Title: Entanglement and the Fermi surface

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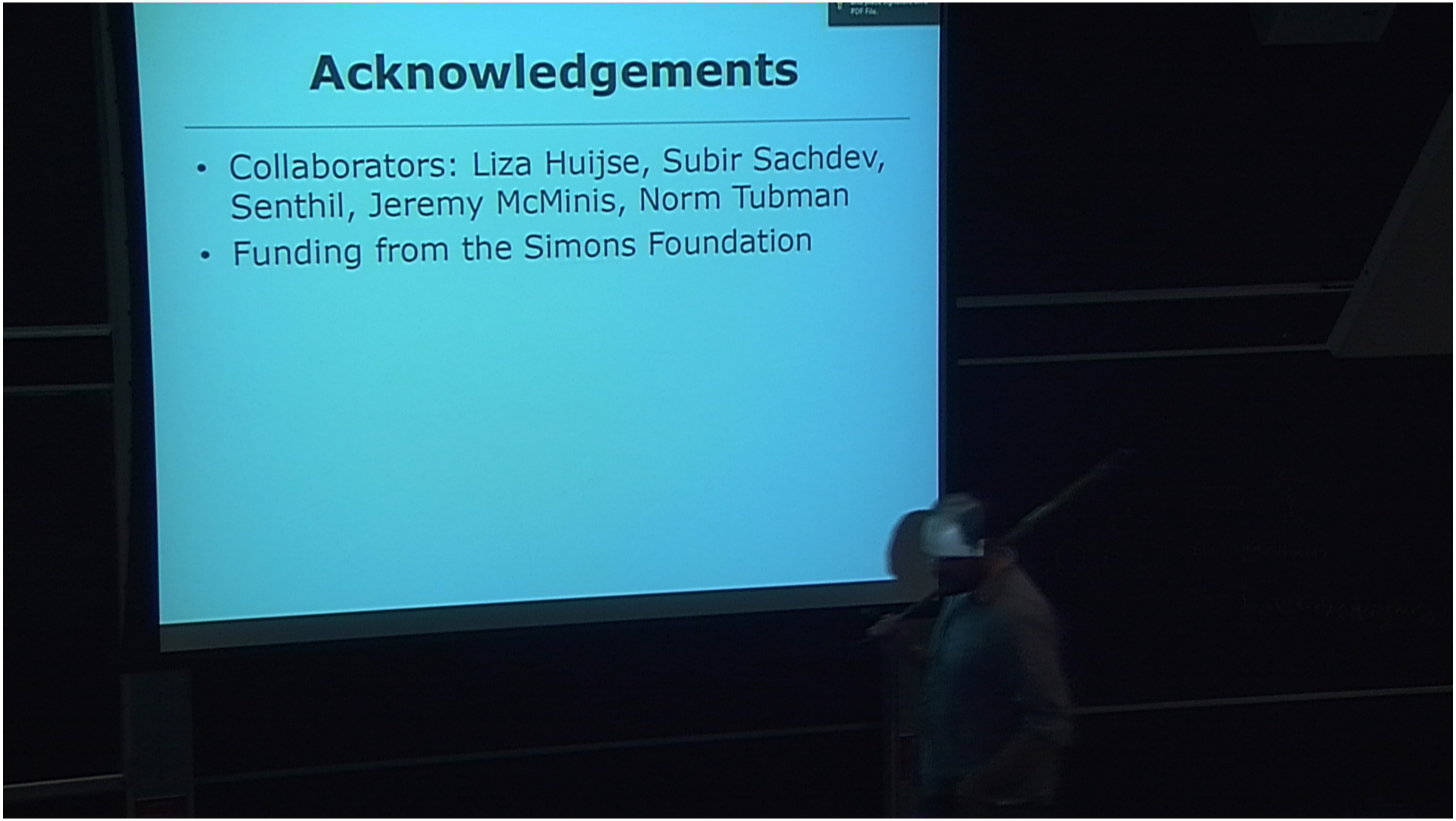
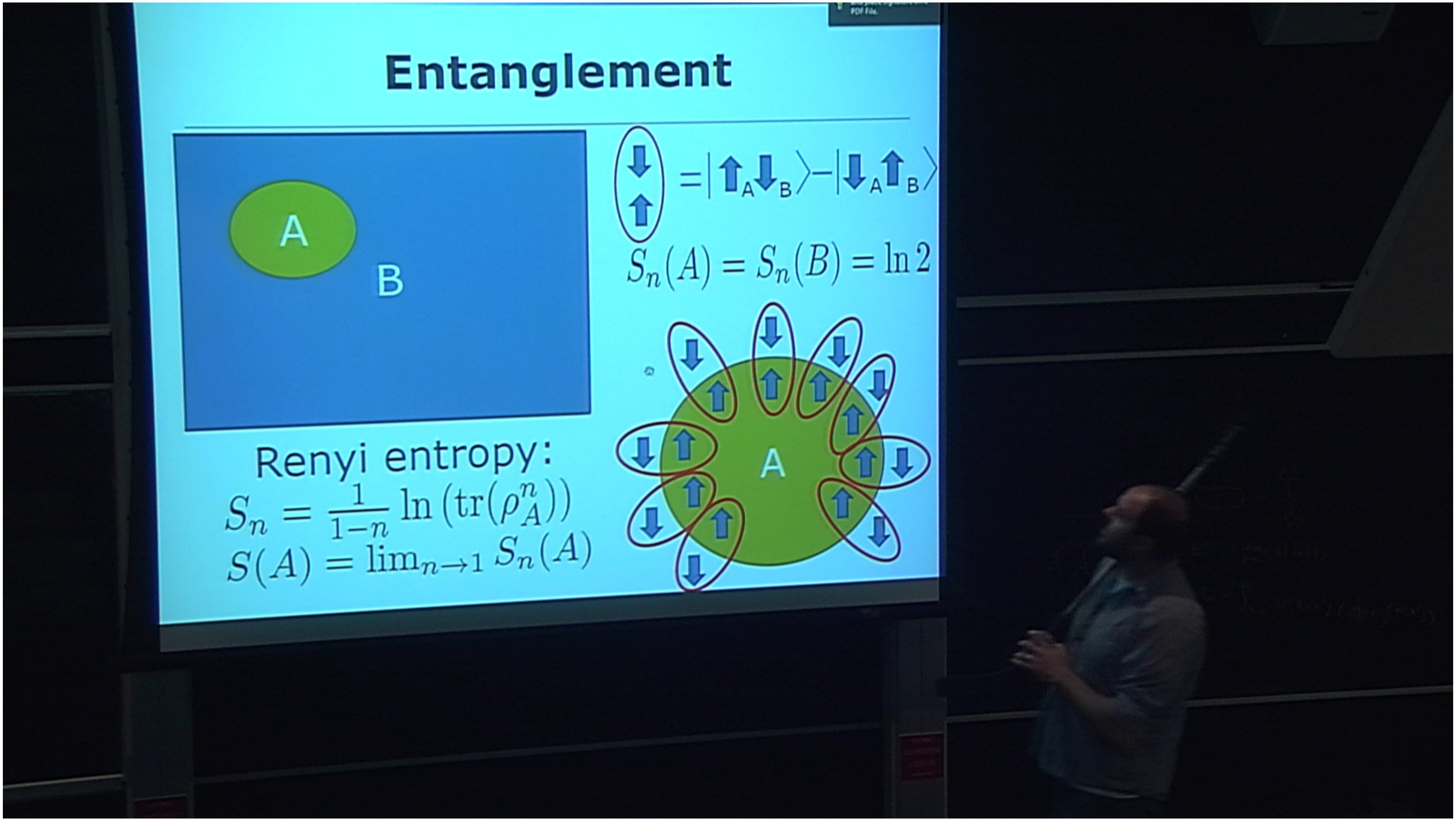
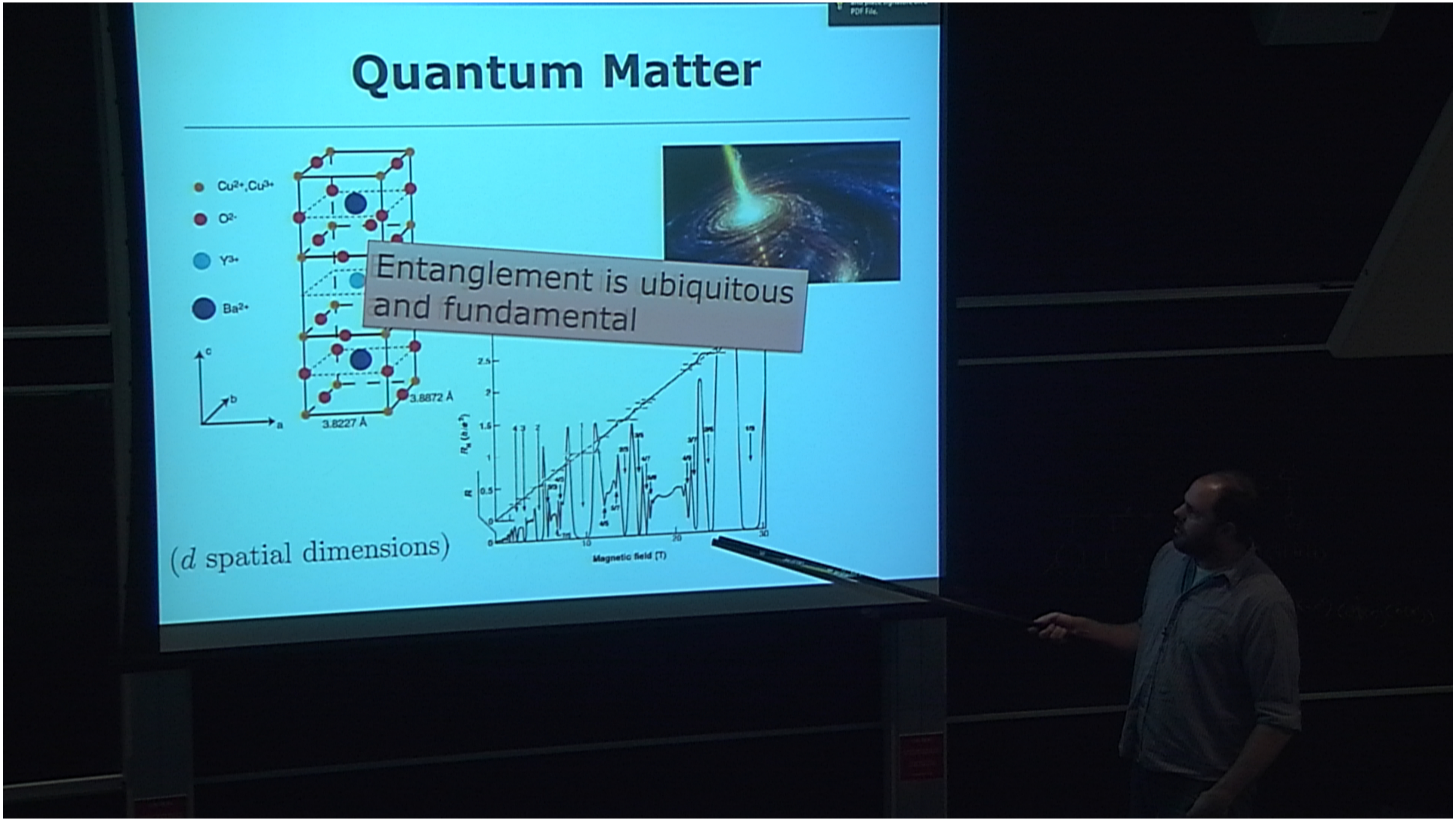
Abstract: In this talk I will describe my work characterizing quantum entanglement in systems with a Fermi surface. This class includes everything from Fermi liquids to exotic spin liquids in frustrated magnets and perhaps even holographic systems. I review my original scaling argument and then describe in detail a number of new precise results on entanglement in Fermi liquids. I will also discuss recent quantum Monte Carlo calculations of Renyi entropies and will argue that we now have a rather complete agreement between theory and numerics for Fermi liquid entanglement. I will also discuss universal crossovers between thermal and entanglement entropy and a class of solvable interacting models where we can prove the universality of the Widom formula for Fermi surface entanglement. If there is time, I will comment on several other topics including fluctuations of conserved quantities and connections to holography.

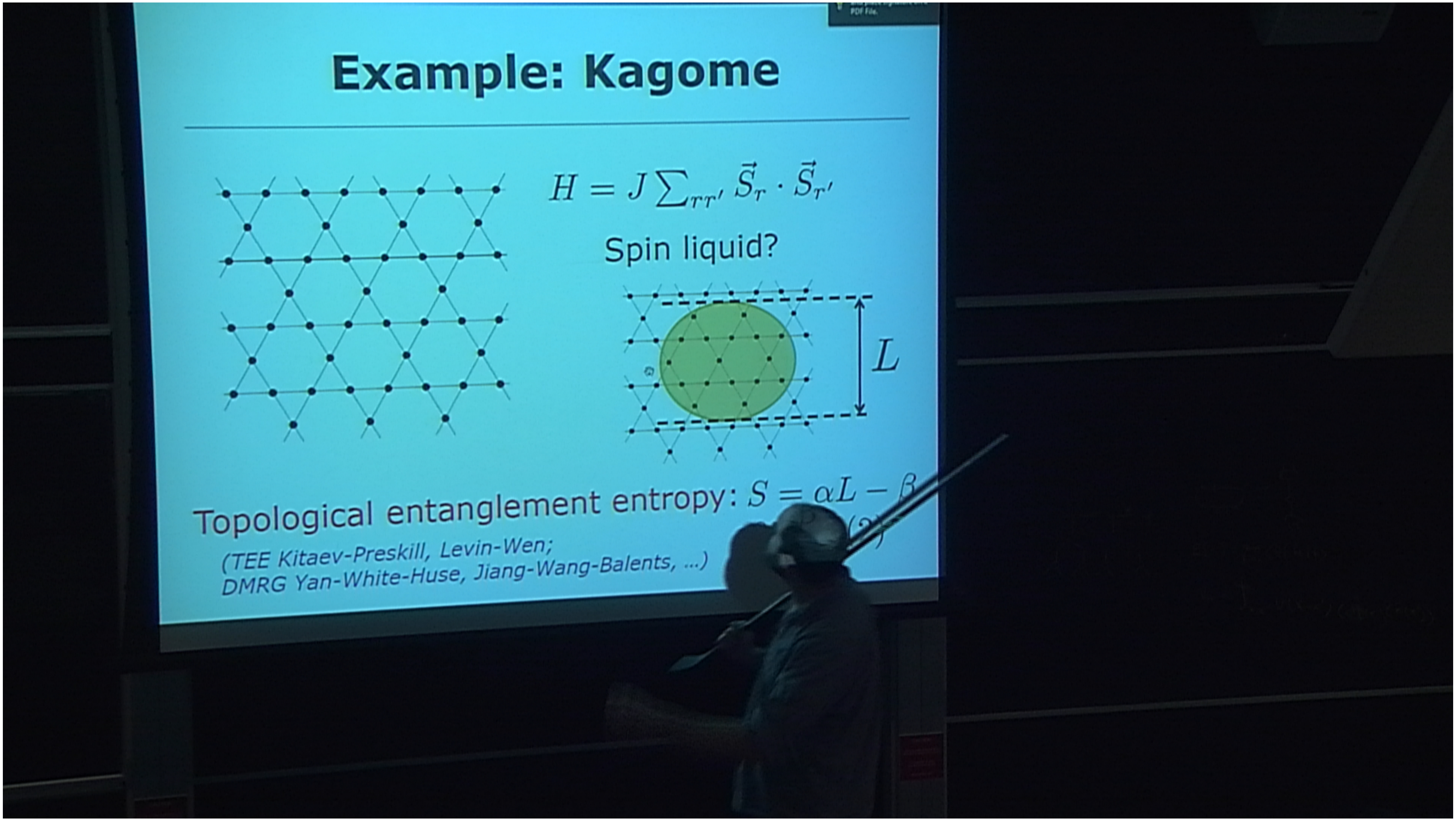

Acknowledgements
• Collaborators: Liza Huijse, Subir Sachdev, Senthil, Jeremy McMinis, Norm Tubman
• Funding from the Simons Foundation

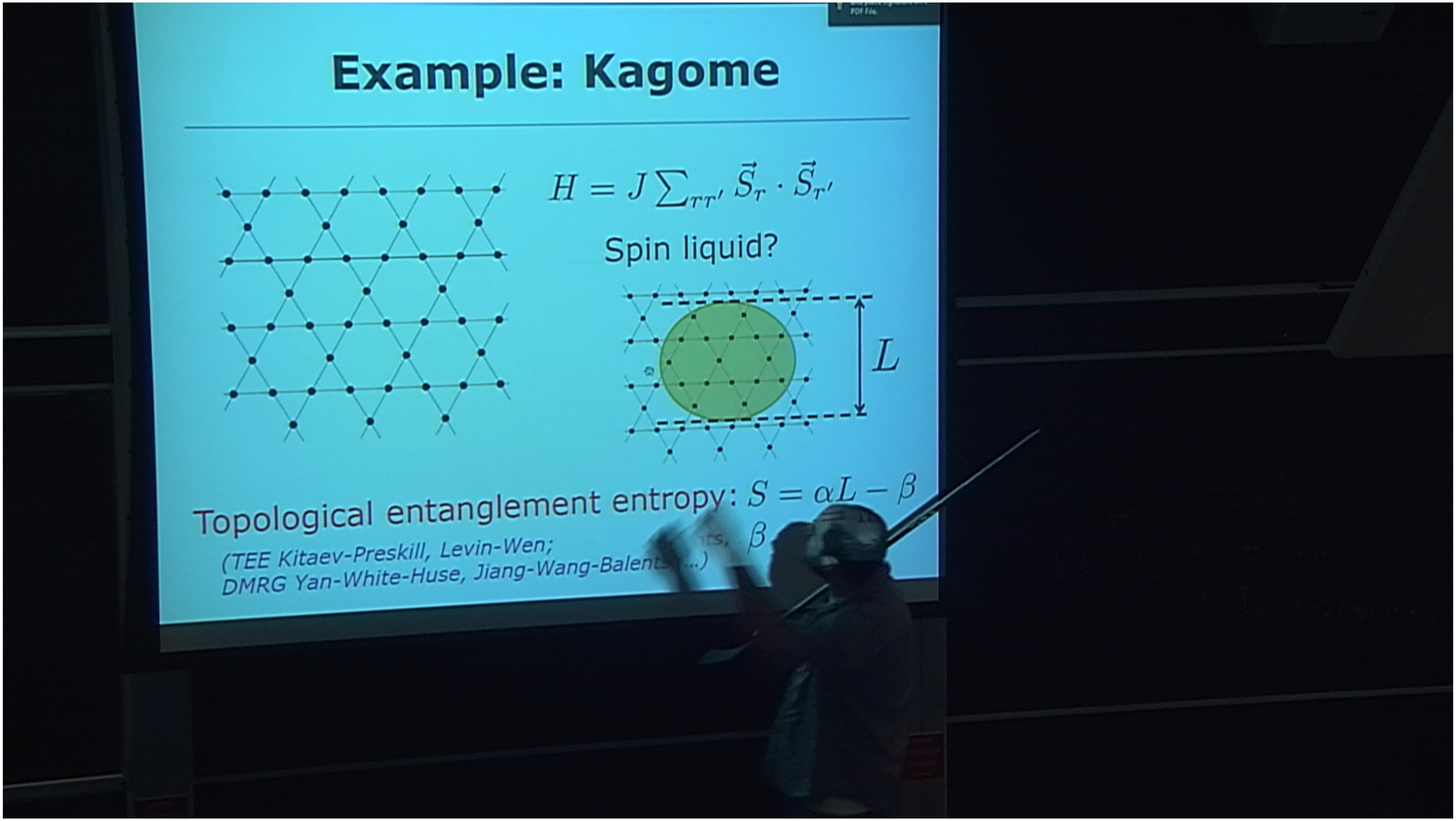


Entanglement
A
B
$= |\uparrow_A \downarrow_B\rangle - |\downarrow_A \uparrow_B\rangle$
$S_n(A) = S_n(B) = \ln 2$
A
Renyi entropy:
$S_n = \frac{1}{1-n} \ln\left(\mathrm{tr}(\rho_A^n)\right)$
$S(A) = \lim_{n\to 1} S_n(A)$



Quantum Matter
Cu2+,Cu3+
O2-
Y3+
Ba2+
3.8872 Å
3.8227 Å
Entanglement is ubiquitous
and fundamental
Magnetic field (T)
(d spatial dimensions)


Example: Kagome
$H = J \sum_{rr'} \vec{S}_r \cdot \vec{S}_{r'}$
Spin liquid?
L
Topological entanglement entropy: $S = \alpha L - \beta$
(TEE Kitaev-Preskill, Levin-Wen;
DMRG Yan-White-Huse, Jiang-Wang-Balents, ...)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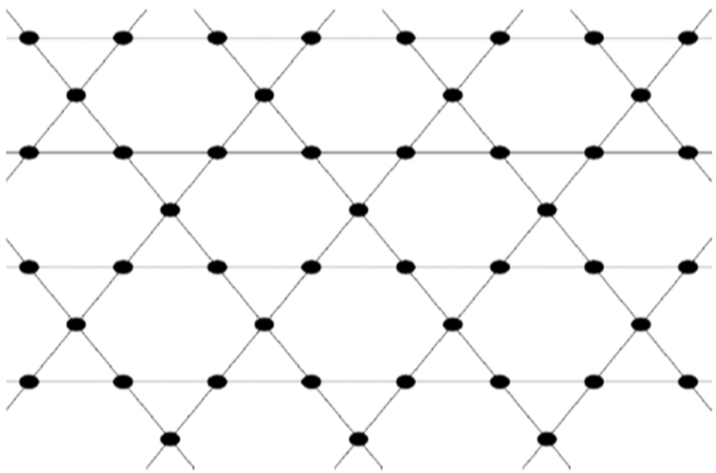
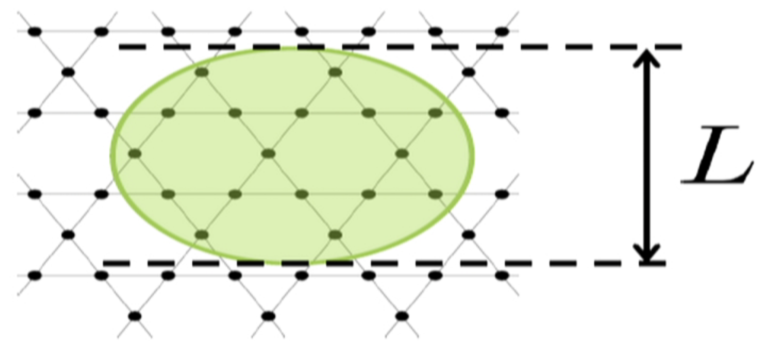


$$H = J \sum_{rr'} \vec{S}_r \cdot \vec{S}_{r'}$$

Spin liquid?



Topological entanglement entropy: $S = \alpha L - \beta$

$\beta = \ln(2)$

(TEE Kitaev-Preskill, Levin-Wen;
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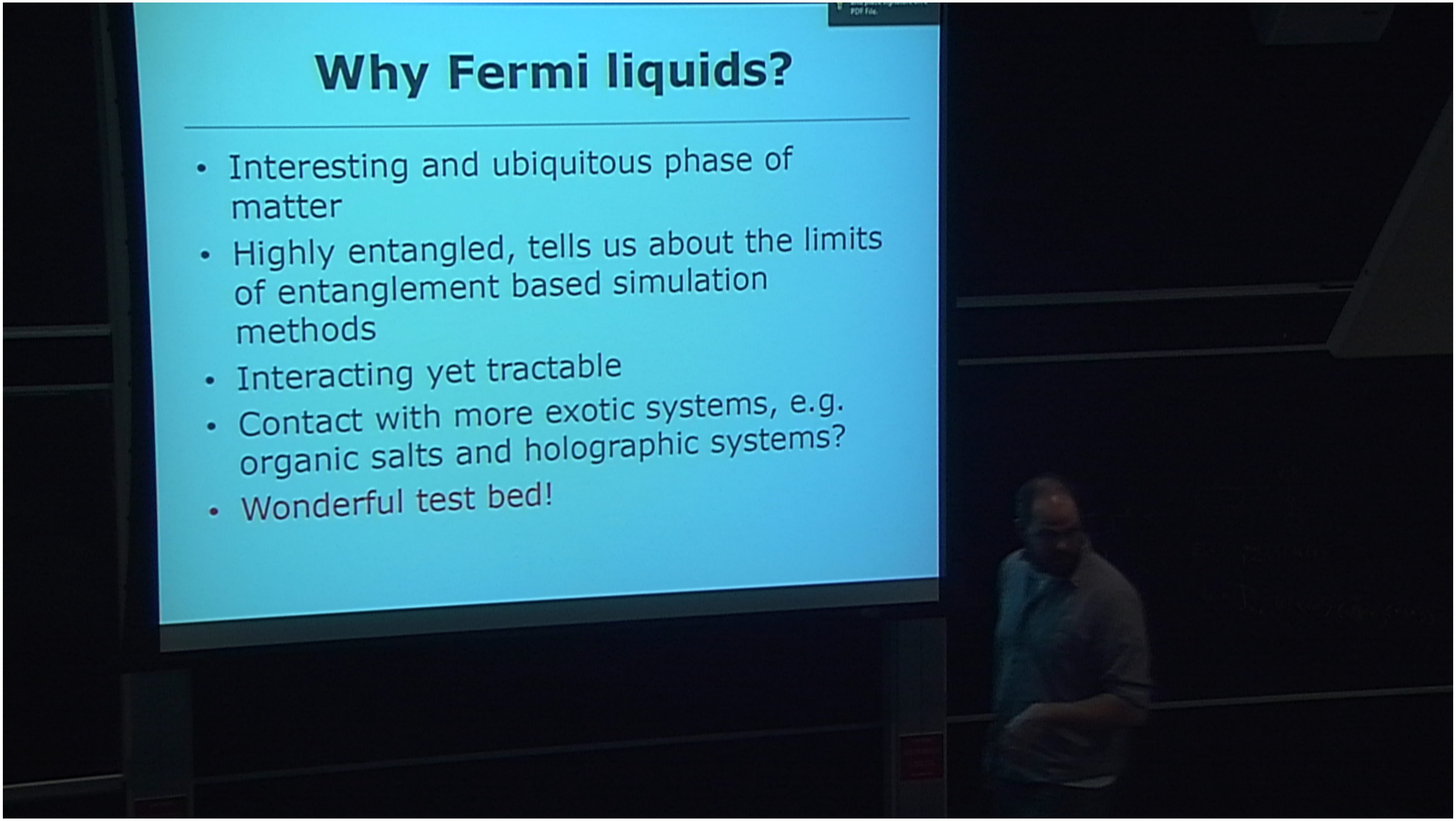
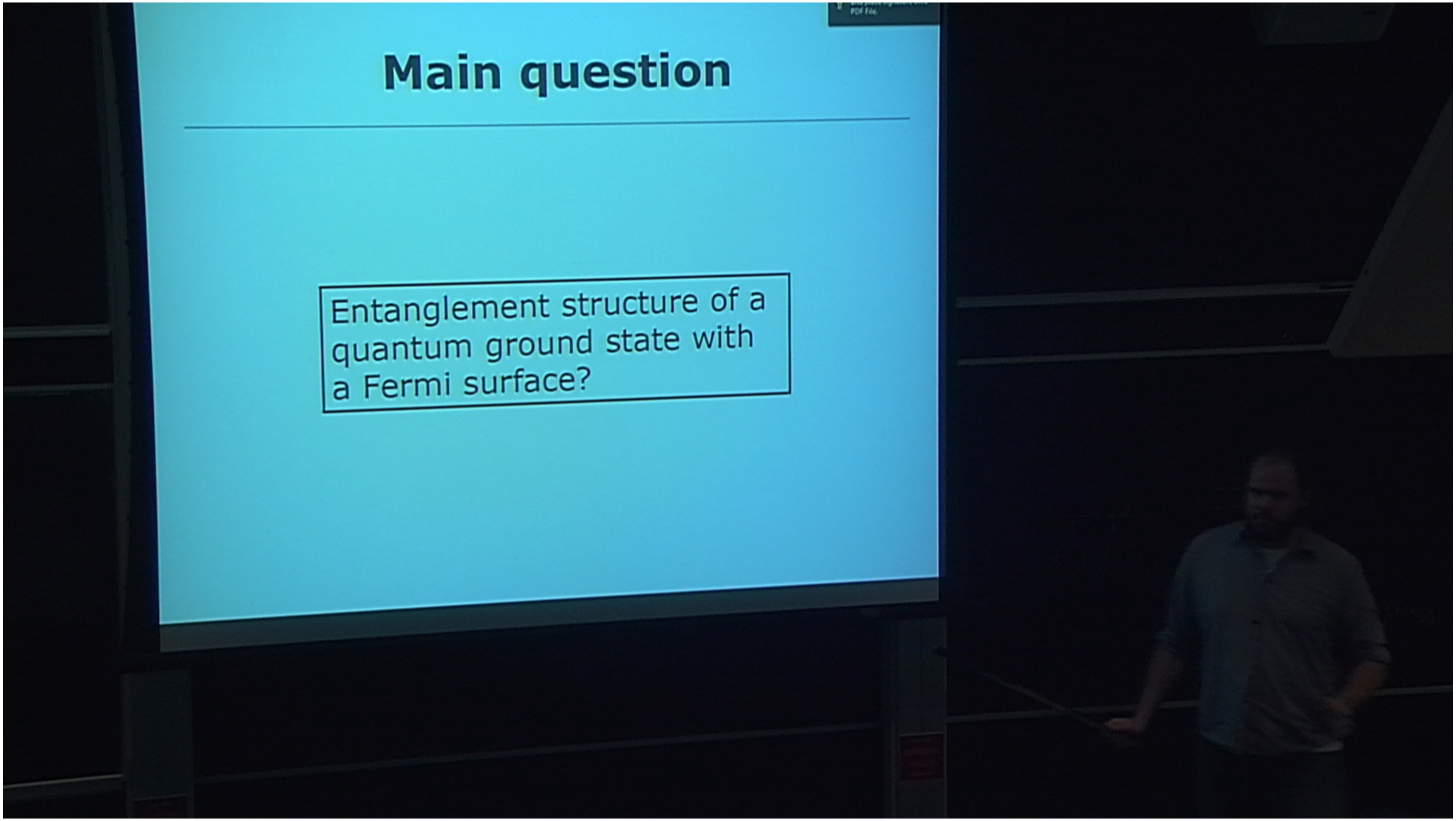
Click on Sign to add text and place signature on a PDF File.

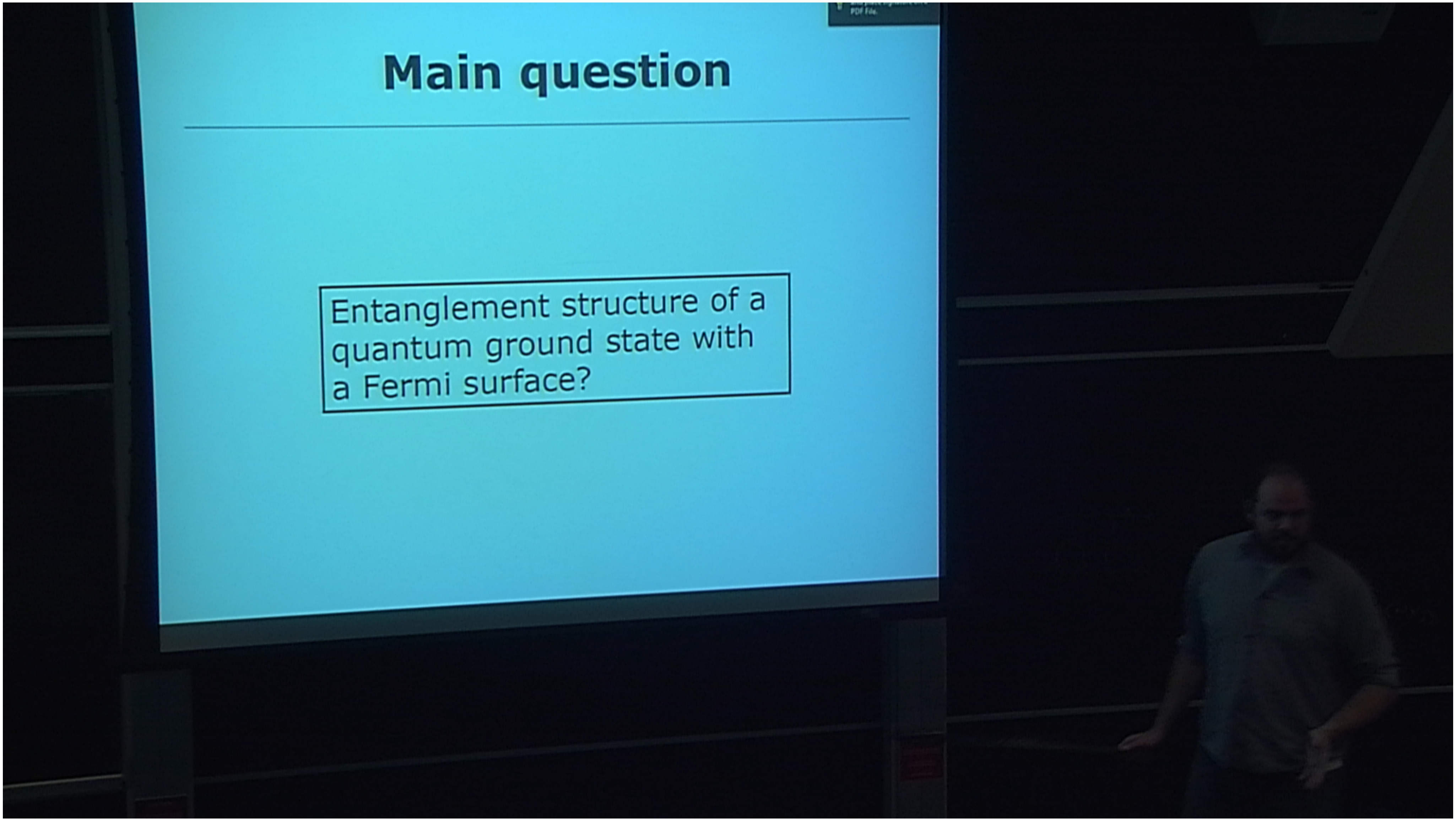
Measuring entanglement

- Possible, in principle, to measure integer Renyi entropy $S_n,\, n > 1$
- Fine print: requires a delicate quantum coherent process involving multiple copies of the system, signal is exponentially small in S_n
- IMPORTANT: classical numerical algorithms also possible, "swap trick"

(QMC Kallin-Isakov-Inglis-Melko-Hastings-...)


Why Fermi liquids?
• Interesting and ubiquitous phase of matter
• Highly entangled, tells us about the limits of entanglement based simulation methods
• Interacting yet tractable
• Contact with more exotic systems, e.g. organic salts and holographic systems?
• Wonderful test bed!


Main question
Entanglement structure of a quantum ground state with a Fermi surface?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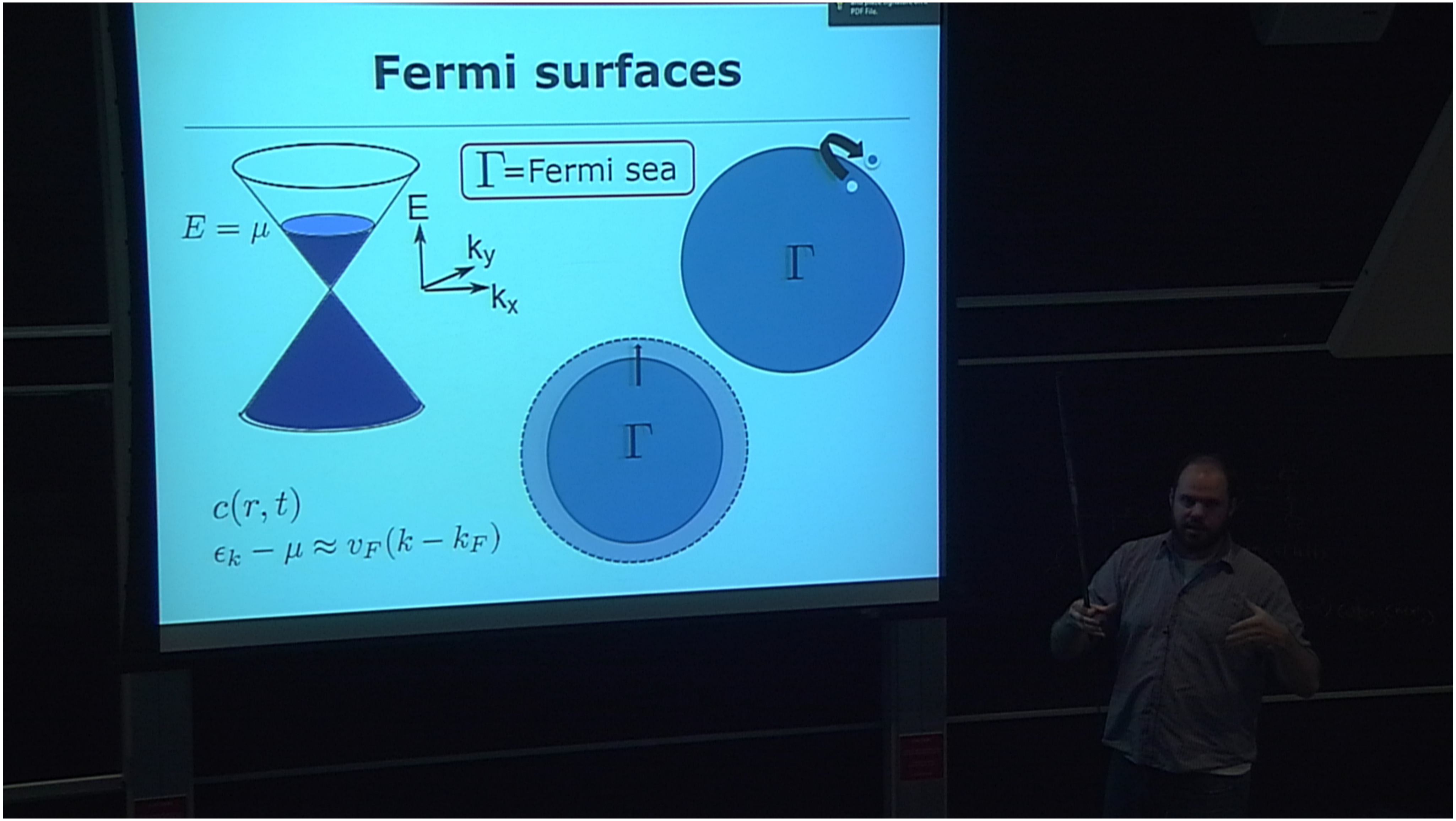
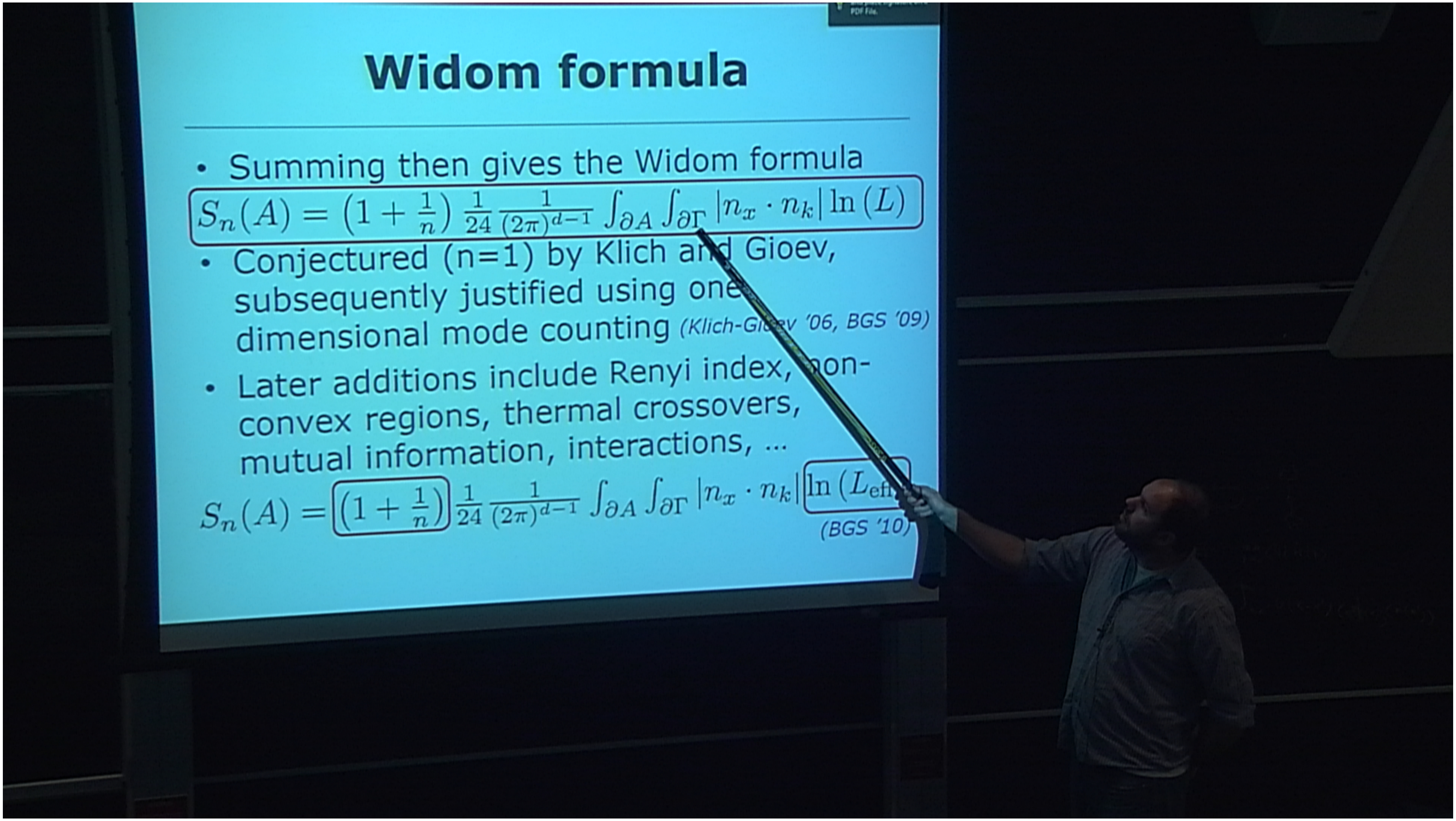
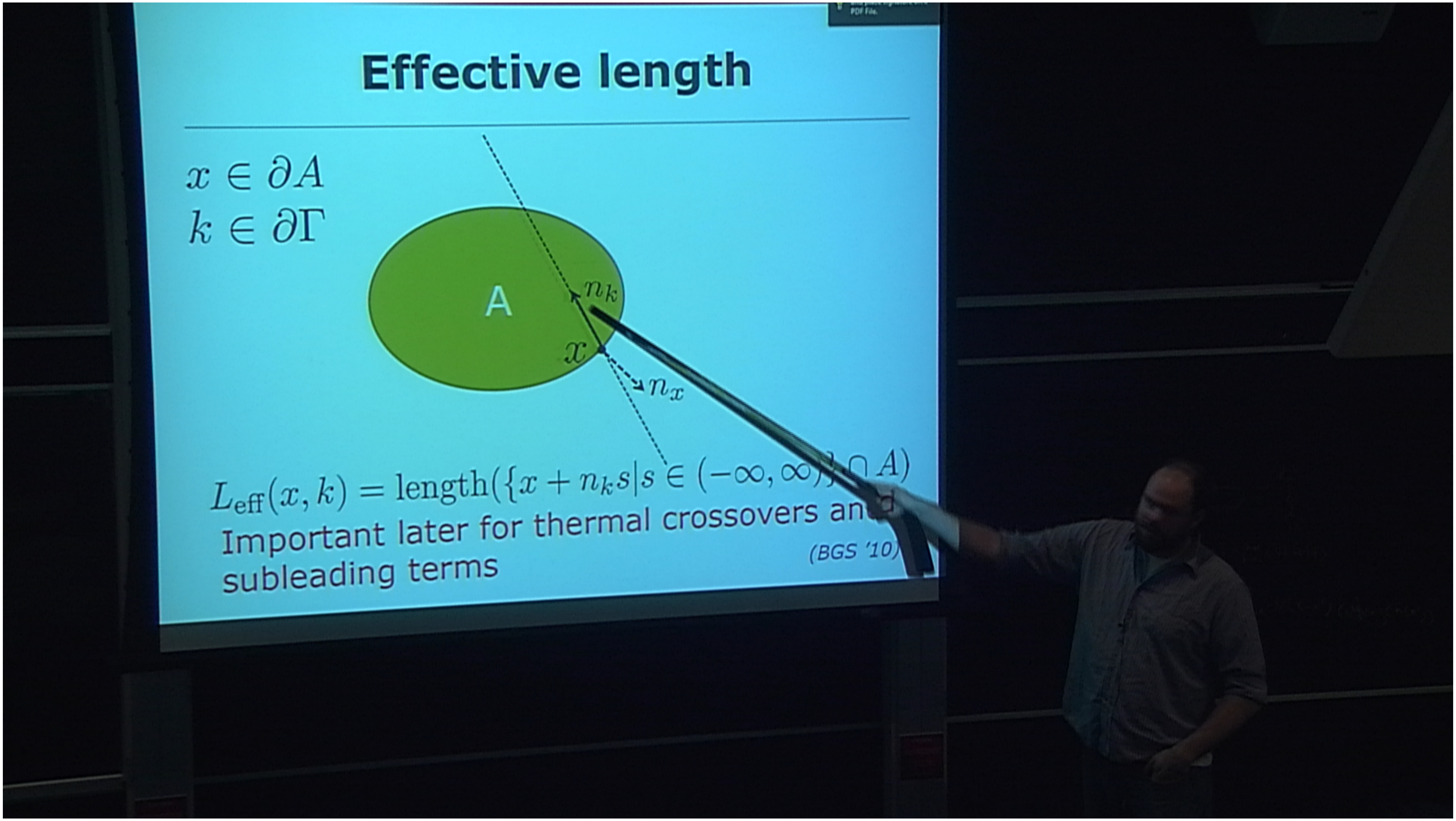
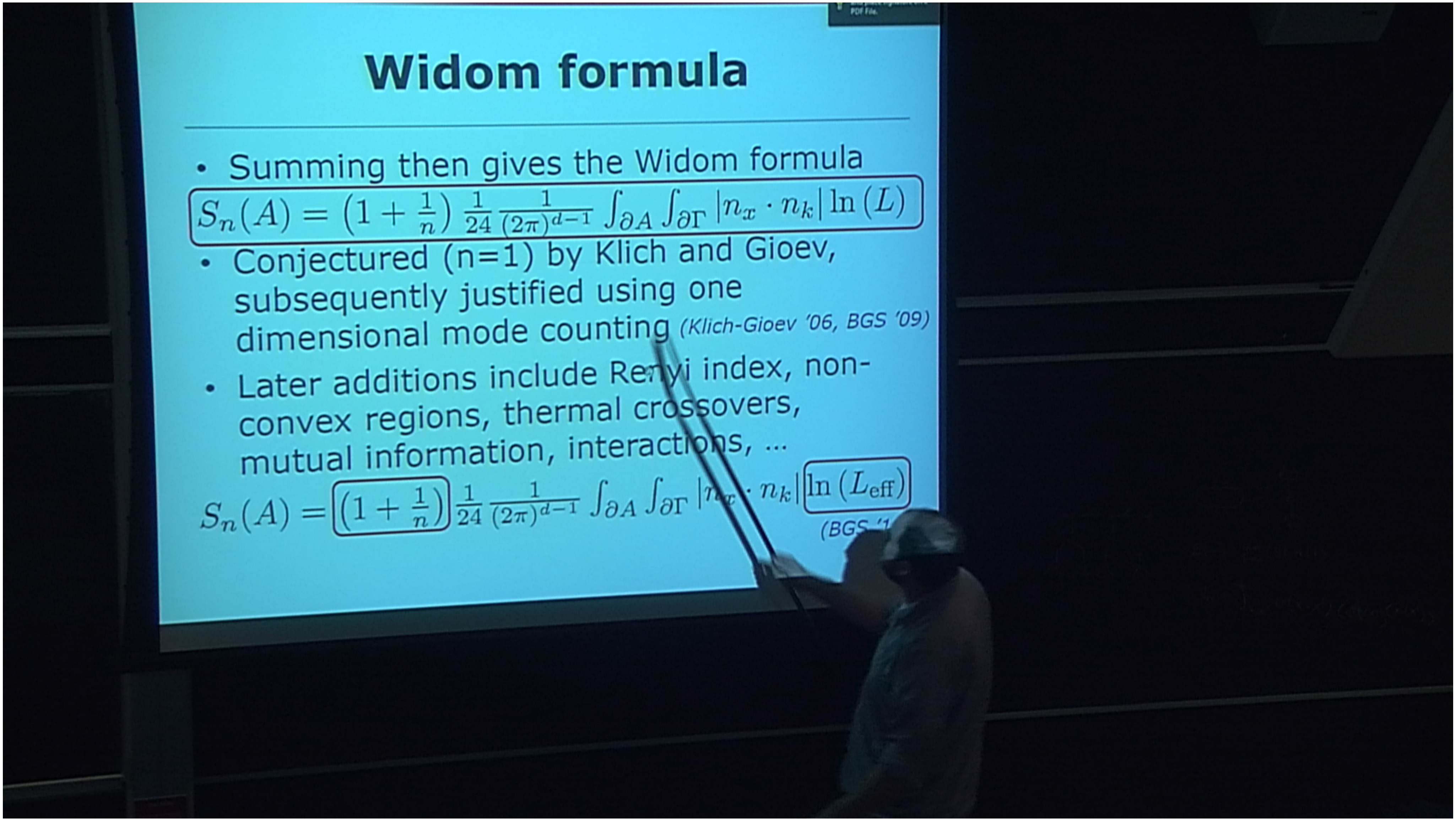

Fermi surfaces
Γ=Fermi sea
$E=\mu$
E
k_y
k_x
Γ
Γ
$c(r,t)$
$\epsilon_k - \mu \approx v_F(k-k_F)$


Widom formula
Summing then gives the Widom formula
$S_n(A) = \left(1+\frac{1}{n}\right)\frac{1}{24}\frac{1}{(2\pi)^{d-1}}\int_{\partial A}\int_{\partial\Gamma}|n_x \cdot n_k| \ln(L)$
Conjectured (n=1) by Klich and Gioev, subsequently justified using one dimensional mode counting (Klich-Gioev '06, BGS '09)
Later additions include Renyi index, non-convex regions, thermal crossovers, mutual information, interactions, …
$S_n(A) = \left(1+\frac{1}{n}\right)\frac{1}{24}\frac{1}{(2\pi)^{d-1}}\int_{\partial A}\int_{\partial\Gamma}|n_x \cdot n_k| \ln(L_{\mathrm{eff}})$
(BGS '10)


Effective length
$x \in \partial A$
$k \in \partial \Gamma$
A
n_k
x
n_x
$L_{\text{eff}}(x,k) = \text{length}(\{x + n_k s | s \in (-\infty, \infty)\} \cap A)$
Important later for thermal crossovers and subleading terms
(BGS '10)



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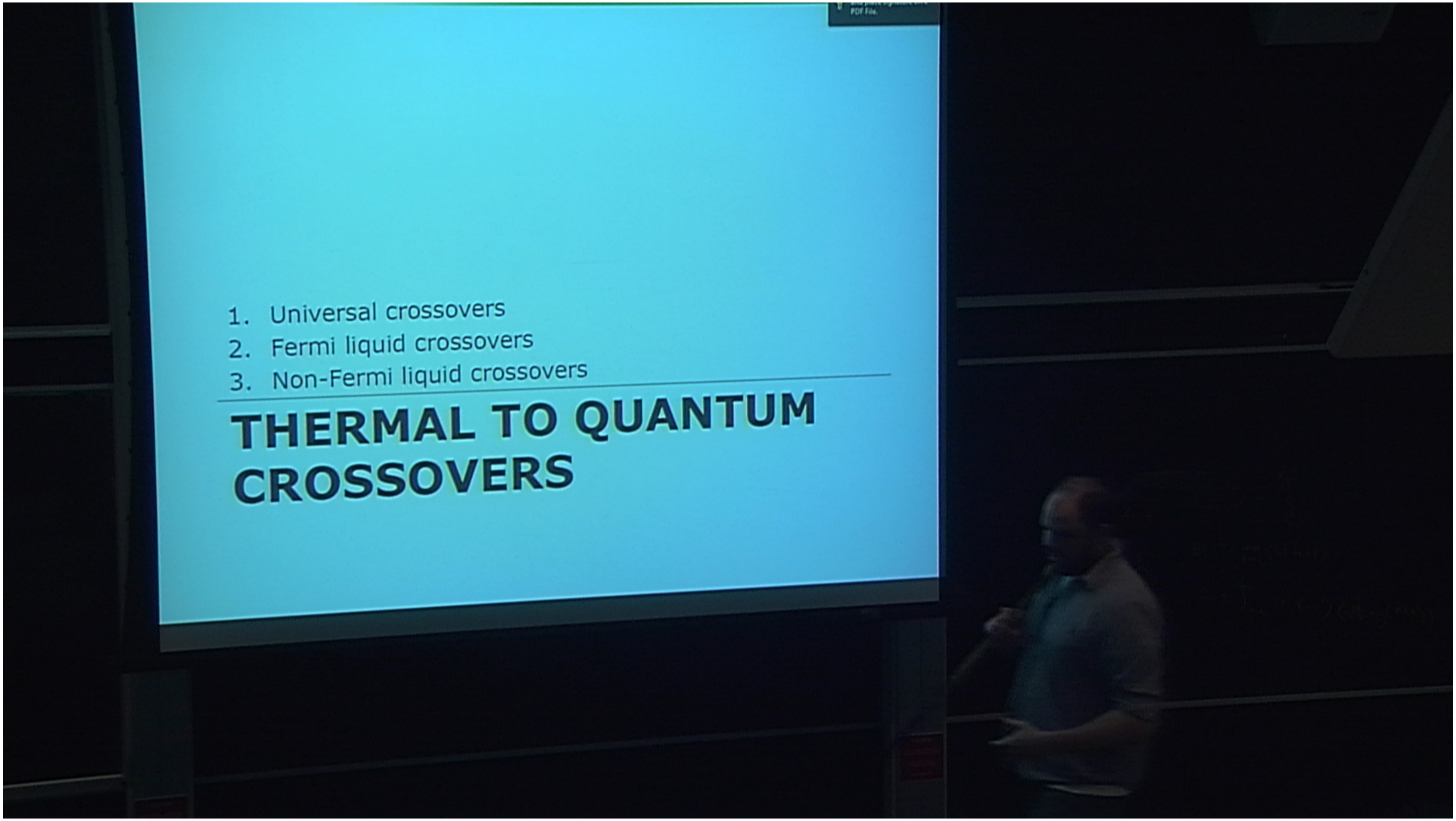
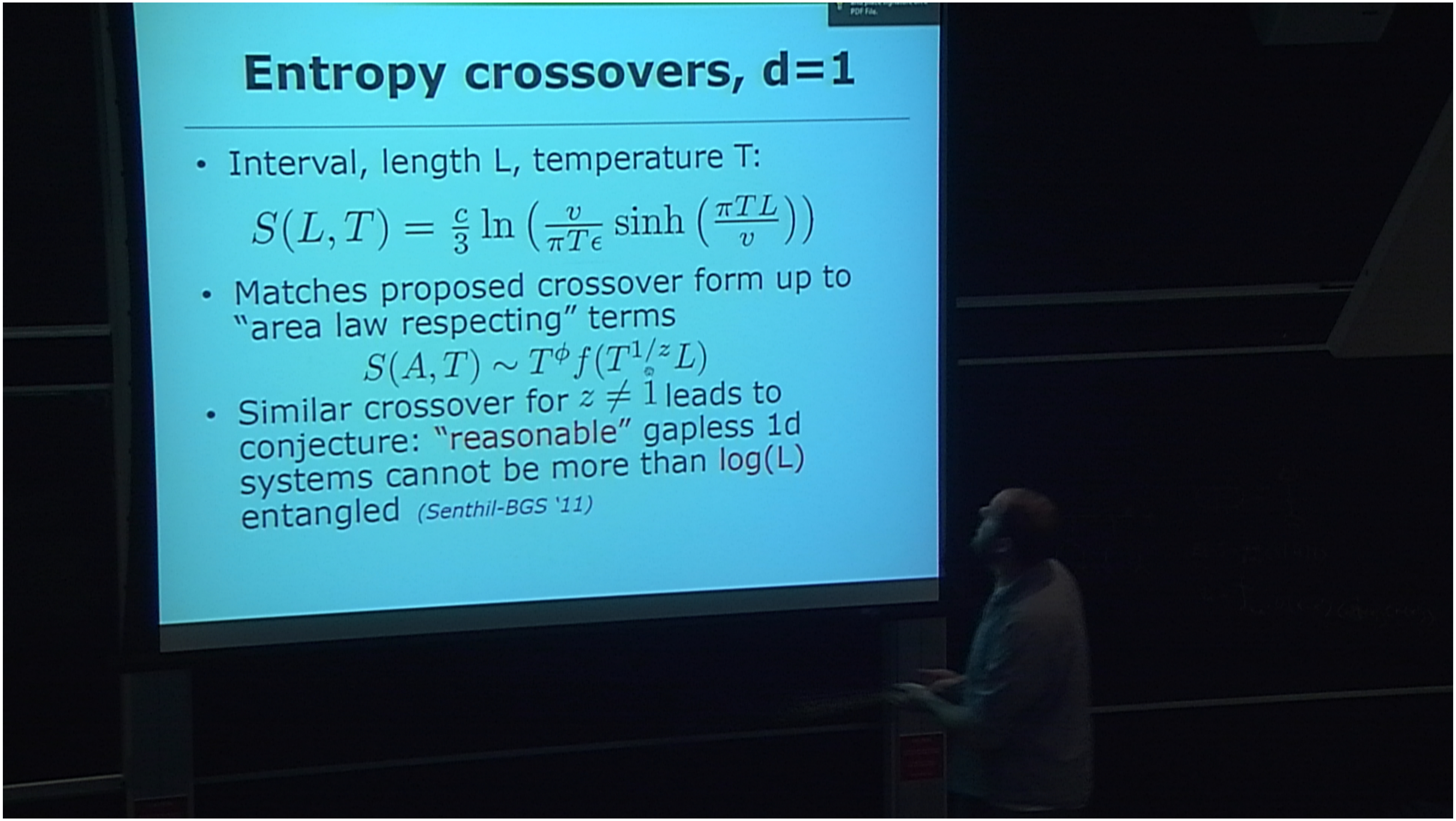
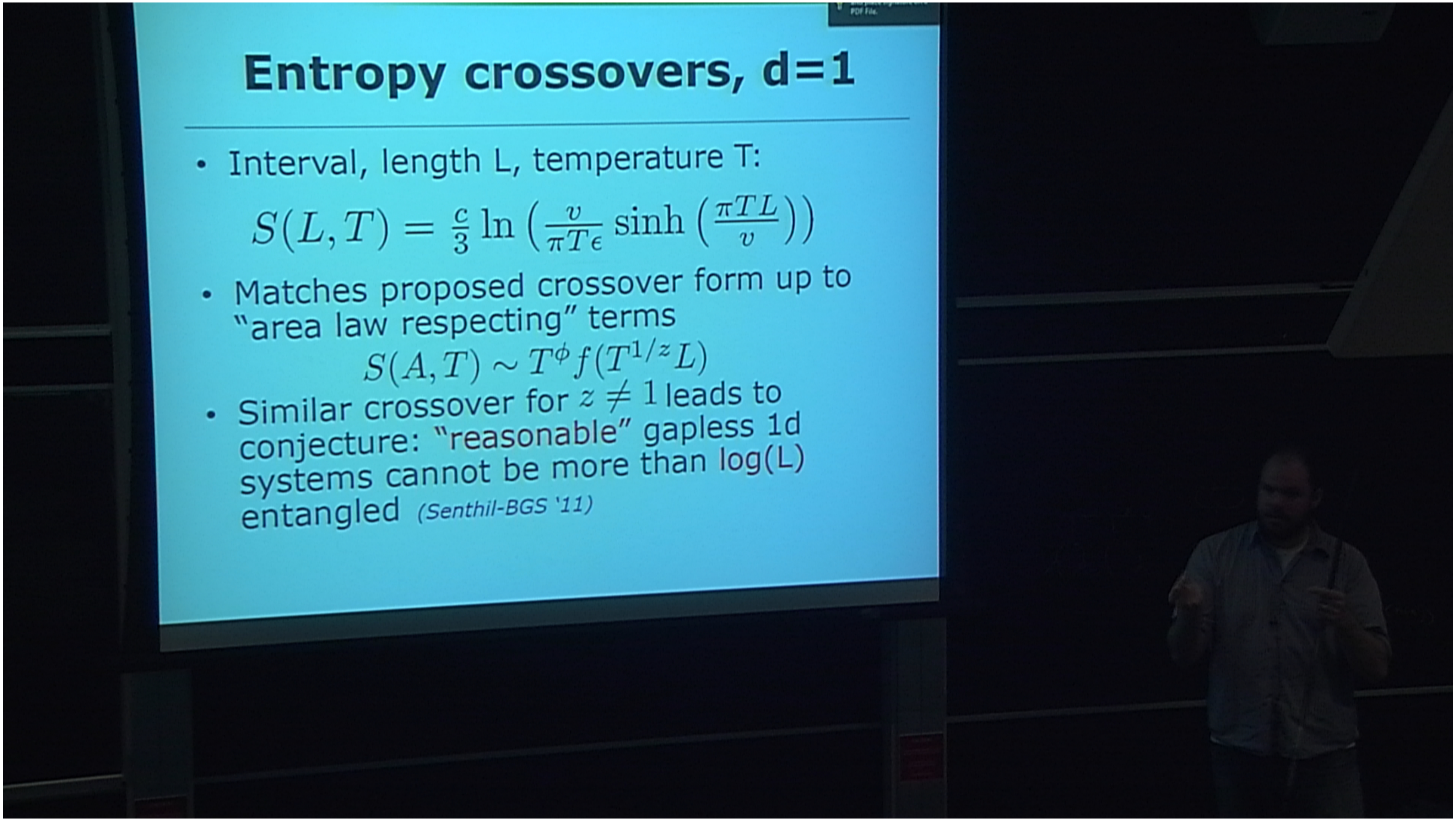

1. Universal crossovers
2. Fermi liquid crossovers
3. Non-Fermi liquid crossovers
THERMAL TO QUANTUM CROSSOVERS


Entropy crossovers, d=1
• Interval, length L, temperature T:
$S(L,T) = \frac{c}{3} \ln\left(\frac{v}{\pi T \epsilon} \sinh\left(\frac{\pi T L}{v}\right)\right)$
• Matches proposed crossover form up to "area law respecting" terms
$S(A,T) \sim T^{\phi} f(T^{1/z} L)$
• Similar crossover for $z \neq 1$ leads to conjecture: "reasonable" gapless 1d systems cannot be more than log(L) entangled
(Senthil-BGS '11)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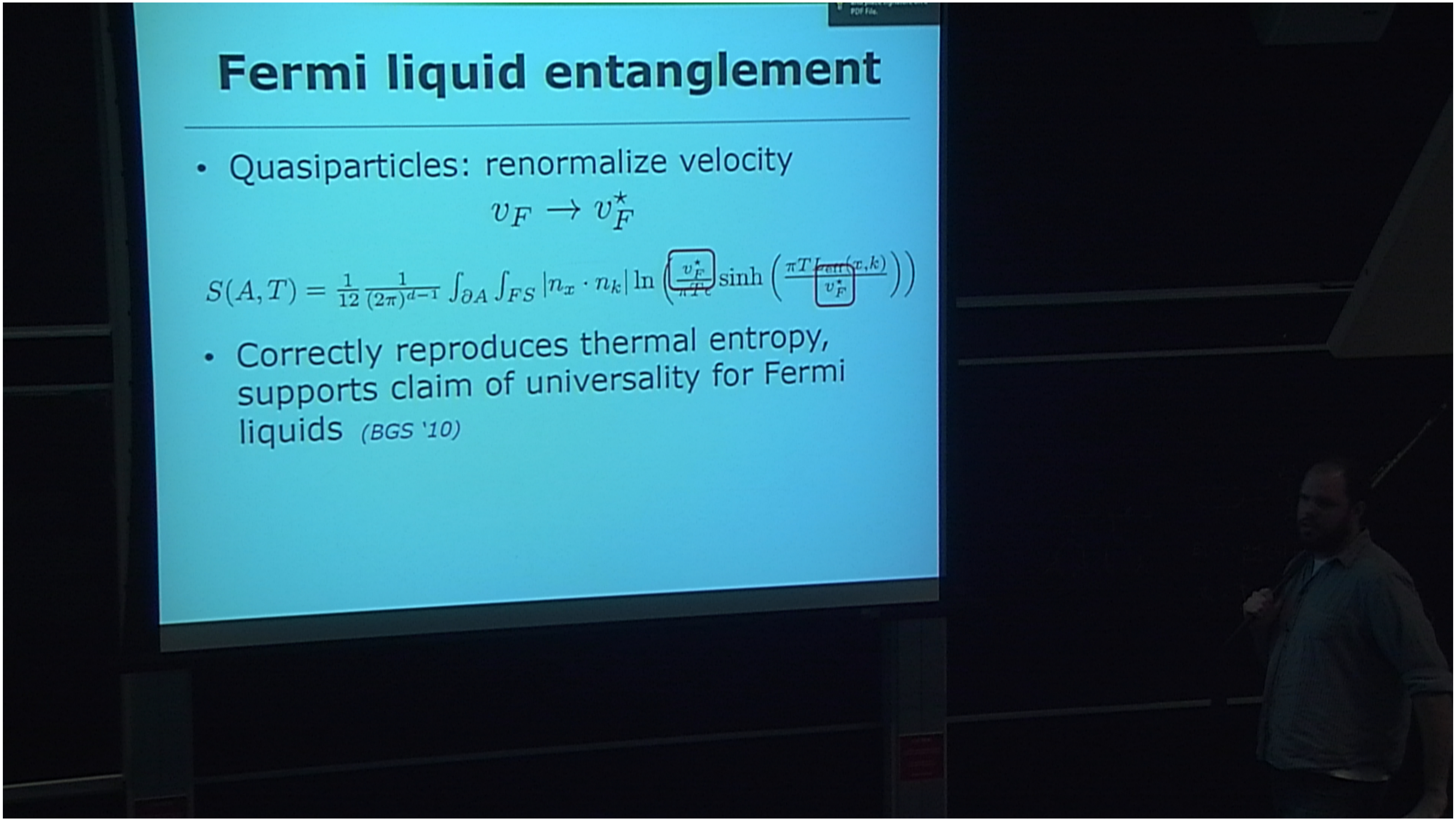
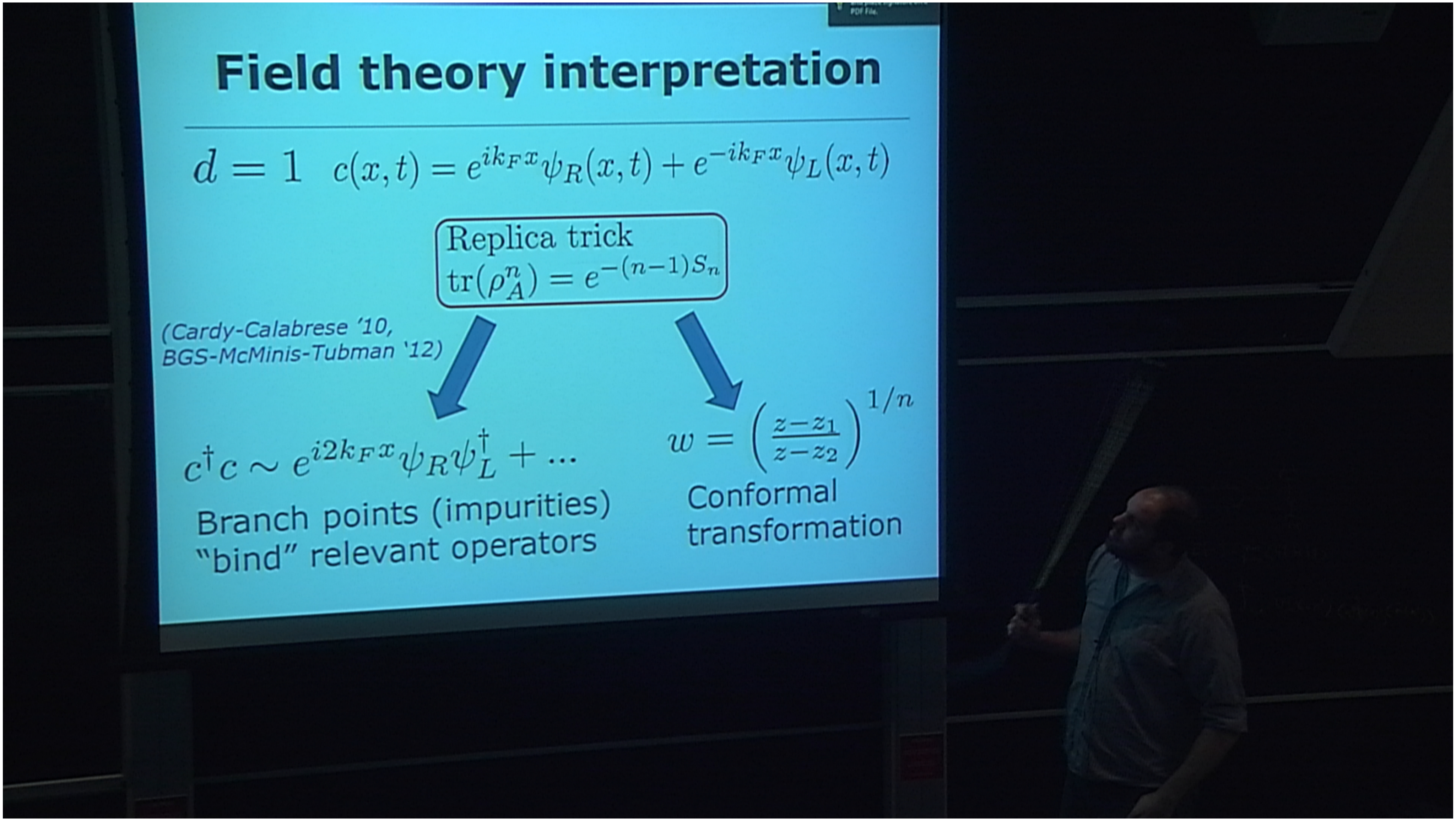

Fermi liquid entanglement
• Quasiparticles: renormalize velocity
$v_F \to v_F^\star$
$S(A,T) = \frac{1}{12}\frac{1}{(2\pi)^{d-1}}\int_{\partial A}\int_{FS}|n_x \cdot n_k| \ln\left(\frac{v_F^\star}{\pi T c}\sinh\left(\frac{\pi T L_{eff}(x,k)}{v_F^\star}\right)\right)$
• Correctly reproduces thermal entropy, supports claim of universality for Fermi liquids (BGS '10)


Field theory interpretation
$d = 1 \quad c(x,t) = e^{ik_F x}\psi_R(x,t) + e^{-ik_F x}\psi_L(x,t)$
Replica trick
$\mathrm{tr}(\rho_A^n) = e^{-(n-1)S_n}$
(Cardy-Calabrese '10,
BGS-McMinis-Tubman '12)
$c^\dagger c \sim e^{i2k_F x}\psi_R\psi_L^\dagger + \ldots$
Branch points (impurities)
"bind" relevant operators
$w = \left(\frac{z-z_1}{z-z_2}\right)^{1/n}$
Conformal
transformation

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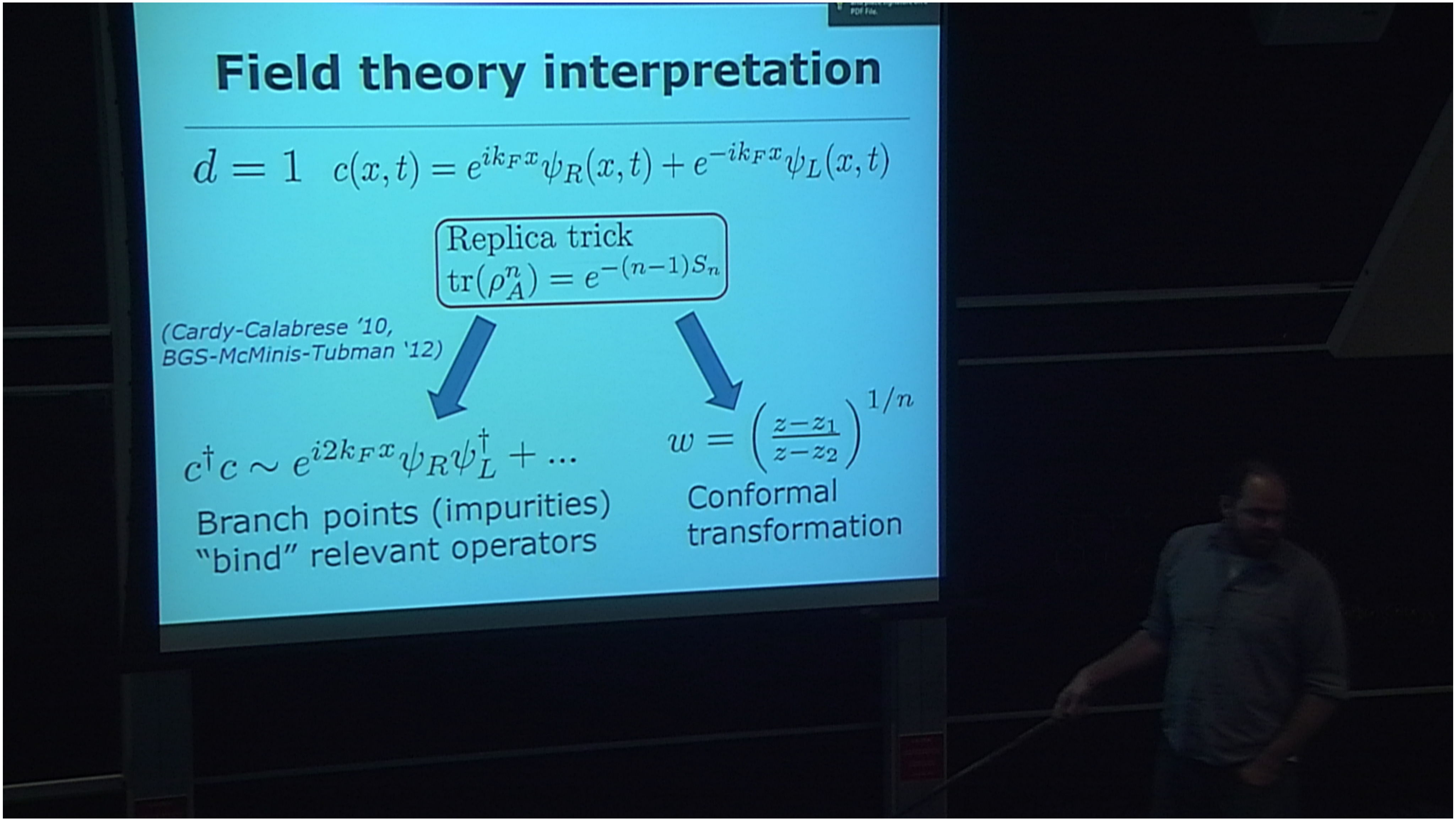
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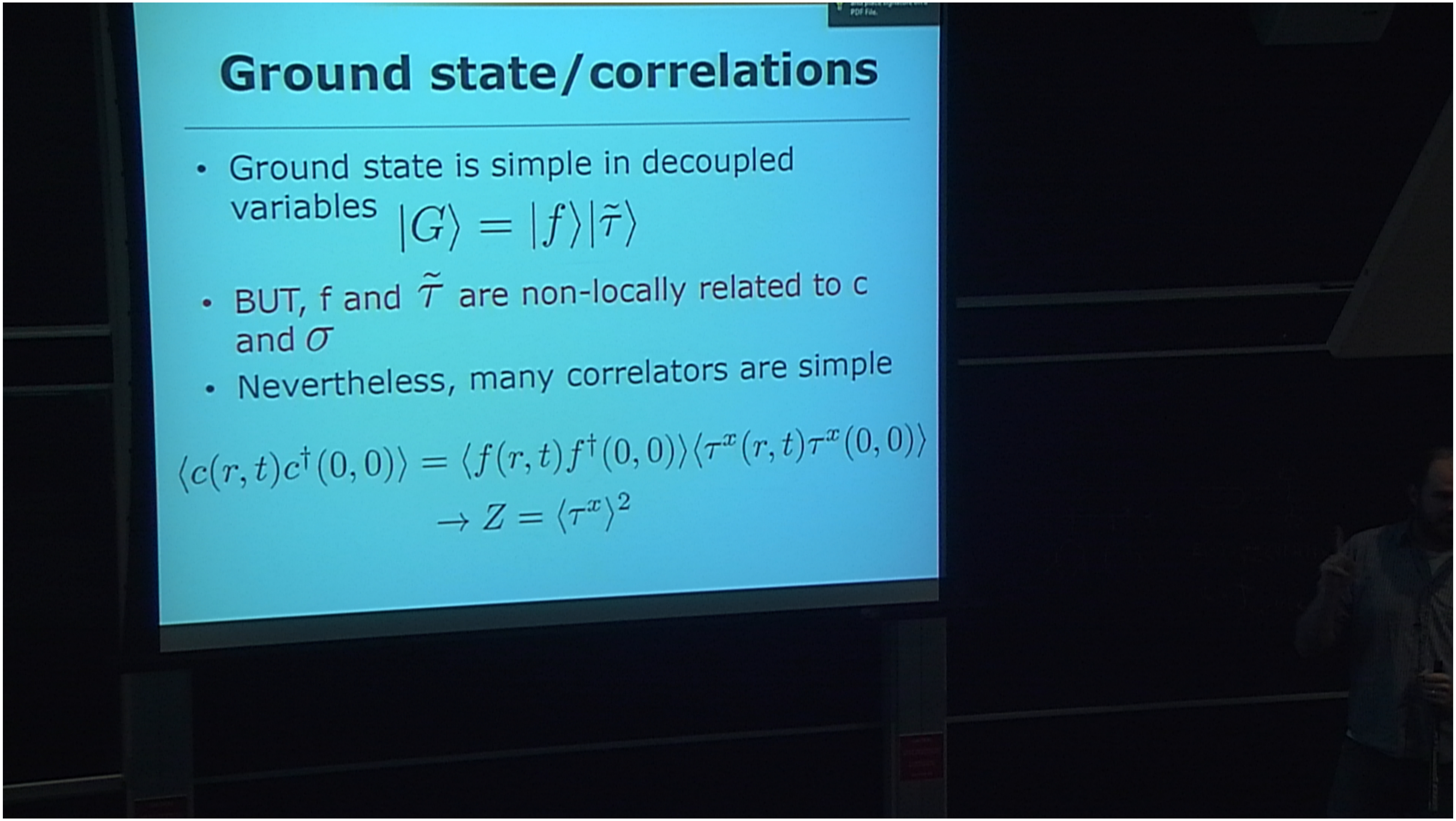
Ground state/correlations
Ground state is simple in decoupled variables $|G\rangle = |f\rangle|\tilde{\tau}\rangle$
BUT, f and $\tilde{\tau}$ are non-locally related to c and σ
Nevertheless, many correlators are simple
$\langle c(r,t)c^\dagger(0,0)\rangle = \langle f(r,t)f^\dagger(0,0)\rangle\langle\tau^x(r,t)\tau^x(0,0)\rangle$
$\rightarrow Z = \langle\tau^x\rangle^2$