Title: Holography for inflation using conformal perturbation theory.

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Abstract: Holographic cosmology maps cosmological time evolution to the inverse RG ï¬,ow of a dual three-dimensional QFT. In cases where this RG ï¬,ow connects two closely separated ï¬•xed points, QFT correlators may be calculated perturbatively in terms of the conformal ï¬•eld theory associated with one of the ï¬•xed points, even when the dual QFT is at strong coupling. Realising slow-roll inï¬,ation in these terms, we show how to derive standard slow-roll inï¬,ationary power spectra and non-Gaussianities through purely holographic calculations. The form of slow-roll inï¬,ationary correlators is seen to be determined by the perturbative breaking of conformal symmetry away from the ï¬•xed point.

Holography for inflation using conformal perturbation theory

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22.11.12

arXiv:1211.4550

with Adam Bzowski & Kostas Skenderis

Introduction

Correlation functions of the primordial perturbations present our best clues to the physics of the early universe.

Observationally, the power spectrum is consistent with a simple power-law

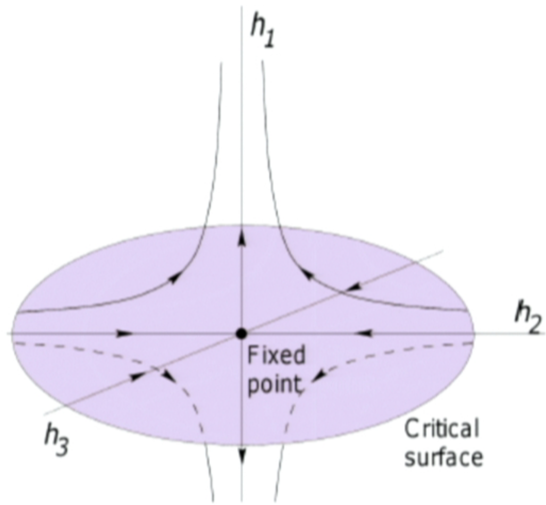
$$\Delta_S^2(q) = \Delta_S^2(q_0)\left(\frac{q}{q_0}\right)^{n_s-1}$$

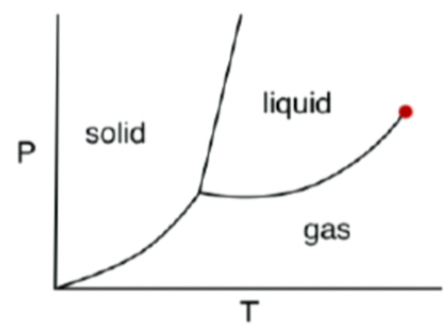
with amplitude $\Delta_S^2(q_0) \approx 10^{-9}$ and spectral tilt $n_s \approx 0.96$, i.e., nearly scale-invariant but with a slight red tilt. (Assuming no tensors or running.)

Introduction

Where else in nature do we typically see power-law scaling of 2-point functions, with non-integer exponents?

Critical phenomena: Systems undergoing continuous phase transition are described by a Euclidean QFT that flows to an IR fixed point. Universal scaling behaviour determined by operator dimensions in fixed point CFT.





Holographic cosmology

Is there a connection?

Holographic cosmology proposes that 4d cosmology admits a dual description in terms of a 3d non-gravitational QFT.

Cosmic time evolution maps to inverse RG flow in the dual QFT:

late times $\leftrightarrow$ UV and early times $\leftrightarrow$ IR.

If the dual QFT is critical, then it will flow to a fixed point in the IR.

Holographically, this is dual to a universe that is asymptotically de Sitter in the far past, i.e., that was inflating. The power-law scaling of 2-pt function in critical QFT translates to power-law scaling of cosmological power spectrum.

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Refining the picture

The RG flow nearest the IR fixed point is dominated by most nearly marginal irrelevant operator.

Let's assume this to be a single scalar operator $\mathcal{O}$ (= single-field inflation) of dimension $\Delta_{IR} = 3 + \lambda_{IR}$, where $0 < \lambda_{IR} \ll 1$.

$$S = S_{CFT} + \int \mathrm{d}^3x\, \phi\mathcal{O}.$$

We will see later that

$$\Delta_S^2 \sim \frac{q^3}{\phi^2 \langle \mathcal{O}\mathcal{O} \rangle} \sim \phi^{-2} q^{-2\lambda_{IR}}, \qquad n_s - 1 = -2\lambda_{IR}$$

i.e., the tilt of the power spectrum on long wavelengths is controlled by the IR dimension of $\mathcal{O}$.

Plan

This talk is based on [1211.4550] with Adam Bzowski and Kostas Skenderis.

- How to derive standard slow-roll inflationary 2- and 3-point functions, both for scalars and tensors, completely from the QFT side!
- The form of slow-roll inflationary correlators is determined by the perturbative breaking of conformal invariance away from fixed point.

Recent related work by Schalm, Shiu & van der Aalst [1211.2157]. See also:

$\langle\gamma\gamma\gamma\rangle$ from CFT: Maldacena & Pimentel '11, Bzowski, PM & Skenderis '11.

dS/CFT story: Strominger '01, Maldacena '02, Larsen, Leigh, van der Schaar '02, Larsen & McNees '04, etc.

Conformal perturbation theory: Ludwig & Cardy '87, A. Zamolodchikov '87.

Plan

1. Perturbative RG flows, calculation of QFT correlators
2. Holographic calculation of inflationary correlators
3. Identifying the dual cosmology

Perturbative RG flows

Starting from the UV fixed point, let's imagine coupling 3d Euclidean CFT to a marginally relevant scalar operator $\mathcal{O}$ of dimension $\Delta = 3 - \lambda$, where $\lambda \ll 1$

$$S = S_{CFT} + \int \mathrm{d}^3x\, \varphi \Lambda^{-\lambda} \mathcal{O},$$

where Λ is UV cutoff, φ is dimensionless coupling.

The β-function may be found by demanding invariance of the partition function under changes of Λ

$$\beta = -\frac{\mathrm{d}\varphi}{\mathrm{d}\ln\Lambda} = -\lambda\varphi + 2\pi C \varphi^2 + O(\varphi^3),$$

where C is the OPE coefficient in the CFT

$$\mathcal{O}(x_1)\mathcal{O}(x_2) = \frac{\alpha}{|x_{12}|^{2\Delta}} + \frac{C}{|x_{12}|^{\Delta}}\mathcal{O}(x_2) + \dots \quad \text{as} \quad |x_{12}| \to 0.$$

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Perturbative RG flows

If C is positive and of order unity, we obtain an RG flow from the UV CFT at $\varphi = 0$ to a nearby IR fixed point at

$$\varphi = \varphi_1 + O(\lambda^2), \qquad \varphi_1 = \frac{\lambda}{2\pi C} \ll 1.$$

(If instead C is negative the fate of the RG flow depends on higher order coefficients in the β-function.)

About IR fixed point

$$\beta = \lambda(\varphi - \varphi_1) + 2\pi C(\varphi - \varphi_1)^2 + O(\varphi - \varphi_1)^3,$$

thus $\Delta_{UV} = 3 - \lambda$ (relevant) while $\Delta_{IR} = 3 + \lambda + O(\lambda^2)$ (irrelevant).

Perturbative RG flows

Since φ is small throughout the flow, we may remove higher order terms in the β-function by a field redefinition $\varphi \to \varphi + O(\varphi^3)$ leaving

$$\beta(\varphi) = -\mathrm{d}\varphi/\mathrm{d}\ln\Lambda = -\lambda\varphi + 2\pi C\varphi^2.$$

Integrating, we find

$$\varphi(\Lambda) = \frac{\varphi_1}{1 + (\varphi_1/\phi\Lambda^\lambda)}$$

where the constant of integration ϕ parametrises the asymptotic behaviour

$$\varphi \to \phi\Lambda^\lambda \qquad \text{as} \qquad \Lambda \to 0.$$

Equivalently, ϕ is the dimensionful renormalised coupling in the UV QFT

$$\int \mathrm{d}^3x\, \Lambda^{-\lambda}\varphi\mathcal{O} \to \int \mathrm{d}^3x\, \phi\mathcal{O}.$$

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Correlators

CFT 2- and 3-point functions are fixed by conformal invariance

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 = \frac{\alpha}{|x_{12}|^{2\Delta}}, \quad \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle_0 = \frac{\alpha C}{|x_{12}|^{\Delta}|x_{23}|^{\Delta}|x_{31}|^{\Delta}}.$$

To evaluate correlators in perturbed theory, however, we need to sum up entire series of CFT correlators with integrated scalar insertions!

$\Rightarrow$ All terms in sum contribute: $\varphi^n \sim \lambda^n$, $\mathcal{I}_n \sim 1/\lambda^n$

Amazingly, this can be achieved. The argument is subtle, but the basic idea is to write a differential equation for $\mathrm{d}\mathcal{I}_n/\mathrm{d}\Lambda$ in terms of $\mathcal{I}_{n-1}$ using the OPE.

Correlators

Correlators may be computed perturbatively in λ starting from those in the CFT (denoted with a subscript zero).

For example,

$$\begin{aligned}\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle &= \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\exp\left(-\int \mathrm{d}^3x\,\varphi\Lambda^{-\lambda}\mathcal{O}\right)\rangle_0 \\ &= \sum_{n=0}^{\infty}\frac{(-\varphi\Lambda^{-\lambda})^n}{n!}\mathcal{I}_n\end{aligned}$$

where

$$\mathcal{I}_n = \int \mathrm{d}^3z_1\ldots\mathrm{d}^3z_n\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(z_1)\ldots\mathcal{O}(z_n)\rangle_0 .$$

To regularise integral, no two operator insertions are allowed to approach closer than Λ.

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Correlators

Let's consider

$$\mathcal{I}_1 = \int \mathrm{d}^3 z \, \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(z)\rangle_0 \, \Theta(|z - x_1| - \Lambda)\Theta(|z - x_2| - \Lambda).$$

Varying wrt the cutoff Λ and using the OPE, $\mathcal{O}\mathcal{O} \sim (C/|x|^{3-\lambda})\mathcal{O}$, find

$$\frac{\mathrm{d}\mathcal{I}_1}{\mathrm{d}\Lambda} = -2(4\pi\Lambda^2)\frac{C}{\Lambda^{3-\lambda}}\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 + \dots$$

for $\Lambda \ll 1$. Integrating,

$$\mathcal{I}_1 = -\frac{8\pi C}{\lambda}(\Lambda^\lambda - b|x_{12}|^\lambda)\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 + \dots$$

- Fix integration constant $b \to 1$ by requiring smooth limit $\lambda \to 0$ (gives logarithm $\Rightarrow$ conformal anomaly). Then, for $\lambda \neq 0$, can send $\Lambda \to 0$.
- Omitted contributions from higher terms in OPE subleading in λ if O is only operator becoming marginal as $\lambda \to 0$.

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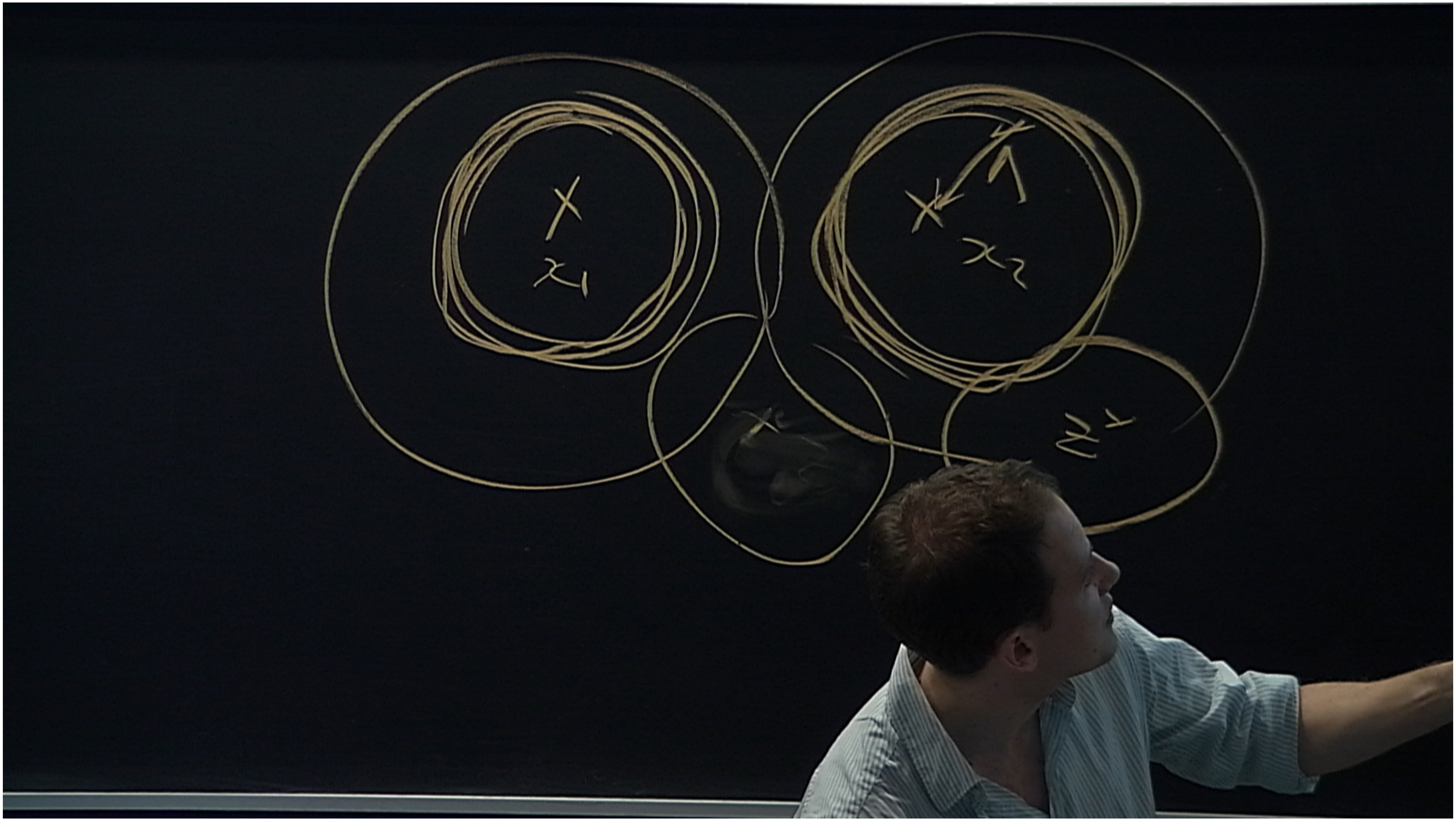
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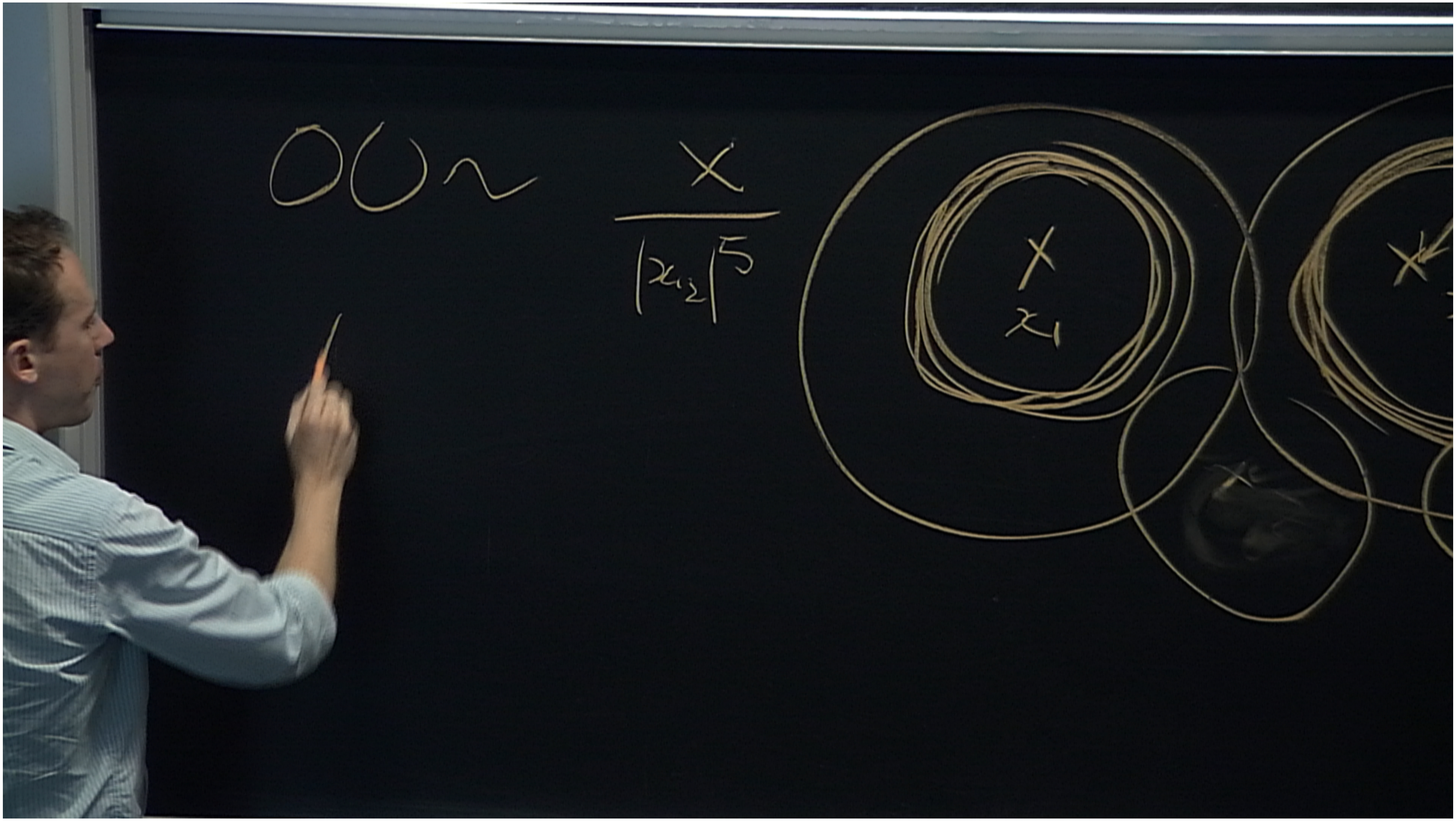
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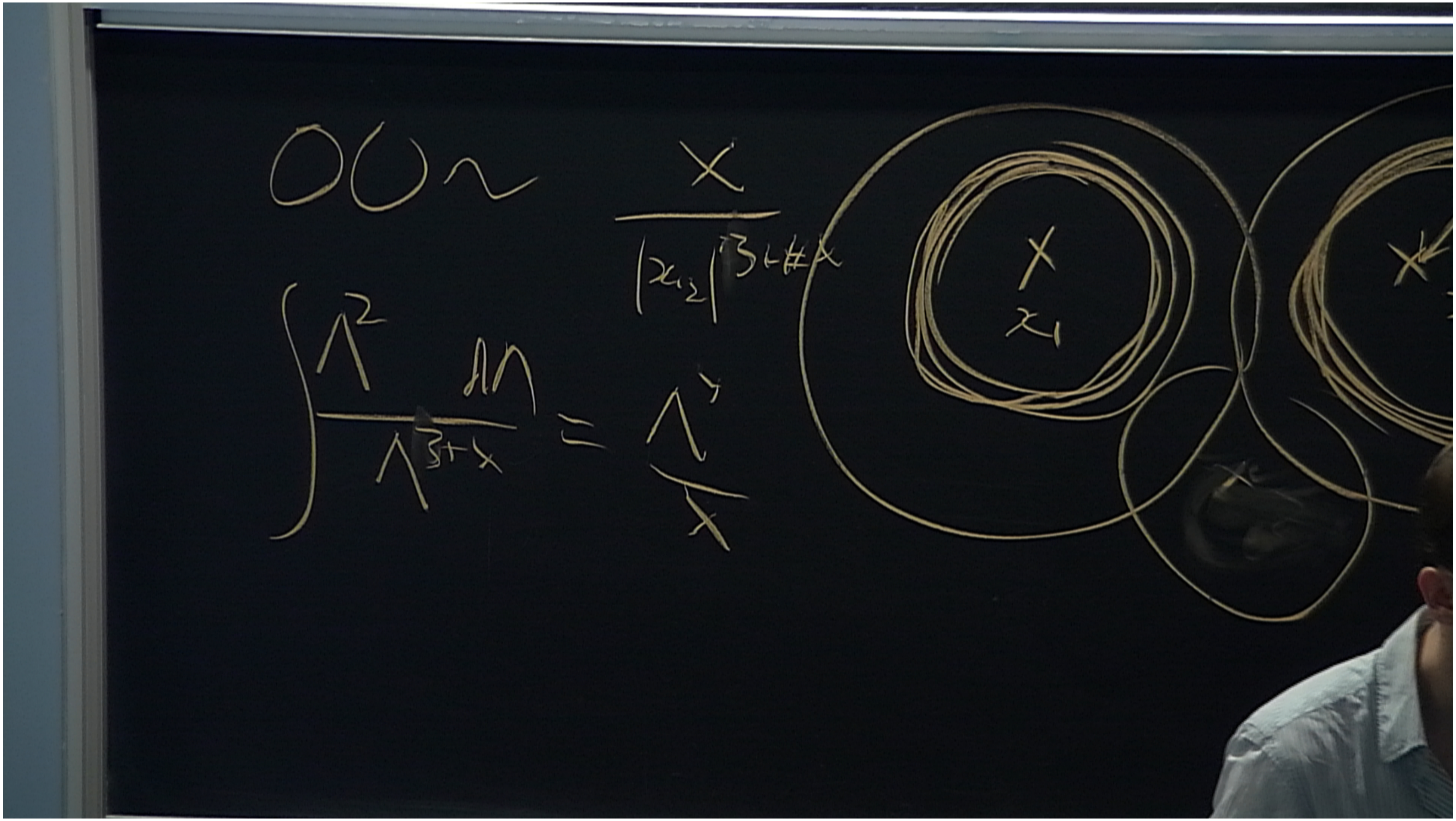
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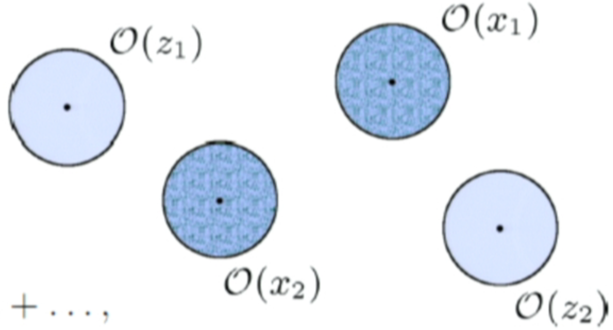

For general case,

$$\frac{\mathrm{d}\mathcal{I}_n}{\mathrm{d}\Lambda} = -4\pi\Lambda^2 B_n \frac{C}{\Lambda^{3-\lambda}} \mathcal{I}_{n-1} + \dots,$$

where combinatorial factor B_n counts pairs of insertions brought together

$$B_n = \binom{n}{2} + 2n = \frac{1}{2}n(n+3).$$

Triple overlaps and higher contribute only at higher order in $\lambda \to$ dilute gas.

Solve recursively, fixing integration constants by $\lambda^n \mathcal{I}_n \to 0$ as $\lambda \to 0$ (i.e., kill leading $1/\lambda^n$ pole), remove cutoff $\Lambda \to 0$.

Correlators

Can write final result as a sum of CFT correlators with shifted dimensions

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{1}{6}\sum_{n=0}^{\infty}(n+3)(n+2)(n+1)(-\frac{\phi}{\varphi_1})^n \langle \mathcal{O}_{\Delta'}(x_1)\mathcal{O}_{\Delta'}(x_2)\rangle_0,$$

where $\Delta' = 3 - \lambda(n+2)/2$. Accurate to leading order in λ.

Resumming the binomial series and Fourier transforming:

$$\langle\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle\rangle = \frac{\pi^2}{12}\alpha q^{3-2\lambda}\left[1+\frac{\phi}{\varphi_1}q^{-\lambda}\right]^{-4}.$$

At large momenta behaves as $q^{-2\lambda}$ reflecting UV dimension $\Delta_{UV} = 3 - \lambda$ while for small momenta as $q^{2\lambda}$ reflecting IR dimension $\Delta_{IR} = 3 + \lambda$.

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$$S_{CFT} + \int \phi O$$

Holography for cosmology

In the first instance, we wish to compute cosmological in-in correlators at tree level (and then order by order in loops). This requires perturbatively quantising small fluctuations about a given background geometry.

On the dual QFT side, this corresponds to working in large-N perturbation theory.

To compute observationally relevant tree-level cosmological correlators, it suffices to establish holographic map via a simple analytic continuation between perturbed FRW cosmologies and holographic RG flows.

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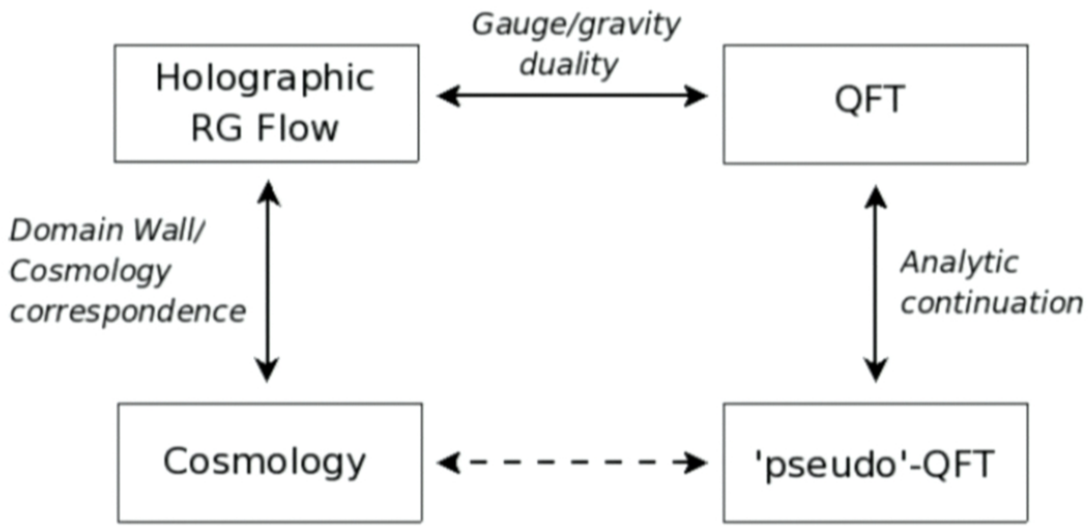

Work with Skenderis '09-'11

Holography for cosmology

Both cosmological and holographic RG flow solutions for background geometry and fluctuations are functions of $\kappa^2 = 8\pi G_N$ and magnitude of 3-momentum on spatial slices $q = \sqrt{\vec{q}^{\,2}}$.

Given a solution for a perturbed holographic RG flow in Euclidean signature, a perturbed FRW cosmological solution in Lorentzian signature is given by

$$\kappa^2 V \to -\kappa^2 V, \qquad q \to -iq.$$

- For explicit proof at quadratic order in gauge-invariant perturbation theory, in case of gravity with minimal scalar and potential, see [1104.3894].
- Bunch-Davies vacuum $\leftrightarrow$ smooth in the interior.

Holography for cosmology

On the dual QFT side, this is equivalent to performing the following analytic continuation on large-N correlators

$$N^2 \to -N^2, \qquad q \to -iq.$$

Thus we first compute correlators in the regular QFT dual to the holographic RG flow, then continue.

As we will be able to check explicitly, this prescription indeed yields the correct cosmological 2- and 3-pt correlators.

Holographic formulae

Complete holographic formulae were calculated in [1011.0452, 1104.3894] with Skenderis. Here, these become:

$$\langle\!\langle \zeta(q)\zeta(-q)\rangle\!\rangle = \frac{1}{2\lambda^2\phi^2\langle\!\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle\!\rangle}, \qquad \langle\!\langle \gamma^{(s)}(q)\gamma^{(s')}(-q)\rangle\!\rangle = \frac{\delta^{ss'}}{A(q)},$$

$$\langle\!\langle \zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle\!\rangle = \frac{\phi\langle\!\langle \mathcal{O}(q_1)\mathcal{O}(q_2)\mathcal{O}(q_3)\rangle\!\rangle - \sum_{j=1}^{3}\langle\!\langle \mathcal{O}(q_j)\mathcal{O}(-q_j)\rangle\!\rangle}{4\lambda^3\phi^4\prod_{j=1}^{3}\langle\!\langle \mathcal{O}(q_j)\mathcal{O}(-q_j)\rangle\!\rangle},$$

$$\langle\!\langle \zeta(q_1)\zeta(q_2)\gamma^{(s_3)}(q_3)\rangle\!\rangle = \frac{-\lambda\phi\langle\!\langle \mathcal{O}(q_1)\mathcal{O}(q_2)T^{(s_3)}(q_3)\rangle\!\rangle + 3\langle\!\langle \mathcal{O}(q)T^{(s)}(-q)\rangle\!\rangle}{2\lambda^3\phi^3\langle\!\langle \mathcal{O}(q_1)\mathcal{O}(-q_1)\rangle\!\rangle\langle\!\langle \mathcal{O}(q_2)\mathcal{O}(-q_2)\rangle\!\rangle A(q_3)},$$

where $A(q) = \sum_{s=\pm}\langle\!\langle T^{(s)}(q)T^{(s)}(-q)\rangle\!\rangle$.

Results

Plugging in our QFT correlators computed via conformal perturbation theory, the results precisely match those of an slow-roll inflationary cosmology with

$$\epsilon_* = \frac{\lambda^4}{8\pi^2 C^2}\left(\frac{q}{q_0}\right)^{2\lambda}\left[1+\left(\frac{q}{q_0}\right)^{\lambda}\right]^{-4} \qquad \eta_* = -\lambda + 2\lambda\left[1+\left(\frac{q}{q_0}\right)^{\lambda}\right]^{-1}$$

Note $\epsilon_* \ll \eta_*$. Find H_* is one to leading order in λ.

We have repackaged the arbitrary dimensionful QFT coupling ϕ into the arbitrary momentum scale $q_0^\lambda = 2\pi C\phi/\lambda$.

Results

Explicitly:

$$\langle\!\langle \zeta(q)\zeta(-q)\rangle\!\rangle = \frac{\kappa^2 H_*^2}{4q^3\epsilon_*}, \qquad \langle\!\langle \gamma^{(s)}(q)\gamma^{(s')}(-q)\rangle\!\rangle = \frac{2\kappa^2 H_*^2}{q^3}\delta^{ss'}$$

$$\langle\!\langle \zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle\!\rangle = \eta_* \sum_{i<j} \langle\!\langle \zeta(q_i)\zeta(-q_i)\rangle\!\rangle \langle\!\langle \zeta(q_j)\zeta(-q_j)\rangle\!\rangle$$

$$\langle\!\langle \zeta(q_1)\zeta(q_2)\gamma^{(\pm)}(q_3)\rangle\!\rangle = \frac{\kappa^4 H_*^4}{16\sqrt{2}\epsilon_* q_3^2}\frac{1}{ac^3}(-a^3+ab+c)(a^3-4ab+8c),$$

where $\kappa^2 = 12/\pi^2\alpha$ and $a = \sum_i q_i$, $b = \sum_{i<j} q_i q_j$, $c = q_1 q_2 q_3$.

- Equilateral piece of $\langle\zeta\zeta\zeta\rangle \sim H_*^4/\epsilon_* \sim \lambda^{-4}$ subleading to local piece above $\sim H_*^4\eta_*/\epsilon_*^2 \sim \lambda^{-7}$, so need higher order calculations to see.
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Identifying the dual cosmology

What is the potential $V(\Phi)$ appearing in the bulk action?

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

Can compute systematically:

❶ Start from 1st order Friedmann equations: (since RG flow monotonic)

$$H = -\frac{1}{2}W(\varphi), \qquad \dot{\varphi} = W'(\varphi), \qquad -2\kappa^2 V = W'^2 - \frac{3}{2}W^2$$

dS asymptopia where $V' = 0$ correspond to either $W' = 0$ or $W'' = 3W/2$.

Only the former correspond to stable RG flows $\Rightarrow$ Choose $W'(0) = 0$.

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Identifying the dual cosmology

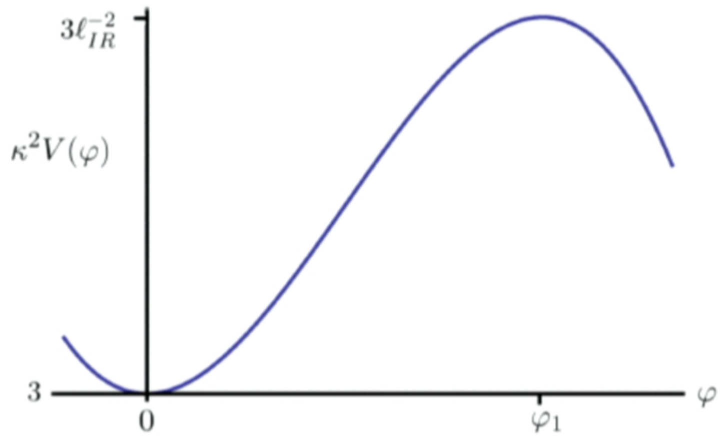
❷ We now Taylor expand: $W(\varphi) = -2 + a_2\varphi^2 + a_3\varphi^3 + O(\varphi^4)$

Fix coefficients via AdS/CFT: a_2 maps to the operator dimension $\Delta = 3 - \lambda$ and a_3 maps to OPE coefficient C.

❸ Our cosmology then derives from the cubic superpotential

$$W(\varphi) = -2 - \frac{1}{2}\lambda\varphi^2 + \frac{2}{3}\pi C\varphi^3 + O(\varphi^4)$$

$V(\varphi)$ is a sextic polynomial describing hilltop inflation:



Identifying the dual cosmology

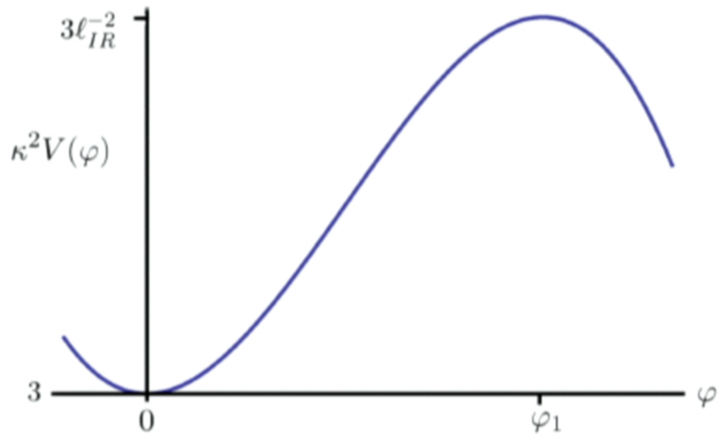
❷ We now Taylor expand: $W(\varphi) = -2 + a_2\varphi^2 + a_3\varphi^3 + O(\varphi^4)$

Fix coefficients via AdS/CFT: a_2 maps to the operator dimension $\Delta = 3 - \lambda$ and a_3 maps to OPE coefficient C.

❸ Our cosmology then derives from the cubic superpotential

$$W(\varphi) = -2 - \frac{1}{2}\lambda\varphi^2 + \frac{2}{3}\pi C\varphi^3 + O(\varphi^4)$$

$V(\varphi)$ is a sextic polynomial describing hilltop inflation:



Identifying the dual cosmology

What is the potential $V(\Phi)$ appearing in the bulk action?

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

Can compute systematically:

❶ Start from 1st order Friedmann equations: (since RG flow monotonic)

$$H = -\frac{1}{2}W(\varphi), \qquad \dot{\varphi} = W'(\varphi), \qquad -2\kappa^2 V = W'^2 - \frac{3}{2}W^2$$

dS asymptopia where $V' = 0$ correspond to either $W' = 0$ or $W'' = 3W/2$.

Only the former correspond to stable RG flows ⇒ Choose $W'(0) = 0$.

Summary

1. We analysed a perturbative RG flow between two closely separated fixed points driven by a nearly marginal scalar operator $\mathcal{O}$.

2. Even though this QFT is strongly interacting, the form of correlators is dictated by the perturbative breaking of conformal symmetry:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)e^{\int \mathrm{d}^3 z \phi \mathcal{O}}\rangle_0$$

   We can calculate and compare answers on both sides of holographic correspondence $\Rightarrow$ sucessful test.

3. Power spectra and non-Gaussianities of the dual slow-roll cosmology given by holographic formulae. The slow-roll parameters ϵ_* and η_* depend on λ and OPE coefficient C.

Further directions

- Different OPE coefficients $\leftrightarrow$ different bulk actions. Classification?
- Can we describe slow-roll with $\epsilon_* \sim \eta_*$?
- Study constraints from broken conformal invariance from bulk perspective.
- Multi-scalar models:

  entropy perturbations $\leftrightarrow$ operator mixing under RG flow.