Title: Dyon condensation in topological Mott insulators

Date: Nov 28, 2012 03:30 PM

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Abstract: We consider quantum phase transitions out of topological Mott insulators in which the ground state of the fractionalized excitations (fermionic spinons) is topologically non-trivial. The spinons in topological Mott insulators are coupled to an emergent compact U(1) gauge field with a so-called "axion" term. We study the confinement transitions from the topological Mott insulator to broken symmetry phases, which may occur via the condensation of dyons. Dyons carry both "electric" and "magnetic" charges, and arise naturally in this system because the monopoles of the emergent U(1) gauge theory acquires gauge charge due to the axion term. It is shown that the dyon condensate, in general, induces simultaneous current and bond orders. When the magnetic transition is driven by dyon condensation, we identify the bond order as valence bond solid order and the current order as scalar spin chirality order. Hence, the confined phase of the topological Mott insulator is an exotic phase where the scalar spin chirality and the valence bond order coexist and appear via a single transition.

If time allows, I will also discuss our recent work on the proximate symmetry-broken phases of Z2 spin liquid on Kagome lattice.

Perimeter Ins (Nov 28th Wed)

DYON CONDENSATION IN TOPOLOGICAL MOTT INSULATORS

Arxiv: 1203.4593, *in press* (*New Journal of Physics*)

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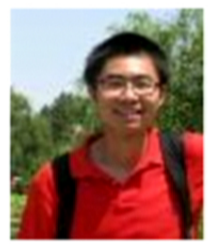
PLAN.

Part 1 : Dyon condensation in TMI

- Topological insulators
- Axion response and Dyon
- Dyon condensation

If time allows....

- Part 2 : Proximate phases of Z2 spin liquid on Kagome lattice (GYC, YML and AV, *in preparation*)
  - Z2 spin liquid in Kagome lattice
  - Proximate phases of Z2 spin liquid



Yuan-Ming Lu

Ashvin Vishwanath

PART1.

I. Topological Insulators

TOPOLOGICAL ORDER

(Ref. X.G Wen, Quantum field theory of many body physics)

Topological phase = gapped

+ no (local) order parameter
+ not a trivial insulator
+ gapless edge/surface state

Example ? Quantum Hall states, topological insulator

"Topological insulator" is well defined
only when time-reversal symmetry is present.

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Intrinsic topological order vs symmetry protected topological order ?

(Ref. Chen, Gu, and Wen 2011)

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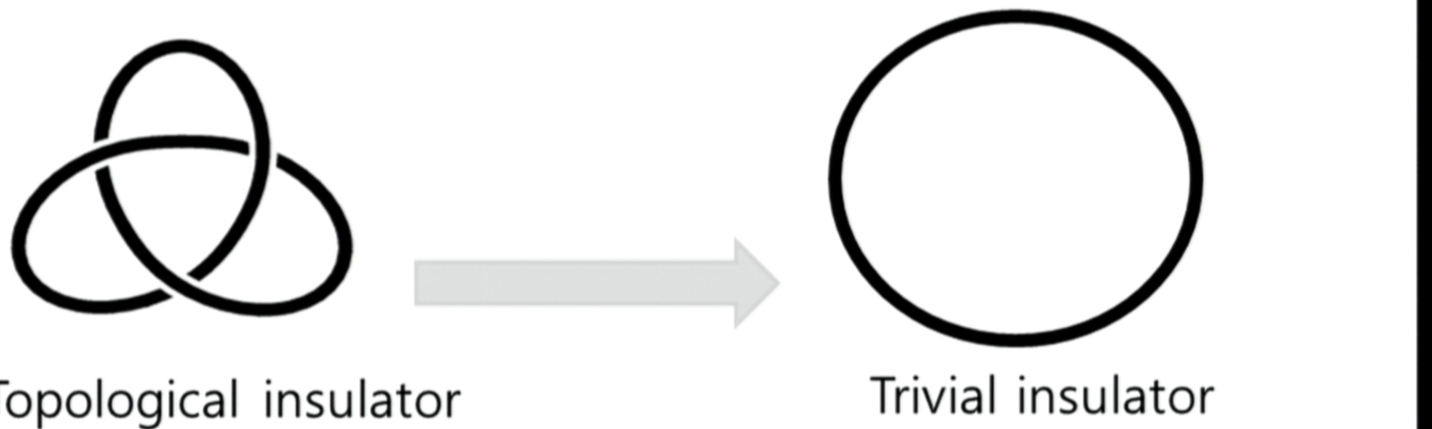
TOPOLOGICAL INSULATOR

= **Time-reversal symmetric ``band" insulator**

+ Z2 classification from the `band topology'



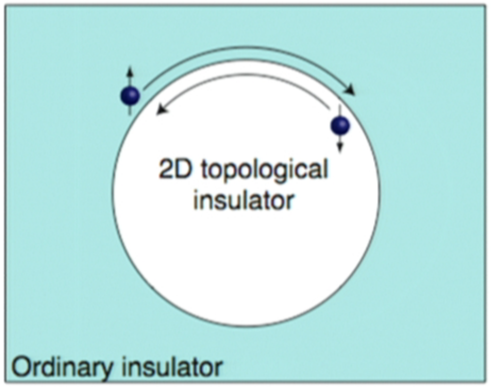
Gapless-ness of edge state is protected by time-reversal symmetry

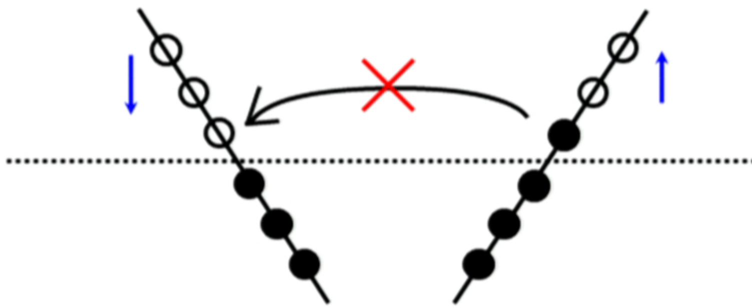
Ref. Kane and Mele (2005), Fu and Kane (2006), Fu, Kane and Mele (2006)
Moore and Balents (2006), Roy (2006), Qi, Hughes, and Zhang ...

Reviews: Hasan and Kane (2010), Hasan and Moore (2011), Qi and Zhang (2011)

2D TOPOLOGICAL INSULATOR

2D topological insulator ≅ two copies of integer QHE





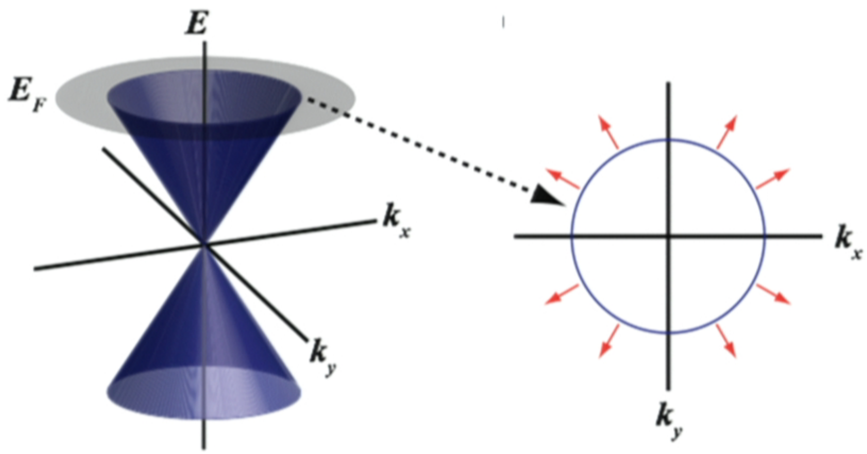
- Z2 - index ≅ integration of Berry curvature in (half) BZ

Ref. Fu and Kane (2006), Essin and Moore (2007)

3D TOPOLOGICAL INSULATOR

Similar Z2 -indices! ≅ single Dirac cone for the non-trivial TI

Gapless surface state protected by time-reversal symmetry!



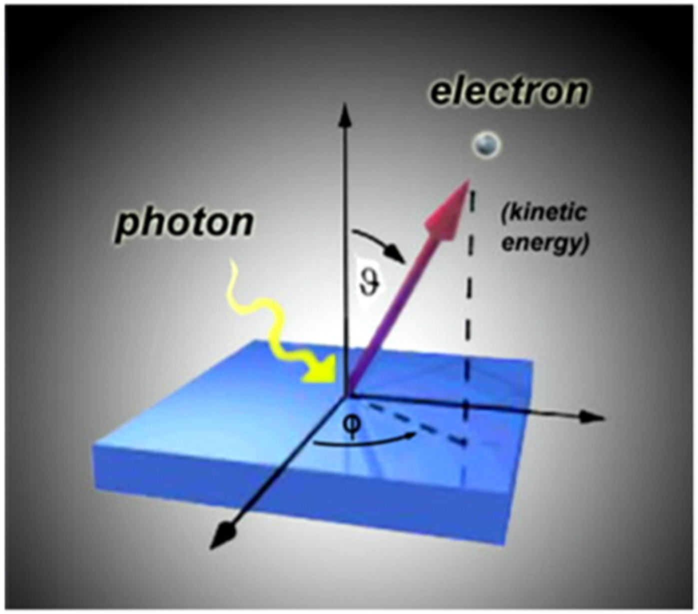
Topological insulator

Z2 -index? Even/odd-ness of the number of Dirac fermions

Ref. Fu, Kane, and Mele (2006)

3D TOPOLOGICAL INSULATOR

Experiment on a topological insulator Bi2Se3 (Hasan's group at Princeton)



What happens if the time-reversal symmetry
is broken (only) on the surface ? Axion !

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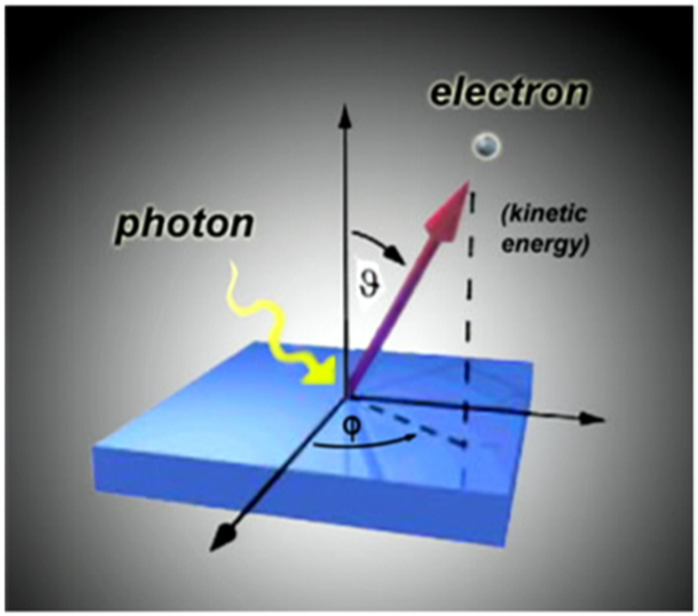
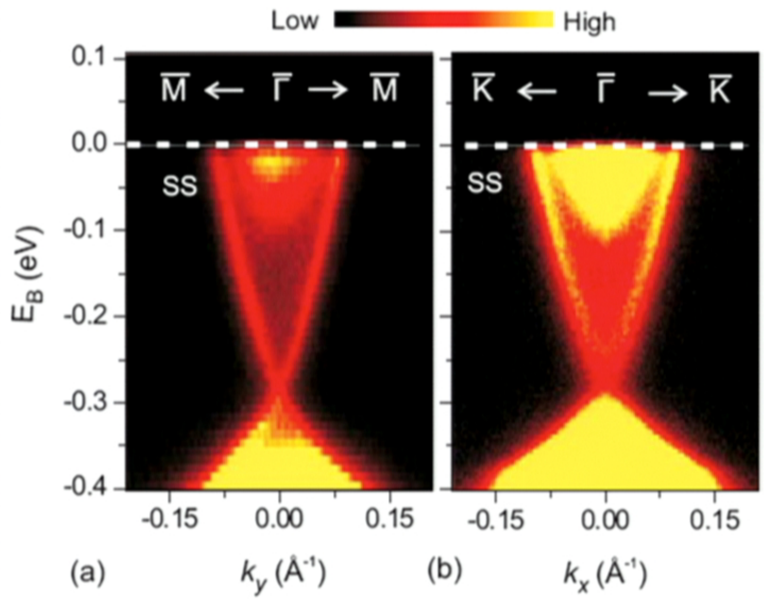




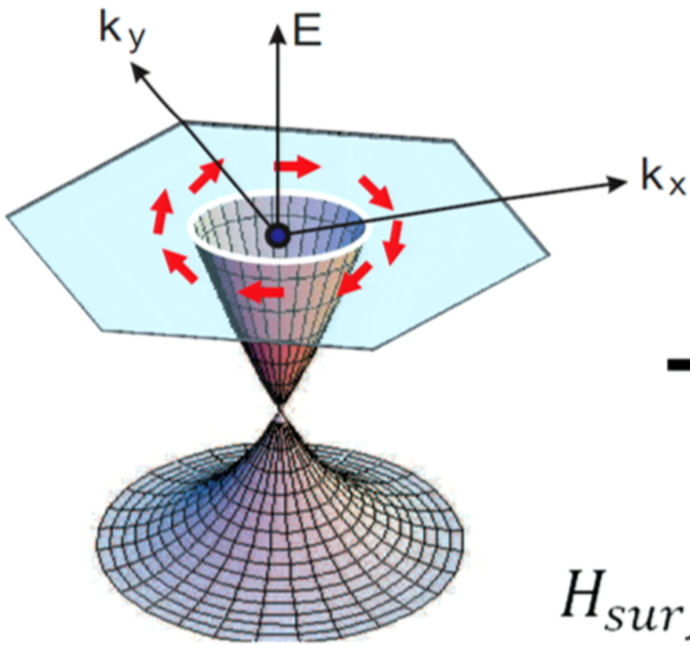
What happens if the time-reversal symmetry is broken (only) on the surface ? Axion !

II. AXION RESPONSE AND DYON

???

GAPPED SURFACE STATE OF TOPOLOGICAL INSULATOR

A. Dirac fermion + magnetization



+

$$H_{surf} = v\,\hat{z}\cdot\vec{\sigma}\times\vec{k} + M\sigma^z$$

$$E = \sqrt{v^2k^2 + M^2}$$

GAPPED SURFACE STATE OF TOPOLOGICAL INSULATOR

B. Half of integer quantum Hall effect Ref. Qi, Hughes, and Zhang (2008)

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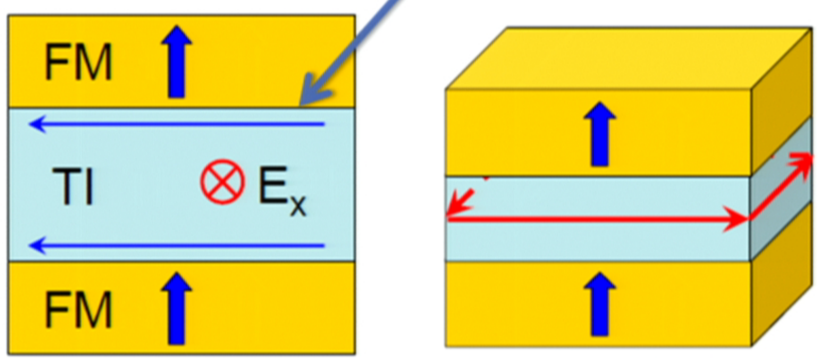
$$\sigma_{xy} = \frac{e^2}{2h}$$



HOW TO CAPTURE THE SURFACE HALL EFFECT?

Gapped surface state ≅ axion response

Axion

$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}.$$

Ref. Fu, Kane, and Mele (2006)
Qi, Hughes, and Zhang (2008)
Essin and Moore (2007)

Surface Quantum Hall effect?

$$\Delta L_{EM} = \frac{\theta}{32\pi^2}\int_M \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} = \frac{\theta}{8\pi^2}\int_M dA\wedge dA$$

$$= \frac{\theta}{8\pi^2}\int_M d(A\wedge dA) = \frac{\theta}{8\pi^2}\oint_{\partial M} A\wedge dA$$

AXION ELECTROMAGNETISM

A. to be consistent with the half of integer quantum Hall effect

$$\Delta L_{EM} = \frac{\theta}{8\pi^2}\oint_{\partial M} A \wedge dA \quad \Longleftrightarrow \quad \sigma_{xy} = \frac{1}{4\pi}\left(= \frac{e^2}{2h}\right)$$

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$\Rightarrow$ We obtain $\theta = \pi$ for Top. Ins.

B. Or we observe

1. Periodic: $\theta \sim \theta + 2\pi$
2. $T: \theta \rightarrow -\theta$

Hence, $\theta = 0$ or $\theta = \pi$

Axion, Dyon, and Witten effect

$$\Delta L_{EM} = \frac{\theta}{32\pi^2}\int_M \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} = \frac{\theta}{8\pi^2}\int_M dA \wedge dA$$

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Monopole's electric charge?

Ref. E Witten (1979), F Wilczek (1982), Goldhaber (1982) t'Hooft (1981), Cardy (1982)

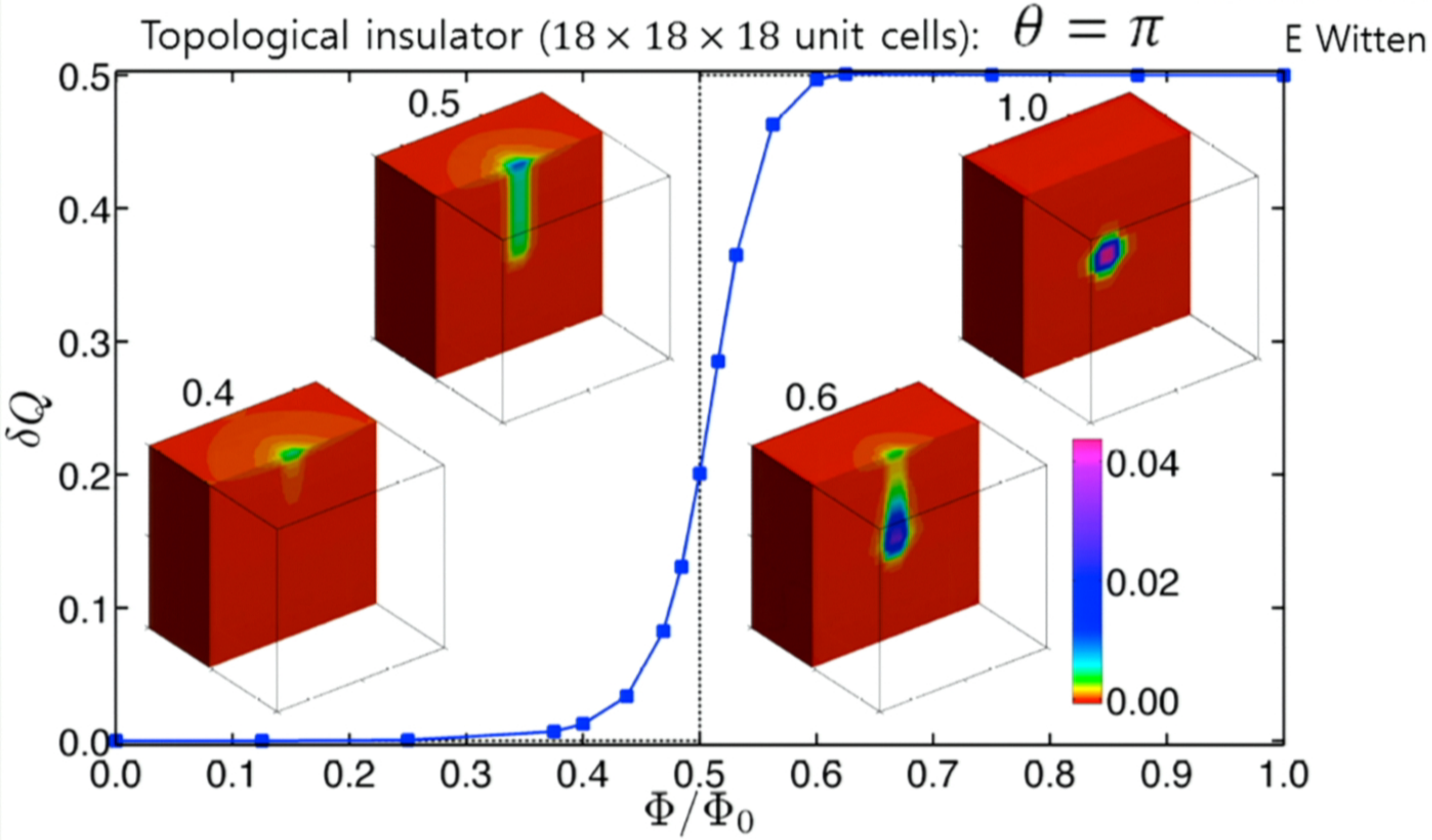
$$Q = \frac{\delta \Delta L_{EM}}{\delta A_0} = \frac{\theta}{4\pi^2} \nabla \cdot \nabla \times \vec{A} = \frac{\theta}{4\pi^2} \nabla \cdot \vec{B}$$

$$= \frac{\theta}{4\pi^2}\rho_m$$ and ρ_m = monopole density

WITTEN EFFECT AND DYON

Rosenberg, Guo, and Franz (2010)

E Witten



WITTEN EFFECT AND DYON

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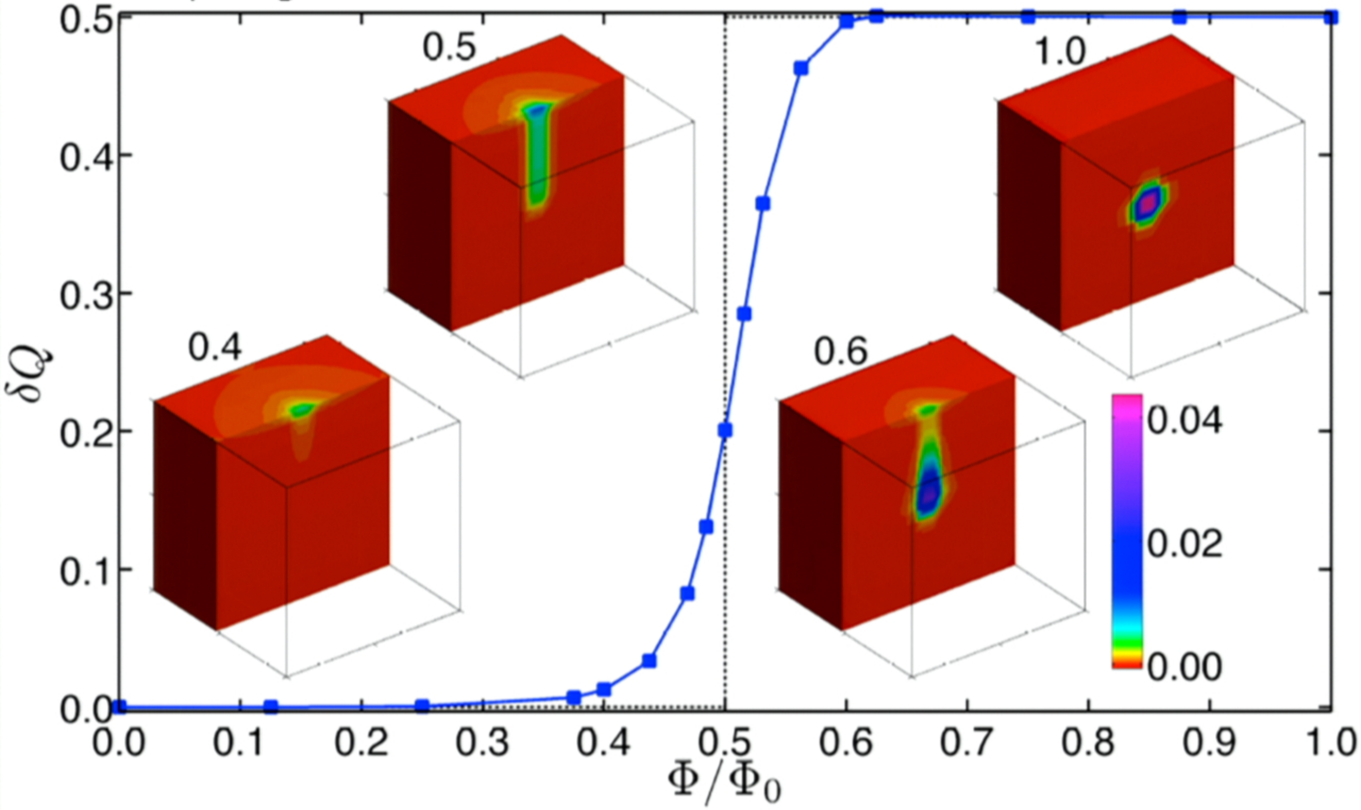
Topological insulator ($18 \times 18 \times 18$ unit cells): $\theta = \pi$



Can we find interesting physics with Dyon & axion ?

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(1) Axion term = a monopole is turned into a Dyon

(2) Dyon = monopole + electric charge ≠ monopole

HEP example: Oblique confinement

Ref. t'Hooft (1981), Cardy (1982), Cardy and Rabinovici (1982), *etc*

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Consider "Dyon condensed" phases

(different from monopole condensed phase)

to demonstrate non-trivial physics induced by axion term

Dyon condensation?

A. Monopole in the gauge theory

= Need a lattice U(1) gauge theory

B. Axion electromagnetism

= Topological Band Structure

= Topological Insulator

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Topological Mott Ins.

A. fractionalized phase

= U(1) spin liquid

= U(1) gauge theory

B. fermionic spinon

in topological band

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Topological Mott Ins.

A. fractionalized phase

= U(1) spin liquid

= U(1) gauge theory

B. fermionic spinon

in topological band

→ Dyon will appear in topological Mott insulator!

TOPOLOGICAL MOTT INSULATOR

Fermionic spinon in the topological band structure

Emergent compact $U(1)$ gauge theory due to fractionalization

e.g. Pesin and Balents (2009)

$$d_{Ri\alpha} = e^{-i\theta_{Ri}} f_{Ri\alpha}$$

See also:
S Bhattacharjee, YB Kim, SS Lee and DH Lee *(2012)*

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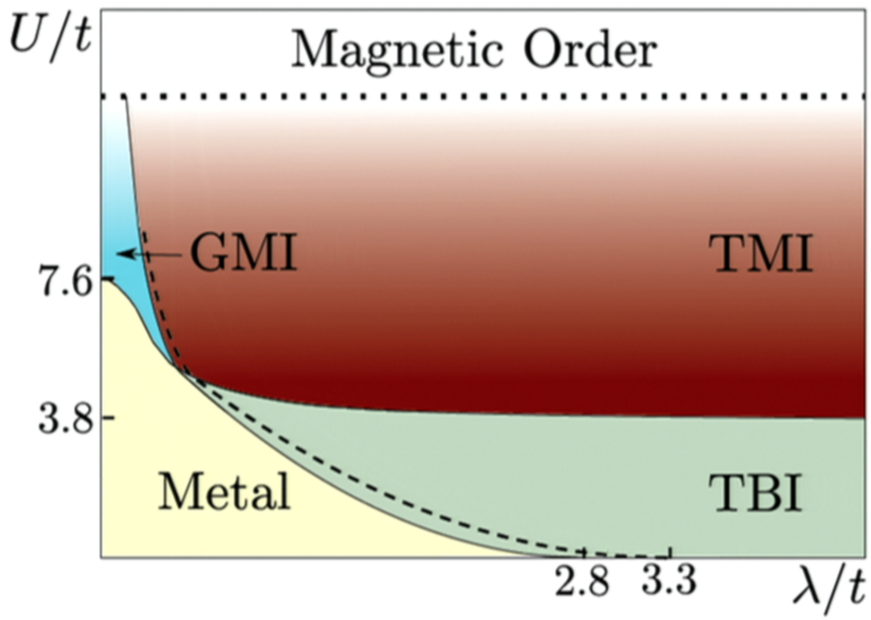

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TOPOLOGICAL MOTT INSULATOR

Topological Mott insulator is

U(1) spin liquid with topological band structure

TOPOLOGICAL MOTT INSULATOR

Topological Mott insulator is

U(1) spin liquid with topological band structure

So,

A. It will have the axion response (from top. Band structure)

B. And a monopole excitation (from U(1) gauge theory)

In this talk, I want to show :

Dyon condensation

(nontrivial physics due to the axion term)

$\neq$ Monopole condensation

Monopole = only magnetic charge

Dyon = magnetic monopole + electric charge

Dyon condensation

vs.

Monopole condensation

PROCEDURE
FOR MONOPOLE CONDENSATION

In the usual U(1) spin liquids

A. Integrate out the gapped matter fields

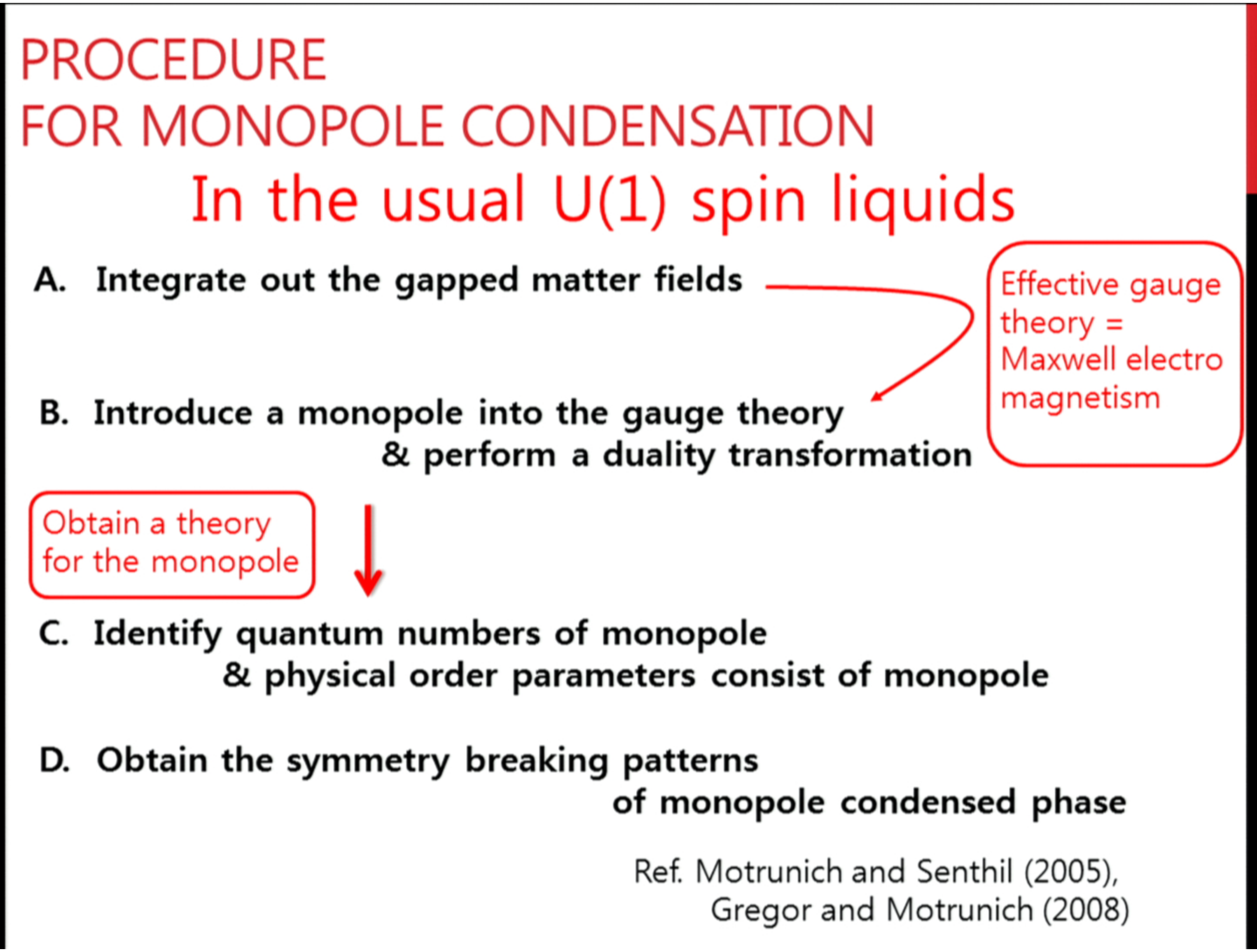
Effective gauge theory = Maxwell electro magnetism

B. Introduce a monopole into the gauge theory
& perform a duality transformation

Obtain a theory for the monopole

C. Identify quantum numbers of monopole
& physical order parameters consist of monopole

Ref. Motrunich and Senthil (2005),
Gregor and Motrunich (2008)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C. Identify quantum numbers of monopole & physical order parameters consist of monopole
D. Obtain the symmetry breaking patterns of monopole condensed phase
Ref. Motrunich and Senthil (2005), Gregor and Motrunich (2008)

MONOPOLE CONDENSED PHASE OF U(1) SPIN LIQUIDS

A. Fractionalization of spins , e.g. CP^1 representation of spins

a. $\vec{S}(r) = \frac{1}{2}\sum_{\alpha\beta} Z^*_\alpha\,(r)[\vec{\sigma}]^{\alpha\beta}\,Z_\beta(r)$, and invariant under $Z \to Z\,e^{i\theta}$

b. U(1) spin liquid: $L(Z_\alpha, Z^*_\beta; A_\mu)$ and A_μ is compact U(1) gauge theory

such that Z_α is gapped.

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B. Integrating out the matter fields = effective gauge theory

$$S_A = \int d\tau d^3\mathbf{r} \frac{1}{4g^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \quad + i\sum_r A_\tau(r) N(-1)^{x+y+z}$$

MONOPOLE CONDENSED PHASE OF U(1) SPIN LIQUIDS

Origin of the staggered charge $\Delta S_A = i \sum_r A_\tau(r) N(-1)^{x+y+z}$?

C. Introduction of monopole field : $\partial_\mu A_\nu - \partial_\nu A_\mu \rightarrow \partial_\mu A_\nu - \partial_\nu A_\mu + S_{\mu\nu}$

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Origin of the staggered charge $\Delta S_A = i\sum_r A_\tau(r) N(-1)^{x+y+z}$?

⇒ the Berry phase of spin $\vec{S}(r)$ with $\vec{S}(r) \sim (-1)^{x+y+z}\,\vec{n}$, and $N = 2S$

Ref. Sachdev and Park (2001), Sachdev and Jalabert (1990)

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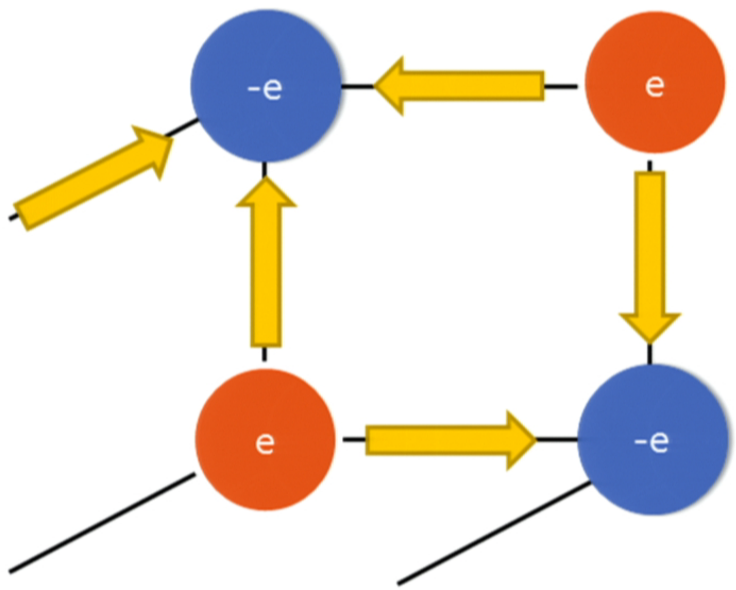
MONOPOLE CONDENSED PHASE OF U(1) SPIN LIQUIDS

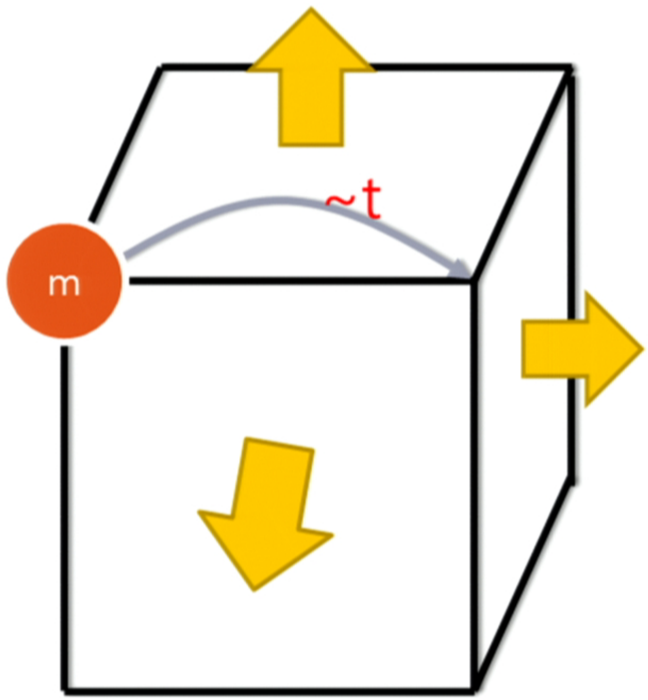
Monopoles in the staggered background charge!

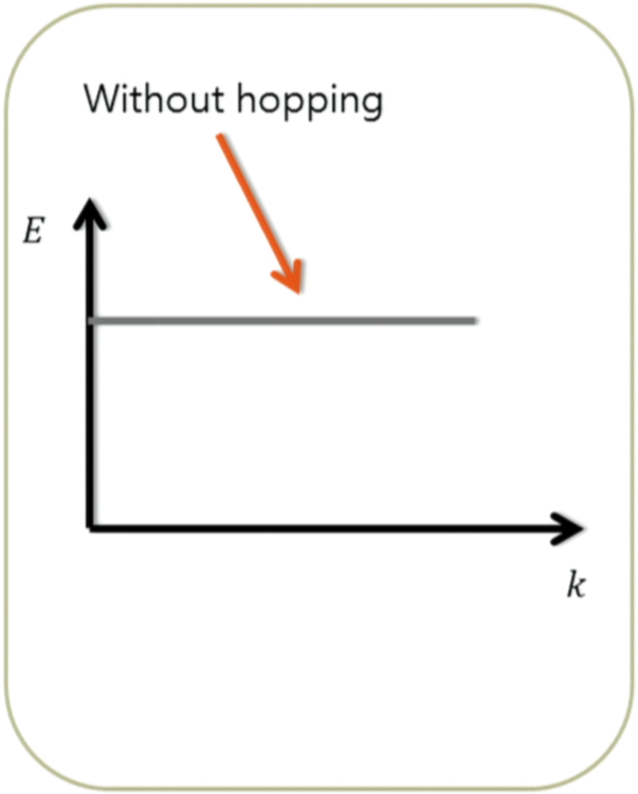
= Patterns of electric field = Berry phase for monopoles!



MONOPOLE CONDENSED PHASE OF U(1) SPIN LIQUID

Solve the hopping problem in the presence of the background fluxes





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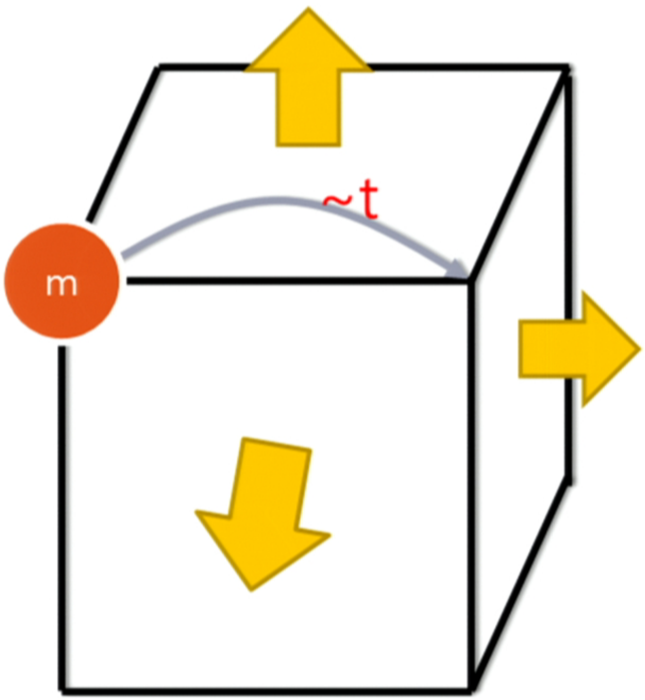

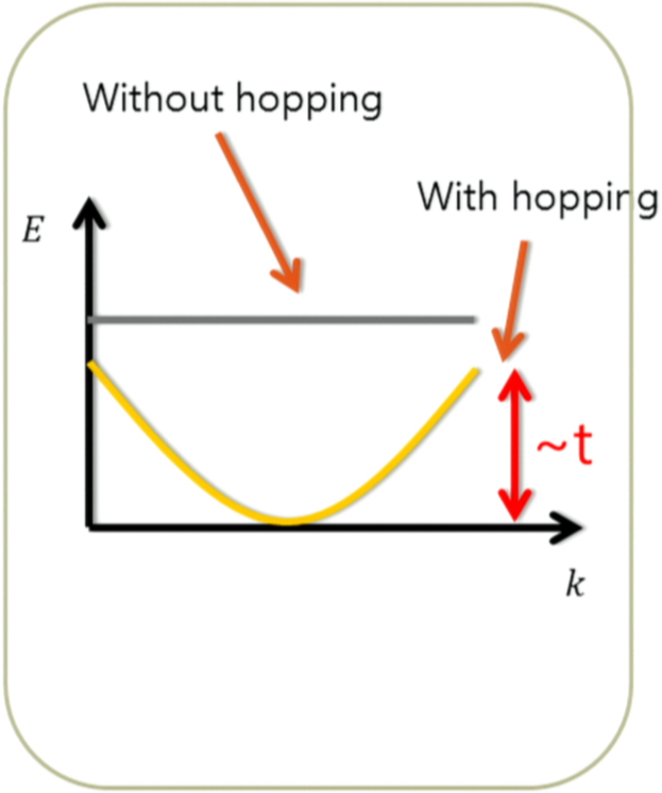

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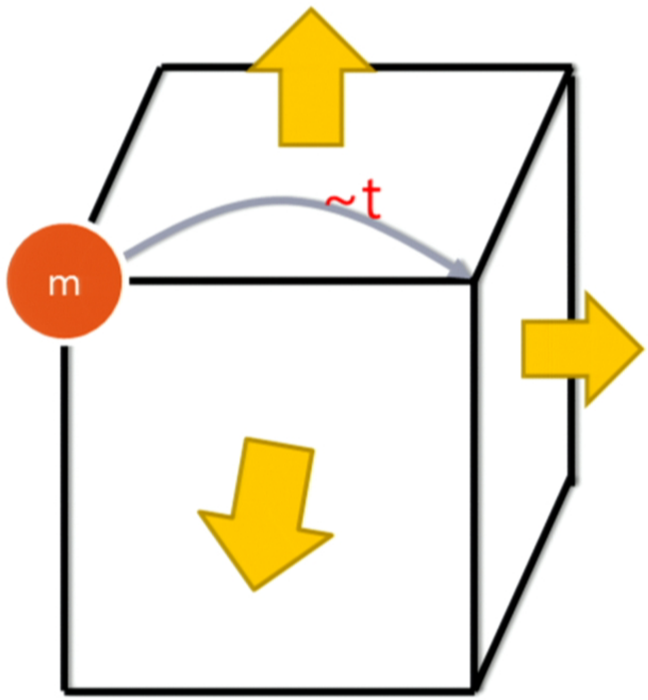

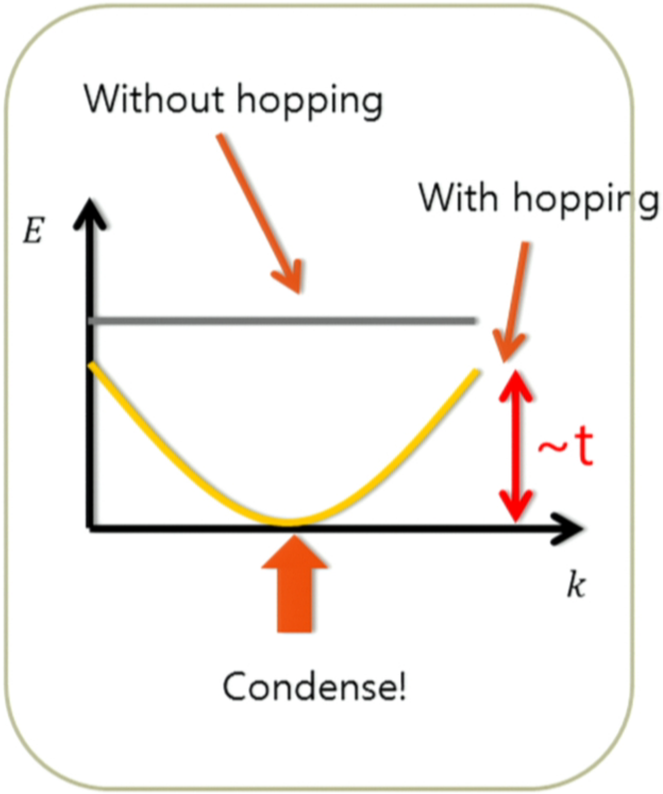

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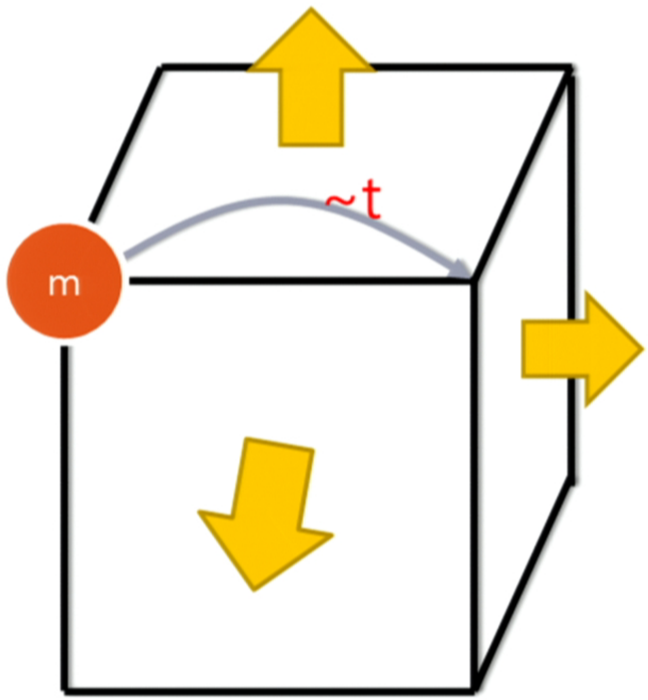
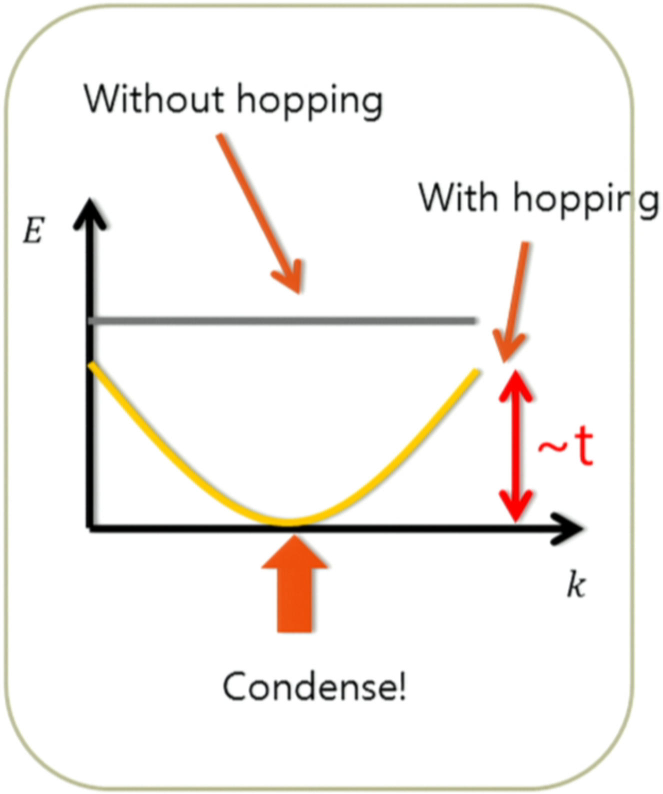




MONOPOLE CONDENSED PHASE OF U(1) SPIN LIQUIDS

Ref. Motrunich and Senthil (2005)
Sachdev and Park (2001),
Sachdev and Jalabert (1990)

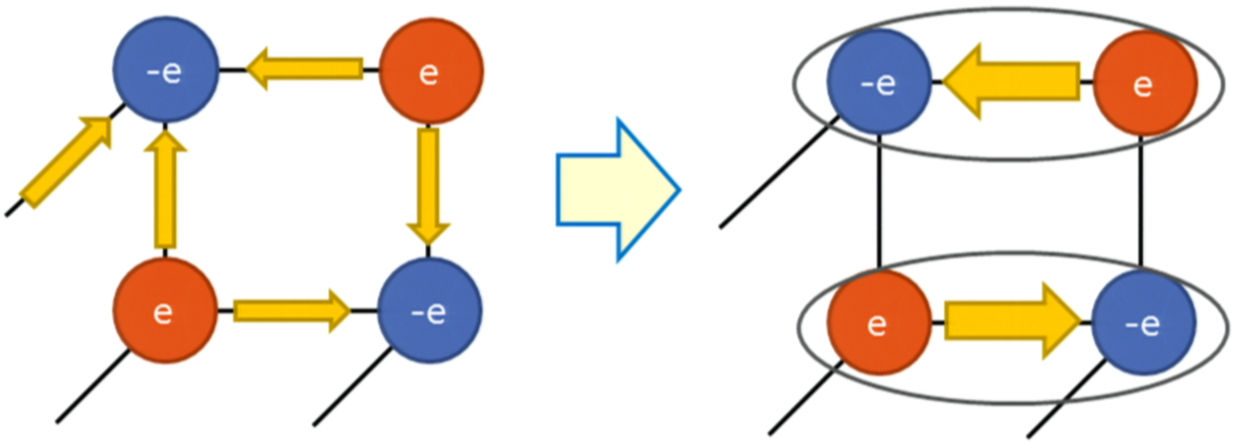
Berry phase for the monopole

= frustrated monopole hopping on the lattice

= minimum of monopole band is at finite momentum

= translational symmetry breaking in the confined phase!

= confined electric flux ($\cong$ bond order or VBS $\propto \langle \vec{S}_i \cdot \vec{S}_j \rangle$)



The monopole condensed phase is VBS!

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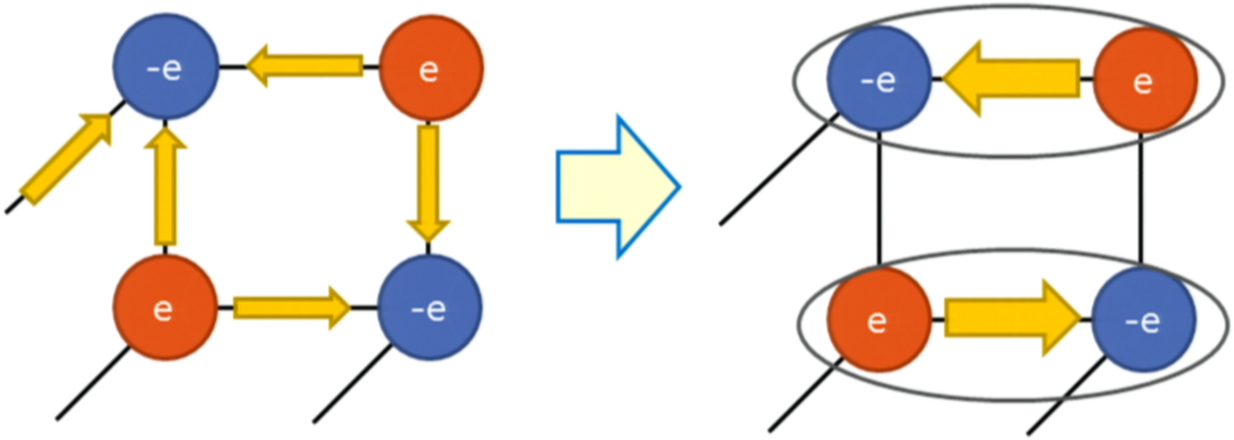

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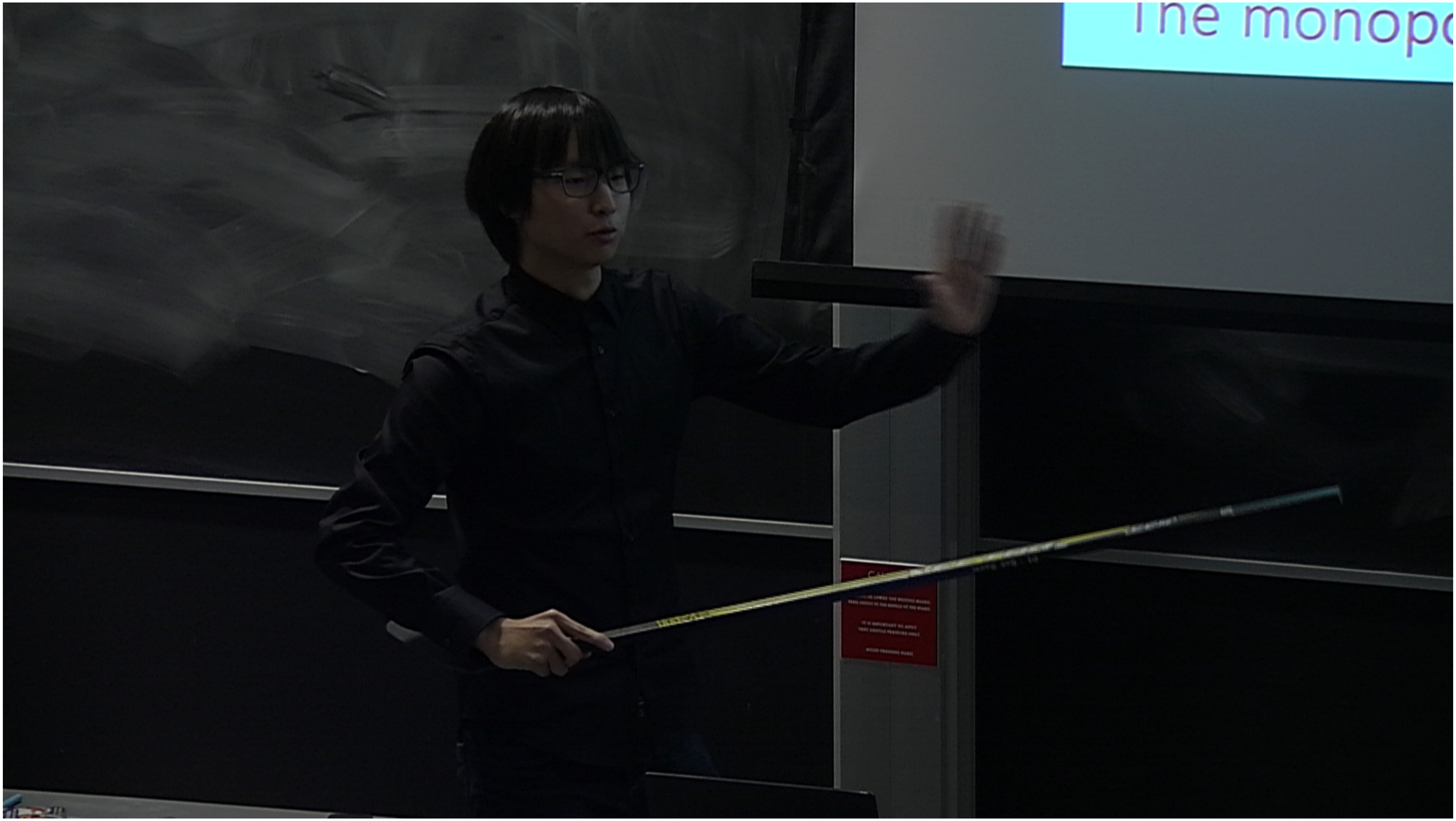
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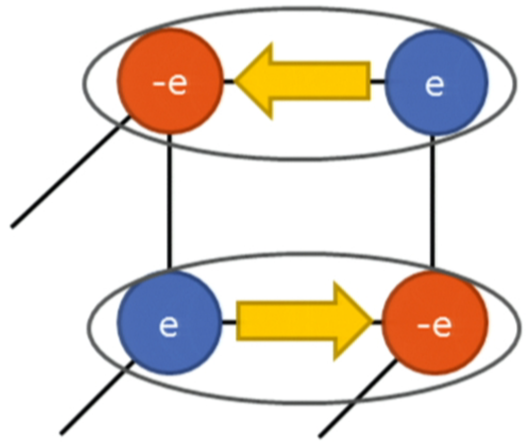
MONOPOLE CONDENSATION

So, monopole condensation in the spin liquid leads us to..

VBS or Bond ordered phase



MONOPOLE CONDENSATION

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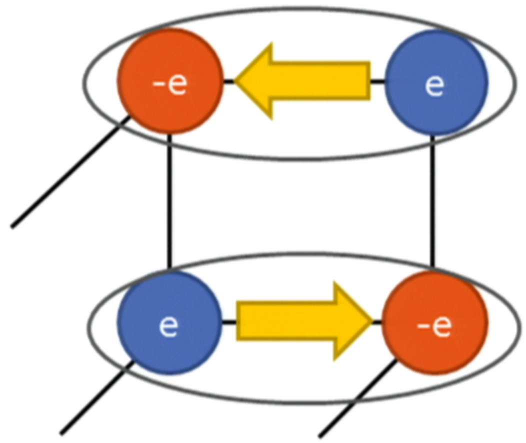

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MONOPOLE CONDENSATION

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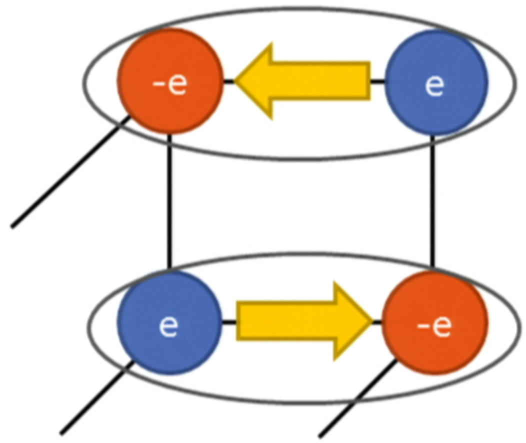

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DYON CONDENSATION IN TOPOLOGICAL MOTT INSULATORS

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U(1) spin liquid with the topological band structure!
= axion response!

PROCEDURE FOR ~~MONOPOLE~~ CONDENSATION

In Topological Mott Ins.

A. Integrate out the gapped matter fields

Effective gauge theory = Maxwell electro magnetism

B. Introduce a ~~monopole~~ into the gauge theory & perform a duality transformation

Obtain a theory for the monopole

C. Identify quantum numbers of ~~monopole~~ & Identify physical order parameters consist of monopole

D. Obtain the symmetry breaking patterns of ~~monopole~~ condensed phase

Ref. Motrunich and Senthil (2005), Gregor and Motrunich (2008)

DUAL GAUGE THEORY AND DYON

A. Integrating out the matter fields = gauge theory + axion term

$$S_A = \int d\tau d^3\mathbf{r} \frac{1}{4g^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{\theta}{8\pi^2}\varepsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho + i\sum_r A_\tau(r)N(-1)^{x+y+z}$$

Maxwell term
~ the photon

Axion term from
topological band
structure

Staggered
background
charge

B. Plug $\partial_\mu A_\nu - \partial_\nu A_\mu \rightarrow \partial_\mu A_\nu - \partial_\nu A_\mu + S_{\mu\nu}$ **into the gauge theory**

The monopole 4-current is: $m_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\partial_\nu S_{\lambda\rho}$

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Coulomb interaction between Dyons

Staggered background charge

DYON CONDENSATION

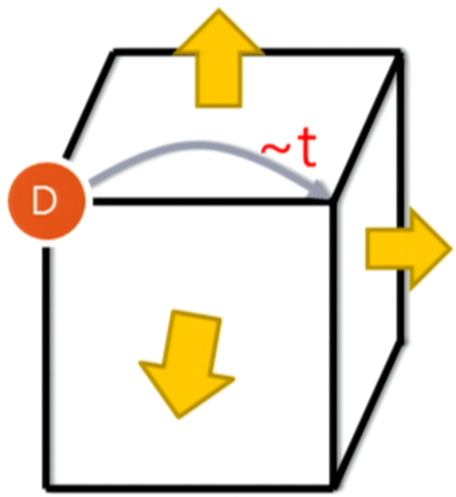
Dyon in the staggered background charge

= Berry phase for Dyon

= minimum of Dyon band is at finite momentum

= translational symmetry breaking

- Dyon = monopole with the electric charge !



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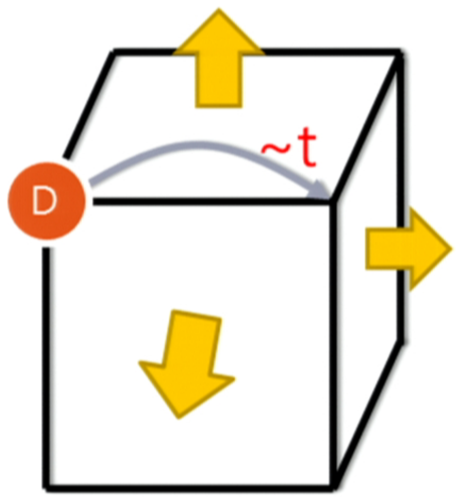

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The same bond order parameter as the usual U(1) spin liquid

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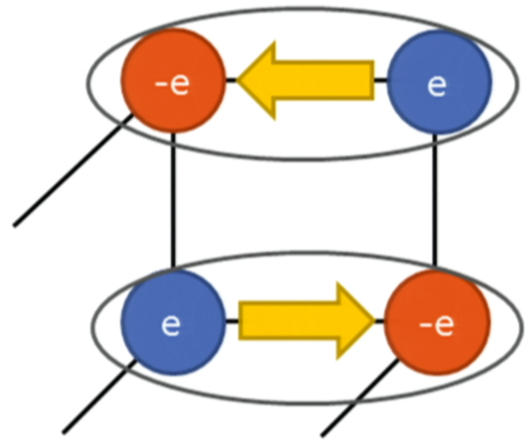
Confined electric flux
= VBS or bond order!



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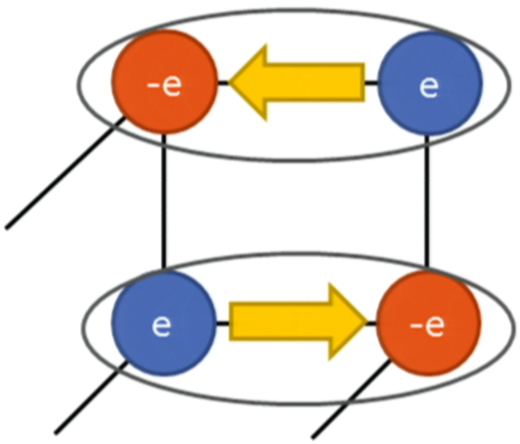

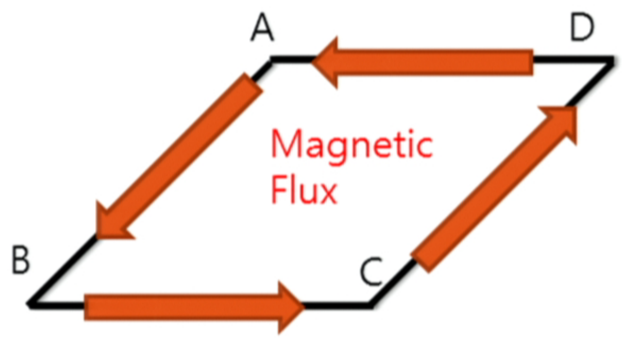

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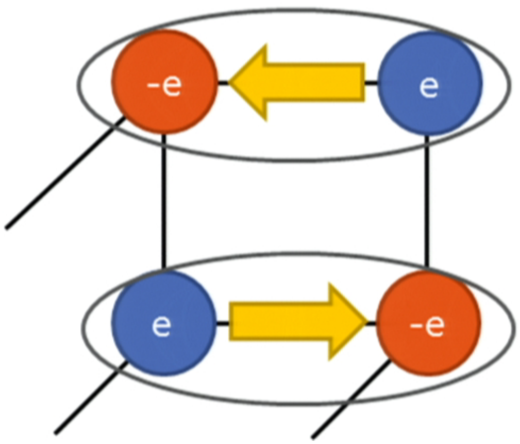
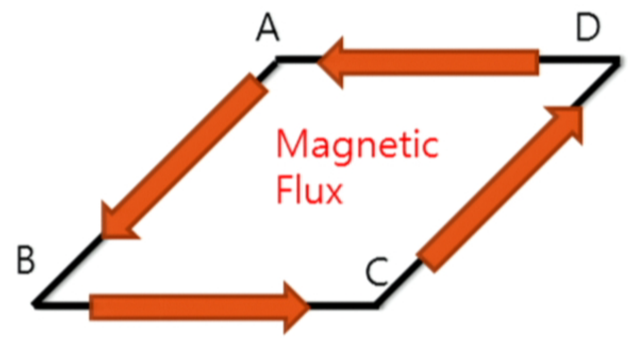


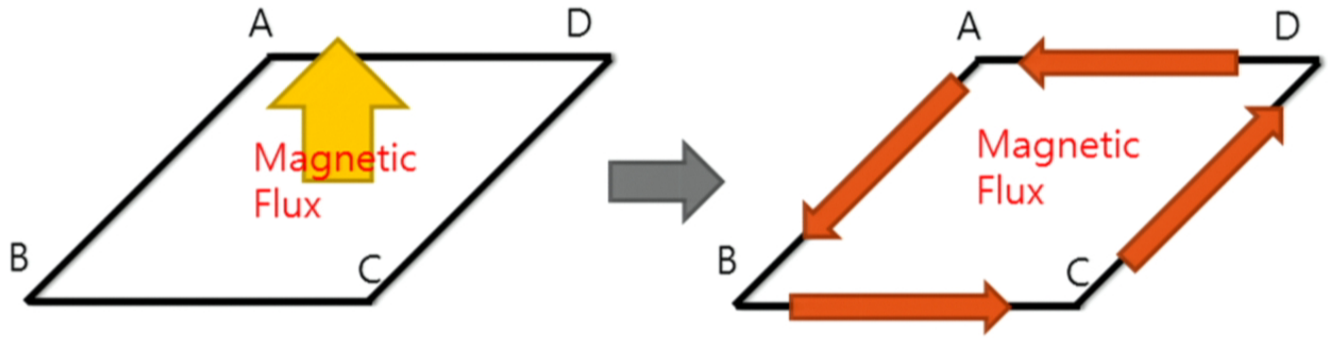


DYON CONDENSATION

Dyon condensed phase

= both electric and magnetic fluxes are confined!

= VBS (electric flux) + ? (magnetic flux)

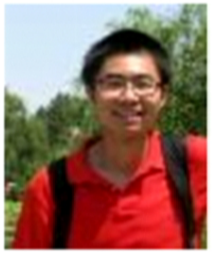


- Magnetic flux? circulating electric current!
- circulating electric current $\cong$ scalar chirality

$$\cong \vec{S}_A \cdot \vec{S}_B \times \vec{S}_C + \vec{S}_B \cdot \vec{S}_C \times \vec{S}_D + \cdots$$

PART2. PROXIMATE PHASES OF Z_2 SPIN LIQUID ON KAGOME LATTICE

(Gil Young Cho, Yuan-Ming Lu and Ashvin Vishwanath, *in preparation*)



Yuan-Ming Lu

Ashvin Vishwanath

Perimeter Ins (Nov 28th Wed)

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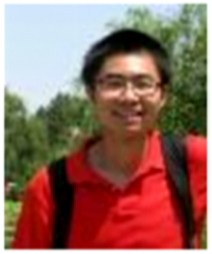

Yuan-Ming Lu

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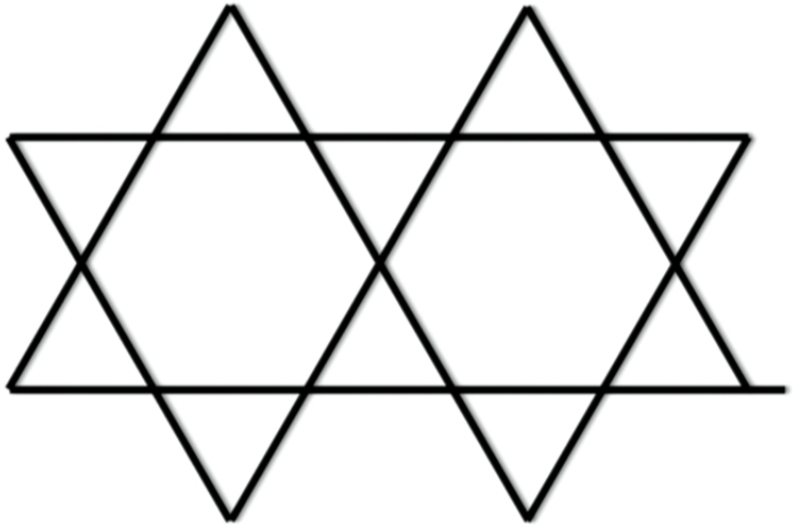
Z_2 SPIN LIQUID ON KAGOME LATTICE

Heisenberg interaction on Kagome lattice : $H = J \sum \vec{S}_i \cdot \vec{S}_j$



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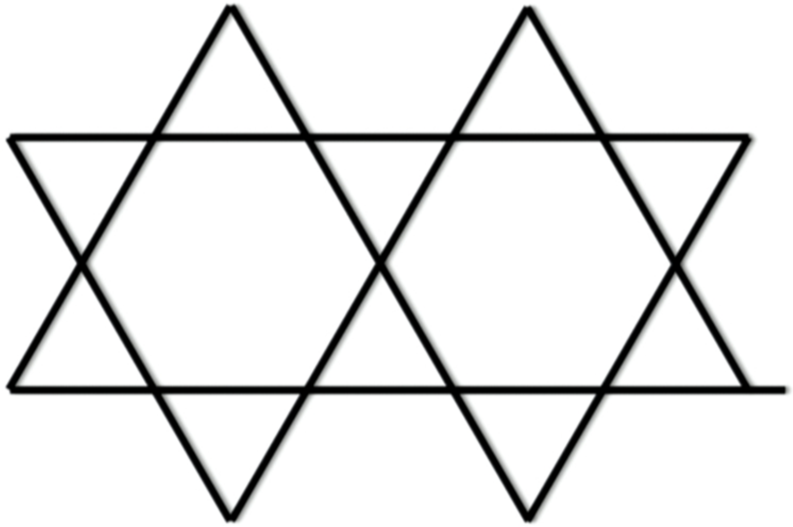

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WHICH SPIN LIQUID ?

Ref. Hastings (2000)
Ran, Hermele, Lee, and Wen (2007)
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Many different spin liquids from the fermionic rep. of spin-1/2

$$\vec{S}(r) = \frac{1}{2} \sum_{\alpha\beta} f_{\alpha}^{*} \vec{\sigma}^{\alpha\beta} f_{\beta}$$

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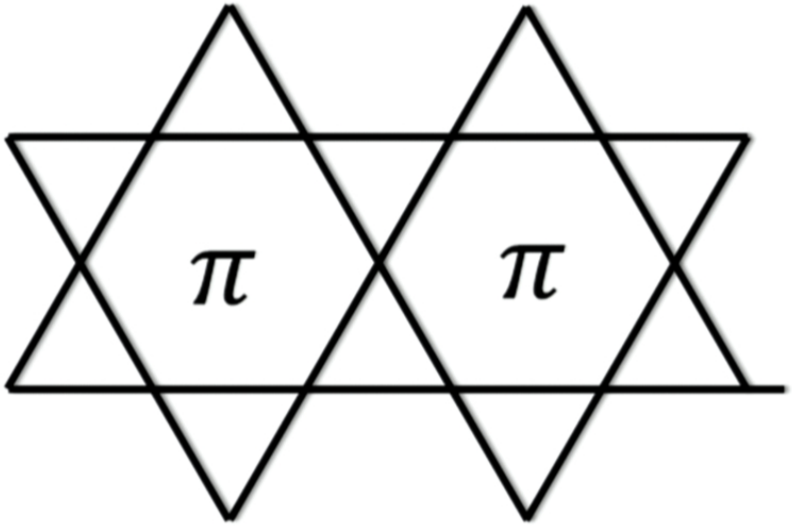

The (relatively) low energy state is, "Algebraic Spin liquid"

$$H = \sum_{ij} t_{ij} f_{i\alpha}^* f_{j\alpha} + h.c.$$

(among the fermionic SL ansatz)

This state features:

A. **Dirac spectrum for fermions**
B. **U(1) Gauge theory**

Is there a Z2 spin liquid near the algebraic spin liquid? Ref. Lu, Ran, and Lee (2011)

What can we tell about this Z2 state?

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Our claim is:

If this Z2 spin liquid is the spin liquid found in DMRG study,

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PROXIMATE PHASES

Special 5-tuplets of masses of Dirac fermions

= WZW term for 5-tuplet masses

= Unconventional second order transitions

Ref. Wiegmann and Abanov (2000), Senthil and Fisher (2006), Grover and Senthil (2008), Ryu, Mudry, Hou, and Chamon (2009), Herbut (2010) *etc.*

Underlying physics of WZW term:

- topological defect in one phase carries the quantum numbers related to the other phase

- Condensation of the defect = destroying one ordering + inducing the other order

Q=0 NON-COLLINEAR MAGNETIC ORDER AND Z2 SPIN LIQUID

consider a 5-tuplet $(\Delta_r, \Delta_i, \vec{V})$ **where** $\vec{V}$ **is vector chirality**

(Δ_r, Δ_i): pairing $\vec{V}$: vector chirality

s-wave paired
algebraic spin liquid
$Z_2[0,\pi]\beta$ state

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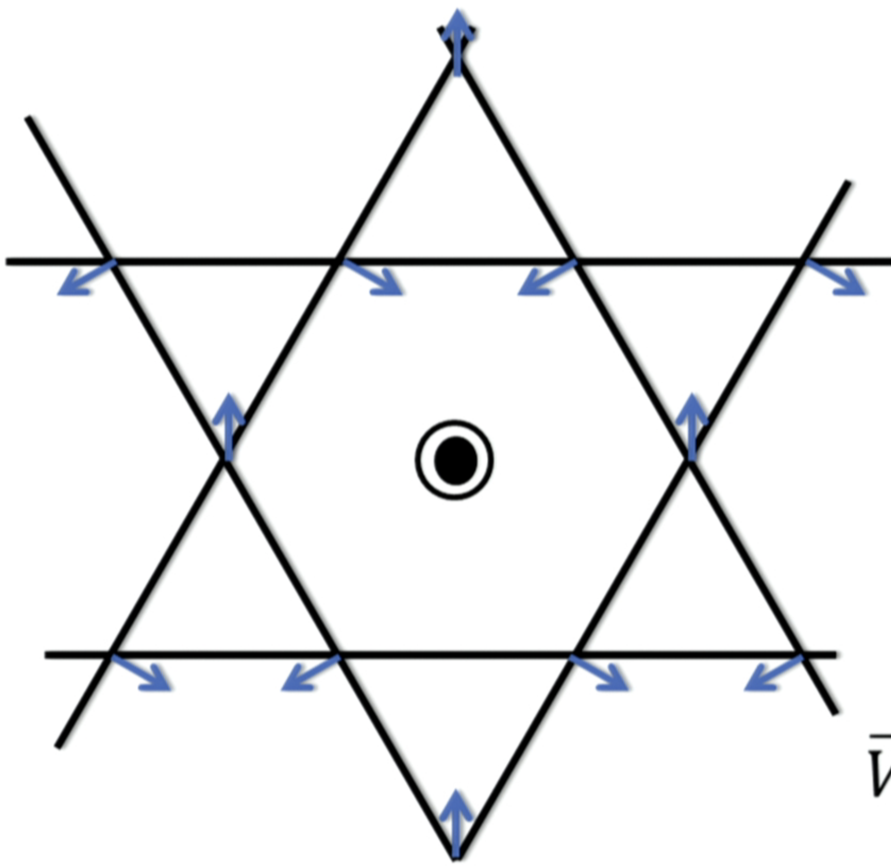
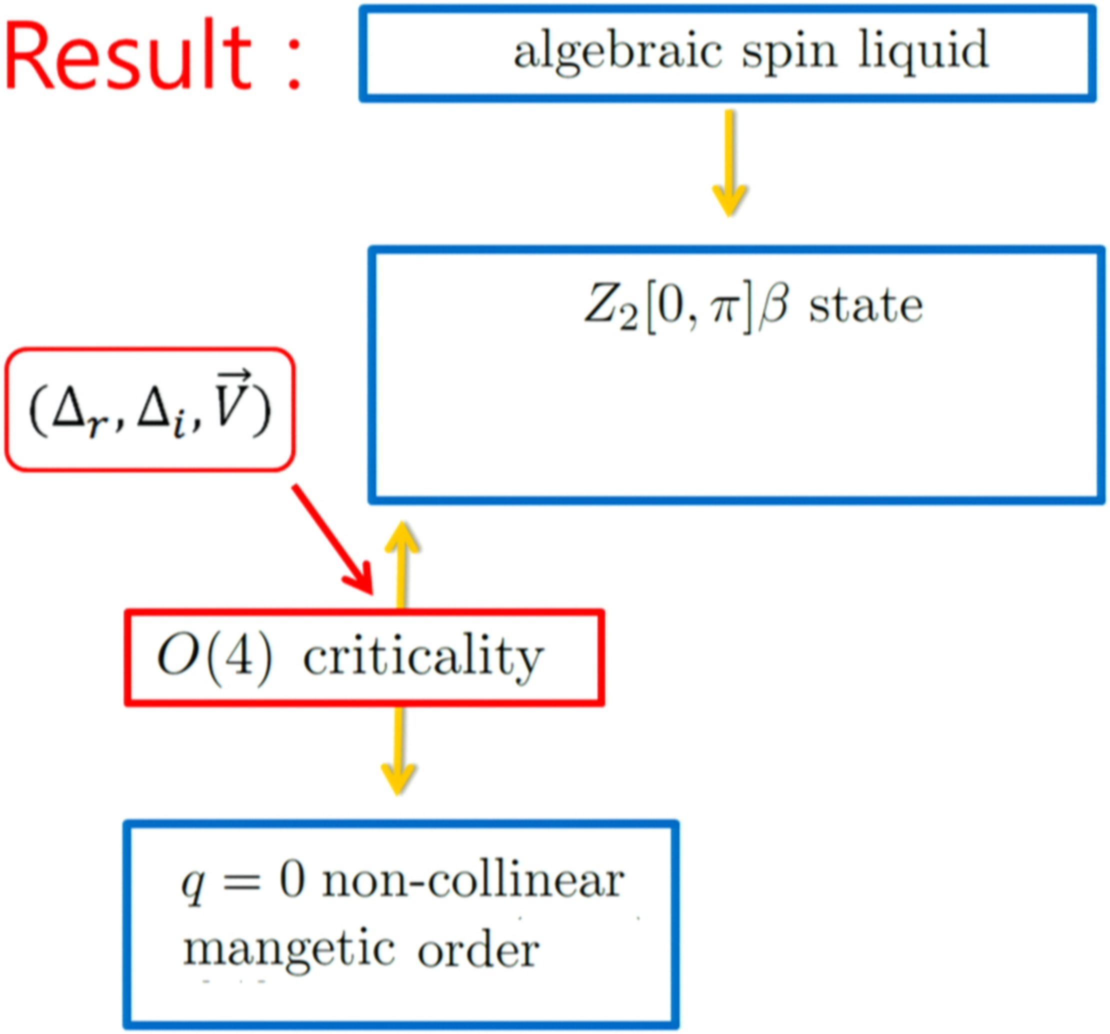


$\hat{n}_r = \uparrow$

$\hat{V} = \odot$

$$\vec{V} \sim \sum_{H} (\vec{S_i} \times \vec{S_j})$$

Result :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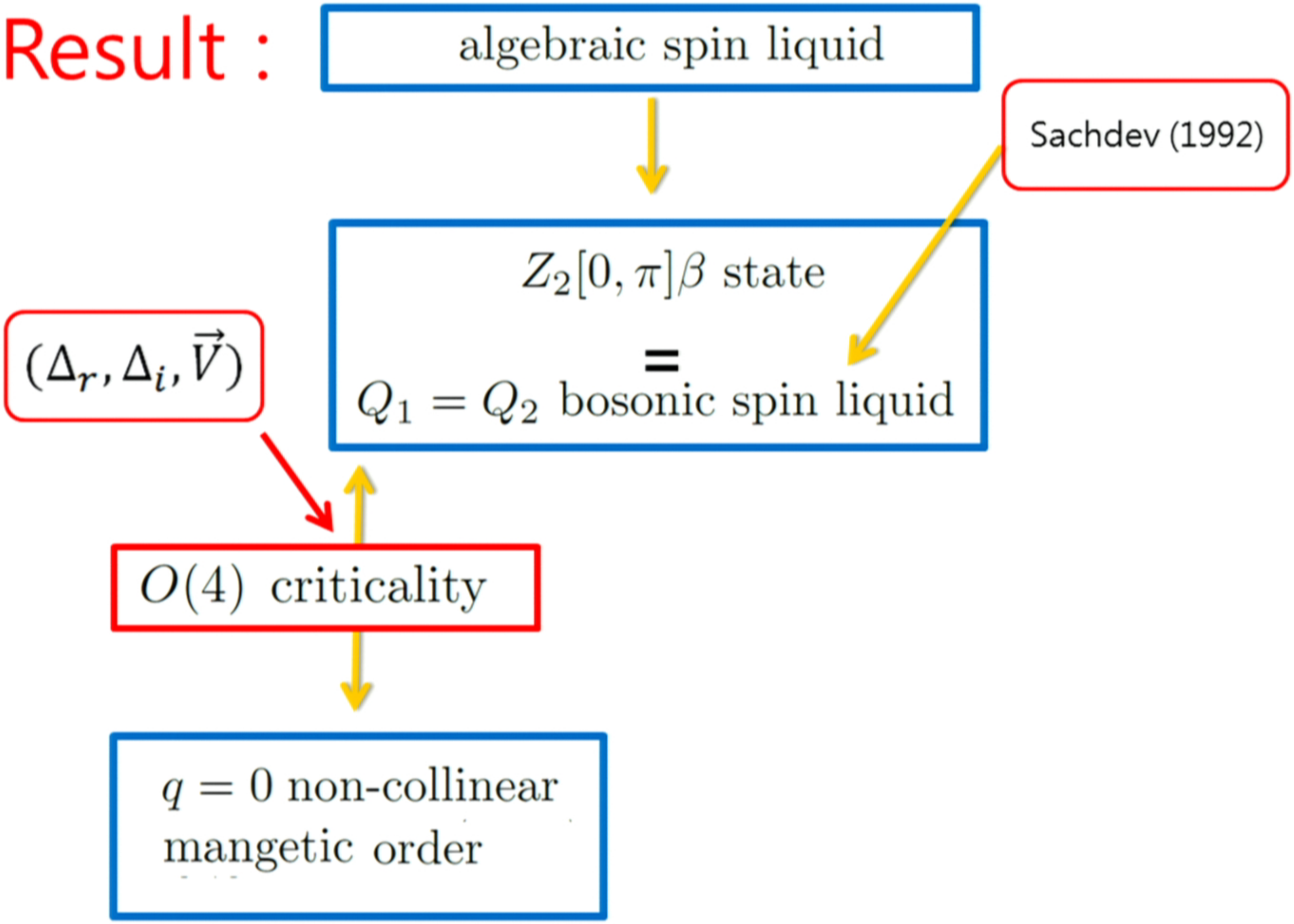

algebraic spin liquid
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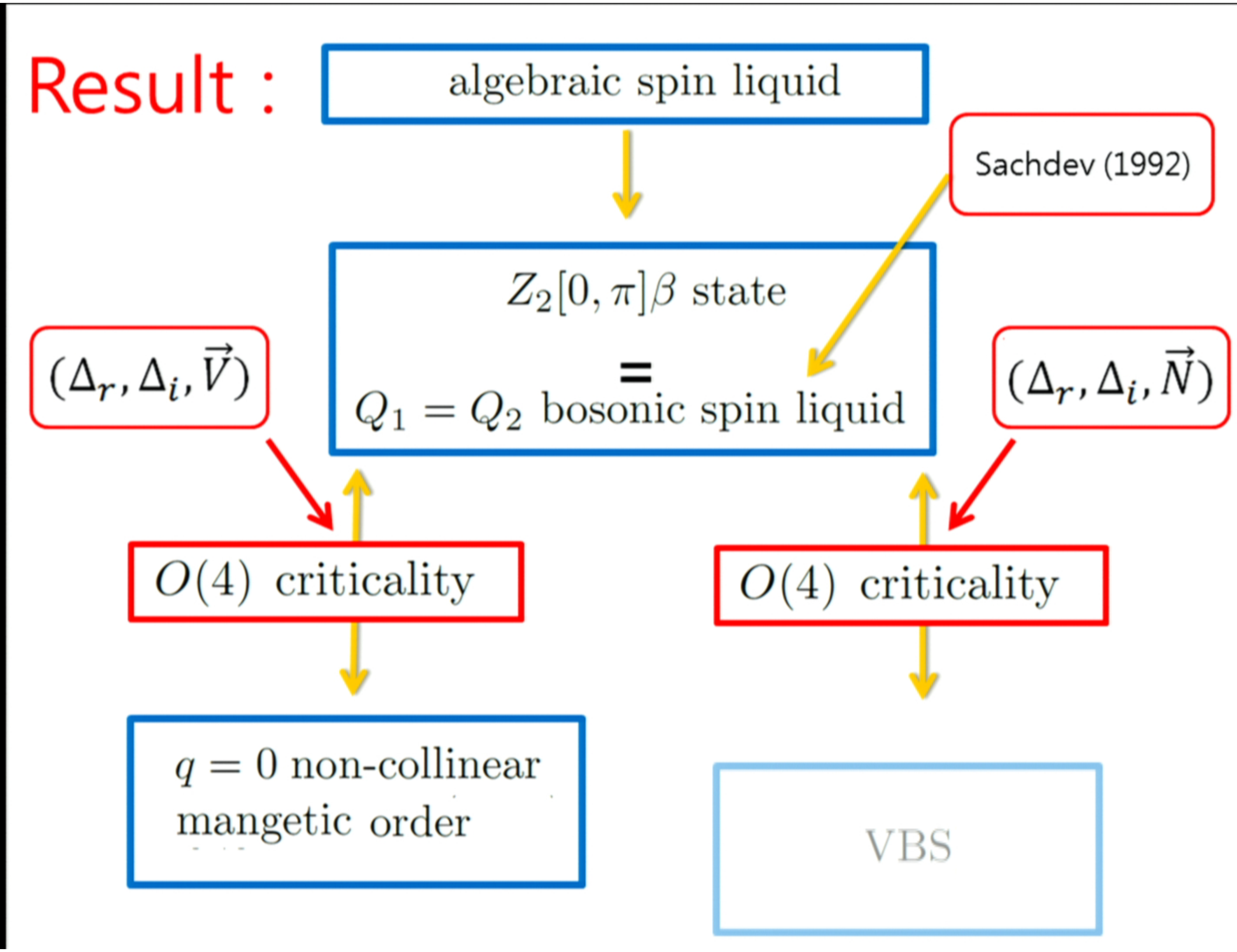
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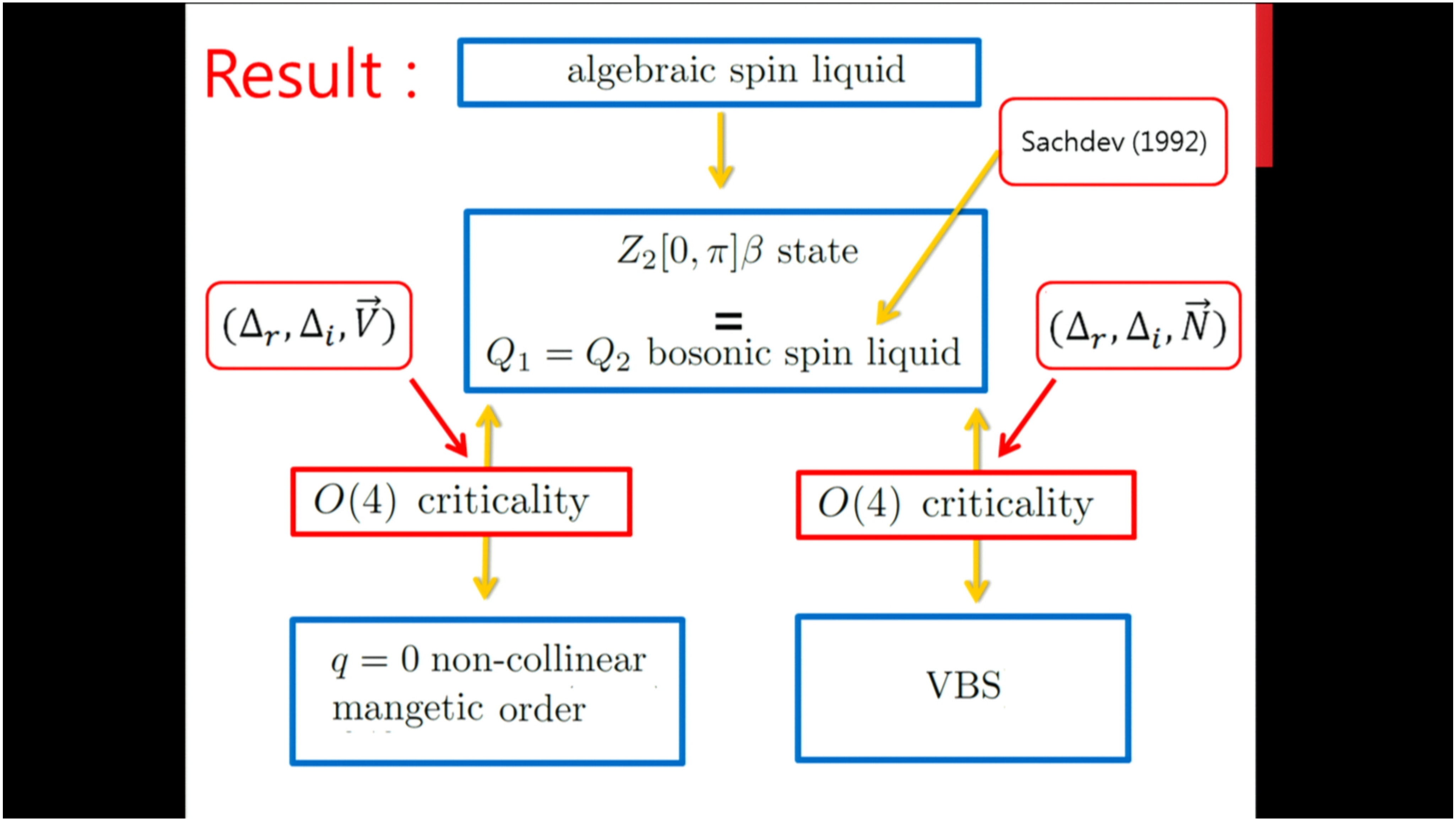
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→ Pairing of Dirac fermions!

Requirement 2: Invariant under the lattice symmetry operation

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Requirement 3: Pairing should gap out the Dirac fermion

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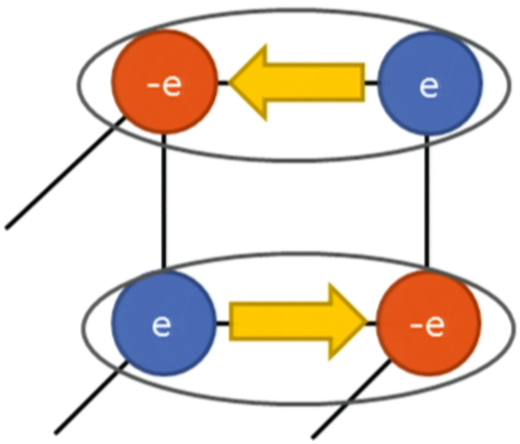

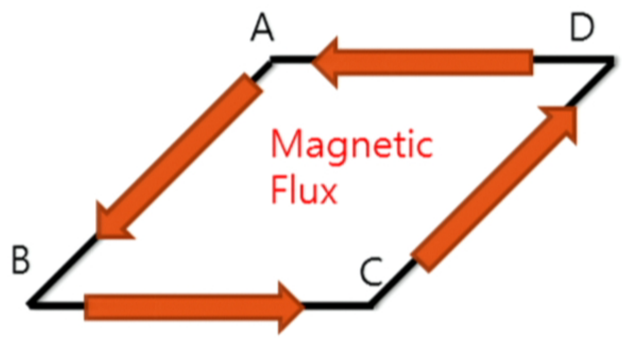

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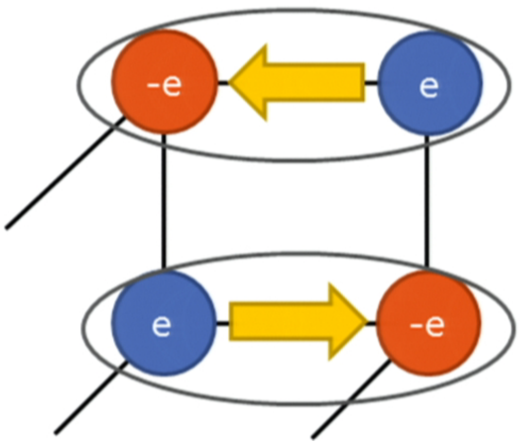

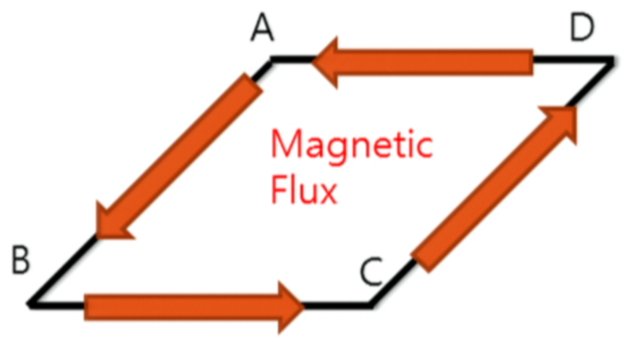

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