Title: What is a Wavefunction?

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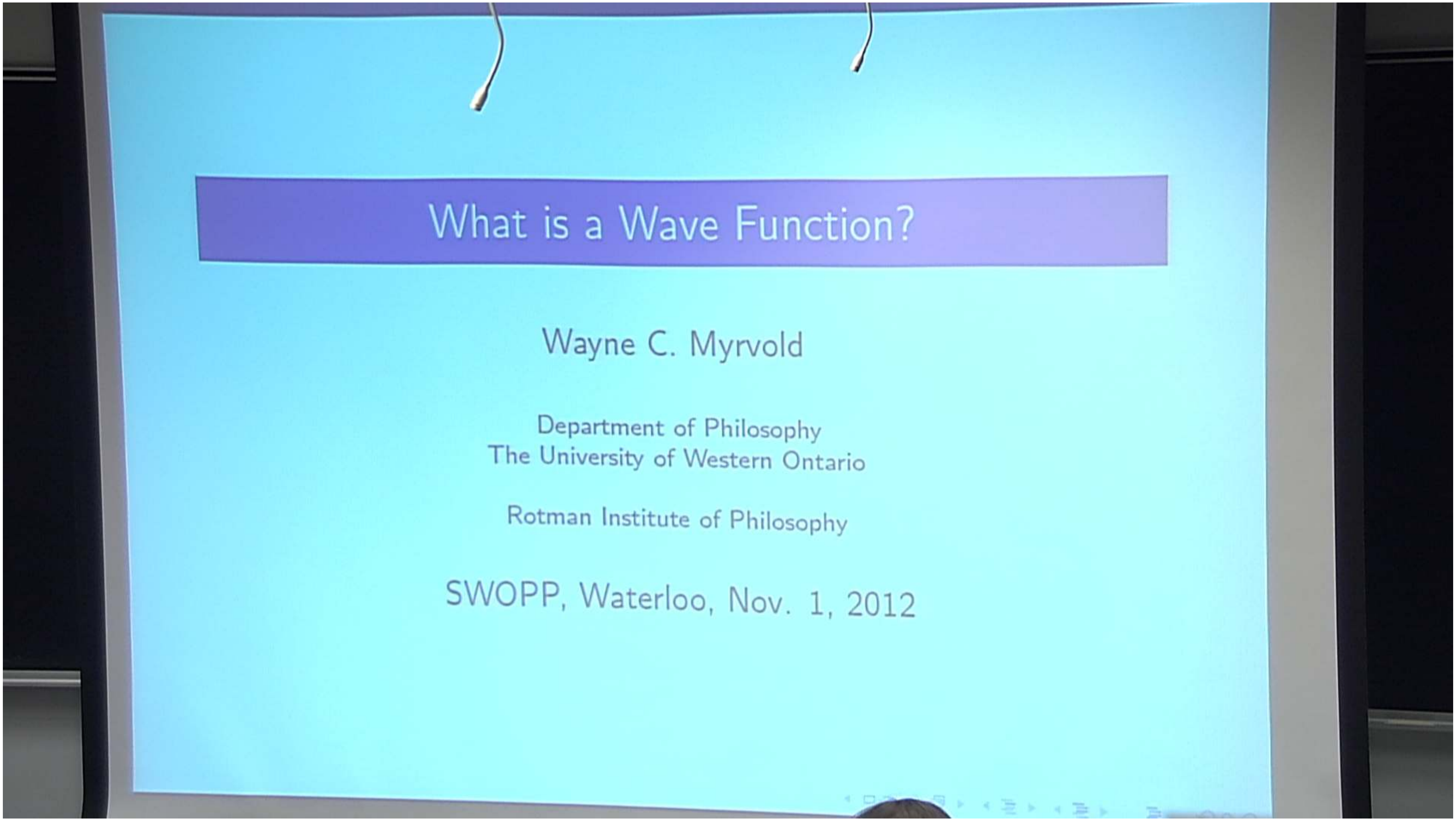
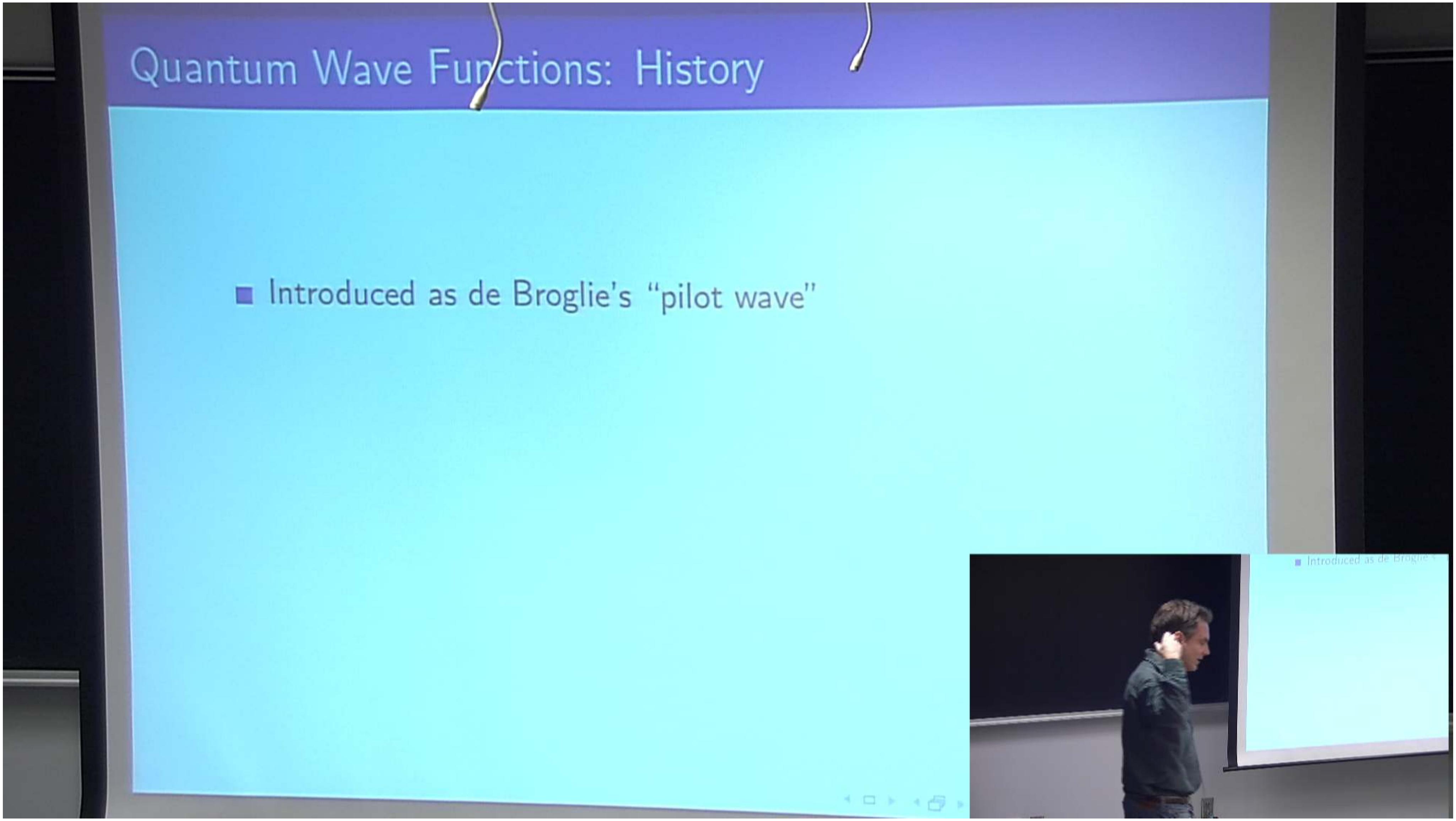
Abstract: The fact that the quantum wavefunction of a many-particle system is a function on a high-dimensional configuration space, rather than on spacetime, has led some to suggest that any realist understanding of quantum mechanics must regard configuration space as more fundamental than spacetime. Worse, it seems that a wavefunction monist ontology cannot help itself to talk of "configuration space" at all, without particles for the configurations to be configurations of. The wavefunction, it might seem, threatens to become a function defined on a high-dimensional space whose relation to spacetime is obscure. I will argue that such worries are misplaced.


What is a Wave Function?
Wayne C. Myrvold
Department of Philosophy
The University of Western Ontario
Rotman Institute of Philosophy
SWOPP, Waterloo, Nov. 1, 2012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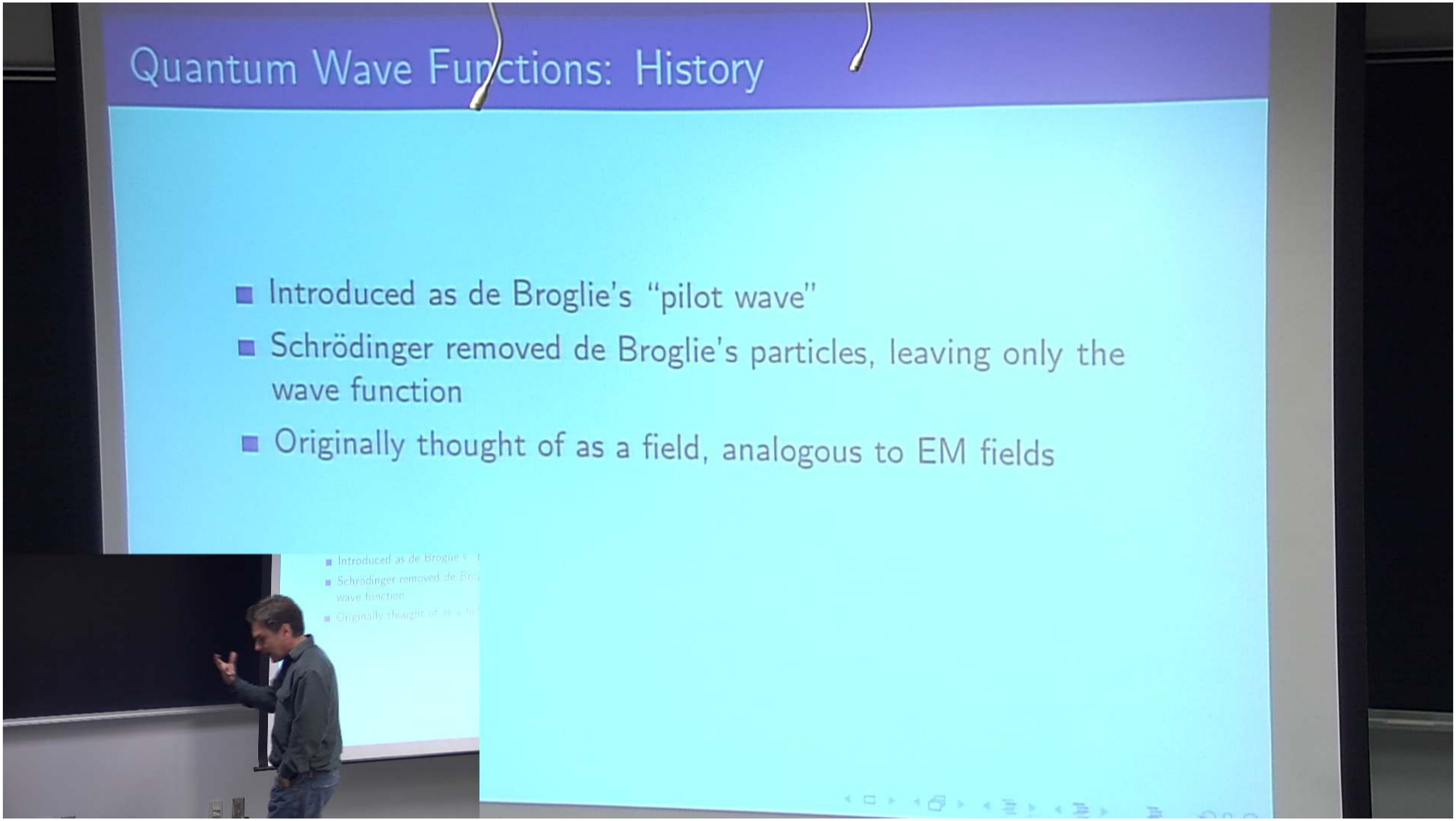
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Biting the Bullet, I

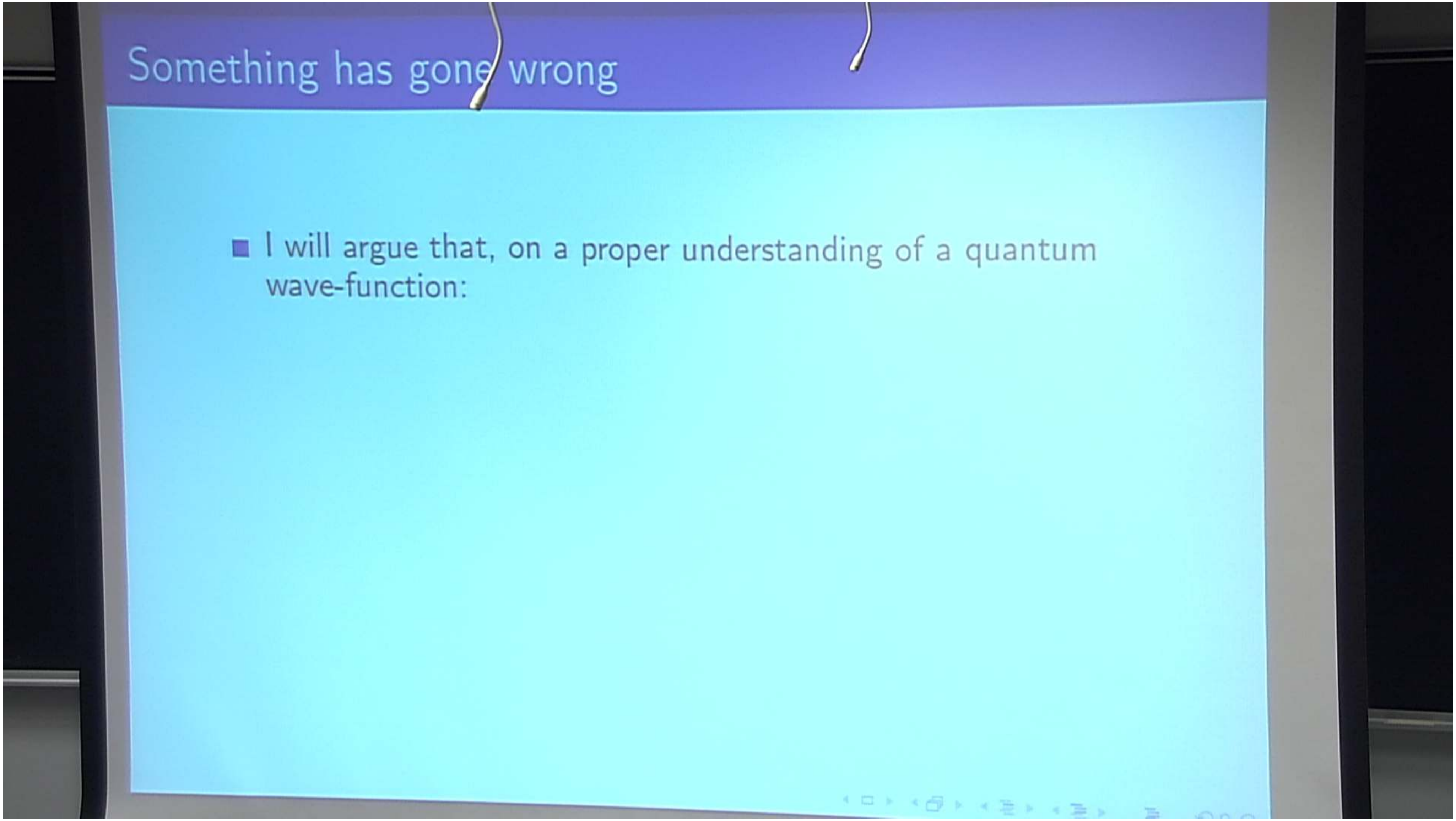
the space *we* live in, the space in which any realistic understanding of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* . . . is *configuration*-space. And whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory.

David Albert, "Elementary Quantum Metaphysics" (1996)

Chewing on the Bullet

any theory whose physical ontology is a complete wavefunction monism automatically inherits a severe interpretational problem: if all there is [is] the wavefunction, an extremely high-dimensional object evolving in some specified way, *how does that account for the low-dimensional world of localized objects that we start out believing in, whose apparent behavior constitutes the explanandum of physics in the first place?*

Tim Maudlin, "Can the world be only wavefunction?" (2010)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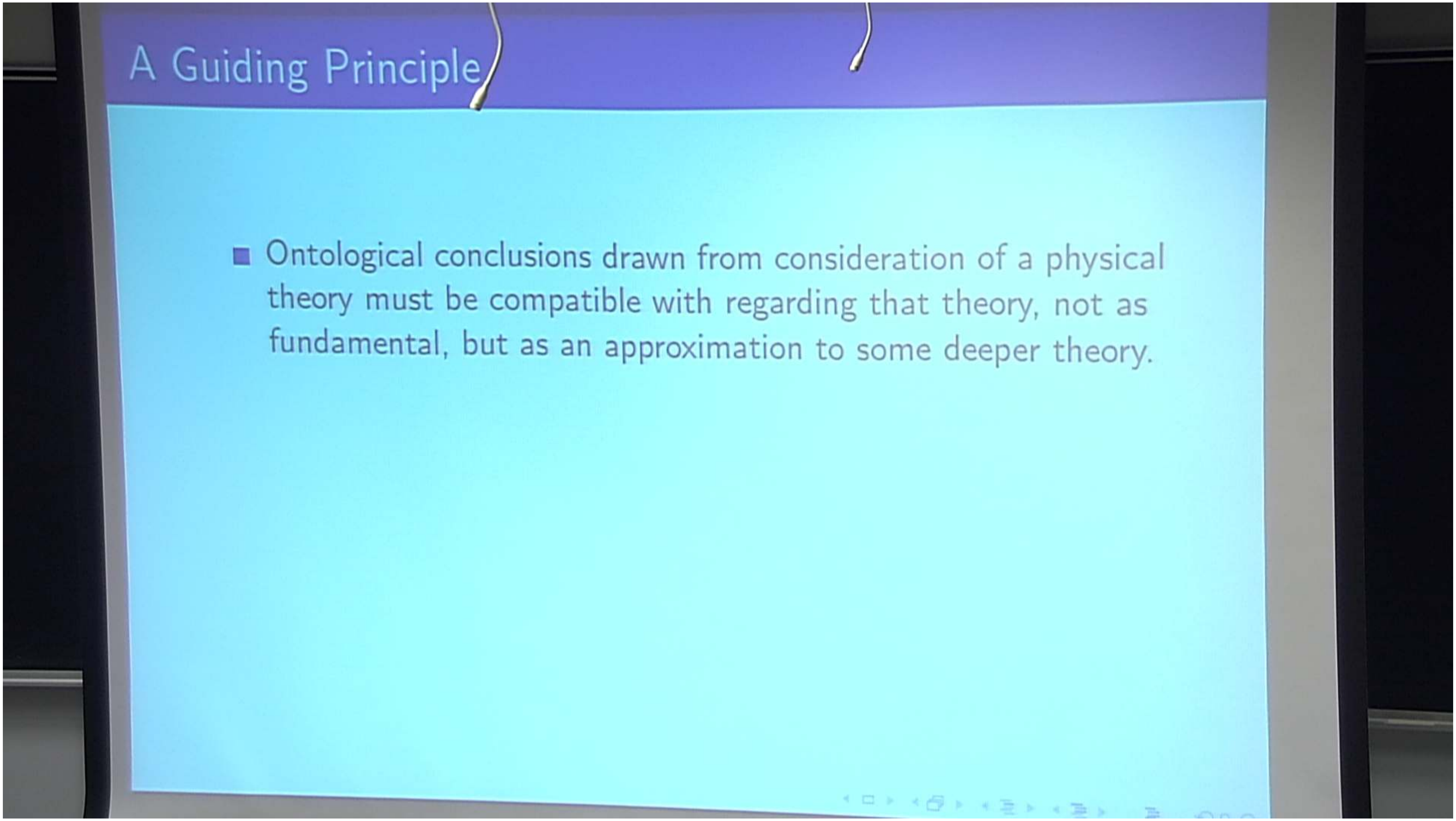
  - even for a single-particle wave function, the value of a wave function at a point cannot be regarded as a local property of that point (not a "local beable")
  - this sheds light relevance for EPR-type arguments, including Einstein's at the 1927 Solvay conference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- Ontological conclusions drawn from consideration of a physical theory must be compatible with regarding that theory, not as fundamental, but as an approximation to some deeper theory.
- In the case of quantum mechanics, we know that it is not fundamental: it should be regarded as a nonrelativistic, low-energy approximation to a quantum field theory.
- How do we get quantum wave functions from a quantum field theory?
- RQFT $\rightarrow$ GQFT $\rightarrow$ QM

A spinless Klein-Gordon Field

- Suppose we have field operators $\hat{\phi}(x)$, satisfying the K-G equation,

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\hat{\phi}(x) + \left(\frac{mc}{\hbar}\right)^2\hat{\phi}(x) = 0,$$

with

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}\left(\hat{a}(k)e^{-ikx} + \hat{b}^\dagger(k)e^{ikx}\right),$$

$$\omega_k = \sqrt{\mathbf{k}^2 + (mc/\hbar)^2}.$$

Newton-Wigner localization

- Pick a reference frame, and define new operators $\hat{\psi}_{NW}(\mathbf{x}, t)$ by

$$\hat{\psi}_{NW}(\mathbf{x}, t) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{k}}{\sqrt{\omega_\mathbf{k}}}\, \hat{a}(\mathbf{k}) e^{i(\mathbf{k}\cdot x - \omega_\mathbf{k} t)}.$$

- We have equal-time commutation relations

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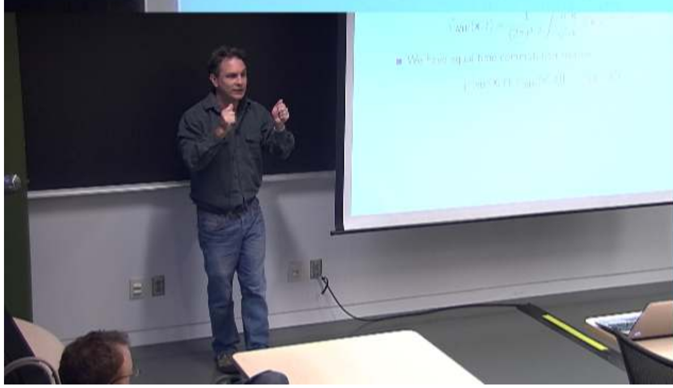

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- If we now define $\hat{\psi}(\mathbf{x}, t)$ by

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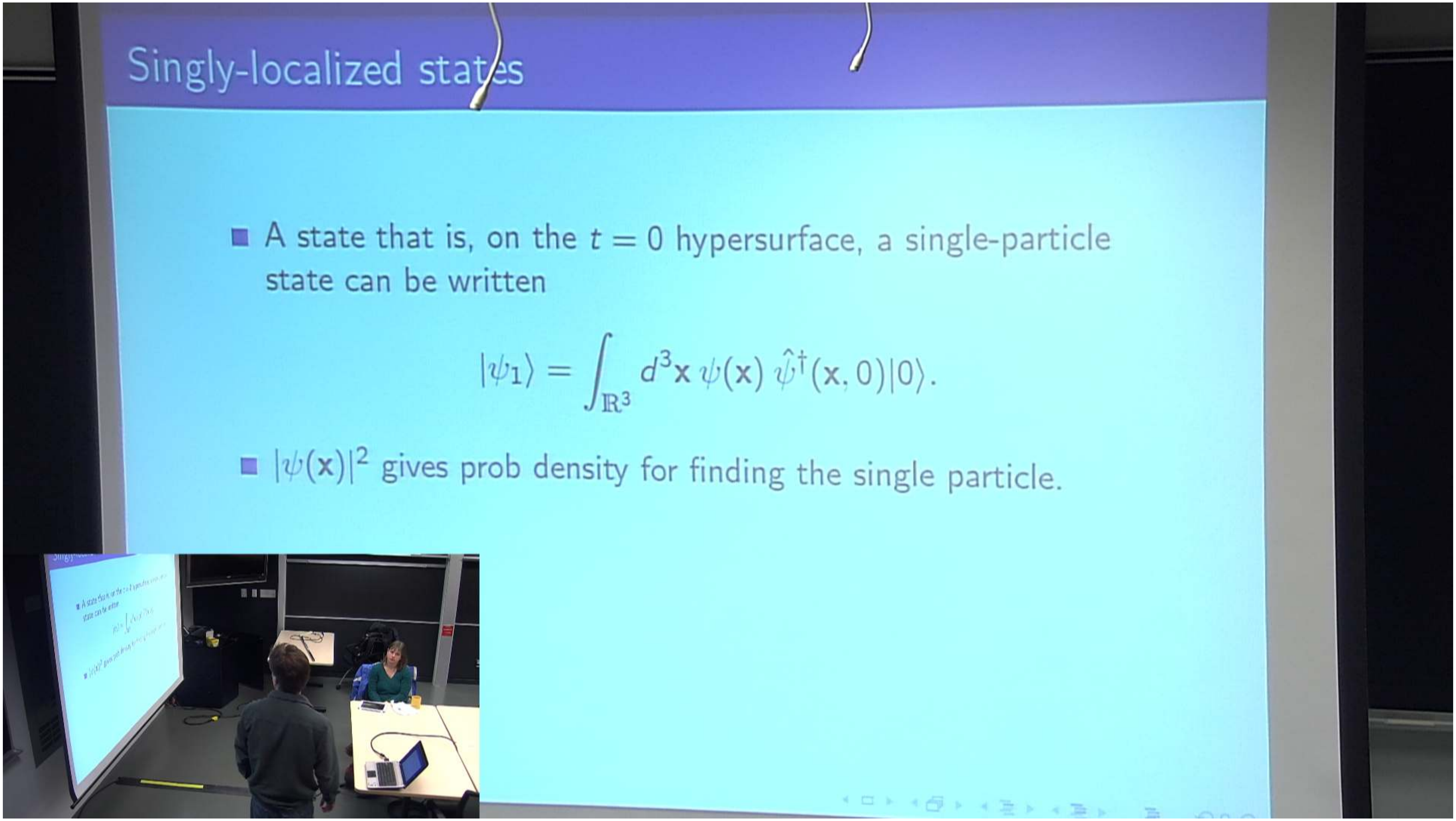
Singly-localized states

- A state that is, on the $t = 0$ hypersurface, a single-particle state can be written

$$|\psi_1\rangle = \int_{\mathbb{R}^3} d^3\mathbf{x}\, \psi(\mathbf{x})\, \hat{\psi}^\dagger(\mathbf{x}, 0)|0\rangle.$$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- Note that this is defined only for a special class of states, satisfying a *global* property of being a single-particle state.
- $\psi(\mathbf{x})$ is *not* a local beable, not an intrinsic attribute of arbitrarily small neighbourhoods of $(\mathbf{x}, 0)$.

Doubly-Localized states

- A state that is, on the $t = 0$ hypersurface, a two-particle state can be written

$$|\psi_2\rangle = \int_{\mathbb{R}^3} d^3\mathbf{x} \int_{\mathbb{R}^3} d^3\mathbf{x}' \, \psi(\mathbf{x}, \mathbf{x}') \, \hat{\psi}^\dagger(\mathbf{x}, 0)\hat{\psi}^\dagger(\mathbf{x}', 0)|0\rangle.$$

- $|\psi(\mathbf{x}, \mathbf{x}')|^2$ gives prob density for finding particles at $\mathbf{x}$, $\mathbf{x}'$.
- Similar remarks holding for n-particle states.

Constructing configuration spaces

- The subspace of n-particle states $|\psi_n\rangle$ is indexed by functions $\psi(\mathbf{x}_1, ..., \mathbf{x}_n)$, defined by

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- That is, we construct a $3n$-dimensional configuration space from our quantum field theory from a subspace of states, and field operators defined on spacetime.

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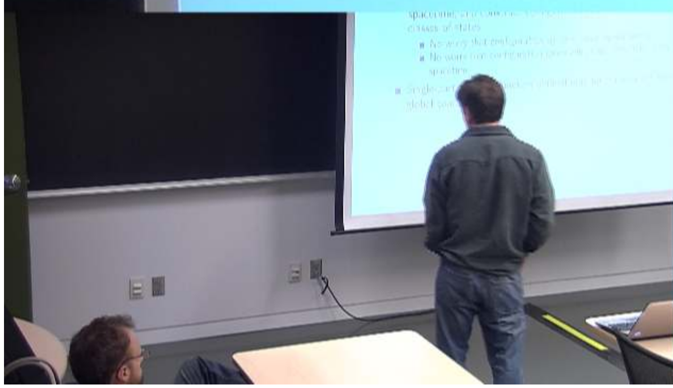

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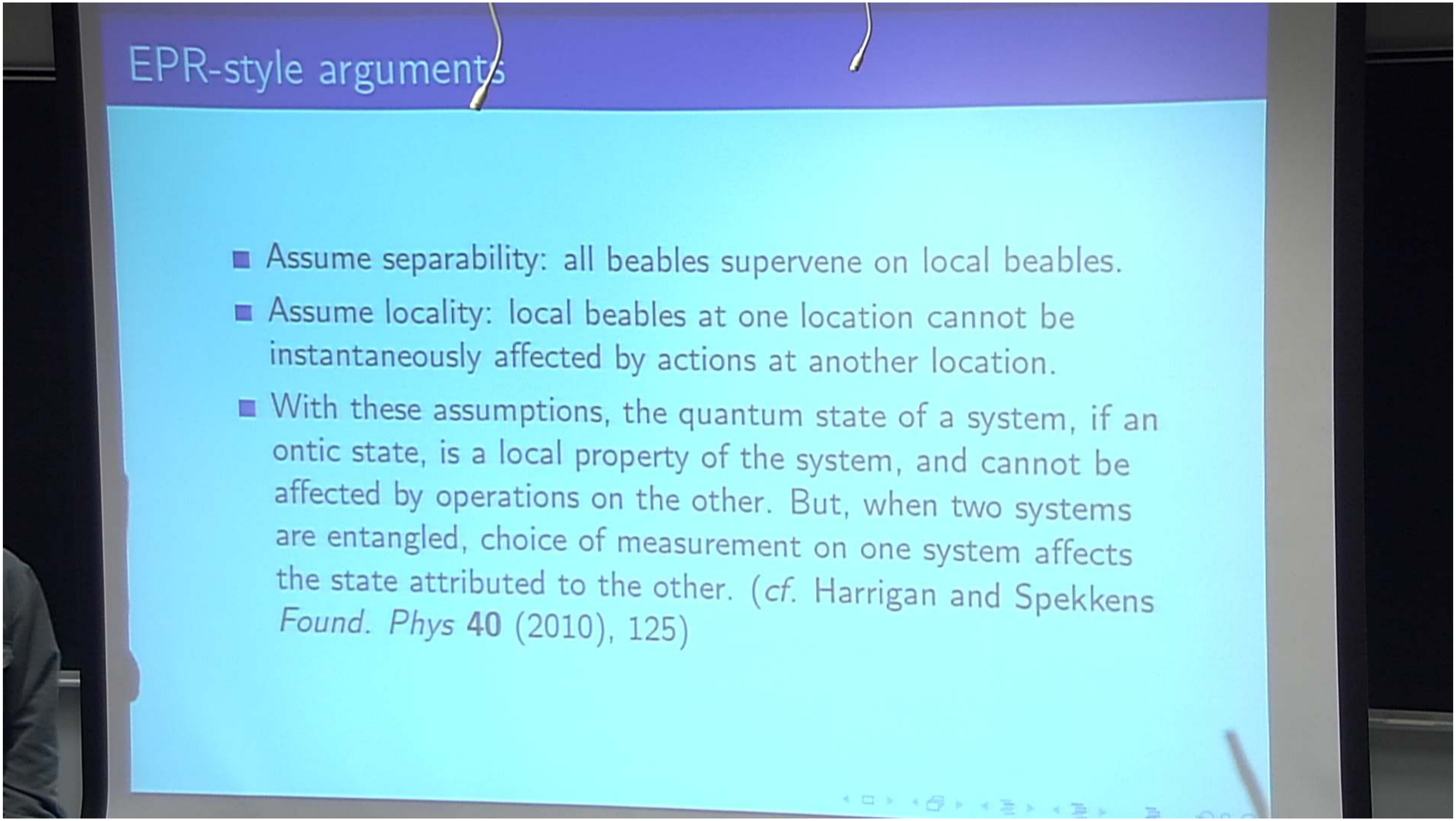
- $\psi(\mathbf{x})$ is coefficient of expansion of $|\mathbf{x}\rangle = \hat{\psi}(\mathbf{x}, 0)|0\rangle$, which is a state in which there is *one particle at* $\mathbf{x}$ *and none anywhere else*.
- Unlike a classical field value, $\psi(\mathbf{x})$ cannot be regarded as an intrinsic property of a small neighbourhood of $\mathbf{x}$.


EPR-style arguments
Assume separability: all beables supervene on local beables.
Assume locality: local beables at one location cannot be instantaneously affected by actions at another location.
With these assumptions, the quantum state of a system, if an ontic state, is a local property of the system, and cannot be affected by operations on the other. But, when two systems are entangled, choice of measurement on one system affects the state attributed to the other. (cf. Harrigan and Spekkens Found. Phys 40 (2010), 125)

Einstein at 5th Solvay Conference, 1927

If $|\psi|^2$ were simply regarded as the probability that at a certain point a given particle is found at a given time, it could happen that *the same* elementary process produces an action *in two or several* places on the screen. But the interpretation, according to which $|\psi|^2$ expresses the probability that *this* particle is found at a given point, assumes an entirely peculiar mechanism of action at a distance, which prevents the wave continuously distributed in space from producing an action in *two* places on the screen.

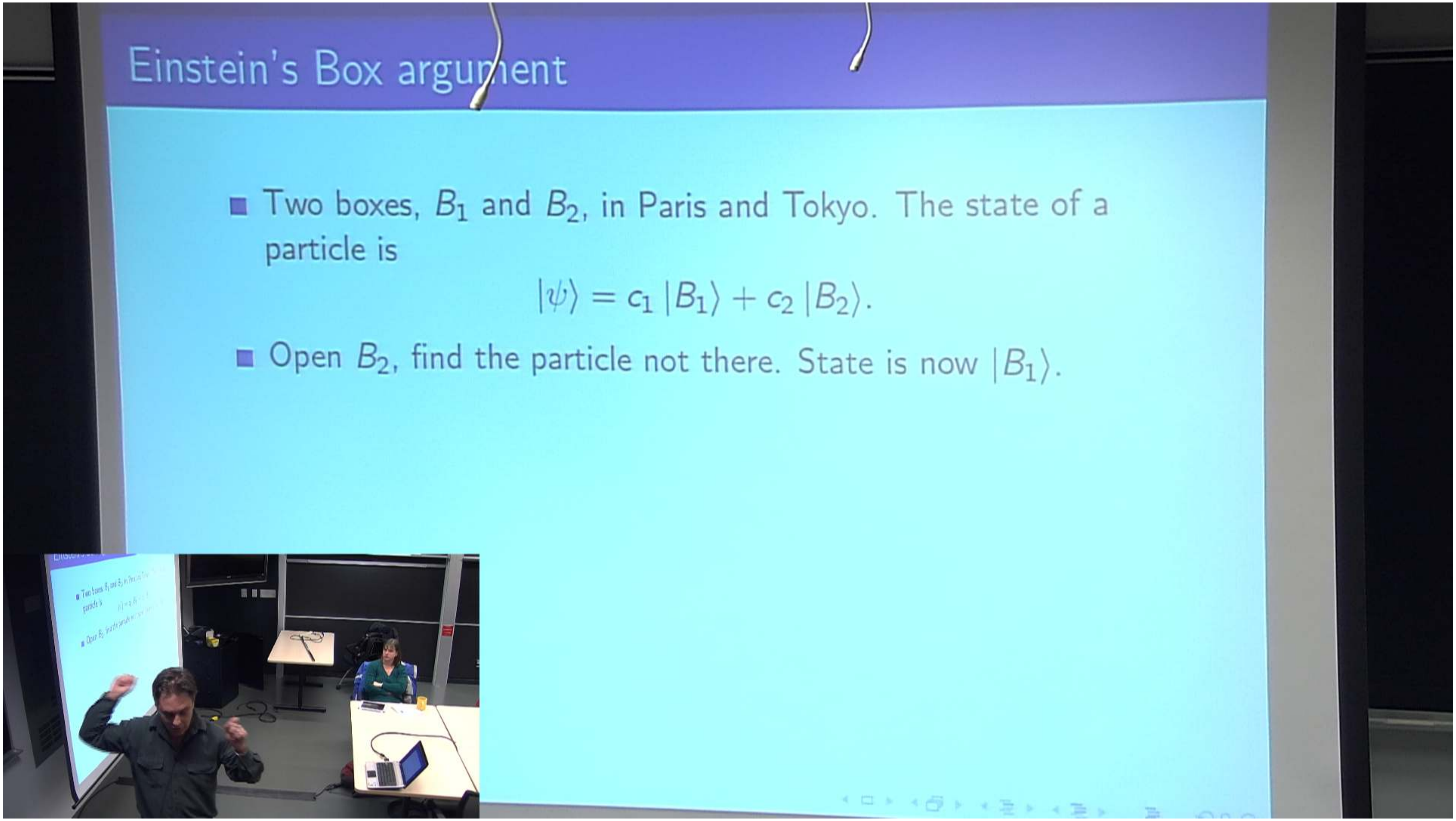
Einstein at 5th Solvay Conference, 1927, cont'd

In my opinion, one can remove this objection only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time one localises the particle during the propagation. I think that Mr de Broglie is right to search in this direction. If one works solely with the Schrödinger waves, interpretation II of $|\psi|^2$ implies to my mind a contradiction with the postulate of relativity.

Einstein's Box argument

- Two boxes, B_1 and B_2, in Paris and Tokyo. The state of a particle is

$$|\psi\rangle = c_1 \, |B_1\rangle + c_2 \, |B_2\rangle.$$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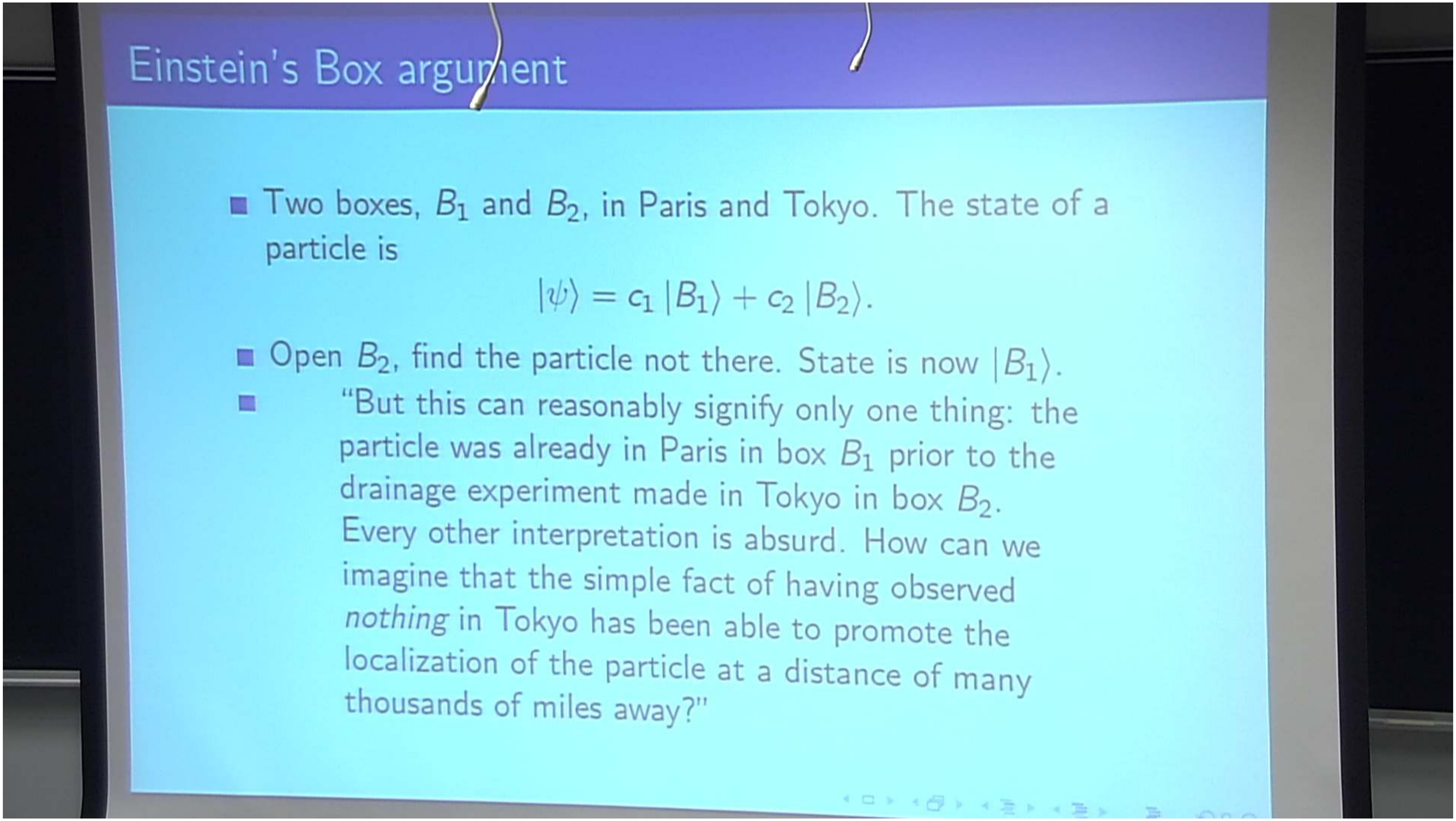
- "But this can reasonably signify only one thing: the particle was already in Paris in box B_1 prior to the drainage experiment made in Tokyo in box B_2. Every other interpretation is absurd. How can we imagine that the simple fact of having observed *nothing* in Tokyo has been able to promote the localization of the particle at a distance of many thousands of miles away?"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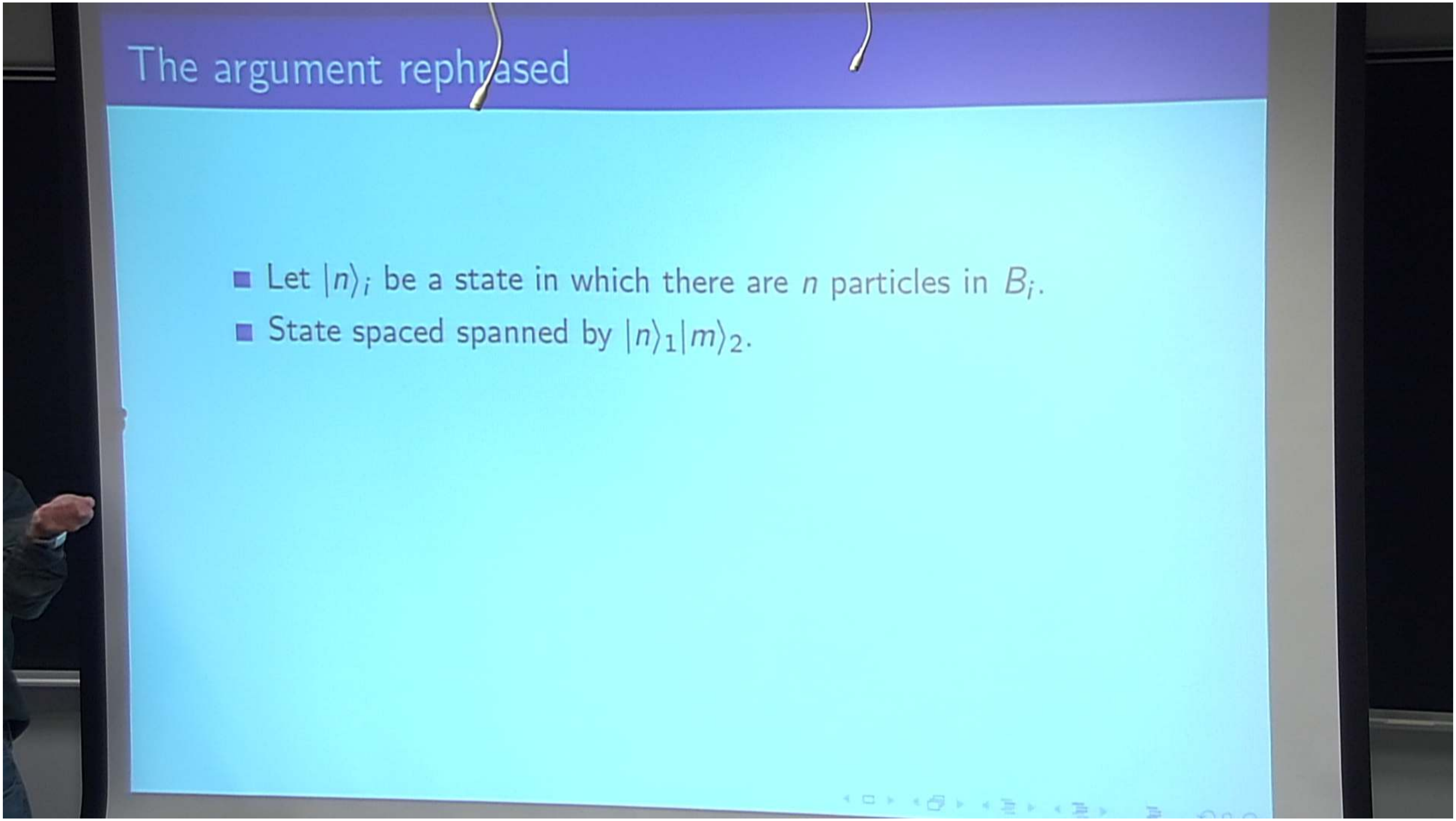
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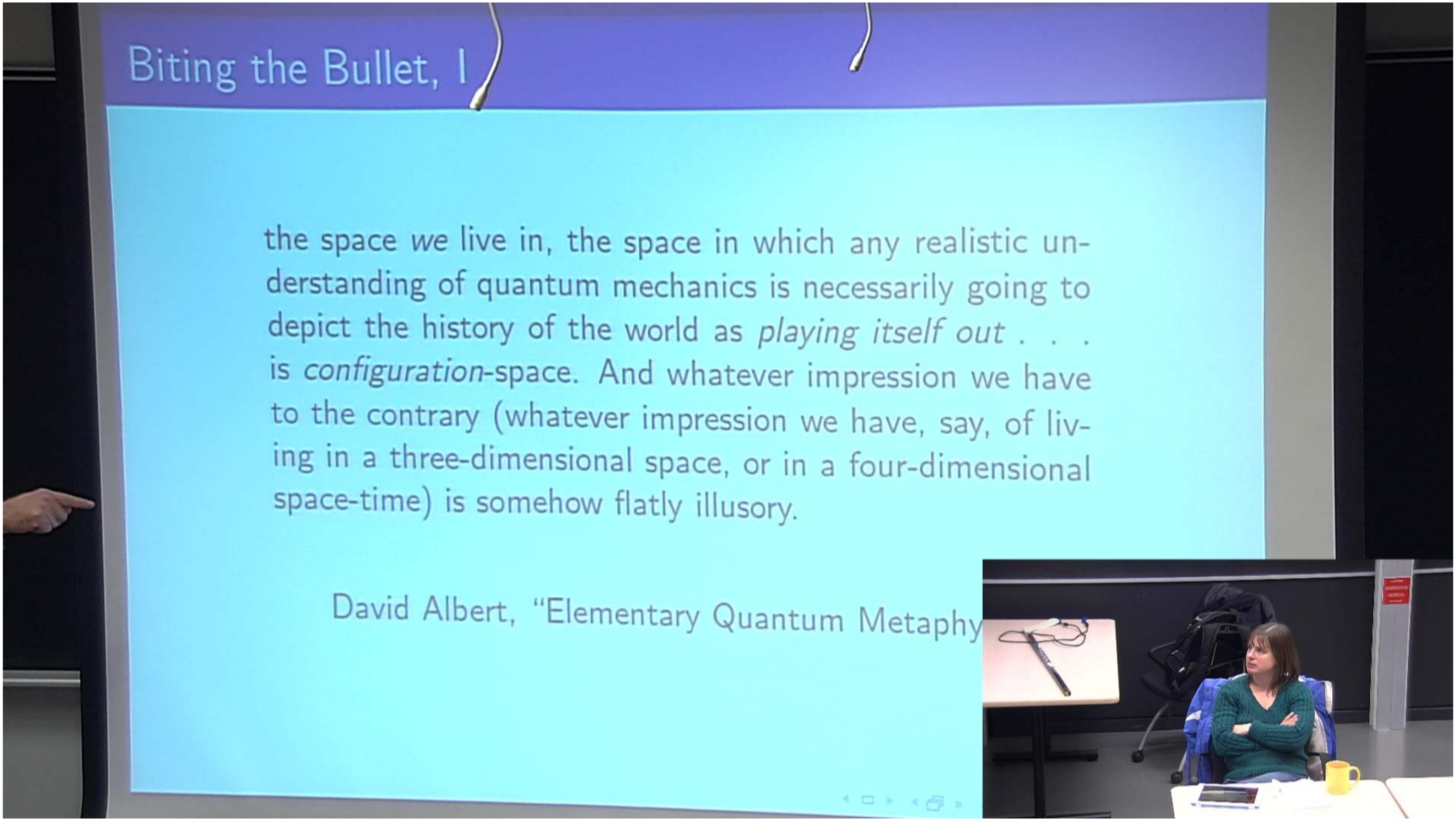
- Makes entanglement of local box-states manifest.
- *Cf.* Norsen, *AJP* **73** (2005), 164; Norton *AJP* **79** (2010), 182.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