Title: Quantification and Evolution of Quantum Correlations

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Abstract:

Quantification and Evolution of Quantum Correlations

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Department of Physics
University of Toronto

Perimeter Institute
for Theoretical Physics
Wednesday, November 28, 2012
4:00pm-5:00pm
The Time Room

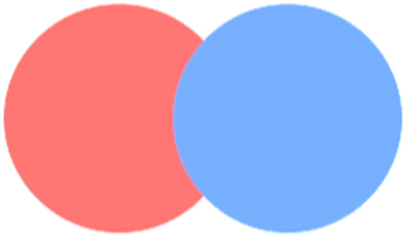
Intro + Outline

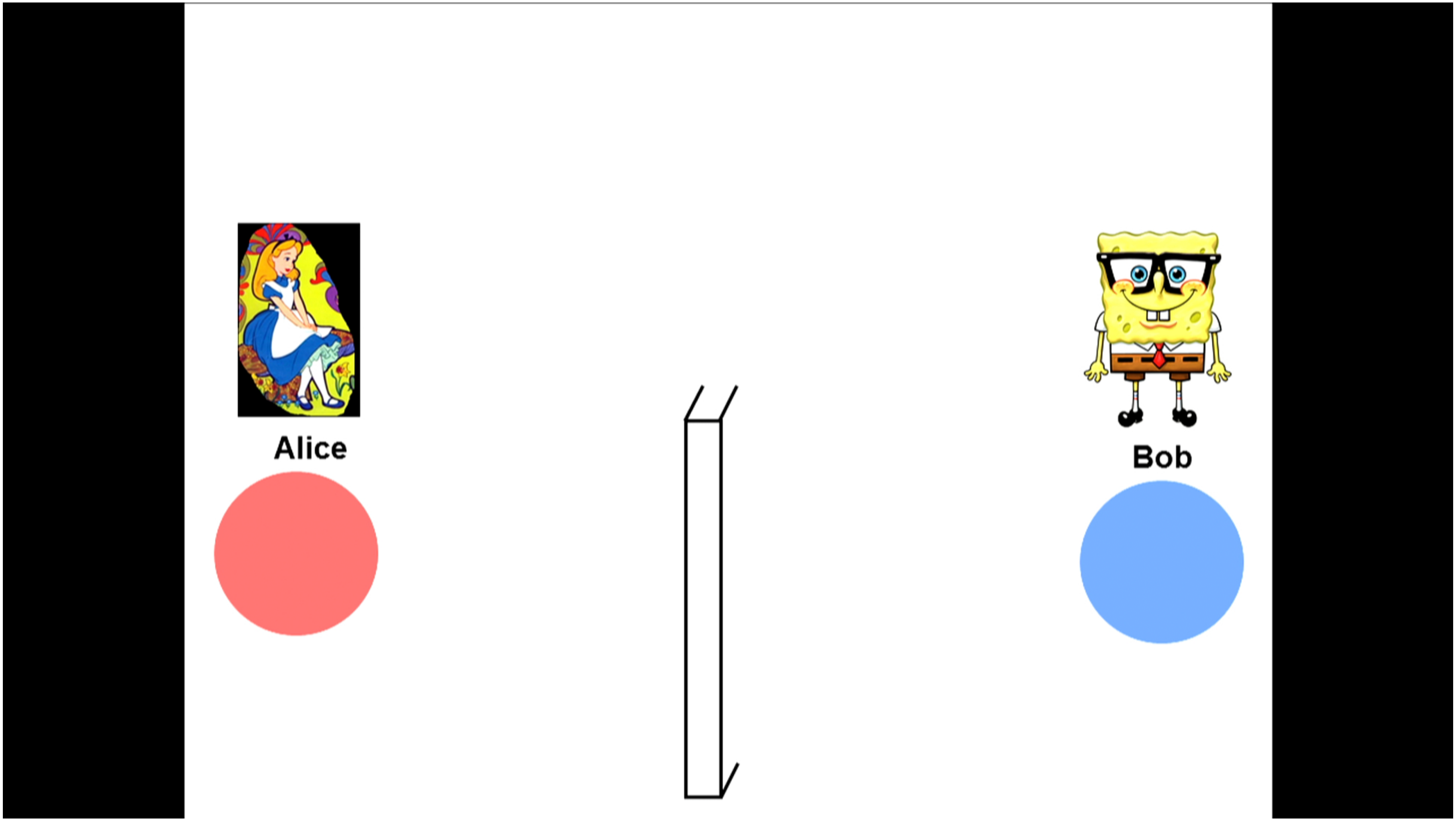
- Correlations, Quantum Correlations

- Why study Quantum Correlations?

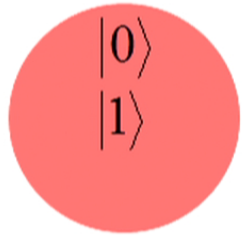
- Different Quantification Methods:
    1) Information-Based
    2) Local Operation
    3) Measurement-Based
    4) Distance Measure
    5) Geometric Representation

- Evolution of Quantum Correlations
    1) Entanglement Sudden Death in Two Qubits
    2) Entanglement Sudden Death in Two Harmonic Oscillators
    3) Correlations in Atomic Spectroscopy



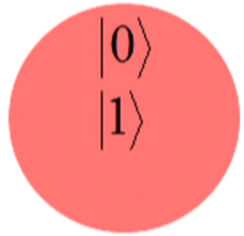

Alice
Bob

State Representation of Quantum Systems



two-level
quantum
system

State Representation of Quantum Systems

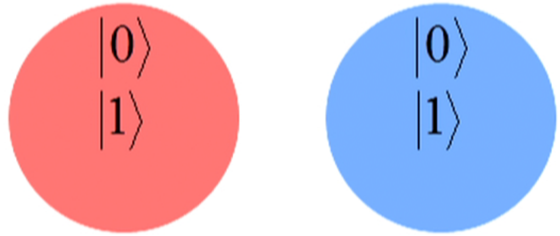


two-level quantum system

Pure State

$$|\psi\rangle = A|0\rangle + B|1\rangle$$

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |A|^2 & AB^* \\ BA^* & |B|^2 \end{pmatrix}$$


$|0\rangle$
$|1\rangle$
$|0\rangle$
$|1\rangle$

System C = System A + System B

$$|\Psi\rangle_C \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

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$$|\Psi\rangle_C \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

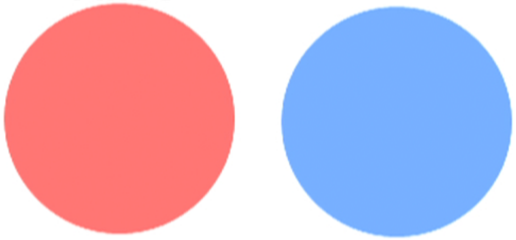
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

Bell State

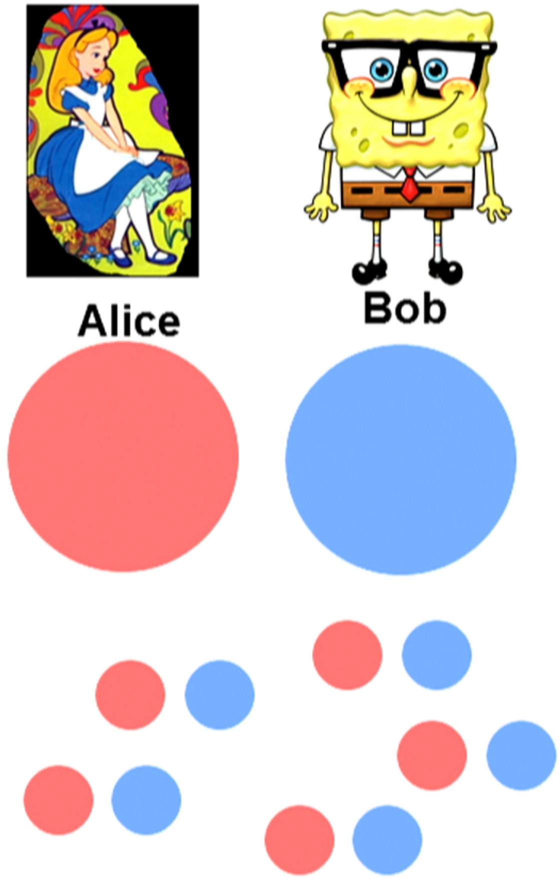
Maximally Entangled
Pure State

Alice

Bob



Bennett *et. al.*, PRA, **53** (4), 2046 (1996).



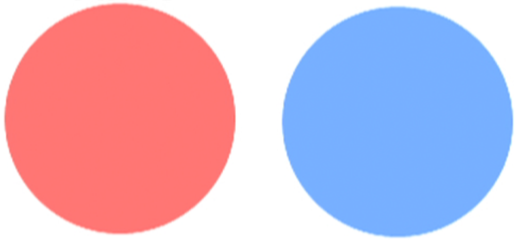
How much entanglement is there between the two qubits?

Look at *m* realizations of the pair of qubits, and ask :

Bennett *et. al.*, PRA, **53** (4), 2046 (1996).

Alice

Bob



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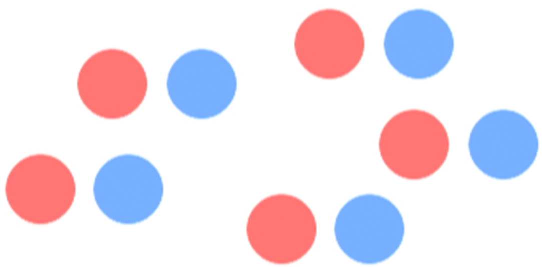

Look at <u>*m*</u> realizations of the pair of qubits, and ask :

By performing local operations and classically communicating with each, how many Bell states (*n*) can Alice and Bob create?

Bennett *et. al.*, PRA, **53** (4), 2046 (1996).

How much information is required to create the state of interest?

W. K. Wootters, Phys. Rev. Lett. **80** (10), 2245 (1998).

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Ebits!!

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$$E(\rho) = S(\rho_{red}^{A}) = S(\rho_{red}^{B})$$

straightforward for *pure* states

not as straightforward for *mixed* states

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$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Monotonic Function of Concurrence

$$E(\rho) = \min \sum_i p_i E(\psi_i)$$

W. K. Wootters, Phys. Rev. Lett. **80** (10), 2245 (1998).

Concurrence is a very useful tool to study entanglement

Asma Al-Qasimi and Daniel F. V. James *Phys Rev A* **77**, 012117 (2008).

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Example: Studies of Entanglement Sudden Death (ESD)

Asma Al-Qasimi and Daniel F. V. James *Phys Rev A* **77**, 012117 (2008).

Mathematical Description of System Investigated and its Dynamics

X-states

$$\hat{\rho}(t) = \begin{pmatrix} a(t) & 0 & 0 & w(t) \\ 0 & b(t) & z(t) & 0 \\ 0 & z^{*}(t) & c(t) & 0 \\ w^{*}(t) & 0 & 0 & d(t) \end{pmatrix}$$

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Master Equation

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + L_1[\hat{\rho}] + L_2[\hat{\rho}]$$

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Unitary evolution

Liouvillian of the two qubits

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}] + L_1[\hat{\rho}] + L_2[\hat{\rho}]$$

$$L_i[\hat{\rho}] = \frac{(\bar{n}+1)\Gamma}{2}\left\{[\hat{\sigma}^i_-,\hat{\rho}\hat{\sigma}^i_+] + [\hat{\sigma}^i_-\hat{\rho},\hat{\sigma}^i_+]\right\} + \frac{\bar{n}\Gamma}{2}\left\{[\hat{\sigma}^i_+,\hat{\rho}\hat{\sigma}^i_-] + [\hat{\sigma}^i_+\hat{\rho},\hat{\sigma}^i_-]\right\}$$

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Depopulation due to stimulated and spontaneous emissions

How to Calculate Concurrence

For mixed density matrix $\hat{\rho}$,
Find eigenvalues, $\{\lambda_i\}$, of the R matrix,

where $R = \hat{\rho}\Lambda\hat{\rho}^{*}\Lambda$ and $\Lambda = \hat{\sigma}_y \otimes \hat{\sigma}_y$.

$$\boxed{C = \max(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0),}$$

$$\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$$

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C = 0 no entanglement
C = 1 maximally entangled

Find $\hat{\rho}(t) = \begin{pmatrix} a(t) & 0 & 0 & w(t) \\ 0 & b(t) & z(t) & 0 \\ 0 & z^*(t) & c(t) & 0 \\ w^*(t) & 0 & 0 & d(t) \end{pmatrix}$

$$C = 2\max\left\{0, |z(t)| - \sqrt{a(t)d(t)}, |w(t)| - \sqrt{b(t)c(t)}\right\}$$

C = 0 equivalent to solving two quartic equations in X.

$$X = e^{-(2\bar{n}+1)\Gamma t}$$

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$$X = e^{-(2\bar{n}+1)\Gamma t}$$

$$t = 0 \Rightarrow X = 1$$

$$t = \infty \Rightarrow X = 0$$

$$\hat{\rho}(0) = \frac{1}{3}\begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-\alpha \end{pmatrix}$$

T. Yu and J. H. Eberly.
Phys. Rev. Lett **93**,
140404 (2004).

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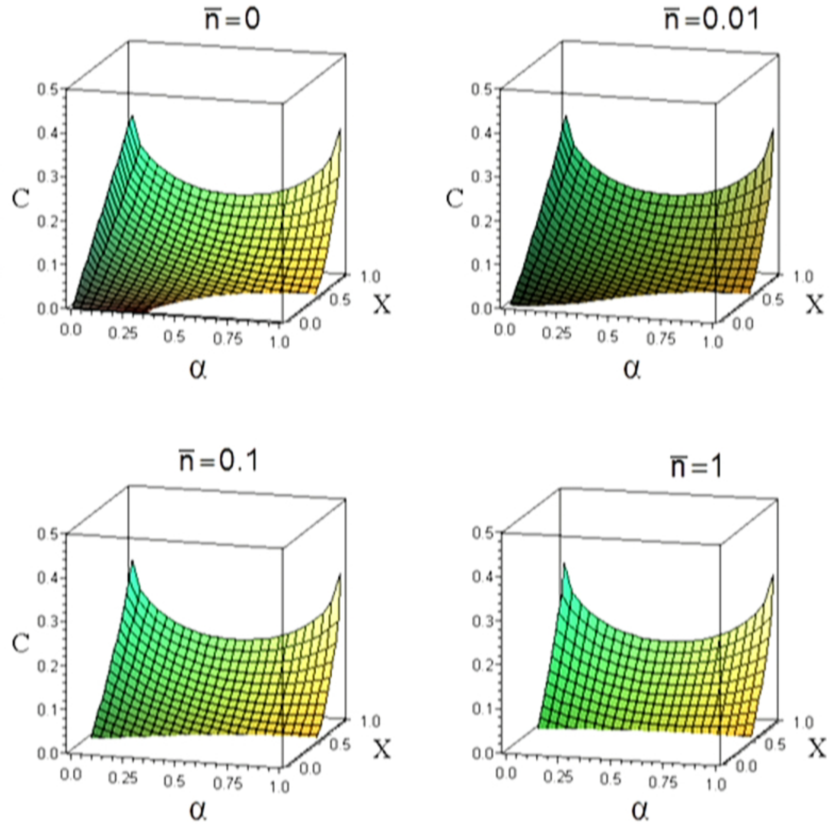

T. Yu and J. H. Eberly.
Phys. Rev. Lett **93**,
140404 (2004).

showed that, at T=0 there is no sudden death for $0 \leq \alpha \leq 1/3$ only.

Partial Transpose Criterion for Separability

ρ = density matrix of total system

transpose over one system: $$\sigma = \sum_A w_A (\rho'_A)^T \otimes \rho''_A$$

$$\sigma_{m\mu,n\nu} \equiv \rho_{n\mu,m\nu}$$

A. Peres, *Phys. Rev. Lett.* **77**, No. 8, 1413-1415 (1996).

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σ has at least one negative eigenvalue → System entangled

Otherwise → System either entangled or separable

A. Peres, *Phys. Rev. Lett.* **77**, No. 8, 1413-1415 (1996).

The weakness in the separability criterion does not prevent it from being useful in some studies.

For example, in the study of the phenomena of Entanglement sudden death (ESD), the <u>nonexistence of ESD</u> *can* be proven with certainty, but *not* its <u>existence.</u>

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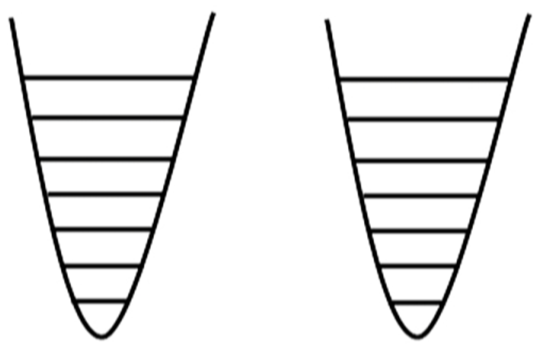

(General) N-photon-two-mode state undergoing dephasing

$$\rho(t) = \sum_{k=0}^{N} \rho_{kk}(0) |N-k,k\rangle\langle N-k,k|$$

$$+ \sum_{\substack{k,m=0 \\ k\neq m}}^{N} \rho_{km}(0) e^{-\frac{1}{2}(k-m)^2 (\Gamma_1+\Gamma_2)t} |N-k,k\rangle\langle N-m,m|$$

For σ_{PT}:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{ e^{i\theta}|N-k,m\rangle - |N-m,k\rangle \right\}$$

$\theta = \mathrm{Arg}(\rho_{km})$

is an eigenvector with eigenvalue:

$$-|\rho_{km}(t)| = -|\rho_{km}(0)|e^{-\frac{1}{2}(k-m)^2(\Gamma_1+\Gamma_2)t}$$

Al-Qasimi and James , *Optics Letters* **34**, 268-270 (2009).

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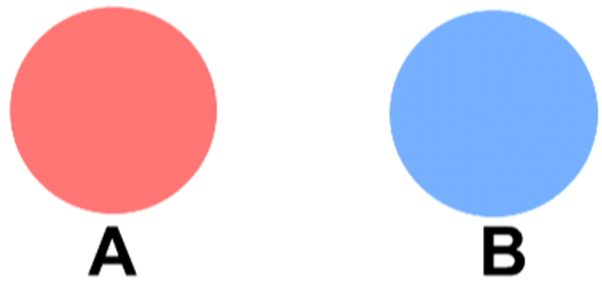
Quantum Discord: Measurement of Quantum Systems Disturbs Them



H. Ollivier and W. H. Zurek, *Phys. Rev. Lett.* **88**, 017901 (2001).

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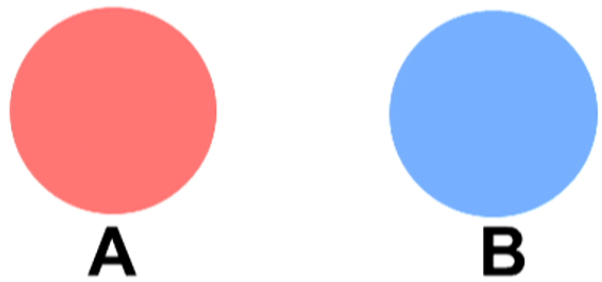

$$I(\rho) = S(\rho^A)+S(\rho^B)-S(\rho)$$

ρ^i = reduced density matrix of system i

Perform a measurement on system B only.

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Calculate $I(\rho)$ before and after the measurement and compare.

↓

$I(\rho)$

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$$Q(\rho) = I(\rho) - C(\rho)$$

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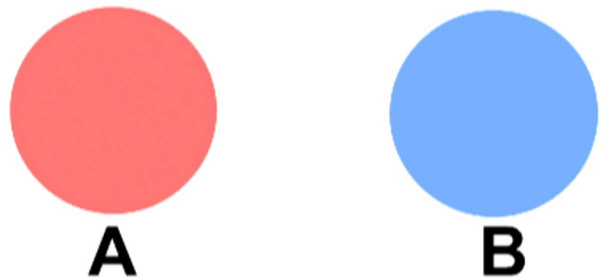

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To obtain final form of $C(\rho)$, maximize over all measurements

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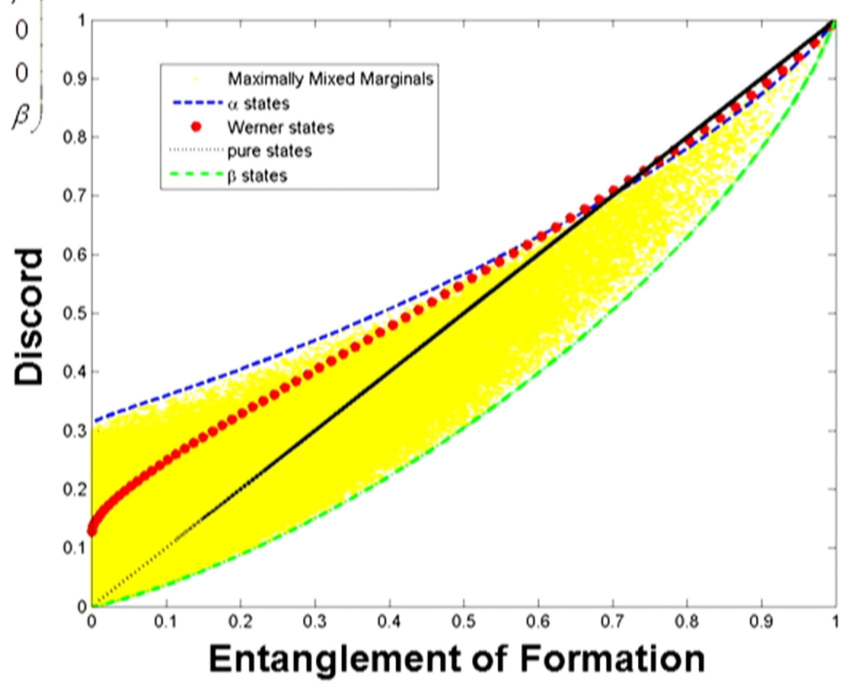
Discord versus Entanglement

$$\hat{\rho}_\beta = \frac{1}{2}\begin{pmatrix} \beta & 0 & 0 & \beta \\ 0 & (1-\beta) & (1-\beta) & 0 \\ 0 & (1-\beta) & (1-\beta) & 0 \\ \beta & 0 & 0 & \beta \end{pmatrix}$$



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$$\rho_{Werner} = \frac{1-z}{4} I + z|\psi\rangle\langle\psi|$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Al-Qasimi and James, *Physical Review A* , **83**, 032101 (2011).

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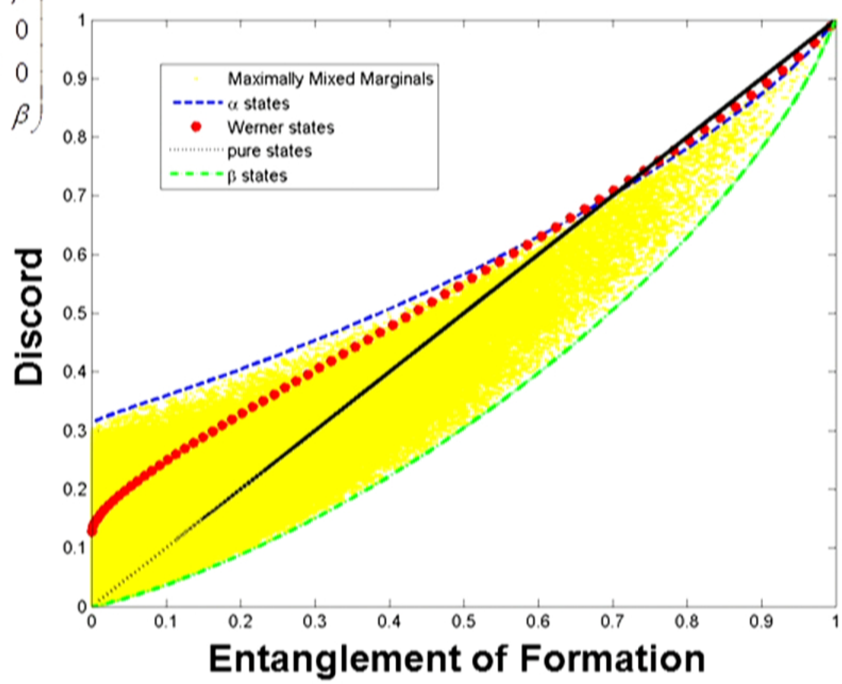

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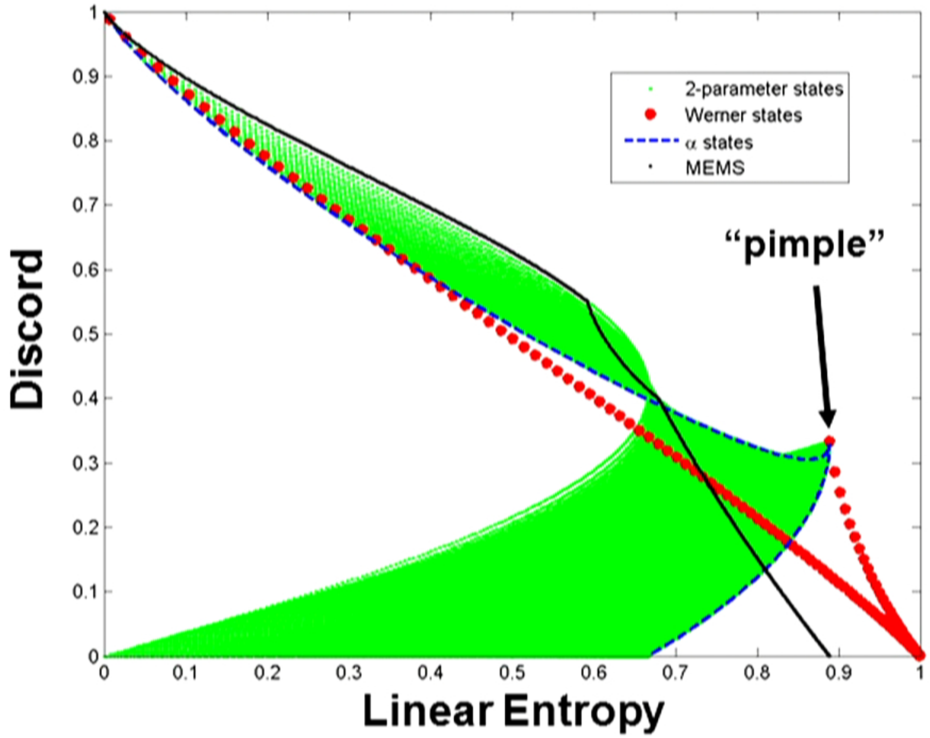
Al-Qasimi and James, *Physical Review A* , **83**, 032101 (2011).

Discord versus Entropy

$$\hat{\rho}_{a,b} = \frac{1}{2}\begin{pmatrix} a & 0 & 0 & a \\ 0 & (1-a-b) & 0 & 0 \\ 0 & 0 & (1-a+b) & 0 \\ a & 0 & 0 & a \end{pmatrix}$$



Spectroscopy Using Atoms

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}|e\rangle\right)$$

In a Ramsey Experiment, by measuring the population of atoms in a certain state, ϕ can be determined.

* Chwalla *et al*., Appl. Phys. B **89**, 483 (2007).

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Correlations in Quantum-Enhanced Atomic Spectroscopy*

2 ions in a trap with ***dephasing*** going on.

* Al-Qasimi, James, Blatt, and White (In Preparation)

Correlations in Quantum-Enhanced Atomic Spectroscopy*

2 ions in a trap with ***dephasing*** going on.

Initial State

$$|\psi\rangle = A|gg\rangle + B|ge\rangle + C|eg\rangle + D|ee\rangle$$

Evolves into the form:

$$\rho = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

* Al-Qasimi, James, Blatt, and White (In Preparation)

Parity $P = 2\,\mathrm{Re}\{z\}$

how well the relative phase is being measured

* Quesada, Al-Qasimi, and James, *J. Mod. Opt.*, **59** (15), 1322 (2012).

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Entanglement $C = 2\max\left\{0, |z| - \sqrt{ad}\right\}$

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$$N_1 = H\left(\frac{1}{2} + \frac{1}{2}\sqrt{(a-d+b-c)^2 + 4z^2}\right)$$

$$H(y) = -y\log_2 y - (1-y)\log_2(1-y)$$

Binary Entropy Function

* Quesada, Al-Qasimi, and James, *J. Mod. Opt.*, **59** (15), 1322 (2012).

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Binary Entropy Function

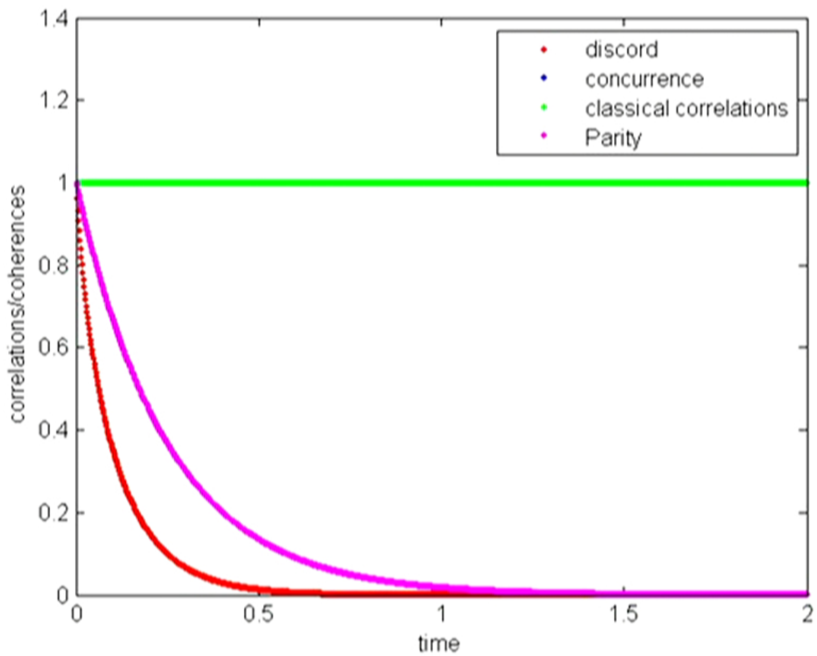
$$N_2 = -a\log_2\left[\frac{a}{a+c}\right] - b\log_2\left[\frac{b}{b+d}\right] - c\log_2\left[\frac{c}{a+c}\right] - d\log_2\left[\frac{d}{b+d}\right]$$

* Quesada, Al-Qasimi, and James, *J. Mod. Opt.*, **59** (15), 1322 (2012).

Examples

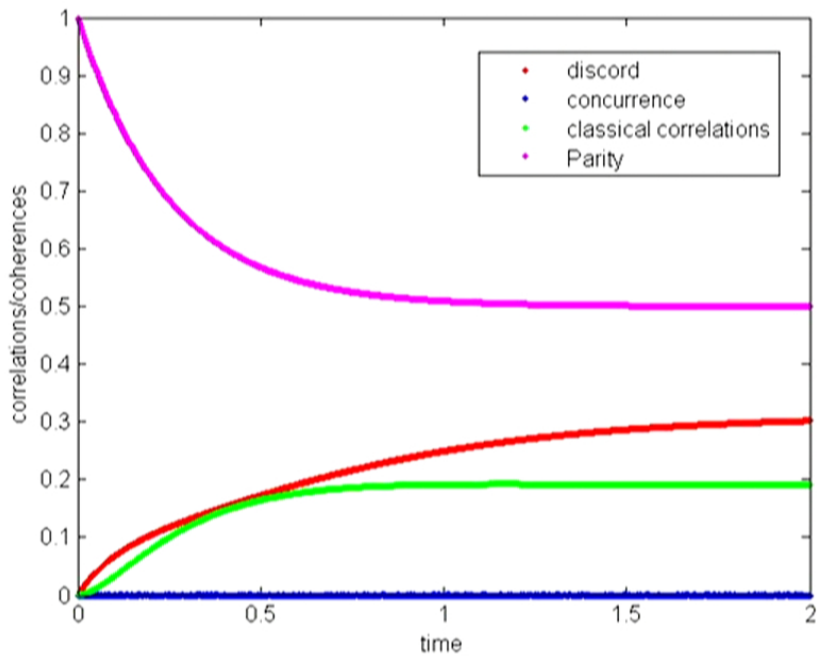
Initial state is a Bell state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

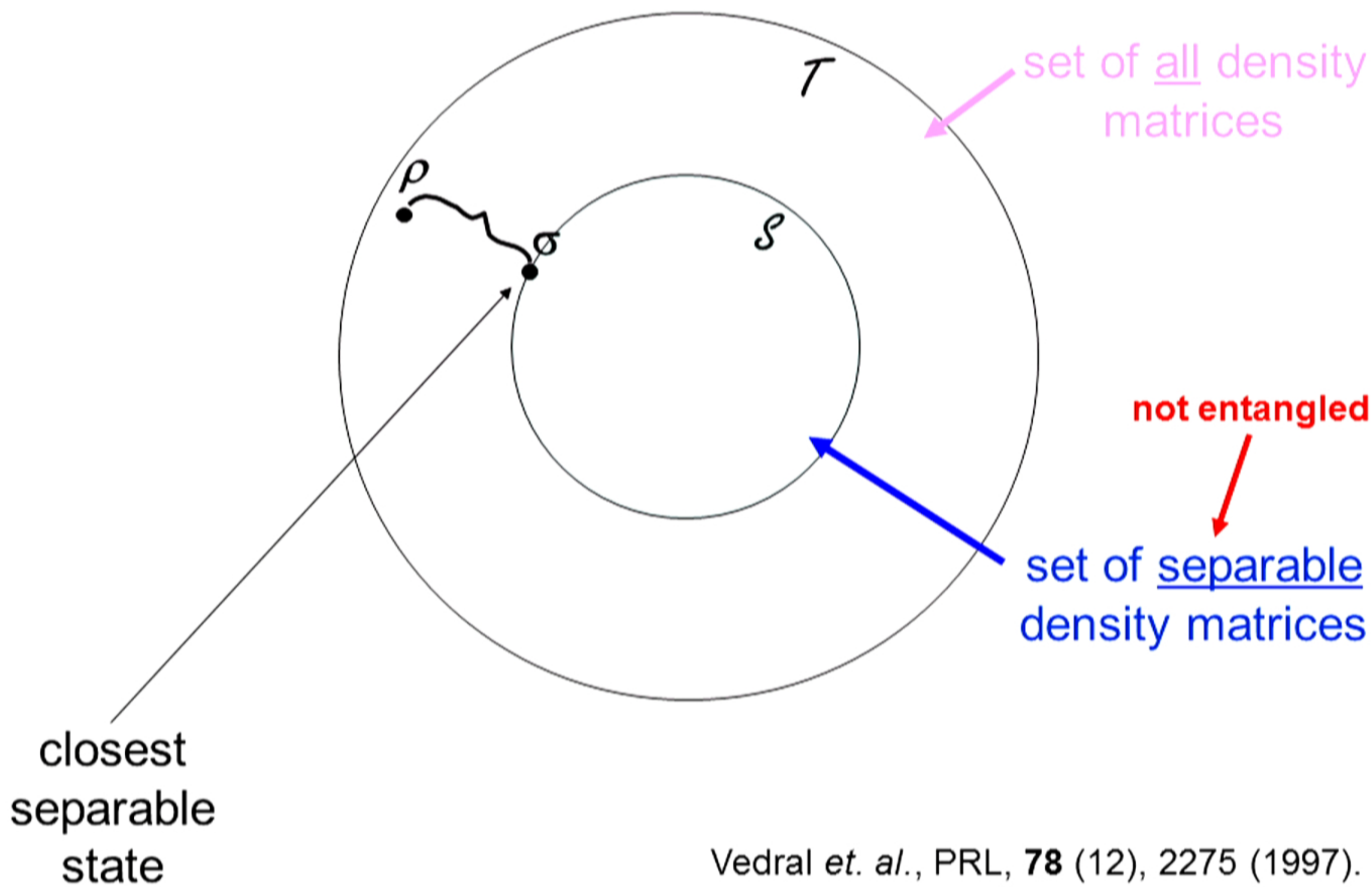


Initial state is a separable state

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

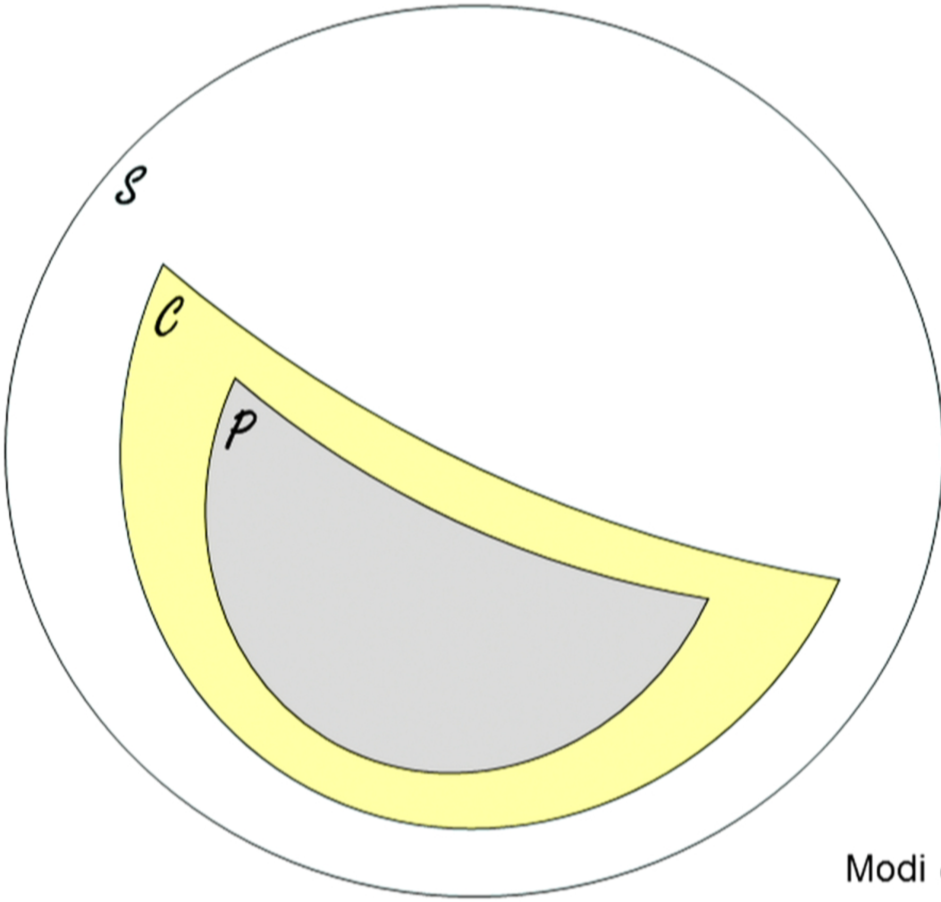


Using 'Distance' to Quantify Correlations



Vedral *et. al.*, PRL, **78** (12), 2275 (1997).

Hierarchy of Separable States



Modi *et. al.*, PRL, **104**, 080501 (2010).

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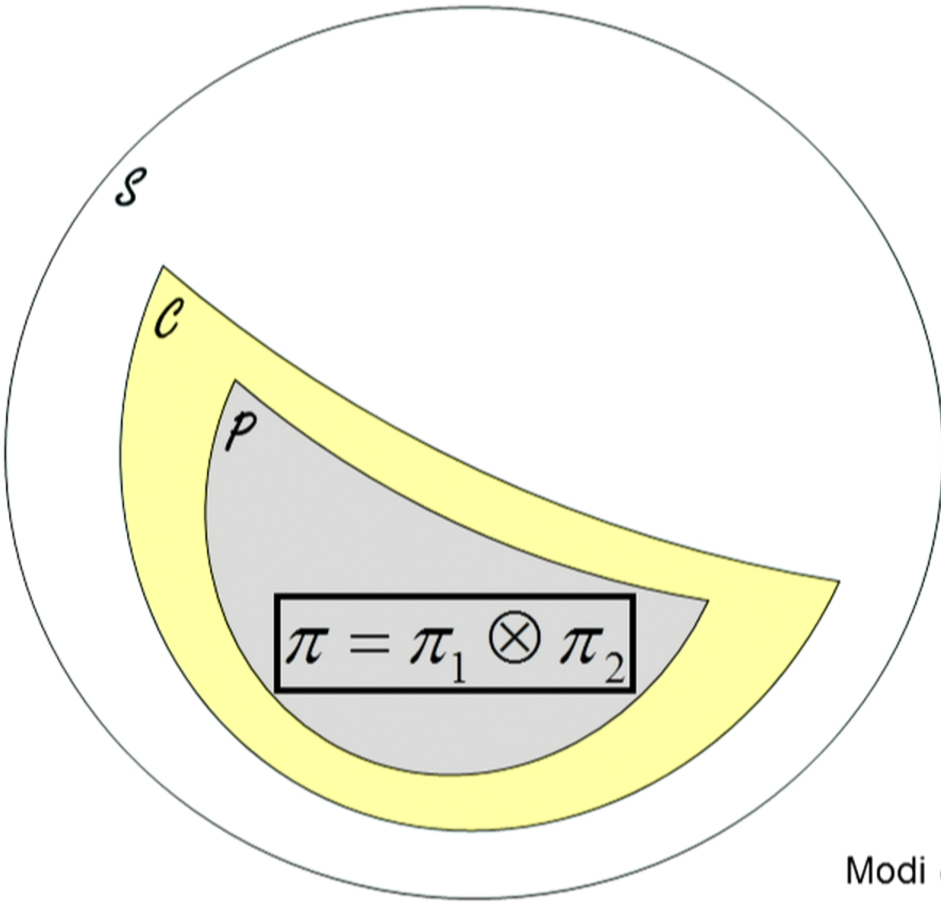

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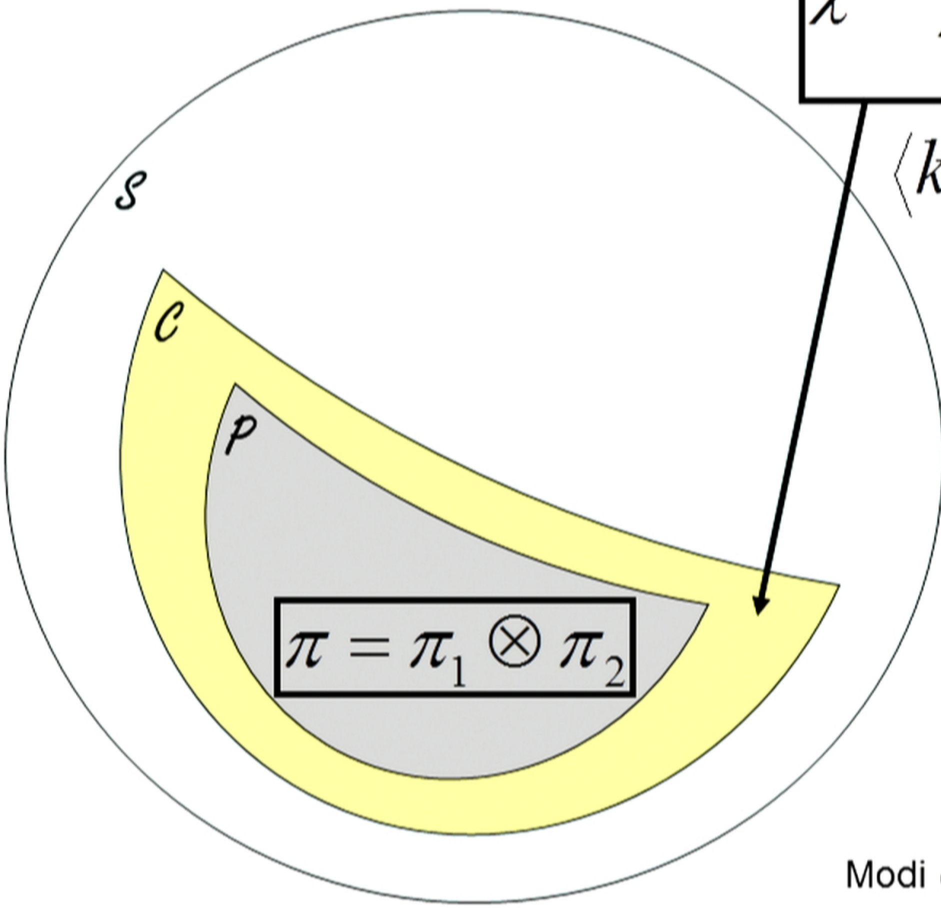
Hierarchy of Separable States

$$\chi = \sum_{k_n} p_{k_1 k_2} |k_1 k_2\rangle\langle k_1 k_2|$$

$$\langle k_1 k_2 | k_1' k_2' \rangle = \delta_{k_1 k_1'} \delta_{k_2 k_2'}$$



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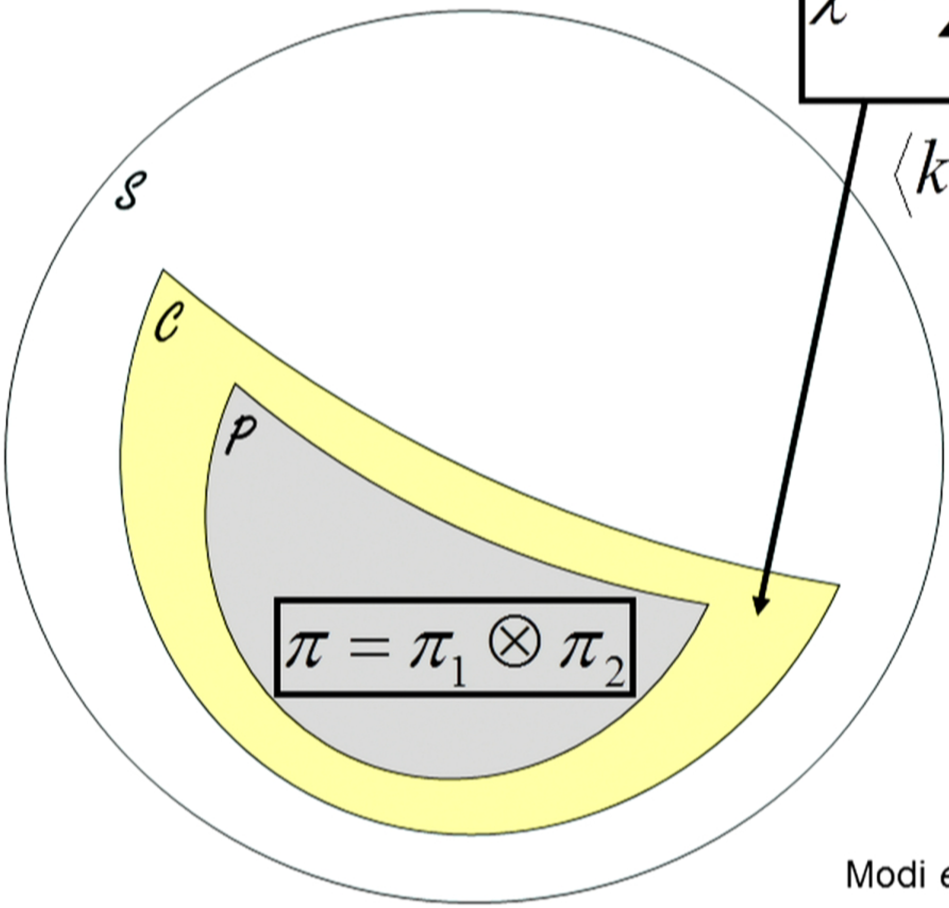

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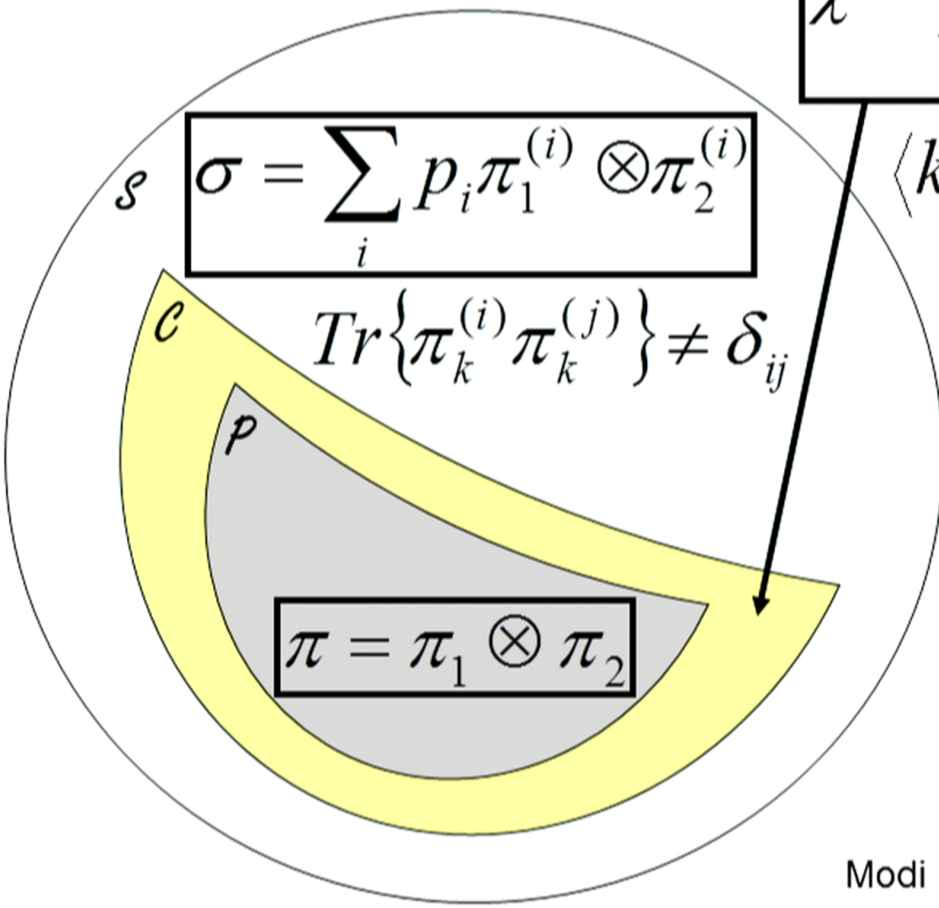

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$$\left\langle k_1 k_2 \middle| k_1' k_2' \right\rangle = \delta_{k_1 k_1'} \delta_{k_2 k_2'}$$

$$\sigma = \sum_i p_i \pi_1^{(i)} \otimes \pi_2^{(i)}$$

$$Tr\left\{\pi_k^{(i)} \pi_k^{(j)}\right\} \neq \delta_{ij}$$

$$\pi = \pi_1 \otimes \pi_2$$

Modi *et. al*., PRL, **104**, 080501 (2010).

Given density matrix ρ

Modi *et. al.*, PRL, **104**, 080501 (2010).

Given density matrix ρ

Entanglement → shortest distance between ρ and a separable state (σ')

Discord → shortest distance between ρ and a classical state (χ')

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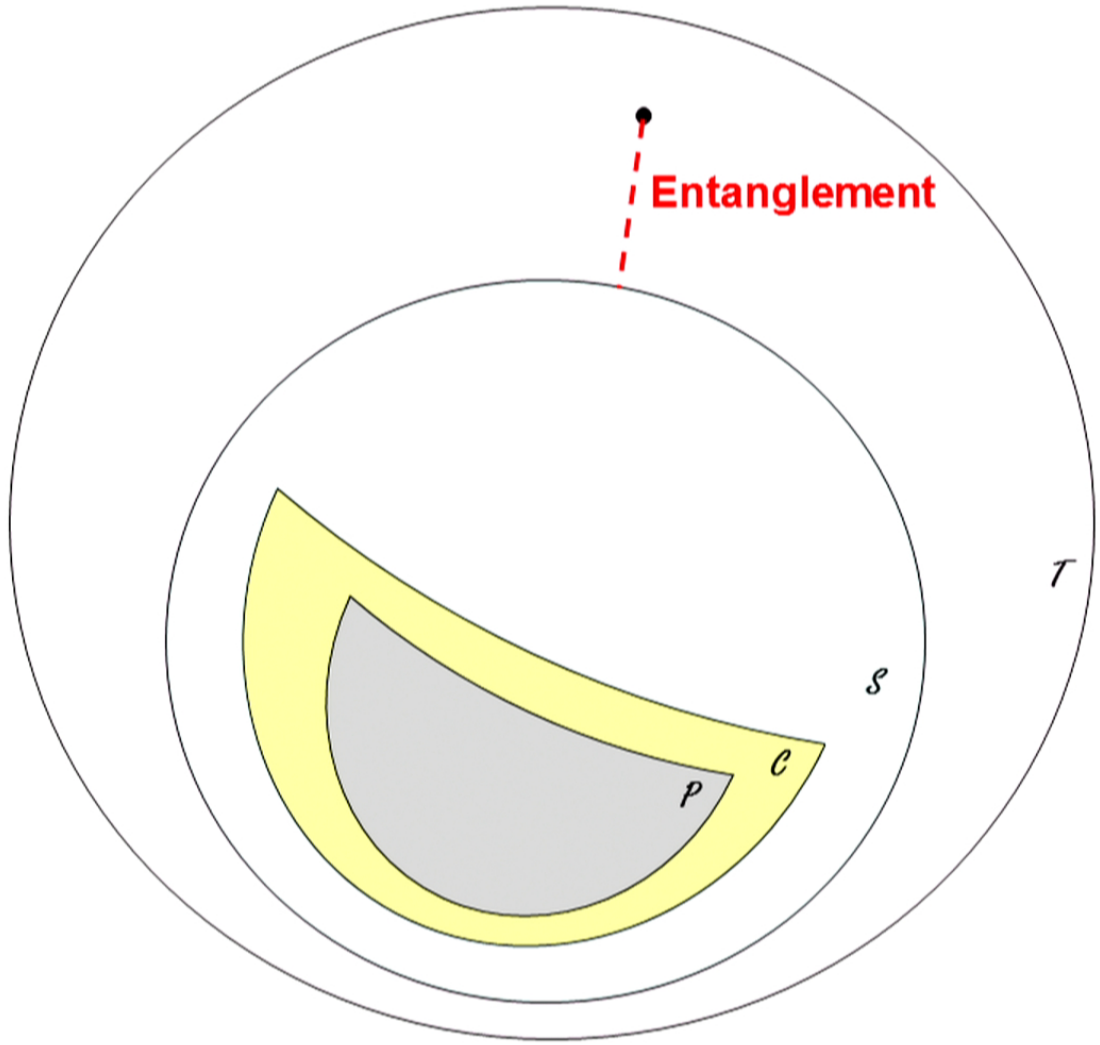
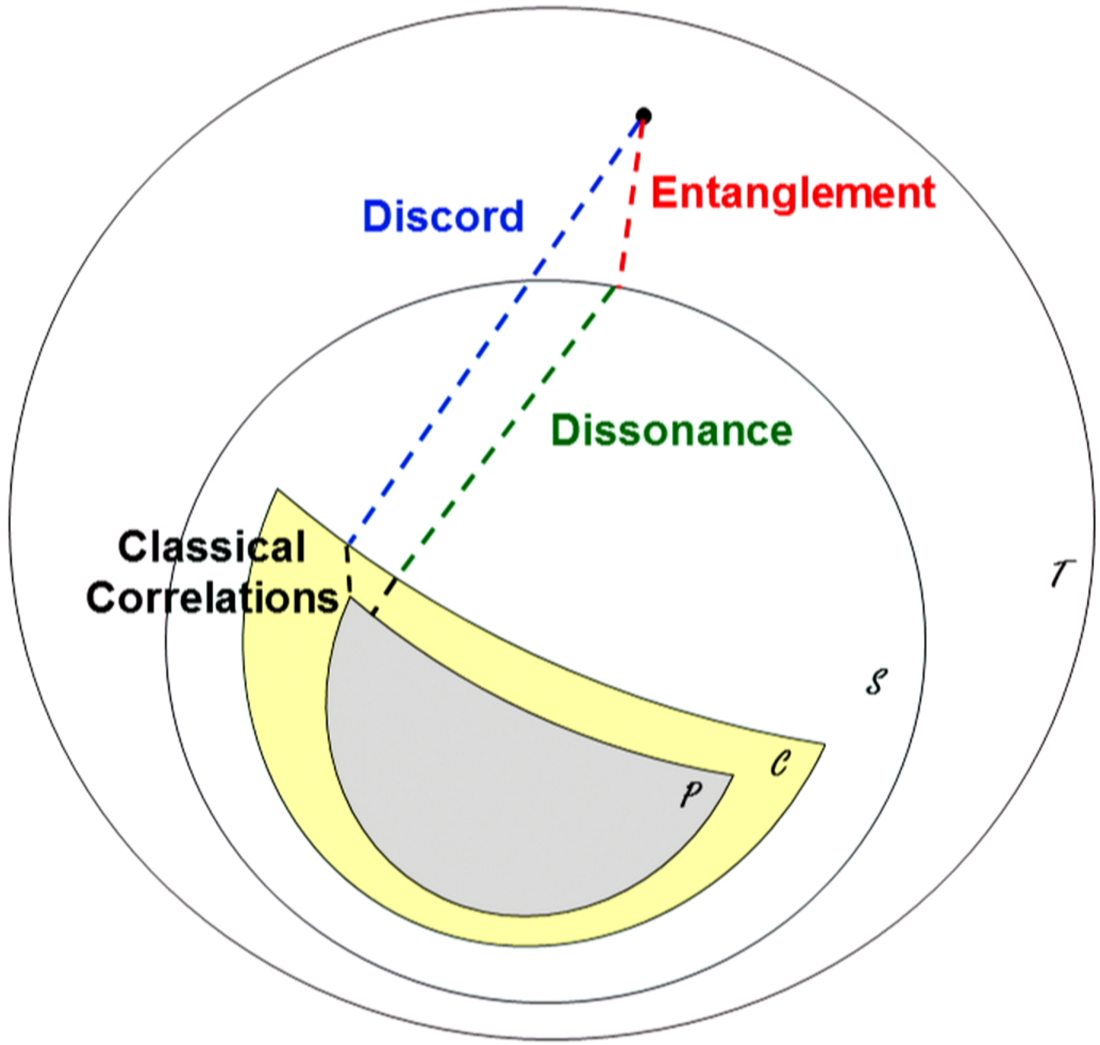
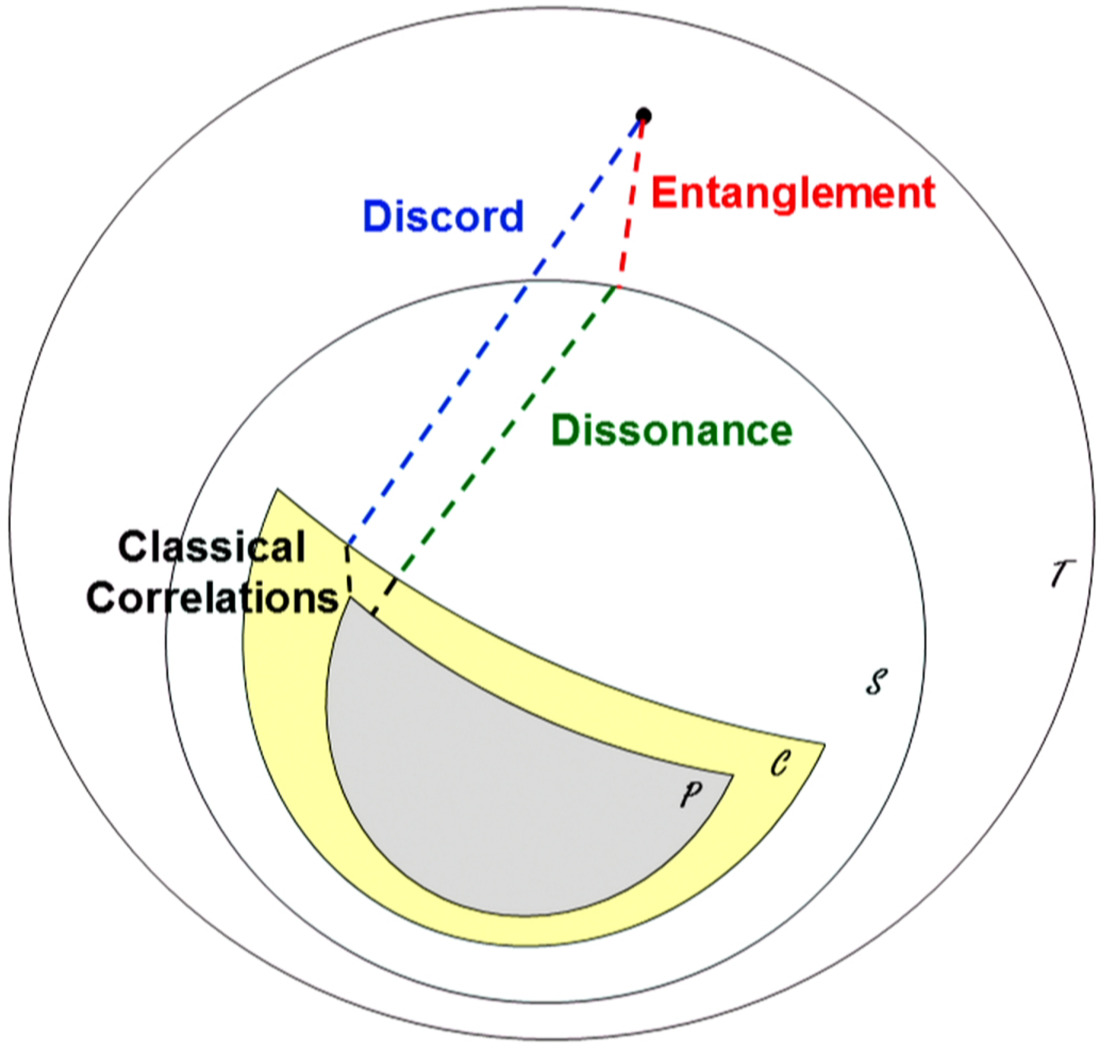
Modi *et. al*., PRL, **104**, 080501 (2010).


Entanglement
T
S
C
P



Discord
Entanglement
Dissonance
Classical
Correlations
$\mathcal{T}$
$\mathcal{S}$
$\mathcal{C}$
$\mathcal{P}$


Discord
Entanglement
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Classical
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$\mathcal{T}$
$\mathcal{S}$
$\mathcal{C}$
$\mathcal{P}$

Measuring the Distance

$$D(\rho \| \sigma)$$

Quantity should be zero when it is known that the correlation is absent.

Quantity should be invariant under local unitary operations.

Quantity cannot increase under
local general measurements + classical communications

$$S(\rho \| \sigma) = Trace\left\{\rho \ln \frac{\rho}{\sigma}\right\}$$

Von Neumann Relative Entropy

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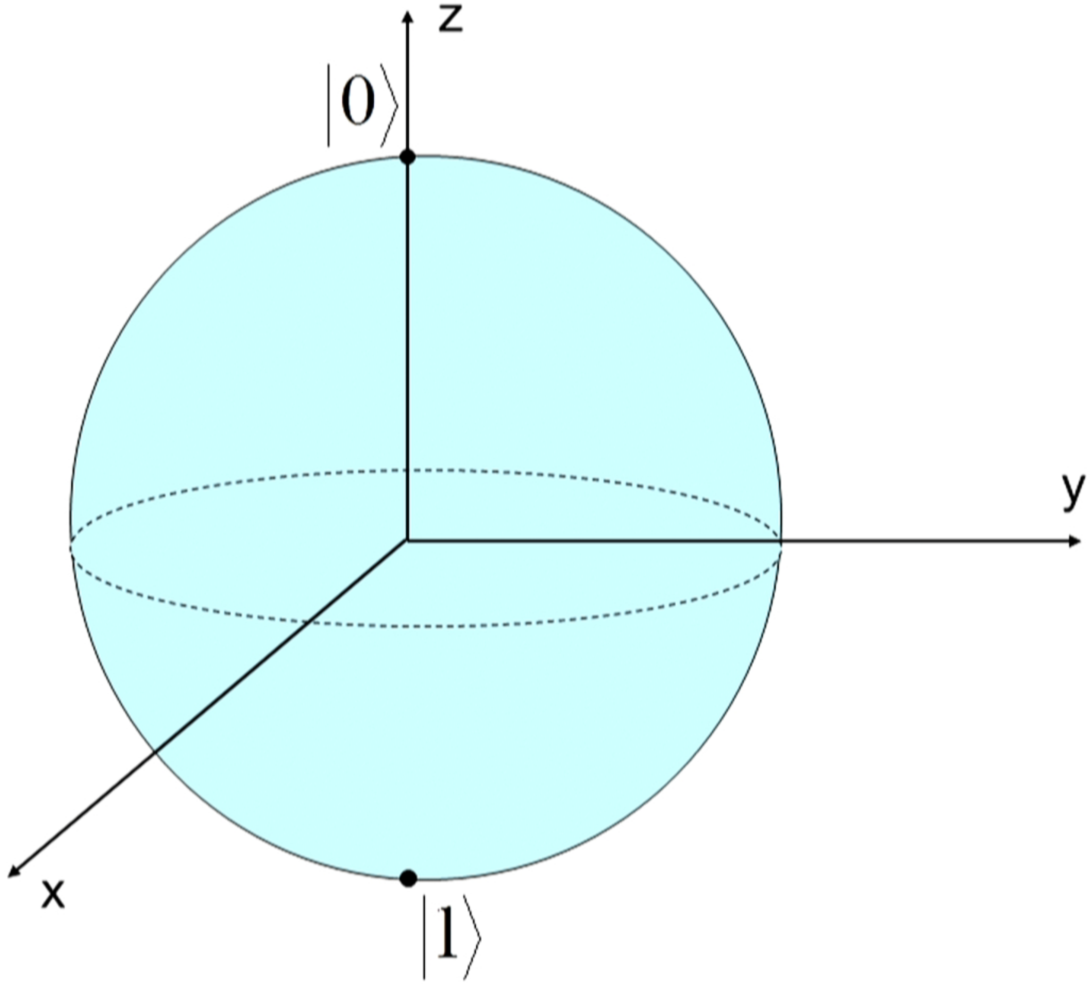
Bloch Sphere Representation of a Qubit



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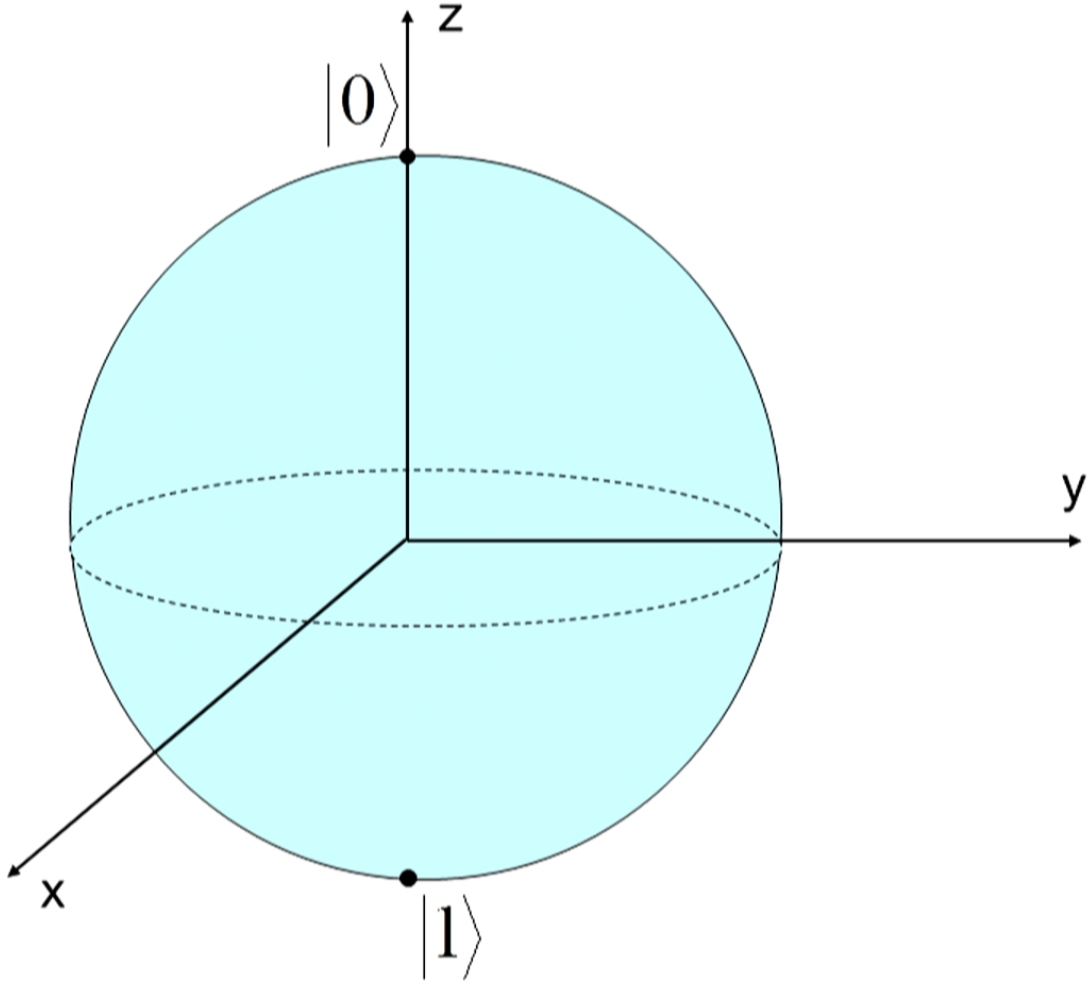

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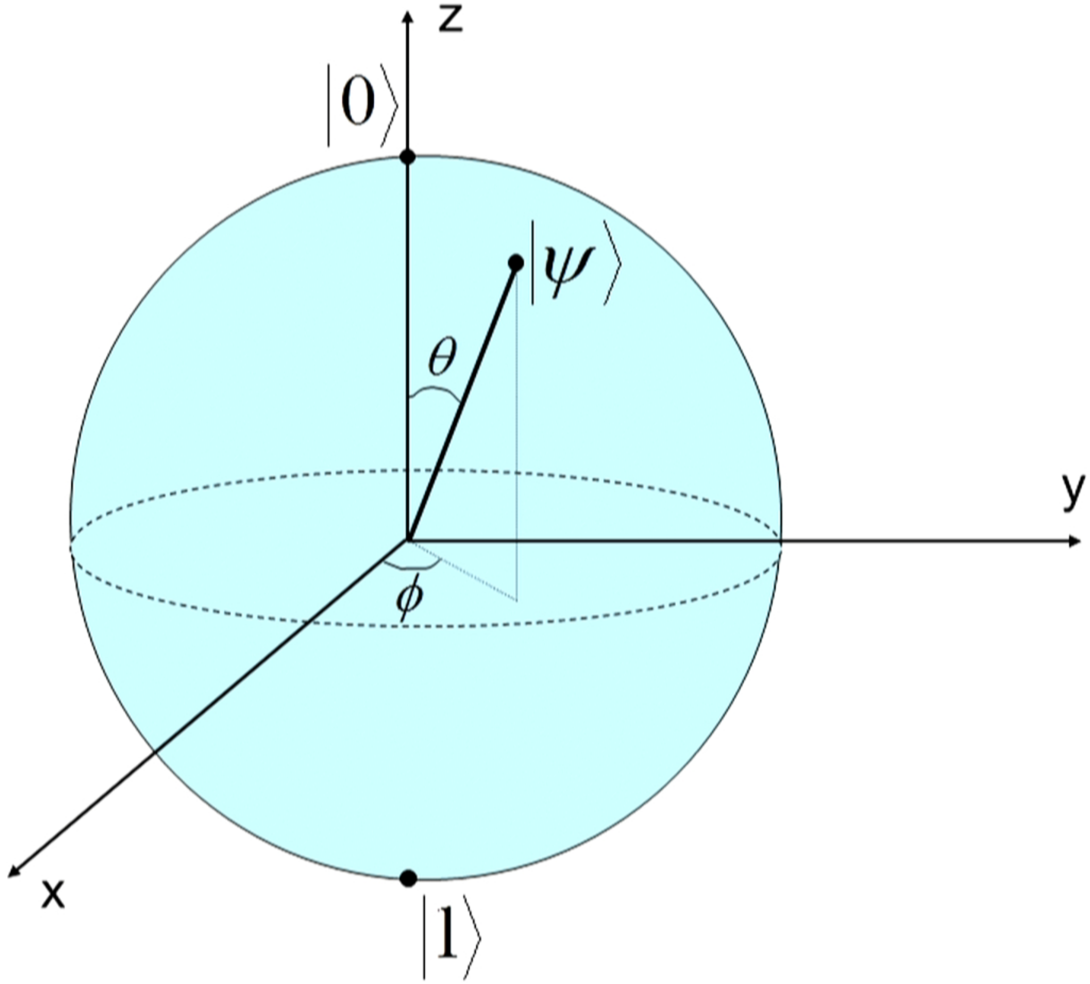

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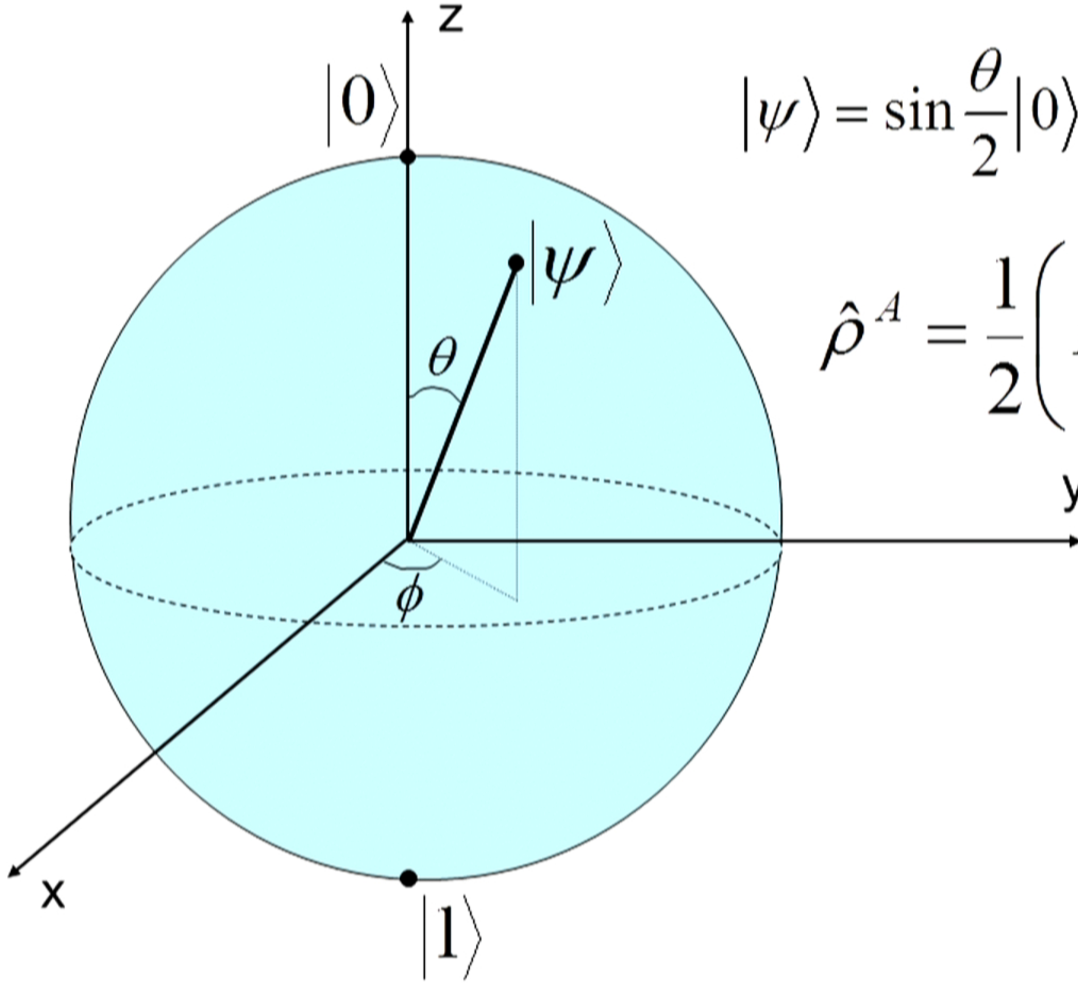
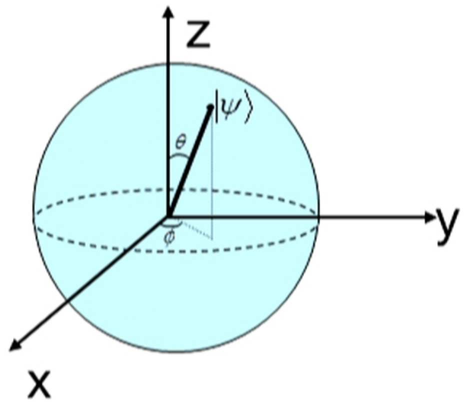
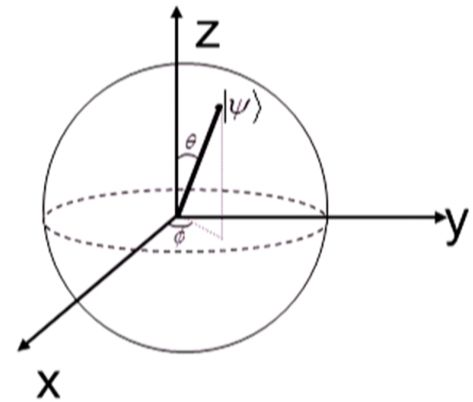


$$|\psi\rangle = \sin\frac{\theta}{2}|0\rangle + e^{i\phi}\cos\frac{\theta}{2}|1\rangle$$

$$\hat{\rho}^{A} = \frac{1}{2}\left(\hat{I} + \sum_{i=1}^{3} A_i \hat{\sigma}_i\right)$$





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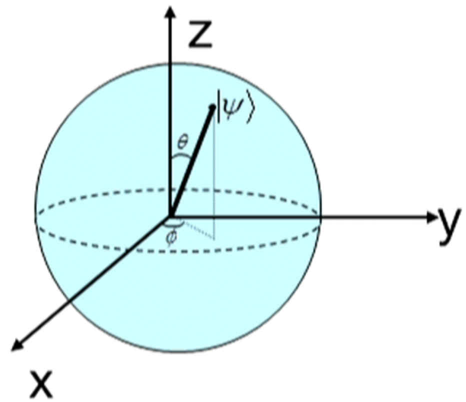

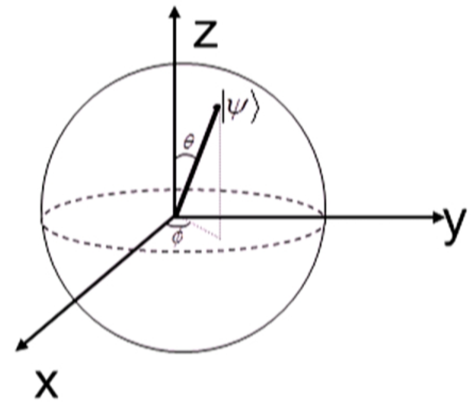

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$$C_{ij} = \left\langle \left(\hat{\sigma}_i - A_i \hat{I} \right) \otimes \left(\hat{\sigma}_j - B_j \hat{I} \right) \right\rangle$$

Cross-Correlation Tensor

$$\hat{\rho}^{AB} = \frac{1}{2}\left(\hat{I} + \sum_{i=1}^{3} A_i \hat{\sigma}_i \right) \otimes \frac{1}{2}\left(\hat{I} + \sum_{j=1}^{3} B_j \hat{\sigma}_j \right) + \frac{1}{4} \sum_{i,j=1}^{3} C_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j$$

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Determinant of C_{ij} tells us about the existence/absence of quantum correlations

1) When Entanglement > 0, $\mathrm{Det}(C_{ij}) < 0$

Al-Qasimi and James (In Preparation)

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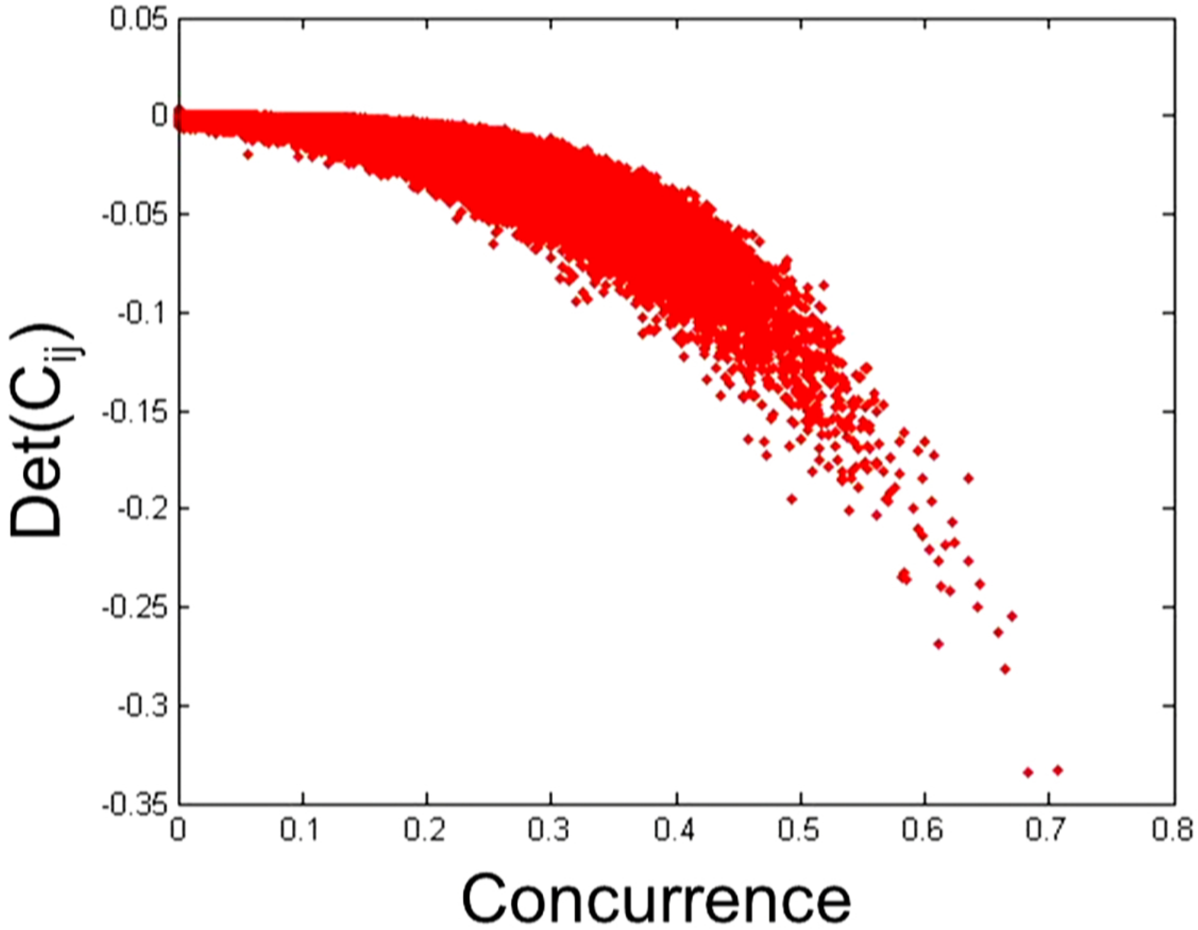
States with $Det(Cij) < 0 \rightarrow$ **Sinister States**

Al-Qasimi and James (In Preparation)


0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Det(C_ij)
Concurrence

Why Sinister?

$$C_{ij} = \sum_{k=1}^{3} s_k a_i^{(k)} b_j^{(k)}$$

Singular Value Decomposition

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If one of the vector sets is LH (and the other RH), Det(Cij) < 0.

Sinister = Latin for *Left*

Conclusions

- Quantify Quantum Correlations (QC) by the amount of information required to create the state.
- Detect QC by performing a local operation
- Quantify QC by the fact that measurements disturb quantum systems.
- Quantify correlations by the distance between the state of interest and the closest state that lacks the quantity of interest.
- Quantify QC by the determinant of the Cross-Correlation Matrix

- Concurrence and Partial Transpose Criterion for Separability to study ESD
- Correlations in Atomic Spectroscopy