Title: Optimal quantum self-tests based on binary nonlocal XOR games

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Abstract: Self-testing a multipartite quantum state means verifying the existence of the state based on the outcomes of unknown or untrusted measurements.

This concept is important in device-independent quantum cryptography.

There are some previously known results on self-testing which involve nonlocal binary XOR games such as the CHSH test and the GHZ paradox. In our work we expand on these results. We provide a general criterion which, when satisfied, guarantees that a given nonlocal binary XOR game is a robust self-test. The error term in this result is quadratic, which is the best possible. In my talk I will explain the conceptual basis for the criterion and offer some examples. This is joint work with Yaoyun Shi (arXiv:1207.1819).

Self-Testing Properties for Binary Nonlocal XOR Games

Carl Miller
Electrical Engineering & Computer Science Department
University of Michigan, Ann Arbor

Intro: Self-Testing

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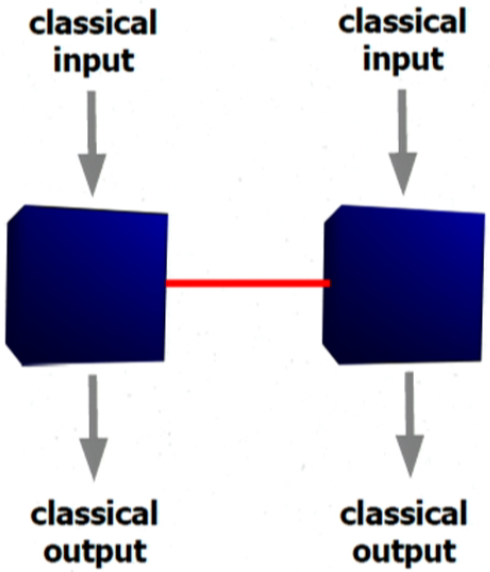

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Let $F(I_1, I_2, O_1, O_2) = \begin{cases} +1 & \text{if } O_1 \oplus O_2 = I_1 \wedge I_2 \\ -1 & \text{otherwise} \end{cases}$

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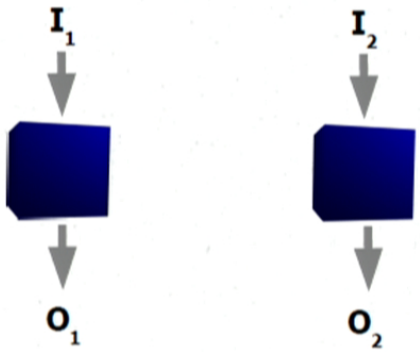

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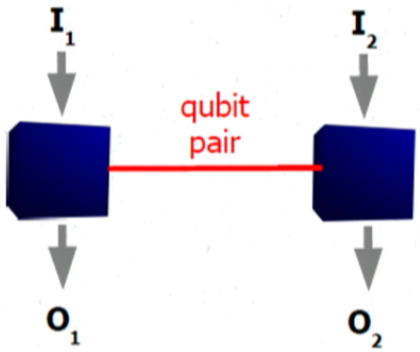

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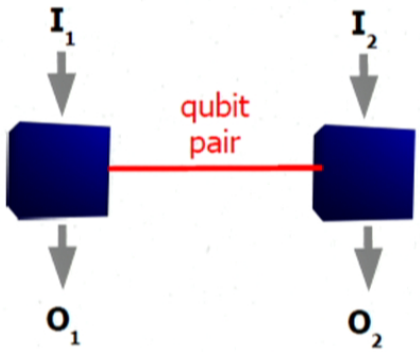

If **E** [F] (on random inputs) is $\sqrt{2}/2$, then the state and measurements are uniquely determined.

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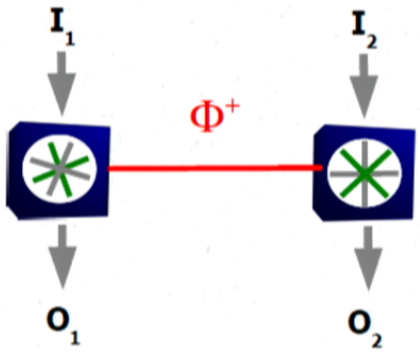

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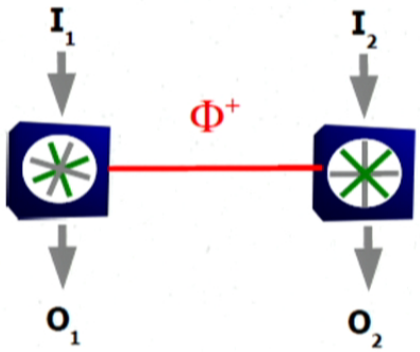

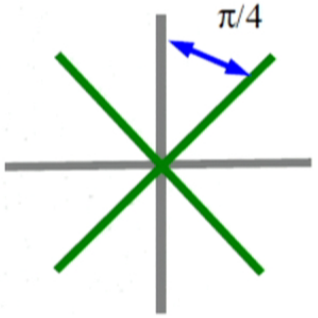

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Overview of Talk

Goal: Begin a general theory for proving self-testing results.

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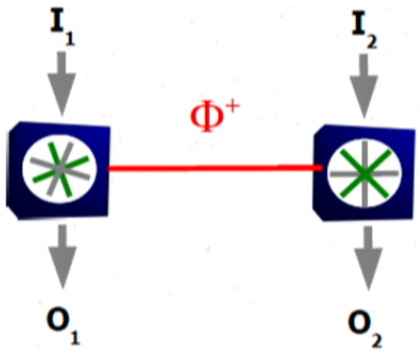

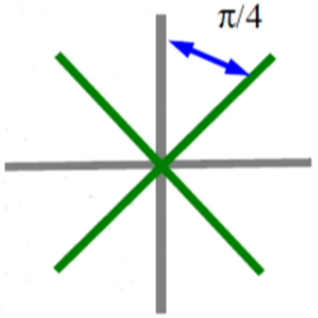

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I. Background

Motivation: Randomness expansion and QKD.

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Key idea: Nonlocal binary XOR games ⟶ sinusoidal functions.

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A criterion for robust self-tests.

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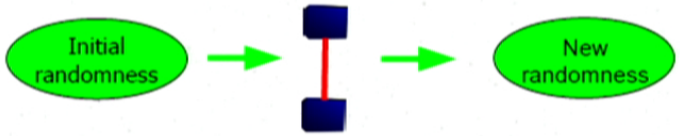
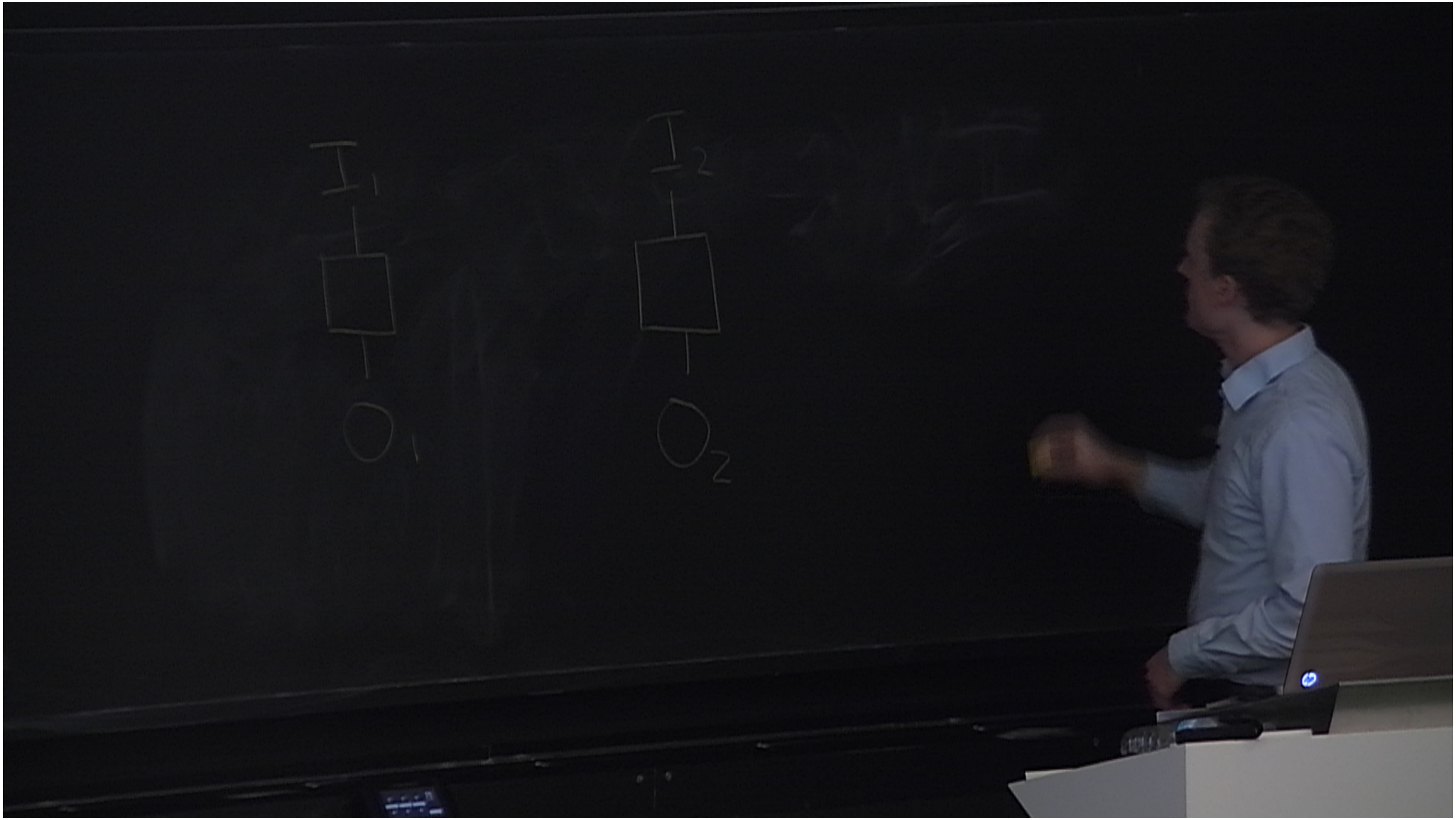
Background

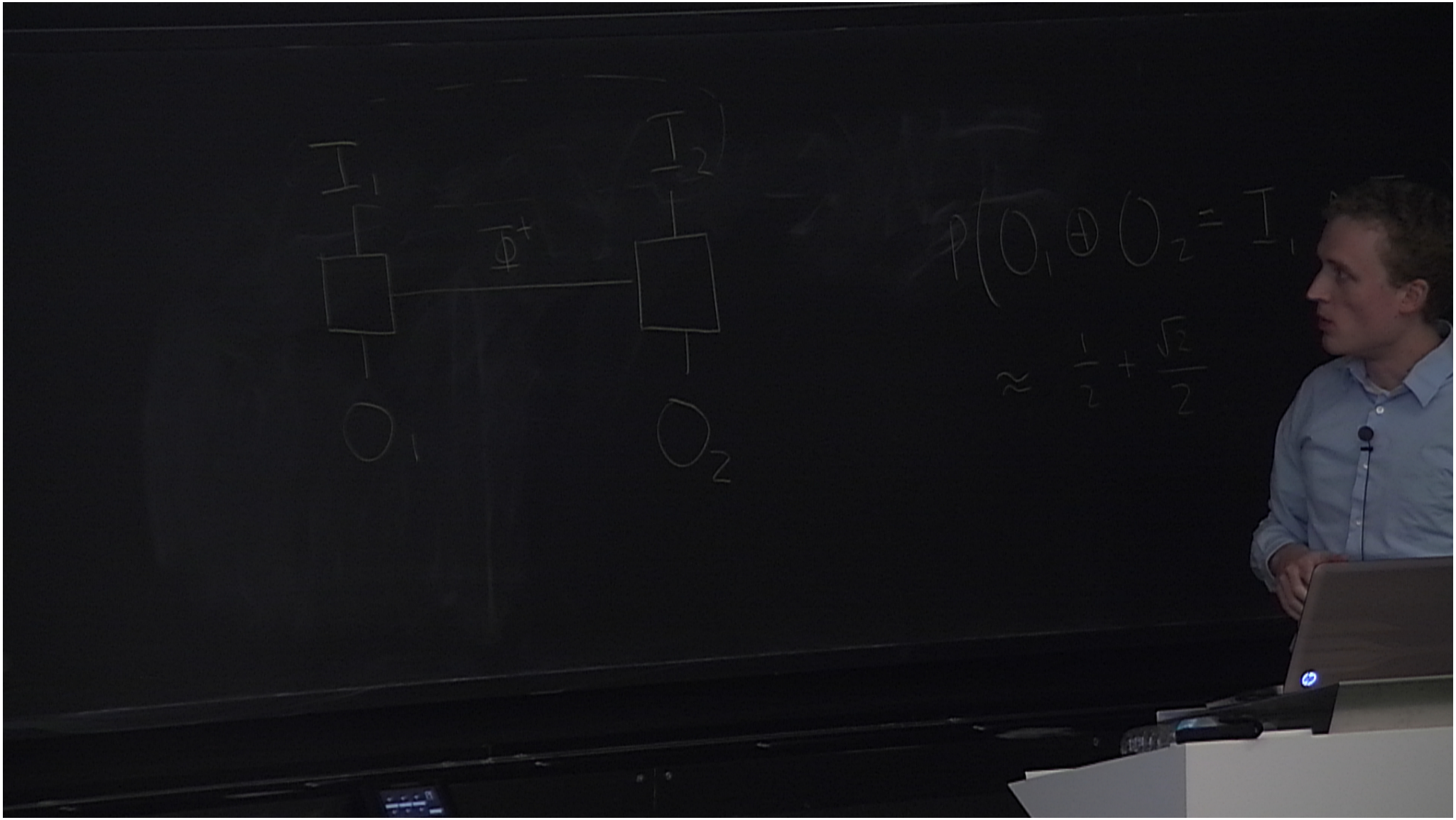
Some motivating problems:

Randomness expansion from an untrusted device.




I1
O1
I2
O2


I1
I2
O1
P(O1 ⊕ O2 = I1 ∧ I2
≈ 1/2 + √2/2


I_1
I_2
Φ^+
O_1
O_2
$P(O_1 \otimes O_2 = I_1$
$\approx \frac{1}{2} + \frac{\sqrt{2}}{2}$

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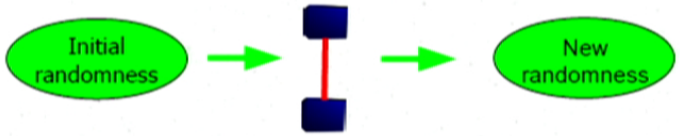

Randomness amplification with an untrusted device.

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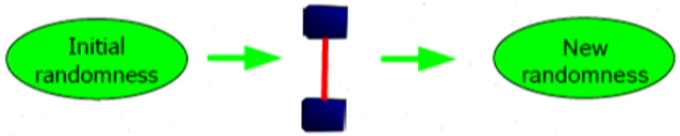
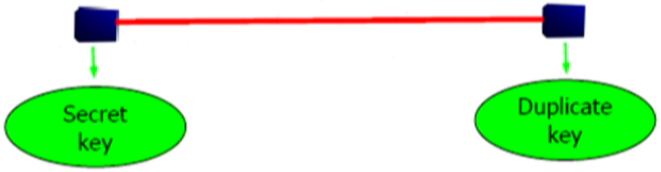


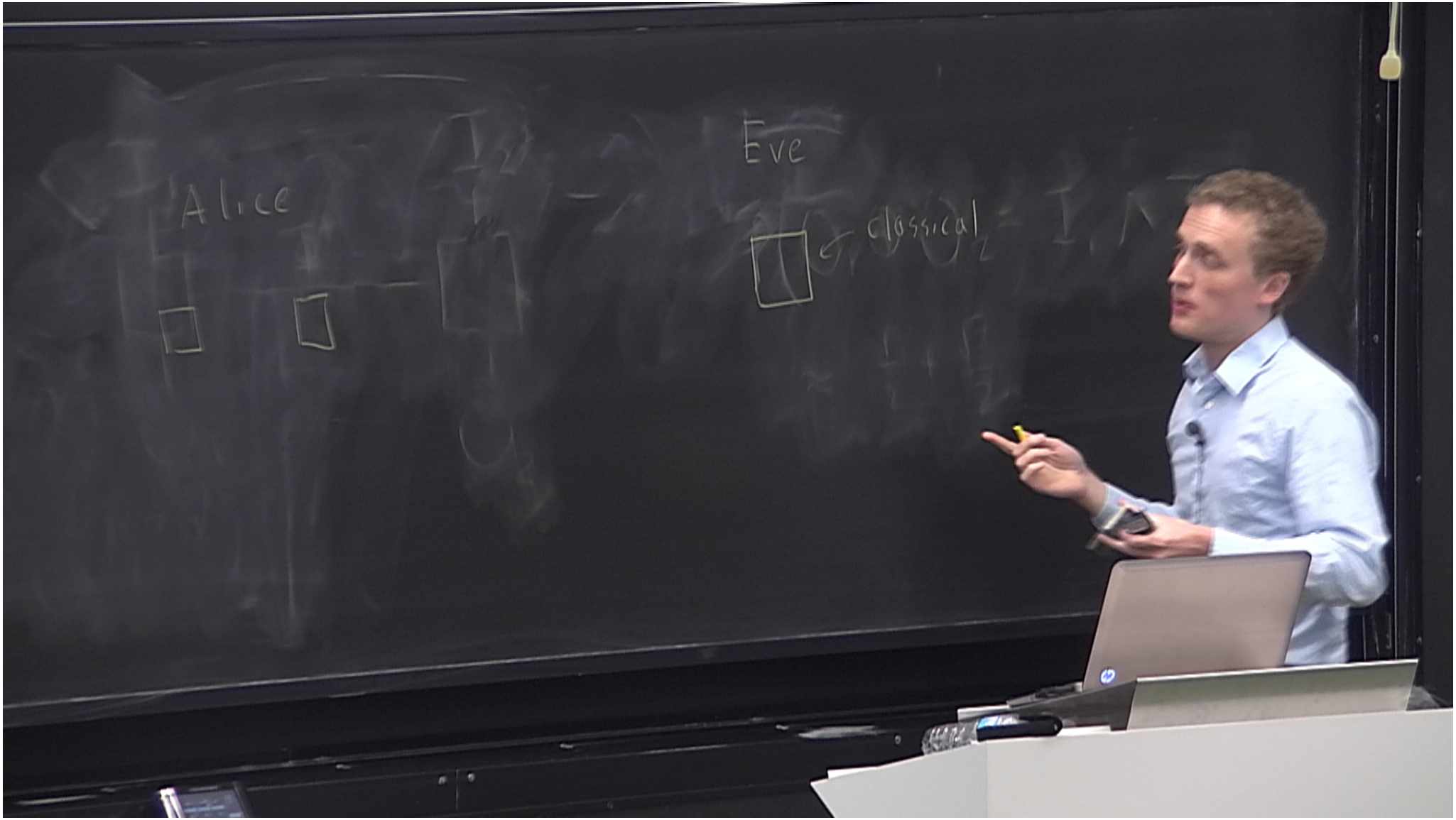
Randomness amplification with an untrusted device.

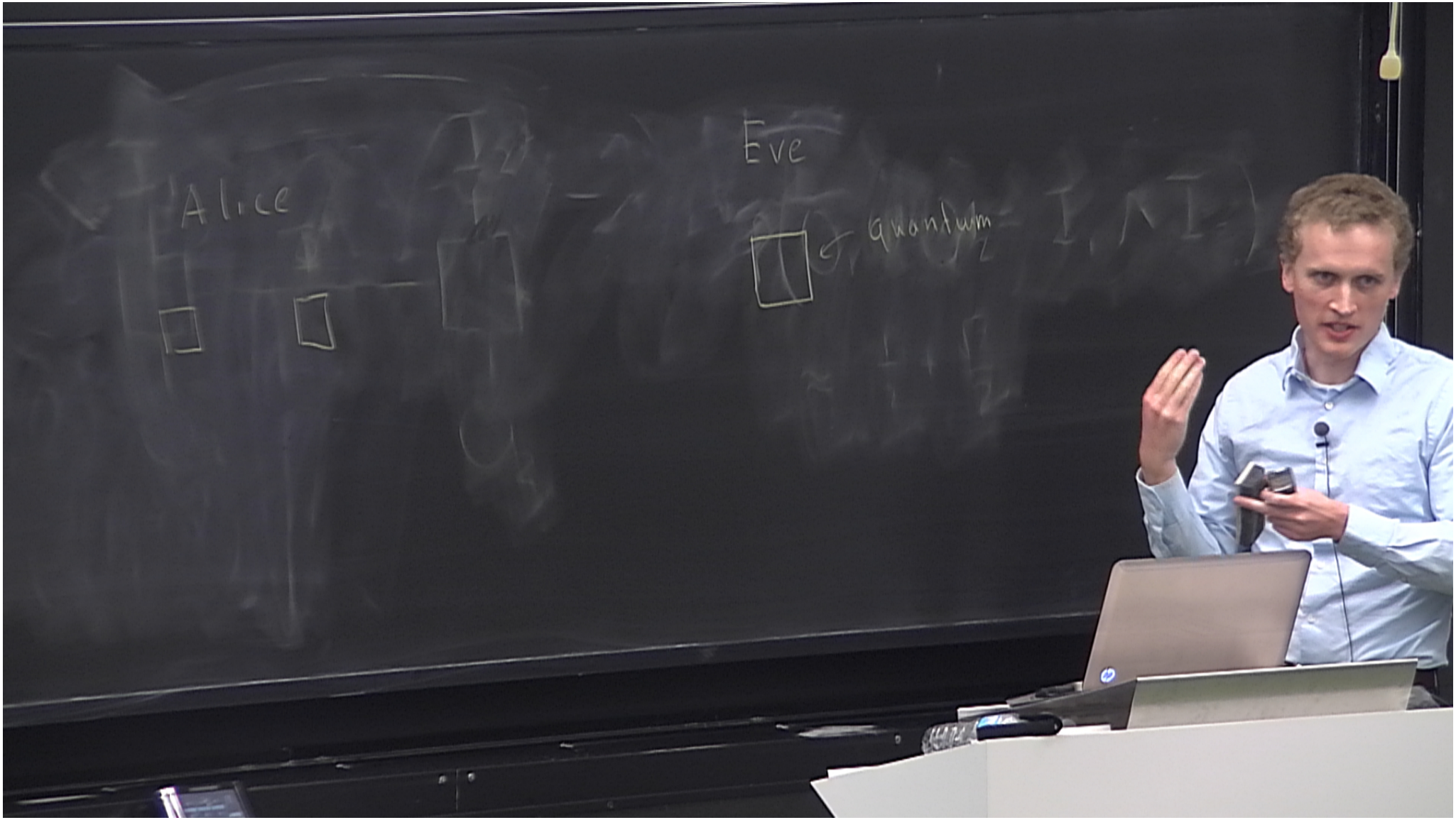
Device-independent QKD.



Background

Self-testing results may be critical for proving **quantum** security of device-independent protocols.


Alice
Eve
classical


Alice
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quantum


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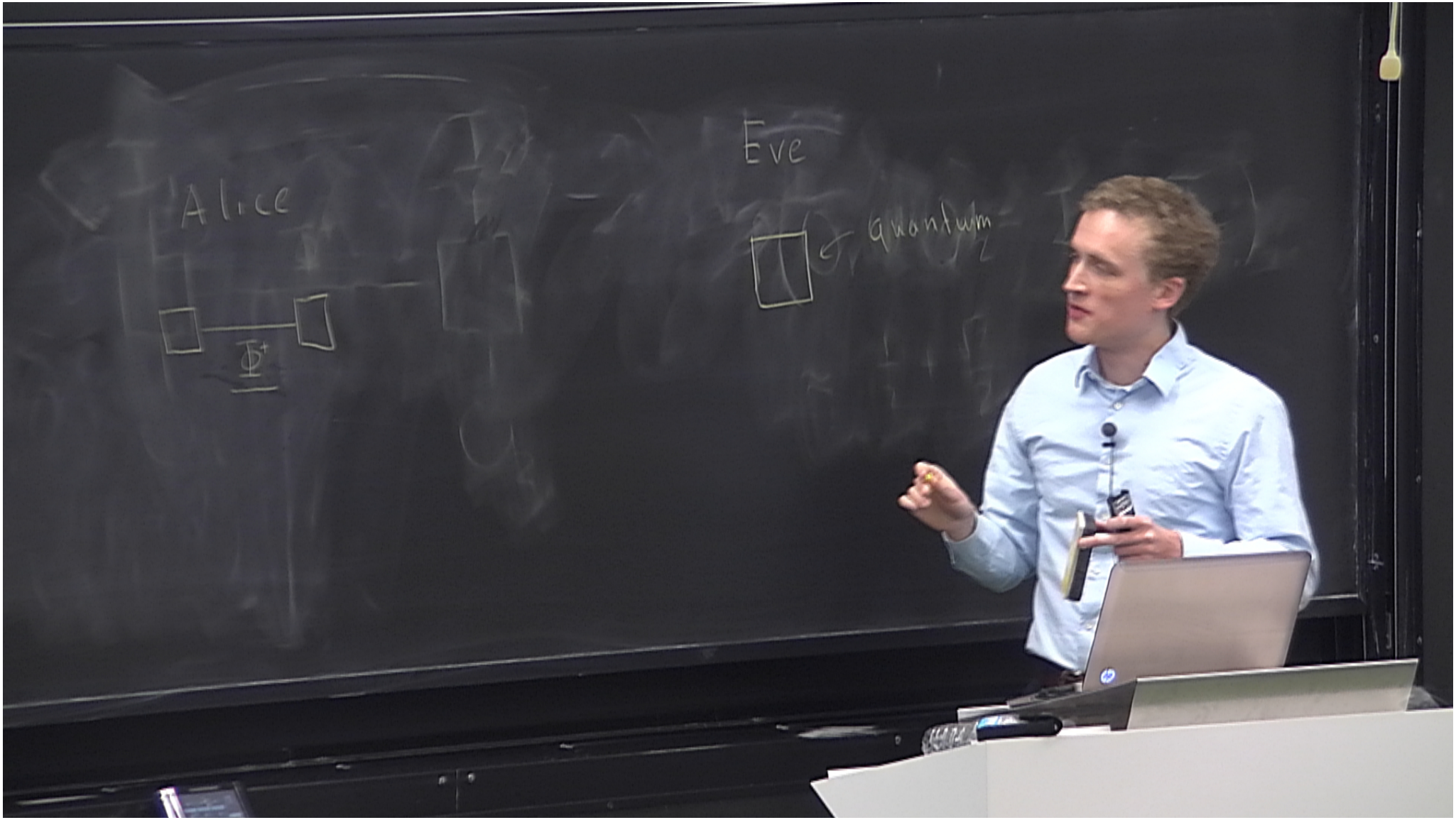
quantum
Φ^+


Alice
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Background

Some important papers:

[Popescu 1992]: Proves an early result on self-testing for CHSH.

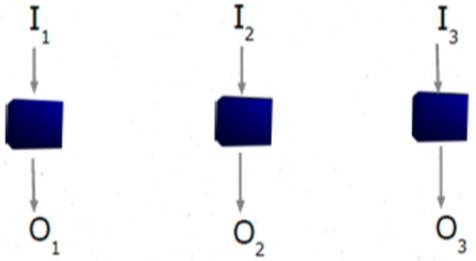
[Mayers, Yao 1998]: Introduces self-testing in the context of QKD.

[Colbeck 2006]: Proves a (non-robust) self-testing result for the GHZ paradox.

[McKague 2012]: Proves a robust self-testing result for CHSH.

Concepts

Binary nonlocal XOR games.



The score depends only on the XOR of the outputs.

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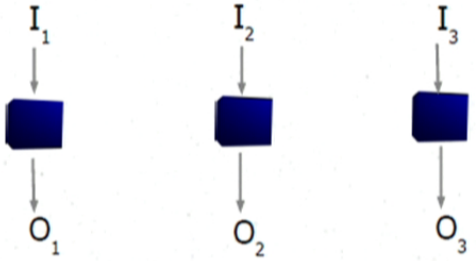
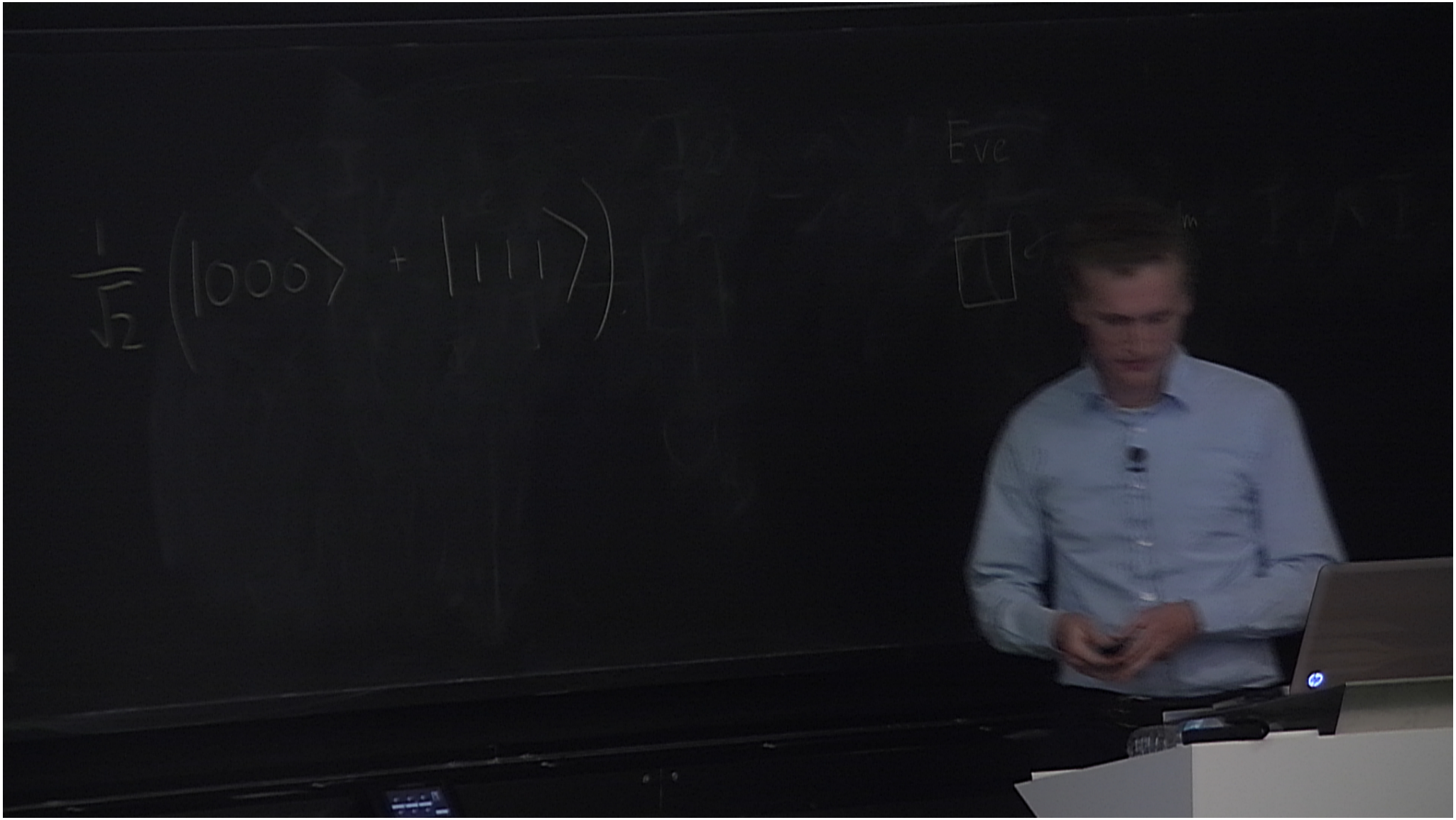


The score depends only on the XOR of the outputs.

Example: The GHZ game.

	Score if $O_1 \oplus O_2 \oplus O_3 = 0$	Score if $O_1 \oplus O_2 \oplus O_3 = 1$
000	-1	+1
110	+1	-1
101	+1	-1
011	+1	-1


$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
Eve

Concepts

Multivariable sinusoidal functions.

To each binary XOR game we associate a function.

$$Z_{CHSH}(\lambda, \theta_1, \theta_2) = (+1)\cos(\lambda) + (+1)\cos(\lambda + \theta_2) + (+1)\cos(\lambda + \theta_1) + (-1)\cos(\lambda + \theta_1 + \theta_2).$$

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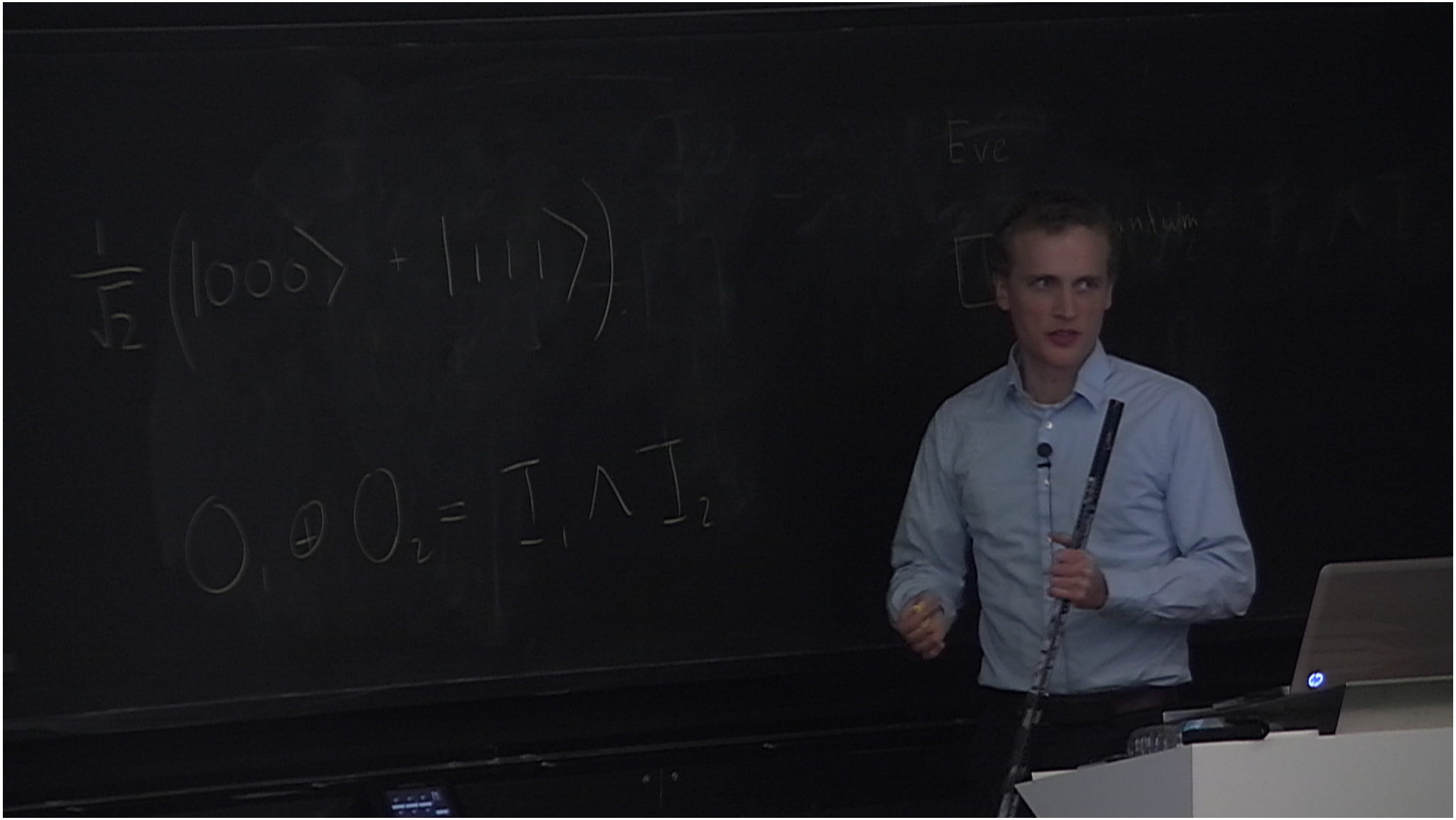
$$Z_{CHSH}(\lambda, \theta_1, \theta_2) = (+1)\cos(\lambda) \quad \leftarrow \text{Input 00}$$
$$+ (+1)\cos(\lambda + \theta_2) \quad \leftarrow \text{Input 01}$$
$$+ (+1)\cos(\lambda + \theta_1) \quad \leftarrow \text{Input 10}$$
$$+ (-1)\cos(\lambda + \theta_1 + \theta_2). \quad \leftarrow \text{Input 11}$$

Score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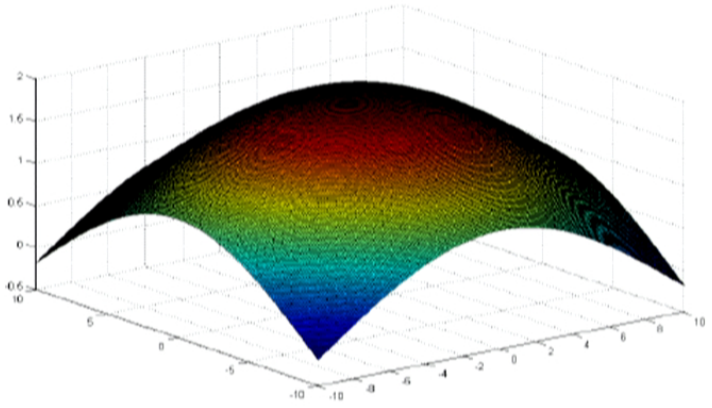
The values of this function correspond to scores achievable by quantum devices.

Concepts

The Hessian.

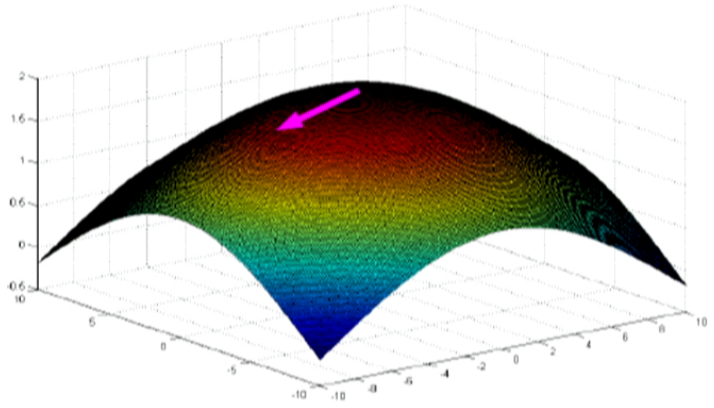


z = F(x,y)

$$H = \begin{bmatrix} \partial^2 F/\partial x^2 & \partial^2 F/\partial x\,\partial y \\ \partial^2 F/\partial x\,\partial y & \partial^2 F/\partial y^2 \end{bmatrix}$$

Concepts

The Hessian.



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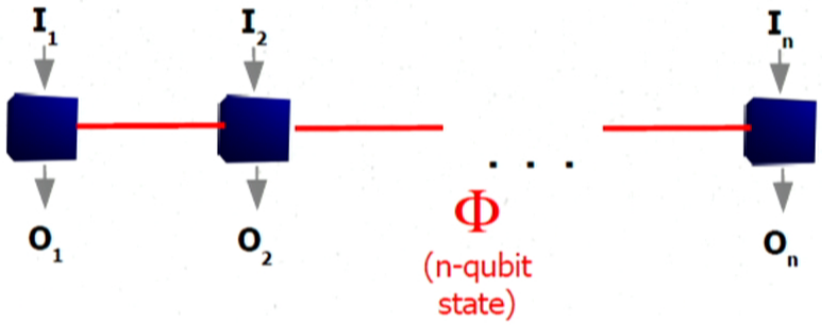
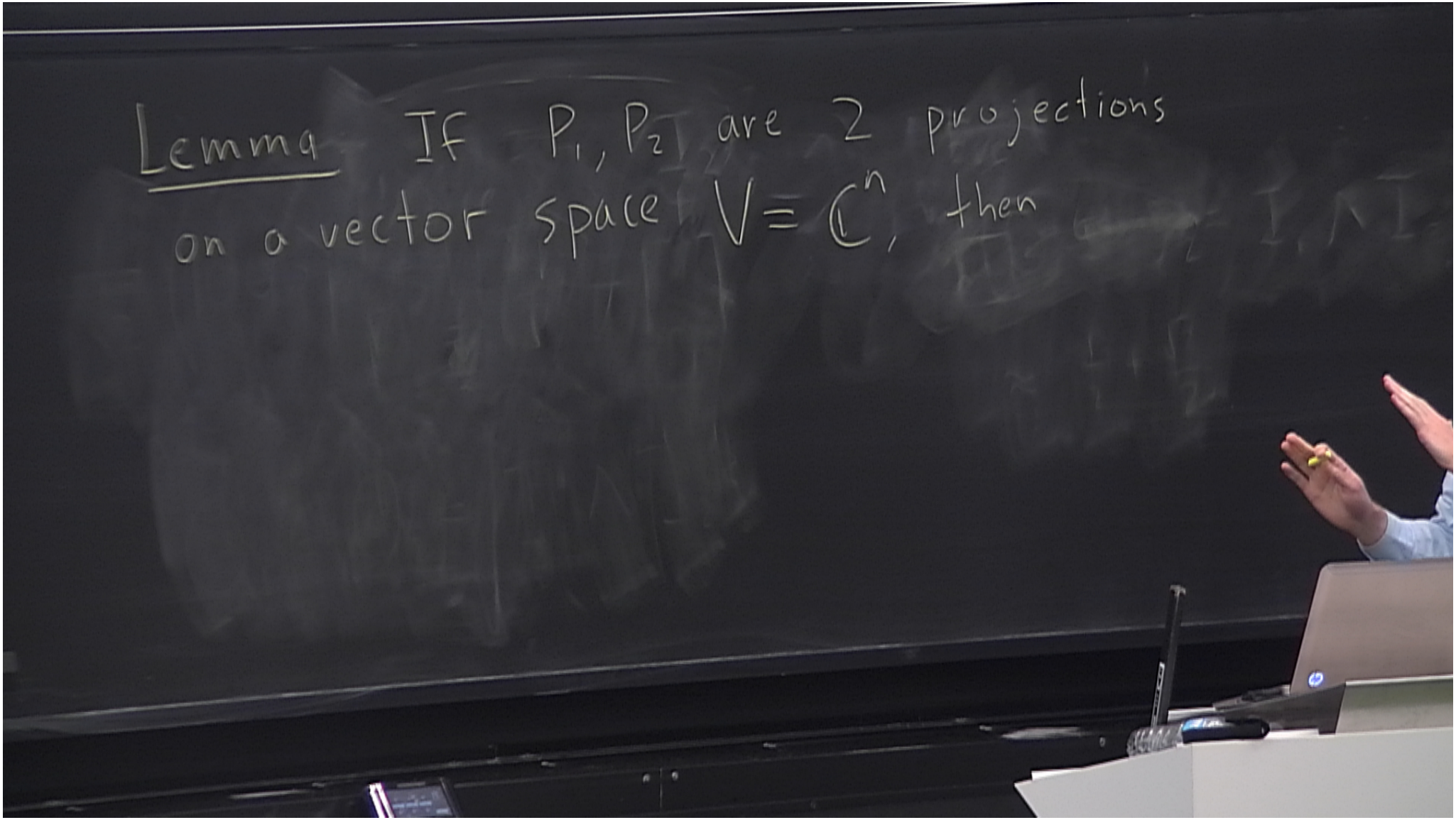
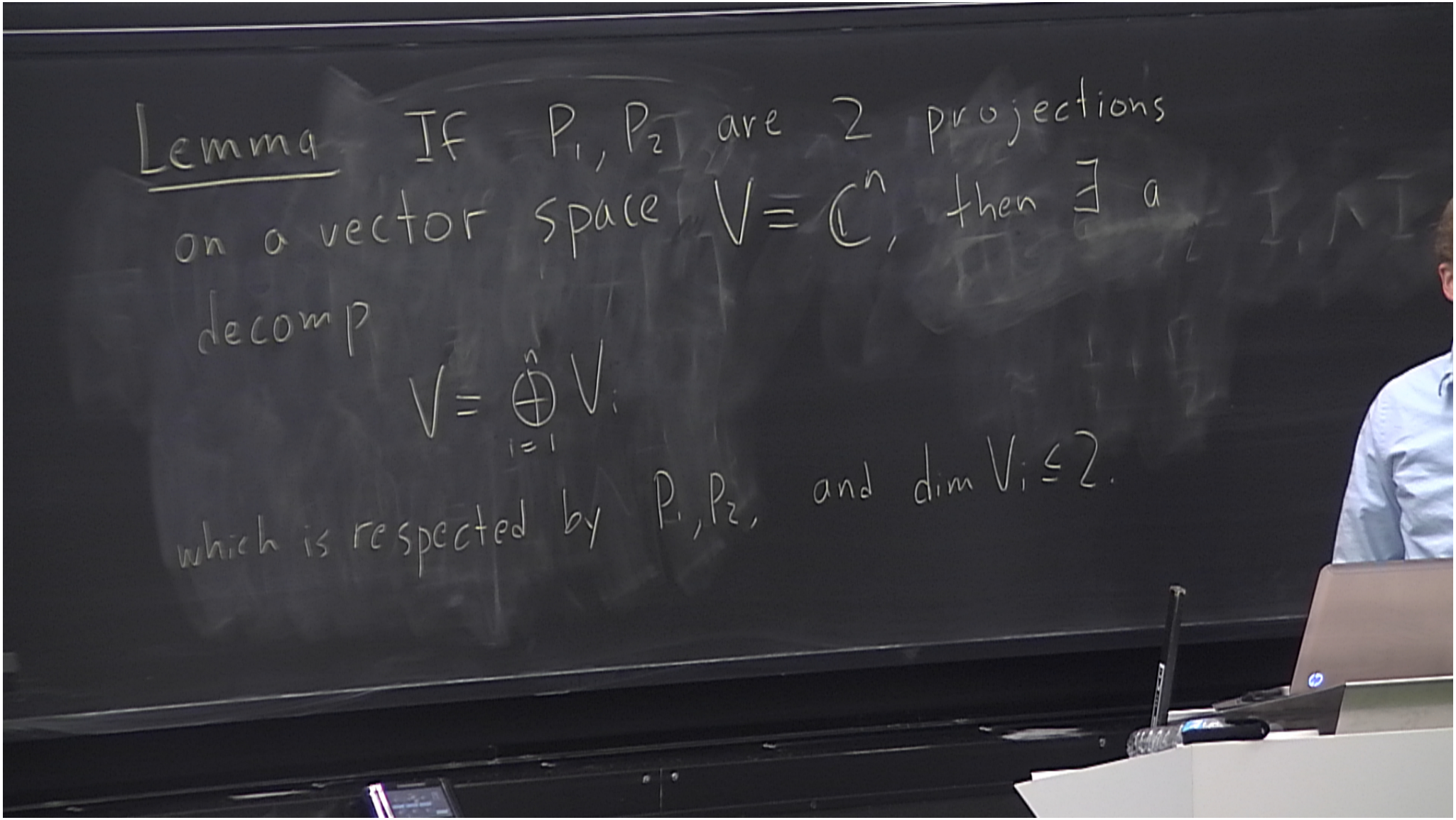
$(v^T H v)$ = second derivative of F in the direction of v

Main result

By Jordan's lemma, any self-testing problem involving binary games can be reduced to **qubit** devices.




Lemma If P_1, P_2 are 2 projections
on a vector space $V = \mathbb{C}^n$, then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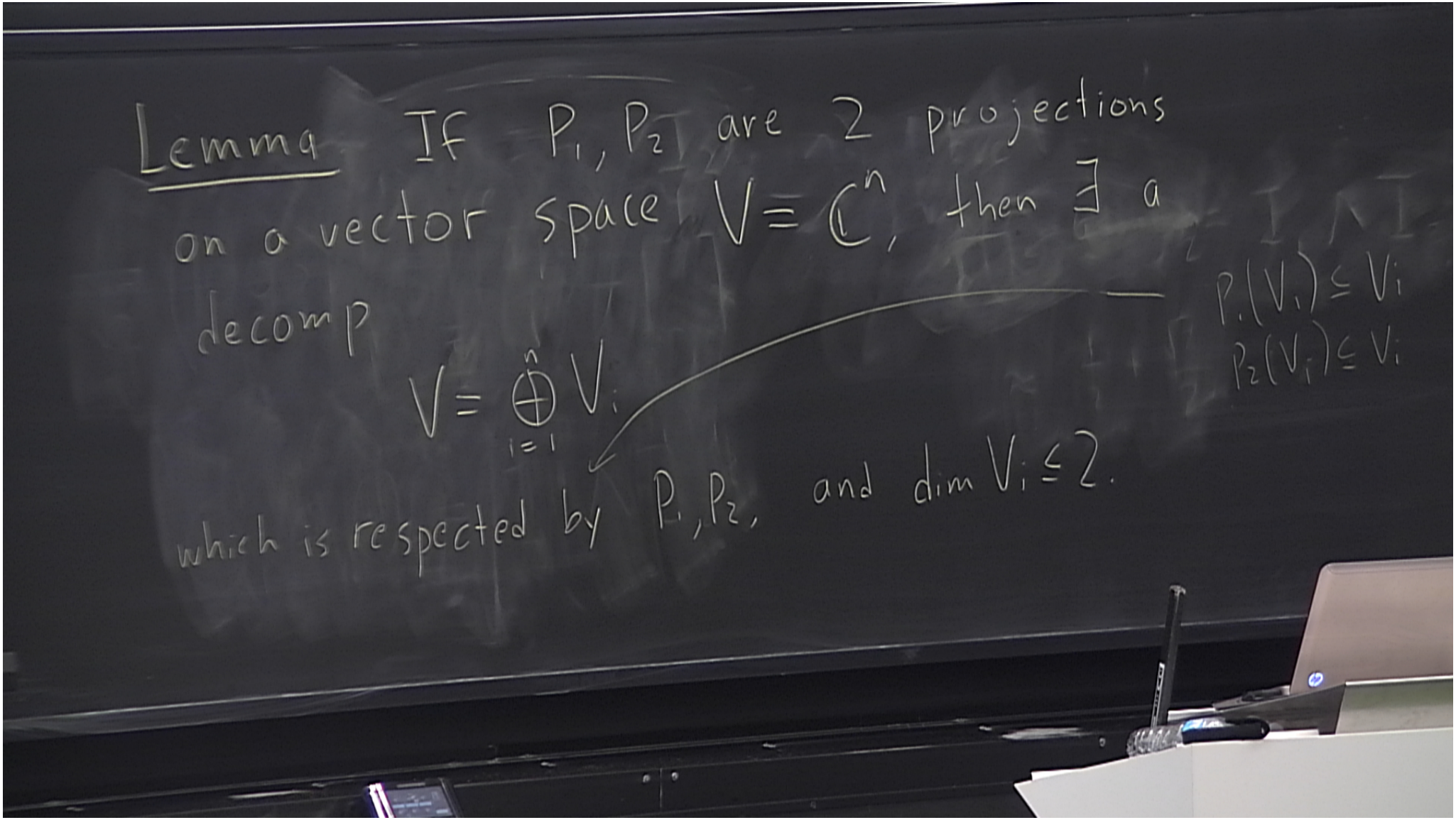
which is respected by P_1, P_2, and $\dim V_i \leq 2$.


Lemma If P_1, P_2 are 2 projections on a vector space $V = \mathbb{C}^n$, then $\exists$ a decomp
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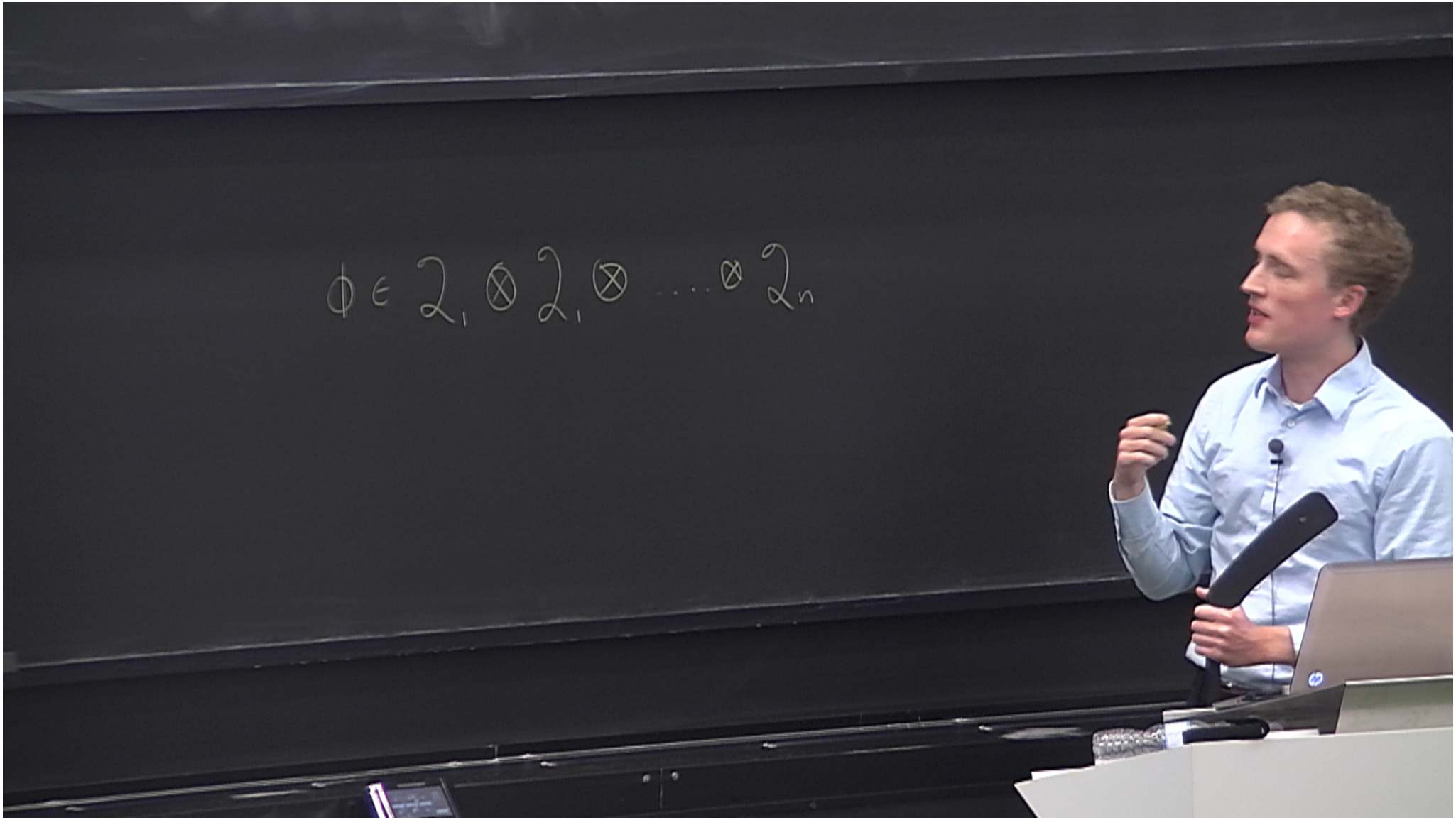
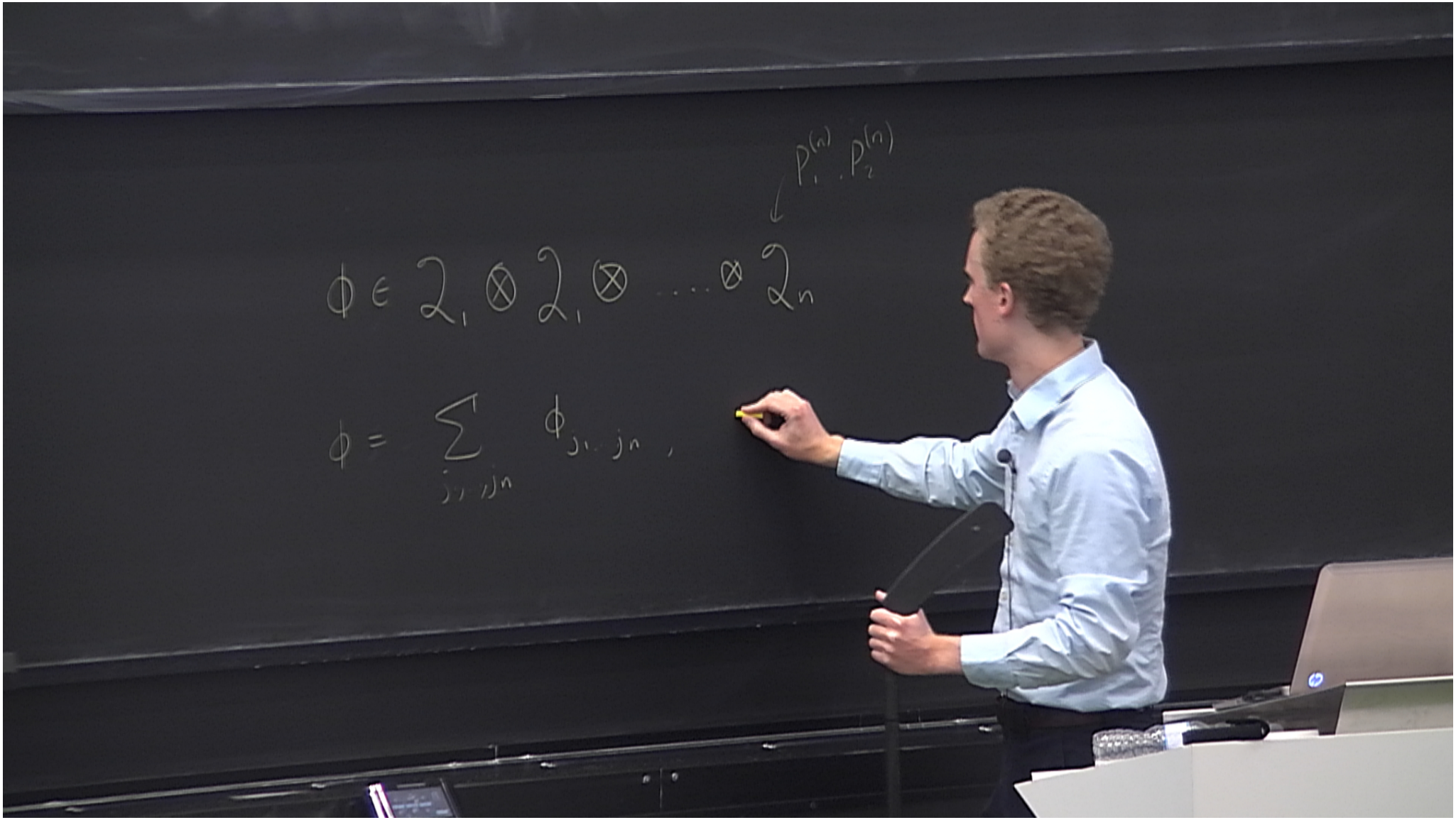
$P_1(V_i) \leq V_i$
$P_2(V_i) \leq V_i$


$\phi \in \mathcal{Z}_1 \otimes \mathcal{Z}_1 \otimes \cdots \otimes \mathcal{Z}_n$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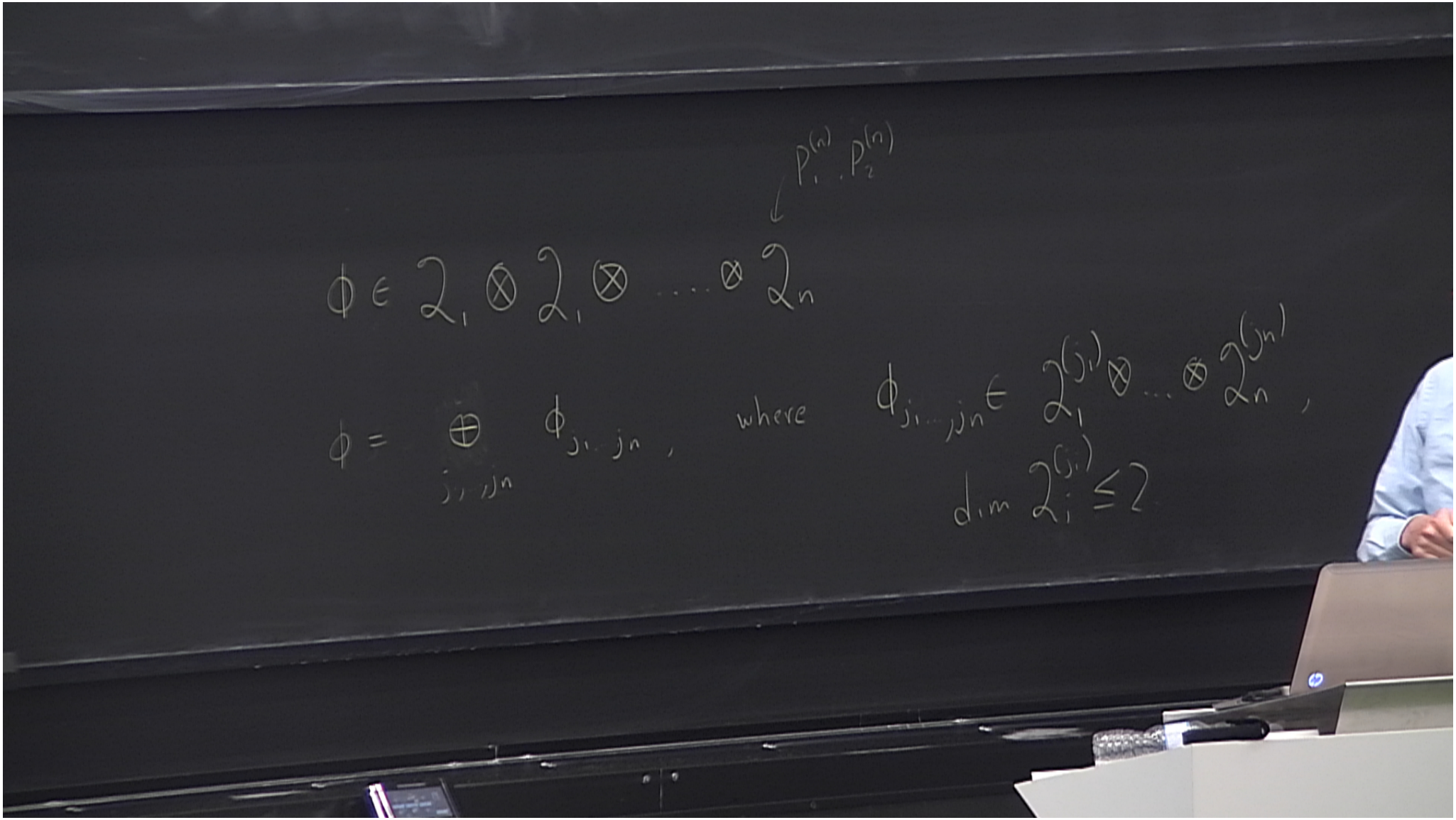
$P_1^{(n)} \cdot P_2^{(n)}$
$\phi = \sum_{j_1 \ldots j_n} \phi_{j_1 \ldots j_n}$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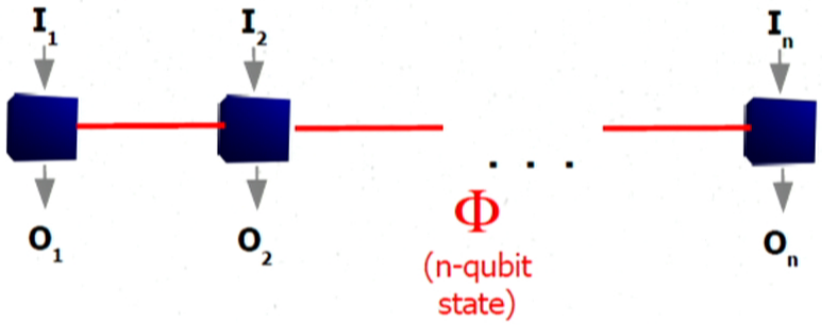
$\phi = \bigoplus_{j_1 \ldots j_n} \phi_{j_1 \ldots j_n}$, where $\phi_{j_1 \ldots j_n} \in \mathcal{Z}_1^{(j_1)} \otimes \cdots \otimes \mathcal{Z}_n^{(j_n)}$,
$\dim \mathcal{Z}_i^{(j_i)} \leq 2$

Main result

By Jordan's lemma, any self-testing problem involving binary games can be reduced to **qubit** devices.



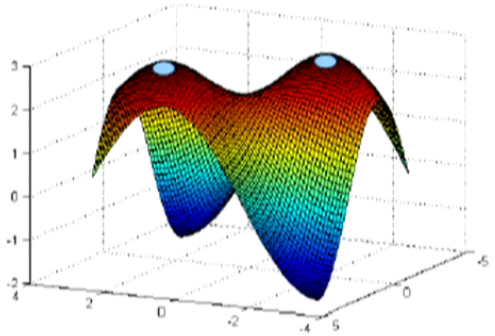
Question: Which binary games have the property that all optimal (qubit) devices behave the same?

Main result

Let S_n be the class of all n-player binary XOR games f such that:

(1) The function Z_f has two maxima in $[-\pi, \pi]^{n+1}$.

(2) The Hessian at each maximum is nonsingular.



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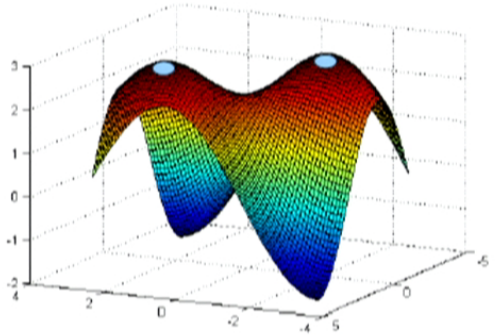

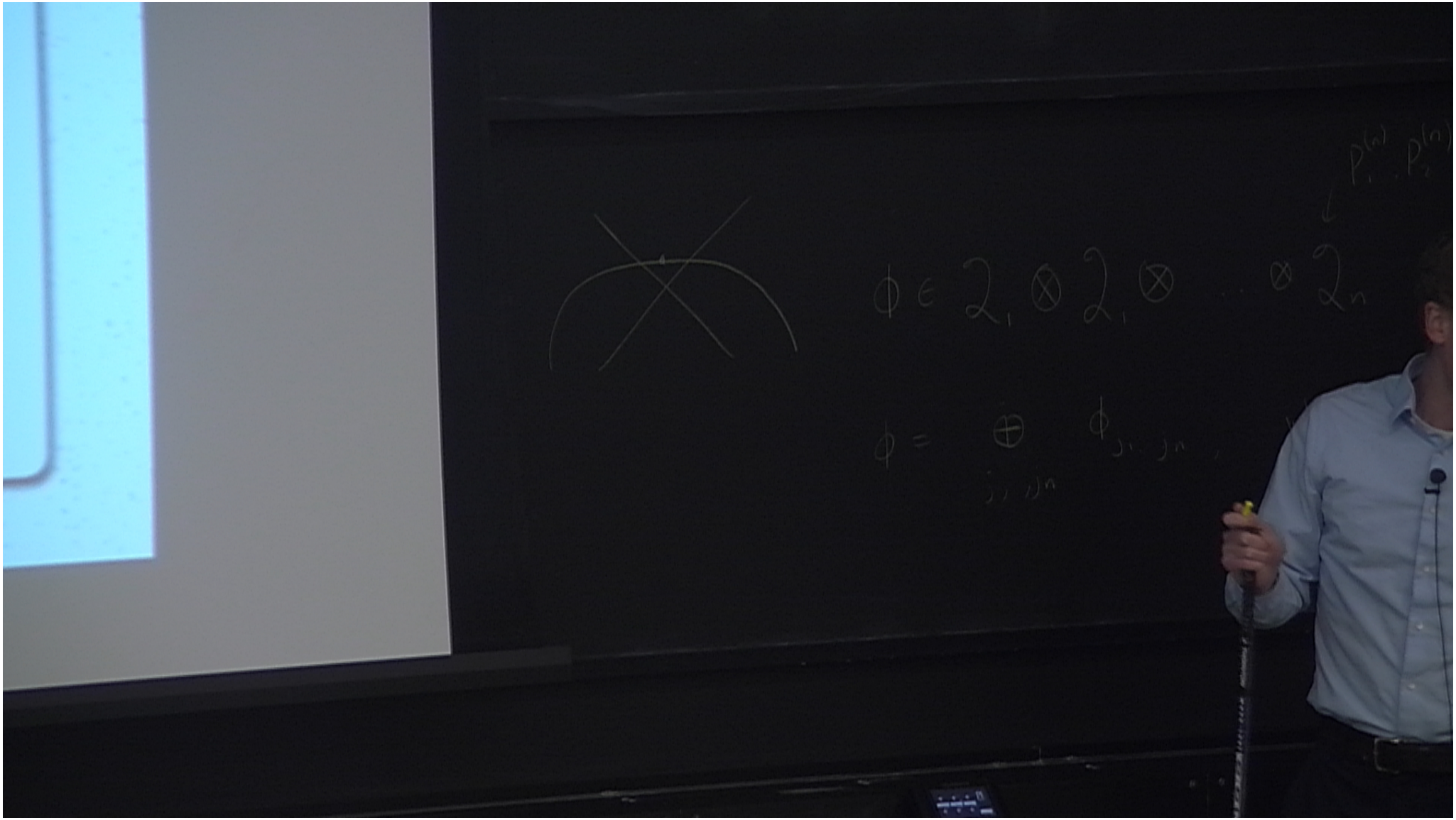
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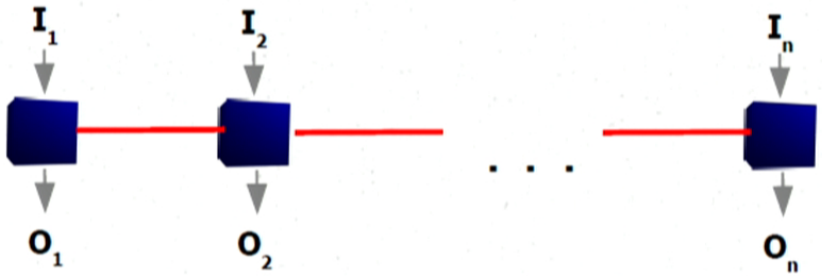
Main result

Proposition 1. Let $f \in S_n$. Then, there is a single optimal qubit device to which all optimal qubit devices are equivalent.



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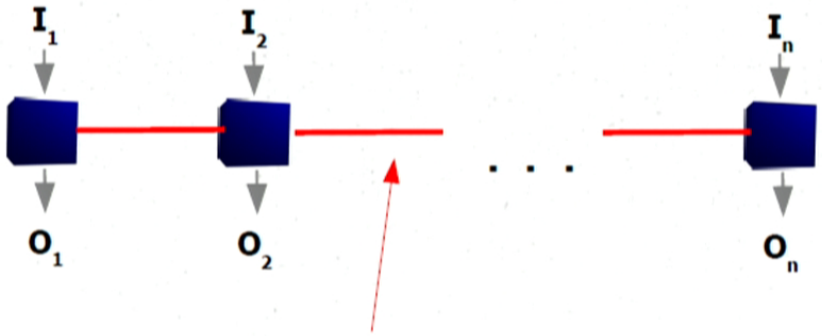

$\Lambda_n = (1/\sqrt{2})[|00\ldots0\rangle + |11\ldots1\rangle]$

(the extended GHZ state)

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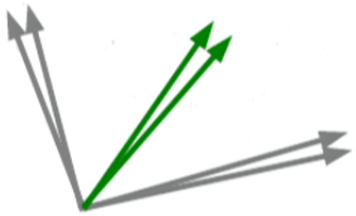
(2) The respective measurement vectors $\{v_i^{jk}\}$, $\{w_i^{jk}\}$ satisfy $|\langle v_i^{jk}, w_i^{jk}\rangle| \geq 1 - K\varepsilon$. (For some constants C, K.)



Applications

The CHSH game.

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The CHSH game. The function

$$Z_{CHSH}(\lambda, \theta_1, \theta_2) = \cos(\lambda) + \cos(\lambda + \theta_2) + \cos(\lambda + \theta_1) - \cos(\lambda + \theta_1 + \theta_2)$$

has two maxima: $(-\pi/4, \pi/2, \pi/2)$ and $(\pi/4, -\pi/2, -\pi/2)$.

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This improves on [McKague 2012] and matches [Reichardt 2012].

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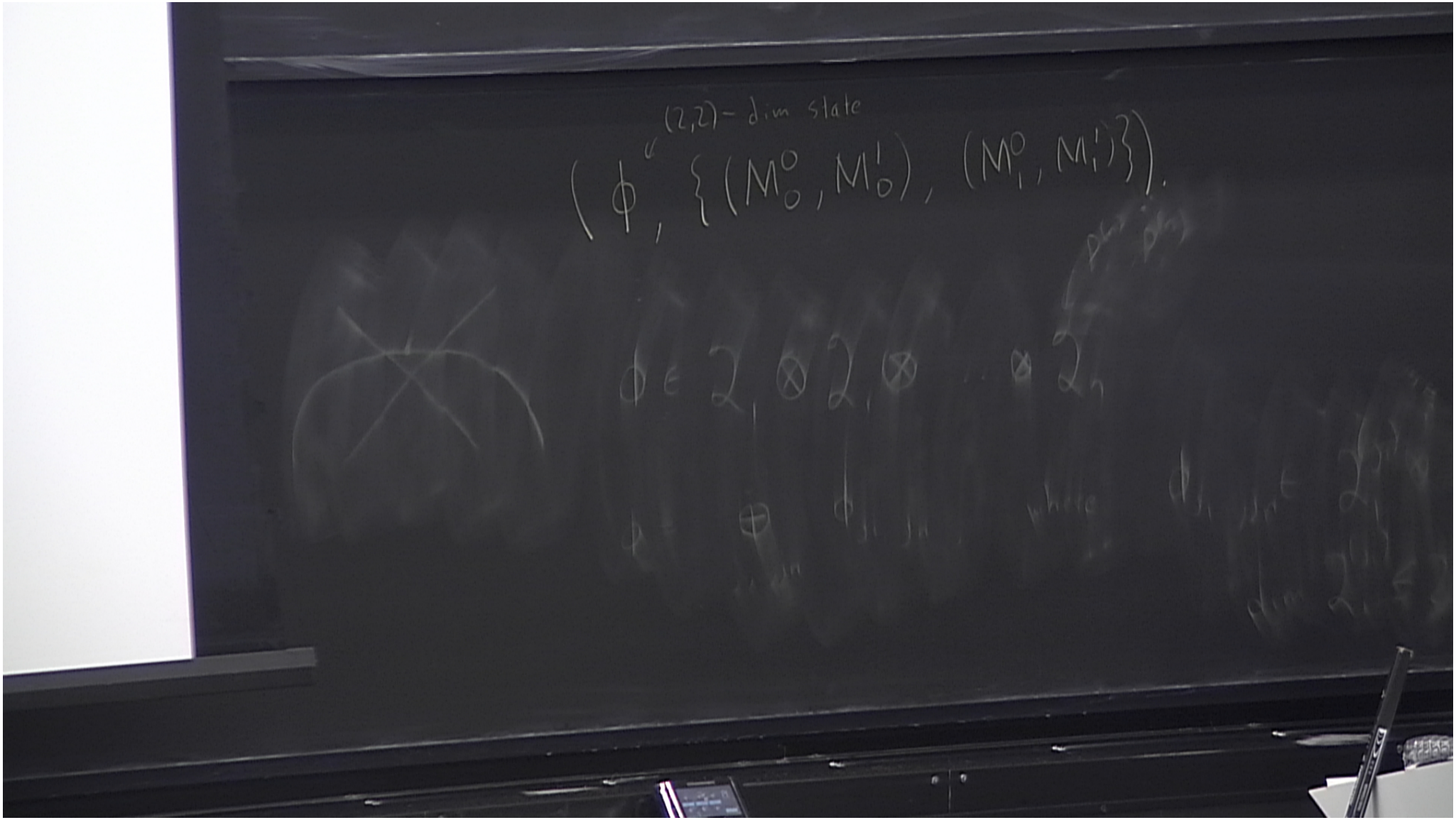
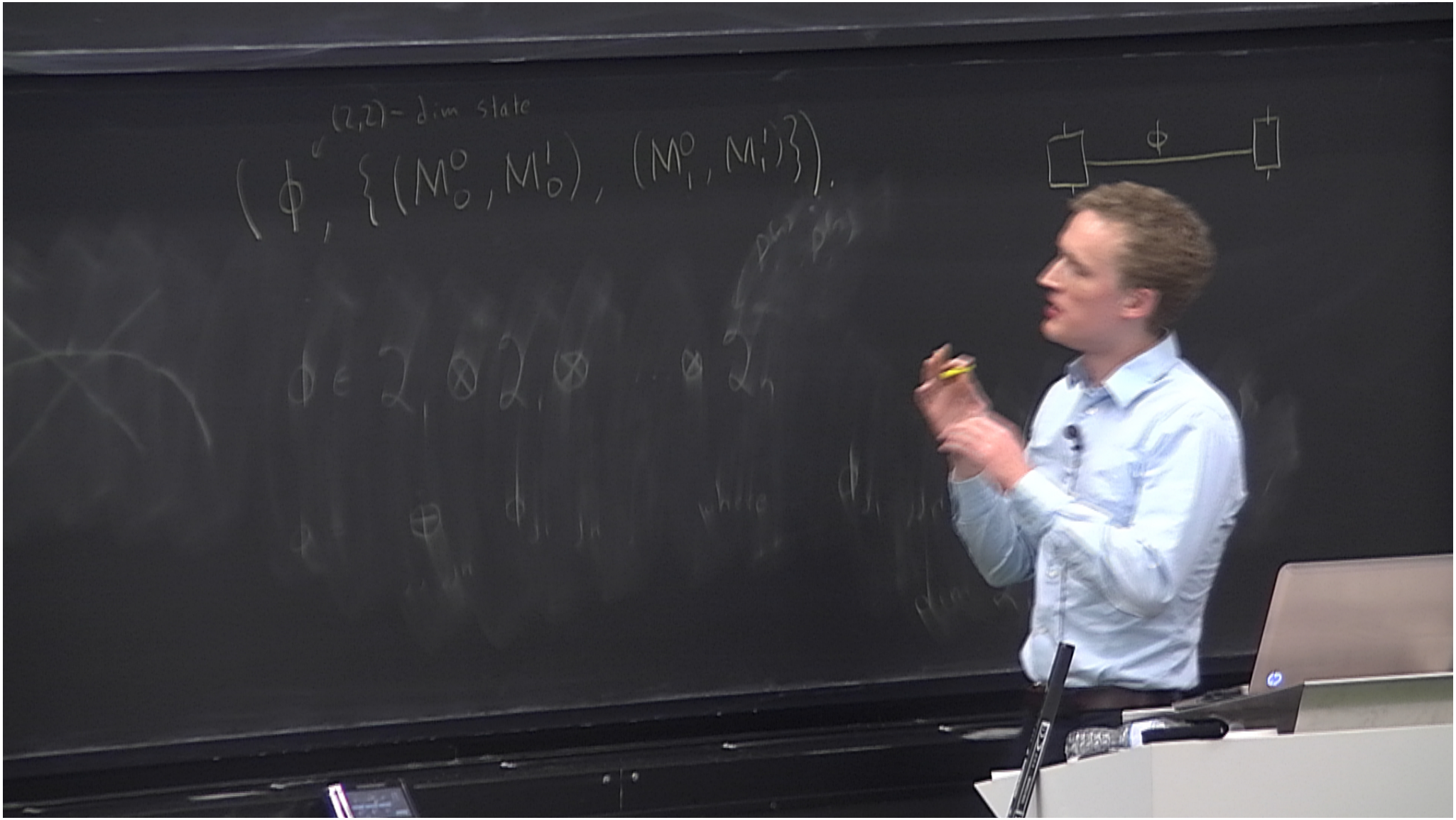

(2,2)- dim state
$(\phi^{(2,2)}, \{(M^0_0, M^1_0), (M^0_1, M^1_1)\})$.


(2,2)-dim state
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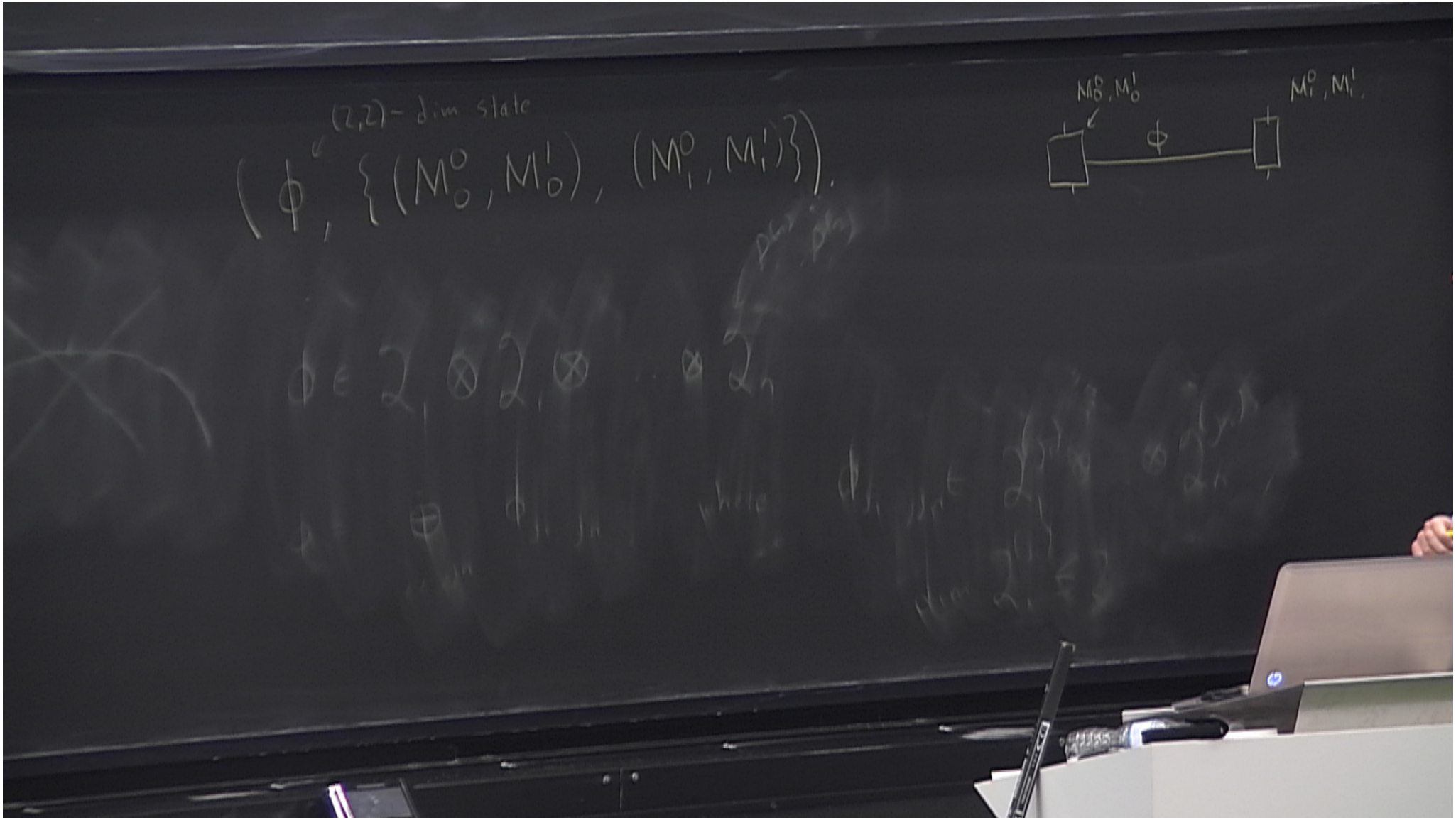
$\phi \in 2_1 \otimes 2_2 \otimes \cdots \otimes 2_n$
ϕ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M^0_0, M^1_0
ϕ
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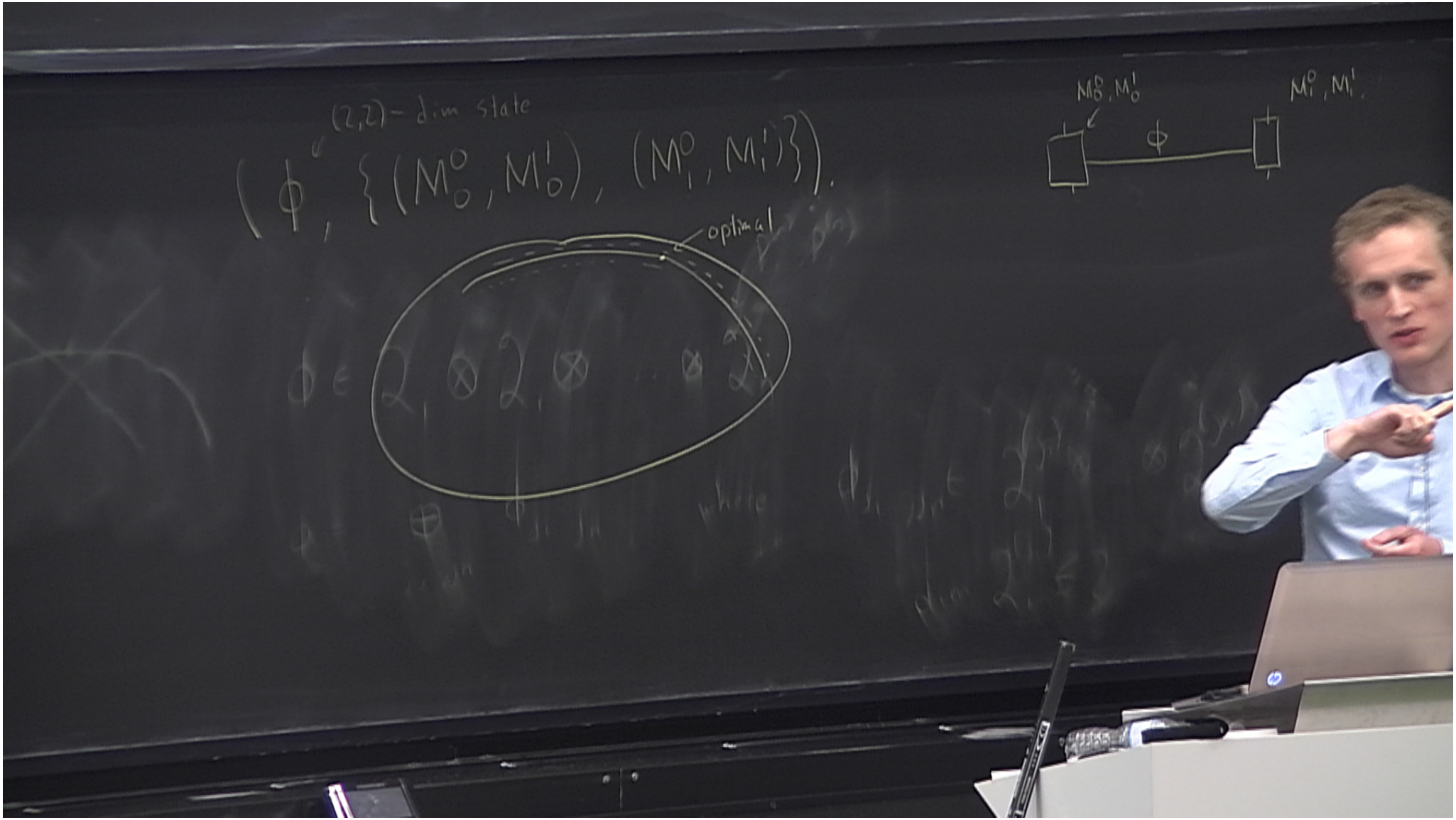
(2,2)-dim state
optimal

Applications

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$$Z_{GHZ}(\lambda, \theta_1, \theta_2) = -\cos(\lambda) + \cos(\lambda + \theta_1 + \theta_2) + \cos(\lambda + \theta_1 + \theta_3) + \cos(\lambda + \theta_2 + \theta_3).$$

The GHZ game is a robust self test. (This is can also be proved from [McKague 2010].)

Applications

Variants of the CHSH game.

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$$Z_{CHSH}(\lambda, \theta_1, \theta_2) = (\alpha)\cos(\lambda) + (\alpha)\cos(\lambda + \theta_2) + \cos(\lambda + \theta_1) - \cos(\lambda + \theta_1 + \theta_2).$$

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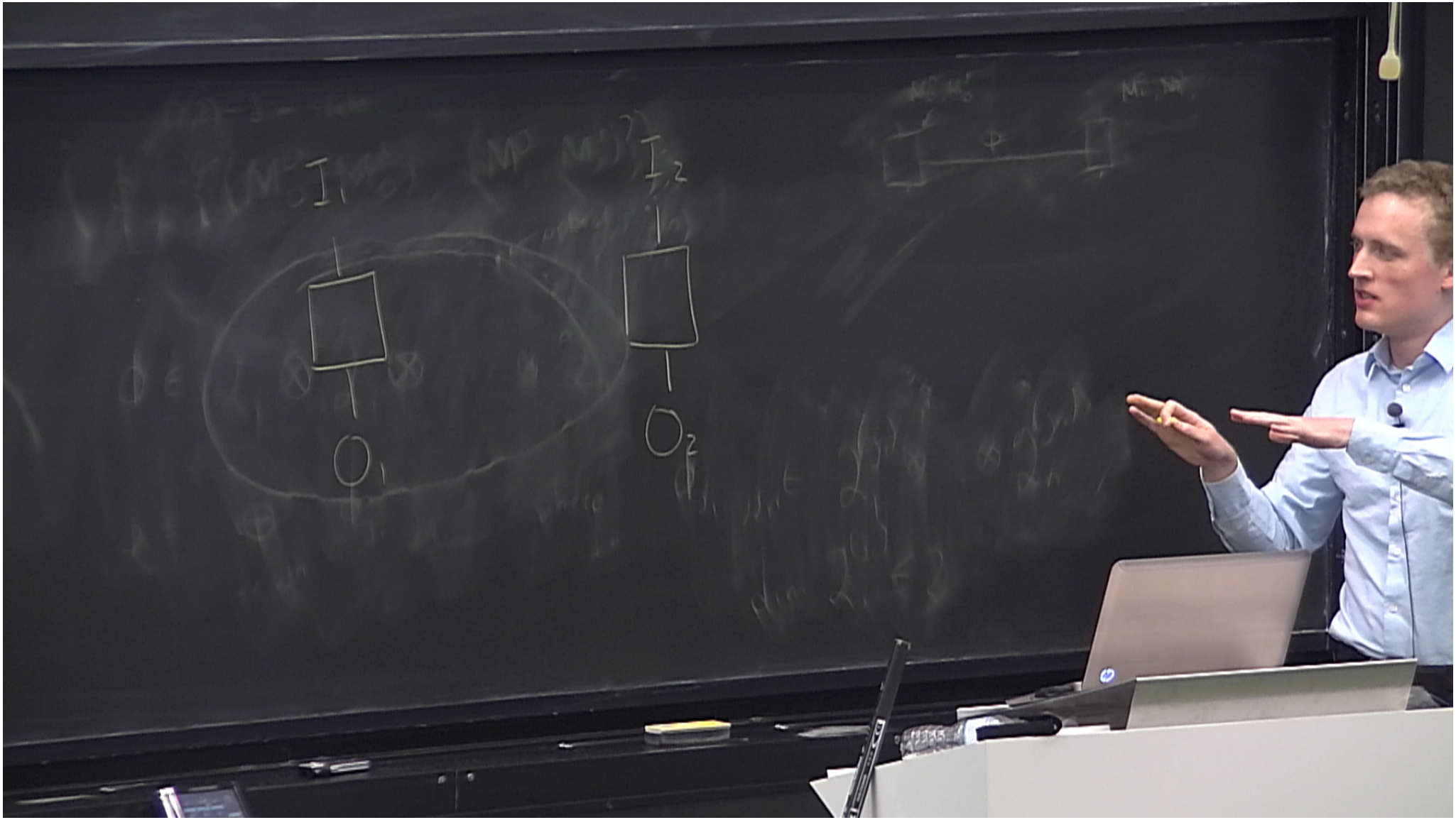
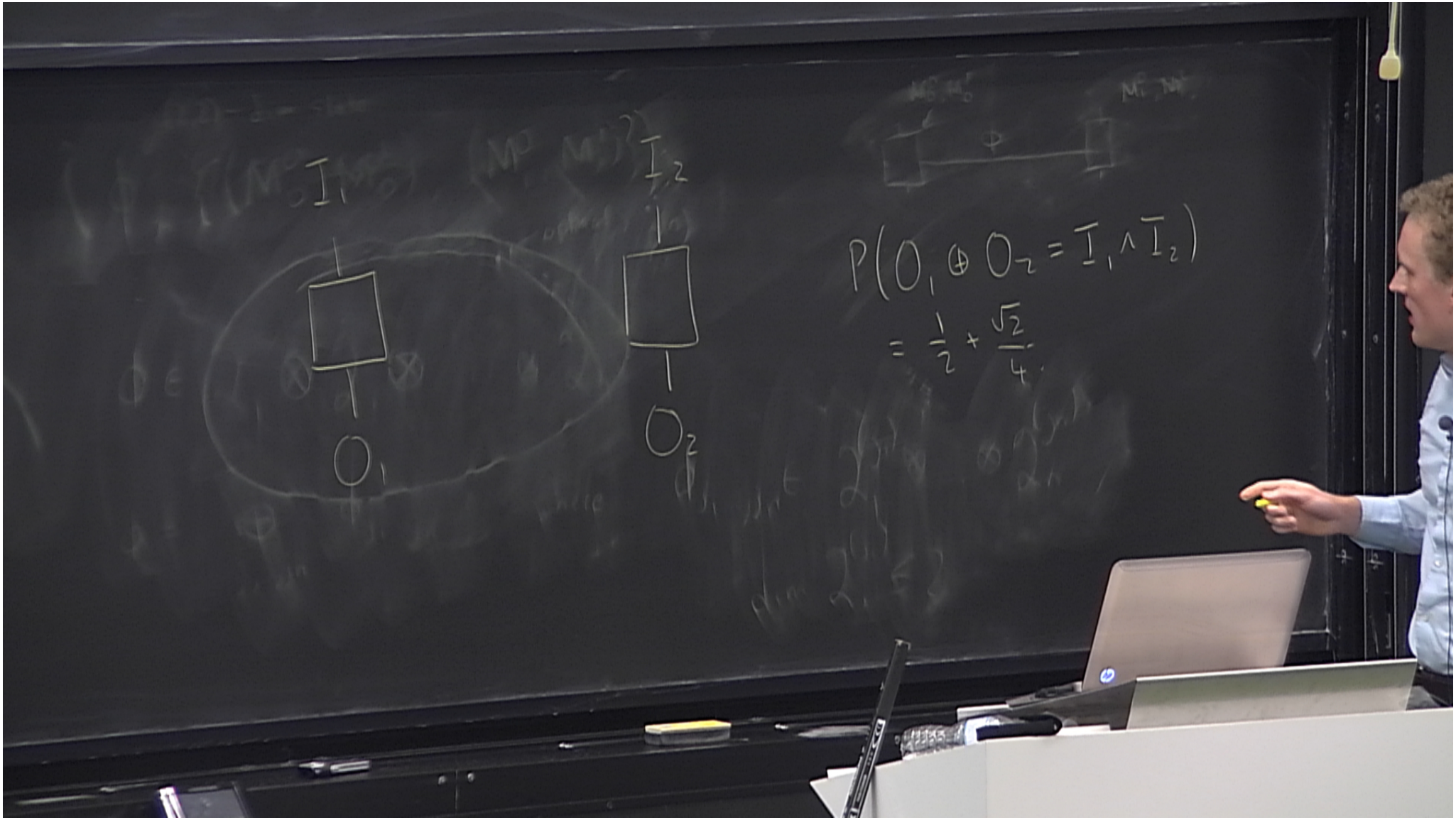
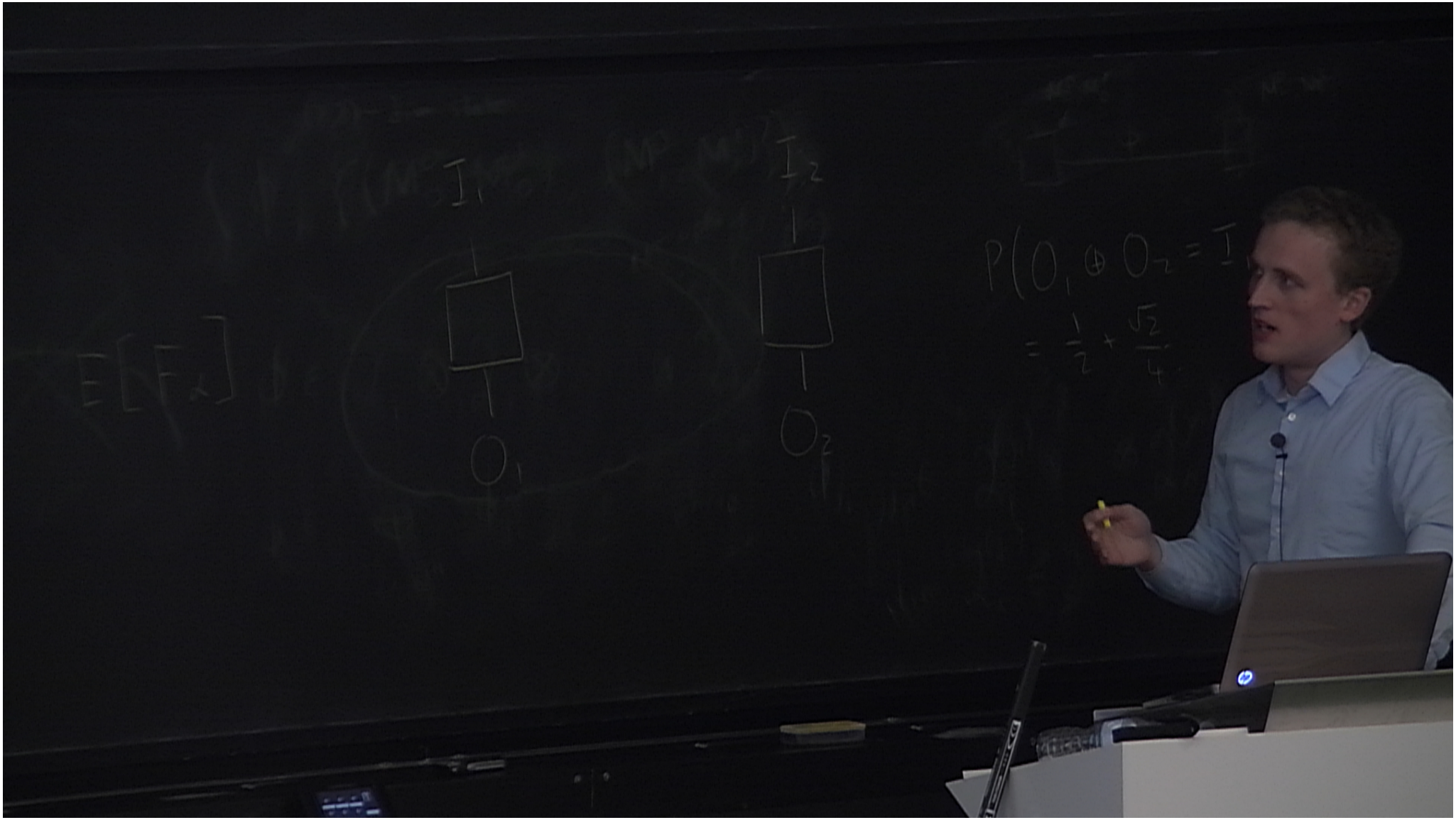
(This game was proposed in [Acin 2012] in the context of randomness expansion.)

By our criterion, this game is a robust self-test.


I_1
I_2
O_1
O_2


I_1
I_2
O_1
O_2
$P(O_1 \oplus O_2 = I_1 \wedge I_2)$
$= \frac{1}{2} + \frac{\sqrt{2}}{4}$


I_1
I_2
O_1
O_2
$P(O_1 \oplus O_2 = I$
$= \frac{1}{2} + \frac{\sqrt{2}}{4}$


I_1
I_2
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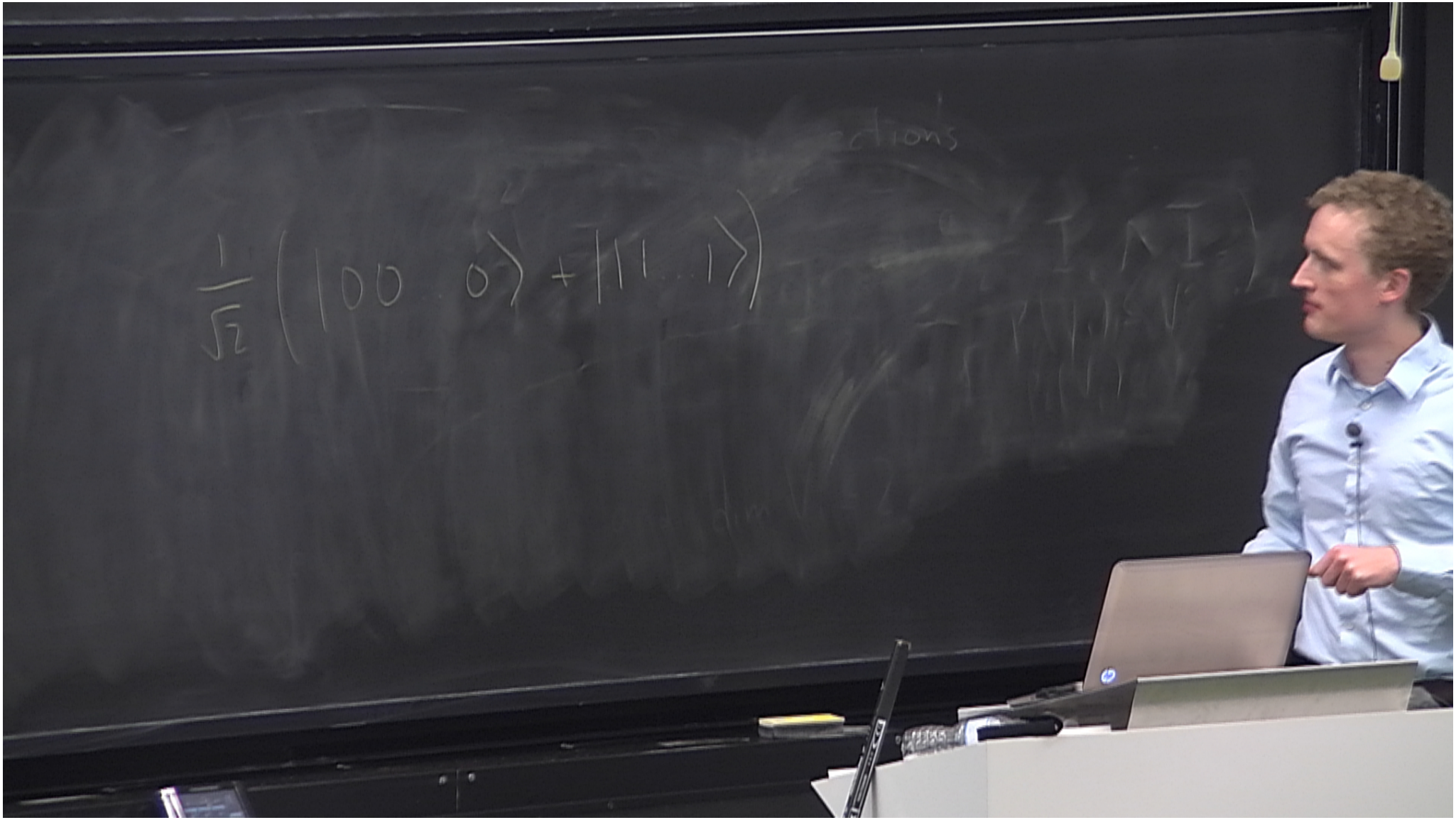
$\frac{3}{4}$

Future Goals

- Prove criteria for robust self-tests of states other than $\{\Lambda_n\}$.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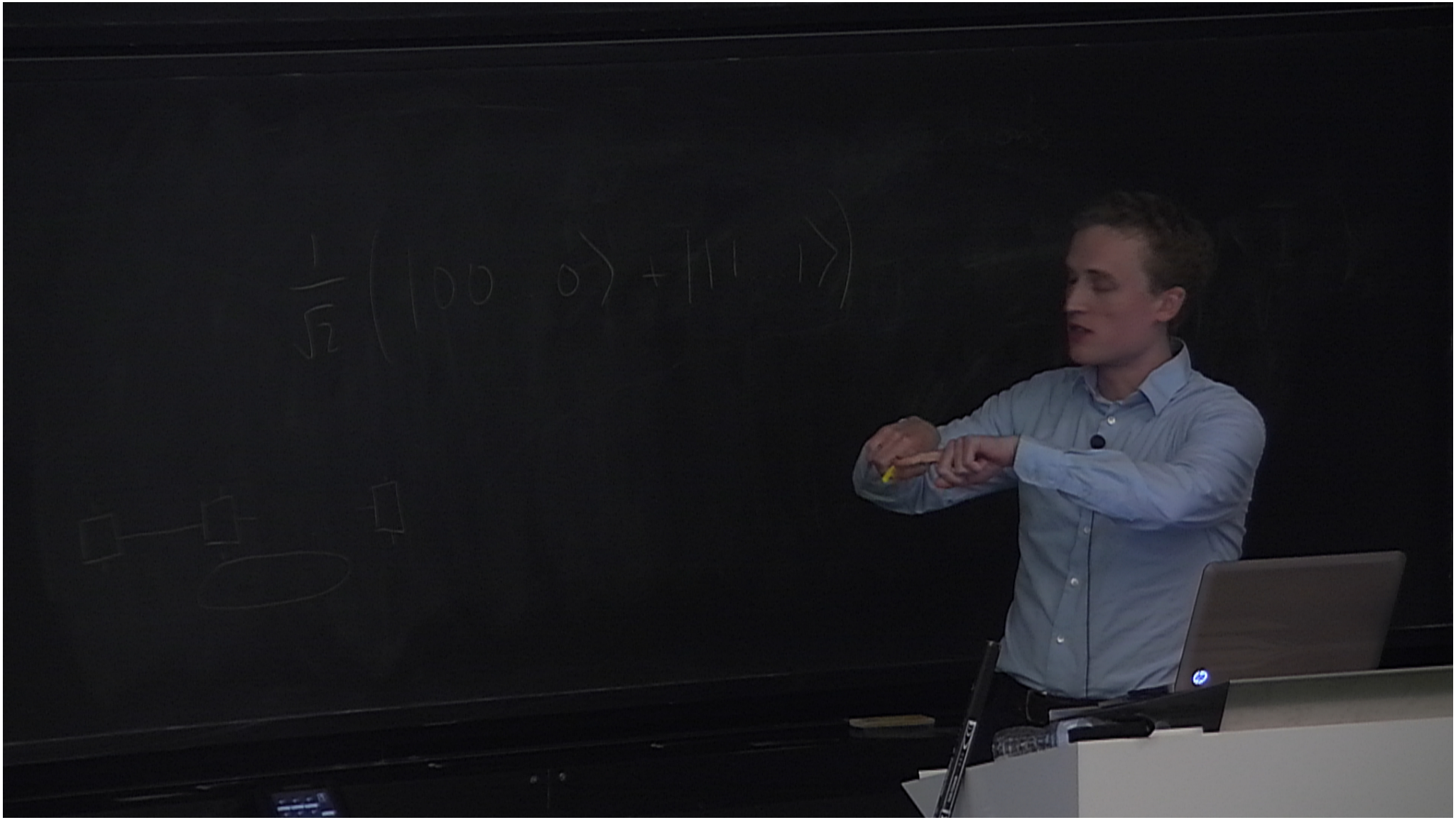
M. McKague. Self-testing graph states. arXiv:1010:1989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- Devise strong security proofs for randomness expansion, randomness amplification, and QKD.

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