Title: An exact solution of the Dirac equation with CP violation.

Date: Nov 20, 2012 11:00 AM

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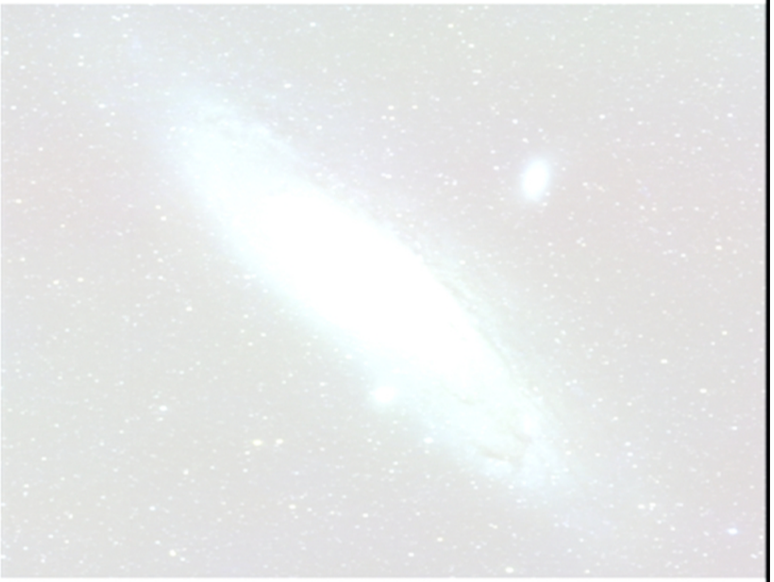
Abstract: After a brief overview of electroweak baryogenesis, I will show how to construct a solution of the Dirac equation for a CP violating kink wall. This solution nicely reduces to the known solution for a CP violating thin (step) wall. The novel solution can be helpful for studies of baryogenesis sources at strong first order phase transitions, which is relevant for electroweak scale baryogenesis studies.

AN EXACT SOLUTION OF THE DIRAC EQUATION WITH CP VIOLATION

Tomislav Prokopec

(ITP & Spinoza Institute, Utrecht U.)

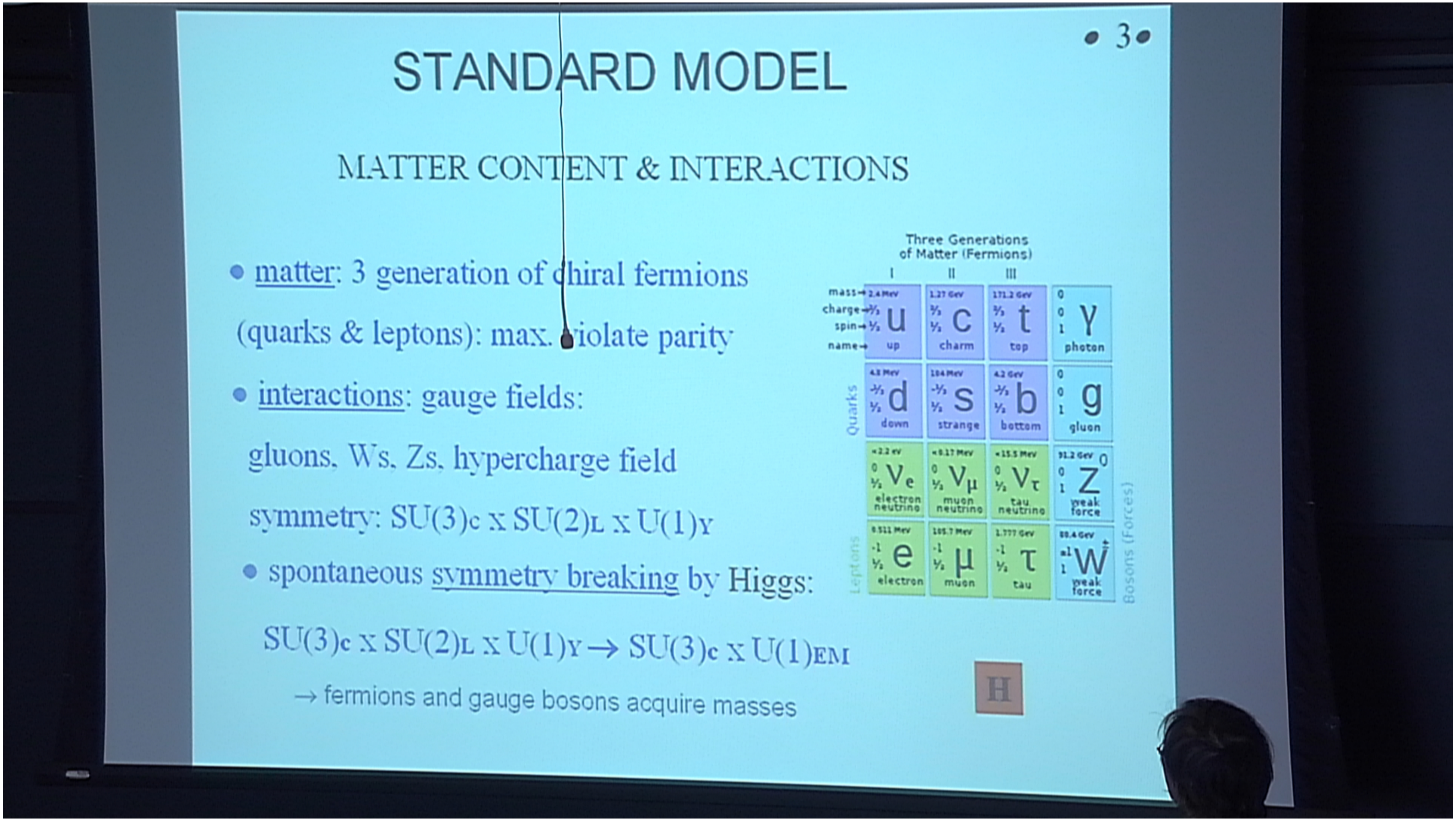
T. Prokopec, Jan Weenink, M.G. Schmidt 2012, in preparation



PI, 20 Nov 2012

OUTLINE

- STANDARD MODEL
  - particle content
- ELECTROWEAK TRANSITION
  - equilibrium considerations
  - dynamics of 1st order transition
- ELECTROWEAK BARYOGENESIS
  - measurements
  - tests
  - particle models
  - calculational techniques



STANDARD MODEL

MATTER CONTENT & INTERACTIONS

- matter: 3 generation of chiral fermions

(quarks & leptons): max. violate parity

- interactions: gauge fields:

gluons, Ws, Zs, hypercharge field

symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y$

- spontaneous symmetry breaking by Higgs:

$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{EM}$

→ fermions and gauge bosons acquire masses

Three Generations of Matter (Fermions)

	I	II	III	
mass→ charge→ spin→ name→	2.4 MeV 2/3 1/2 u up	1.27 GeV 2/3 1/2 c charm	171.2 GeV 2/3 1/2 t top	0 0 1 γ photon
Quarks	4.8 MeV -1/3 1/2 d down	104 MeV -1/3 1/2 s strange	4.2 GeV -1/3 1/2 b bottom	0 0 1 g gluon
	<2.2 eV 0 1/2 ν_e electron neutrino	<0.17 MeV 0 1/2 ν_μ muon neutrino	<15.5 MeV 0 1/2 ν_τ tau neutrino	91.2 GeV 0 1 Z^0 weak force
Leptons	0.511 MeV -1 1/2 e electron	105.7 MeV -1 1/2 μ muon	1.777 GeV -1 1/2 τ tau	80.4 GeV ±1 1 $W^\pm$ weak force

Bosons (Forces)

H

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  (quarks & leptons): max. violate parity
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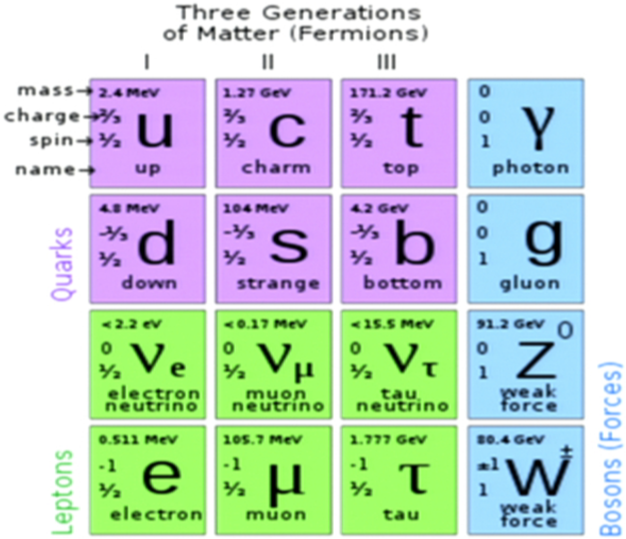

Three Generations of Matter (Fermions)

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Bosons (Forces)

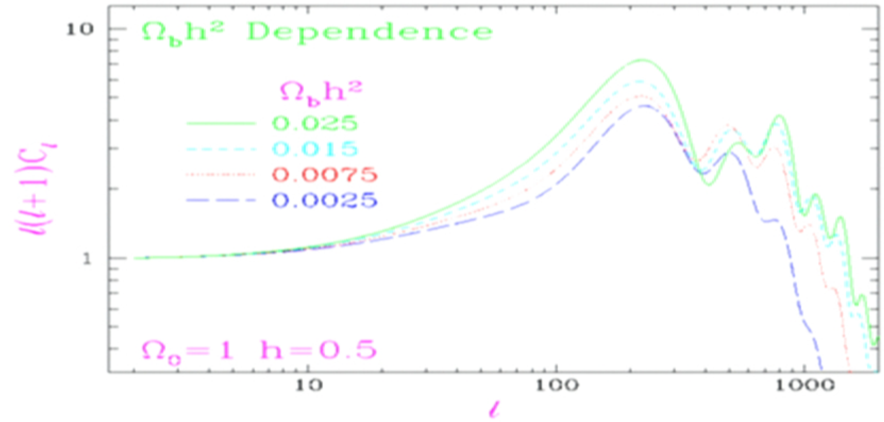
H

OBSERVED MATTER-ANTIMATTER ASYMMETRY

- ***The ratio of the baryon and photon number densities:***

$$\eta_B = \frac{n_B}{n_\gamma} = (6.2 \pm 0.2) \times 10^{-10}$$

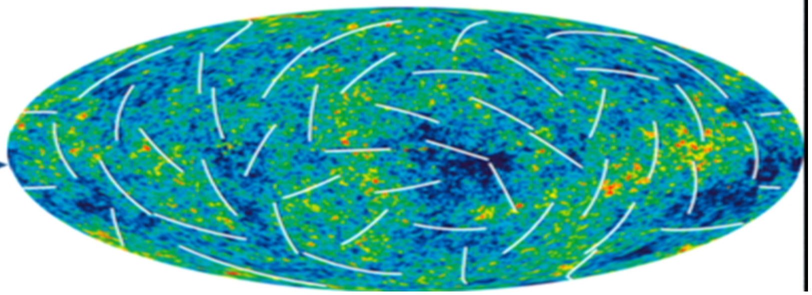
- nucleosynthesis constraint, cmbr+LSS measurements



baryons: increase compression (odd) peaks, decrease rarefaction peaks

Based on temperature anisotropies in CMB

(WMAP 2008)

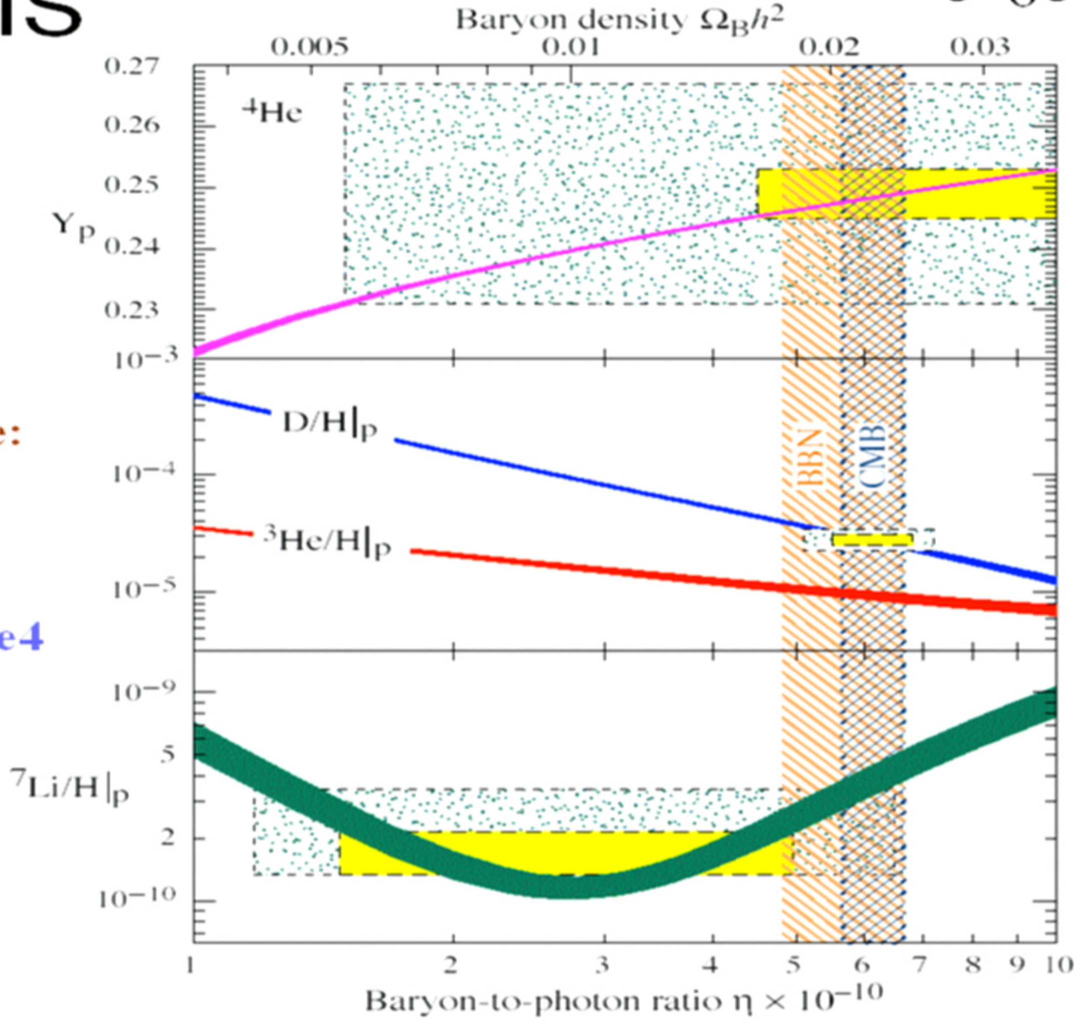


NUCLEOSYNTESIS (BBN)

- synthesis of nucleons in the Universe:

yields a constraint on η_B in a reasonable agreement with CMB

- some tension with: Li6, Li7 and He4



Olive & Sarkar, PDG

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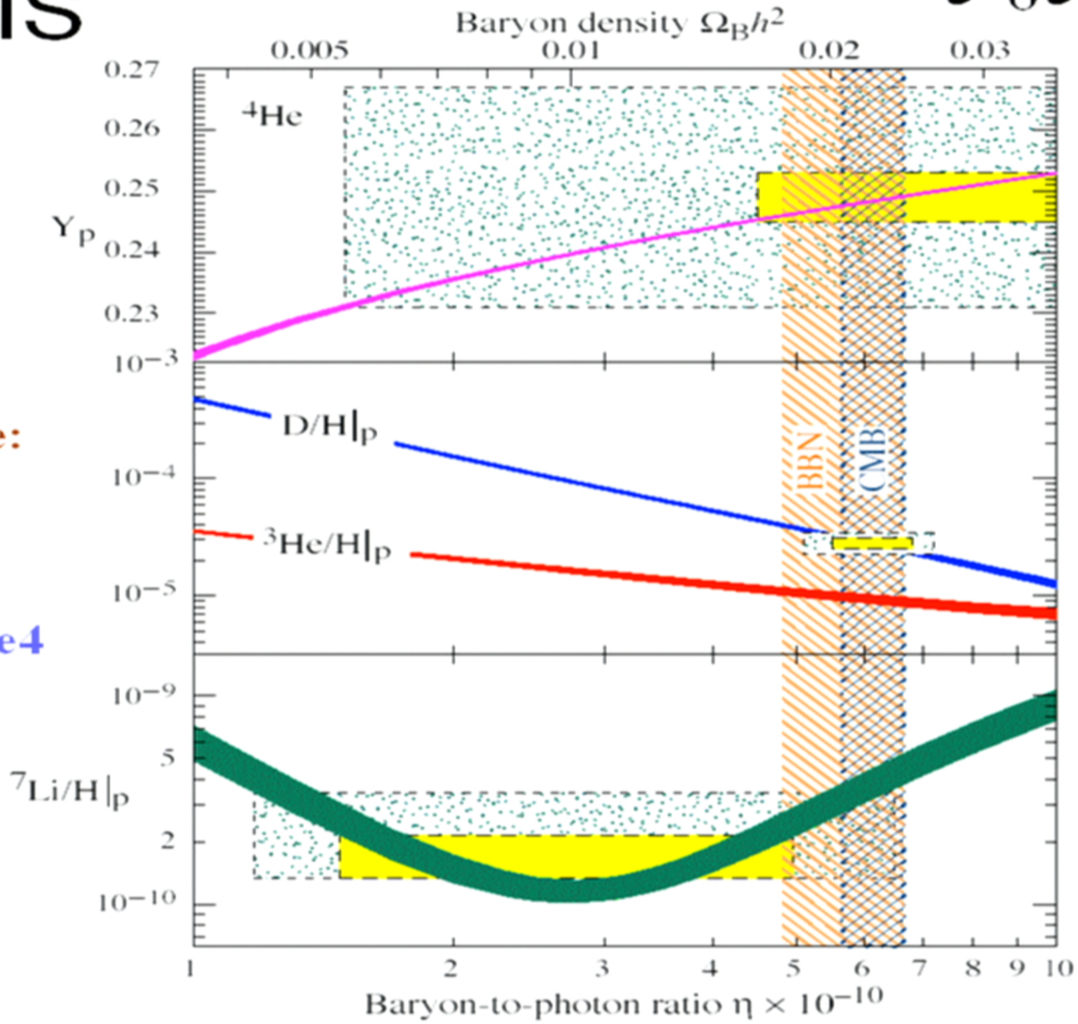

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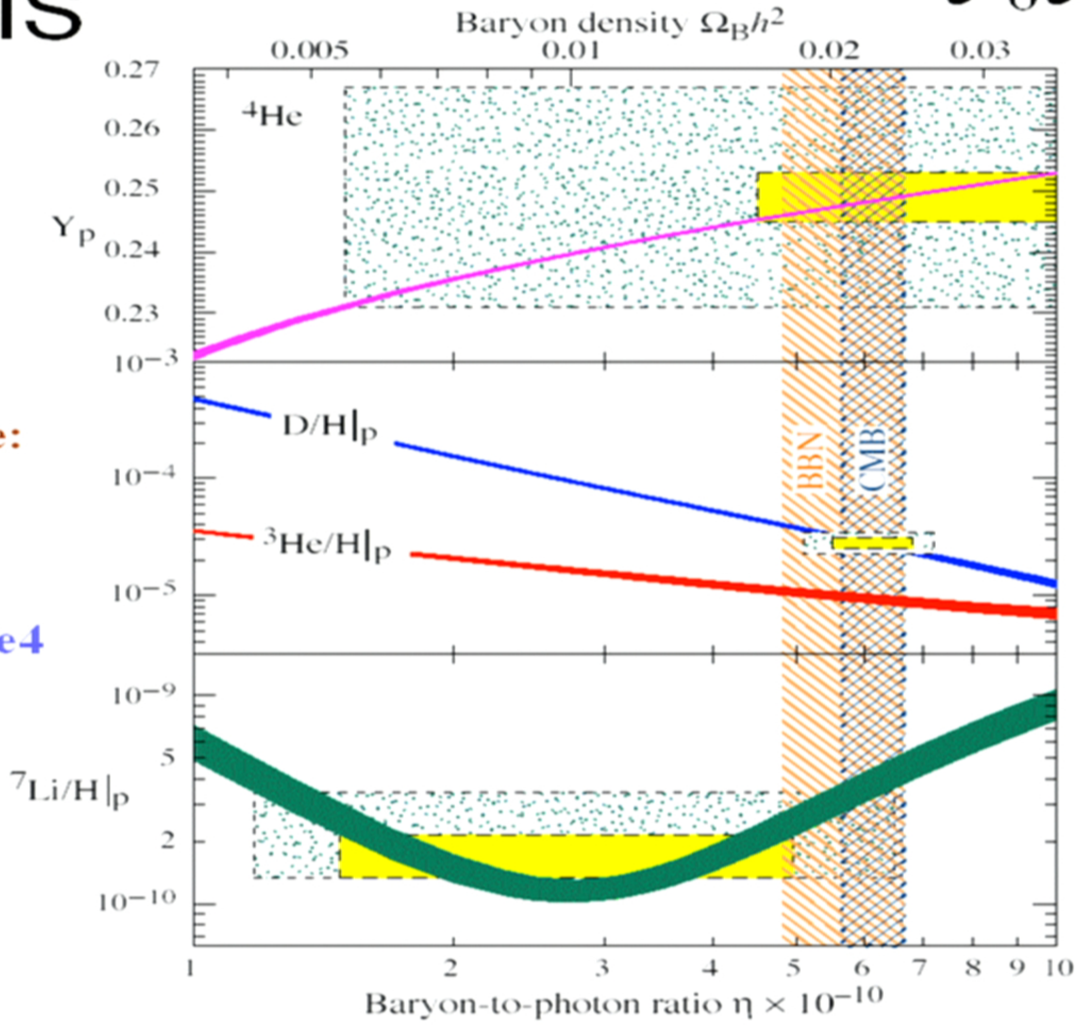


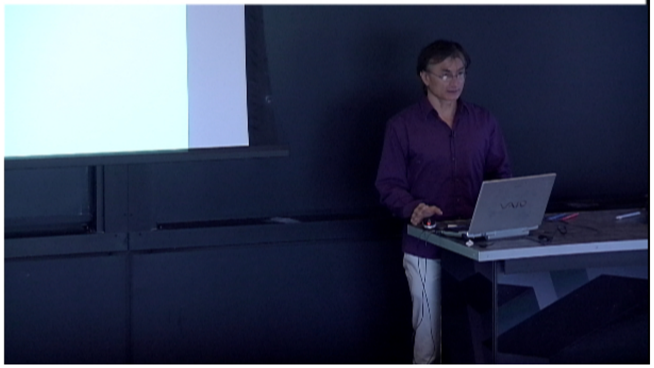
Olive & Sarkar, PDG

SAKHAROVS CONDITIONS

Sakharov (1967) establisheed the criteria for dynamical baryogenesis:

1. B-violation
2. C & CP violation
   - processes including baryons and antibaryons are not equally fast
3. Departure from equilibrium
   - CPT symmetry implies equal number of particles and antiparticles.

ELECTROWEAK TRANSITION AND BARYOGENESIS



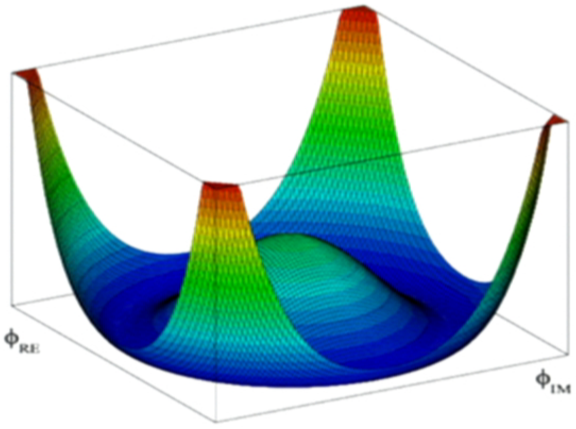
EQUILIBRIUM ASPECTS OF THE TRANSITION

- Higgs potential at T=0:

$$V(H) = -m_H^2 H^+ H + \lambda (H^+ H)^2$$

$$H \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = \frac{m_H}{\sqrt{\lambda}} = 246\,\mathrm{GeV}$$

→ LHC: **m_H** = 126 GeV



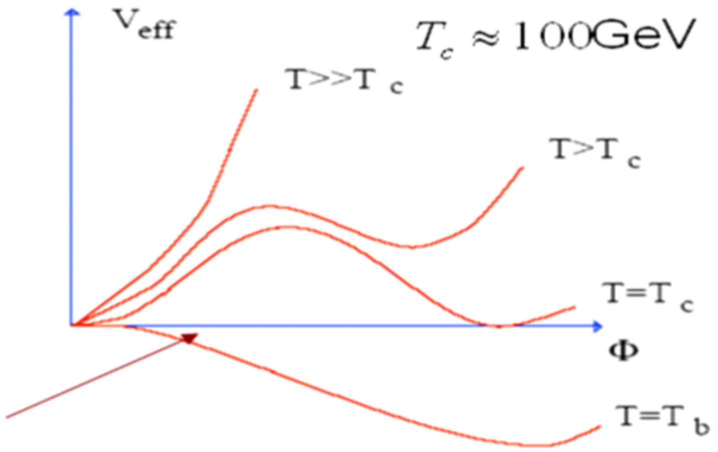
- Higgs potential at T≠0:

$$V_T(H) = m_0^2 \left(1 - \frac{T^2}{T_c^2} \right) H^2 - E(T) H^3 + \lambda_T H^4$$

→ E: regulates the strength of the transition: <u>bosons</u>

→ Shaposhnikov's condition: $\Delta\phi/T_c > 1$

→ bubbles of 1st order transition form @ $T_c > T_n > T_b$



EQUILIBRIUM CONSIDERATIONS

EW TRANSITION in MSM & MSSM

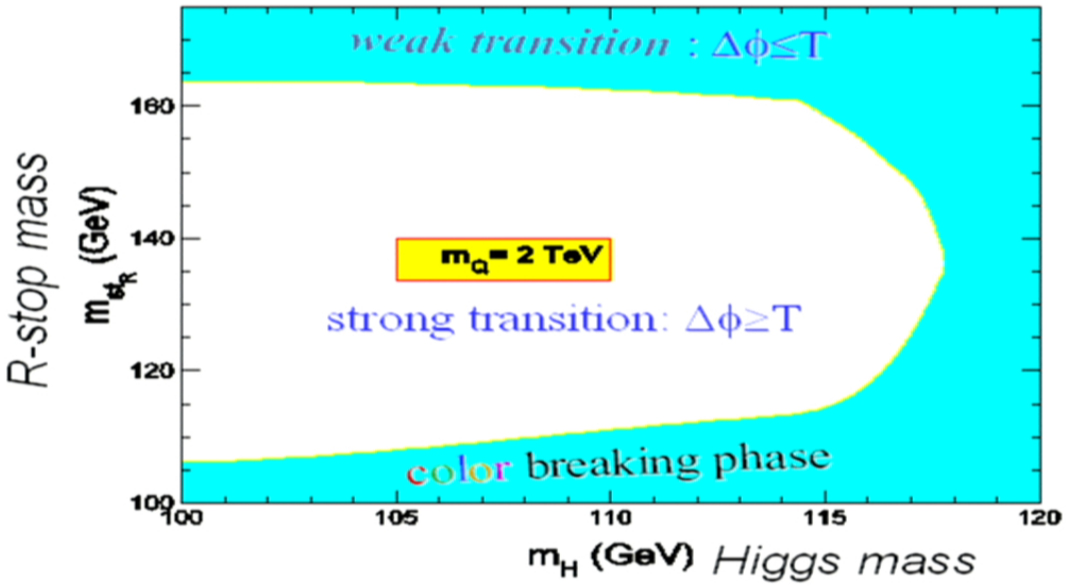
Kajantie, Laine, Rummukainen, Shaposhnikov 1996

- for Higgs mass >72GeV transition is a **crossover**
- LHC result for the Higgs mass ~126GeV: rules out BAU in MSM and MSSM

Strong first order phase transition in MSSM

- allowed "triangle" for MSSM (white):

Carena, Quiros, Seco, Wagner, 2000; Quiros 2001



- the "triangle" extends by ~5GeV for large m_Q>>TeV

Carena, Nardini, Quiros, Wagner, 2008

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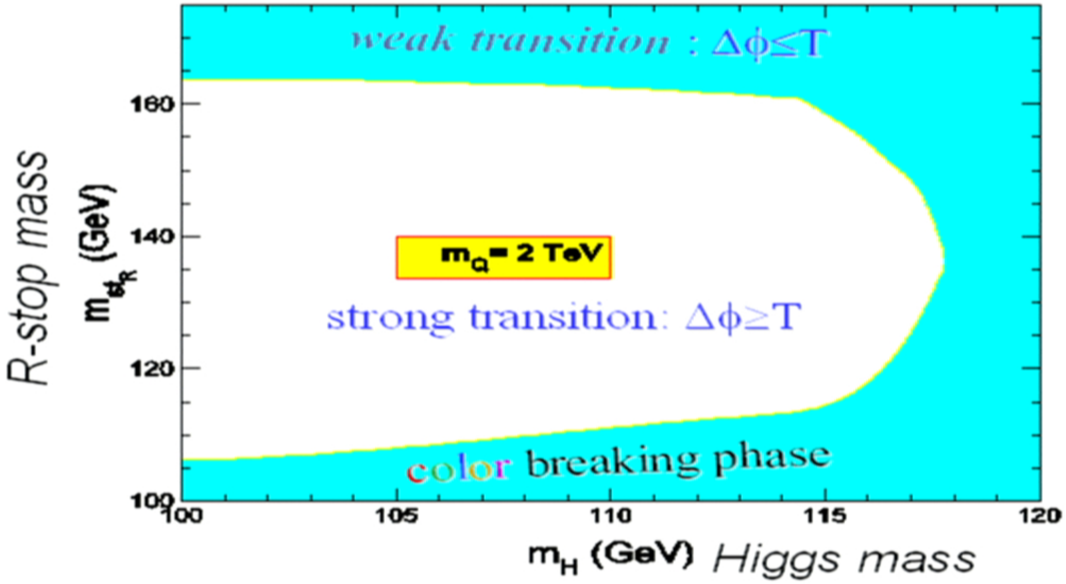


- the "triangle" extends by ~5GeV for large m_Q>>TeV

Carena, Nardini, Quiros, Wagner, 2008

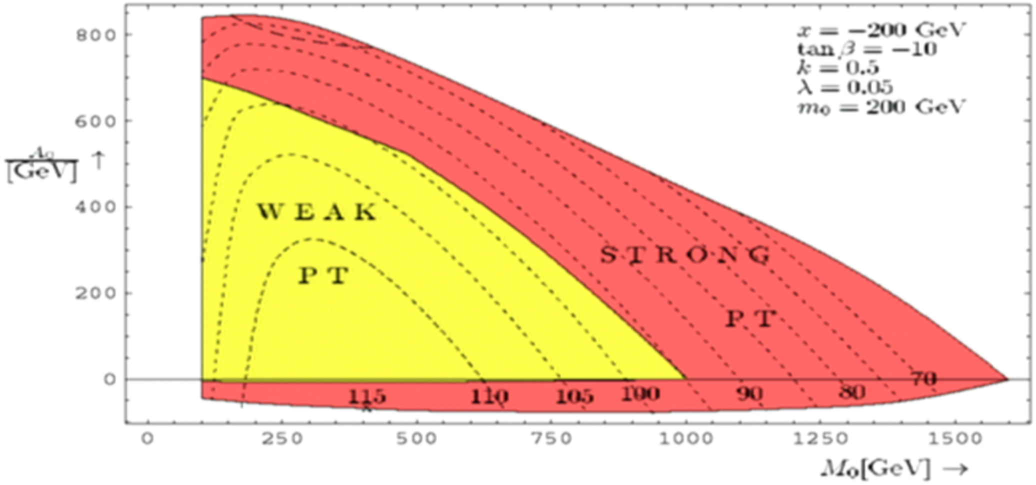
EQUILIBRIUM IN MODELS WITH SINGLETS

- singlet model without discrete symmetries

nMSSM

$$W = \lambda S H_1 H_2 + \frac{k}{3} S^3 + \mu H_1 H_2 + r S$$

$$W_{nMSSM} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + \frac{m_{12}^2}{\lambda} \hat{S}$$



Huber,Schmidt 2000

Menon, Morrissey, Wagner 2004

Huber, Konstandin, Prokopec, Schmidt 2006

WHY IS ELECTROWEAK BARYOGENESIS INTERESTING?

MODELS ARE TESTABLE AT ACCELERATORS (LHC, ILC)

- New (scalar) particles at LHC (Higgs, ..)
- New sources of CP violation (EDM experiments)
- Gravitational waves produced at the EW phase transition
- Magnetic fields produced at the EW phase transition (indirect)

CENTRAL QUESTION: **Does the explanation for the origin of matter-antimatter asymmetry lies at energies ~1TeV accessible by LHC, ILC?**

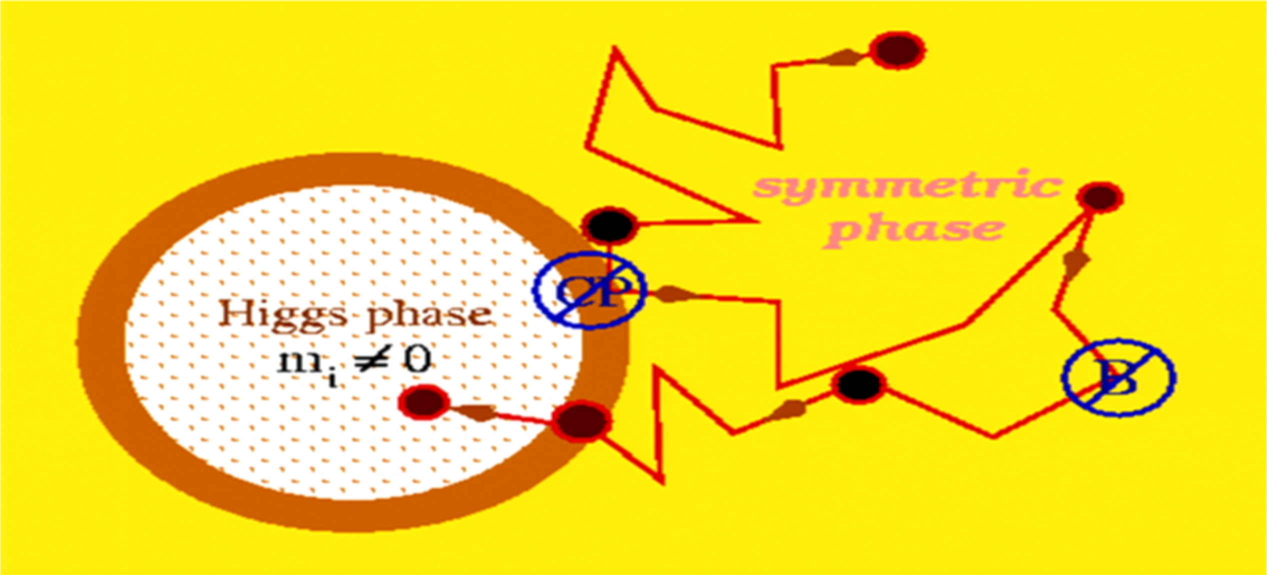
ALTERNATIVE: **LEPTOGENESIS**

→ problem: GUT scale physics, not directly accessible to experiments

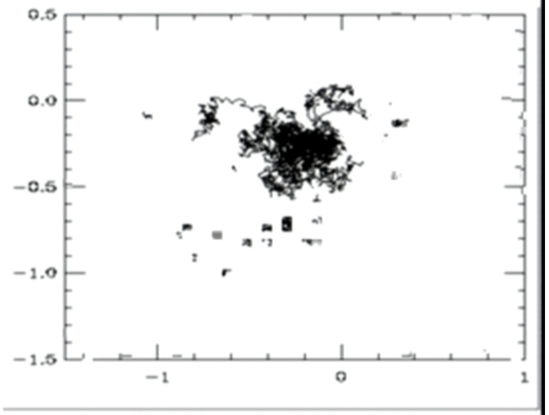
ELECTROWEAK BARYOGENESIS AT A STRONG 1st ORDER TRANSITION

CHARGE TRANSPORT

Cohen,Kaplan,Nelson 1991



- diffusion: ink in water



$$j_s = j_0\delta(\vec{x}-\vec{x}_0)\delta(t-t_0)$$

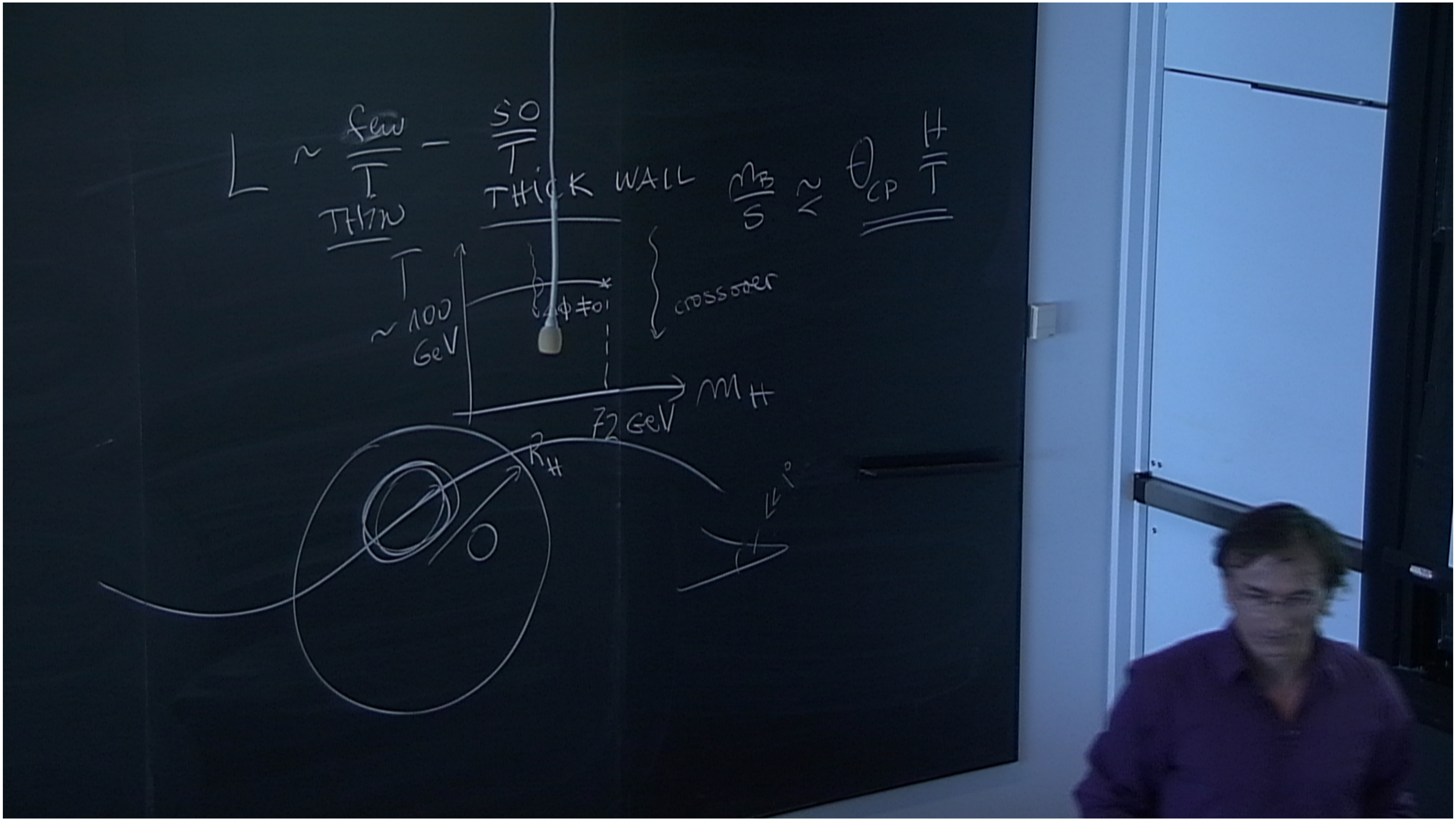
$$D\nabla^2 n - \partial_t n = j_s$$

⊙ expanding bubbles of higgs phase

⊙ CP violation on bubble walls

⊙ B violation in symmetric phase

bb p6!



$L \sim \frac{\text{few}}{T} - \frac{50}{T}$
THIN
THICK WALL
$\frac{n_B}{s} \lesssim \theta_{CP} \frac{H}{T}$
T
~100 GeV
$\Delta\phi \neq 0$
crossover
m_H
72 GeV
R_H

SUPERSIMETRY & MSSM

- each particle of the standard model has a supersymmetric partner with a different spin (& statistic):

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \Leftrightarrow \tilde{Q}_L = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L, \tilde{u}_R, \tilde{d}_R \text{ (squarks)}$$

$$l_L = \begin{pmatrix} e \\ \nu \end{pmatrix}_L, e_R \Leftrightarrow \tilde{l}_L = \begin{pmatrix} \tilde{e} \\ \tilde{\nu} \end{pmatrix}_L, \tilde{e}_R \text{ (sleptons)}$$

$$H_1 = \begin{pmatrix} h_1^+ \\ h_1^0 \end{pmatrix}, H_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix} \Leftrightarrow \tilde{H}_1 = \begin{pmatrix} \tilde{h}_1^+ \\ \tilde{h}_1^0 \end{pmatrix}, \tilde{H}_2 = \begin{pmatrix} \tilde{h}_2^+ \\ \tilde{h}_2^0 \end{pmatrix} \text{ (higgsinos)}$$

$$W^\pm \Leftrightarrow \tilde{W}^\pm \text{ (winos)}$$

$$\gamma, Z^0 \Leftrightarrow \tilde{\gamma}, \tilde{Z}^0 \text{ (photino, zino)}$$

$$g \Leftrightarrow \tilde{g} \text{ (gluinos)}$$

charginos

&

neutralinos

NB: in contrast to SM, MSSM has 2 complex Higgs doublets

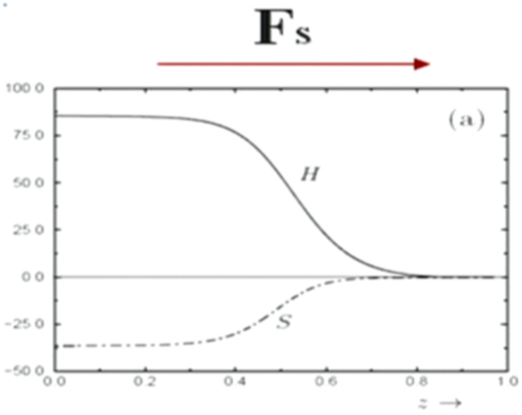
SEMICLASSICAL FORCE

Joyce, Prokopec, Turok, 1994

Kainulainen, Prokopec, Schmidt, Weinstock 2001; Rangarajan 2003

- **semi-classical force**: consequence of a difference in energy between fermionic particles & anti-particles (s = spin = ± , $m=|m|e^{i\theta}$, c=1):

bb p8: $$E_{s\pm} = E_0 \pm \hbar s \frac{|m|^2\,\theta'}{2E_0\sqrt{p_\perp^2 + |m|^2}}, \quad E_0 = \sqrt{\vec{p}^{\,2} + |m|^2}$$



- **semi-classical force** :

$$\vec{F}_{s\pm} = -\vec{\nabla} E_{s\pm},$$

- **semi-classical force in** kinetic theory:

$$\left(\partial_{\mathbf{t}} + \vec{v}\cdot\partial_{\vec{\mathbf{x}}} + \vec{F}_{s\pm}\cdot\partial_{\vec{\mathbf{p}}}\right) f_{s\pm} = C[f_{s\pm}], \qquad \vec{v}_s = \partial_{\vec{p}} E_s$$

NB: **applicable for energetic particles for which gradient expansion applies**:

$$\hbar c \|\nabla\| << E$$

→ **met for typical thermal particles with E~T, since** $\|\nabla\| \sim 1/L_w$ **and LwT>>1.**

SEMICLASSICAL FORCE FOR CHARGINOS

Joyce, Prokopec, Turok, 1994

Kainulainen, Prokopec, Schmidt, Weinstock 2001

LAGRANGIAN

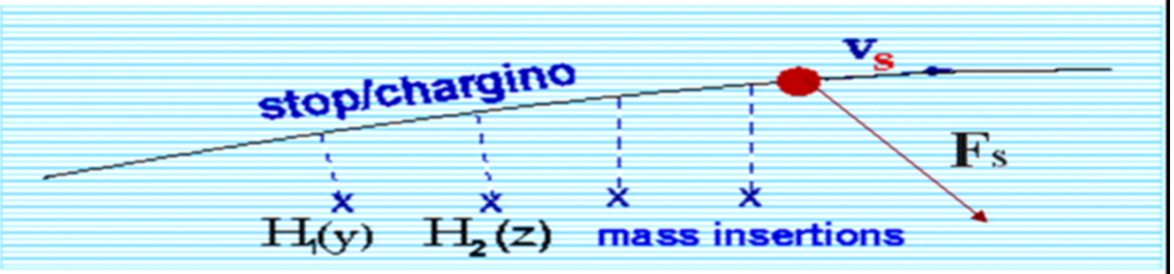
$$\mathcal{L}_{\rm mass} = \overline{\Psi}_R M \Psi_L + {\rm h.c.}$$

$$\Psi_R = \begin{pmatrix} \tilde{W}^+_R \\ \tilde{h}^+_{1,R} \end{pmatrix}, \; \Psi_L = \begin{pmatrix} \tilde{W}^+_L \\ \tilde{h}^+_{2,L} \end{pmatrix}$$

mass matrix: $M = \begin{pmatrix} m_2 & gH_2^* \\ gH_1^* & \mu \end{pmatrix}$

Higgs vev's: $H_1 = h_1 e^{i\theta_1}$ $\quad H_2 = h_2 e^{i\theta_2}$

- μ, m_2: soft susy breaking parameters



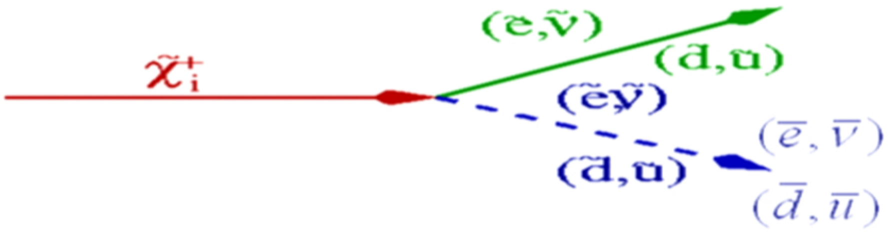
- The presence of a propagating bubble wall (Higgs condensate) induces chargino flavour oscillations (1st order in gradients), analogous to neutrino flavour oscillations, and semiclassical force (2nd order in gradients)

CHARGINOS MEDIATED BARYOGENESIS

Carena, Moreno, Quiros, Seco, Wagner (2000)
Cline, Joyce, Kainulainen (2000)
Konstandin, Prokopec, Schmidt, Seco (2005)

- Charginos decay into quarks & leptons via weak strength interactions:

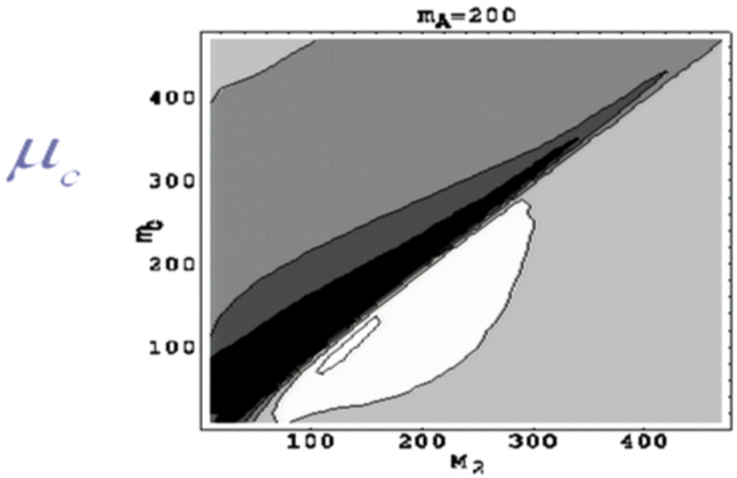
- Sphalerons bias production of a net baryon number, which diffuses into the broken phase

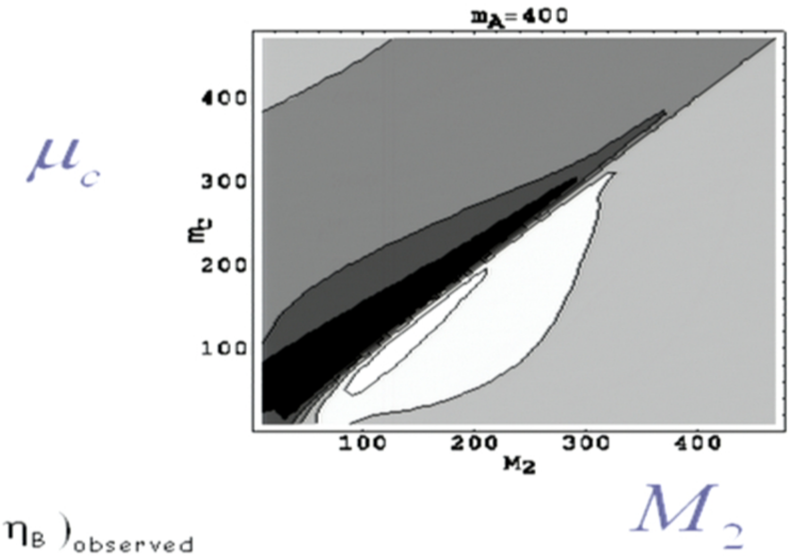


We solve the relevant diffusion equations. We use the system of equations initially proposed by Huet and Nelson 1995, and refined in the work of Carena, Moreno, Quiros, Seco, Wagner 2000; Balazs et al 2004.

CHARGINO BARYOGENESIS IN MSSM (3)

Konstandin, Prokopec, Schmidt, Seco (2005)





black regions mean $\eta_b = \frac{n_B}{s} > 10^{-10} \sim (\eta_B)_{observed}$

Baryon asymmetry from charginos with maximum CP violation assumed

The current measurements of the electron electric dipole moment

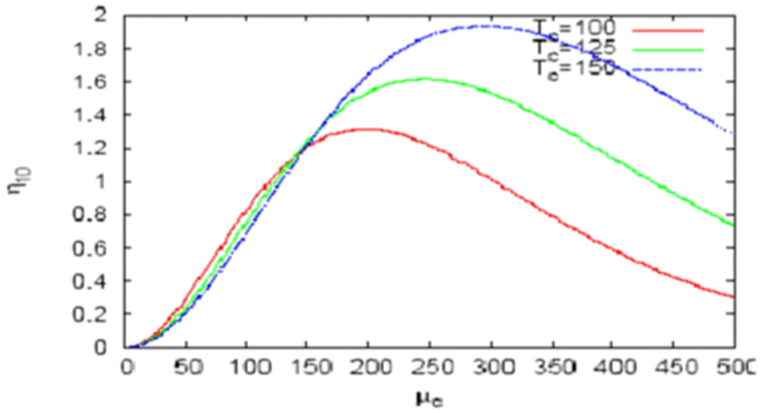
$|d_e| \leq 1.6\times10^{-27}\,\mathrm{ecm}$ Regan et al, Phys. Rev. Lett. 88:071805, 2002
Romalis et al., 2002

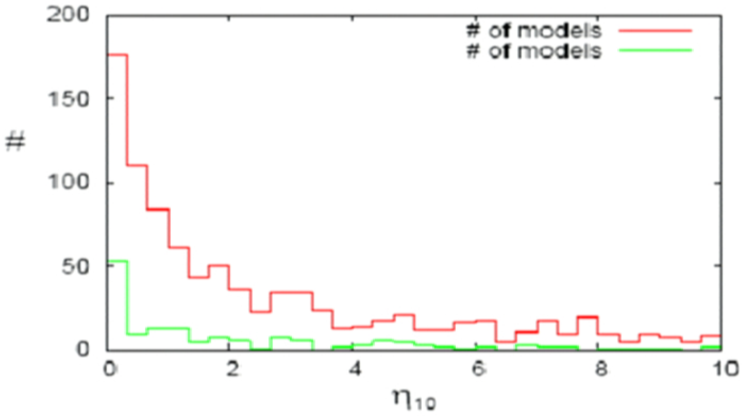
constrain the CP violating phase to be < 0.1, implying that charginos (& neutralinos) cannot produce enough baryons to explain the BAU (unless there are fortuitous cancellations of the MSSM contributions to the EDM).

CHARGINO BARYOGENESIS IN nMSSM

$W = \lambda H_1 H_2 - mS/\lambda + ..$

Menon, Morrissay, Wagner (2004)
Huber, Konstandin, Prokopec, Schmidt (2006)

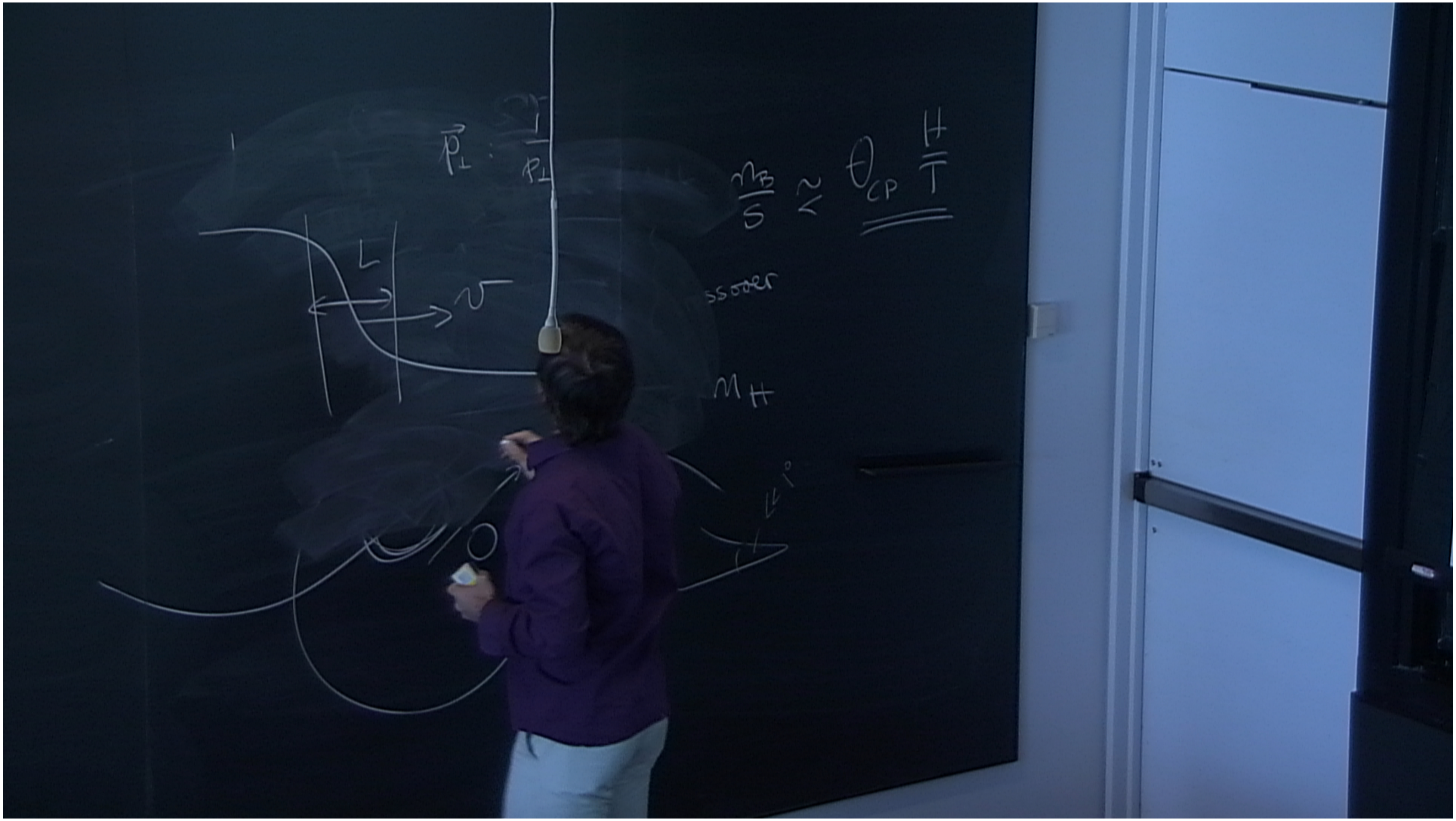




BG (~50%)
BG+EDM

Baryogenesis in nMSSM is efficient ($\eta_{10}|_{obs}=0.9$)!

- Because of the additional scalar singlet S, PT transition and CP violation are much stronger than in the MSSM


$\vec{p}_\perp : \frac{2\pi}{p_\perp}$
L
v
ssover
$\frac{n_B}{s} \lesssim \theta_{CP} \frac{H}{T}$
m_H

KINK WALL WITH CP VIOLATION

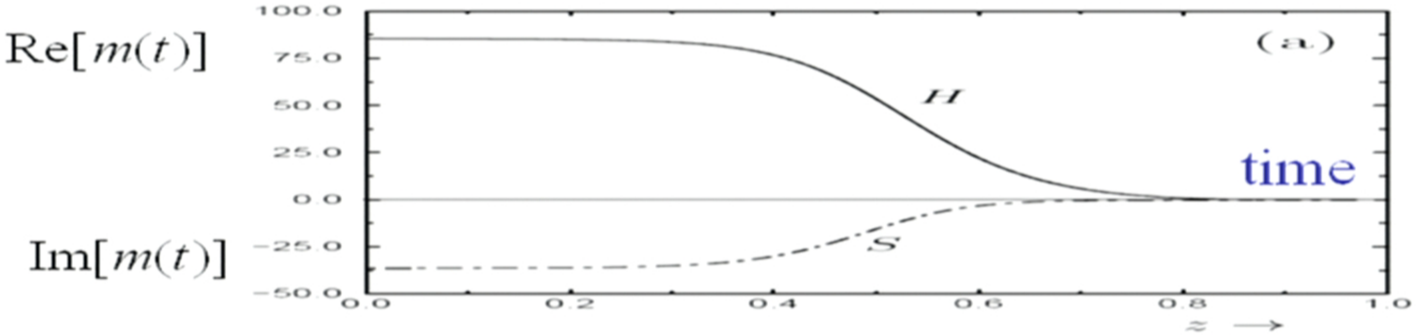
T. Prokopec, Jan Weenink, M.G. Schmidt 2012, in preparation

- Use time dependent case: particle creation ⇔ reflection/transmission

  helicity conservation ⇔ spin (⊥) conservation

- FERMION KINK MASS TERM:

$$m(t) = m_1 + m_2 \tanh(-\gamma t) \quad m_1 = m_{1R} + i m_{1I} \quad m_2 = m_{2R} + i m_{2I}$$



NB1: can remove m_{2I} by global fermion rotation

NB2: by varying $\gamma = 1/\tau$ can change `wall' thickness.

- CP even case ($m_I = 0$) has been studied by Ayala, Jalilian-Marian, McLerran&Vischer (1994)
- CP violating case ($m_I \neq 0$) was studied pert. by Funakubo, Kakuto, Otsuki, Takenaga&Toyoda

EARLY TIME SOLUTIONS

- POSITIVE FREQUENCY SOLUTIONS FOR $u_{\pm h}$ (early times):

$$u_{+h}(t) \equiv u_{+h}^{(1)}(t) = \sqrt{\frac{\omega_- + (m_{1R} + m_{2R})}{2\omega_-}} \times z^{\alpha}(1-z)^{\beta} \times {}_2F_1(a_+, b_+; c; z) \xrightarrow{t\to-\infty} u_{+h0}e^{-i\omega_- t}$$

$$u_{-h}(t) \equiv u_{-h}^{(1)}(t) = -\underbrace{\frac{hk - im_I}{\sqrt{k^2 + m_I^2}}}_{CP\text{ violation}} \times \sqrt{\frac{\omega_- - (m_{1R} + m_{2R})}{2\omega_-}} \times z^{\alpha}(1-z)^{\beta} \times {}_2F_1(a_-, b_-; c; z) \xrightarrow{t\to-\infty} u_{-h0}e^{-i\omega_- t}$$

- (One can prove that) these solutions are properly normalized: $|u_{+h}|^2 + |u_{-h}|^2 = 1$
- CP violation is in the phase factor in front of u_{-h} (in the rel. phase between u_{-h} and u_{+h})
- The solutions are valid both when CP violating phase is small (<<1) or large (~π).

[in contrast to the works of Funakubo, Kakuto, Otsuki, Takenaga & Toyoda]

- When t→∞ (z→1), the above solutions become linear combinations of late time solutions. This follows from the identity:

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \times {}_2F_1(a,b;a+b+a-c;1-z) + \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \times (1-z)^{c-a-b} \times {}_2F_1(c-a,c-b;c+1-a-b;1-z)$$

PARTICLE PRODUCTION

- BOGOLIUBOV TRANSFORMATION LINKS EARLY AND LATE TIME SOLNS:

$$u_{\pm h}(t)=\alpha_{\pm h}\tilde{u}^{(1)}_{\pm h}(t)+\beta_{\pm h}\tilde{u}^{(2)}_{\pm h}(t)$$

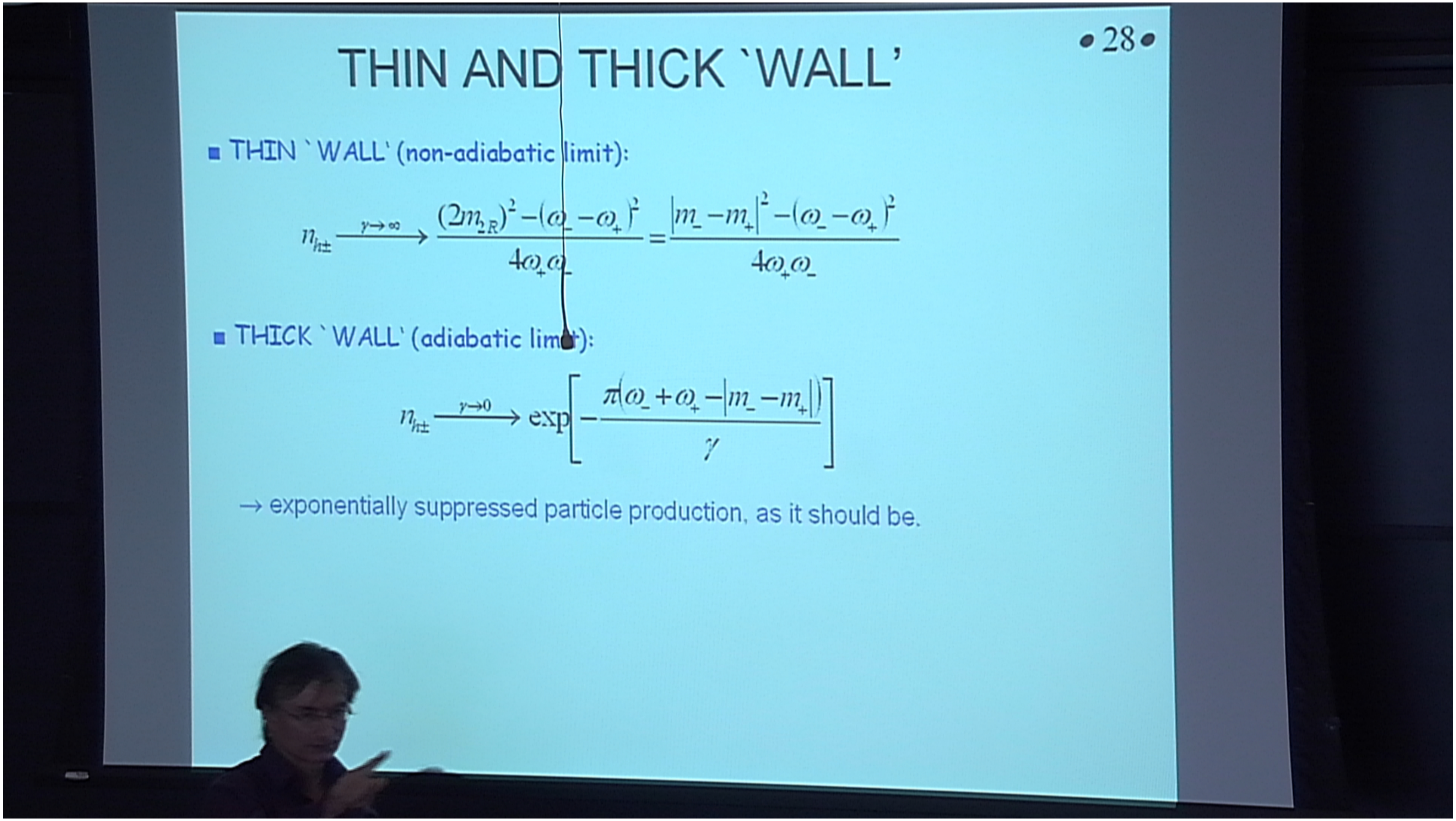
$$\alpha_{\pm h}=\sqrt{\frac{\omega_+\left[\omega_-\pm(m_{1R}+m_{2R})\right]}{\omega_-\left[\omega_+\pm(m_{1R}-m_{2R})\right]}}\times\frac{\Gamma(c)\Gamma(a_\pm+b_\pm-c)}{\Gamma(a_\pm)\Gamma(b_\pm)}$$

$$\beta_{\pm h}=\pm\sqrt{\frac{\omega_+\left[\omega_-\pm(m_{1R}+m_{2R})\right]}{\omega_-\left[\omega_+\pm(m_{1R}-m_{2R})\right]}}\times\frac{\Gamma(c)\Gamma(c-a_\pm-b_\pm)}{\Gamma(c-a_\pm)\Gamma(a-b_\pm)}$$

- PARTICLE NUMBER :

$$n_{h\pm}\equiv|\beta_{h\pm}|^2=\frac{\sinh\left(\frac{\pi[\omega_- -\omega_+ +2m_{2R}]}{2\gamma}\right)\sinh\left(\frac{\pi[\omega_+ -\omega_- +2m_{2R}]}{2\gamma}\right)}{\sinh\left(\frac{\pi\omega_+}{\gamma}\right)\sinh\left(\frac{\pi\omega_-}{\gamma}\right)},\quad 0\le n_{h\pm}\le 1$$

- Check: one can show that: $|\alpha_{h\pm}|^2+|\beta_{h\pm}|^2=1$ as it should be.
- Particle production is EVEN under CP; the phase of β contains CP odd information.



THIN AND THICK `WALL'

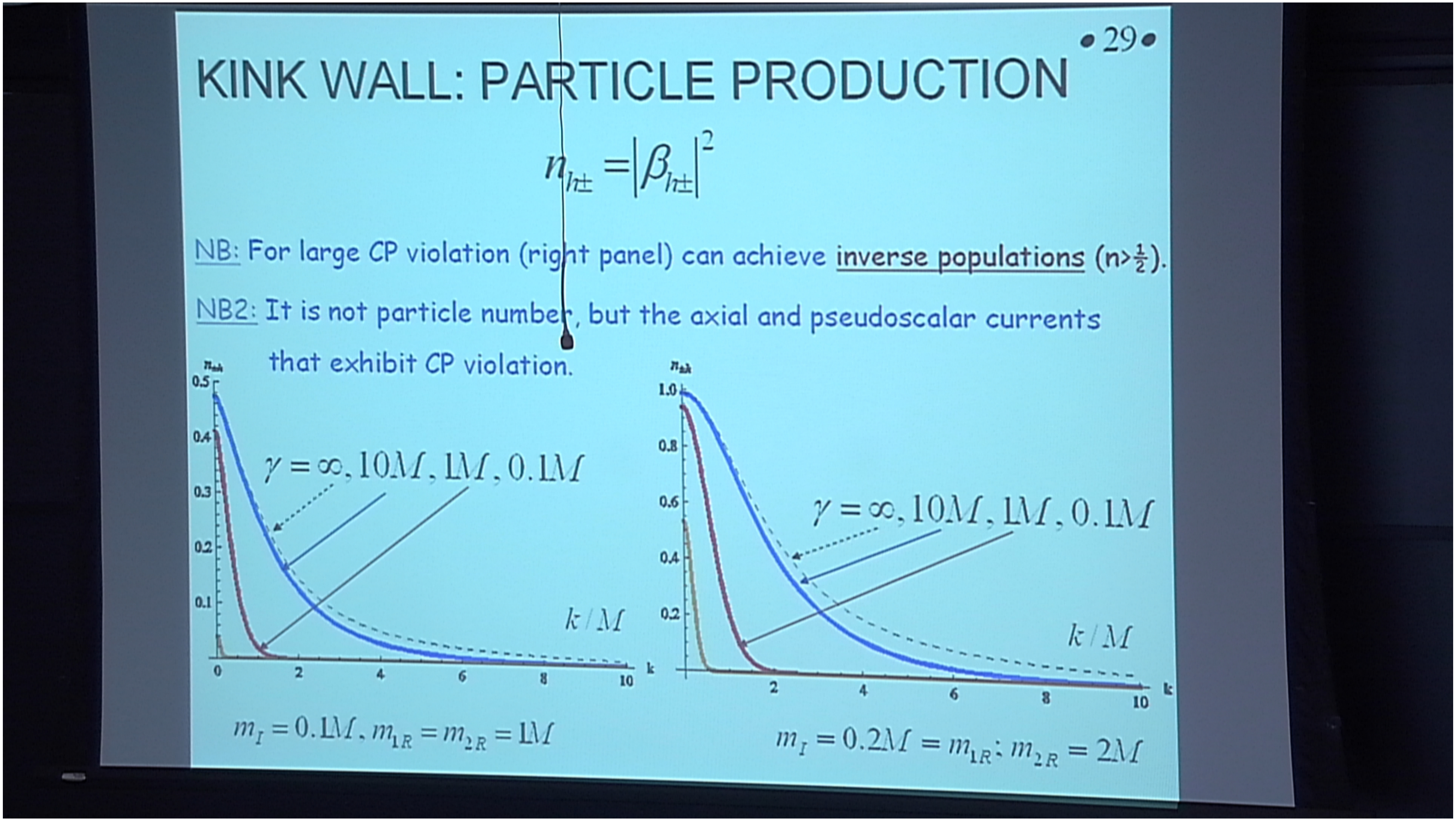
- THIN `WALL' (non-adiabatic limit):

$$n_{h\pm} \xrightarrow{\gamma\to\infty} \frac{(2m_{2R})^2-(\omega_- - \omega_+)^2}{4\omega_+\omega_-} = \frac{|m_- - m_+|^2-(\omega_- - \omega_+)^2}{4\omega_+\omega_-}$$

- THICK `WALL' (adiabatic limit):

$$n_{h\pm} \xrightarrow{\gamma\to 0} \exp\left[-\frac{\pi(\omega_- + \omega_+ - |m_- - m_+|)}{\gamma}\right]$$

→ exponentially suppressed particle production, as it should be.



KINK WALL: PARTICLE PRODUCTION

$$n_{h\pm} = \left|\beta_{h\pm}\right|^2$$

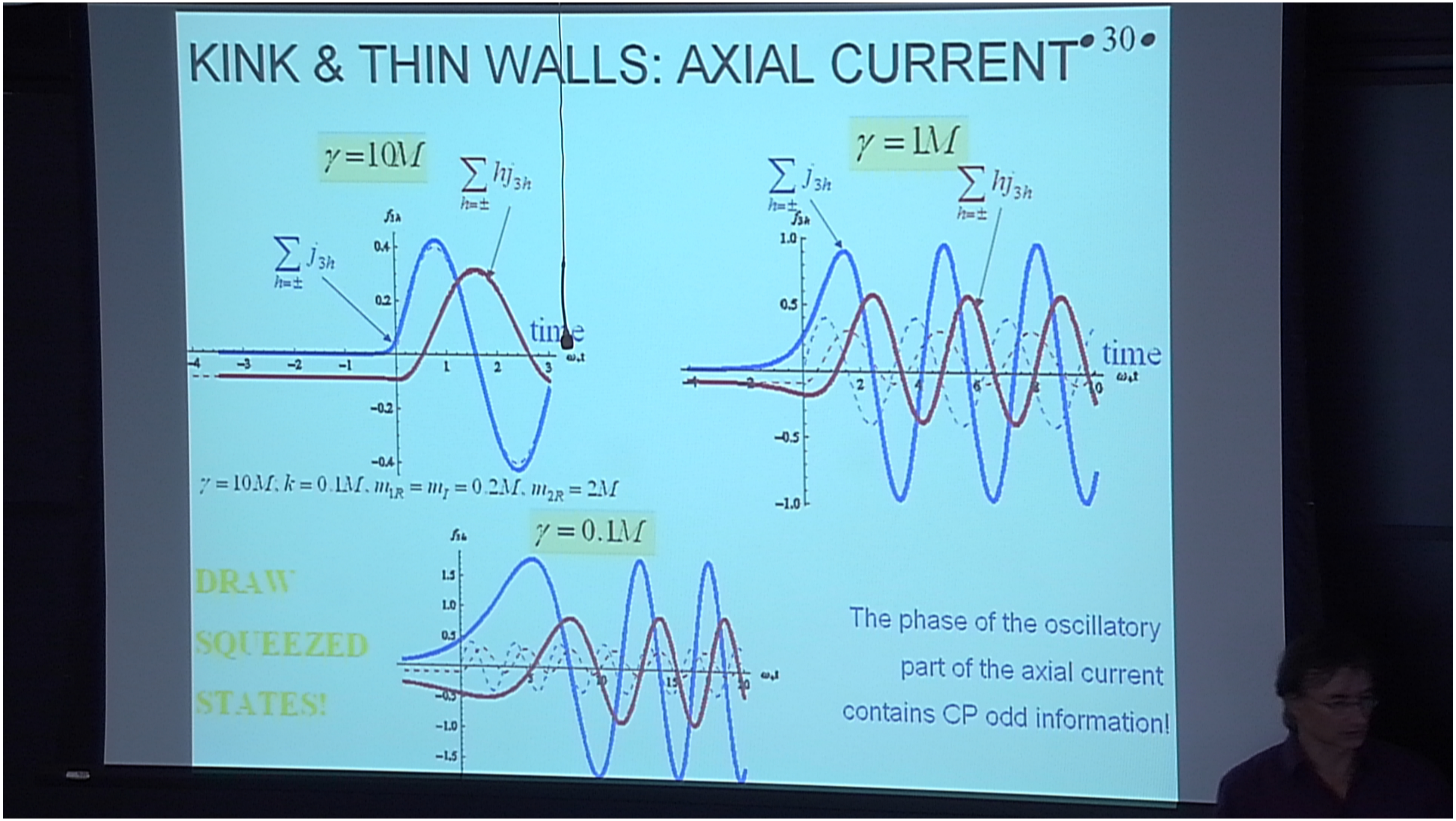
NB: For large CP violation (right panel) can achieve inverse populations ($n > \frac{1}{2}$).

NB2: It is not particle number, but the axial and pseudoscalar currents that exhibit CP violation.

$m_I = 0.1M,\ m_{1R} = m_{2R} = 1M$

$m_I = 0.2M = m_{1R};\ m_{2R} = 2M$

KINK & THIN WALLS: AXIAL CURRENT



$\gamma = 10M, k = 0.1M, m_{1R} = m_I = 0.2M, m_{2R} = 2M$

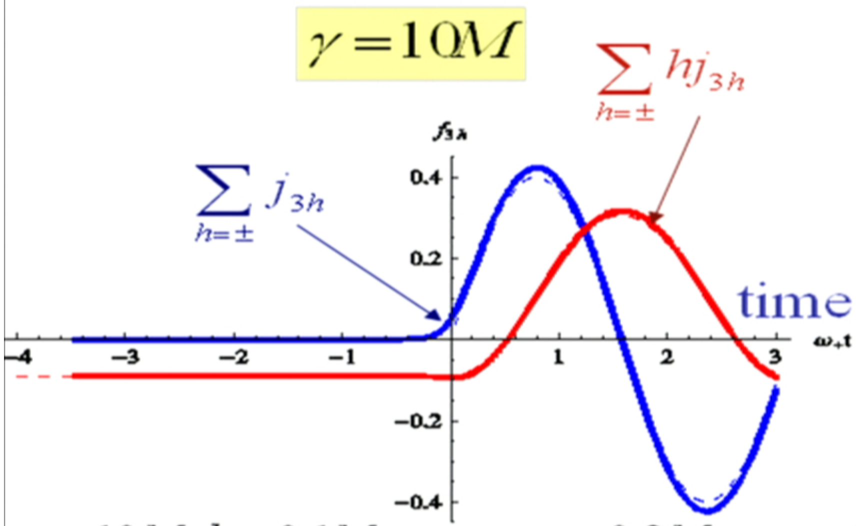
DRAW

SQUEEZED

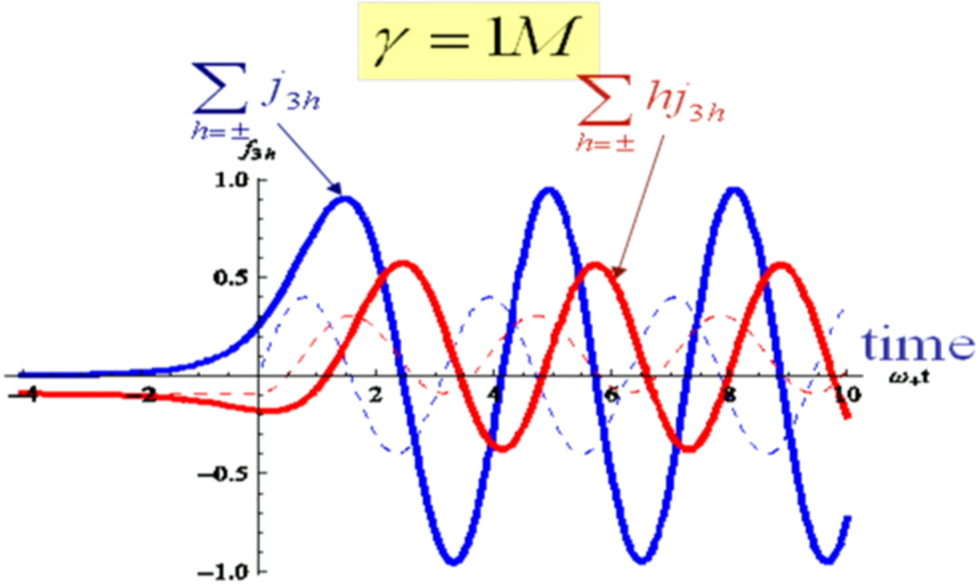
STATES!

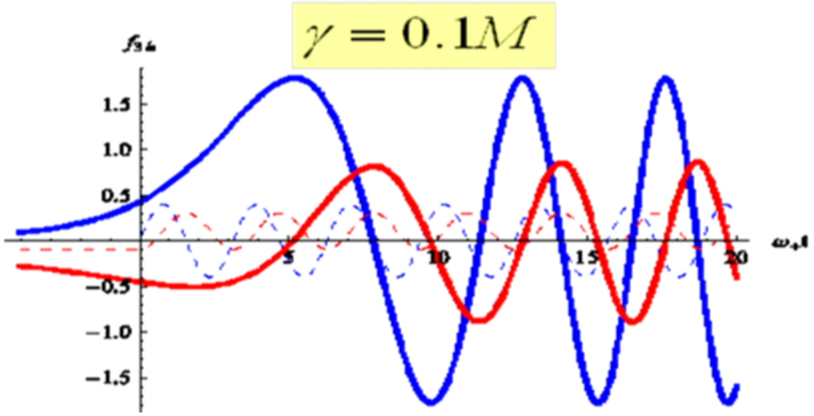
The phase of the oscillatory part of the axial current contains CP odd information!

KINK & THIN WALLS: AXIAL CURRENT



$\gamma = 10M;\ k = 0.1M,\ m_{1R} = m_I = 0.2M,\ m_{2R} = 2M$





DRAW

SQUEEZED

STATES!

The phase of the oscillatory part of the axial current contains CP odd information!

DISCUSSION & CONCLUSIONS

- We understand well how to determine the strength of the EW transition

  → Based on this: we have ruled out baryogenesis in MSM, MSSM.
  Models with additional scalar singlets and general 2HD models are still viable.

- Baryon production at the EW scale baryogenesis:

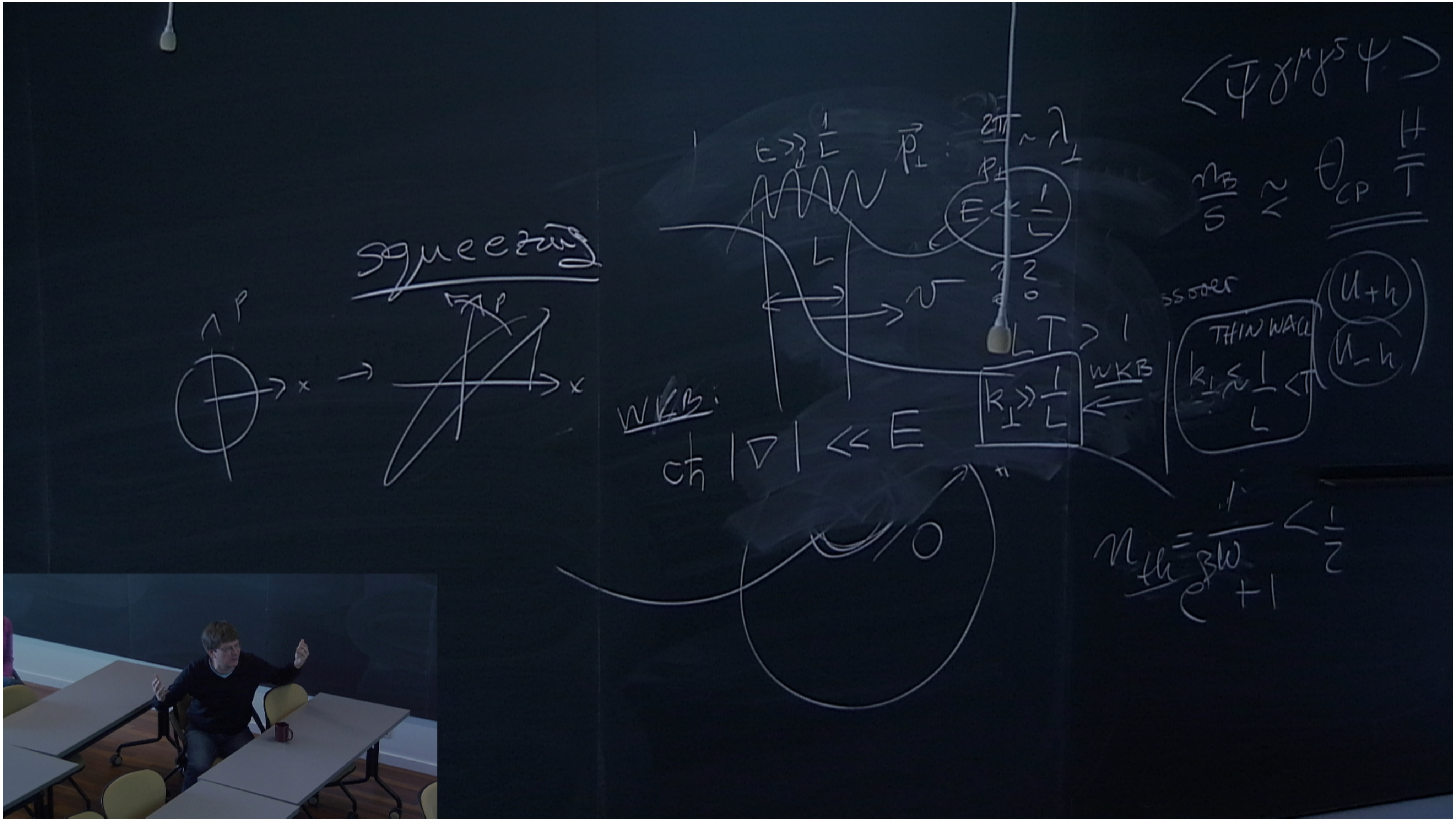
  (1) well understood from semiclassical force and flavor mixing (charginos, neutralinos) in gradient expansion;

  (2) poor understanding of sources from quantum reflection (thin wall)

  (3) Need a better understanding of transport of mixing flavors and resonant baryon production; also of dynamics of the transition.

- Discovery of the neutrino masses & PMNS mass matrix lead to a surge of activities in leptogenesis: production of leptons from L and CP violating decays of heavy majorana neutrinos at a GUT scale.

  PROBLEM: In contrast to the EW scale baryogenesis, leptogenesis may not be testable (because it involves GUT scale physics).



squeezing

WKB:

$c\hbar\,|\nabla| \ll E$

$E \gg \frac{1}{L}$

$\vec{p}_L : \frac{2\pi}{p_L} \sim \lambda_L$

$E \ll \frac{1}{L}$

$LT > 1$

$k_\perp \gg \frac{1}{L}$ WKB

THIN WALL

$k_\perp \lesssim \frac{1}{L} < T$

$\langle \bar{\Psi} \gamma^\mu \gamma^5 \psi \rangle$

$\frac{n_B}{s} \lesssim \theta_{CP} \frac{H}{T}$

$U_+ h$

$U_- h$

$n_{th} = \frac{1}{e^{\beta\omega}+1} < \frac{1}{2}$