Title: What's an unknown POVM, and what does a two qubit state look like?

Date: Nov 06, 2012 03:30 PM

URL: http://pirsa.org/12110068

Abstract: If probabilities represent knowledge, what is an "unknown
probability"? De Finetti's theorem licenses the view that it is simply a
convenient metaphor for a certain class of knowledge about a series of
events. There are quantum versions for "unknown states" and
"unknown
channels". I will explain how "unknown measurements" can be
rehabilitated
too.

I will then move to a totally different topic. The Bloch sphere is handy
for representing qubit states, but the equivalent for two qubits is
15-dimensional! I will advocate instead drawing the set of states that Bob
can steer Alice to, the "steering ellipsoid". I will show how
entanglement
and discord look from this perspective, and outline a geometric
classification of separable two qubit states.

What's an unknown POVM, and what does a two qubit state look like?

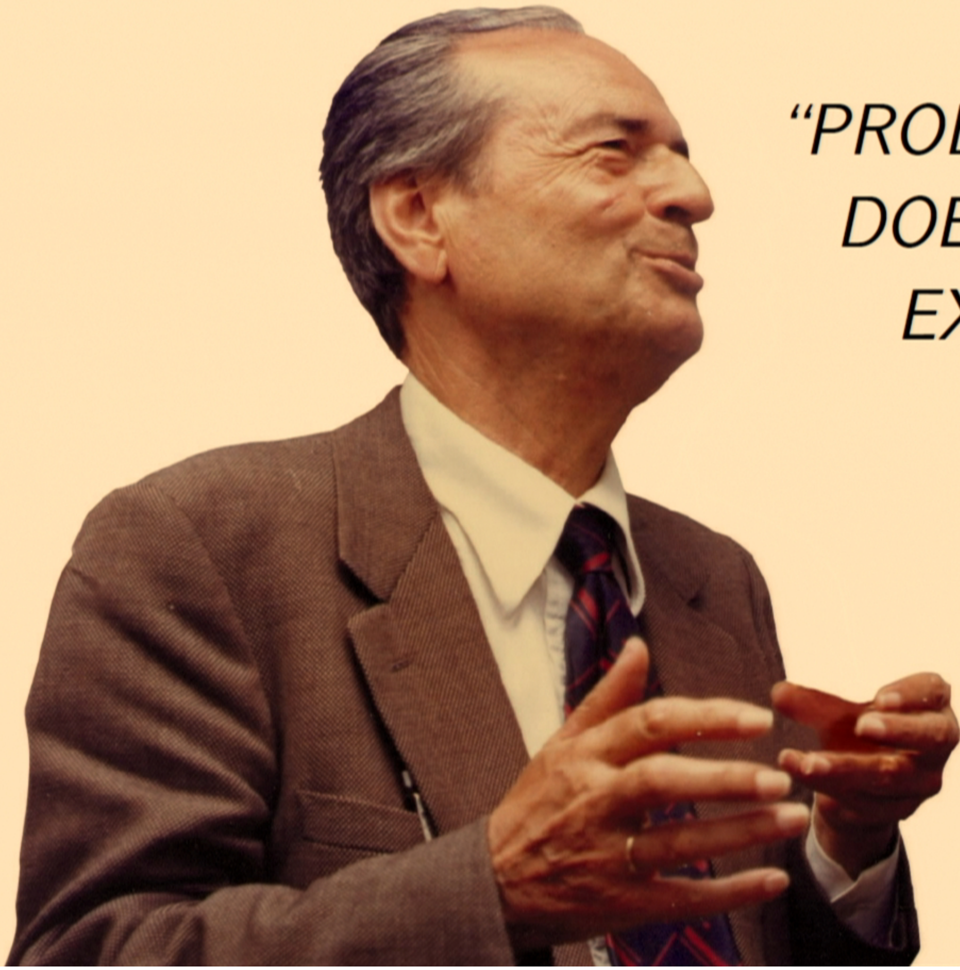
Matthew Pusey
Imperial College London
m@physics.org
Office 268 till Nov 22

What's an unkown POVM?

Matthew Pusey
Jon Barrett
Matt Leifer

Bruno de Finetti

Bruno de Finetti



"PROBABILITY DOES NOT EXIST"

Exchangeable sequence

Say $P_1, P_2, \ldots$ is *exchangeable* if each P_n is symmetric, and $P_n(x_1, \ldots, x_n) = \sum_{x_{n+1}=1}^{d} P_{n+1}(x_1, \cdots, x_n, x_{n+1})$.

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De Finetti theorem

If $P_1, P_2, \ldots$ is exchangeable then

$$P_n(x_1, \ldots, x_n) = \int d\mu(p) p(x_1) \cdots p(x_n),$$

where μ is a unique probability measure on the set of probability distributions, independent of n.

Example

$$P_1 = \begin{pmatrix} P_H \\ P_T \end{pmatrix}, \quad P_2 = \begin{pmatrix} P_{HH} & P_{HT} \\ P_{TH} & P_{TT} \end{pmatrix},$$

$$P_3 = \left\{ \begin{pmatrix} P_{HHH} & P_{HTH} \\ P_{THH} & P_{TTH} \end{pmatrix}, \begin{pmatrix} P_{HHT} & P_{HTT} \\ P_{THT} & P_{TTT} \end{pmatrix} \right\}, \ldots$$

Example

$$P_1 = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix},$$

$$P_3 = \left\{ \begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1/2 \end{pmatrix} \right\}, \ldots$$

Updating $\mu(p)$

$$P_n(x_2, \ldots, x_n | x_1) = \frac{P_n(x_1, x_2, \ldots, x_n)}{\sum_{x'_2, \ldots, x'_n} P_n(x_1, x'_2, \ldots, x'_n)}$$

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$$= \frac{P_n(x_1, x_2, \ldots, x_n)}{P_1(x_1)}$$

$$\mu(p) \to \frac{\mu(p) p(x_1)}{\int d\mu(p') p'(x_1)}$$

Quantum Bayesianism

QUANTUM STATES DO NOT EXIST

De Finetti theorem

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Exchangeable sequences of quantum states

Say $\rho_1, \rho_2, \ldots$ is *exchangeable* if each ρ_n is symmetric, and $\rho_n = \mathrm{tr}_{n+1}\, \rho_{n+1}$.

De Finetti for quantum states[1]

If $\rho_1, \rho_2, \ldots$ is exchangeable then

$$\rho_n = \int d\mu(\rho) \rho^{\otimes n},$$

where μ is a unique probability measure on the set of quantum states, independent of n.

[1] Hudson & Moody, Wahrschein. verw. Geb. 33, 343 (1976). Caves, Fuchs & Schack, quant-ph/0104088.

De Finetti for quantum channels[2]

If $\Phi_1, \Phi_2, \ldots$ is exchangeable then

$$\Phi_n = \int d\mu(\Phi)\Phi^{\otimes n},$$

where μ is a unique probability measure on the set of quantum channels, independent of n.

[2] Fuchs, Schack & Scudo, quant-ph/0307198

Exchangeable sequences of POVMs

Say $\{E_1^i\}, \{E_2^{ij}\}, \{E_3^{ijk}\}\ldots$ is *exchangeable* if each POVM is symmetric (permuting systems permutes the upper indices), and $E_n^{ij\ldots k} \otimes I = \sum_l E_{n+1}^{ij\ldots kl}$.

De Finetti for POVMs

If $\{E_1^i\}, \{E_2^{ij}\}, \{E_3^{ijk}\} \ldots$ is exchangeable,

$$E_n^{ij\ldots k} = \int d\mu(E) E^i \otimes E^j \otimes \cdots \otimes E^k$$

where μ is a unique probability measure on the set of single-system POVMs, independent of n.

Proof sketch

1. View POVMs as channels that output classical states.
2. Show exchangeable POVMs are exchangeable channels, and apply existence part of de Finetti for channels.

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4. Apply uniqueness part of de Finetti for quantum states to $\Phi_n(\rho^{\otimes n})$.

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