Title: Chern-Simons Contact Terms

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Abstract: Chern-Simons contact terms constitute new observables in three-dimensional quantum field theory. In N=2 supersymmetric theories with an R-symmetry, they lead to a superconformal anomaly. This understanding clarifies several puzzles surrounding the S3 partition function of these theories. In particular, it leads to a proof of the F-maximization principle. Chern-Simons contact terms can be computed exactly using localization and lead to new tests of proposed dualities.

Chern-Simons Contact Terms

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Based on arXiv:1205.4142, 1206.5218 with
C. Closset, G. Festuccia, Z. Komargodski, N. Seiberg

Outline

In this talk I will discuss QFT in three Euclidean dimensions

- Chern-Simons contact terms (no SUSY)
- Currents in $\mathcal{N} = 2$ theories with a $U(1)_R$ symmetry
- Supersymmetric Chern-Simons contact terms
- Anomaly: superconformal invariance vs. compact $U(1)_R$
- Partition function of $\mathcal{N} = 2$ theories on S^3
- F-maximization
- Tests of duality
- Conclusions

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Contact Terms

Contact terms are correlation functions at coincident points:

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \cdots + c\delta^{(3)}(x) + c'\partial^2\delta^{(3)}(x) + \cdots$$

It is helpful to couple $\mathcal{O}(x)$ to a classical source $\lambda(x)$ and specify a Lagrangian for dynamical and background fields:

$$\mathcal{L} = \cdots + \lambda(x)\mathcal{O}(x) + \frac{c}{2}\lambda(x)^2 + \frac{c'}{2}\lambda(x)\partial^2\lambda(x) + \cdots$$

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Typically, contact terms are not universal (arbitrary):

- Reflect short-distance physics
- Depend on the regularization scheme
- Modified by adding local counterterms
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Contact Terms (cont.)

Sometimes contact terms are meaningful. This happens when a physical principle (e.g. symmetry) restricts the allowed counterterms.

- Seagull term in scalar quantum electrodynamics
- Trace anomaly in two-dimensional CFT
  - T^μ_μ is a redundant operator; its correlation functions vanish at separated points.
  - Imposing $\partial^\mu T_{\mu\nu} = 0$ forces T^μ_μ to have non-trivial contact terms.
  - Couple $T_{\mu\nu}$ to a background metric $g_{\mu\nu}$. Then conservation = background diffeomorphism invariance.
  - Contact terms $\langle T^\mu_\mu \rangle \sim cR$ cannot by changed by adding diffeomorphism-invariant counterterms.

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Chern-Simons Contact Terms

Consider a three-dimensional QFT with a global $U(1)$ symmetry. Assume that the $U(1)$ is compact. Couple the conserved current j_μ to a classical background gauge field a_μ.

A contact term in the two-point function

$$\langle j_\mu(x) j_\nu(0) \rangle = \cdots + \frac{i\kappa}{2\pi} \varepsilon_{\mu\nu\rho} \partial^\rho \delta^{(3)}(x)$$

corresponds to a Chern-Simons term for a_μ in the effective action (free energy):

$$F[a] = \cdots + \frac{i\kappa}{4\pi} \int d^3x \, \varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

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Chern-Simons Contact Terms (cont.)

- In order to shift the contact term $\kappa \longrightarrow \kappa + \delta\kappa$, we add a counterterm to the UV Lagrangian:

$$\delta\mathcal{L} = \frac{i\delta\kappa}{4\pi}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho$$

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- Counterterms summarize local physics at short distances. They should make sense on any three-manifold and for any configuration of the background field a_μ.
- Since the $U(1)$ is compact, $\delta\kappa \in \mathbb{Z}$. The ambiguity in κ is quantized and $\kappa \bmod (1)$ is a physical observable.

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- Since the $U(1)$ is compact, $\delta\kappa \in \mathbb{Z}$. The ambiguity in κ is quantized and $\kappa \bmod (1)$ is a physical observable.
- The Chern-Simons term is defined by extending a_μ to a (spin) four-manifold. The quantization of $\delta\kappa$ follows from requiring that answer not depend on how this is done.
- Sometimes we need a counterterm with fixed fractional part for consistency. The integer part remains arbitrary.

The Current Two-Point Function

Current conservation restricts

$$\langle j_\mu(p) j_\nu(-p)\rangle = \tau\left(\frac{p^2}{\mu^2}\right)\frac{p_\mu p_\nu - p^2\delta_{\mu\nu}}{16|p|} + \kappa\left(\frac{p^2}{\mu^2}\right)\frac{\varepsilon_{\mu\nu\rho}p^\rho}{2\pi}$$

- Two dimensionless structure functions.
- In a CFT τ and κ are constants.
- Unitarity implies $\tau \geq 0$.
- $\tau\left(\frac{p^2}{\mu^2}\right)$ always corresponds to separated points.
- The p-dependence of $\kappa\left(\frac{p^2}{\mu^2}\right)$ corresponds to separated points.
- Shifting $\kappa\left(\frac{p^2}{\mu^2}\right)$ by a constant amounts to adding a contact term.

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Interesting Observables

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- $\tau\left(\frac{p^2}{\mu^2}\right)$ is always a good physical observable.
- The p-dependence of $\kappa\left(\frac{p^2}{\mu^2}\right)$ is physical, but the constant piece is more subtle. Define

$$\kappa_{\rm UV} = \lim_{p\to\infty}\kappa\left(\frac{p^2}{\mu^2}\right) \quad , \qquad \kappa_{\rm IR} = \lim_{p\to 0}\kappa\left(\frac{p^2}{\mu^2}\right)$$

- We can change $\kappa_{\rm UV}, \kappa_{\rm IR}$ by adding a counterterm, but $\kappa_{\rm UV} - \kappa_{\rm IR}$ is a physical observable.
- If the $U(1)$ symmetry is compact, the ambiguity is quantized and $\kappa_{\rm UV} \bmod (1)$, $\kappa_{\rm IR} \bmod (1)$ are physical.

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Example 1: Free Fermion

A massive Dirac Fermion has a global $U(1)$ symmetry under which the fermion has change $+1$. Computing $\kappa\left(\frac{p^2}{m^2}\right)$ we find

$$\kappa_{\mathrm{UV}} - \kappa_{\mathrm{IR}} = \frac{1}{2}\mathrm{sign}(m)$$

The IR theory is completely empty; the theory is fully gapped. The effective Lagrangian is a Chern-Simons term for the background $U(1)$ gauge field, with level κ_{IR}.

Since the system is fully gapped, consistency demands that $\kappa_{\mathrm{IR}} \in \mathbb{Z}$. Therefore we must add a UV counterterm such that [Redlich]

$$\kappa_{\mathrm{UV}} = \frac{1}{2} + (\mathrm{integer})$$

The fact that $\kappa_{\mathrm{UV}} \neq 0$ is sometimes called a ‘parity anomaly.’

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$$\kappa_{\rm UV} = \frac{1}{2} + ({\rm integer})$$

The fact that $\kappa_{\rm UV} \neq 0$ is sometimes called a 'parity anomaly.'

Example 3: Non-Trivial RG Flow

Consider an RG flow with two crossover scales $m \ll M$:

- The UV theory above the scale M is free.
- At intermediate energies $m \ll E \ll M$, the theory is approximately a CFT.
- The IR theory below the scale m is fully gapped (no topological degrees of freedom).

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We can use our knowledge of $\kappa_{\rm UV}, \kappa_{\rm IR}$ to learn about $\kappa_{\rm CFT}$:

- $\kappa_{\rm UV} = \kappa(M^2 \ll p^2)$ is fixed up to an integer by the number of fermions and topological contributions.
- $\kappa_{\rm CFT} = \kappa(m^2 \ll p^2 \ll M^2)$ is an intrinsic observable of the CFT, up to an integer that knows about UV or IR.
- $\kappa_{\rm IR} = \kappa(p^2 \ll m^2)$ must be an integer.

Gravitational Chern-Simons Contact Terms

We can repeat the previous discussion for contact terms in the two-point function of the energy-momentum tensor $T_{\mu\nu}$:

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \cdots - \frac{i\kappa_g}{192\pi}\varepsilon_{\mu\rho\lambda}\partial^\lambda(\partial_\nu\partial_\sigma - \partial^2\delta_{\nu\sigma})\delta^{(3)}(x) + \cdots$$

This contact term corresponds to a Chern-Simons term for the background metric $g_{\mu\nu}$

$$\frac{i\kappa_g}{192\pi}\int \sqrt{g}\, d^3x\, \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\omega_\mu\partial_\nu\omega_\rho + \frac{2}{3}\omega_\mu\omega_\nu\omega_\rho\right)$$

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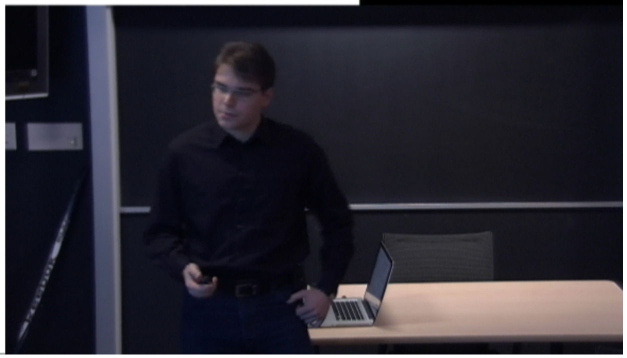
Currents in $\mathcal{N} = 2$ Supersymmetry

Distinguish $U(1)$ flavor symmetries and $U(1)_R$ symmetries.

- A flavor current j_μ is embedded in a real linear multiplet $\mathcal{J} = (j_\mu, K, J) + (\text{fermions})$.
- We can couple it to a background gauge field a_μ in a vector superfield $\mathcal{V} = (a_\mu, \sigma, D) + (\text{gauginos})$, which can take arbitrary values.



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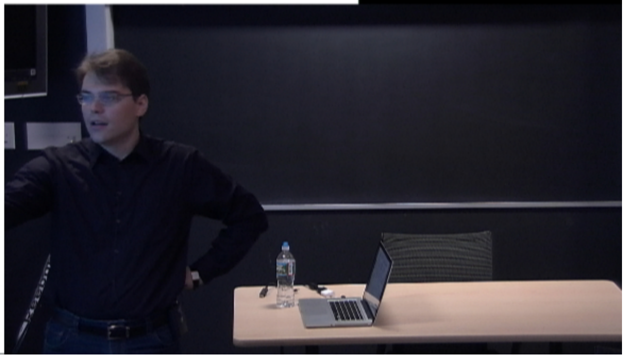

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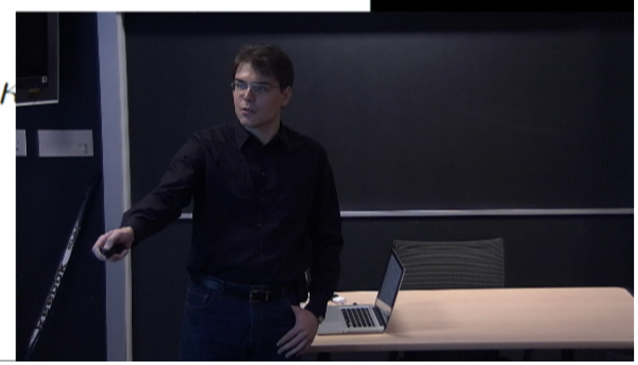
The supersymmetric flavor-flavor Chern-Simons term is

$$\mathcal{L}_{ff}=\frac{\kappa_{ff}}{4\pi}\left(i\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho-2\sigma D\right)+(\text{fermions}) \ .$$

- The flavor-flavor Chern-Simons contact term
in $\langle j_\mu(x)j_\nu(0)\rangle$ and $\langle J(x)K(0)\rangle$.
- As before, $\kappa_{ff} \bmod (1)$ is physical.



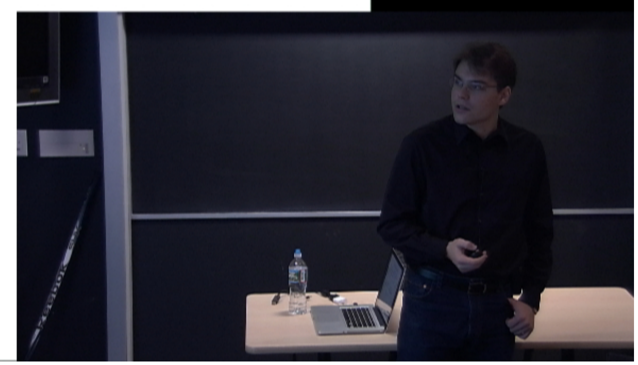
R-Symmetry Currents

- In a three-dimensional $\mathcal{N} = 2$ theory, a $U(1)_R$ current is in a supermultiplet with the SUSY current and the energy-momentum tensor [TD, Seiberg]

$$\mathcal{R} = (j_\mu^{(R)}, S_{\alpha\mu}, T_{\mu\nu}, J^{(Z)}, j_\mu^{(Z)})$$

- The appropriate background fields are the fields in the three-dimensional, new-minimal supergravity multiplet

$$\mathcal{H} = (A_\mu, \Psi_{\alpha\mu}, g_{\mu\nu}, H, C_\mu) \ , \quad V_\mu = -\varepsilon_{\mu\nu\rho}\partial^\nu C^\rho$$



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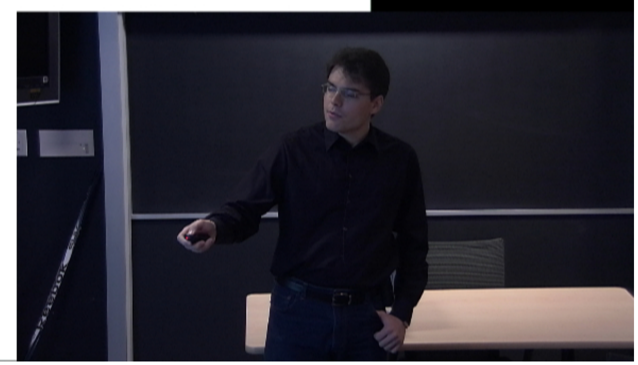

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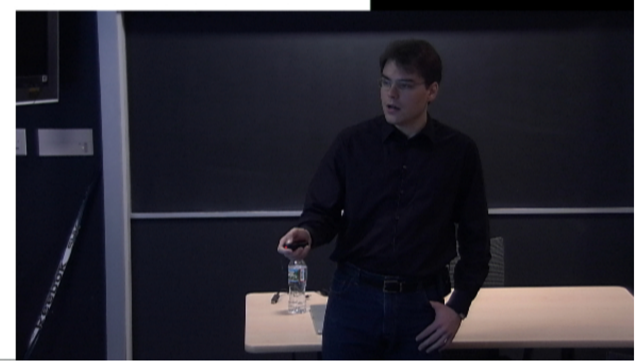

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More Chern-Simons Contact Terms

Using the background fields that couple to the $\mathcal{R}$-multiplet, we can construct three more Chern-Simons terms:

- Gravitational: $\omega \wedge d\omega + \cdots$
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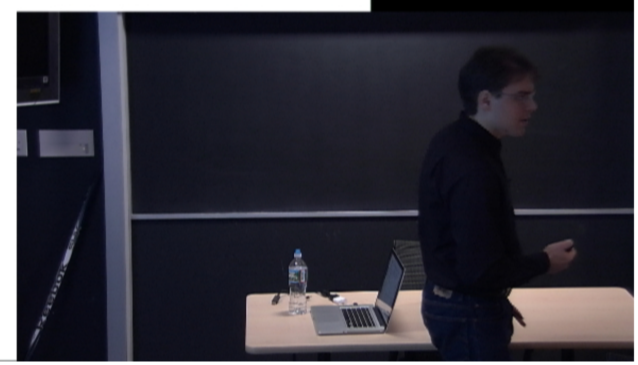

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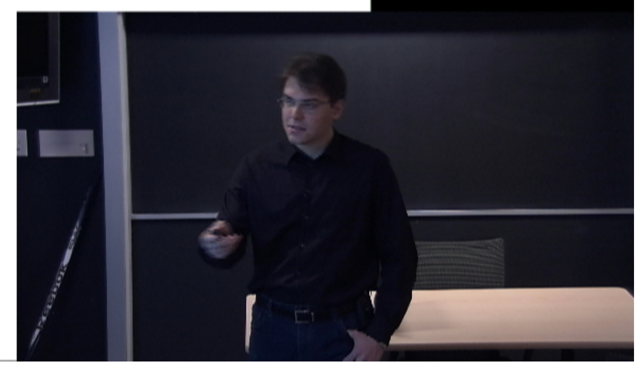

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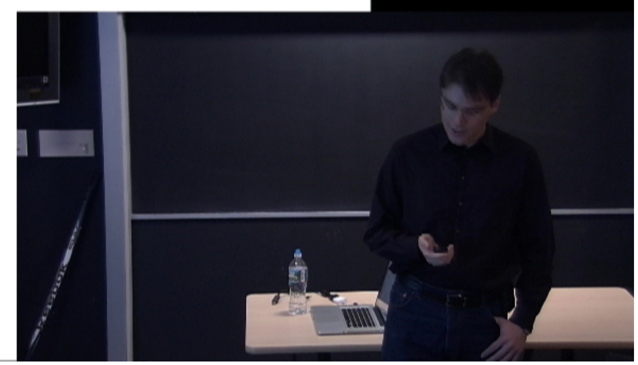
The Gravitational Chern-Simons term can be completed to a superconformal expression [Rocek, van Nieuwenhuizen]

$$\mathcal{L}_g = \frac{\kappa_g}{192\pi} i\varepsilon^{\mu\nu\rho}\Big(\mathrm{Tr}\left(\omega_\mu \partial_\nu \omega_\rho + \frac{2}{3}\omega_\mu\omega_\nu\omega_\rho\right) + 4\big(A_\mu - \frac{3}{2}V_\mu\big)\partial_\nu\big(A_\rho - \frac{3}{2}V_\rho\big)\Big) + (\text{fermions})$$

Non-Superconformal Chern-Simons Terms

- A contact term in the two-point function $\langle j_\mu(x) j_\nu^{(R)}(0)\rangle$ is captured by the SUSY flavor-R Chern-Simons term

$$\mathcal{L}_{fr} = -\frac{\kappa_{fr}}{2\pi}\left(i\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu\left(A_\rho - \frac{1}{2}V_\rho\right) + \frac{1}{4}\sigma R - DH + \cdots\right)$$



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- The flavor-R term is not conformally invariant:
  - The fields $A_\mu - \frac{1}{2}V_\mu$, H and the Ricci scalar R do not arise in conformal supergravity. (Not gauge invariant.)
  - These fields couple to redundant operators in the CFT. For example R couples to T^μ_μ.
- The R-R term is also not conformally invariant:

$$\mathcal{L}_{rr} = -\frac{\kappa_{rr}}{4\pi}\left(i\varepsilon^{\mu\nu\rho}\big(A_\mu - \frac{1}{2}V_\mu\big)\partial_\nu \big(A_\rho - \frac{1}{2}V_\rho\big) + \frac{1}{2}HR + \cdots\right)$$

Summary of SUSY Chern-Simons Terms

We studied four different Chern-Simons terms:

- Flavor-Flavor: $a \wedge da + \cdots$
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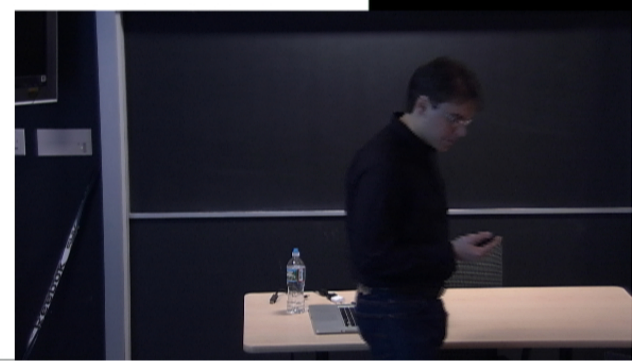
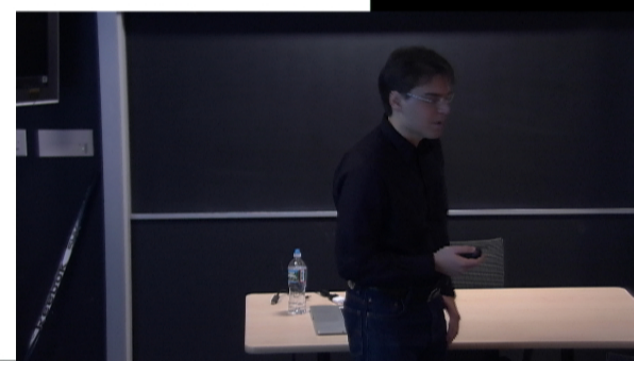


A New Anomaly

If the $U(1)$ flavor and the $U(1)_R$ symmetries are compact, the fractional parts of the various Chern-Simons contact terms are physical observables. They cannot be removed.

The flavor-flavor and gravitational terms are superconformal and can be non-zero in a superconformal theory.



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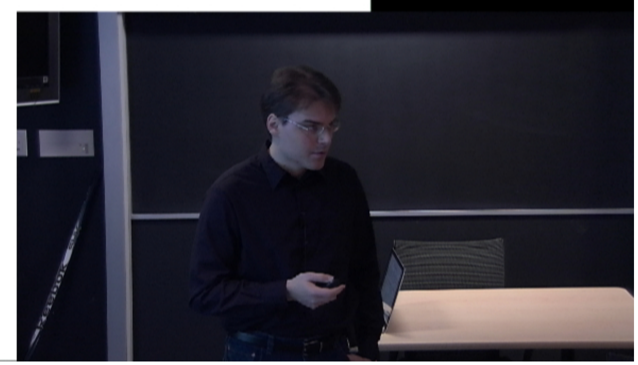

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The flavor-R and R-R terms are supersymmetric but not conformal. They should not arise in a superconformal theory. However, if they have fractional parts they cannot be removed.

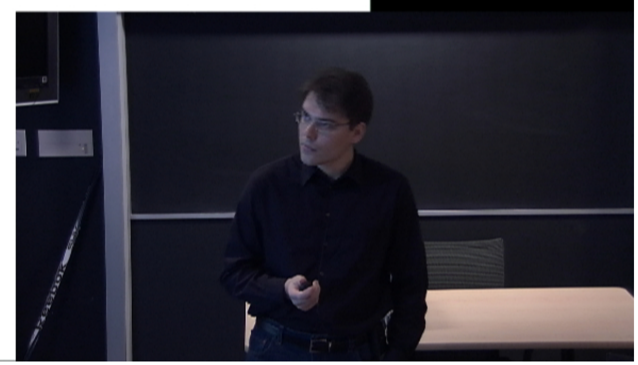
What should we do with these non-conformal terms?

A New Anomaly (cont.)

An anomaly arises when we cannot impose different physical requirements at the same time. In an SCFT we would like:

- Supersymmetry
- Conformal invariance
- Compactness of all flavor and R-symmetries

If the flavor-R and R-R terms have fractional parts, then we must give up one of these requirements.



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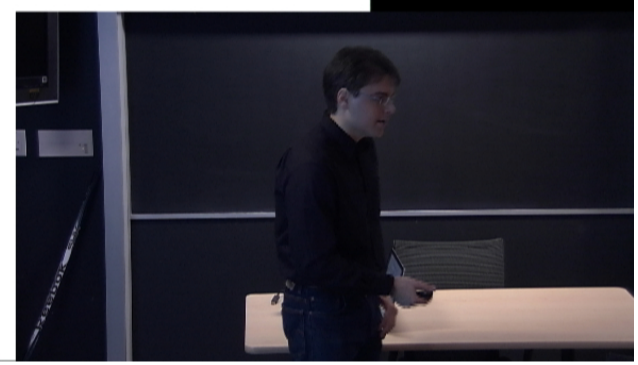

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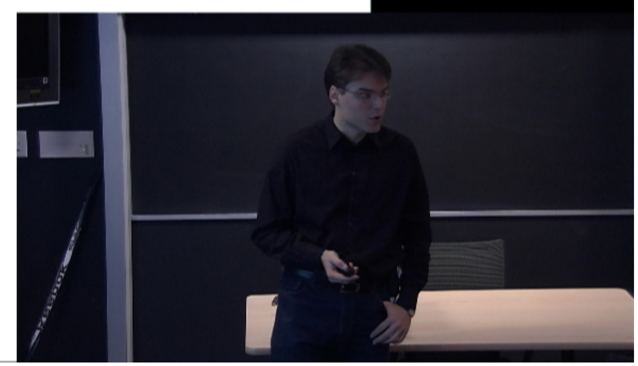

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This is a $U(1)$ gauge theory with N_f flavors and a dynamical Chern-Simons term at level k, which flows to a CFT in the IR.

Starting with zero contact terms in the UV, explicit two-loop computations uncover Chern-Simons contact terms in the IR:

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We also expect the gravitational and R-R terms to be nonzero.

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Using the supergravity perspective, we fix the metric on the S^3. The other supergravity fields are completely fixed, if we want to preserve four supercharges:

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- The choice of R-symmetry is not unique. It can mix with a $U(1)$ flavor symmetry:

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- Using localization, we can exactly compute the partition function $Z = e^{-F}$ on S^3 as a function of $\sigma = m + \frac{it}{r}$. The answer is a holomorphic function of σ.
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Resolution: Chern-Simons Contact Terms

- In flat space, we found four supersymmetric Chern-Simons contact terms. Two of them are not superconformal.
- If we do not add bare counterterms to remove them, the superconformal field theory has non-conformal Chern-Simons terms for the background fields.
- Substituting the complex background fields H, σ and D, these non-conformal terms give rise to complex Z.
- The phase of the partition function is computable using the flat-space values of the contact terms. It agrees with localization computations on S^3.
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F-Maximization

Explore the free energy $F(t)$ as a function of the R-symmetry parameter t near the superconformal point $t = t_*$.

- The real part is an extremum [Jafferis]. The Imaginary part violates conformal invariance due to κ_{fr}:

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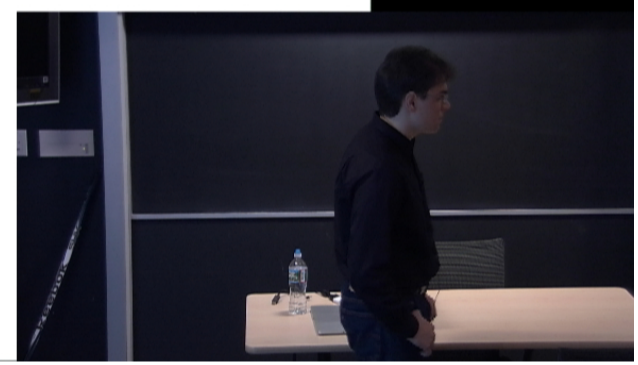

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