Title: Fractional Chern Insulators

Date: Nov 29, 2012 11:00 AM

URL: http://pirsa.org/12110080

Abstract: Fractional Chern insulators (FCIs) are topologically ordered states of interacting fermions that share their universal properties with fractional quantum Hall states in Landau levels. FCIs have been found numerically in a variety of two-dimensional lattice models upon partially filling an almost dispersionless band with nontrivial topological character with repulsively interacting fermions. I will show how FCIs emerge in bands with Chern number C=1 and C=2 and in Z_2 topological insulators, where the latter are accompanied by a spontaneous breaking of time-reversal symmetry. Further, I will discuss the relevance of the noncommutative quantum geometry of the flat topological band to the stability of FCIs and how they can be distinguished from phases of spontaneously broken point group symmetry, such as charge density waves.

Fractional Chern Insulators

Titus Neupert
Condensed Matter Theory Group
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Christopher Mudry, PSI
Claudio Chamon, Boston U
Shinsei Ryu, U Illinois Urbana
Luiz Santos, Perimeter
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Outline

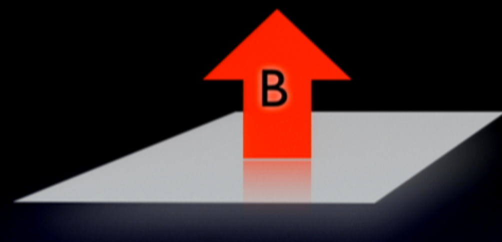
1. Chern insulators
   - Haldane's model
   - noncommutative geometry

2. Fractional Chern insulators (FCI)
   - C=1
   - C=2

   exact diagonalization results

3. spontaneous formation of FCI in Z_2 topological insulators

Quantum Hall effect: **Landau levels**



energy scale $\omega_c = \dfrac{eB}{m}$

length scale $\ell_B^2 = \dfrac{\hbar c}{eB}$

degeneracy $N = \dfrac{A}{2\pi \ell_B^2} = \dfrac{\Phi}{\Phi_0}$

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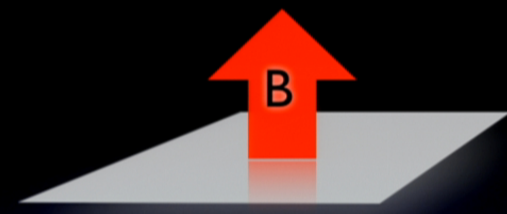

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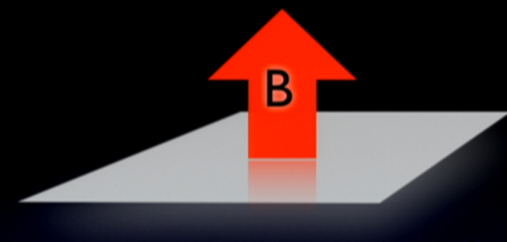

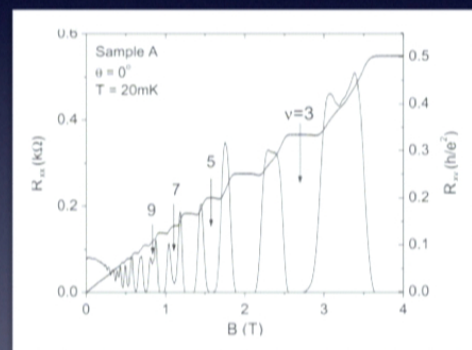

Integer quantum Hall effect $\sigma_{xy} = \dfrac{e^2}{h} n$

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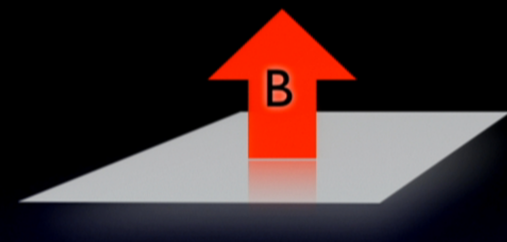
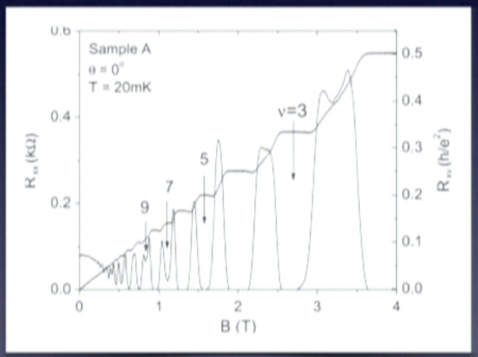
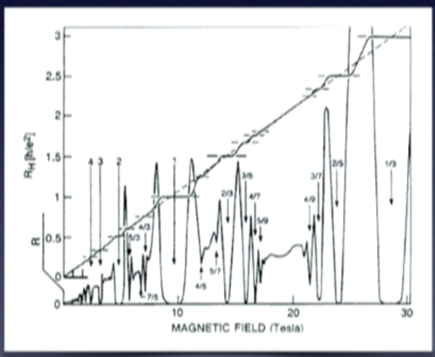
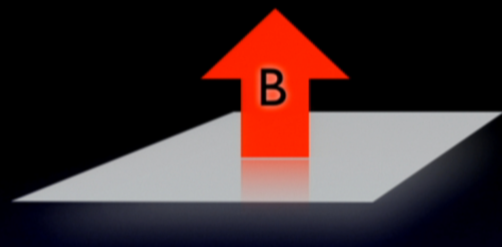




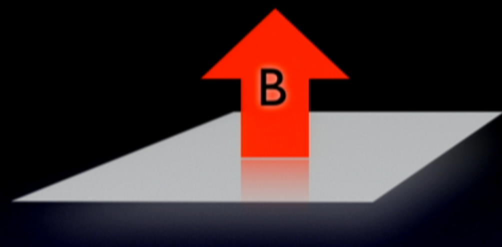


Fractional quantum Hall effect $\sigma_{xy} = \dfrac{e^2}{h}\nu$



Are there other systems in which a quantum Hall effect appears?

Can we dispose of the large magnetic field?



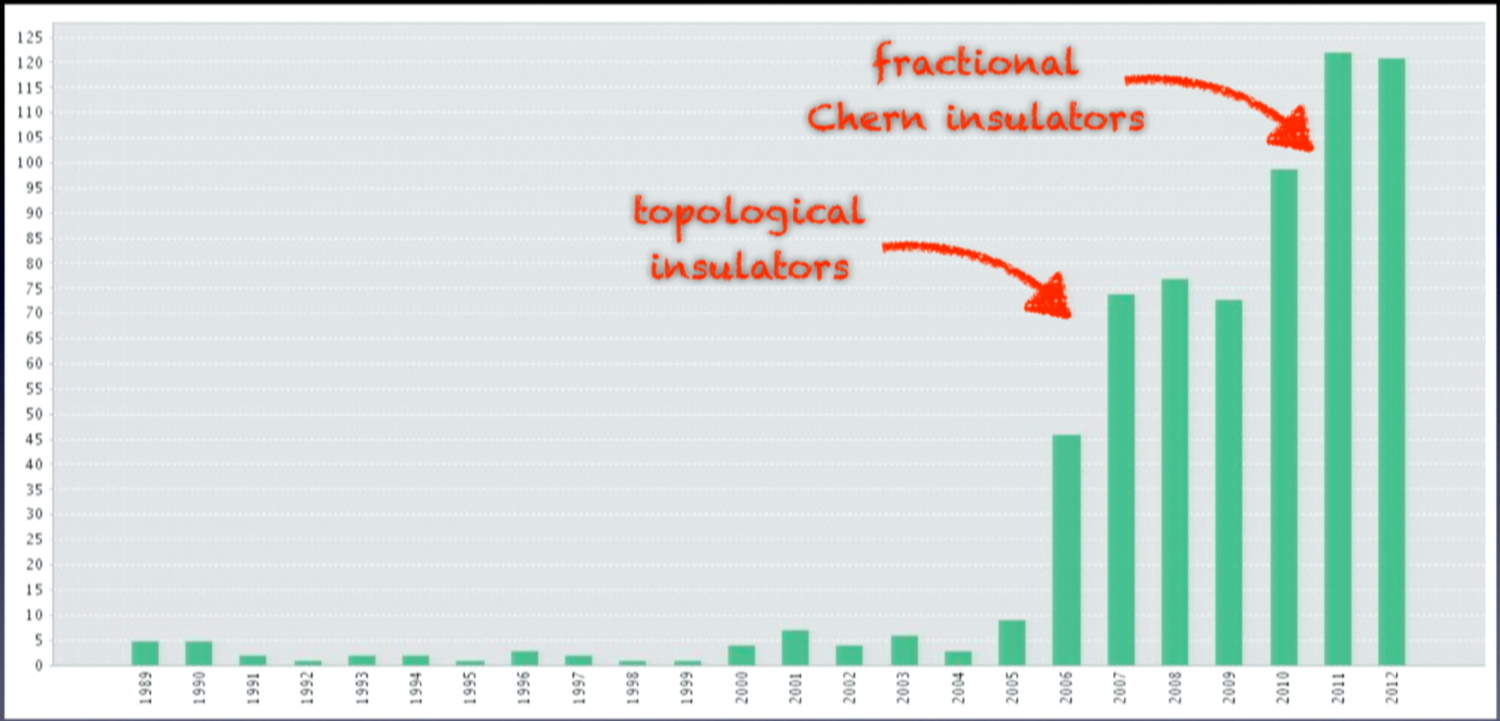
Answer for integer quantum Hall effect:

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

Are there other systems in which a quantum Hall effect appears? **Yes**

Can we dispose of the large magnetic field? **Yes**

Citations: **Haldane's model**



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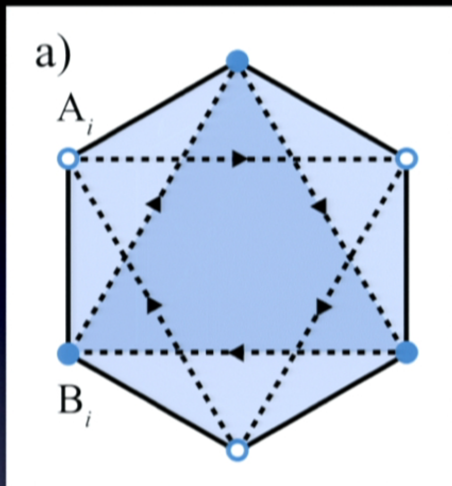
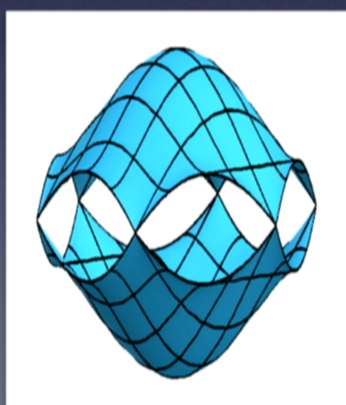
Chern insulator: **Haldane's model**





spinless fermions on honeycomb lattice

complex NNN hoppings open mass gap in graphene spectrum

"Chern insulator" $\sigma_{xy} = \frac{e^2}{h} C$

$C = \pm 1$

$\ell_B \to \mathfrak{a}$

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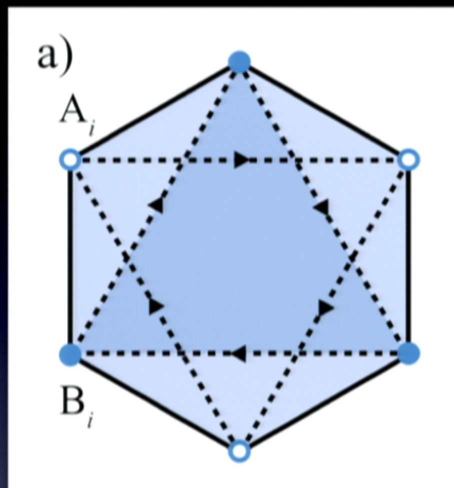

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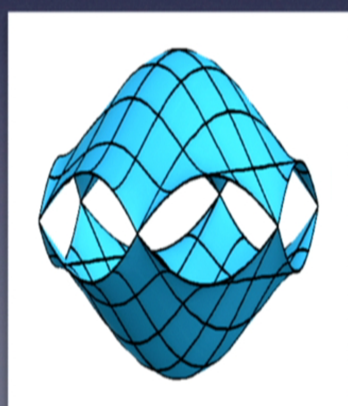

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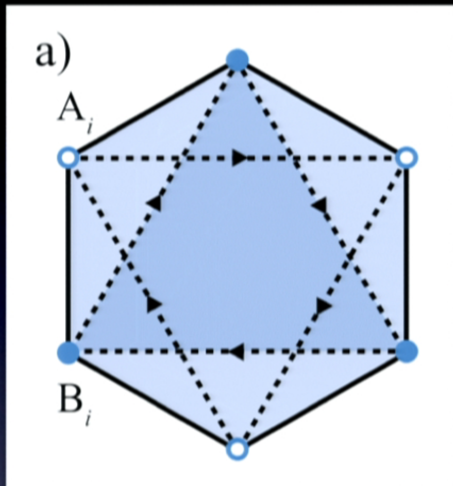

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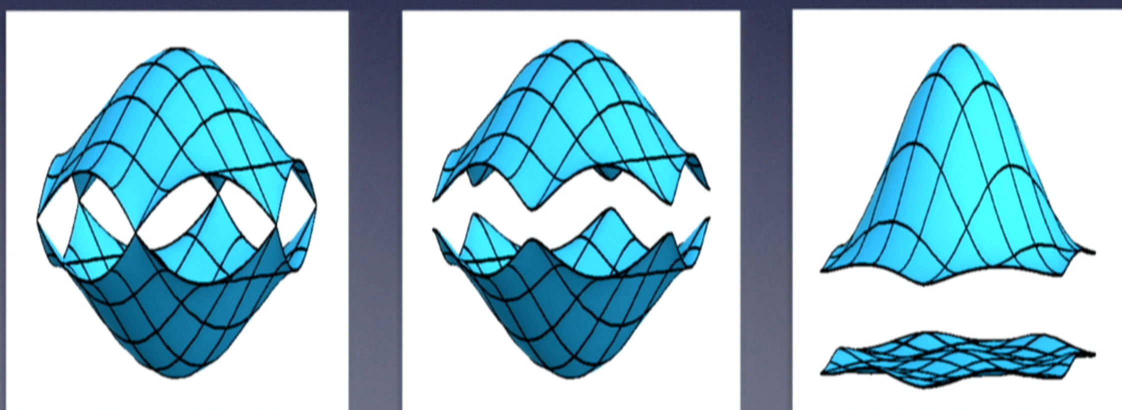

$\ell_B \to \mathfrak{a}$

Just like a Landau level?

3 functions that are constant in the Landau level

... but not in a generic Chern insulator

1. Dispersion $\varepsilon(\boldsymbol{k})$
2. Quantum metric $g_{\mu\nu}(\boldsymbol{k})$
3. Berry curvature $F_{\mu\nu}(\boldsymbol{k})$

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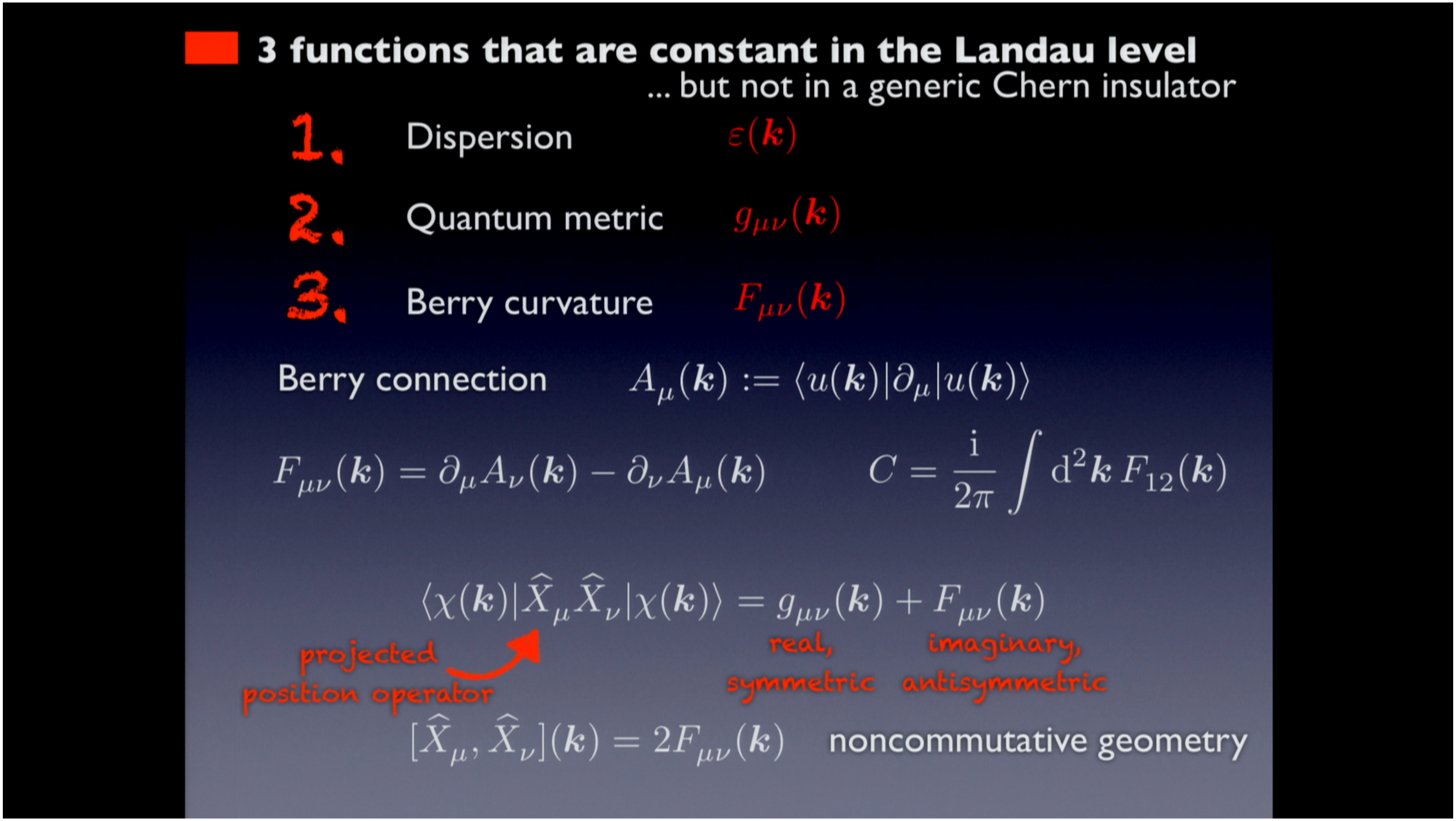
Berry connection $A_\mu(\boldsymbol{k}) := \langle u(\boldsymbol{k})|\partial_\mu|u(\boldsymbol{k})\rangle$

$$F_{\mu\nu}(\boldsymbol{k}) = \partial_\mu A_\nu(\boldsymbol{k}) - \partial_\nu A_\mu(\boldsymbol{k}) \qquad C = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}^2\boldsymbol{k}\, F_{12}(\boldsymbol{k})$$



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$C = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}^2\boldsymbol{k}\, F_{12}(\boldsymbol{k})$
$\langle\chi(\boldsymbol{k})|\widehat{X}_\mu\widehat{X}_\nu|\chi(\boldsymbol{k})\rangle = g_{\mu\nu}(\boldsymbol{k}) + F_{\mu\nu}(\boldsymbol{k})$
projected position operator
real, symmetric
imaginary, antisymmetric
$[\widehat{X}_\mu, \widehat{X}_\nu](\boldsymbol{k}) = 2F_{\mu\nu}(\boldsymbol{k})$
noncommutative geometry

Density algebra

Landau level

$$[\widehat{\rho}_{\boldsymbol{q}}, \widehat{\rho}_{\boldsymbol{q}'}] = -2\mathrm{i}\sin\left(\frac{\ell_{\mathrm{B}}^2}{2}\boldsymbol{q}\wedge\boldsymbol{q}'\right)\widehat{\rho}_{\boldsymbol{q}+\boldsymbol{q}'} \qquad \widehat{\rho}_{\boldsymbol{q}} = e^{\mathrm{i}\boldsymbol{q}\cdot\widehat{\boldsymbol{X}}}$$

S. M. Girvin et al., Phys. Rev. Lett. **54**, 581 (1985)

Chern insulator

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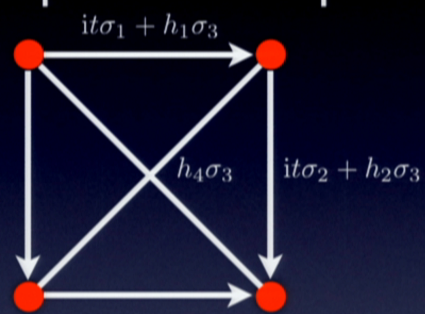
S. M. Girvin et al., Phys. Rev. Lett. **54**, 581 (1985)

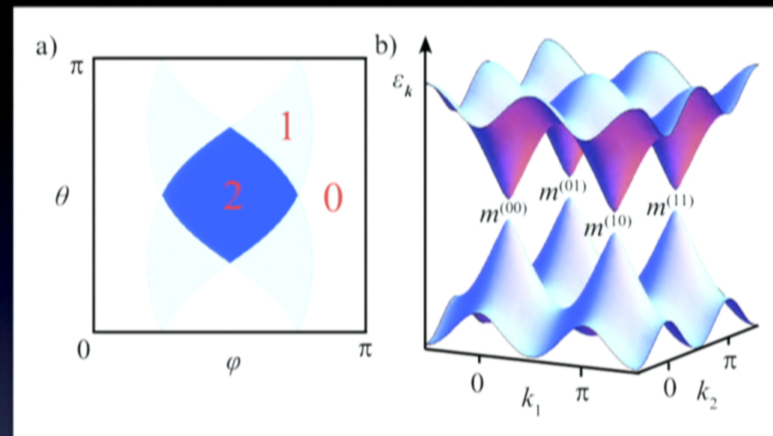
Chern insulator

Fractional Chern insulator

Model

square lattice,
spin 1/2 d.o.f. per site



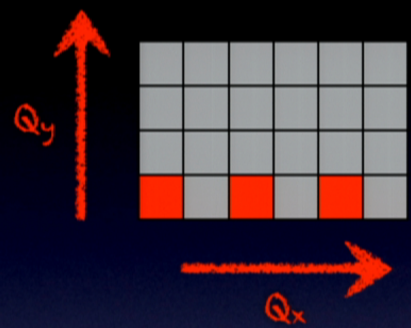


$$m^{(ij)} = (-1)^i h_1 + (-1)^j h_2 + (-1)^{i+j}\, 2\, h_4$$

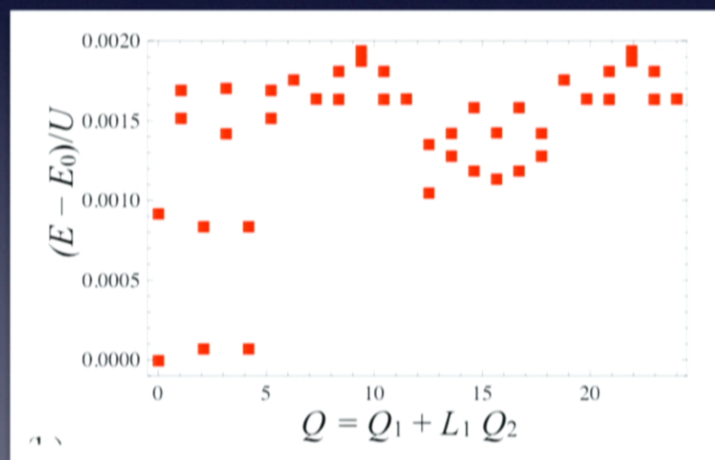
A. G. Grushin et al., arXiv:1207.4097

Fractional Chern insulator

C = 1 at 1/3 and 2/3 filling



4 x 6 lattice
many-body
c.o.m. BZ



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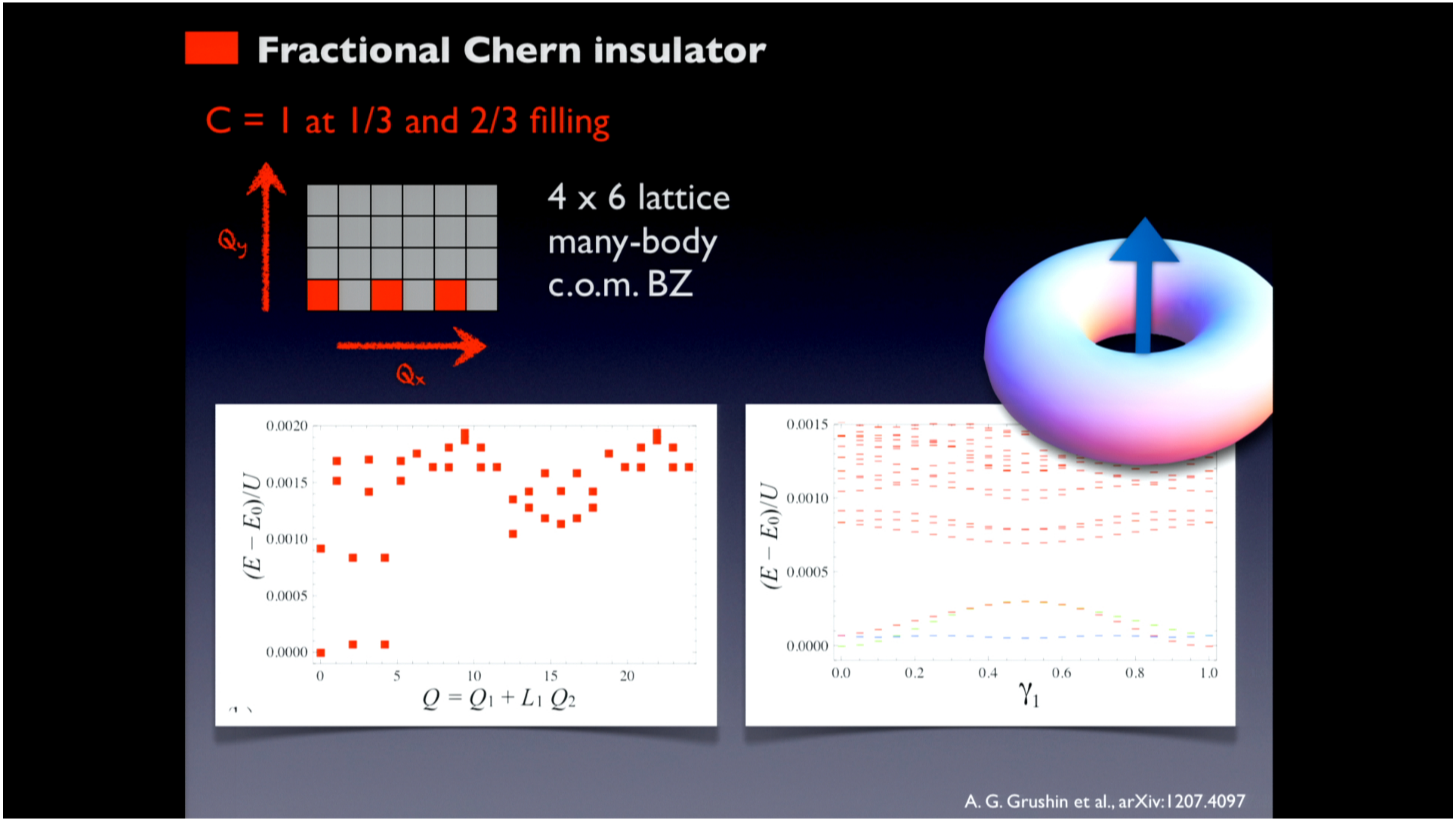
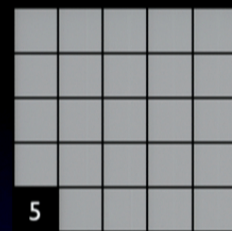

Fractional Chern insulator
C = 1 at 1/3 and 2/3 filling
Qy
Qx
4 x 6 lattice
many-body
c.o.m. BZ
0.0020
0.0015
0.0010
0.0005
0.0000
(E − E0)/U
0 5 10 15 20
Q = Q1 + L1 Q2
0.0015
0.0010
0.0005
0.0000
(E − E0)/U
0.0 0.2 0.4 0.6 0.8 1.0
γ1
A. G. Grushin et al., arXiv:1207.4097

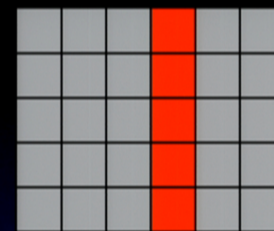
Fractional Chern insulator

C = 2 at 1/5 and 4/5 filling

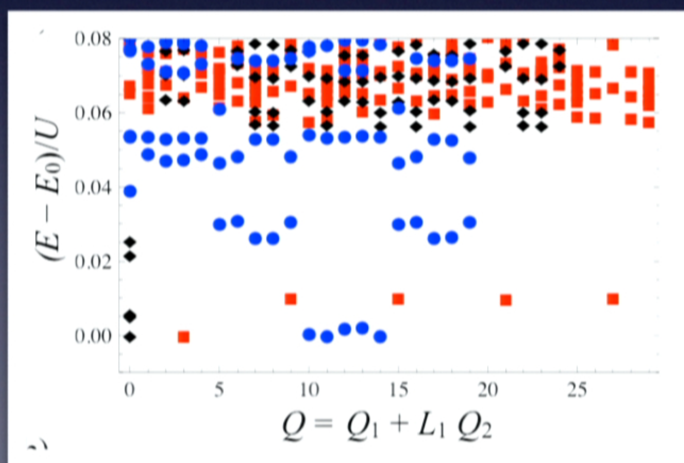
5 x 4 lattice



5 x 5 lattice



5 x 6 lattice



A. G. Grushin et al., arXiv:1207.4097

Fractional Chern insulator

Nature of incompressible states: FQH states

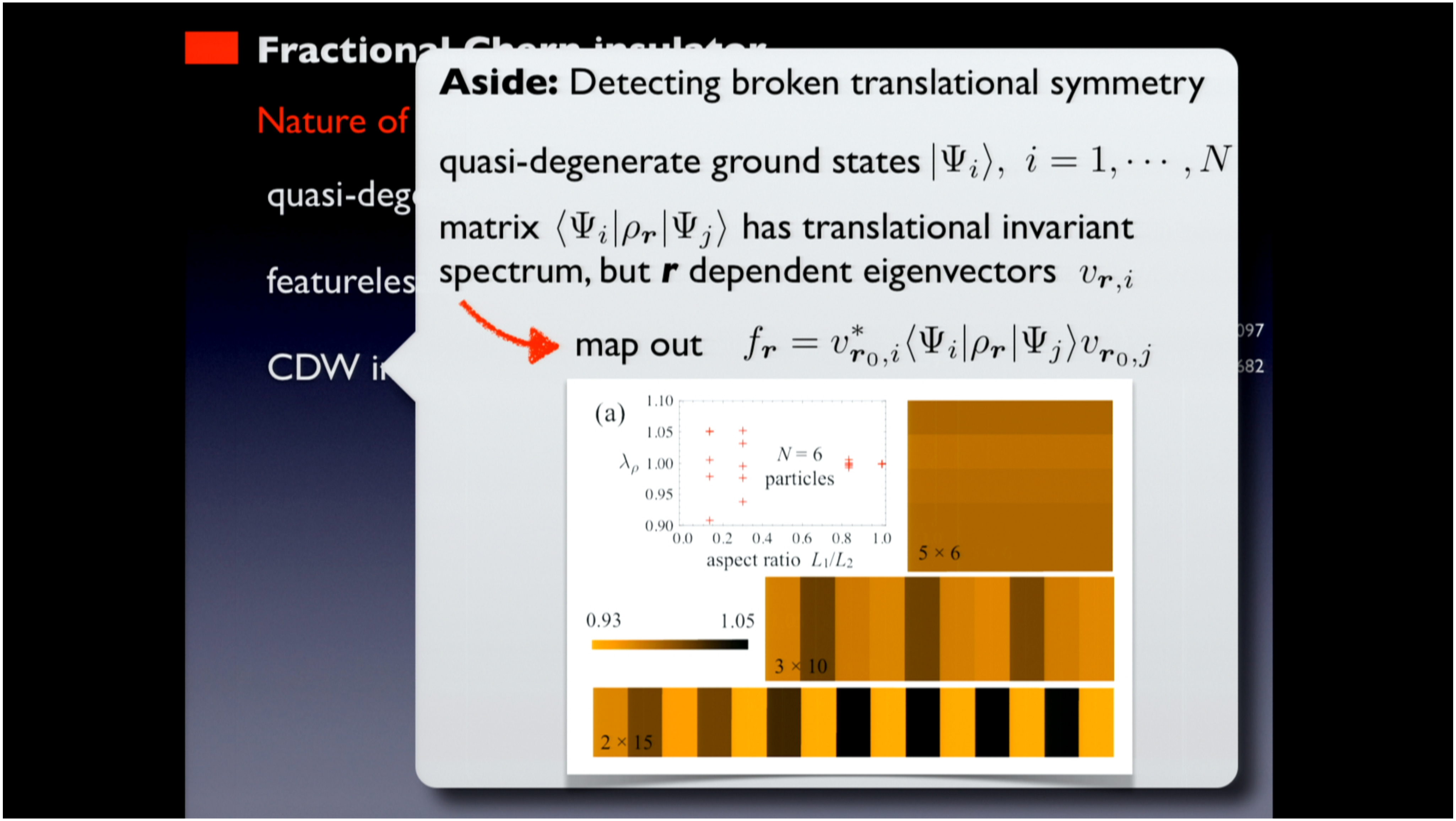
quasi-degeneracy of topologically ordered states ν^{-g}

featureless

CDW in thin-torus limit

A. G. Grushin et al., arXiv:1207.4097

B. A. Bernevig et al., arXiv:1204.5682


Fractional Chern insulator
Nature of
quasi-deg
featureles
CDW i
Aside: Detecting broken translational symmetry
quasi-degenerate ground states $|\Psi_i\rangle,\ i = 1, \cdots, N$
matrix $\langle \Psi_i | \rho_{\boldsymbol{r}} | \Psi_j \rangle$ has translational invariant spectrum, but $\boldsymbol{r}$ dependent eigenvectors $v_{\boldsymbol{r},i}$
map out $f_{\boldsymbol{r}} = v^*_{\boldsymbol{r}_0,i} \langle \Psi_i | \rho_{\boldsymbol{r}} | \Psi_j \rangle v_{\boldsymbol{r}_0,j}$
(a)
λ_ρ
1.10
1.05
1.00
0.95
0.90
0.0 0.2 0.4 0.6 0.8 1.0
aspect ratio L_1/L_2
$N = 6$ particles
5×6
0.93
1.05
3×10
2×15
097
682

Fractional

Nature of

quasi-dege

featureles

CDW in t

many-bo

Aside: Computing the many-body Chern number

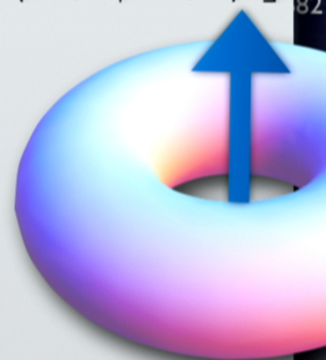
Thouless: Average over boundary conditions

$$|\Psi(\gamma_1, \gamma_2)\rangle, \qquad (\gamma_1, \gamma_2) \in [0, 2\pi]^2$$

$$C = -\frac{\mathrm{i}}{2\pi} \int_0^{2\pi} \mathrm{d}\gamma_1 \int_0^{2\pi} \mathrm{d}\gamma_2 \left[\left\langle \frac{\partial \Psi}{\partial \gamma_1} \middle| \frac{\partial \Psi}{\partial \gamma_2} \right\rangle - \left\langle \frac{\partial \Psi}{\partial \gamma_2} \middle| \frac{\partial \Psi}{\partial \gamma_1} \right\rangle \right]$$

97

82



Fractional Chern insulator

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A. G. Grushin et al., arXiv:1207.4097
B.A. Bernevig et al., arXiv:1204.5682

many-body Chern number (Hall conductivity) 1/3 and 2/5

T. Neupert et al., Phys. Rev. B **86** 165133 (2012)

level counting in the entanglement spectrum

B.A. Bernevig et al., Phys. Rev. B **85** 075128 (2012)

mapping to Landau level Laughlin state
with high overlap, continuous deformation

X.-L. Qi, Phys.Rev. Lett. **107**, 126803 (2011)
Y.-L. Wu, et al., Phys. Rev. B **86**, 085129 (2012)
Z. Liu et al., arXiv:1209.5310.
Y.-L. Wu, et al., arXiv:1210.6356.

Fractional Chern insulator

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CDW i

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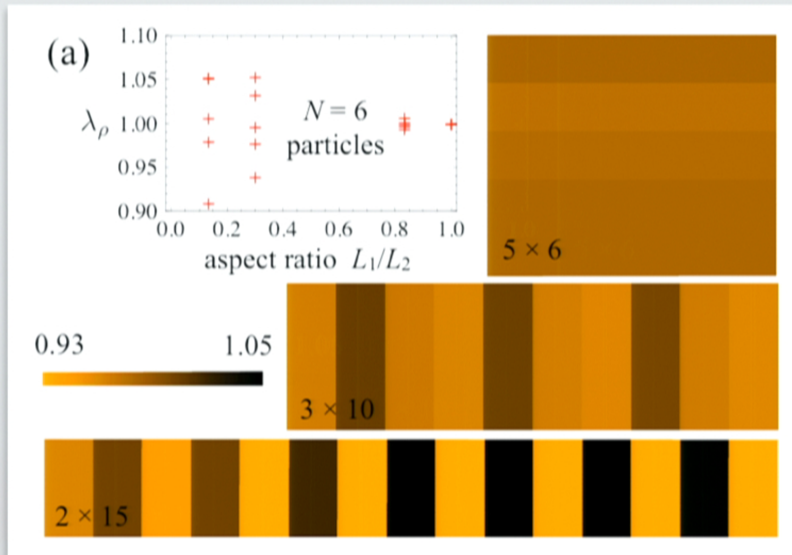

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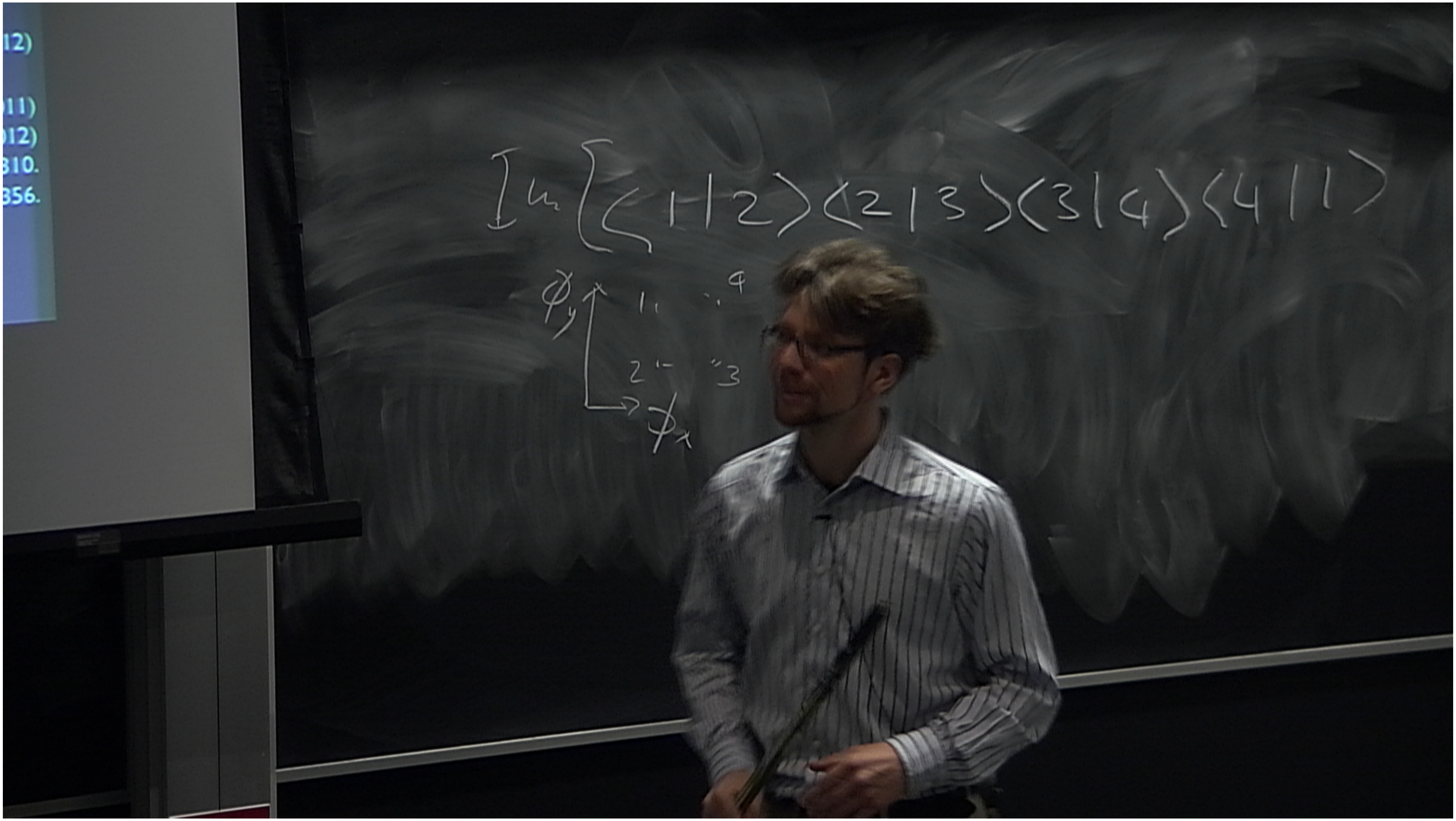
12)
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310.
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Im{⟨1|2⟩⟨2|3⟩⟨3|4⟩⟨4|1⟩
φy
φx
1
2
3
4

Fractional Chern insulator

Abelian states compatible with hierarchical CS description

non-Abelian states (Moore-Read, Z_k Read-Rezayi) with 3-body and 4-body interaction

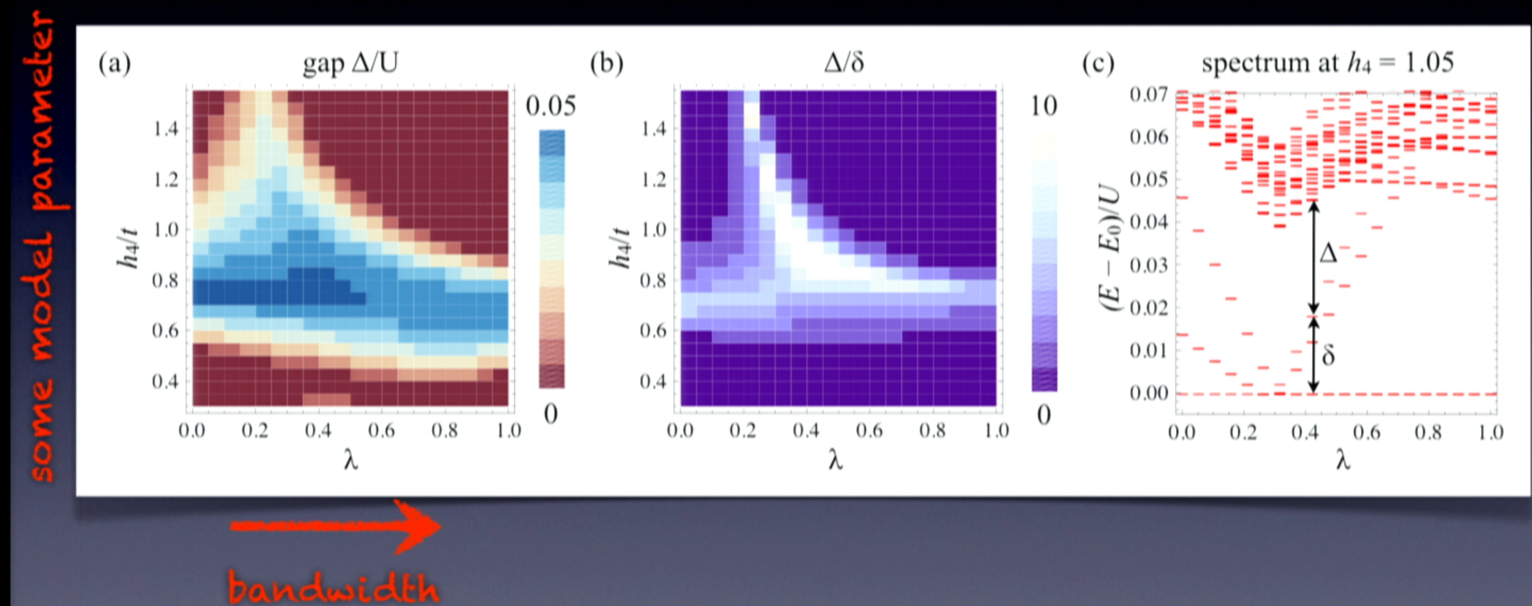
B.A. Bernevig et al., Phys. Rev. B **85**, 075128 (2012)
Y.-F. Wang et al., Phys Rev. Lett **108** 126805
Y.-L. Wu et al., Phys. Rev. B **85**, 075116 (2012)

series of states at higher Chern number $\nu = \frac{k}{1+2C}$

A.M. Läuchli arXiv:1207.6094
A. Sterdyniak et al., arXiv:1207.6385

Fractional Chern insulator: **Effect of bandwidth**

$\varepsilon(\boldsymbol{k})$ $g_{\mu\nu}(\boldsymbol{k})$ $F_{\mu\nu}(\boldsymbol{k})$ The flatter the better?



Fractional Chern insulator: **Effect of quantum distance**

- Projected interaction:

overlaps of single-particle states

$$H := \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4} V_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4} \chi^\dagger_{\boldsymbol{k}_1} \chi^\dagger_{\boldsymbol{k}_2} \chi_{\boldsymbol{k}_3} \chi_{\boldsymbol{k}_4}$$

$$V_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4} = v_{\boldsymbol{k}_1-\boldsymbol{k}_3} \langle \chi_{\boldsymbol{k}_1} | \chi_{\boldsymbol{k}_3} \rangle \langle \chi_{\boldsymbol{k}_2} | \chi_{\boldsymbol{k}_4} \rangle \delta_{\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_3+\boldsymbol{k}_4}$$

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Quantum distance enters interaction

$$d(\boldsymbol{k}, \boldsymbol{k}') := \sqrt{1 - |\langle \chi(\boldsymbol{k}) | \chi(\boldsymbol{k}') \rangle|}$$

connects to Fubini-Study metric

$$d(\boldsymbol{k}_1, \boldsymbol{k}_2) = \inf_{\gamma_{1,2}} \int_{\gamma_{1,2}} \mathrm{d}\ell \sqrt{g_{\mu\nu}(\boldsymbol{k}) \frac{\mathrm{d}k^\mu}{\mathrm{d}\ell} \frac{\mathrm{d}k^\nu}{\mathrm{d}\ell}}$$

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   - Haldane's model
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2. Fractional Chern insulators (FCI)
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   exact diagonalization results

3. spontaneous formation of FCI in Z_2 topological insulators

(2+1)D topological insulator

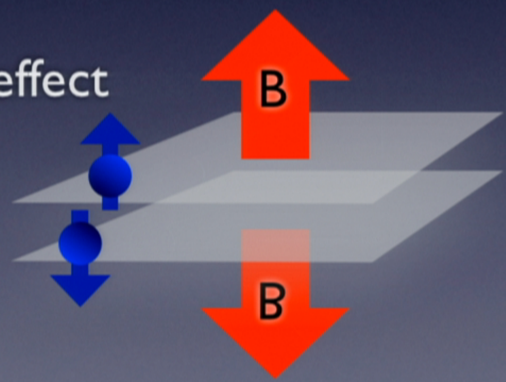
topological band insulators in (2+1)D: Z IQHE (class A)
Z_2 TI (class AII)

Gedanken experiment: filled Landau level with opposite chirality (magnetic field) for up-spin and down-spin electrons

time-reversal symmetric

with S_z conservation: quantum spin Hall effect

$$\sigma_{\rm H} = 0 \qquad \sigma_{\rm sH} = \frac{e}{2\pi}$$

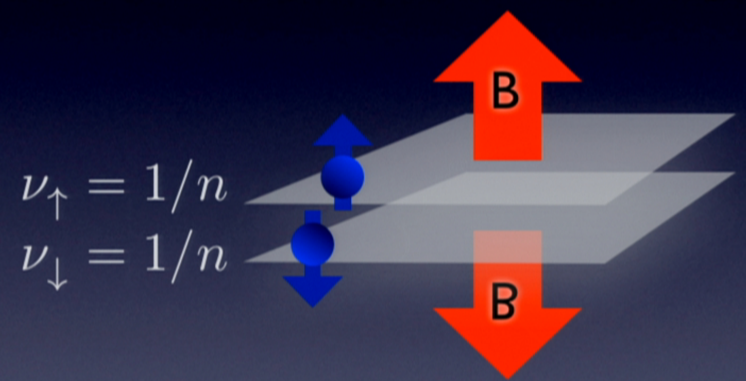


Physical realization?

Fractional topological insulator

Gedanken experiment:
take two **fractional** quantum Hall layers with opposite field with S_z conservation

M. Levin et al., Phys. Rev. Lett. **103**, 196803 (2009)



Is this there a physical setting for the fractional quantum spin Hall effect to appear?

Fractional topological insulator

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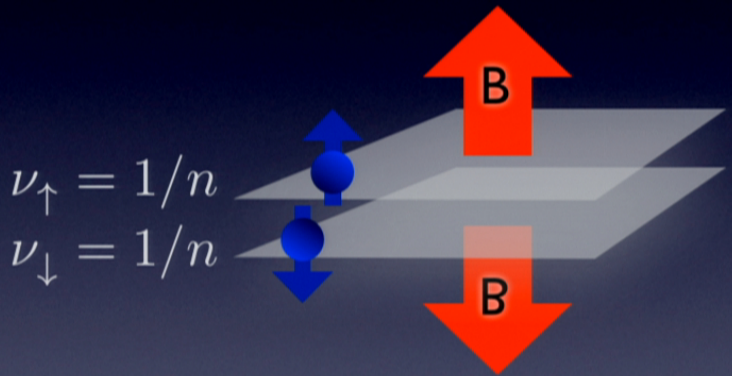

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?, but ...

T. Neupert et al., Phys. Rev. B **84**, 165107 (2011)

Fractional topological insulator

Need

Quantum spin Hall insulator (TI)	band gap	Δ
reasonably flat nontrivial Kramers pair of band	band width	W
add short-range repulsive interactions	interaction	U

with hierarchy of energy scales

T. Neupert et al., Phys. Rev. B **84**, 165107 (2011)

$$\Delta \gg U \gg W$$

Model

$$H = \mathrm{CI}_\downarrow + \mathrm{CI}^*_\uparrow + U\sum_i n_i^\downarrow n_i^\uparrow + V\sum_{(i,j)} \left(n_i^\downarrow n_j^\downarrow + n_i^\uparrow n_j^\uparrow + 2\lambda n_i^\downarrow n_j^\uparrow\right)$$

Fractional topological insulator

Result

Flatband Stoner magnetism preempts formation of fractional topological insulator

T. Neupert et al., Phys. Rev. B **84**, 165107 (2011)

$$\sigma_{\mathrm{H}} = \frac{e^2}{h}\int \mathrm{d}^2\boldsymbol{k}\ [n_\uparrow(\boldsymbol{k})\, F_\uparrow(\boldsymbol{k}) + n_\downarrow(\boldsymbol{k})\, F_\downarrow(\boldsymbol{k})]$$

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1/3 → 2/3

-1/3 → 0

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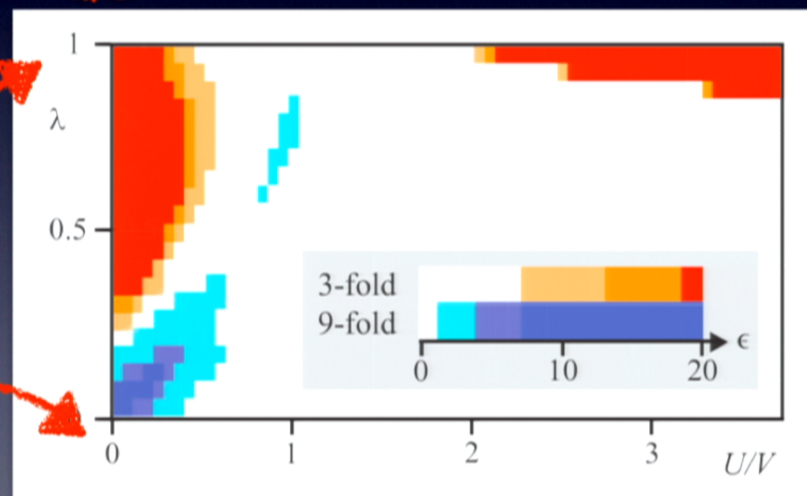
spin-isotropic interaction

only intra-species repulsion



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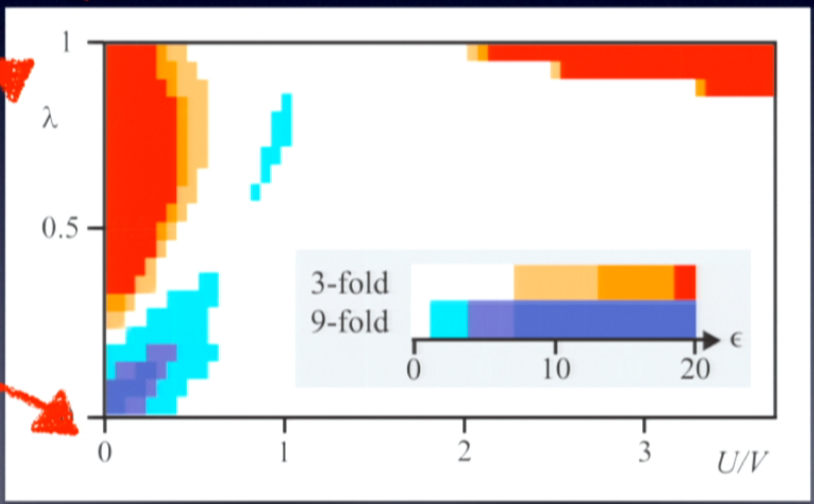

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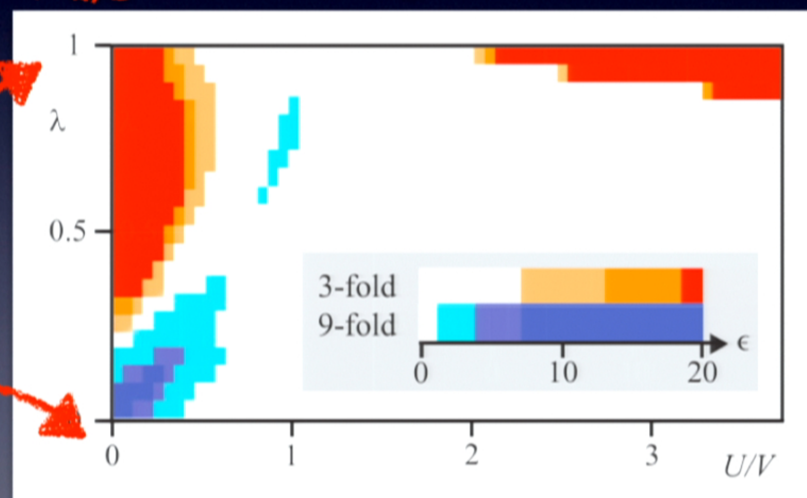

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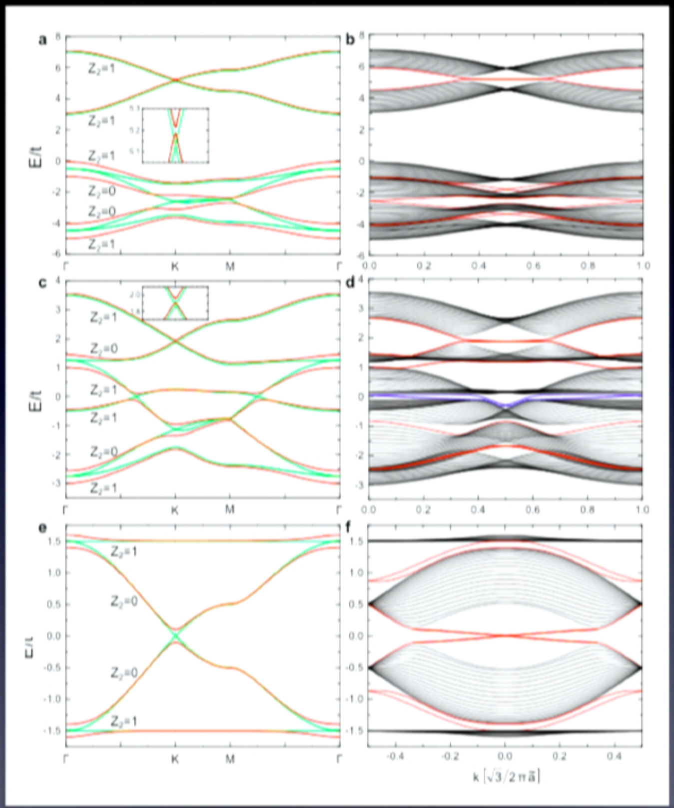
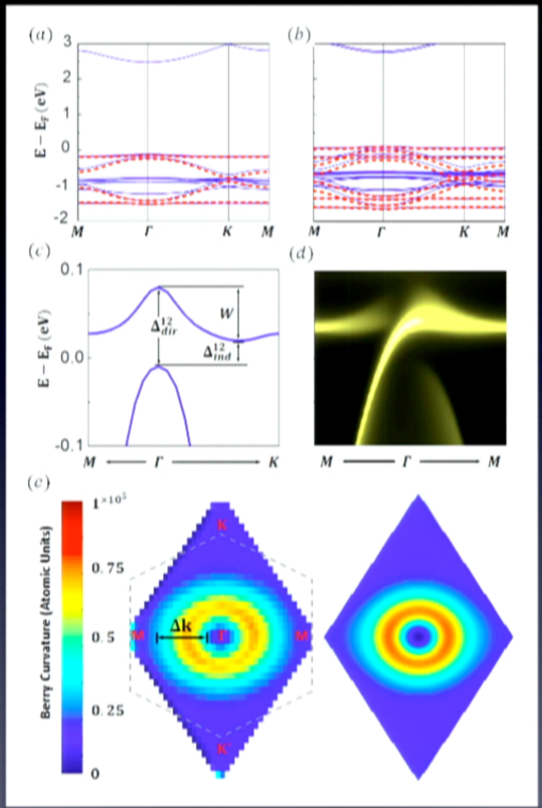
Flatband topological insulators: **Towards a physical system**



models for transition metal oxide heterostructures

D. Xiao et al., Nature Comm. **2**, 596 (2011)



2D indium-phenylene organometallic framework

Zheng Liu et al., arXiv:1210.1826

Fractional Chern insulators: **Why do we care?**

What is at the heart of the FQHE?

many-body gap $\approx U$ gives high-temperature effect

at 1/2 filling: full magnetization gives quantum anomalous Hall effect.

low dissipative conductor at room temperature

T. Neupert et al., Phys. Rev. Lett. **108** 046806 (2011)

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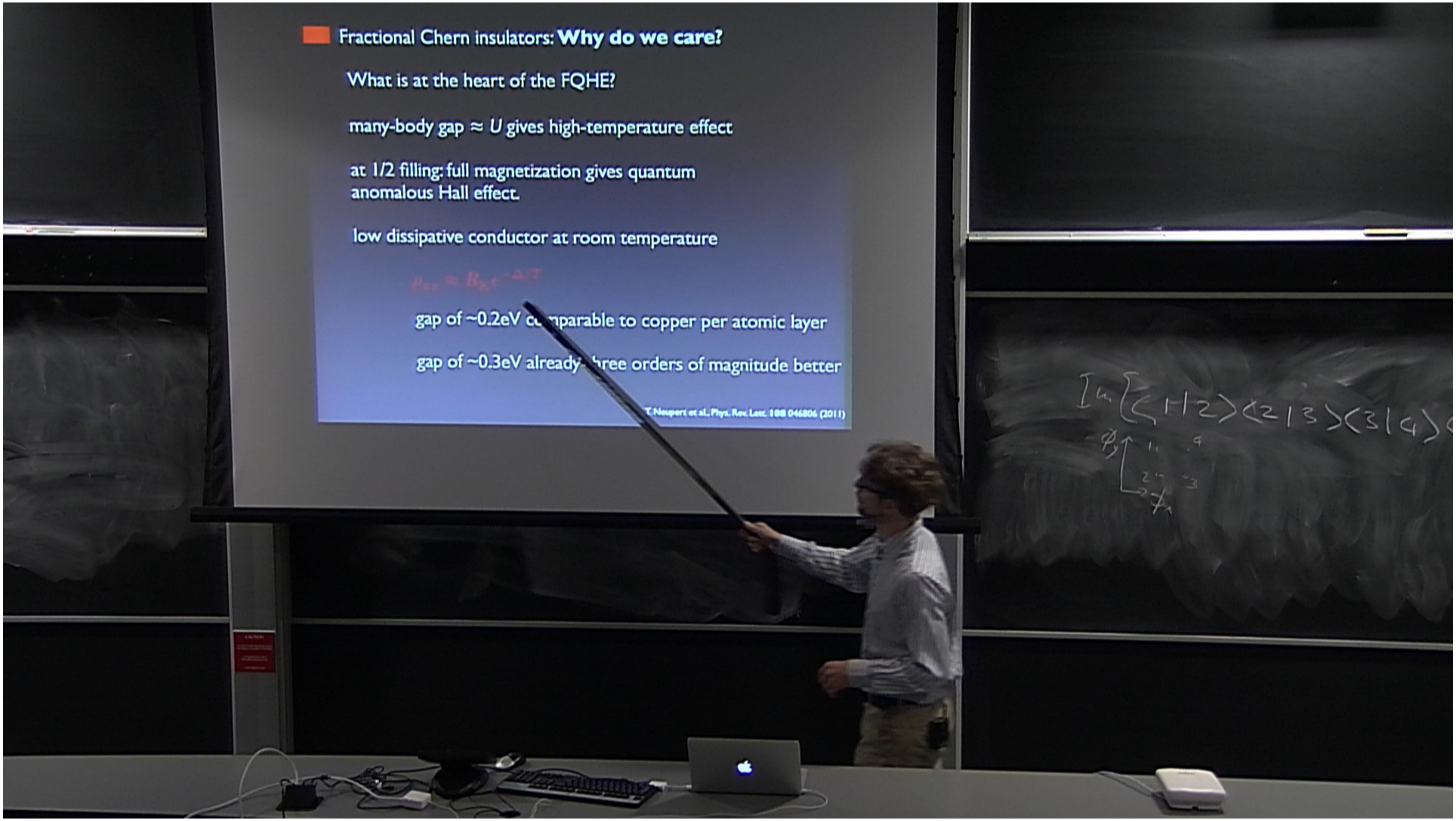
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