

Title: Twisted quantum double model of topological phases

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URL: <http://pirsa.org/12110079>

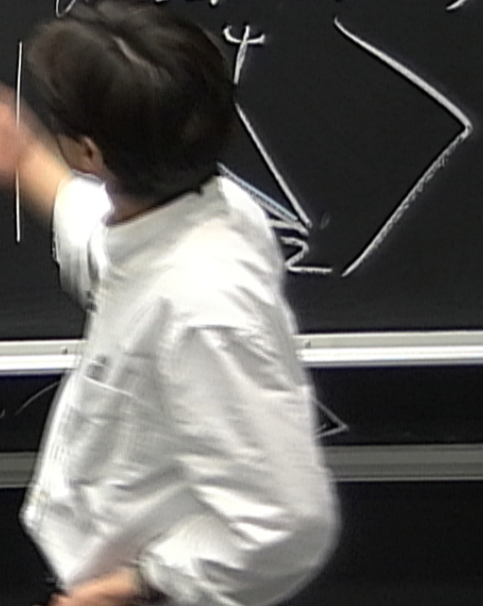
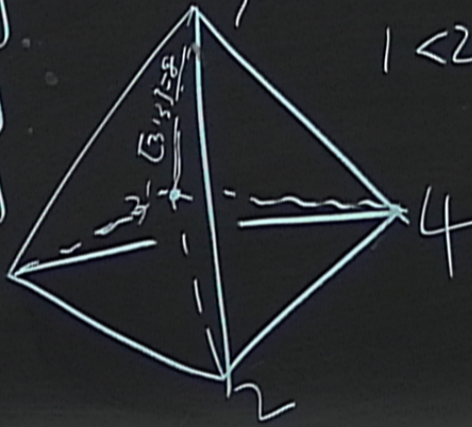
Abstract:

$$A_v = \frac{1}{|G|} \sum_{[v',v] \in G} A_v^g$$

$$A_B^g \left| \begin{array}{c} \text{tetrahedron} \\ \text{vertices } 1, 2, 3, 4 \end{array} \right\rangle = \int_{[3'3], g} \alpha([12], [23'], [3'3]) \alpha([23'], [3'3], [34]) \alpha([13'], [3'3], [34])^{-1}$$

$$1 < 2 < 3' < 3 < 4$$

$$\begin{aligned} [13'] \cdot [3'3] &= [13] \\ [23'] \cdot [3'3] &= [23] \\ [3'3] \cdot [34] &= [3'4] \end{aligned}$$



$$H_{G, \alpha}$$

$$H_{G, \alpha'}$$

$$\alpha \sim \alpha'$$

$$\alpha' = \alpha \delta \beta$$

$$\beta^{(t)} = \beta(x, t)^t$$

$$\alpha^{(t)} = \alpha \delta \beta^{(t)}$$

$$\beta(x, y, z) \beta \colon G^2 \rightarrow U(1)$$

$$\delta \beta(x, y, z) = \frac{\beta(y, z) \beta(x, yz)}{\beta(xy, z) \beta(x, y)}$$

$$H_{G, \alpha}$$

$$H_{G, \alpha'}$$

$$\alpha \sim \alpha'$$

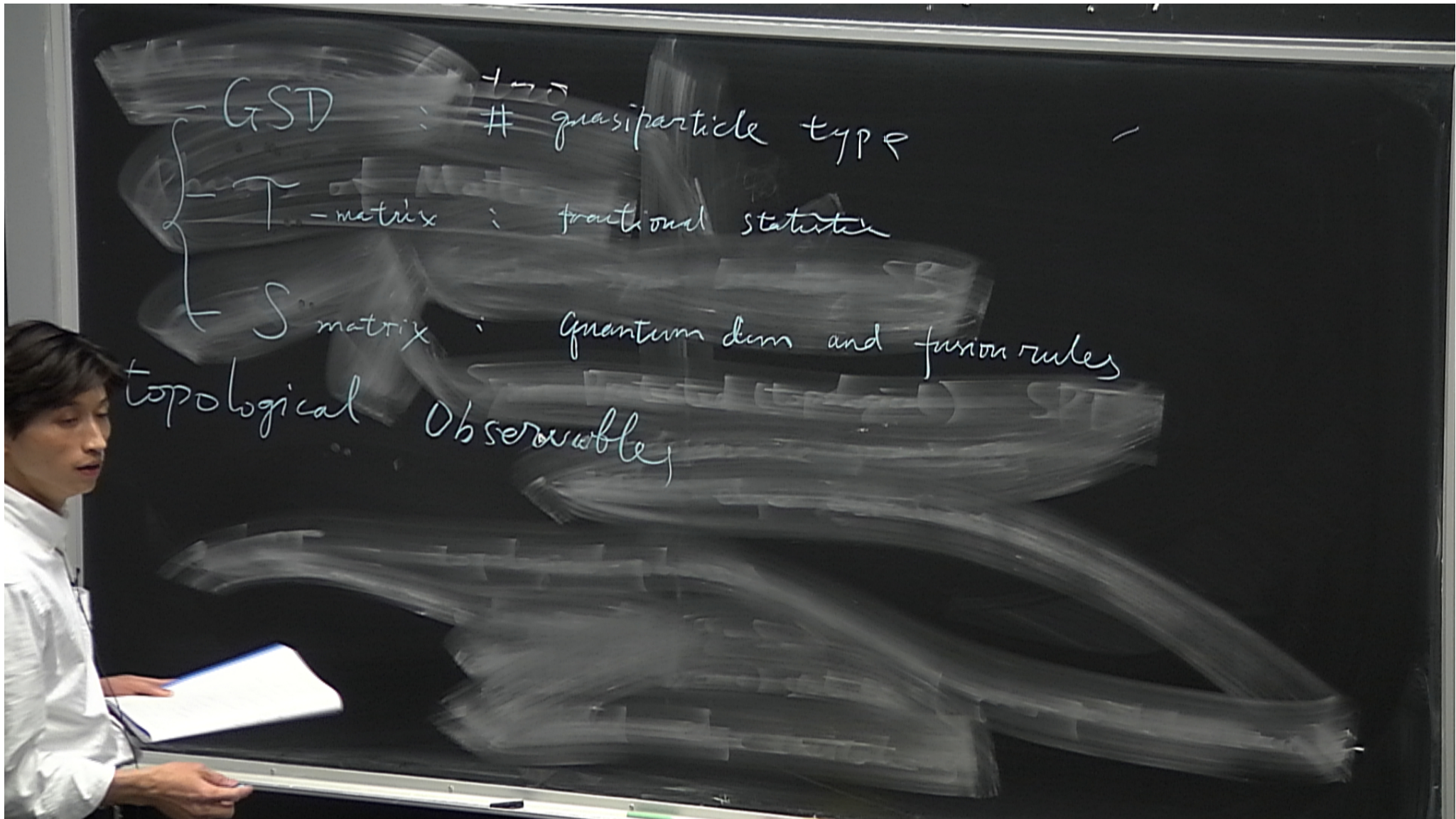
$$\alpha' = \alpha \delta \beta$$

$$\beta^{(t)} = \beta(x, t)^t$$

$$\alpha^{(t)} = \alpha \delta \beta^{(t)}$$

$$0 \leq t \leq 1, \quad \beta(y, z)^\beta : G^2 \rightarrow U(1)$$

$$\delta \beta(x, y, z) = \frac{\beta(y, z) \beta(x, yz)}{\beta(xy, z) \beta(x, y)}$$



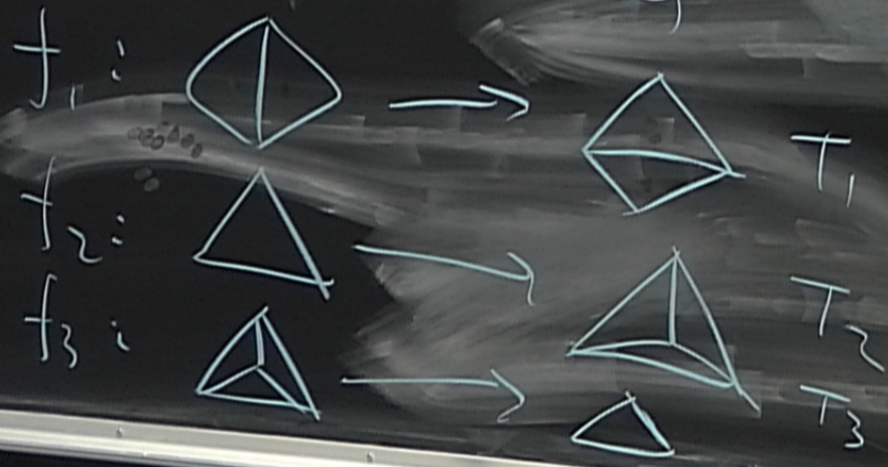
- GSD : $\#$ quasiparticle type
- T-matrix : fractional statistics
- S-matrix : quantum dimension and fusion rules
topological observable

- GSD : $\#$ quasiparticle type

- T-matrix : fractional statistics

- S-matrix : quantum dim and fusion rules

topological observable



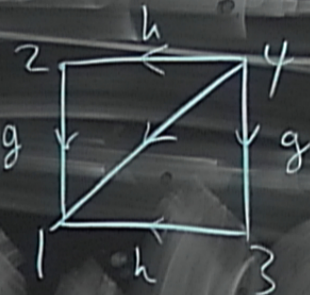
$$P_T^0 = \prod_{f \in T} B_f \prod_{v \in T} A_v$$

$$P_T^0 |\psi\rangle = |\psi\rangle$$

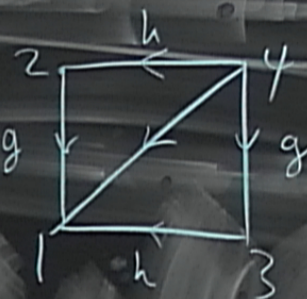
$$T_i P_T^0 = P_T^0 T_i$$

$$\langle T_i \phi | T_i \psi \rangle = \langle \phi | \psi \rangle$$

$$G_{SD} = \text{tr} P_T^0$$



$$GSD = \text{tr} P_T^0$$



$$P_T^0 = \frac{1}{|G|} \sum_{x \in G} A^x$$

$|g, h\rangle$

$$\begin{aligned} A^x |g, h\rangle &= \frac{\beta_g(x^{-1}, x_h x^{-1})}{\beta_g(h, x^{-1})} |x_g x^{-1}, x_h x^{-1}\rangle \\ &= \eta_g^x(h, x) |x_g x^{-1}, x_h x^{-1}\rangle \end{aligned}$$

$$\beta_a(b, c) = \frac{\alpha(a, b, c) \alpha(b, c, c^{-1} b^{-1} a b c)}{\alpha(b, b^{-1} a b, c)}$$

twisted 2-cocycle

$$\beta_a(b, c) = \frac{\alpha(a, b, c) \alpha(b, c, c^{-1} b^{-1} a b c)}{\alpha(b, b^{-1} a b, c)}$$

twisted 2-cocycle

$$\delta \beta_a(x, y, z) = \frac{\beta_{x^{-1} a x}(y, z) \beta_a(x, y z)}{\beta_a(x y, z) \beta_a(x, y)} = 1$$

$$\beta_a(b, c) = \frac{\alpha(a, b, c) \alpha(b, c, c^{-1} b^{-1} a b c)}{\alpha(b, b^{-1} a b, c)}$$

twisted 2-cocycle

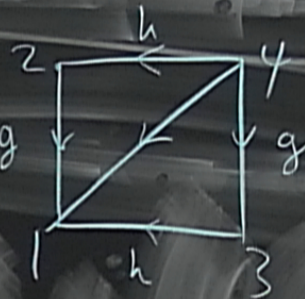
$$\tilde{\delta} \beta_a(x, y, z) = \frac{\beta_{x^{-1} a x}(y, z) \beta_a(x, yz)}{\beta_a(xz, z) \beta_a(x, y)} = 1$$

$$\mathcal{H}^0 = \text{span} \left\{ \frac{1}{|G|} \sum_{x \in G} \eta^g(h, x) |xg\bar{x}^{-1}, xh\bar{x}^{-1}\rangle \mid g \in G, h \in Z_g \right\}$$

H

$\{y \in G \mid yg = gy\}$
 $h \in Z_g$

$$GSD = \text{tr} P_T^0$$



$$P_T^0 = \frac{1}{|G|} \sum_{x \in G} A^x$$

$|g, h\rangle$

$$gh = hg$$

$$A^x |g, h\rangle = \frac{\beta_g(x^{-1}, x_h x^{-1})}{\beta_g(h, x^{-1})} |x_g x^{-1}, x_h x^{-1}\rangle$$

$$= \eta_g^g(h, x) |x_g x^{-1}, x_h x^{-1}\rangle$$



twisted 2-cocycle

$$\tilde{\delta} \beta_a(x, y, z) = \frac{\beta_{x^{-1}ax}(y, z) \beta_a(x, yz)}{\beta_a(x, y) \beta_a(x, yz)} = 1$$

$$\mathcal{H}^0 = \text{span} \left\{ \frac{1}{|G|} \sum_{x \in G} \eta^g(h, x) |xgx^{-1}, xhx^{-1}\rangle \mid g \in G, h \in Z_g \right\}$$

GST $\text{Hom}(Z, G)$

$\{y \in G \mid yg = gy\}$
 Z_g

$$\alpha(g_1, g_2, g_3) = \alpha(g_1, g_2, g_3) \alpha(g_0 g_1, g_2, g_3)^{-1}$$

$$\alpha(g_0, g_1, g_2, g_3) \alpha(g_0, g_1, g_2 g_3)^{-1}$$

$$\alpha(g_0, g_1, g_2) = 1$$

$$A_\nu = \frac{1}{|G|} \sum_{g \in G} A_\nu^g$$

$$GSD = \frac{1}{|G|} \sum_{g, h, x \in G} \delta_{gh, hg} \delta_{gx, xg} \delta_{hx, xh} \eta^g(h, x)$$

$$= \sum_{A=1}^{r(G)} r(Z_1^A, \beta_{g^A})$$

$$Z_n$$

$$=$$

$$B_f \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle = \delta_{[v_1, v_2], [v_2, v_3], [v_3, v_1], 1} \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle$$

$$A_L = \frac{1}{|G|} \sum A_L^g$$

$$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx, xg} \delta_{hx, xh} \eta^g(h, x)$$

$$= \sum_{A=1}^{r(G)} r(Z^A, \beta_{g^A})$$

$$Z_n = \sum_A \#(\beta_{g^A} \text{-irrep of } Z^A)$$

$$\tilde{p}(x) \tilde{p}(y) = \beta_{g^A}$$

$$B_f \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle = \delta_{[v_1, v_2], [v_2, v_3], [v_3, v_1], 1} \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle$$

$$A_\nu = \frac{1}{|G|} \sum A_\nu^g$$

$$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx, xg} \delta_{hx, xh} \eta^g(h, x)$$

$$= \sum_{A=1}^{N(G)} r(Z^A, \beta_{g^A})$$

$$\begin{aligned} & \textcircled{Z_n} \\ r(Z_n) &= \textcircled{n} \end{aligned}$$

$$= \sum (\beta_{g^A} \text{-irrep of } Z^A)$$

$$\tilde{p}(x) \tilde{p}(y) = \beta_a(x, y) \tilde{p}(xy) \\ x, y \in Z_a$$

$$B_f \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle = \delta[v_1, v_2] \cdot [v_2, v_3] [v_3, v_1] \cdot 1$$

$$\beta_a(b, c) = \frac{\alpha(a, b, c) \alpha(b, c, c^{-1}b^{-1}abc)}{\alpha(b, b^{-1}ab, c)}$$

twisted 2-cocycle

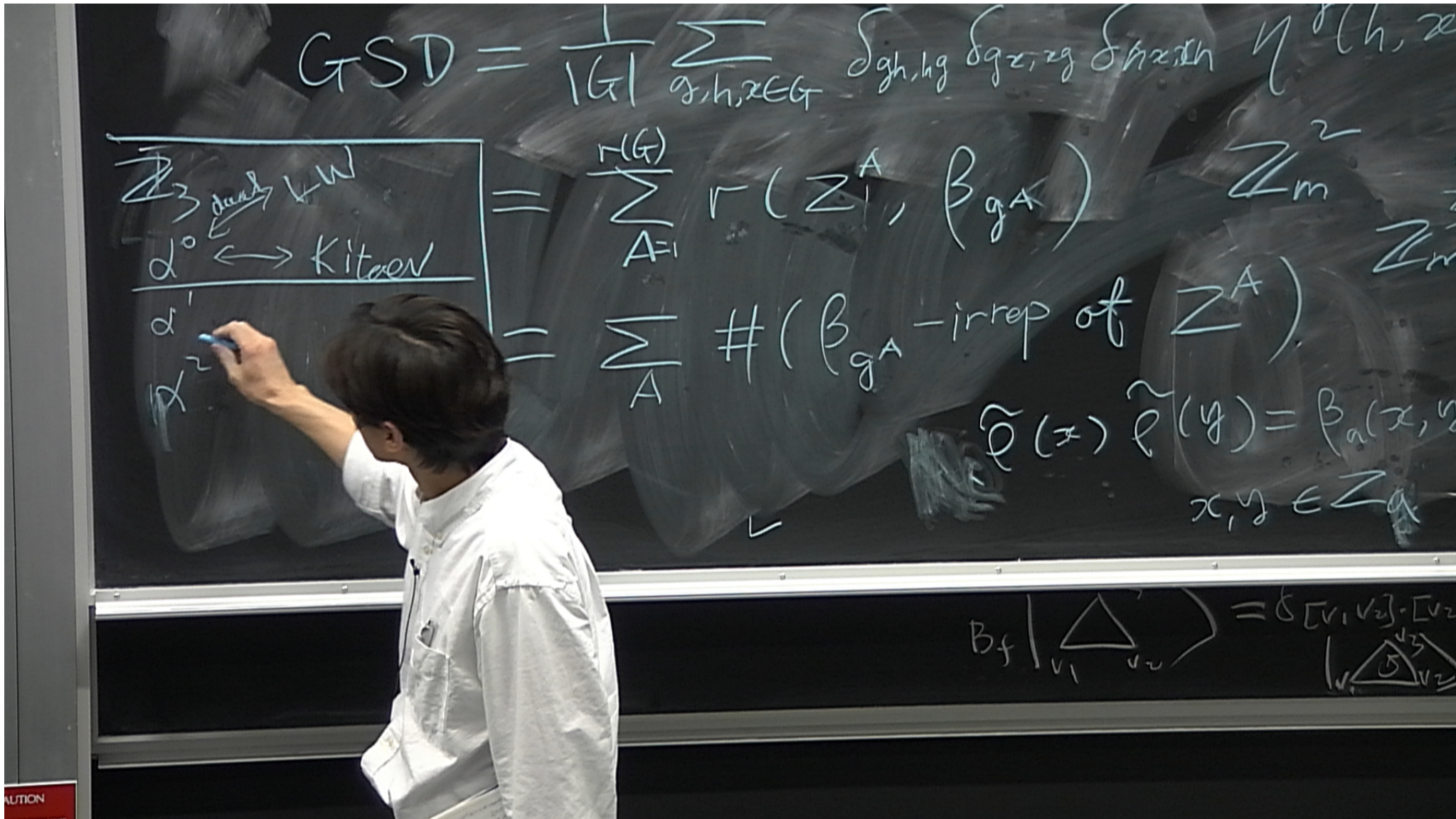
$$\delta \beta_a(x, y, z) = \frac{\beta_{x^{-1}ax}(y, z) \beta_a(x, yz)}{\beta_a(xyz, z) \beta_a(x, y)} = 1$$

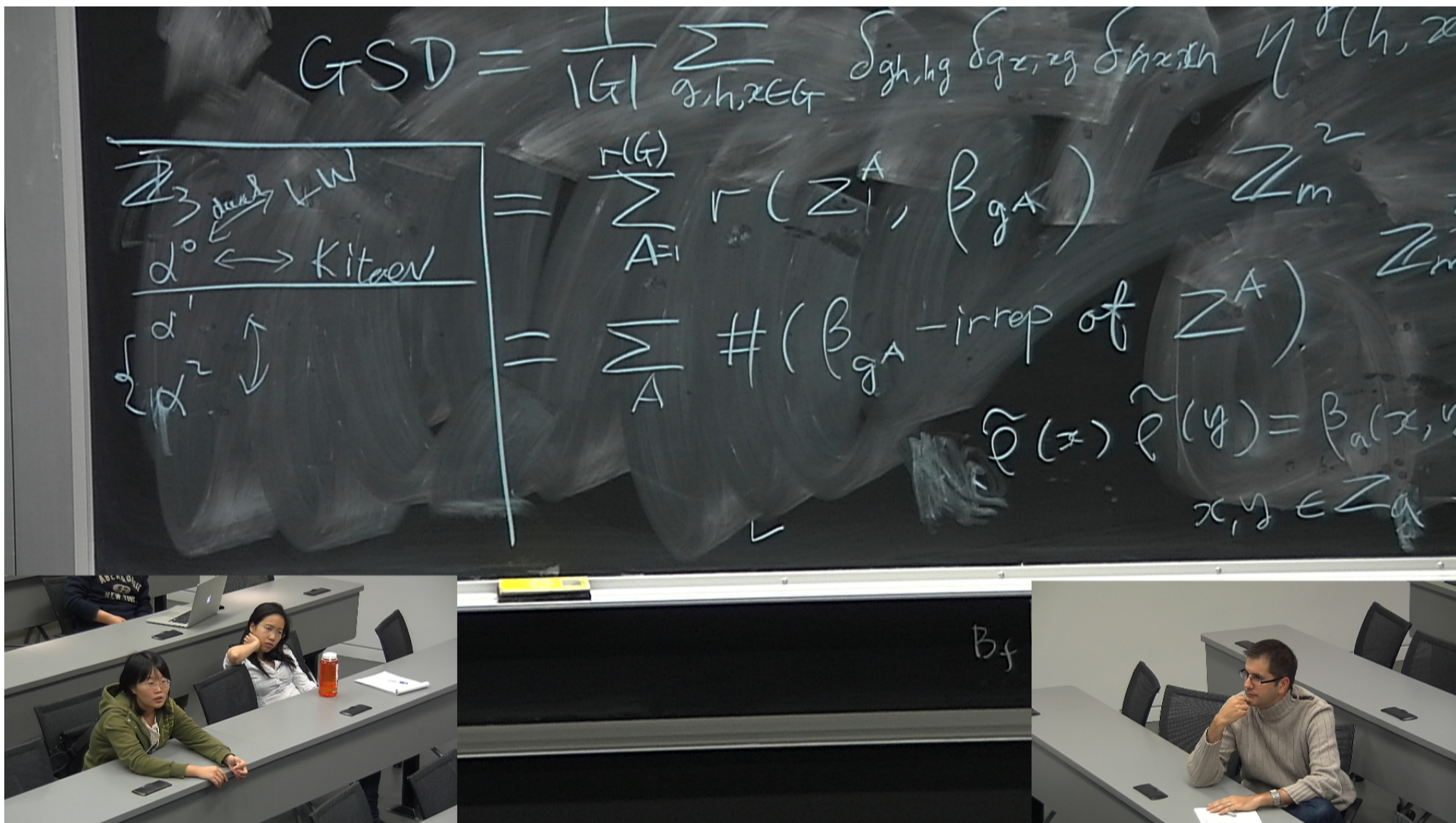
$$\mathcal{H}^0 = \text{span} \left\{ \frac{1}{|G|} \sum_{x \in G} \eta^g(h, x) |xgx^{-1}, xhx^{-1}\rangle \mid g \in G, h \in Z \right\}$$

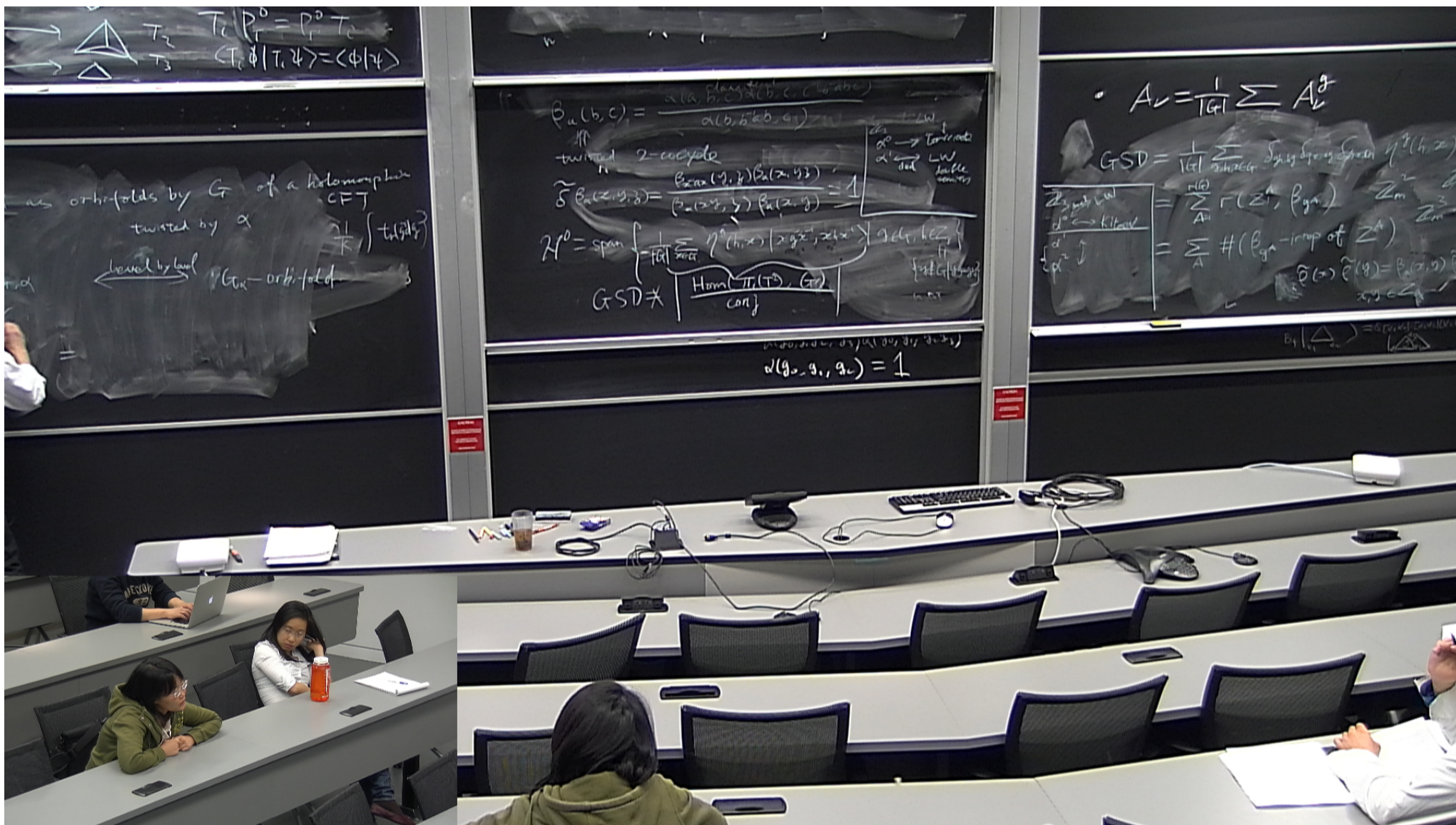
$$GSD \cong \frac{\text{Hom}(\pi, (T^2)^*, G)}{\text{conj}}$$

$$\begin{array}{c} \mathcal{H}^0 \\ \alpha^0 \rightarrow \text{toric code} \\ \alpha^1 \end{array}$$









RCT as orbifolds by G of a holomorphic CFT
 twisted by α

$$\int dG, \alpha$$

level by level

$$GSD$$

=

G, α -orbifold

$$\frac{1}{K} \int \text{tr} g dg$$

RCFT as orbifolds by G of a holomorphic CFT
 twisted by α

$H(G, \alpha)$

level by level

G_{α} -orbifold

GSD

$|g, h\rangle =$

$|A, \mu\rangle$

$\equiv$

primary fields

$h \square g$

$$\frac{1}{k} \int \text{tr} g dg$$

$$A_\nu = \frac{1}{|G|} \sum A_\nu^g$$

$$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx,xg} \delta_{hx,xh} \eta^g(h,x)$$

$$\begin{aligned} \frac{\mathbb{Z}_3 \xrightarrow{\text{dual}} \text{LW}}{d^0 \leftrightarrow \text{Kitaev}} &= \sum_{A=1}^{r(G)} r(Z^A, \beta_{g^A}) \quad \mathbb{Z}_m^2 \quad \mathbb{Z}_{g,h} \\ &= \sum_A \#(\beta_{g^A} \text{-irrep of } Z^A) \quad \mathbb{Z}_m^3 \\ &\quad \tilde{\rho}(x) \tilde{\rho}(y) = \beta_a(x,y) \tilde{\rho}(xy) \\ &\quad x,y \in \mathbb{Z}_a \end{aligned}$$

$$B_f \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle = \delta_{[v_1, v_2], [v_2, v_3], [v_3, v_1], 1} \left| \begin{array}{c} \triangle \\ v_1 \quad v_2 \end{array} \right\rangle$$

RCFT as orbifolds by G of a holomorphic CFT
 twisted by α

$$H_{G, \alpha}$$

level by level

$$\frac{1}{k} \int \text{tr}(\dot{g} dg)$$

$$GSD$$

G_{α} -orbifold

$$|q, h\rangle =$$

$\Rightarrow$
 $\longleftrightarrow$

primary fields

$$|A, \mu\rangle$$

$$\left(\begin{array}{c} \square \\ q \end{array} \right)$$

$$\chi_{\mu}^A$$

RCFT as orbifolds by G of a holomorphic CFT
 twisted by α

$H_{G,\alpha}$

level by level

$$\frac{1}{k} \int \text{tr}(\dot{g} dg)$$

G SD

G_{α} -orbifold

primary fields



$$\chi_{\mu}^g = \frac{1}{N|G|} \sum_{g \in G} \sum_{h \in Z_g} \chi_{\mu}(h) \chi_{\mu}(g)$$

$$|A, \mu\rangle = \frac{1}{N|G|} \sum_{g \in G} \sum_{h \in Z_g} \hat{\chi}_{\mu}^g(h) |g, h\rangle$$

RCFT as orbifolds by G of a holomorphic CFT
 twisted by α

$$H_{G, \alpha}$$

level by level

$$\frac{1}{k} \int \text{tr}(\dot{g} dg)$$

$$GSD$$

G_{α} -orbifold

$$|g, h\rangle =$$

$$|A, \mu\rangle$$

primary fields

$$\left(\begin{array}{c} h \\ \square \\ g \end{array} \right)$$

$$K_A = \frac{1}{N|G|} \sum_{g \in G} \sum_{h \in Z_g} \chi_{\mu}(h) \left(\begin{array}{c} h \\ \square \\ g \end{array} \right)$$

$$|A, \mu\rangle = \frac{1}{N|G|} \sum_{g \in G} \sum_{h \in Z_g} \hat{\chi}_{\mu}^g(h) |g, h\rangle$$