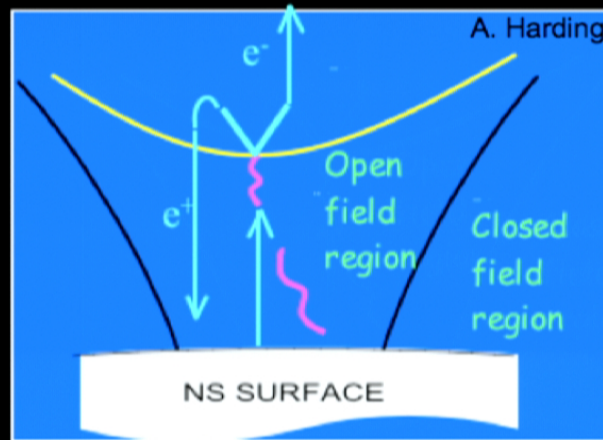
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Faraday disk: unipolar induction



Arons & Scharleman 79, Muslimov & Harding 03

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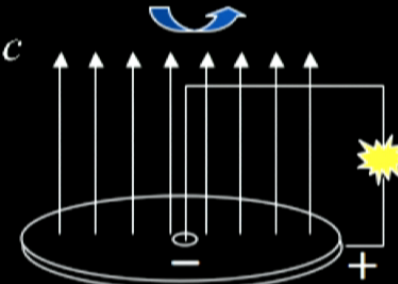
Plasma source

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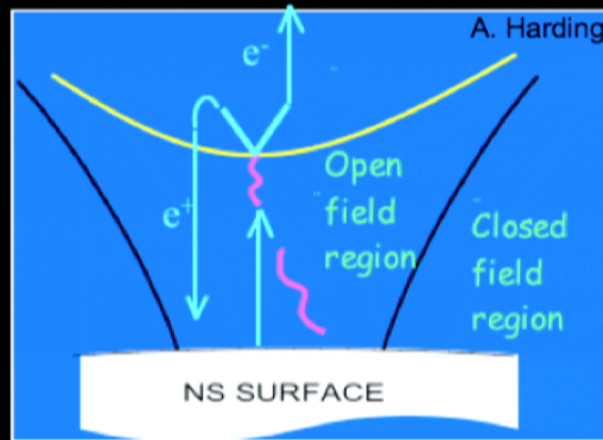
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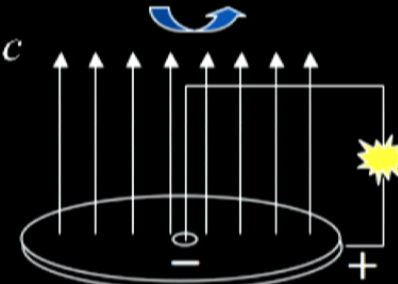
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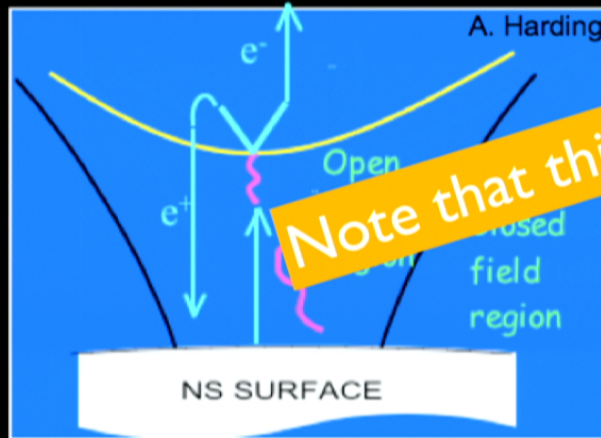
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Arons & Scharleman 79, Muslimov & Harding 03

Electrostatic accelerator, non-MHD region

Note that this is a local model!

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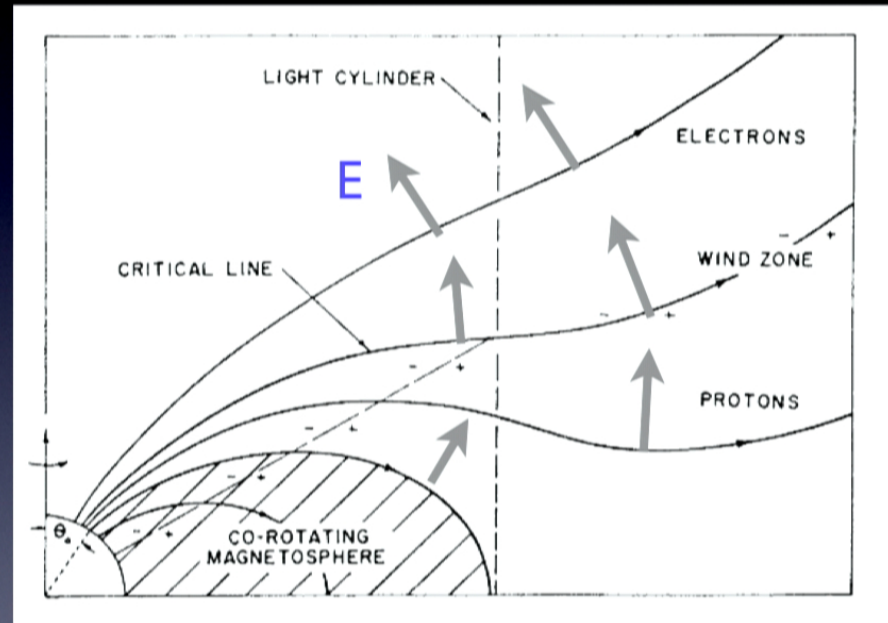
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After pair formation front -- enough plasma to use MHD.

Pulsars: energy loss

$$\rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c}$$

- Corotation electric field
- Sweepback of B field due to poloidal current
- $E \times B \rightarrow$ Poynting flux
- Electromagnetic energy loss

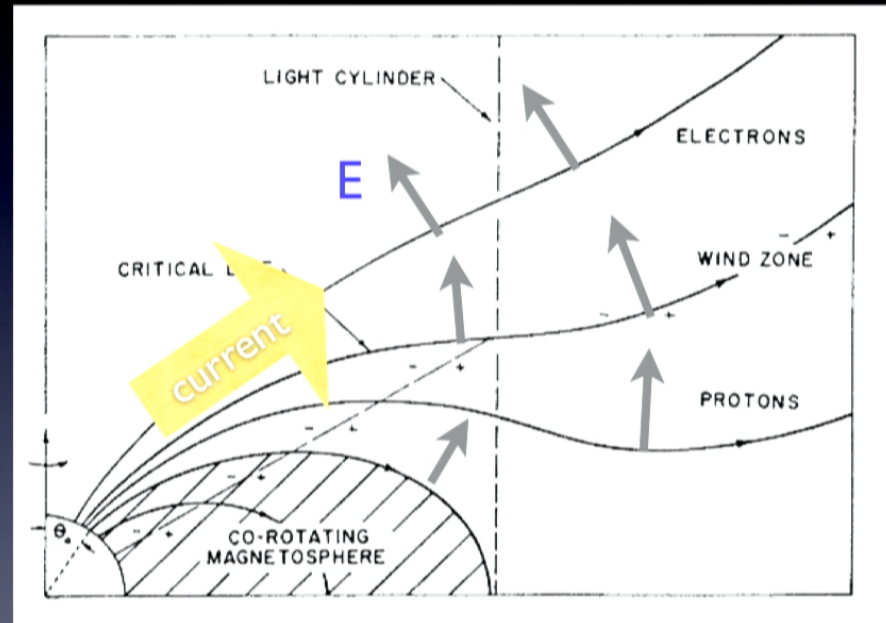


Goldreich & Julian 1969

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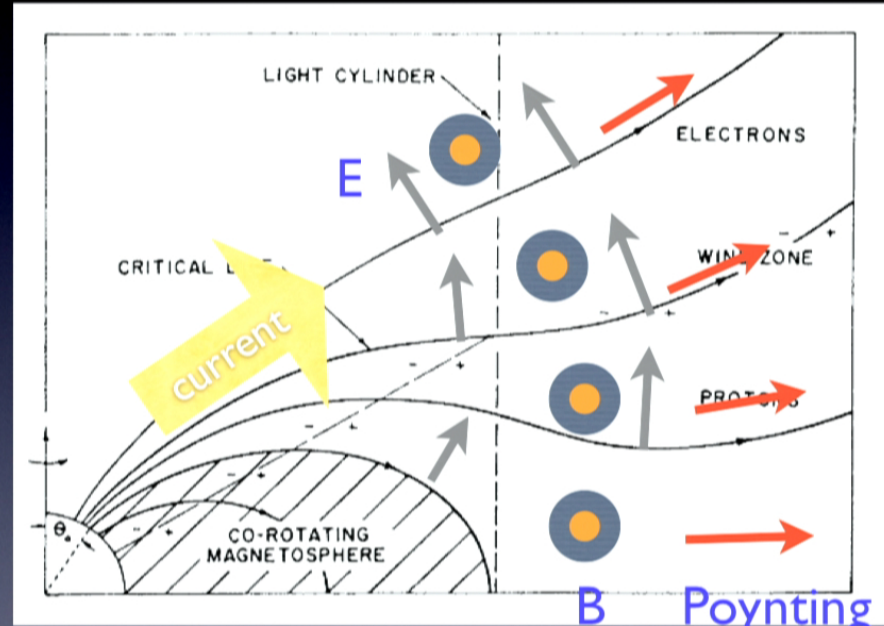
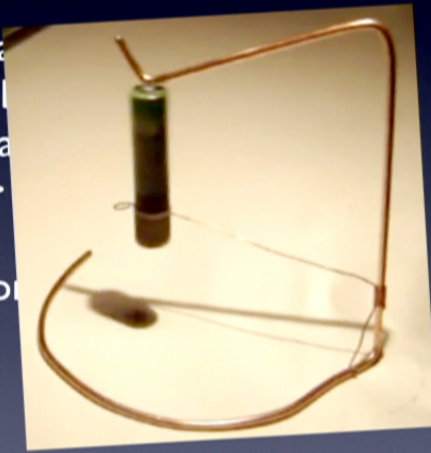


Goldreich & Julian 1969

Pulsars: energy loss

$$\rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c}$$

- Corotating
- Sweeping
- poloidal
- $E \times B \rightarrow$
- Electron



B Poynting
Goldreich & Julian 1969

Magnetospheric cartoon

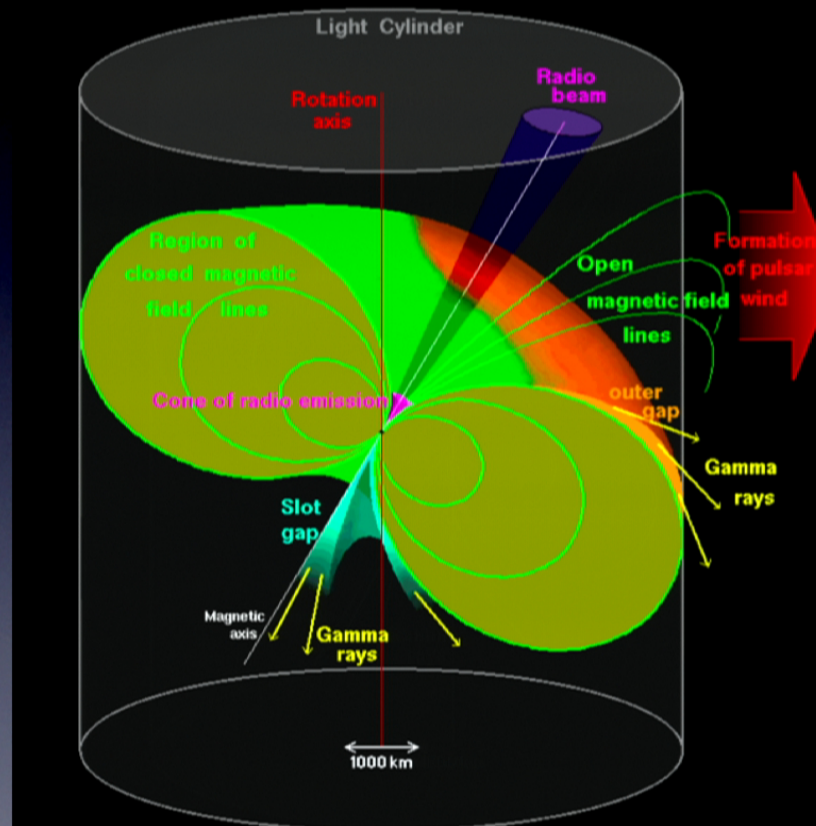
Open + closed
(corotating) zones

Light Cylinder

Sweepback (part
due to dB/dt , part
due to current)

Current modifies
the field

Can we model this?



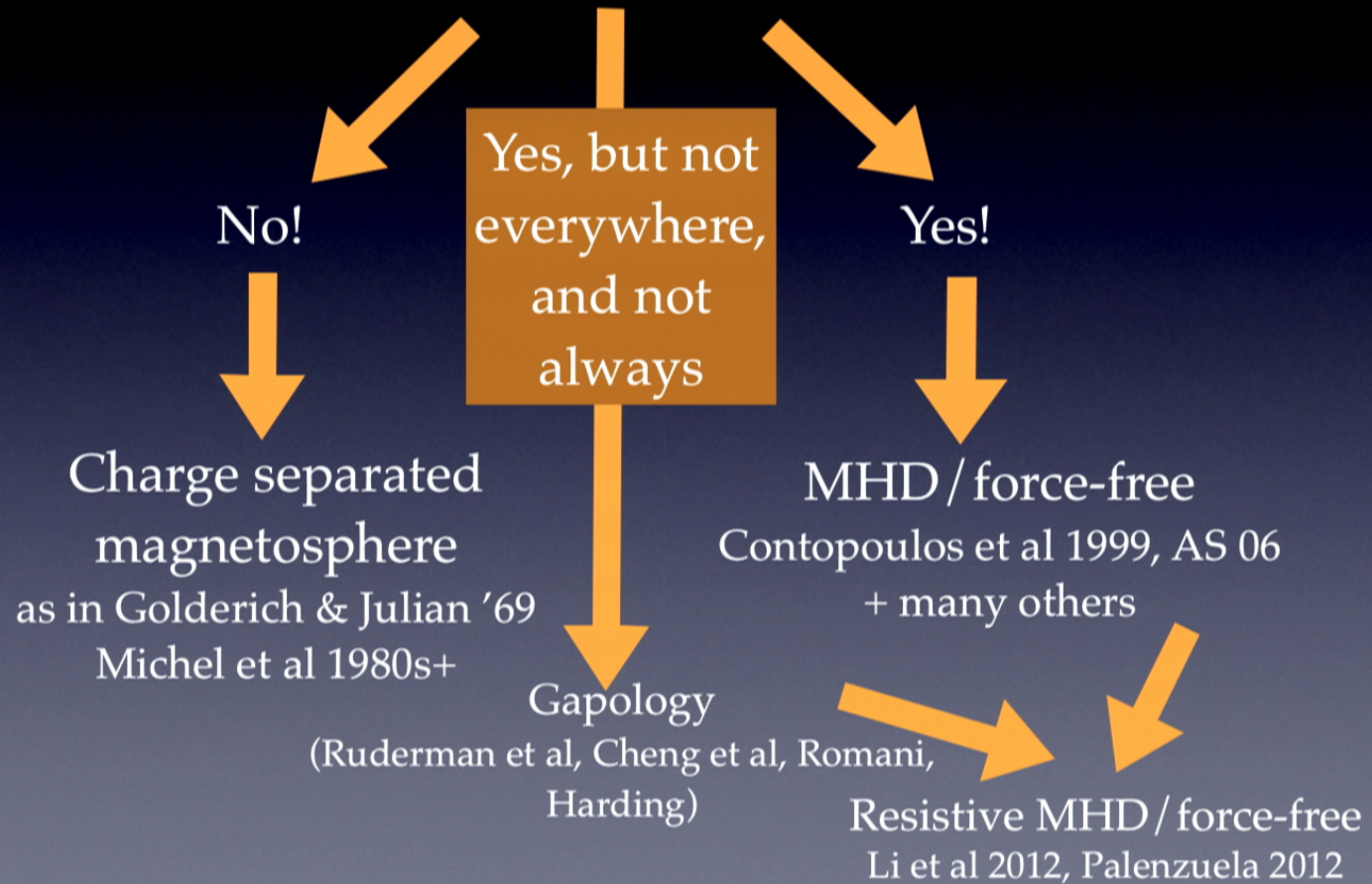
MODELING: TWO PATHS

Is there dense ($n \gg n_{GJ}$) plasma in the magnetosphere?



MODELING: TWO PATHS

Is there dense ($n \gg n_{GJ}$) plasma in the magnetosphere?



Magnetospheric models

	Vacuum	Space charge limited	Space charge limited+pairs	Abundant plasma
Field	Rotating vacuum dipole (RVD)	?	Assume RVD	Force-free
Acceleration	wild	gaps	Slot / Outer gaps	none / re-connection?
Spin down	$\frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta$?	?	$\frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \theta)$

Ostriker & Gunn 70

Goldreich & Julian 69
Michel 85, 00; AS
+Arons 02

Arons 78, Cheng et al
86; Romani et al;
Harding et al; Hirovani;

Contopoulos 99;
Gruzinov 05;
Timokhin 06;
AS 06

Magnetospheric models

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Field	Rotating vacuum dipole (RVD)	?	Assume RVD	Force-free
Acceleration	wild	gaps	Slot / Outer gaps	none / re-connection?
Spin down	$\frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta$?	?	$\frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \theta)$

Ostriker & Gunn 70

Goldreich & Julian 69
Michel 85, 00; AS
+Arons 02

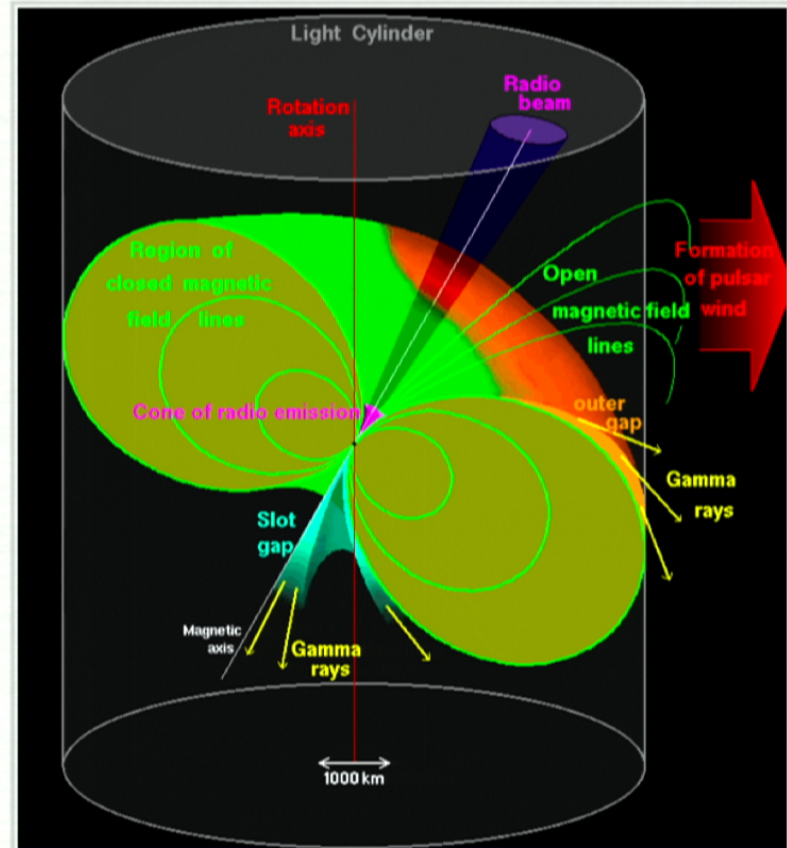
Arons 78, Cheng et al
86; Romani et al;
Harding et al; Hirovani;

Contopoulos 99;
Gruzinov 05;
Timokhin 06;
AS 06

Magnetospheric shape

Several ways of modeling, depending on charge supply:

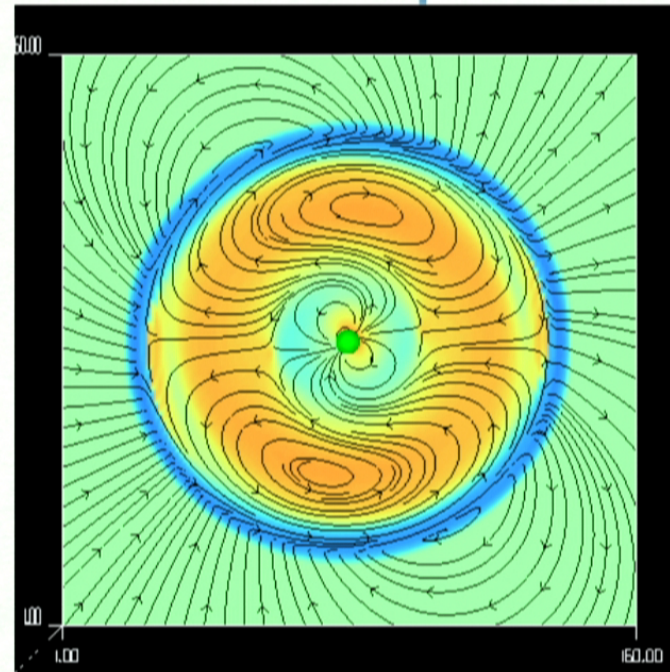
- * Vacuum rotator
- * Ab-initio particle
- * Full RMHD
- * Force-free variants
- * “Pulsar equation”



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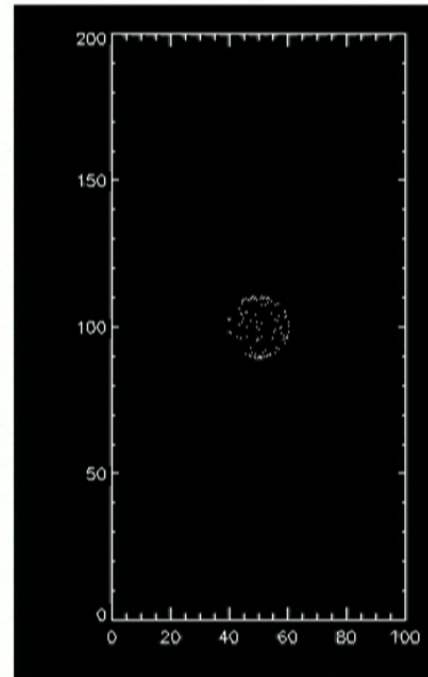
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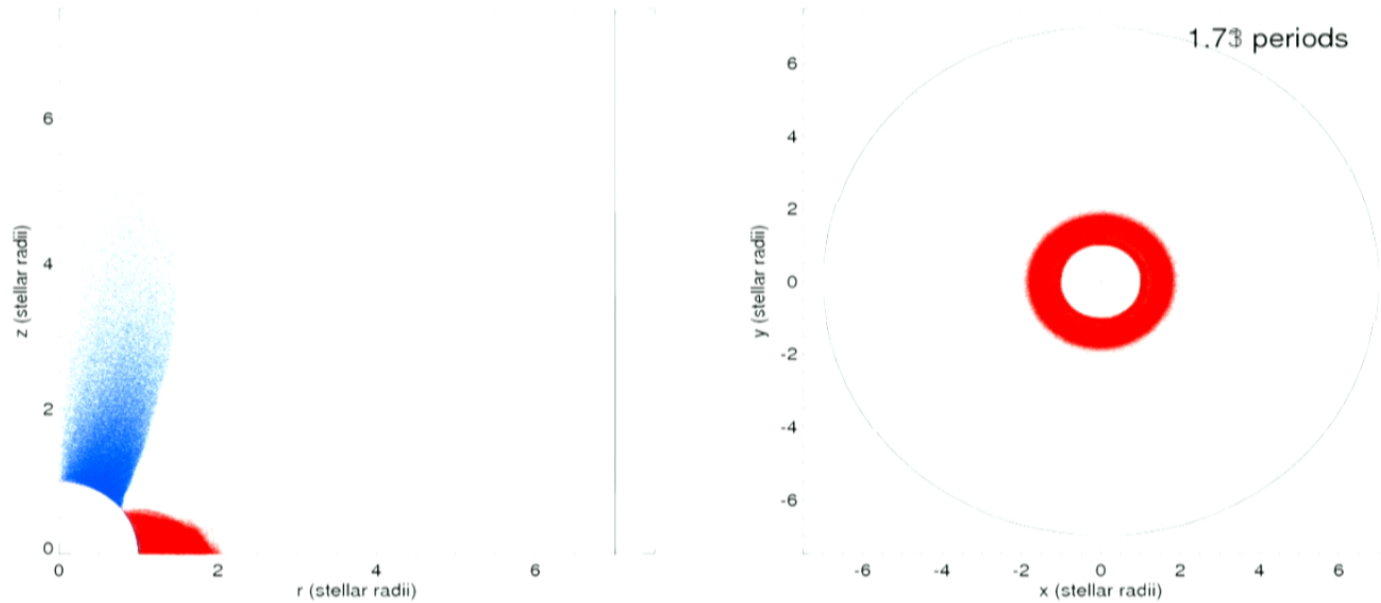
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AS & Arons 02;
Michel et al 84, 01

Non-axisymmetric instabilities



Disk-Torus Electrosphere

Michel et al '84-01

Diocotron instability

AS & Arons 02;

Petri et al 02-

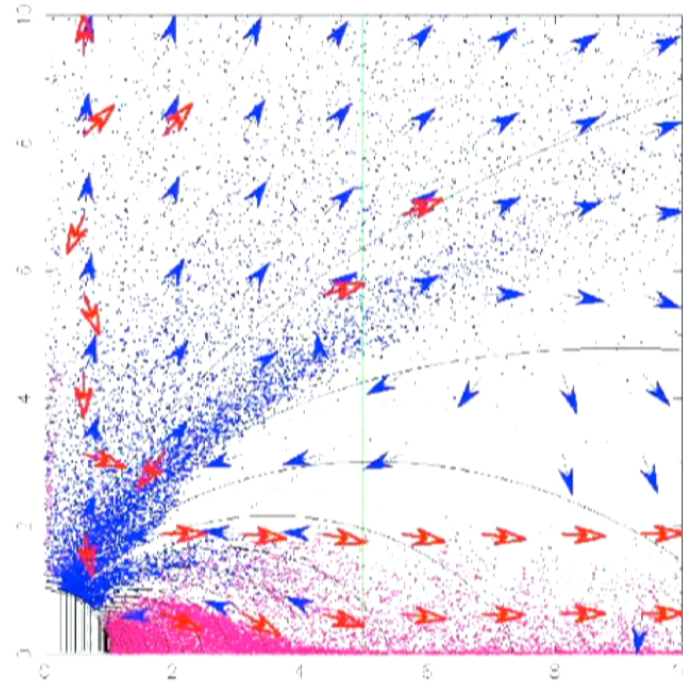
Belyaev & AS, in prep



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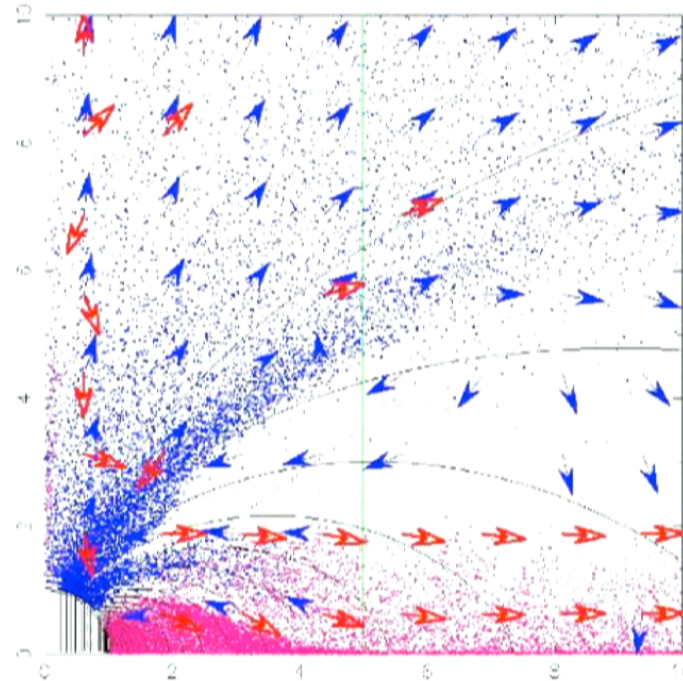


Yuki & Shibata 12

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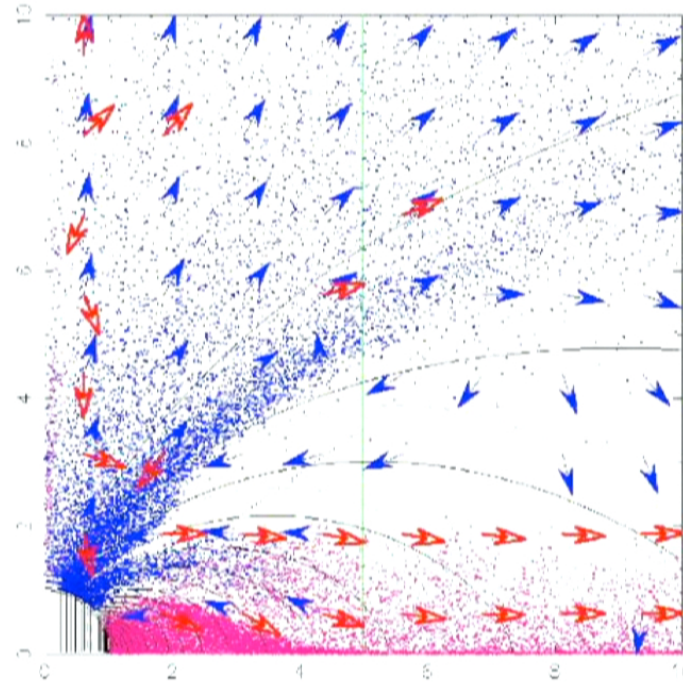


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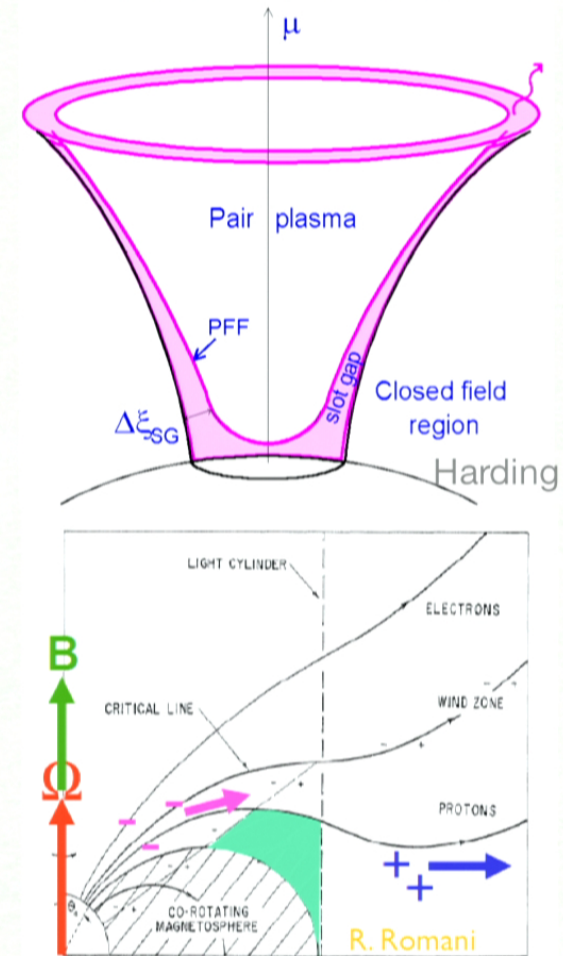


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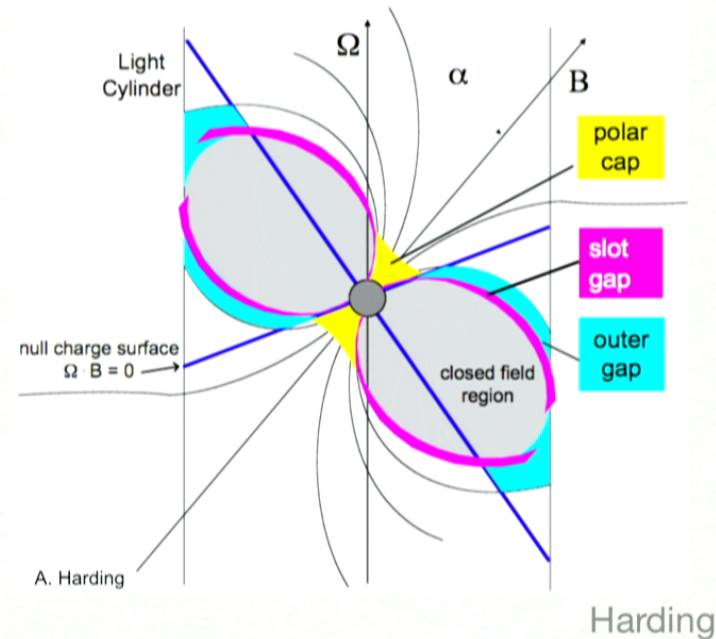
Several ways of modeling, depending on charge supply:

- * Vacuum rotator
- * **Particle/Vacuum+**
- * Full RMHD
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- * “Pulsar equation”



Gaps in understanding of gaps...

- * Acceleration due to charge starvation
- * Gaps imply space-charge separated background flow, even though pairs are created.
- * PWNe require pair densities \gg minimum charge separated density.

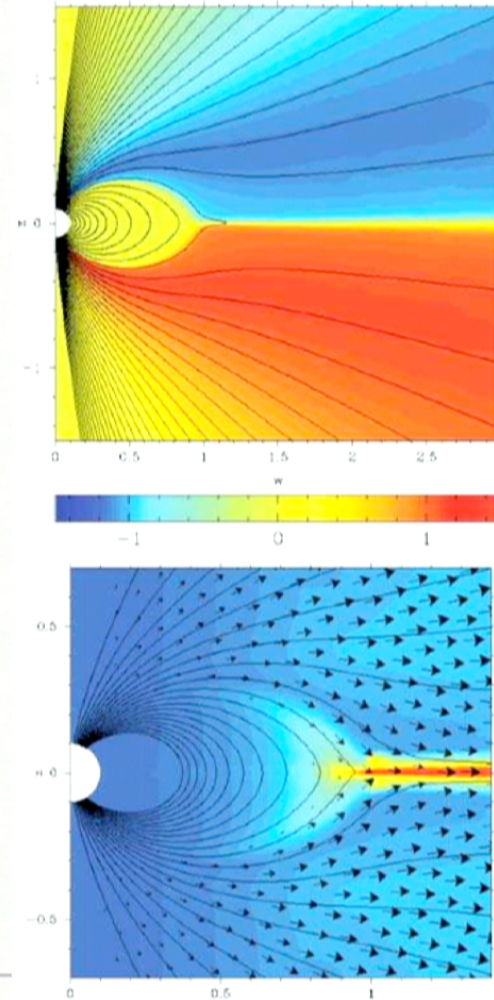


Gap models are best developed but are not self-consistent.

Magnetospheric shape

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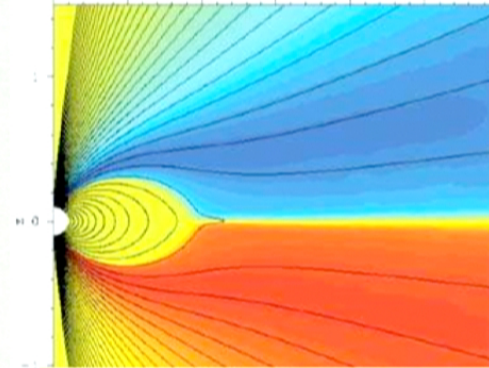


Komissarov 06

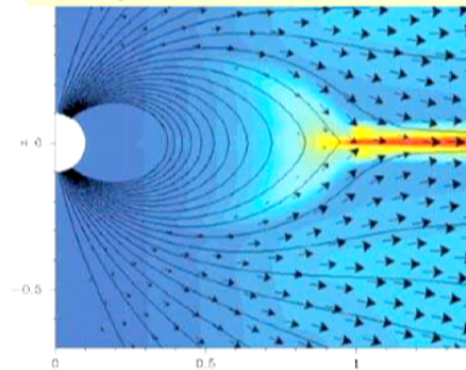
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High magnetization near the star is difficult to simulate with MHD. Require simplification: force-free



Komissarov 06

Conditions:

$$v \simeq c, \quad v_a \simeq c,$$

Equations:

$$\nabla_\beta \left(T_{(m)}^{\alpha\beta} + T_{(f)}^{\alpha\beta} \right) = 0$$

$$\nabla_\beta {}^*F^{\alpha\beta} = 0$$

$$\nabla_\alpha (nu^\alpha) = 0$$

$$F_{\nu\mu} u^\mu = 0 \quad \text{- perfect conductivity}$$

$$T_{(f)}^{\alpha\beta} = F^{\alpha\gamma} F_{\gamma}^{\beta} - \frac{1}{4} (F_{\mu\nu} F^{\mu\nu}) g^{\alpha\beta}$$

-stress-energy-momentum of
electromagnetic field

$$T_{(m)}^{\alpha\beta} = wu^\alpha u^\beta + pg^{\alpha\beta}$$

-stress-energy-momentum of matter

Full MHD vs force-free

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$$v \simeq c, \quad v_a \simeq c,$$

Equations:

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Conditions:

$$T_{(m)}^{\alpha\beta} \ll T_{(ef)}^{\alpha\beta}$$

Equations:

$$\nabla_\mu T_{(f)}^{\nu\mu} = 0$$

$$\nabla_\beta {}^*F^{\alpha\beta} = 0$$

$$F_{\mu\nu} {}^*F^{\mu\nu} = 0$$

$$F_{\mu\nu} F^{\mu\nu} > 0$$

or

$$\mathbf{E} \cdot \mathbf{B} = 0$$

$$B^2 - E^2 > 0$$

(Komissarov 2002)

Force-free equations

Full RMHD equations become stiff for high magnetization

$$mn \frac{\partial \gamma \vec{v}}{\partial t} = \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} \approx 0$$

Derive dynamical set of equations by ignoring particle inertia but retaining plasma charges and currents.

$$\left. \begin{aligned} \frac{1}{c} \frac{\partial \vec{E}}{\partial t} &= \nabla \times \vec{B} - \frac{4\pi}{c} \vec{j} \\ \frac{1}{c} \frac{\partial \vec{B}}{\partial t} &= -\nabla \times \vec{E} \\ \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} &= 0 \\ \frac{\partial}{\partial t} \vec{E} \cdot \vec{B} &= 0 \end{aligned} \right\} \vec{j} = \frac{c}{4\pi} (\nabla \cdot \vec{E}) \frac{\vec{E} \times \vec{B}}{B^2} + \frac{c \vec{B} (\vec{B} \cdot \nabla \times \vec{B} - \vec{E} \cdot \nabla \times \vec{E})}{4\pi B^2}$$

"Force-free MHD"

Gruzinov 99, Blandford 01

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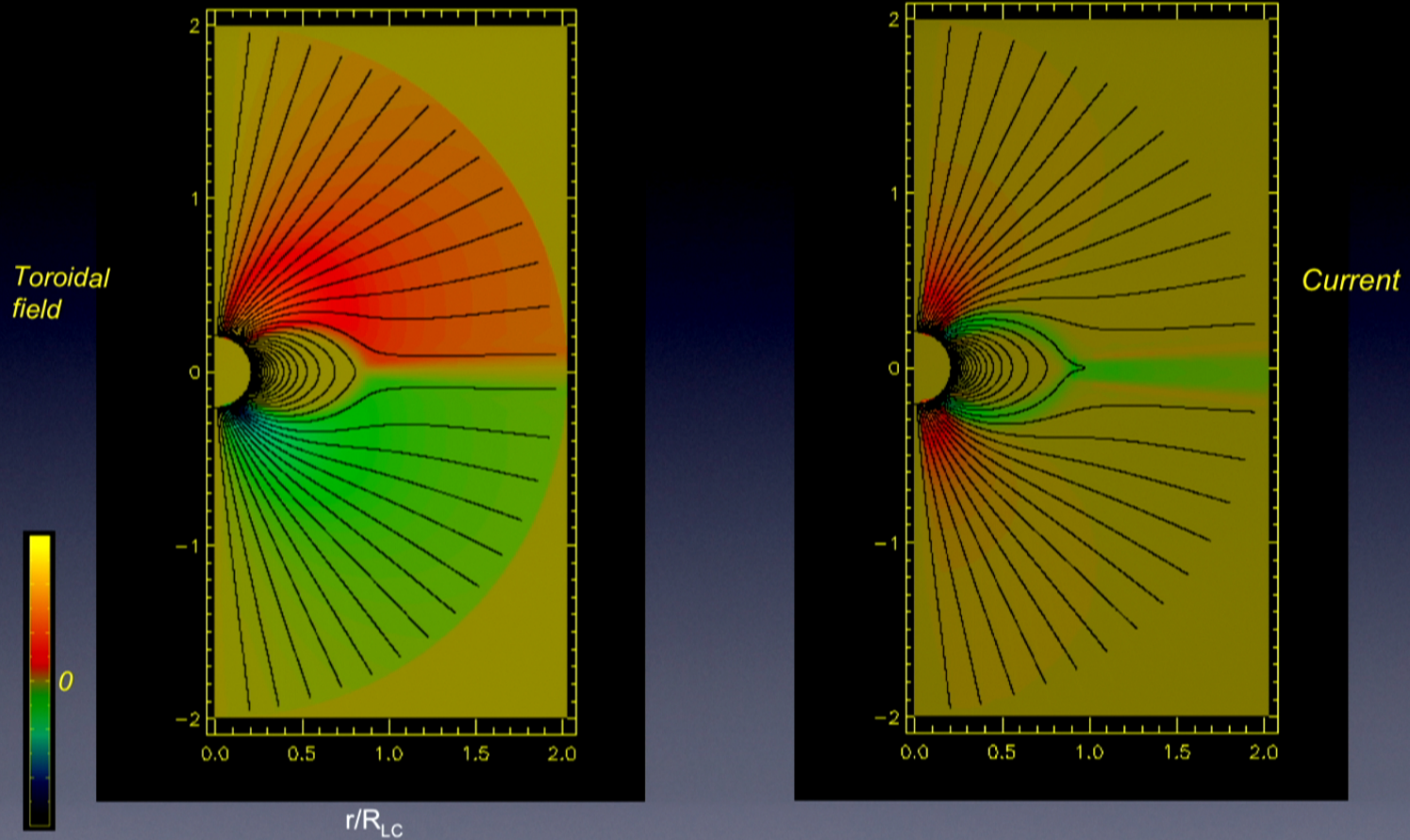
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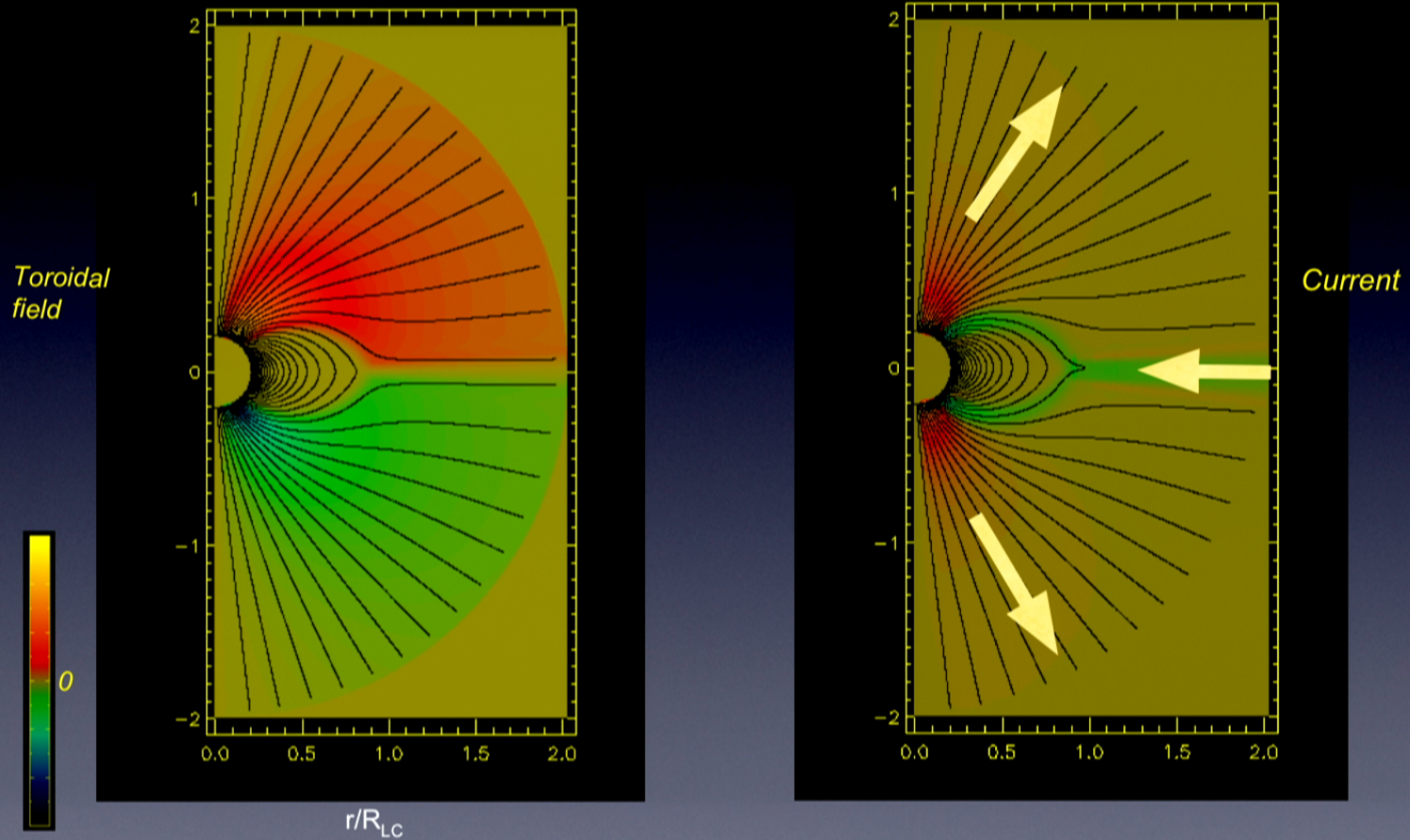
Gruzinov 99, Blandford 01

Aligned rotator: plasma magnetosphere



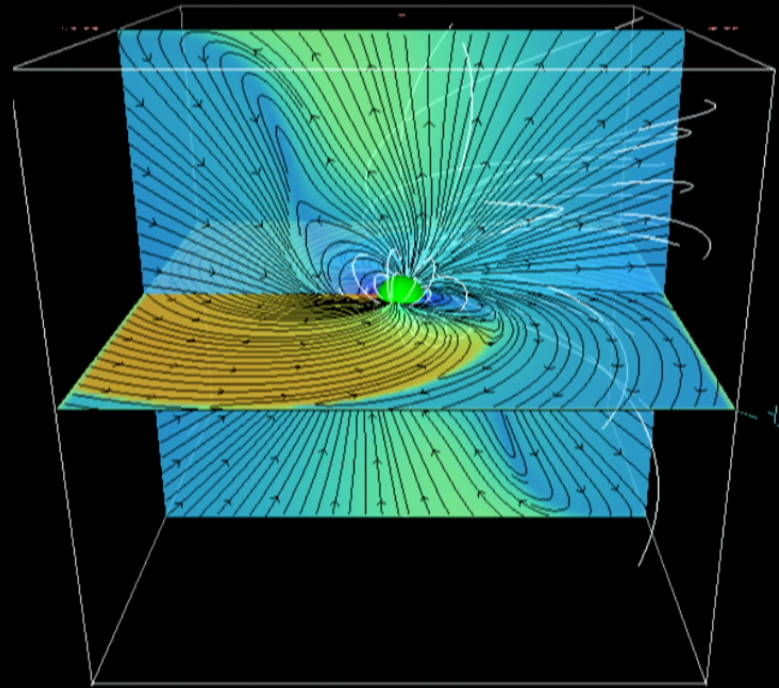
Properties: current sheet, split-monopolar asymptotics; closed-open lines; Y-point; null charge surface is not very interesting. Now at least 6 groups can do this (recently, Yu 11, Parfrey 11, Petri 12, Palenzuela 12, in addition to AS 06, McKinney 06, Kalapotharakis 09)

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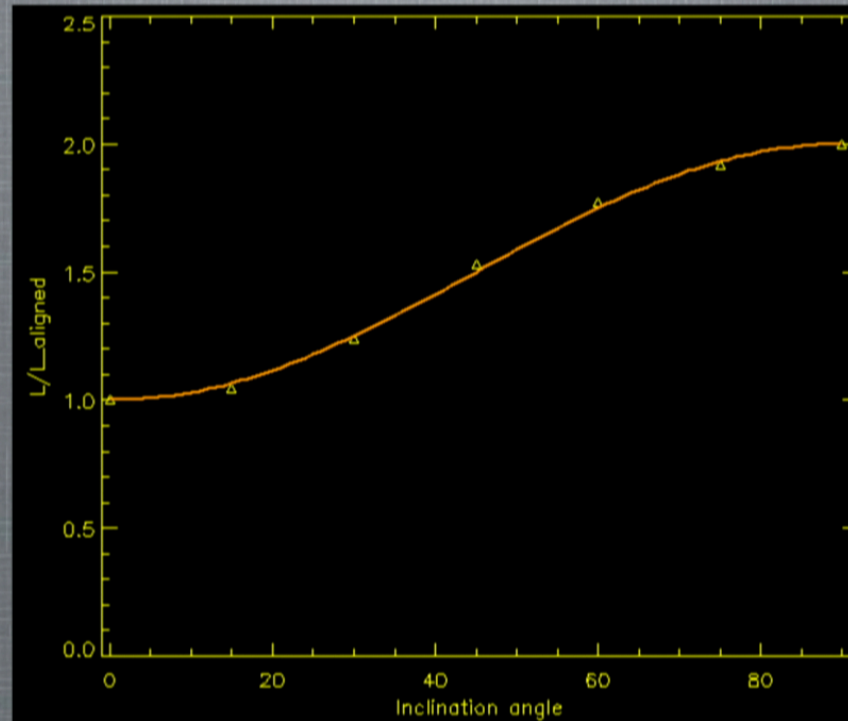
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Oblique rotator: force-free



A.S. 2006

SPIN-DOWN POWER



Spin-down of oblique rotator

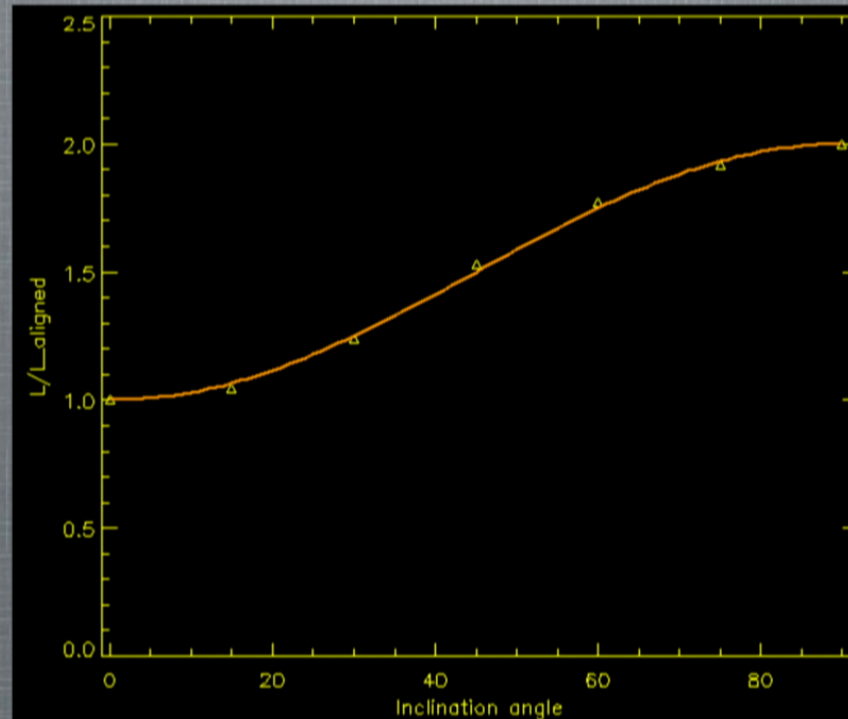
NB: this is a fit!

$$\dot{E} = \frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \theta)$$

$$\dot{E}_{vac} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta$$

A.S.'06; also confirmed by Kalapotharakos & Contopoulos 09

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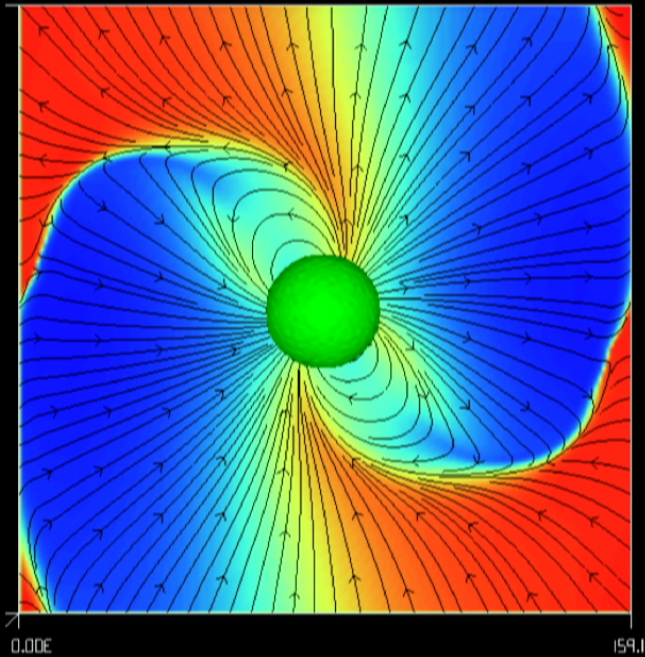
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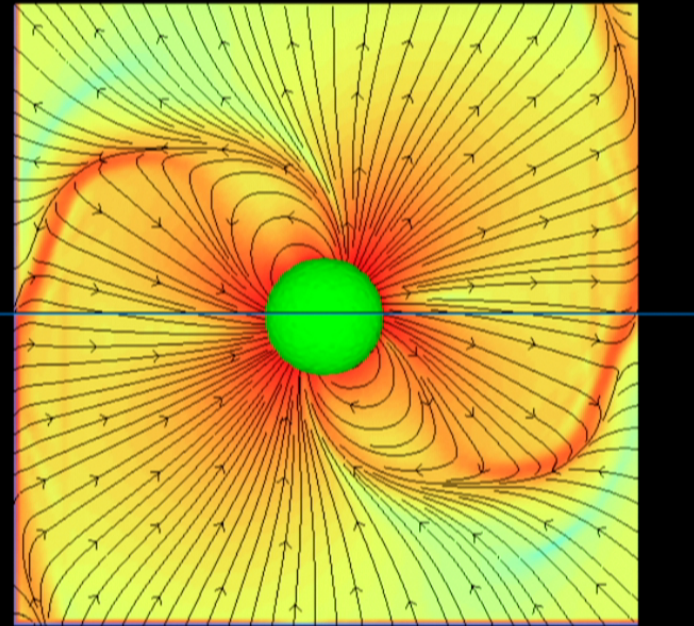
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IN COROTATING FRAME

60 degree inclination



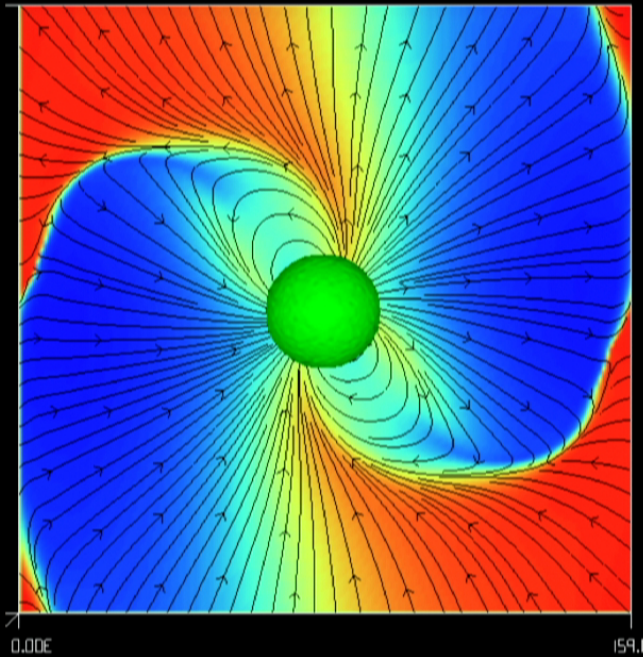
Force-free



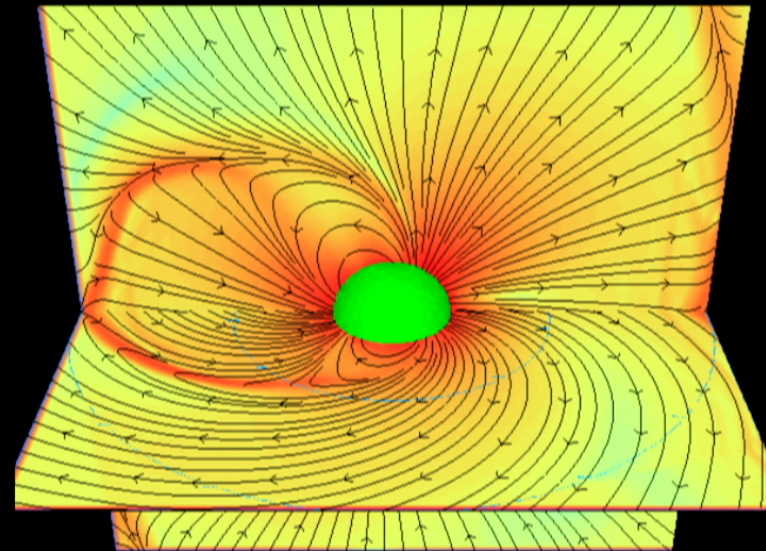
Force-free current density

IN COROTATING FRAME

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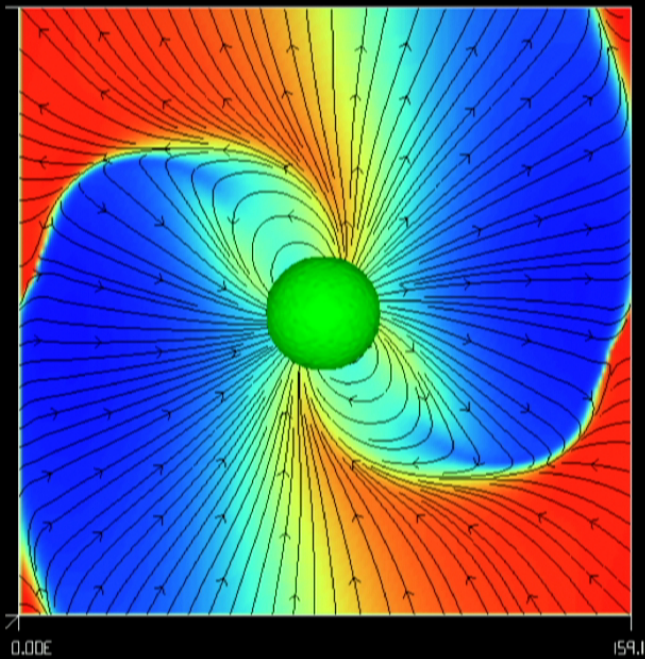
Force-free



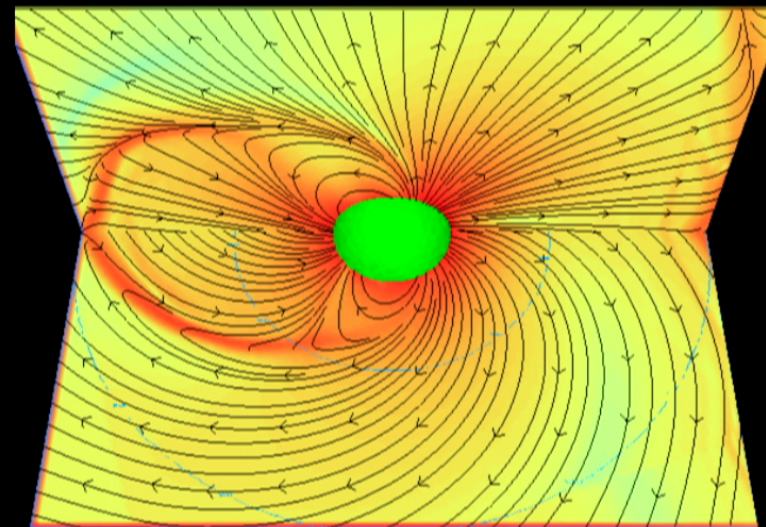
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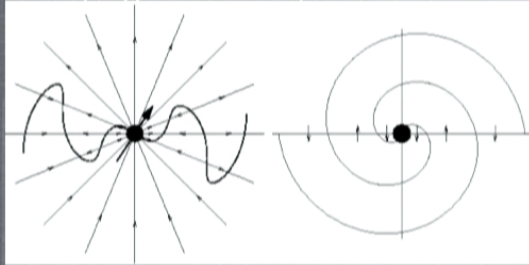


Force-free current density

IMPLICATIONS

Force-free pulsar solution

Split-monopole:
(Bogovalov 1999)



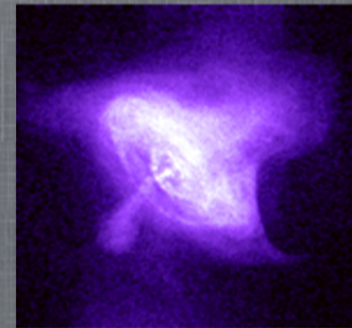
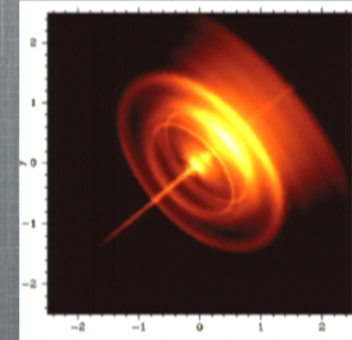
Heliospheric current sheet

Simulations of PWN require latitude-dependent energy injection, which is naturally provided by the model with reconnecting reversing fields in the equatorial zone.

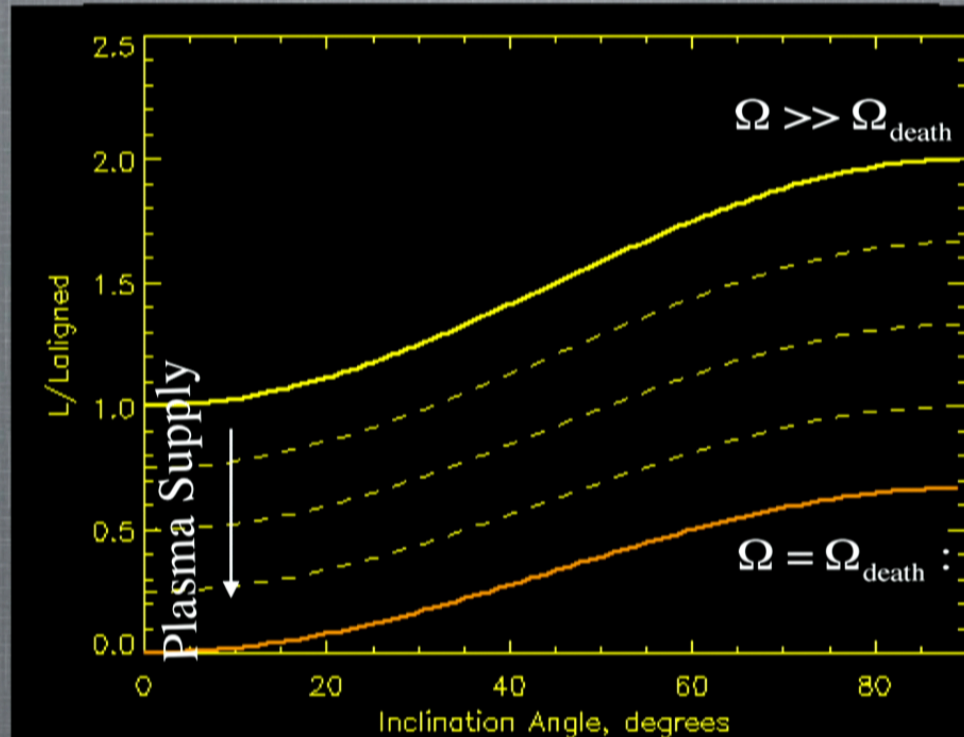
$$B_{\varphi} \propto \sin\theta \left(1 - \frac{2\theta}{\pi}\right)$$

Poynting flux is distributed as $\sin^2\theta$ in force-free models (θ -polar angle)

Komissarov & Lyubarsky

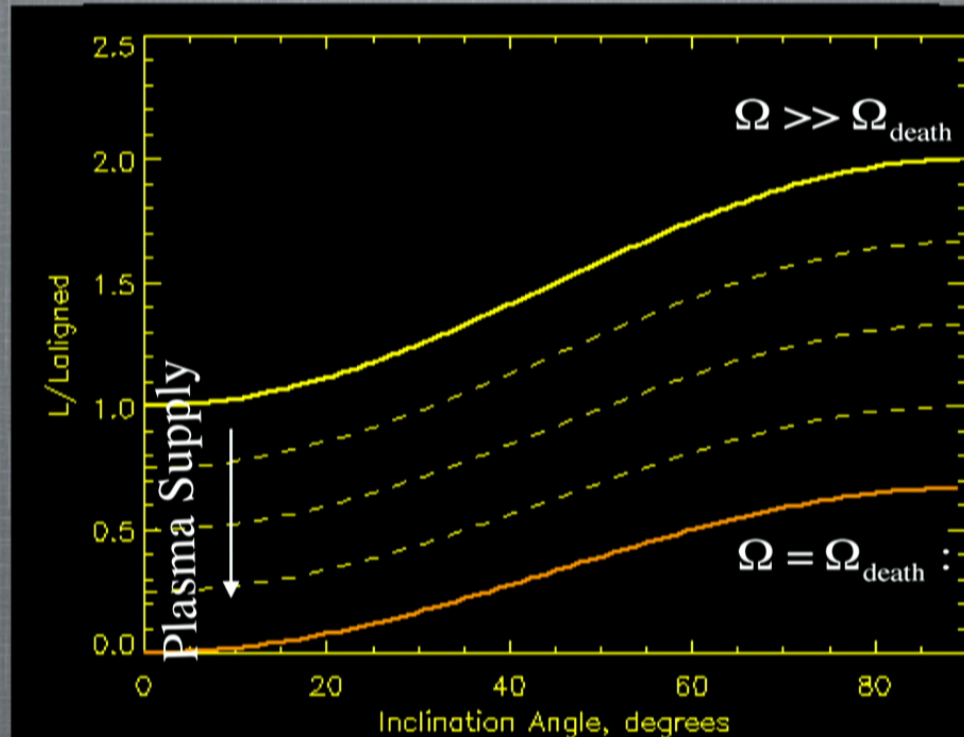


SPIN-DOWN POWER



We expect a continuum of solutions that depend on plasma supply. These can be characterized by the presence of accelerating E field, or resistivity.

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Resistive force-free

- There is a continuum of solutions between vacuum and ideal conducting force-free magnetosphere if plasma is not perfect everywhere.

- Can parameterize these with resistivity in the proper frame.

- Nice feature: re-emergence of parallel E field.

Ohm's law in the proper frame:

$$\vec{j}_{\text{fluid}} = \sigma \vec{E}_{\text{fluid}}$$

In lab frame:

$$\vec{j} = \frac{\rho c \vec{E} \times \vec{B}}{B^2 + E_0^2} + \frac{(-\beta_{\parallel} \rho c + \sqrt{\frac{B^2 + E_0^2}{B_0^2 + E_0^2}} (1 - \beta_{\parallel}^2) \sigma E_0) (B_0 \vec{B} + E_0 \vec{E})}{B^2 + E_0^2}$$

$$B_0^2 = \frac{\vec{B}^2 - \vec{E}^2 + \sqrt{(\vec{B}^2 - \vec{E}^2)^2 + 4(\vec{E} \cdot \vec{B})^2}}{2},$$

$$E_0 = \sqrt{B_0^2 - \vec{B}^2 + \vec{E}^2},$$

$$B_0 = \text{sign}(\vec{E} \cdot \vec{B}) \sqrt{B_0^2},$$

cf. Lyutikov 03

Gruzinov 07-11

Li, AS, Tchekhovskoy, 2011

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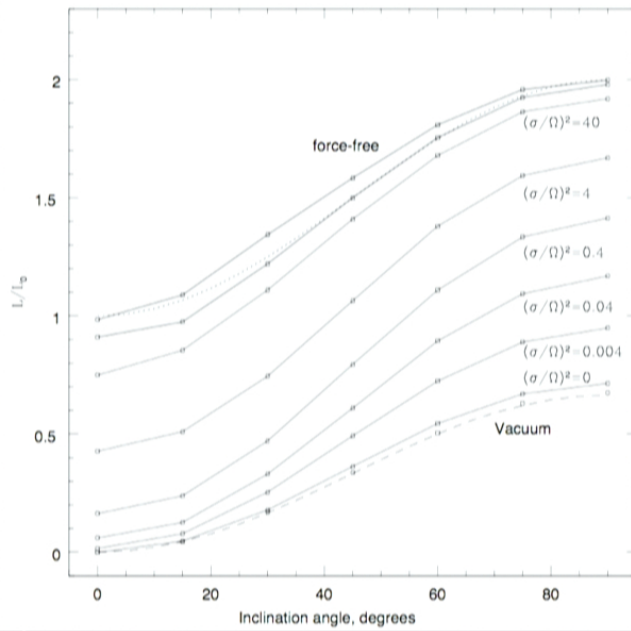
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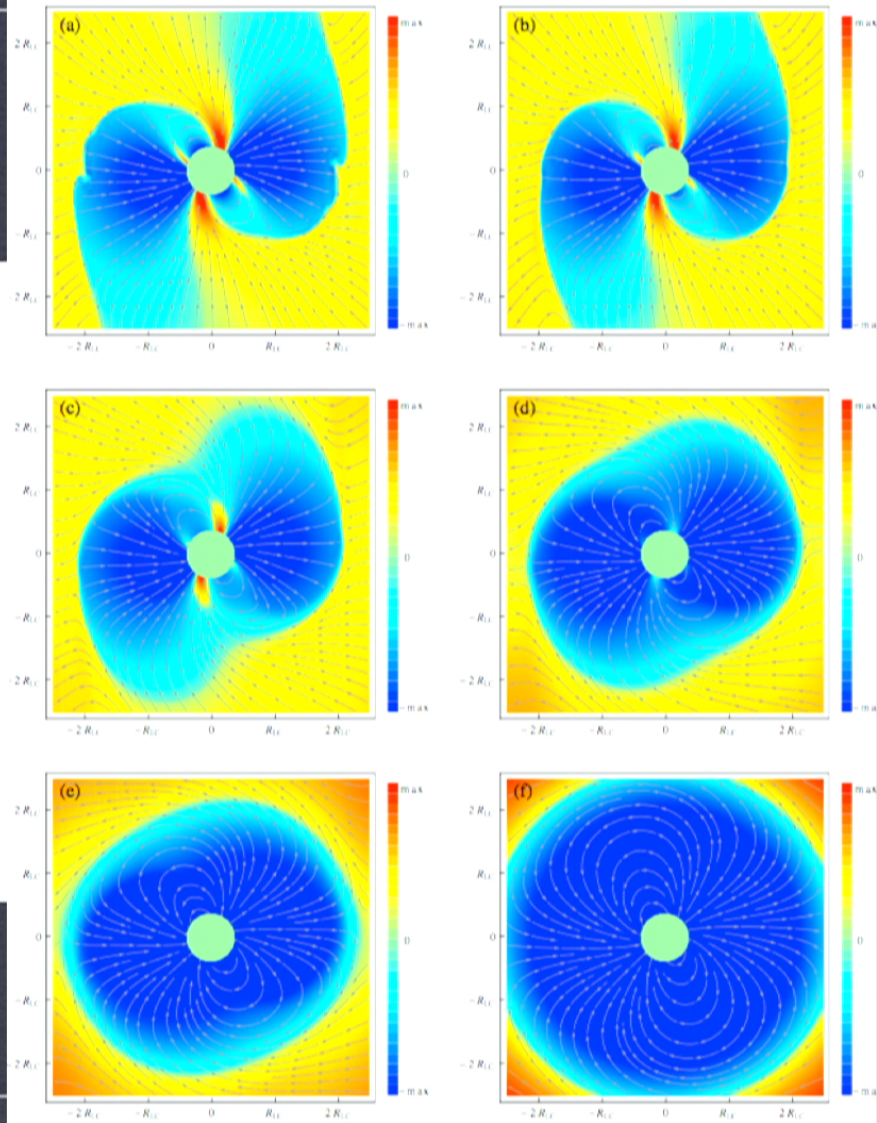
Li, AS, Tchekhovskoy, 2011
also, Kalapotharakos et al 11

Resistive: Vary σ/Ω



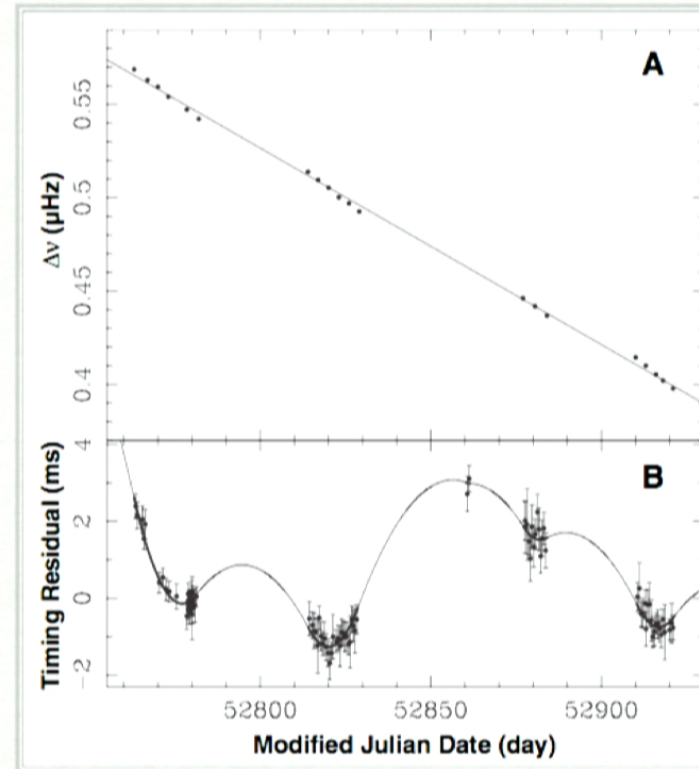
Spin-down power

Li, AS, Tchekhovskoy 12



Application: intermittent pulsars

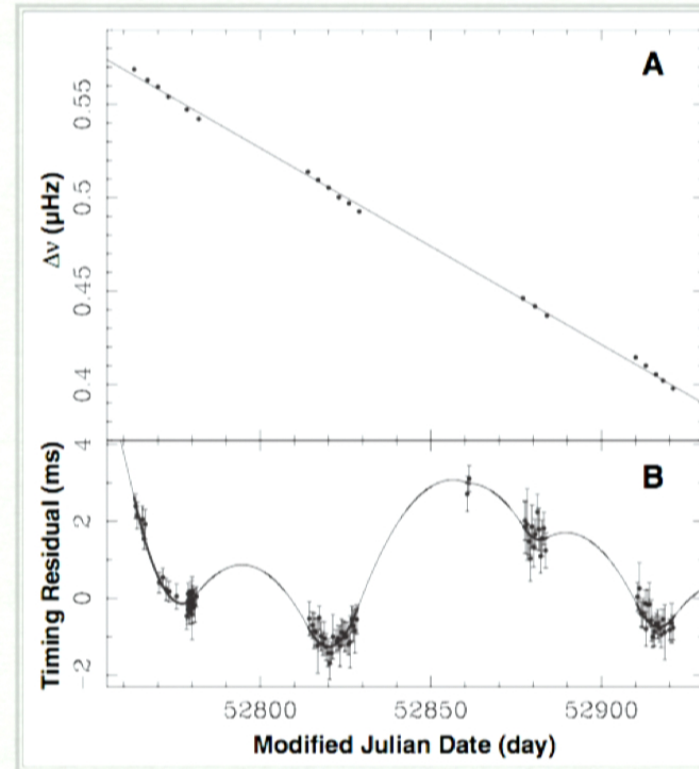
- ✱ Intermittent pulsars display changes in spin-down power when they are ON and OFF in radio by factor >1.5
- ✱ One possibility: conducting closed zone, vacuum-like open zone; Interrupted plasma production



Kramer et al 06

Application: intermittent pulsars

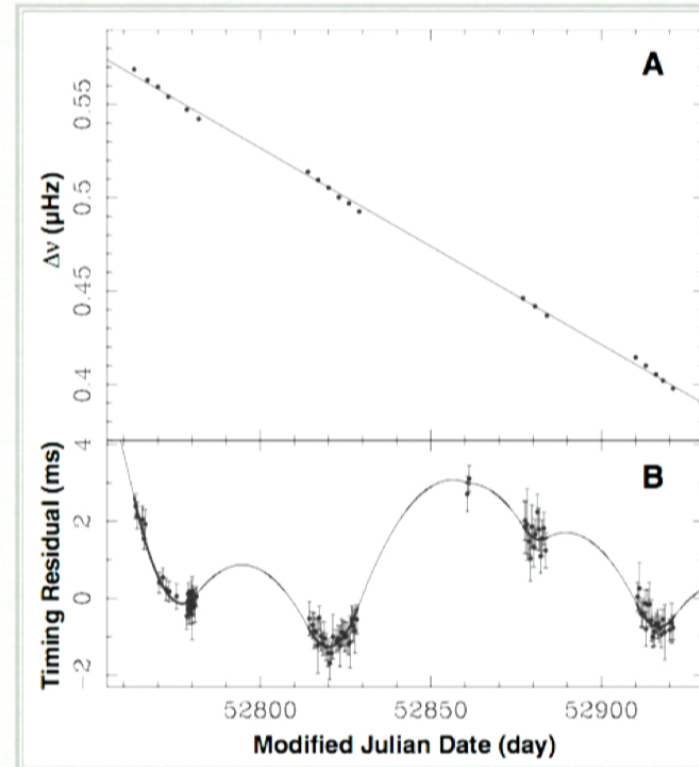
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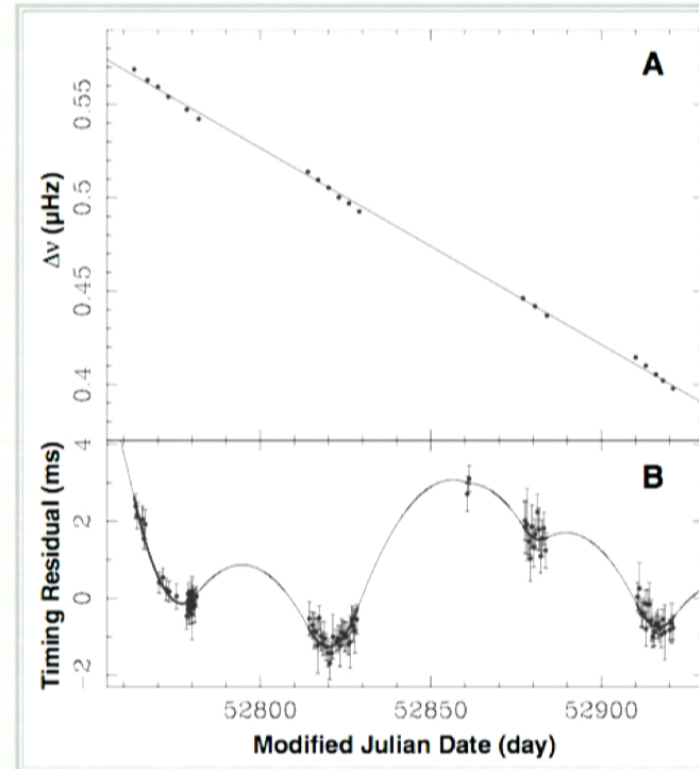
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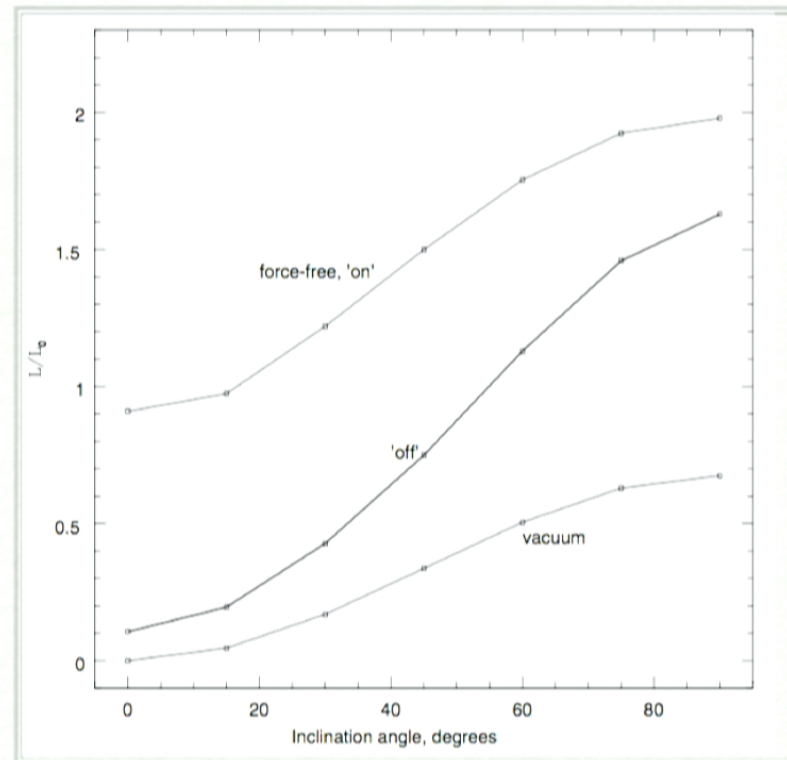
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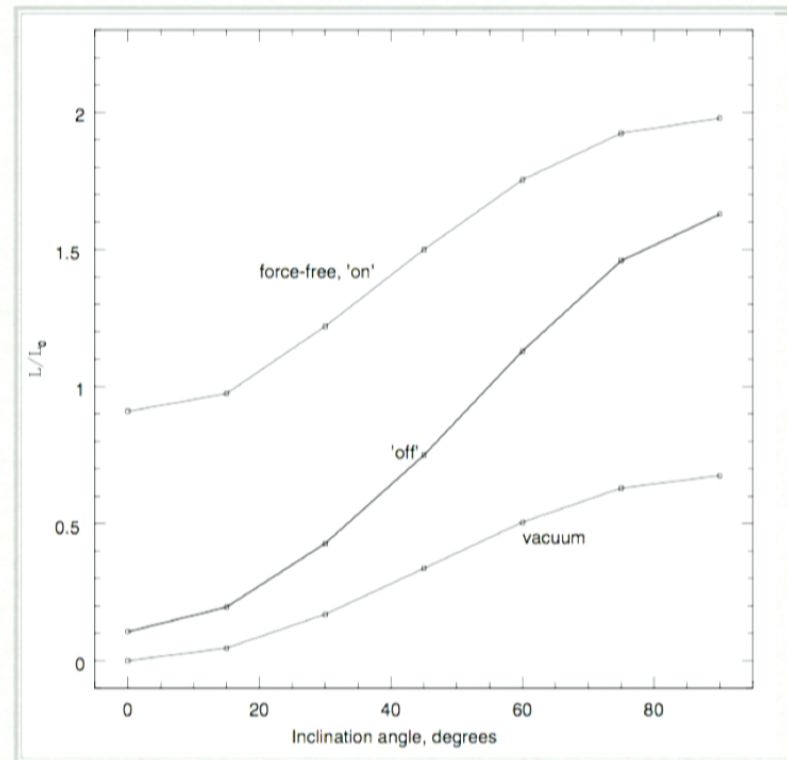
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Li et al 12

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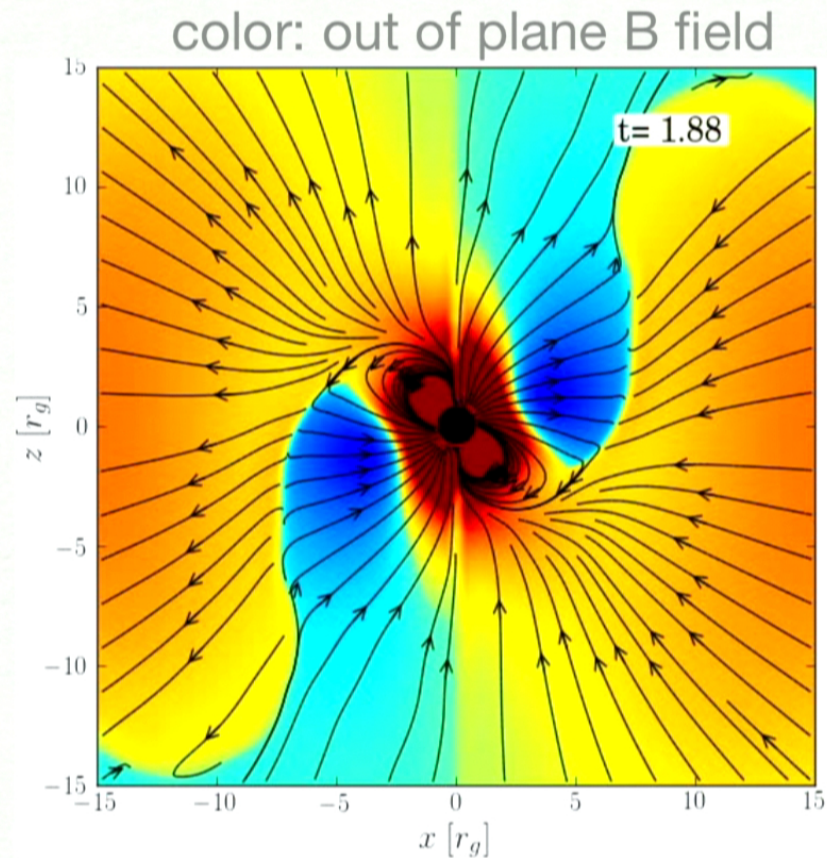
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- * Oblique rotator can now be studied in ideal MHD
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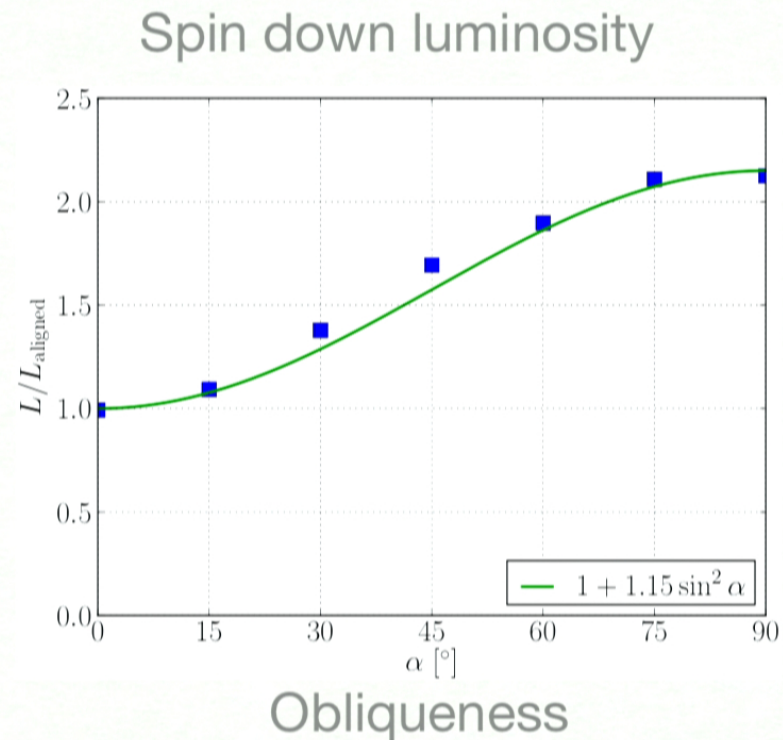
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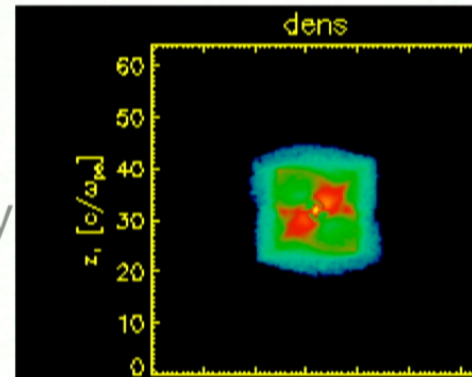
Variation with angle is similar to force-free

Recent advances:

Vacuum field + test particles

- * Full particle modeling with PIC (particle-in-cell) in 3D (AS in prep).

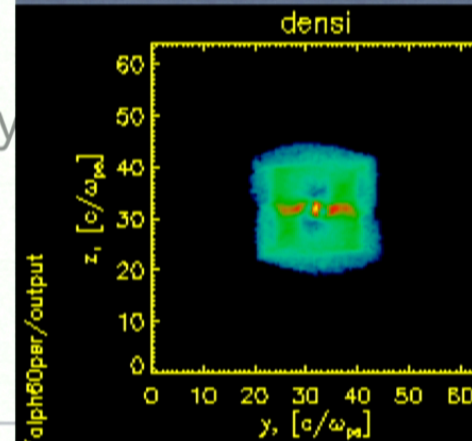
Full density



- * Idea: supply enough neutral plasma and "let it figure it out."

- * $E = E_{\text{plasma}} + E_{\text{Deutsch}}$;
 $B = B_{\text{plasma}} + B_{\text{Deutsch}}$

Ion density



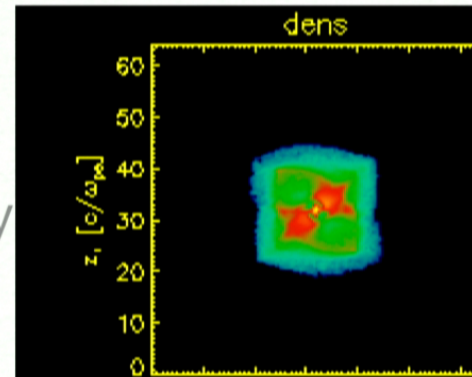
Step 1: Test electrons & ions become distributed in different locations -- no clear sheet

Recent advances:

Vacuum field + test particles

- * Full particle modeling with PIC (particle-in-cell) in 3D (AS in prep).

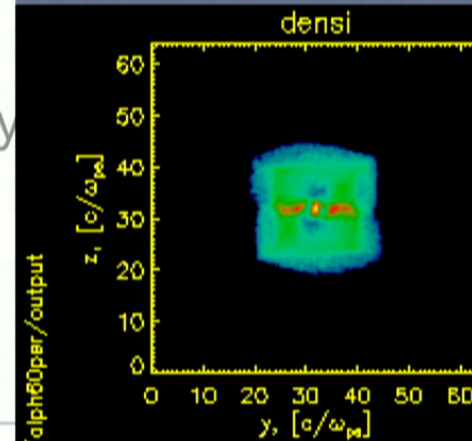
Full density



- * Idea: supply enough neutral plasma and “let it figure it out.”

- * $E = E_{\text{plasma}} + E_{\text{Deutsch}}$;
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Ion density



<--- 7 R_{LC} --->

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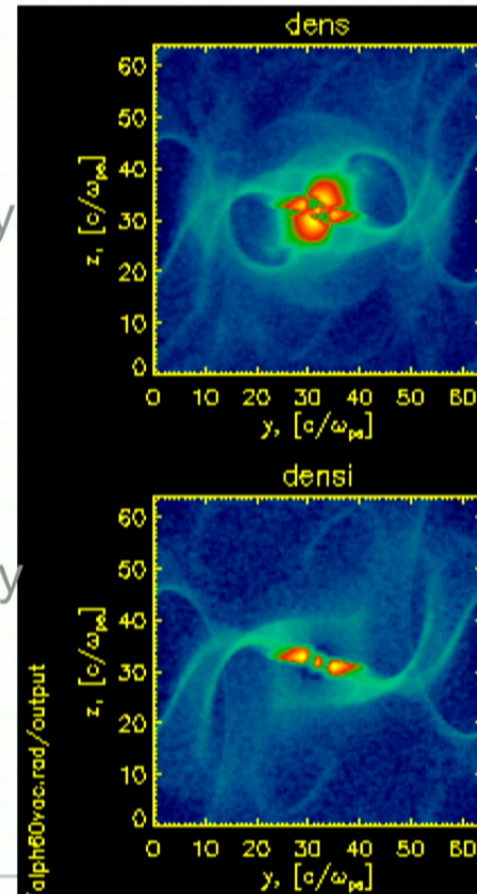
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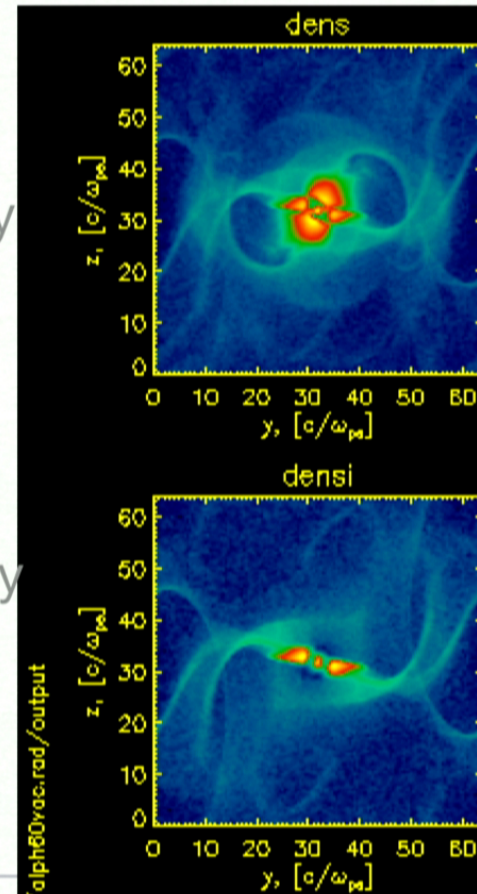
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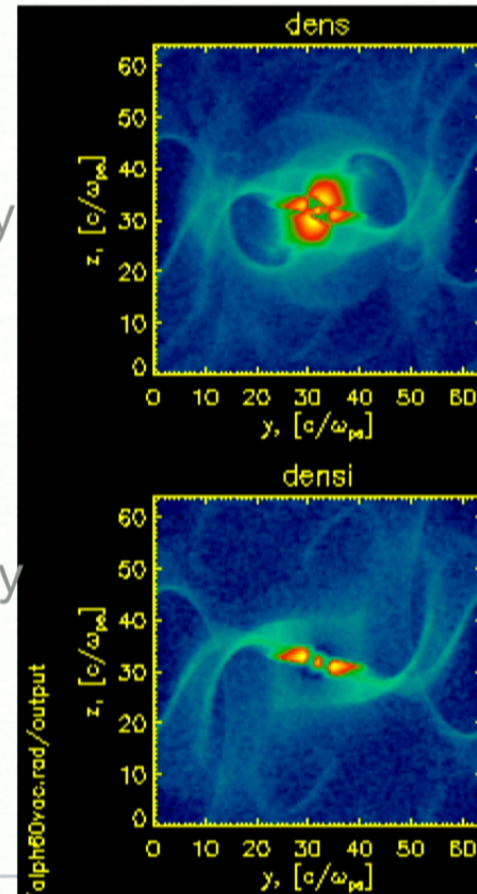
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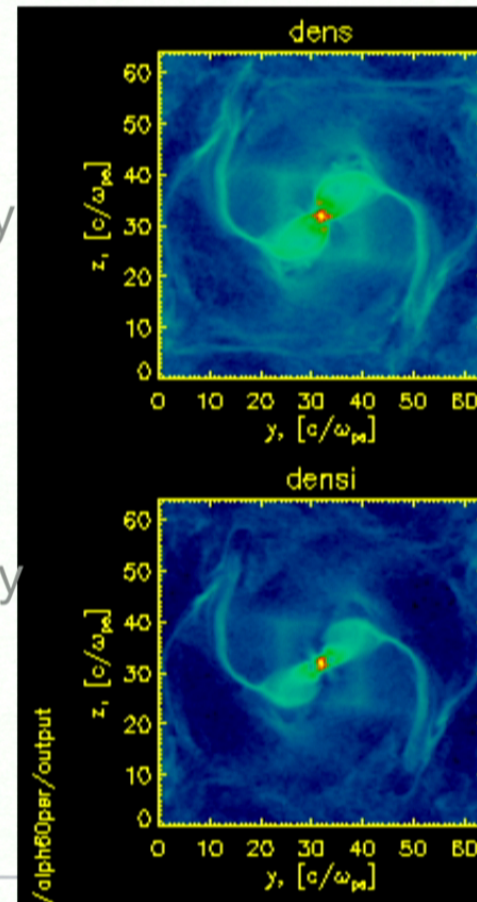
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Full density

- * Turn on plasma back reaction:

Ion density

Electrons & ions now are together and form a current sheet. Approaches force-free.

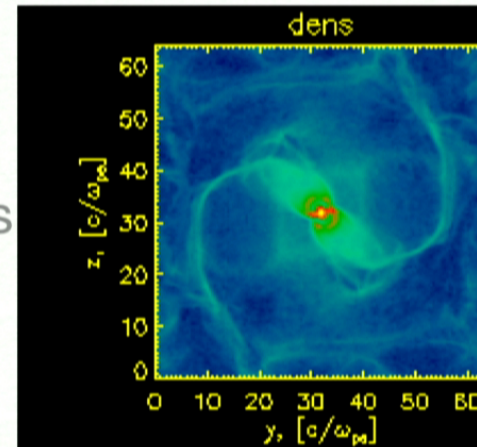


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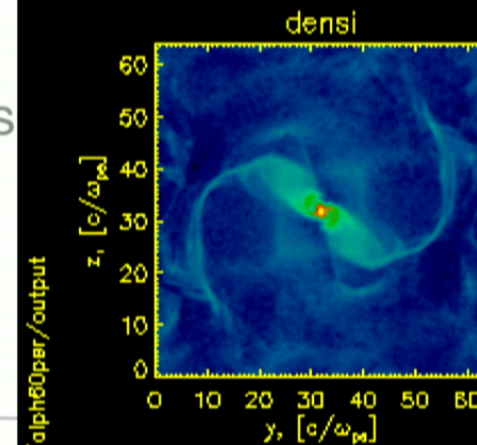
Vacuum field + plasma

- * In principle, such a solution has everything: acceleration, plasma velocity, reconnection.
- * In practice the resolution will always be limited.
- * Also, the aligned rotator part depends on good BCs on the star -- still in progress.

Full
dens



Ion
dens

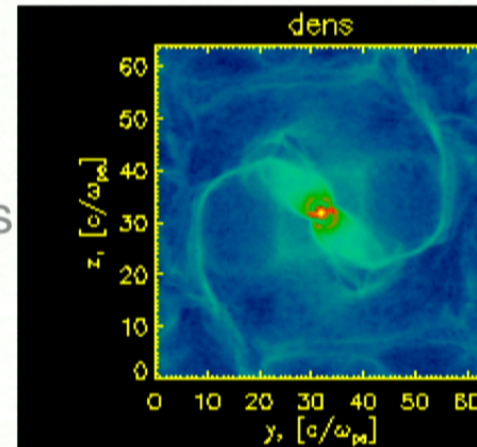


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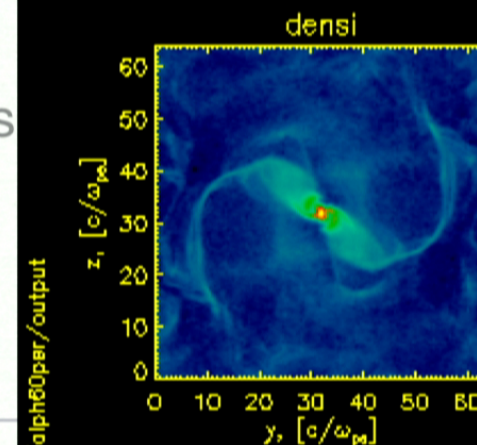
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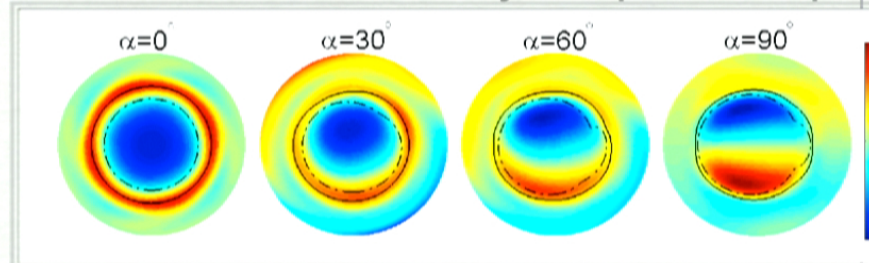


alpha60per/output

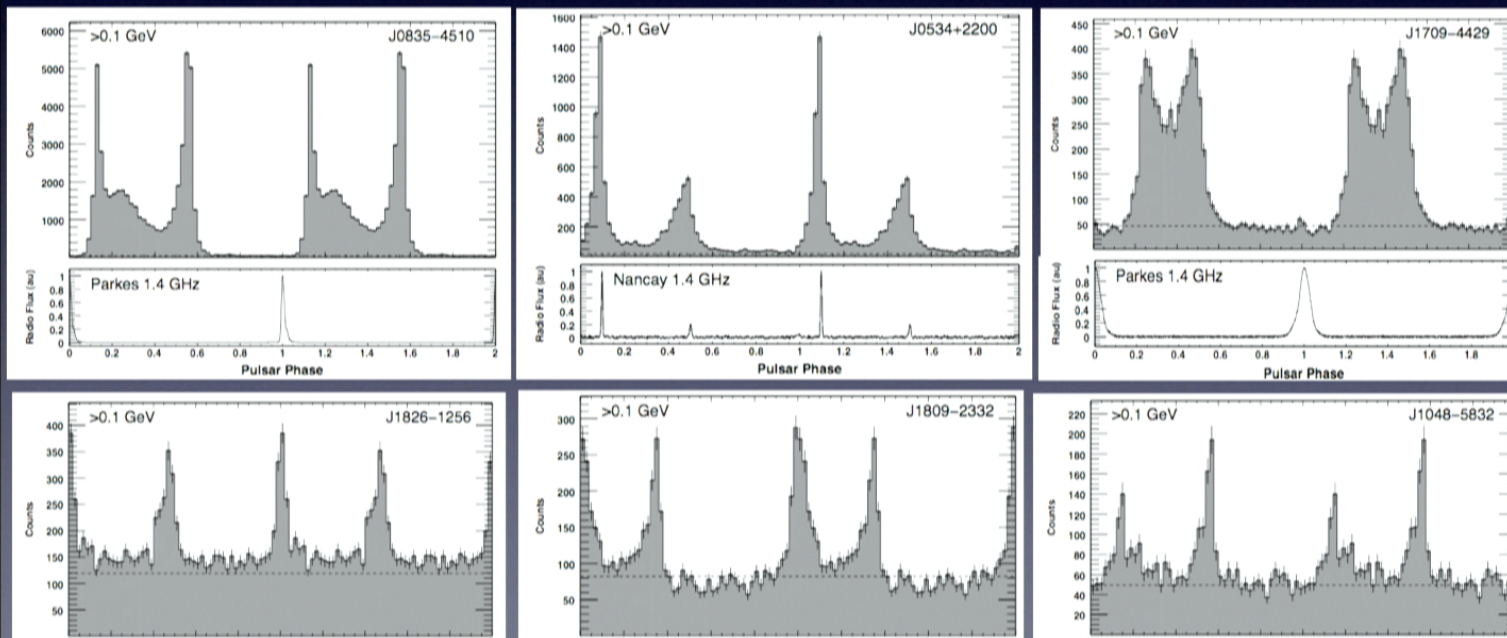
Where does emission come from?

- * With force-free shape of the magnetosphere at hand, emission physics can be studied again.
- * Most progress so far is on the geometry of the emission so far (gamma-rays).
- * Does the emission come from gap regions? Inside or outside LC?

Current density on polar cap



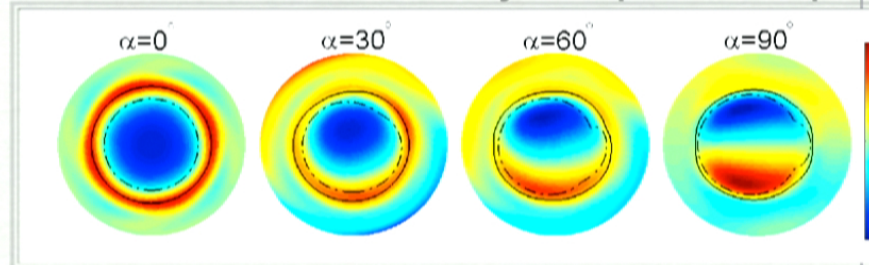
Gamma-ray emission from pulsars



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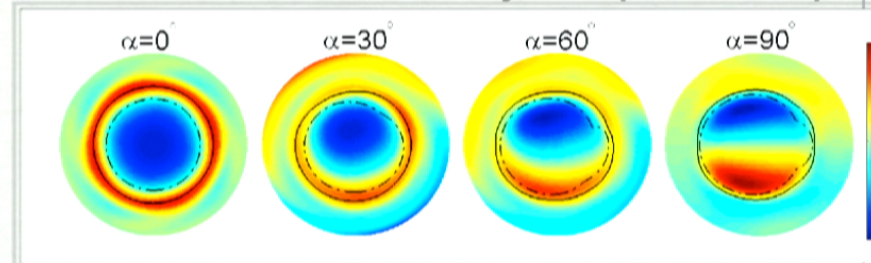
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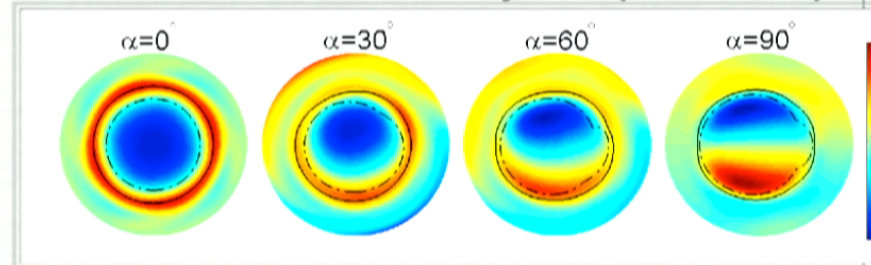
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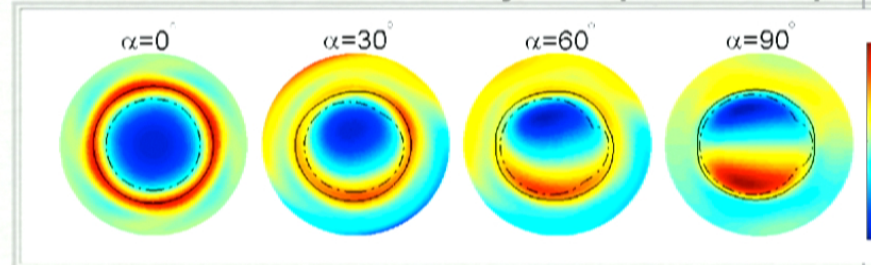
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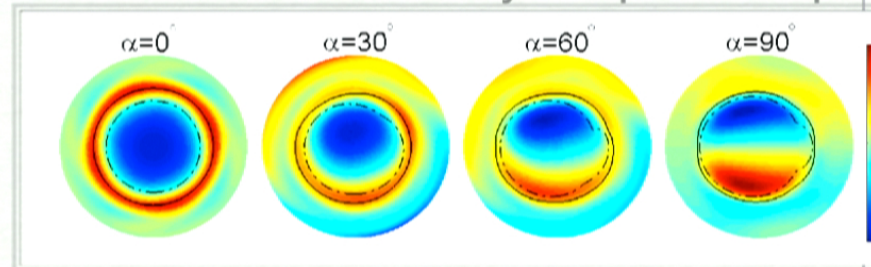
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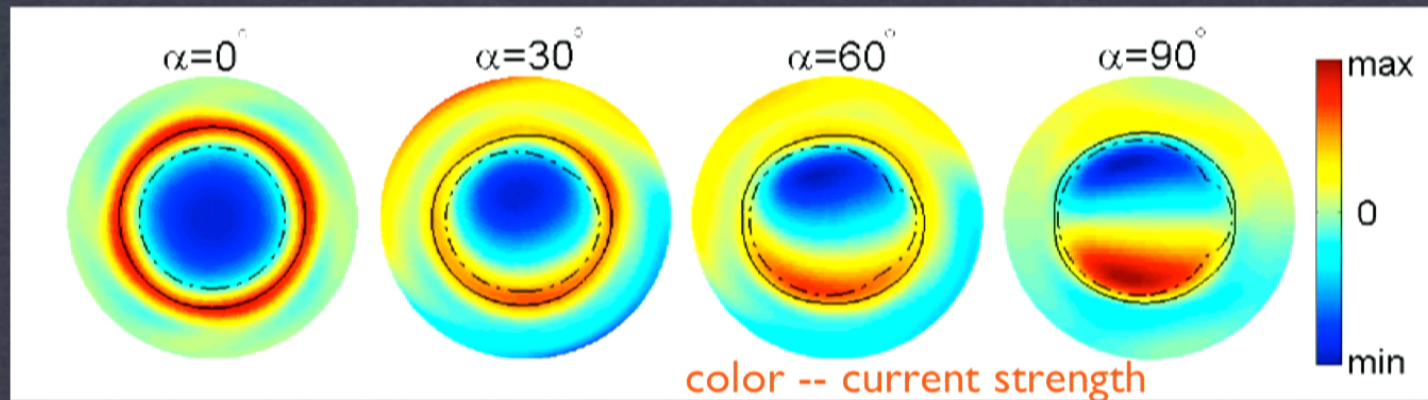
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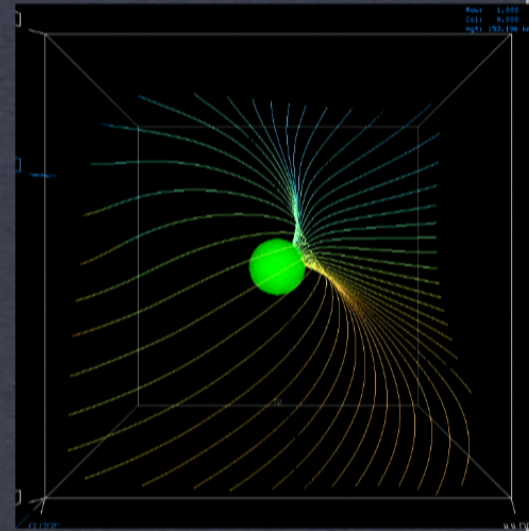
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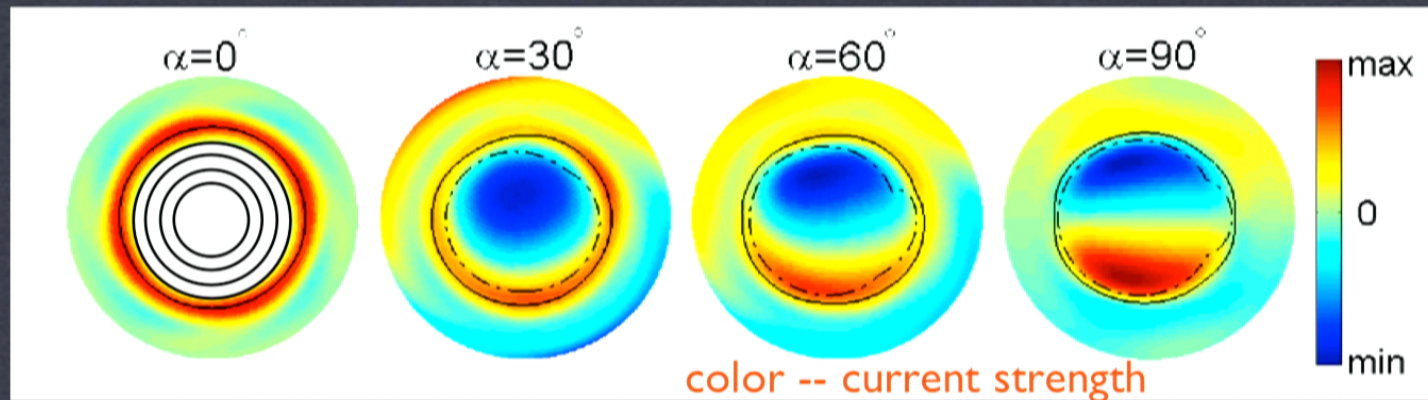
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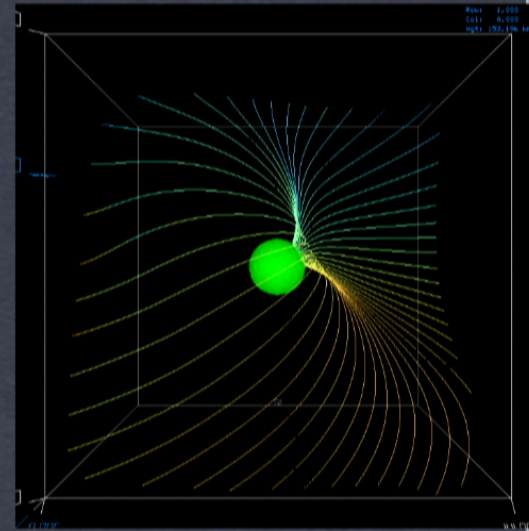
- Select flux tubes that map into rings on the polar caps. The rings are congruent to the edge of the polar cap.
- While ad-hoc, the point is to study the geometry of the possible emission zone.
- Emission is along field lines, with aberration and time delay added



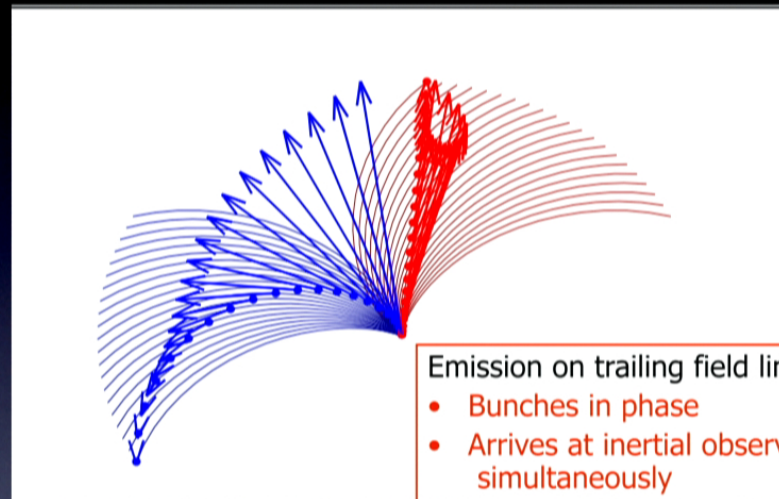
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Caustic formation:



Emission on trailing field lines

- Bunches in phase
- Arrives at inertial observer simultaneously

Emission on leading field lines

- Spreads out in phase
- Arrives at inertial observer at different times

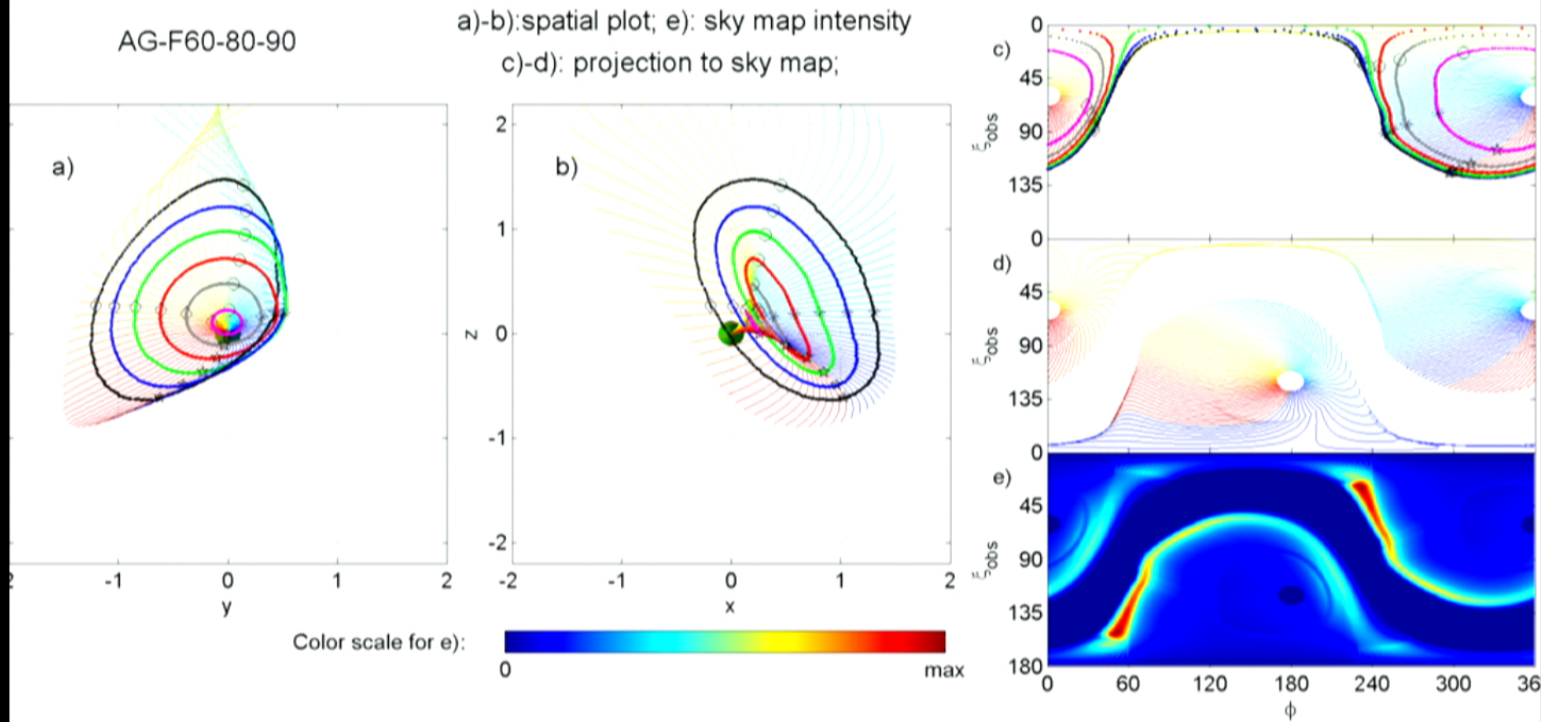
Caustic emission

- Dipole magnetic field
- Outer edge of open volume

Harding 07

Emission from different flux tubes

Bai & A. S. 2010



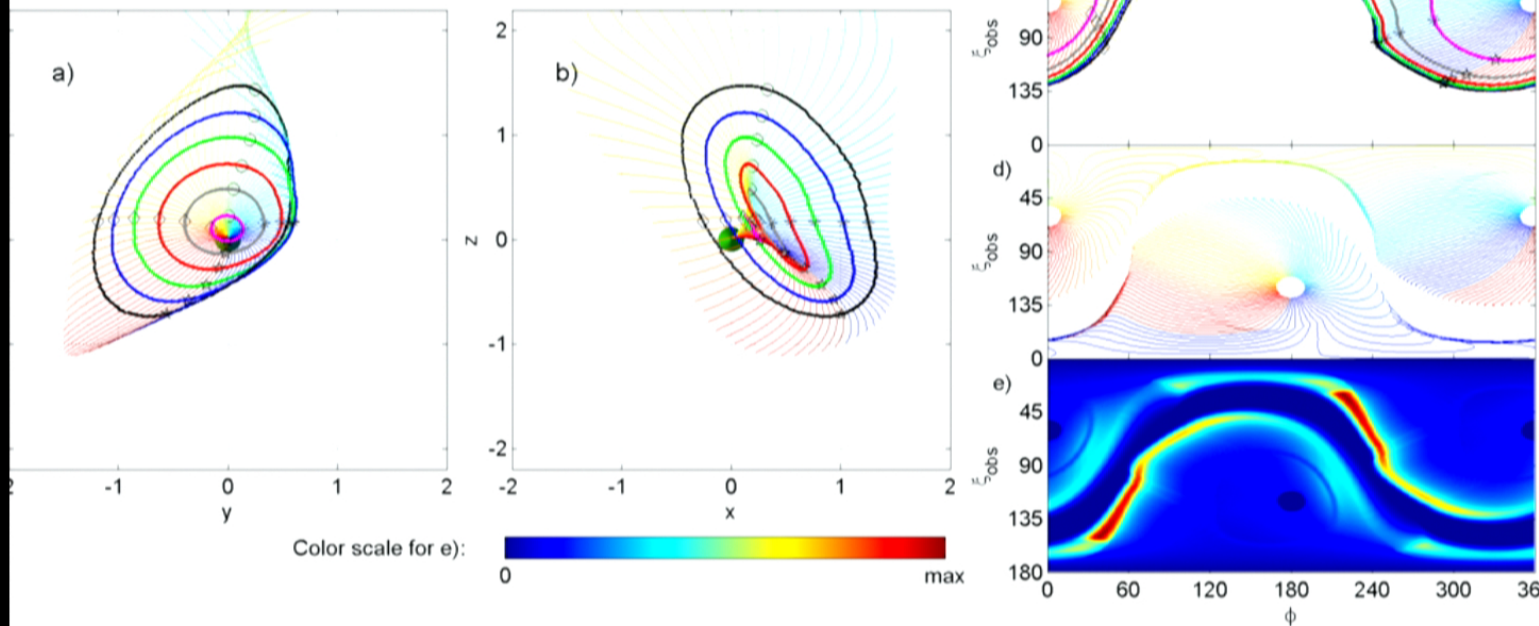
Emission from two poles merges on some flux tubes: what's special about them?

Emission from different flux tubes

Bai & A. S. 2010

AG-F60-85-90

a)-b): spatial plot; e): sky map intensity
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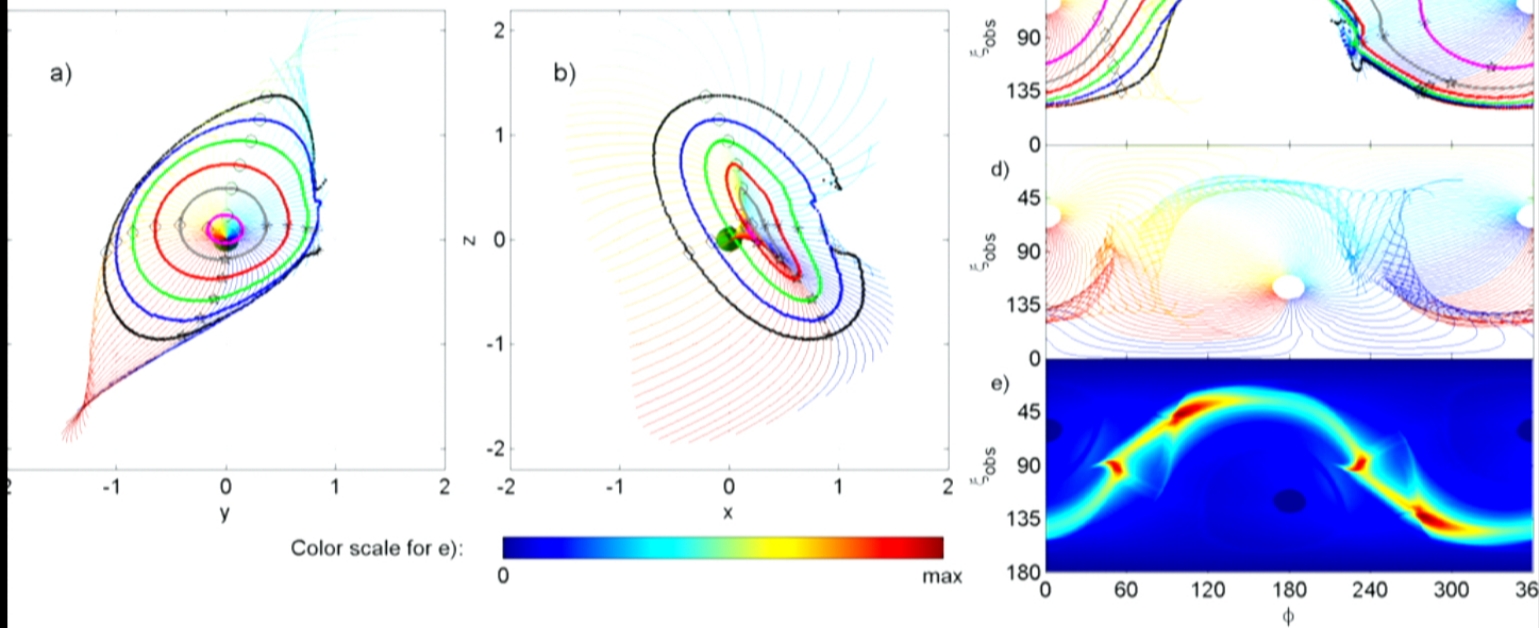
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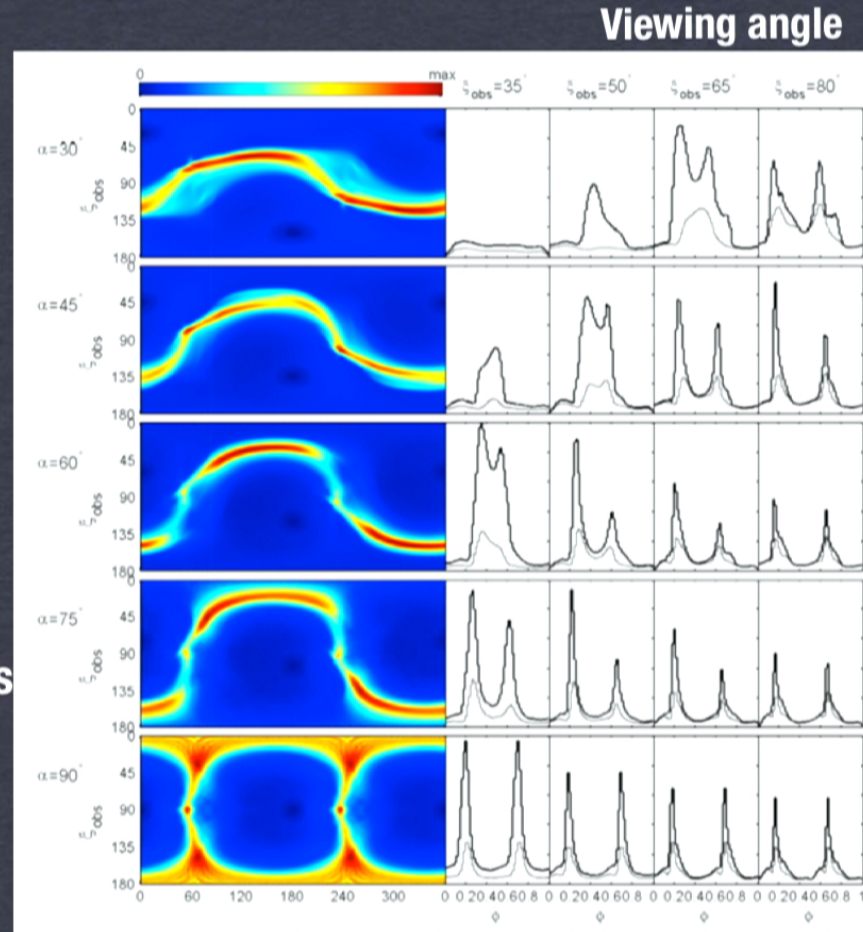
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Force-free light curves

**“Separatrix Layer”
model is a real
contender. It’s not
just outer gap vs
slot gap anymore!**

**Inclination
angle**

**Most of the
emission in FF
model accumulates
beyond $0.9 R_{lc}$**



Double peak profiles very common.

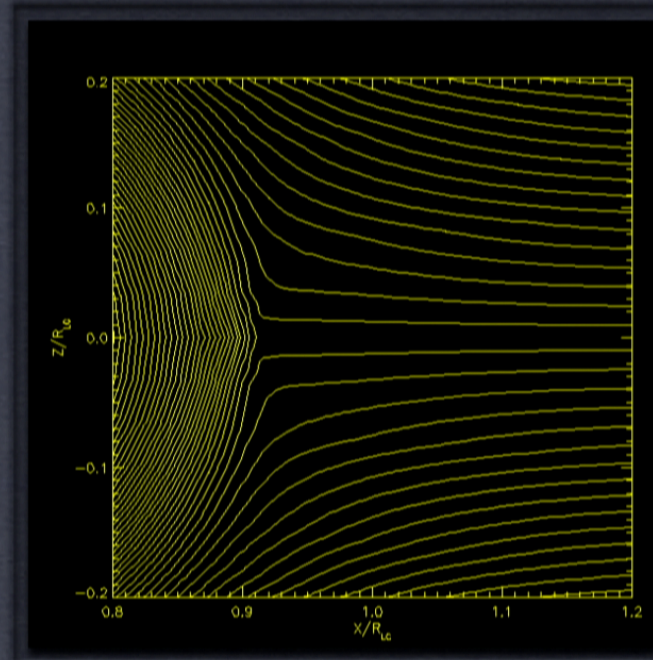
Bai & AS, 2010

Source of emission

- ✦ Emission is geometrically associated with the current sheet
- ✦ What is the acceleration and radiation mechanism in current sheet?

Most likely culprit -- relativistic reconnection. This is different from conventional picture of accelerating gaps starved of plasma and curvature emission

- ✦ Boosted synchrotron from heated plasma can work

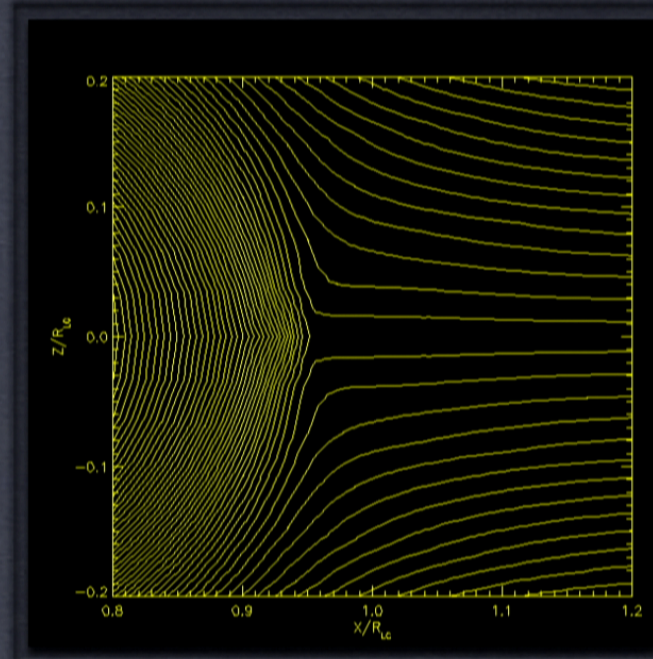


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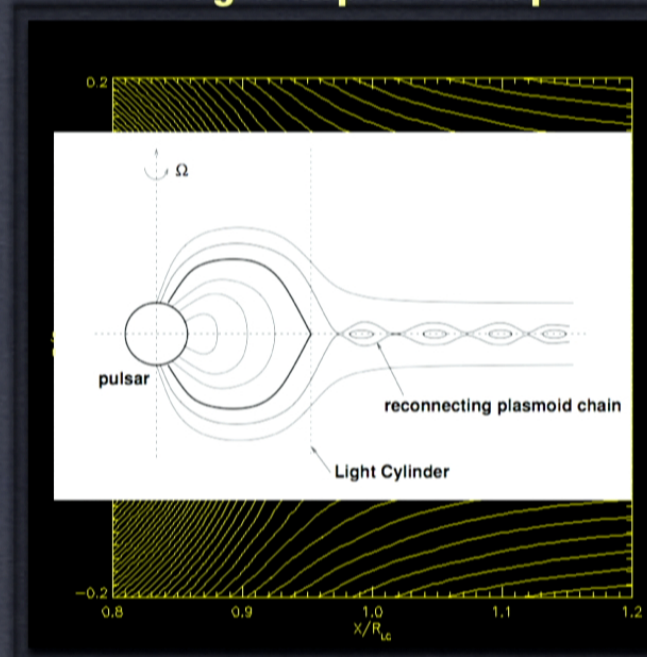
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Reconnection controls magnetospheric shape!



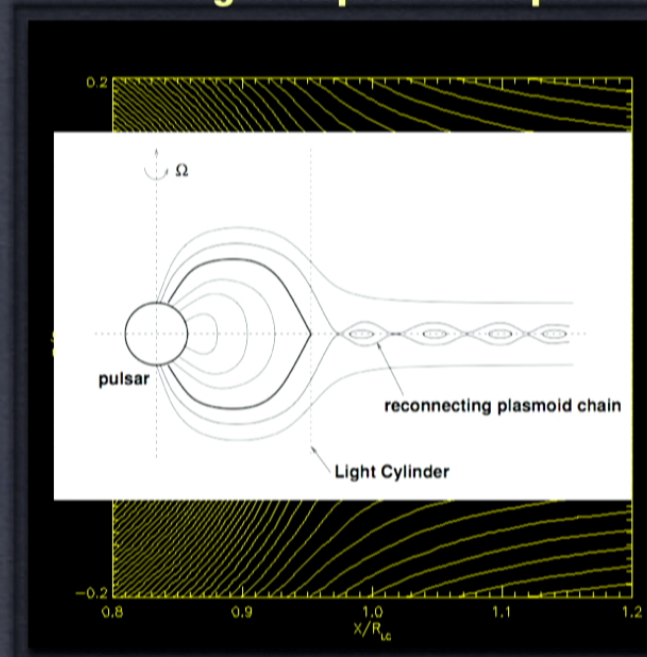
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- * Geometrically these models are being contrasted with gamma-ray observations (Separatrix Layer vs Gaps).
- * More realistic models with 3D RMHD, cascade physics and full PIC are advancing -- expect cool results in the next few years. Benefit a lot from Moore's law.
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