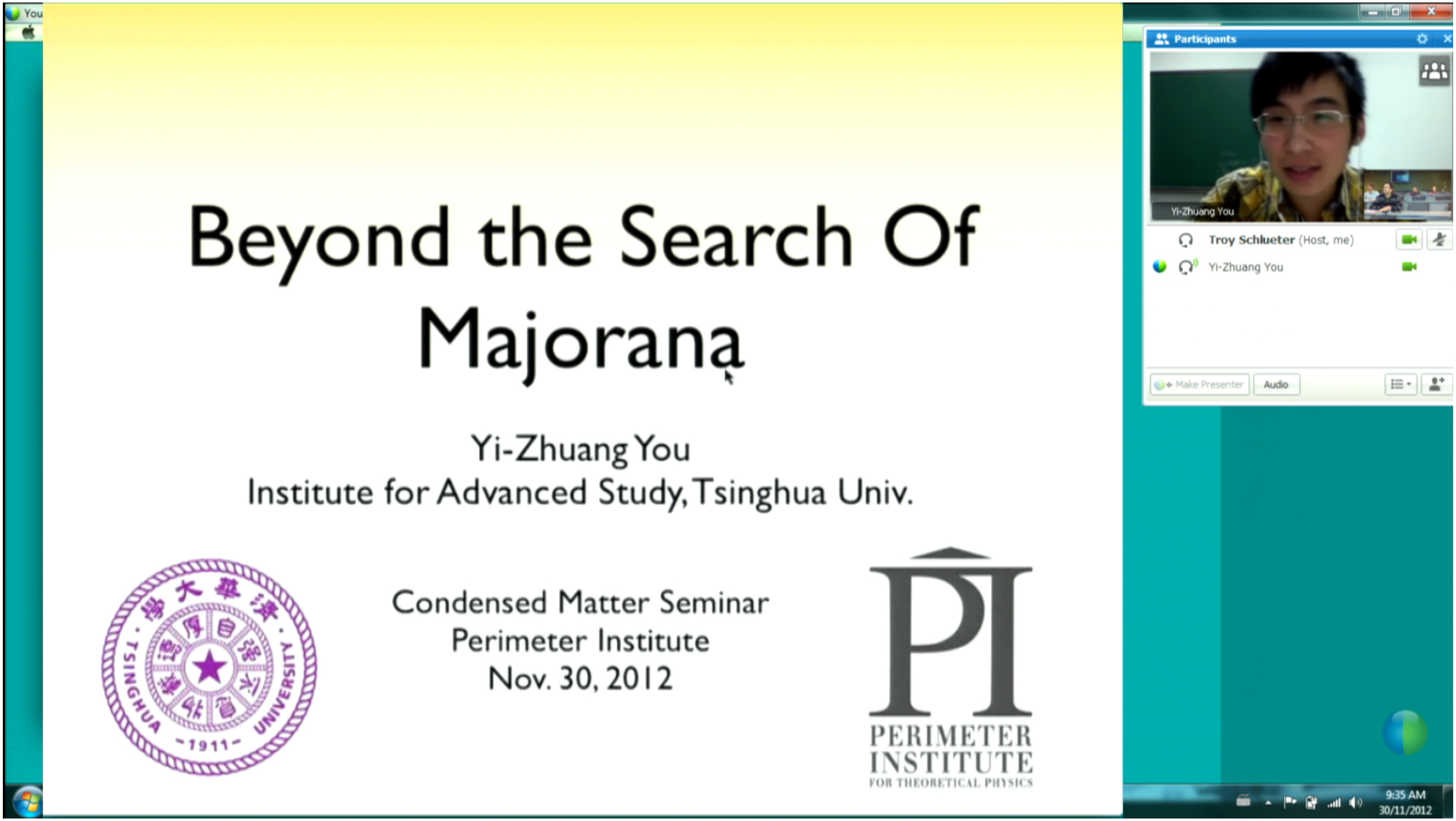
Title: Beyond the Search for Majorana

Date: Nov 30, 2012 09:30 AM

URL: http://pirsa.org/12110088

Abstract: The search for Majorana zero-modes in condensed matter system has attract increasing research interests recently. Looking for Majorna zero-mode is actually looking for topologically protected ground state degeneracy. The topological degeneracies on closed manifolds have been used to discover/define topological order in many-body systems, which contain excitations with fractional statistics. In this talk, I will present our recent work on new types of topological degeneracy induced by condensing anyons along a line in 2D topological ordered states. Such topological degeneracy can be viewed as carried by each end of the line-defect, which is a generalization of Majorana zero-modes. The ends of line-defects carry projective non-Abelian statistics even though they are produced by condensation of Abelian anyons, and braiding them allow us to perform fault tolerant quantum computations.


Beyond the Search Of Majorana
Yi-Zhuang You
Institute for Advanced Study, Tsinghua Univ.
Condensed Matter Seminar
Perimeter Institute
Nov. 30, 2012
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The Search of Majorana

- Majorana Zero-Modes in Condensed Matter System
- Fermion Models
  - 1D: Kitaev Majorana Chain (spinless p-wave SC)
  - 2D: p+ip superconductor with π-vortex
- Non-Abelian statistics → Topological Quantum Computation (against decoherence)
- Practical Realizations J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
  - TI-FM-SC interface
  - SOC wire with magnetic field and pairing
  - ν=5/2 FHQ
  - 3D TI + s-wave SC + FM/vortices

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Z_2 Plaquette Model

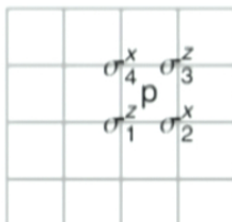
X.-G. Wen, Phys. Rev. Lett. **90**, 016803 (2003).

- Hilbert Space
  - Square lattice of qubits
  - Each qubit (spin): $|0\rangle, |1\rangle$
  - Qubit operator

$$\begin{matrix} & |0\rangle & |1\rangle \end{matrix}$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(0 \quad 1$$



- Hamiltonian
  - 4-qubit Interaction

$$H_0 = -\sum_p O_p$$

Plaquette operator

$$O_p = \sigma_1^z \, \sigma_2^x \, \sigma_3^z \, \sigma_4^x$$

- They all commute

$$\forall\, p, p' : O_p \, O_{p'} = O_{p'}$$

- Beyond Majorana zero-mode
  - Why in the Fermion system? → Boson/Spin system?
    - By lattice dislocations in Z_2 plaquette model (H. Bombin, 2010)
  - Why each pair associated to 2 fold? → 3,4,5… fold?
    - By anyon condensation (You, Wen, 2012; You, Jian, Wen, 2012)
    - And many other approaches …

Participants

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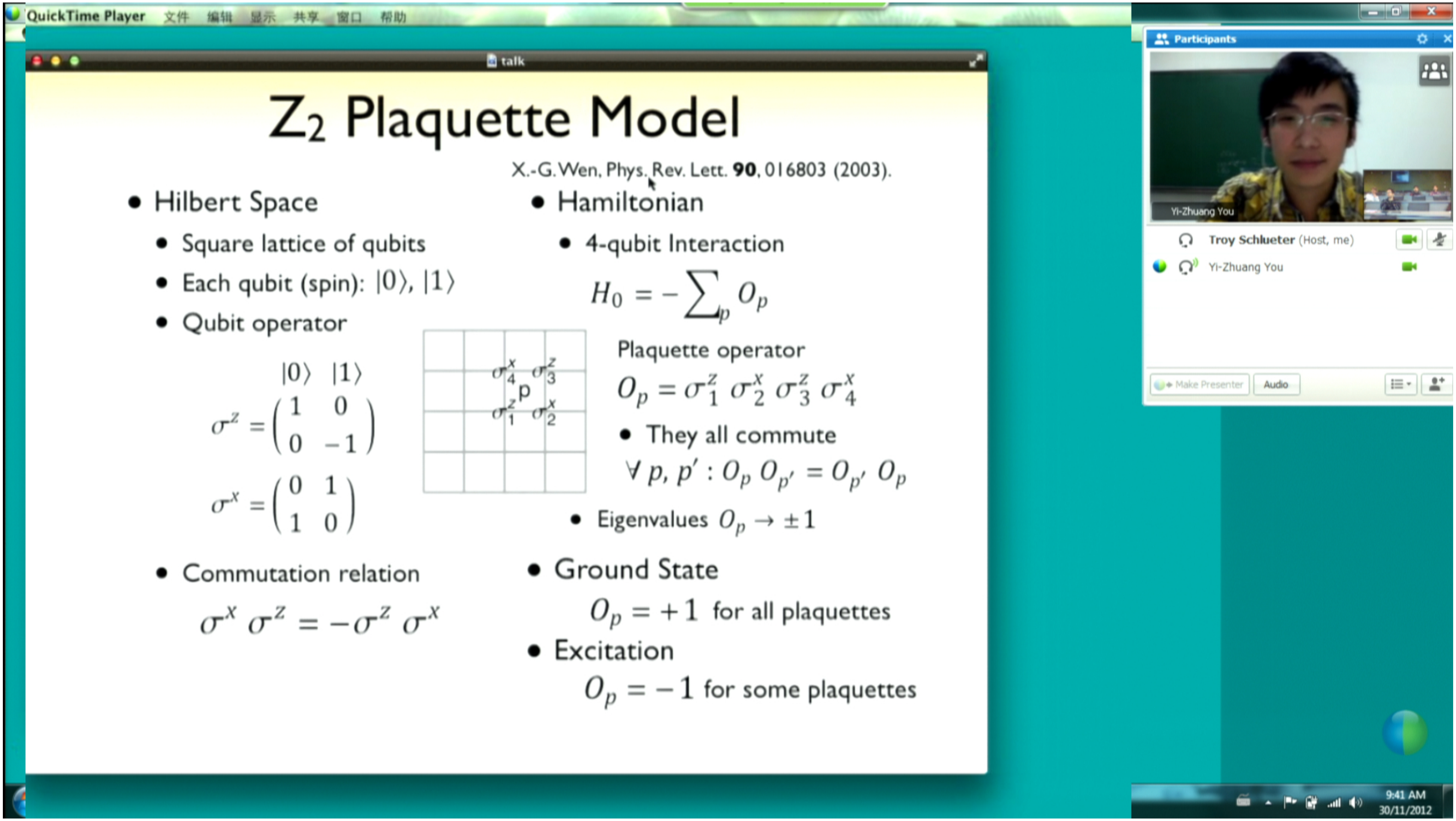
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talk
Z_2 Plaquette Model
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Hilbert Space
Square lattice of qubits
Each qubit (spin): $|0\rangle, |1\rangle$
Qubit operator
$|0\rangle \quad |1\rangle$
$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Commutation relation
$\sigma^x \sigma^z = -\sigma^z \sigma^x$
σ_4^x σ_3^z p σ_1^z σ_2^x
Hamiltonian
4-qubit Interaction
$H_0 = -\sum_p O_p$
Plaquette operator
$O_p = \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$
They all commute
$\forall p, p' : O_p O_{p'} = O_{p'} O_p$
Eigenvalues $O_p \to \pm 1$
Ground State
$O_p = +1$ for all plaquettes
Excitation
$O_p = -1$ for some plaquettes
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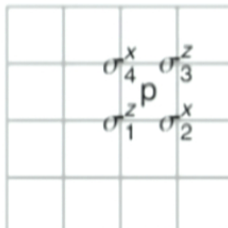


  - Commutation relation

$$\sigma^x \sigma^z = -\sigma^z \sigma^x$$

- Hamiltonian
  - 4-qubit Interaction

$$H_0 = -\sum_p O_p$$

Plaquette operator

$$O_p = \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$$

  - They all commute

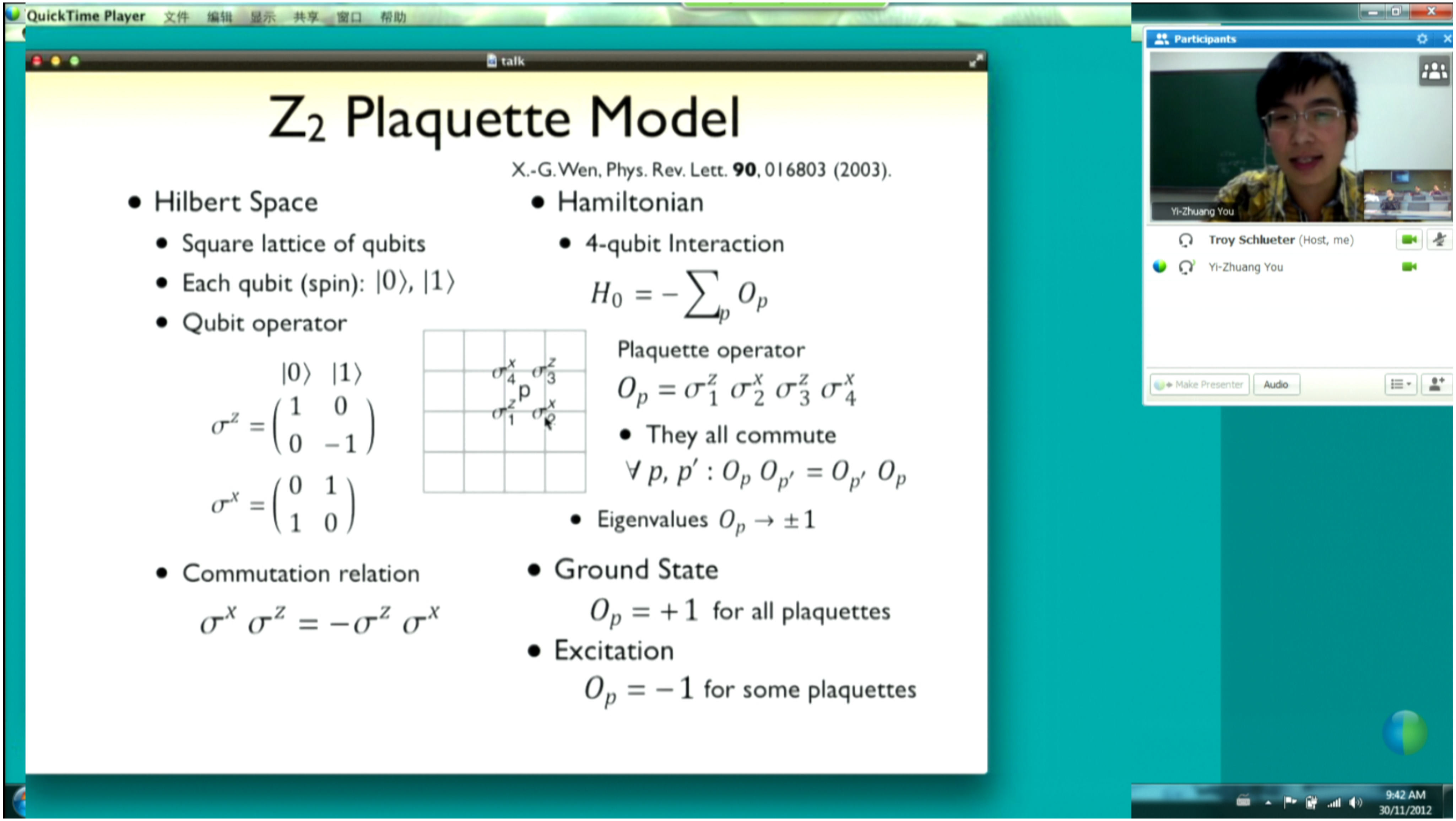
$$\forall\, p, p' : O_p O_{p'} = O_{p'} O_p$$

  - Eigenvalues $O_p \to \pm 1$
- Ground State

$$O_p = +1 \text{ for all plaquettes}$$

- Excitation

$$O_p = -1 \text{ for some plaquettes}$$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talk
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Hamiltonian
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$H_0 = -\sum_p O_p$
Plaquette operator
$O_p = \sigma^z_1 \sigma^x_2 \sigma^z_3 \sigma^x_4$
They all commute
$\forall\, p, p' : O_p O_{p'} = O_{p'} O_p$
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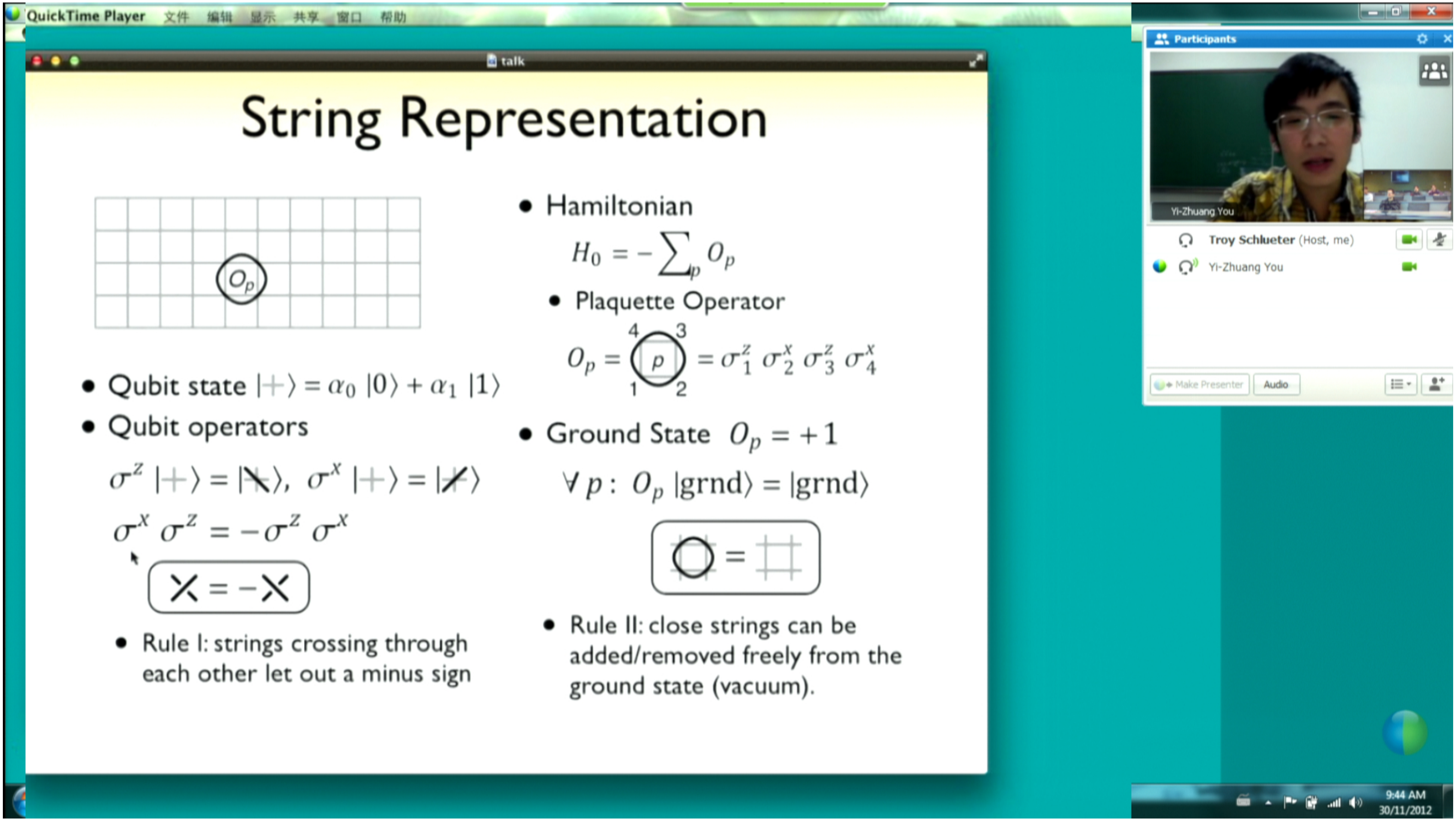
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talk
String Representation
Hamiltonian
$H_0 = -\sum_p O_p$
Plaquette Operator
$O_p = $ [p] $= \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$
4 3 1 2
Qubit state $|+\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$
Qubit operators
$\sigma^z |+\rangle = |\setminus\rangle, \sigma^x |+\rangle = |/\rangle$
$\sigma^x \sigma^z = -\sigma^z \sigma^x$
Rule I: strings crossing through each other let out a minus sign
Ground State $O_p = +1$
$\forall p: O_p |\mathrm{grnd}\rangle = |\mathrm{grnd}\rangle$
Rule II: close strings can be added/removed freely from the ground state (vacuum).
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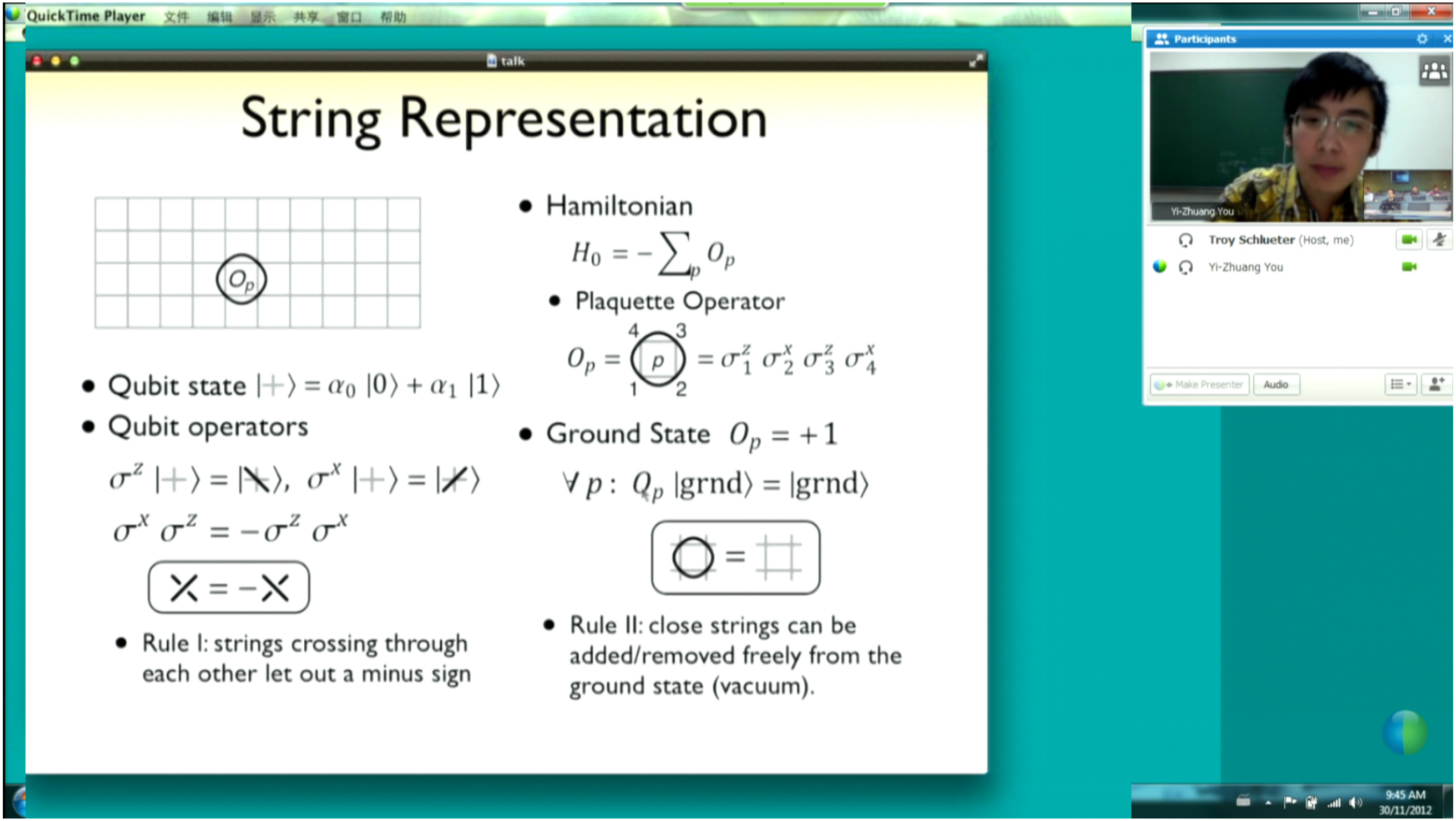
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talk
String Representation
O_p
• Qubit state $|+\rangle = \alpha_0\,|0\rangle + \alpha_1\,|1\rangle$
• Qubit operators
$\sigma^z\,|+\rangle = |\diagdown\rangle,\ \sigma^x\,|+\rangle = |\diagup\rangle$
$\sigma^x\,\sigma^z = -\sigma^z\,\sigma^x$
• Rule I: strings crossing through each other let out a minus sign
• Hamiltonian
$H_0 = -\sum_p O_p$
• Plaquette Operator
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• Ground State $O_p = +1$
$\forall\, p:\ O_p\,|\mathrm{grnd}\rangle = |\mathrm{grnd}\rangle$
• Rule II: close strings can be added/removed freely from the ground state (vacuum).
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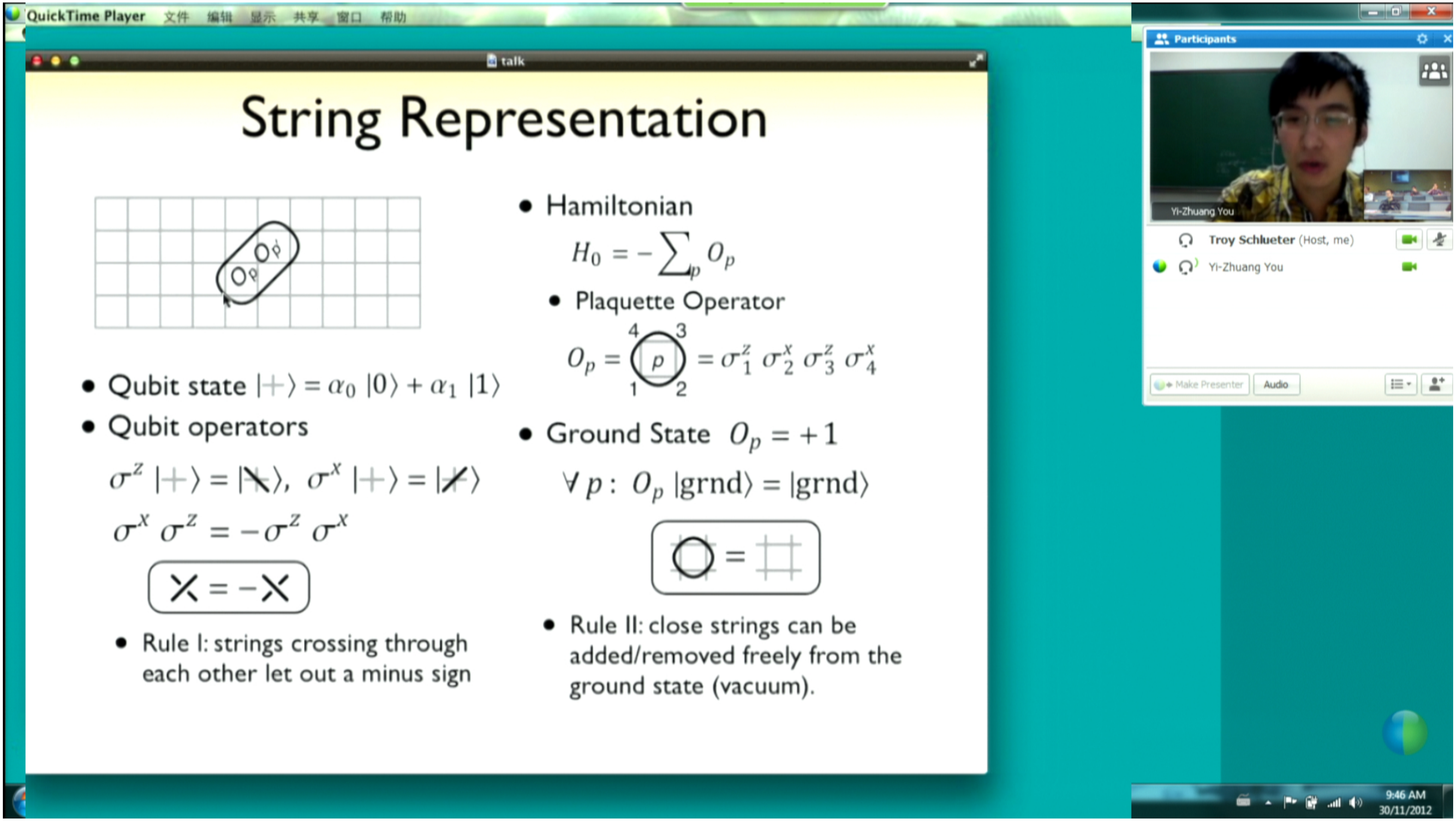
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$= -$
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Hamiltonian
$H_0 = -\sum_p O_p$
Plaquette Operator
$O_p = \; p \; = \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$
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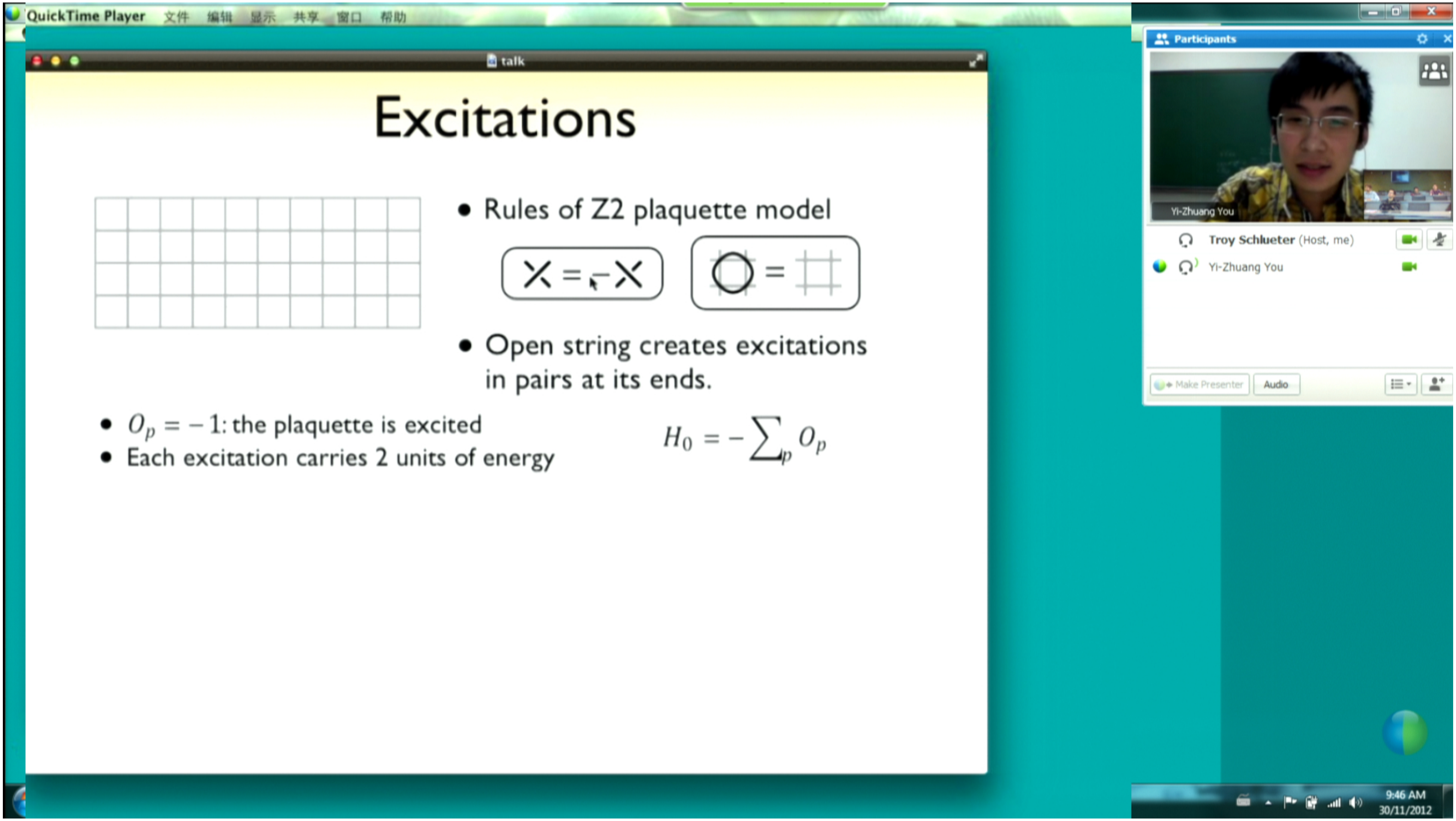
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talk
Excitations
Rules of Z2 plaquette model
Open string creates excitations in pairs at its ends.
$O_p = -1$: the plaquette is excited
Each excitation carries 2 units of energy
$H_0 = -\sum_p O_p$
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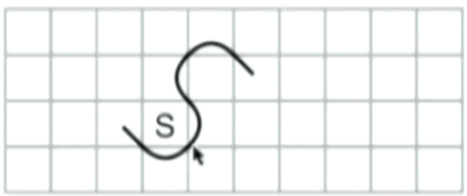
Excitations



$S\,|\mathrm{grnd}\rangle$

- Rules of Z2 plaquette model

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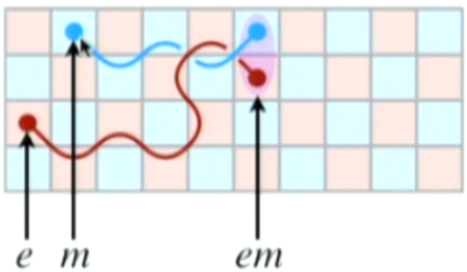
$$H_0 = -\sum_p O_p$$

Excitations

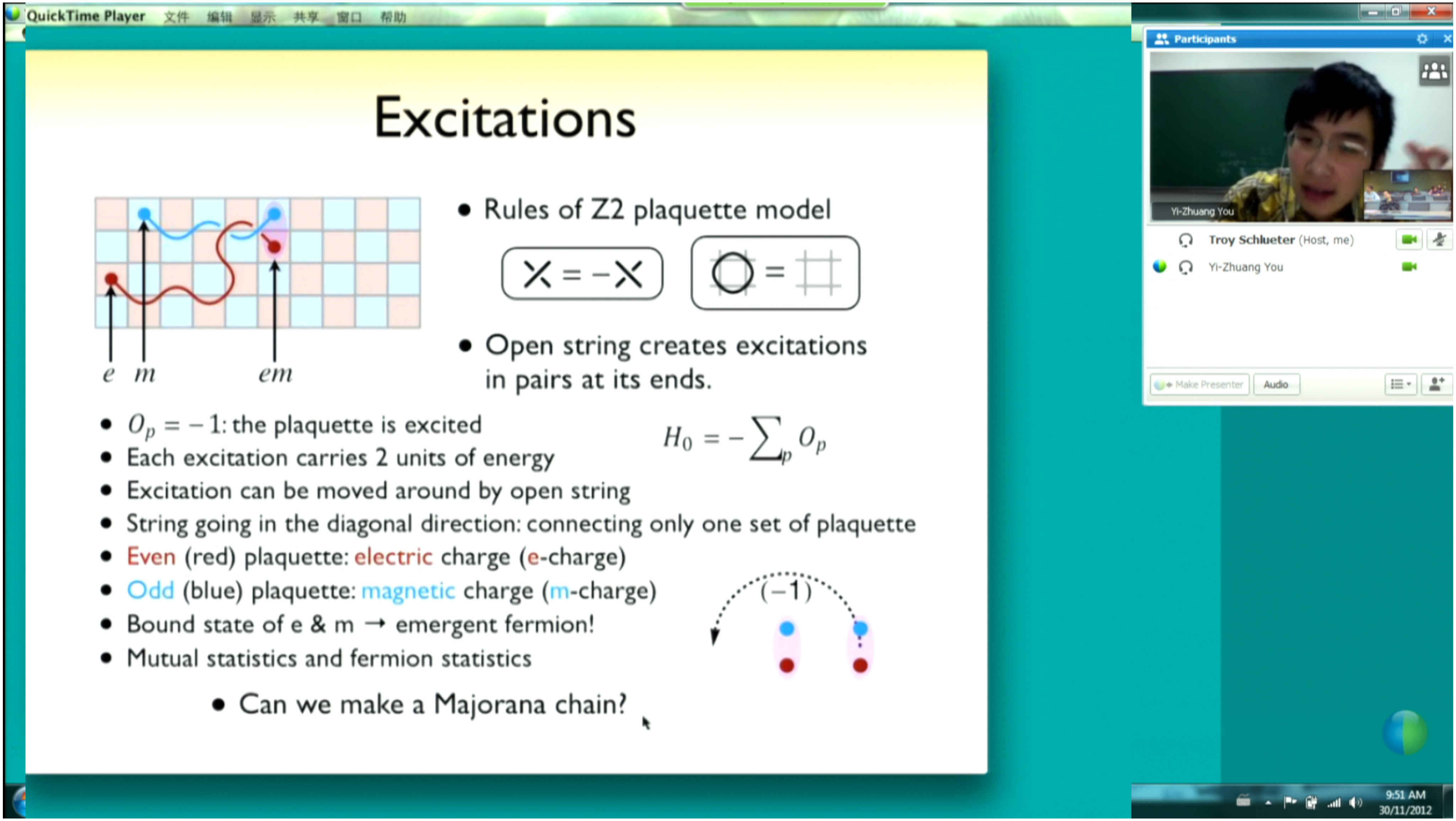


- Rules of Z2 plaquette model

- Open string creates excitations in pairs at its ends.

$$H_0 = -\sum_p O_p$$

- $O_p = -1$: the plaquette is excited
- Each excitation carries 2 units of energy
- Excitation can be moved around by open string
- String going in the diagonal direction: connecting only one set of plaquette
- Even (red) plaquette: electric charge (e-charge)
- Odd (blue) plaquette: magnetic charge (m-charge)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Excitations
e m em
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• Excitation can be moved around by open string
• String going in the diagonal direction: connecting only one set of plaquette
• Even (red) plaquette: electric charge (e-charge)
• Odd (blue) plaquette: magnetic charge (m-charge)
• Bound state of e & m → emergent fermion!
• Mutual statistics and fermion statistics
(−1)
• Can we make a Majorana chain?
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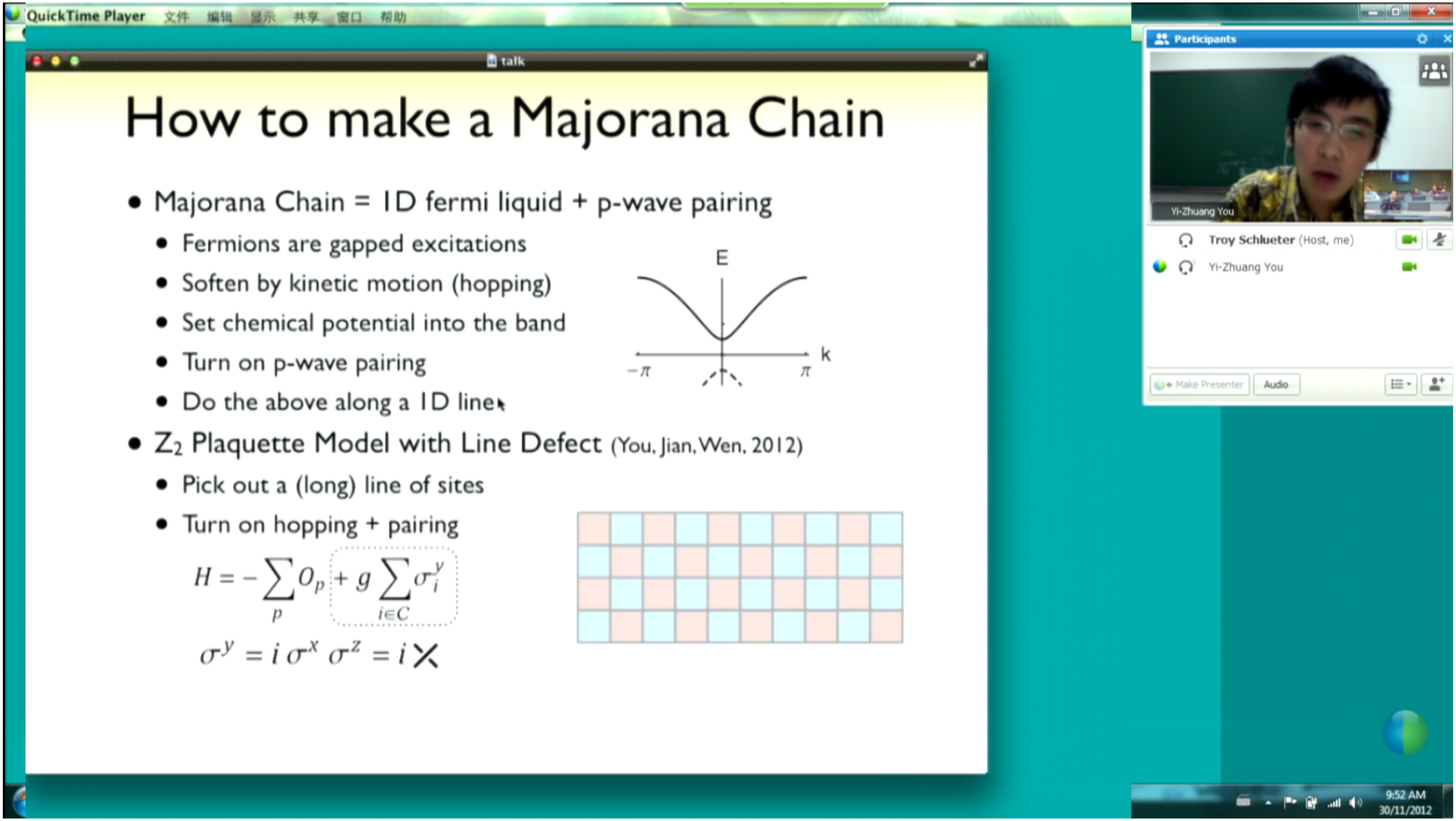
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talk
How to make a Majorana Chain
Majorana Chain = 1D fermi liquid + p-wave pairing
Fermions are gapped excitations
Soften by kinetic motion (hopping)
Set chemical potential into the band
Turn on p-wave pairing
Do the above along a 1D line
E
k
$-\pi$
π
Z_2 Plaquette Model with Line Defect (You, Jian, Wen, 2012)
Pick out a (long) line of sites
Turn on hopping + pairing
$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$
$\sigma^y = i\,\sigma^x\,\sigma^z = i$
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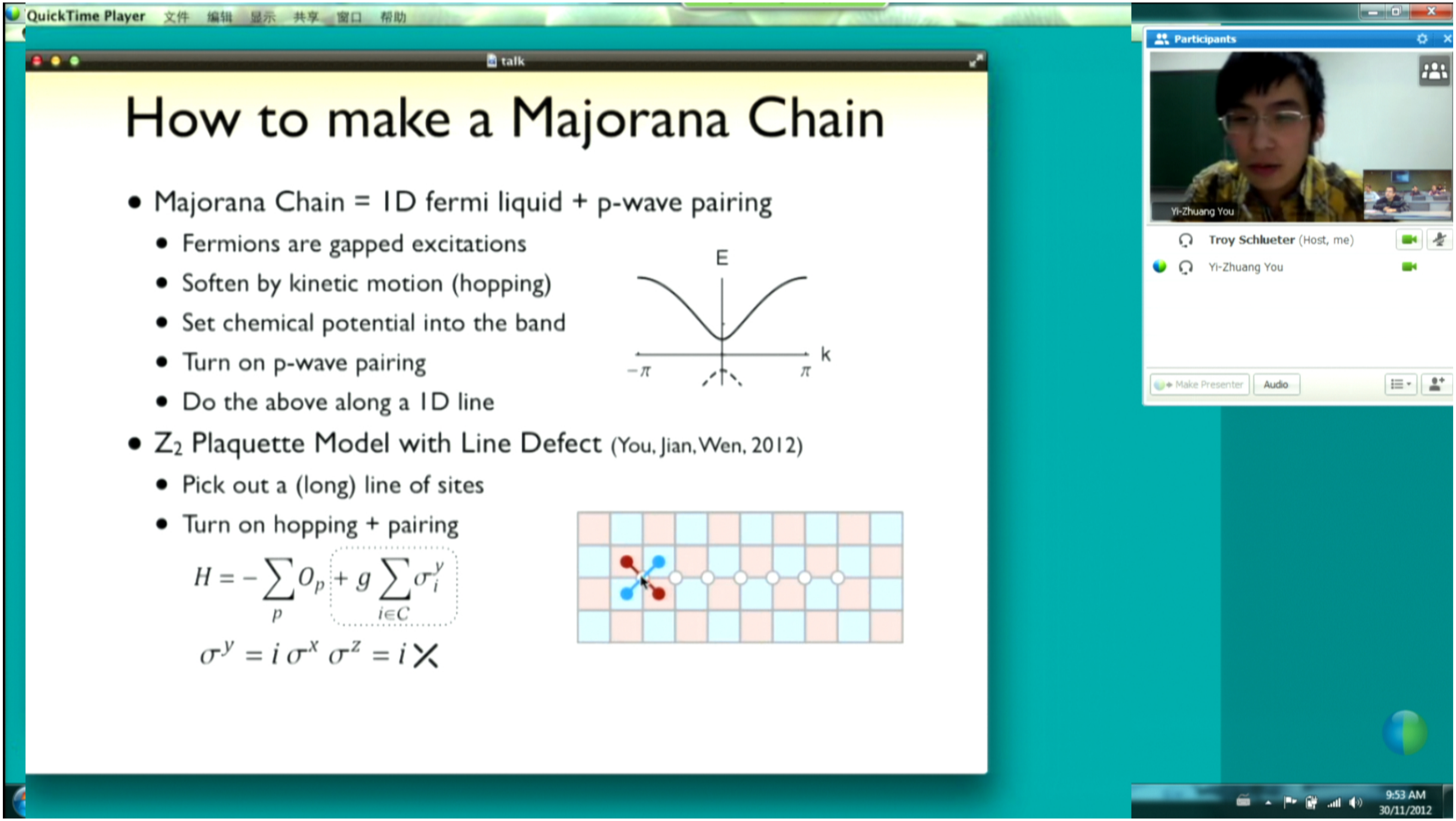
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How to make a Majorana Chain

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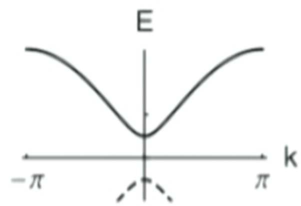
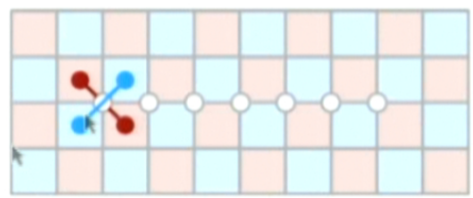


- Z_2 Plaquette Model with Line Defect (You, Jian, Wen, 2012)
  - Pick out a (long) line of sites
  - Turn on hopping + pairing

$$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$$

$$\sigma^y = i\,\sigma^x \sigma^z = i \times$$



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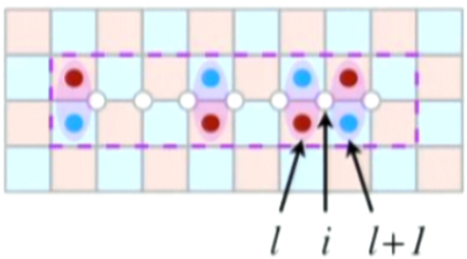
Fermion Condensation



- Plaquette Model with Line Defect

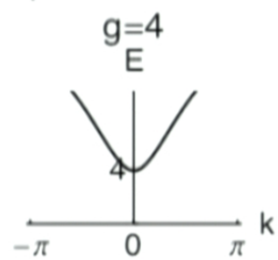
$$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$$

$$\sigma^y = c_{l+1}^\dagger c_l^\dagger + c_{l+1}^\dagger c_l + h.c.$$

- Effective Hamiltonian along the defect line
  - Kitaev Majorana chain model (A. Kitaev, 2001)

$$H_C = g \sum_l (c_{l+1}^\dagger c_l^\dagger + c_{l+1}^\dagger c_l + h.c.) + 4 \sum_l c_l^\dagger c_l$$

  - g term: drives fermion pairing and hopping
  - Small g ($g < g_c = 2$), trivial, no zero mode
  - Large g ($g > g_c = 2$), fermion condensation
  - Majorana zero-modes: 2-fold degeneracy



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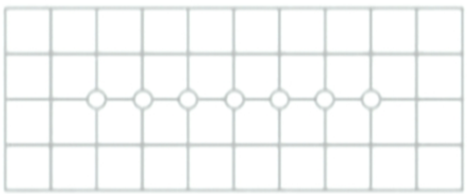
From the Other Limit



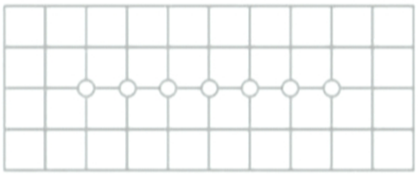
- Plaquette Model with Line Defect

$$H = -\sum_{p} O_p + g \sum_{i \in C} \sigma_i^y$$

$$\sigma^y = i\,\sigma^x\,\sigma^z = i \times = \begin{cases} -1 & \circ \\ +1 & \bullet \end{cases}$$

- Large g limit ($g \to \infty$)
  - g is like an external Zeeman field applying to the defect sites
  - $g \to \infty$: polarize the qubits to $\sigma^y = -1$ state
  - Completely quench the qubit degrees of freedom (unable to couple to the other qubits)
  - As if these sites were removed
  - For large but finite g, perturbation effect will sew up the branch-cut
  - Lattice dislocations!

From the Other Limit



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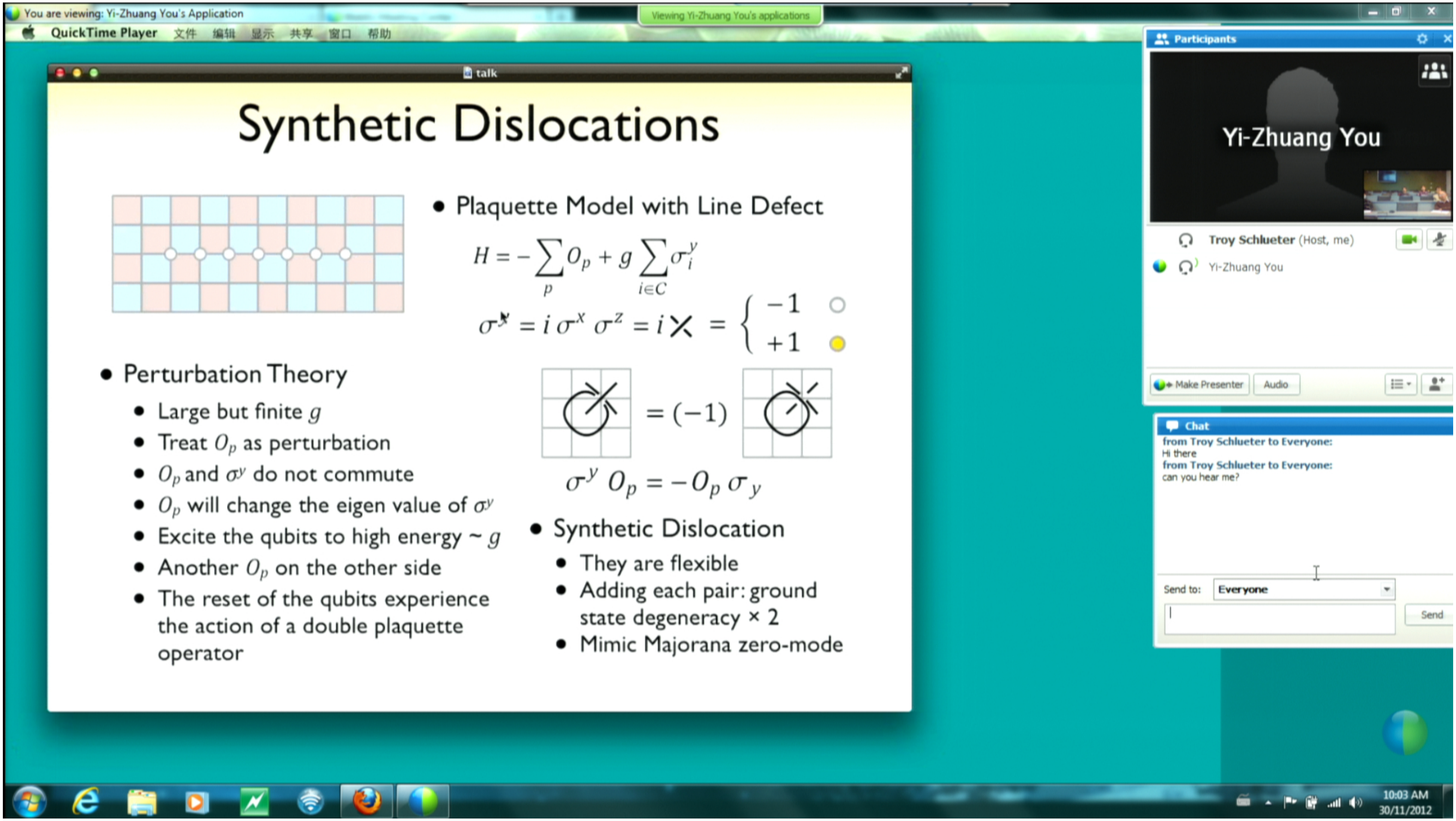
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Synthetic Dislocations
• Plaquette Model with Line Defect
$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$
$\sigma^y = i\,\sigma^x\,\sigma^z = i \times = \begin{cases} -1 \\ +1 \end{cases}$
$= (-1)$
$\sigma^y\, O_p = -O_p\, \sigma_y$
• Perturbation Theory
• Large but finite g
• Treat O_p as perturbation
• O_p and σ^y do not commute
• O_p will change the eigen value of σ^y
• Excite the qubits to high energy ~ g
• Another O_p on the other side
• The reset of the qubits experience the action of a double plaquette operator
• Synthetic Dislocation
• They are flexible
• Adding each pair: ground state degeneracy × 2
• Mimic Majorana zero-mode
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Troy Schlueter (Host, me)
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from Troy Schlueter to Everyone:
can you hear me?
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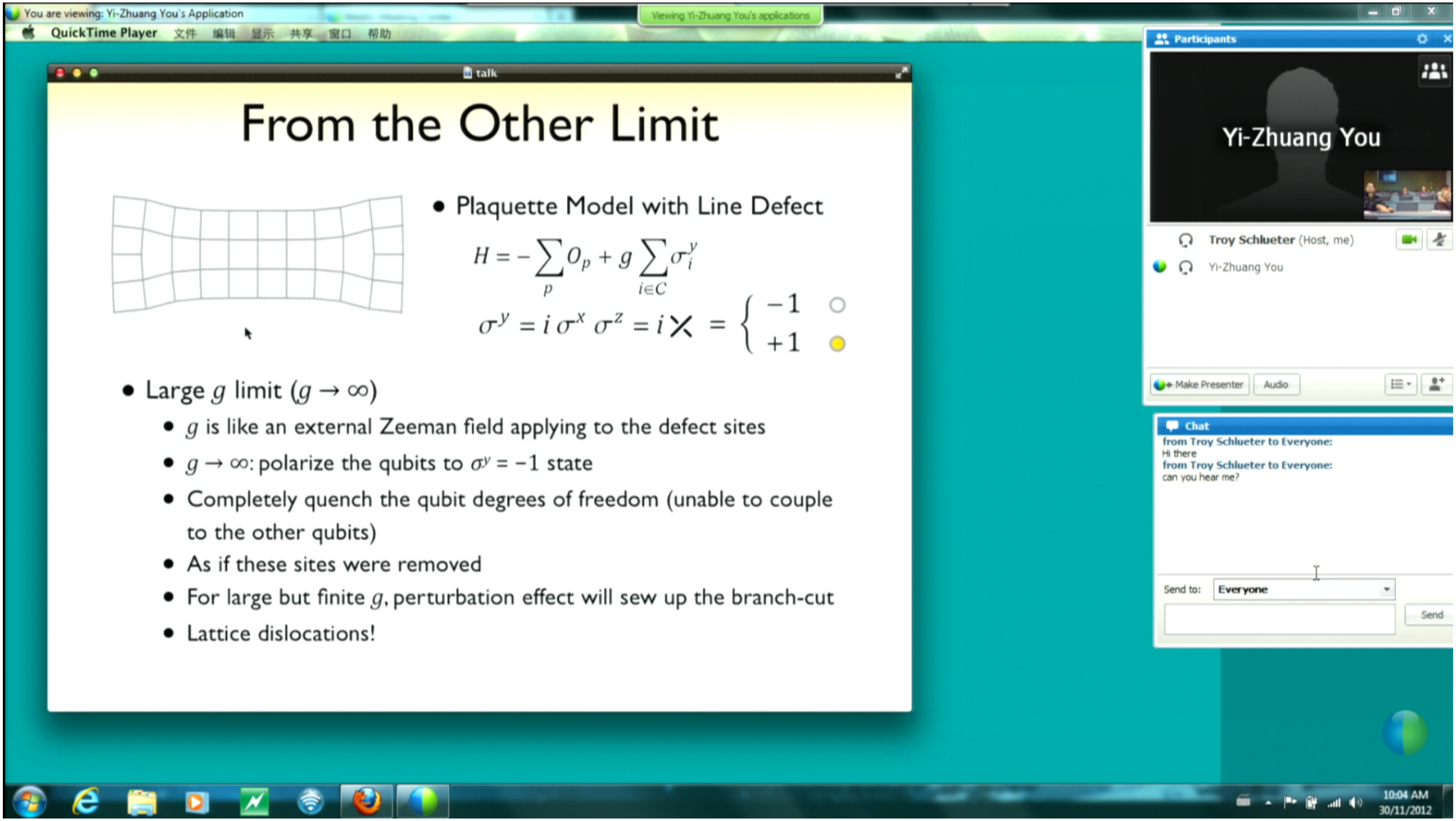
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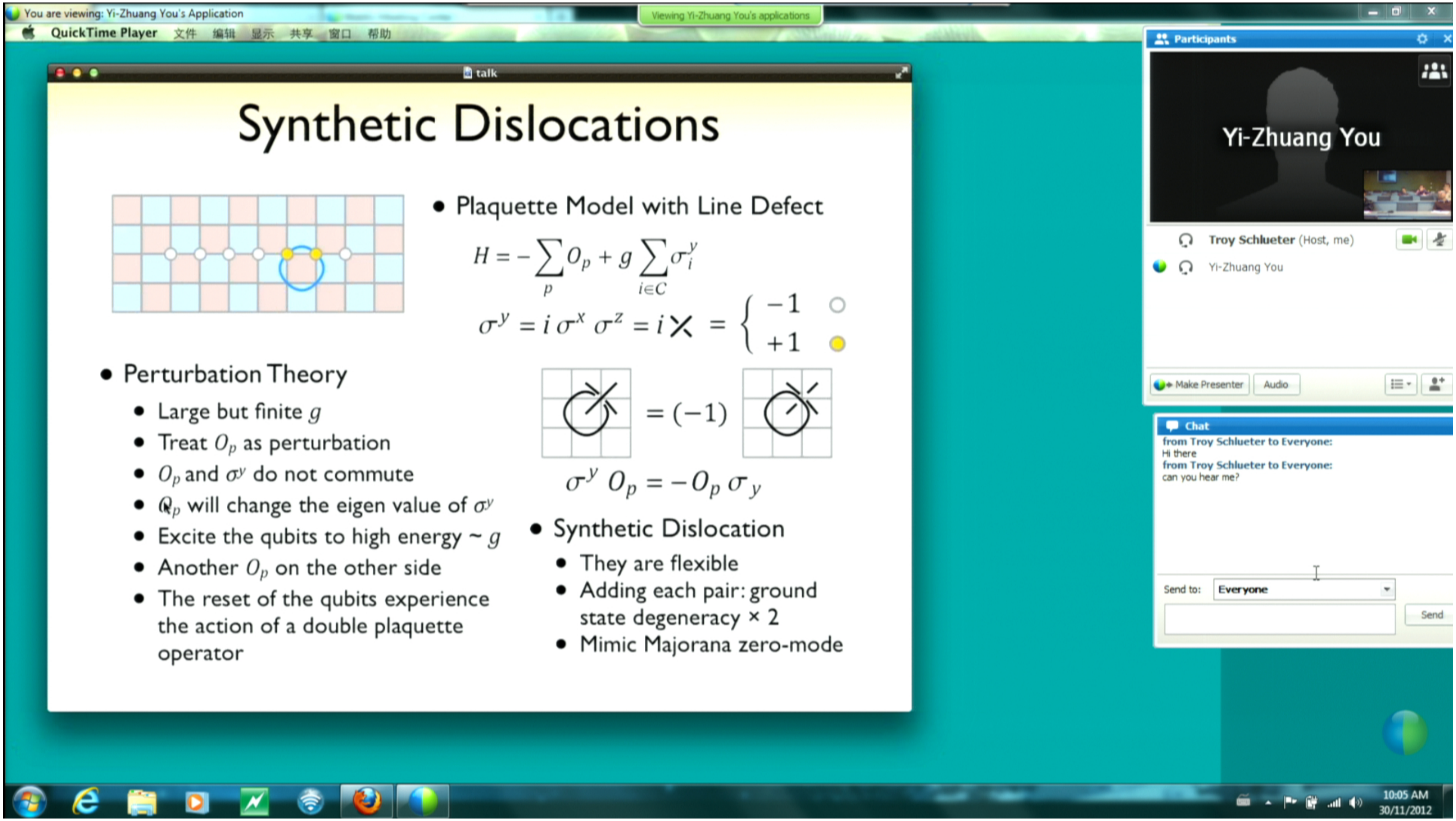
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Synthetic Dislocations
Plaquette Model with Line Defect
$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$
$\sigma^y = i\,\sigma^x\,\sigma^z = i$ ✕ $= \begin{cases} -1 \\ +1 \end{cases}$
$= (-1)$
$\sigma^y\,O_p = -O_p\,\sigma_y$
Perturbation Theory
Large but finite g
Treat O_p as perturbation
O_p and σ^y do not commute
O_p will change the eigen value of σ^y
Excite the qubits to high energy ~ g
Another O_p on the other side
The reset of the qubits experience the action of a double plaquette operator
Synthetic Dislocation
They are flexible
Adding each pair: ground state degeneracy × 2
Mimic Majorana zero-mode
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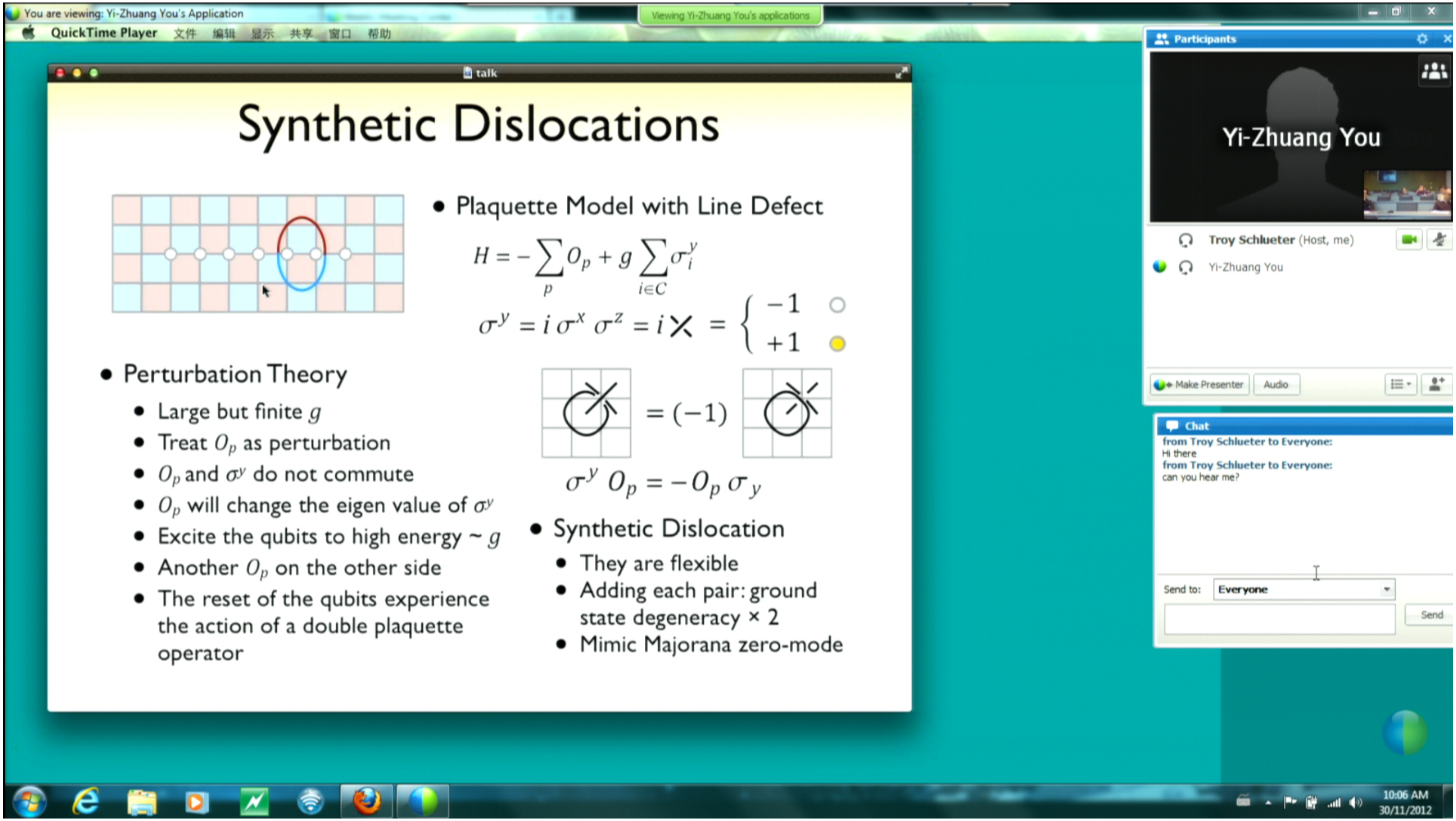
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Synthetic Dislocations
• Plaquette Model with Line Defect
$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$
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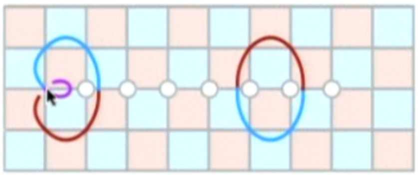
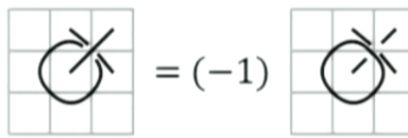
Synthetic Dislocations



- Plaquette Model with Line Defect

$$H = -\sum_p O_p + g \sum_{i \in C} \sigma_i^y$$

$$\sigma^y = i\,\sigma^x\,\sigma^z = i \times = \begin{cases} -1 & \circ \\ +1 & \bullet \end{cases}$$



$$\sigma^y\, O_p = -O_p\, \sigma_y$$

- Perturbation Theory
  - Large but finite g
  - Treat O_p as perturbation
  - O_p and σ^y do not commute
  - O_p will change the eigen value of σ^y
  - Excite the qubits to high energy ~ g
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  - They are flexible
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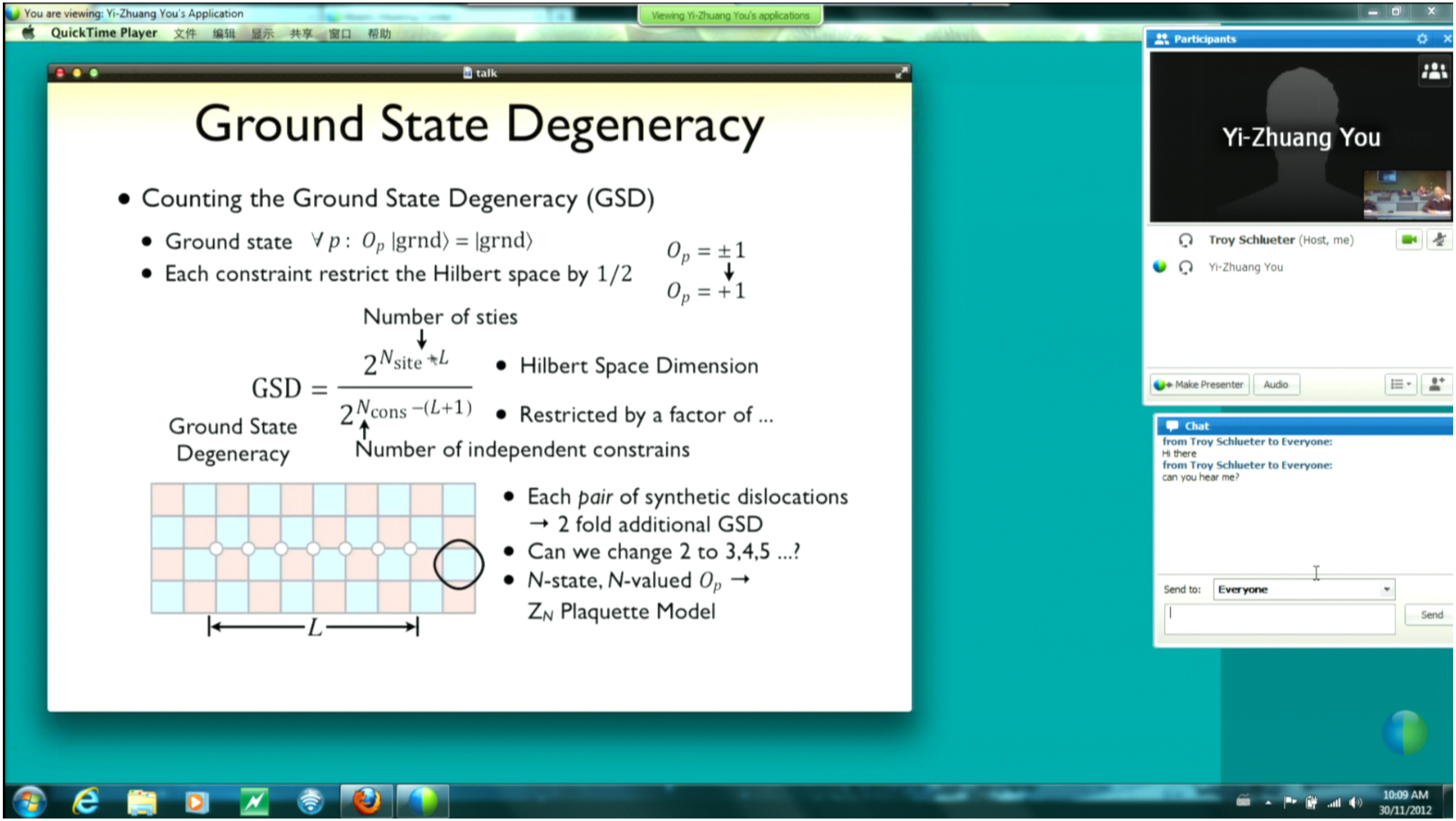
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talk
Ground State Degeneracy
• Counting the Ground State Degeneracy (GSD)
• Ground state $\forall p: O_p |\text{grnd}\rangle = |\text{grnd}\rangle$
• Each constraint restrict the Hilbert space by 1/2
$O_p = \pm 1$
$O_p = +1$
Number of sties
$\text{GSD} = \frac{2^{N_{\text{site}}+L}}{2^{N_{\text{cons}}-(L+1)}}$
• Hilbert Space Dimension
• Restricted by a factor of ...
Ground State Degeneracy
Number of independent constrains
L
• Each *pair* of synthetic dislocations → 2 fold additional GSD
• Can we change 2 to 3,4,5 ...?
• *N*-state, *N*-valued O_p → Z_N Plaquette Model
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Ground State Degeneracy

- Counting the Ground State Degeneracy (GSD)
  - Ground state $\forall p: \; O_p |\text{grnd}\rangle = |\text{grnd}\rangle$
  - Each constraint restrict the Hilbert space by $1/2$

$$O_p = \pm 1 \;\downarrow\; O_p = +1$$

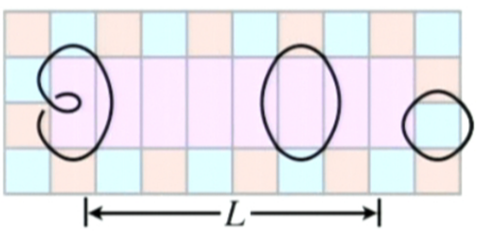
Number of sties

$$\text{GSD} = \frac{2^{N_{\text{site}} - L}}{2^{N_{\text{cons}} - (L+1)}}$$

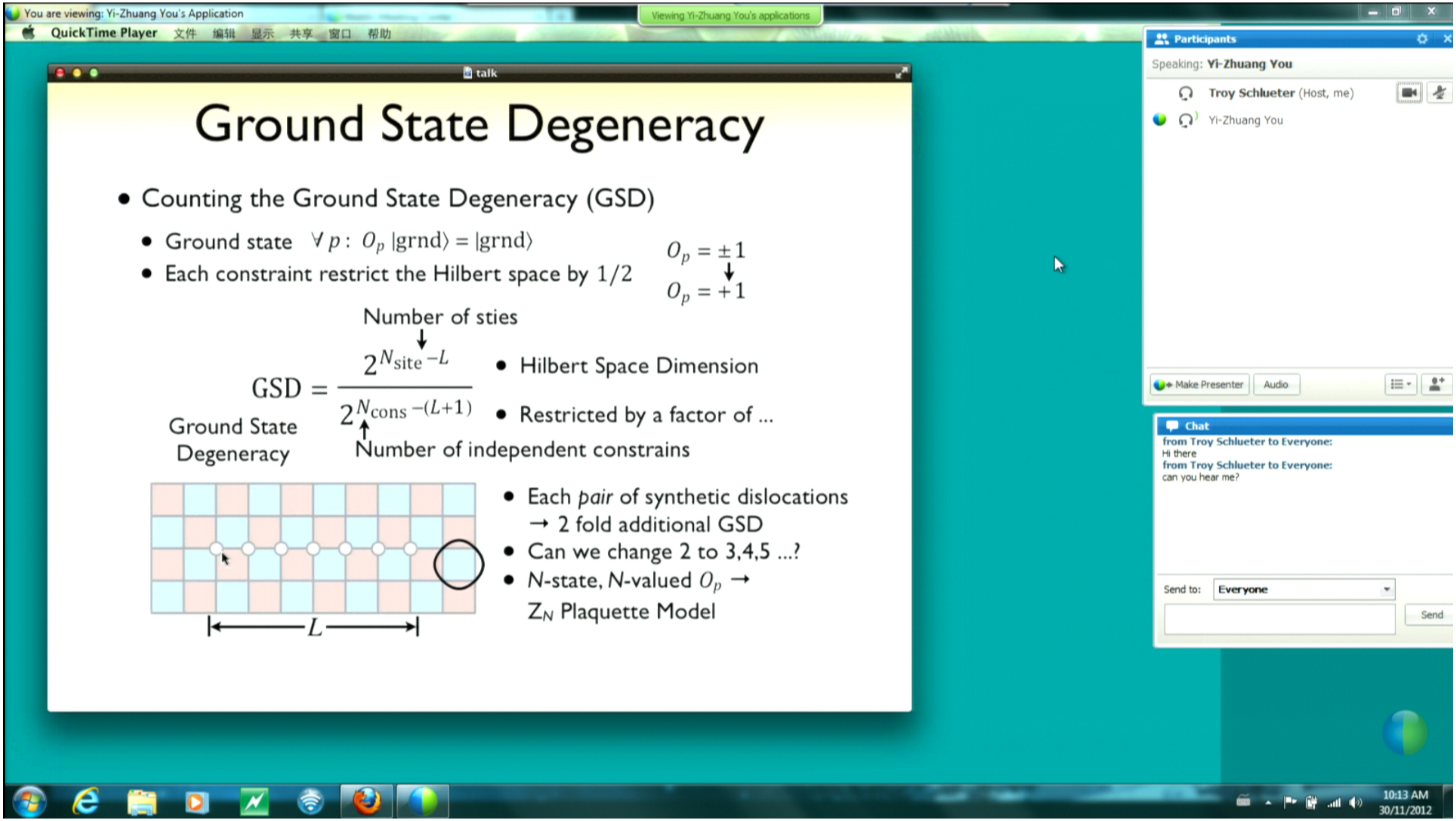
Ground State Degeneracy

Number of independent constrains

- Hilbert Space Dimension
- Restricted by a factor of ...



- Each *pair* of synthetic dislocations → 2 fold additional GSD
- Can we change 2 to 3,4,5 ...?
- *N*-state, *N*-valued O_p → Z_N Plaquette Model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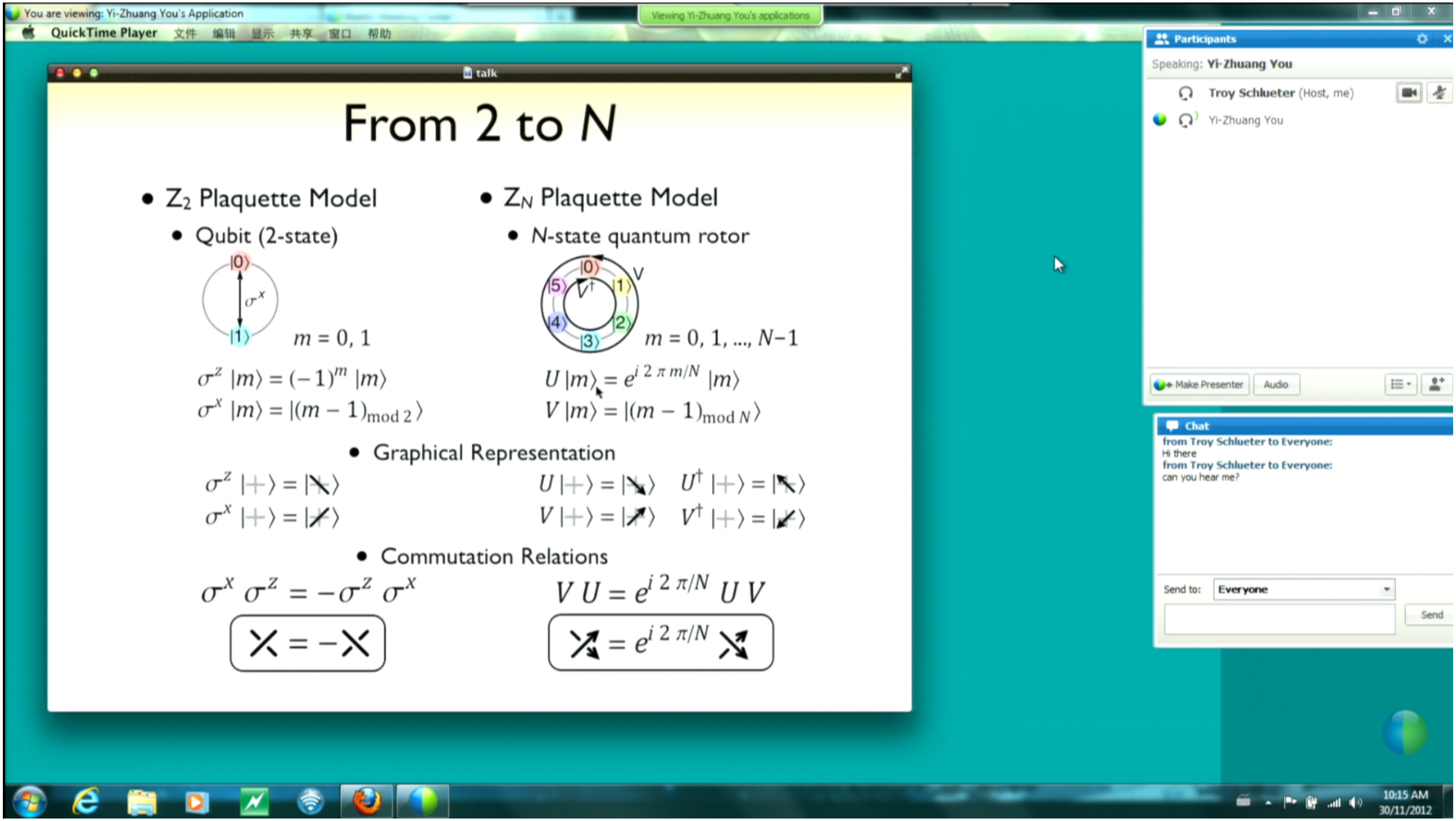
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From 2 to N
• Z_2 Plaquette Model
• Qubit (2-state)
$m = 0, 1$
$\sigma^z\, |m\rangle = (-1)^m\, |m\rangle$
$\sigma^x\, |m\rangle = |(m-1)_{\text{mod } 2}\rangle$
• Z_N Plaquette Model
• N-state quantum rotor
$m = 0, 1, \ldots, N-1$
$U\, |m\rangle = e^{i\, 2\, \pi\, m/N}\, |m\rangle$
$V\, |m\rangle = |(m-1)_{\text{mod } N}\rangle$
• Graphical Representation
• Commutation Relations
$\sigma^x\, \sigma^z = -\sigma^z\, \sigma^x$
$V\, U = e^{i\, 2\, \pi/N}\, U\, V$
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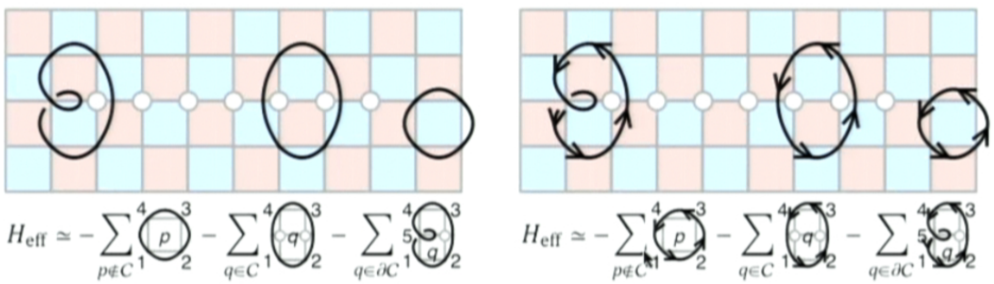
Z_N Plaquette Model

- Z_2 Plaquette Model
- Z_N Plaquette Model
  - Effective Hamiltonian



- Associated Ground State Degeneracy
  - Each pair: 2 fold

$$\mathrm{GSD} = \frac{2^{N_{\rm site}}}{2^{N_{\rm cons}}} \times 2$$

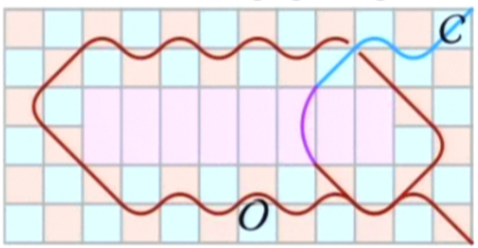
  - Each pair: N fold

$$\mathrm{GSD} = \frac{N^{N_{\rm site}}}{N^{N_{\rm cons}}} \times N$$

- Majorana Zero-Mode
  quantum dimension $\sqrt{2}$
  with non-Abelian statistics
- Beyond Majorana Zero-Mode
  higher quantum dimension $\sqrt{N}$
  with non-Abelian statistics

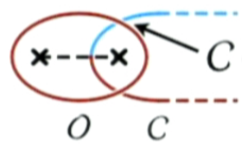
Is the Degeneracy Topological

- Majorana Chain
  - Two degenerated ground states are differed by their fermion parity
  - Add one fermion by fermion charging string C



  - Detect the fermion parity by large enclosing loop O
  - Both are closed strings (open boundary condition)

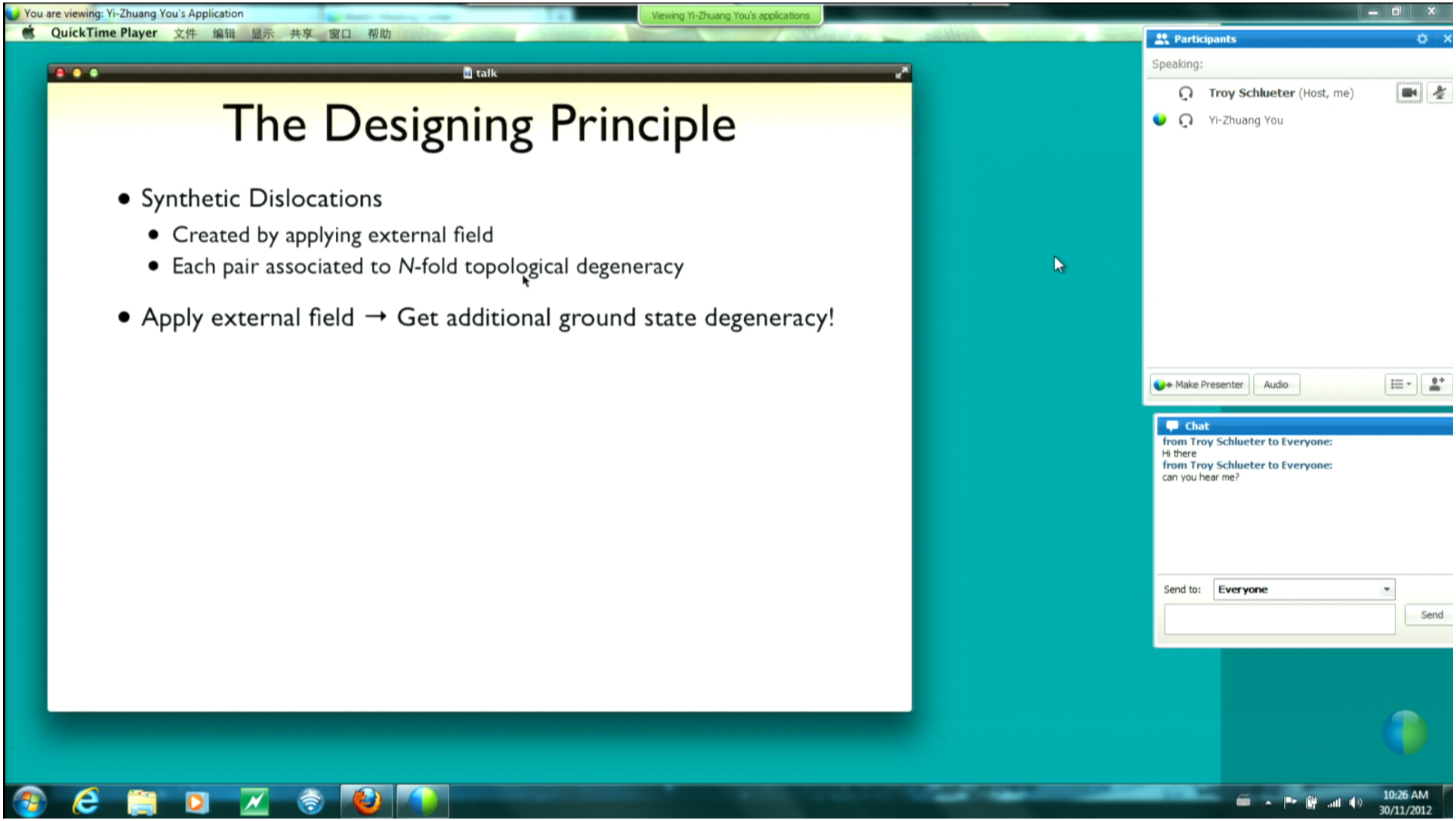
- Ground states
  - Connected by C
  - Labeled by O
  - Ground state manifold = representation space of the loop algebra



$$CO = -OC$$

$$O = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

  - 2-dim Rep. = 2 fold GSD
- Stable against any local perturbation!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The Designing Principle
• Synthetic Dislocations
• Created by applying external field
• Each pair associated to N-fold topological degeneracy
• Apply external field → Get additional ground state degeneracy!
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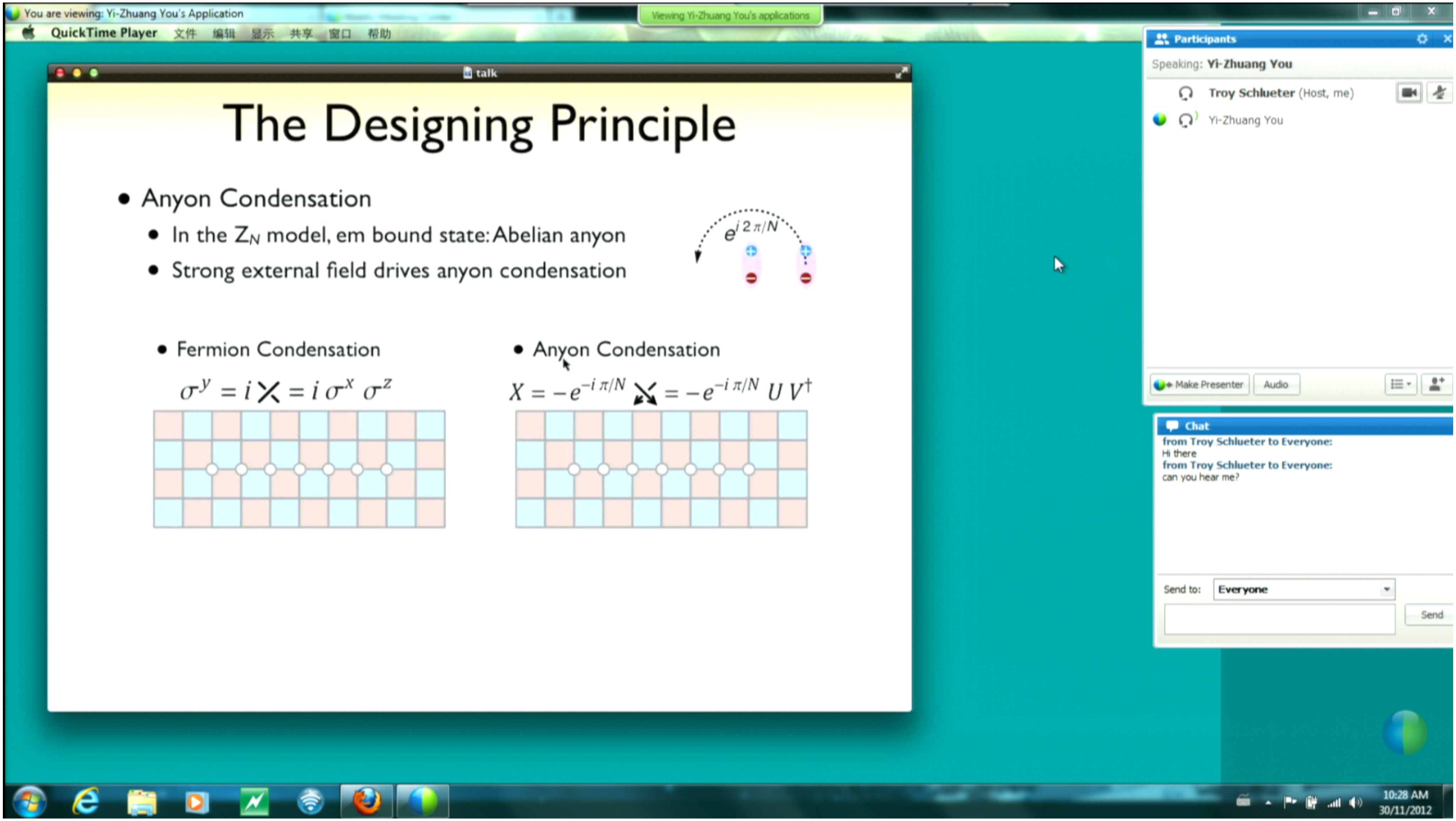
The Designing Principle
Anyon Condensation
In the Z_N model, em bound state: Abelian anyon
Strong external field drives anyon condensation
e^{i 2π/N}
Fermion Condensation
σ^y = i X = i σ^x σ^z
Anyon Condensation
X = −e^{−iπ/N} X = −e^{−iπ/N} U V†
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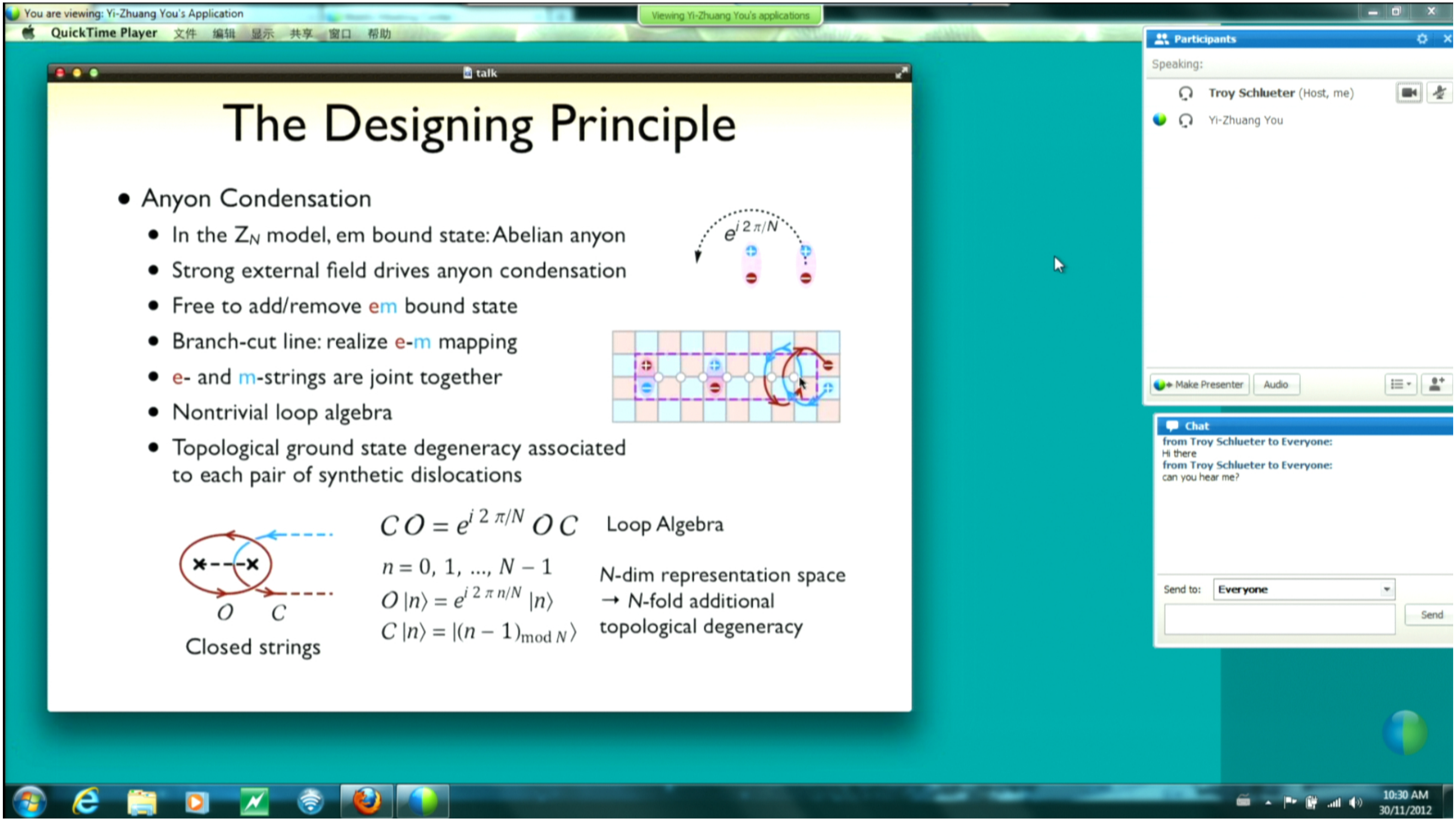
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The Designing Principle
Anyon Condensation
In the Z_N model, em bound state: Abelian anyon
Strong external field drives anyon condensation
Free to add/remove em bound state
Branch-cut line: realize e-m mapping
e- and m-strings are joint together
Nontrivial loop algebra
Topological ground state degeneracy associated to each pair of synthetic dislocations
$e^{i\,2\pi/N}$
O C
Closed strings
$C\,O = e^{i\,2\pi/N}\,O\,C$
Loop Algebra
$n = 0, 1, \ldots, N-1$
$O\,|n\rangle = e^{i\,2\pi\,n/N}\,|n\rangle$
$C\,|n\rangle = |(n-1)_{\mathrm{mod}\,N}\rangle$
N-dim representation space
→ N-fold additional topological degeneracy
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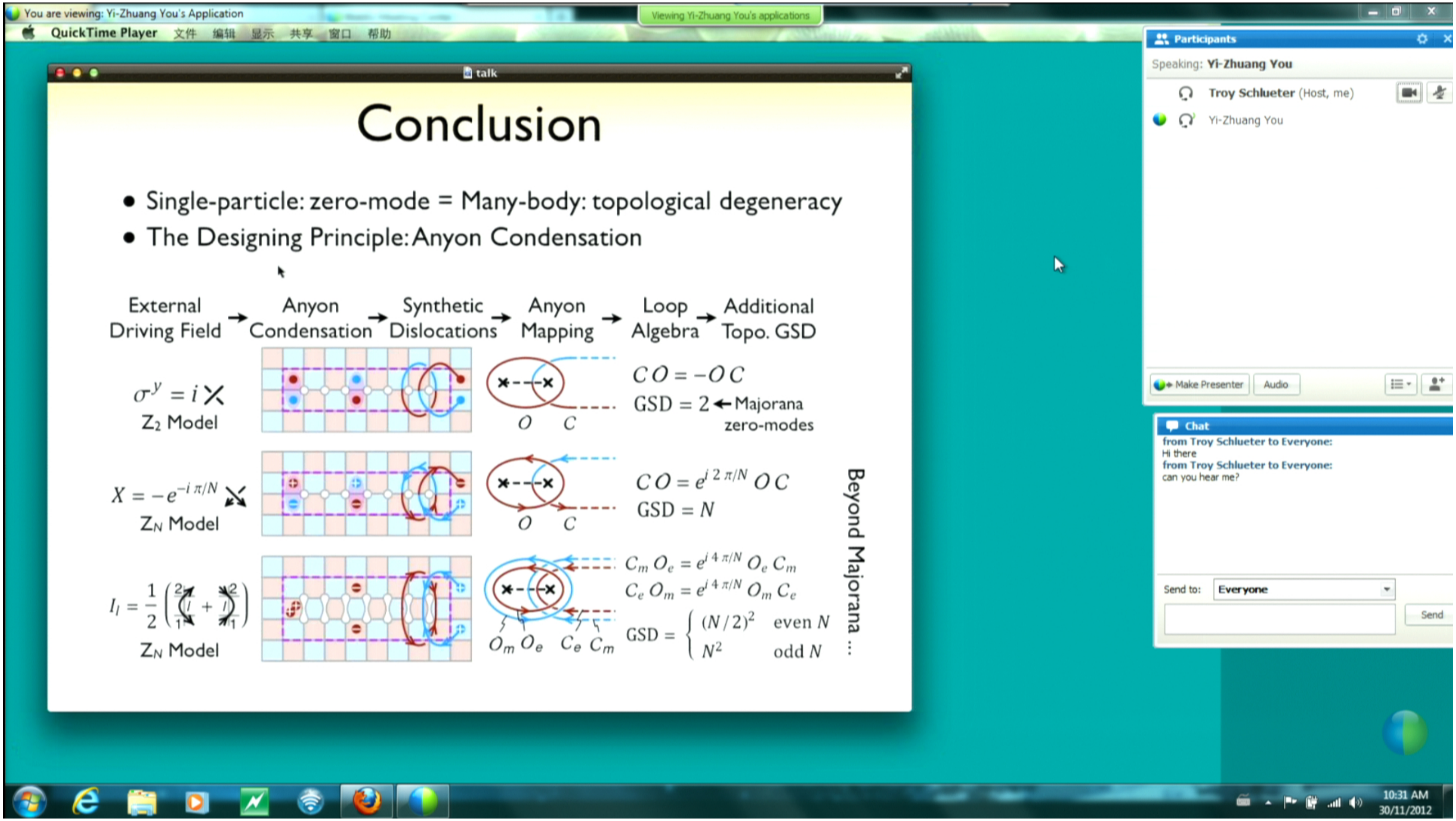
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Conclusion
Single-particle: zero-mode = Many-body: topological degeneracy
The Designing Principle: Anyon Condensation
External Driving Field → Anyon Condensation → Synthetic Dislocations → Anyon Mapping → Loop Algebra → Additional Topo. GSD
$\sigma^y = i$
Z_2 Model
$CO = -OC$
GSD = 2 ← Majorana zero-modes
$X = -e^{-i\pi/N}$
Z_N Model
$CO = e^{i2\pi/N}OC$
GSD = N
$I_l = \frac{1}{2}(\ldots + \ldots)$
Z_N Model
$C_m O_e = e^{i4\pi/N} O_e C_m$
$C_e O_m = e^{i4\pi/N} O_m C_e$
GSD = $(N/2)^2$ even N; N^2 odd N
O C
O_m O_e C_e C_m
Beyond Majorana ...
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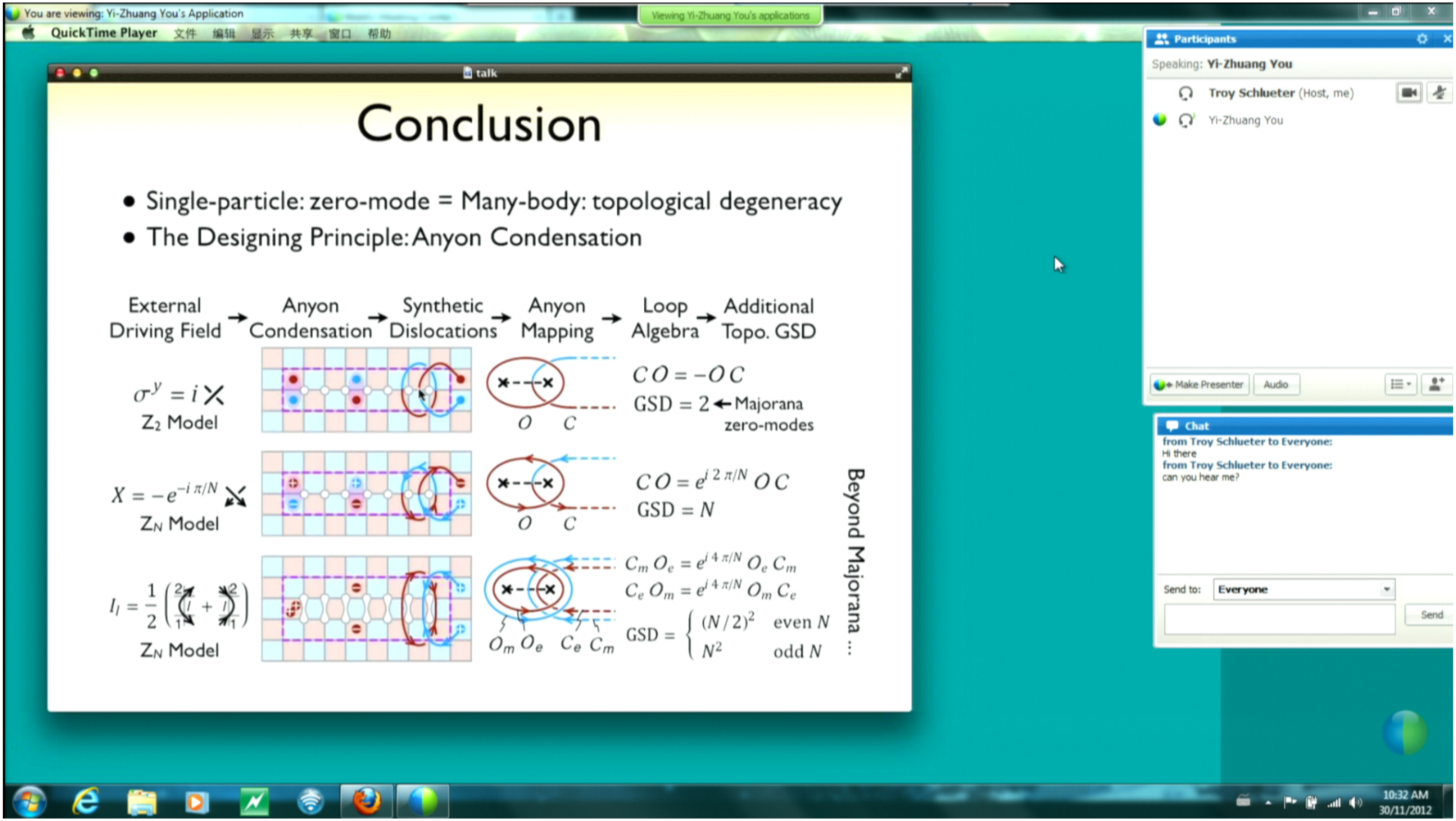
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Conclusion
• Single-particle: zero-mode = Many-body: topological degeneracy
• The Designing Principle: Anyon Condensation
External Driving Field → Anyon Condensation → Synthetic Dislocations → Anyon Mapping → Loop Algebra → Additional Topo. GSD
$\sigma^y = i$
Z2 Model
$X = -e^{-i\pi/N}$
ZN Model
$I_l = \frac{1}{2}$
ZN Model
$C\,O = -O\,C$
GSD = 2 ← Majorana zero-modes
$C\,O = e^{i\,2\pi/N}\,O\,C$
GSD = N
$C_m\,O_e = e^{i\,4\pi/N}\,O_e\,C_m$
$C_e\,O_m = e^{i\,4\pi/N}\,O_m\,C_e$
$\mathrm{GSD} = \begin{cases} (N/2)^2 & \text{even } N \\ N^2 & \text{odd } N \end{cases}$
Beyond Majorana ...
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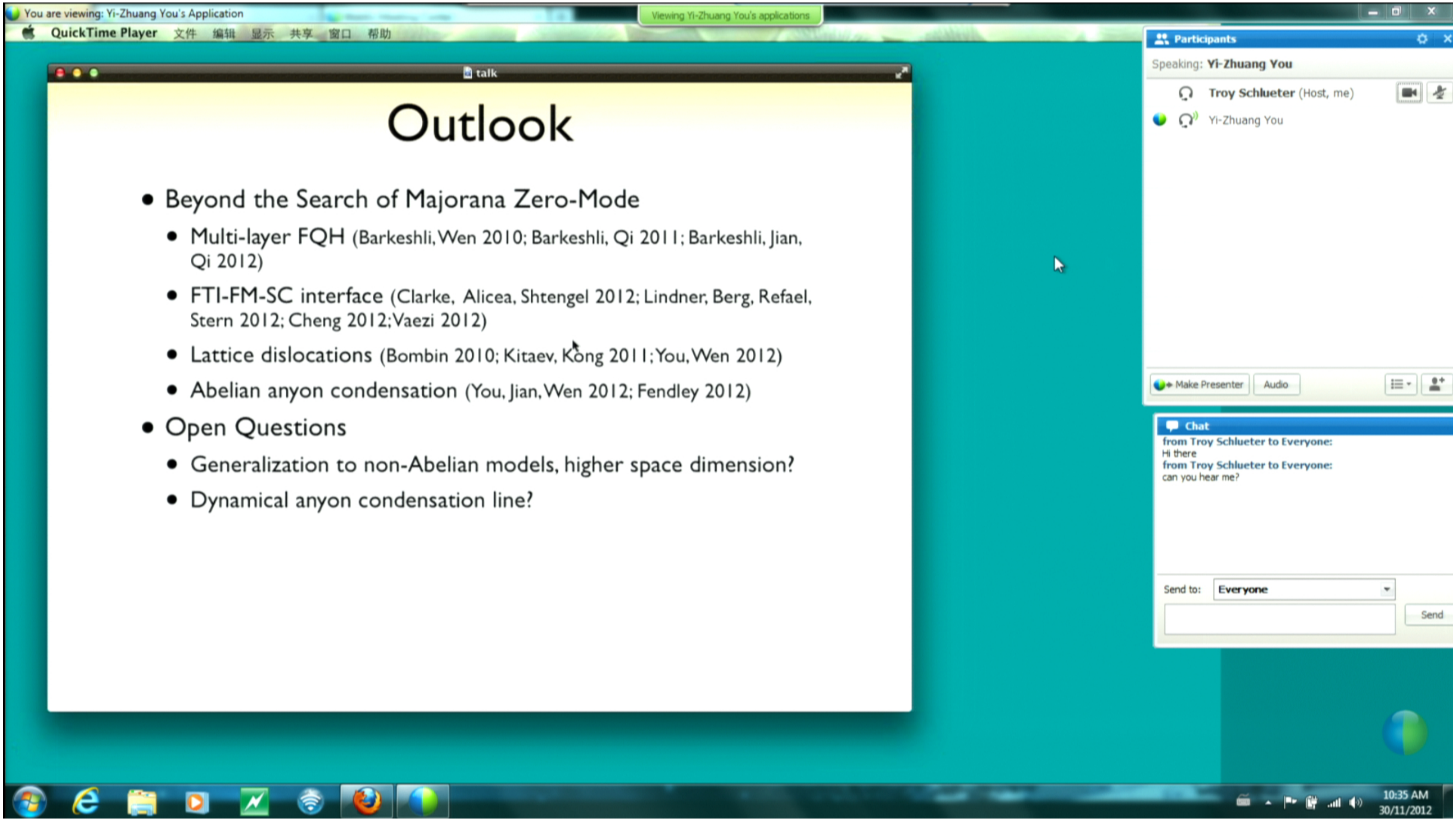
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Outlook
Beyond the Search of Majorana Zero-Mode
Multi-layer FQH (Barkeshli, Wen 2010; Barkeshli, Qi 2011; Barkeshli, Jian, Qi 2012)
FTI-FM-SC interface (Clarke, Alicea, Shtengel 2012; Lindner, Berg, Refael, Stern 2012; Cheng 2012; Vaezi 2012)
Lattice dislocations (Bombin 2010; Kitaev, Kong 2011; You, Wen 2012)
Abelian anyon condensation (You, Jian, Wen 2012; Fendley 2012)
Open Questions
Generalization to non-Abelian models, higher space dimension?
Dynamical anyon condensation line?
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Outlook

- Beyond the Search of Majorana Zero-Mode
  - Multi-layer FQH (Barkeshli, Wen 2010; Barkeshli, Qi 2011; Barkeshli, Jian, Qi 2012)
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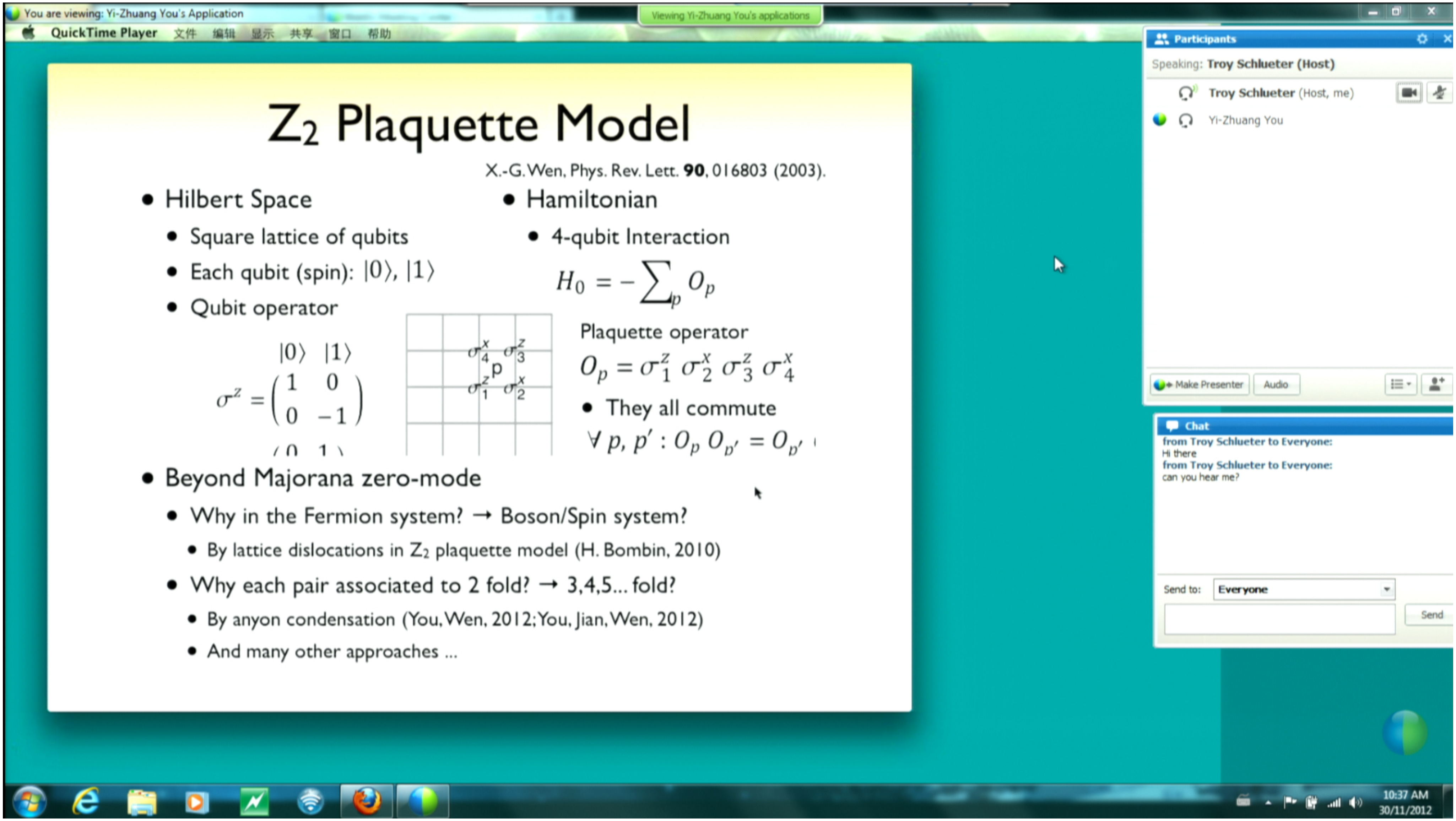
- Open Questions
  - Generalization to non-Abelian models, higher space dimension?
  - Dynamical anyon condensation line?

Thanks for your attention!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Z2 Plaquette Model
X.-G. Wen, Phys. Rev. Lett. 90, 016803 (2003).
Hilbert Space
Square lattice of qubits
Each qubit (spin): |0⟩, |1⟩
Qubit operator
|0⟩ |1⟩
$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Hamiltonian
4-qubit Interaction
$H_0 = -\sum_p O_p$
Plaquette operator
$O_p = \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$
They all commute
$\forall p, p' : O_p O_{p'} = O_{p'}$
Beyond Majorana zero-mode
Why in the Fermion system? → Boson/Spin system?
By lattice dislocations in Z2 plaquette model (H. Bombin, 2010)
Why each pair associated to 2 fold? → 3,4,5... fold?
By anyon condensation (You, Wen, 2012; You, Jian, Wen, 2012)
And many other approaches ...
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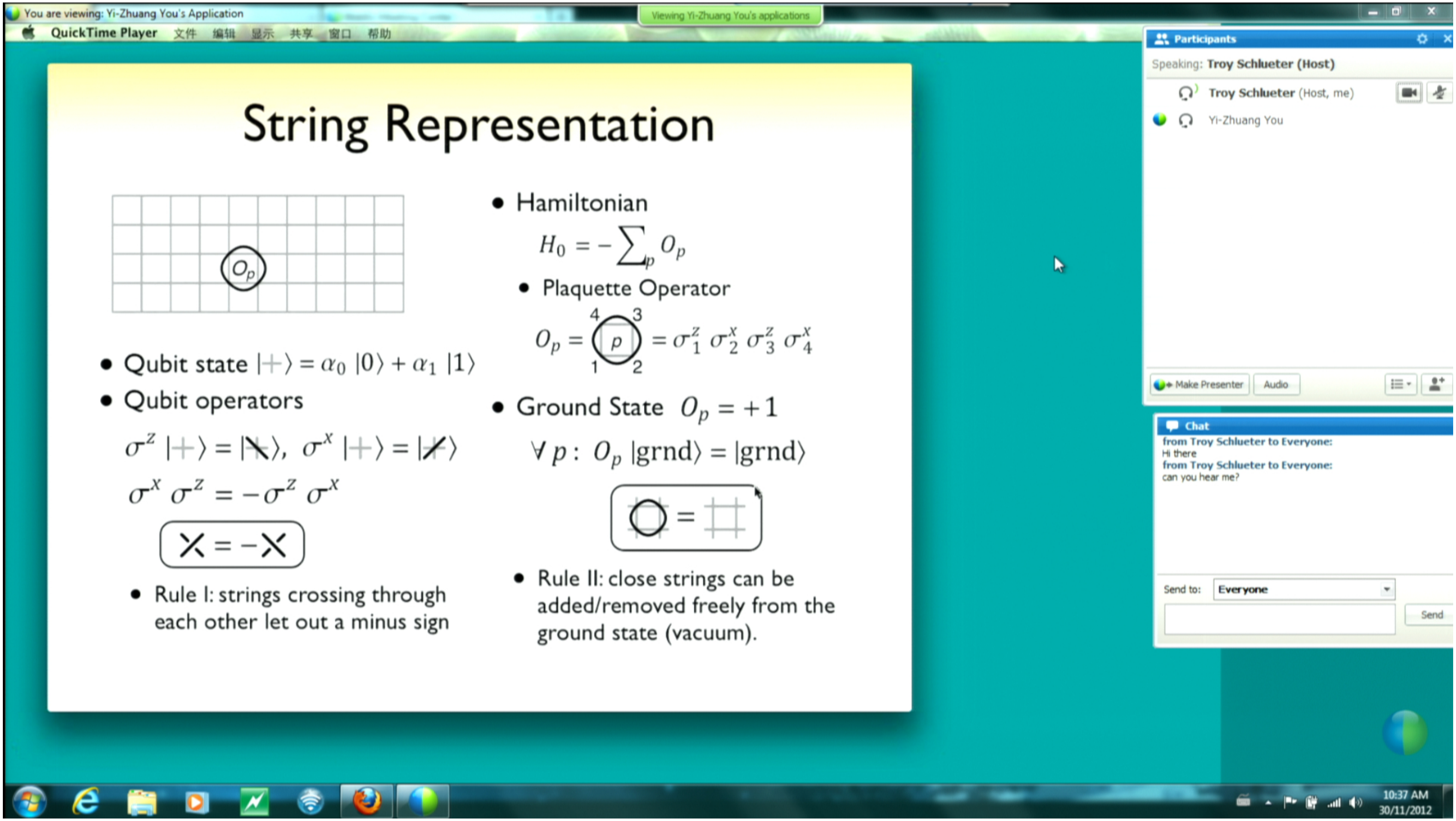
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String Representation
O_p
• Hamiltonian
$H_0 = -\sum_p O_p$
• Plaquette Operator
$O_p = $ (p) $= \sigma_1^z \sigma_2^x \sigma_3^z \sigma_4^x$
1 2 3 4
• Qubit state $|+\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$
• Qubit operators
$\sigma^z |+\rangle = |\diagdown\rangle, \ \sigma^x |+\rangle = |\diagup\rangle$
$\sigma^x \sigma^z = -\sigma^z \sigma^x$
• Rule I: strings crossing through each other let out a minus sign
• Ground State $O_p = +1$
$\forall p: \ O_p |\text{grnd}\rangle = |\text{grnd}\rangle$
• Rule II: close strings can be added/removed freely from the ground state (vacuum).
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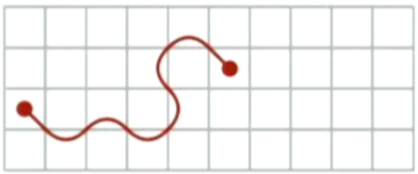
Excitations

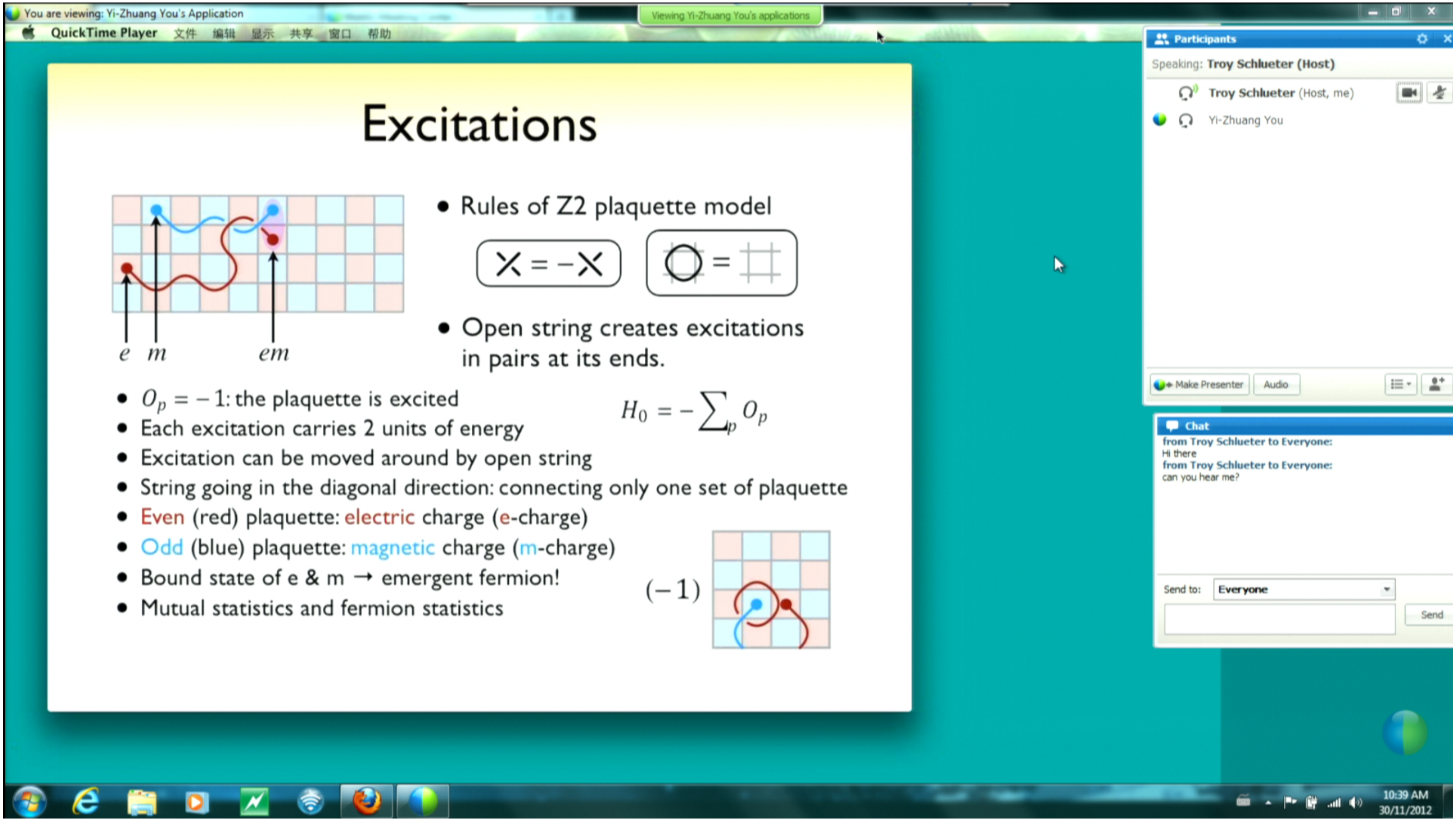


- Rules of Z2 plaquette model

- Open string creates excitations in pairs at its ends.

$$H_0 = -\sum_p O_p$$

- $O_p = -1$: the plaquette is excited
- Each excitation carries 2 units of energy
- Excitation can be moved around by open string
- String going in the diagonal direction: connecting only one set of plaquette
- Even (red) plaquette: electric charge (e-charge)
- Odd (blue) plaquette: magnetic charge (m-charge)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Each excitation carries 2 units of energy
Excitation can be moved around by open string
String going in the diagonal direction: connecting only one set of plaquette
Even (red) plaquette: electric charge (e-charge)
Odd (blue) plaquette: magnetic charge (m-charge)
Bound state of e & m → emergent fermion!
Mutual statistics and fermion statistics
$H_0 = -\sum_p O_p$
(-1)
Participants
Speaking: Troy Schlueter (Host)
Troy Schlueter (Host, me)
Yi-Zhuang You
Make Presenter
Audio
Chat
from Troy Schlueter to Everyone:
Hi there
from Troy Schlueter to Everyone:
can you hear me?
Send to: Everyone
Send
10:39 AM
30/11/2012