Title: Principle of relativity for quantum theory

Date: Nov 20, 2012 03:30 PM

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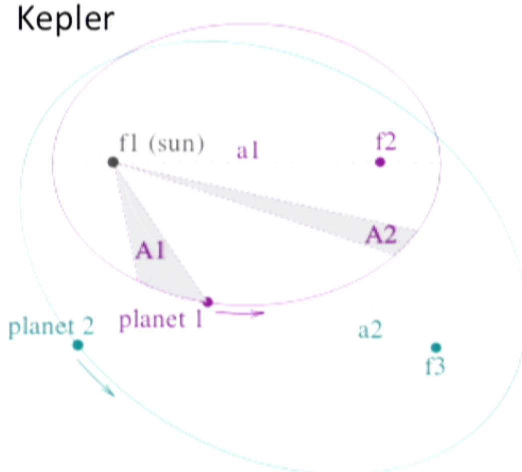
Abstract: In a generic quantum experiment we have a given set of devices analyzing some physical property of a system. To each device involved in the experiment we associate a set of random outcomes corresponding to the possible values of the variable analyzed by the device. Devices have apertures that permit physical systems to pass through them. Each aperture is labelled as "input" or "output" depending on whether it is assumed that the aperture lets the system go inside or outside the device. Assuming a particular input/output structure for the devices involved in a generic experiment is equivalent to assume a particular causal structure for the space-time events constituted by the outcomes happening on devices. The joint probability distribution of these outcomes is usually predicted assuming an absolutely defined input/output structure of devices. This means that all observers of the experiment agree on whether an aperture is labelled as "input" or "output". In this talk we show that the mathematical formalism of quantum theory permits to predict the joint probability distribution of outcomes in a generic experiment in such a way that the input/output structure is indeed relative to an observer. This means that two observers of the same experiment can predict the joint probability distribution of outcomes assuming different input/output labels for the apertures. Since input/output structure is the causal structure of the space-time events constituting the outcomes involved in the experiment we conclude that in quantum theory, the causal structure of events may not be regarded as absolute but rather as relative to the observer. We finally point out that properly extending this concept to the cosmological domain could shed light on the problem of dark energy.

Motivation

Ad hoc laws:

Kepler



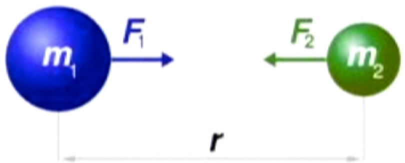
Lorentz

$$\mathrm{E}' = \gamma\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) - (\gamma - 1)\left(\mathbf{E}\cdot\hat{\mathbf{v}}\right)\hat{\mathbf{v}}$$

$$\mathrm{B}' = \gamma\left(\mathrm{B} - \frac{\mathbf{v}\times\mathbf{E}}{c^2}\right) - (\gamma - 1)\left(\mathrm{B}\cdot\hat{\mathbf{v}}\right)\hat{\mathbf{v}}$$

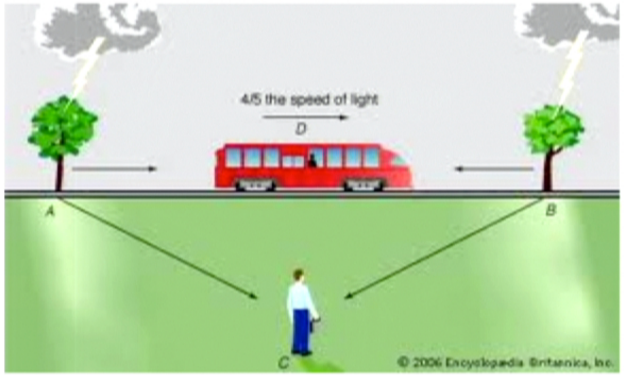
Basic principles:

Newton



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Einstein



Motivation

Quantum Theory:

Systems → Complex Hilb. Spaces

States → Density Matrices

Measurements → POVMs

Evolutions → CPTP maps

Probabilities → Born's rule

Basic principles:

?

- Quantum Logic : Birkhoff, Von Neumann, Mackey, Piron, Jauch, Mielnik…
- Realistic Interpretations : Bohm…
- Multiverse : Everett..
- Collapsing models : GRW, Von Neumann…
- Consistent histories : Hartle...
- Operationalism : Ludwig, Hardy, Chiribella, D'ariano, Perinotti, Masanes, Mueller, Bruckner…

Could a foundational principle for quantum theory be formulated questioning the role of causal structure? [Hardy (2006) Leifner, Spekkens (2011) Oreshkov, Costa, Bruckner (2011)]

Outline of the talk:

- Define causal structure of probabilistic outcomes of a generic quantum experiment in an operational way.
- State the main result and how this directly leads to the principle of relativity of causal structure.
- Give a detailed sketch of the proof of the result.
- Speculate about extending the above principle from the quantum domain to the cosmological domain.

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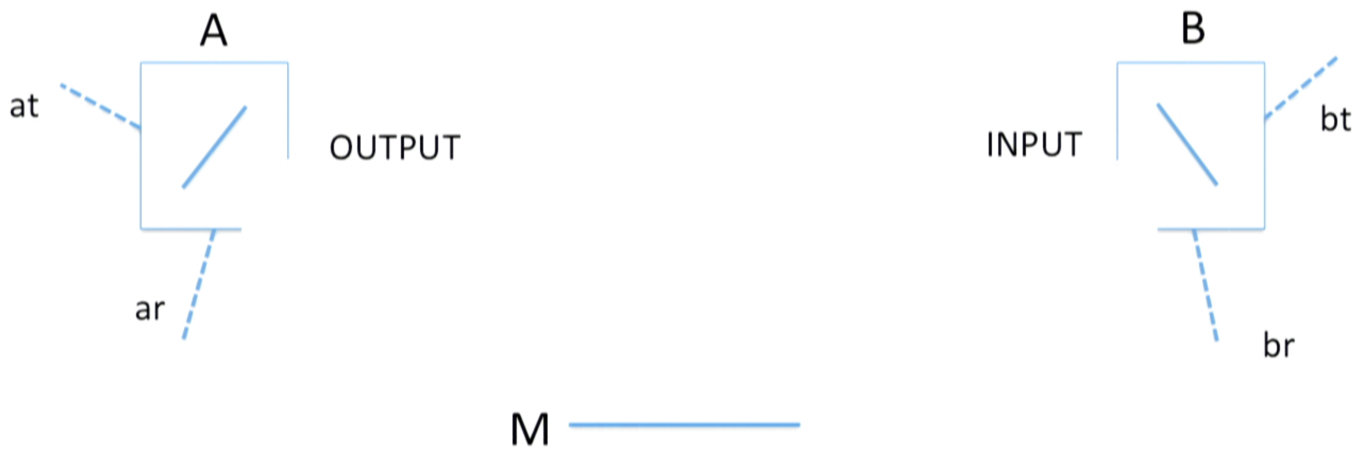
Causal structure in Quantum Theory



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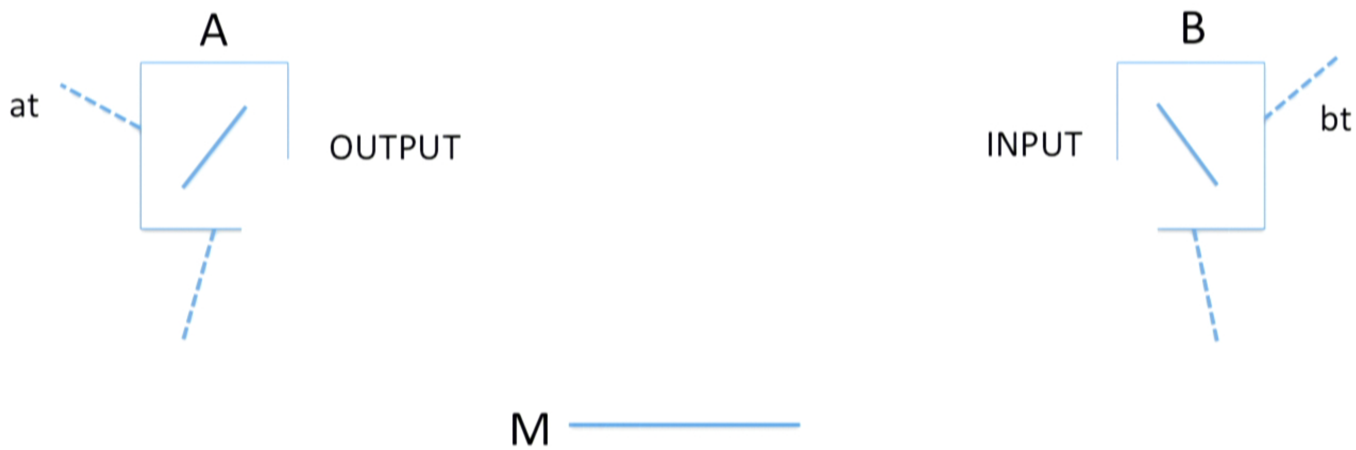

at, bt are two events in space-time such that at causes bt

at is in the preparation ensemble {ar,at}, {p(ar), 1-p(ar)} represented by a density matrix

bt is an outcome of a measurement {br,bt} represented by a PVM (POVM)

M is represented by a unitary matrix (CPTP map)

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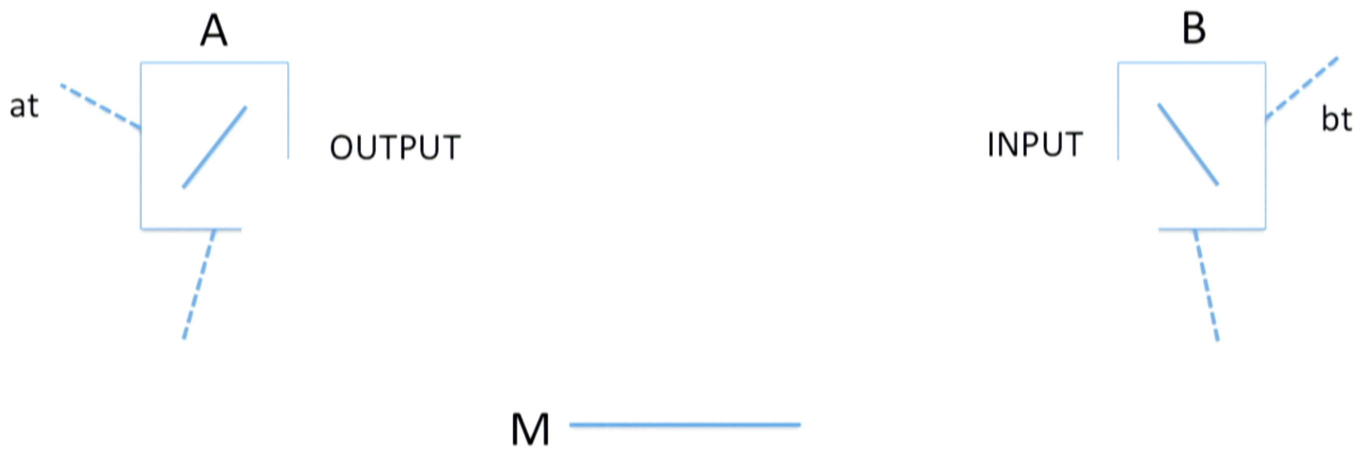

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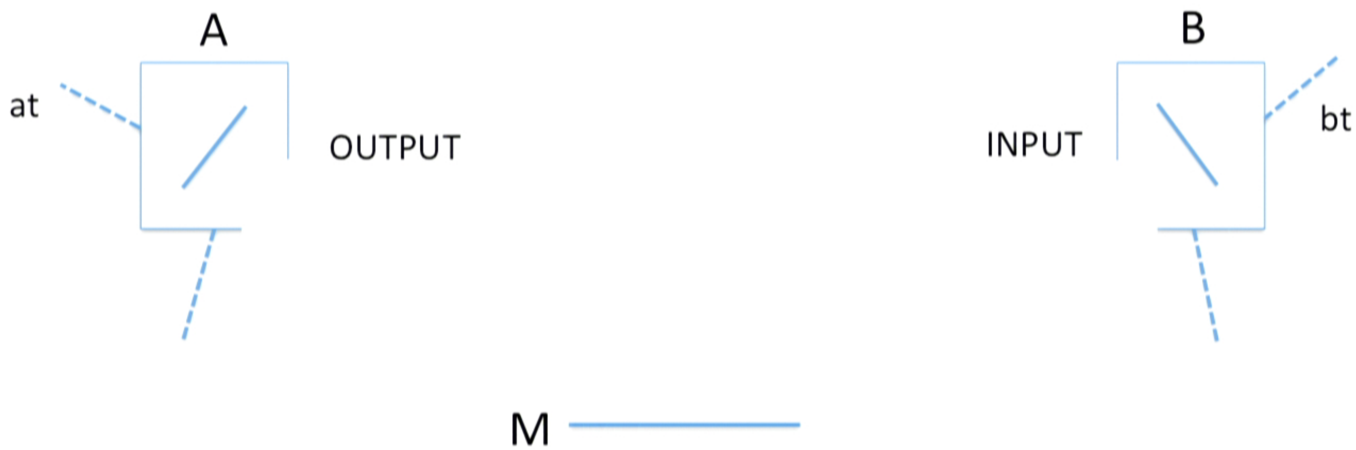

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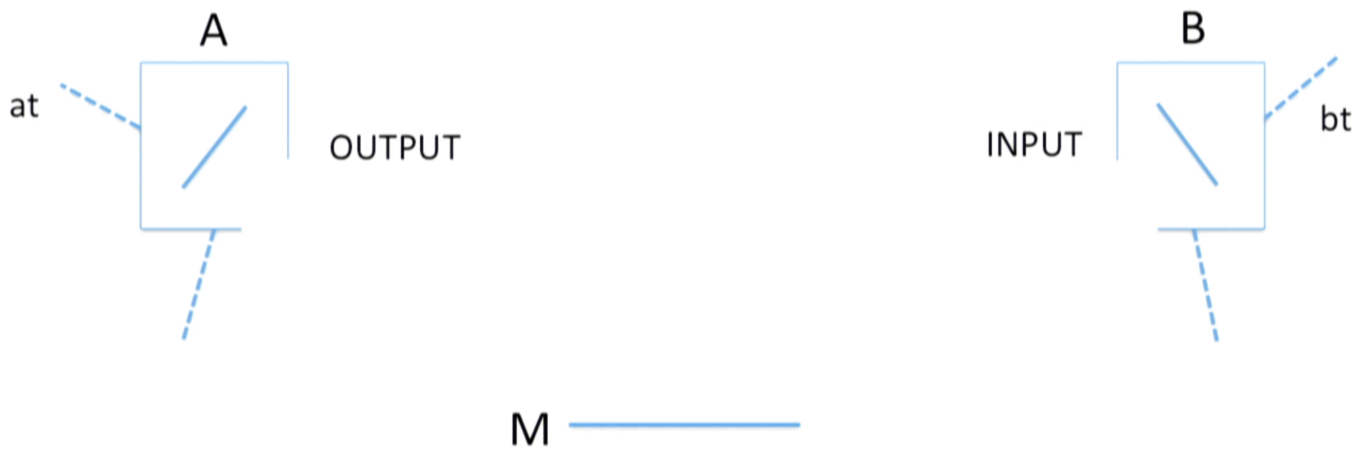

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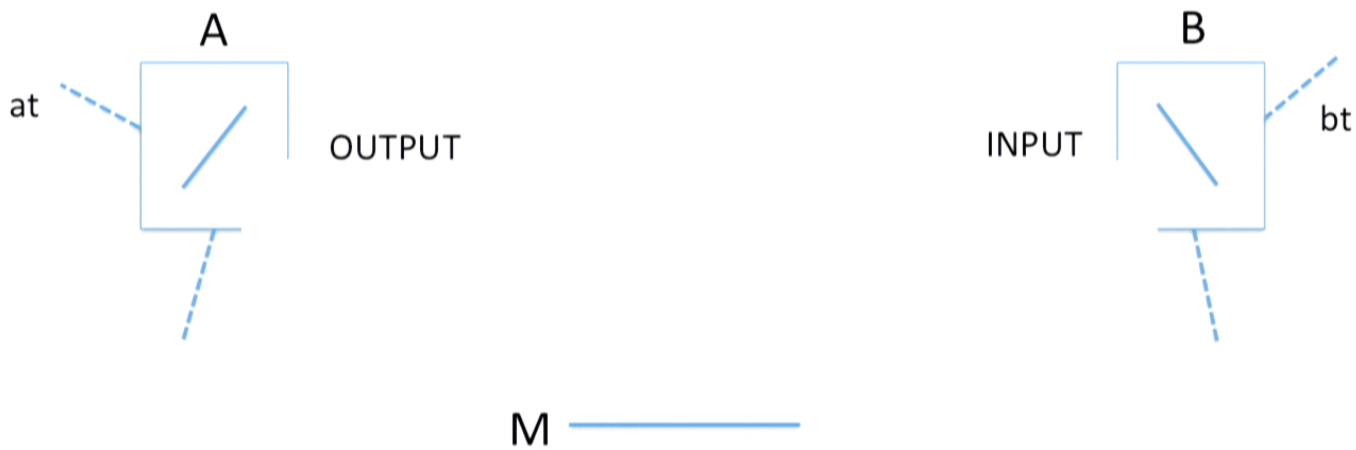

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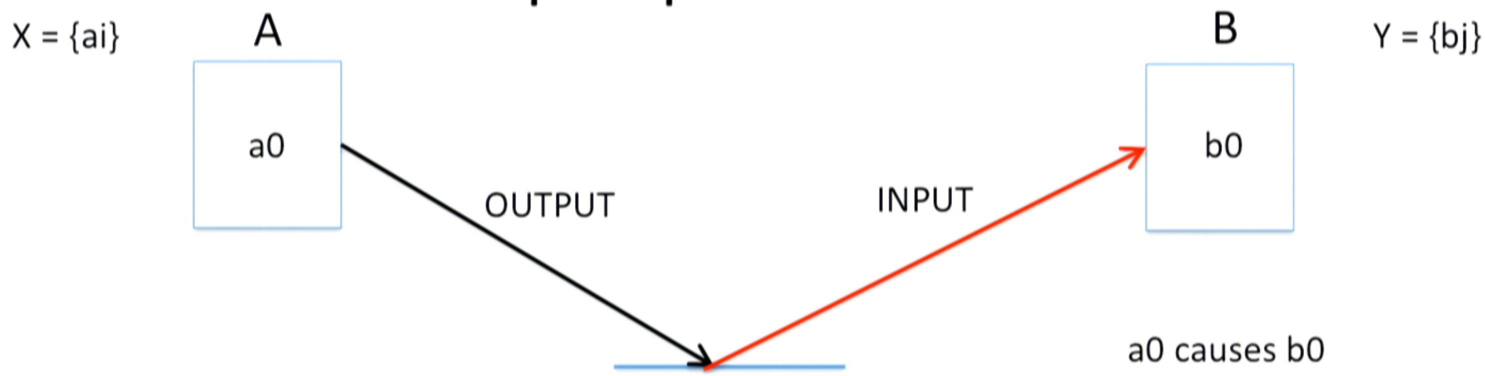
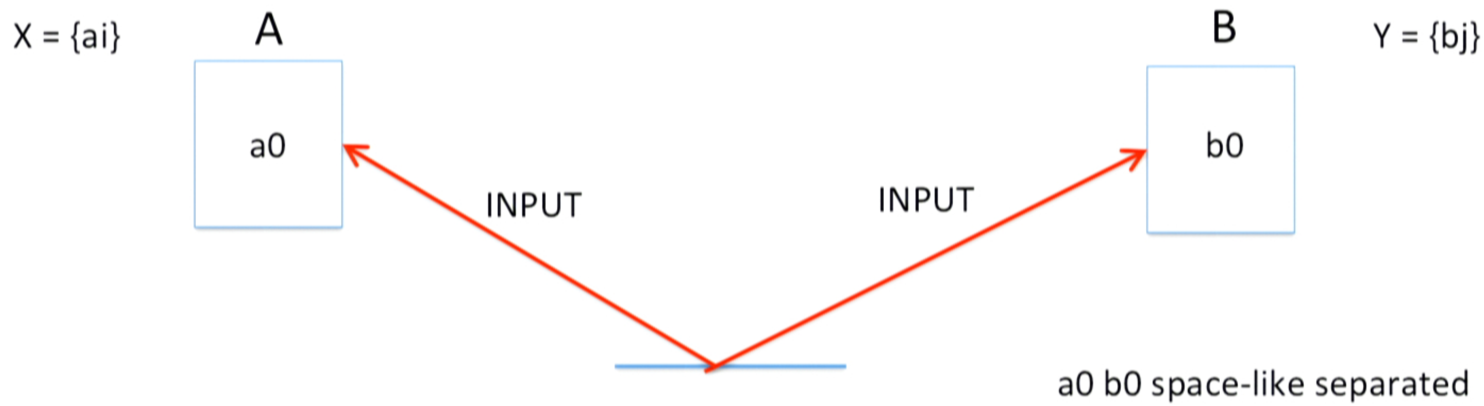
Causal structure from an "operational" perspective





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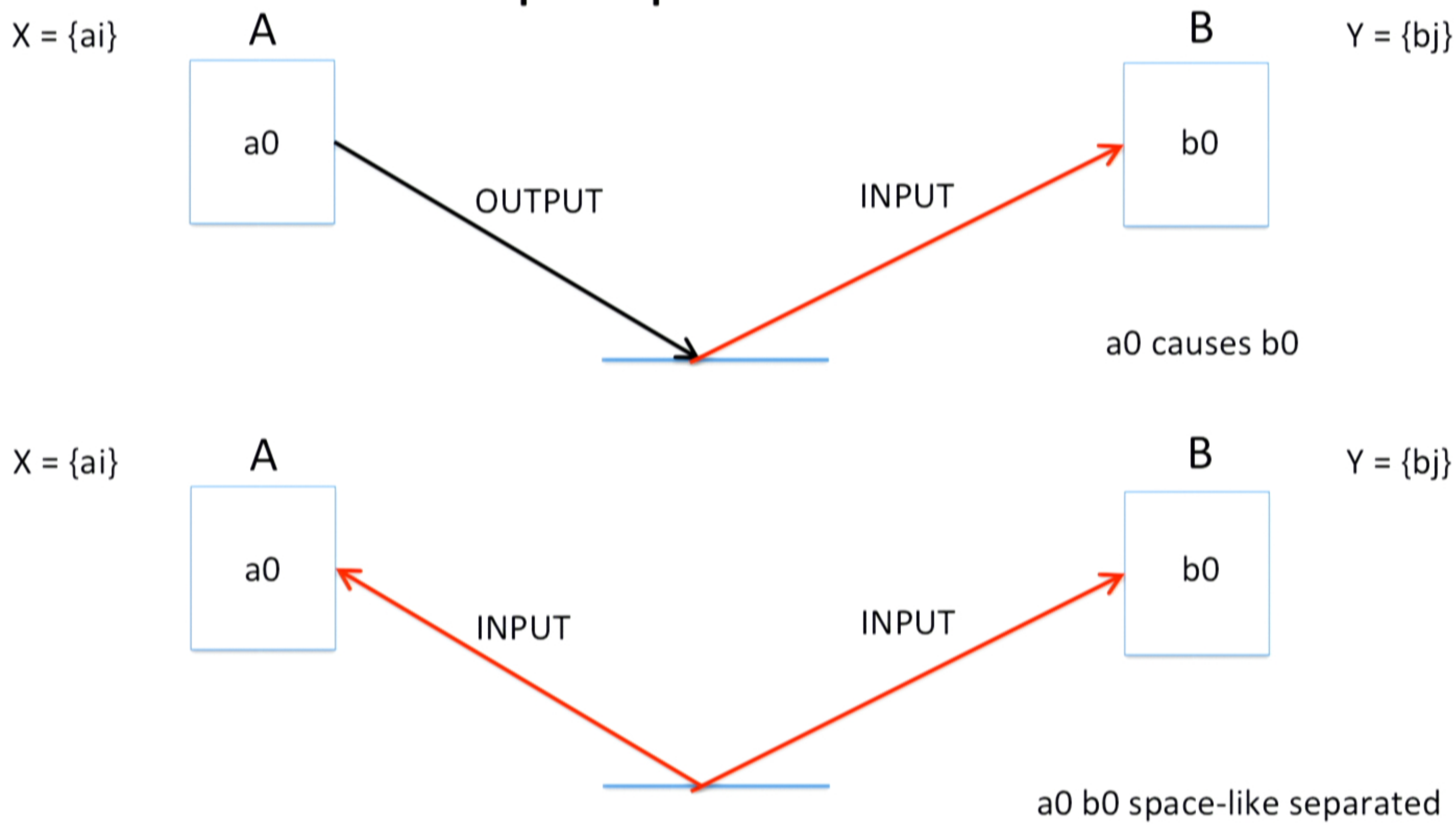

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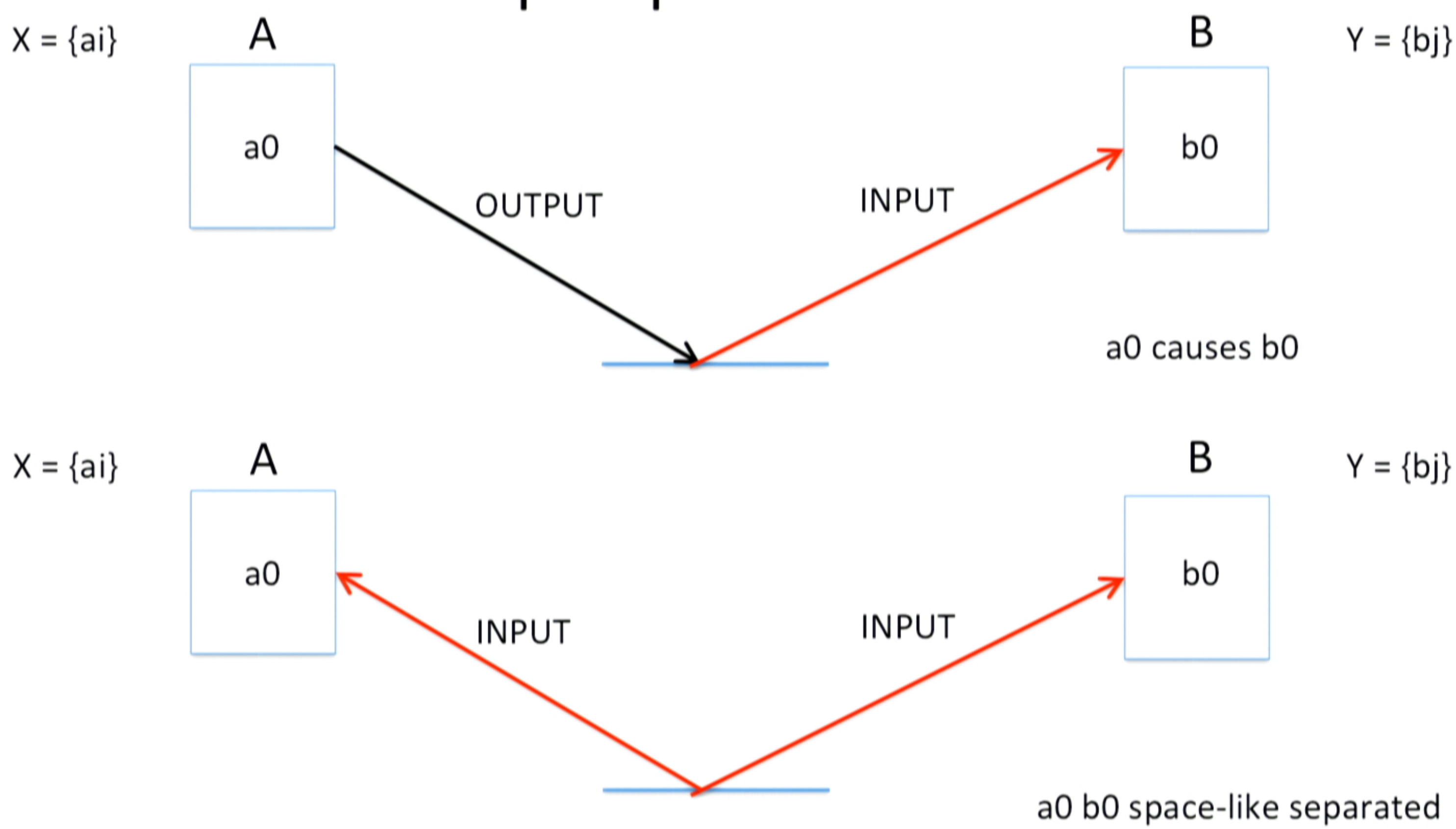

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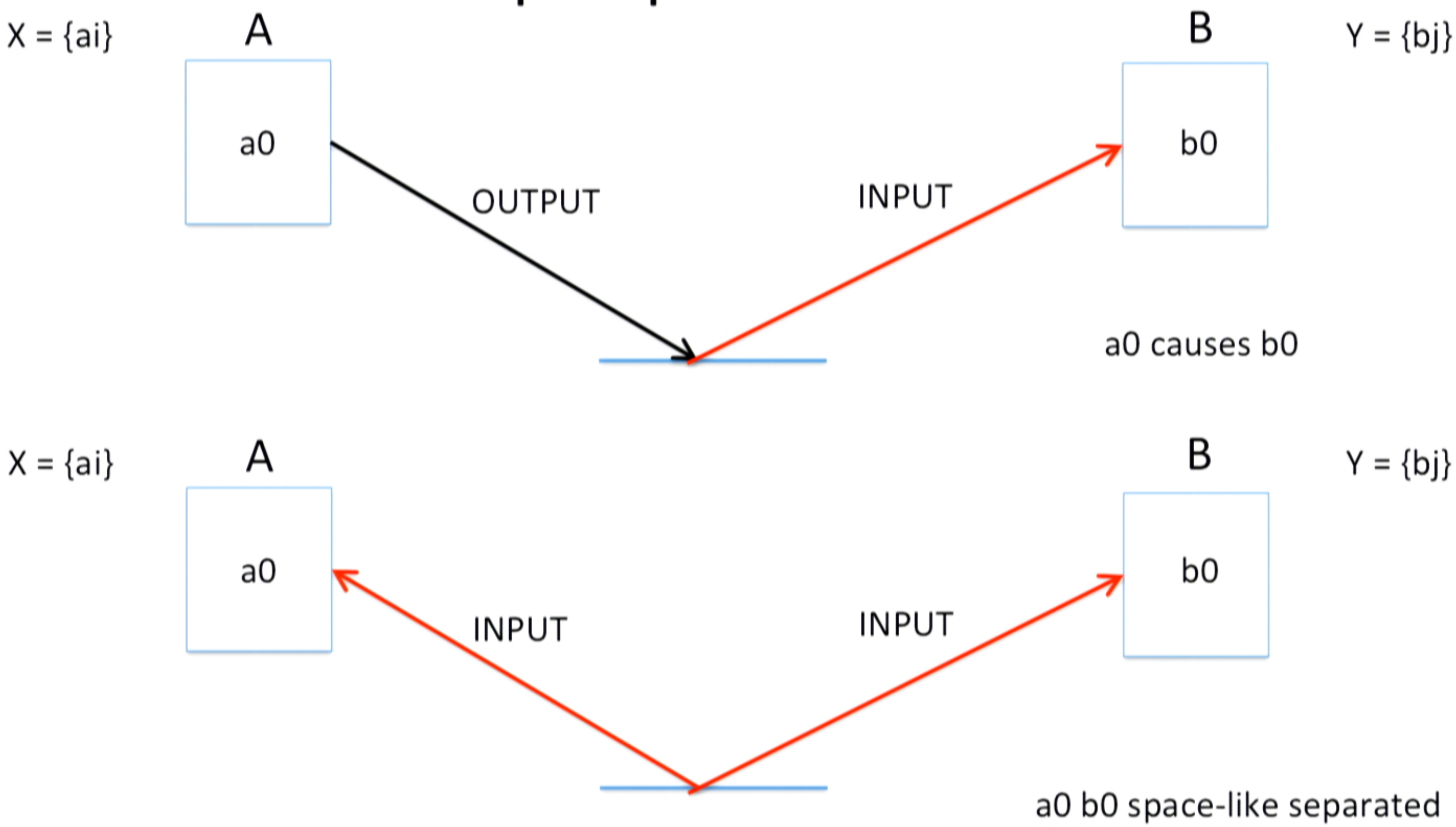
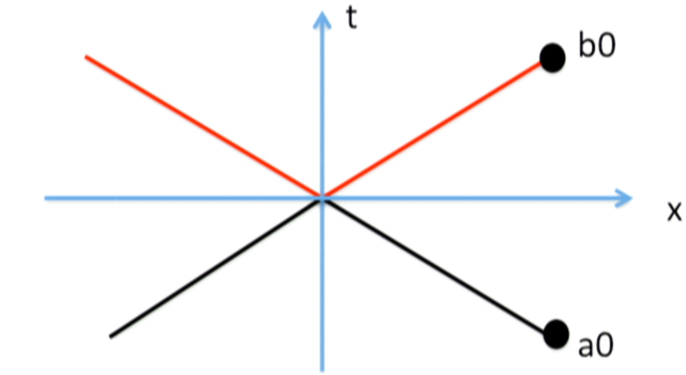
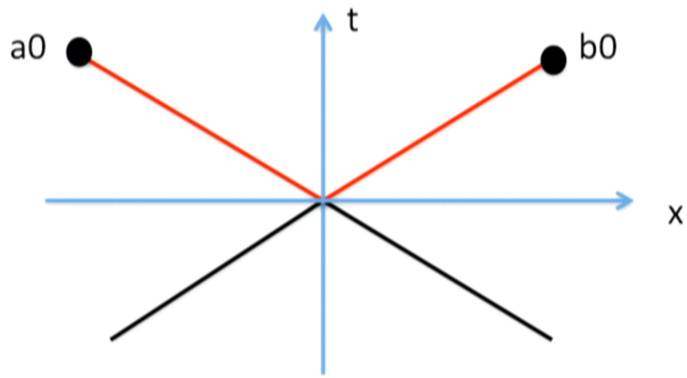


Space-time and causal structure

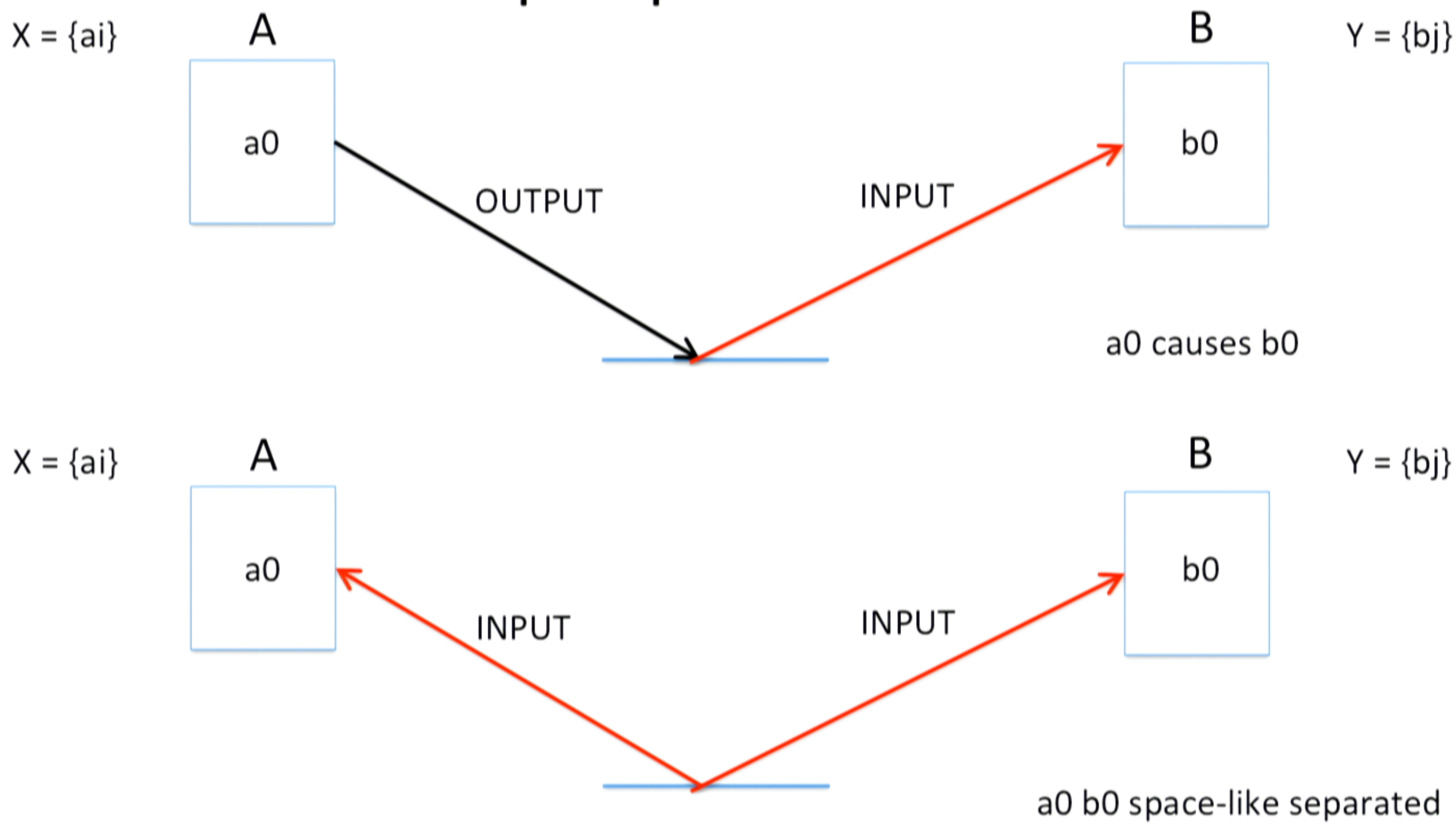


a0 causes b0



a0 b0 space-like separated

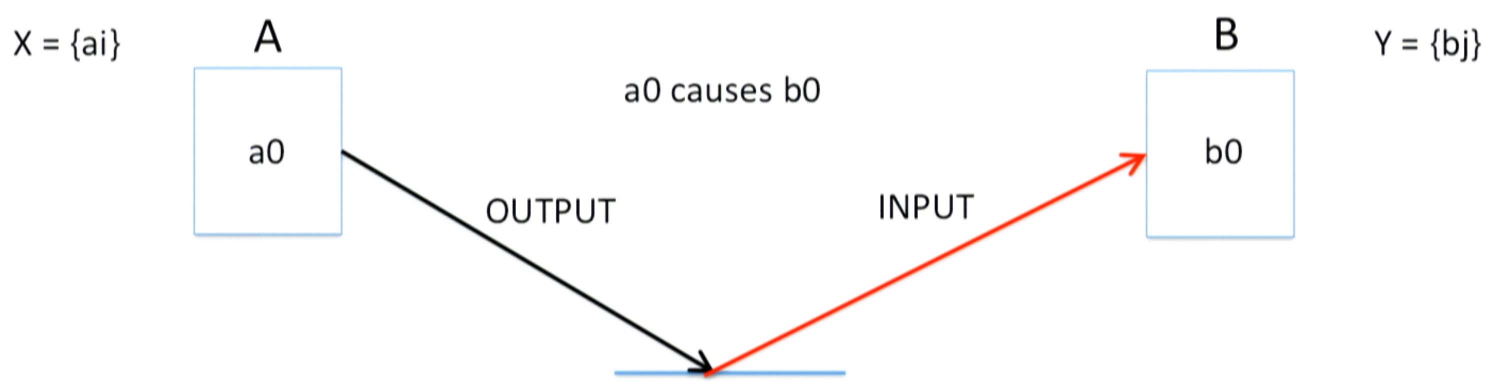
Causal structure from an "operational" perspective



In every physical process it is assumed that causal ordering of events is an ABSOLUTE property.

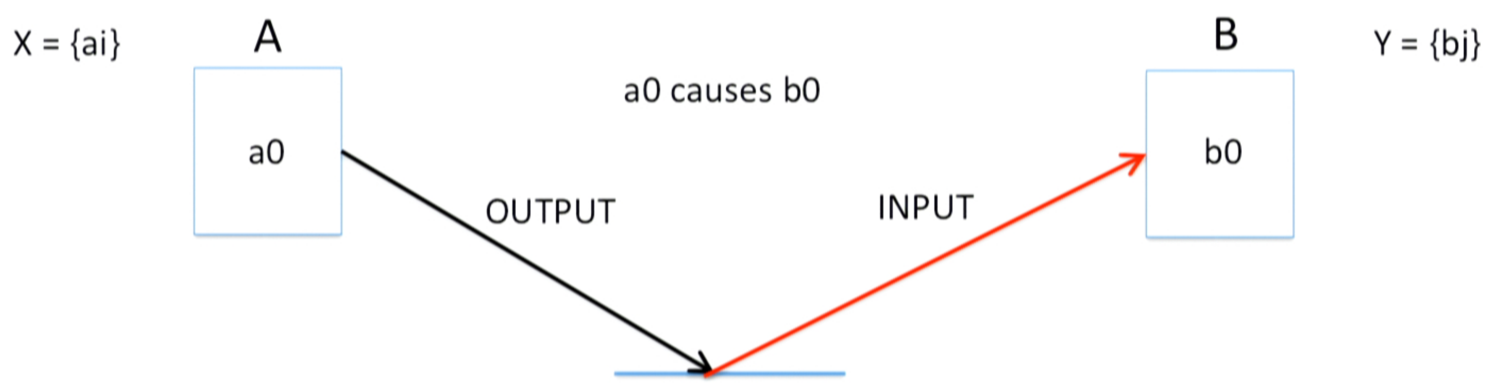
All observers of a process agree on whether two events are one the cause of the other or space-like separated.

INPUT-OUTPUT STRUCTURE is regarded as absolute



Main Result

In a generic quantum experiment, INPUT-OUTPUT structure may be regarded as an OBSERVER DEPENDENT property.This means that two observers of the same probabilistic quantum experiment can assume a different INPUT-OUTPUT structure and predict the same joint probabilities of outcomes.



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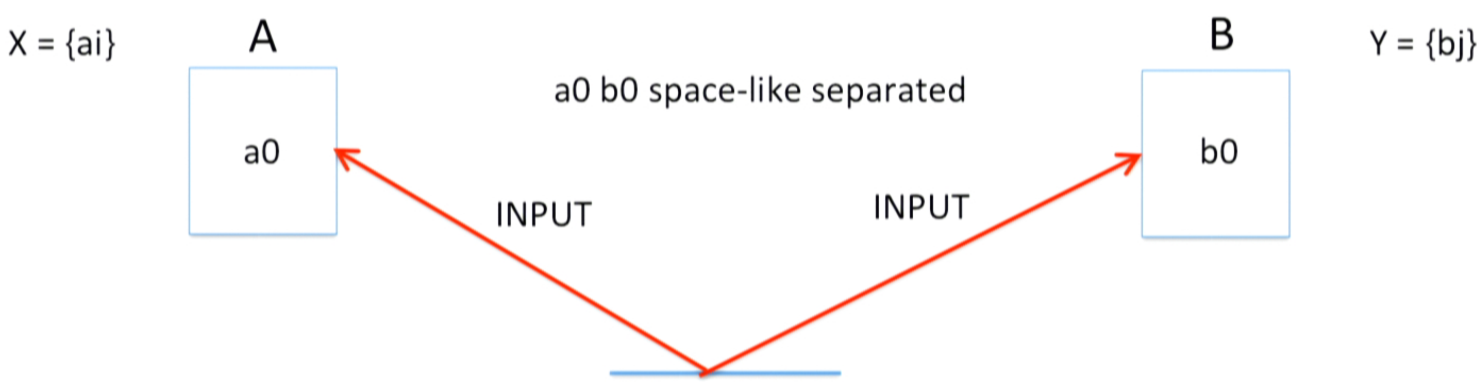


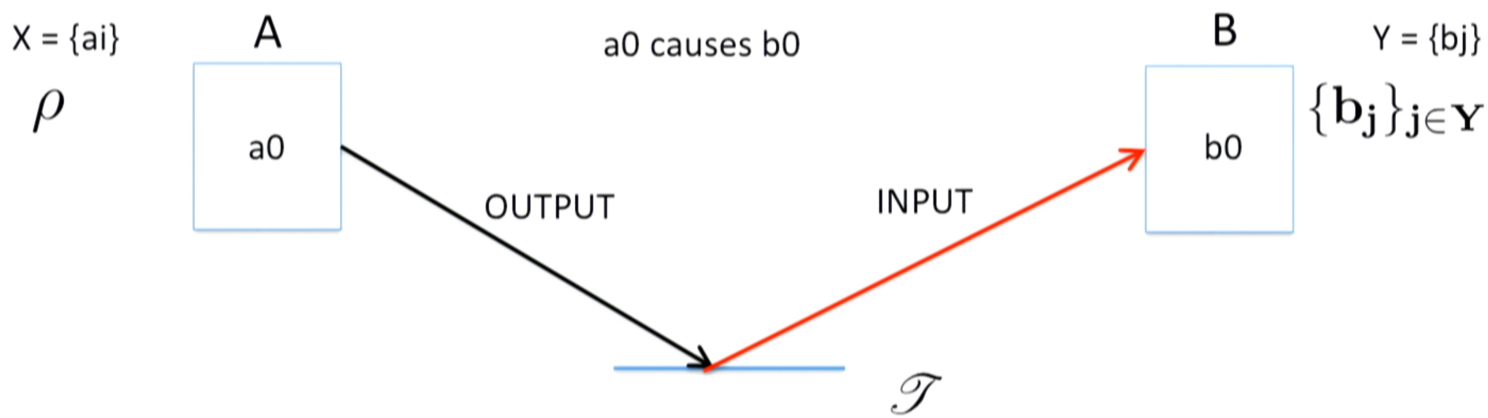
What we learn from it?

Since INPUT-OUTPUT structure = causal structure of outcomes

Principle of relativity of causal structure:

Two observers of the same probabilistic quantum experiment can assume a different causal structure for the space-time events constituting random outcomes and predict the same joint probability distributions.

Proof: part 1



$$\rho = \sum_{i\in A} \mathrm{Tr}[\mathbf{a_i}\rho]\,\frac{\sqrt{\rho}\,\mathbf{a_i}\sqrt{\rho}}{\mathrm{Tr}[\mathbf{a_i}\rho]} \qquad \{\mathbf{a_i}\}_{i\in X} \ \text{POVM}$$

$$\mathscr{T}(\rho) = \mathrm{Tr}_A[\sum_{m,ef,cd} K^m_{ef} K^{m*}_{cd}\sqrt{\rho}|c\rangle_{A\,A}\langle f|\sqrt{\rho}\otimes|e\rangle_{B\,B}\langle d|]$$

$\{\mathbf{b_j}\}_{\mathbf{j}\in\mathbf{Y}}$ POVM

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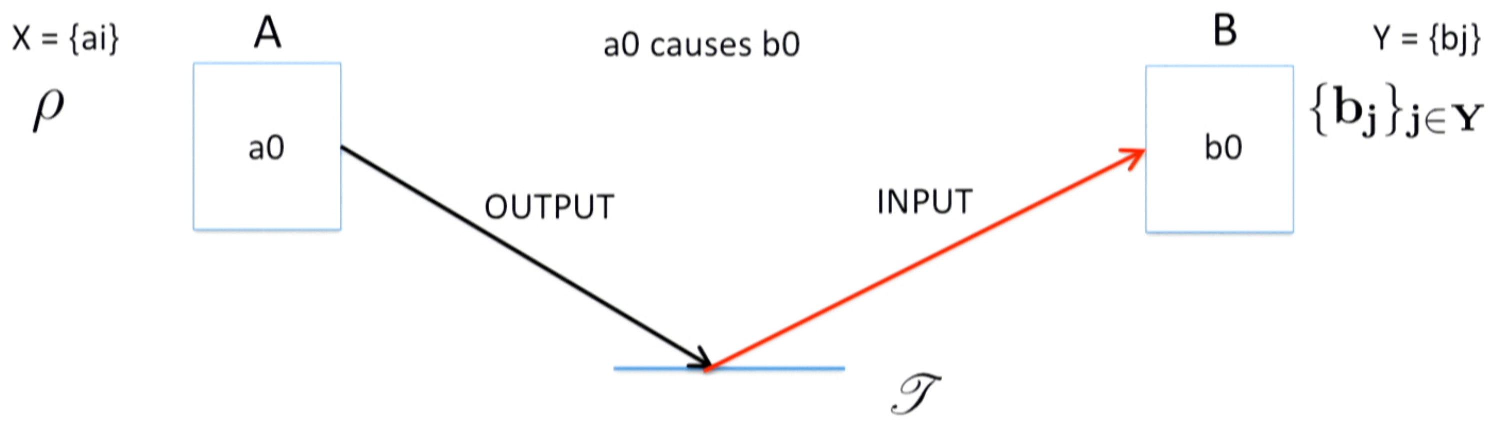

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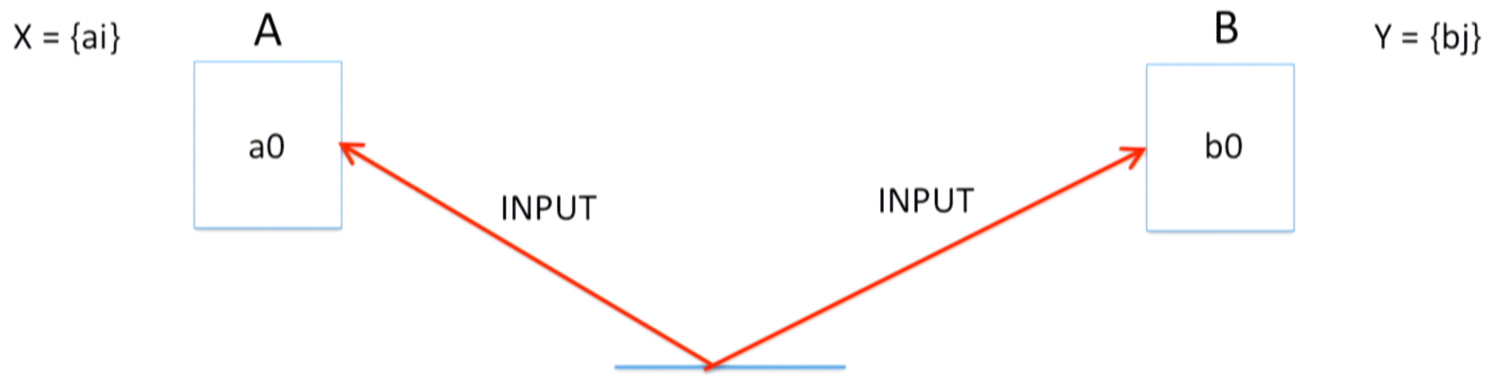
Proof: part 2

Define the operator $\mathscr{T}_\rho := \sqrt{\rho} \otimes I_B [\sum_m (K^m \otimes K^{m\dagger})] \sqrt{\rho} \otimes I_B$

where $\mathscr{T}(\rho) = \mathrm{Tr}_A[\mathscr{T}_\rho]$



$$p_1(a_{i_0}, b_{j_0}) = \mathrm{Tr}_B[\mathbf{b_{j_0}} \mathrm{Tr}_{\mathbf{A}}[\mathscr{T}_\rho \mathbf{a_{i_0}}]]$$

$$\tau_{AB} = \mathscr{T}_\rho^{T_A} \quad \mathbf{a_{i_0}}' = \mathbf{a_{i_0}}^T$$

$$p_2(a_{i_0}, b_{j_0}) = \mathrm{Tr}_{AB}[\mathbf{a_{i_0}}^T \otimes \mathbf{b_{j_0}} \mathscr{T}_\rho^{T_A}]$$

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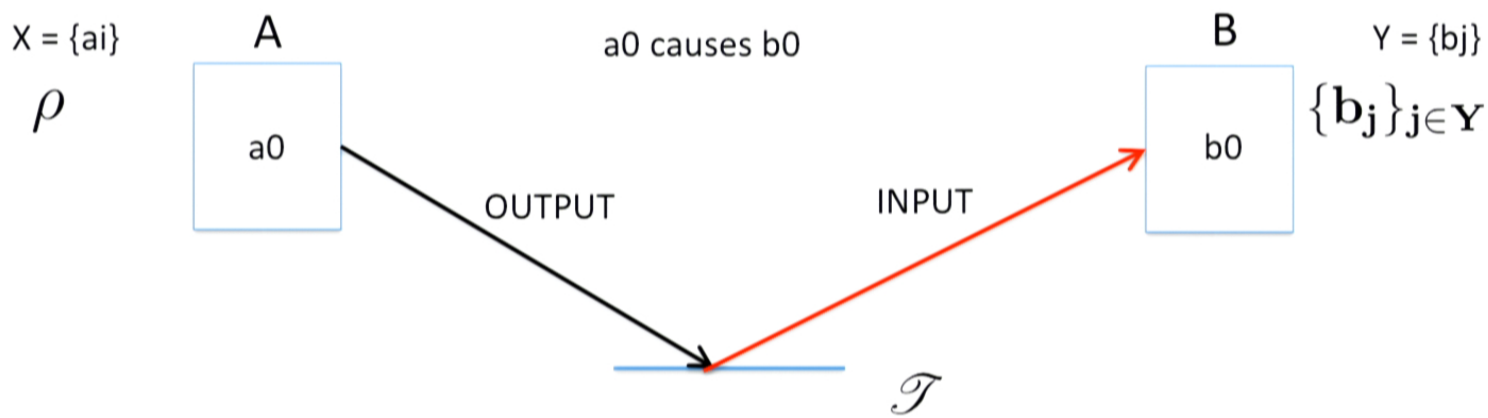

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$\{\mathbf{b_j}\}_{\mathbf{j}\in\mathbf{Y}}$ POVM

Proof: part 2

Transformation rule: If a system s with hilbert space Hs is an input for an observer and an output for another observer, the operators involving Hs used by one observer are the transpose on Hs of those used by the other observer.

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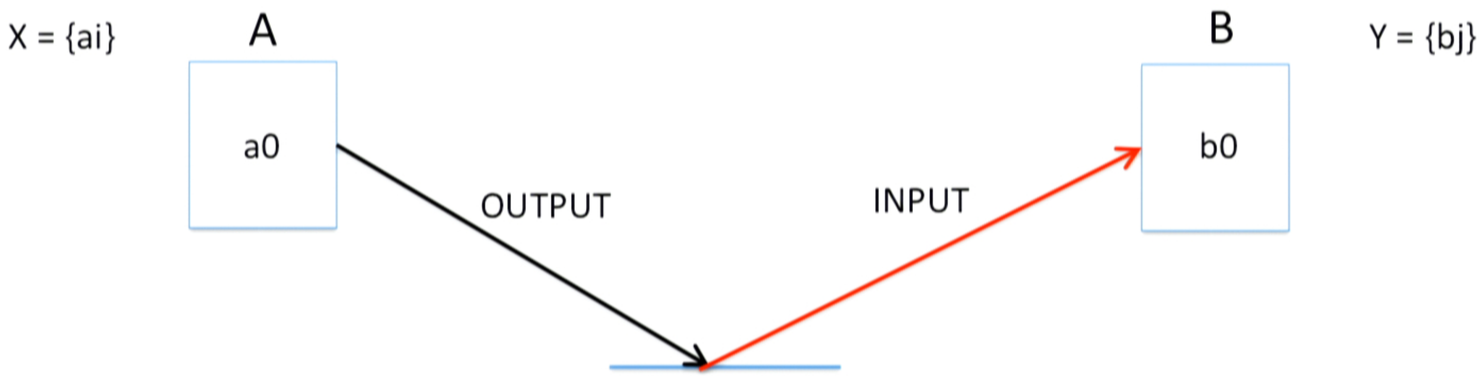

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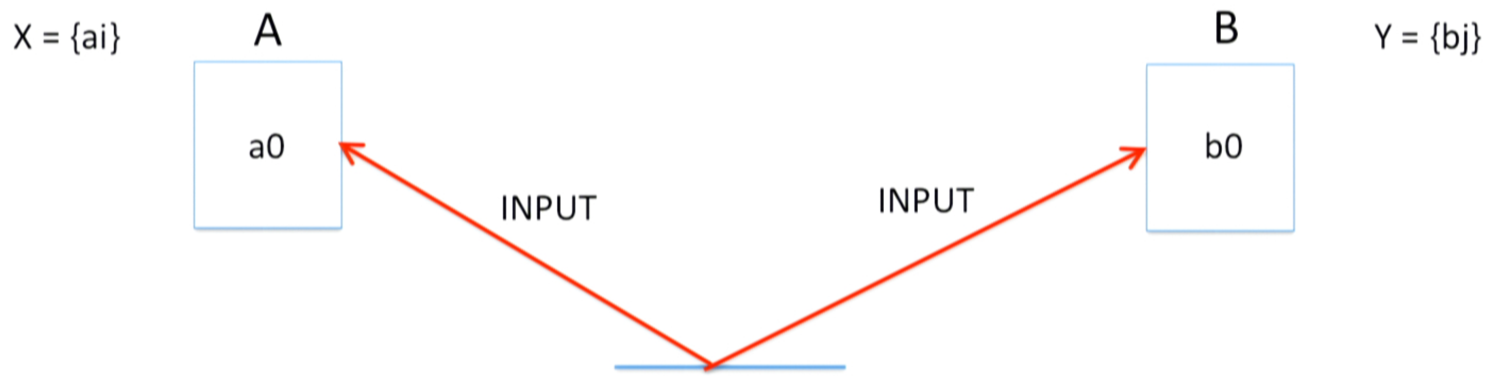

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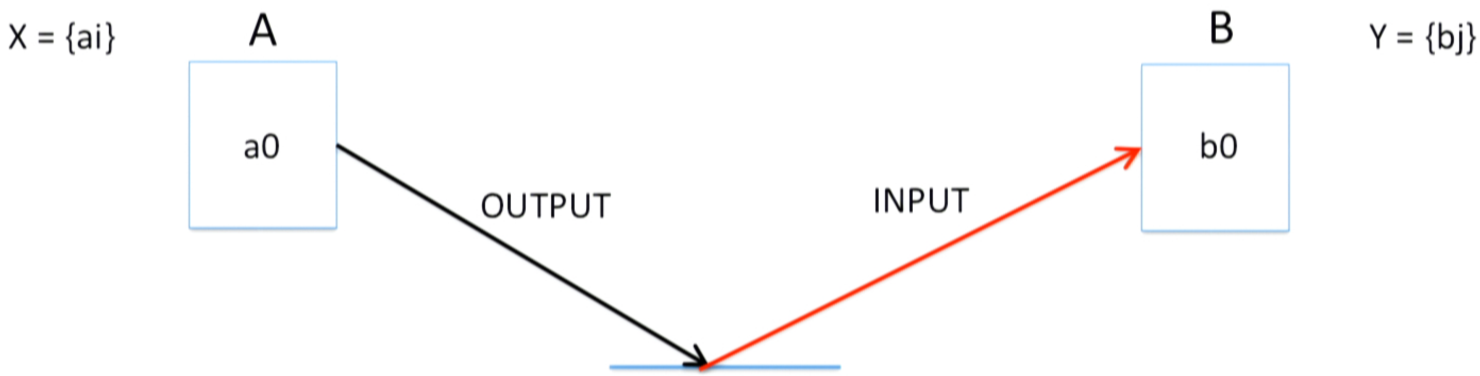

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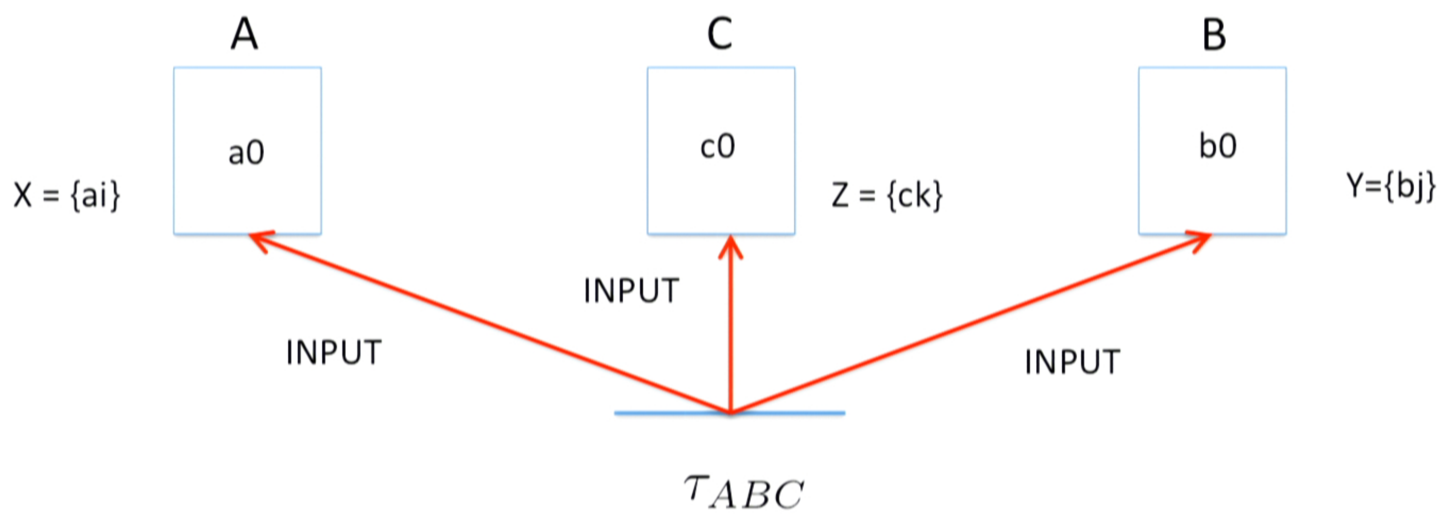
Proof: part 3

What if more devices are involved?



Two foundational principles for quantum theory

- **Causality:** Input/output structure of devices involved in an experiment determines the causal structure of outcomes that happen on devices

- **Relativity of Causal Structure:** Two observers looking at the same quantum experiment and assuming a different causal structure for the outcomes happening on the devices cannot become aware of differences in their respective probabilistic predictions

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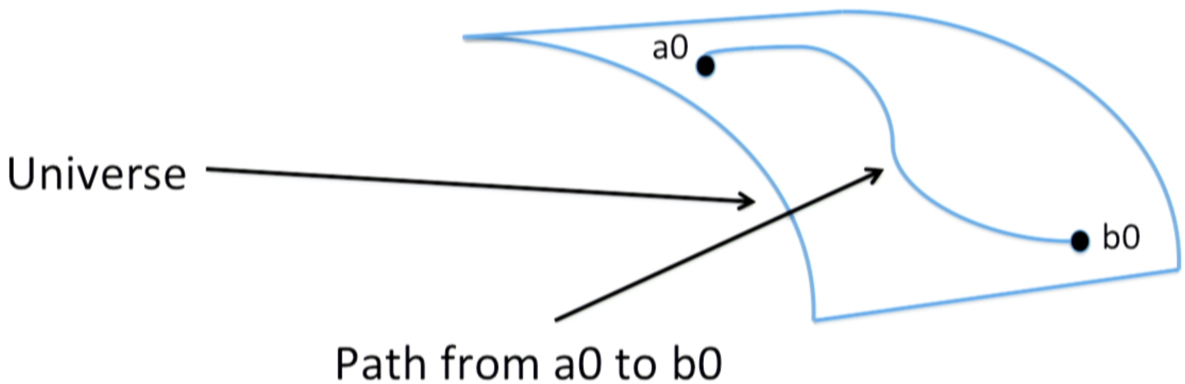
Extending relativity of causal structure to universal principle

General relativity theory:



If space-like path then a0 and b0 are space-like separated

If time-like path then it can be that a0 and b0 are one the cause of the other

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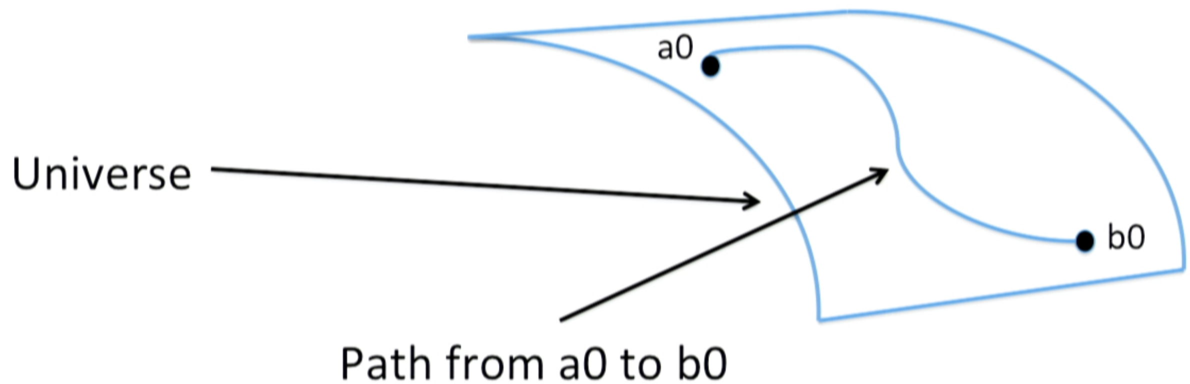

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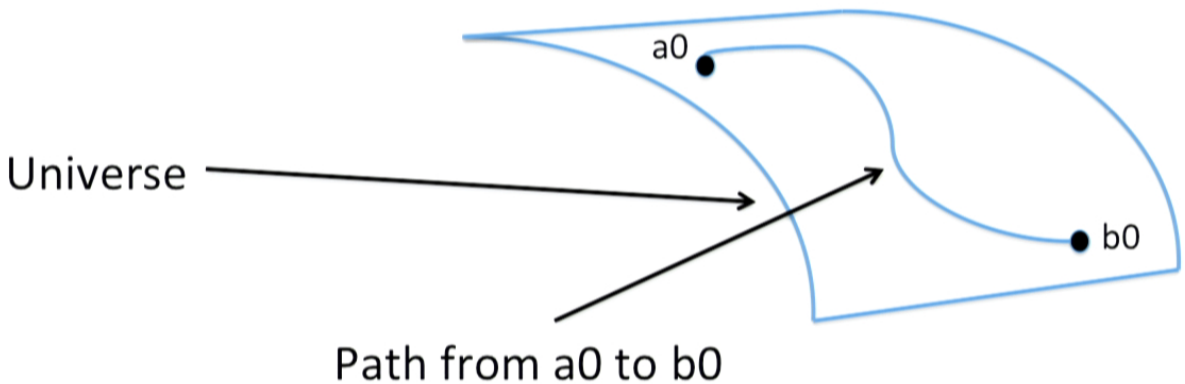

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Vanishing of the covariant derivative (what determines the path)

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0$$

Levi-Civita connection (tensor needed to write the above equation)

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

Einstein equation (determining the metric and the L-C connection)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = kT_{\mu\nu}$$

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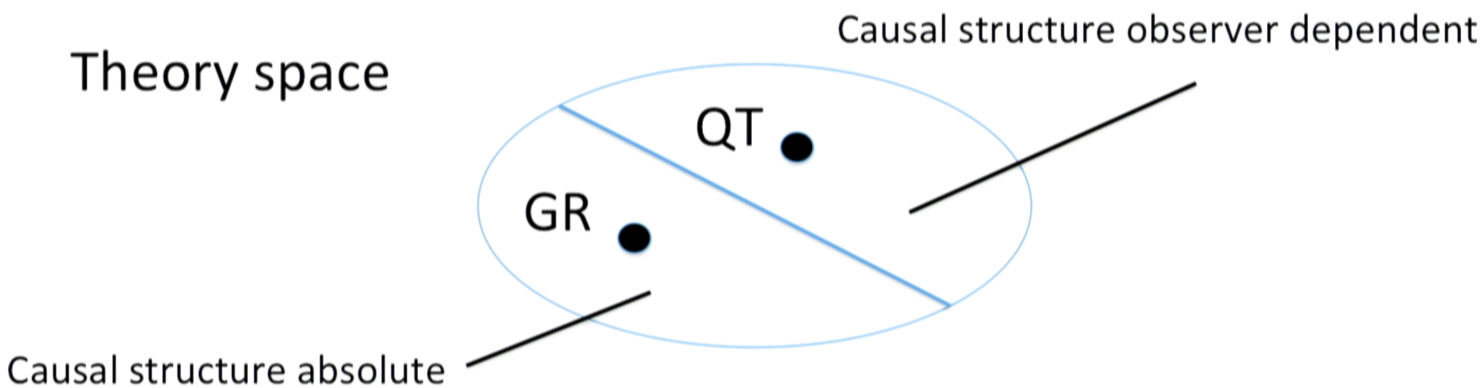
- In QT, the causal structure of (probabilistic) events may be regarded as an observer dependent property

- In GR, the causal ordering of events is related via Einstein's equations to a physical objective quantity giving rise to a unique metric tensor. Causal structure is an absolute property.



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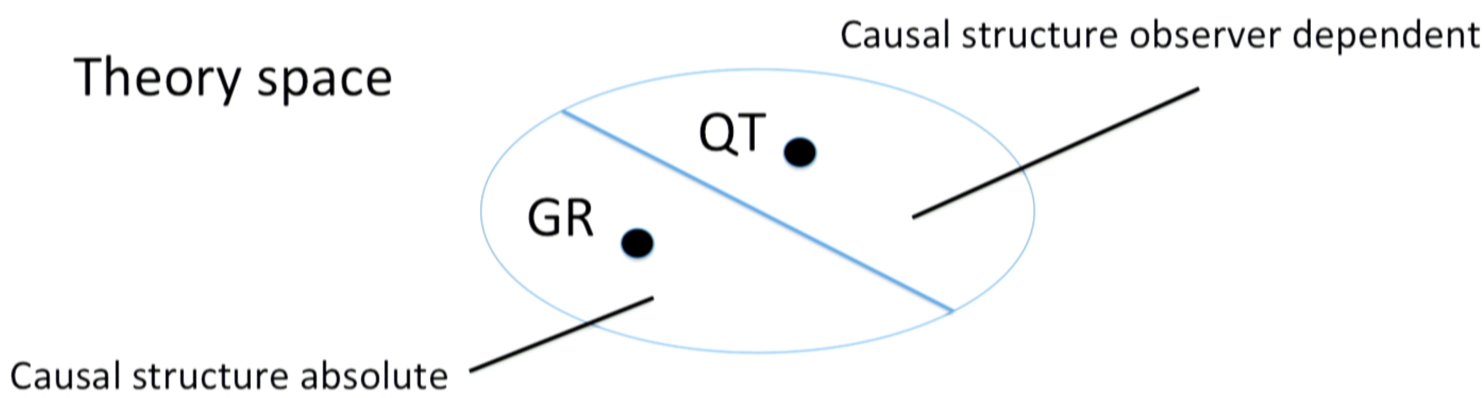
Theory space



Extending relativity of causal structure to universal principle

Dark energy:

Measuring the curvature of space-time

Direct method: Measuring energy density from direct observations of galaxies and clusters

Indirect method: Anisotropies of the Cosmic Microwave Background

The indirect method leads to the conclusion that the universe is flat; the direct method leads to the conclusion that energy density is only the 30% of the amount needed for the universe to be flat

We have 70% of missing energy

If we attribute the missing energy to vacuum energy we overestimate the missing energy for 120 ORDERS OF MAGNITUDE!