Title: Massive gravitons, the cosmological constant and new directions in gravity

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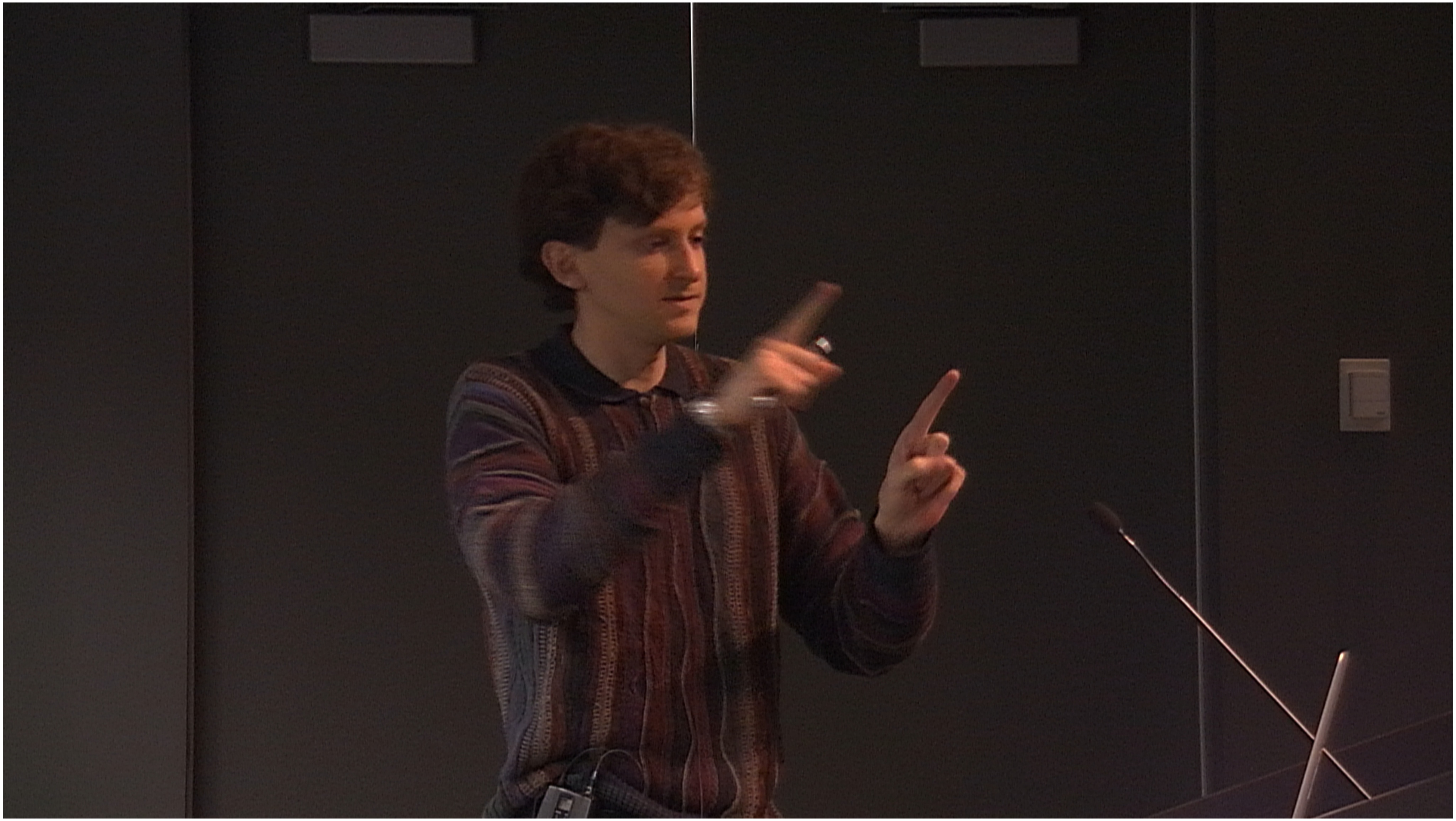
Abstract: The solutions to
the cosmological constant problems may involve modifying the very long-range
dynamics of gravity by adding new degrees of freedom. As an example of a conservative and minimal
such modification, we consider the possibility that the graviton has a very
small mass. Massive gravity has received
renewed interest due to recent advances which have resolved its traditional
problems. This kind of modification has
some peculiar and unexpected features, and it points us towards a universe
which looks quite unfamiliar.
Support
for this colloquium is provided by The Templeton Frontiers Program.



Massive gravitons, the cosmological constant and new directions in gravity

Kurt Hinterbichler (Perimeter Institute)

PI Colloquium, November 14, 2012

General Relativity

Einstein-Hilbert action: dynamical variable is the spacetime metric $g_{\mu\nu}$, and matter fields ψ

$$S = \frac{M_P^2}{2} \int d^4x \, \sqrt{-g}R + \mathcal{L}_M(\psi, g)$$

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Equations of motion for the metric -- Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{M_P} T_{\mu\nu}$$

Energy and momentum

second derivatives of the metric

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_M}{\delta g^{\mu\nu}}$$

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The redundancy is diffeomorphism invariance: $x^\mu \to f^\mu(x)$

$$g_{\mu\nu}(x) \to \frac{\partial f^\alpha}{\partial x^\mu} \frac{\partial f^\beta}{\partial x^\nu} g_{\alpha\beta}\left(f(x)\right)$$

GR is interacting massless spin-2

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Expand in fluctuations around flat space: $g_{\mu\nu} \sim \eta_{\mu\nu} + \frac{1}{M_P^2} h_{\mu\nu}$

$$S = \int d^4x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h + \frac{1}{M_{Pl}} h_{\mu\nu}T^{\mu\nu}$$

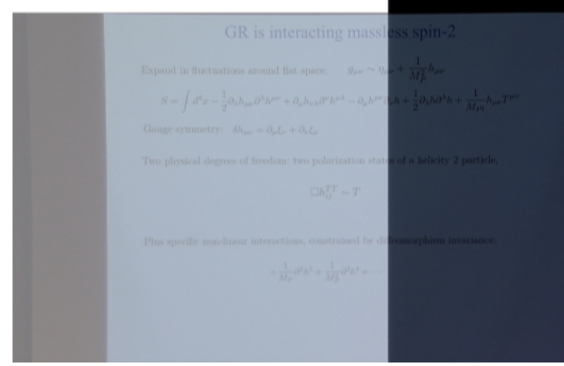
Gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Two physical degrees of freedom: two polarization states of a helicity 2 particle,

$$\Box h_{ij}^{TT} \sim T$$

Plus specific non-linear interactions, constrained by diffeomorphism invariance:

$$+\frac{1}{M_P}\partial^2 h^3 + \frac{1}{M_P^2}\partial^2 h^4 + \cdots$$



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Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
η_N	Nordtvedt effect	9×10^{-4}	lunar laser ranging
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent (see Equation (58))

Will, Living Rev. Relativity, 9, (2006)

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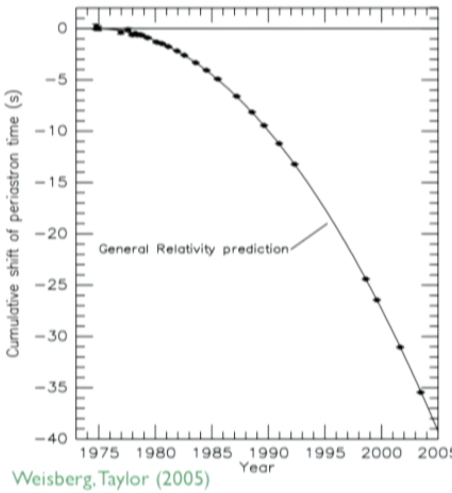

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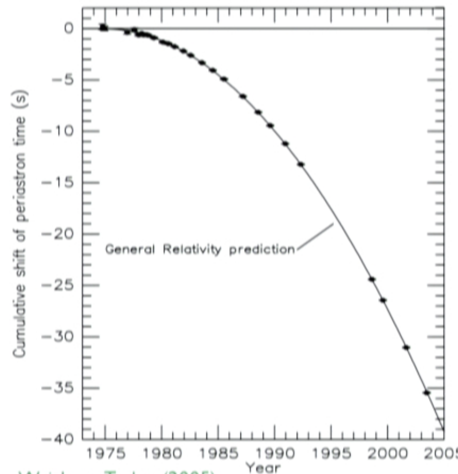

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Data agrees with GR, but all tests of the linear regime

Apply to cosmology

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$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3M_P^2} \sum_i \rho_i$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

Possibilities:

1) If we can measure the geometry $a(t)$ and the matter contents, we can *test* GR,

2) If we *assume* GR and measure the geometry $a(t)$, we can determine the matter content,

3) If we *assume* GR and measure the matter, we can determine the geometry $a(t)$.

Measuring the geometry

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Luminosity distance:

$$d_L^2 = \frac{L}{4\pi F}$$

Instrinsic luminosity

Observed flux

If the intrinsic luminosity of a source is known, the luminosity distance can be found by measuring the apparent luminosity.

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Calculate the Luminosity distance from the geometry:

$$d_L^2 = (1+z)\int_0^z \frac{dz'}{H(z')} \quad , \qquad 1+z = \frac{a_{obs}}{a_{em}}$$

If we can find $d_L(z)$ for a bunch of objects over a range of z, we can find $a(t)$

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For small z, this is Hubble's law:

$$H_{obs} d_L = z + \frac{1}{2}(1-q_{obs})z^2 - \frac{1}{6}\left(1 + q_{obs} - 3q_{obs}^2 + j_{obs} + \frac{k}{H_{obs}^2 a_{obs}^2}\right) z^3 + \cdots$$

$$q \equiv \frac{1}{H^2 a}\frac{d^2 a}{dt}, \quad j \equiv \frac{1}{H^3 a}\frac{d^3 a}{dt}, \quad \ldots$$

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Alter right hand side:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{m_P^2} \left[T_{\mu\nu} - m_P^2 \Lambda g_{\mu\nu} \right]$$ "Mysterious dark energy"

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Or alter left hand side:

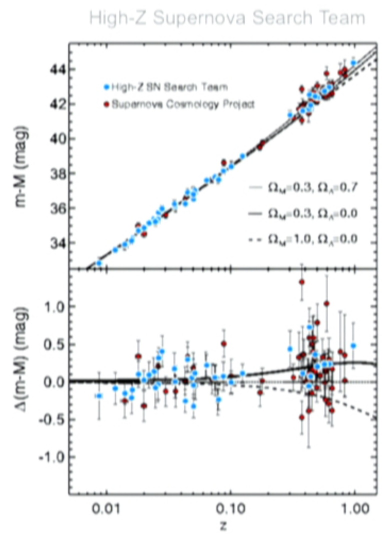
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{m_P^2} T_{\mu\nu}$$ "Modified gravity"

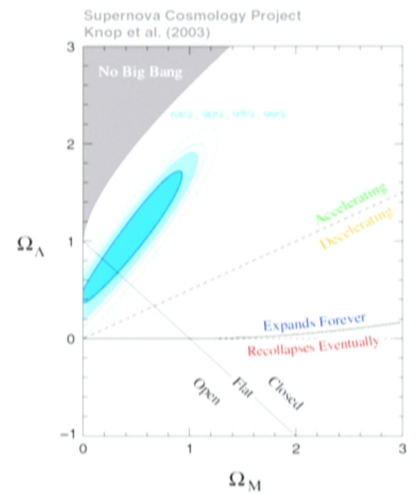
Supernova data

2011 Nobel Prize

Fit to a two component universe with matter w=0 and vacuum energy w=-1 (assuming GR),

$$p = w\rho, \quad \rho \sim a^{-3(1+w)} \qquad H(z)^2 = \sum_i \frac{\rho_0}{3M_P^2}(1+z)^{3(1+w_i)}$$





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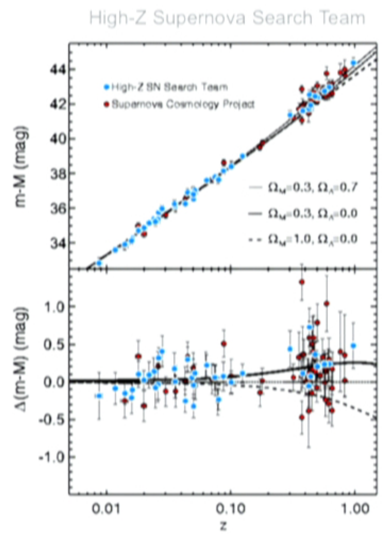

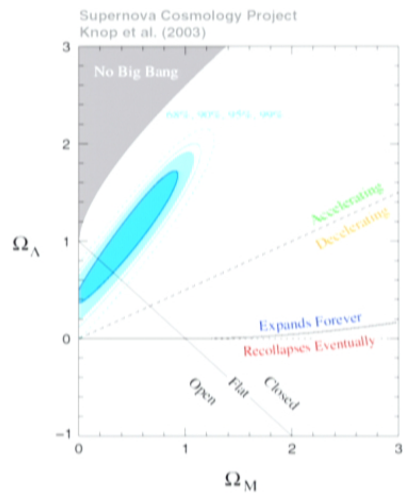

$$\boxed{\text{Data shows } \Lambda M_{\rm Pl}^2 \sim \rho_M}$$

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Observed CC is sum of everything:

$$\Lambda_{\text{observed}} = \Lambda_{\text{Bare}} + \Lambda_{\text{quantum}} + \Lambda_{\text{phase}} + \cdots$$

The cosmological constant problem

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$$

$$\frac{\Lambda}{M_P^2} \sim 10^{-122}$$

Really small

The cosmological constant problem

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} \qquad\qquad \frac{\Lambda}{M_P^2} \sim 10^{-122}$$

Really small

Two aspects to the problem:

- existence of the small number (naturalness)
- stability under quantum corrections (technical naturalness)

$$\Lambda_{\text{observed}} = \Lambda_{\text{Bare}} + \Lambda_{\text{quantum}} + \Lambda_{\text{phase}} + \cdots$$

Old CC problem: why is the CC zero and not large (i.e. planck or electroweak scale?)

New CC problem: why is the CC non-zero and ~ matter density today

Possibilities

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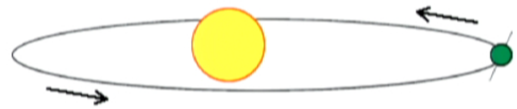

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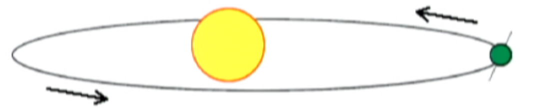

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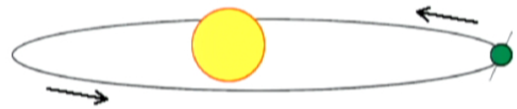

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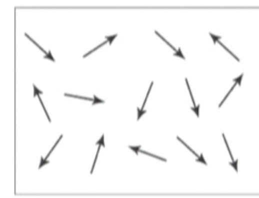

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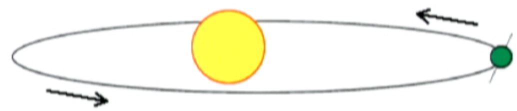

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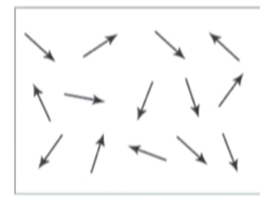

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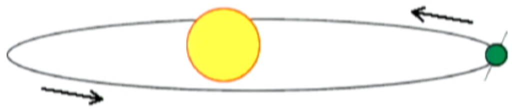

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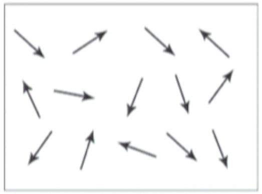

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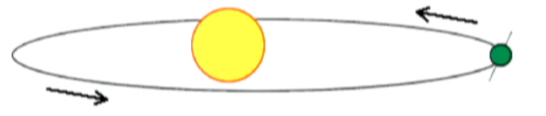

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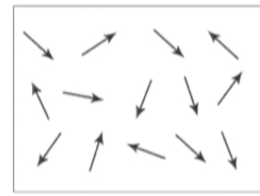

- Sheer luck

- Calculation wrong

$2+2=5$

Zoo of modified gravity

Any proposed theory should have something to say on two fronts:

1) Why is the universe accelerating at such a small rate?

2) Why does a large CC not curve the universe?

- Should be an infrared modification, to say something about the cosmological constant without messing up solar system tests of gravity

- GR is the unique theory of an interacting massless helicity-2 at low energies → to modify gravity is to change the degrees of freedom

Historical case of modified gravity vs. dark energy

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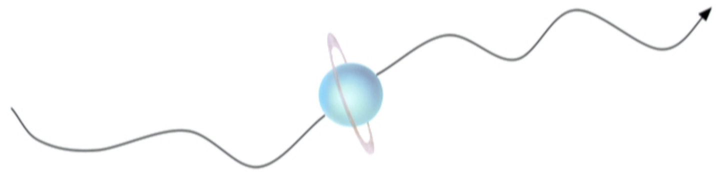
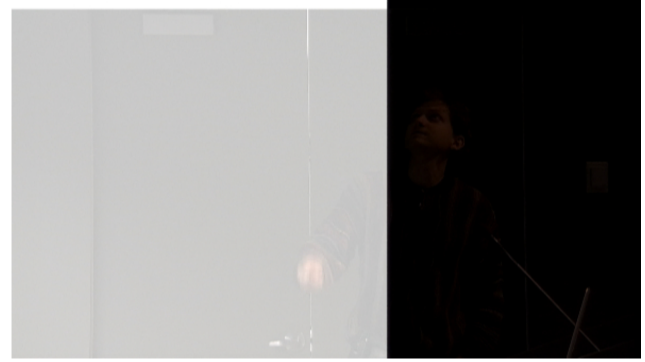
Mid 1800's: Newtonian gravity is the accepted theory, Uranus has a discrepancy in its orbit





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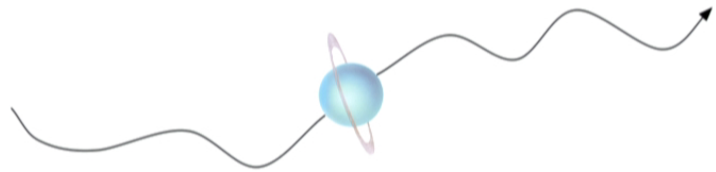
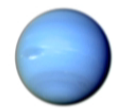


Le Verrier discovers Neptune (with the tip of his pen)



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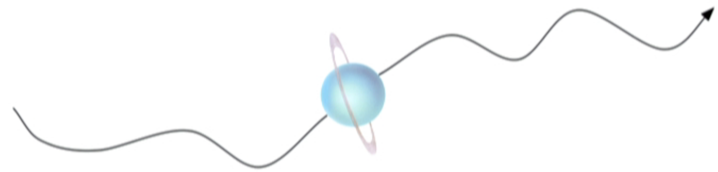
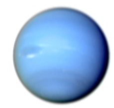
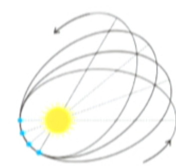


Le Verrier discovers Neptune (with the tip of his pen)



Mercury has a discrepancy in its orbit: perihelion precession by 43" per century



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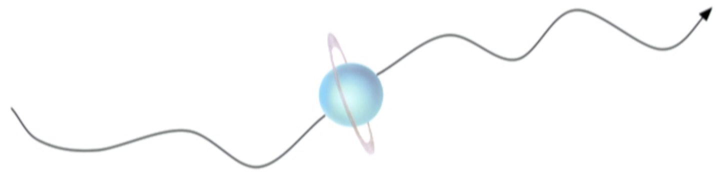

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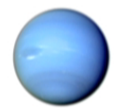

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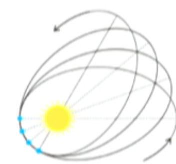

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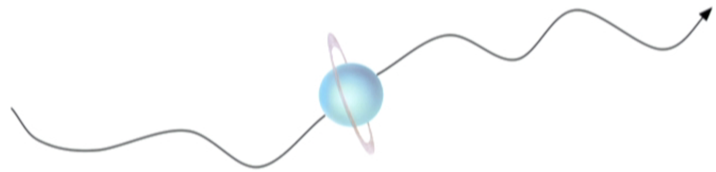

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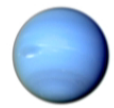

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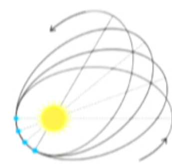

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Vulcan never found: precession later explained by GR

Lorentz invariance: degrees of freedom & interactions

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- Yang-mills is the only way for helicity-1's to interact at low energies
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First thought: make the graviton massive

Massive particle obeys $(\Box - m^2) = 0$

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IR modification scale

$$V(r) \sim \frac{M}{M_P^2}\frac{1}{r}e^{-mr}, \quad m \sim H$$

Extra DOF: 5 massive spin states as opposed to 2 helicity states

Massive gravity: other motivations

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2) It gives general lessons about GR:
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- Appreciation for why GR is special

3) It shows us new mechanisms: massive gravity is a deformation of GR $\rightarrow$ pathologies should go away as mass term goes to zero $\rightarrow$ new mechanisms for curing pathologies

Massive graviton: linear theory

Massive spin 2 particle: 5 degrees of freedom (as opposed to 2 for massless helicity 2)

Fierz-Pauli action: Fierz, Pauli (1939)

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

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Equations of motion: $(\Box - m^2)h_{\mu\nu} = 0, \quad \partial^\mu h_{\mu\nu} = 0, \quad h = 0$

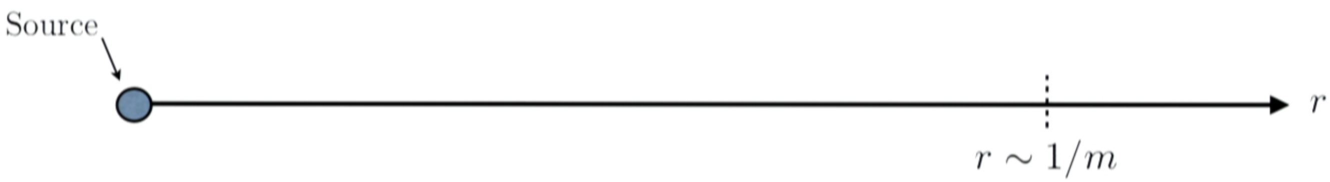
Linear solutions around sources

Amplitude for interaction of two conserved sources:

$$\mathcal{A} \sim \frac{1}{M_P} \int d^4p \frac{1}{p^2+m^2}\left[T'^{\mu\nu}(p)T_{\mu\nu}(p) - \frac{1}{3}T'(p)T(p)\right]$$

For GR this would be 1/2

Newtonian Potential: $\phi_N = -\frac{4}{3}\frac{GM}{r}e^{-mr}$



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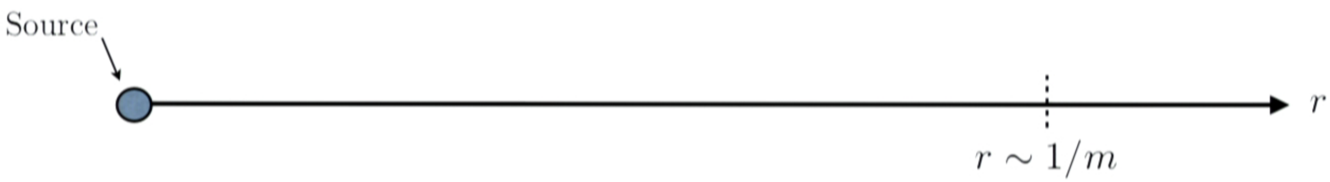

Massless gravity vs. massless limit of massive gravity: the vDVZ discontinuity

van Dam, Veltman, and Zakharov (1970)

	$m \to 0$	$m = 0$
Newtonian potential	$\phi_N = -\frac{4}{3}\frac{GM}{r}$	$\phi_N = -\frac{GM}{r}$
Light bending angle (at impact parameter b)	$\alpha = \frac{4GM}{b}$	$\alpha = \frac{4GM}{b}$

Non-linearities

Take interactions to be those of GR: $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

$$S = \frac{M_P^2}{2} \int d^4x \left[(\sqrt{-g}R) - \sqrt{-\eta}\frac{1}{4}m^2 \eta^{\mu\alpha}\eta^{\nu\beta} \left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right) \right]$$

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Non-linearity expansion of the potential: $\phi(r) \quad = \quad \phi_0(r) + \epsilon\phi_1(r) + \epsilon^2\phi_2(r) + \cdots$

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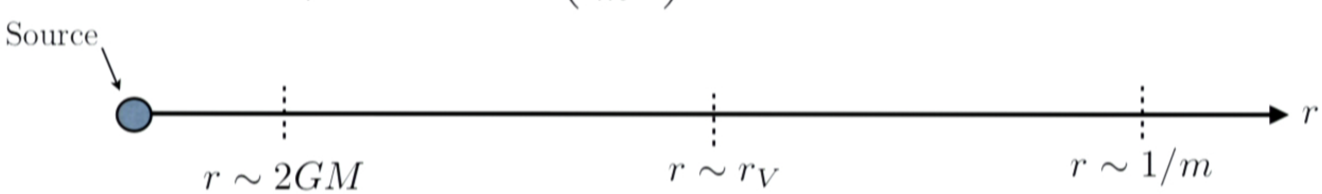
Non-linearity become important at the Vainshtein radius: Vainshtein (1972)

$$\epsilon \sim \left(\frac{r_V}{r}\right)^5, \qquad r_V \equiv \left(\frac{GM}{m^4}\right)^{1/5}$$



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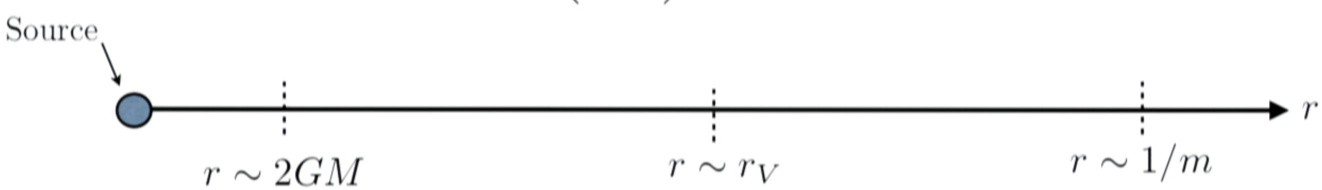

vDVZ discontinuity could possibly be cured by non-linearities

The Boulware-Deser ghost

Boulware, Deser (1972)

ADM variables:
$$\begin{aligned} g_{00} &= -N^2 + g^{ij} N_i N_j, \\ g_{0i} &= N_i, \\ g_{ij} &= g_{ij}. \end{aligned}$$

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In massive GR, they are auxiliary variables

Phase space DOF = 6 spatial metric + 6 canonical momentum - 0 constraints = 12 $\rightarrow$ 6 real space DOF

Extra non-linear D.O.F. is the Boulware-Deser ghost

Hamiltonian is unbounded.

Helicity analysis

Introduce Stükelberg fields: $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu \partial_\nu \phi$

$$h_{\mu\nu} \xrightarrow[\text{relativistic limit } m \to 0]{} \begin{cases} h_{\mu\nu} \sim \text{helicity } \pm 2 & \text{2 DOF} \\ A_\mu \sim \text{helicity } \pm 1 & \text{2 DOF} \\ \phi \sim \text{helicity } 0 & \text{1 DOF} \end{cases}$$

5 DOF

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5 DOF → 2 DOF, 2 DOF, 1 DOF

Canonically normalize $A_\mu \sim \frac{1}{m}\hat{A}_\mu, \quad \phi \sim \frac{1}{m^2}\hat{\phi}$ massless limit

$$\mathcal{L}_{m=0} - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 2\left(h_{\mu\nu}\partial^\mu \partial^\nu \hat{\phi} - h\partial^2 \hat{\phi}\right) + \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}$$

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Diagonalize kinetic terms $h_{\mu\nu} = h'_{\mu\nu} + \hat{\phi}\,\eta_{\mu\nu}$

This is the vDVZ discontinuity: scalar fifth force

$$\mathcal{L}_{m=0}(h') - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 3\,\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{1}{M_P}\hat{\phi}T$$

Interaction terms

$$\frac{M_P^2}{2}\int d^4x\,\left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h)\right],$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
V_5(g,h) &= +f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle \\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5, \\
&\vdots
\end{aligned}$$

The effective field theory

Arkani-Hamed, Georgi and Schwartz (2003)
Creminelli, Nicolis, Pappuchi, Trincherini (2005)

After replacement $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu\partial_\nu\phi + \cdots$ there are interaction terms:

$$m^2 M_P^2 h^{n_h} (\partial A)^{n_A} (\partial^2\phi)^{n_\phi} \sim \Lambda_\lambda^{4-n_h-2n_A-3n_\phi} \hat{h}^{n_h} (\partial\hat{A})^{n_A} (\partial^2\hat{\phi})^{n_\phi}$$

Various strong coupling scales:
The larger $\boldsymbol{\lambda}$, the smaller the scale

$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$$

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The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_P m^4)^{1/5}$$

This is the (UV) strong coupling scale of the theory

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After replacement $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu \partial_\nu \phi + \cdots$ there are interaction terms:

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Various strong coupling scales:
The larger $\boldsymbol{\lambda}$, the smaller the scale

$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$$

The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \qquad \Lambda_5 = (M_P m^4)^{1/5}$$

This is the (UV) strong coupling scale of the theory

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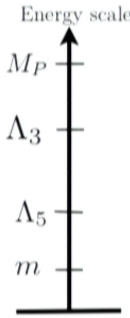
Scalar self-interactions responsible for Vainshtein radius, and display the BD ghost.

Key insight: Can choose the interactions, order by order in h, so that the scalar self-interactions appear in total derivative combinations.

The Λ_3 theory (dRGT theory)

de Rham, Gabadadze (2010)

The leading operators now carry the scale $\Lambda_3 \equiv (M_P m^2)^{1/3}$ $\quad \sim \frac{\hat{h}(\partial^2\hat{\phi})^n}{M_P^{n+1} m^{2n+2}}$



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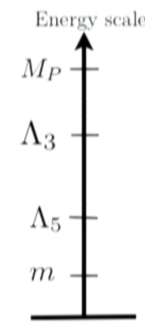
$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}^{\mu\nu}\left[-4X^{(1)}_{\mu\nu}(\hat{\phi}) + \frac{4(6c_3-1)}{\Lambda_3^3}X^{(2)}_{\mu\nu}(\hat{\phi}) + \frac{16(8d_5+c_3)}{\Lambda_3^6}X^{(3)}_{\mu\nu}(\hat{\phi})\right] + \frac{1}{M_P}\hat{h}_{\mu\nu}T^{\mu\nu}$$



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X tensors:

$$X^{(n)}_{\mu\nu} = \frac{1}{n+1}\frac{\delta}{\delta\Pi_{\mu\nu}}\mathcal{L}^{\rm TD}_{n+1}(\Pi)$$

$$\begin{aligned}
X^{(0)}_{\mu\nu} &= \eta_{\mu\nu} \qquad\qquad (\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi)\\
X^{(1)}_{\mu\nu} &= [\Pi]\,\eta_{\mu\nu} - \Pi_{\mu\nu}\\
X^{(2)}_{\mu\nu} &= \left([\Pi]^2 - [\Pi^2]\right)\eta_{\mu\nu} - 2\,[\Pi]\,\Pi_{\mu\nu} + 2\Pi^2_{\mu\nu}\\
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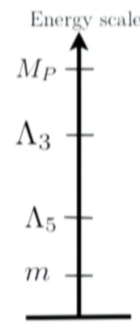

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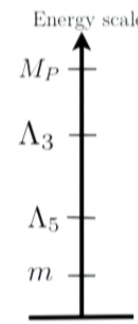

They have the following properties, which ensures that the decoupling limit is ghost free

$$\partial^\mu X^{(n)}_{\mu\nu} = 0$$

$X^{(n)}_{ij}$ has at most two time derivatives,

$X^{(n)}_{0i}$ has at most one time derivative,

$X^{(n)}_{00}$ has no time derivatives.

Galileons

Luty, Porrati, Rattazzi (2003)
Nicolis, Rattazzi, Trincherini (2008)

Diagonalize: $\hat{h}_{\mu\nu} \rightarrow \hat{h}_{\mu\nu} + \hat{\phi}\hat{h}_{\mu\nu} + \frac{2(6c_3 - 1)}{\Lambda_3^3}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}$

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta}$$
$$-3(\partial\hat{\phi})^2 + \frac{6(6c_3-1)}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} - 4\frac{(6c_3-1)^2 - 4(8d_5+c_3)}{\Lambda_3^6}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2]\right)$$
$$-\frac{40(6c_3-1)(8d_5+c_3)}{\Lambda_3^9}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3 - 3[\hat{\Pi}^2][\hat{\Pi}] + 2[\hat{\Pi}^3]\right)$$

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Longitudinal mode is described by Galileon interactions:

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- Equations of motion are second order (no ghost)
- Wess-Zumino terms for shifts of the field and its derivative $\phi(x) \to \phi(x) + c + c_\mu x^\mu$ — Goon, KH, Joyce, Trodden (2012)
- Not renormalized at any loop (no quantum corrections in the decoupling limit) — Luty, Porrati, Rattazzi (2003); Nicolis, Rattazzi (2004); KH, Trodden, Wesely (2010)

Recovering GR in the solar system: Vainshtein Mechanism

Nicolis, Rattazzi (2004)

$$\mathcal{L} = -3(\partial\hat{\phi})^2 - \frac{1}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{1}{M_4}\hat{\phi}T$$

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Solution around point source of mass M:

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Kinetic terms are enhanced, which means that, after canonical normalization, the coupling to δT is suppressed. The non-linear coupling scale is also raised.

This is known as a Screening mechanism

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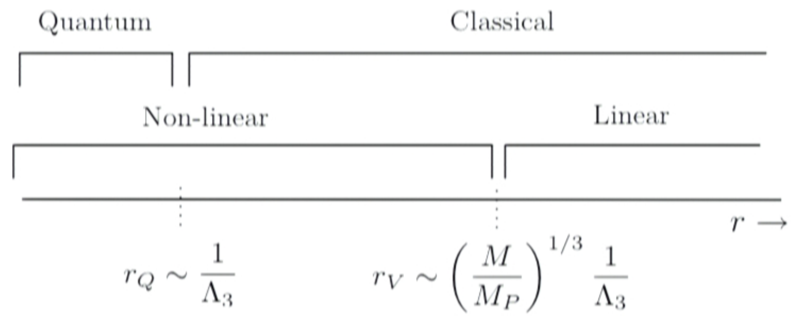
“Good” massive gravity



- Higher cutoff
- Free of the Boulware-Deser ghost, to all orders beyond the decoupling limit Hassan, Rosen (2011)
- Possesses a screening mechanism in the non-linear regime, which is under control quantum mechanically, and restores continuity with GR as m approaches 0.

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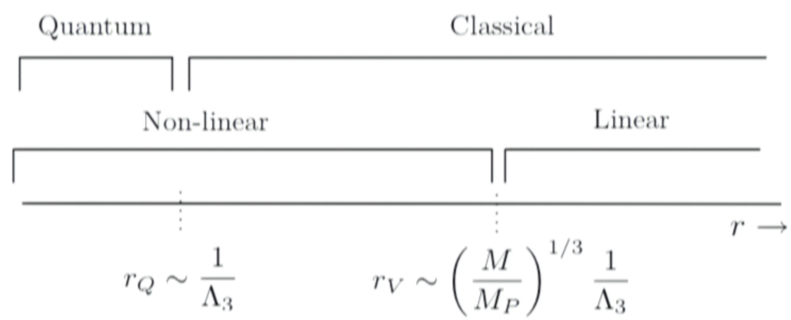

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dRGT massive gravity

de Rham, Gabadadze, Tolley (2011)

The theory can be re-summed:

$$\frac{M_P^{D-2}}{2}\int d^Dx\,\sqrt{-g}\left[R-\frac{m^2}{4}\sum_{n=0}^{D}\beta_n S_n(\sqrt{g^{-1}\eta})\right]$$

Characteristic Polynomials

$$S_n(M)=\frac{1}{n!(D-n)!}\,\tilde{\epsilon}_{A_1A_2\cdots A_D}\,\tilde{\epsilon}^{B_1B_2\cdots B_D}\,M^{A_1}_{\ B_1}\cdots M^{A_n}_{\ B_n}\,\delta^{A_{n+1}}_{\ B_{n+1}}\cdots\delta^{A_D}_{\ B_D}$$

$$\begin{aligned}
S_0(M) &= 1\,,\\
S_1(M) &= [M]\,,\\
S_2(M) &= \frac{1}{2!}\left([M]^2-[M^2]\right)\,,\\
S_3(M) &= \frac{1}{3!}\left([M]^3-3[M][M^2]+2[M^3]\right)\,,\\
&\ \ \vdots\\
S_D(M) &= \det M\,,
\end{aligned}$$

Vielbein formulation of ghost-free massive gravity

KH, Rachel Rosen (arXiv:1203.5783)

Or in terms of vierbeins $g_{\mu\nu} = e_\mu{}^A e_\nu{}^B \eta_{AB}$

$$\frac{M_P^{D-2}}{2} \int d^D x \; |e| R[e] - m^2 \sum_n a_n \int \epsilon_{A_1 \cdots A_D} e^{A_1} \wedge \cdots \wedge e^{A_n} \wedge 1^{A_{n+1}} \wedge \cdots \wedge 1^{A_n}$$

Ghost-free mass terms are simply all possible ways of wedging a vierbein and background vierbein:

$$\begin{aligned}
&\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4} \\
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Some features of the theory

Theoretical:

- Consistent, ghost free effective field theory propagating a single massive graviton

CC problem:

- Exist self accelerating solutions, in the absence of a CC (acceleration is caused by graviton mass $m \sim H$) de Rham, Gabadadze, Heisenberg, Pirtskhalava (2010) Gumrukcuoglu, Lin, Mukohyama (2011)

- A small graviton mass is protected from large quantum corrections (diff invariance restored as $m \to 0$)

- Screening of a large CC (works for linear theory, non-linear (?)) Dvali, Gabadadze, Shifman (2002) Arkani-Hamed, Dimopolous, Dvali Gabadadze (2002) Dvali, Hoffman, Khoury (2007)

- Vainshtein mechanism hides 5-th force from experiments

Phenomenology/new signals: D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley (2011)

- Forbids strictly flat homogeneous, isotropic FRW solutions $\to$ signals of anisotropy and/or inhomogeneity should be starting to become important as CC dominates

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Phenomenology/new signals: D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley (2011)

- Forbids strictly flat homogeneous, isotropic FRW solutions $\rightarrow$ signals of anisotropy and/or inhomogeneity should be starting to become important as CC dominates

Superluminality: secret Lorentz violation

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Nicolis, Rattazzi, Trincherini (2008)

$$\pi = \pi_0(r) + \varphi$$

$$\mathcal{S}_\varphi = \frac{1}{2}\int d^4x \left[K_t(r)(\partial_t\varphi)^2 - K_r(r)(\partial_r\varphi)^2 - K_\Omega(r)(\partial_\Omega\varphi)^2\right]$$

Speeds of radial and angular perturbations: $c_r^2 = \frac{K_r}{K_t}$ $\quad c_\Omega^2 = \frac{K_\Omega}{K_t}$

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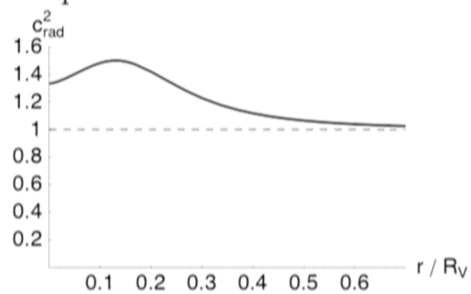

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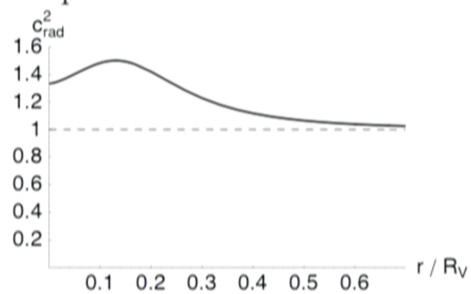

In a Lorentz invariant quantum field theory, we have: $[\phi(x), \phi(y)] = 0, \quad (x-y)^\mu$ spacelike

This is an operator statement, independent of the state (background)

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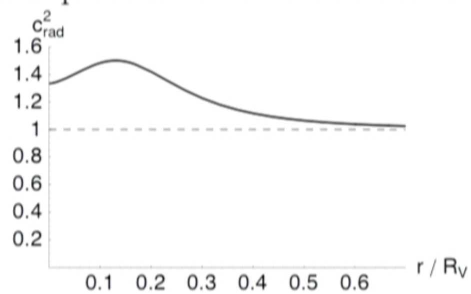

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Suggests that the UV completion (if it exists) is not a standard, local, Lorentz invariant quantum field theory.

Summary and open issues

- Λ_3 massive gravity is the best behaved IR modification of gravity proposed so far
- $\sim$ 40 year old problem of the Boulware-Deser ghost has been solved
- Makes use of galileons, scalar theories with interesting and promising properties
- New signals for cosmology/potential for model building
- Still some underlying topological structure yet to be articulated
- Still the issue of UV completion: if it exists, it's unlike the theories we are used to

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