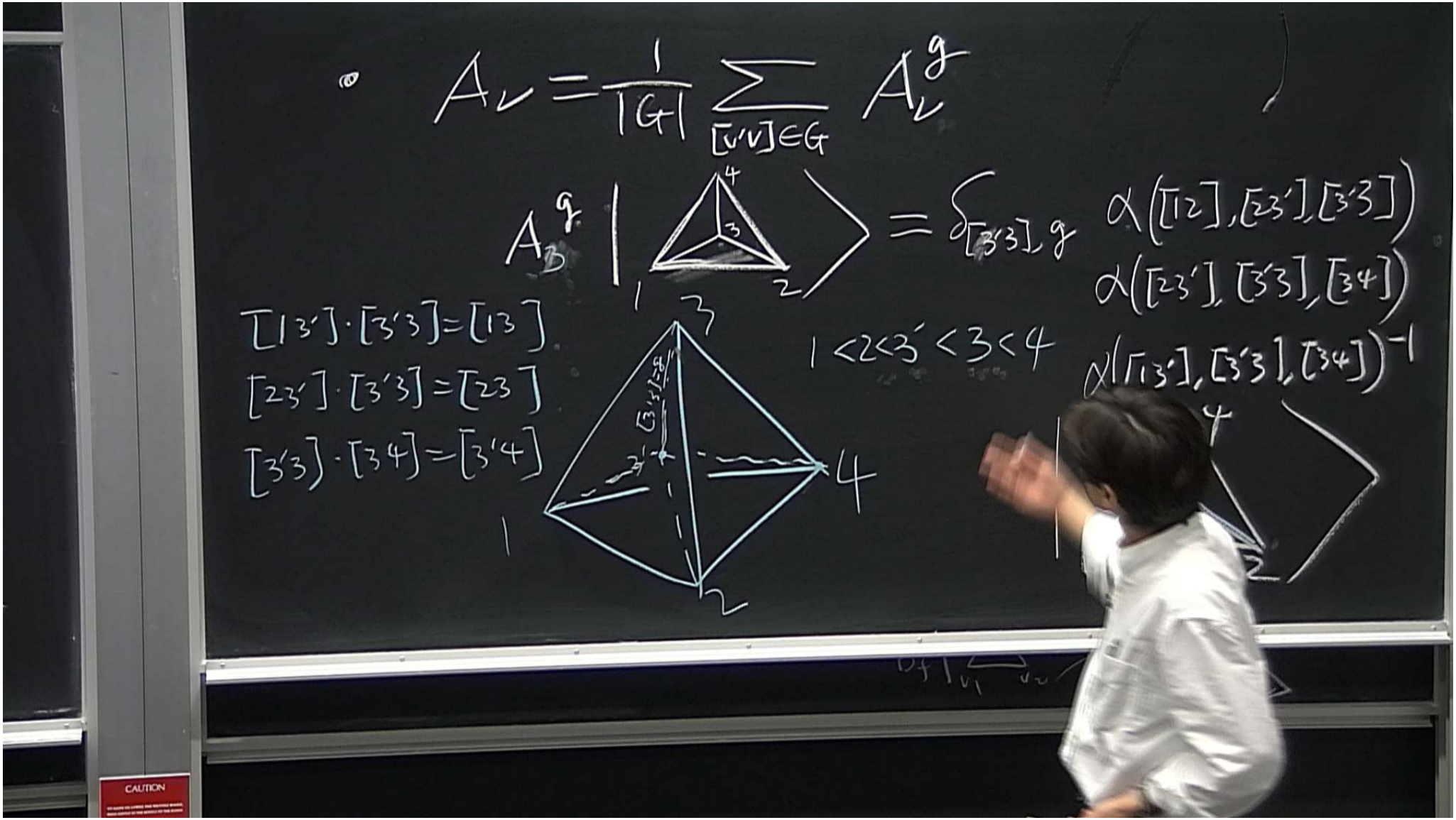
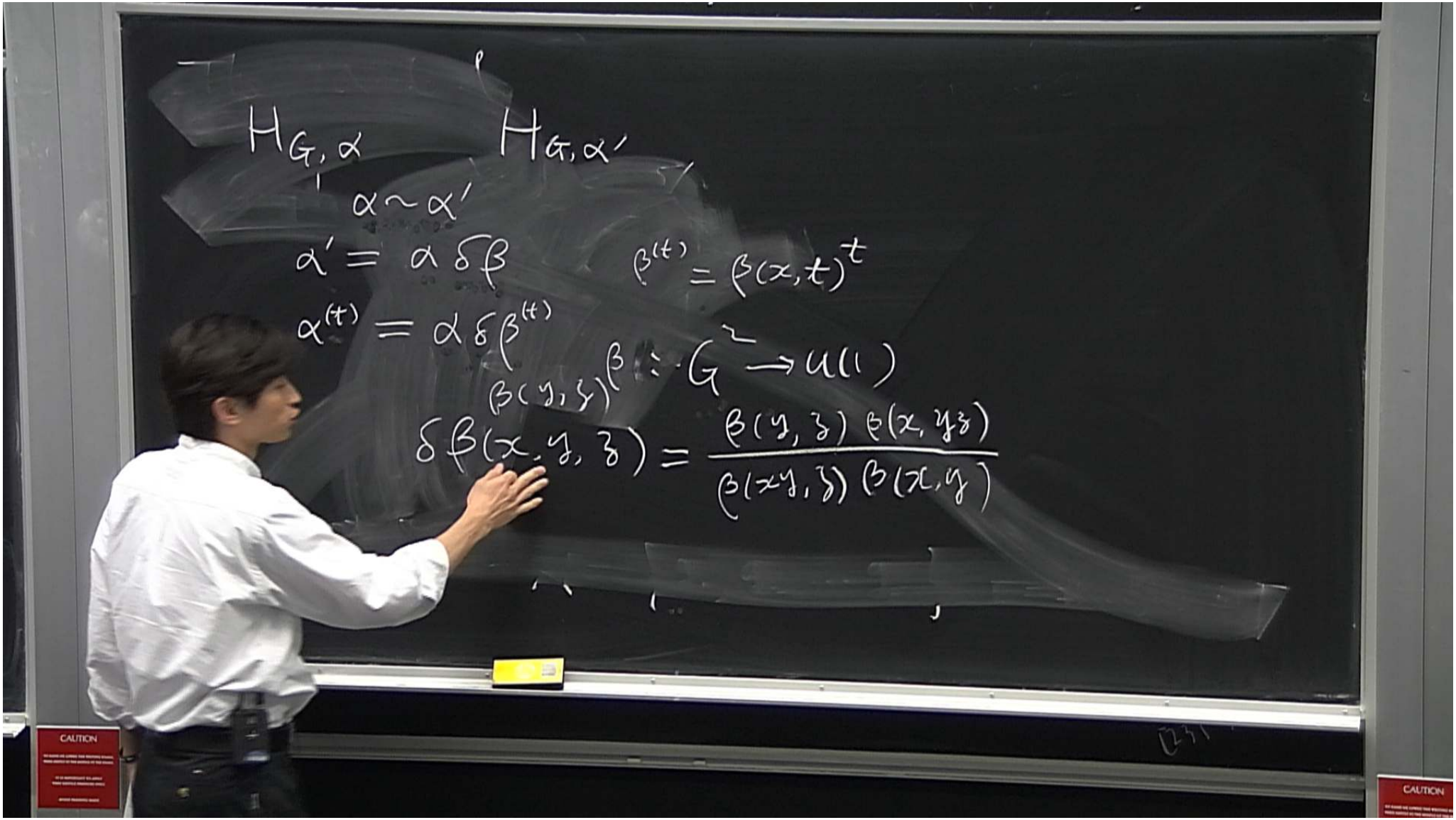
Title: Twisted quantum double model of topological phases

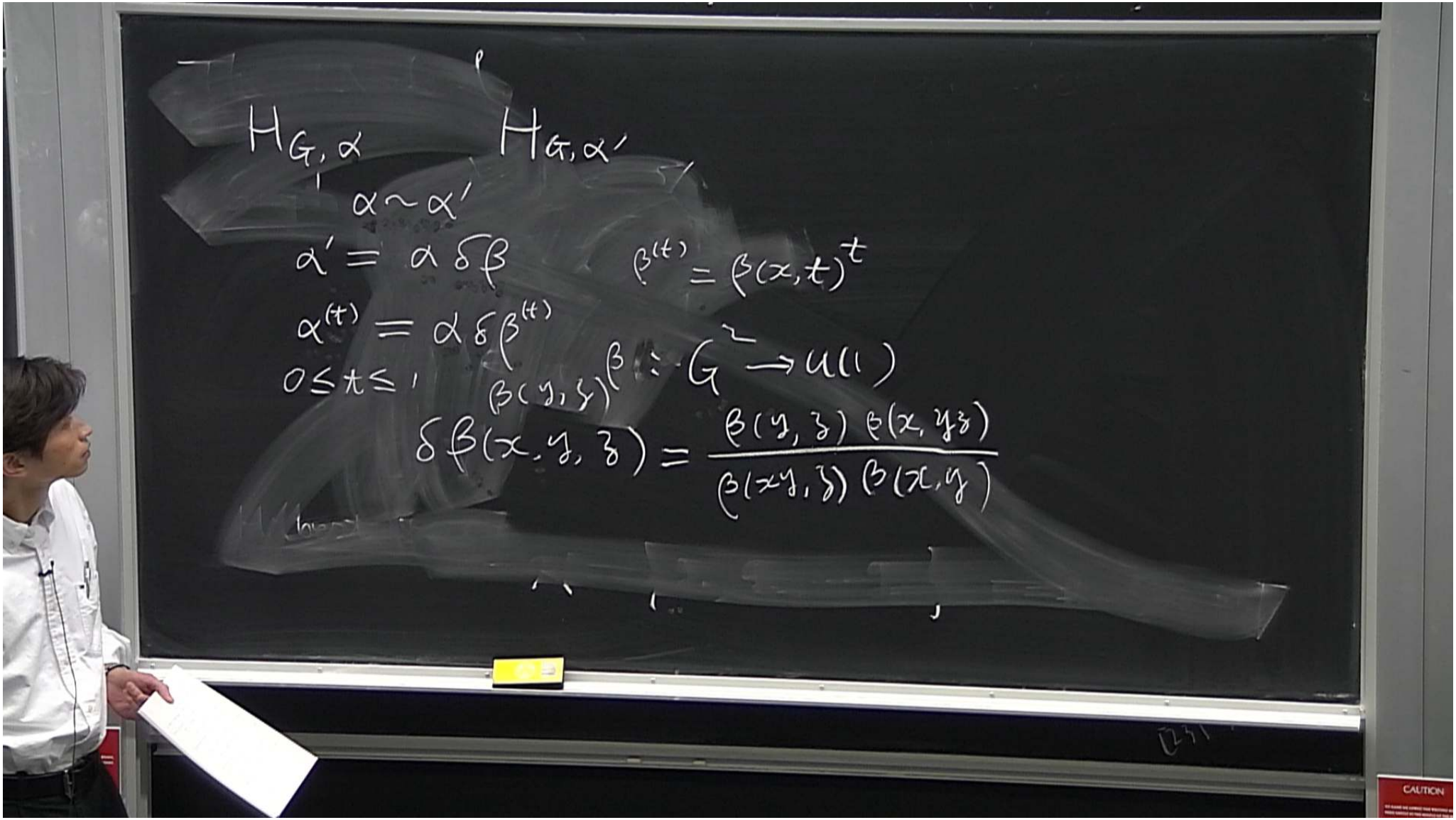
Date: Nov 21, 2012 03:15 PM

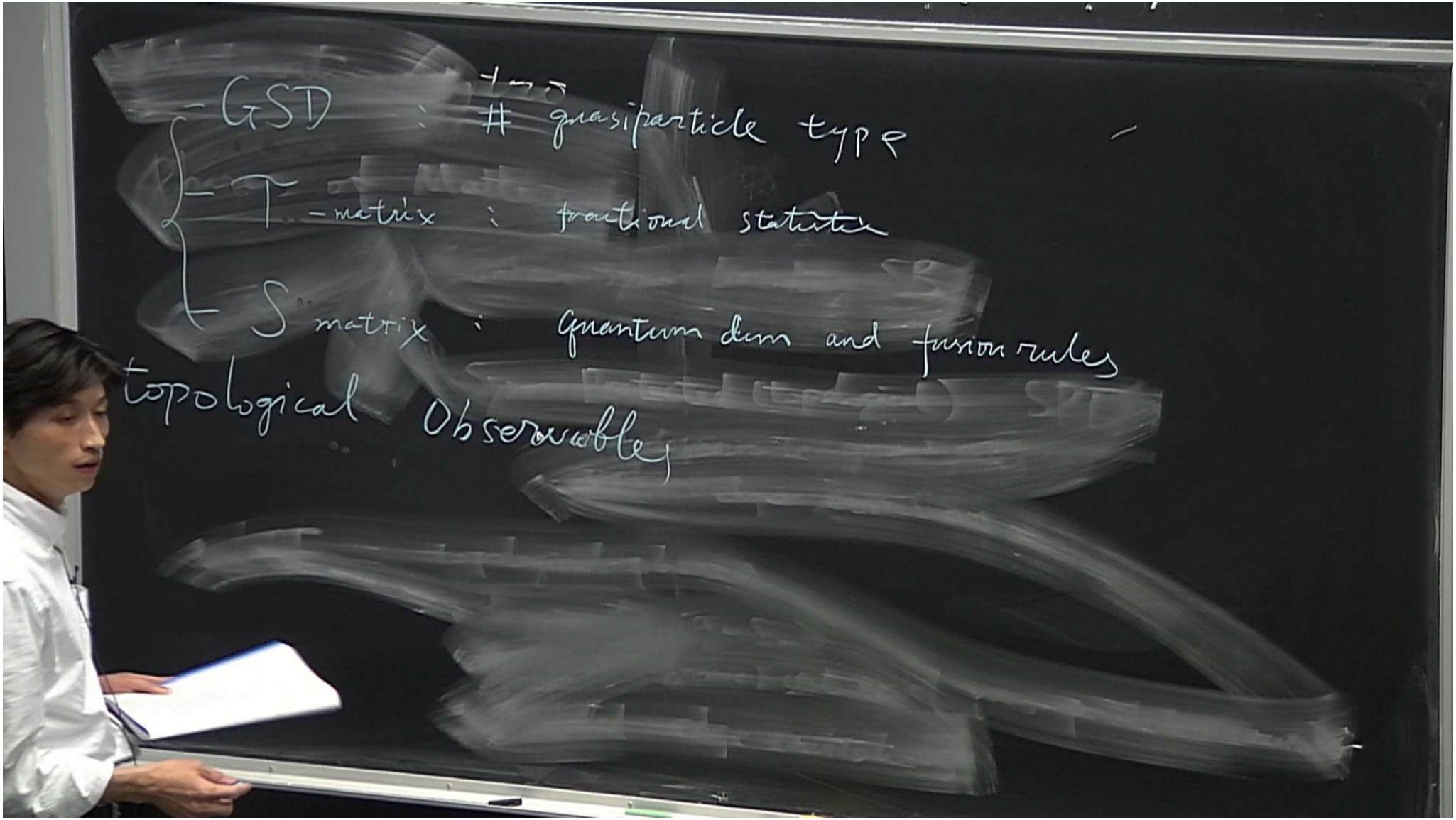
URL: http://pirsa.org/12110079

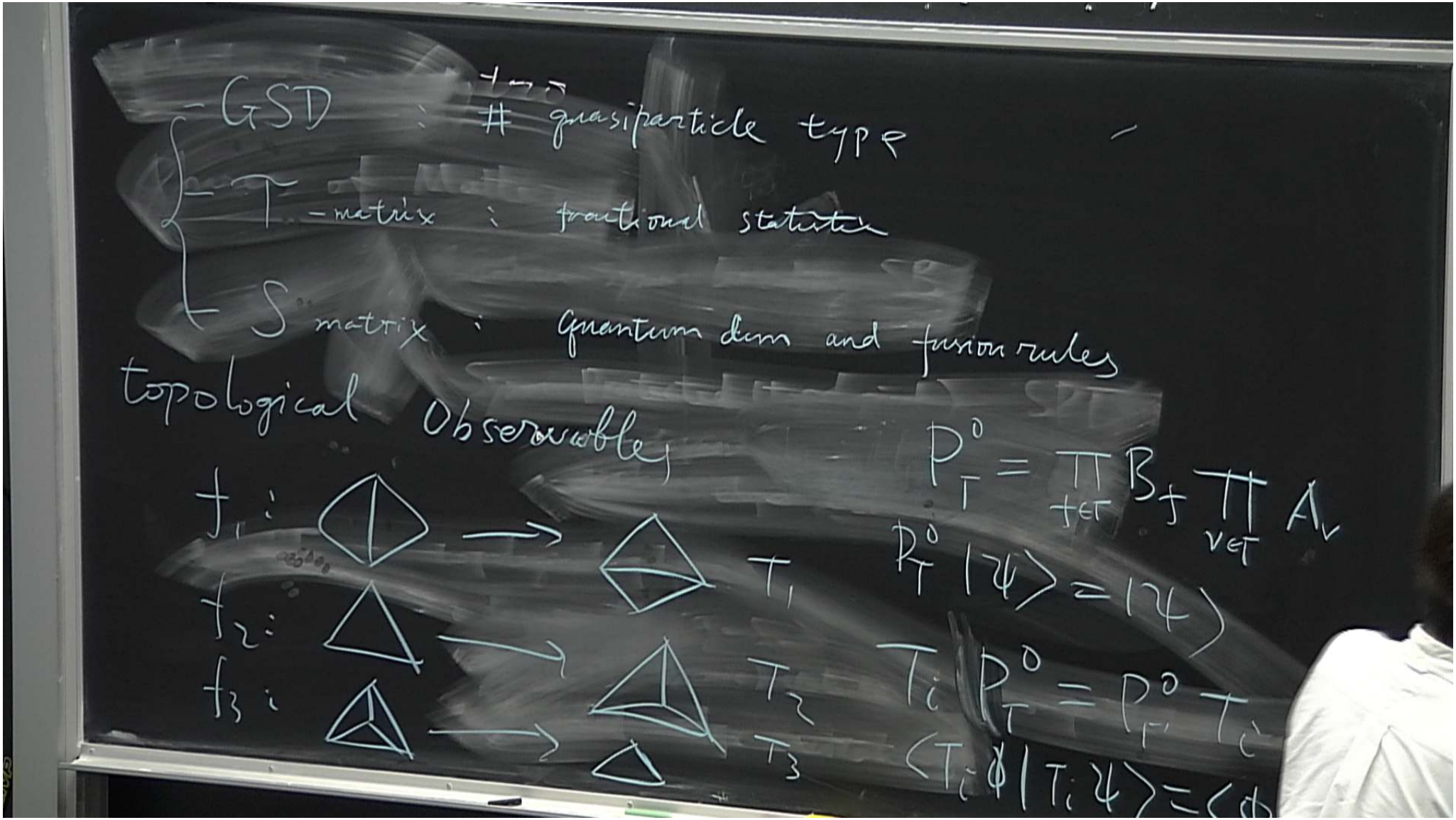
Abstract:

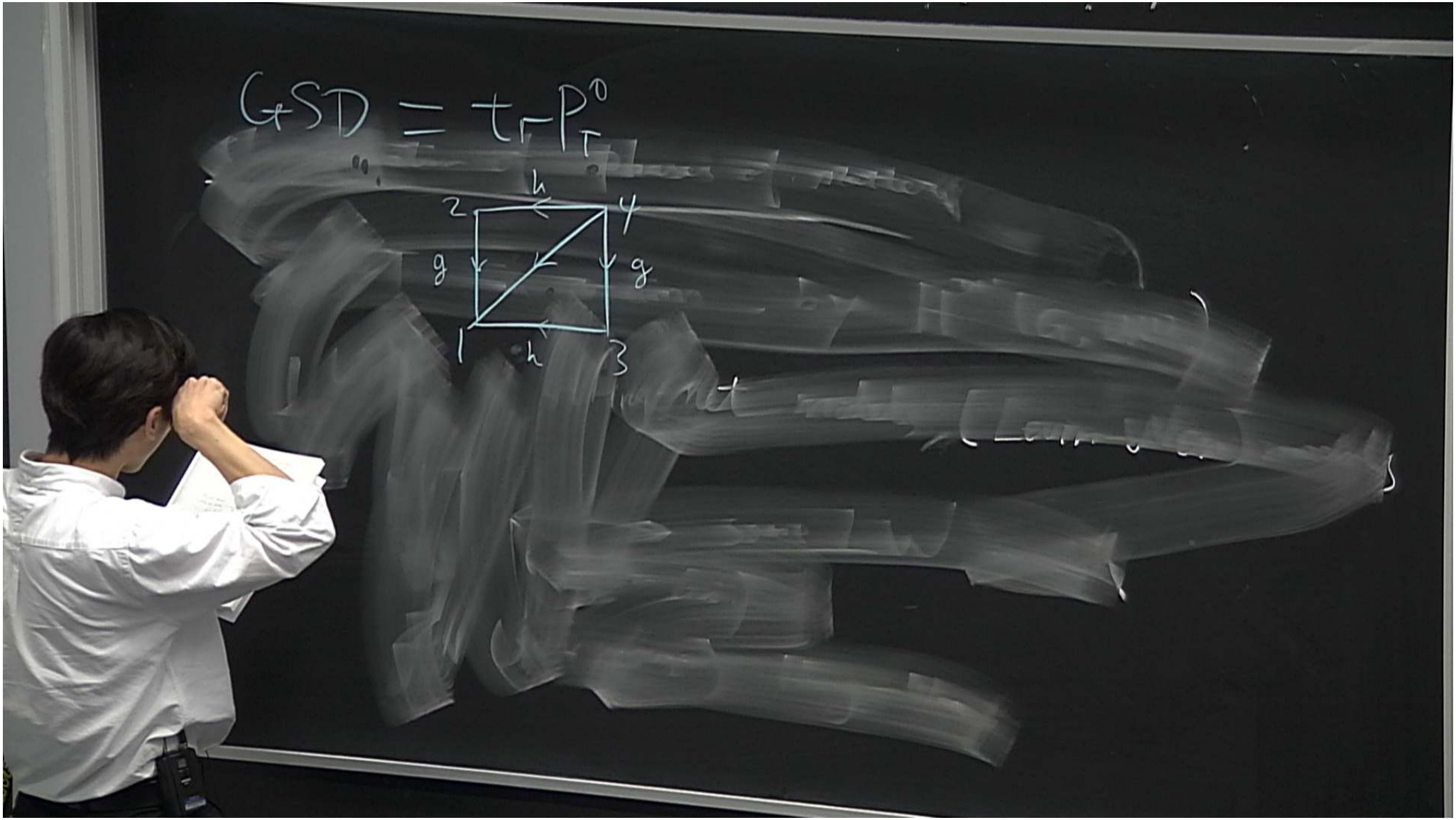

$A_L = \frac{1}{|G|} \sum_{[V,V] \in G} A_L^g$
$A_B^g \left| \right\rangle = \delta_{[B'3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$[13'] \cdot [3'3] = [13]$
$[23'] \cdot [3'3] = [23]$
$[3'3] \cdot [34] = [3'4]$
$1 < 2 < 3' < 3 < 4$
CAUTION

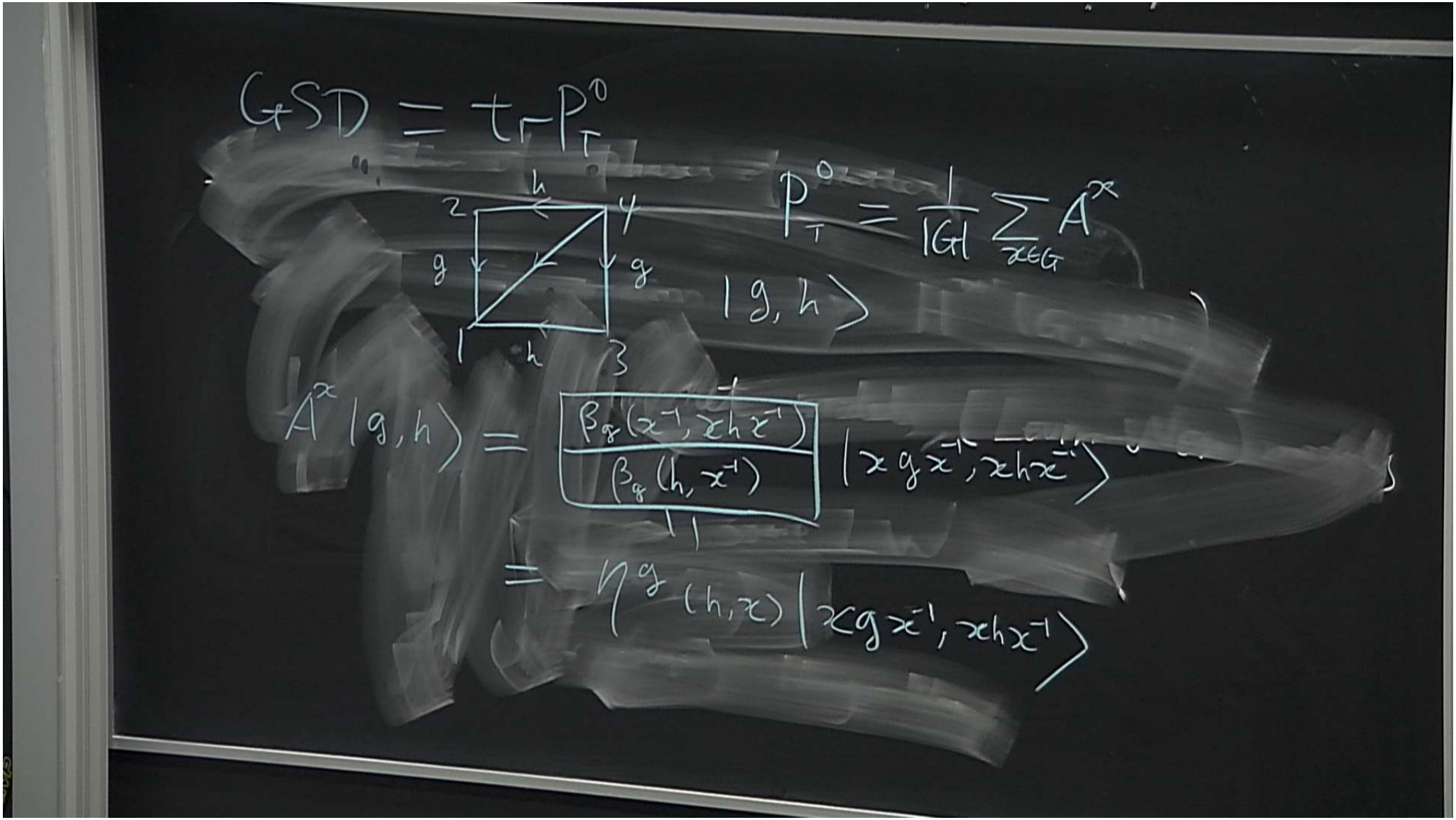

$H_{G,\alpha}$ $H_{G,\alpha'}$
$\alpha \sim \alpha'$
$\alpha' = \alpha\,\delta\beta$
$\alpha^{(t)} = \alpha\,\delta\beta^{(t)}$
$\beta^{(t)} = \beta(x,t)^t$
$\beta : G^2 \to U(1)$
$\delta\beta(x,y,z) = \frac{\beta(y,z)\,\beta(x,yz)}{\beta(xy,z)\,\beta(x,y)}$
CAUTION
CAUTION

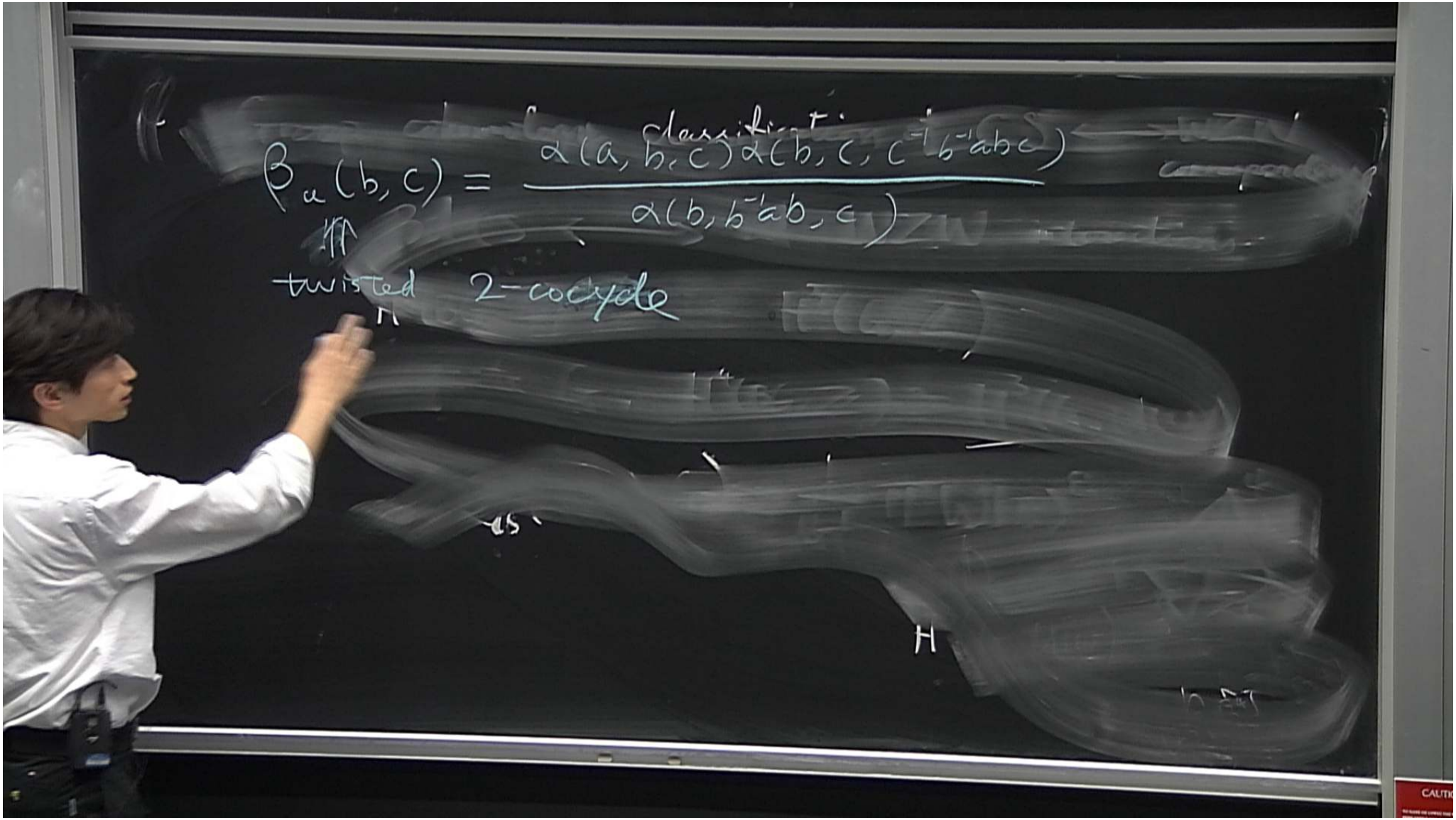

$H^{G,\alpha}$ $H^{G,\alpha'}$
$\alpha \sim \alpha'$
$\alpha' = \alpha \delta\beta$
$\beta^{(t)} = \beta(x,t)^t$
$\alpha^{(t)} = \alpha \delta\beta^{(t)}$
$0 \le t \le 1$
$\beta(y,z)$
$\beta : G^2 \to U(1)$
$\delta\beta(x,y,z) = \frac{\beta(y,z)\,\beta(x,yz)}{\beta(xy,z)\,\beta(x,y)}$
CAUTION

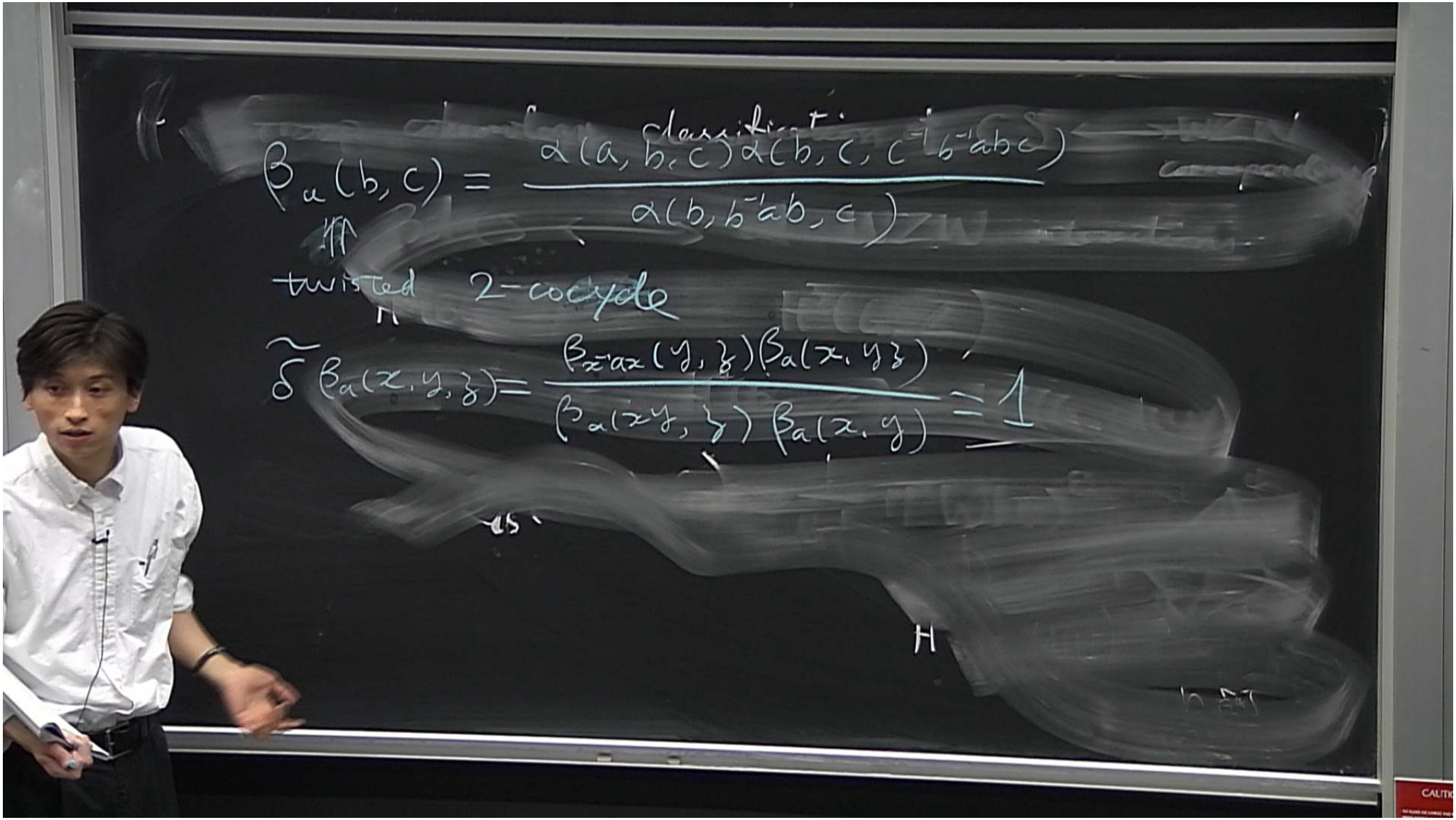

GSD : # quasiparticle type
T-matrix : fractional statistics
S matrix : Quantum dim and fusion rules
topological Observables

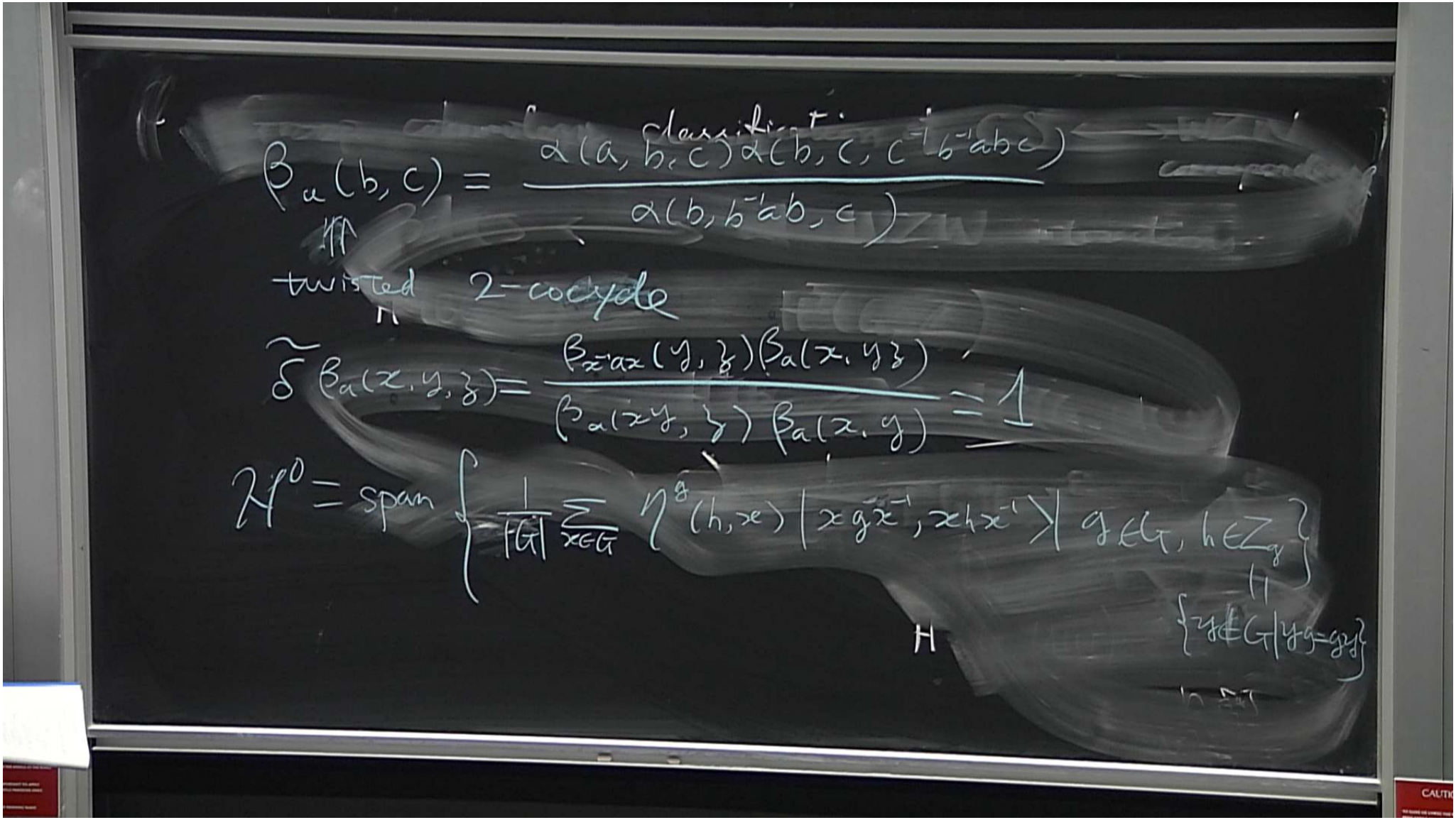

GSD : # quasiparticle type
T-matrix : fractional statistics
S matrix : Quantum dim and fusion rules
topological Observables
$P_T^0 = \prod_{t \in T} B_t \prod_{v \in T} A_v$
$P_T^0 |\psi\rangle = |\psi\rangle$
$T_i P_T^0 = P_T^0 T_i$
$\langle T_i \phi | T_i \psi \rangle = \langle \phi$
$T_1$
$T_2$
$T_3$

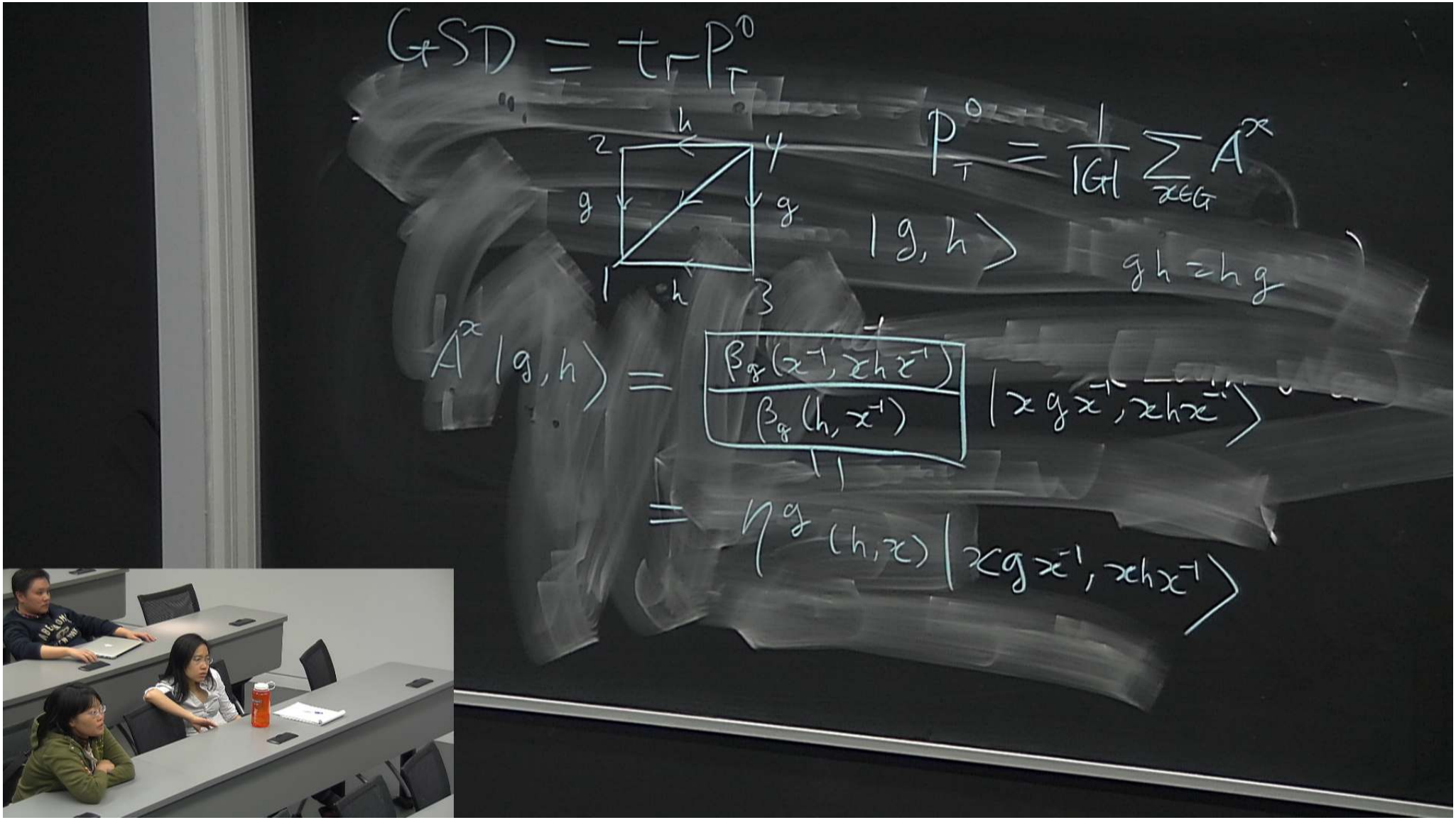

GSD = tr P^0
h
2
4
g
g
1
h
3

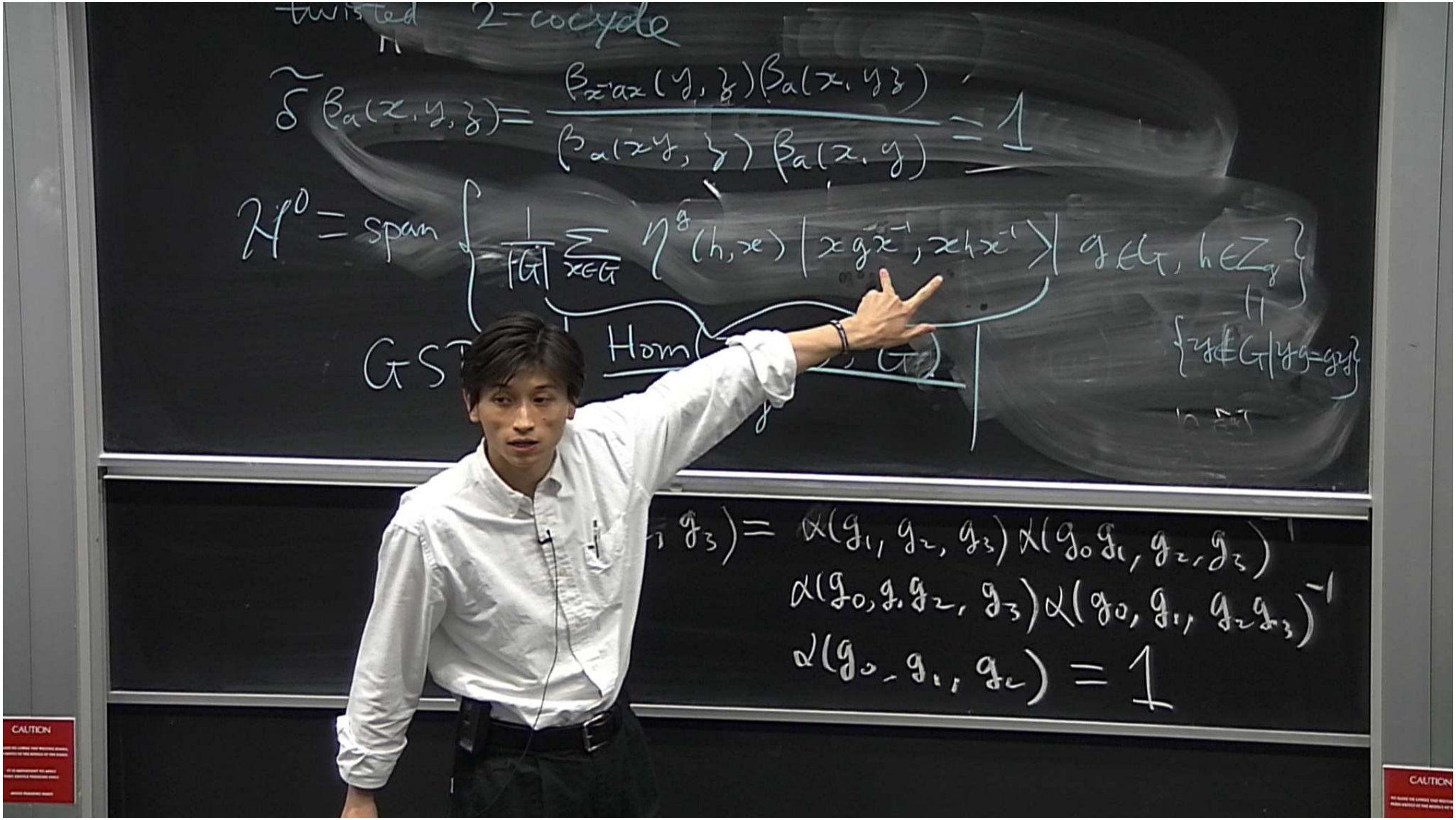

GSD = tr P_Γ^0
P_Γ^0 = 1/|G| Σ_{x∈G} A^x
1 h 2
g y g
h 3
|g, h⟩
A^x |g, h⟩ = [β_g(x^{-1}, xhx^{-1}) / β_g(h, x^{-1})] |xgx^{-1}, xhx^{-1}⟩
= η^g(h, x) |xgx^{-1}, xhx^{-1}⟩

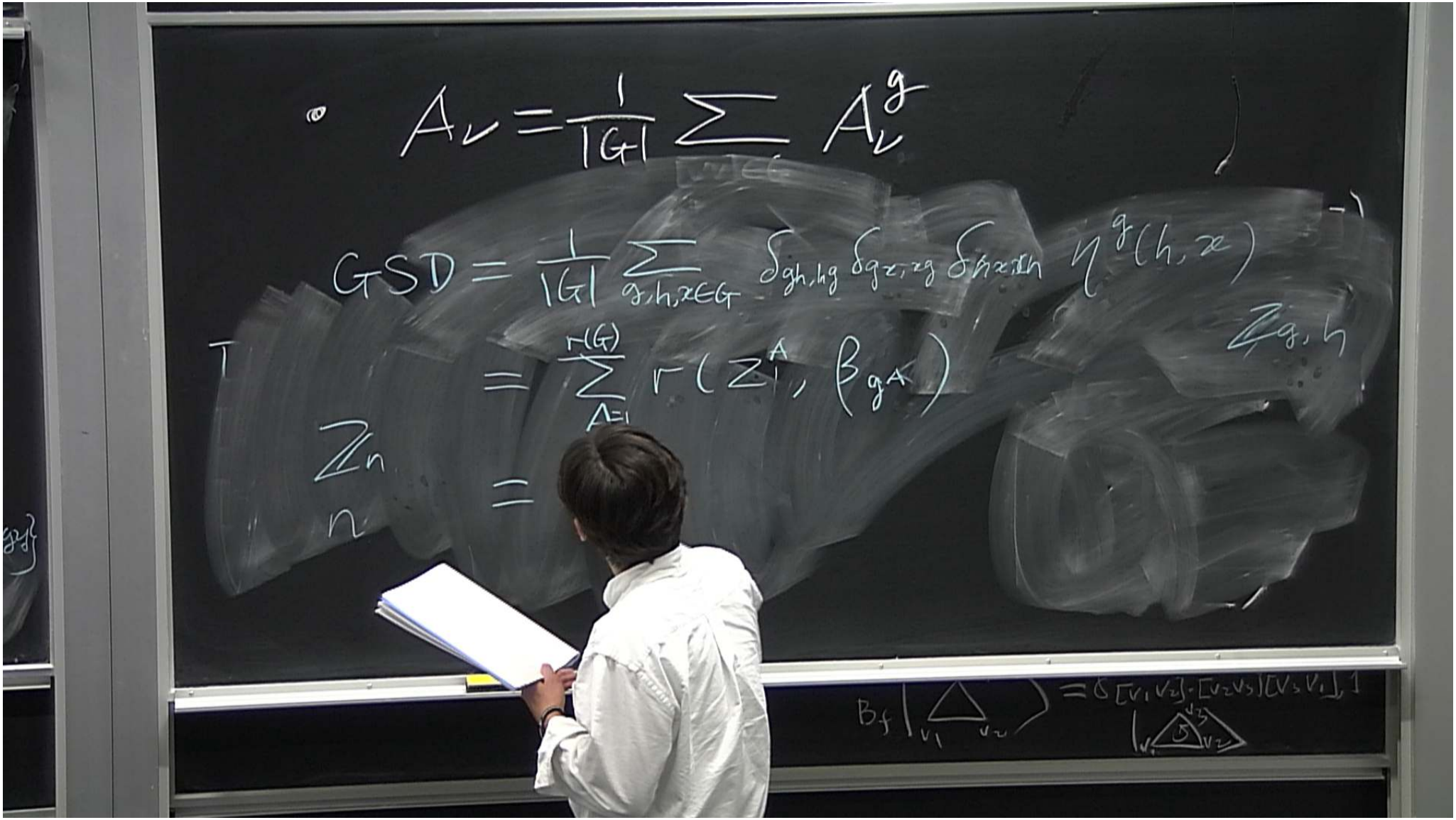

$\beta_a(b,c) = \frac{\alpha(a,b,c)\alpha(b,c,c^{-1}b^{-1}abc)}{\alpha(b,b^{-1}ab,c)}$
twisted 2-cocycle
H

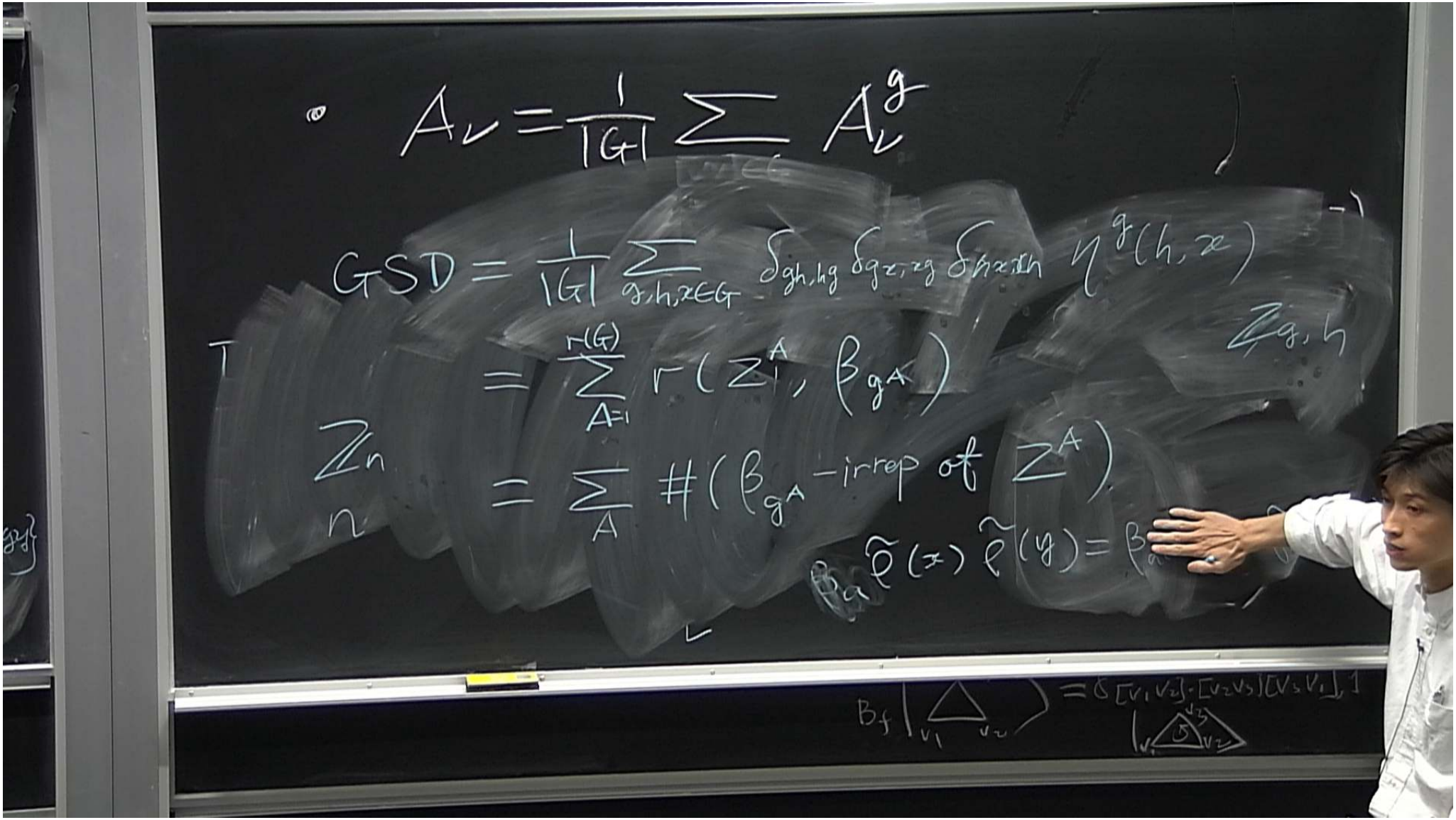

$\beta_a(b,c) = \frac{\alpha(a,b,c)\alpha(b,c,c^{-1}b^{-1}abc)}{\alpha(b,b^{-1}ab,c)}$
classifying
twisted 2-cocycle
$\tilde{\delta}\beta_a(x,y,z) = \frac{\beta_{x^{-1}ax}(y,z)\beta_a(x,yz)}{\beta_a(xy,z)\beta_a(x,y)} = 1$
H

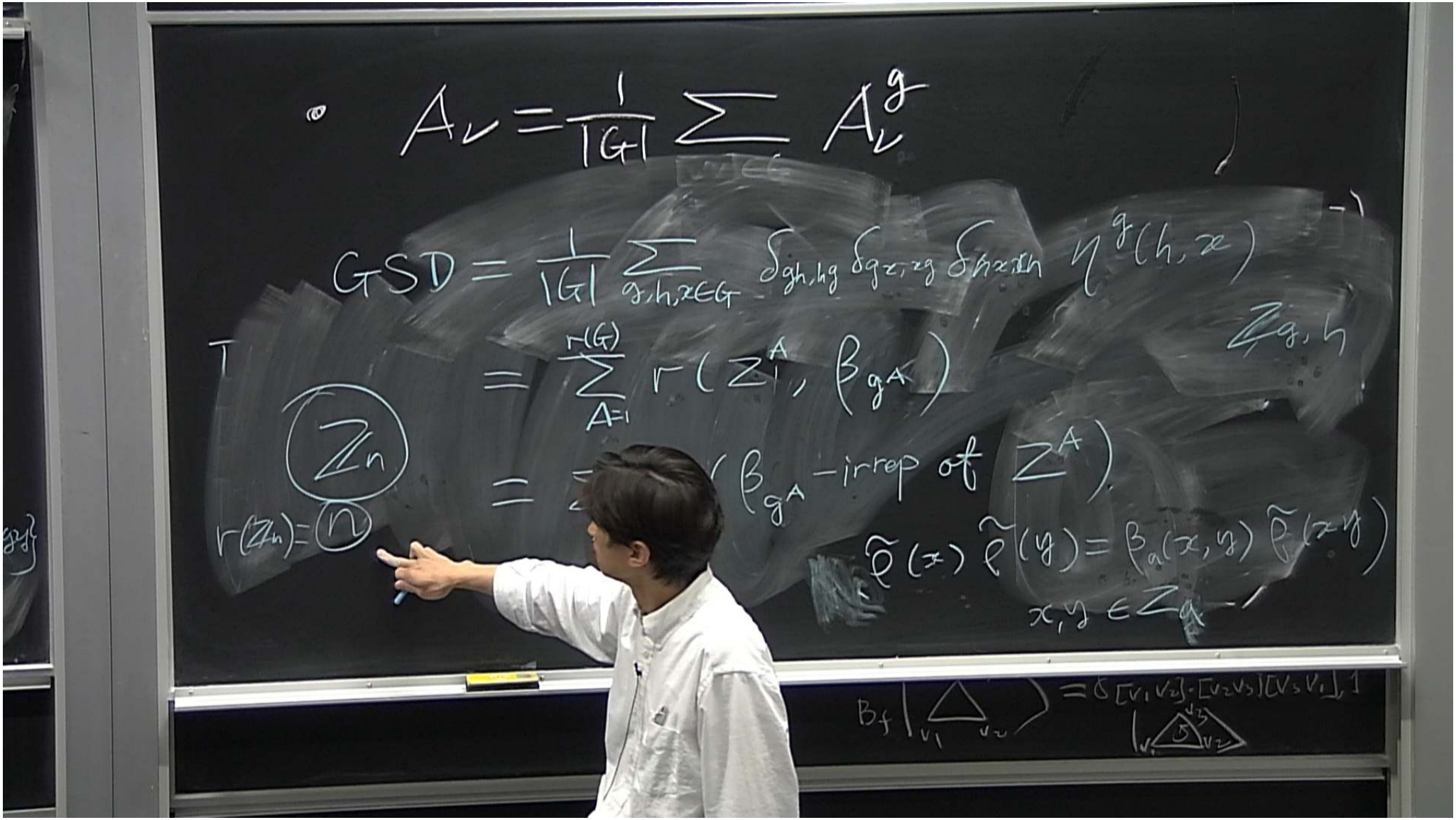

$\beta_a(b,c) = \frac{\alpha(a,b,c)\alpha(b,c,c^{-1}b^{-1}abc)}{\alpha(b,b^{-1}ab,c)}$
twisted 2-cocycle
$\tilde{\delta}\beta_a(x,y,z) = \frac{\beta_{x^{-1}ax}(y,z)\beta_a(x,yz)}{\beta_a(xy,z)\beta_a(x,y)} = 1$
$\mathcal{H}^0 = \mathrm{span}\left\{ \frac{1}{|G|}\sum_{x\in G} \eta^g(h,x) \,|\, xgx^{-1}, xhx^{-1}\rangle \,|\, g\in G, h\in Z_g \right\}$
$\{y\in G \,|\, yg=gy\}$

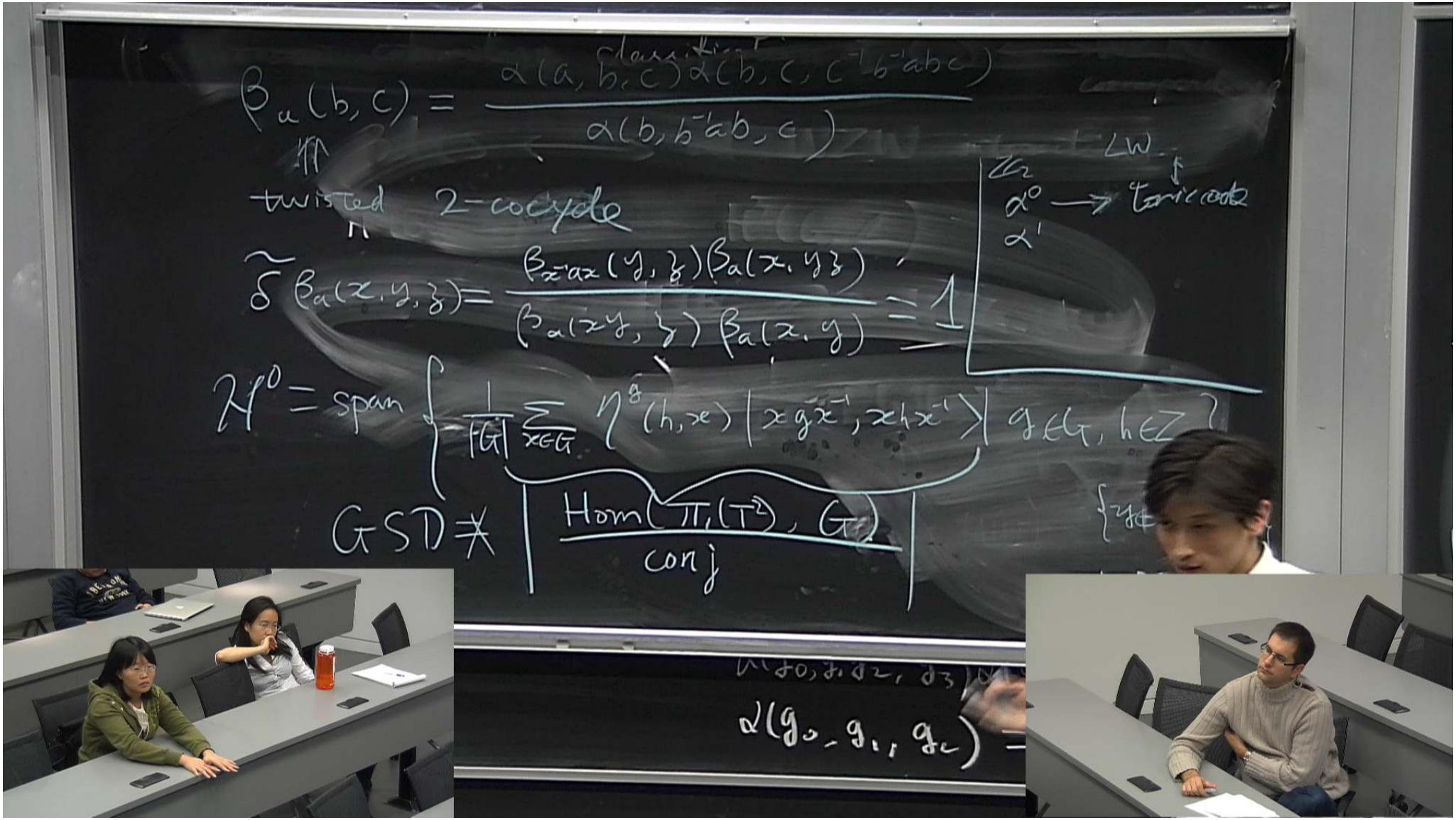

GSD = tr P_T^0
P_T^0 = 1/|G| Σ_{x∈G} A^x
|g, h⟩
gh = hg
A^x |g, h⟩ = [β_g(x^{-1}, xhx^{-1}) / β_g(h, x^{-1})] |xgx^{-1}, xhx^{-1}⟩
= η^g(h, x) |xgx^{-1}, xhx^{-1}⟩

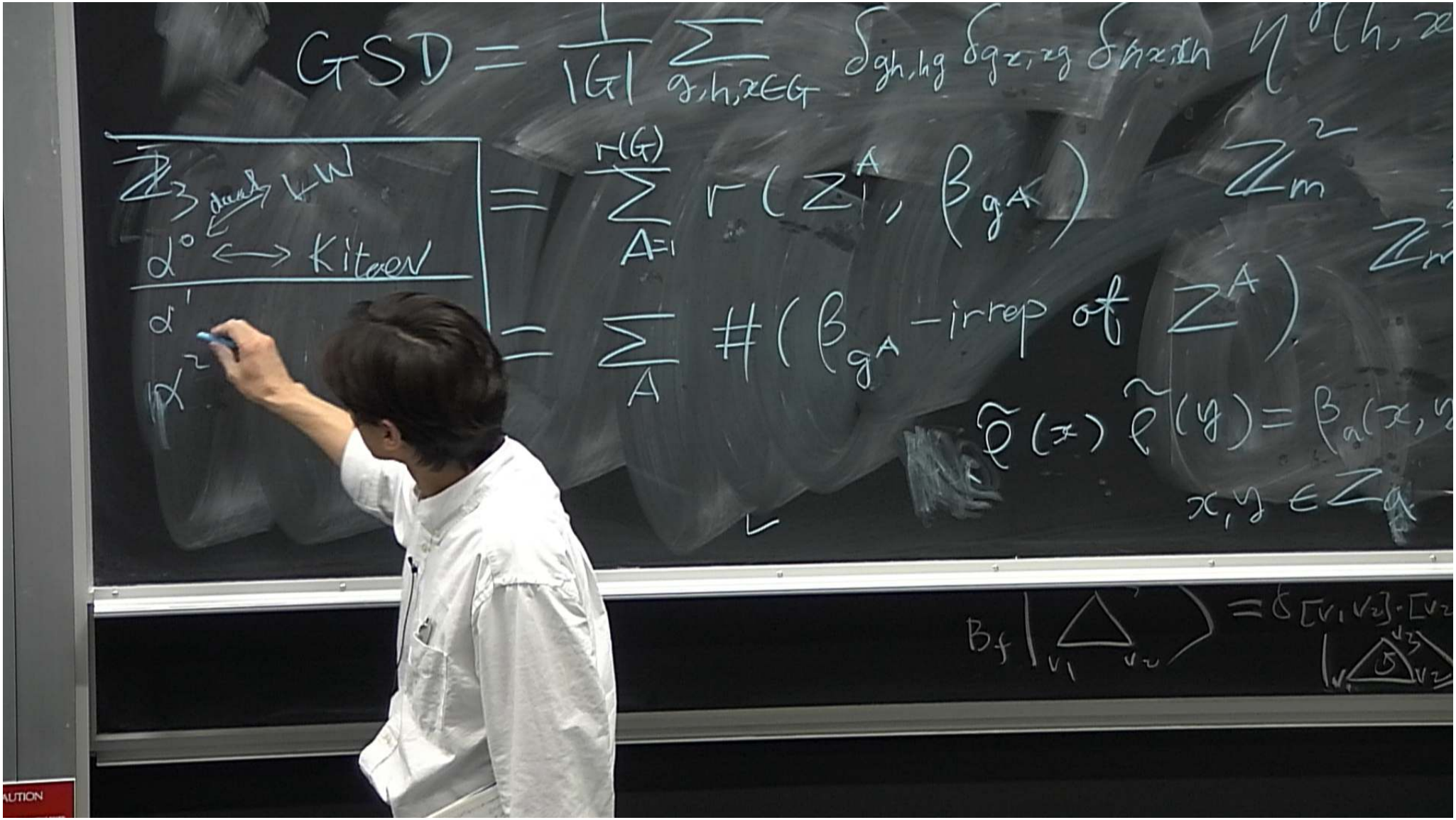

twisted 2-cocycle
$\widetilde{\delta}\beta_\alpha(x,y,z) = \frac{\beta_{x^{-1}ax}(y,z)\beta_\alpha(x,yz)}{\beta_\alpha(xy,z)\beta_\alpha(x,y)} = 1$
$\mathcal{H}^0 = \mathrm{span}\left\{ \frac{1}{|G|}\sum_{x\in G} \eta^g(h,x) \,|\, xgx^{-1}, xhx^{-1}\rangle \,|\, g\in G, h\in Z_g \right\}$
$\{x\in G \mid yg=gy\}$
$\alpha(g_1,g_2,g_3)\alpha(g_0g_1,g_2,g_3)$
$\alpha(g_0,g_1g_2,g_3)\alpha(g_0,g_1,g_2g_3)^{-1}$
$\alpha(g_0,g_1,g_2) = 1$
CAUTION
CAUTION

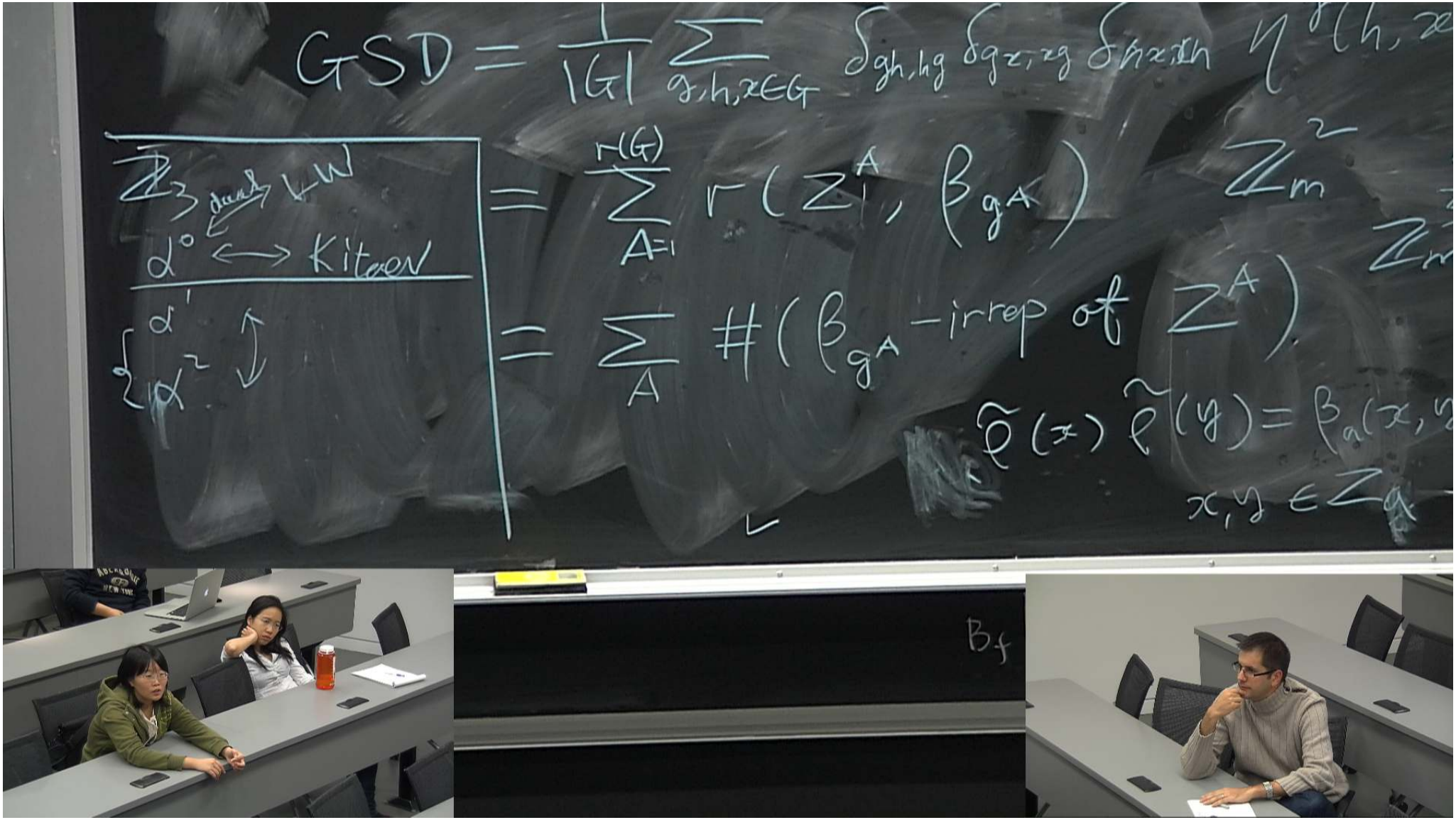

$A_\nu = \frac{1}{|G|} \sum A_\nu^g$
$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx,xg} \delta_{hxh,x} \eta^g(h,x)$
$= \sum_{A=1}^{r(G)} r(Z^A, \beta_{gA})$
$Z_{g,h}$
$Z_n$
$n$
$B_f$

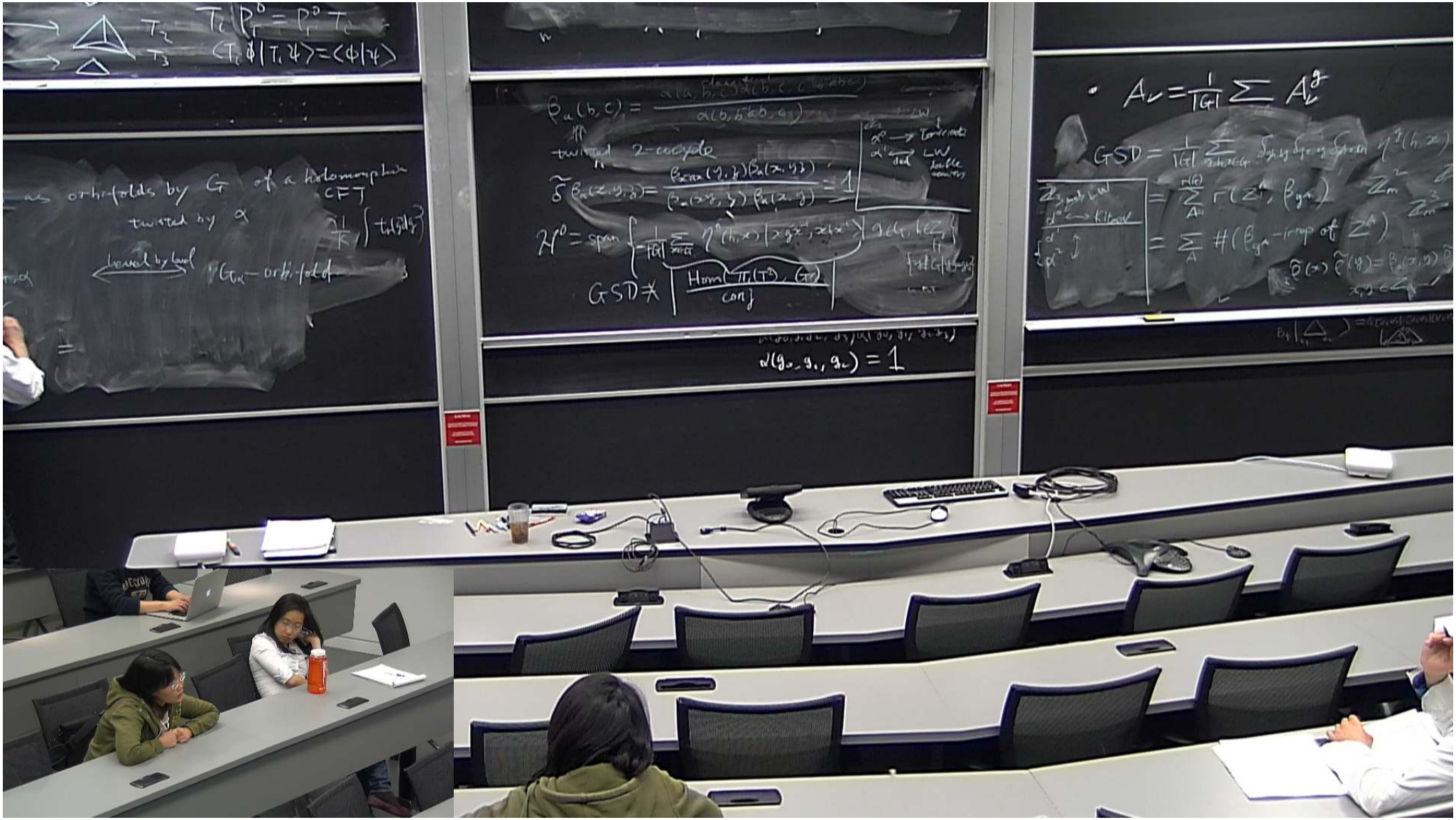

$A_\nu = \frac{1}{|G|} \sum_g A_\nu^g$
$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx,xg} \delta_{hxh,x} \eta^g(h,x)$
$= \sum_{A=1}^{r(G)} r(Z^A, \beta_{gA})$
$Z_n$
$n = \sum_A \#(\beta_{gA}\text{-irrep of } Z^A)$
$Z_{g,h}$
$B_f$

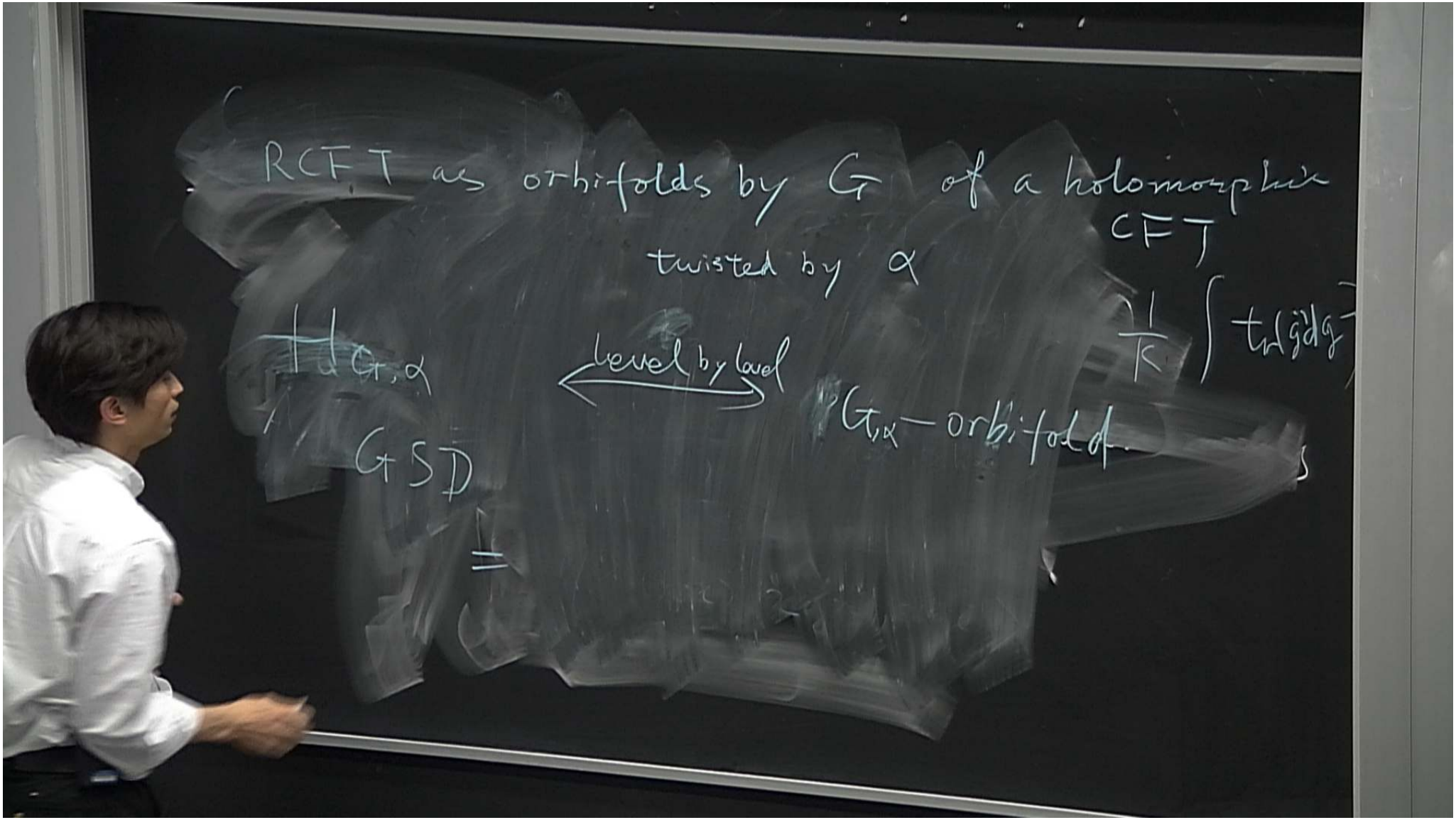

$A_\nu = \frac{1}{|G|} \sum_g A_\nu^g$
$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx,xg} \delta_{hxh,x} \eta^g(h,x)$
$= \sum_{A=1}^{r(G)} r(Z_i^A, \beta_{gA})$
$Z_n$
$r(Z_n) = n$
$\beta_{gA}$ - irrep of $Z^A$
$\tilde{e}(x)\tilde{e}(y) = \beta_a(x,y)\tilde{e}(xy)$
$x, y \in Z_q$

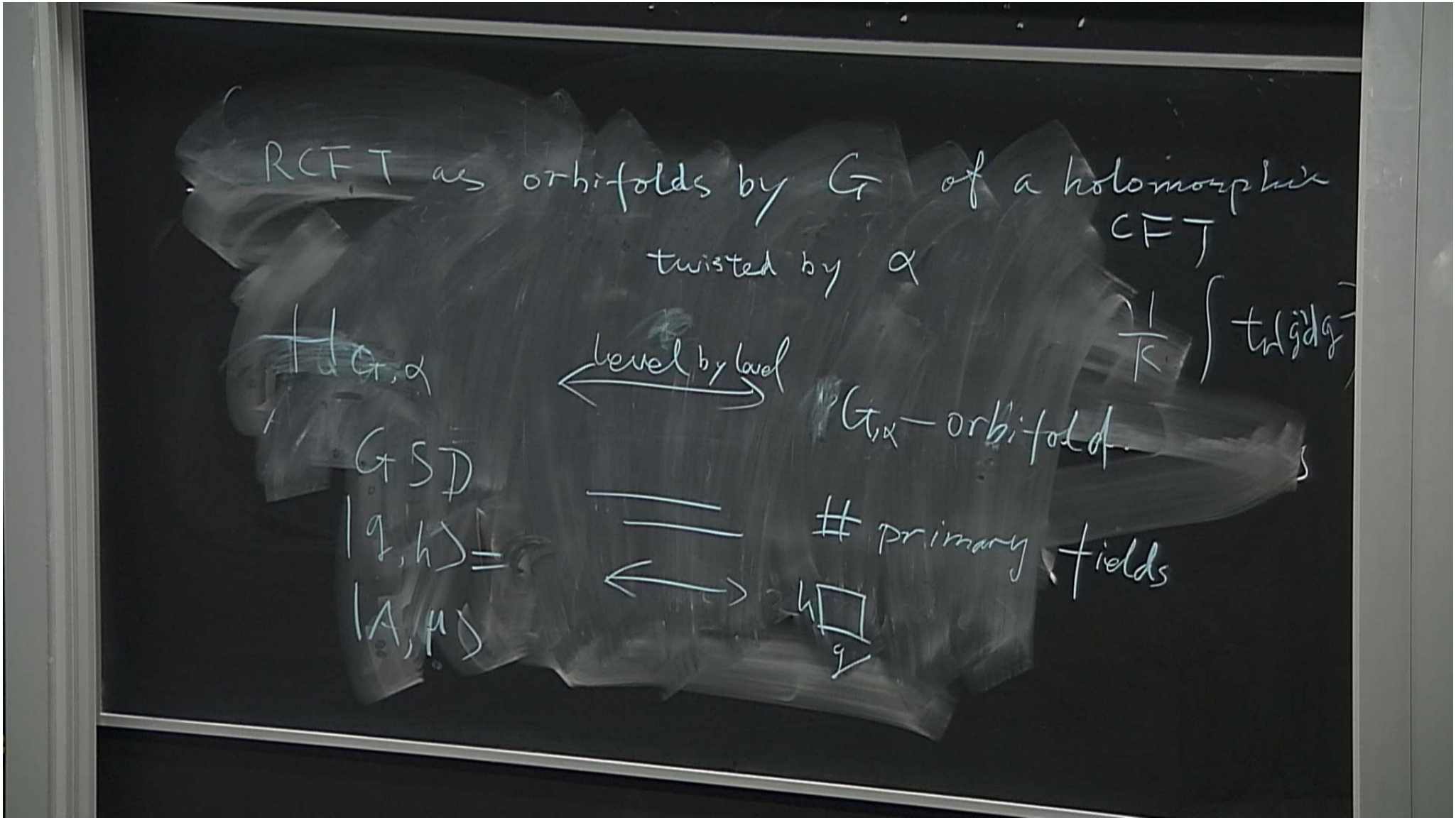

twisted 2-cocycle
GSD ≠
Hom(π, (T²), G)
conj

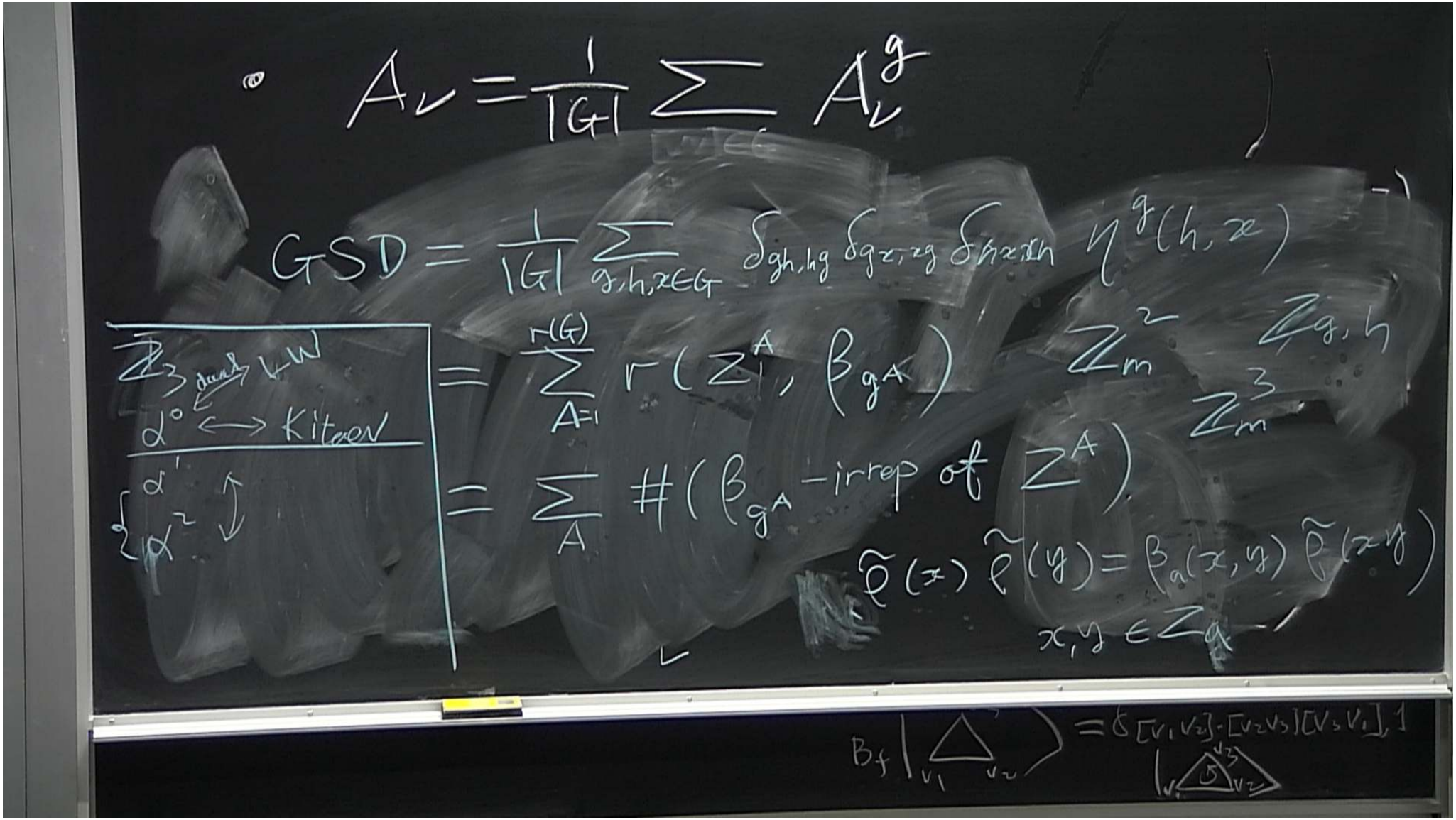

GSD = 1/|G| Σ_{g,h,x∈G} δ_{gh,hg} δ_{gx,xg} δ_{hxh,x}
Z_3 double LW
d° ⟷ Kitaev
d¹
d²
= Σ_{A=1}^{n(G)} r(Z^A, β_{g^A})
= Σ_A #(β_{g^A}-irrep of Z^A)
Z_m^2
Z_m
ρ̃(x) ρ̃(y) = β_a(x,
x, y ∈ Z_q
B_f

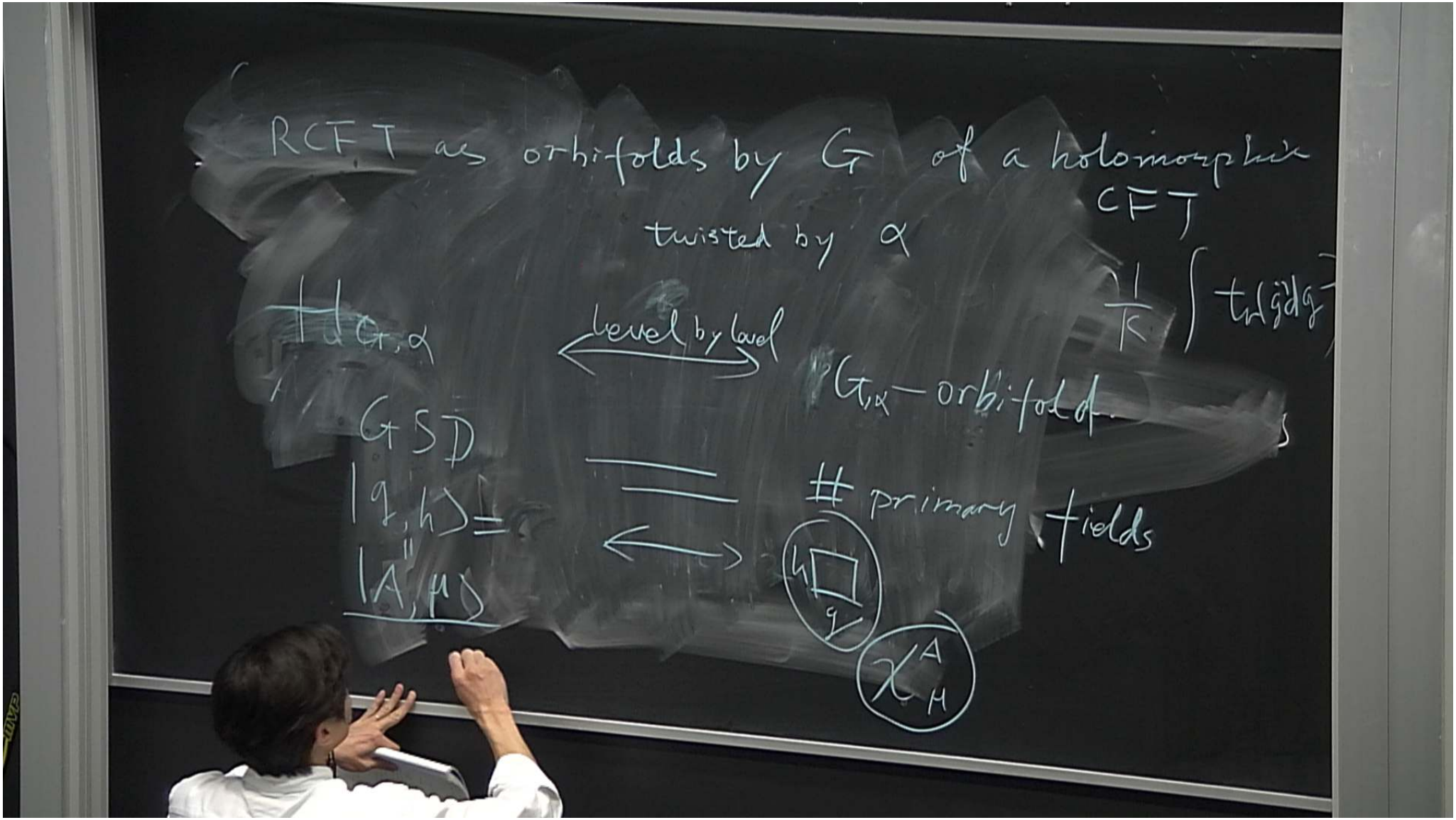

GSD = 1/|G| Σ_{g,h,x∈G} δ_{gh,hg} δ_{gx,xg} δ_{hxh,xh} η^g(h,x
Z_3 dual LW
α° ⟷ Kitaev
α
α^-1
= Σ_{A=1}^{r(G)} r(Z^A, β_{gA})
= Σ_A #(β_{gA}-irrep of Z^A)
Z_m^2
Z_m
ρ̃(x) ρ̃(y) = β_a(x,y
x,y ∈ Z_q
Bf

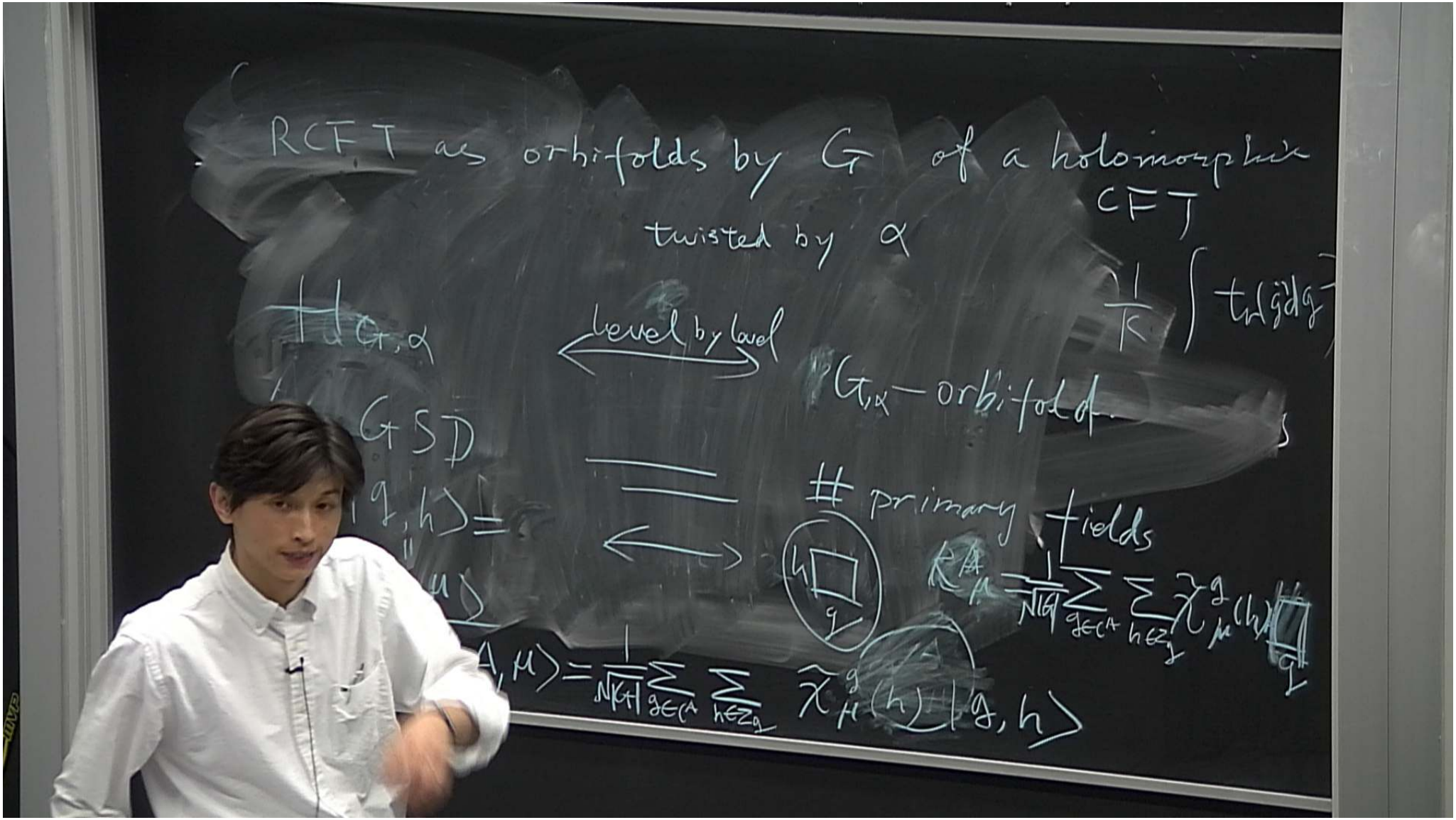

twisted by α
twisted 2-cocycle
GSD =
GSD ≠
ω(g_0, g_1, g_2) = 1

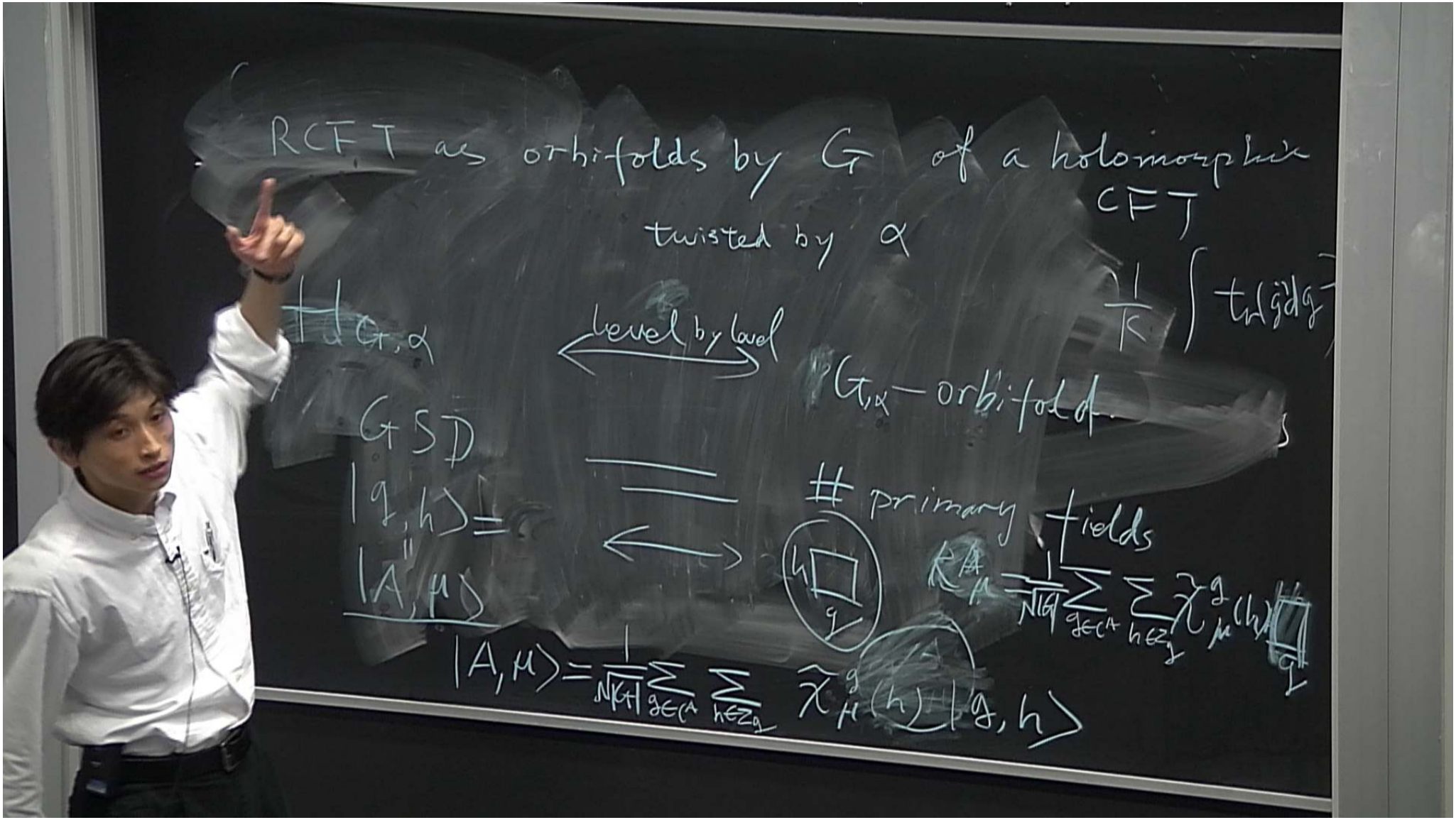

RCFT as orbifolds by G of a holomorphic CFT
twisted by α
level by level
GSD
=


RCFT as orbifolds by G of a holomorphic CFT
twisted by α
level by level
GSD
# primary fields


$A_\nu = \frac{1}{|G|} \sum_g A_\nu^g$
$GSD = \frac{1}{|G|} \sum_{g,h,x \in G} \delta_{gh,hg} \delta_{gx,xg} \delta_{hxh,x} \eta^g(h,x)$
$\mathbb{Z}_3$ dual to LW
$\alpha^0 \longleftrightarrow$ Kitaev
$= \sum_{A=1}^{r(G)} r(Z^A, \beta_{gA})$
$= \sum_A \#(\beta_{gA}\text{-irrep of } Z^A)$
$Z_m^2$
$Z_{g,h}$
$Z_m^3$
$\tilde{\rho}(x)\tilde{\rho}(y) = \beta_a(x,y)\tilde{\rho}(xy)$
$x,y \in Z_q$
$B_f$


RCFT as orbi-folds by G of a holomorphic CFT
twisted by α
level by level
GSD
# primary fields


RCFT as orbifolds by G of a holomorphic CFT
twisted by α
level by level
G-orbifold
GSD
# primary fields


RCFT as orbi-folds by G of a holomorphic CFT
twisted by α
level by level
G-orbi-fold
GSD
# primary fields