Title: Fractional quantum Hall effect and tunable interactions

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Abstract: <span>In this talk I will review some existing experimental methods, as well as a few recent theoretical proposals, to tune the interactions in a number of low-dimensional systems exhibiting the fractional quantum Hall effect (FQHE). The materials in question include GaAs wide quantum wells and multilayer graphene, where the tunability of the electron-electron interactions can be achieved via modifying the band structure, dielectric environment of the sample, by tilting the magnetic field or varying the mass tensor, and by mixing of electronic subbands and Landau levels.

Because the interesting topological (and in particular, non-Abelian) states arise solely due to strong interactions, the ability to tune them is essential for ``designing'' more robust FQHE states. Furthermore, I will argue that some of these mechanisms can also be used to probe the subtle aspects of FQHE physics, such as the breaking of particle-hole symmetry between the Moore-Read Pfaffian and anti-Pfaffian states, and the transition between FQHE fluids and broken-symmetry states due to the fluctuation of the intrinsic geometric degree of freedom.</span>

# Fractional quantum Hall effect and tunable interactions

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Princeton University

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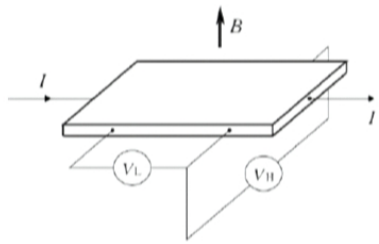
Perimeter, 11/29/2012

PRINCETON UNIVERSITY

# Outline

- Introduction to fractional quantum Hall effect: old history and recent developments
- Tunability of the effective electron-electron interactions in various materials supporting FQHE and why it is useful
- Wide quantum wells and mixing of subbands/Landau levels: a probe for particle-hole symmetry breaking between the Moore-Read Pfaffian and ``anti-Pfaffian" states

# Fractional quantum Hall effect



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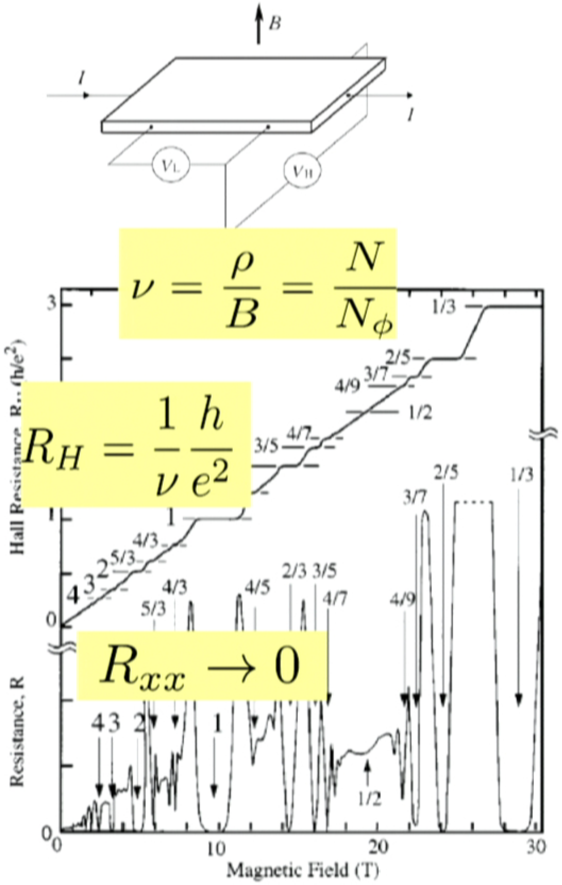
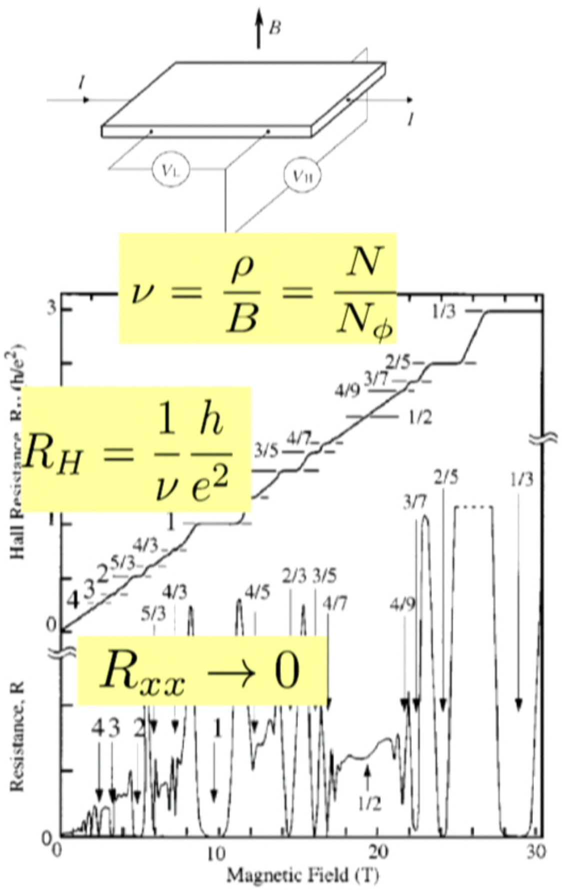



$$\nu = \frac{\rho}{B} = \frac{N}{N_\phi}$$

$$R_H = \frac{1}{\nu}\frac{h}{e^2}$$

$$R_{xx} \to 0$$

# Fractional quantum Hall effect

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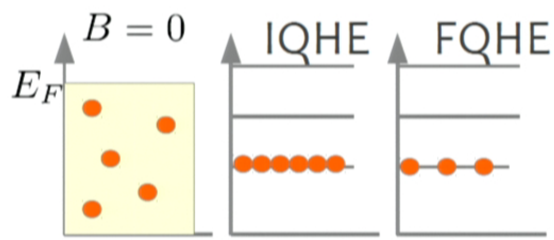
- Integer vs Fractional Hall Effect




$$H = \text{const} + \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

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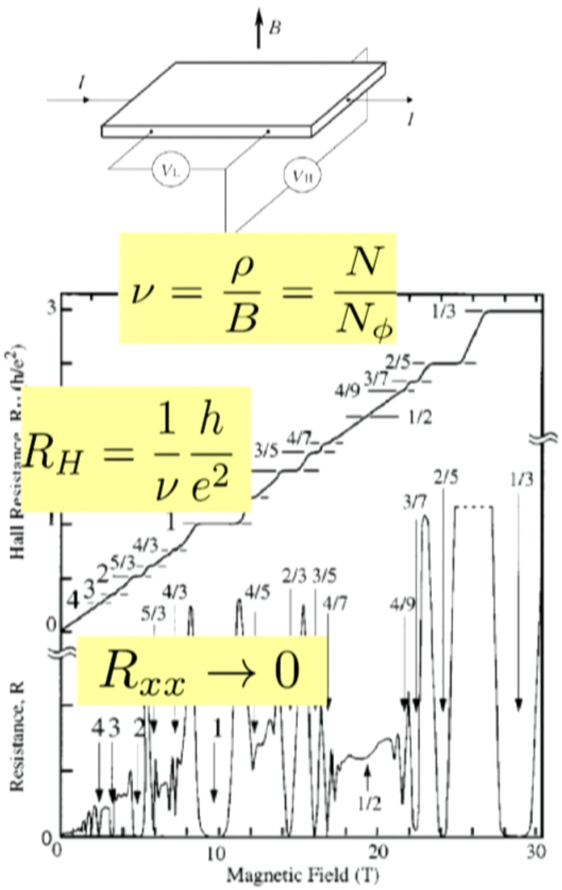
# Fractional quantum Hall effect

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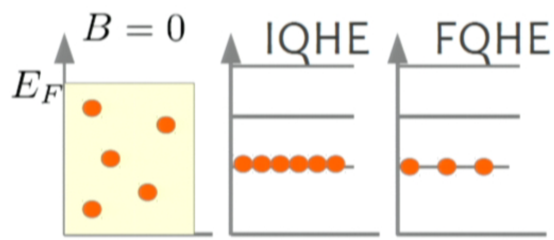


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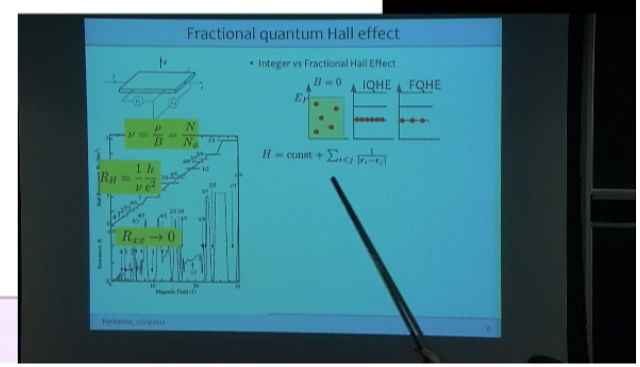

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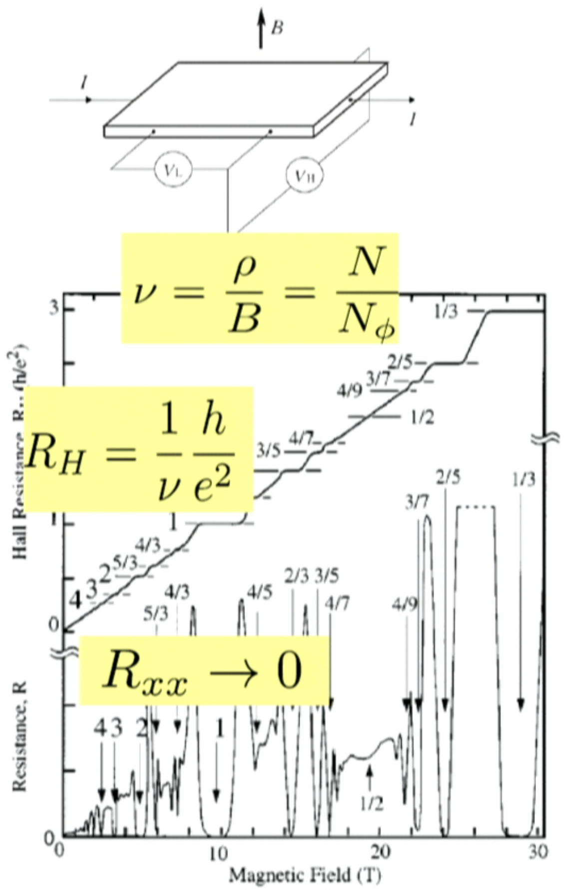


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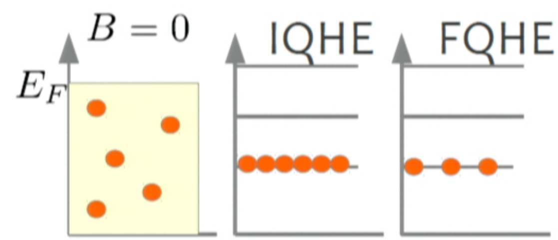



$$H = \text{const} + \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

- Trial wavefunctions

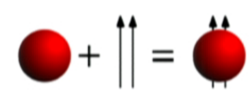
$$\Psi = P(z_1, z_2, ..., z_N)\exp(-\sum_i |z_i|^2/4\ell_B^2)$$

because one-body states are

$$\phi_m(\text{“}z = x + iy\text{”}) \sim z^m \exp(-|z|^2/4\ell_B^2)$$

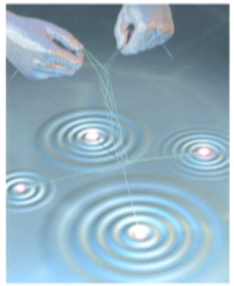
- Laughlin wavefunction: $\Psi_{1/3} = \prod_{i<j}(z_i - z_j)^3$

- “Composite fermions” (Jain) or Haldane/Halperin hierarchy

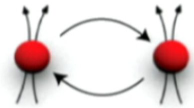


# Topological order [Wen]

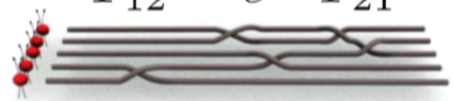
- Gapped many-body states, degenerate on torus, fractionalized excitations – "anyons"



Sci Am '06



$$\Psi_{12} = e^{i\phi}\Psi_{21}$$



- Chern-Simons field theory $\mathcal{L}_0 = \frac{\hbar}{4\pi pq}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - J^\mu(a_\mu - \frac{pe}{\hbar}A_\mu)$

  [Zhang, Hansson, Kivelson]

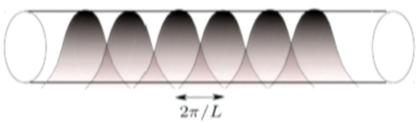
  ``composite bosons"

$$J^\mu = \frac{1}{2\pi pq}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda \qquad J^0 = \frac{e}{2\pi\hbar q}B \qquad (\nu = \frac{p}{q})$$

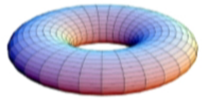
- Incompressible fluids in the bulk, chiral gapless modes at the edge [Wen]
- Girvin-MacDonald-Platzman algebra ('85): $[\rho_{\mathbf{q}}, \rho_{\mathbf{q}'}] = 2i\sin(\frac{1}{2}\mathbf{q}\times\mathbf{q}'\ell_B^2)\rho_{\mathbf{q}+\mathbf{q}'}$
- However, TFT has no knowledge of energetics (cannot compute the energy gap), and does not obviously reproduce GMP algebra

# Numerical approaches: Exact diagonalization

- Richness of boundary conditions (disk, cylinder, sphere, torus) gives complementary insights into the physics







- Recipe: $H = \sum_{i<j} \sum_{m=1,3,\ldots} V_m P_{ij}^m$ (finite matrix – diagonalize it; **DiagHam**)
- Model wavefunctions/Hamiltonians, overlaps, entanglement spectrum [Li,Haldane '08]

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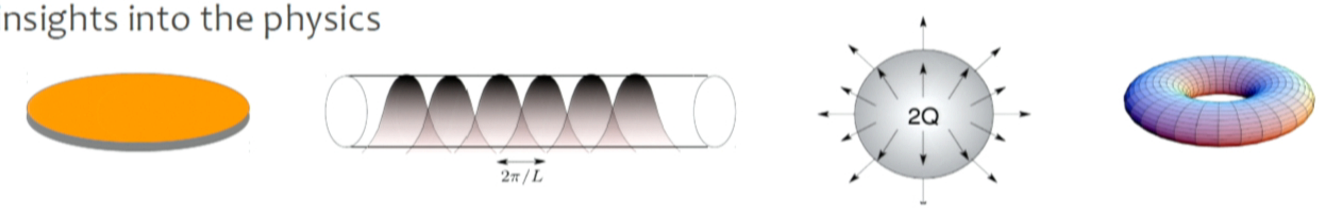


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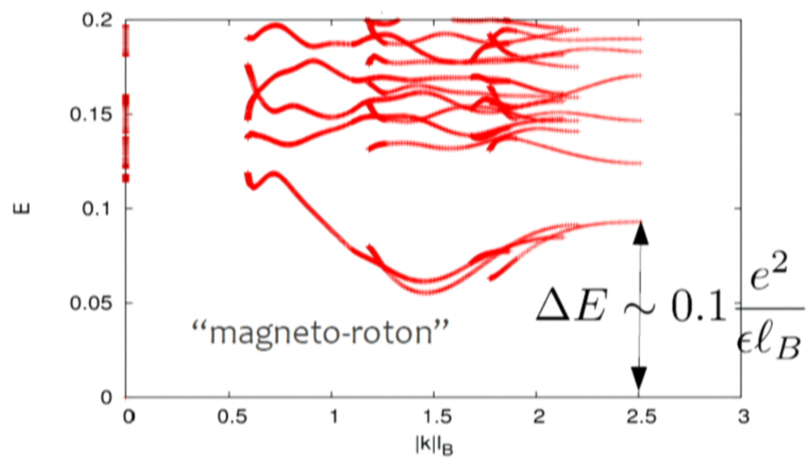
- Model wavefunctions/Hamiltonians, overlaps, entanglement spectrum [Li,Haldane '08]
- Among hard problems in physics, FQHE is the one where ED has been truly useful

Laughlin 1/3 state
**(for 4 particles!)**



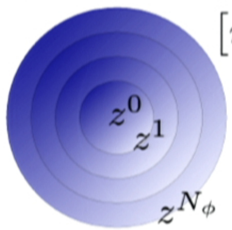
# Analytical knowledge about many-body WFs

- Trial wavefunctions can sometimes be expressed as correlators of conformal field theories; CFT can motivate new trial wavefunctions

$$\langle \prod_i \mathcal{V}_{\alpha_i}(z_i) \rangle = \prod_{j<k} (z_j - z_k)^{\alpha_j \alpha_k} \qquad \mathcal{V}_\alpha(z) = e^{i\alpha\phi(z)} \qquad \langle \phi(z)\phi(z') \rangle = -ln(z - z')$$

- They are highly structured (little entanglement) and relate to the Jack polynomials [Bernevig,Haldane '08]

$$(z_1 - z_2)^3 = (z_1^3 - z_2^3) - 3(z_1^2 z_2 - z_1 z_2^2)$$




$[n_0, n_1, \ldots, n_{N_\phi}]$

1100000011 ← do not exist
1010000101
...
1001001001 ← root: $\mathcal{A}\{z_1^0 z_2^3 z_3^6 z_4^9\}$
0110001001
0110000110
0101001010

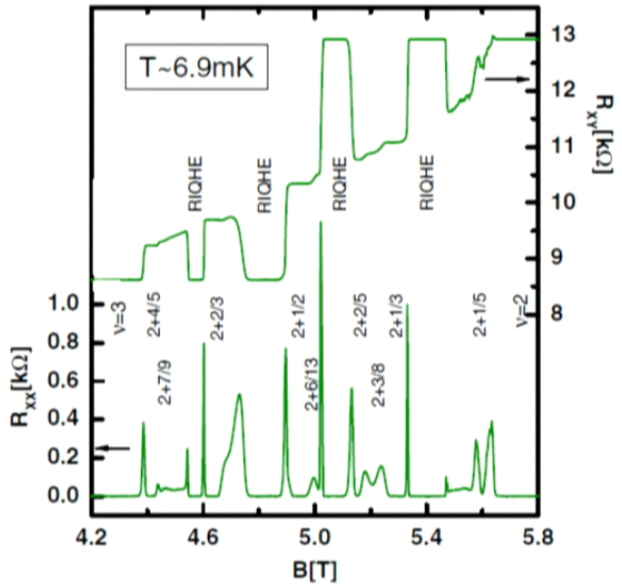
- The wavefunction encodes clustering conditions: how it vanishes when clusters of particles are brought to the same point ("pattern of zeros" [Wen, Wang '08])

- Their decomposition in Fock space can be computed recursively, without the need for diagonalization

- Jacks are related to matrix-product-state description [Zalatel, Mong, Pollmann '12]

# Non-Abelian physics in higher Landau levels

Kumar *et al*, '10




- **Pfaffian** state $\nu = 2 + 1/2$ [Moore, Read '91]

$$\Psi_{\mathrm{Pf}} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2$$

- example of a non-Abelian Ising phase
- Possible other states in the **parafermion** series [Read, Rezayi '99] e.g. $\nu = 2 + 2/5$
- Non-Abelian hierarchy states? $\nu = 2 + 6/13$
- Other candidates: BS states [Bonderson, Slingerland '08], "bipartite CF" [Jain *et al*, '11]

- There is not as much theoretical understanding of FQHE in higher LLs
- Main obstacles: **LL mixing, many compressible phases** (stripes, bubbles) so the incompressible states tend to sit very close to phase transitions
- But at least for $\nu = 2 + 1/2$ the evidence seems pretty solid

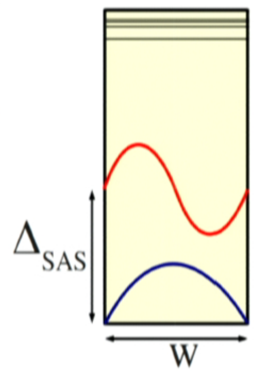
# Latest developments and open problems

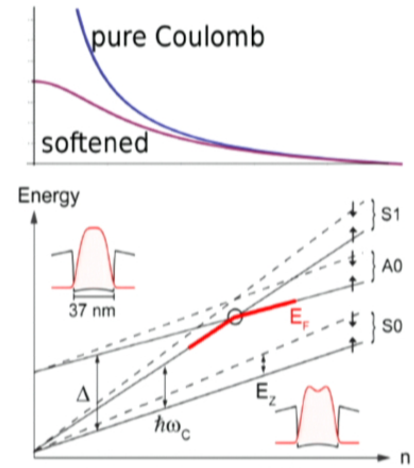
- Experimental proof that 5/2 is **non-Abelian** – interferometry experiments
- Particle-hole symmetry breaking: **Pfaffian vs. ``anti-Pfaffian"** (more on this later)
- Is **2+2/5** non-Abelian? (Read-Rezayi level 3 state with Fibonacci anyons)
- Reconciliation between **edge physics** (predicted to be universal) and experiments
- **Spin** and **multicomponent** physics in general not very well known (notable exception: excitonic superfluid in quantum Hall bilayer at total filling 1 [Eisenstein group])
- Rich internal symmetry gives rise to **unexplained hierarchy of FQH states in graphene** [Yacoby group, '11]
- Is there an analog of 5/2 state in graphene?
- Interplay of topology and **quantum geometry** in FQHE [Haldane, '11]
- Coexistence of **topological order** and **broken symmetry**? [Xia *et al.*, '11]
- How does FQHE fit into the general framework of strongly correlated systems? (relation to Hubbard model, matrix-product-state/tensor network description)
- Realization in **lattice** systems with topological flat bands? [Haldane, '88]

# How does one tune the interaction in GaAs?

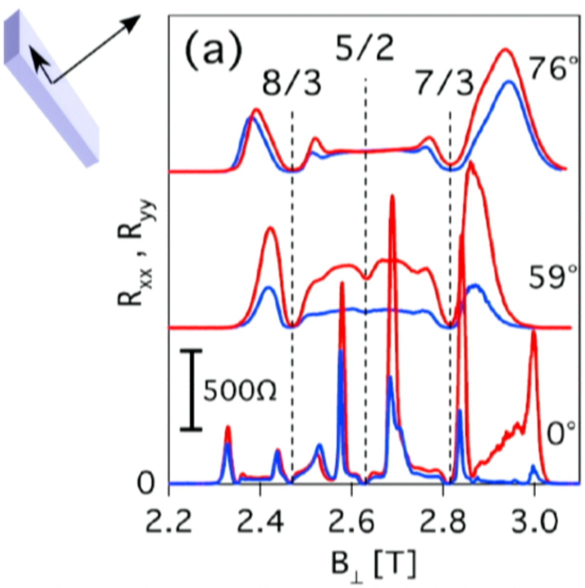
- Play with the **sample width** in z-direction

  (softens the Coulomb repulsion, may be favorable for non-Abelian states [Peterson et al., '08]) but mixes in higher subbands




[Shayegan group, '11]



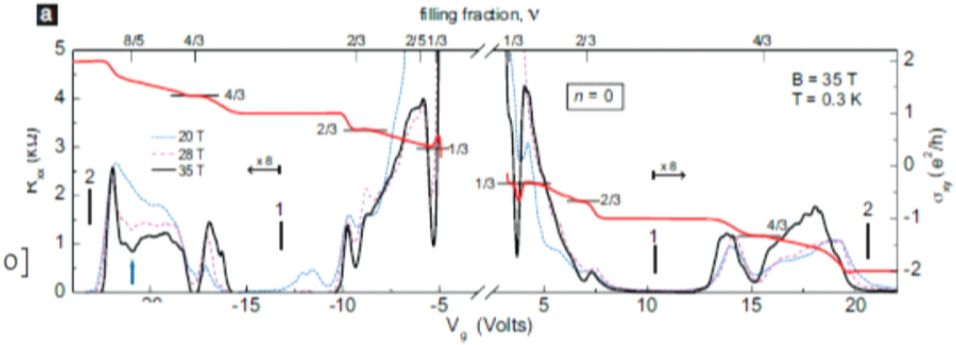

J. Xia et al., PRL **105**, 176807 (2010)

- **Tilt** the magnetic field

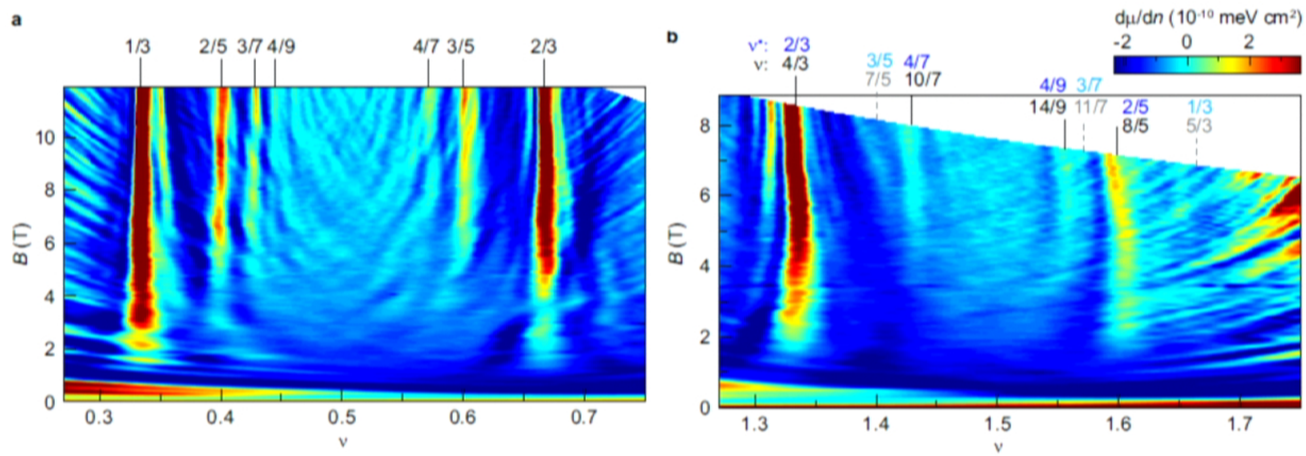
  (produces anisotropy, enhances LL mixing [Bishara, Nayak, '08; Wojs *et al.*, '10; Rezayi and Simon, '11])

# FQHE in Dirac materials

- First observed in 2009 [Eva Andrei group, P. Kim group]
- Expected in bilayer/trilayer graphene, topological insulators...
- Intriguing symmetry breaking effects




[P. Kim group, '10]

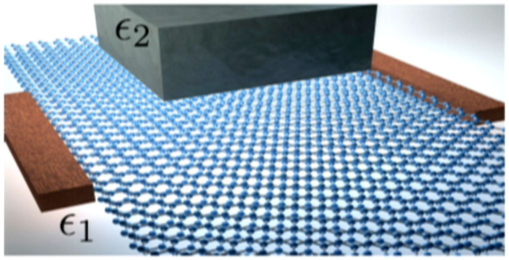



[Yacoby group,'11]

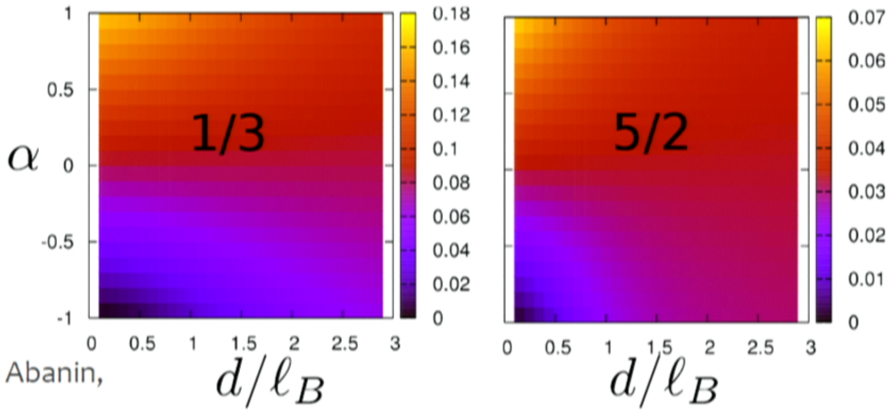
# Tunable interactions in graphene

- Because of their exposed surface, graphene-like materials allow for modification of the effective interaction via dielectric screening [ZP, R. Thomale, and D. Abanin, '11]

$$V(r) = \frac{e^2}{\epsilon_1 r} + \alpha \frac{e^2}{\epsilon_1 \sqrt{r^2 + d^2}} \qquad \left(\alpha = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2}\right)$$




- This setup allows for a broad tunability of the excitation gaps of Abelian and non-Abelian states




[ZP, R. Thomale, and D. Abanin, PRL **107**, 176602 (2011)]

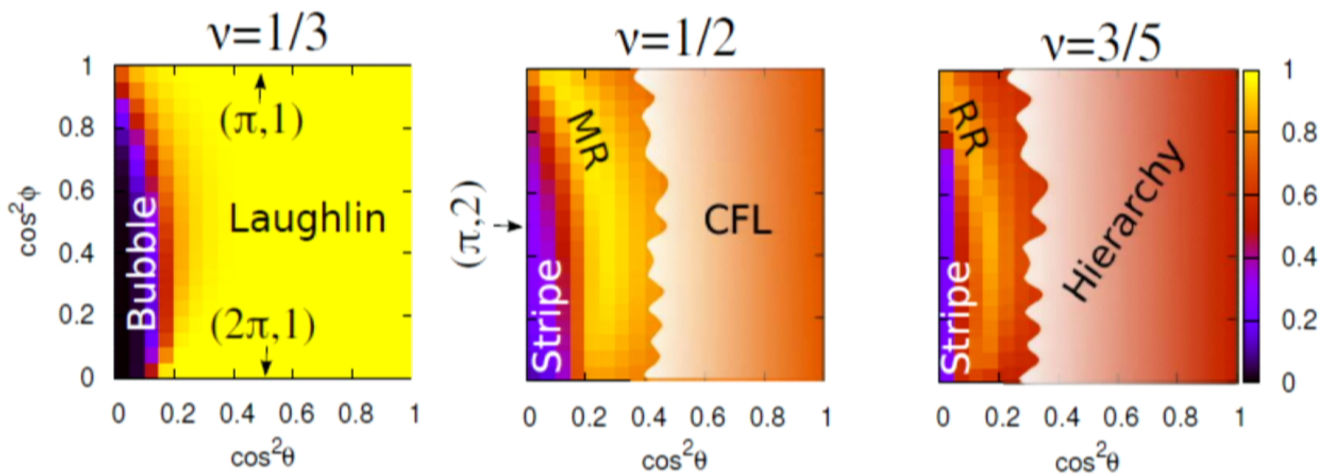
# Tunability via band structure in Dirac materials

- Consider massive fermions with Berry phase $\pi$ or $2\pi$

$$H_{\lambda\pi} = \begin{bmatrix} \Delta & \mathcal{M}_\lambda (p_x + i p_y)^\lambda \\ \mathcal{M}_\lambda (p_x - i p_y)^\lambda & -\Delta \end{bmatrix}, \ \lambda = 1, 2$$

- Consequences for the many-body phases as $\Delta/\ell_B$ is varied: $1/q \to 1/q \times |F(q)|^2$
- In the most general case:

$$F_n(q) = \cos^2\theta \mathcal{L}_{|n|}(\frac{q^2\ell_B^2}{2}) + \sin^2\theta\cos^2\phi\mathcal{L}_{|n|+1}(\frac{q^2\ell_B^2}{2}) + \sin^2\theta\sin^2\phi\mathcal{L}_{|n|+2}(\frac{q^2\ell_B^2}{2})$$




ZP, D. Abanin, Y. Barlas, and R. Bhatt, PRB **84**, 241306(R) (2011); NJP **14**, 025009 (2012).

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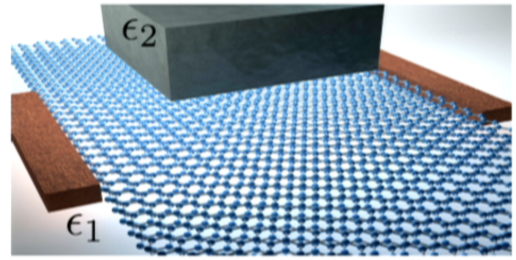


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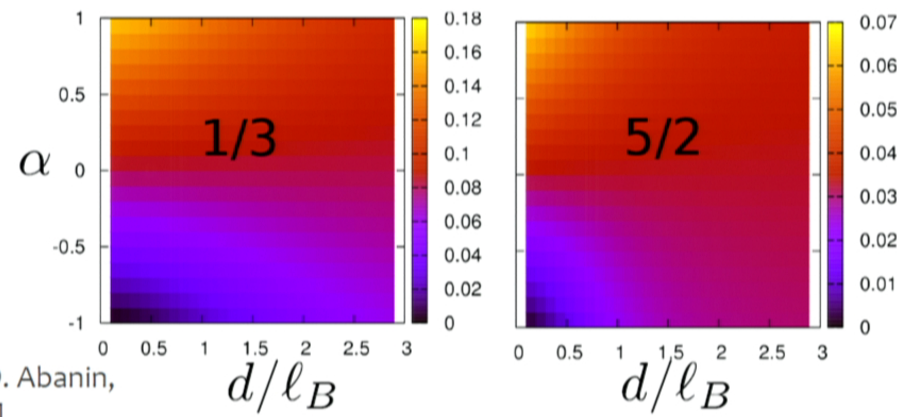
Charge gaps

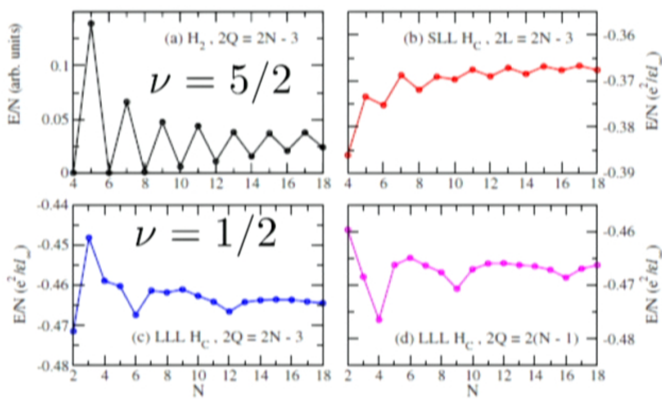


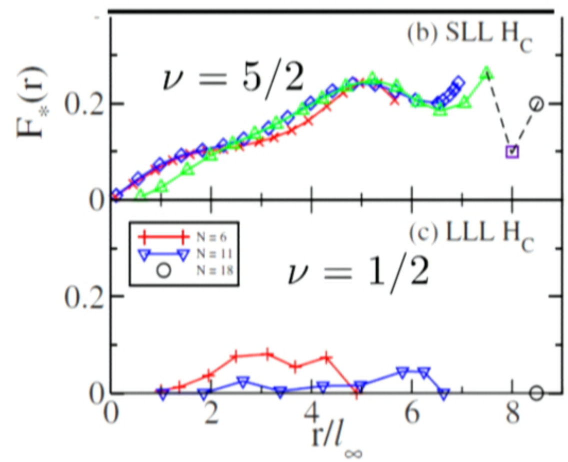

[ZP, R. Thomale, and D. Abanin, PRL **107**, 176602 (2011)]

# $\nu = 5/2$ in wide quantum wells

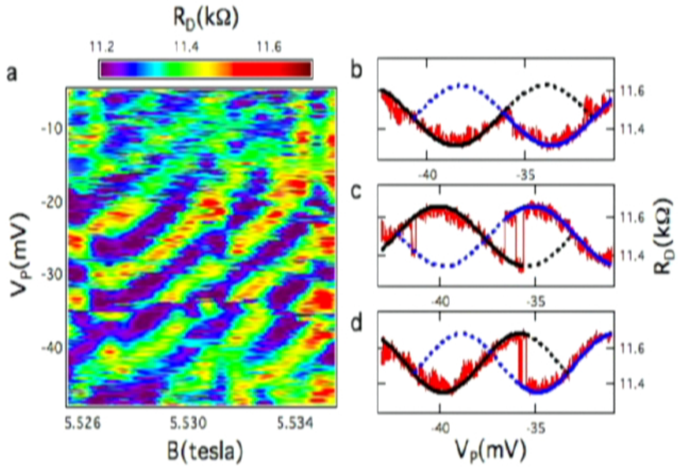
# Theoretical evidence for $\nu = 5/2$

- The ground state is **polarized** [Morf, '98] and in the vicinity of stripe/Fermi liquid phase [Rezayi,Haldane '00]. Finite width is good for the incompressible phase [Peterson, Jolicoeur, Das Sarma '08]
- Adiabatic connection between the ground state of model Hamiltonian and the realistic Hamiltonian [Moller, Simon '08, Storni, Morf, Das Sarma '10] $\lambda H_{model} + (1-\lambda) H_{real}$
- Localization of quasiholes and braiding [Prodan, Haldane '10, Storni, Morf '11]. Neutral fermion mode [Moller, Wojs, Cooper '11, Feiguin, Bonderson, Nayak '11]
- **Even-odd** effect, SC order parameter [Lu, Das Sarma, Park '10]

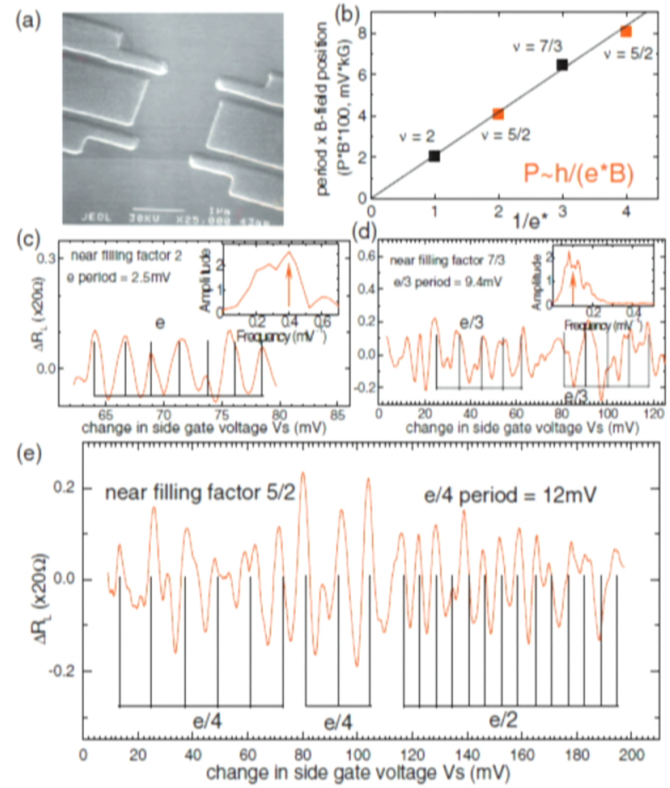


# Experiments and the non-Abelian statistics



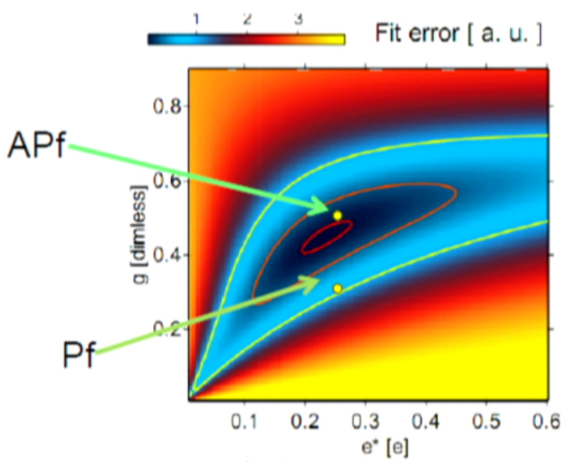

W. Kang group, '11



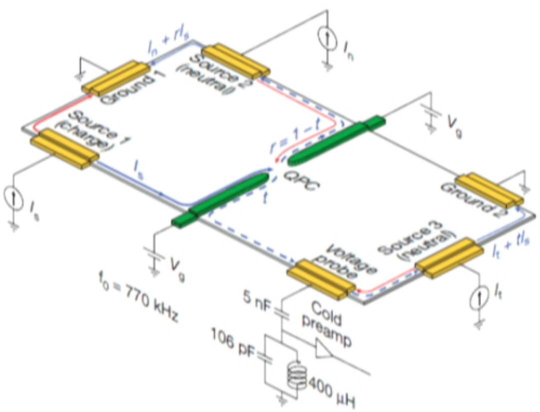

Willett, '10

# Two possible candidates for $\nu = 5/2$

- Particle-hole conjugate of Pfaffian is a different state – the **"anti-Pfaffian"** [Levin *et al.*, S.-S. Lee *et al.*, '07]
- Both states are non-Abelian in the bulk, but APf has different edge physics (backward propagating edge mode)
- In a single LL, the interaction is p-h symmetric – only LL mixing breaks it (the effects of LL mixing have been tricky to calculate and simulate ([Bishara, Nayak '09, Wojs, Toke, Jain '10, Rezayi,Simon '11])




C. Marcus group, '08



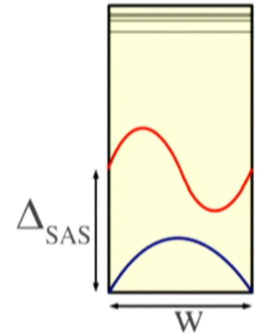

M. Heiblum group, '10

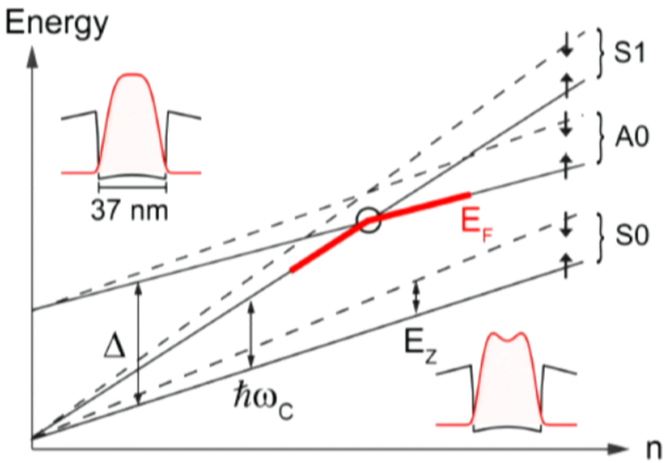
# Wide quantum wells

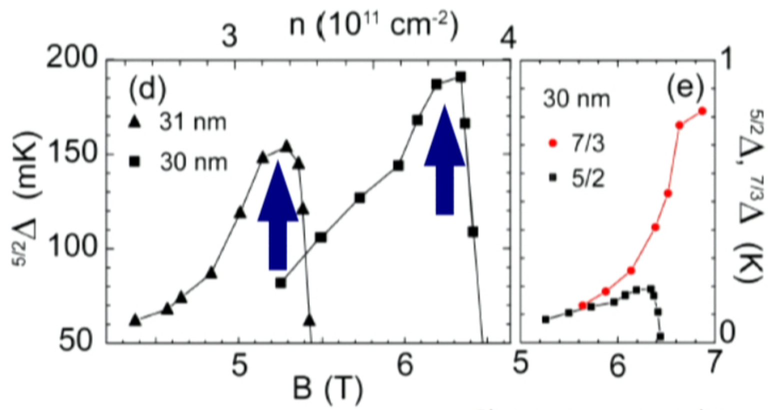
Tuning the density – crossing between S1 and A0 subbands

Subbands are labeled by $(\sigma, n)$

$$S \sim \sin(\pi z/w), A \sim \sin(2\pi z/w), \ldots$$
$$n = \text{ Landau level index}$$



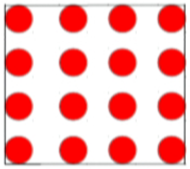




Shayegan group, '11

# $\nu = 5/2$ on the torus

- Magnetic translation algebra for a single electron: $T(\vec{a}) = \exp(i\vec{a}\vec{K}/\hbar)$
  where $K_\alpha = \Pi_\alpha - \frac{\hbar}{\ell_B^2}(\hat{z}\times\vec{r})_\alpha$ and $T(\vec{a})T(\vec{b}) = T(\vec{b})T(\vec{a})\exp(-i\hat{z}\cdot(\vec{a}\times\vec{b}))$
- Projective representations quantization of flux: $|\vec{a}\times\vec{b}| = 2\pi\ell_B^2 N_\phi$
- Emergent symmetries in the many-body case [Haldane, '85]
- Spectrum is classified by $\vec{k} = (s, t \in 0, ..., N-1)$ $N = gcd(N_e, N_\phi)$



- Every eigenvalue is at least $q$-fold degenerate $\nu = N_e/N_\phi = p/q, (p,q) = 1$

# $\nu = 5/2$ on the torus

- Magnetic translation algebra for a single electron: $T(\vec{a}) = \exp(i\vec{a}\vec{K}/\hbar)$
  where $K_\alpha = \Pi_\alpha - \frac{\hbar}{\ell_B^2}(\hat{z}\times\vec{r})_\alpha$ and $T(\vec{a})T(\vec{b}) = T(\vec{b})T(\vec{a})\exp(-i\hat{z}\cdot(\vec{a}\times\vec{b}))$
- Projective representations quantization of flux: $|\vec{a}\times\vec{b}| = 2\pi\ell_B^2 N_\phi$
- Emergent symmetries in the many-body case [Haldane, '85]
- Spectrum is classified by $\vec{k} = (s, t \in 0, ..., N-1)$ $N = gcd(N_e, N_\phi)$
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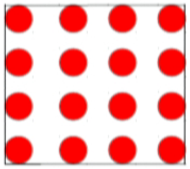
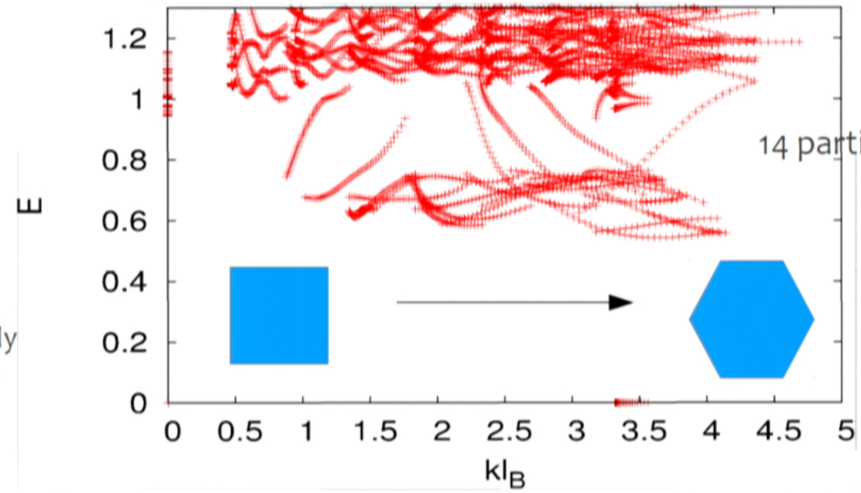
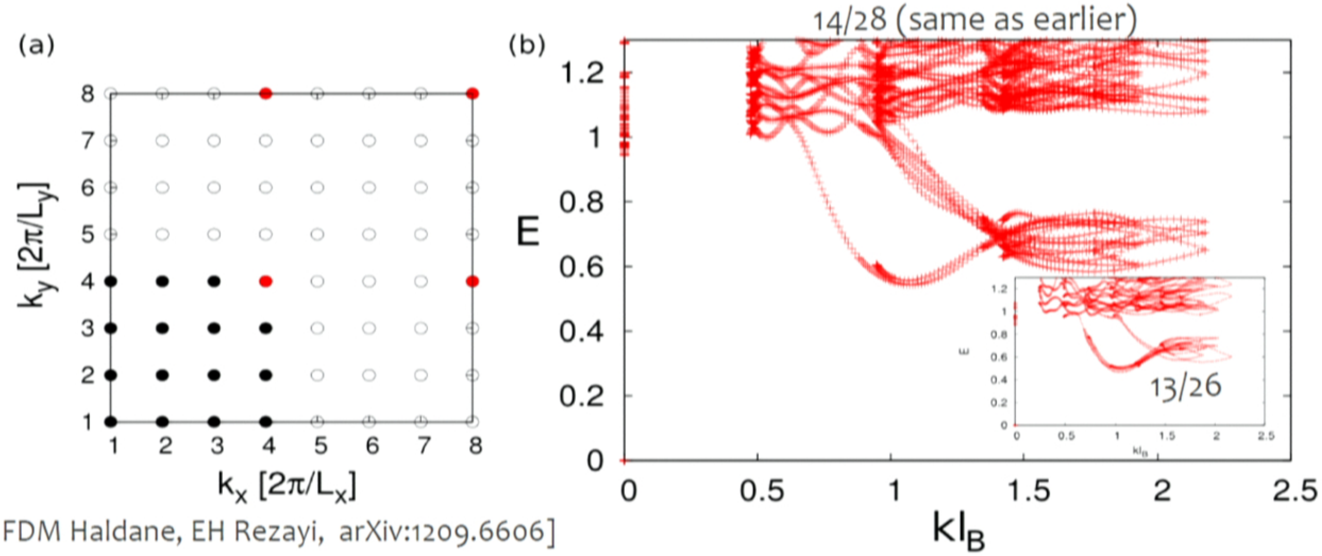





aspect ratio=1 but quasicontinuum of momenta because the angle is adiabatically varied between square and hexagon!

# $\nu = 5/2$ on the torus – the correct way

- Because it is paired, one should view $\nu = 1/2 = 2/4$
- Leads to a "quartered" many-body Brillouin zone scheme
- No essential difference between even and odd particle numbers
- Nice resolution of the collective modes – both fermionic ("neutral fermion") and magneto-roton for a fixed number of particles




[ZP, FDM Haldane, EH Rezayi, arXiv:1209.6606]

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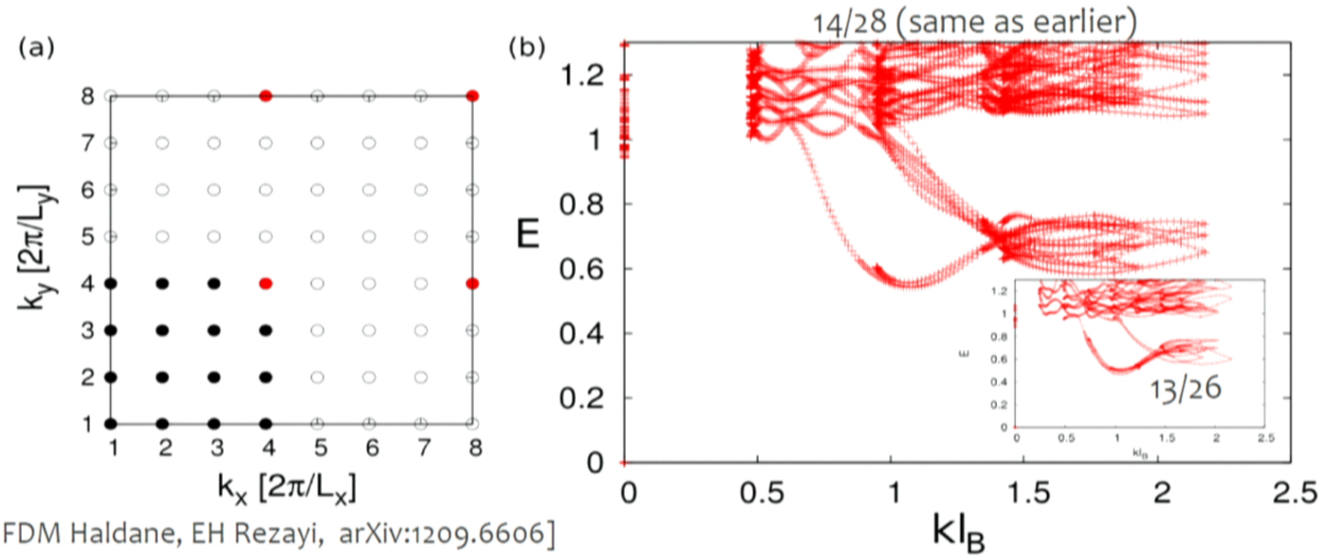


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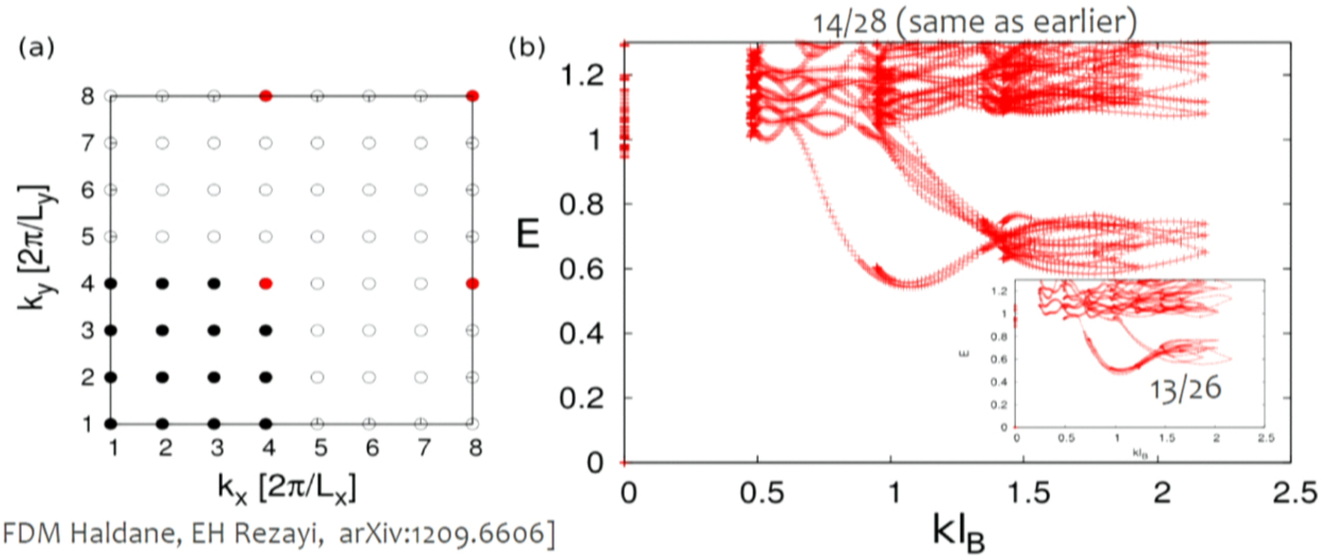
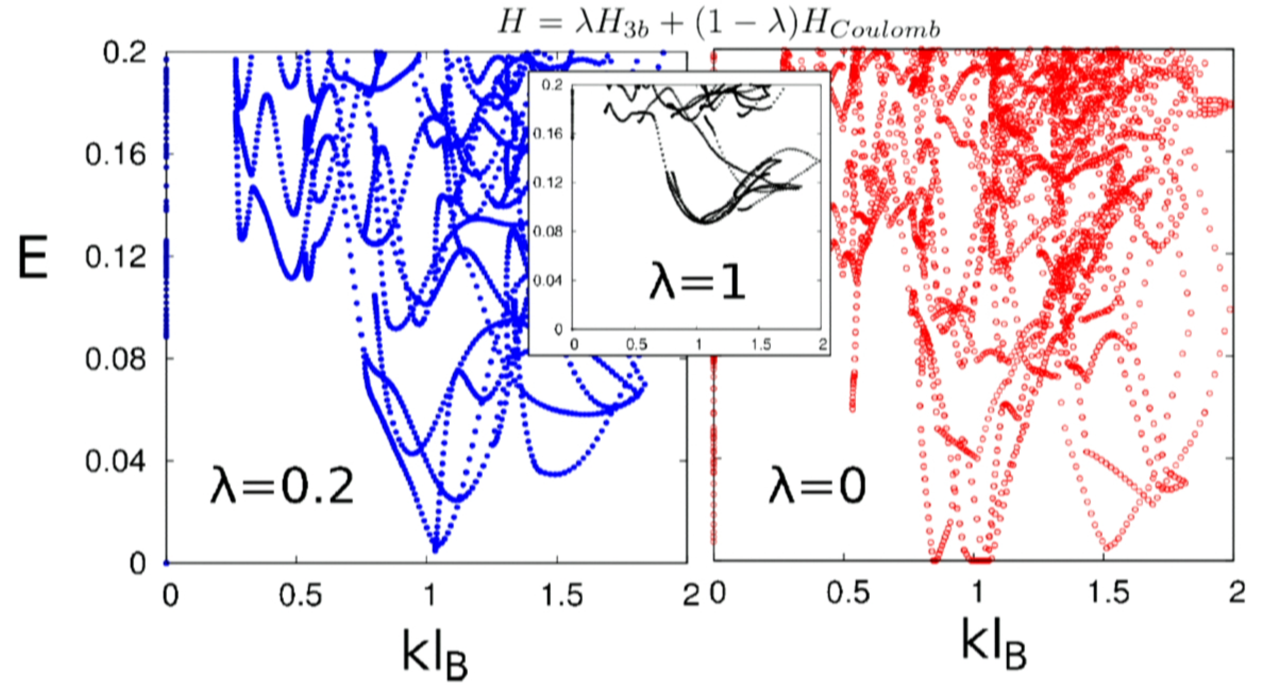



[ZP, FDM Haldane, EH Rezayi, arXiv:1209.6606]

# Transition to the Fermi liquid state

- Hamiltonian interpolates between 3-body and Coulomb: $H = \lambda H_{3b} + (1-\lambda) H_{Coulomb}$
- Collapse of the neutral fermion branch and the ground state moves from $k=0$ to $k_F \sim \ell_B^{-1}$; no closing of the gap in the first excited Landau level
- This is the first time we can study the transition directly (no problem of shift etc.)

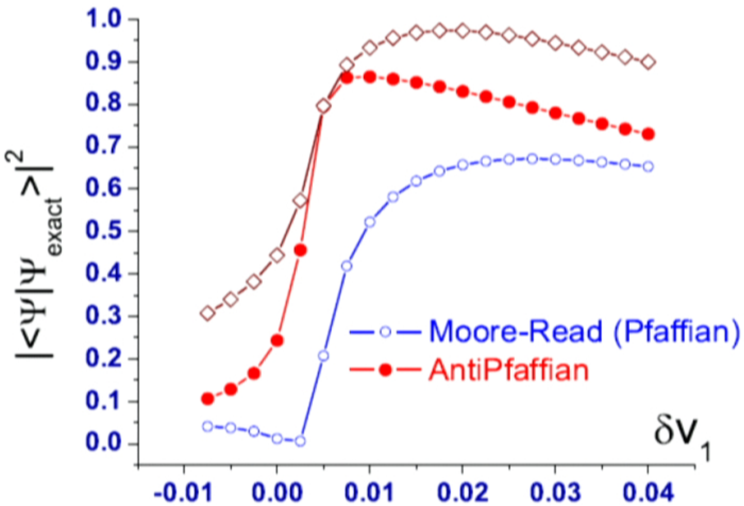


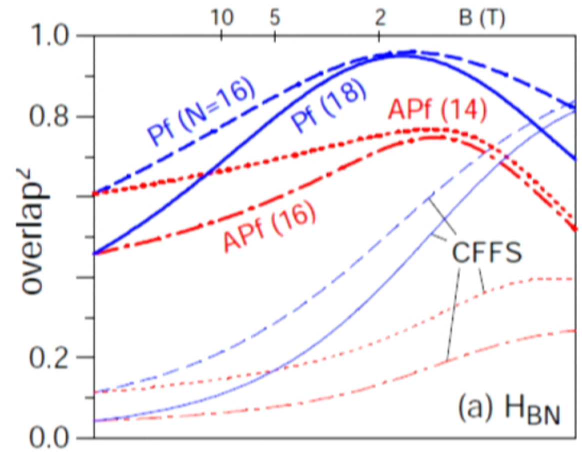

[ZP, FDM Haldane, EH Rezayi, arXiv:1209.6606]

# Previous attempts to study P-H symmetry breaking

- Consider $H_{3b} + \overline{H}_{3b}$ – 2-body Hamiltonian with a ``Mexican hat" structure – spontaneous breaking of particle-hole symmetry in TD limit? [Peterson et al, '08]

- Study of Landau-level mixing and breaking of P-H symmetry in the spherical geometry [Wojs et al, '10] using perturbative interaction [Bishara, Nayak '09]
  **problem: Pf and APf have different shifts!**



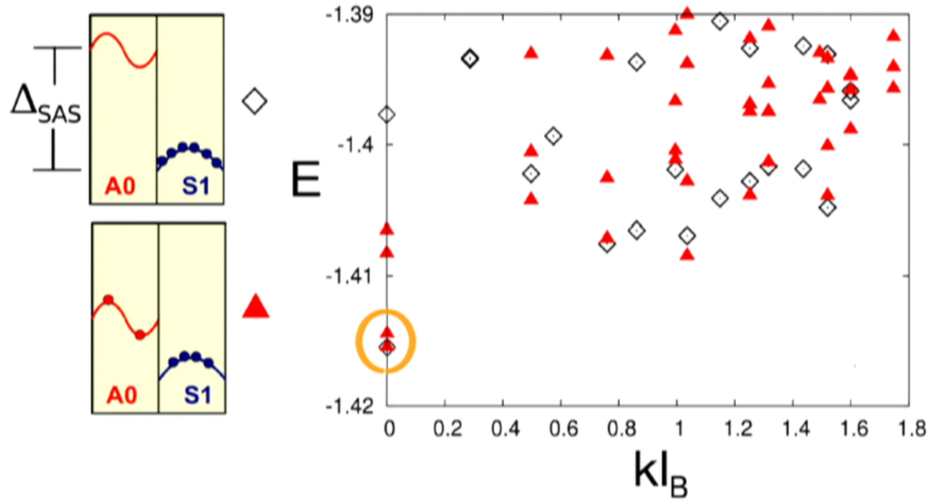




- Study on the torus [Rezayi,Simon '11]

**problem: Pf and APf have very large mutual overlap! (finite size effect)**

# A clean way to detect P-H symmetry breaking

- Previously we saw that even and odd $N$ on torus yield the same physics
- In some odd cases, the ground state of projected Coulomb interaction (without symmetry breaking) at 5/2 is found to be an exact doublet
- This only occurs when there is hexagonal symmetry and $N \neq 6m+1$
- The effect is due to particle-hole symmetry and point group symmetry
- Infinitesimal amount of symmetry breaking field splits the doublet



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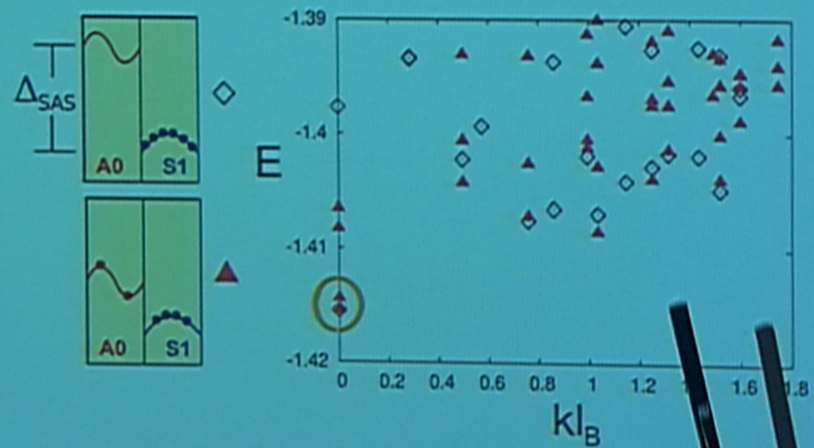
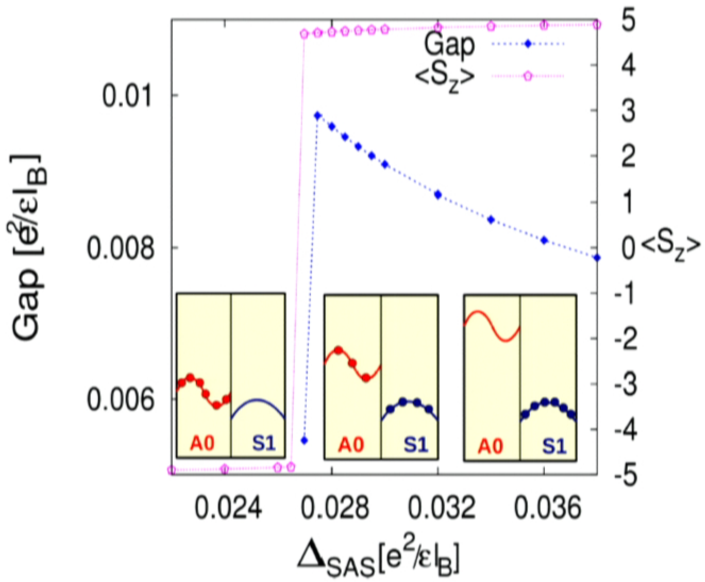
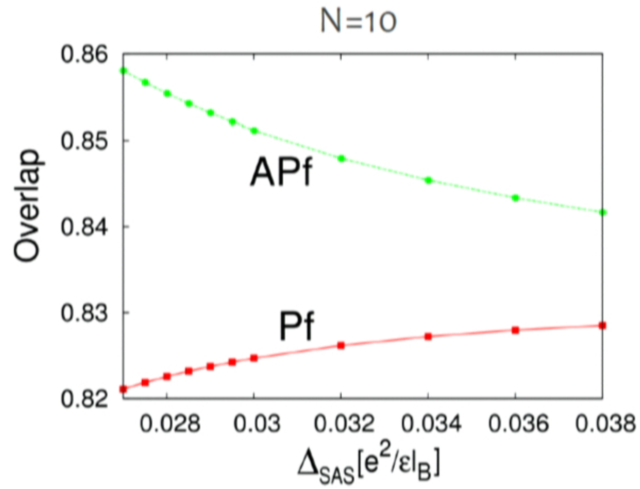


# Particle-hole symmetry breaking: Pf vs APf

- When the doublet occurs, Pf and APf must be orthogonal due to symmetry!
- Angular momentum of Pf measured relative to APf is $\Delta M = 2N_{pair} \; mod \; n$
- If N=3,5,9,11 etc., <Pf|APf>=0 for symmetry reasons! (can also formally construct a "twist" operator that distinguishes Pf from APf)
- for N=11, exact ground state has high overlap with APf and zero with Pf

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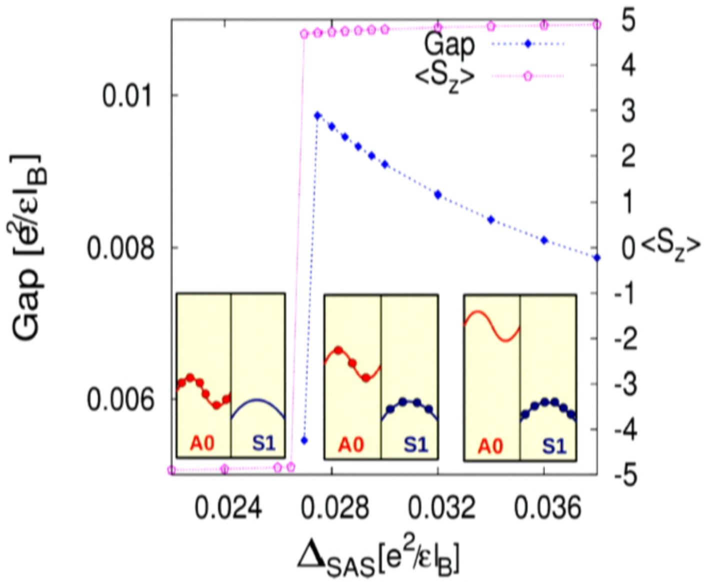


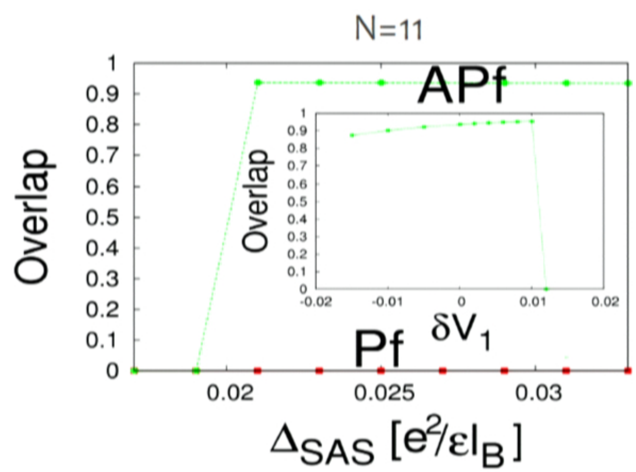


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# Conclusions

- Since FQHE arises solely due to electron-electron interactions, in order to learn about it one should be able to **tune the interaction**
- **Graphene-type materials** could be important from this point of view because their 2D electron gas is fully exposed to external probes
- GaAs wide quantum wells support FQHE and recently it became possible to tune the effect of mixing between different subbands and Landau levels in them
- At half filling, **symmetry breaking selects anti-Pfaffian** to describe the ground state of realistic systems
- Similar outcome occurs for heterostructures and narrow wells where APf also wins
- In special finite systems, **symmetry can be useful even when we study topological properties!**
- Subband mixing also leads to an **enhancement of the excitation gap** prior to the transition to the compressible phase