Title: Quantum Signatures of Chaos

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Abstract:

Quantum Signatures of Chaos

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Introduction

"There's no such thing as quantum chaos!"

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• Classical chaos

Characterized by **sensitivity to small perturbations**:
Divergence of initially close trajectories in phase space.
Rate of divergence quantified by the Lyapunov exponent.

• Quantum chaos

How can there be chaos in the quantum regime?

- Uncertainty principle does not allow well-defined trajectories in phase space
- Evolution of the wave function described by the Schrodinger equation is linear and unitary. Where is the sensitivity to perturbations?
- How can we observe chaos in the quantum world?

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- **Quantum chaos:** study of quantum systems that are classically chaotic to explore the connections between quantum and classical mechanics

- Old field, new questions: effect of classical chaos on **entanglement, decoherence, fidelity decay, tomography**

- Relevant to implementation of realistic quantum computing algorithms

- Relatively few experiments: challenges of state preparation, quantum control and quantum state reconstruction

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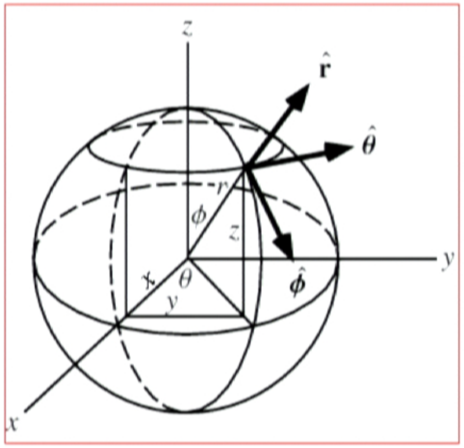
A Kicked Top

Consider a point on a sphere which is the tip of a vector **L.**

At regular intervals, apply a **rotation** about the **y axis** by an angle **p**.

Also apply a **rotation** about the **x axis** by an angle $\boldsymbol{\theta} \mathbf{L_x}$ that depends on its x coordinate $\mathbf{L_x}$.

$$H = \frac{\kappa}{2j\tau} L_x^2 + pL_y \sum_n \delta(t - n\tau)$$



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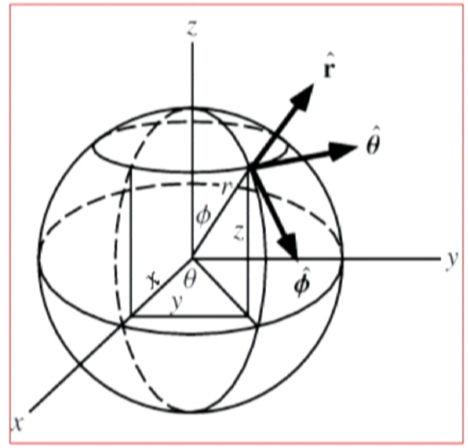

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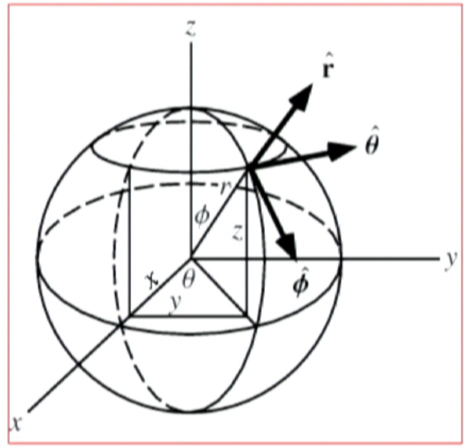

$$H = \frac{\kappa}{2j\tau} L_x^2 + pL_y \sum_n \delta(t - n\tau)$$

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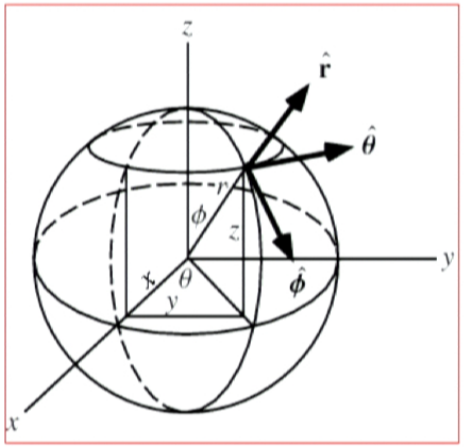
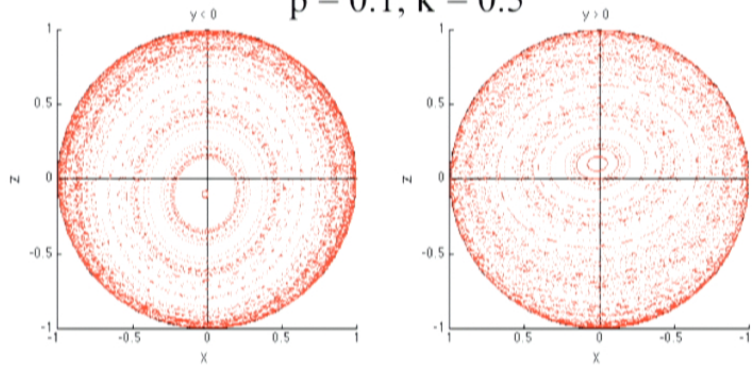


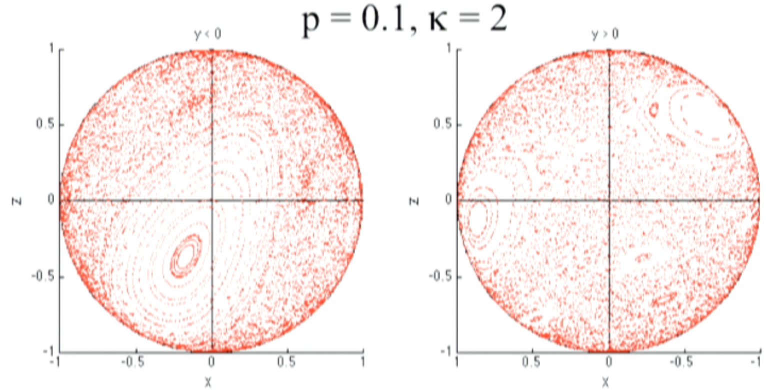
Classical Chaos

Different initial vectors will trace out different trajectories on the sphere.

p = 0.1, κ = 0.5



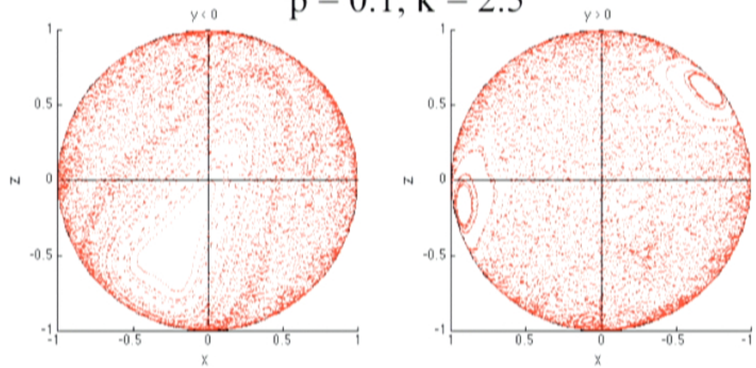
p = 0.1, κ = 2



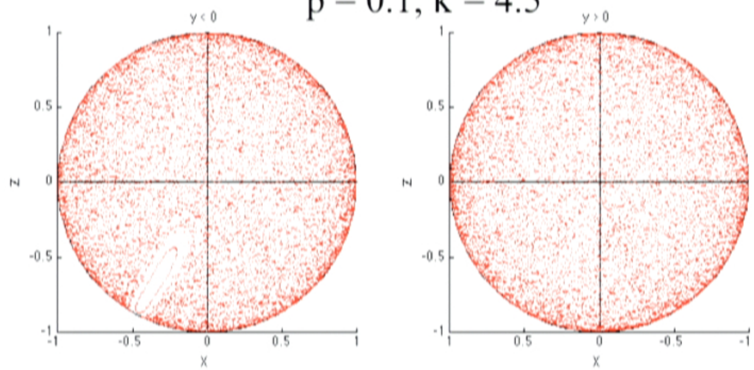
Classical Chaos

Dynamics eventually becomes completely chaotic as κ is increased

$p = 0.1, \kappa = 2.5$



$p = 0.1, \kappa = 4.5$



Quantum Kicked Top

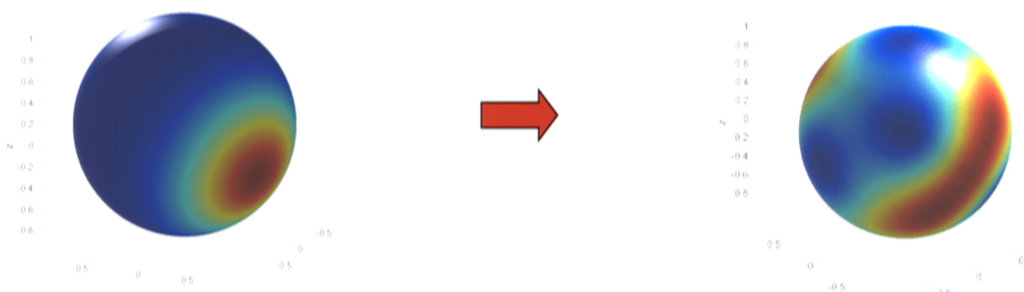
Start with a spin coherent state (rotation of state |j,j>)

$$|\theta,\phi\rangle = \exp\left[i\theta\left(J_x\cos\phi - J_y\sin\phi\right)\right]|j,j\rangle$$

$$\Delta J_i \Delta J_k = \frac{j}{2}$$

Apply the same rotations around the x and y axes

$$H = \frac{\kappa}{2j\tau}J_x^2 + pJ_y\sum_n \delta(t-n\tau)$$



H. Frahm, H. J. Mikeska, Z. Phys. B: Condens. Matter 60, 117 (1985)
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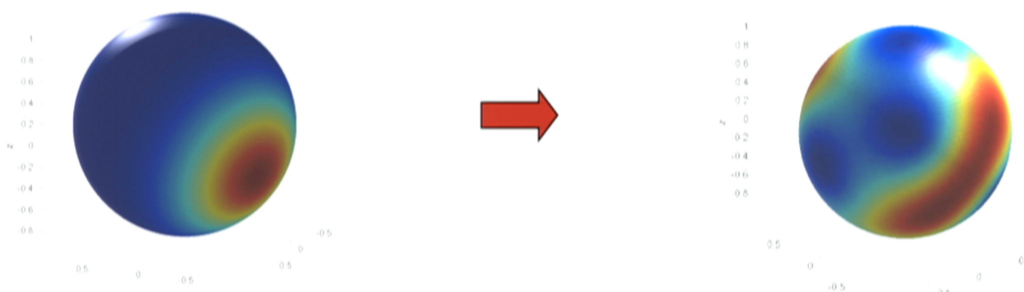

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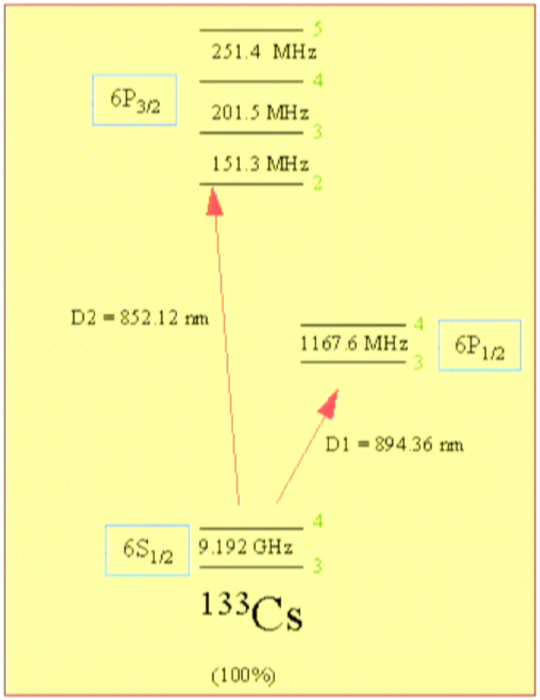
Experimental Setting

Cold Cesium atoms interacting with lasers and magnetic fields

-rich toolbox for quantum control
rich internal structure of atoms allow many knobs: intensities, detunings, magnetic fields

-clean model system
can control noise by laser detuning

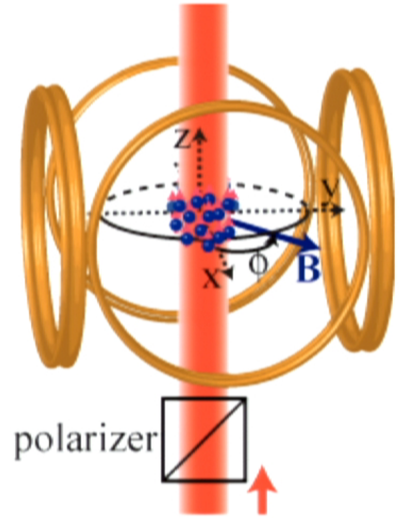
-Powerful measurement tools
Faraday rotation of probe field allows complete measurement of atomic spin (electron + nuclear) state



Experimental Setting

Initial State Preparation

- Prepare atoms in spin down state by cooling and optical pumping
- Apply magnetic field to rotate state on the Bloch sphere to an arbitrary **spin coherent state**



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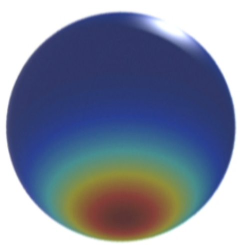
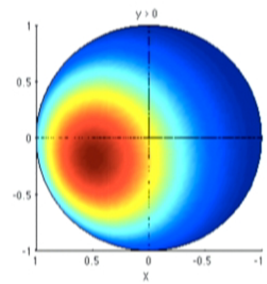




$$Q(\theta,\phi) = \frac{2F+1}{4\pi} Tr\left[\rho|\theta,\phi\rangle\langle\theta,\phi|\right]$$

S. Chaudhury et al, Phys. Rev. Lett. 99, 163002 (2007)

Applying the QKT Hamiltonian

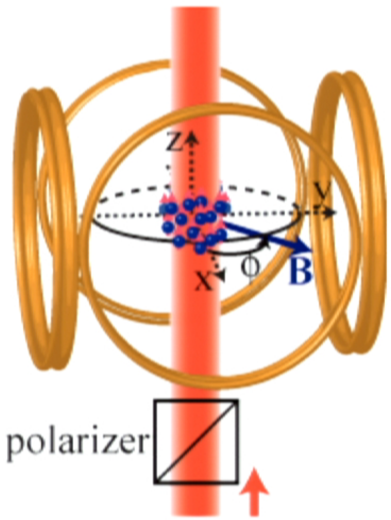
$$H = \frac{\kappa}{2j\tau} J_x^2 + pJ_y \sum_n \delta(t - n\tau)$$

- Interaction of a laser field with **cesium atoms**

$$H_{int} = -\mathbf{E}^* \cdot \vec{\vec{\alpha}} \cdot \mathbf{E}$$

- Linearly polarized light gives a **non-linear term**

$$H_{int} = \frac{2}{3} U_0 I - \alpha \frac{\gamma_s}{\hbar} \left| \vec{\varepsilon}_p \bullet \mathbf{F} \right|^2$$



γ_s : photon scattering rate (depends on intensity, detuning)

α : depends on atomic species, laser frequency

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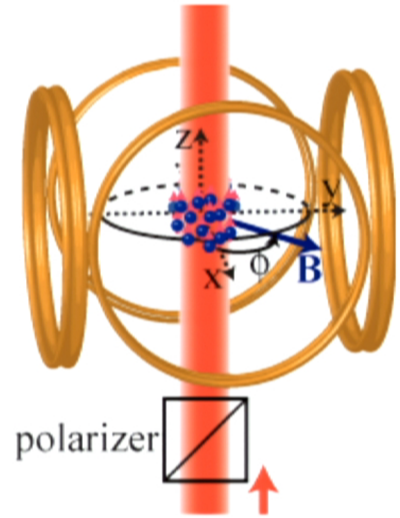
$$H_{\text{int}} = -\alpha \frac{\gamma_s}{\hbar} \left| \vec{\varepsilon}_p \bullet \mathbf{F} \right|^2 + g_F \mu_B \mathbf{B}(t) \cdot \mathbf{F}$$

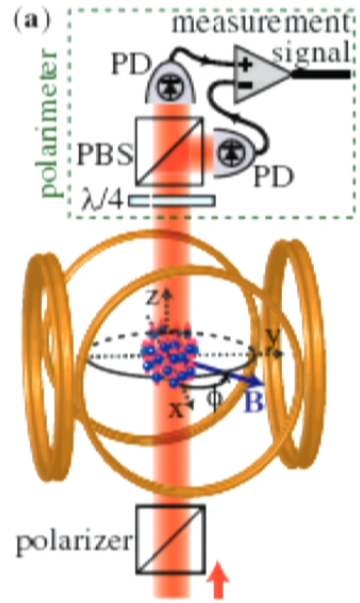


- Quantum kicked top can be realized by using a periodically pulsed magnetic field

γ_s : photon scattering rate (depends on intensity, detuning)
α : depends on atomic species, laser frequency

Measurement of the Atomic Spin State

- Apply known unitary to atoms
- Measure spin dependent polarization rotation of probe laser field. Meter for $\langle F_z \rangle$
- Use the continuous measurement signal $\langle F_z(t) \rangle$ to estimate the initial quantum state
- Single shot state tomography ! (almost)

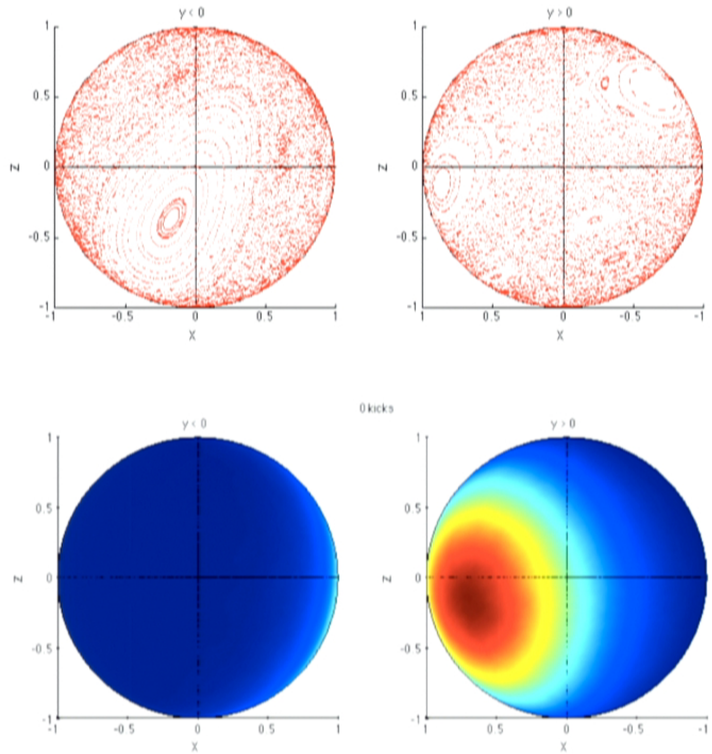


A. Silberfarb, P. S. Jessen, and I. H. Deutsch, PRL. **95**, 030402 (2005)
G. A. Smith, A. Silberfarb, I. H. Deutsch, and P. S. Jessen, PRL **97**, 180403 (2006)

Classical Dynamics in a Regular Island

Evolution of a classical distribution $P(\theta,\varphi)$ centered on a regular island

The islands are classically isolated



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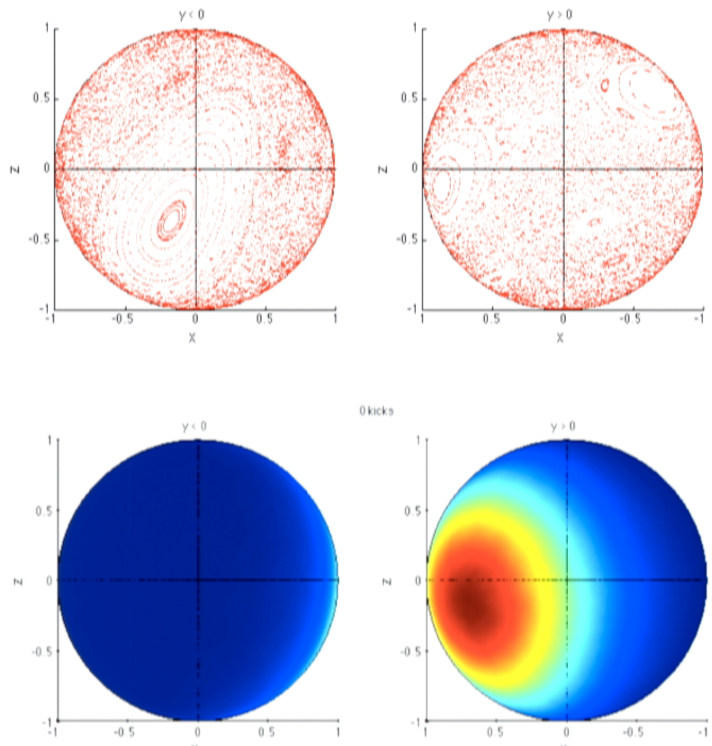

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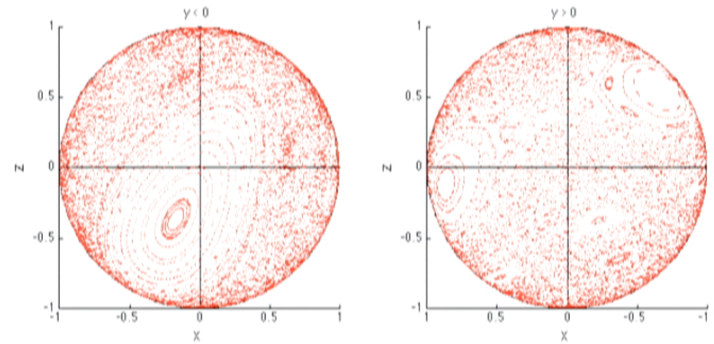
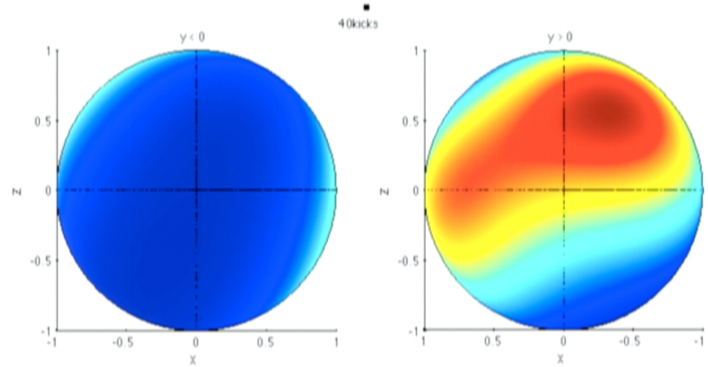
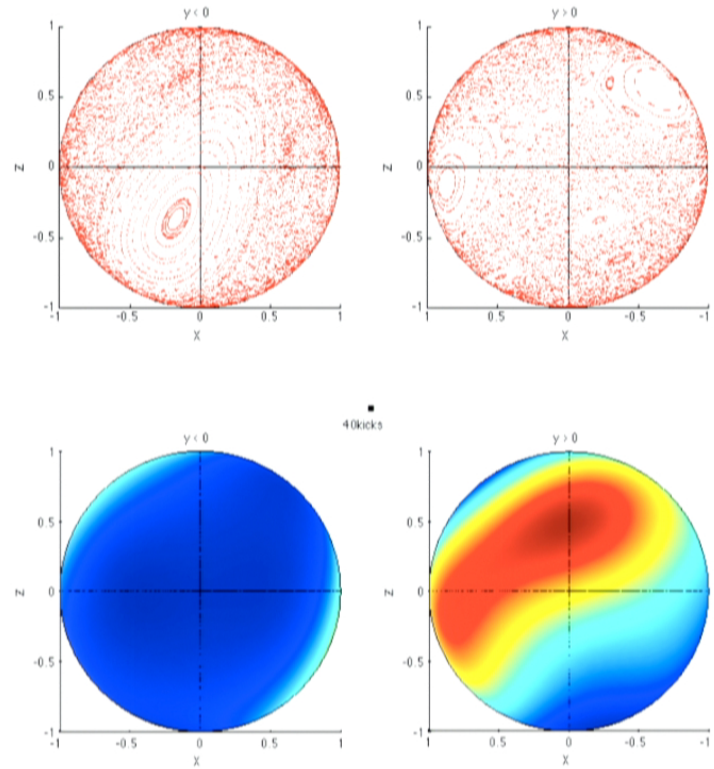


Evolution of a quantum wave function centered on a regular island

Simulation with full quantum master equation



Dynamical Tunneling: The Motion Picture



Evolution of a quantum wave function centered on a regular island

Experimental Data: Direct Phase Space Monitoring !

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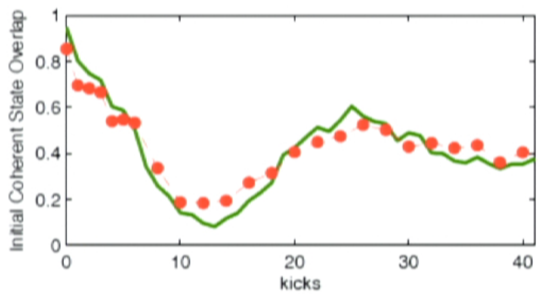
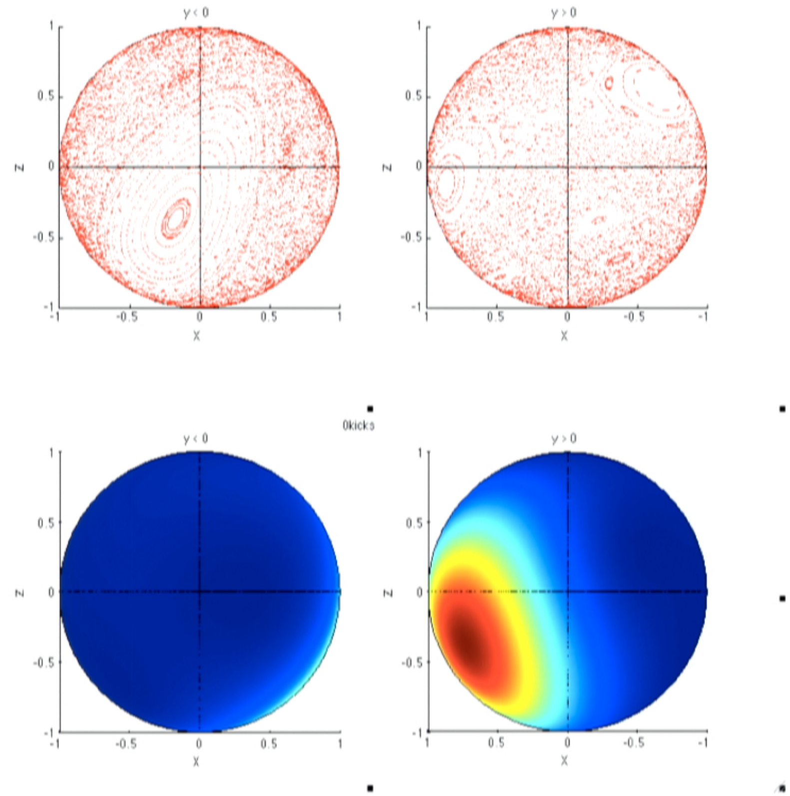
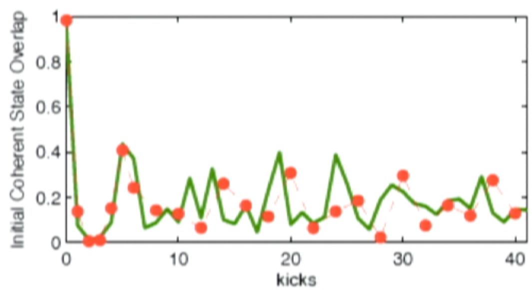




Evolution of a quantum wave function centered on a regular island

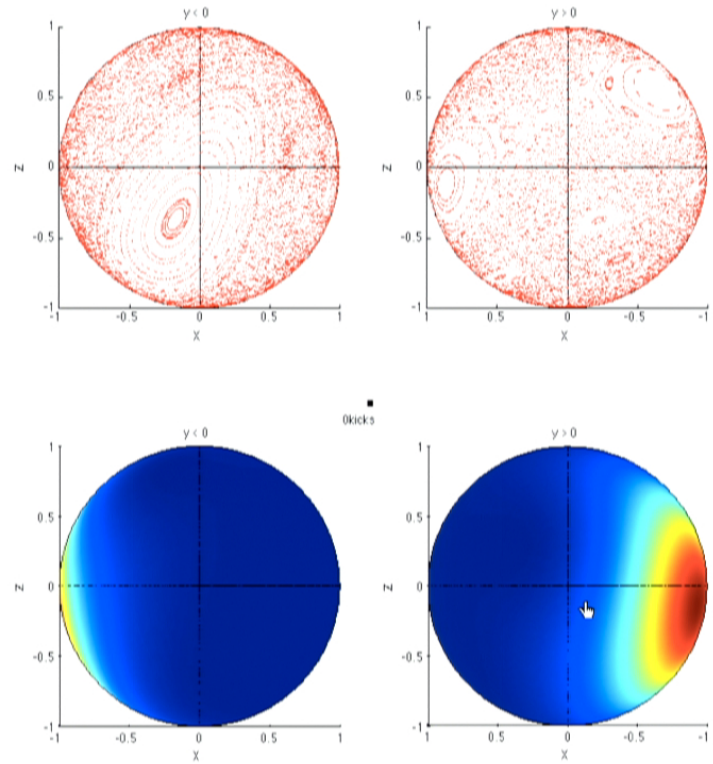
Experimental Data: Direct phase space monitoring of dynamical tunneling!

Quantum Dynamics in the Chaotic Sea

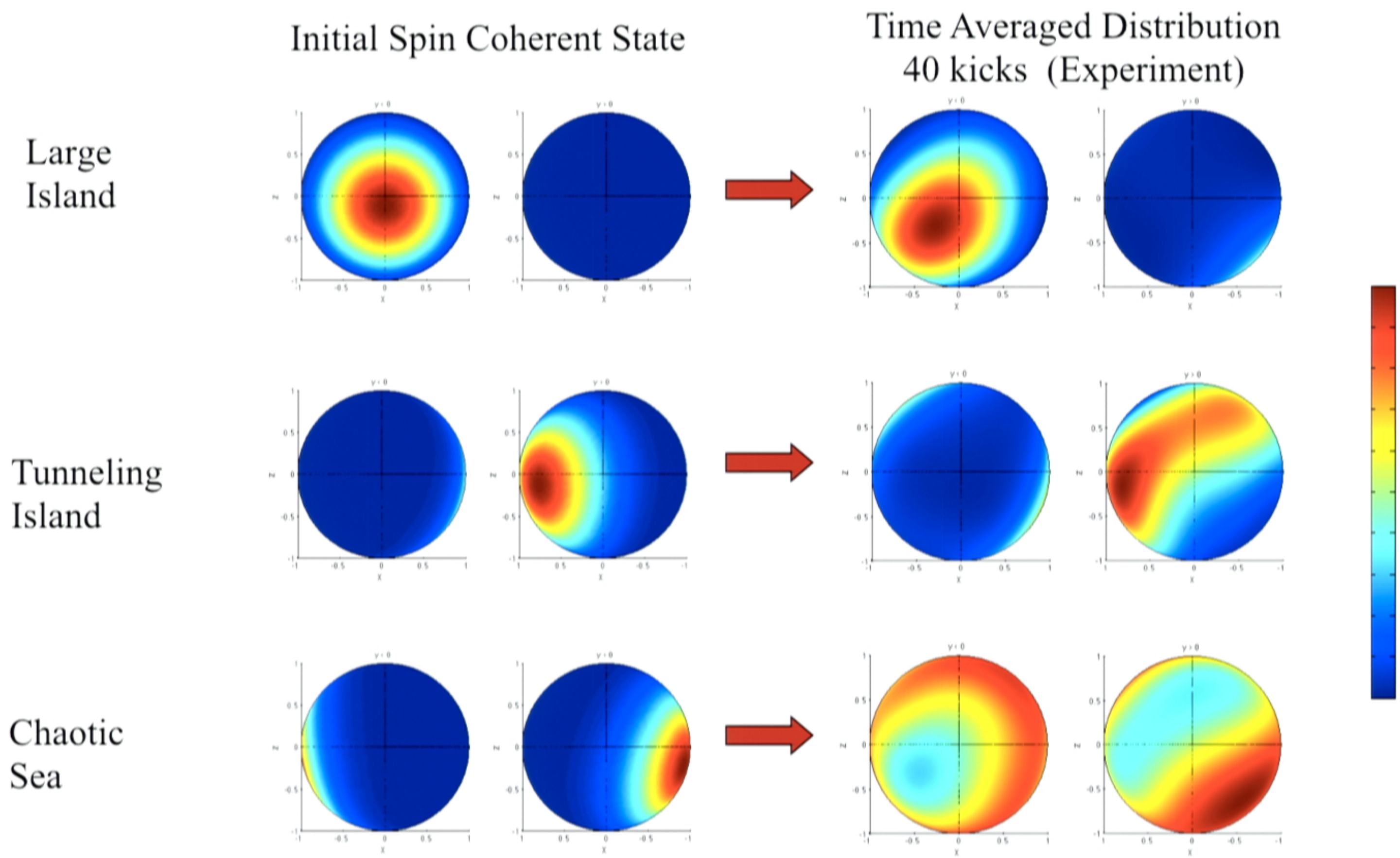


Evolution of a quantum wave function centered in the chaotic sea

Experimental Data: Direct phase space monitoring of rapid spreading in chaotic sea



Comparison of Different Initial States



Signatures of classical phase space structures

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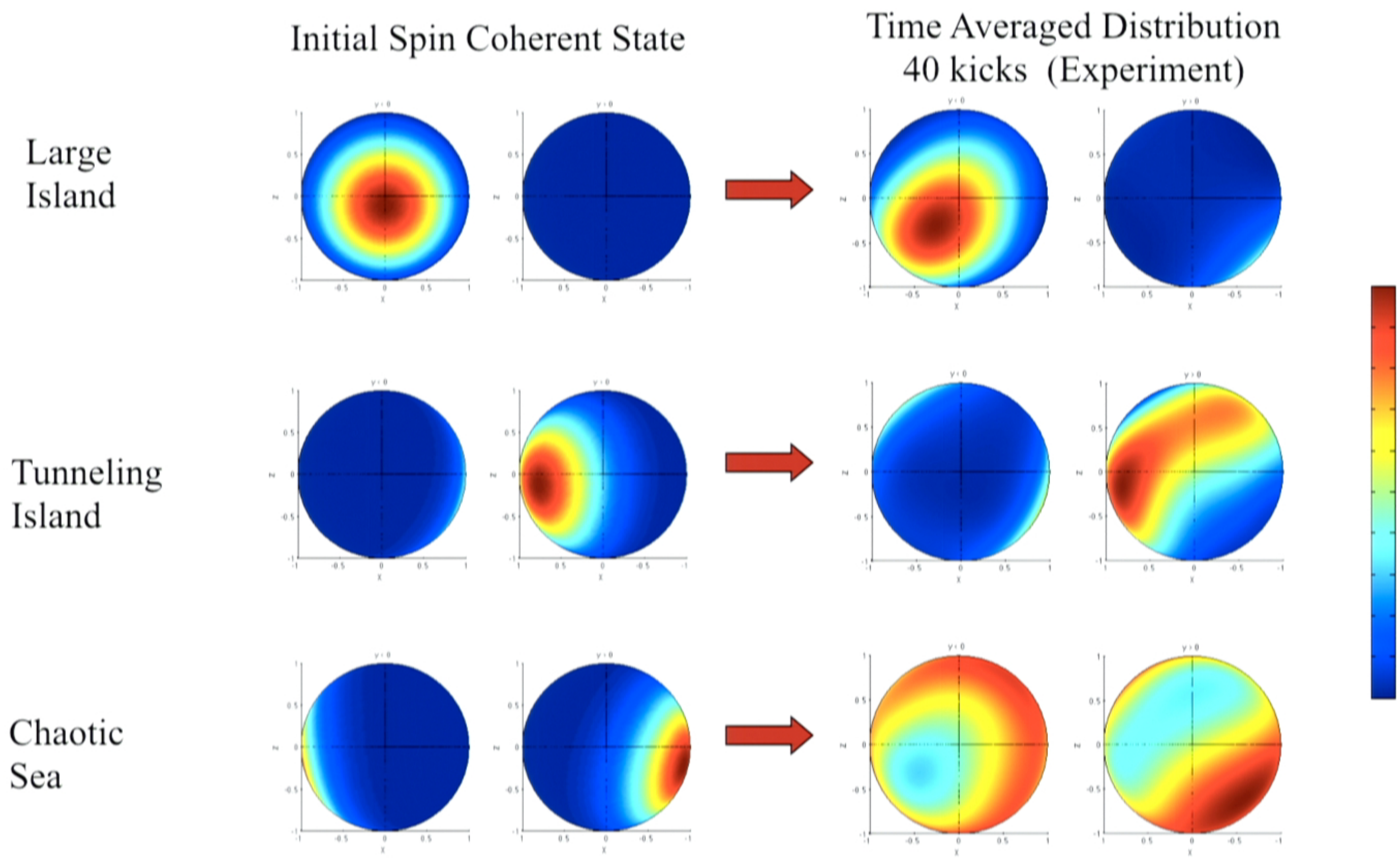
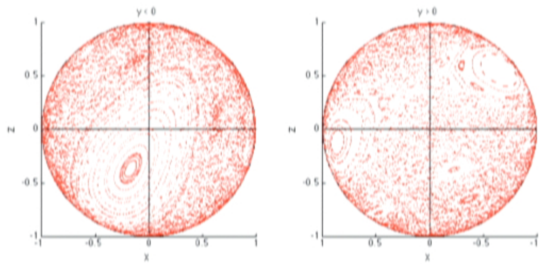


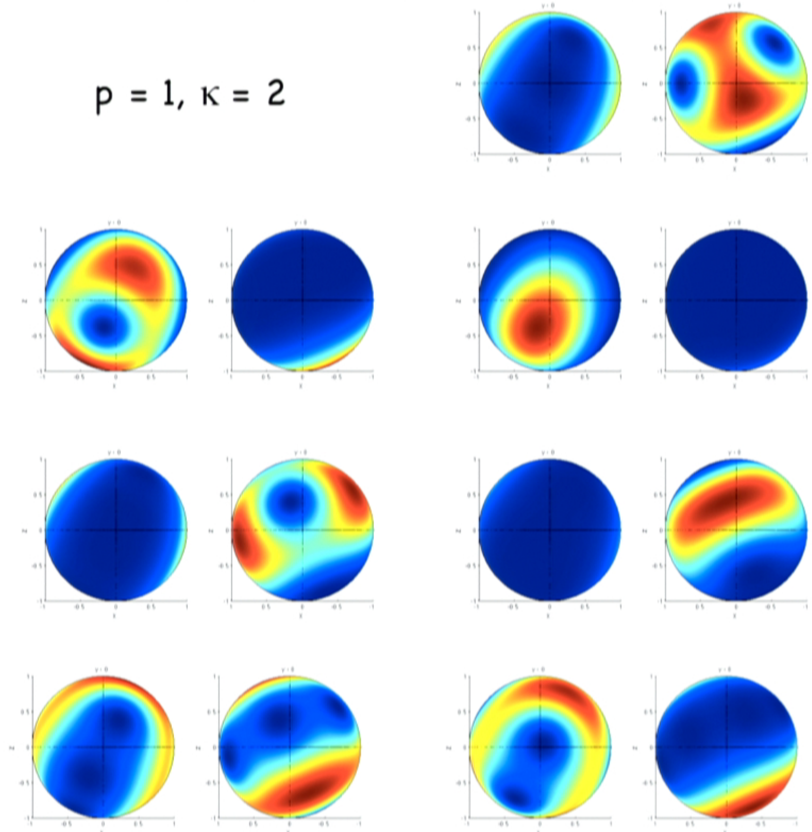
Signatures of classical phase space structures

Floquet Eigenstates

$$U = e^{-i\frac{\kappa}{2j}J_x^2}\, e^{-i\frac{\pi}{2}J_y}$$

p = 1, κ = 2



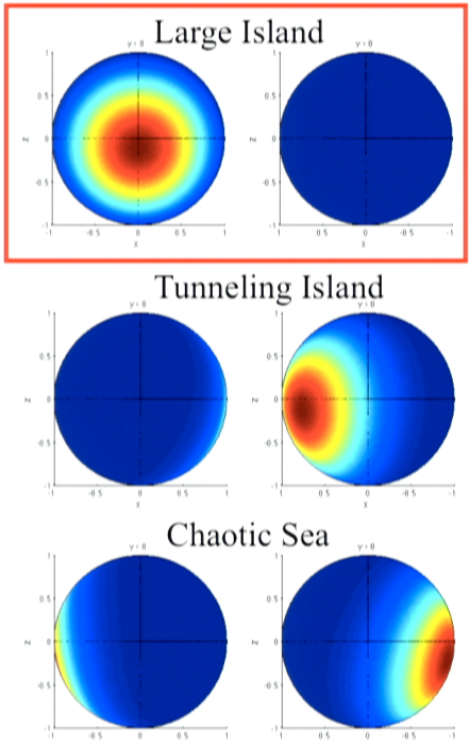


Floquet eigenstates reflect classical phase space structures

S.G, R. Stock, P. Jessen, R. Lal and A. Silberfarb, PRA 78, 042318 (2008)

Floquet Eigenstates

Initial spin coherent state



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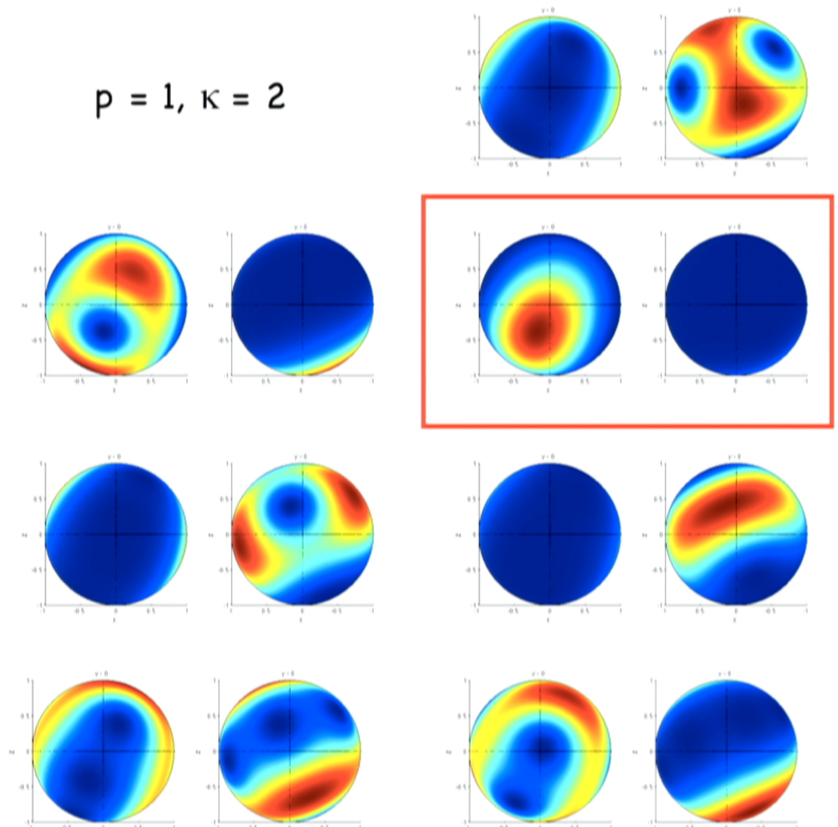
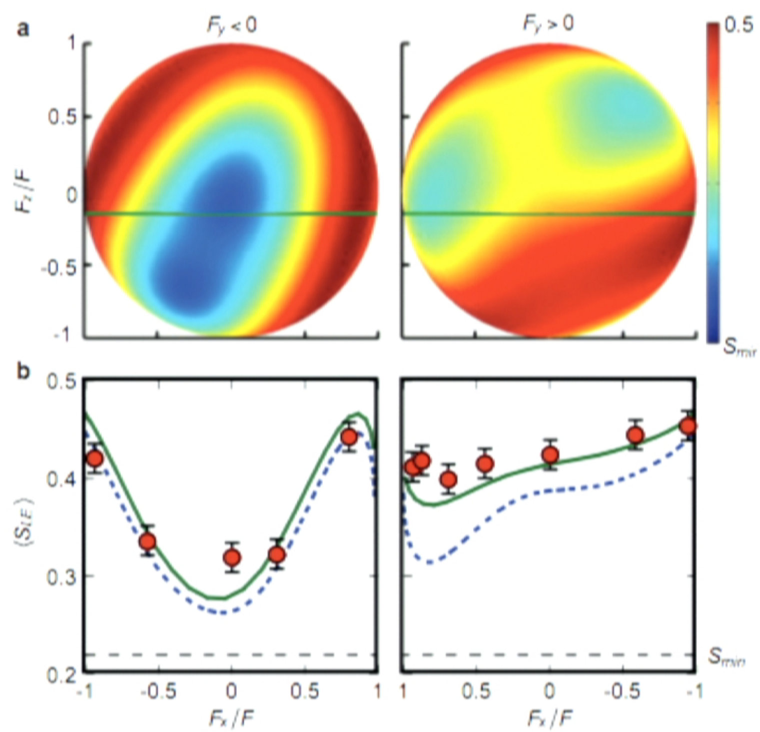


Dynamics depends on support of the initial state on regular vs chaotic eigenstates

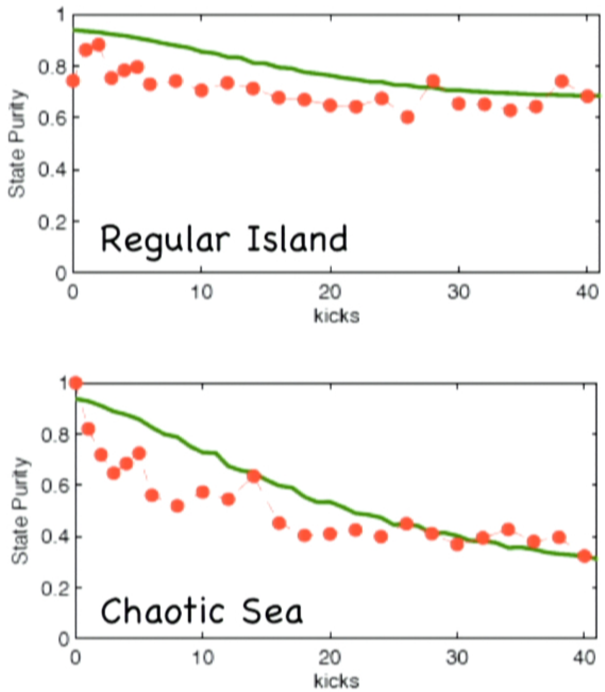
Time Averaged Electron-Nuclear Entanglement

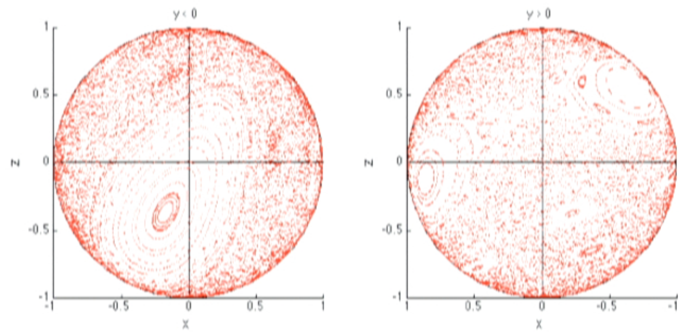
Initial state scanned across green line (Avg over 40 kicks)



Shows the boundary between regular and chaotic dynamics

Purity decay





• For the same scattering parameter, the **rate of purity decay** is faster for an initial state in a chaotic region than one in a regular region.

(W. H. Zurek, J. P. Paz, PRL 72, 2508 (1994))

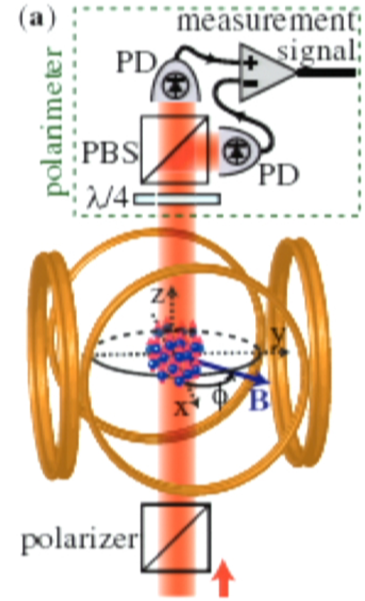
Fidelity of tomography

- Apply known unitary *U* to unknown state

- Measure time evolution of an observable.
Example: $\langle F_z \rangle$

- Use the continuous measurement signal $\langle F_z(t) \rangle$ to estimate the unknown initial state

- **How does the regular versus chaotic nature of *U* affect the estimation process ?**



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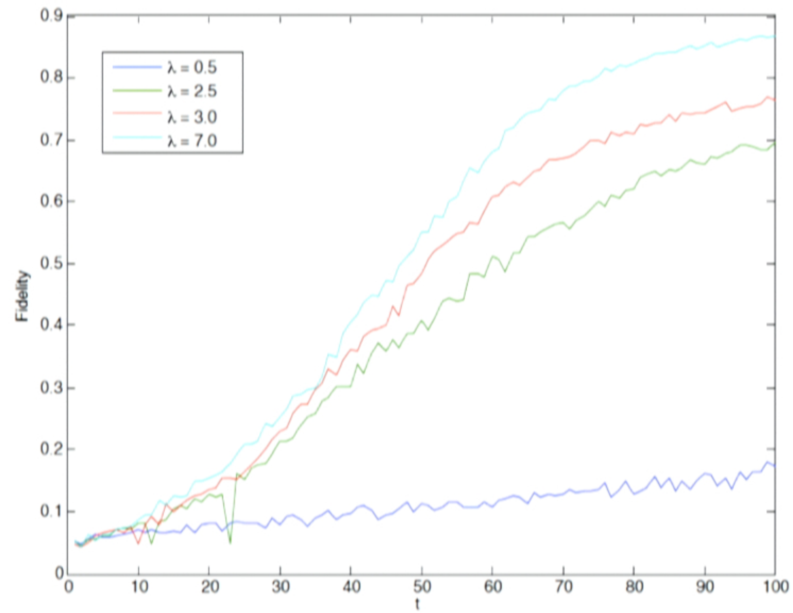
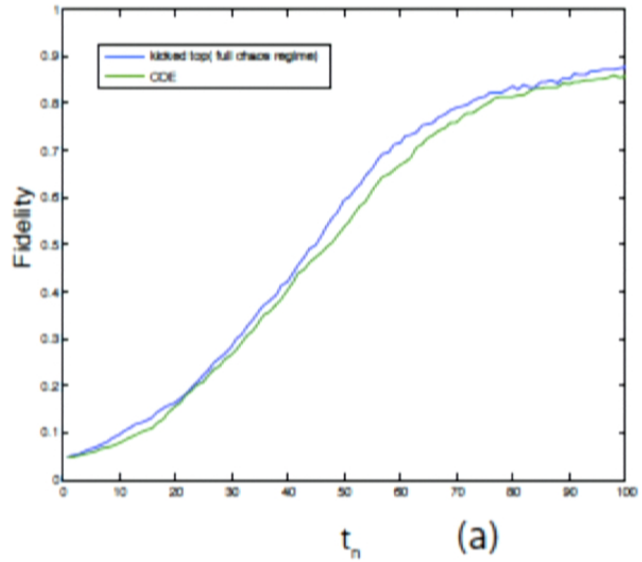
Fidelity of tomography: A new signature of chaos



- Average fidelity of reconstruction of 100 states for F=10.
- The rate of increase in fidelity increases with the degree of chaos of applied unitary U.

See PI Quantum Foundations Seminar by Dr. Vaibhav Madhok on Dec 11.

Fidelity of tomography: A new signature of chaos



A completely chaotic map has statistical properties described by a **Random Matrix** picked from an appropriate ensemble, depending on symmetries of the Hamiltonian.

Fidelity of tomography is confirmed by random matrix theory calculations

O. Bohigas, M. J. Giannoni, and C. Schmit, Phys. Rev. Lett **52**, 1 (1984.)
F. Haake, Quantum Signatures of Chaos (Spring-Verlag, Berlin, 1991.)

Exploring chaos with trapped ions

- Consider two ions in an anisotropic harmonic trap:

$$U = m\omega_0^2(X^2+Y^2) + m\omega_z^2 Z^2 + m\omega_0^2(x^2+y^2) + m\omega_z^2 z^2 + \frac{ke^2}{r}$$

R: Center-of-mass coordinate
r: Relative motion coordinate

Separating out the relative motion,

$$U = m\omega_0^2\rho^2 + m\omega_z^2 z^2 + \frac{ke^2}{\sqrt{\rho^2+z^2}}$$

- Hamiltonian system
- Nonlinear coupling between continuous variables

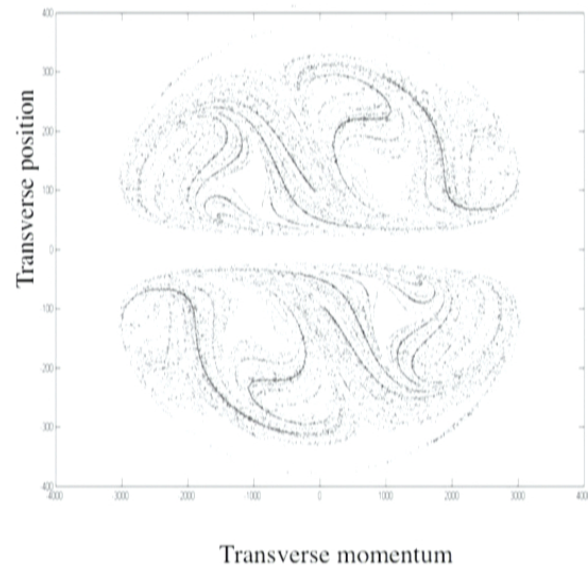
C. F. Roos et al., PRA **77**, 040302 (2008)

Exploring chaos with trapped ions

- **Classical chaos**

$$U = m\omega_0^2\rho^2 + m\omega_z^2 z^2 + \frac{ke^2}{\sqrt{\rho^2 + z^2}}$$

- Solve classical equations of motion
- Explore dynamics in experimentally accessible regimes
- Poincaré section shows a mixed phase space of regular and chaotic trajectories



Exploring chaos with trapped ions

- **Classical chaos**

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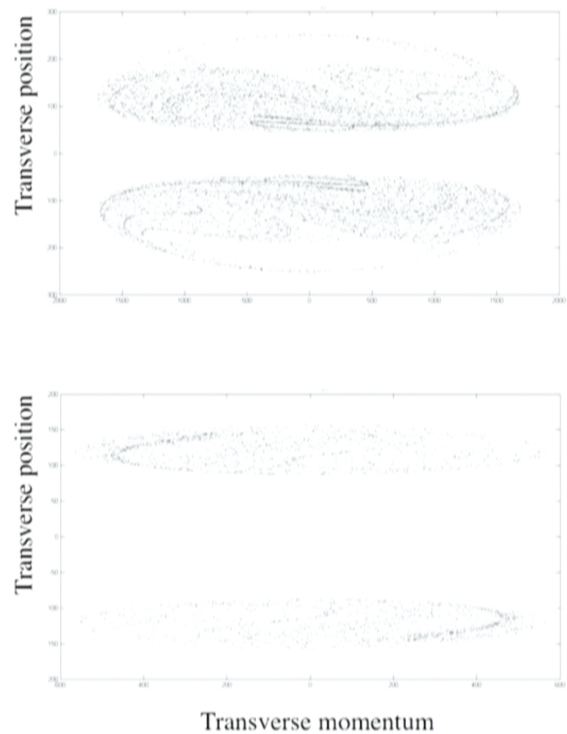
- Solve classical equations of motion
- Explore dynamics in experimentally accessible regimes
- Chaos increases with decreasing energy

Next steps:

- Characterize chaos more carefully (Lypunov exponents)
- Simulate quantum dynamics and explore phonon-phonon entanglement
- Include internal structure of ions



Summary

Cold atoms/ions interacting with light and magnetic fields provide a good testbed to explore quantum chaos.

- Direct observation of **dynamical tunneling.**

- Observation of **signatures of chaos** in **entanglement** dynamics in a deeply quantum regime.

- A new signature of chaos: **fidelity of tomography**

- Possibility of exploring new types of chaotic behaviour with trapped ions

Joint work with…

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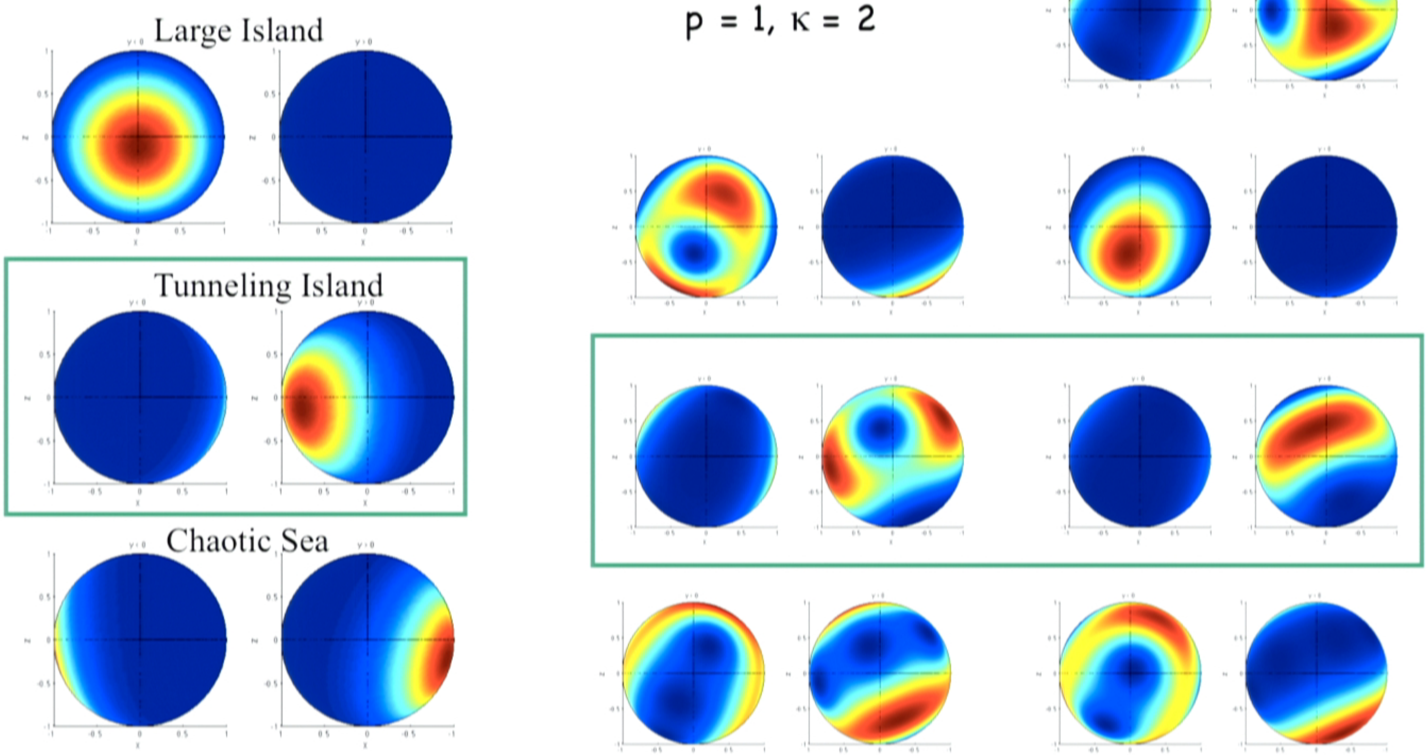
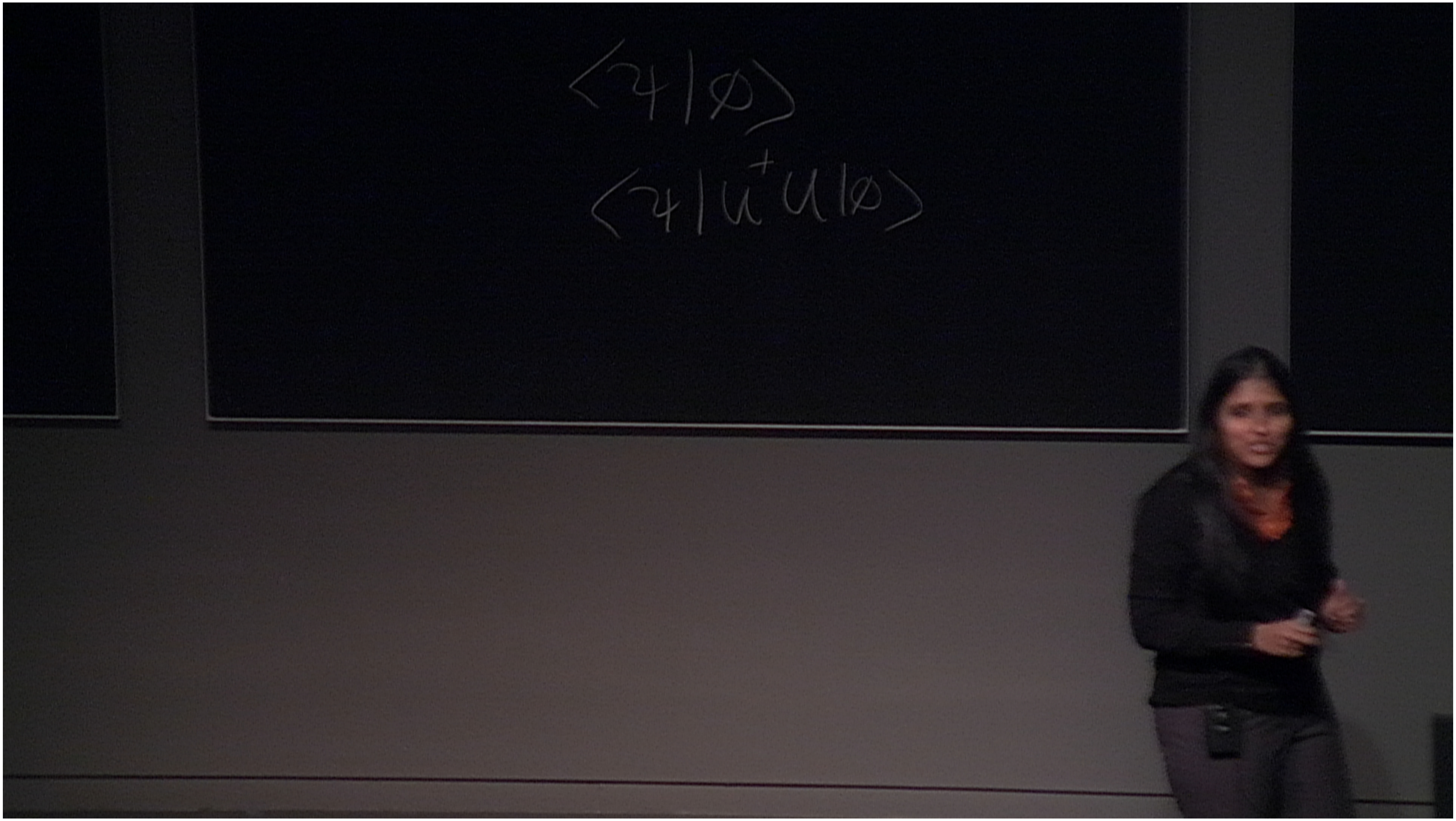
Floquet Eigenstates

Initial spin coherent state



Dynamics depends on support of the initial state on regular vs chaotic eigenstates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