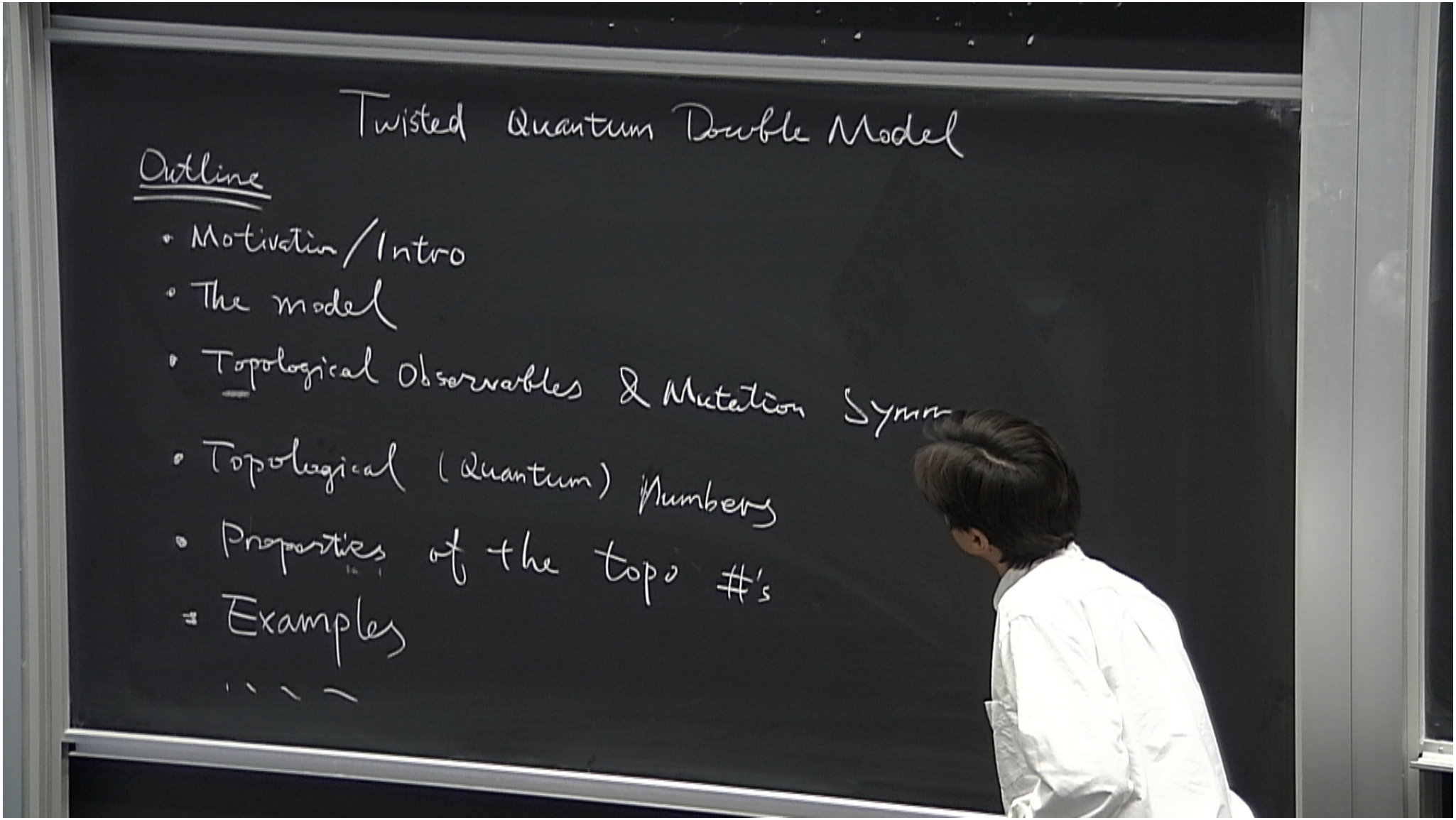
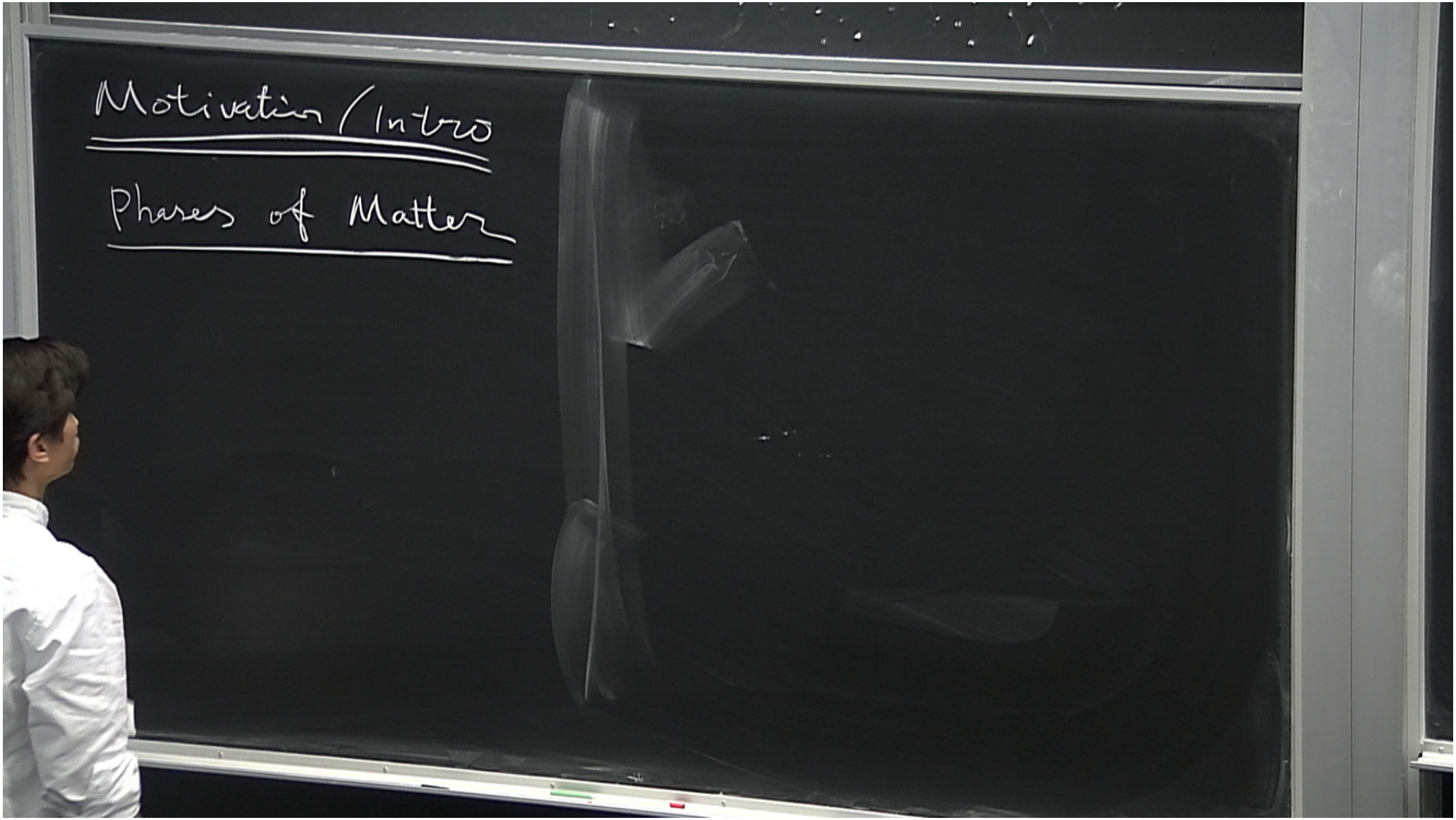
Title: Twisted quantum double model of topological phases

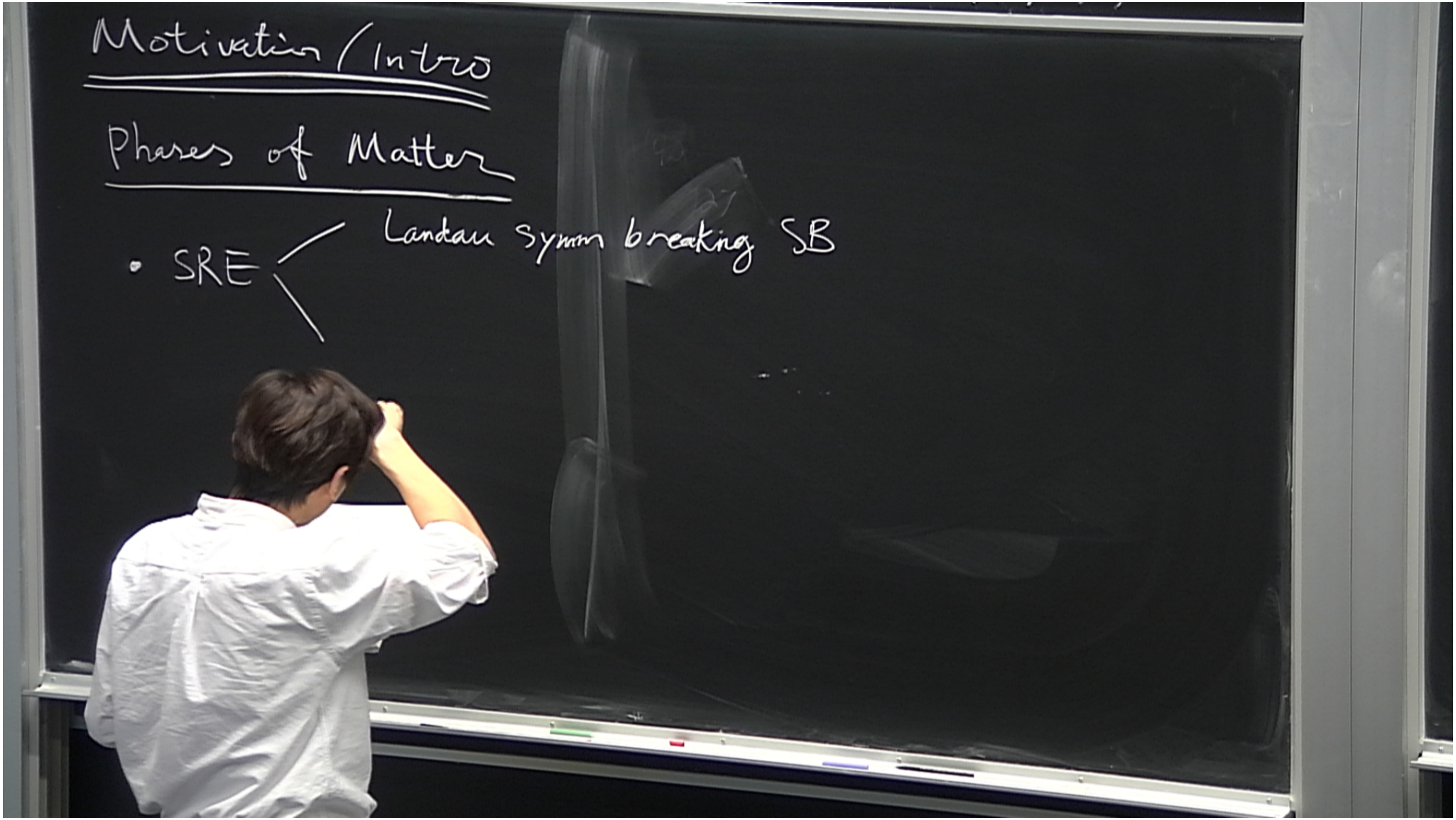
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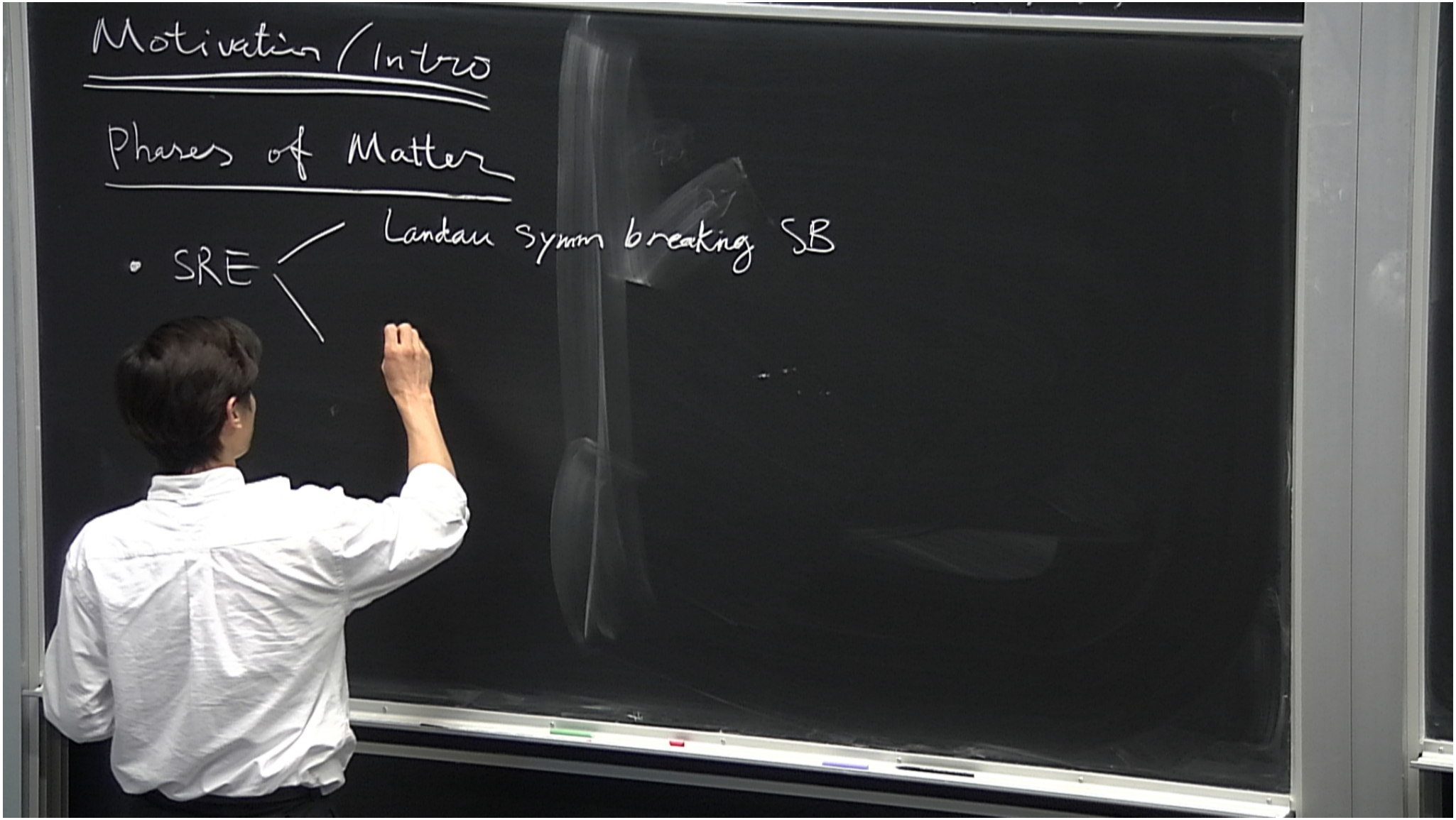
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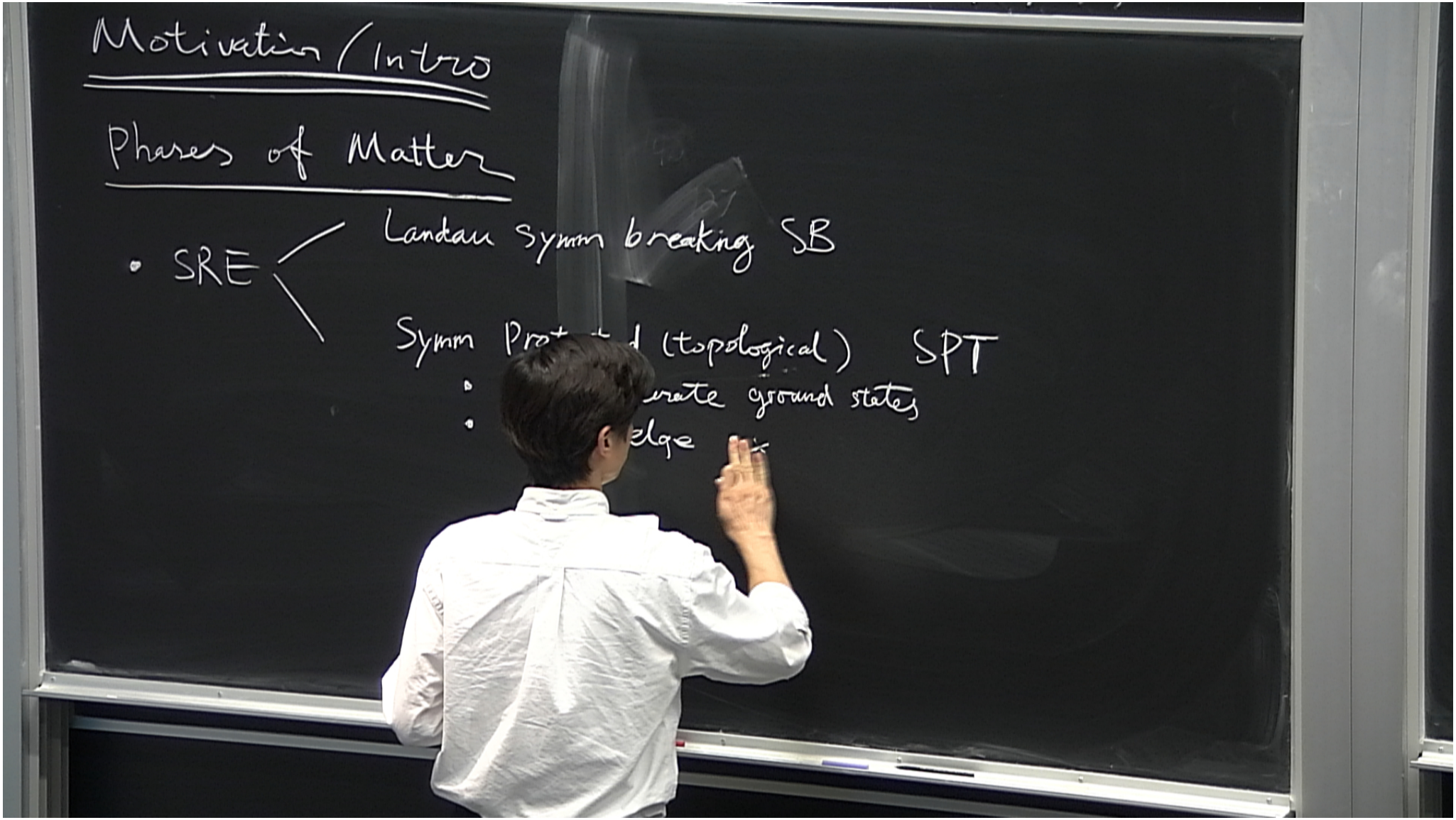
Abstract:

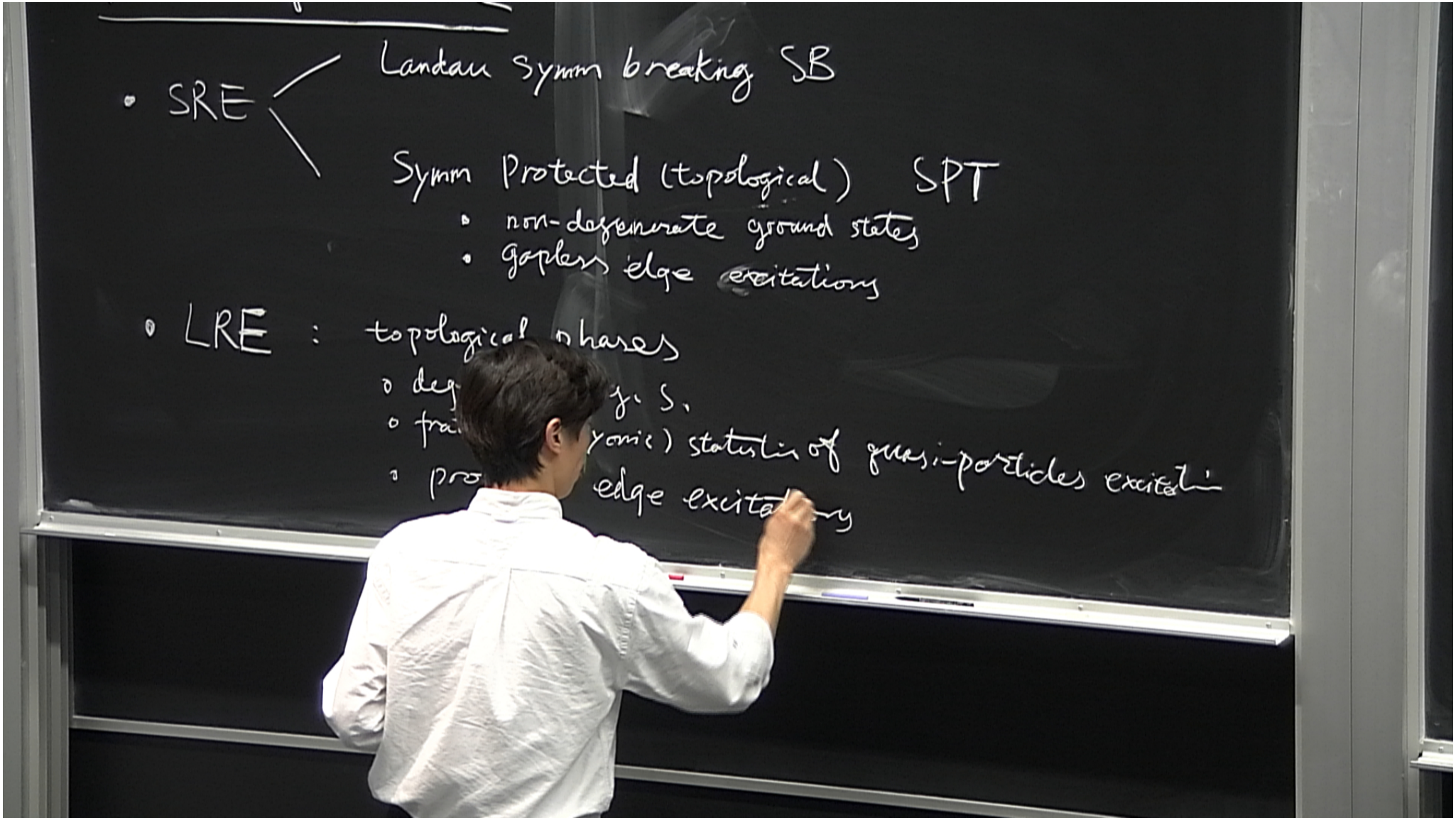

Twisted Quantum Double Model
Outline
• Motivation / Intro
• The model
• Topological Observables & Mutation Symm
• Topological (Quantum) Numbers
• Properties of the top. #'s
• Examples

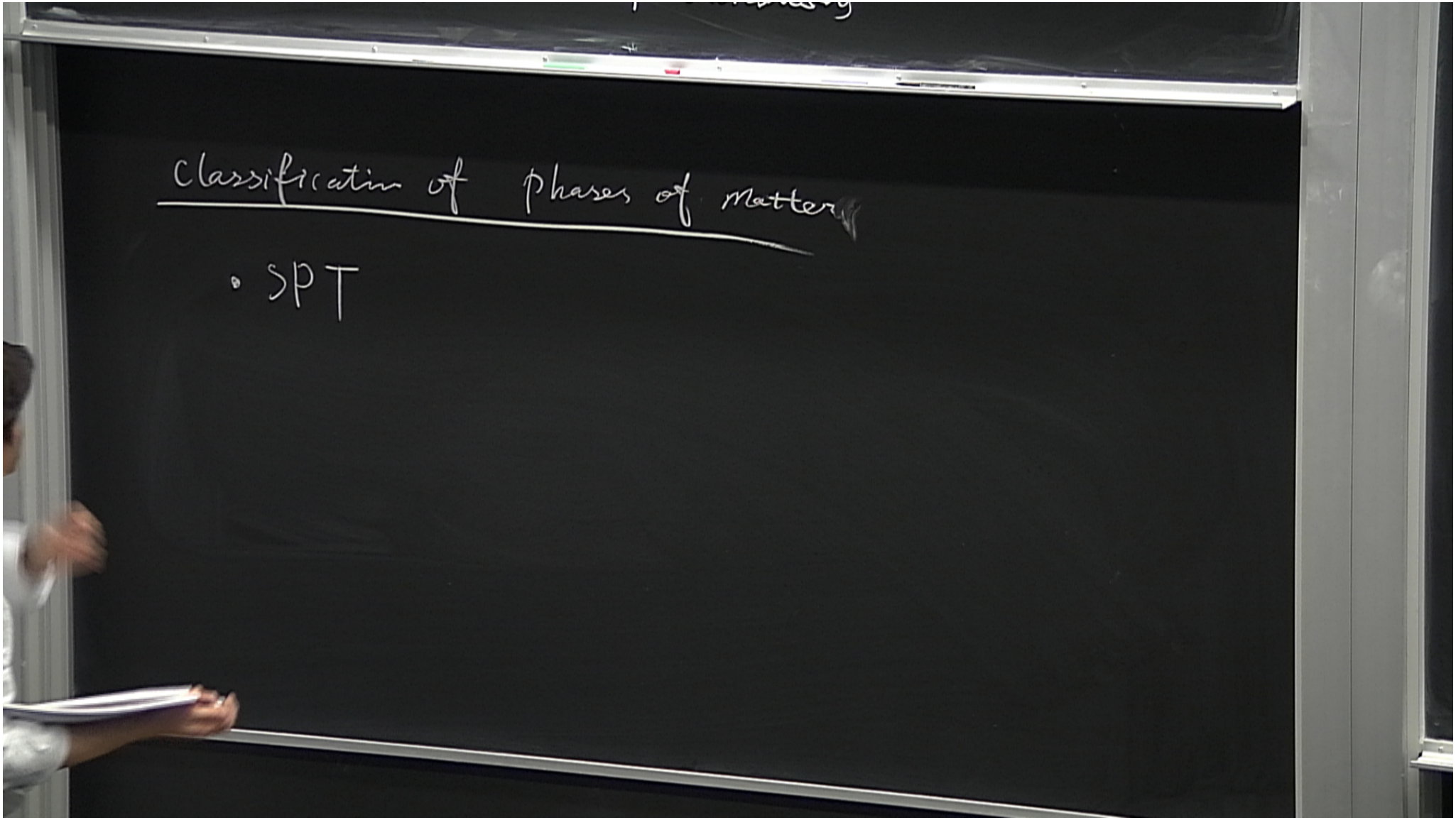

Motivation / Intro
Phases of Matter

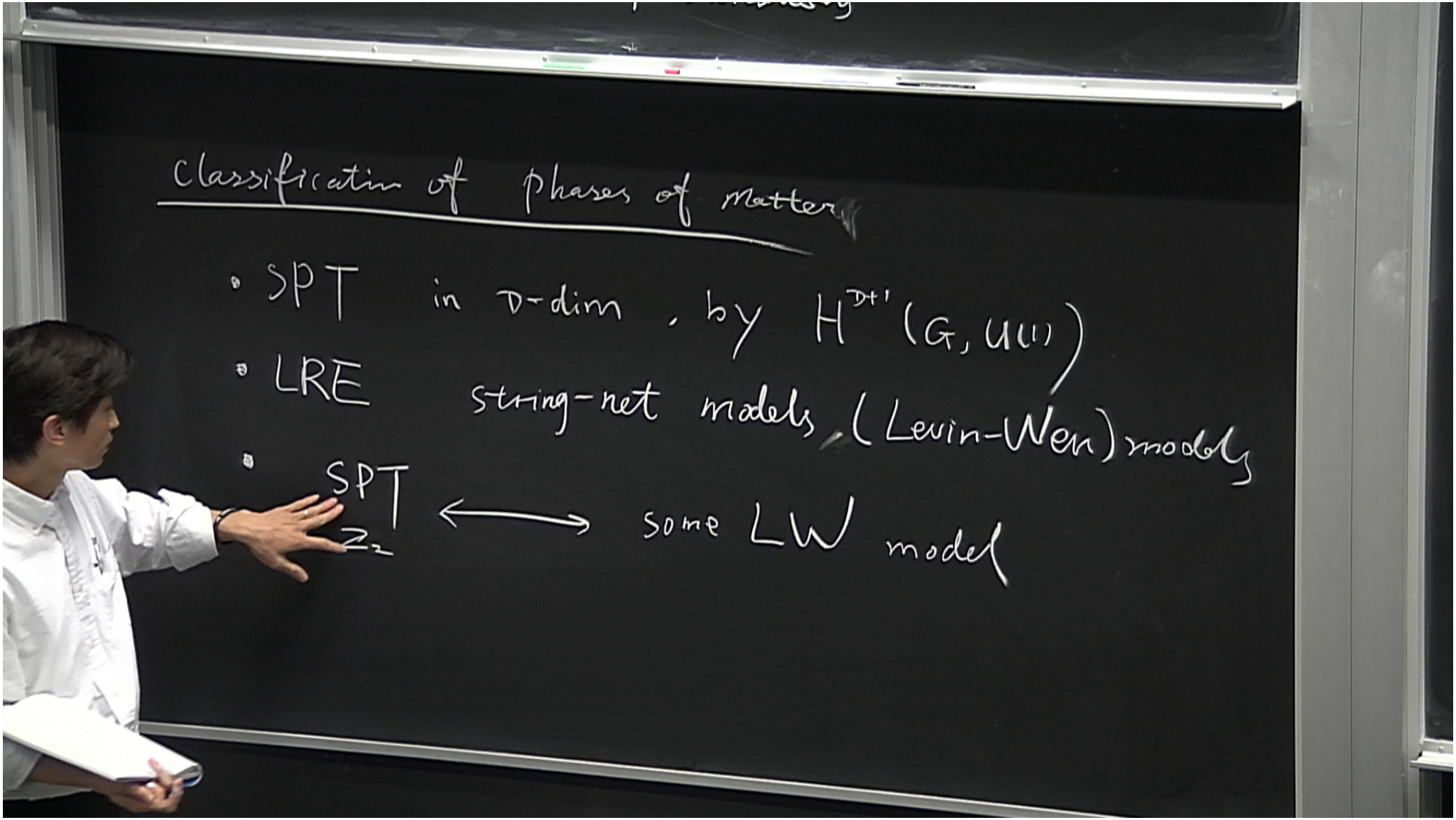

Motivation / Intro
Phases of Matter
• SRE
Landau symm breaking SB

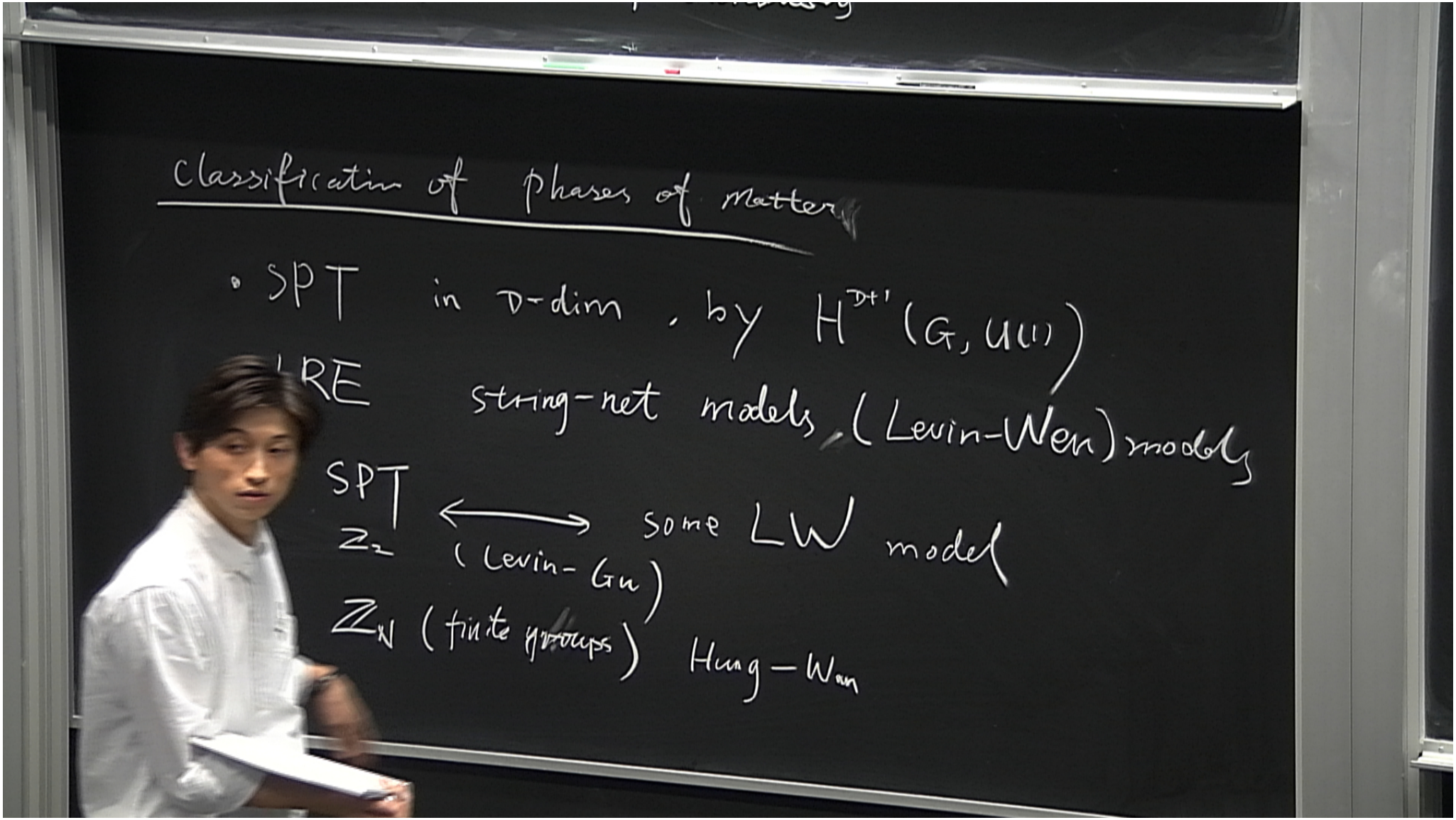

Motivation / Intro
Phases of Matter
SRE
Landau symm breaking SB

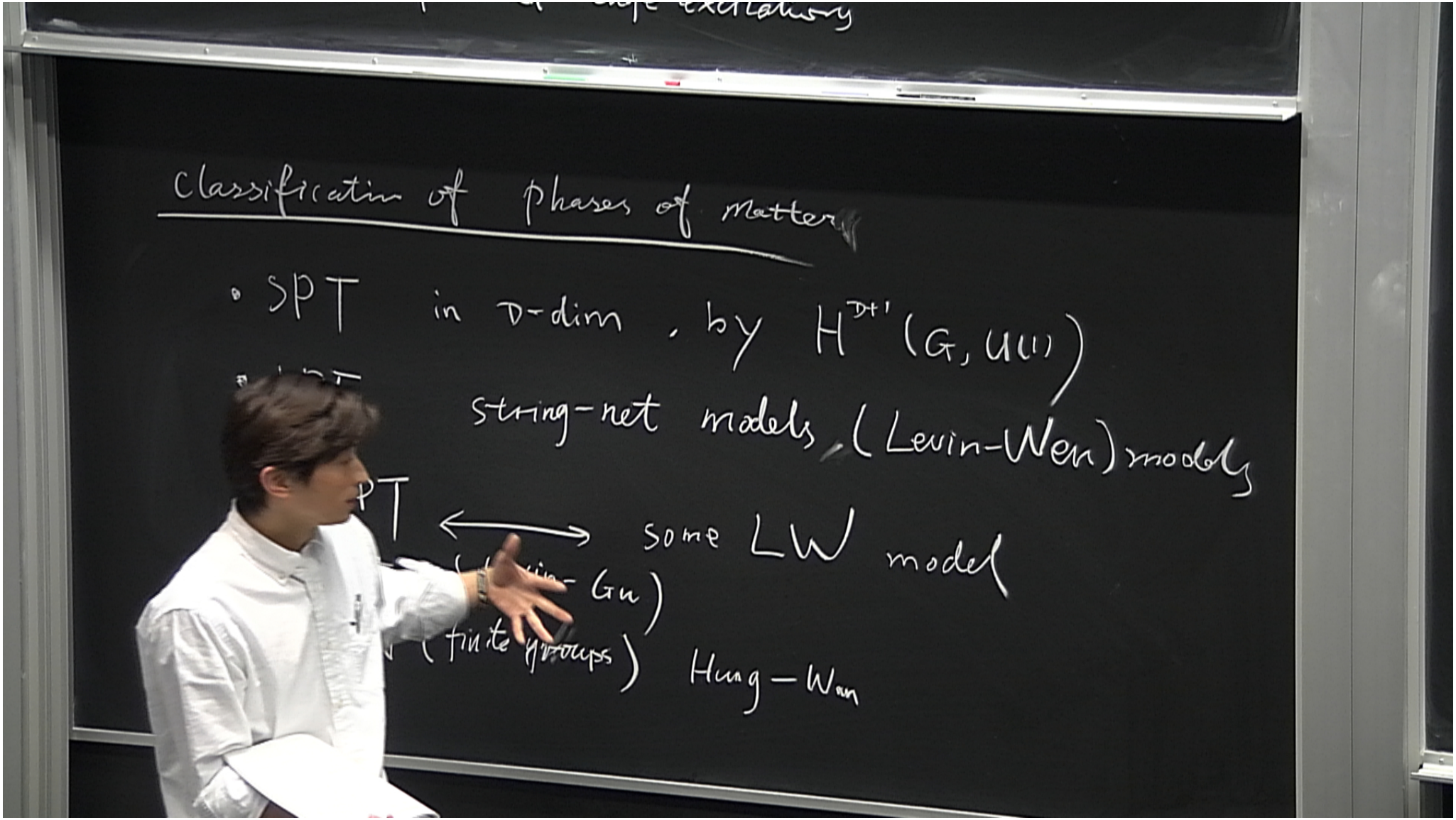

Motivation / Intro
Phases of Matter
SRE
Landau symm breaking SB
Symm
(topological) SPT
ground states

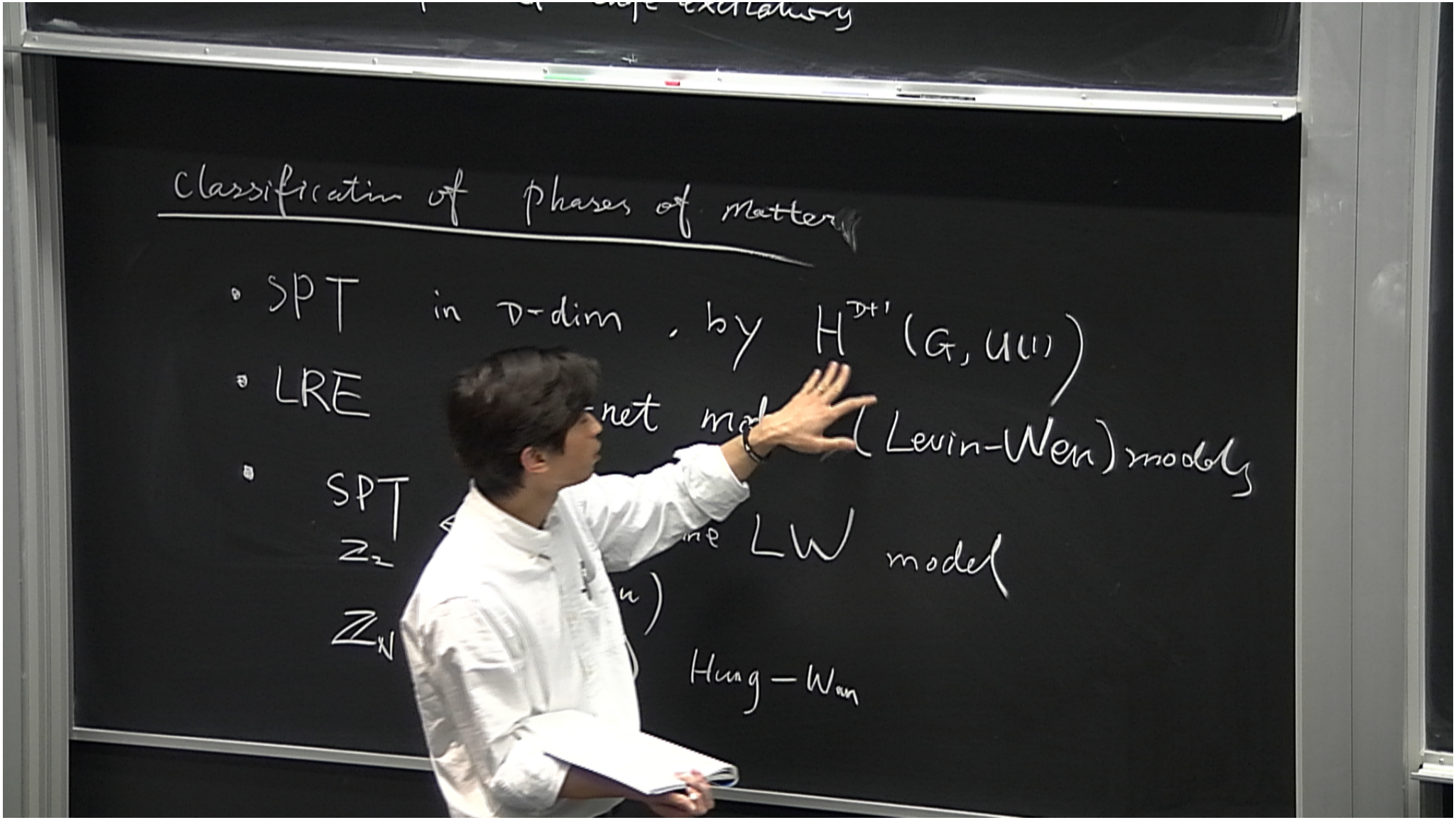

• SRE
Landau symm breaking SB
Symm Protected (topological) SPT
• non-degenerate ground states
• gapless edge excitations
• LRE : topological phases
• edge excitations

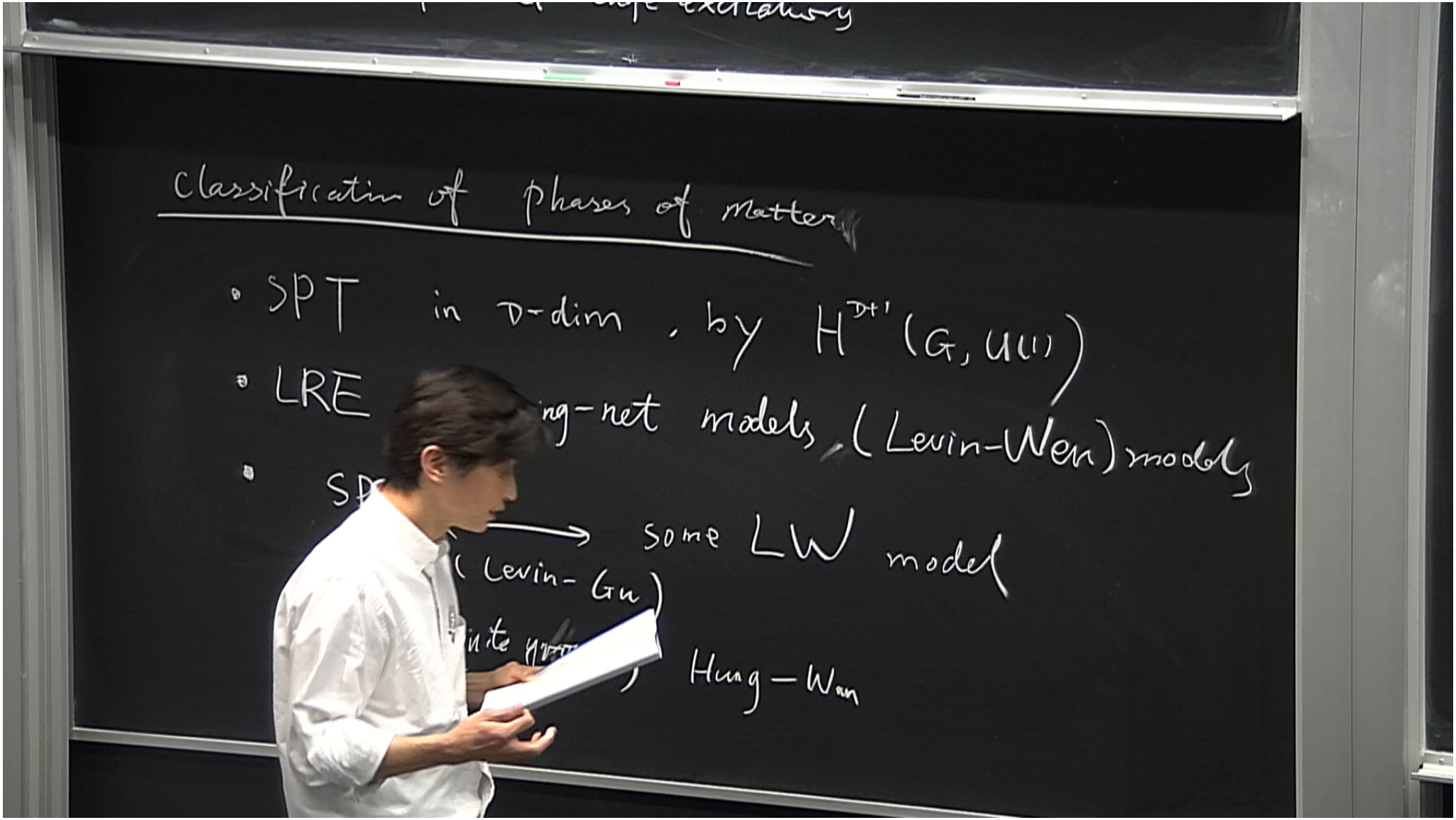

Classification of phases of matter
• SPT

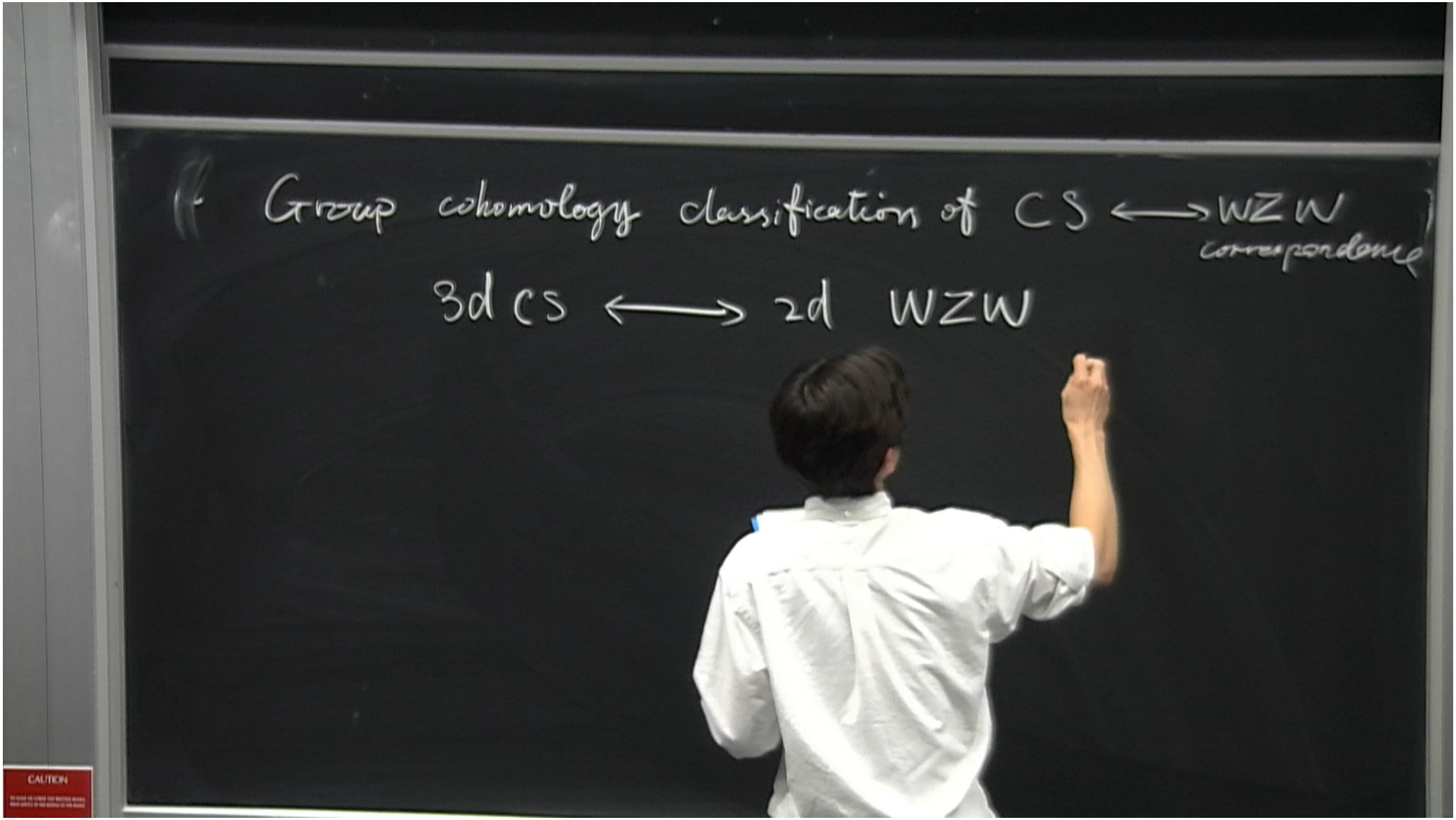

Classification of phases of matter
• SPT in D-dim. by $H^{D+1}(G, U(1))$
• LRE string-net models, (Levin-Wen) models
• SPT_{Z_2} ⟷ some LW model

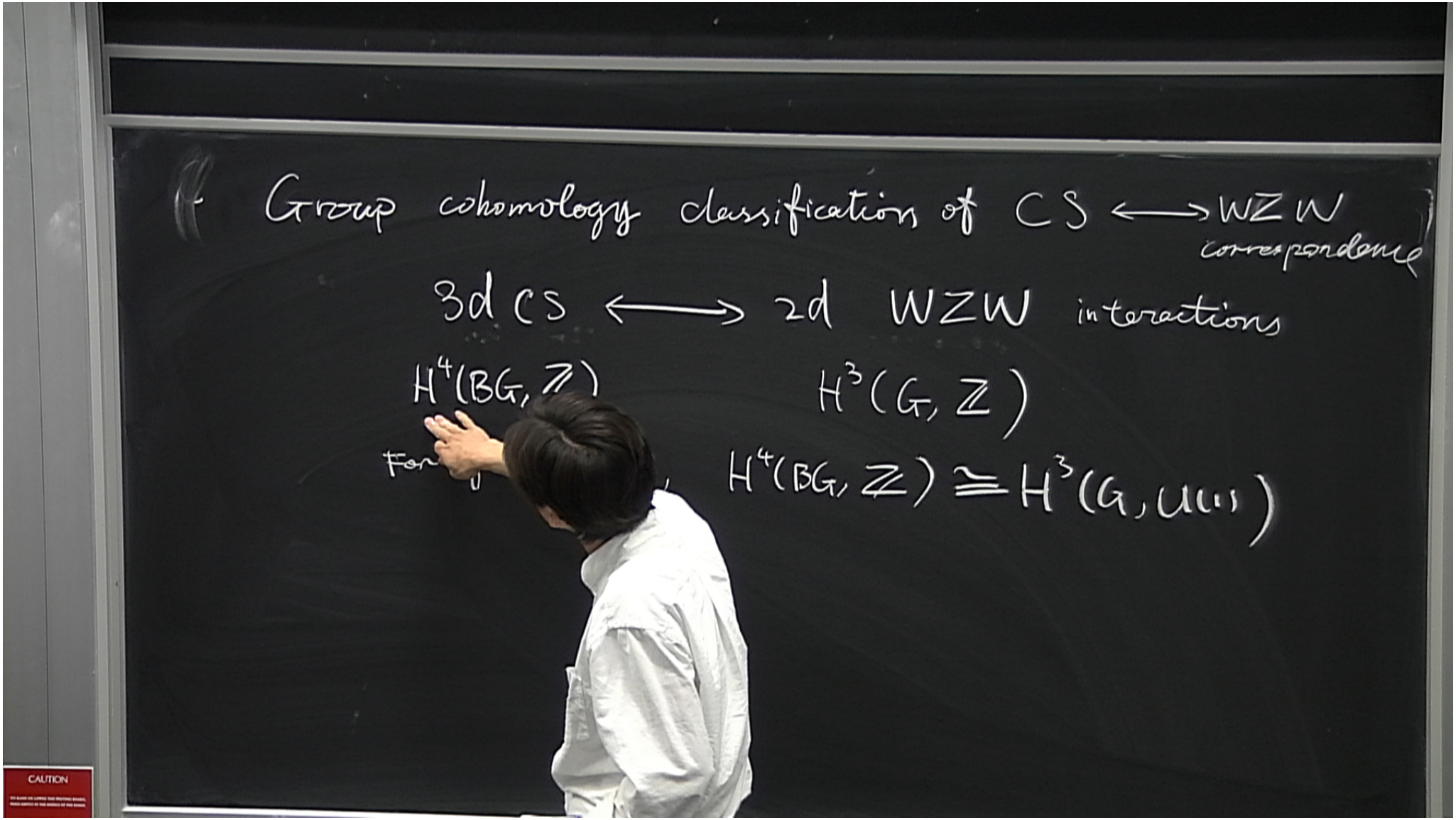

Classification of phases of matter
• SPT in D-dim . by $H^{D+1}(G, U(1))$
RE string-net models (Levin-Wen) models
SPT
Z_2 ⟷ some LW model
(Levin-Gu)
Z_N (finite groups) Hung-Wan


Classification of phases of matter
• SPT in D-dim . by $H^{D+1}(G, U(1))$
String-net models, (Levin-Wen) models
PT ⟷ some LW model
(finite groups) Hung-Wan


Classification of phases of matter
• SPT in D-dim . by $H^{D+1}(G, U(1))$
• LRE
• SPT
Z_2
Z_N
LW model
Hung-Wen


Classification of phases of matter
• SPT in D-dim . by $H^{D+1}(G, U(1))$
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some LW model
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Hung-Wan


Group cohomology classification of CS ⟷ WZW
correspondence
3d CS ⟷ 2d WZW
CAUTION


Group cohomology classification of CS ⟷ WZW correspondence
3d CS ⟷ 2d WZW interactions
$H^4(BG, \mathbb{Z})$
$H^3(G, \mathbb{Z})$
$H^4(BG, \mathbb{Z}) \cong H^3(G, U(1))$
CAUTION

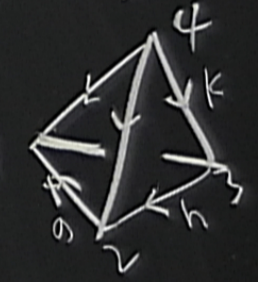
Group cohomology classification of CS $\longleftrightarrow$ WZW correspondence

3d CS $\longleftrightarrow$ 2d WZW interactions

$H^4(BG, \mathbb{Z})$ $H^3(G, \mathbb{Z})$

For finite G, $H^4(BG, \mathbb{Z}) \cong H^3(G, U(1))$

$\frac{1}{|G|} \prod_i W(T_i)^{\varepsilon_i}$



CAUTION

Group cohomology classification of CS $\longleftrightarrow$ WZW correspondence

3d CS $\longleftrightarrow$ 2d WZW interactions

$H^4(BG, \mathbb{Z})$ $\qquad$ $H^3(G, \mathbb{Z})$

For finite G, $H^4(BG, \mathbb{Z}) \cong H^3(G, U(1))$

$$\mathcal{Z}_{LS}(T(M), G) = \frac{1}{|G|} \prod_i W(T_i)^{\varepsilon_i}$$

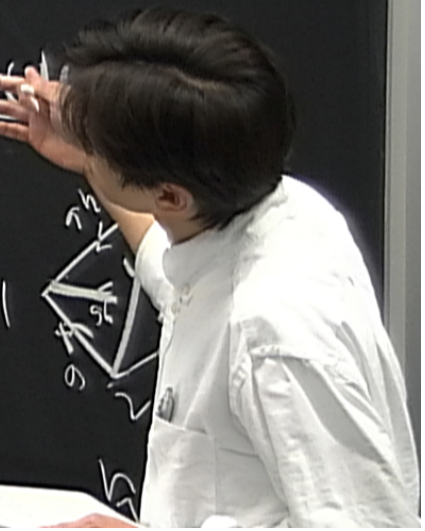
Group cohomology classification of CS $\longleftrightarrow$ WZW correspondence

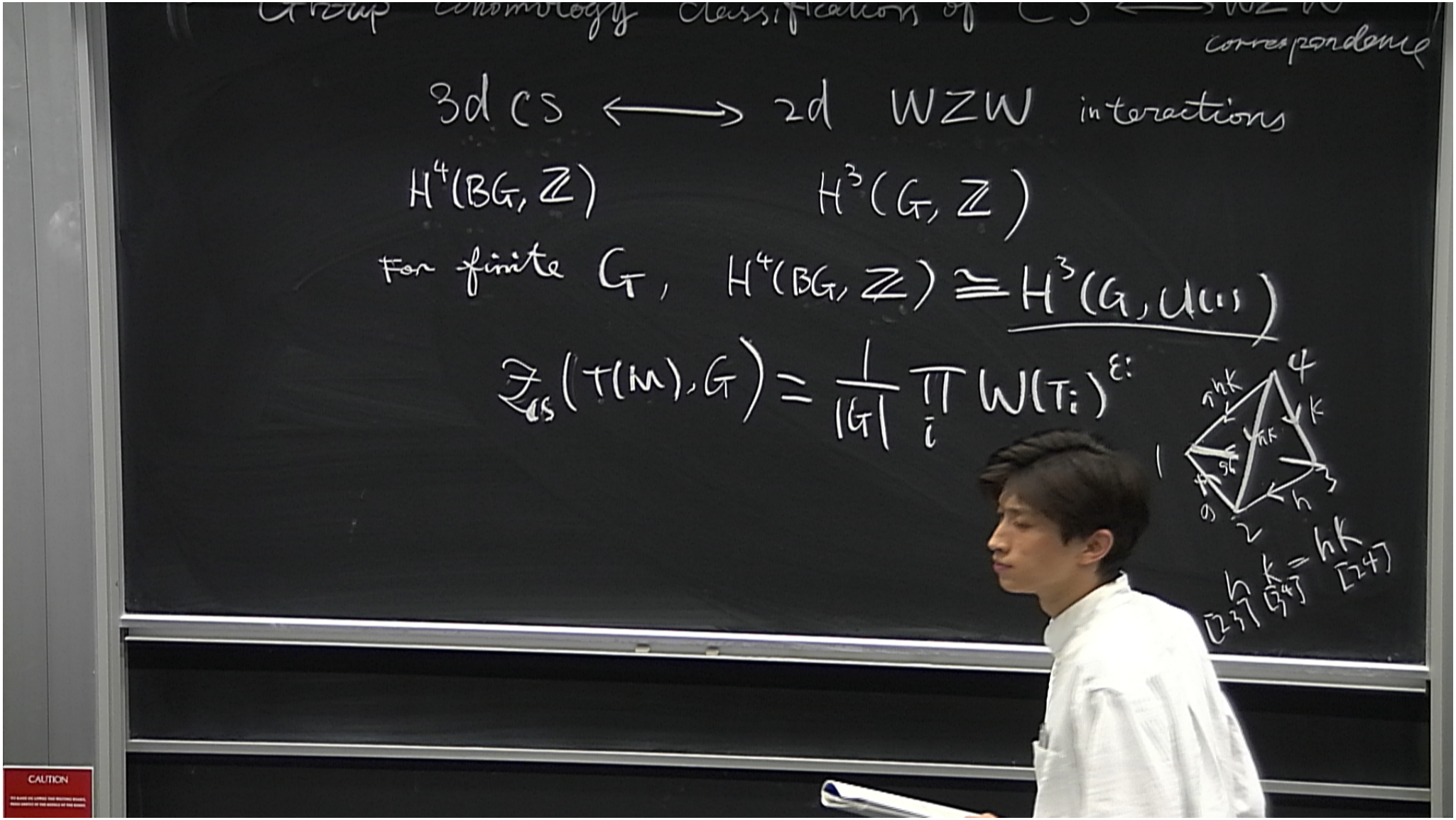
3d CS $\longleftrightarrow$ 2d WZW interactions

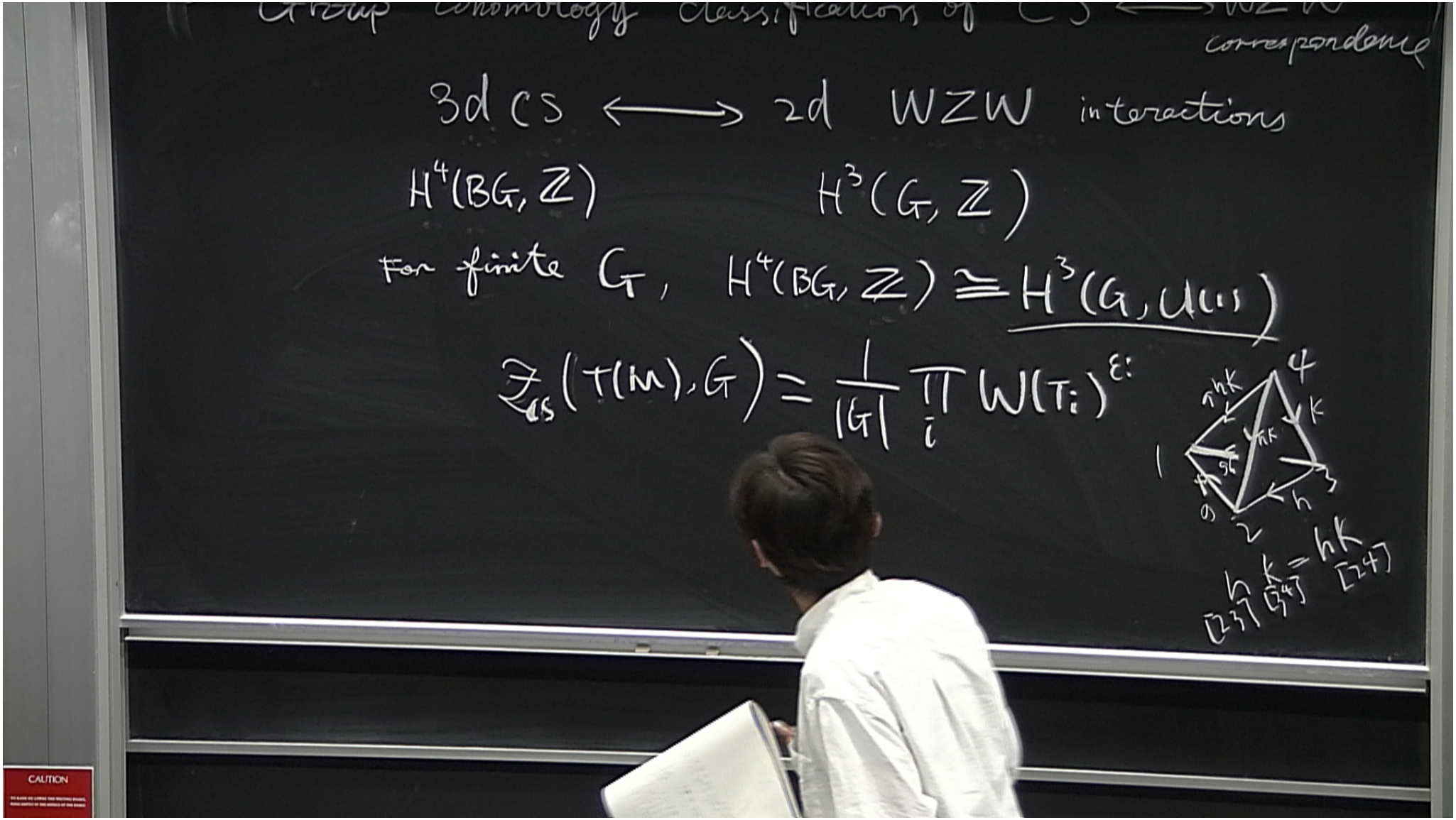
$H^4(BG, \mathbb{Z})$ $\qquad$ $H^3(G, \mathbb{Z})$

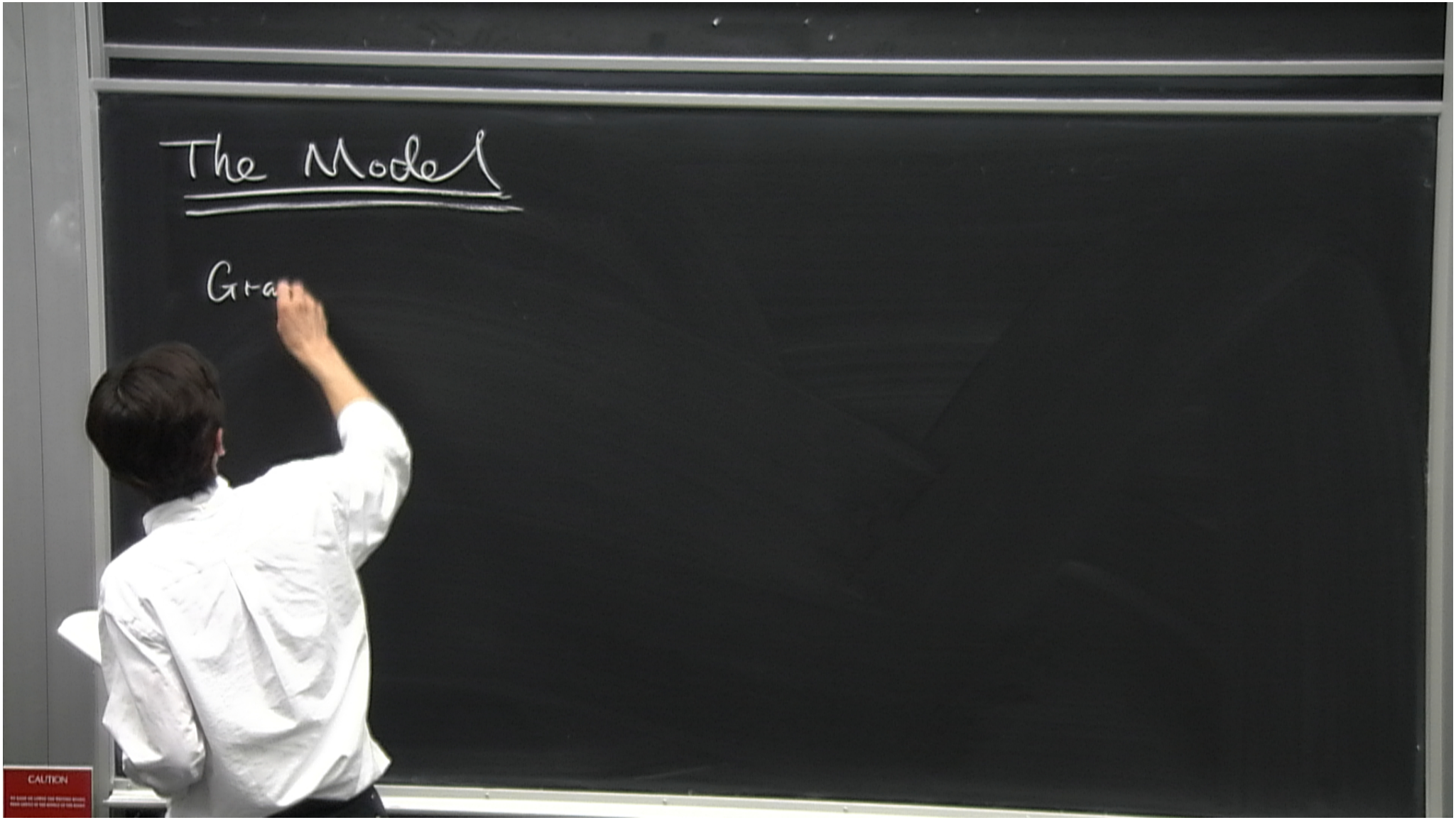
For finite G, $H^4(BG, \mathbb{Z}) \cong H^3(G$

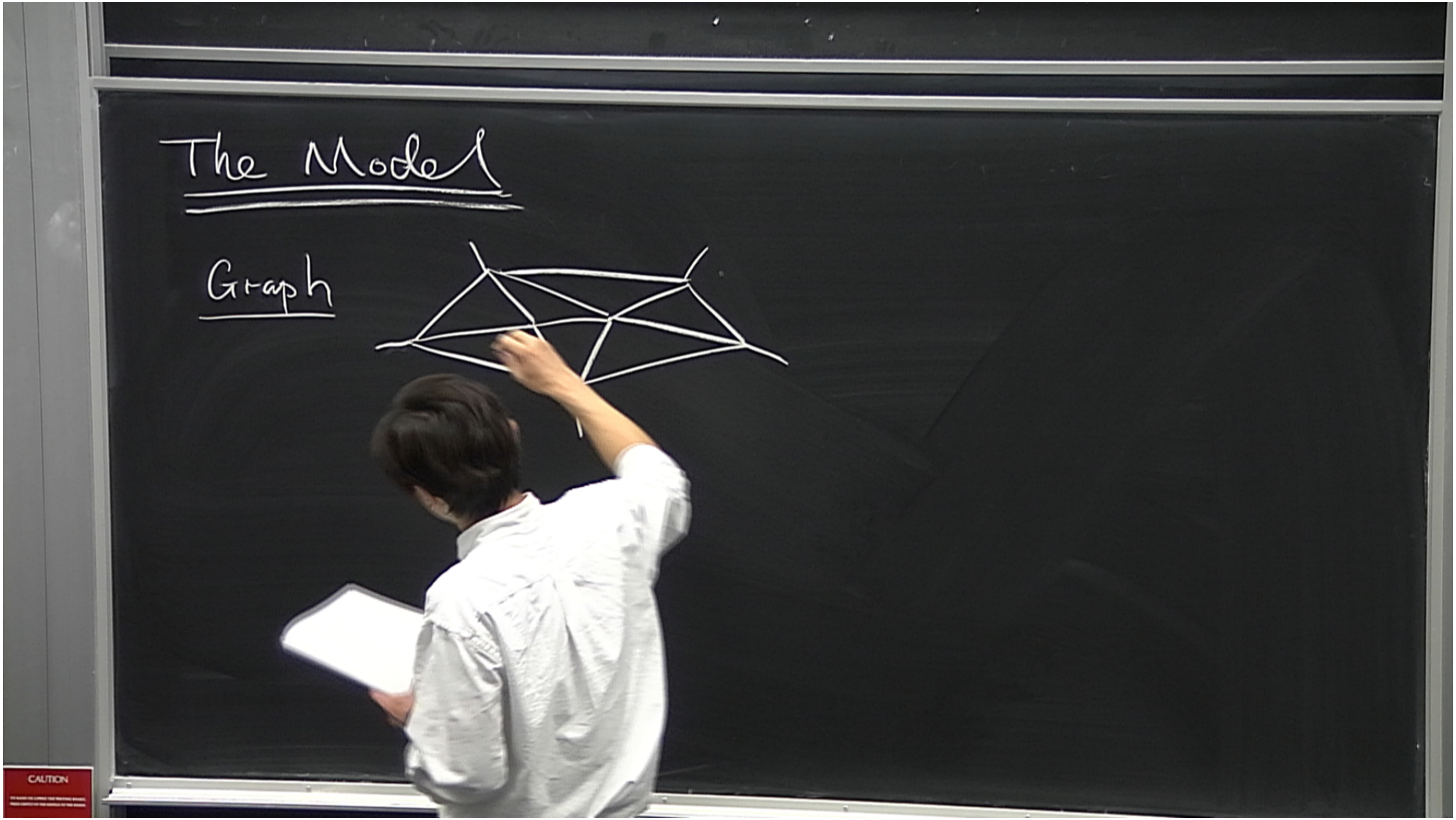
$$Z_{CS}(T(M), G) = \frac{1}{|G|} \prod_i W(T_i)^{\varepsilon_i}$$

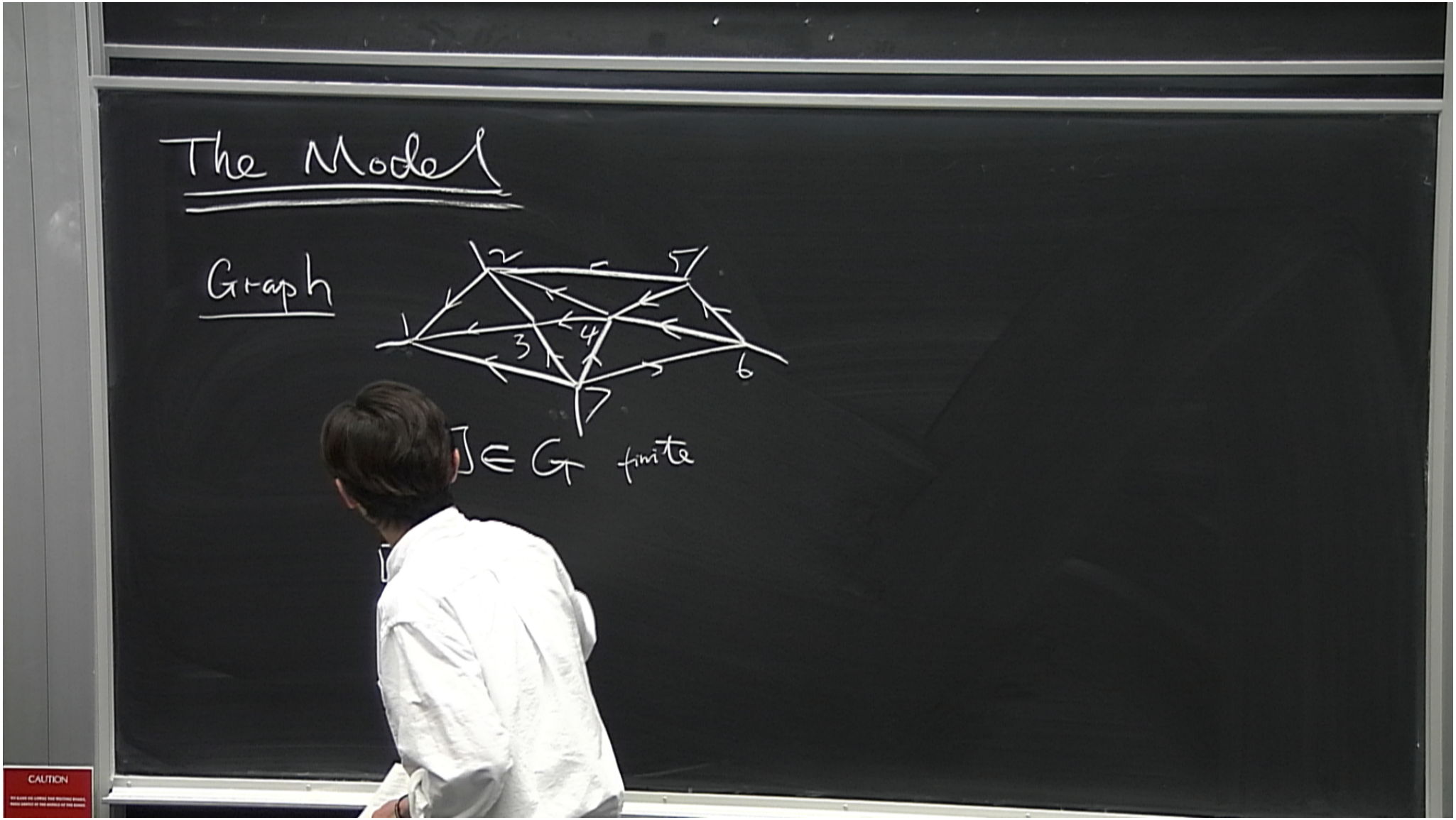


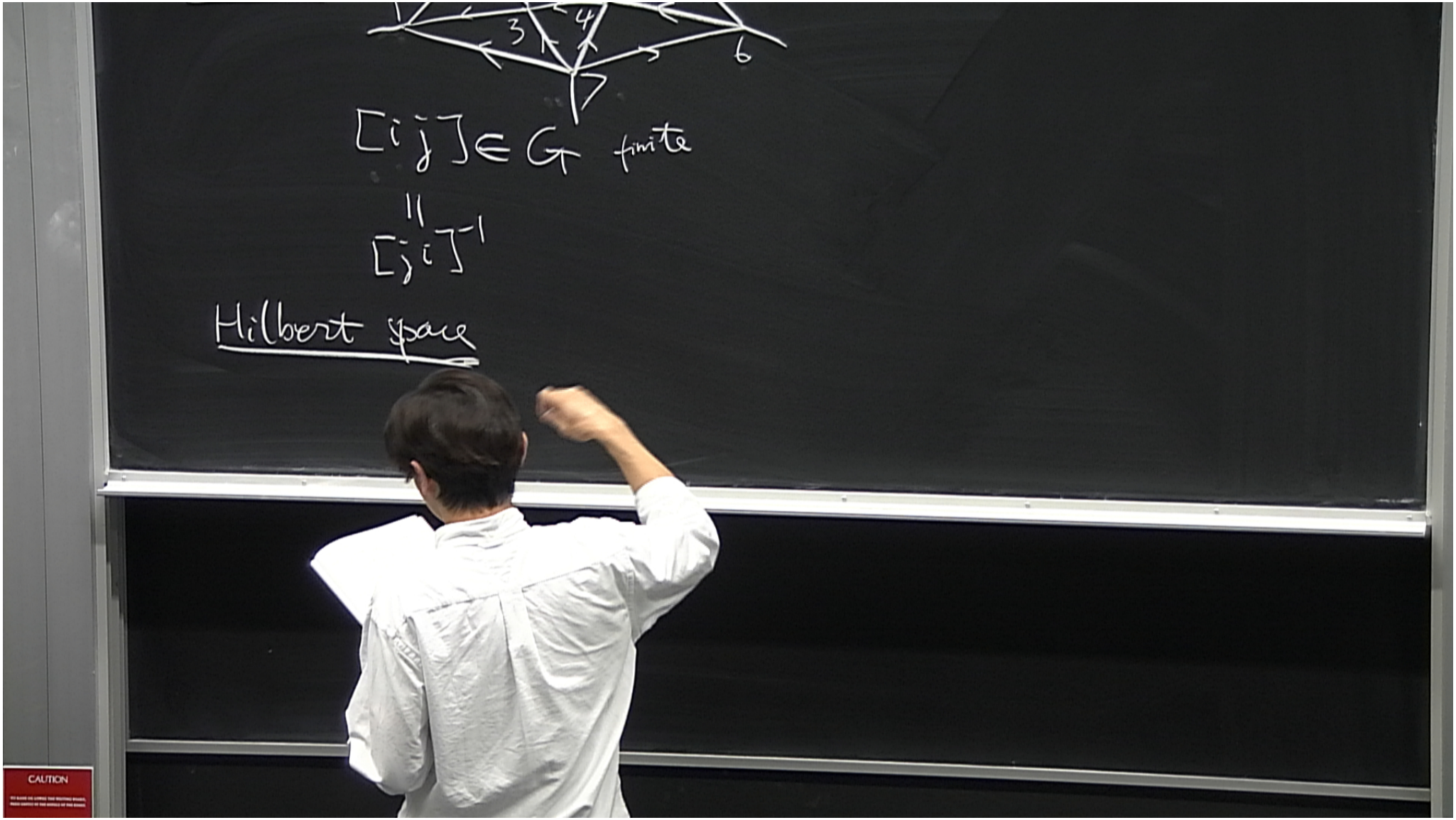

correspondence
3d CS ⟷ 2d WZW interactions
$H^4(BG,\mathbb{Z})$ $H^3(G,\mathbb{Z})$
For finite G, $H^4(BG,\mathbb{Z}) \cong H^3(G,U(1))$
$Z_{CS}(T(M),G) = \frac{1}{|G|}\prod_i W(T_i)^{\varepsilon_i}$
CAUTION

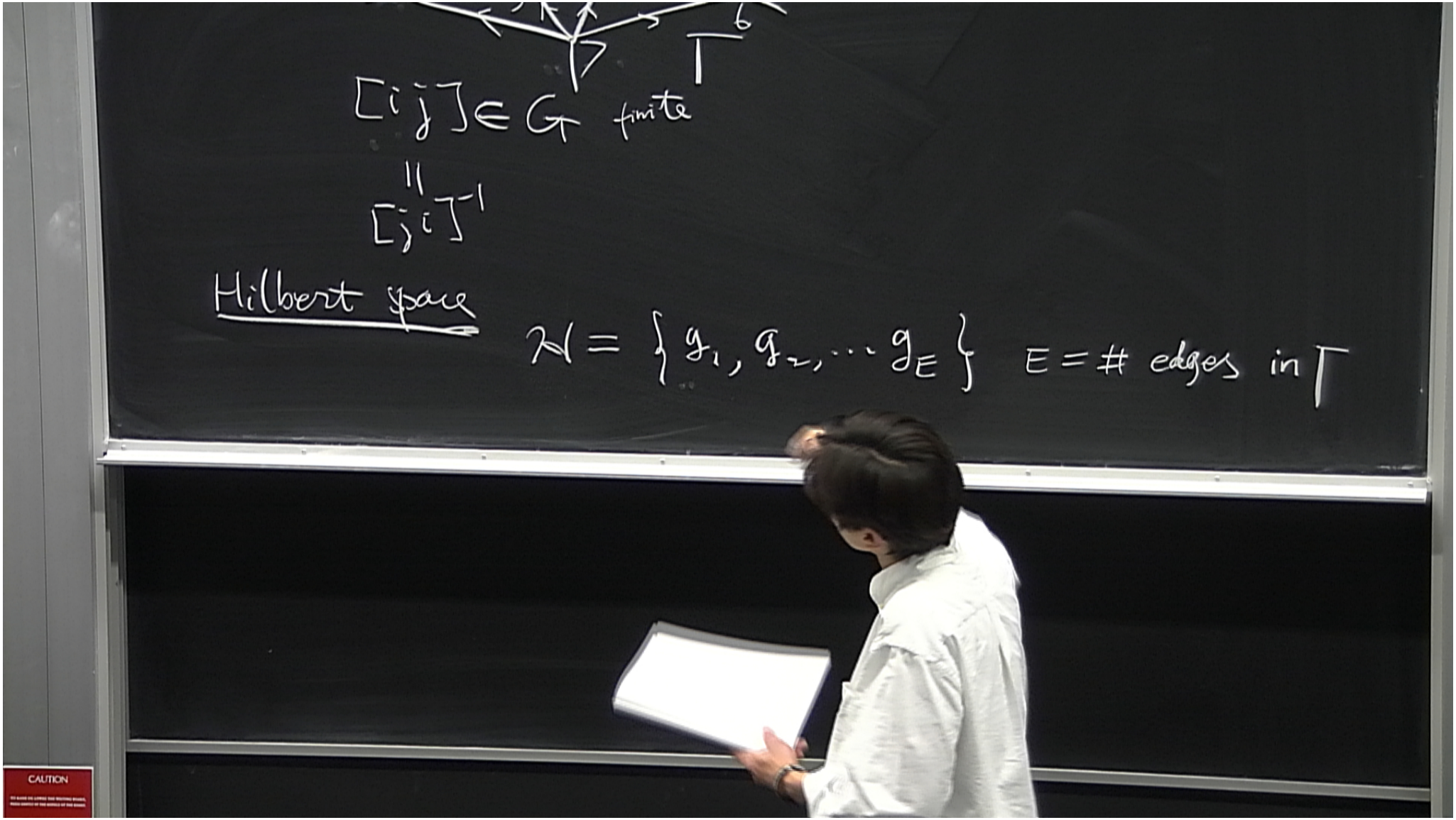

correspondence
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CAUTION

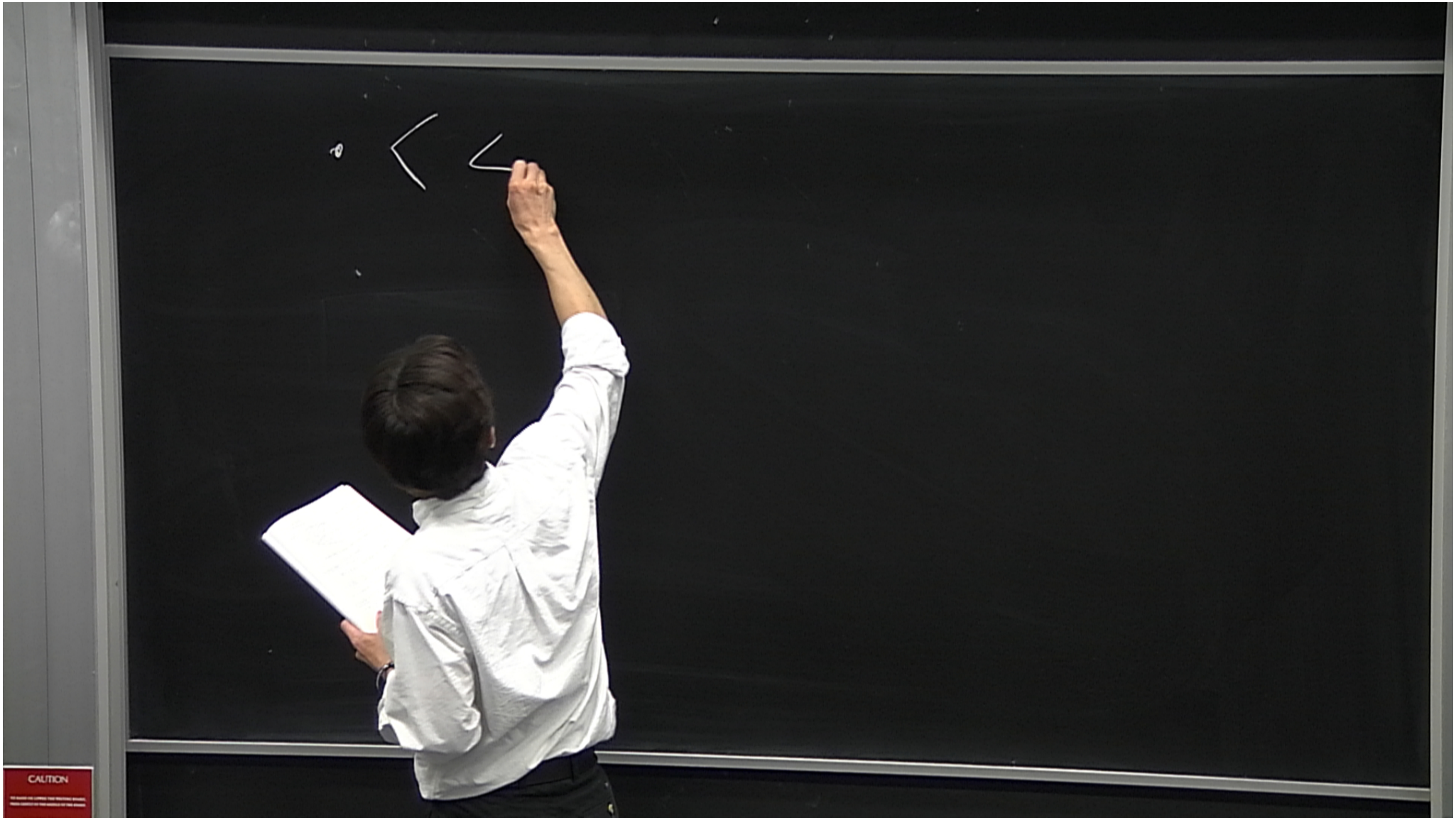

The Model
Gra
CAUTION

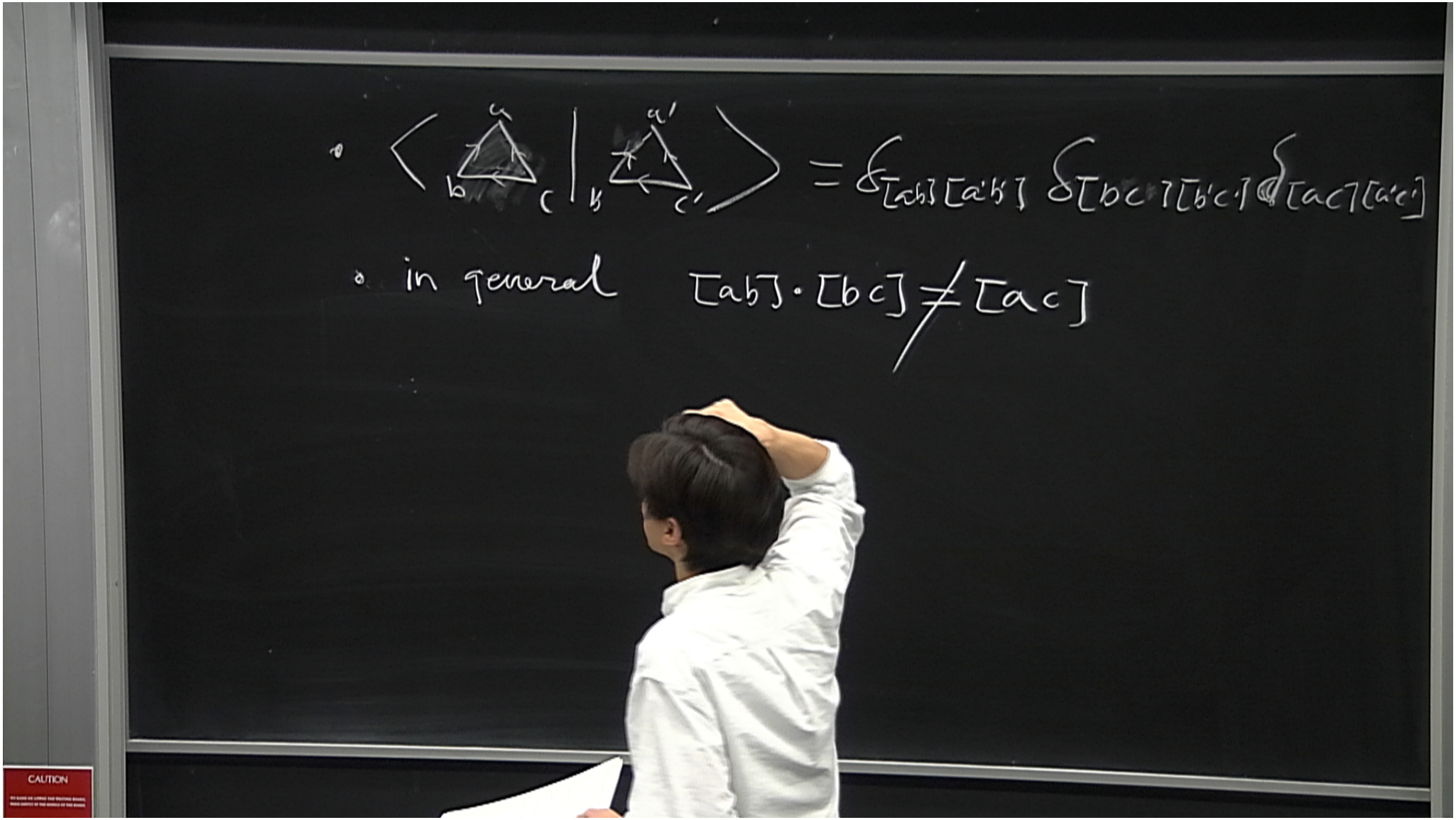

The Model
Graph
CAUTION

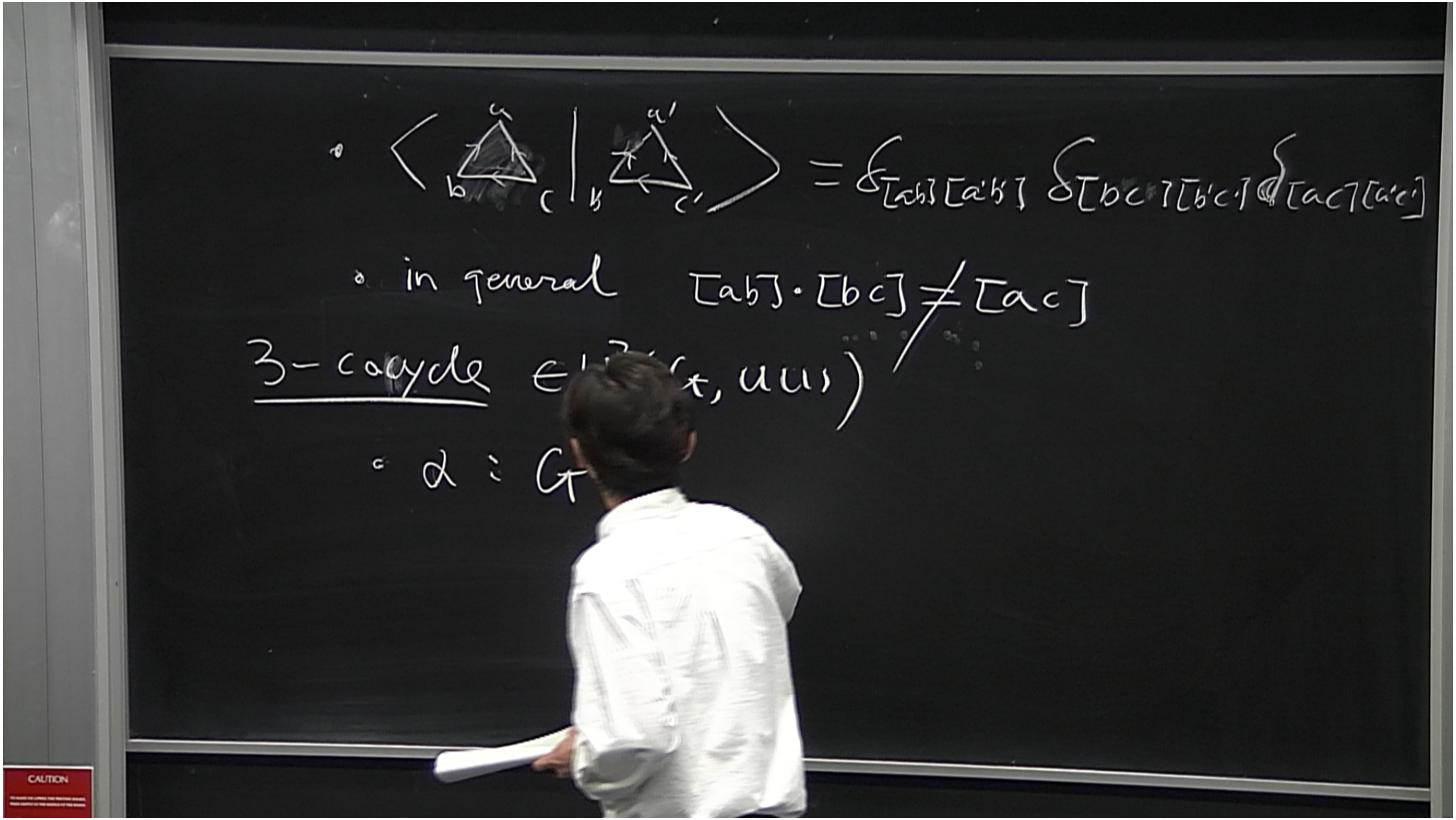

The Model
Graph
1
2
3
4
5
6
7
finite
CAUTION

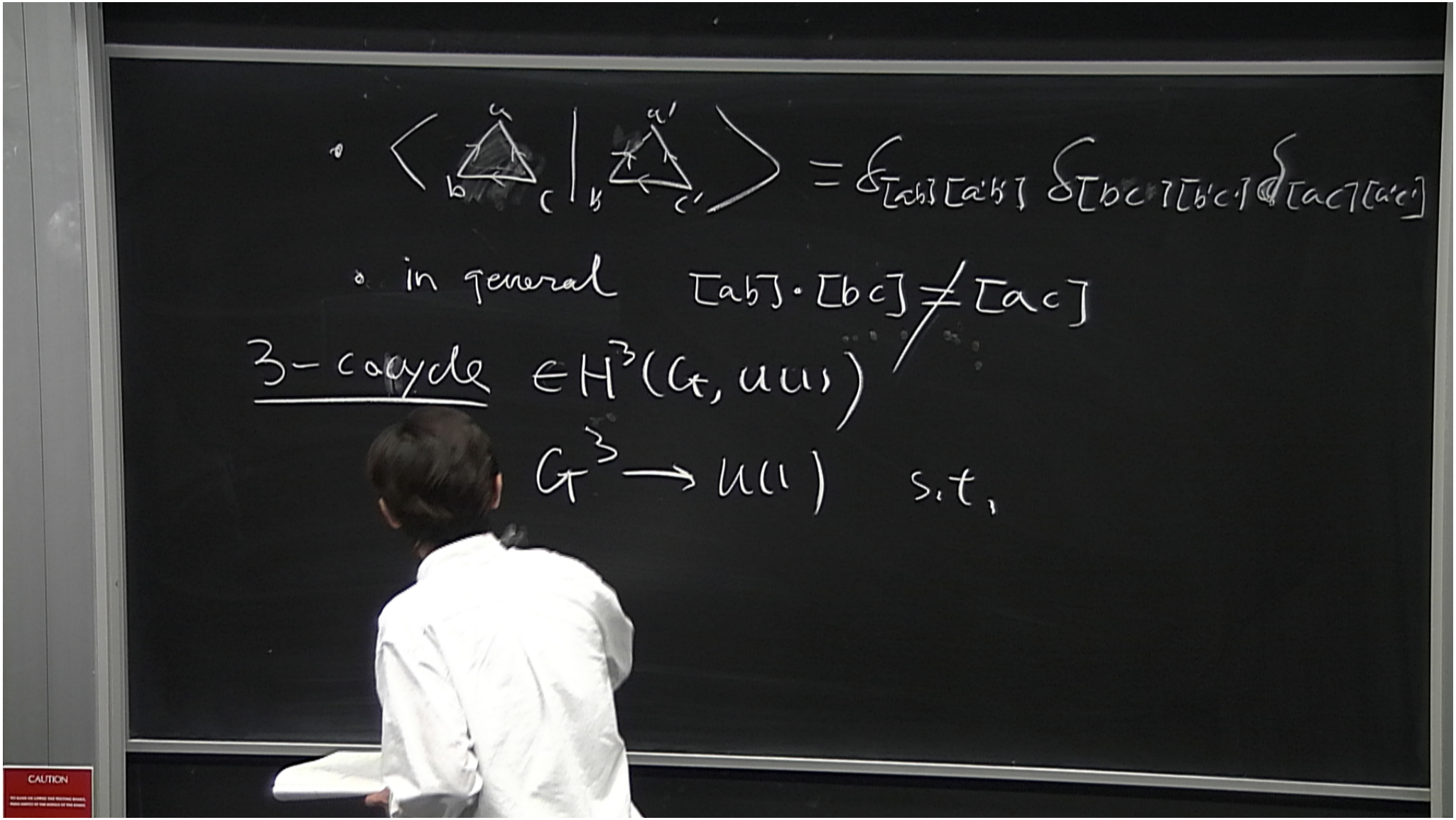

3
4
6
7
[i j] ∈ G finite
||
[j i]^-1
Hilbert space
CAUTION

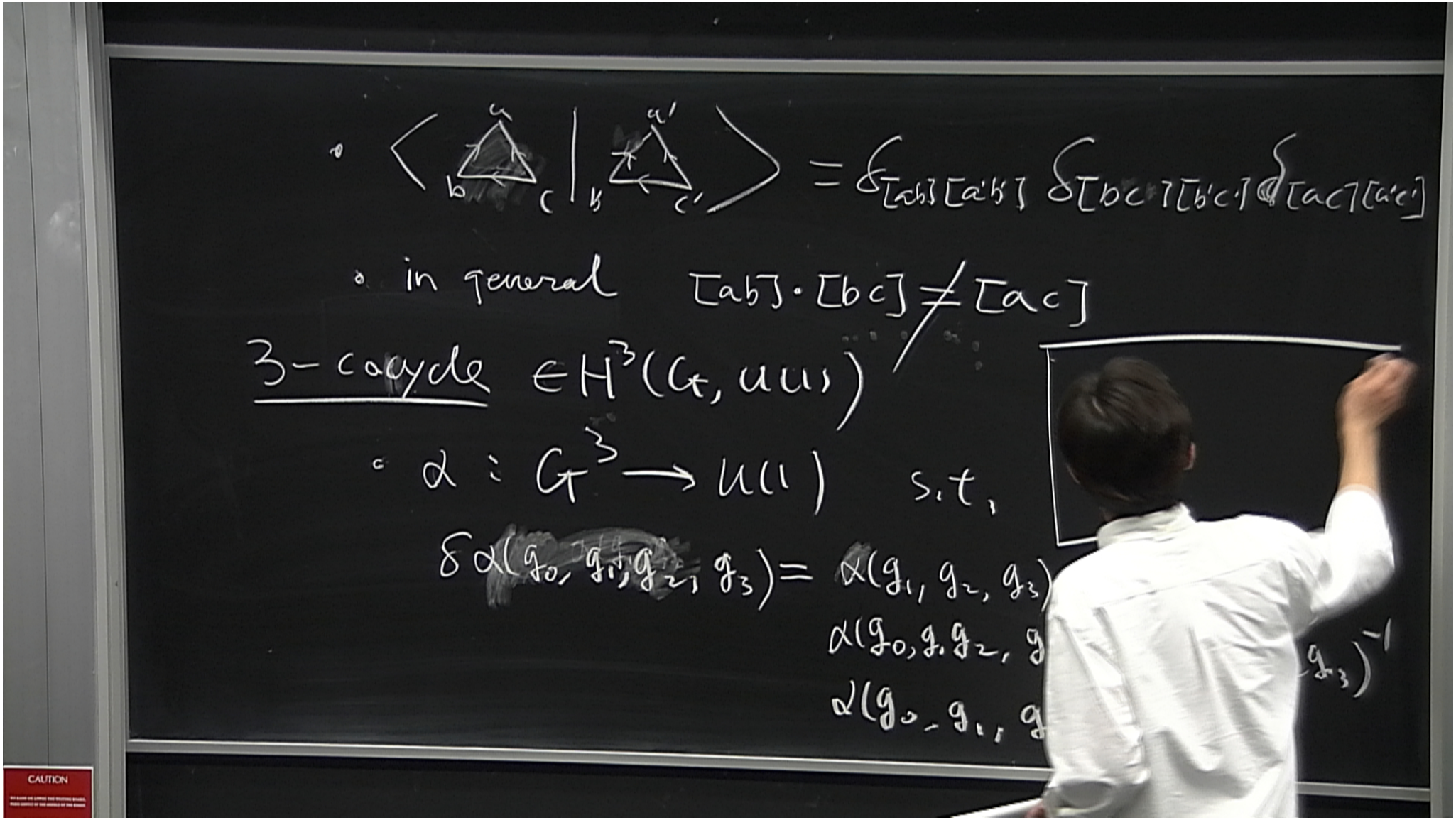

[ij] ∈ G finite
=
[ji]^-1
Hilbert space
ℋ = { g_1, g_2, ... g_E } E = # edges in Γ
CAUTION

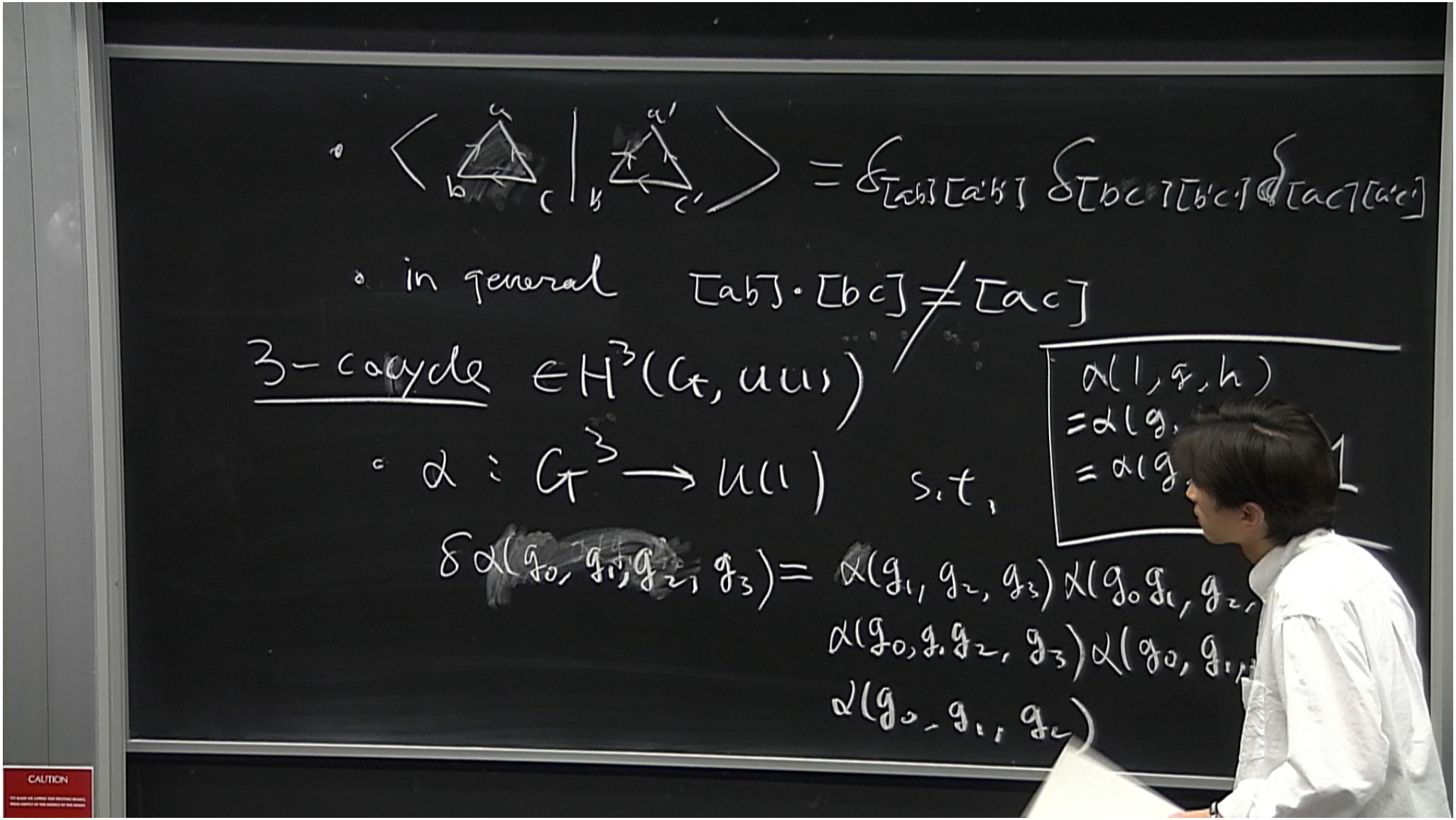

CAUTION

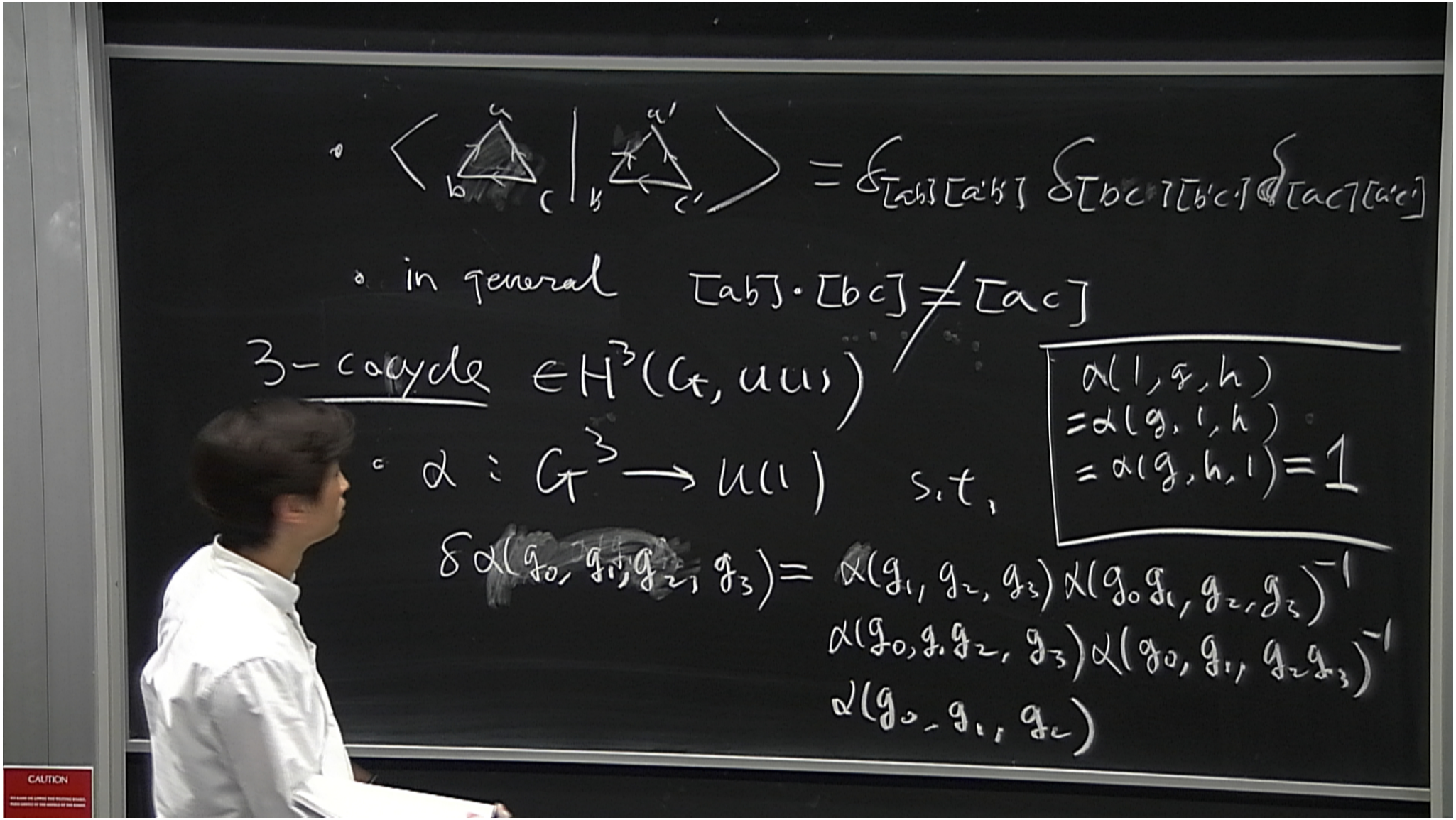

$\langle$ a b c | a' b' c' $\rangle$ = $\delta_{[ab][a'b']}\,\delta_{[bc][b'c']}\,\delta_{[ac][a'c']}$
in general $[ab]\cdot[bc] \neq [ac]$
CAUTION

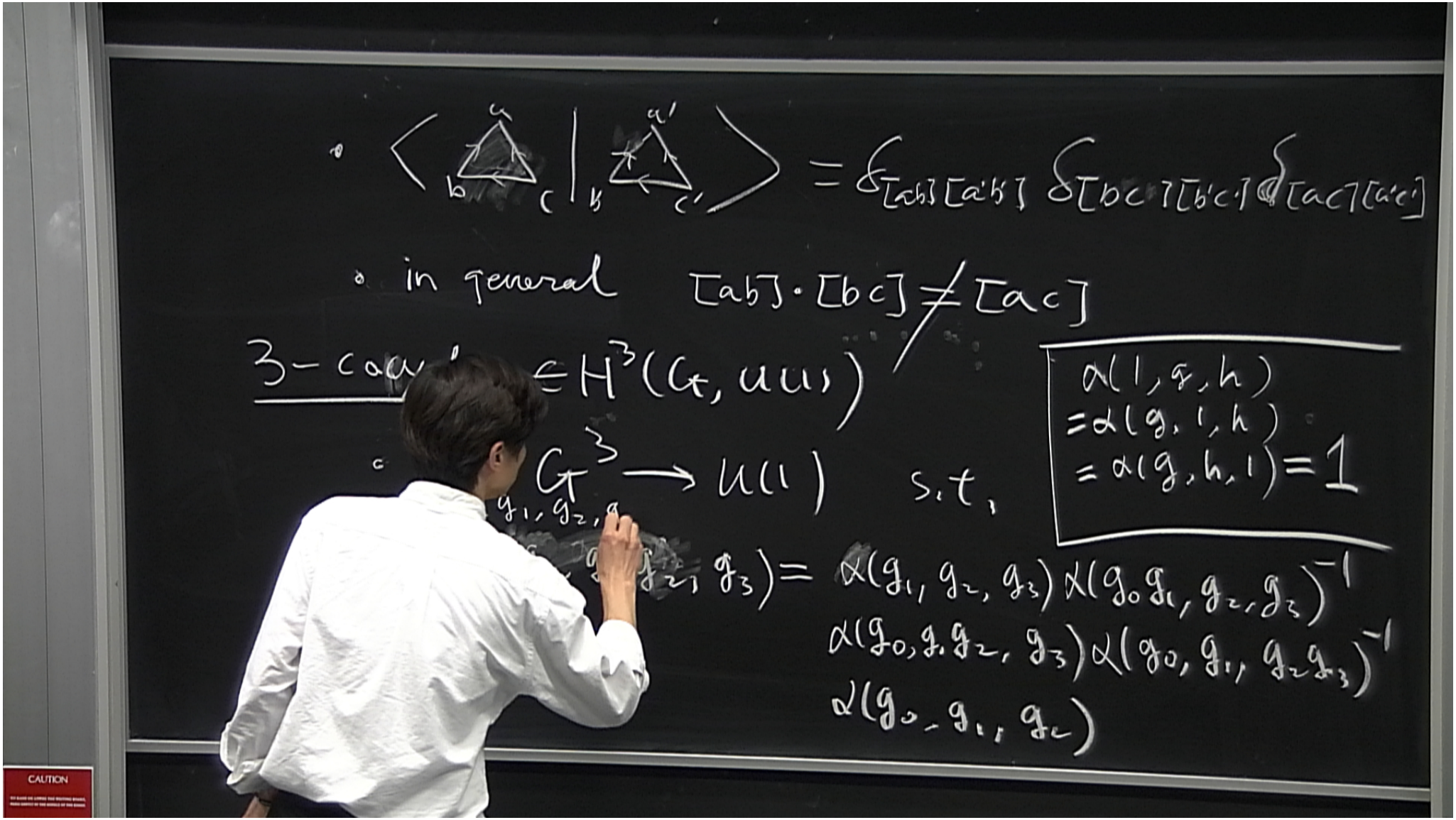

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in general $[ab]\cdot[bc] \neq [ac]$
3-cocycle
$\alpha : G$
CAUTION

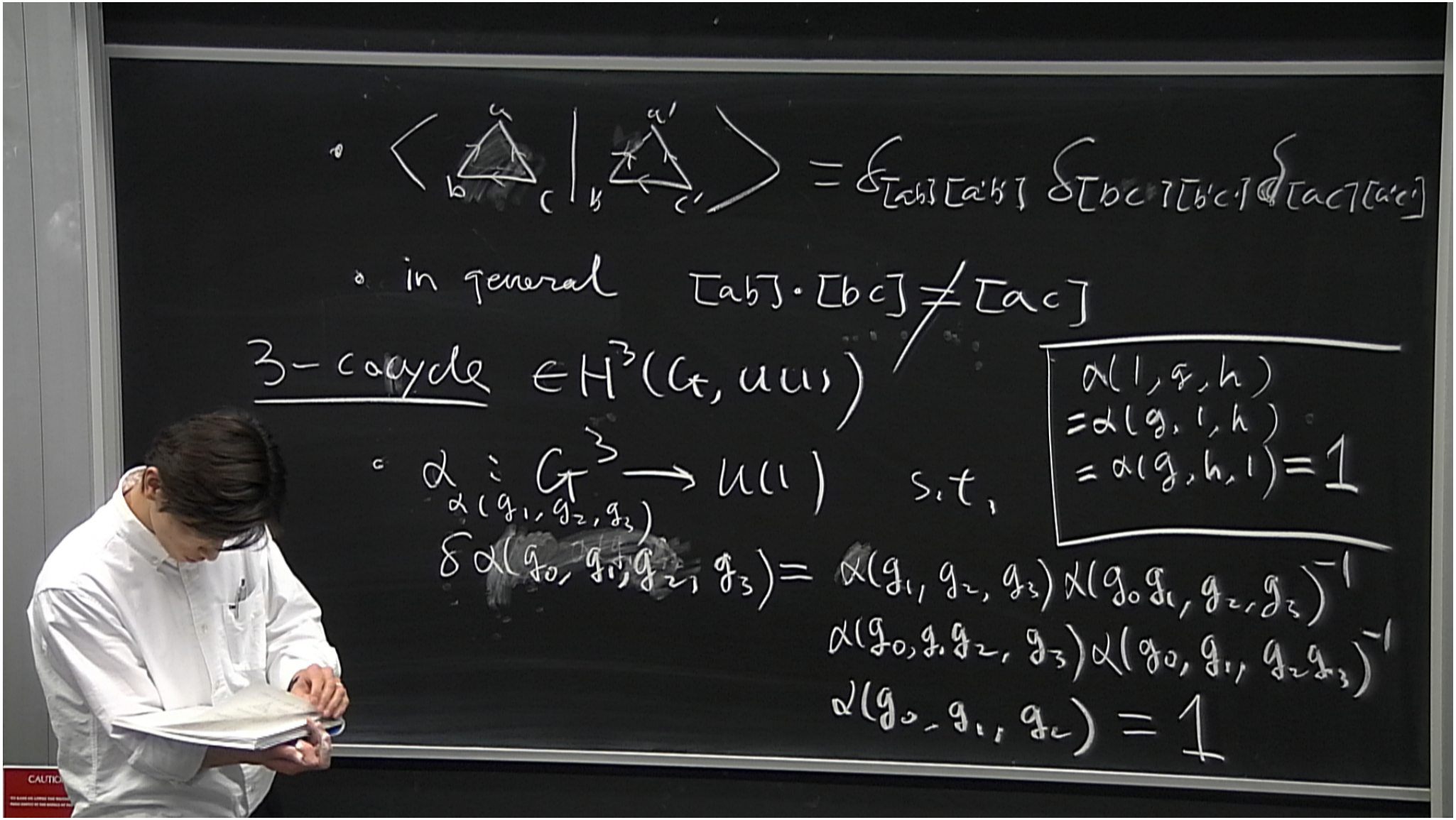

a
b
c
a'
b'
c'
$= \delta_{[abc][a'b'c']}\,\delta_{[bc][b'c']}\,\delta_{[ac][a'c']}$
• in general $[ab]\cdot[bc] \neq [ac]$
3-cocycle $\in H^3(G, U(1))$
$G^3 \to U(1)$ s.t.
CAUTION

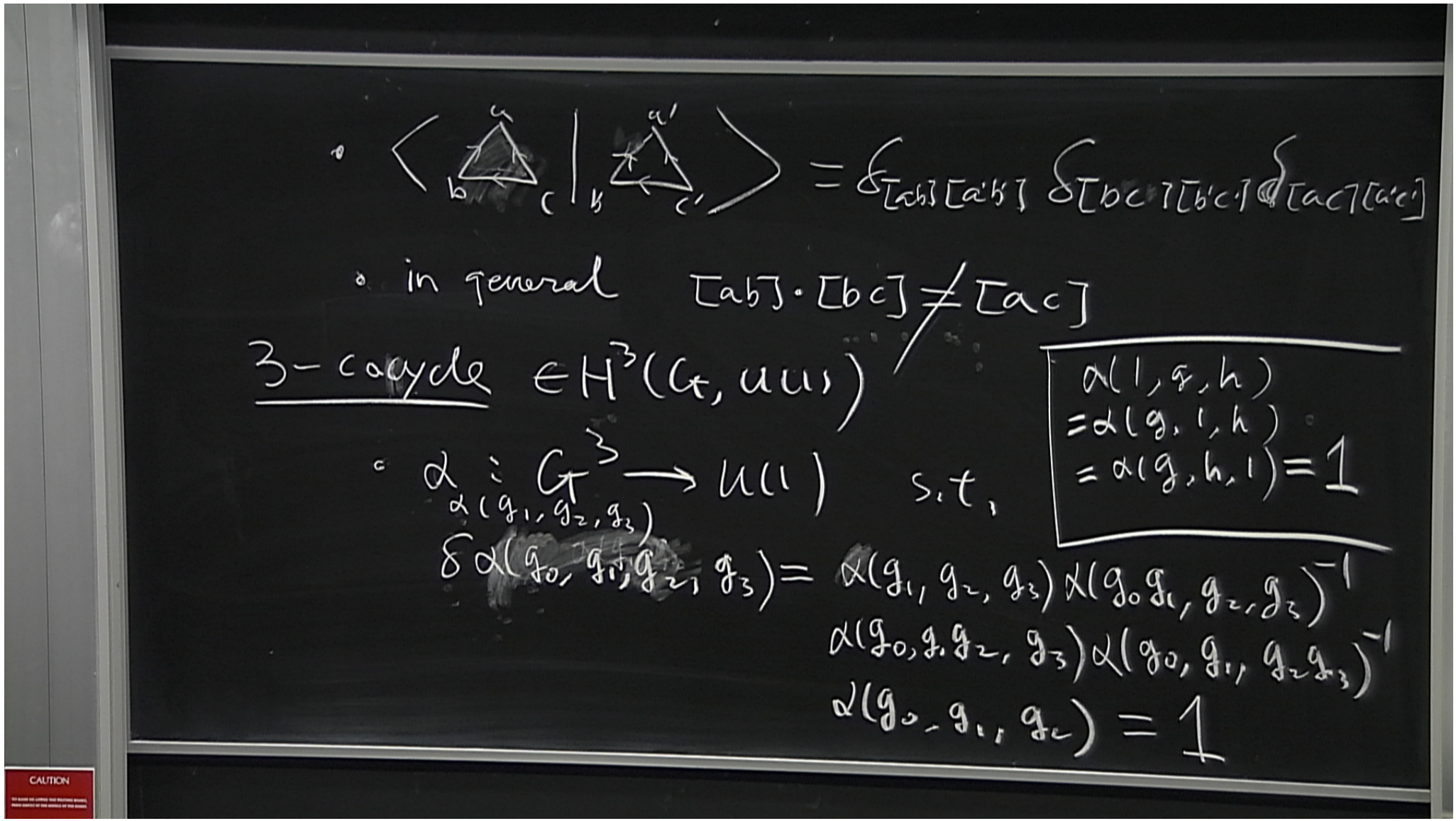

$\langle$ a b c | a' b' c' $\rangle$ $= \delta_{[abc][a'b'c']}\,\delta_{[bc][b'c']}\,\delta_{[ac][a'c']}$
in general $[ab]\cdot[bc] \neq [ac]$
3-cocycle $\in H^3(G, U(1))$
$\alpha : G^3 \to U(1)$ s.t.
$\delta\alpha(g_0, g_1, g_2, g_3) = \alpha(g_1, g_2, g_3)$
$\alpha(g_0, g_1 g_2, g$
$\alpha(g_0, g_1, g$
$g_3)^{-1}$
CAUTION

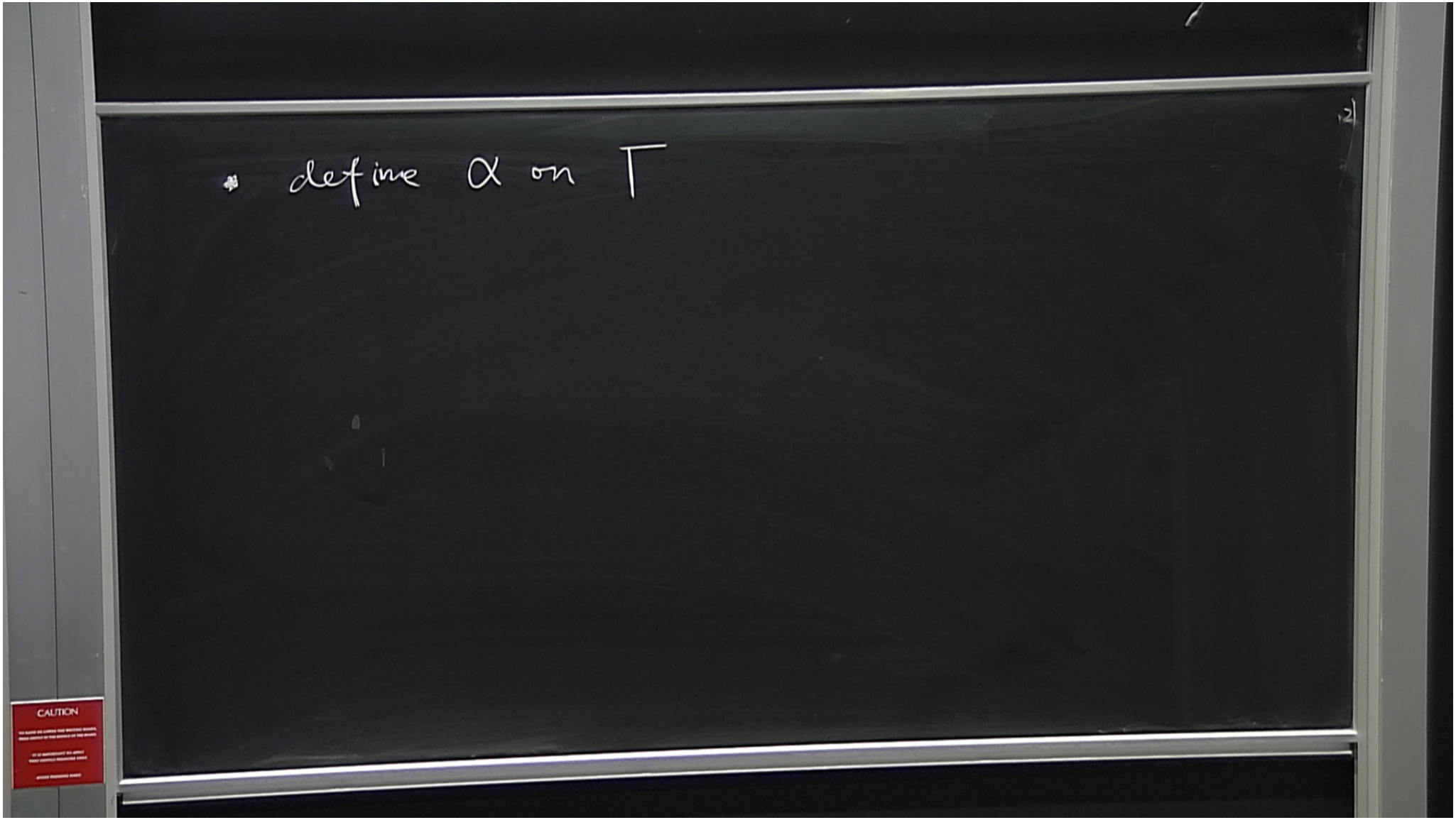

a
b
c
a'
b'
c'
$\langle \; | \; \rangle = \delta_{[abc][a'b'c']} \delta_{[bc][b'c']} \delta_{[ac][a'c']}$
• in general $[ab] \cdot [bc] \neq [ac]$
3-cocycle $\in H^3(G, U(1))$
• $\alpha : G^3 \to U(1)$ s.t.
$\delta\alpha(g_0, g_1, g_2, g_3) = \alpha(g_1, g_2, g_3)\,\alpha(g_0 g_1, g_2,$
$\alpha(g_0, g_1 g_2, g_3)\,\alpha(g_0, g_1,$
$\alpha(g_0, g_1, g_2)$
$\alpha(1, g, h)$
$= \alpha(g,$
$= \alpha(g,$
1
CAUTION

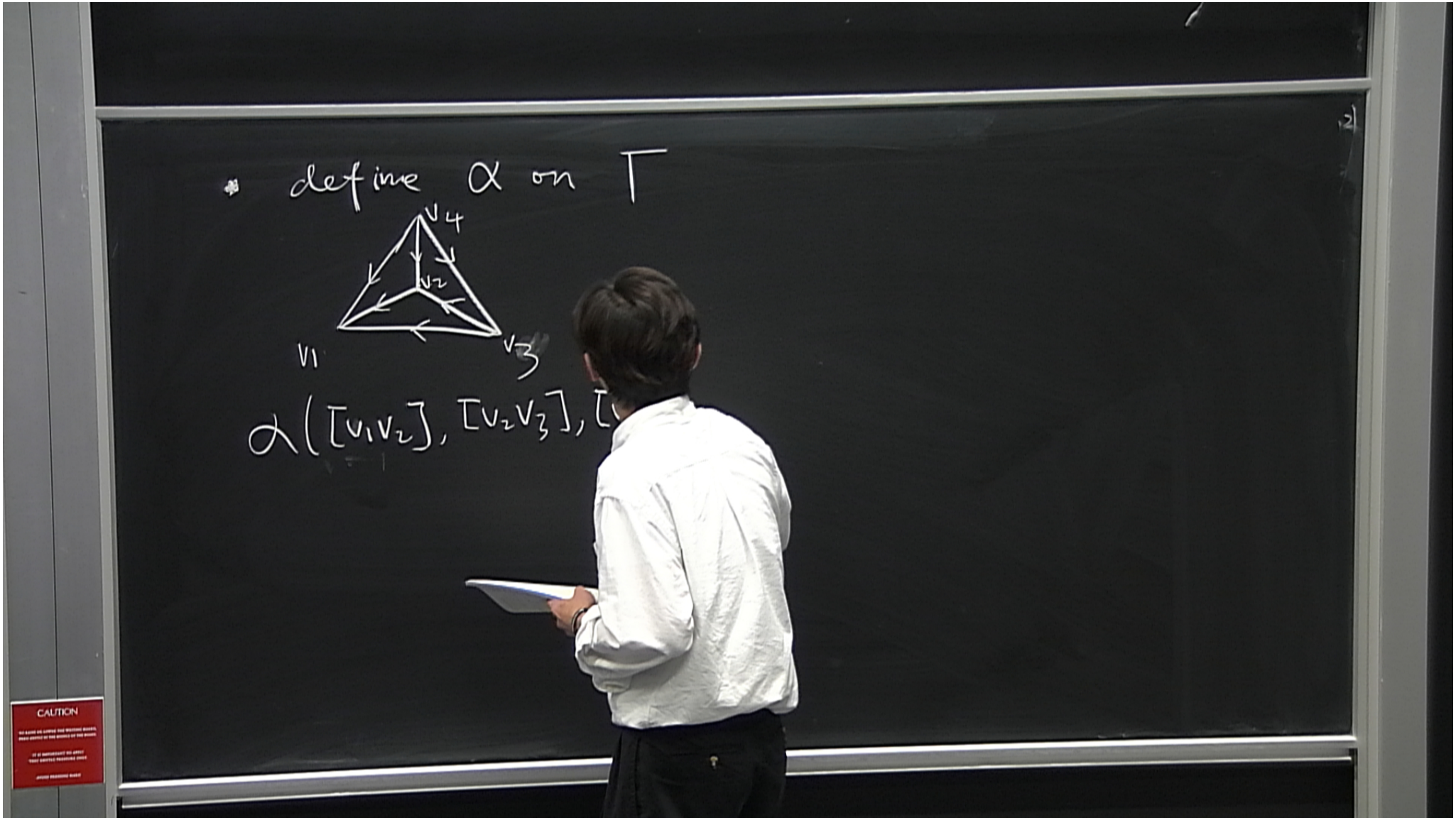

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3-cocycle $\in H^3(G, U(1))$
$\alpha : G^3 \to U(1)$ s.t.
$\alpha(1,g,h) = \alpha(g,1,h) = \alpha(g,h,1) = 1$
$\delta\alpha(g_0,g_1,g_2,g_3) = \alpha(g_1,g_2,g_3)\,\alpha(g_0g_1,g_2,g_3)^{-1}\,\alpha(g_0,g_1g_2,g_3)\,\alpha(g_0,g_1,g_2g_3)^{-1}\,\alpha(g_0,g_1,g_2)$
CAUTION

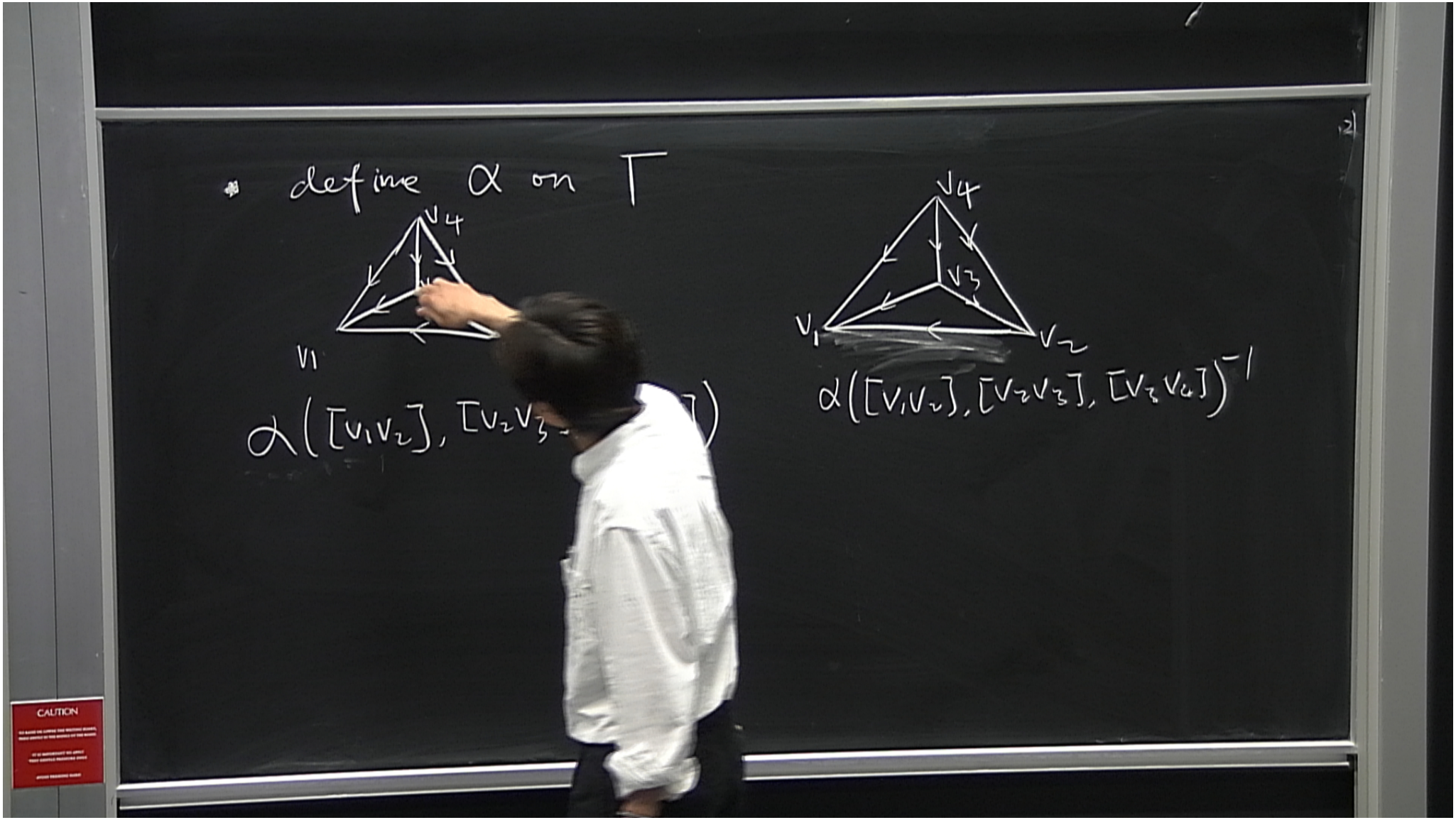

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3-cocycle $\in H^3(G, U(1))$
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$\alpha(1,g,h) = \alpha(g,1,h) = \alpha(g,h,1) = 1$
$\alpha(g_1,g_2,g_3)\,\alpha(g_0g_1,g_2,g_3)^{-1}$
$\alpha(g_0,g_1g_2,g_3)\,\alpha(g_0,g_1,g_2g_3)^{-1}$
$\alpha(g_0,g_1,g_2)$
CAUTION

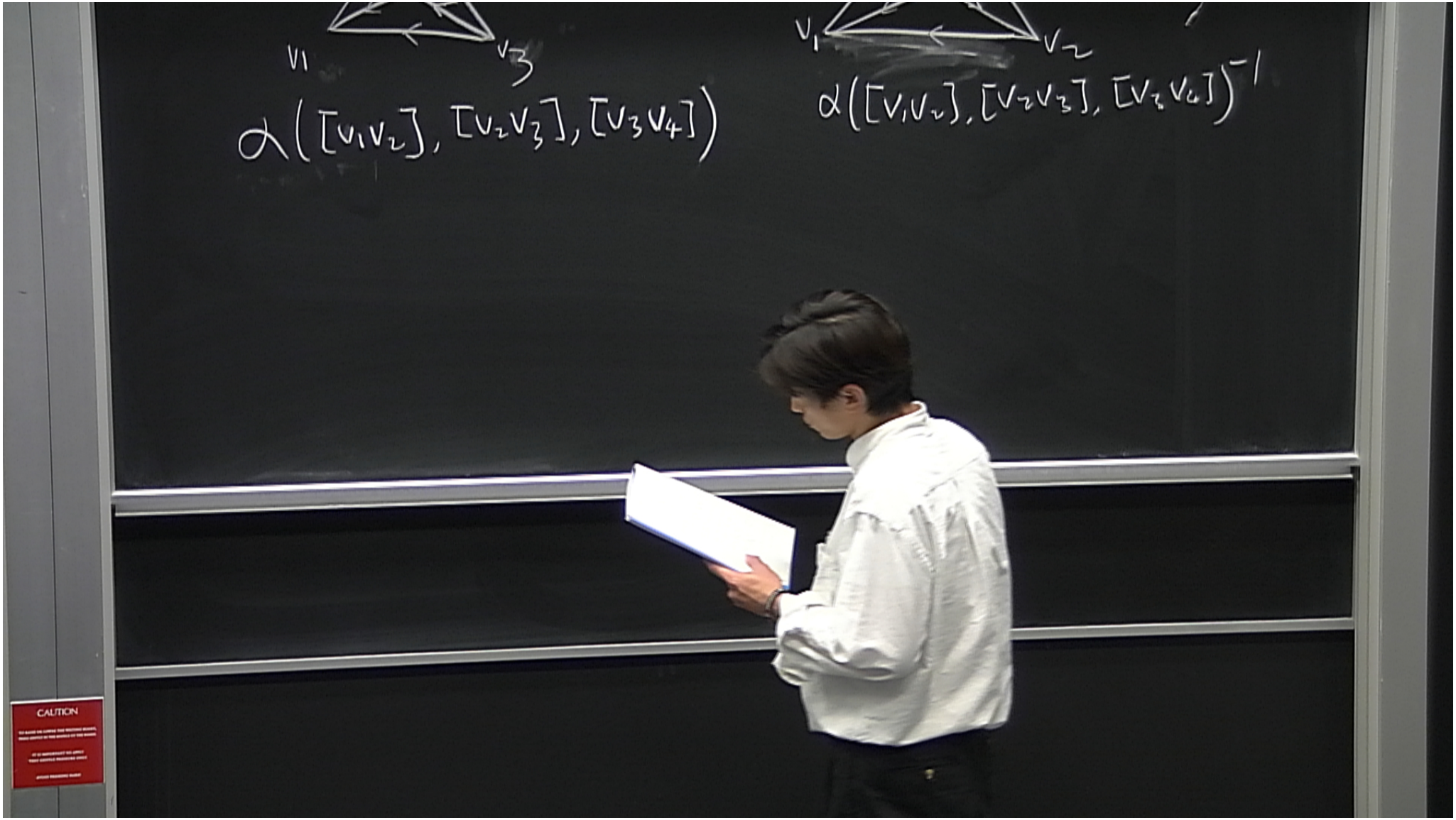

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3-cocycle $\in H^3(G, U(1))$
$\alpha : G^3 \longrightarrow U(1)$ s.t.
$\alpha(g_1, g_2, g_3)$
$\alpha(1, g, h) = \alpha(g, 1, h) = \alpha(g, h, 1) = 1$
$\delta\alpha(g_0, g_1, g_2, g_3) = \alpha(g_1, g_2, g_3)\,\alpha(g_0 g_1, g_2, g_3)^{-1}$
$\alpha(g_0, g_1 g_2, g_3)\,\alpha(g_0, g_1, g_2 g_3)^{-1}$
$\alpha(g_0, g_1, g_2) = 1$

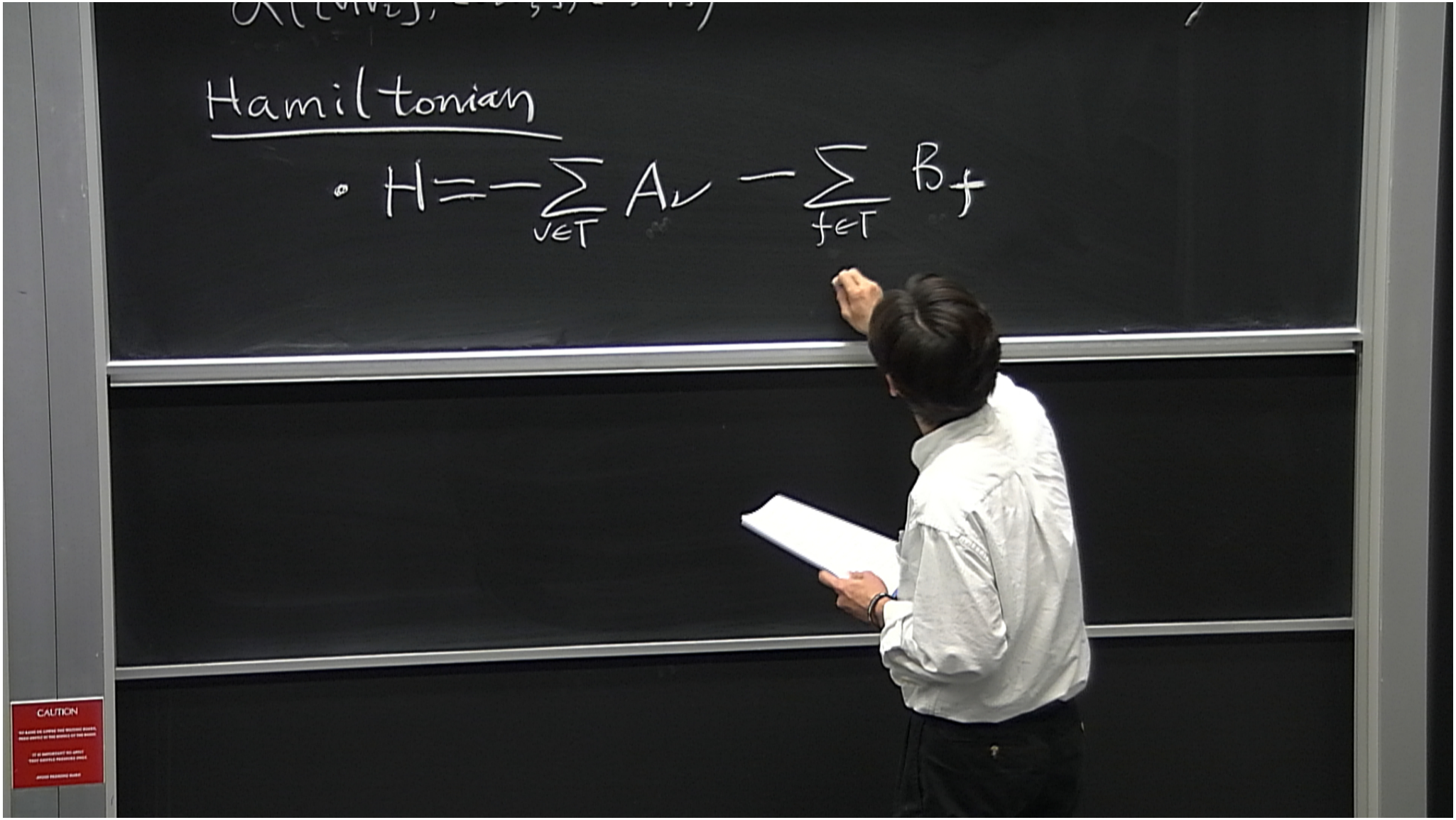

a
b
c
a'
b'
c'
$\langle \; | \; \rangle = \delta_{[abc][a'b'c']}\,\delta_{[bcd][b'c'd']}\,\delta_{[acd][a'c'd']}$
• in general $[ab]\cdot[bc] \neq [ac]$
3-cocycle $\in H^3(G, U(1))$
$\alpha : G^3 \to U(1)$ s.t.
$\alpha(g_1, g_2, g_3)$
$\alpha(1, g, h) = \alpha(g, 1, h) = \alpha(g, h, 1) = 1$
$\delta\alpha(g_0, g_1, g_2, g_3) = \alpha(g_1, g_2, g_3)\,\alpha(g_0 g_1, g_2, g_3)^{-1}$
$\alpha(g_0, g_1 g_2, g_3)\,\alpha(g_0, g_1, g_2 g_3)^{-1}$
$\alpha(g_0, g_1, g_2) = 1$

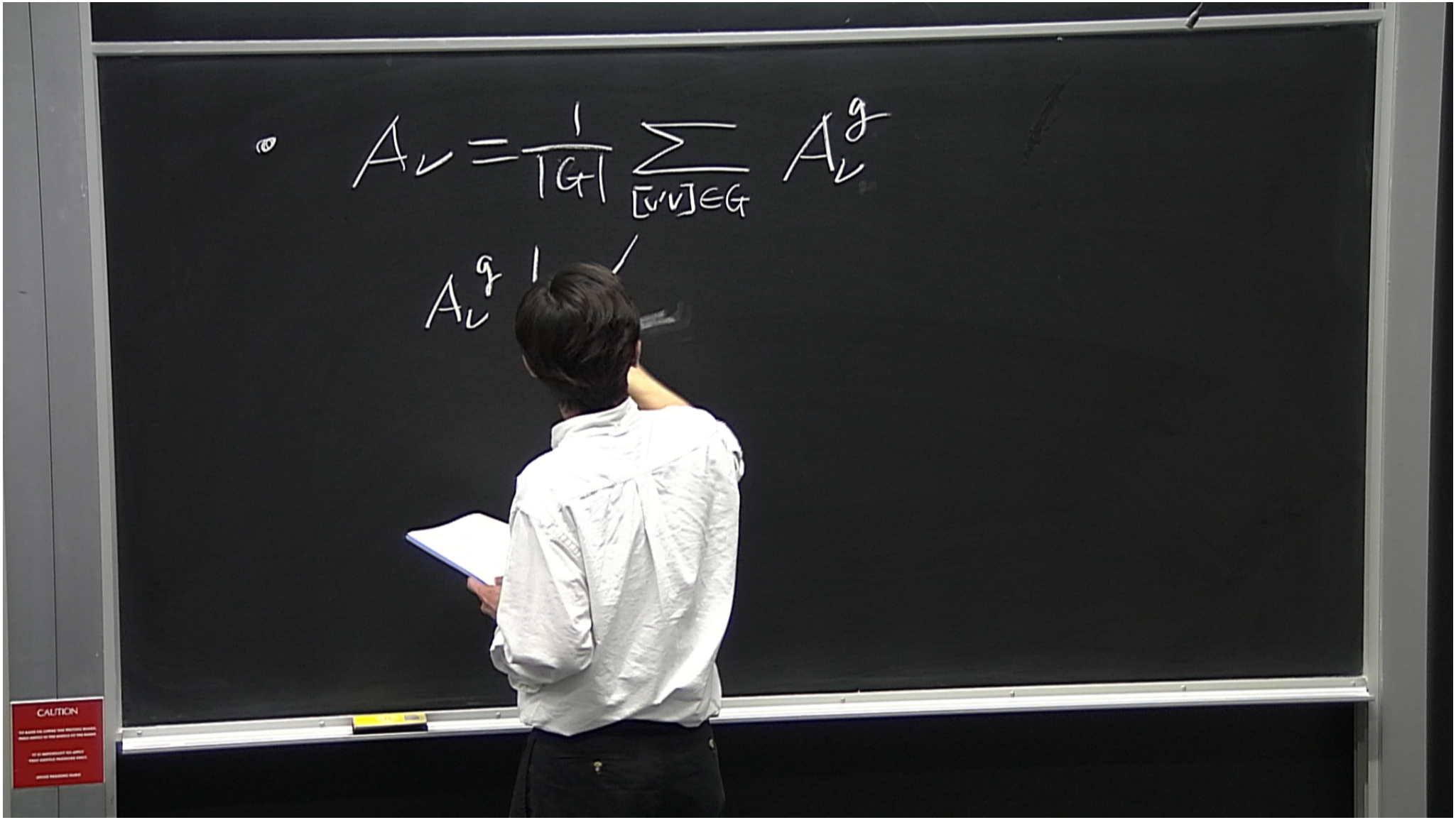

define α on Γ
CAUTION


define α on Γ
v4
v2
v1
v3
α([v1v2], [v2v3], [


define α on Γ
v4
v1
v4
v3
v1
v2
α([v1v2], [v2v3
α([v1v2], [v2v3], [v3v4])^-1
CAUTION

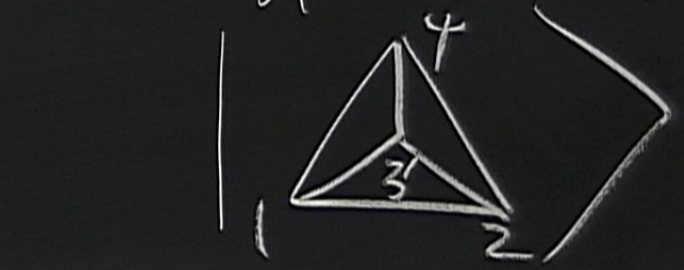

$\alpha([v_1v_2],[v_2v_3],[v_3v_4])$
$\alpha([v_1v_2],[v_2v_3],[v_3v_4])^{-1}$
CAUTION

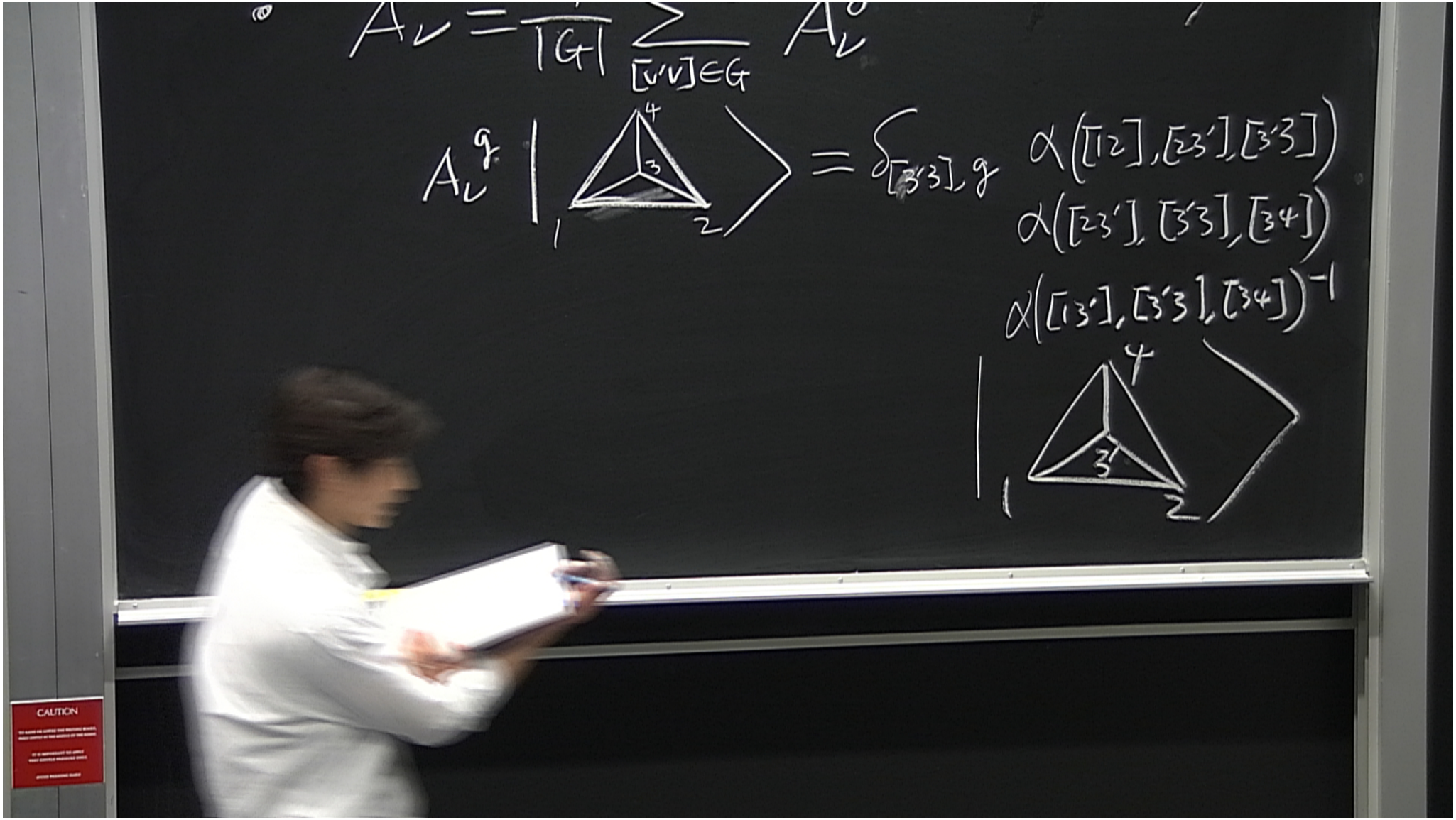

Hamiltonian
$H = -\sum_{v \in T} A_v - \sum_{f \in T} B_f$
CAUTION


$A_\nu = \frac{1}{|G|} \sum_{[v,v] \in G} A_\nu^g$
A_ν^g
CAUTION

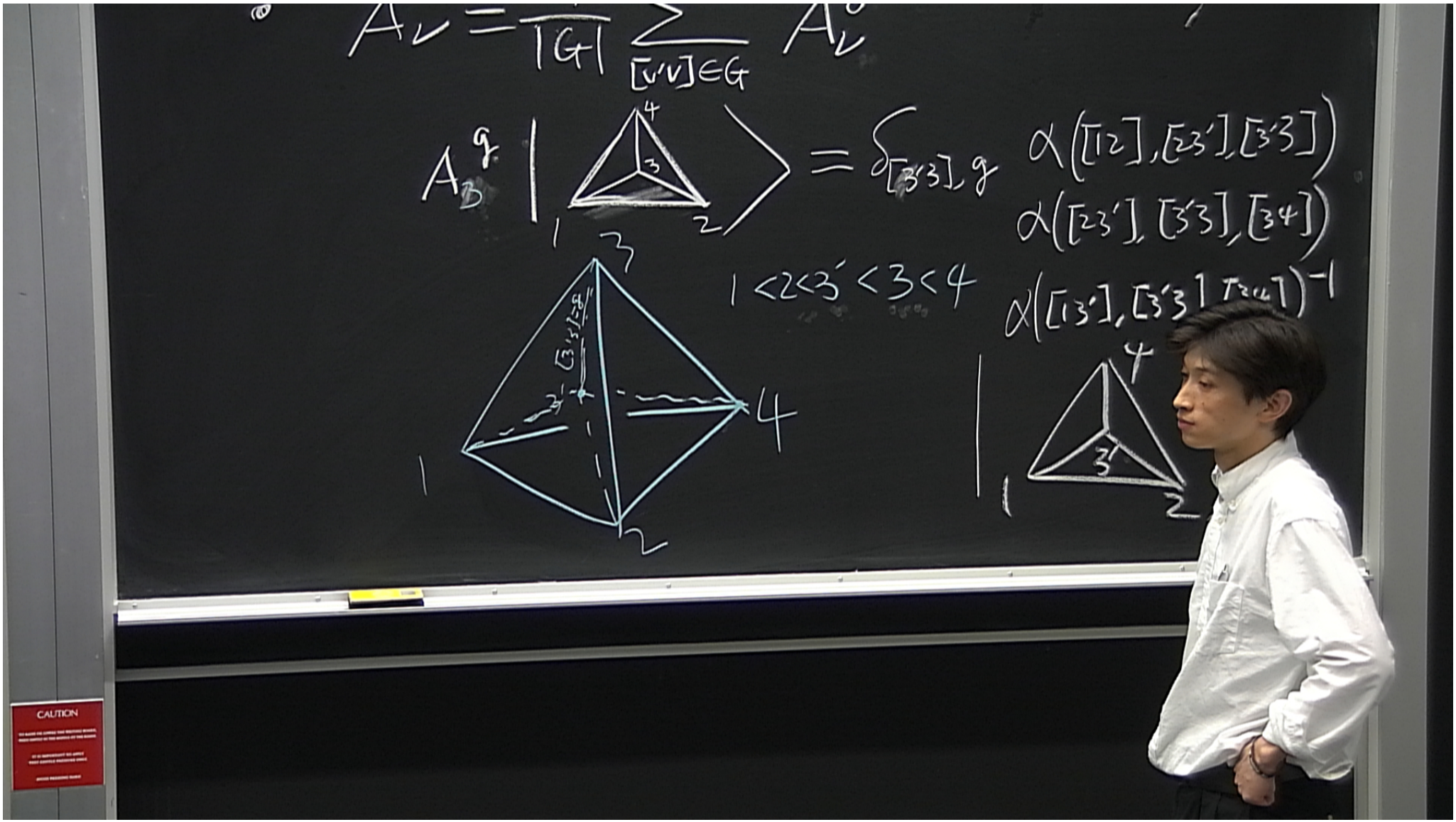
$$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^g$$

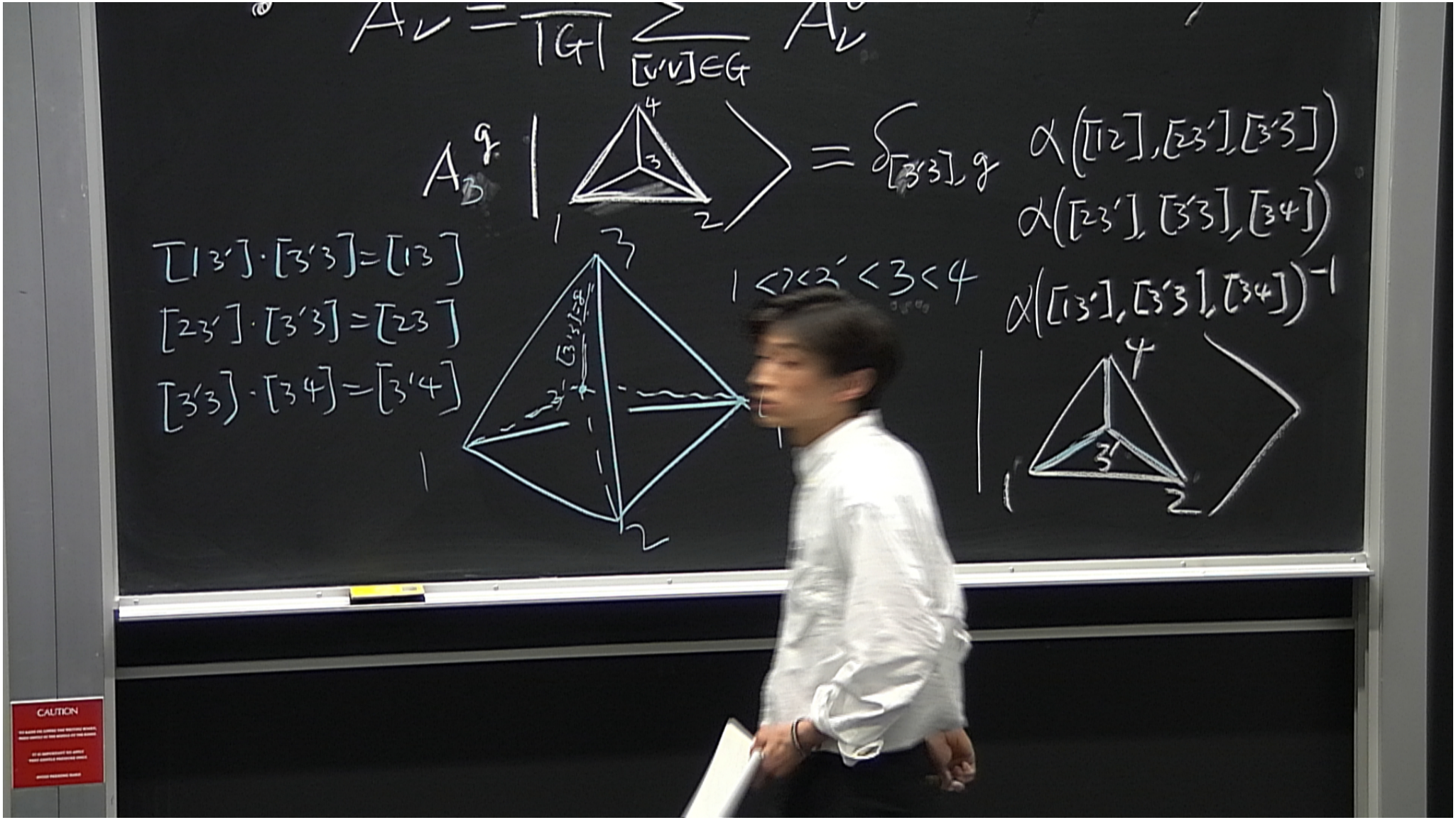
$$A_L^g \left| \right. = \delta_{[3,3],g} \; \alpha([12],[23'],[3'3]) \; \alpha([23'],[3'3],[34]) \; \alpha([13'],[3'3],[34])^{-1}$$

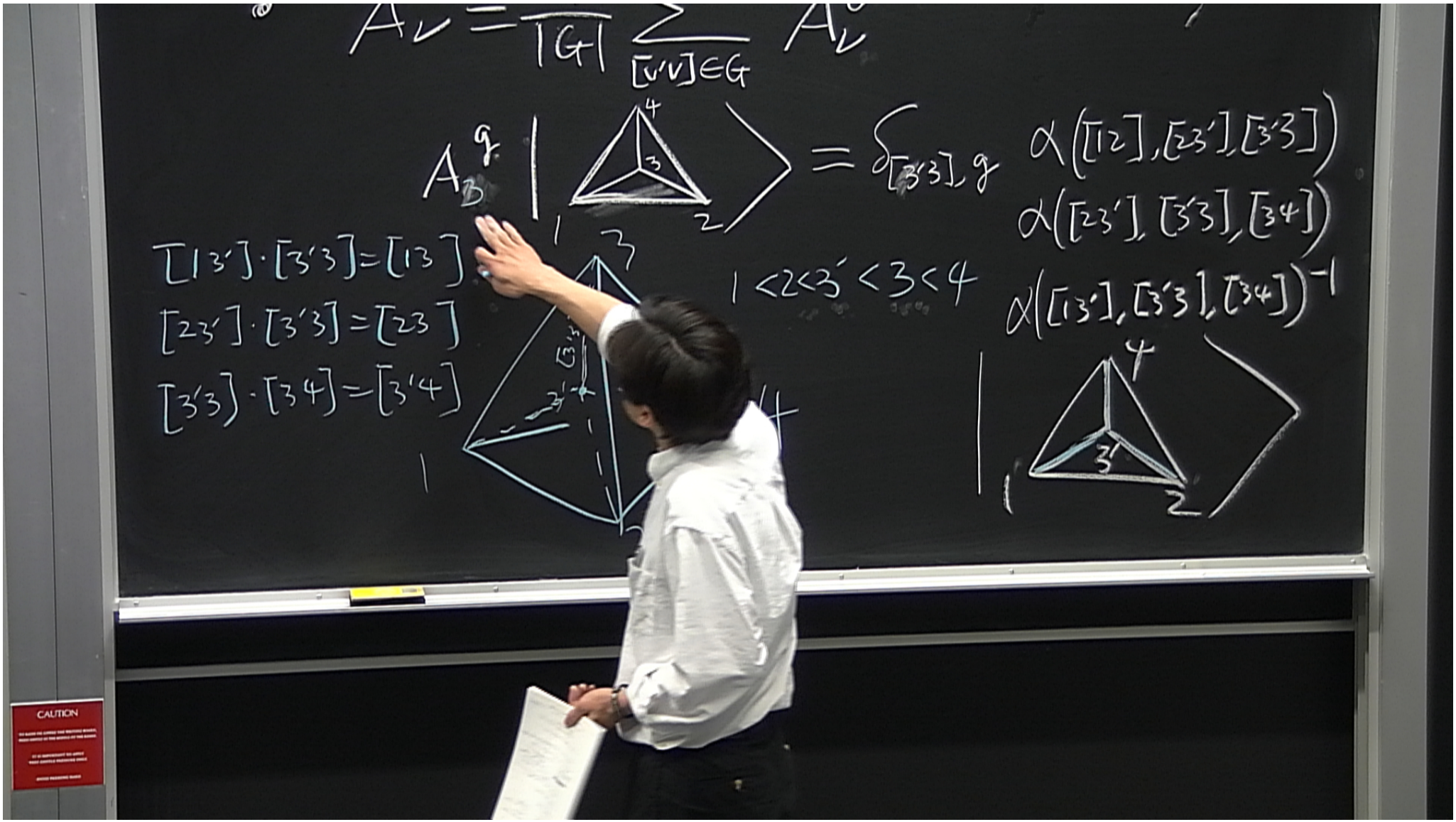


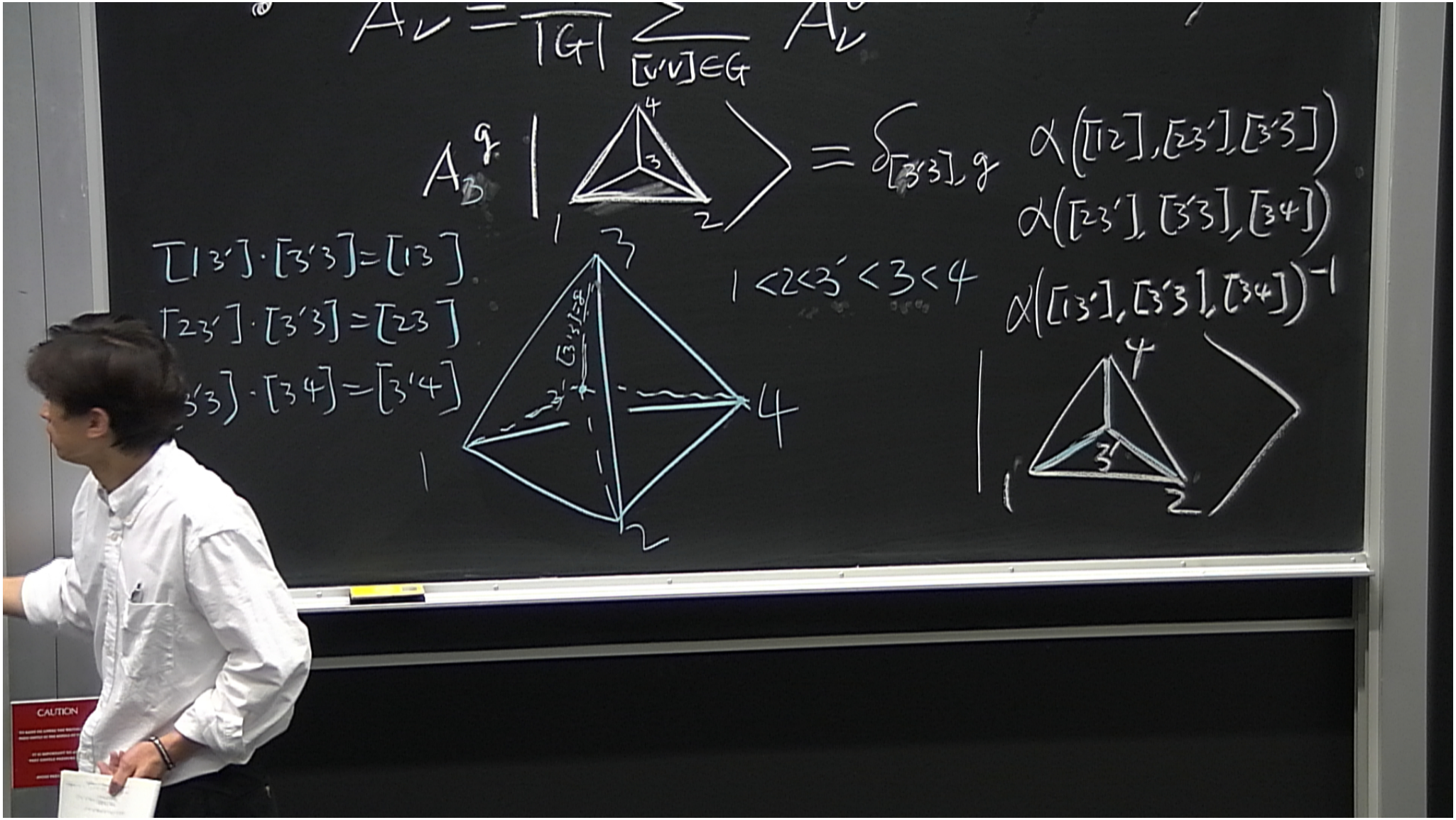

$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^g$
$A_L^g \left| \right\rangle = \delta_{[3,3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$\left| \right\rangle$
CAUTION

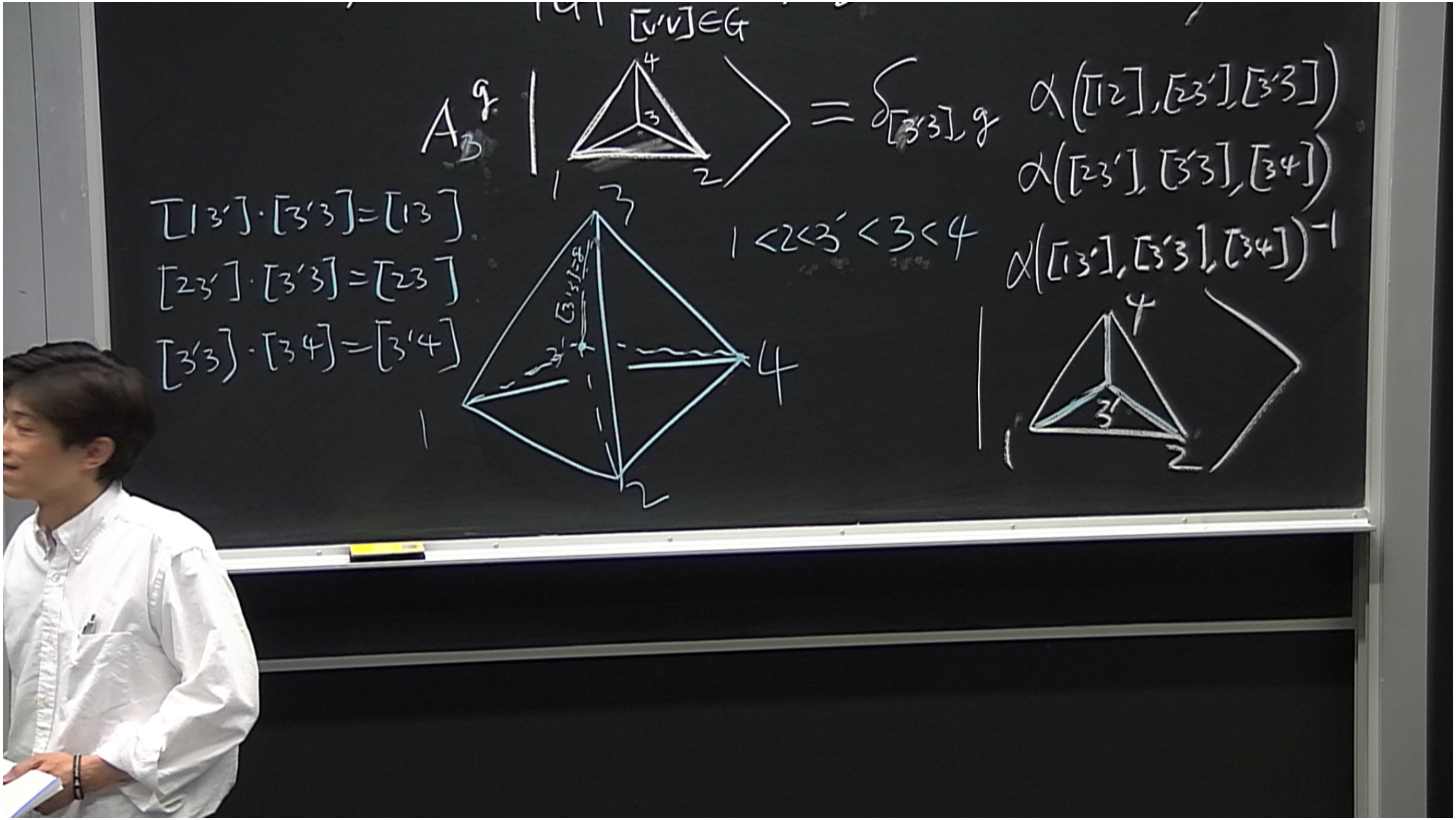

5 min


$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^0$
$A_B^g \left| \right\rangle = \delta_{[3,3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$1<2<3'<3<4$
CAUTION


$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^v$
$A_B^g \left| \right\rangle = \delta_{[3'3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$[13'] \cdot [3'3] = [13]$
$[23'] \cdot [3'3] = [23]$
$[3'3] \cdot [34] = [3'4]$
$1 < 2 < 2' < 3 < 4$
CAUTION


$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^v$
$A_B^g \left| \right\rangle = \delta_{[3'3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$[13'] \cdot [3'3] = [13]$
$[23'] \cdot [3'3] = [23]$
$[3'3] \cdot [34] = [3'4]$
$1 < 2 < 3' < 3 < 4$
CAUTION


$A_L = \frac{1}{|G|} \sum_{[v,v] \in G} A_L^{[v,v]}$
$A_B^g \left| \right\rangle = \delta_{[3,3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$[13'] \cdot [3'3] = [13]$
$[23'] \cdot [3'3] = [23]$
$[3'3] \cdot [34] = [3'4]$
$1 < 2 < 3' < 3 < 4$
CAUTION


[u,v]∈G
A_B^g
$= \delta_{[3'3],g}$
$\alpha([12],[23'],[3'3])$
$\alpha([23'],[3'3],[34])$
$\alpha([13'],[3'3],[34])^{-1}$
$[13'] \cdot [3'3] = [13]$
$[23'] \cdot [3'3] = [23]$
$[3'3] \cdot [34] = [3'4]$
$1 < 2 < 3' < 3 < 4$