

Title: Universal driven dynamics near phase transitions : Kibble-Zurek ramps with and without an order parameter

Date: Nov 29, 2012 04:00 PM

URL: <http://pirsa.org/12110095>

Abstract: Near a critical point, the equilibrium relaxation time of a system diverges and any change of control parameters leads to non-equilibrium behavior. The Kibble-Zurek (KZ) problem is to determine the evolution of the system when the change is slow. In this talk, I will introduce a non-equilibrium scaling limit in which these evolutions are universal and define a KZ universality classification with exponents and scaling functions. I will illustrate the physics accessible in this scaling limit in simple classical and quantum model theories with symmetry-breaking transitions.

I will then turn to the KZ problem near quantum phase transitions without a local order parameter.

First, I will introduce the necessary background through the example of the Ising gauge theory/generalized toric code. Using duality and the scaling theory developed in the first part of the talk, I will then argue that the late time dynamics exhibits a slow coarsening of the string-net that is condensed in the starting topologically ordered state. I will also discuss a time dependent amplification of the energy splitting between topologically degenerate states on closed manifolds and the dangerous irrelevance of gapped modes. Finally, I will extend these ideas to the non-abelian $SU(2)_k$ ordered phases of the relevant Levin-Wen models.



Universal driven dynamics near phase transitions:

Kibble-Zurek ramps with and without an order parameter

Anushya Chandran

F. J. Burnell

A. Erez

V. Khemani

S. S. Gubser

S. L. Sondhi





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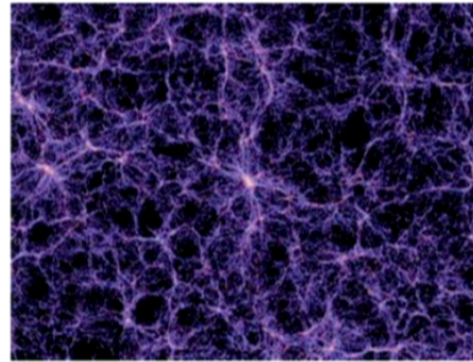
S. L. Sondhi



The Kibble-Zurek (KZ) mechanism



Expanding
universe →



Kibble, JPA 9, 1387 (1976)



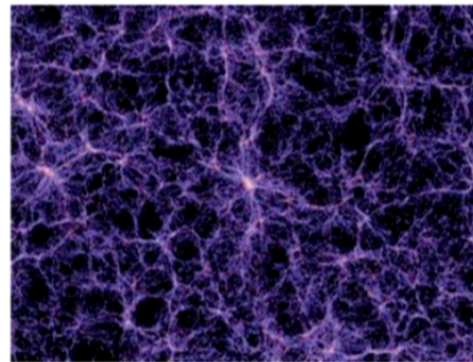
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The Kibble-Zurek (KZ) mechanism

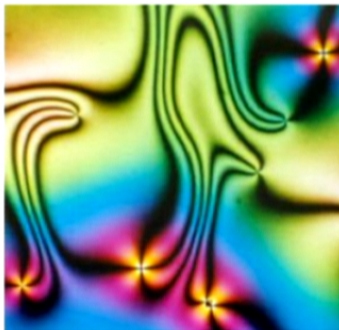


Expanding
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Kibble, JPA 9, 1387 (1976)

Laboratory



Zurek, Nature 317, 6037 (1985)
Zurek, Physics Reports (1996)
Images courtesy Oleg Lavrentovich

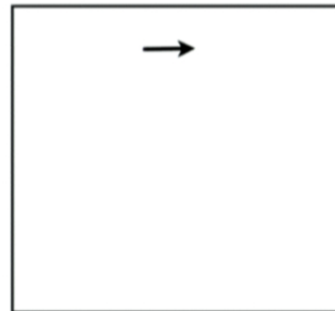
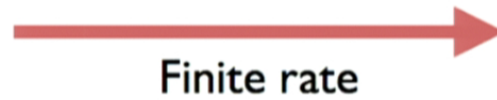
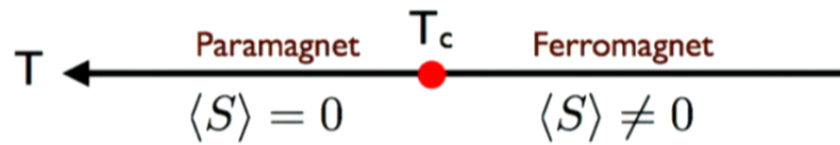
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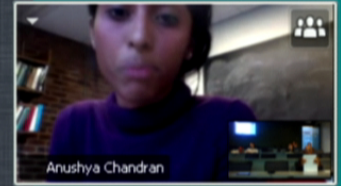


Why should defects form?

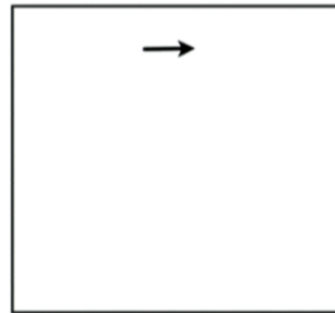
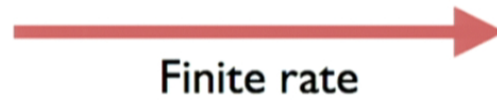
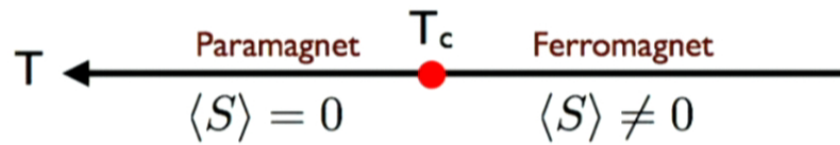


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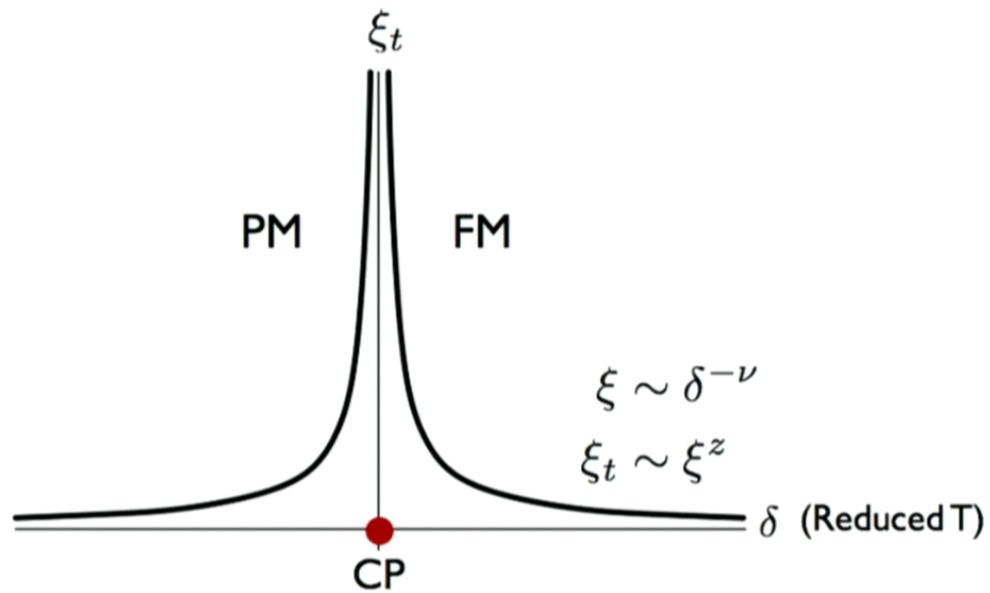
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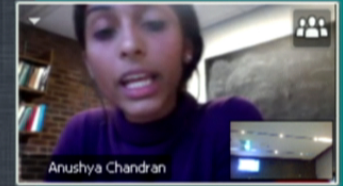
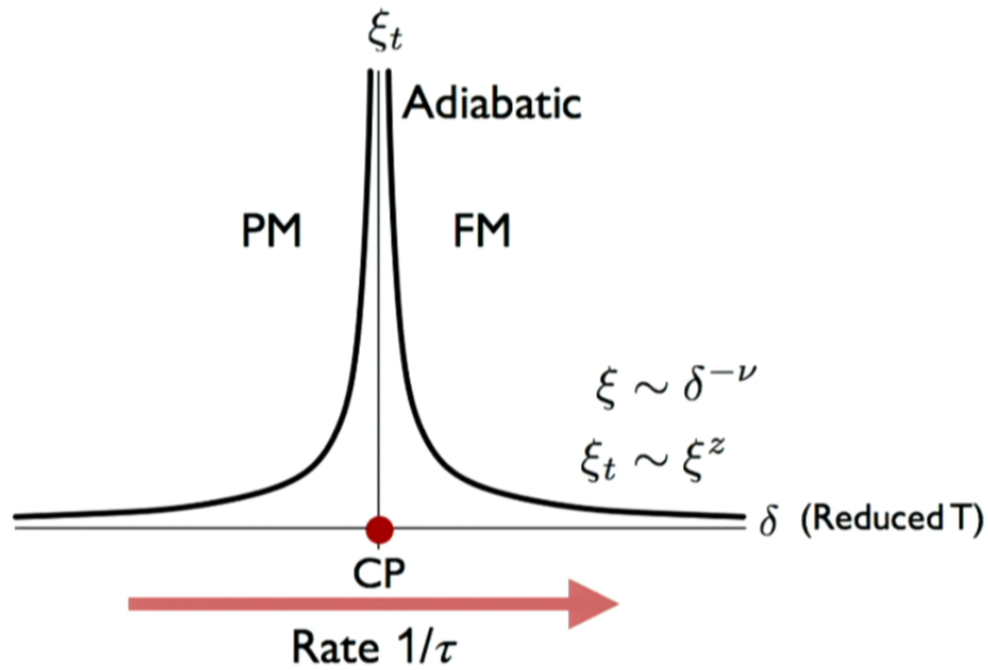
Zurek's argument



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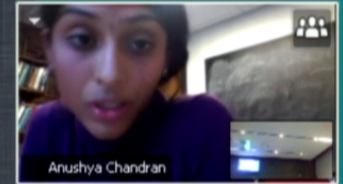
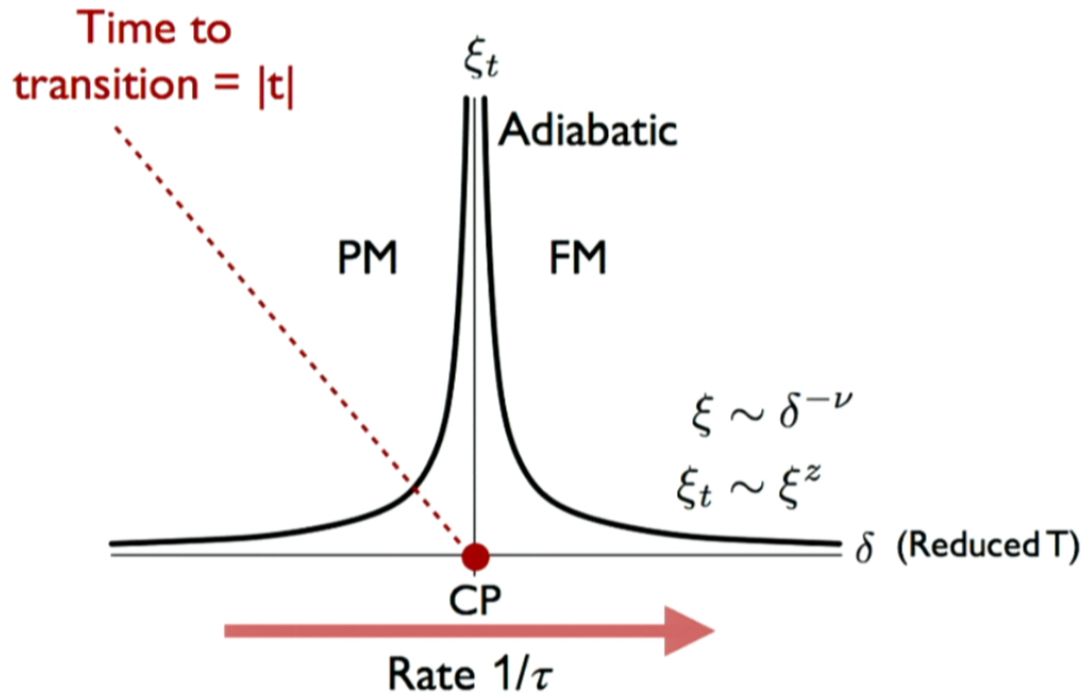


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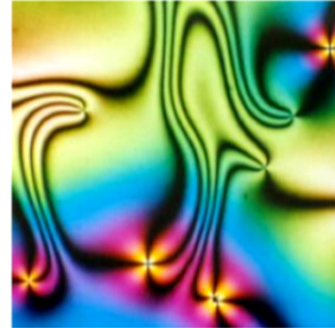


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Universal defect scaling

- Defect density $\sim \tau \frac{-d\nu}{\nu z + 1}$



- Liquid crystals, superconductors, cold atoms..
- Quantum, critical lines, excess energy..

Biroli et al., PRE 85, 050101 (2010)
De Grandi et al., PRB 84, 224303 (2011)
Kolodrubetz et al., PRL 109, 015701 (2012)

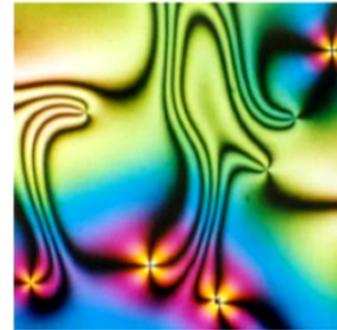
Chuang et al., Science 251, 1336 (1991)
Ducci et al., PRL 83, 5210 (1999)
Monaco et al., PRL 96, 180604 (2006)
Bloch et al., RMP 80, 885 (2008)
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Outline

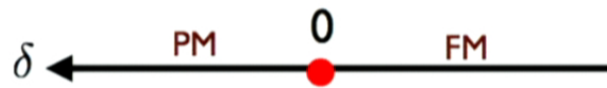
- Universal content in the KZ mechanism
 - Scaling limit
 - Exponents, scaling functions
 - Classical example with order parameter
- KZ without an order parameter
 - Topological orders (TO): Ising and $SU(2)_2$
 - (2+1)D transverse field Ising model (TFIM)
 - String-net coarsening



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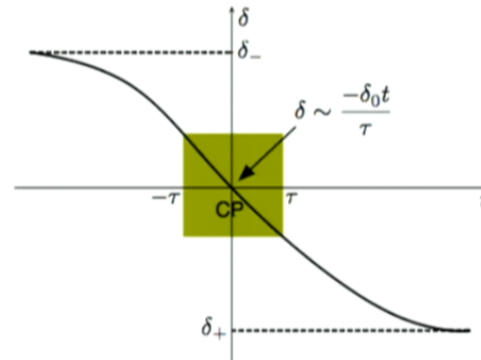
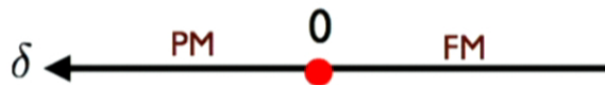
Generalizing the setup



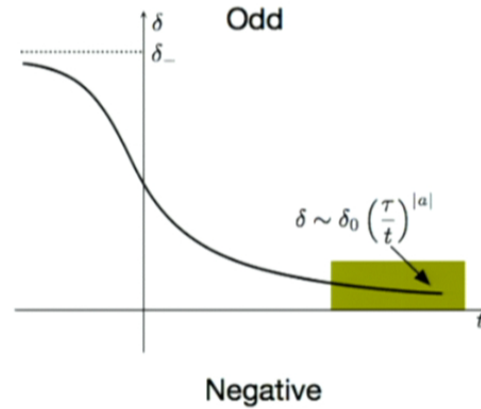
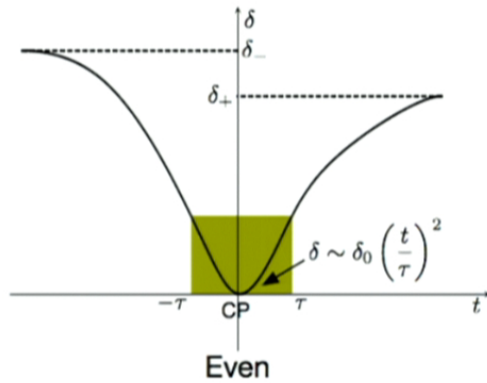
Chandran et al, PRB 86, 064304

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Generalizing the setup



Non-linear protocols



Chandran et al, PRB 86, 064304

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Out-of-equilibrium scales

$$t_K = \tau^{\frac{\nu z}{\nu z + 1}}$$

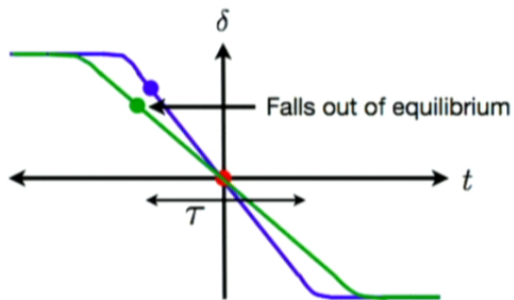
$$l_K = t_K^{1/z}$$



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The KZ scaling limit



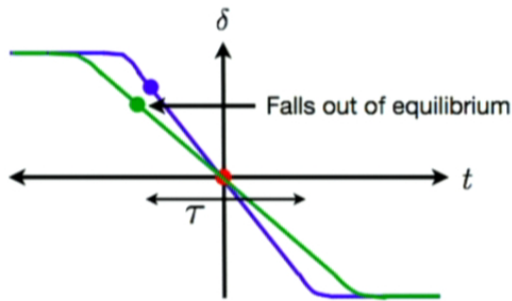
As ramp time $\tau \rightarrow \infty$



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The KZ scaling limit



As ramp time $\tau \rightarrow \infty$

Fall out of equilibrium
closer to CP

l_K and t_K diverging

Limit $\tau \rightarrow \infty$ holding $\frac{x}{l_K}, \frac{t}{t_K}$ fixed



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Universal content

- Exponents & scaling functions

$$\lim_{\substack{\tau \rightarrow \infty \\ x/l_K, t/t_K \text{ fixed}}} [\quad](x, t; \tau) = \frac{1}{l_K^\Delta} f \left(\frac{x}{l_K}, \frac{t}{t_K} \right)$$

- ▶ Correlation functions
- ▶ Thermodynamic quantities: energy, entropy
- ▶ Defect density: Fixed positive t/t_K



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The KZ universality class

Equilibrium critical theory +

Ramp protocol near
critical point

↑
Order parameter

Dimension

Conserved quantities

↑
Leading behavior

$$\delta(t) \sim (t/\tau)^a$$



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The KZ universality class

Equilibrium critical theory +

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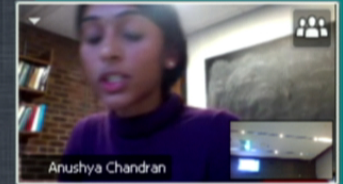
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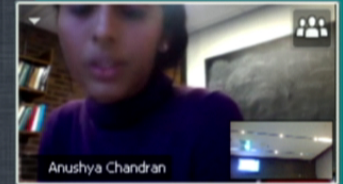
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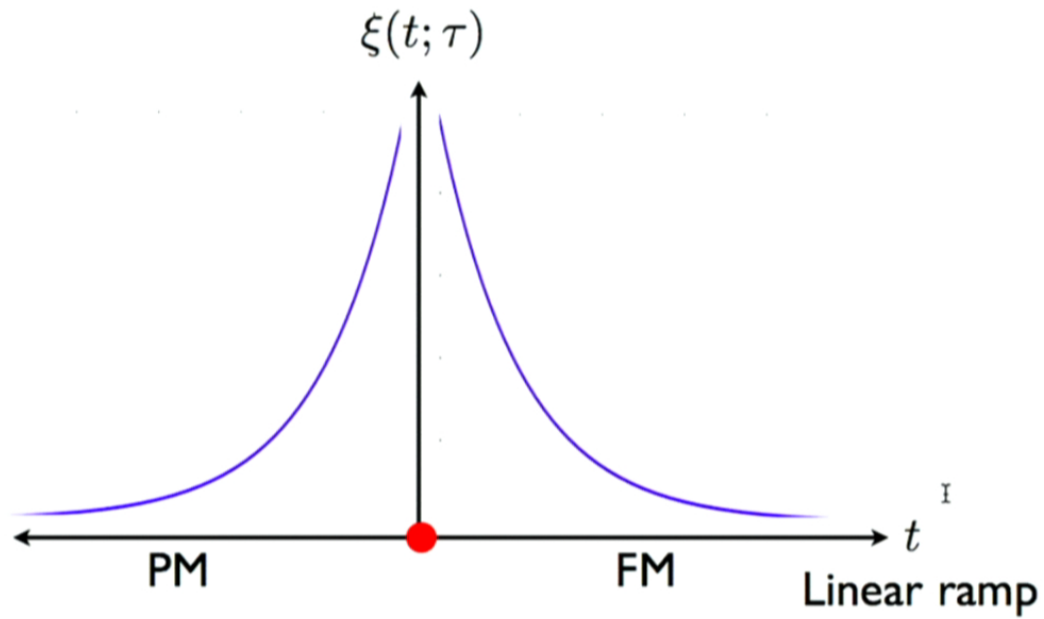
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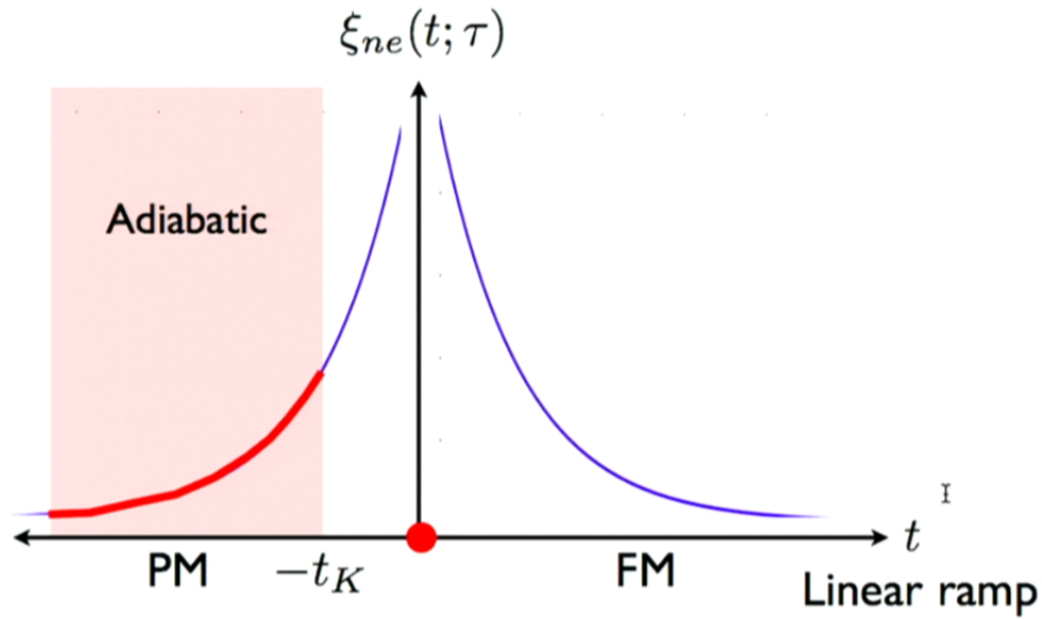
Three regimes



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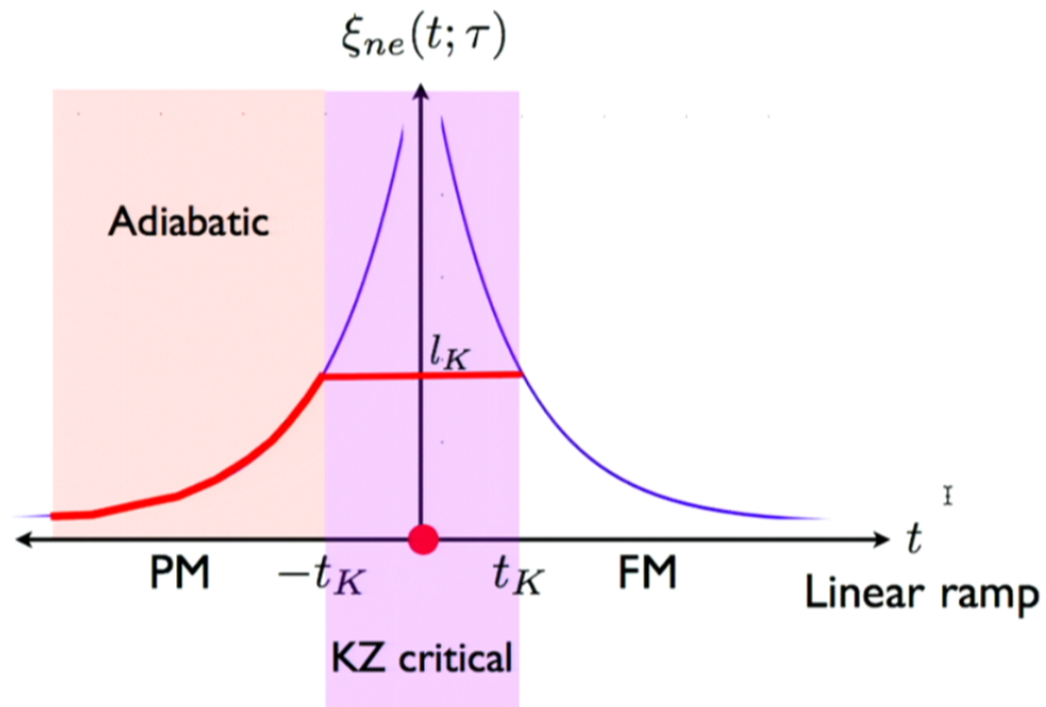
Three regimes



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Three regimes

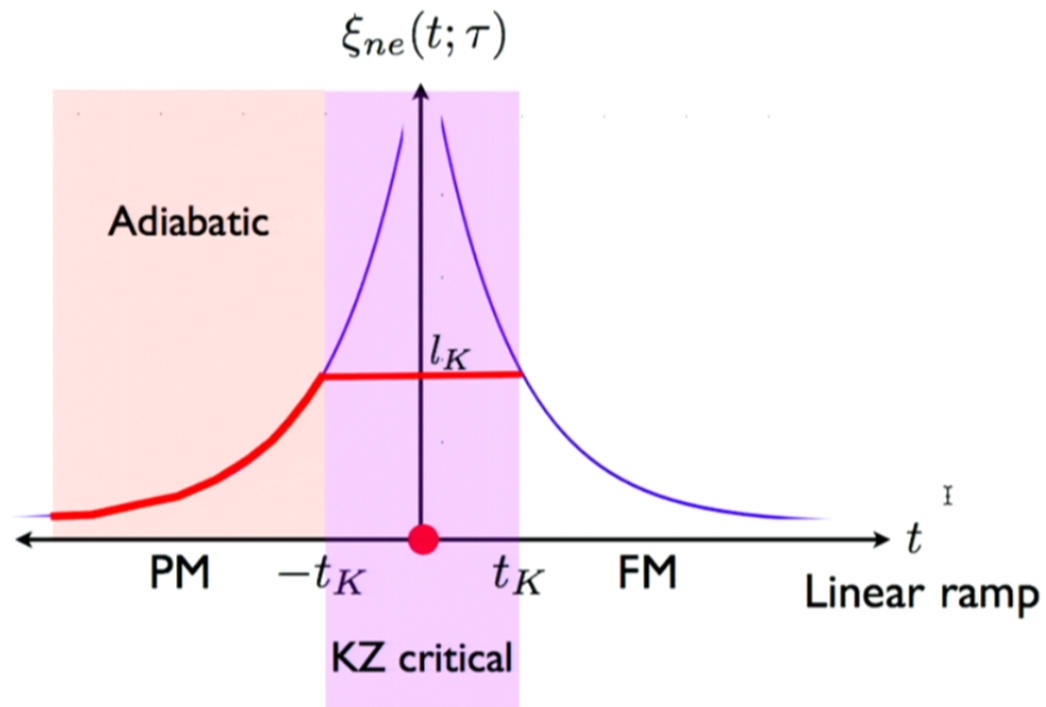


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Three regimes

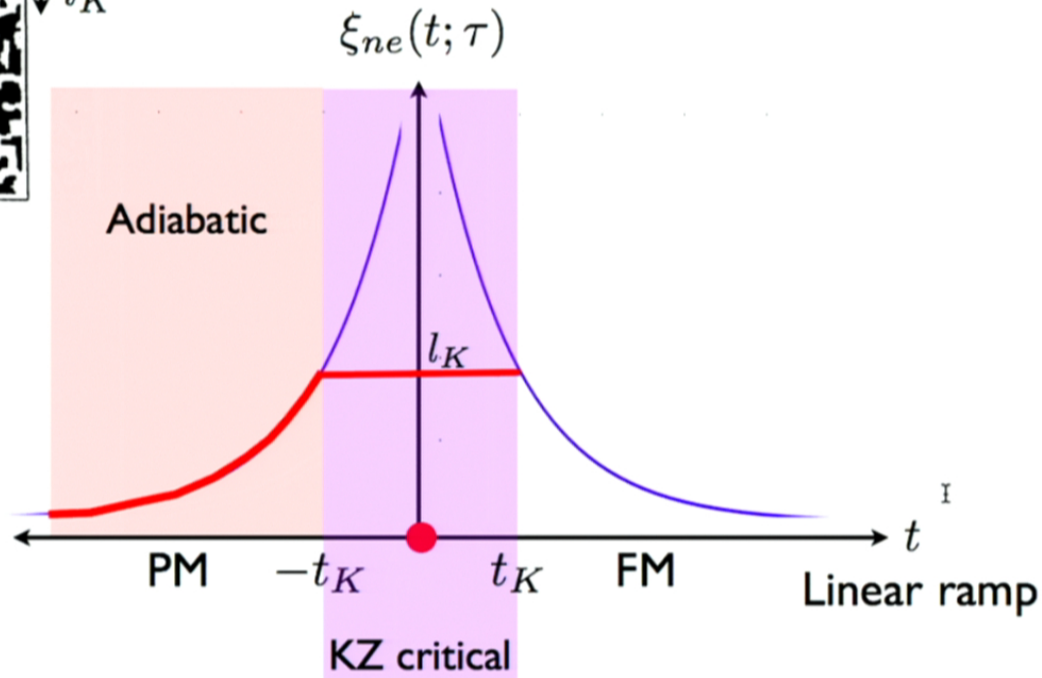


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Three regimes



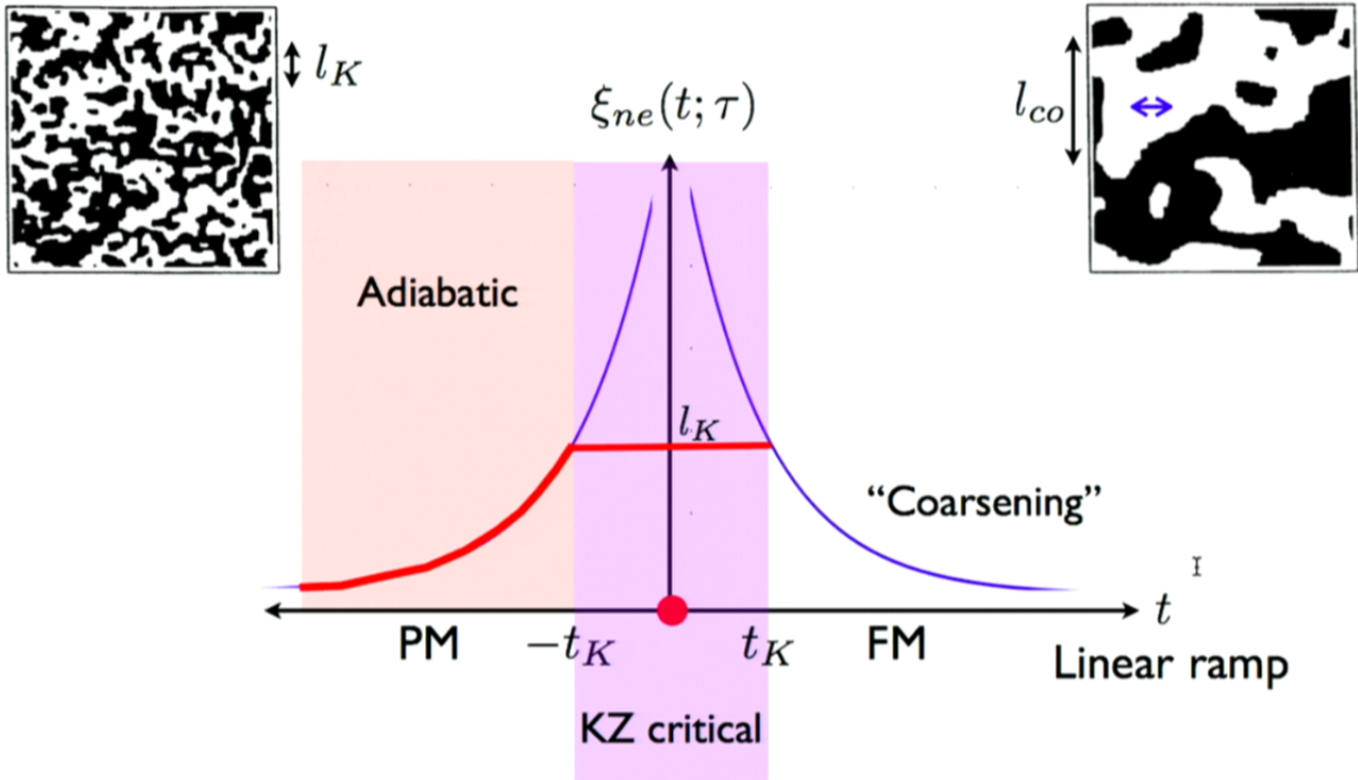
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Three regimes

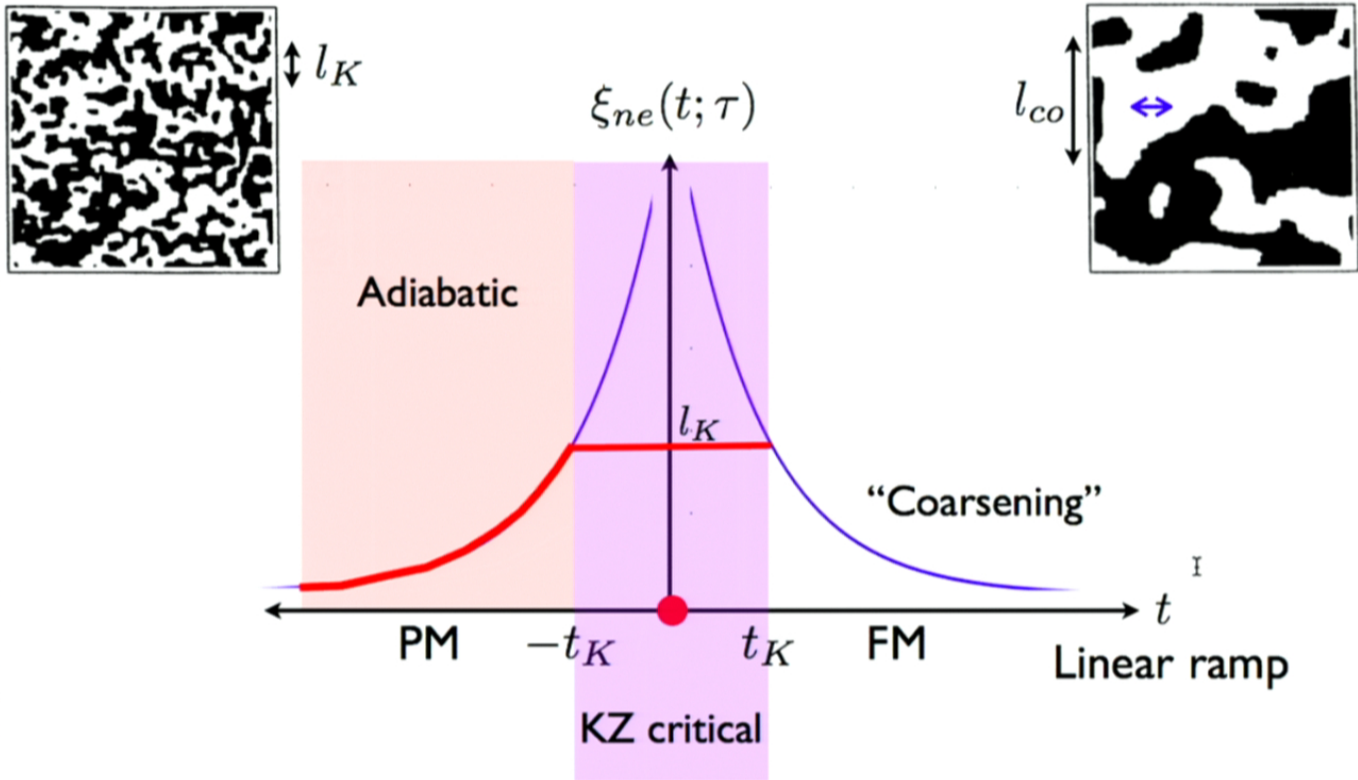


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Three regimes



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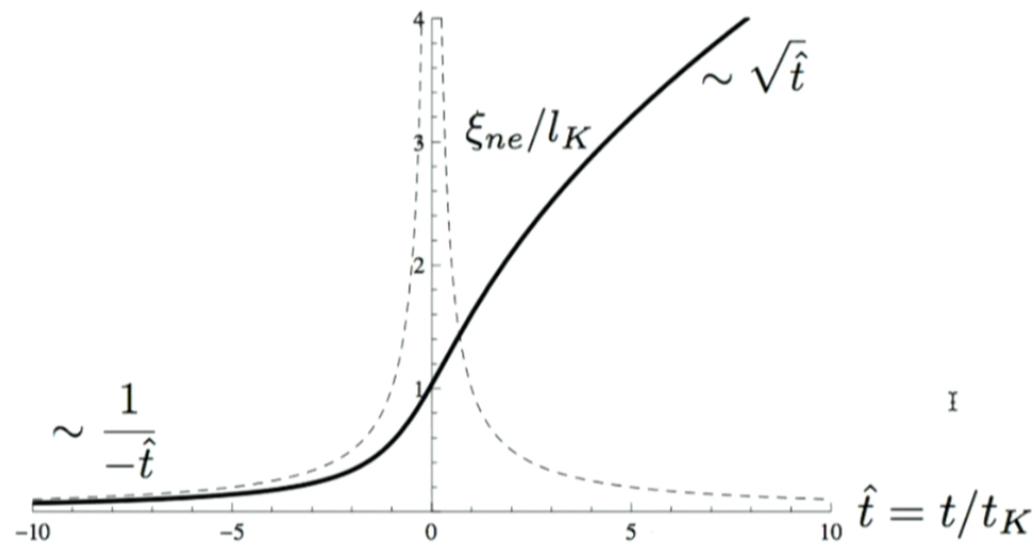


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Scaling function in $O(N)$ in $d=3$

$$\xi_{ne}(t; \tau) \sim l_K \mathcal{L}_{ne} \left(\frac{t}{t_K} \right)$$

Classical $N \rightarrow \infty$
Relaxational

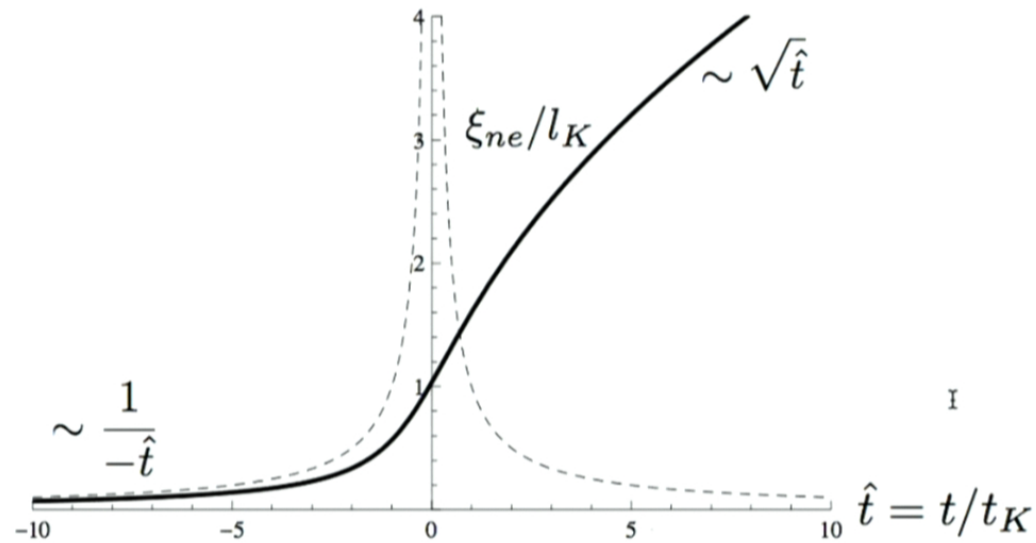


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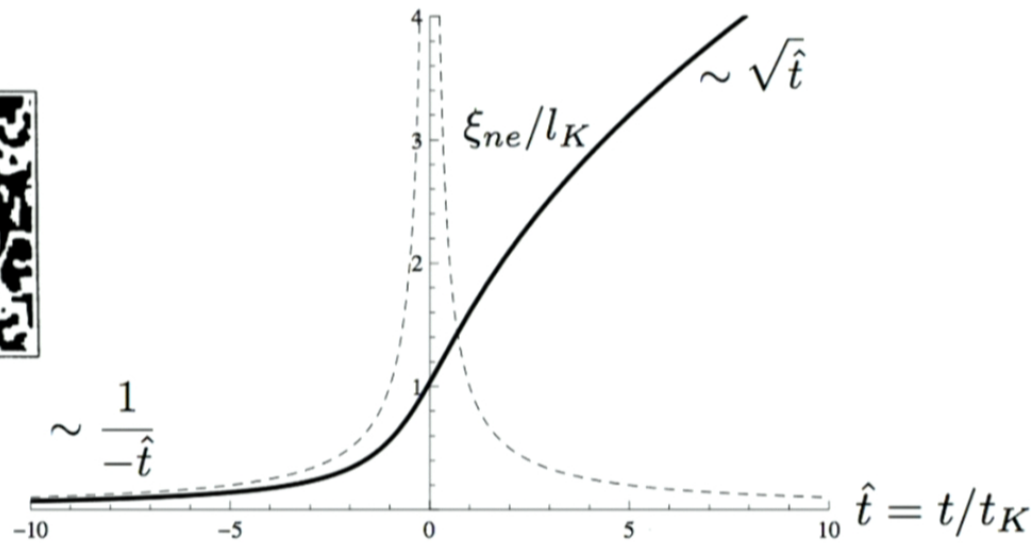


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Classical $N \rightarrow \infty$
Relaxational



Adiabatic

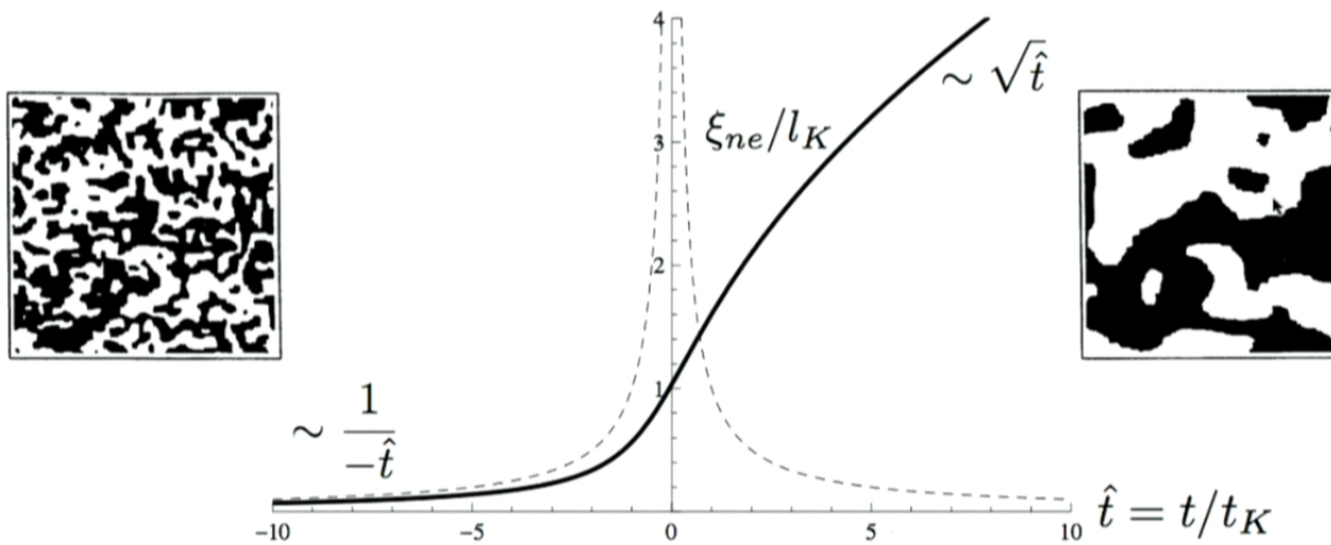


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Adiabatic

Coarsening



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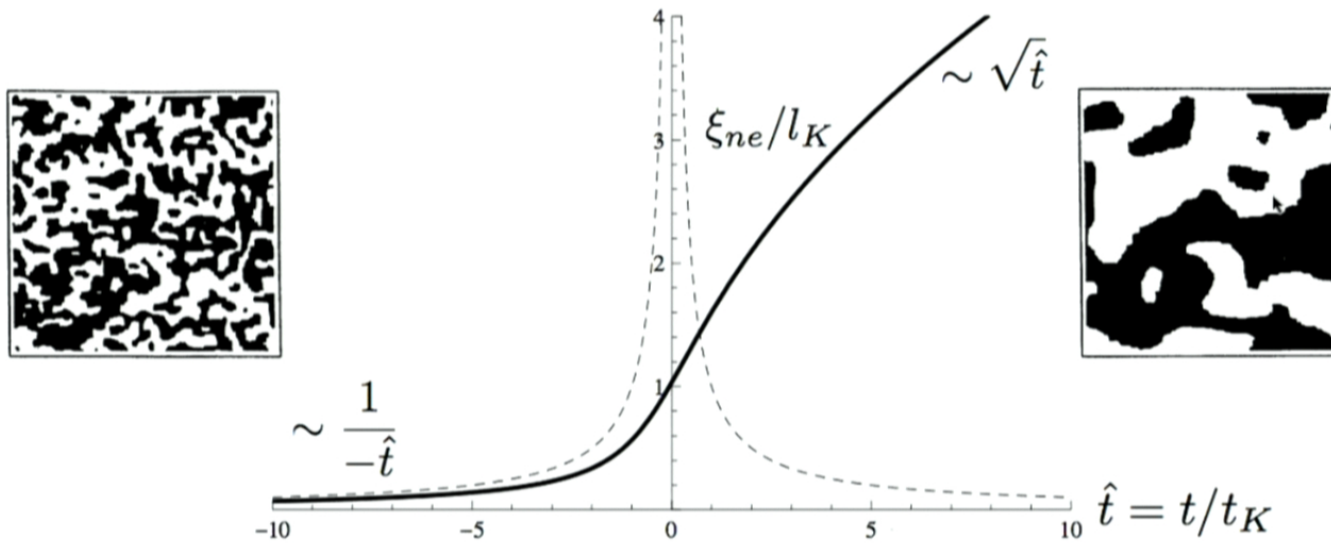
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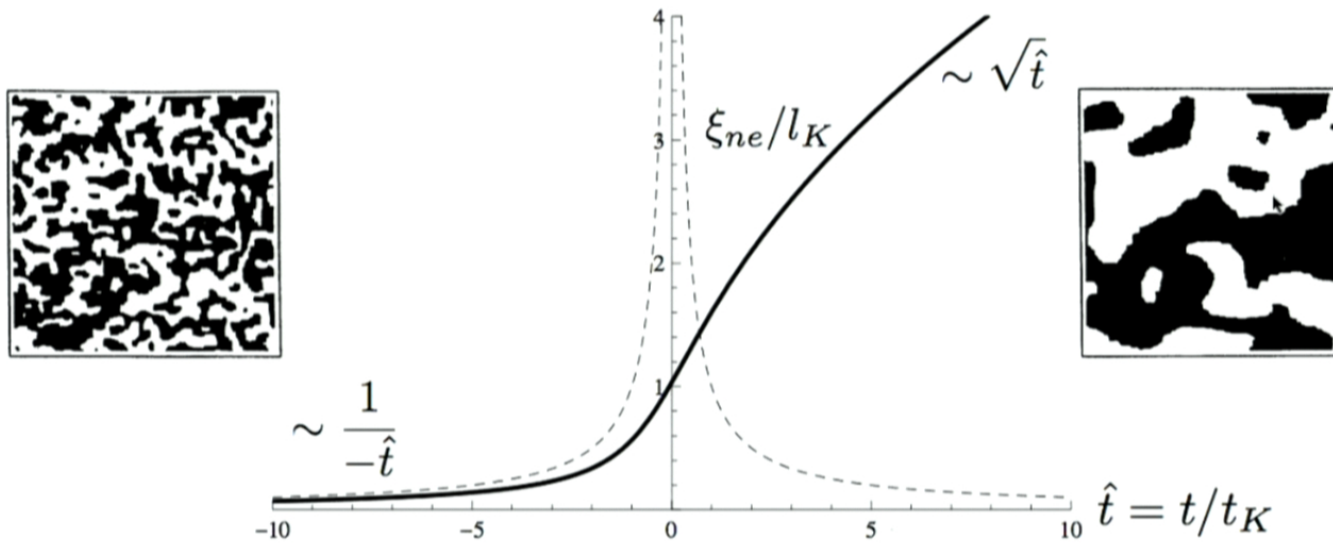
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Scaling function in $O(N)$ in $d=3$

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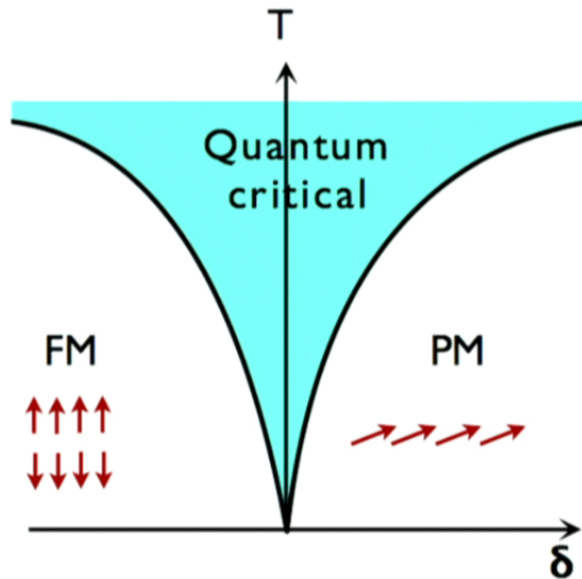
Adiabatic

Coarsening

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Quantum vs KZ critical



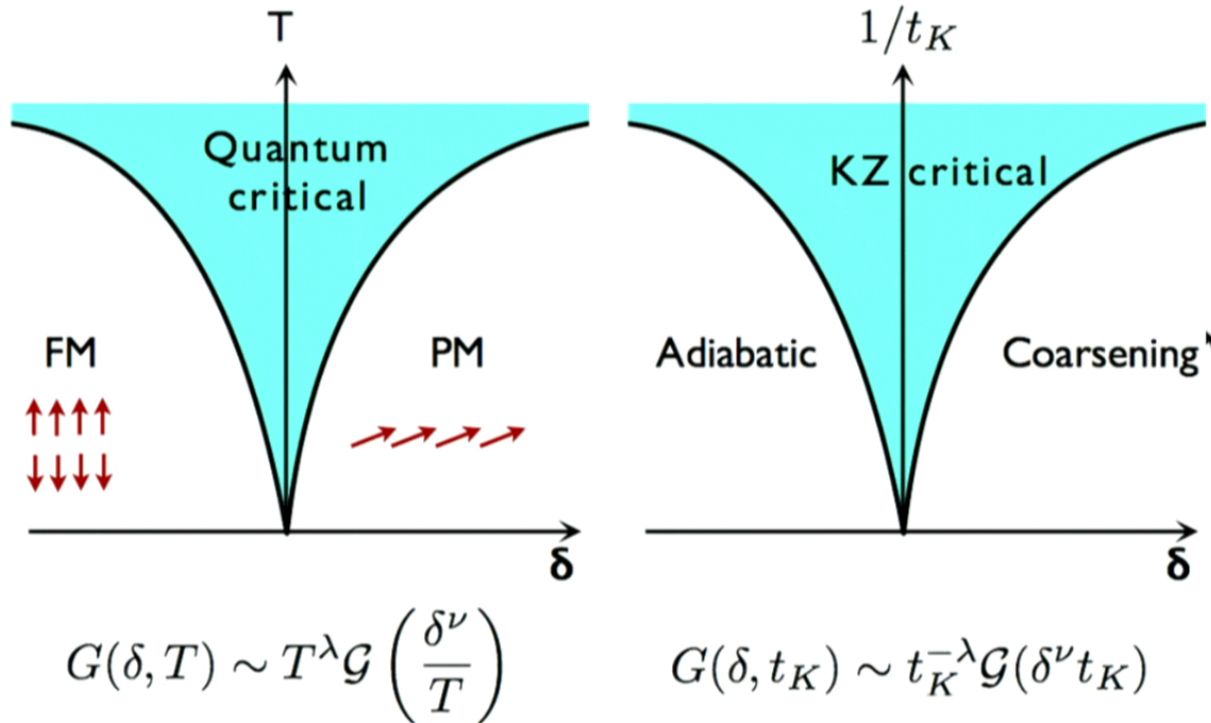
$$G(\delta, T) \sim T^\lambda \mathcal{G}\left(\frac{\delta^\nu}{T}\right)$$



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Quantum vs KZ critical



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Outline

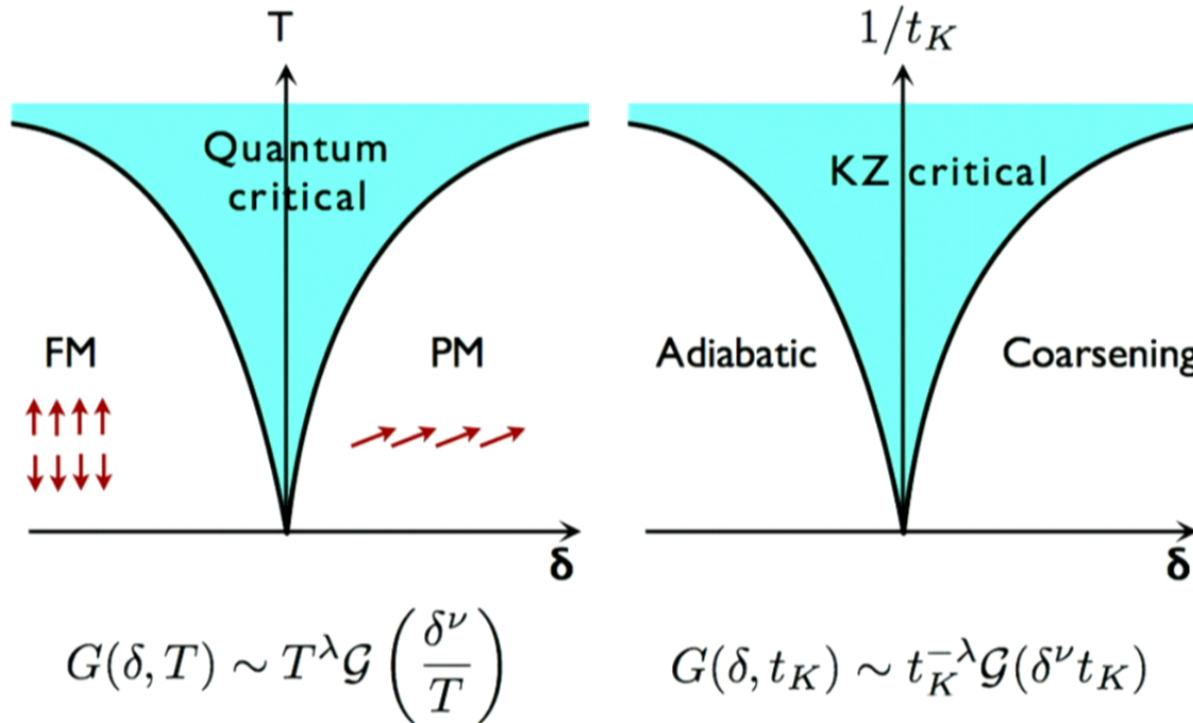
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 - Scaling limit
 - Exponents, scaling functions
 - Classical example
- **KZ without an order parameter**
 - Ising gauge theory
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 - String-net coarsening
 - Generalizations



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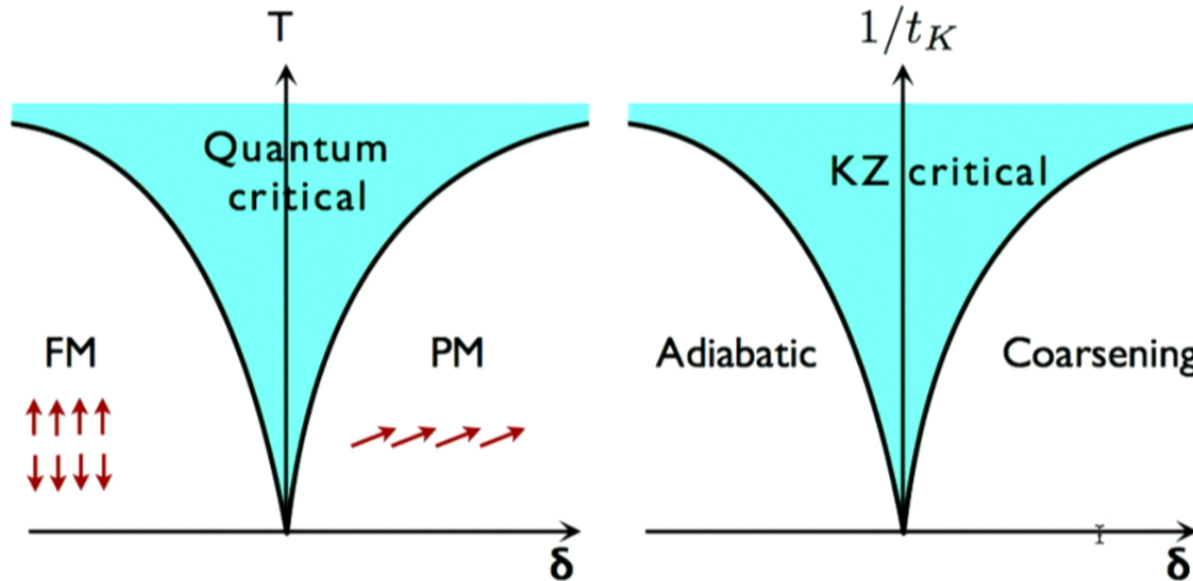
Quantum vs KZ critical



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Quantum vs KZ critical



$$G(\delta, T) \sim T^\lambda \mathcal{G}\left(\frac{\delta^\nu}{T}\right)$$

$$G(\delta, t_K) \sim t_K^{-\lambda} \mathcal{G}(\delta^\nu t_K)$$



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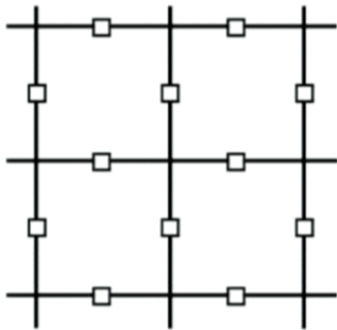


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Pure Ising gauge theory

\square : σ (Gauge)



Wegner, J. Math Phys, 12
(1971)
Fradkin & Shenker, PRD
19, 3682 (1979)

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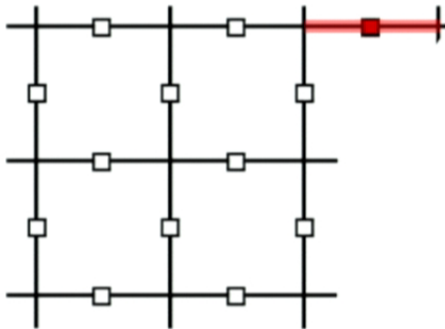


Pure Ising gauge theory

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Electric field :

$$\sigma_l^x = -1$$

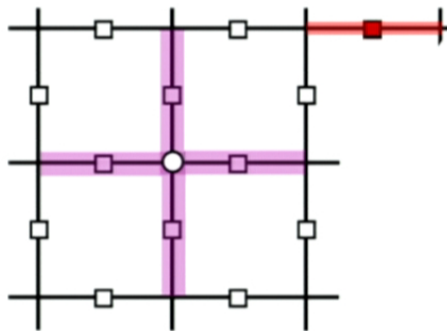


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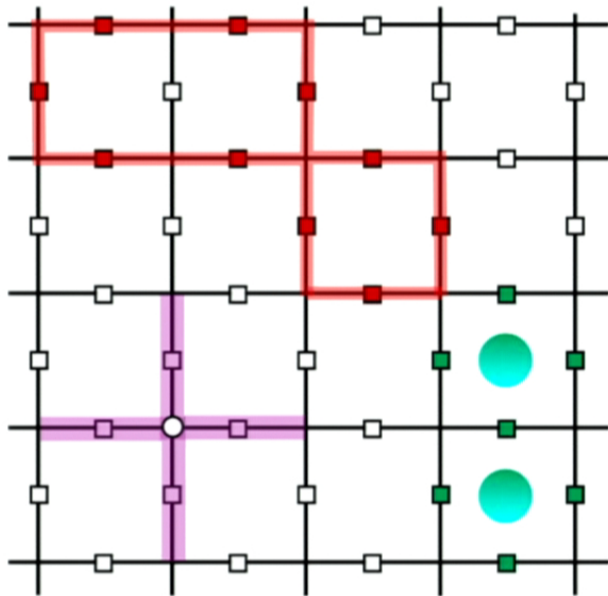
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Vortex :

$$B_p = \prod_{l \in \partial p} \sigma_l^z \equiv -1$$

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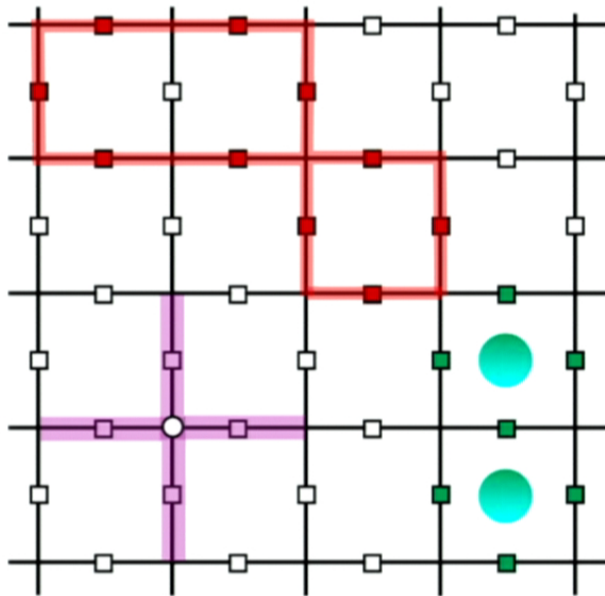
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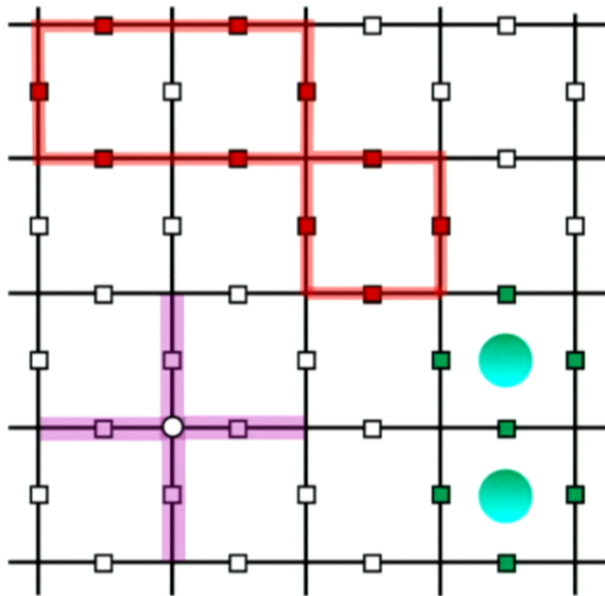
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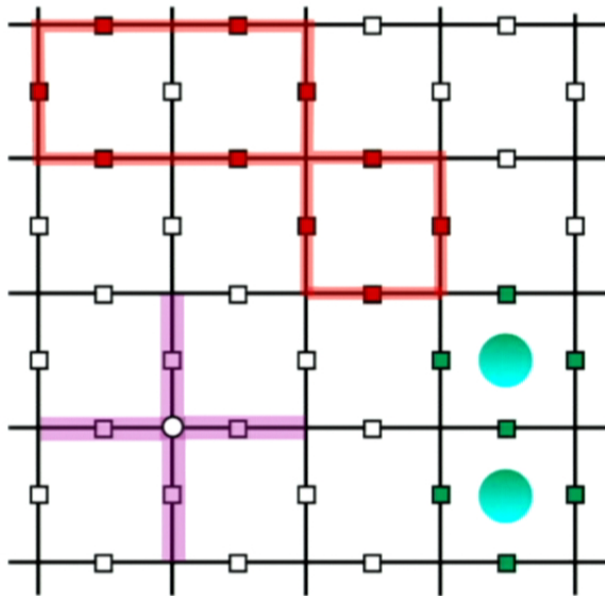
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$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$



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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

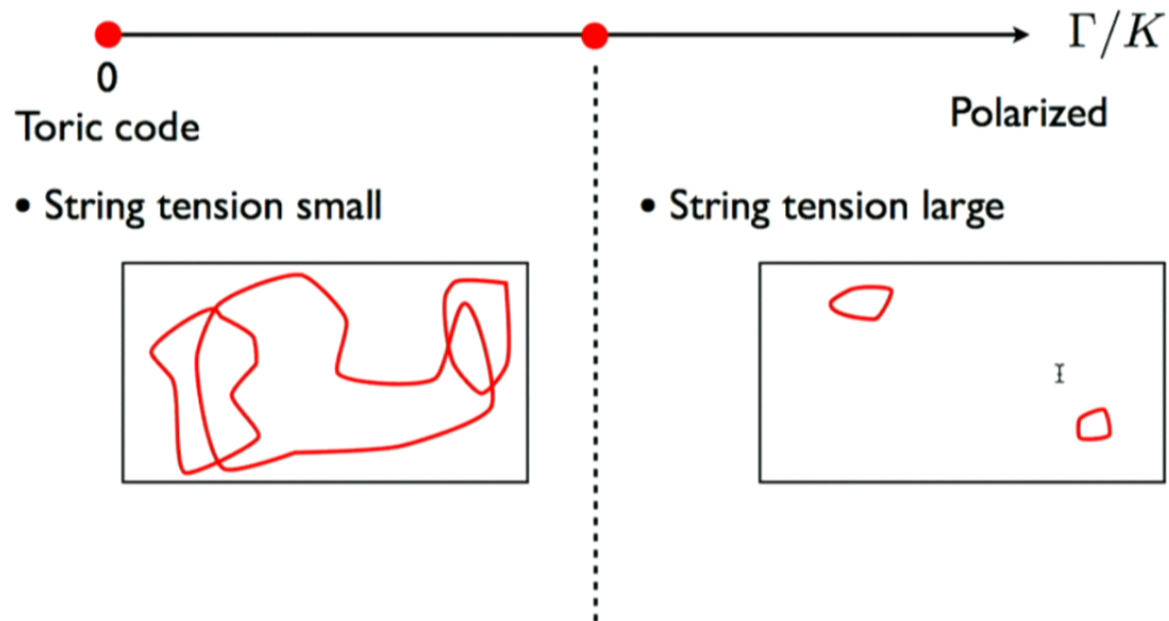


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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

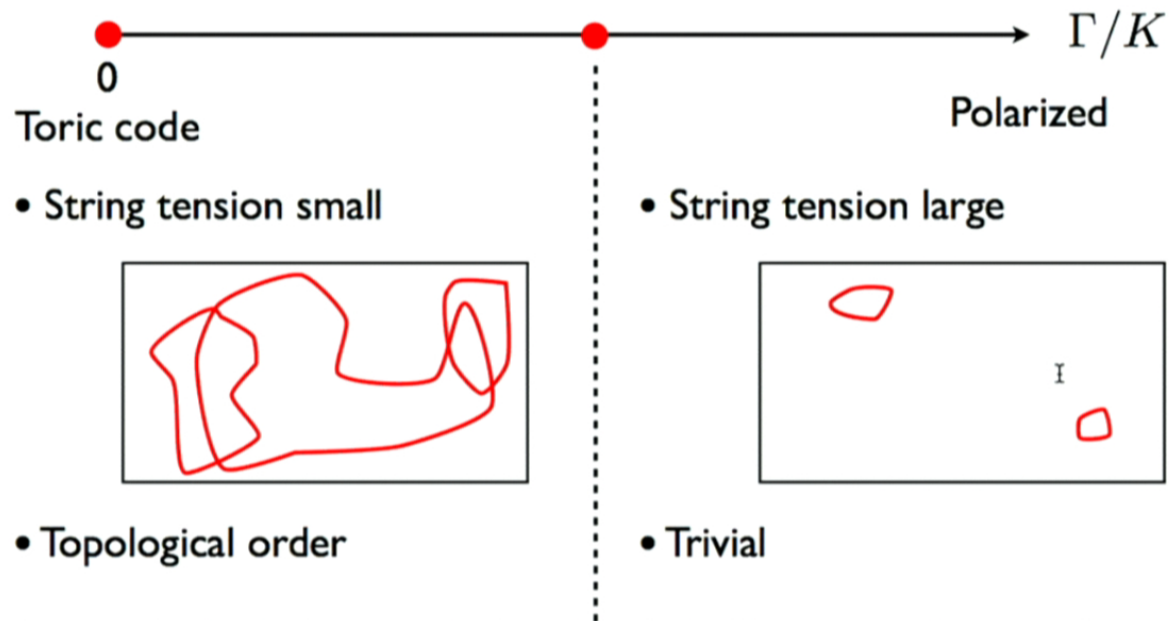


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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

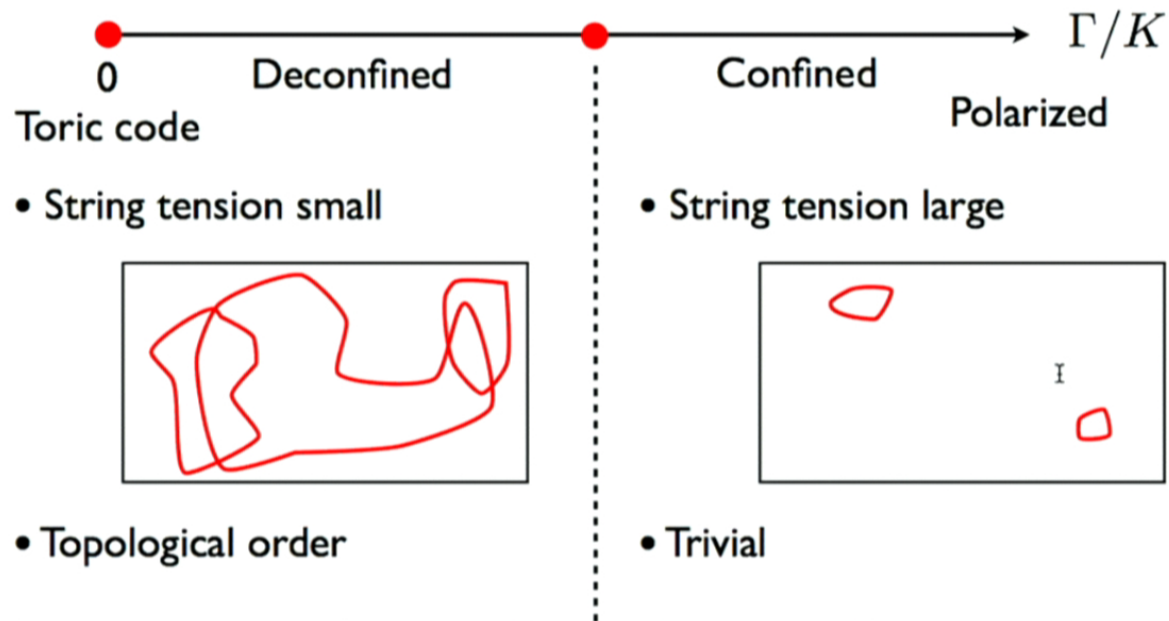


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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

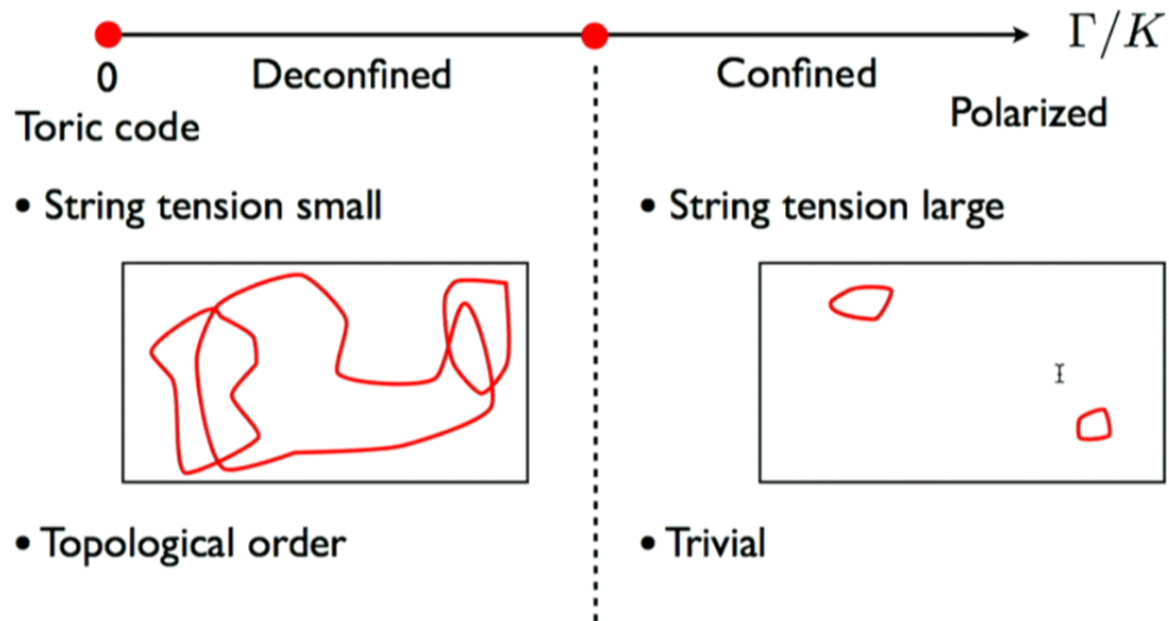


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Phase diagram

$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

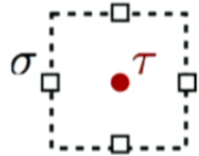


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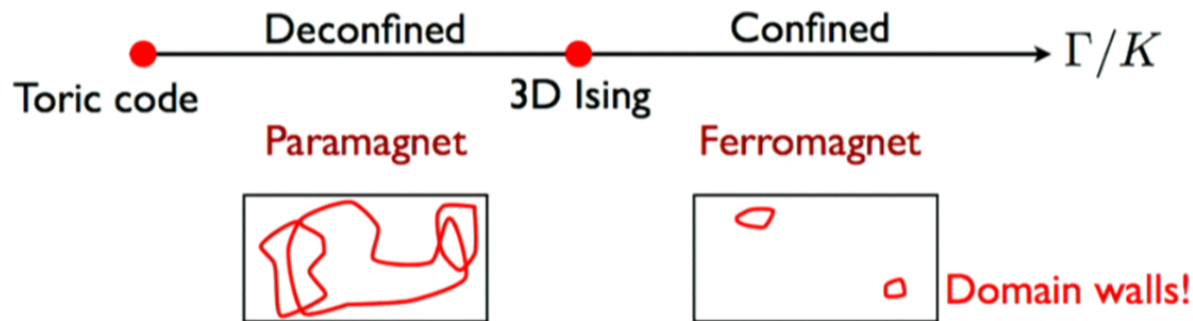
Duality

- Maps the gauge theory to the (2+1)D TFIM



$$H = -K \sum_p B_p + -\Gamma \sum_l \sigma_l^x$$

$$H_{TFIM} = -K \sum_{\bar{s}} \tau_{\bar{s}}^x - \Gamma \sum_{\bar{s}, \bar{s}'} \tau_{\bar{s}}^z \tau_{\bar{s}'}^z$$



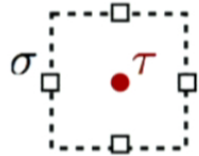
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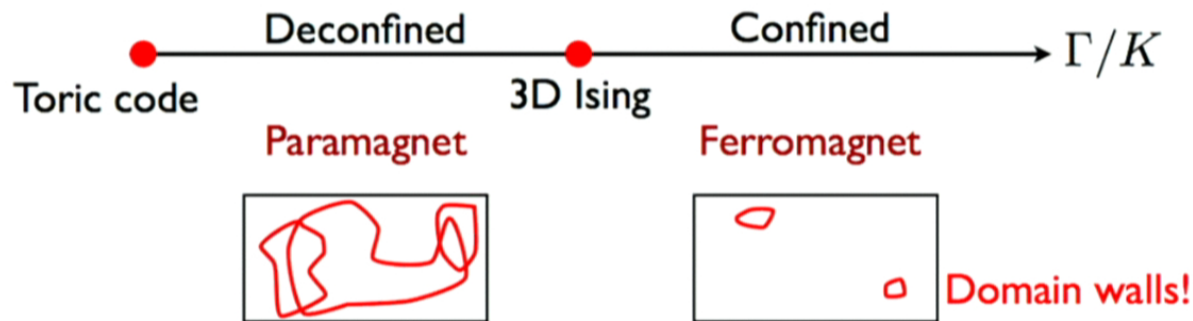
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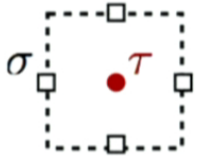


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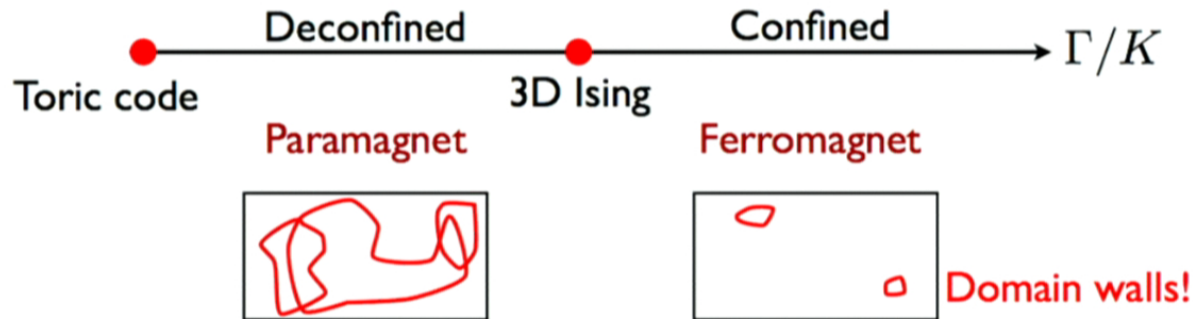
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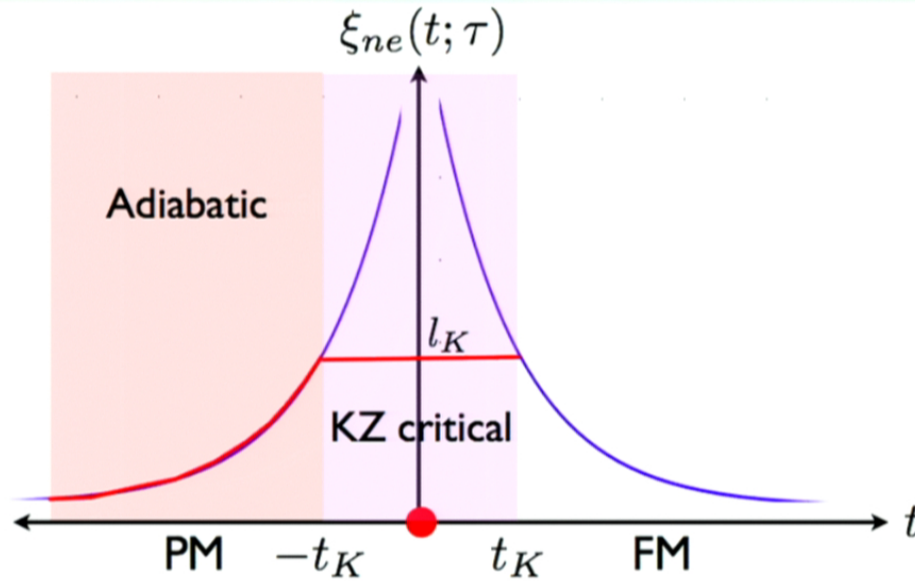
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KZ in the (2+1)D TFIM



- Many-body gap closes at $t = 0$
- Defect density in the ordered phase

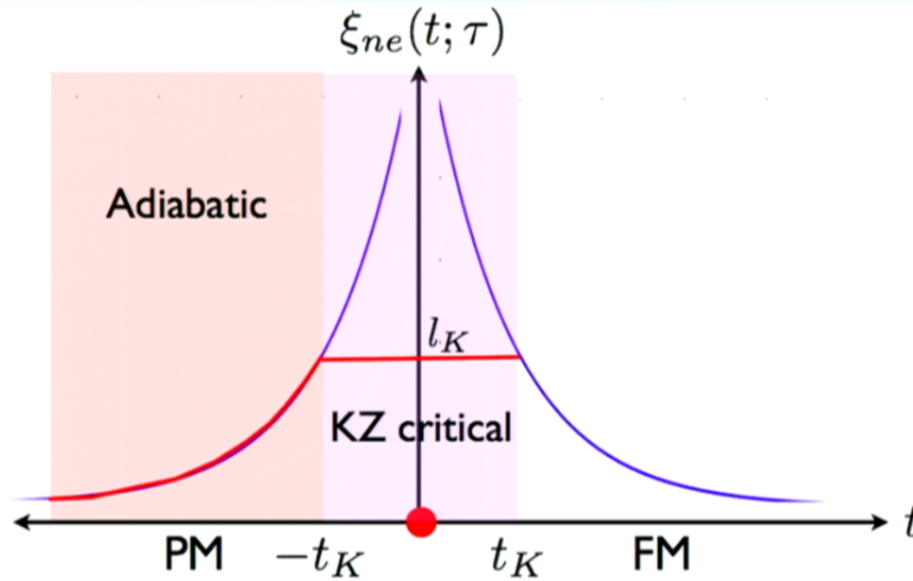
Dziarmaga, PRL 95, 245701 (2005)
Polkovnikov, PRB 72, 161201 (2005)
Zurek et. al, PRL 95, 105701 (2005)



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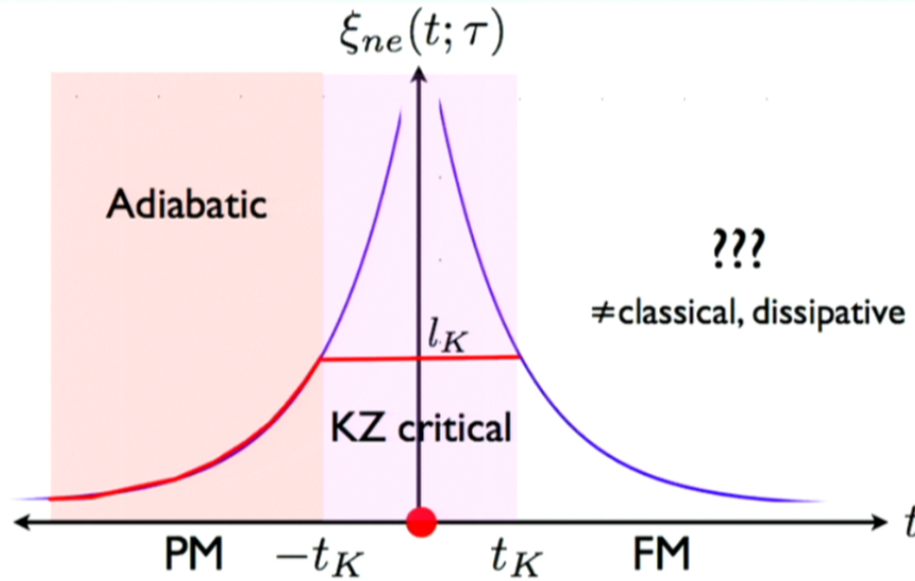
Dziarmaga, PRL 95, 245701 (2005)
Polkovnikov, PRB 72, 161201 (2005)
Zurek et. al, PRL 95, 105701 (2005)



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KZ in the (2+1)D TFIM



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Dziarmaga, PRL 95, 245701 (2005)
Polkovnikov, PRB 72, 161201 (2005)
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Driven isolated

- Increase energy density above ground state (q)

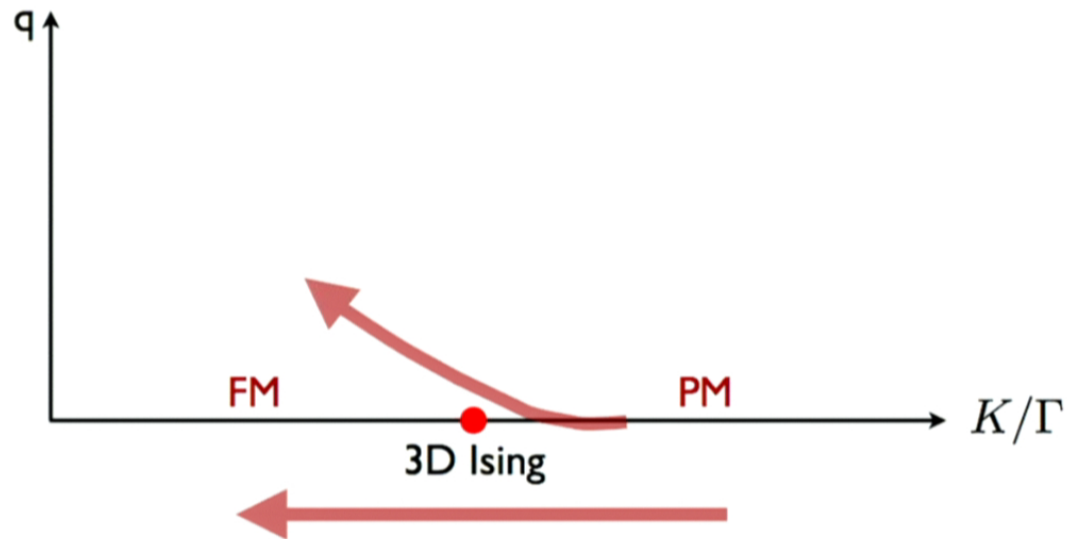


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Driven isolated

- Increase energy density above ground state (q)

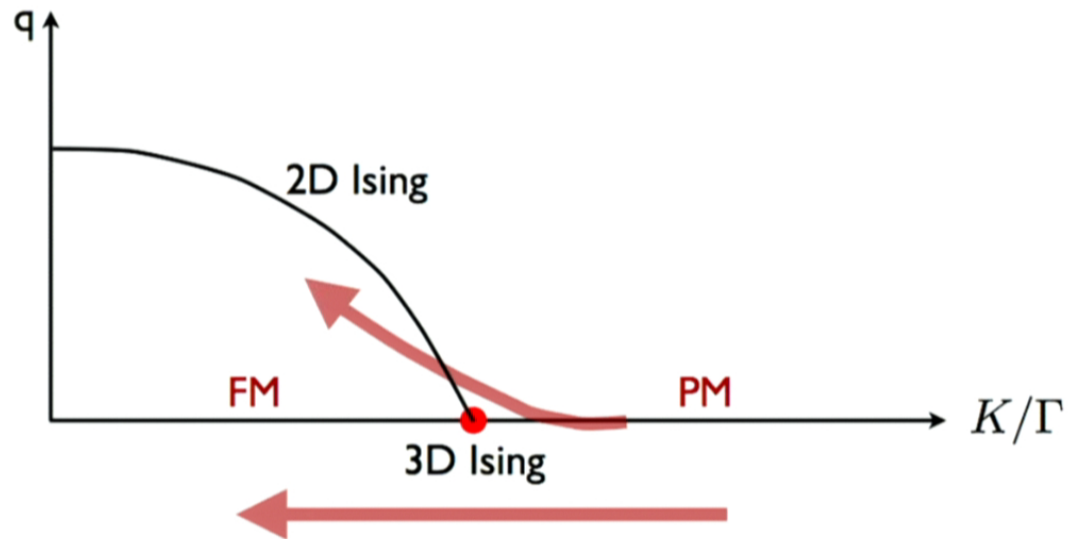


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Driven isolated

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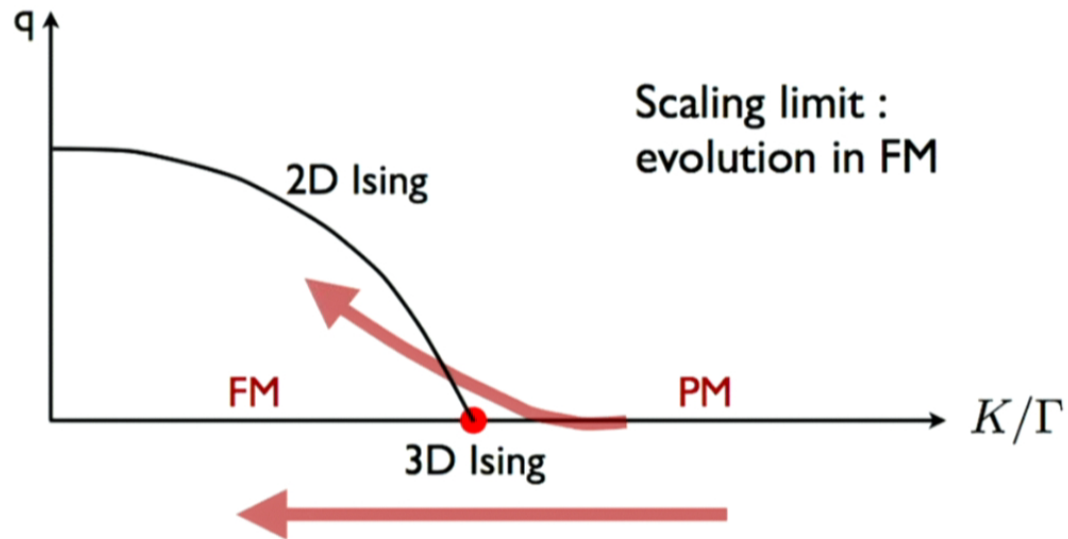


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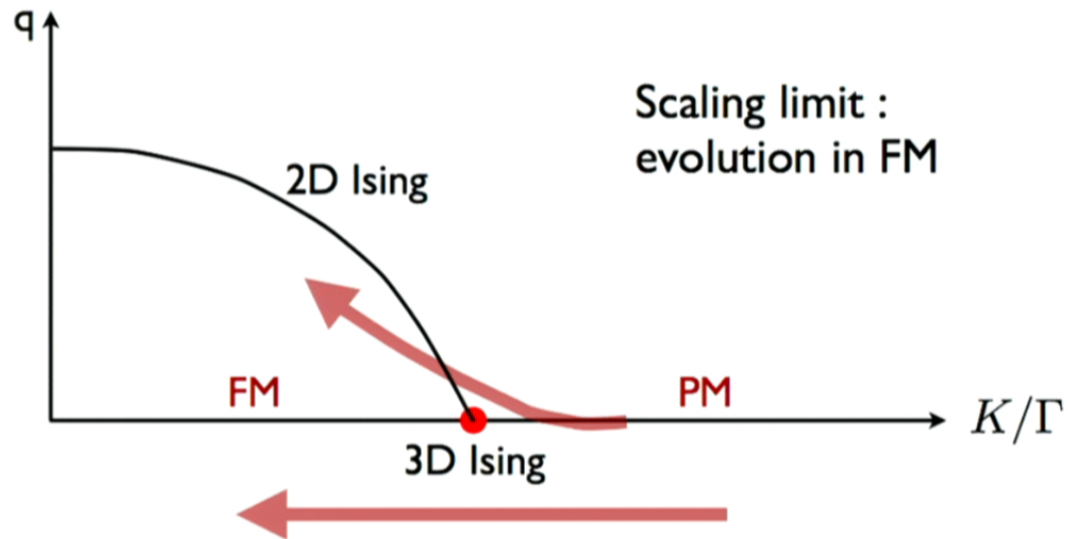


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Driven isolated

- Increase energy density above ground state (q)



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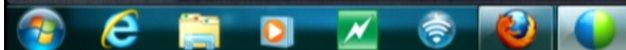
How is long-range order approached?

- Classical, dissipative
 - Bigger domains convert smaller ones (Bray, 1994)
- Energy conserved
 - Conversion generates heat
 - Entropically driven
- Quantum coherence
 - Scattering processes
 - Large-N

Adiabatic, KZ critical, Coarsening



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How is long-range order approached?

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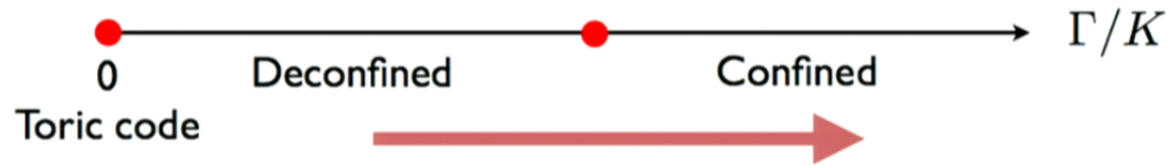
Adiabatic, KZ critical, Coarsening



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Translating dynamics

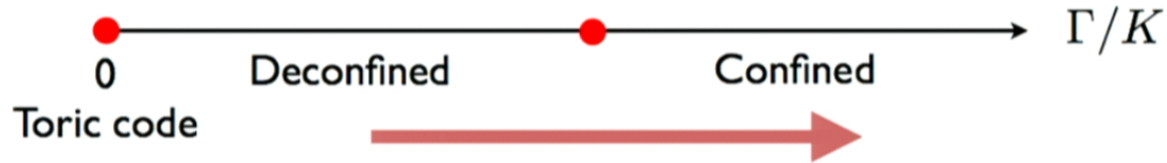


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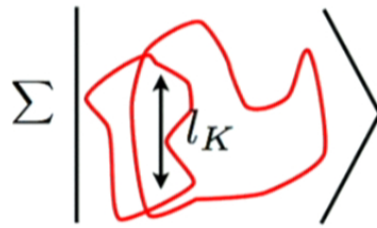


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Translating dynamics



$$t/t_K \rightarrow -\infty$$



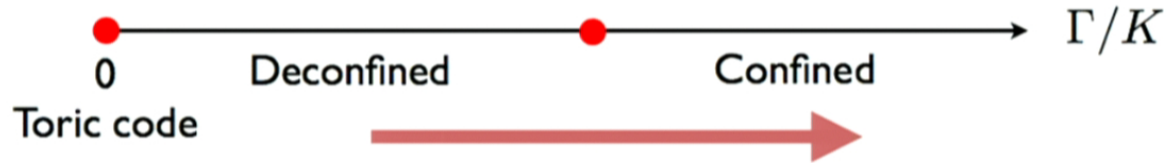
$$t/t_K \approx 1$$



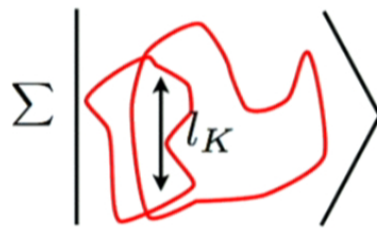
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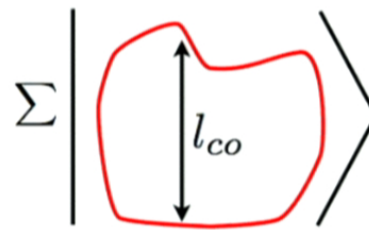
Translating dynamics



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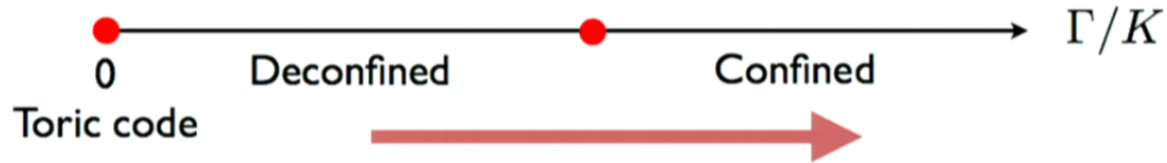
$$t/t_K \rightarrow \infty$$



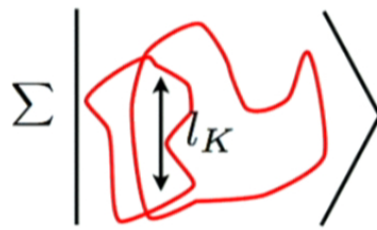
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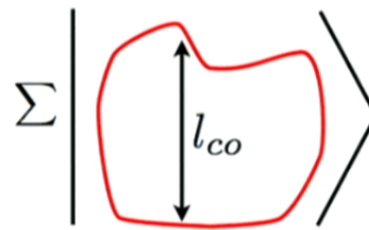
Translating dynamics



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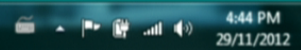


$$t/t_K \rightarrow \infty$$

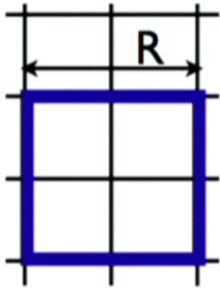
String net coarsening!



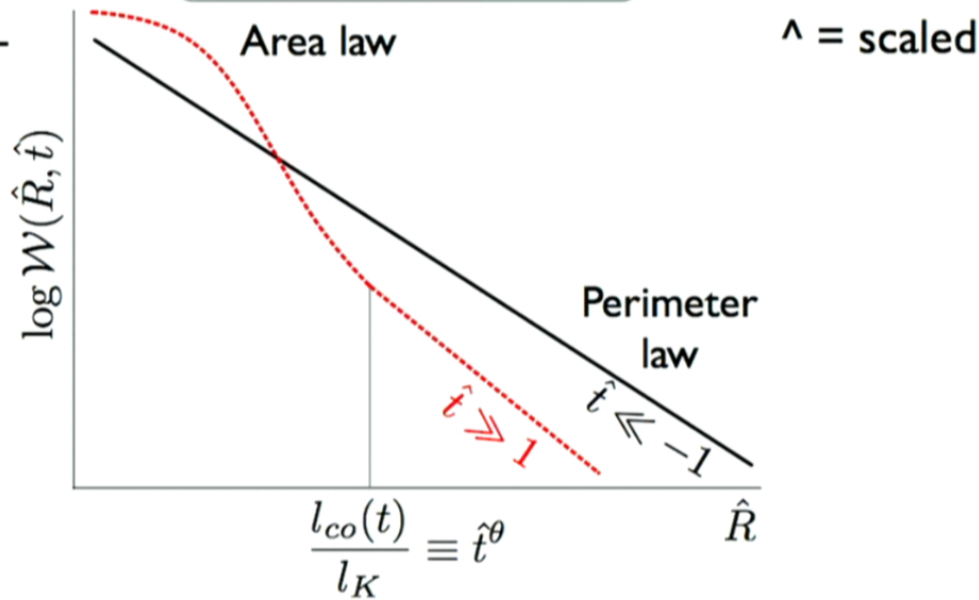
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Wilson loop



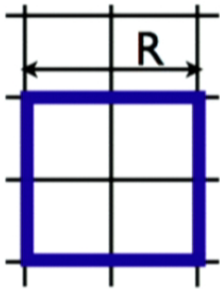
$$W(R, t; \tau) = \prod_{l \in C} \sigma_l^z$$
$$\sim \mathcal{W}(\hat{R}, \hat{t})$$



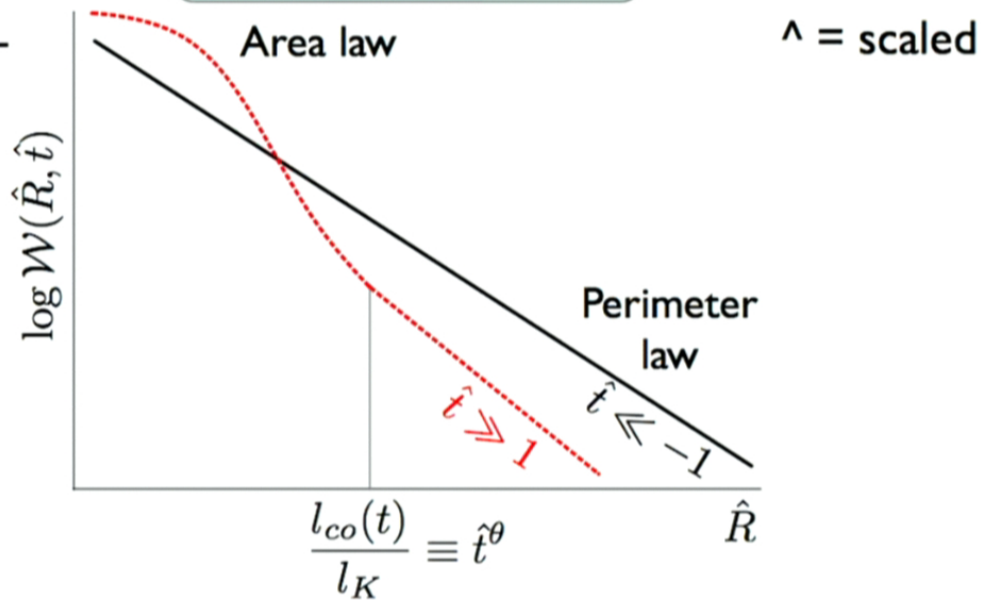
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Wilson loop



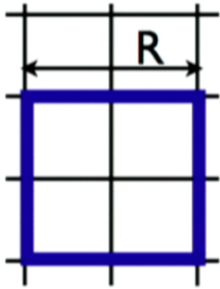
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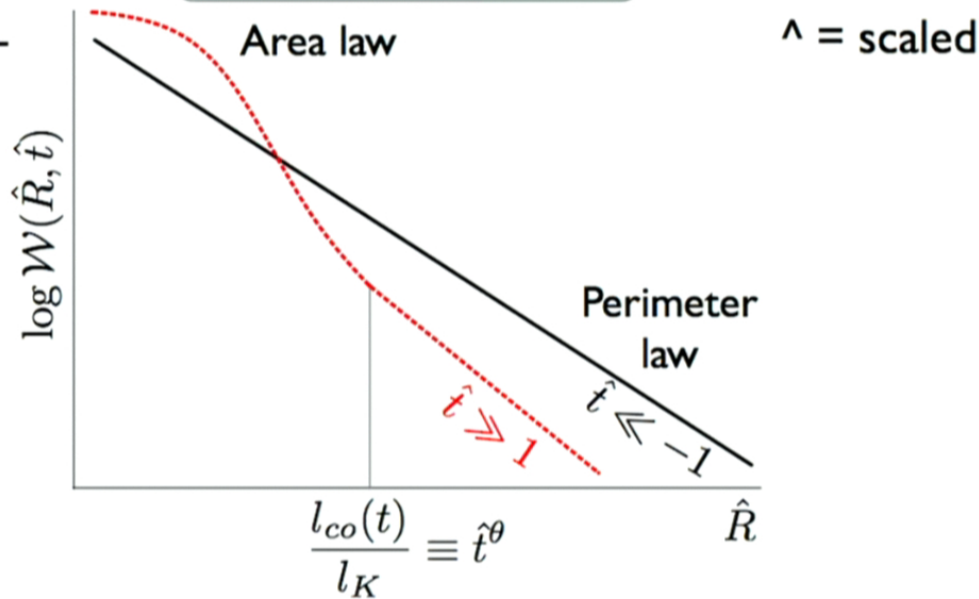
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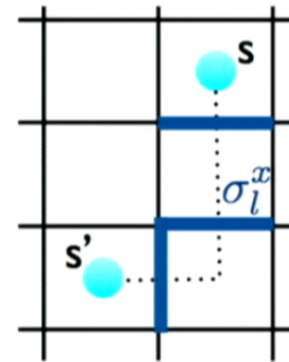


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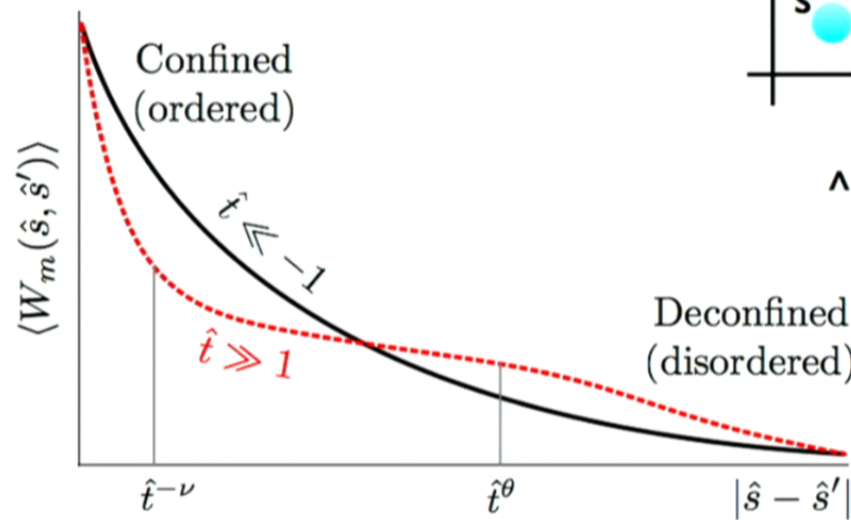
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Vortex pair creation

- $B_p = -1$: vortex
- Confined phase: vortex condensate
- $\langle W_m \rangle \neq 0$



$\wedge = \text{scaled}$

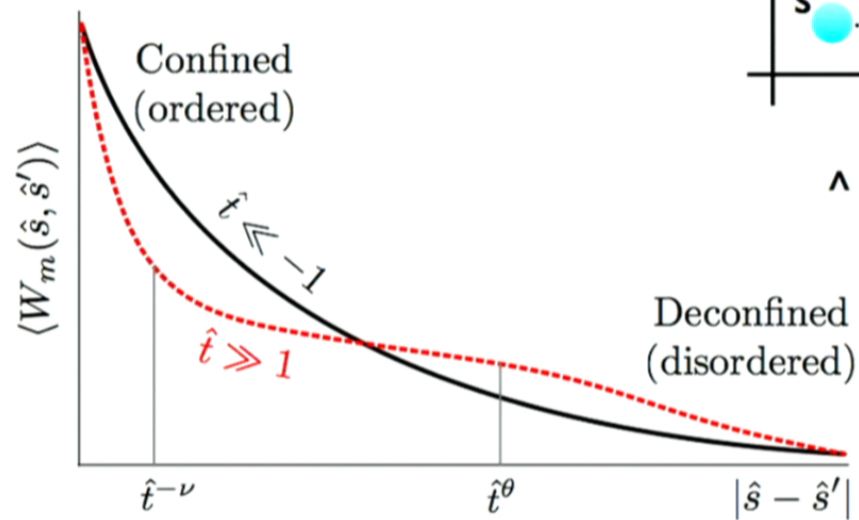
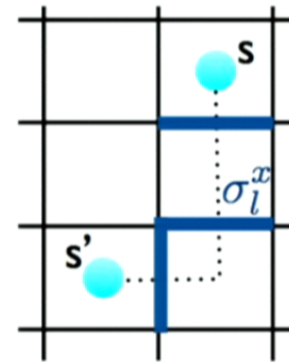


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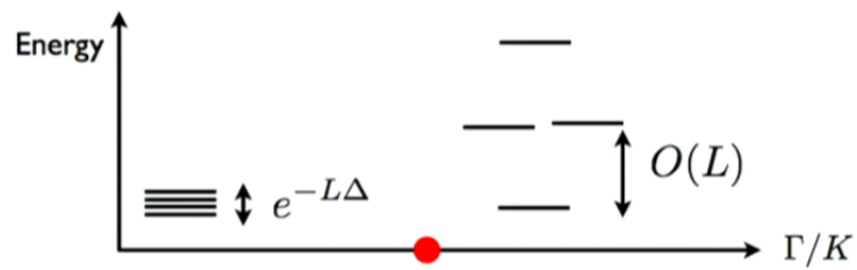


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Comments

- Topological degeneracy

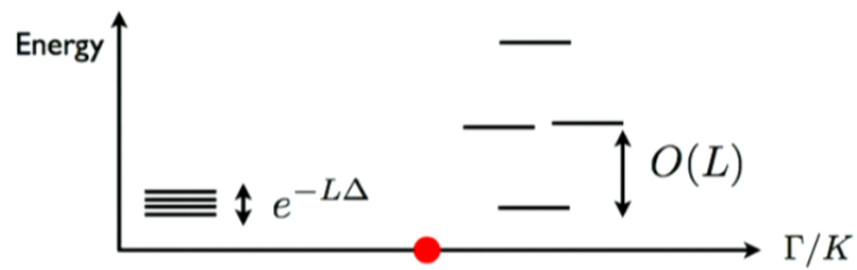


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Comments

- Topological degeneracy

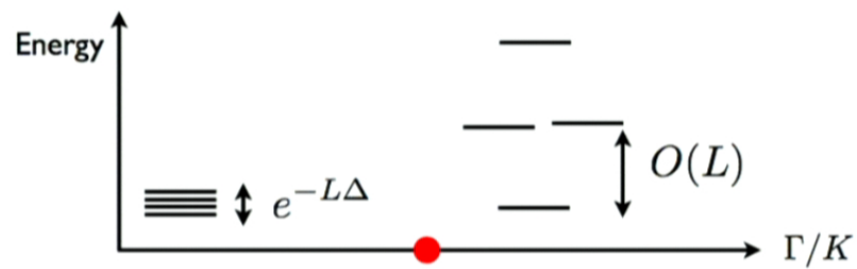


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Comments

- Topological degeneracy



- Energy at late times
 - $O(L^2)$: KZ physics
 - $O(L)$: Initial topological sector



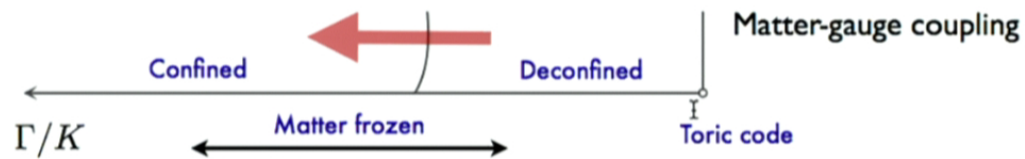
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Comments

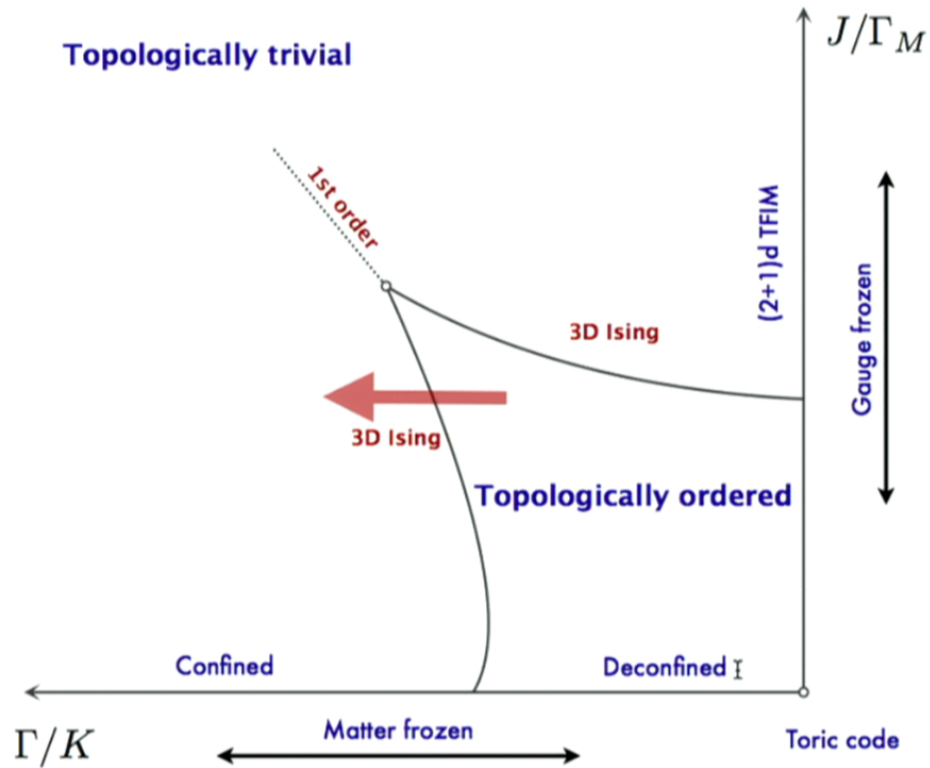
- Density of matter $\sim \exp(-l_K)$
 - Coarsening stops
 - $\exp(l_K) \gg l_K$
- Dangerously irrelevant



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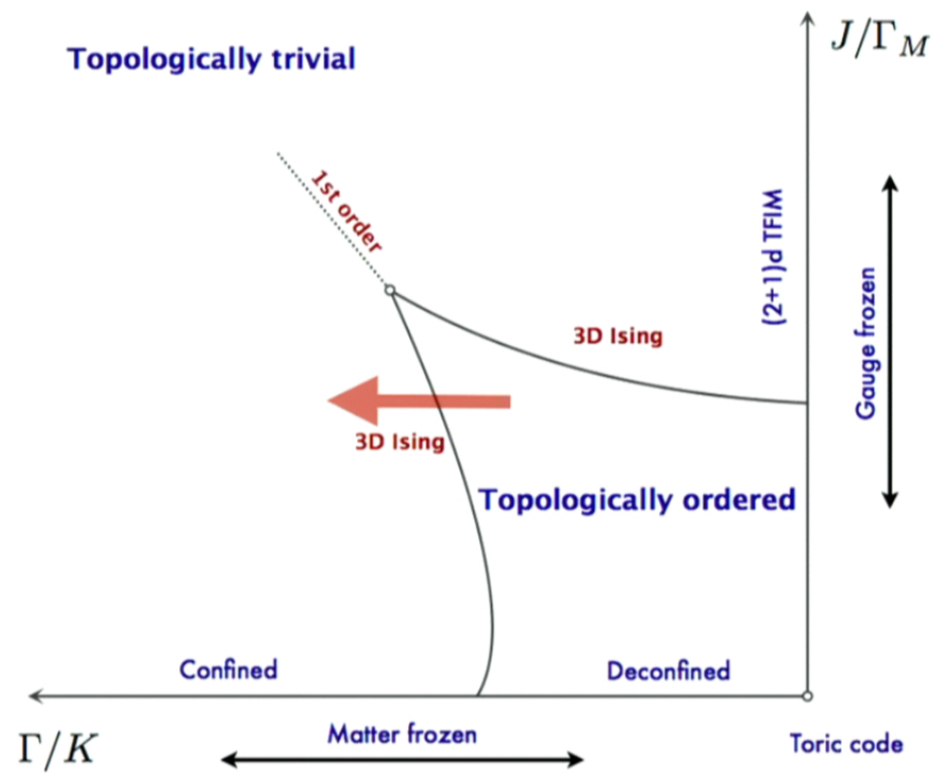
Comments



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Comments



Phase
 $U(2)_2 \times \overline{SU(2)}_2$



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Summary

- Established scaling framework
- Applied to
 - Symmetry breaking transitions
 - Topological order reducing transitions
- Coarsening at late times in a quantum system
 - (2+1)D TFIM
 - Large-N
 - String-net coarsening



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Related work

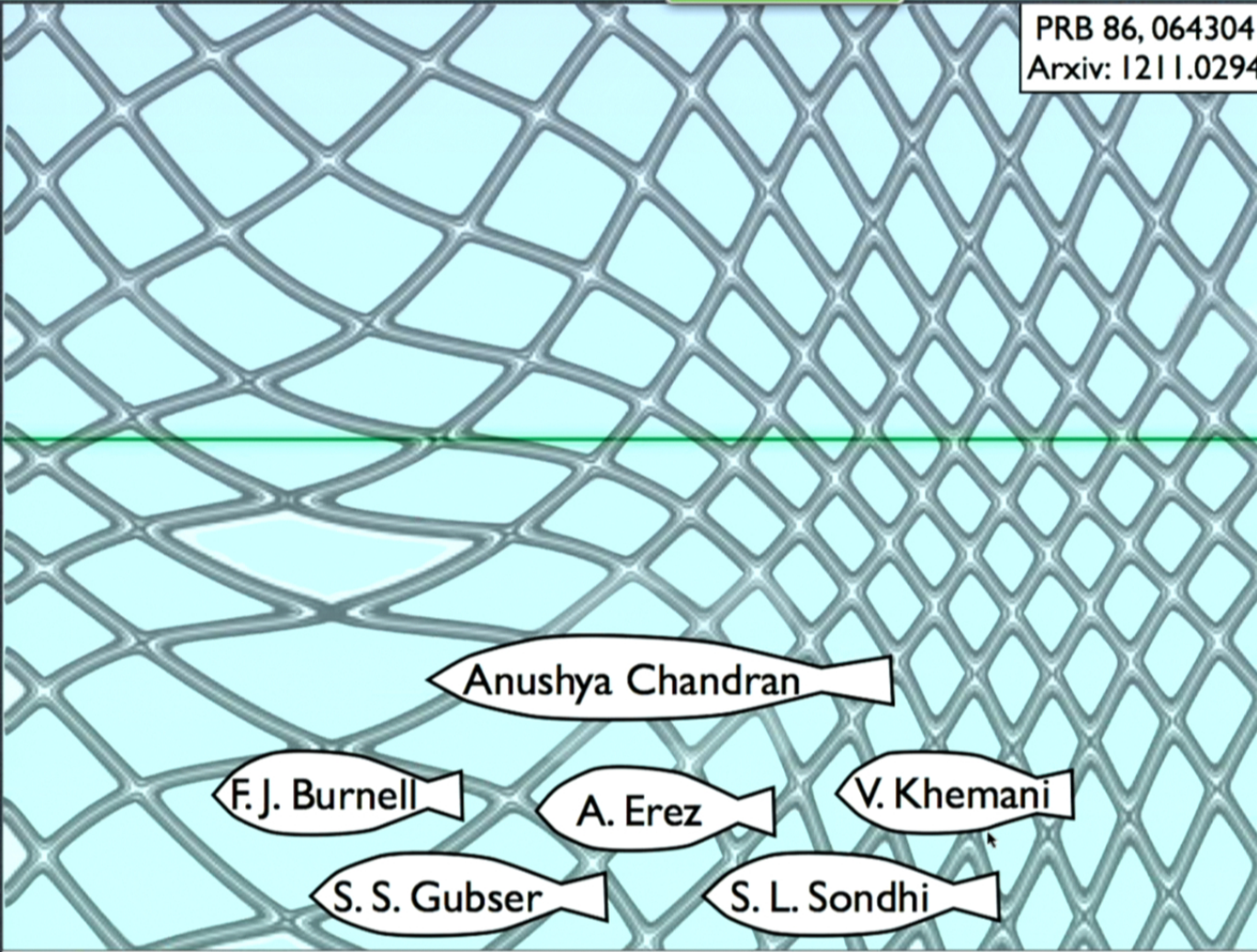
- Hysteresis
 - Symmetry-breaking ramp
 - Universal hysteresis loops
 - Experiments
- Asymptotic approach to critical point
 - Anomalous dimensions
- AdS/CFT correspondence
 - Holographic dictionary



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PRB 86, 064304
Arxiv: 1211.0294



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