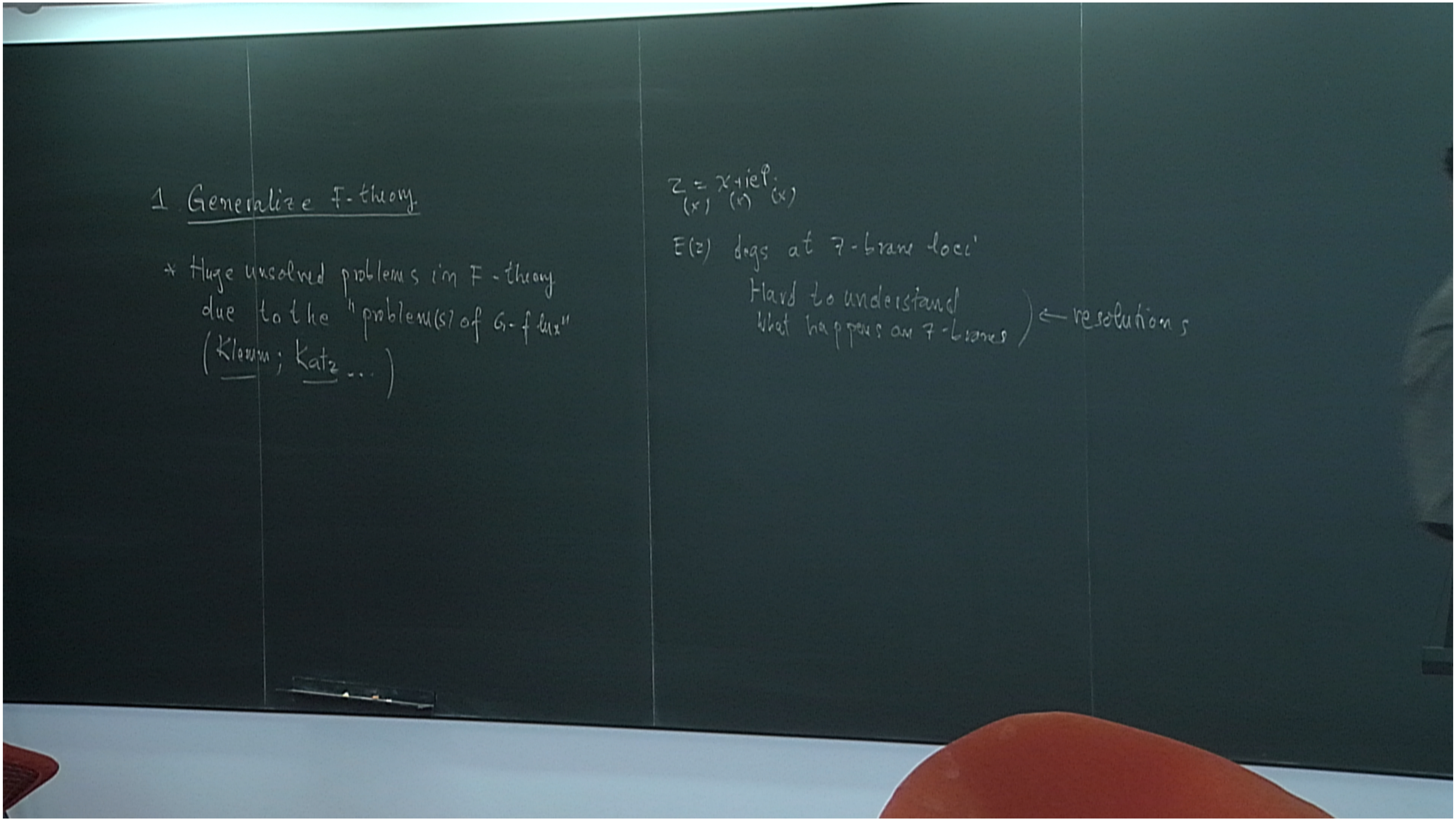
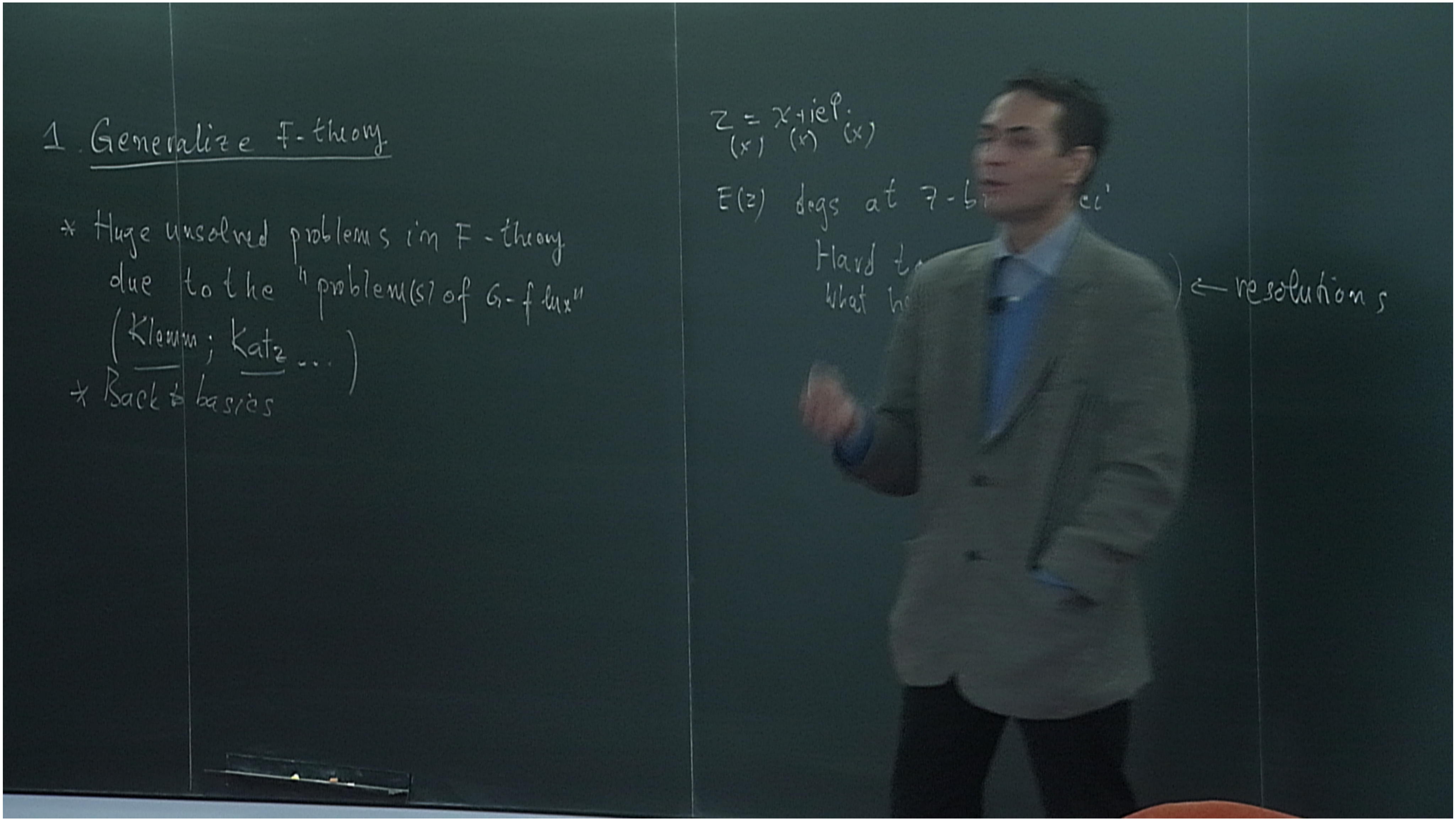
Title: Geometric algebra techniques in flux compactifications

Date: Nov 09, 2012 11:00 AM

URL: http://pirsa.org/12110058

Abstract: Using techniques originating in a certain approach to Clifford bundles known as "geometric algebra", I discuss a geometric reformulation of constrained generalized Killing spinor equations which proves to be particularly effective in the study and classification of supersymmetric flux compactifications of string and M-theory. As an application, I discuss the most general N=2 compactifications of M-theory to three dimensions, which were never studied in full generality before. I also touch upon the connection of such techniques with a certain variant of the quantization of spin systems.


1. Generalize F-theory
* Huge unsolved problems in F-theory
due to the "problem(s) of G-flux"
(Klemm; Katz ...)
E(z) degs at 7-brane loci
Hard to understand
What happens on 7-branes
← resolutions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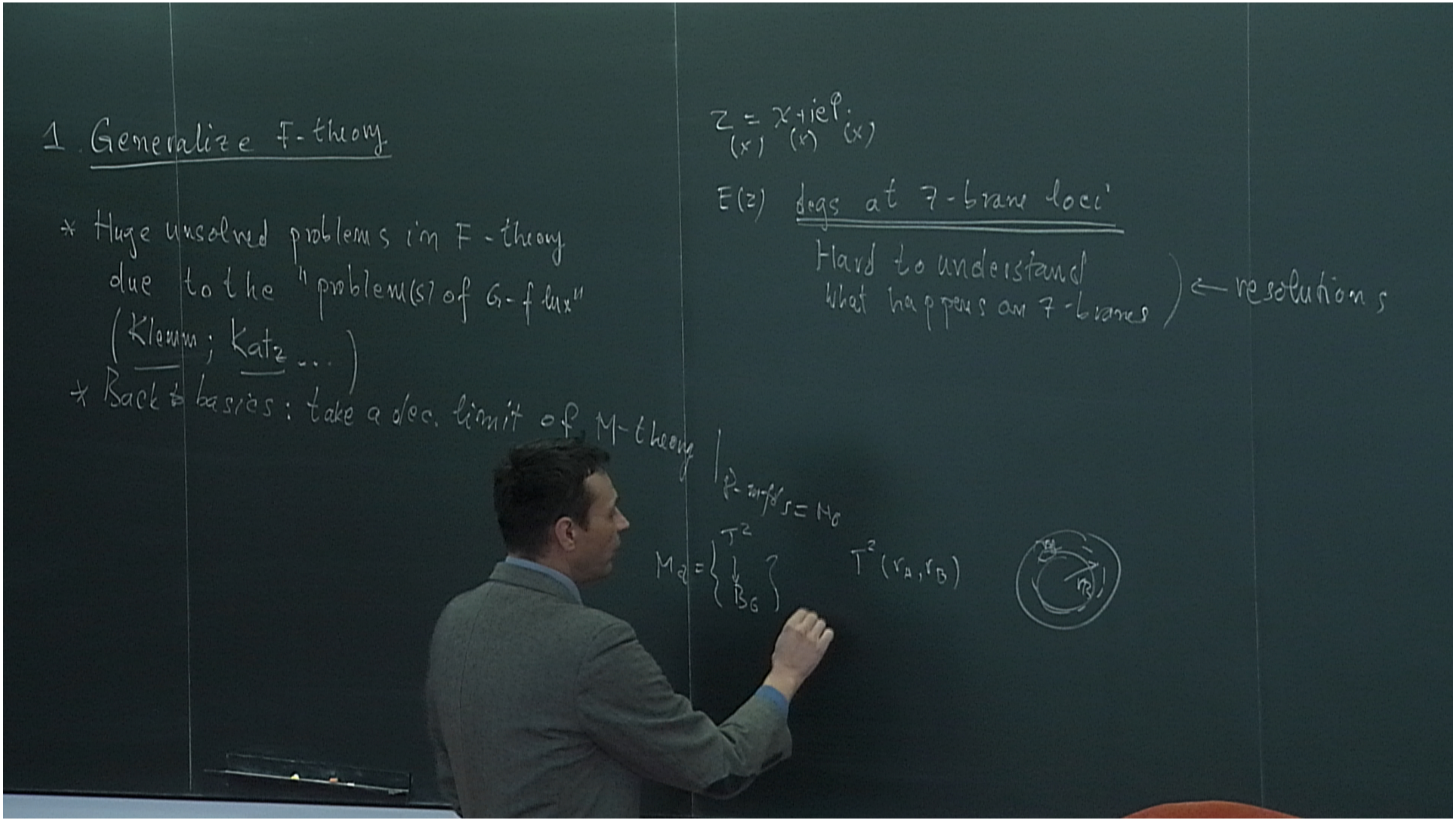
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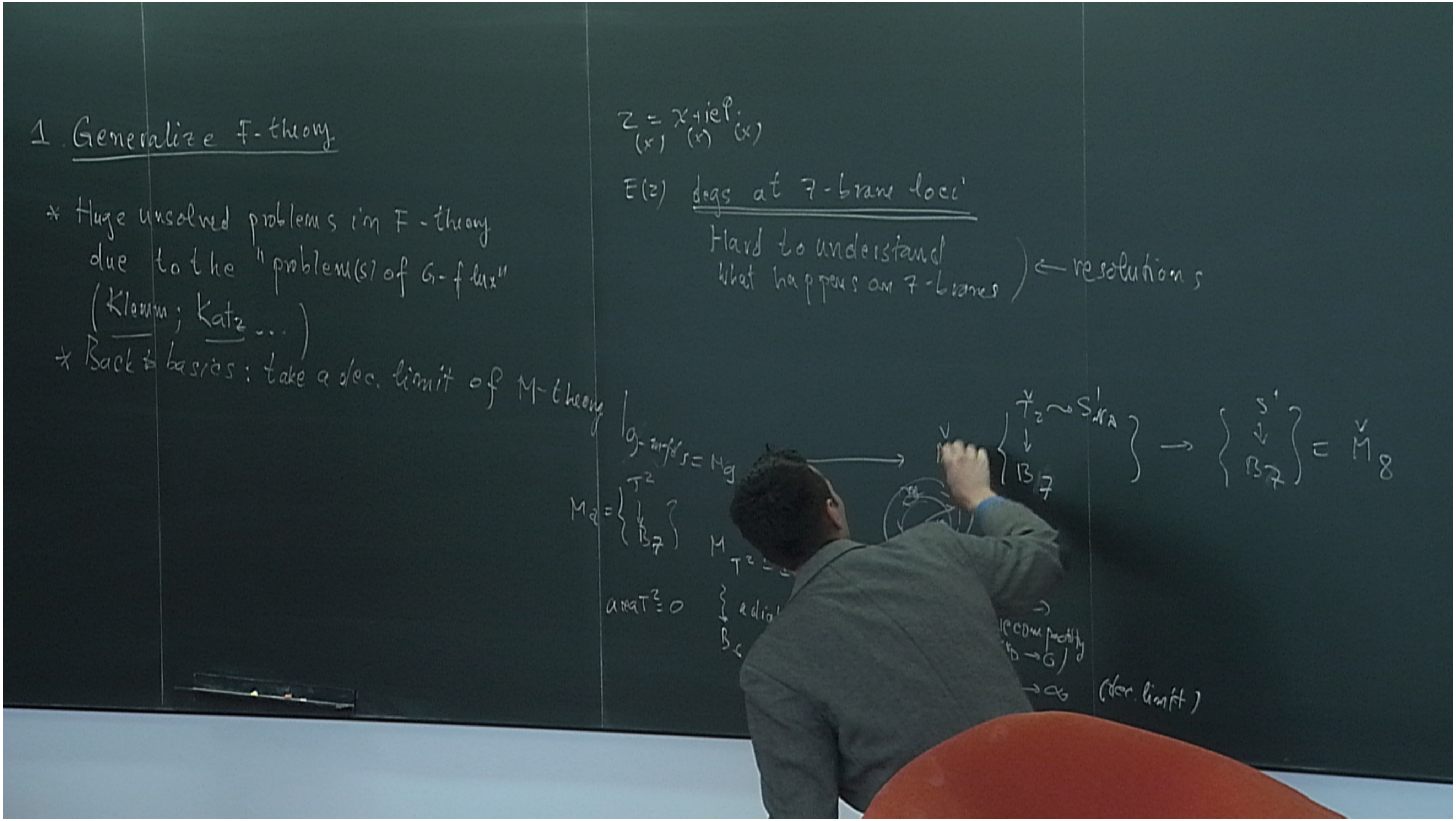
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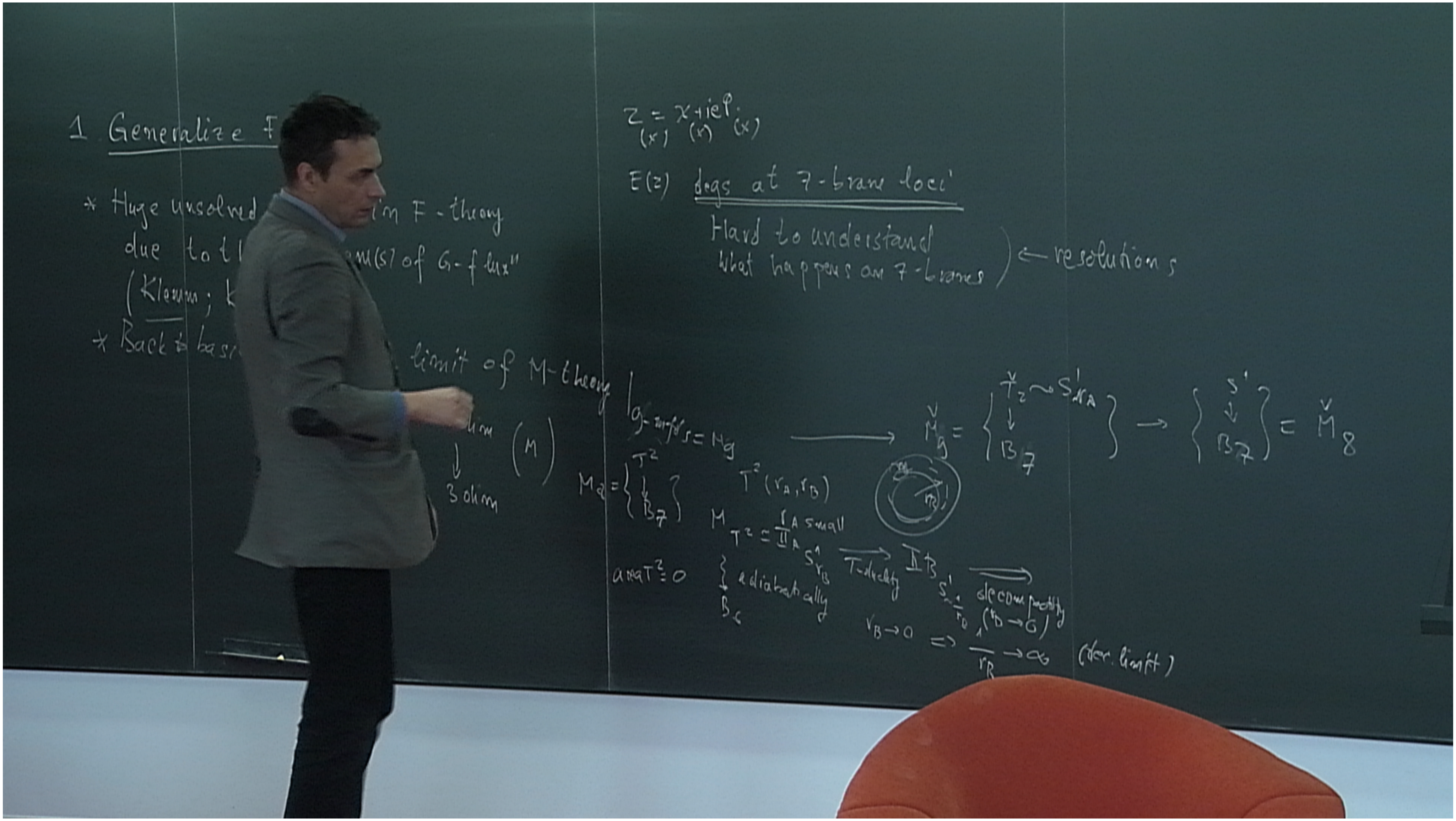
(dec. limit)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T-duality
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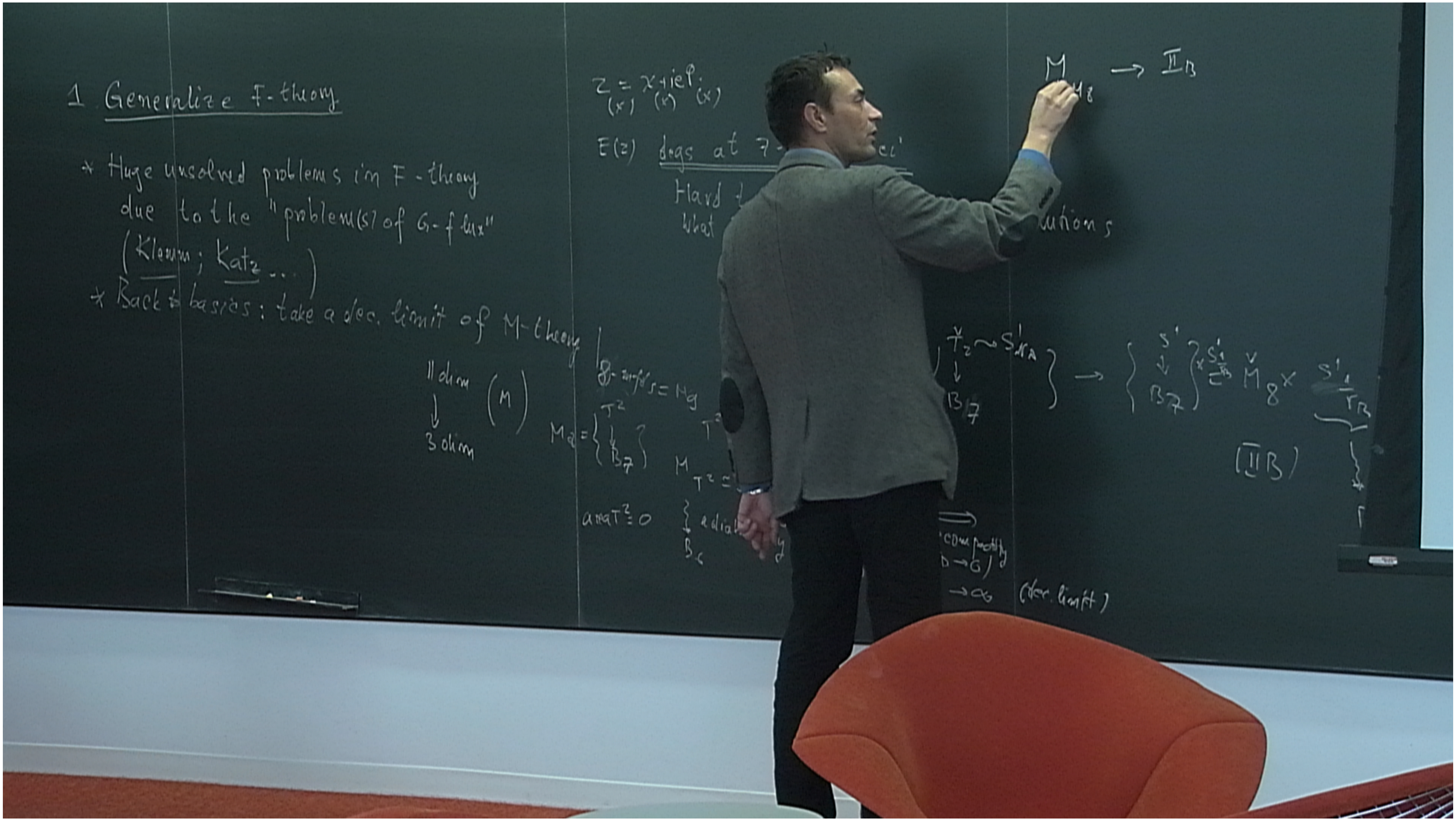
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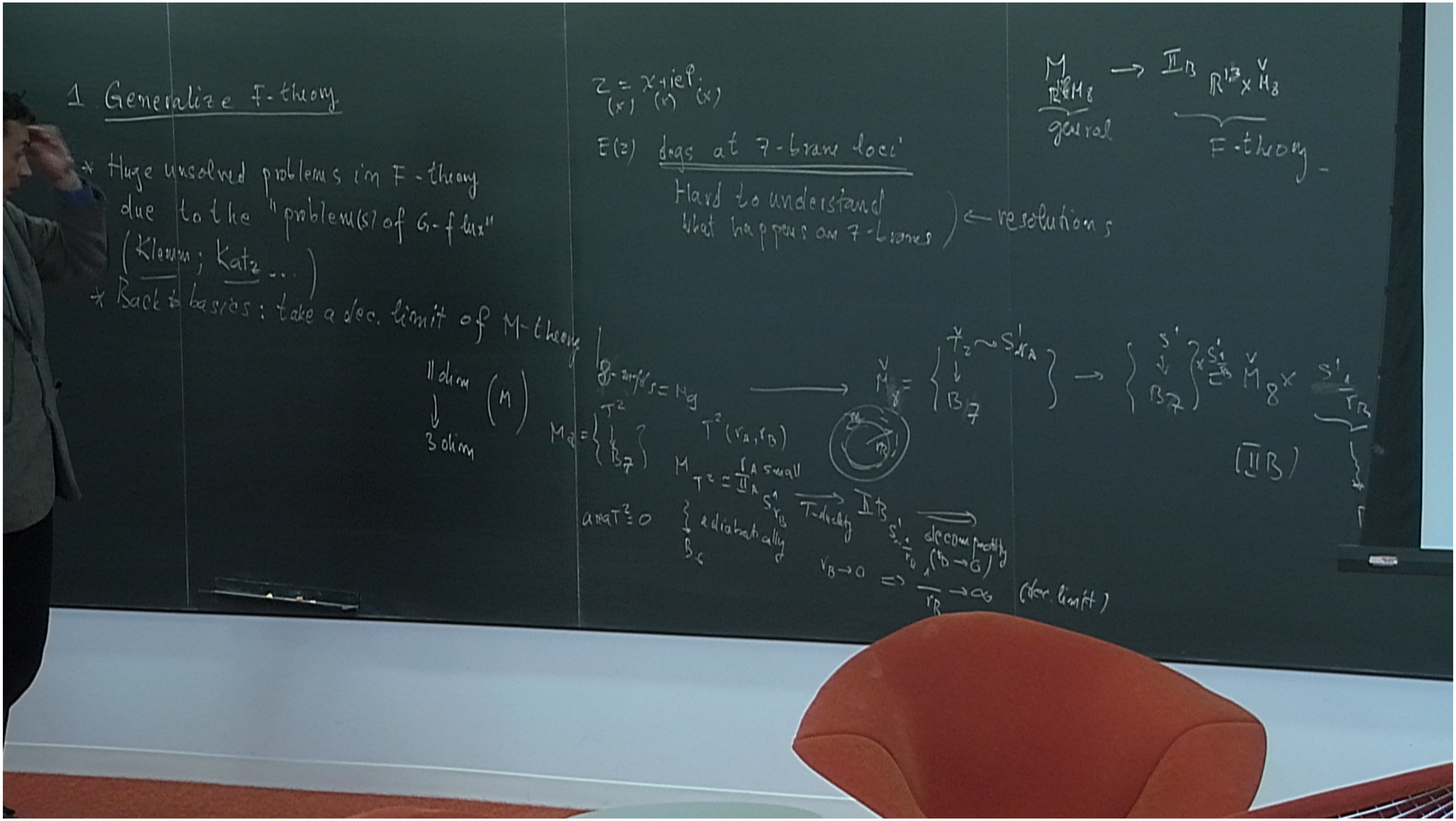
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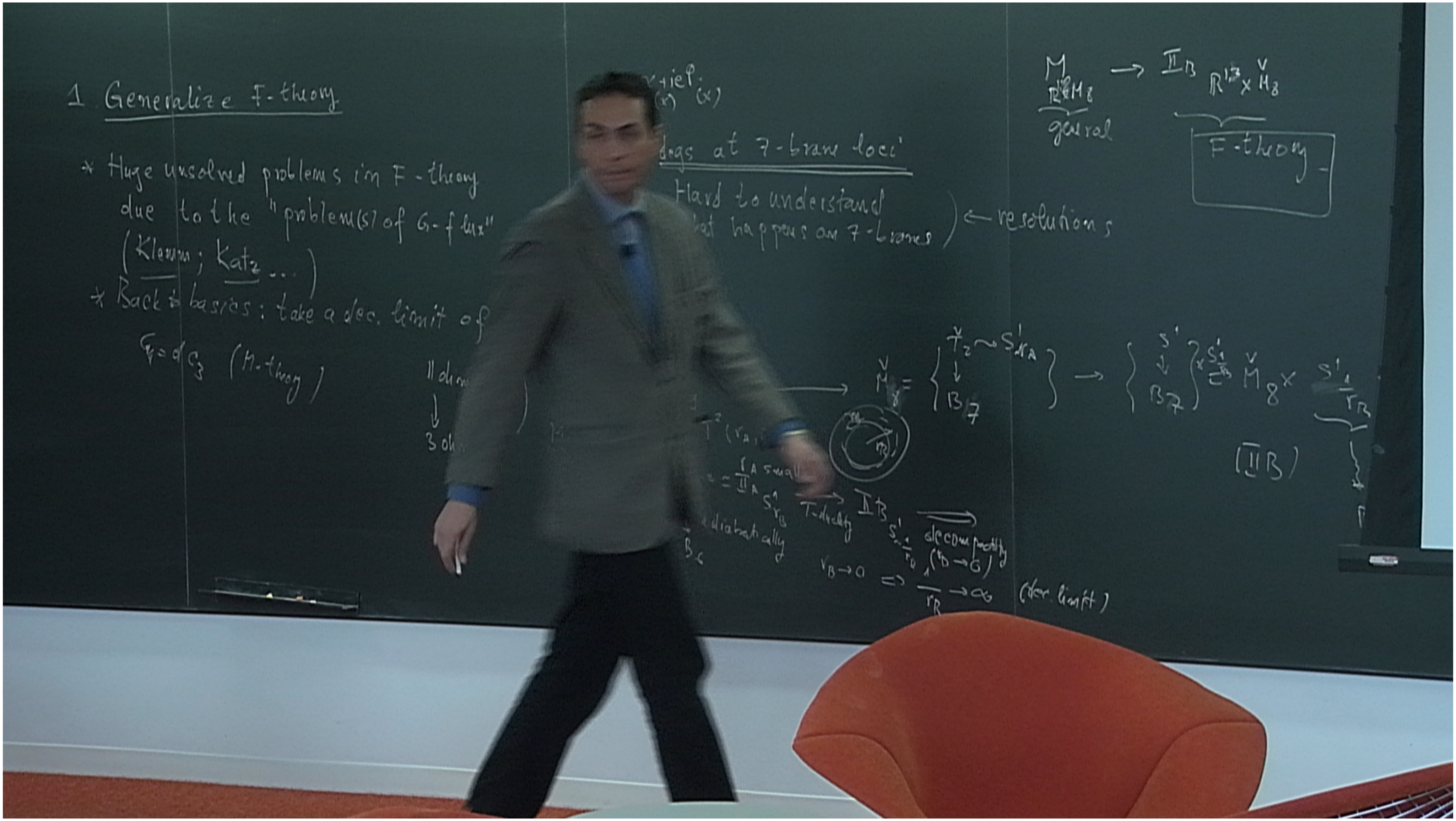
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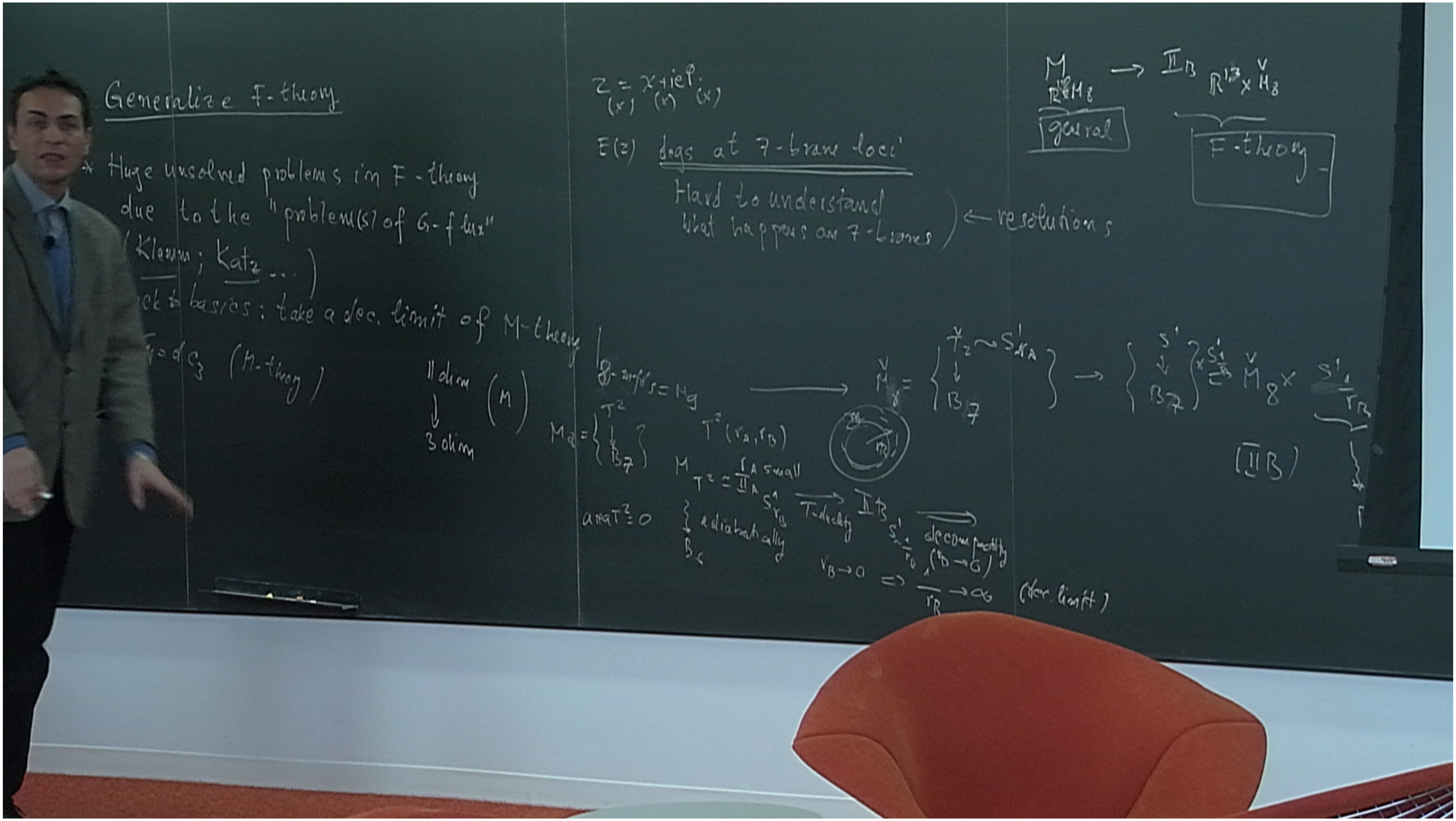
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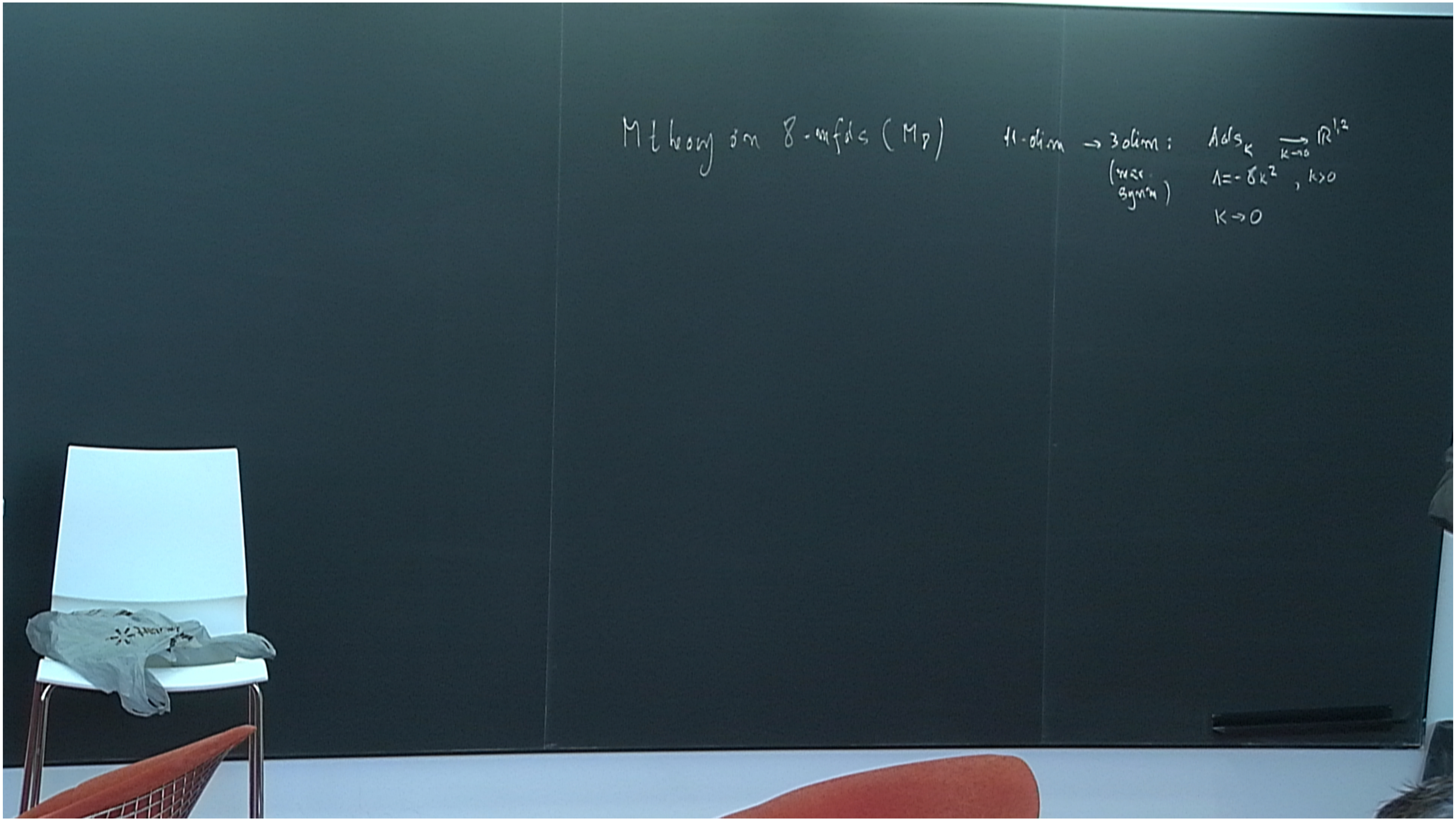
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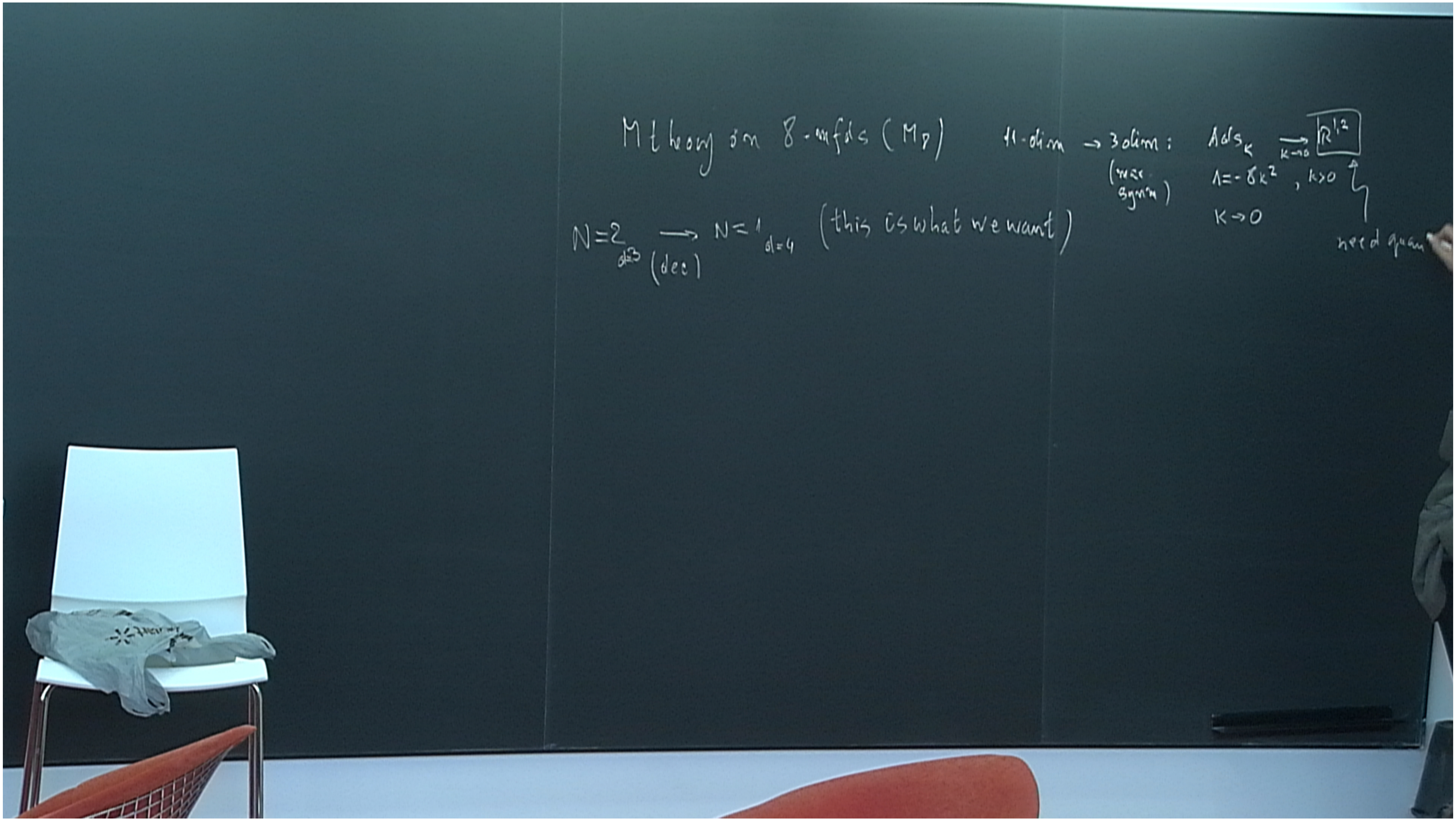
11-dim → 3 dim :
(max. symm)
$AdS_K \xrightarrow[K\to 0]{} \mathbb{R}^{1,2}$
$\Lambda = -8K^2$, $K>0$
$K \to 0$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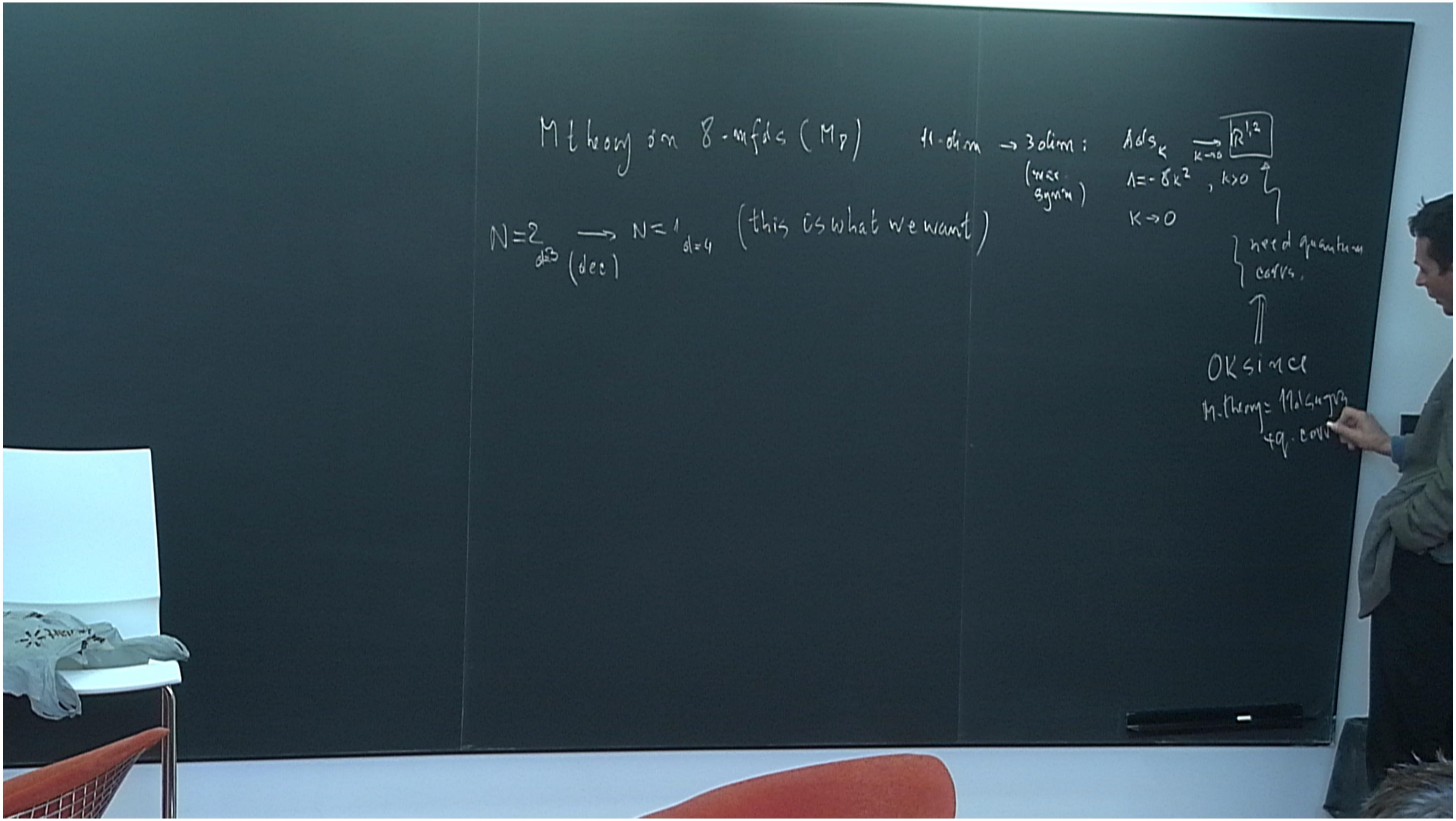
need quan
N=2 d=3 → N=1 d=4 (this is what we want)
(dec)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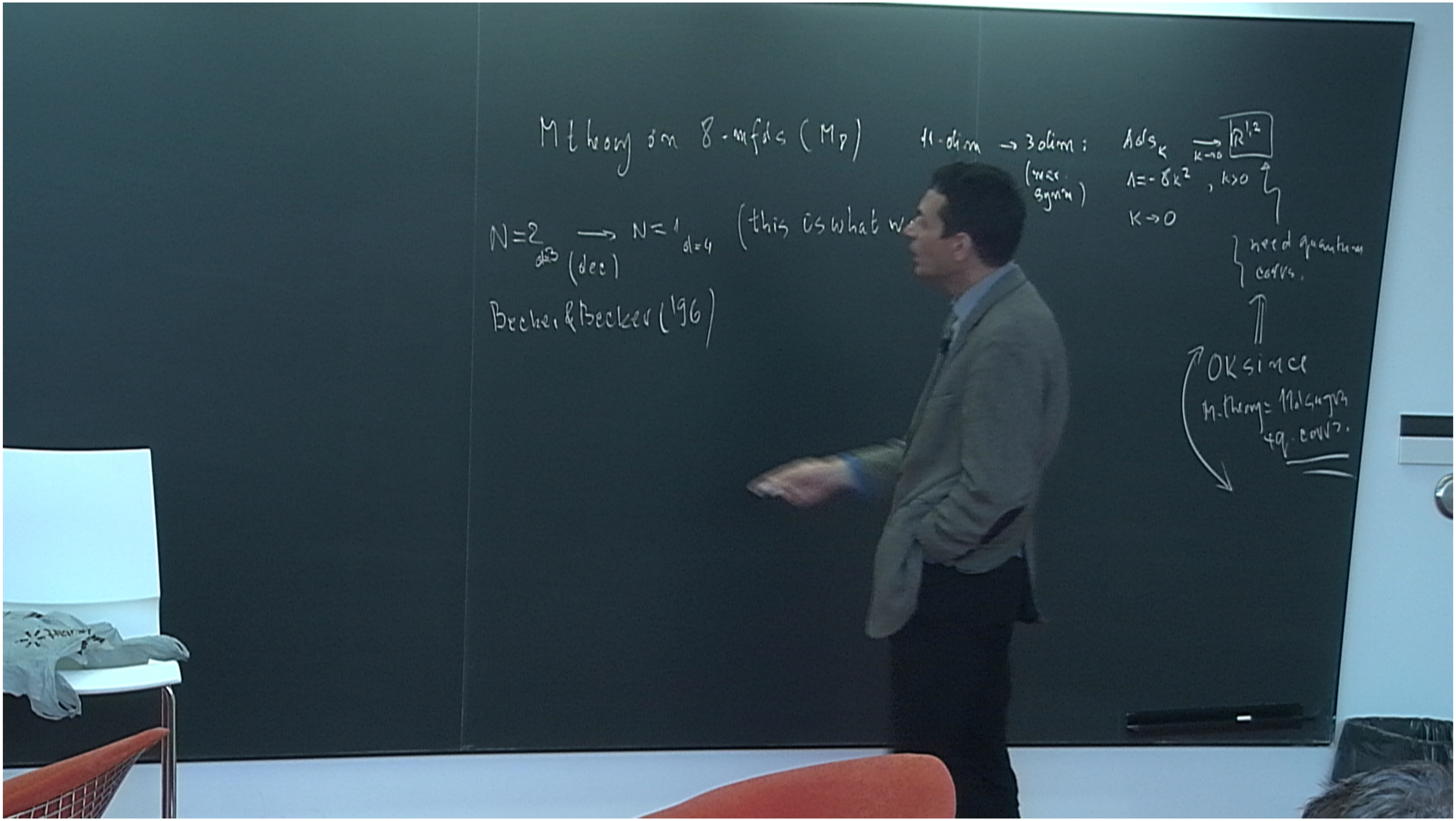
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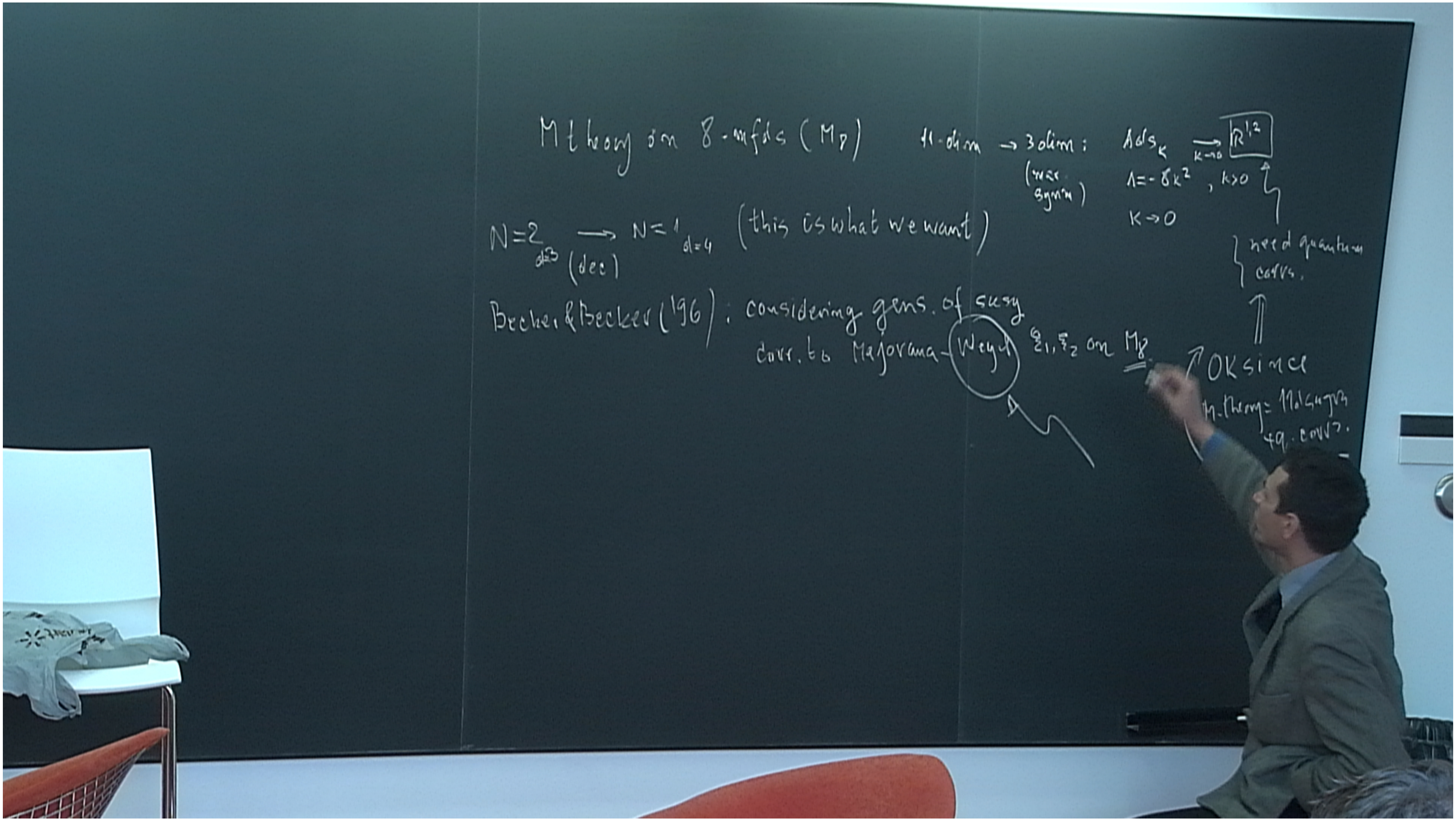
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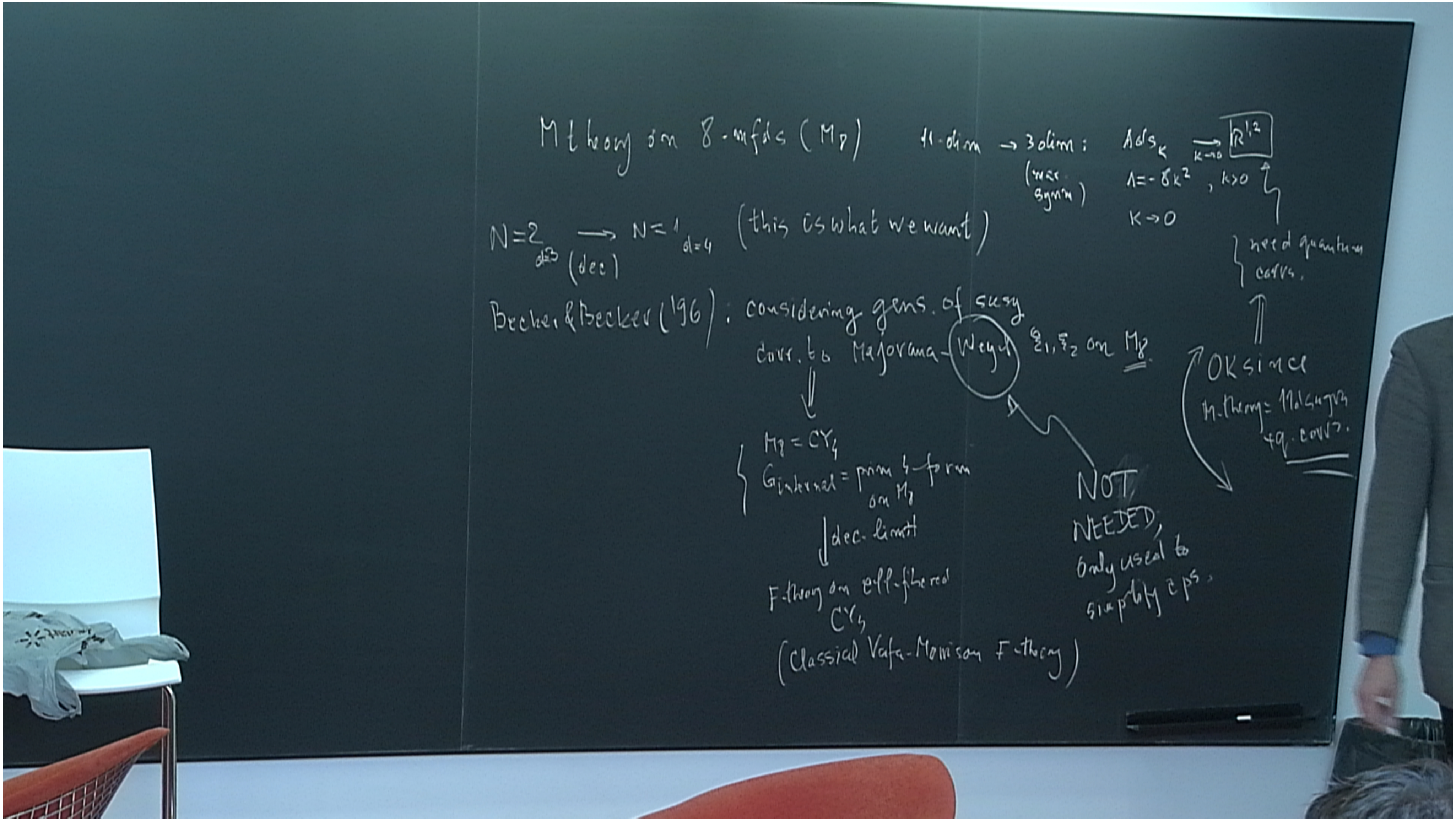
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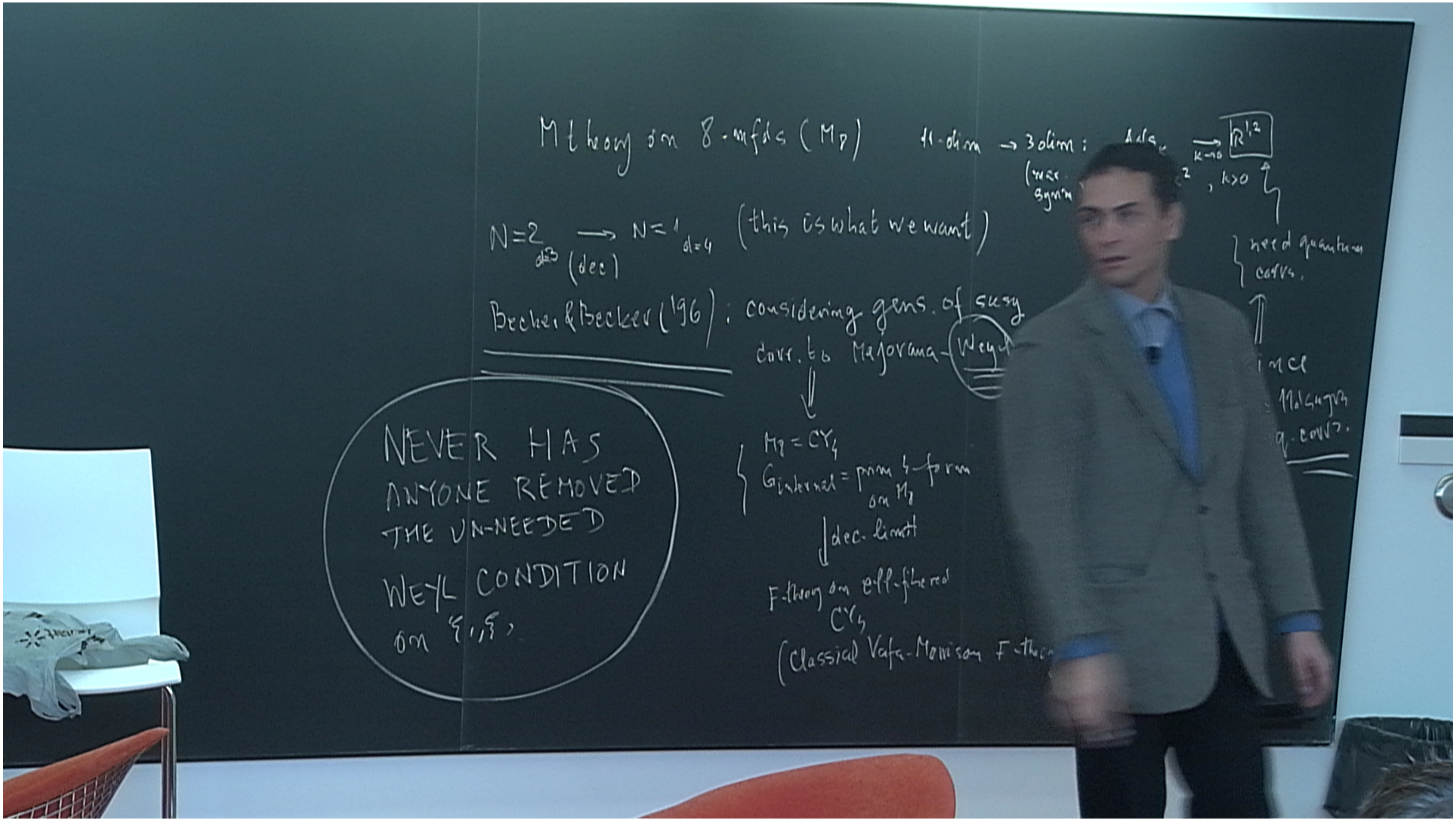
M-theory = 11d sugra + q. corrs.
NOT NEEDED,
only used to simplify eqs.
dec. limit
F-theory on ell-fibered
(Classical Vafa-Morrison F-theory)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M8 = CY4
dec. limit
F-theory on ell-fibered CY4
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NEVER HAS
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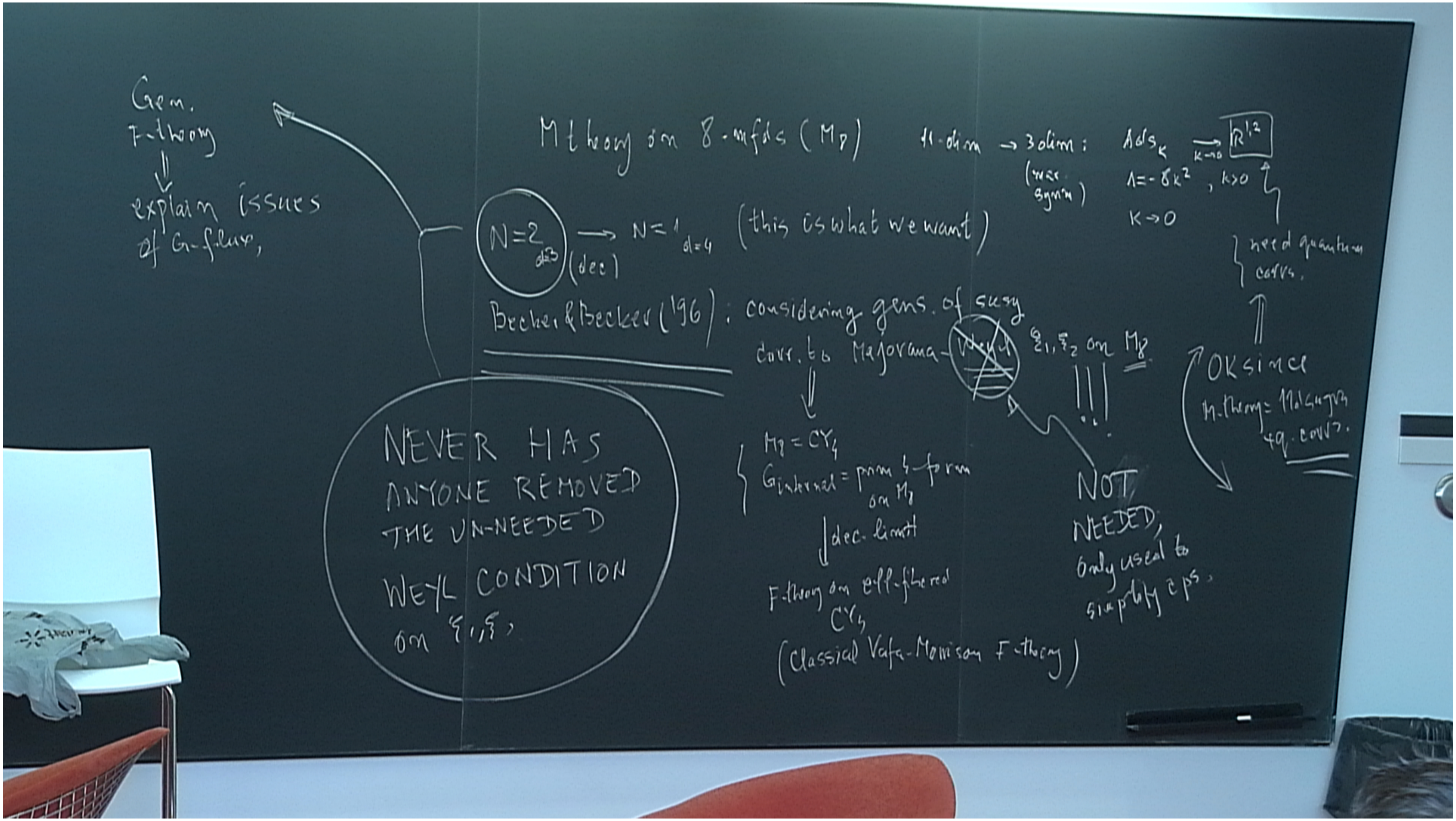

Gen. F-theory
explain issues of G-flux,
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!!!
$M_8 = CY_4$
$G_{internal}$ = prim 4-form on M_8
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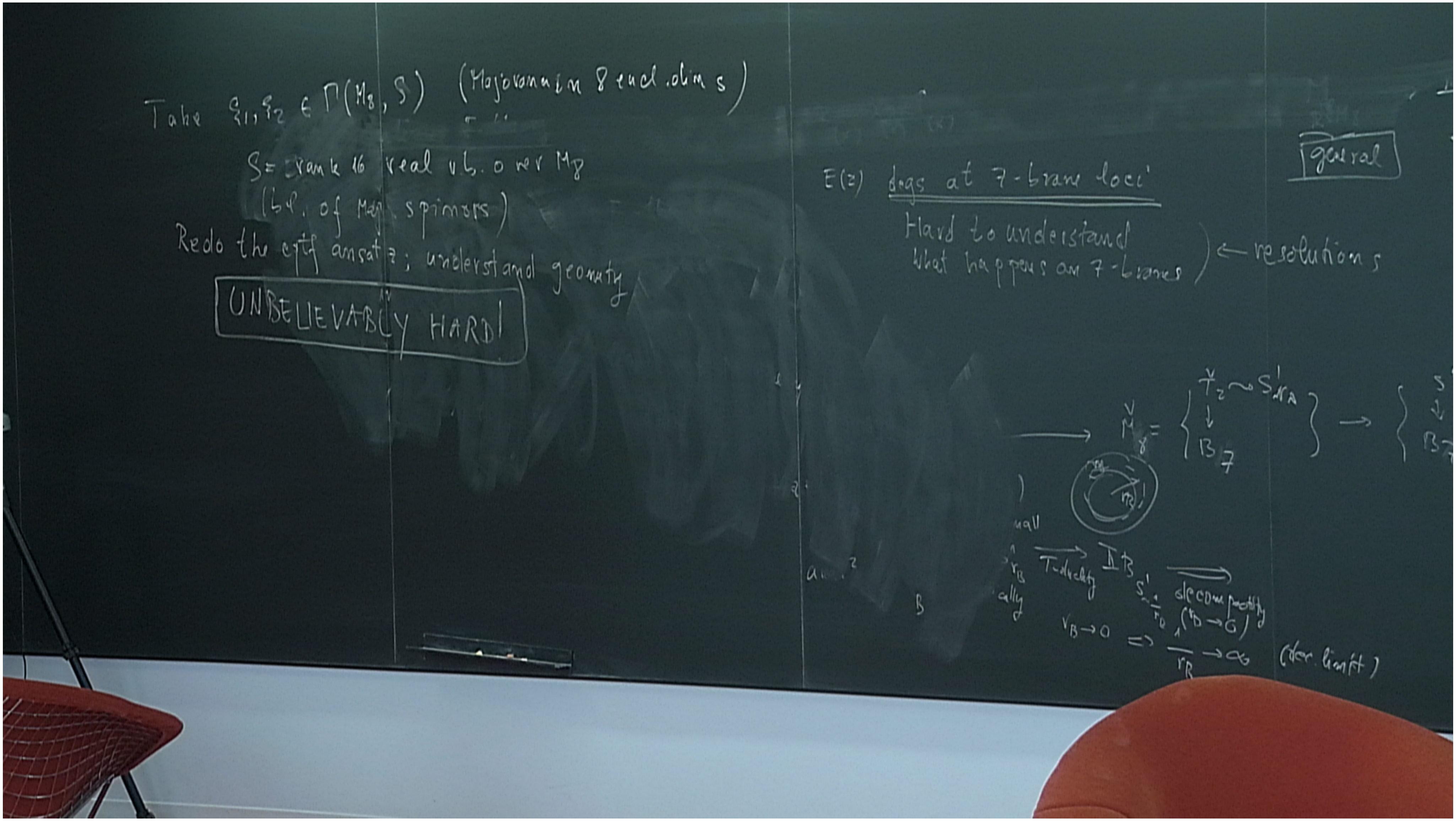
Take $\xi_1, \xi_2 \in \Gamma(M_8, S)$ (Majorana in 8 eucl. dims)
S = rank 16 real v.b. over M_8
(b.l. of M_8 spinors)
Redo the cptf ansatz; understand geometry
UNBELIEVABLY HARD!
E(z) degs at 7-brane loci
general
Hard to understand
What happens on 7-branes
← resolutions
$\check{T}_2 \sim S^1_{R_A}$
$\check{M}_8 = \{ B_7 \}$
T-duality
IIB
$r_B \to 0 \Rightarrow \frac{1}{r_B} \to \infty$
(der. limit)

Warped compactifications of M-theory on an 8-manifold

We consider 11-dim SUGRA on a connected 11-manifold $\hat{M}$ whose first two Stiefel-Whitney classes vanish. ($\hat{w}_1(\hat{M}) = 0 \Leftrightarrow \hat{M}$ is orientable, $\hat{w}_2(\hat{M})=0 \Leftrightarrow \hat{M}$ admits spin structures.)

The fields of 11-dim SUGRA: a Lorentzian metric $\hat{g}$ (taken of signature $(-,+,\ldots,+)$), a 3-form potential $\hat{C}$ with 4-form field strength $\hat{G} = d\hat{C}$ and a gravitino $\hat{\Psi}_M$.

Supersymmetric backgrounds are those for which the supersymmetry variation of the gravitino vanishes

$$(\delta_{\hat{\eta}} \hat{\Psi})(X) := \hat{\mathcal{D}}_X \hat{\eta} = 0$$

The component form is obtained by taking $X = \hat{e}_M$ for a local orthonormal frame of $T\hat{M}$

$$\boxed{\delta_{\hat{\eta}} \hat{\Psi}_M := \hat{\mathcal{D}}_M \hat{\eta} = 0}$$

parallel transport equation for $\hat{\eta}$ with respect to the supercovariant connection $\hat{\mathcal{D}}$

$$\hat{\mathcal{D}}_M := \hat{\nabla}^S_M - \frac{1}{288}\left(\hat{G}_{NPQR}\hat{\Gamma}^{NPQR}{}_M - 8\hat{G}_{MNPQ}\hat{\Gamma}^{NPQ}\right)$$

$$\hat{\nabla}^S_M = \partial_M + \frac{1}{4}\hat{\omega}_{MNP}\hat{\Gamma}^{NP}$$

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The warped product ansatz

We consider backgrounds $\hat{M} = M_3 \times M_8$, with metric of the form:

Ansatz for metric:
$$d\hat{s}^2_{11} = e^{2\Delta} ds^2_{11}$$
$$ds^2_{11} = ds^2_3 + ds^2_8 = g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n$$
$$\mu, \nu = 0, 1, 2 \quad m, n = 3, \dots, 10$$

Ansatz for flux: $\hat{G} = e^{3\Delta} G$ with $G = \mathrm{vol}_3 \wedge f + F$

F=4-form f=1-form on M_8

vol_3 = volume form of $(M_3, g_{\mu\nu})$

Ansatz for susy generator: $\hat{\eta} = e^{+\frac{\Delta}{2}} \eta$ with $\eta \in \Gamma(S)$

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Rescaling and decomposition of gamma matrices

$$\hat{\Gamma}_\mu = e^\Delta (\gamma_\mu \otimes_{\mathbb{R}} \gamma_9) \quad , \quad \hat{\Gamma}_m = e^\Delta (1 \otimes_{\mathbb{R}} \gamma_m)$$

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∇^S_μ the spin connection of $(M_3, g_{\mu\nu})$ and:

$$\bar{Q} = \frac{1}{2}\gamma^n \partial_n \Delta + \frac{1}{6} f_n \gamma^n \gamma_9 - \frac{1}{288} F_{pqrs}\gamma^{pqrs}$$

$$\tilde{D}_m = \nabla^S_m - \frac{1}{12} f_n \gamma^n{}_m \gamma_9 - \frac{1}{6} f_m \gamma_9 - \frac{1}{288}(F_{pqrs}\gamma^{pqrs}{}_m - 8F_{mpqr}\gamma^{pqr})$$

Ansatz for the decomposition of η: $\qquad \eta = \psi \otimes \xi$

Majorana spinor fields on M_3 , respectively M_8

The case of maximally symmetric M_3

$\nabla^S_\mu \psi + \kappa \gamma_\mu \psi = 0$ — Killing spinor equation for $(M_3, g_{\mu\nu})$ $\quad \psi \in \Gamma(S_3)$

integrability condition $\Longrightarrow \Lambda = -8\kappa^2$ — For $\Lambda = 0$ ($\kappa = 0$), M_3 = Minkowski space

For $\Lambda < 0$ ($\kappa \neq 0$), $M_3 = \mathrm{AdS}_3$

$\Longrightarrow$ $D_m \xi = 0$, $\quad Q\xi = 0$ — supersymmetry conditions

ξ is a *Q-constrained generalized Killing spinor*

The susy condition decomposes into external and internal parts:

$$\hat{\mathcal{D}}_\mu \hat{\eta} = 0 \Longleftrightarrow \mathcal{D}_\mu \eta = 0 \qquad \mathcal{D}_\mu = \nabla^S_\mu \otimes_{\mathbb{R}} 1 + \gamma_\mu \otimes_{\mathbb{R}} (\gamma_9 \tilde{Q})$$

$$\hat{\mathcal{D}}_m \hat{\eta} = 0 \Longleftrightarrow \mathcal{D}_m \eta = 0 \qquad \mathcal{D}_m = 1 \otimes_{\mathbb{R}} \tilde{D}_m$$

∇^S_μ the spin connection of $(M_3, g_{\mu\nu})$ and:

$$\tilde{Q} = \frac{1}{2}\gamma^n \partial_n \Delta + \frac{1}{6} f_n \gamma^n \gamma_9 - \frac{1}{288} F_{pqrs}\gamma^{pqrs}$$

$$\tilde{D}_m = \nabla^S_m - \frac{1}{12} f_n \gamma^n{}_m \gamma_9 - \frac{1}{6} f_m \gamma_9 - \frac{1}{288}(F_{pqrs}\gamma^{pqrs}{}_m - 8 F_{mpqr}\gamma^{pqr})$$

Ansatz for the decomposition of η: $\eta = \psi \otimes \xi$

Majorana spinor fields on M_3 , respectively M_8

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integrability condition $\Longrightarrow \Lambda = -8\kappa^2$ For $\Lambda = 0$ ($\kappa = 0$), M_3 = Minkowski space

For $\Lambda < 0$ ($\kappa \neq 0$), $M_3 = \mathrm{AdS}_3$

$\Longrightarrow$ $D_m \xi = 0$, $Q\xi = 0$ supersymmetry conditions

ξ is a *Q-constrained generalized Killing spinor*

$\xi_1^T \gamma_{\mu_1 \ldots \mu_k} \xi_2 = \mathcal{B}(\xi_1, \gamma_{\mu_1 \ldots \mu_k} \xi_2) = \omega_{\mu_1 \ldots \mu_k}(\xi_1, \xi_2)$

Take $\xi_1, \xi_2 \in \Gamma(M_8, S)$ (Majorana in 8 eucl. dims)

S = rank 16 real v.b. over M_8 (b.f. of Maj. spinors)

$(S \otimes S, \cdot) \simeq (\mathrm{End}(S), \circ)$ $E(z)$

$\stackrel{\vee}{=}$ Fierz isomf.

$\Lambda T^*(M_8)$

Redo the eqtf ansatz; understand geometry

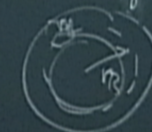
UNBELIEVABLY HARD!

dogs at 7-brane loci

Hard to understand what happens on 7-branes) ← resolution

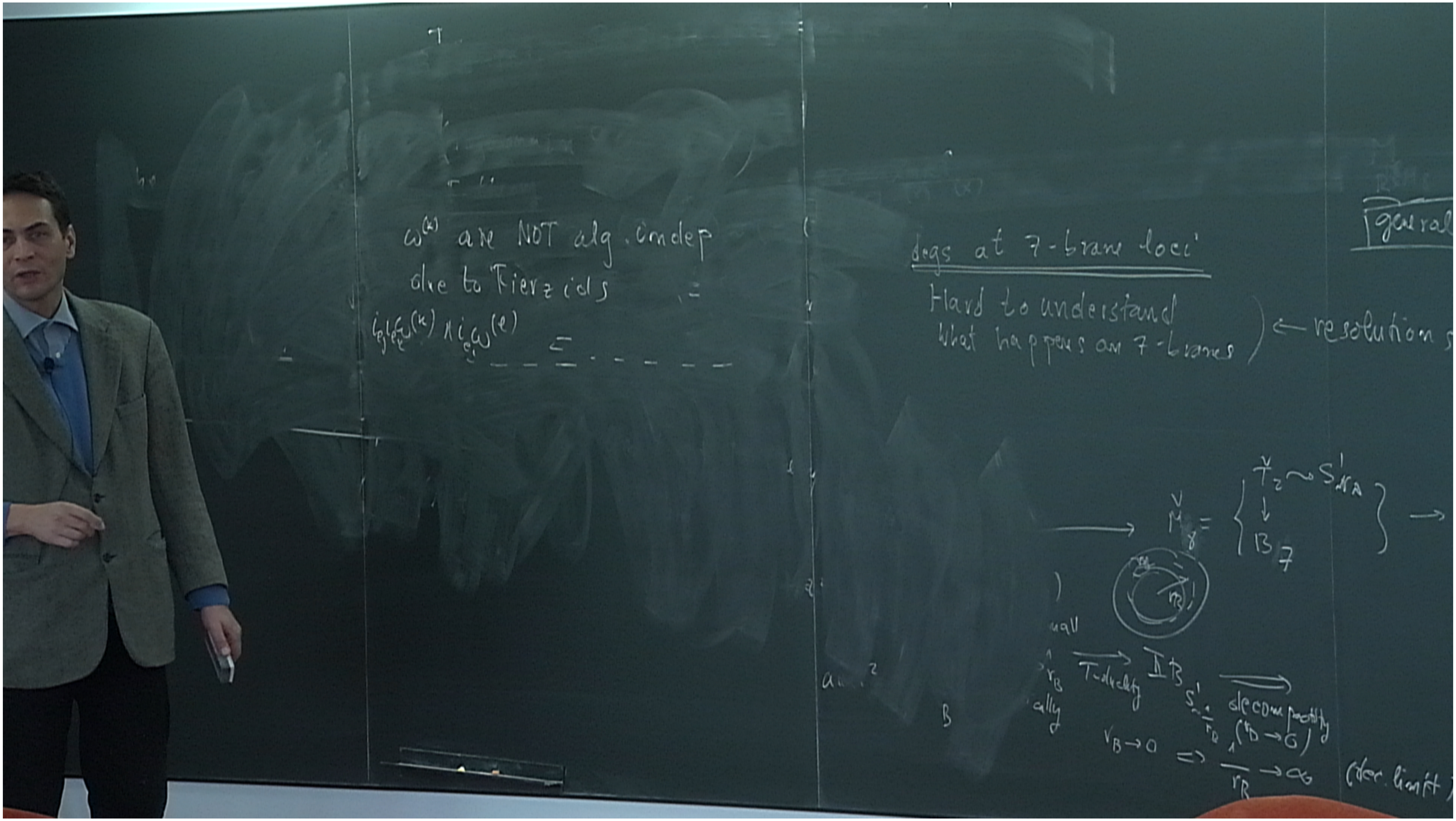
general

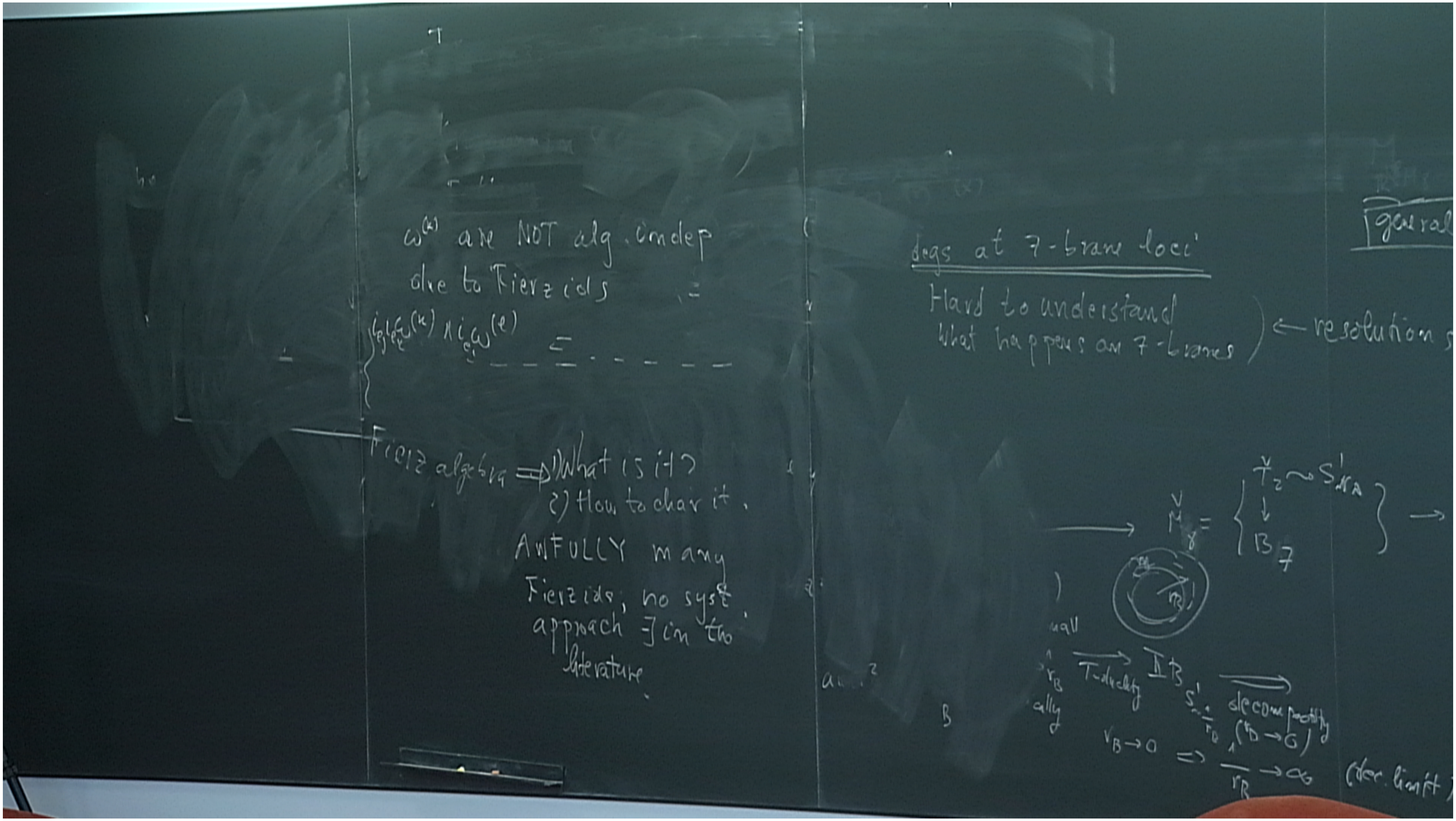
$\check{M}_8 = \left\{ \begin{array}{l} \check{T}_2 \sim S^1_{r_A} \\ \downarrow \\ B_7 \end{array} \right\} \longrightarrow$

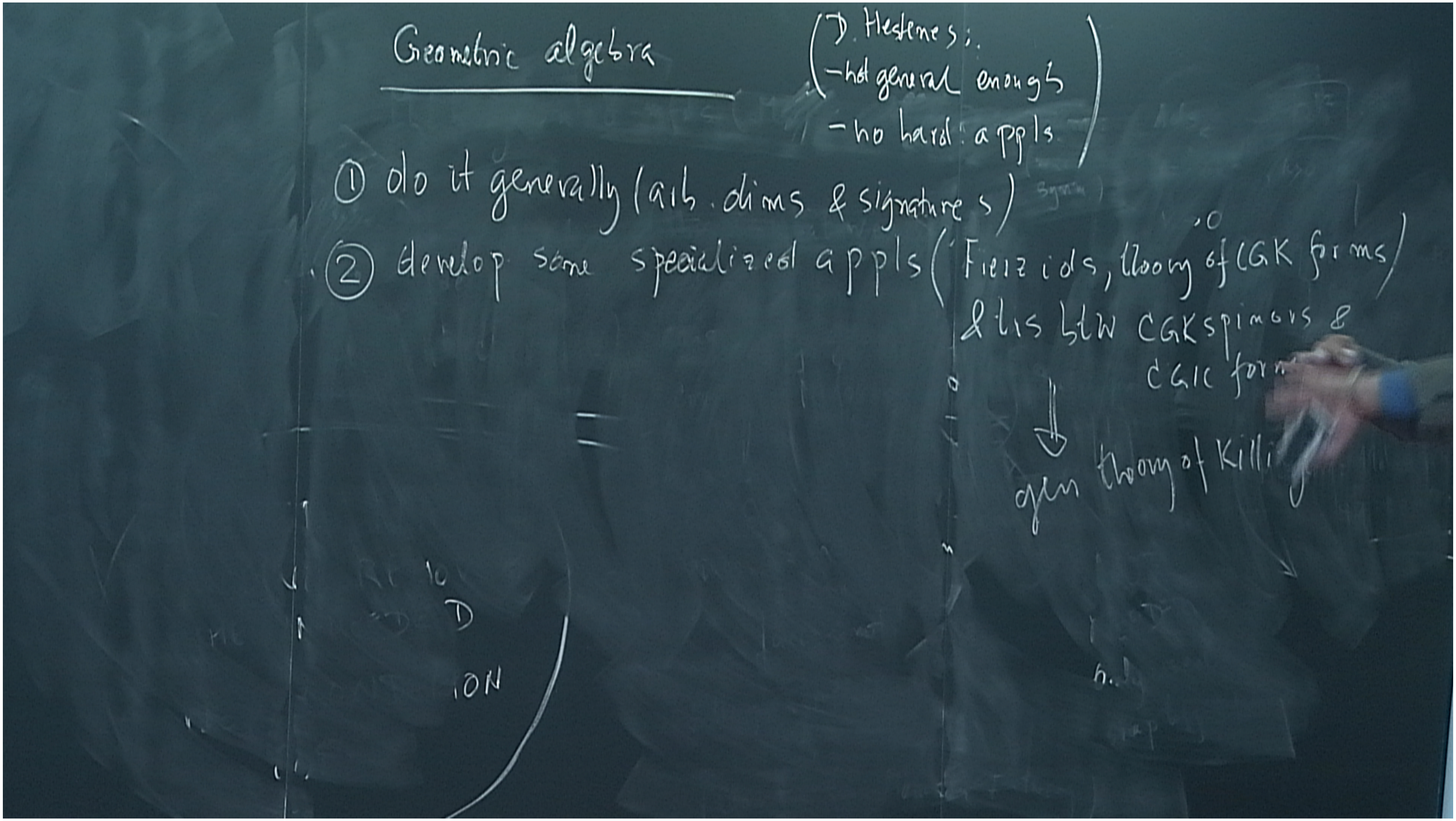


$\xrightarrow{\text{T-duality}}$ IIB $\xrightarrow[\text{decompactify}]{S^1_{r_B} \; (r_B \to 0)}$

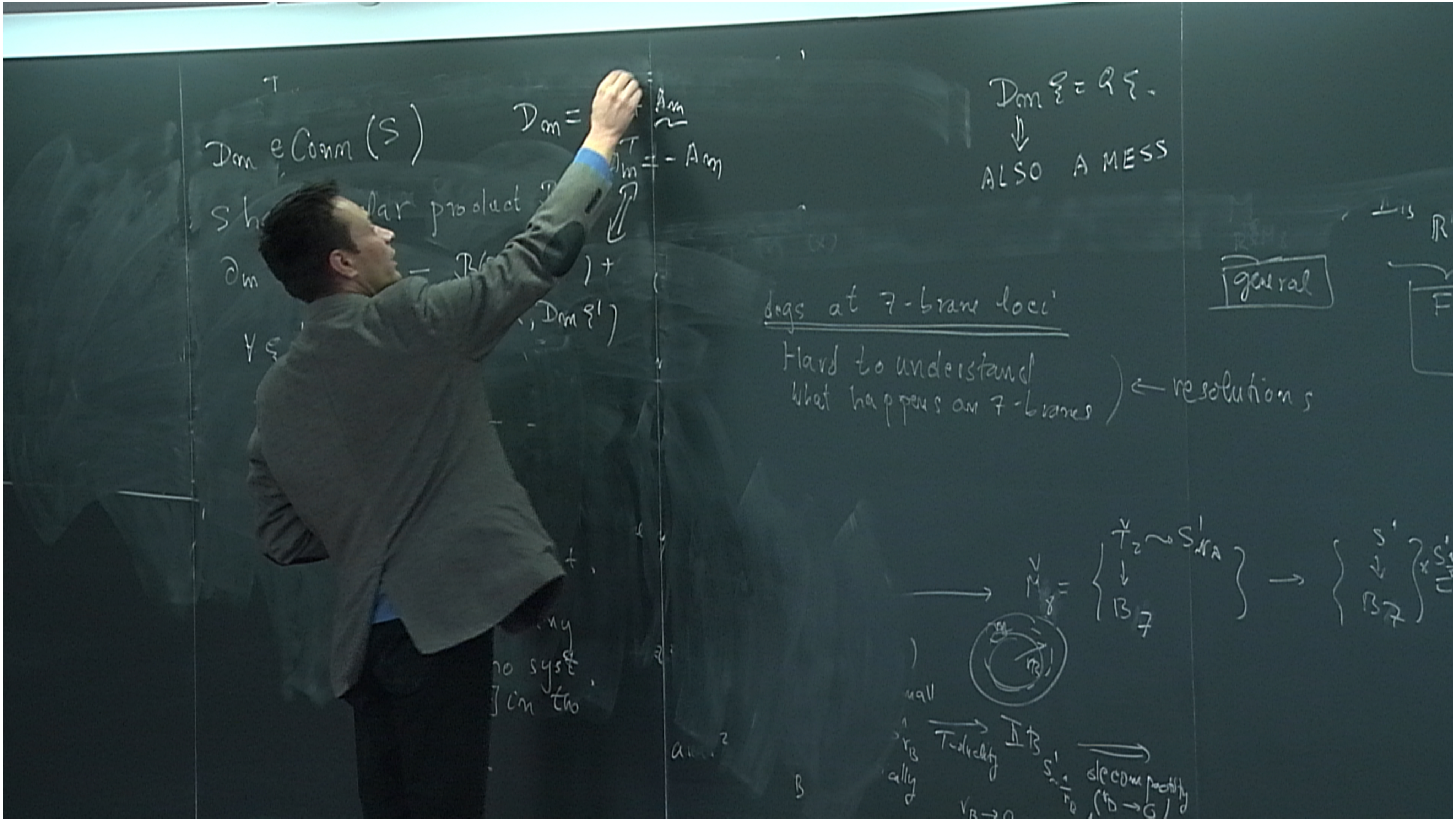
$r_B \to 0 \Rightarrow \frac{1}{r_B} \to \infty$ (dec. limit)

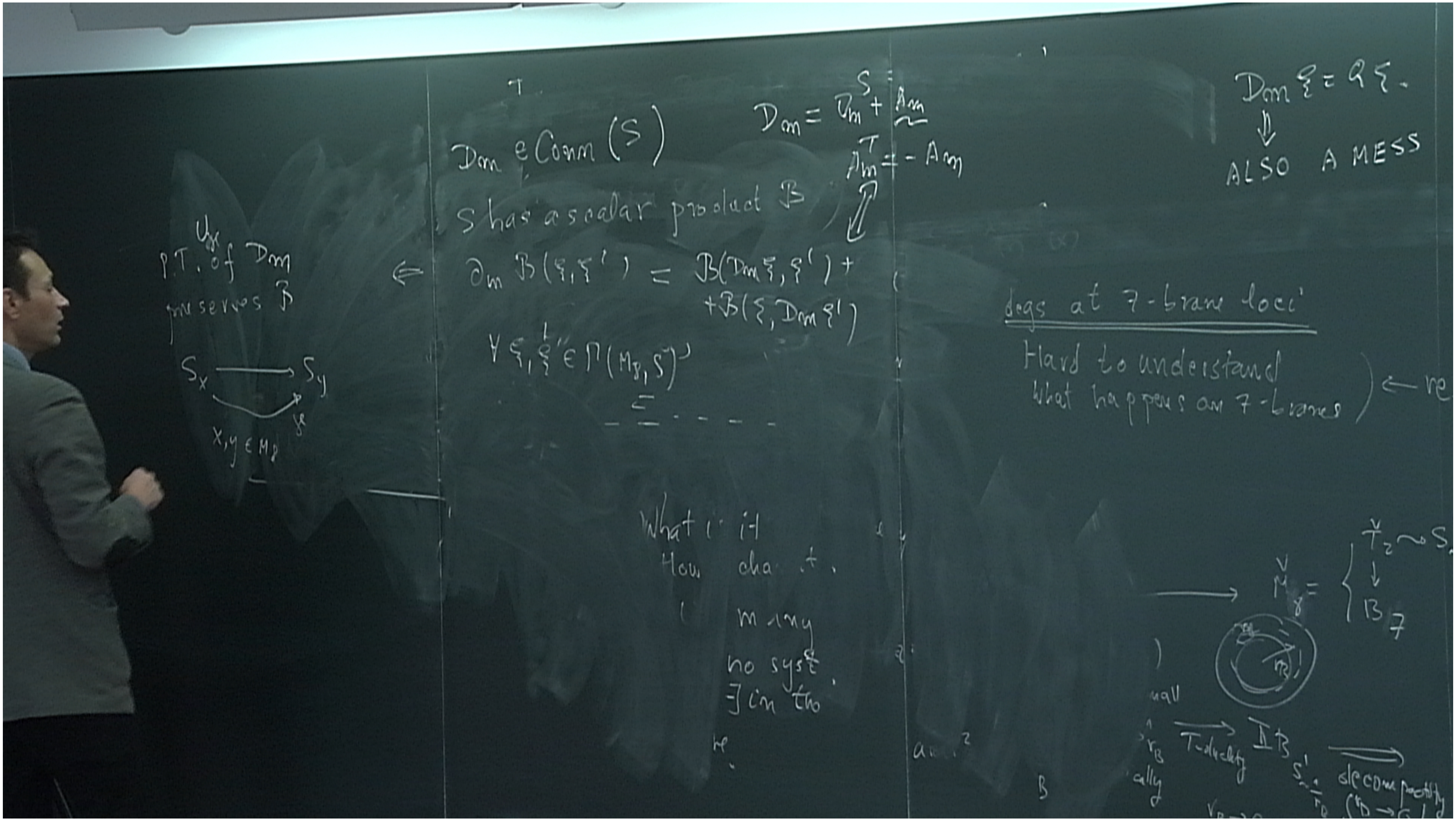

ω^(α) are NOT alg. indep
due to Fierz ids
degs at 7-brane loci
Hard to understand
what happens on 7-branes
← resolution
general
T-duality
decompactify
(dec. limit

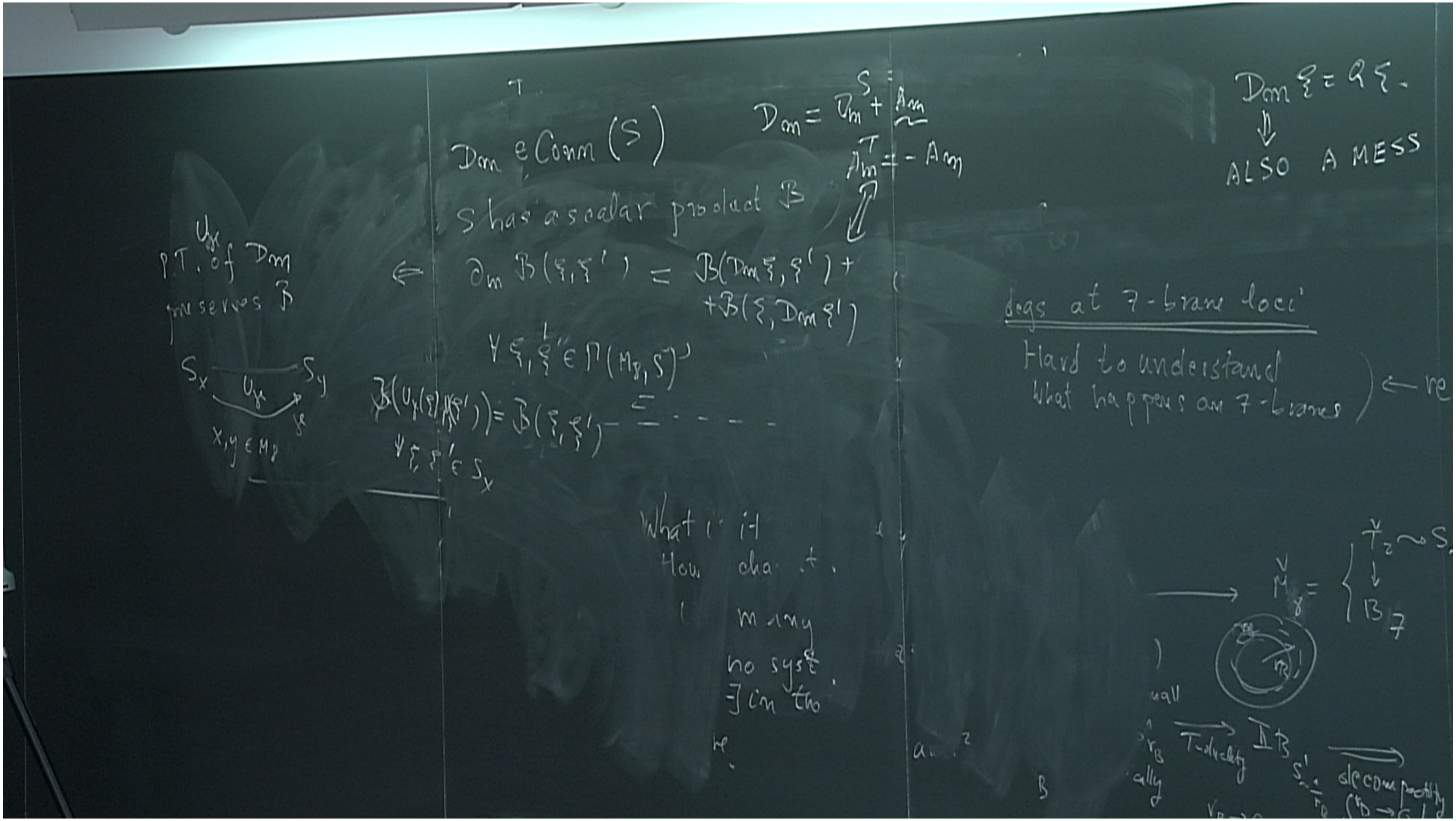

ω are NOT alg. indep
due to Fierz ids
Fierz algebra ⇒ 1) What is it?
2) How to char it.
AWFULLY many
Fierz ids; no syst.
approach ∃ in the
literature.
degs at 7-brane loci
Hard to understand
what happens on 7-branes
← resolution
general
T-duality
IIB
decompactify
(dec. limit)

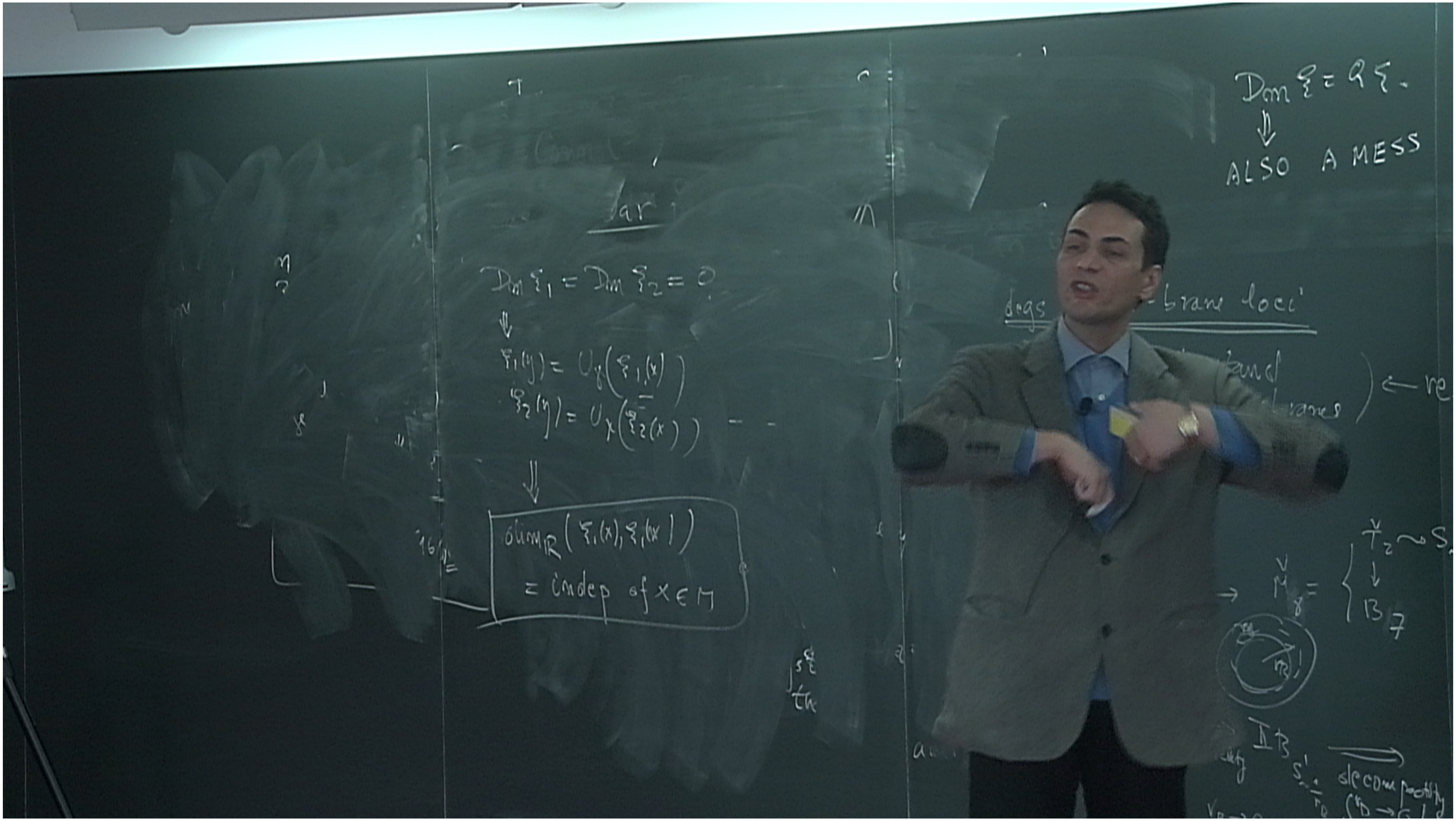


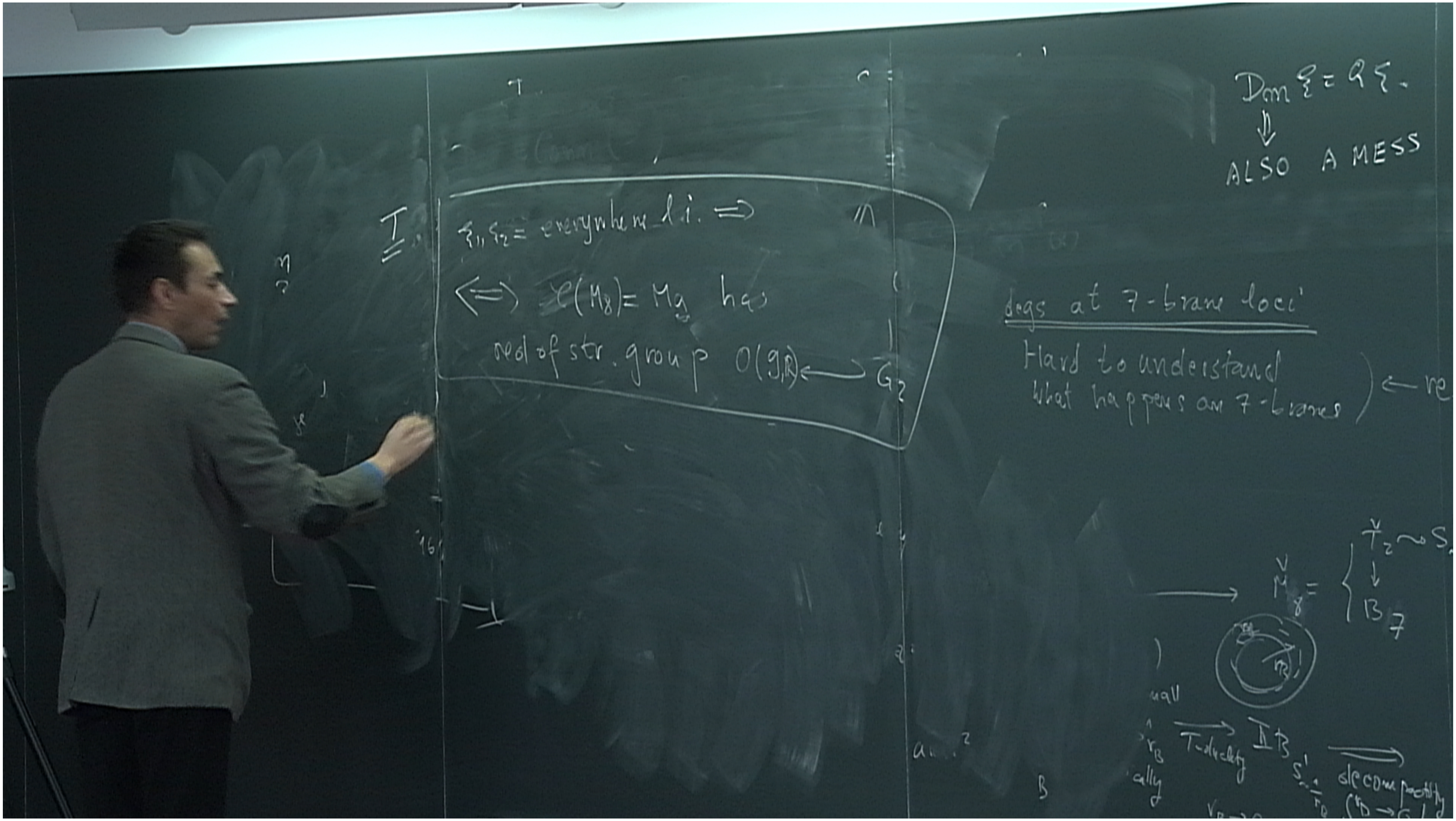
Geometric algebra
D. Hestenes:
- not general enough
- no hard appls
① do it generally (arb. dims & signatures)
② develop some specialized appls (Fierz ids, theory of CGK forms
& lis btw CGK spinors &
CGK for
gen theory of Killi

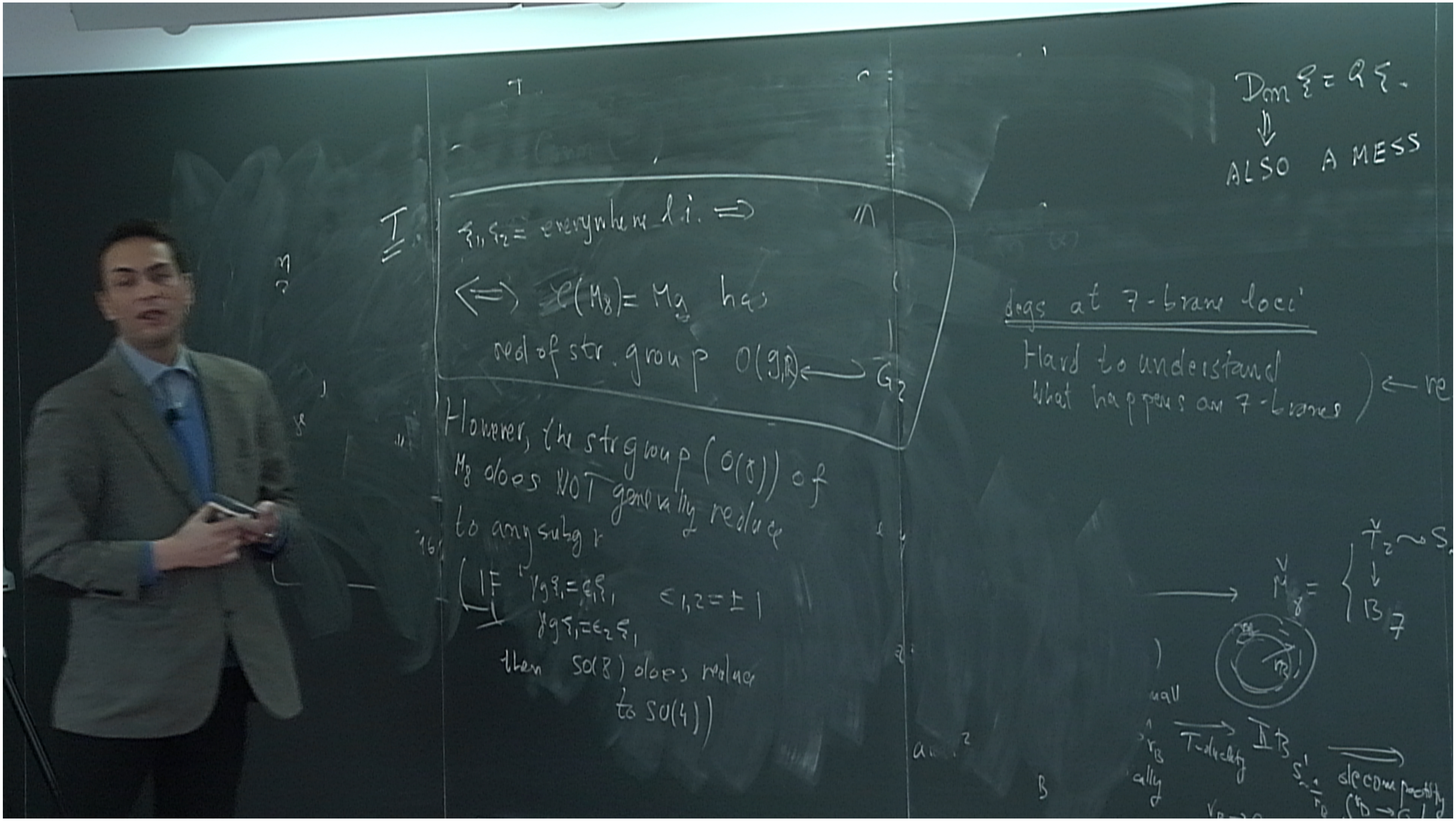

ALSO A MESS
Hard to understand
what happens on 7-branes
resolutions
general
T-duality

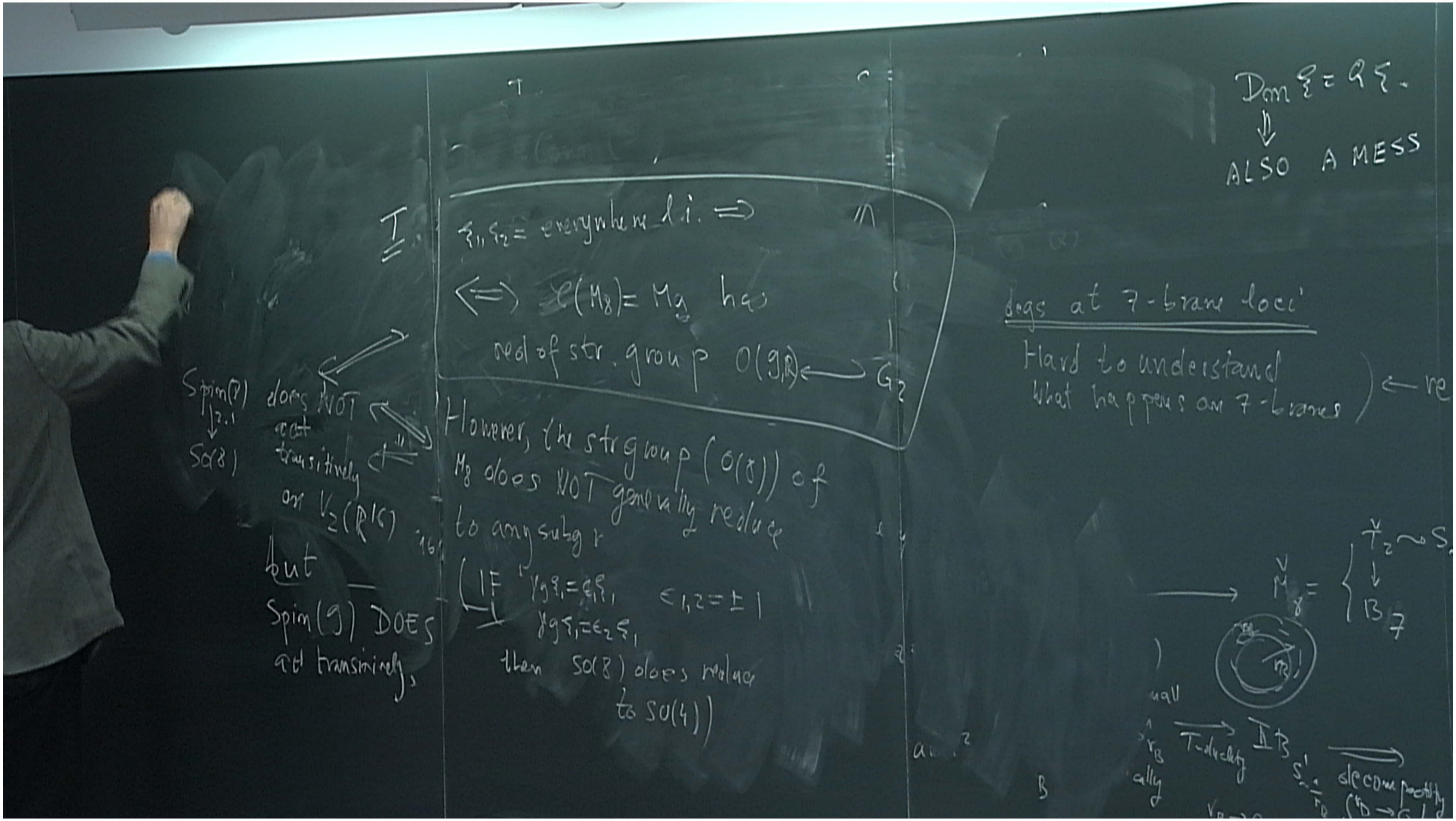

P.T. of Dm preserves B
Dm ∈ Conn (S)
S has a scalar product B
degs at 7-brane loci
Hard to understand what happens on 7-branes
ALSO A MESS
What is it
T-duality
decompactify


Dm ∈ Conn (S)
Dm = ∂m + Am
S has a scalar product B
P.T. of Dm preserves B
⇐ ∂m B(ξ, ξ') = B(Dm ξ, ξ') + B(ξ, Dm ξ')
∀ ξ, ξ' ∈ Γ(M8, S)
B(Ux(ξ), Ux(ξ')) = B(ξ, ξ')
∀ ξ, ξ' ∈ Sx
x, y ∈ M8
Dm ξ = Q ξ
ALSO A MESS
dogs at 7-brane loci
Hard to understand
What happens on 7-branes
What is it
How
many
no syst
T-duality
IIB
decompactify

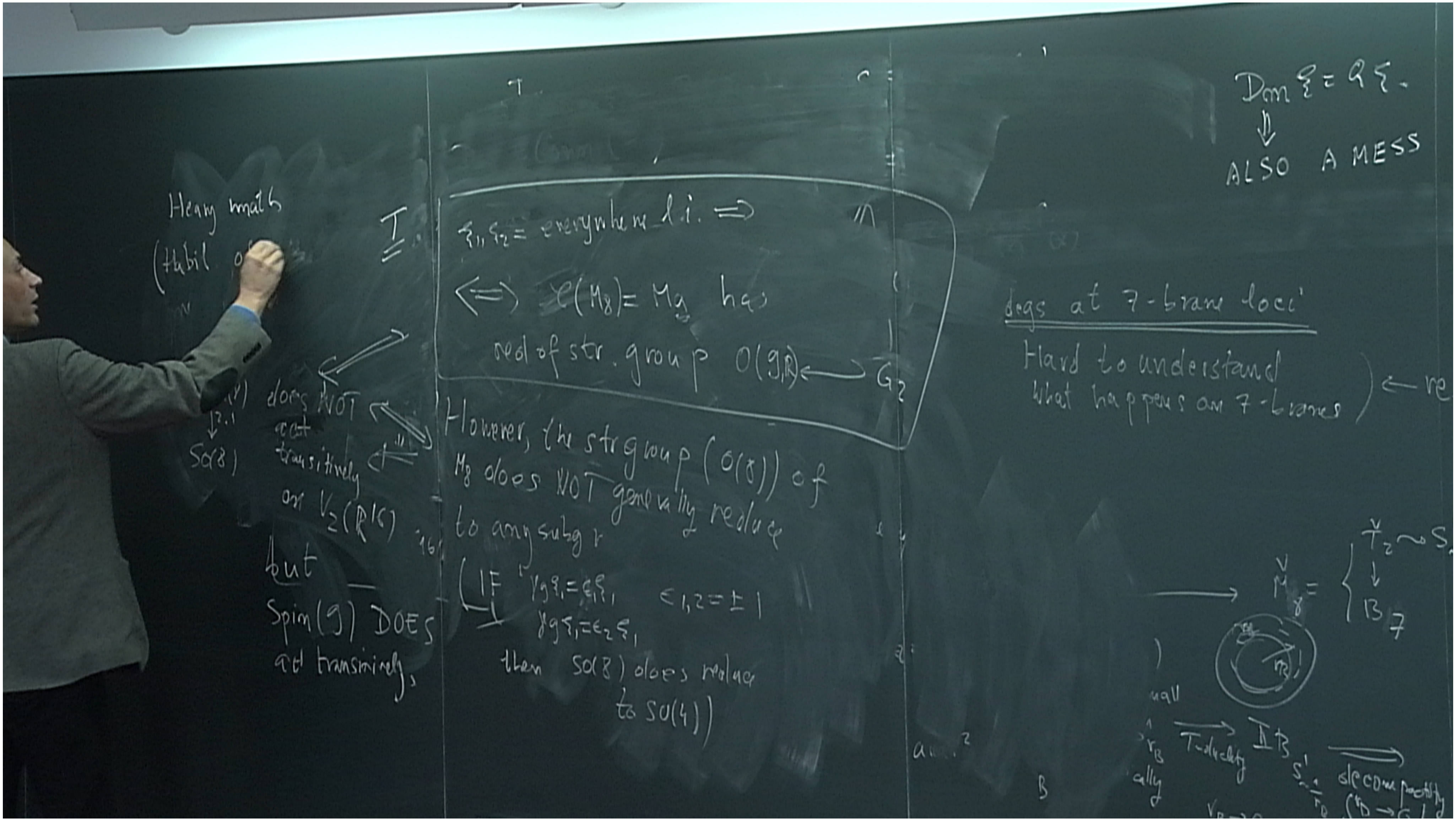

Dm ξ = Q ξ
ALSO A MESS
Dm ξ1 = Dm ξ2 = 0
= indep of x ∈ M
brane loci

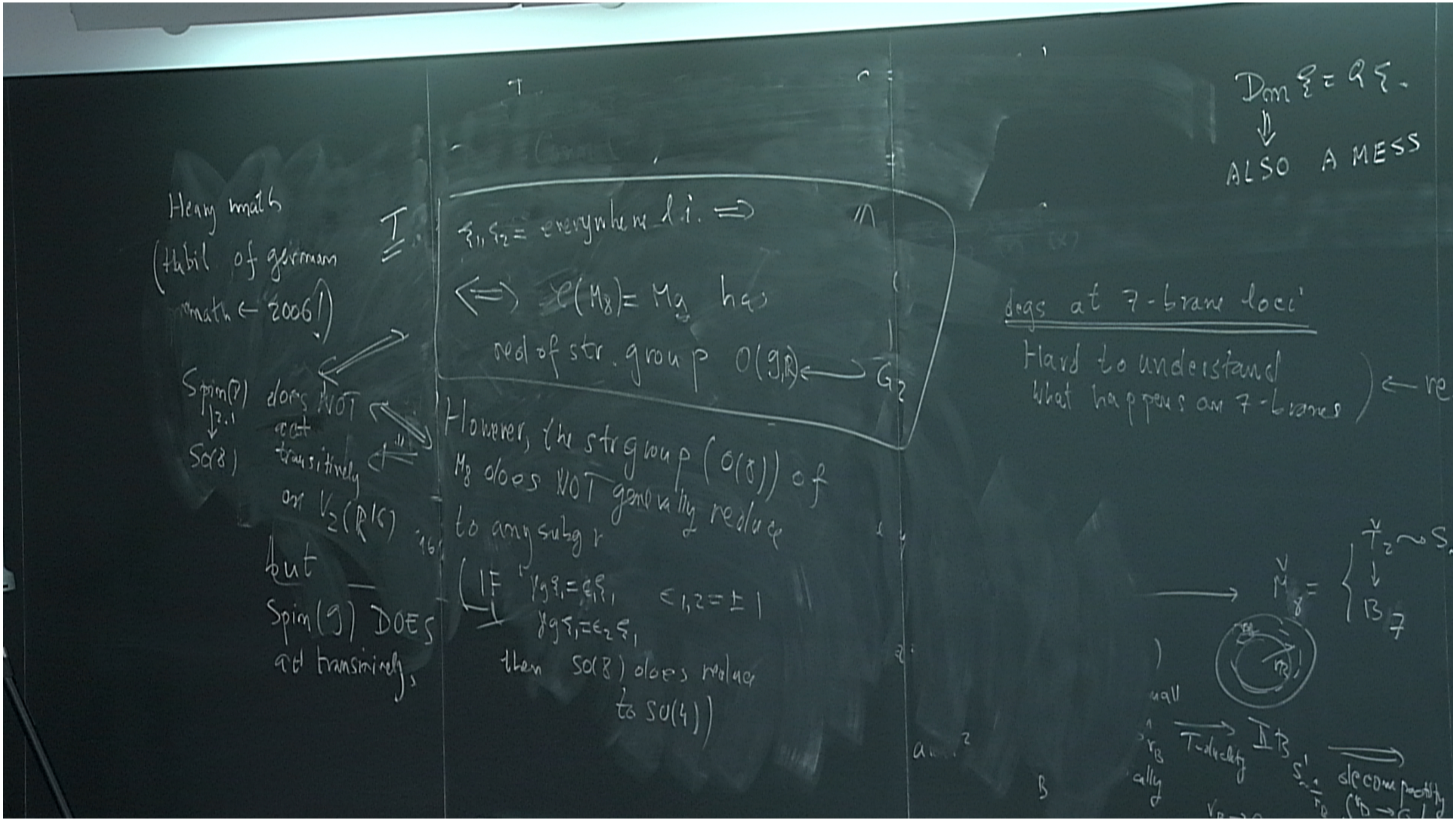

ALSO A MESS
Hard to understand
what happens on 7-branes
T-duality

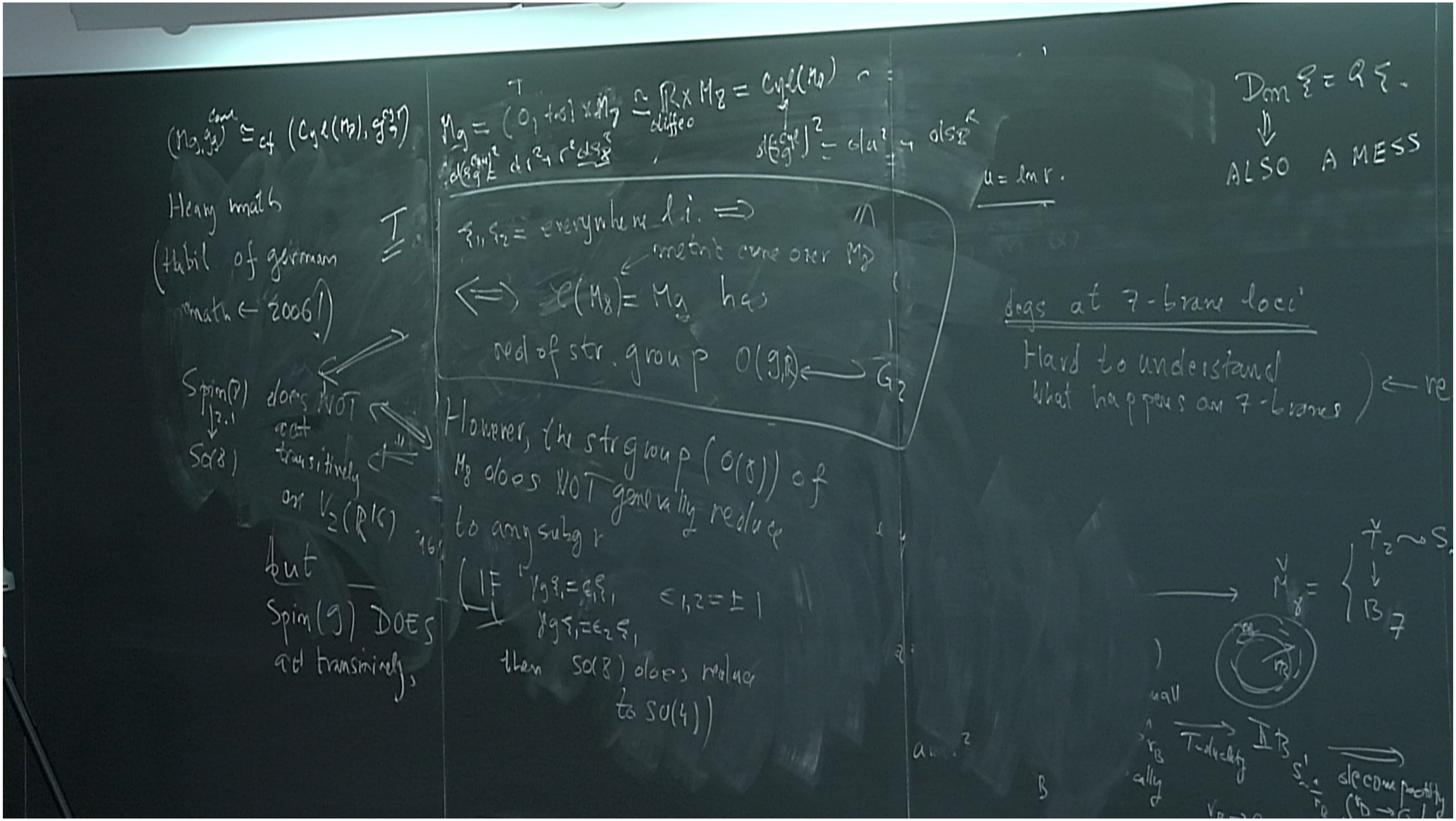

D ξ = Q ξ
ALSO A MESS
ξ1, ξ2 = everywhere l.i. ⟹
⟺ e(M8) = M8 has
red of str. group O(8,R) ⟵ G2
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
(IF γ9 ξ1 = ε1 ξ1, ε1,2 = ±1
γ9 ξ1 = ε2 ξ1
then SO(8) does reduce
to SU(4))
degs at 7-brane loci
Hard to understand
what happens on 7-branes
T-duality
decompactify

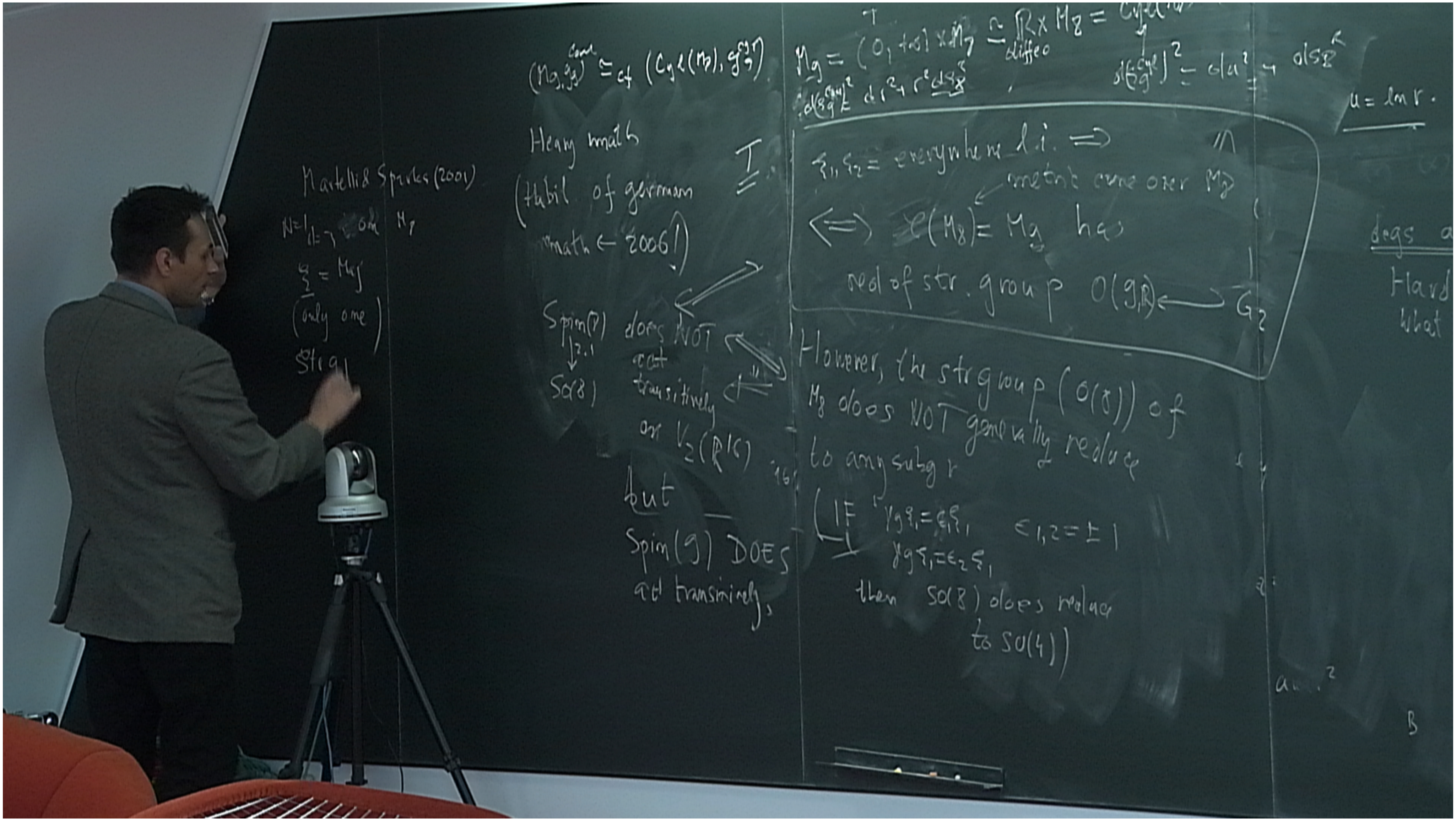


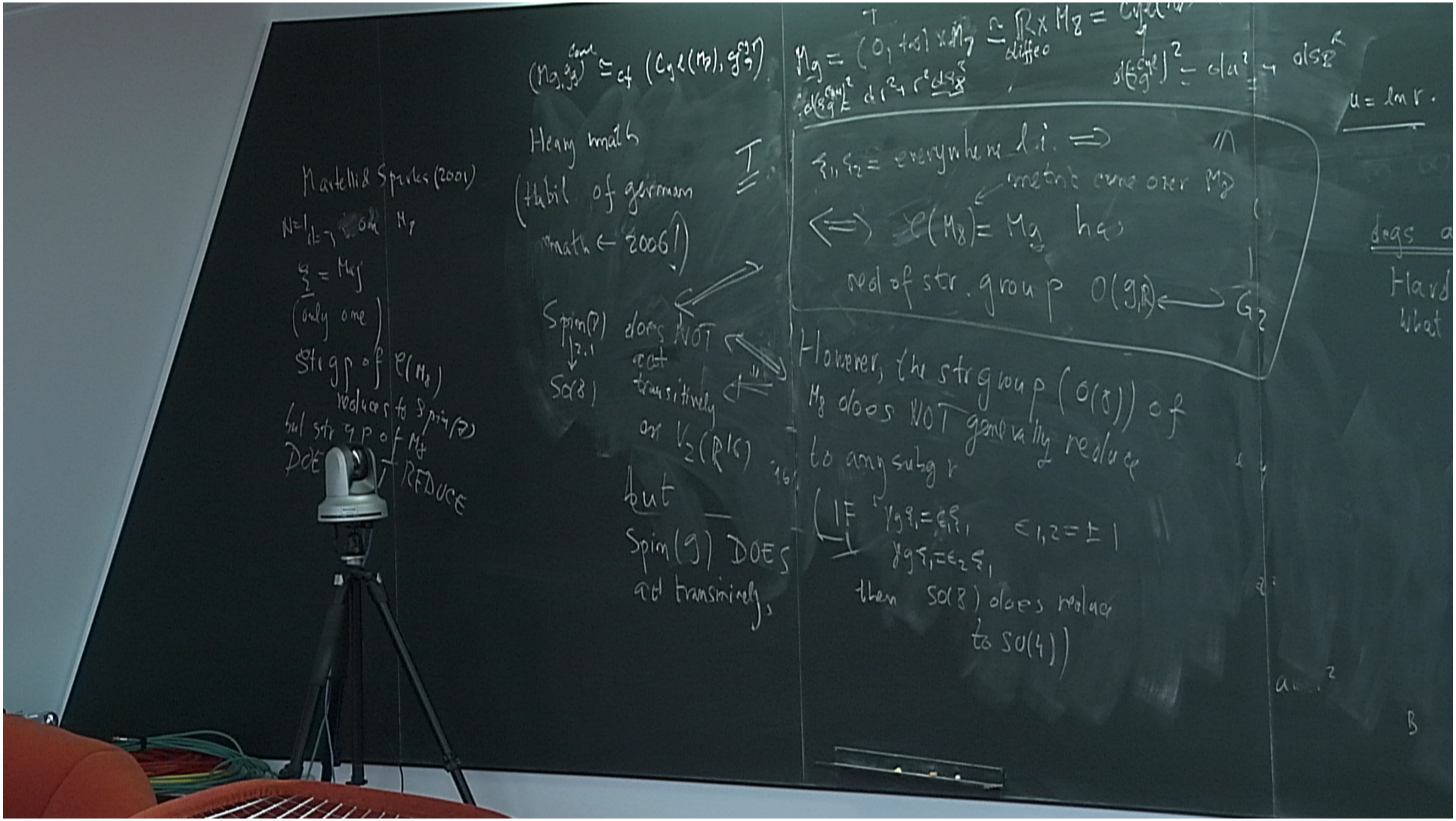
Dm ξ = Q ξ.
ALSO A MESS
I.
ξ1, ξ2 = everywhere l.i. ⟹
⟸ e(M8) = M8 has
red of str. group O(8,R) ⟵ G2
Spin(8) does NOT
act
transitively
on V2(R16)
SO(8)
but
Spin(9) DOES
act transitively,
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
(IF γ9ξ1 = ε1ξ1, ε1,2 = ±1
γ9ξ1 = ε2ξ1
then SO(8) does reduce
to SO(4))
degs at 7-brane loci
Hard to understand
what happens on 7-branes
← re
T-duality
IIB
decompactify

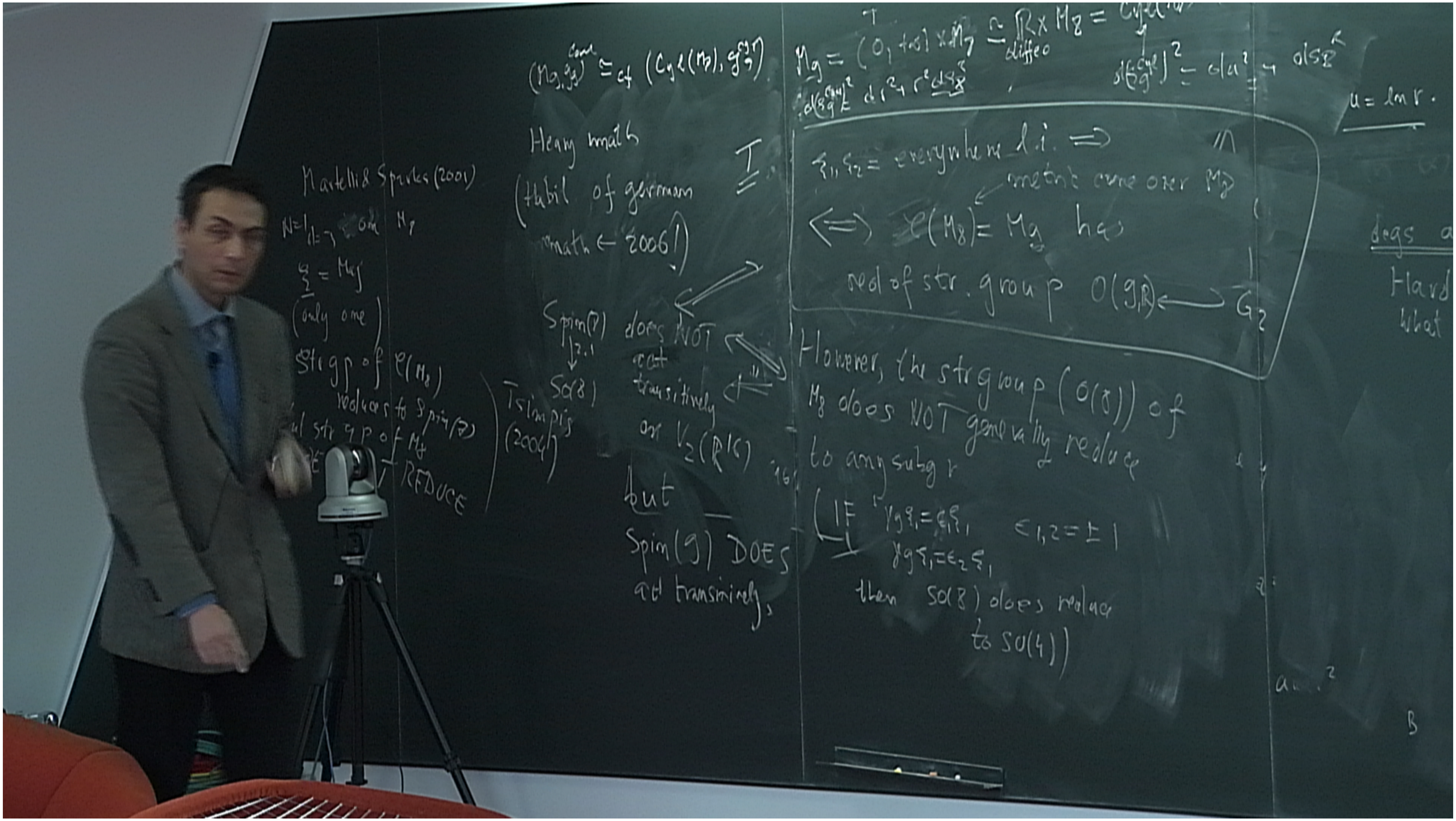

Dom ξ = Q ξ.
ALSO A MESS
Heavy math
I.
ξ1, ξ2 = everywhere l.i. ⟹
⟺ e(M8) = M8 has
red of str. group O(8,R) ⟵ G2
degs at 7-brane loci
Hard to understand
what happens on 7-branes
does NOT
act
transitively
on V2(R^16)
SO(8)
but
Spin(9) DOES
act transitively,
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
(IF γ9ξ1 = ε1ξ1, ε1,2 = ± 1
γ9ξ1 = ε2ξ1
then SO(8) does reduce
to SO(4))
T-duality
IIB
decompactify


Heavy math
(Habil of german
math ← 2006!)
Spin(7) does NOT
act
transitively
on $V_2(\mathbb{R}^{16})$
SO(8)
but
Spin(9) DOES
act transitively,
I.
ξ_1, ξ_2 = everywhere l.i. ⟹
⟸ $\mathcal{E}(M_8) = M_8$ has
red of str. group O(9,R) ← G_2
However, the str group (O(8)) of
M_8 does NOT generally reduce
to any subgr
(IF $\gamma_9 \xi_1 = \epsilon_1 \xi_1$ $\epsilon_{1,2} = \pm 1$
$\gamma_9 \xi_1 = \epsilon_2 \xi_1$
then SO(8) does reduce
to SU(4))
$D_m \xi = Q \xi$.
⇓
ALSO A MESS
degs at 7-brane loci
Hard to understand
what happens on 7-branes
← re
T-duality
IIB
decompactify


Heavy math
(tabil of german
math ← 2006!)
Spin(7) does NOT act transitively
but
Spin(9) DOES act transitively,
metric cone over M7
red of str. group O(9,R)
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
then SO(8) does reduce
to SU(4))
u = ln r.
Dm ξ = Q ξ.
ALSO A MESS
degs at 7-brane loci
Hard to understand
what happens on 7-branes
T-duality
decompactify


Martelli & Sparks (2001)
(only one)
Heavy math
(Habil. of german
math ← 2006!)
Spin(7) does NOT
act
transitively
but
Spin(9) DOES
act transitively
ξ₁, ξ₂ = everywhere l.i. ⟹
metric cone over M₇
red. of str. group O(9,R) ⟵ G₂
However, the str group (O(8)) of
M₈ does NOT generally reduce
to any subgr
then SO(8) does reduce
to SO(4))
u = ln r.
diffeo


Martelli & Sparks (2001)
Heavy math
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
then SO(8) does reduce
to SU(4))
Spin(9) DOES
act transitively
but
REDUCE


Martelli & Sparks (2001)
Heavy math
red of str. group O(9,R) ⟷ G2
Spin(9) DOES
act transitively
Tsimpis (2006)
However, the str group (O(8)) of
M8 does NOT generally reduce
to any subgr
then SO(8) does reduce
to SU(4)
u = ln r.

in 8-dimensions or 9-dimensions, it is enough to give all relations up to rank 4

Hodge dualisation rules

8 dim:

$$\gamma^{a_1\ldots a_k}\gamma^{(9)} = \frac{(-1)^{\frac{k(k-1)}{2}}}{(8-k)!}\epsilon^{a_1\ldots a_k}{}_{b_1\ldots b_{8-k}}\gamma^{b_1\ldots b_{8-k}}$$

$$(*_8\Omega^{(8-k)})_{a_1\ldots a_k} = \frac{(-1)^k}{k!}\epsilon_{a_1\ldots a_k}{}^{b_1\ldots b_{8-k}}\Omega^{(8-k)}_{b_1\ldots b_{8-k}}$$

9 dim:

$$\gamma^{A_1\ldots A_k} = \frac{(-1)^{\frac{k(k-1)}{2}}}{(9-k)!}\epsilon^{A_1\ldots A_k}{}_{B_1\ldots B_{9-k}}\gamma^{B_1\ldots B_{9-k}}$$

$$(*A^{(9-k)})_{A_1\ldots A_k} = \frac{1}{k!}\epsilon_{A_1\ldots A_k}{}^{B_1\ldots B_{9-k}}A^{(9-k)}_{B_1\ldots B_{9-k}}$$

$$\boxed{D_m\xi = 0 \quad , \quad Q\xi = 0}$$

$$D_m = \nabla^S_m + A_m$$

The connection A_m and the endomorphism Q in 9 dimensions are isomorphic with:

$$\check{A}_m = \frac{1}{4}e_m \lrcorner F + \frac{1}{4}e_m \wedge (f \wedge \theta)$$

$$\check{Q} = \frac{1}{2}d\Delta - \frac{1}{6}f \wedge \theta - \frac{1}{12}F$$

where $\theta = \frac{\partial}{\partial r}$ corresponds to $\theta_n = \delta_{n9}$

The transpose of Q is isomorphic with:

$$\tau(\check{Q}) = \frac{1}{2}d\Delta + \frac{1}{6}f \wedge \theta - \frac{1}{12}F$$

where the main anti-automorphism τ (the 'reversion') has the following action on a k-form:

$$\tau(\check{\omega}^{(k)}) = (-1)^{\frac{k(k-1)}{2}}\check{\omega}^{(k)}$$

$$\downarrow$$

$$\check{\omega}^{(k)}(\xi, \xi') := \frac{1}{k!}\mathscr{B}(\xi, \gamma_{a_1 \dots a_k}(\xi'))e^{a_1 \dots a_k}$$

$$\downarrow$$

$$\mathscr{B}(\xi, \gamma_{a_1 \dots a_k}(\xi')) = \check{\omega}_{a_1 \dots a_k}$$

Given the nontrivial bilinear forms in our case

$$V_1^m = \xi_1^T \gamma^m \xi_1 \qquad V_2^m = \xi_2^T \gamma^m \xi_2 \qquad V_3^m = \xi_1^T \gamma^m \xi_2$$

$$K^{mn} = \xi_1^T \gamma^{mn} \xi_2$$

$$\psi^{mnp} = \xi_1^T \gamma^{mnp} \xi_2$$

$$\phi_1^{mnpq} = \xi_1^T \gamma^{mnpq} \xi_1 \qquad \phi_2^{mnpq} = \xi_2^T \gamma^{mnpq} \xi_2 \qquad \phi_3^{mnpq} = \xi_1^T \gamma^{mnpq} \xi_2$$

$$(||\xi_1|| = ||\xi_2|| = 1, \quad \langle \xi_1, \xi_2 \rangle = 0)$$

one can choose a basis for the Killing algebra:

$$\check{E}_{12} = \frac{1}{16}(V_3 + K + \psi + \phi_3)$$

$$\check{E}_{21} = \frac{1}{16}(V_3 - K - \psi + \phi_3) = \tau(\check{E}_{12})$$

$$\check{E}_{11} = \frac{1}{16}(1 + V_1 + \phi_1)$$

$$\check{E}_{22} = \frac{1}{16}(1 + V_2 + \phi_2),$$

} generators ($\check{E}_{12}$, $\check{E}_{21}$)

The entire system of constraints reduces to:

$$\begin{cases} \breve{Q} \diamond \breve{E}_{12} = \breve{E}_{12} \diamond \tau(\breve{Q}) = 0 \\ \breve{D}_m \breve{E}_{12} = 0 \\ \breve{E}_{12} \diamond \breve{E}_{12} = 0 \\ \breve{E}_{12} \diamond \tau(\breve{E}_{12}) \diamond \breve{E}_{12} = \breve{E}_{12} \end{cases}$$

Expansion for $\omega \in \Omega^p(M)$ and all $\eta \in \Omega^q(M)$ with $p \leq q$:

$$\omega \diamond \eta = \sum_{k=0}^{p} (-1)^{k(p-k)+[k/2]} \omega \Delta_k \eta$$

$$\eta \diamond \omega = (-1)^{pq} (-1)^{k(p-k+1)+[k/2]} \omega \Delta_k \eta$$

generalized products Δ_k are the homogeneous components of $\diamond$

$$\Delta_k = \frac{1}{k!} \wedge_k$$

$\wedge_k$ are the *contracted wedge products*

$$\omega \wedge_0 \eta = \omega \wedge \eta \quad , \quad \omega \wedge_{k+1} \eta = g^{mn} (e_m \lrcorner \omega) \wedge_k (e_n \lrcorner \eta)$$

Using Ricci (a Mathematica package for tensor computations)

In[56]:=
```
Eg2 = GenProd[2, F, Wedge[V1, ψ], tangent] // BasisExpand // TensorSimplify; //.
 {Null → ""}
NewDummy[Eg2][L[k], L[l], L[m], L[n]] // TensorSimplify
Hold[GenProd[2, F, Wedge[V1, ψ], tangent]]
```

Out[56]//OutputForm=

Out[57]//OutputForm=

$$
\begin{aligned}
&F_{mnpq}\, V1^{p}\, \psi_{kl}{}^{q} - F_{lnpq}\, V1^{p}\, \psi_{km}{}^{q} + F_{lmpq}\, V1^{p}\, \psi_{kn}{}^{q} - \\
&\frac{1}{2} F_{mnpq}\, V1_{l}\, \psi_{k}{}^{pq} + \frac{1}{2} F_{lnpq}\, V1_{m}\, \psi_{k}{}^{pq} - \frac{1}{2} F_{lmpq}\, V1_{n}\, \psi_{k}{}^{pq} + \\
&F_{knpq}\, V1^{p}\, \psi_{lm}{}^{q} - F_{kmpq}\, V1^{p}\, \psi_{ln}{}^{q} + \frac{1}{2} F_{mnpq}\, V1_{k}\, \psi_{l}{}^{pq} - \\
&\frac{1}{2} F_{knpq}\, V1_{m}\, \psi_{l}{}^{pq} + \frac{1}{2} F_{kmpq}\, V1_{n}\, \psi_{l}{}^{pq} + F_{klpq}\, V1^{p}\, \psi_{mn}{}^{q} - \\
&\frac{1}{2} F_{lnpq}\, V1_{k}\, \psi_{m}{}^{pq} + \frac{1}{2} F_{knpq}\, V1_{l}\, \psi_{m}{}^{pq} - \frac{1}{2} F_{klpq}\, V1_{n}\, \psi_{m}{}^{pq} + \\
&\frac{1}{2} F_{lmpq}\, V1_{k}\, \psi_{n}{}^{pq} - \frac{1}{2} F_{kmpq}\, V1_{l}\, \psi_{n}{}^{pq} + \frac{1}{2} F_{klpq}\, V1_{m}\, \psi_{n}{}^{pq}
\end{aligned}
$$

Out[58]//OutputForm=
```
Hold[GenProd[2, F, V1 ^ ψ, tangent]]
```

$$F\Delta_2(V_1 \wedge \psi)$$

Algebraic constraints separated on ranks: $\check{Q} \diamond \check{E}_{12} \mp \check{E}_{12} \diamond \tau(\check{Q}) = 0$

$$\left.\begin{aligned}
(f \wedge \theta) \lrcorner K &= 0 \\
(d\Delta) \lrcorner K + \frac{1}{3}(f \wedge \theta) \lrcorner \psi - \frac{1}{6}\psi \lrcorner F &= 0 \\
\frac{1}{3} K \lrcorner (f \wedge \theta) - \frac{1}{6} F \boldsymbol{\Delta}_3 \phi_3 + (d\Delta) \wedge V_3 &= 0 \\
(d\Delta) \lrcorner \phi_3 - \frac{1}{3} V_3 \wedge f \wedge \theta + \frac{1}{6} V_3 \lrcorner F - \frac{1}{6} *(F \boldsymbol{\Delta}_1 \phi_3) + \frac{1}{3} *(f \wedge \theta \wedge \phi_3) &= 0 \\
(d\Delta) \wedge \psi - \frac{1}{3} f \wedge \theta \wedge K - \frac{1}{6} K \boldsymbol{\Delta}_1 F - \frac{1}{3} *(f \wedge \theta \wedge \psi) + \frac{1}{6} *(F \boldsymbol{\Delta}_1 \psi) &= 0
\end{aligned}\right\}$$

$$\left.\begin{aligned}
-\frac{1}{6} F \lrcorner \phi_3 + (d\Delta) \lrcorner V_3 &= 0 \\
\frac{1}{3} V_1 \lrcorner (f \wedge \theta) - \frac{1}{6} *(F \wedge \phi_3) &= 0 \\
(d\Delta) \lrcorner \psi + \frac{1}{3}(f \wedge \theta) \boldsymbol{\Delta}_1 K + \frac{1}{6} K \lrcorner F + \frac{1}{6} *(F \wedge \psi) &= 0 \\
\frac{1}{3}(f \wedge \theta) \boldsymbol{\Delta}_1 \psi + \frac{1}{6} \psi \boldsymbol{\Delta}_2 F + \frac{1}{6} *(K \wedge F) + (d\Delta) \wedge K &= 0 \\
\frac{1}{3}(f \wedge \theta) \boldsymbol{\Delta}_1 \phi_3 + \frac{1}{6} F \boldsymbol{\Delta}_2 \phi_3 + \frac{1}{6} *(F \wedge V_3) - *((d\Delta) \wedge \phi_3) &= 0
\end{aligned}\right\}$$

The differential constraints obtained, separated on ranks:

$$d\check{E}_{12} = e^m \wedge \nabla_m \check{E}_{12} = -e^m \wedge [\check{A}_m, \check{E}_{12}]_{-,\diamond}$$

$$\begin{cases} dV_3 = \frac{1}{2}\phi_3 \boldsymbol{\Delta}_3 F - (f \wedge \theta) \lrcorner \phi_3 \\ dK = (f \wedge \theta)\boldsymbol{\Delta}_1 \psi + \psi \boldsymbol{\Delta}_2 F \\ d\psi = \frac{3}{2} F \boldsymbol{\Delta}_1 K + \frac{1}{2} F \boldsymbol{\Delta}_3 (*\psi) + 2*(f \wedge \theta \wedge \psi) - f \wedge \theta \wedge K \\ d\phi_3 = -2F \wedge V_3 + \frac{1}{2} e^m \wedge *(((e_m \lrcorner F)\boldsymbol{\Delta}_1 \phi_3) + \frac{1}{2} e^m \wedge *(((e_m \wedge f \wedge \theta)\boldsymbol{\Delta}_1 \phi_3) \end{cases}$$

Fierz relations for the generators of the algebra

$$\check{E}_{12} \diamond \check{E}_{12} = 0 \quad \left(\Longleftrightarrow \tau(\check{E}_{12}) \diamond \tau(\check{E}_{12}) = 0 \right)$$

$$\check{E}_{12} \diamond \tau(\check{E}_{12}) \diamond \check{E}_{12} = \check{E}_{12} \quad \left(\Longleftrightarrow \tau(\check{E}_{12}) \diamond \check{E}_{12} \diamond \tau(\check{E}_{12}) = \check{E}_{12} \right)$$

all quadratic Fierz relations for all the basis elements $\check{E}_{ij}$ for $i, j = 1, 2$:

$$(\mathrm{F1}): \ \check{E}_{12} \diamond \check{E}_{12} = 0, \qquad (\mathrm{F2}): \ \check{E}_{12} \diamond \check{E}_{21} = \check{E}_{21},$$

$$(\mathrm{F3}): \ \check{E}_{12} \diamond \check{E}_{22} = \check{E}_{12}, \qquad (\mathrm{F4}): \ \check{E}_{12} \diamond \check{E}_{11} = 0,$$

$$(\mathrm{F5}): \ \check{E}_{11} \diamond \check{E}_{11} = \check{E}_{11}, \qquad (\mathrm{F6}): \ \check{E}_{11} \diamond \check{E}_{12} = \check{E}_{12},$$

$$(\mathrm{F7}): \ \check{E}_{11} \diamond \check{E}_{21} = 0, \qquad (\mathrm{F8}): \ \check{E}_{11} \diamond \check{E}_{22} = 0,$$

$$(\mathrm{F9}): \ \check{E}_{21} \diamond \check{E}_{12} = \check{E}_{22}, \qquad (\mathrm{F10}): \ \check{E}_{21} \diamond \check{E}_{11} = \check{E}_{21},$$

$$(\mathrm{F11}): \ \check{E}_{21} \diamond \check{E}_{21} = 0, \qquad (\mathrm{F12}): \ \check{E}_{21} \diamond \check{E}_{22} = 0,$$

$$(\mathrm{F13}): \ \check{E}_{12} \diamond \check{E}_{11} = 0, \qquad (\mathrm{F14}): \ \check{E}_{22} \diamond \check{E}_{12} = 0,$$

$$(\mathrm{F15}): \ \check{E}_{22} \diamond \check{E}_{21} = \check{E}_{21}, \qquad (\mathrm{F16}): \ \check{E}_{22} \diamond \check{E}_{22} = \check{E}_{22},$$

Independent constraints:

$$K = V_1 \wedge V_3$$

$$\phi_3 = V_1 \wedge \psi$$

$$V_2 = -V_1$$

$$\| V_1 \|^2 = 1, \quad \| V_3 \|^2 = 1, \quad \| \psi \|^2 = 7,$$

$$\langle V_1, V_3 \rangle = 0, \quad V_1 \lrcorner \psi = 0, \quad V_3 \lrcorner \psi = 0,$$

$$\psi \Delta_1 \psi = 6 * (V_1 \wedge V_3 \wedge \psi)$$

$$\phi_1 = -V_3 \wedge \psi - *(V_1 \wedge V_3 \wedge \psi)$$

$$\phi_2 = V_3 \wedge \psi - *(V_1 \wedge V_3 \wedge \psi)$$

independent forms V_1, V_3 and ψ

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independent forms V_1, V_3 and ψ

Other relations (non-independent constraits) involving the dependent forms ϕ_1 and ϕ_2

$$
\begin{aligned}
\| \phi_1 \|^2 = 14 \ , &\qquad \| \phi_2 \|^2 = 14, \\
V_1 \lrcorner \phi_1 = 0 \ , &\qquad V_1 \lrcorner \phi_2 = 0 \ , \\
V_3 \lrcorner \phi_1 = -\psi \ , &\qquad V_3 \lrcorner \phi_2 = \psi \ , \\
*(V_1 \wedge \phi_1) = \phi_1 \ , &\qquad *(V_1 \wedge \phi_2) = -\phi_2 \ , \\
*(V_1 \wedge V_3 \wedge \phi_1) &= *(V_1 \wedge V_3 \wedge \phi_2) = -\psi, \\
*(V_3 \wedge \phi_1) = V_1 \wedge \psi \ , &\qquad *(V_3 \wedge \phi_2) = V_1 \wedge \psi \\
\psi \lrcorner \phi_1 = *(V_1 \wedge \phi_1 \wedge \psi) = 7V_3 \ , &\qquad \psi \lrcorner \phi_2 = -*(V_1 \wedge \phi_2 \wedge \psi) = -7V_3 \ , \\
*(\psi \wedge \phi_1) = -7V_1 \wedge V_3, &\qquad *(\psi \wedge \phi_2) = -7V_1 \wedge V_3, \\
V_1 \wedge (\psi \lrcorner \phi_1) &= *(\phi_1 \wedge \psi) = 7V_1 \wedge V_3 \ , \\
V_1 \wedge (\psi \lrcorner \phi_2) &= *(\phi_2 \wedge \psi) = -7V_1 \wedge V_3 \ , \\
\langle \phi_1, \phi_2 \rangle &= 0 \ , \\
*(\phi_1 \wedge \phi_1) = 14V_1 \ , \quad *(\phi_2 \wedge \phi_2) &= -14V_1 \ , \quad *(\phi_1 \wedge \phi_2) = 0 \ , \\
*(\psi \Delta_1 \phi_1) = 6V_1 \wedge \psi \ , &\qquad *(\psi \Delta_1 \phi_2) = -6V_1 \wedge \psi \ , \\
\phi_1 \Delta_3 \phi_2 = 0, \quad \phi_1 \Delta_2 \phi_2 &= 0 \ , \quad *(\phi_1 \Delta_1 \phi_2) = 0. \\
\phi_1 \Delta_2 \phi_1 = -12\phi_1, &\qquad \phi_2 \Delta_2 \phi_2 = -12\phi_2, \\
\psi \Delta_2 \phi_1 = -6\psi \ , &\qquad \psi \Delta_2 \phi_2 = -6\psi
\end{aligned}
$$

Other relations (non-independent constraits) involving the dependent forms K and ϕ_3

$$\| K \|^2 = 1, \quad \| \phi_3 \|^2 = 7, \qquad \langle \phi_1, \phi_3 \rangle = \langle \phi_2, \phi_3 \rangle = 0$$

$$K \lrcorner \phi_1 = 0 \ , \quad K \lrcorner \phi_2 = 0 \ , \qquad K \lrcorner \phi_3 = 0 \ , \quad K \lrcorner \psi = 0 \ ,$$

$$*(\phi_2 \Delta_1 \phi_3) = -6\psi \ , \qquad *(\phi_1 \Delta_1 \phi_3) = 6\psi$$

$$V_1 \lrcorner \phi_3 = \psi \ , \qquad V_3 \lrcorner \phi_3 = 0 \ ,$$

$$*(\psi \Delta_1 \phi_1) = 6\phi_3 \ , \qquad *(\psi \Delta_1 \phi_2) = -6\phi_3 \ ,$$

$$\begin{aligned} \psi \Delta_2 \phi_3 &= 0 \ , \\ \psi \lrcorner \phi_3 &= -7V_1 \ , \\ \phi_3 \Delta_2 \phi_3 &= \psi \Delta_1 \psi \\ \psi \Delta_1 \psi &= \phi_3 \Delta_2 \phi_3 , \end{aligned}$$

$$\phi_1 \Delta_3 \phi_3 = 7V_1 \wedge V_3 \ , \quad \phi_2 \Delta_3 \phi_3 = -7V_1 \wedge V_3 \ , \quad \phi_3 \Delta_3 \phi_3 = 0 \ .$$

Relations between all form bilinears written with Clifford product:

V_1	V_2	V_3	K	ψ
$V_1 \diamond V_1 = -1$	$V_2 \diamond V_1 = -1$	$V_3 \diamond V_1 = -K$	$K \diamond V_1 = -V_3$	$\psi \diamond V_1 = -\phi_3$
$V_1 \diamond V_2 = -1$	$V_2 \diamond V_2 = 1$	$V_3 \diamond V_2 = K$	$K \diamond V_2 = V_3$	$\psi \diamond V_2 = \phi_3$
$V_1 \diamond V_3 = -K$	$V_2 \diamond V_3 = -K$	$V_3 \diamond V_3 = 1$	$K \diamond V_3 = V_1$	$\psi \diamond V_3 = \frac{1}{2}(\phi_1 - \phi_2)$
$V_1 \diamond K = V_3$	$V_2 \diamond K = -V_3$	$V_3 \diamond K = -V_1$	$K \diamond K = -1$	$\psi \diamond K = -\frac{1}{2}(\phi_1 + \phi_2)$
$V_1 \diamond \psi = \phi_3$	$V_2 \diamond \psi = -\phi_3$	$V_3 \diamond \psi = \frac{1}{2}(\phi_2 - \phi_1)$	$K \diamond \psi = -\frac{1}{2}(\phi_1 + \phi_2)$	$\psi \diamond \psi = -7 - 3(\phi_1 + \phi_2)$
$V_1 \diamond \phi_1 = \phi_1$	$V_2 \diamond \phi_1 = -\phi_1$	$V_3 \diamond \phi_1 = -\psi + \phi_3$	$K \diamond \phi_1 = \psi - \phi_3$	$\psi \diamond \phi_1 = -7V_3 + 7K + 6\psi - 6\phi_3$
$V_1 \diamond \phi_2 = -\phi_2$	$V_2 \diamond \phi_2 = \phi_2$	$V_3 \diamond \phi_2 = \psi + \phi_3$	$K \diamond \phi_2 = \psi + \phi_3$	$\psi \diamond \phi_2 = 7V_3 + 7K + 6\psi + 6\phi_3$
$V_1 \diamond \phi_3 = \psi$	$V_2 \diamond \phi_3 = -\psi$	$V_3 \diamond \phi_3 = \frac{1}{2}(\phi_1 + \phi_2)$	$K \diamond \phi_3 = \frac{1}{2}(\phi_1 - \phi_2)$	$\psi \diamond \phi_3 = 7V_1 + 3(\phi_1 - \phi_2)$

ϕ_1	ϕ_2	ϕ_3
$\phi_1 \diamond V_1 = \phi_1$	$\phi_2 \diamond V_1 = -\phi_2$	$\phi_3 \diamond V_1 = -\psi$
$\phi_1 \diamond V_2 = -\phi_1$	$\phi_2 \diamond V_2 = \phi_2$	$\phi_3 \diamond V_2 = \psi$
$\phi_1 \diamond V_3 = \psi + \frac{1}{2}(\phi_1 - \phi_2)$	$\phi_2 \diamond V_3 = -\psi + \phi_3$	$\phi_3 \diamond V_3 = \frac{1}{2}(\phi_1 + \phi_2)$
$\phi_1 \diamond K = \psi + \frac{1}{2}(\phi_1 - \phi_2)$	$\phi_2 \diamond K = \psi - \phi_3$	$\phi_3 \diamond K = \frac{1}{2}(\phi_2 - \phi_1)$
$\phi_1 \diamond \psi = 7V_3 + 7K + 6\psi + 6\phi_3$	$\phi_2 \diamond \psi = -7V_3 + 7K + 6\psi - 6\phi_3$	$\phi_3 \diamond \psi = -7V_1 - 3(\phi_1 - \phi_2)$
$\phi_1 \diamond \phi_1 = 14 + 14V_1 + 12\phi_1$	$\phi_2 \diamond \phi_1 = 0$	$\phi_3 \diamond \phi_1 = 7V_3 - 7K - 6\psi + 6\phi_3$
$\phi_1 \diamond \phi_2 = 0$	$\phi_2 \diamond \phi_2 = 14 - 14V_1 + 12\phi_2$	$\phi_3 \diamond \phi_2 = -7V_3 - 7K - 6\psi$ − [illegible] ϕ_3
$\phi_1 \diamond \phi_3 = 7V_3 + 7K + 6\psi + 6V_3$	$\phi_2 \diamond \phi_3 = 7V_3 - 7K - 6\psi + 6\phi_3$	$\phi_3 \diamond \phi_3 = 7 + 3(\phi_1 + \phi_2)$

Other relations (non-independent constraits) involving the dependent forms K and ϕ_3

$$\| K \|^2 = 1, \quad \| \phi_3 \|^2 = 7, \qquad \langle \phi_1, \phi_3 \rangle = \langle \phi_2, \phi_3 \rangle = 0$$

$$K \lrcorner \phi_1 = 0 \ , \quad K \lrcorner \phi_2 = 0 \ , \qquad K \lrcorner \phi_3 = 0 \ , \quad K \lrcorner \psi = 0 \ ,$$

$$*(\phi_2 \Delta_1 \phi_3) = -6\psi \ , \qquad *(\phi_1 \Delta_1 \phi_3) = 6\psi$$

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$$\psi \Delta_2 \phi_3 = 0 \ ,$$

$$\psi \lrcorner \phi_3 = -7V_1 \ ,$$

$$\phi_3 \Delta_2 \phi_3 = \psi \Delta_1 \psi$$

$$\psi \Delta_1 \psi = \phi_3 \Delta_2 \phi_3 ,$$

$$\phi_1 \Delta_3 \phi_3 = 7V_1 \wedge V_3 \ , \quad \phi_2 \Delta_3 \phi_3 = -7V_1 \wedge V_3 \ , \quad \phi_3 \Delta_3 \phi_3 = 0 \ .$$

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$$\psi \Delta_2 \phi_3 = 0 \ ,$$

$$\psi \lrcorner \phi_3 = -7V_1 \ ,$$

$$\phi_3 \Delta_2 \phi_3 = \psi \Delta_1 \psi$$

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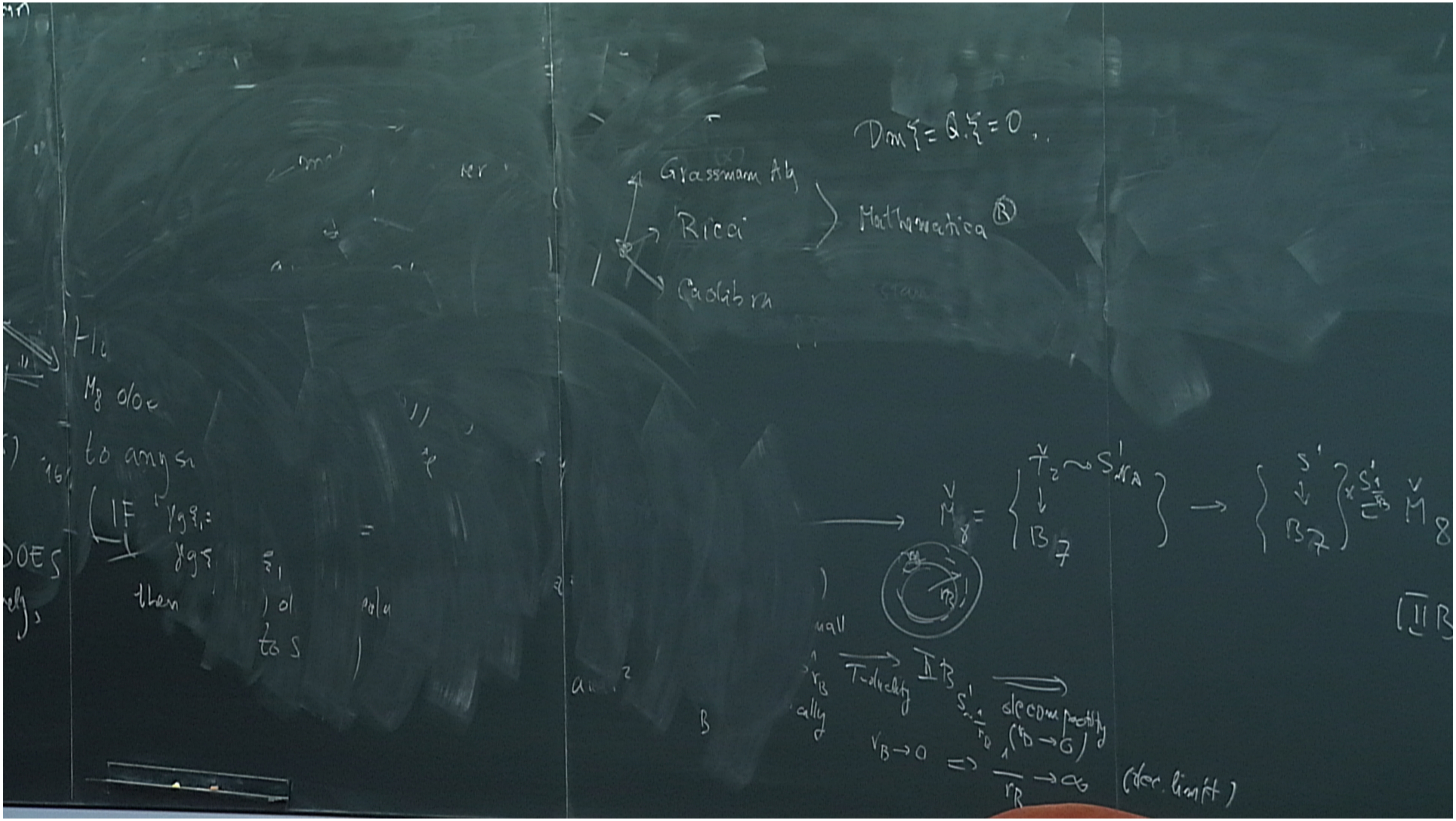
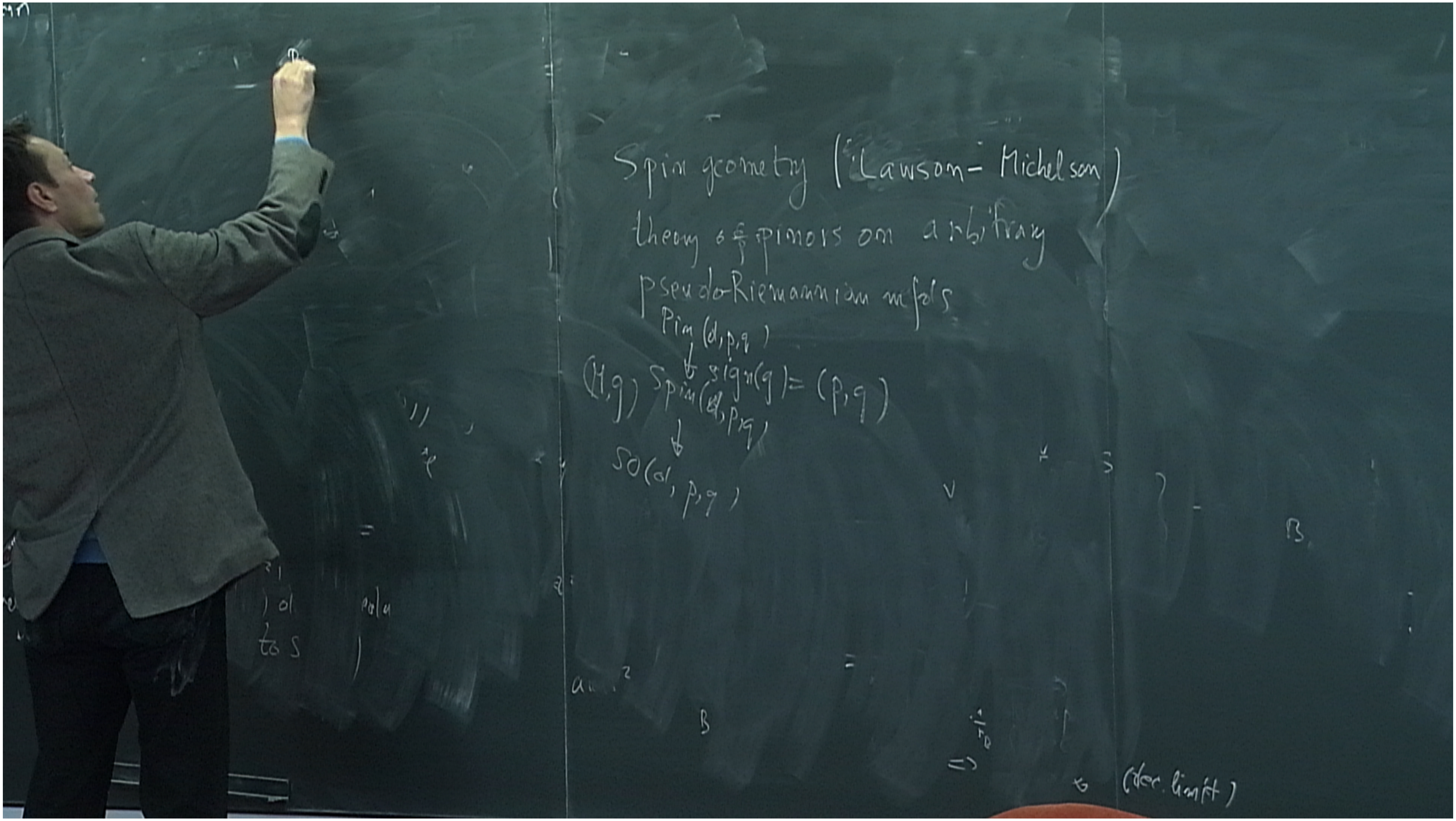
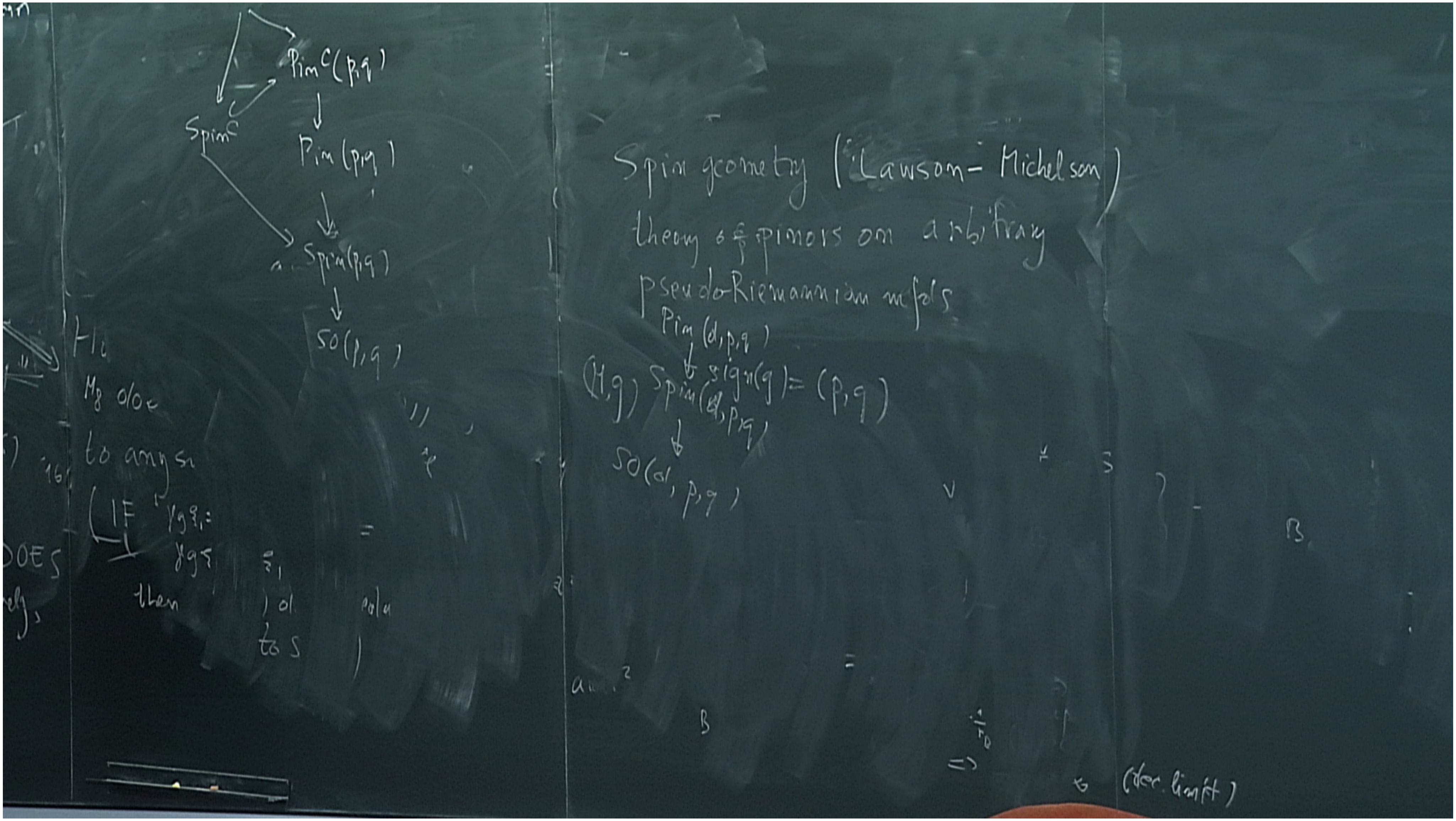
The differential constraints obtained, separated on ranks:

$$d\check{E}_{12} = e^m \wedge \nabla_m \check{E}_{12} = -e^m \wedge [\check{A}_m, \check{E}_{12}]_{-,\diamond}$$

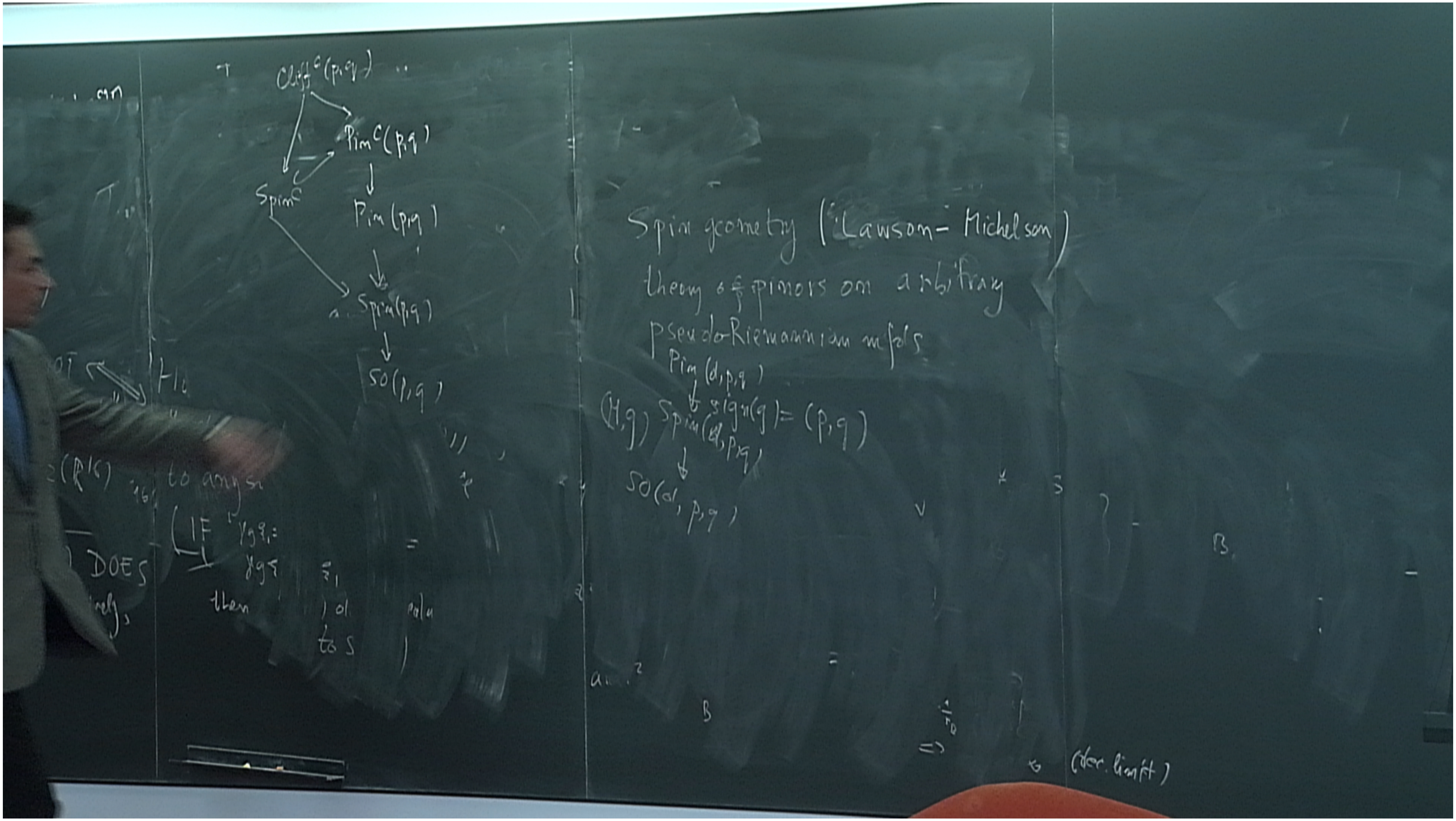
$$\begin{cases} dV_3 = \frac{1}{2}\phi_3 \mathbf{\Delta}_3 F - (f \wedge \theta) \lrcorner \phi_3 \\ dK = (f \wedge \theta)\mathbf{\Delta}_1 \psi + \psi \mathbf{\Delta}_2 F \\ d\psi = \frac{3}{2} F \mathbf{\Delta}_1 K + \frac{1}{2} F \mathbf{\Delta}_3 (*\psi) + 2*(f \wedge \theta \wedge \psi) - f \wedge \theta \wedge K \\ d\phi_3 = -2F \wedge V_3 + \frac{1}{2} e^m \wedge *(((e_m \lrcorner F)\mathbf{\Delta}_1 \phi_3) + \frac{1}{2} e^m \wedge *(((e_m \wedge f \wedge \theta)\mathbf{\Delta}_1 \phi_3) \end{cases}$$

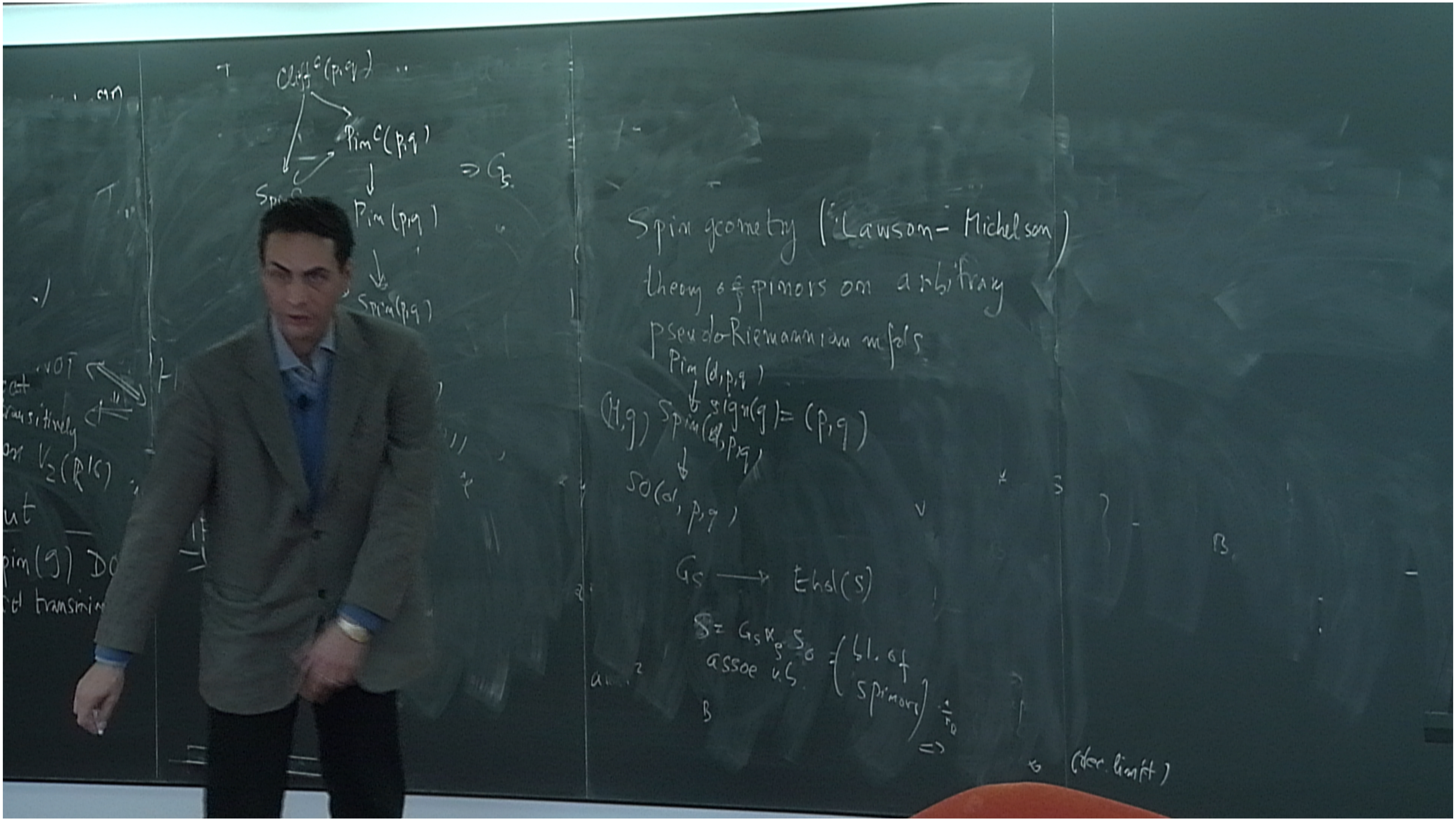

Grassmann Alg
Ricci
Cadabra
Mathematica®
T-duality
IIB
decompactify
(dec. limit)

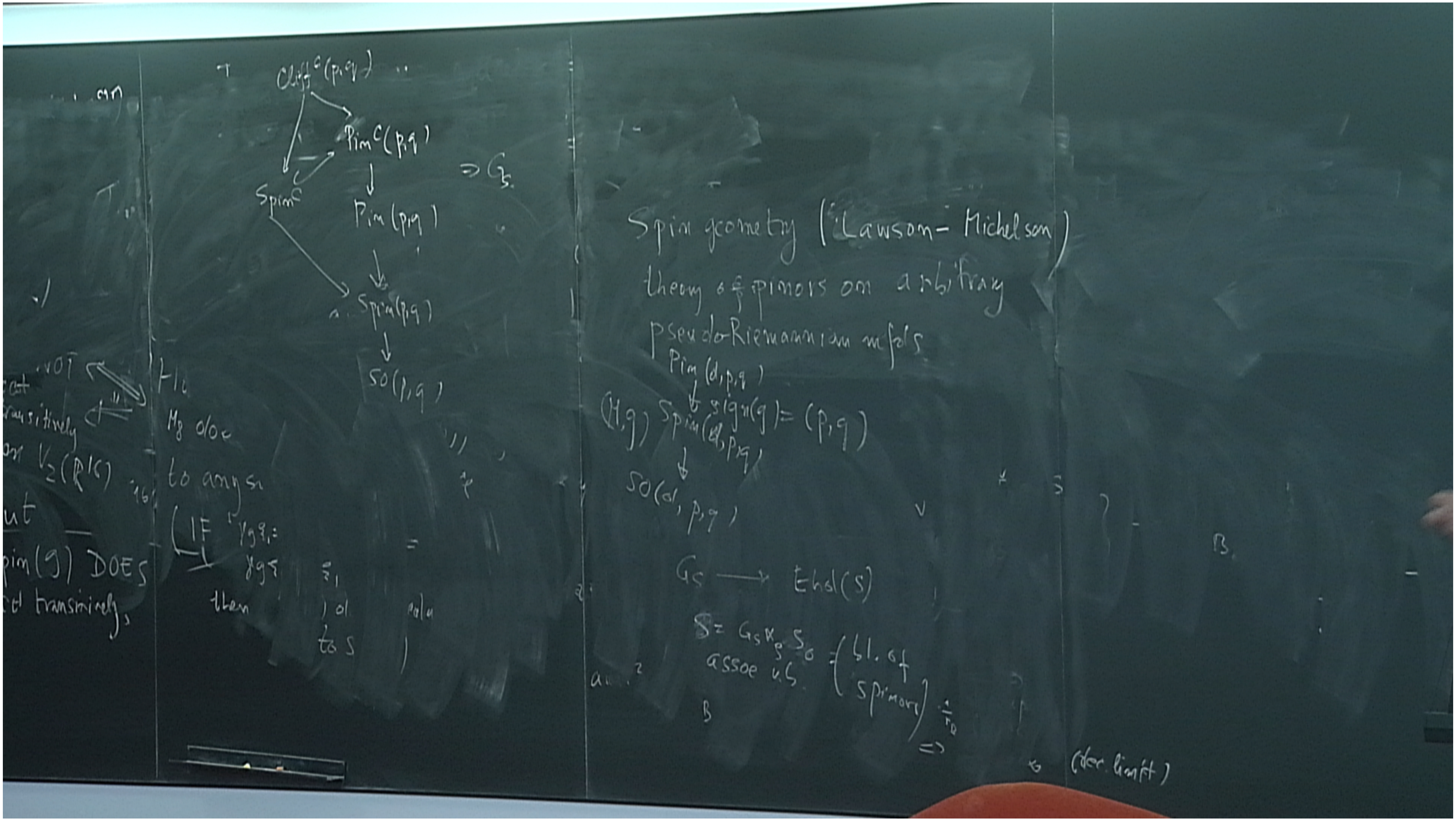

Spin geometry (Lawson-Michelson)
theory of spinors on arbitrary
pseudo-Riemannian mfds
Pin(d,p,q)
(M,g) Spin(d,p,q) sign(g) = (p,q)
SO(d,p,q)
(dec. limit)

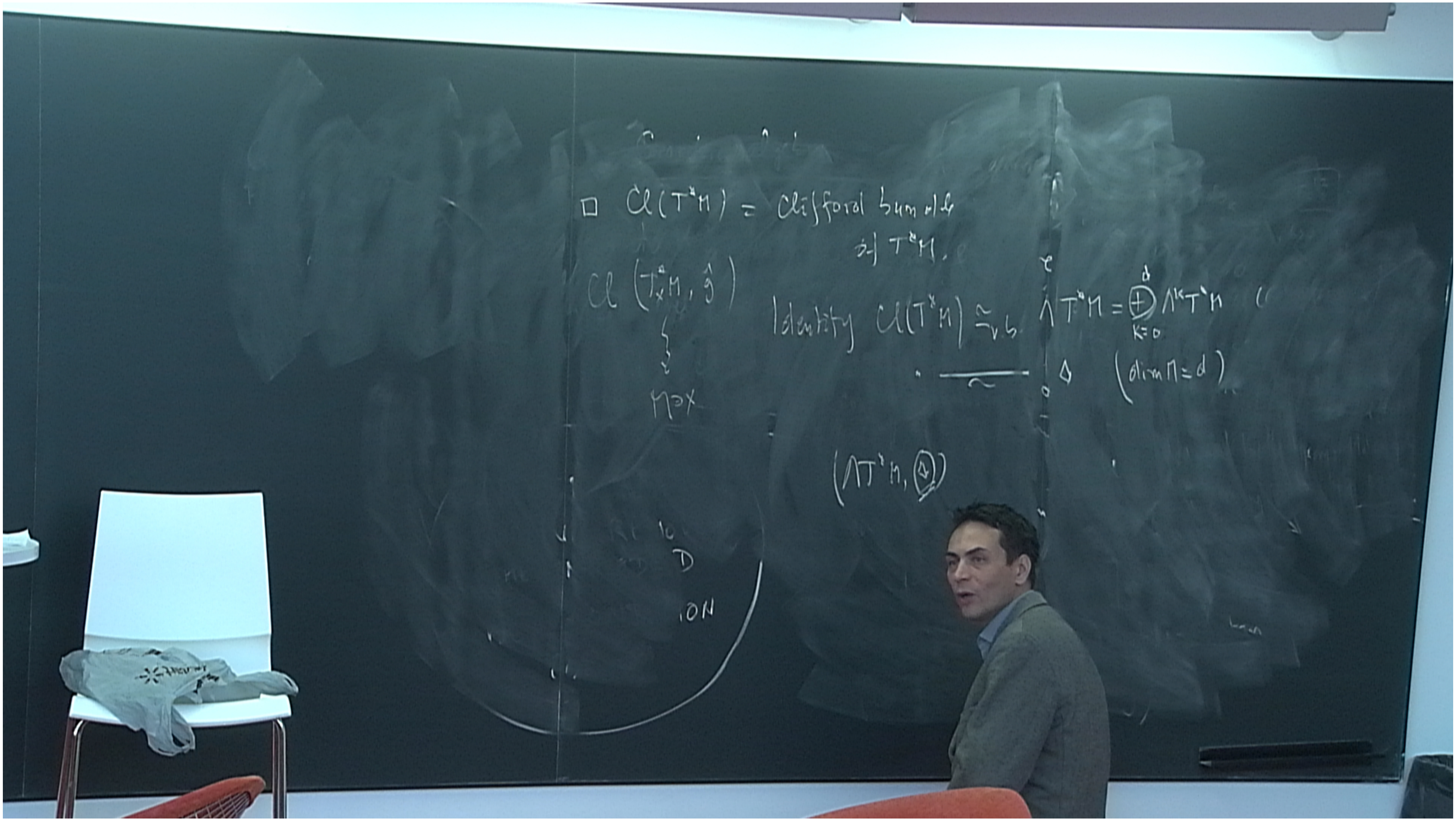


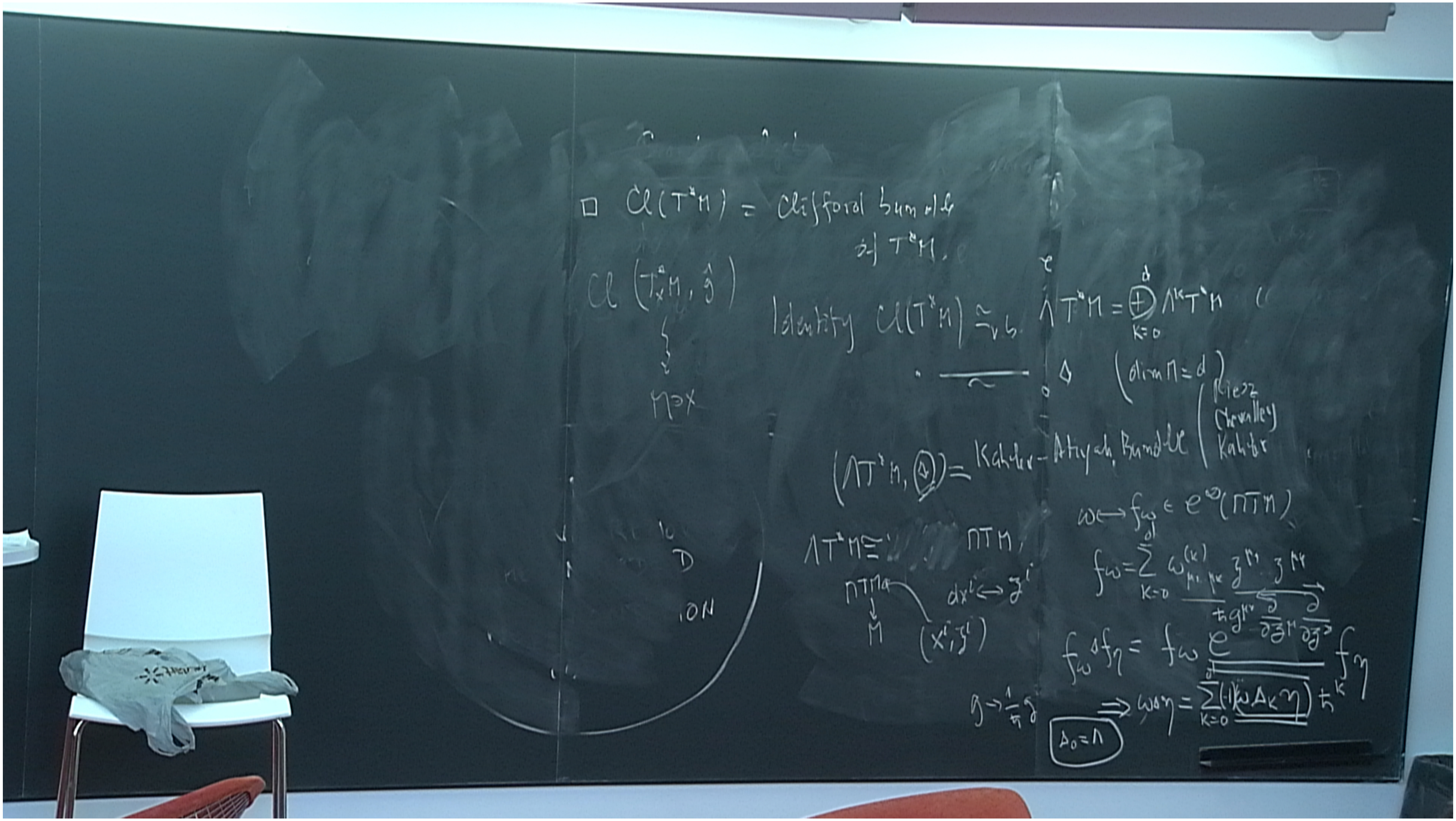
$Pin^c(p,q)$
$Spin^c$
$Pin(p,q)$
$Spin(p,q)$
$SO(p,q)$
Spin geometry (Lawson–Michelson)
theory of spinors on arbitrary
pseudoRiemannian mfds
$Pin(d,p,q)$
(M,g) $Spin(d,p,q)$ $sign(g) = (p,q)$
$SO(d,p,q)$
to any
then
(dec. limit)

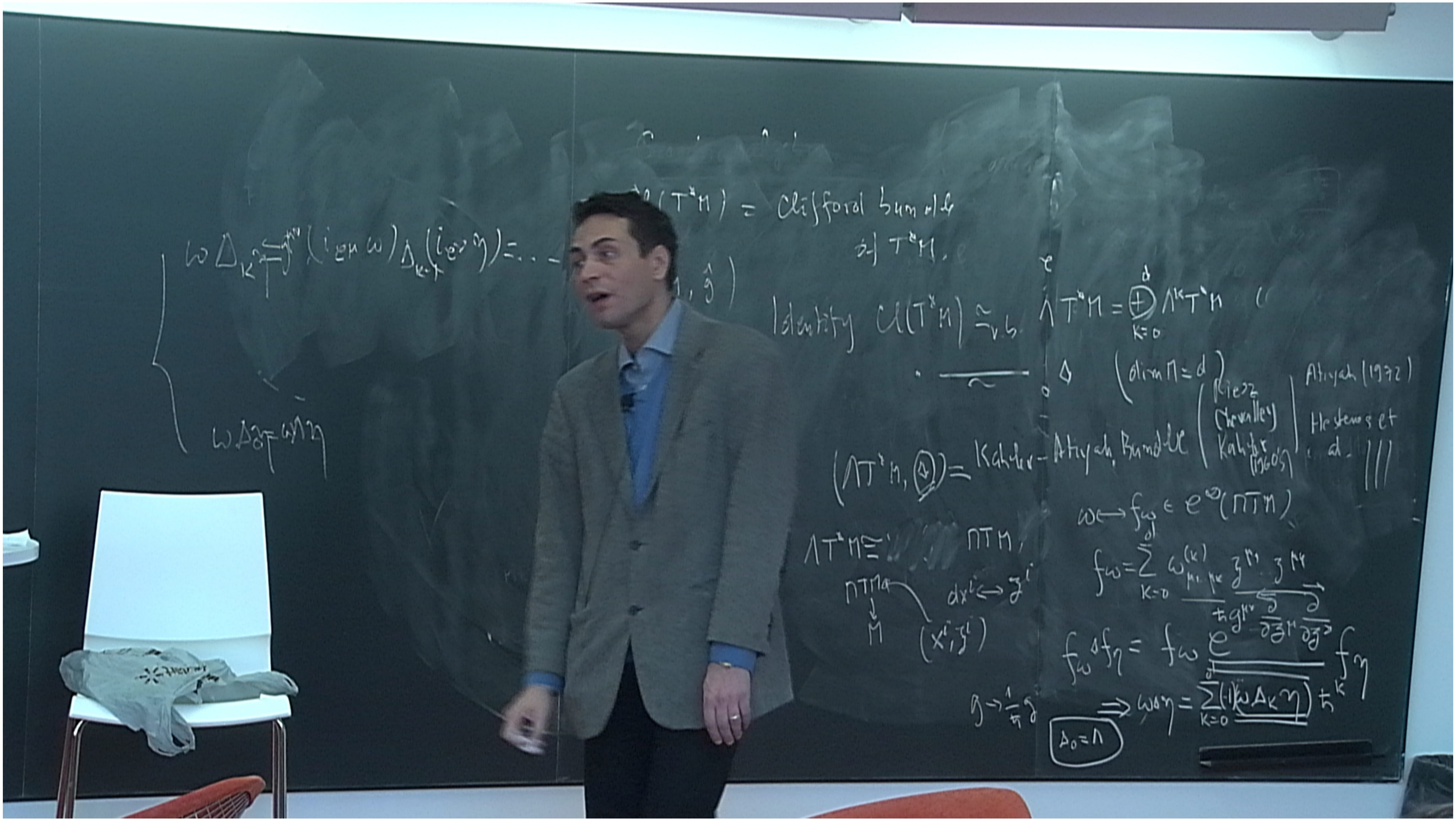

Pin^c(p,q)
Spin^c
Pin(p,q)
Spin(p,q)
SO(p,q)
Spin geometry (Lawson–Michelson)
theory of spinors on arbitrary
pseudoRiemannian mfd's
Pin(d,p,q)
sign(g) = (p,q)
Spin(d,p,q)
SO(d,p,q)
DOES
then
(der. limit)

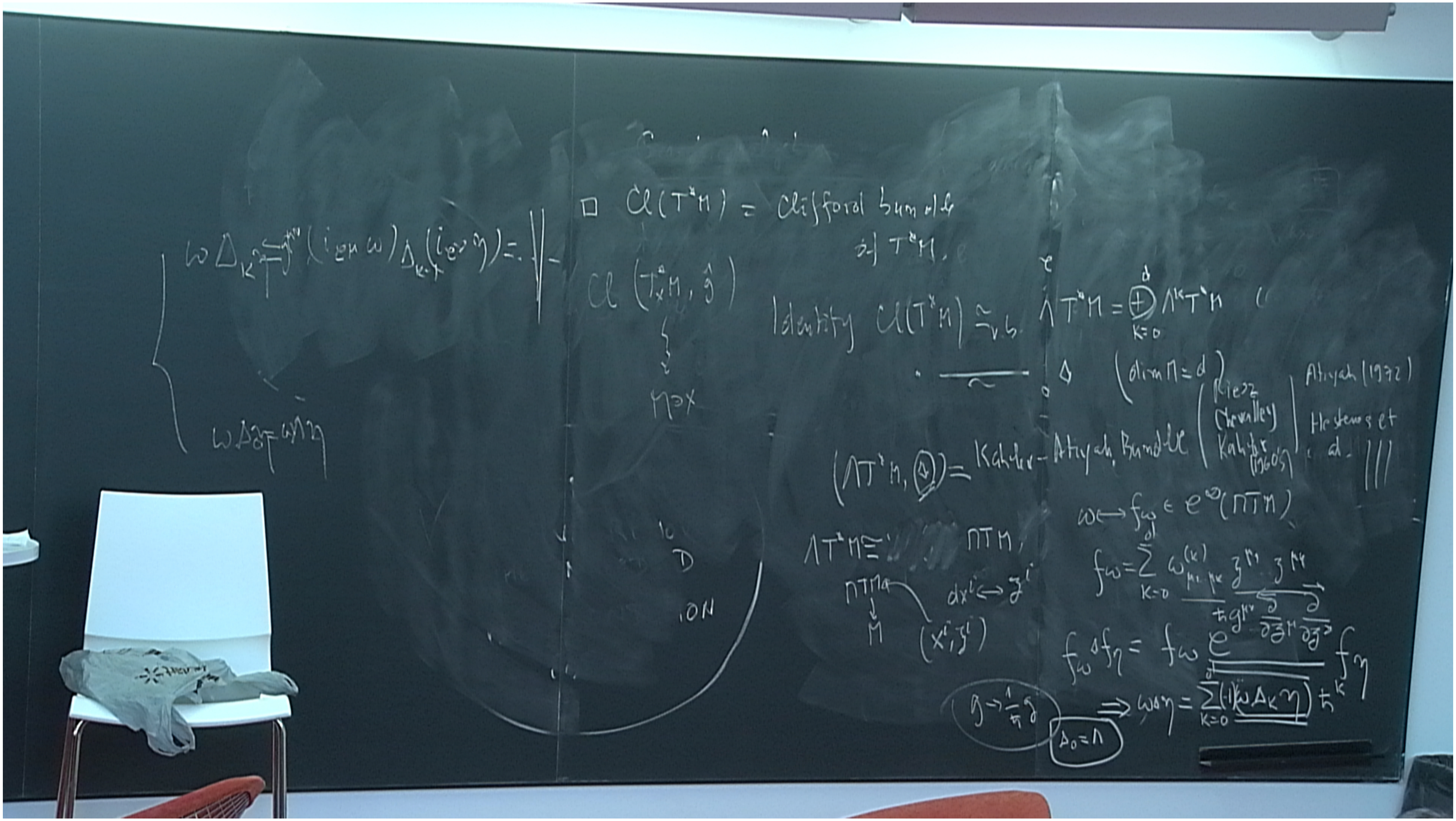

Spin geometry (Lawson–Michelson)
theory of spinors on arbitrary
pseudoRiemannian mfds
Pin(d,p,q)
sign(g) = (p,q)
Spin(d,p,q)
SO(d,p,q)
Pim(p,q)
Spin(p,q)
End(S)

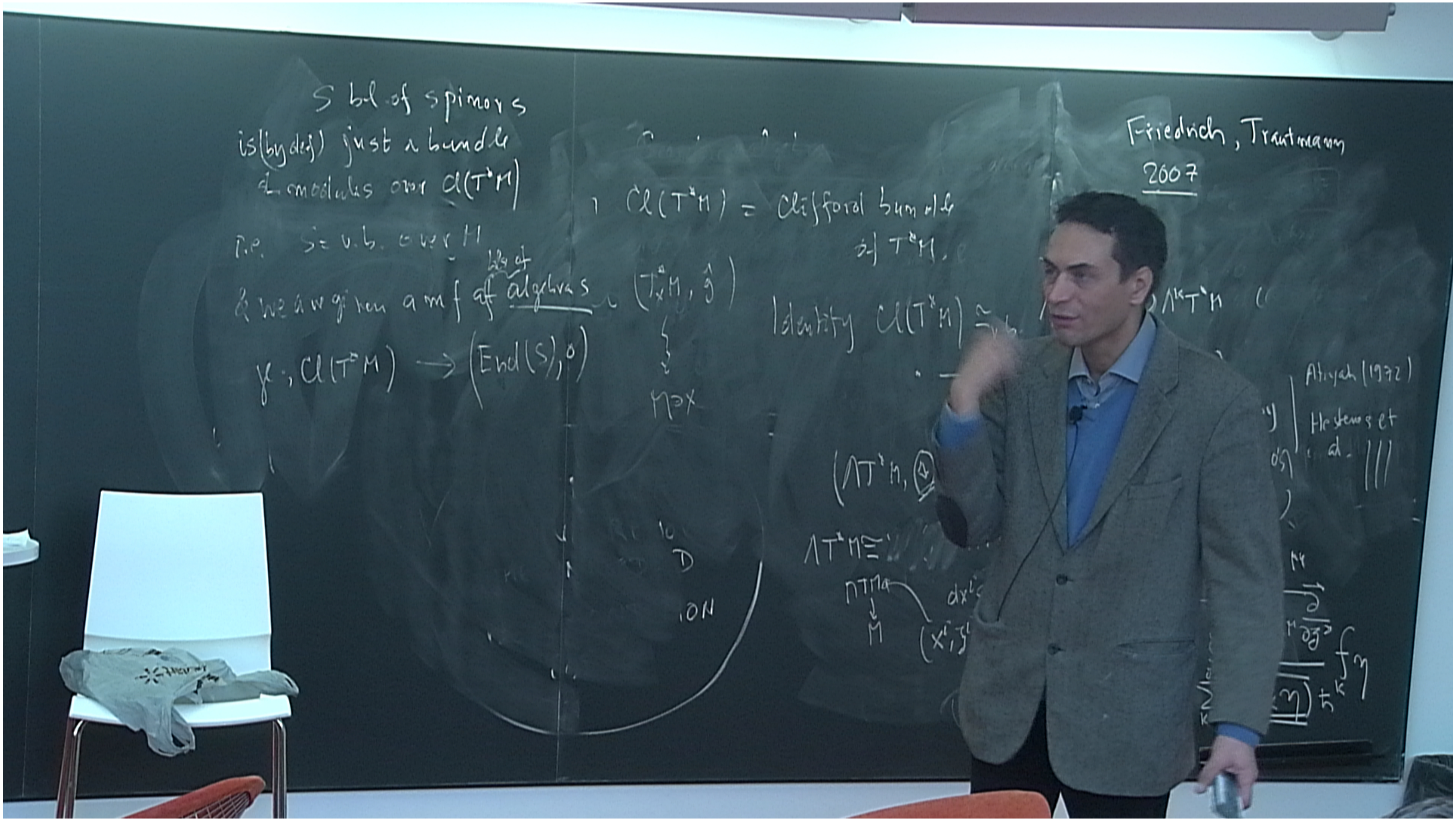

Spin geometry (Lawson-Michelson)
theory of spinors on arbitrary
pseudo-Riemannian mfds
Pin(d,p,q)
Spin(d,p,q)
SO(d,p,q)
sign(g) = (p,q)
Pin^c(p,q)
Pin(p,q)
Spin(p,q)
SO(p,q)
Spin^c
Gs → End(S)
assoc v.S.
spinors
pim(g) DOES
then

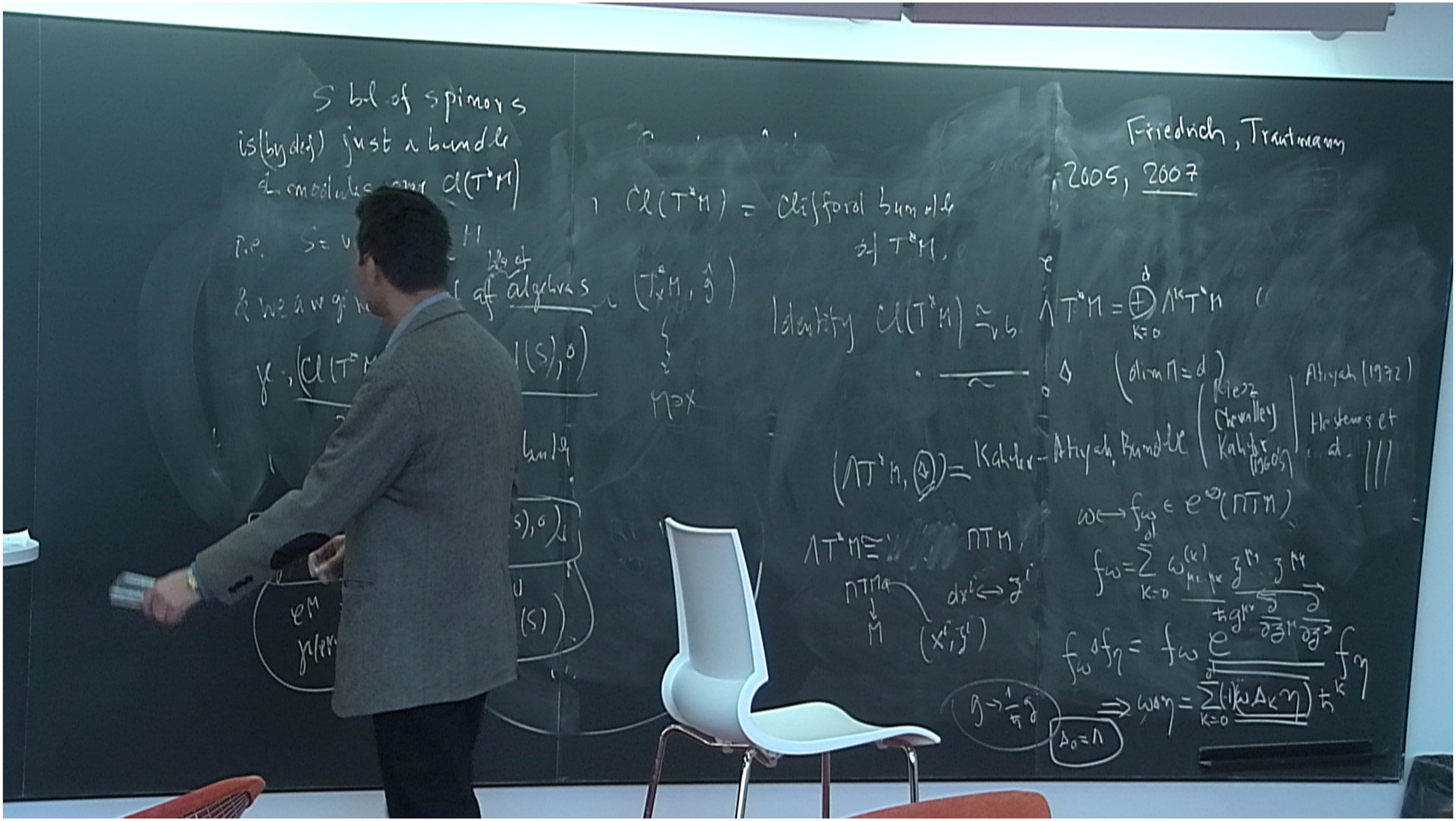

Cl(T*M) = Clifford bundle
of T*M.
Cl(T*ₓM, ĝ)
M∋x
Identify Cl(T*M) ≃v.s. ΛT*M = ⊕ Λ^k T*M
k=0
(dim M = d)
(ΛT*M, ⊕)

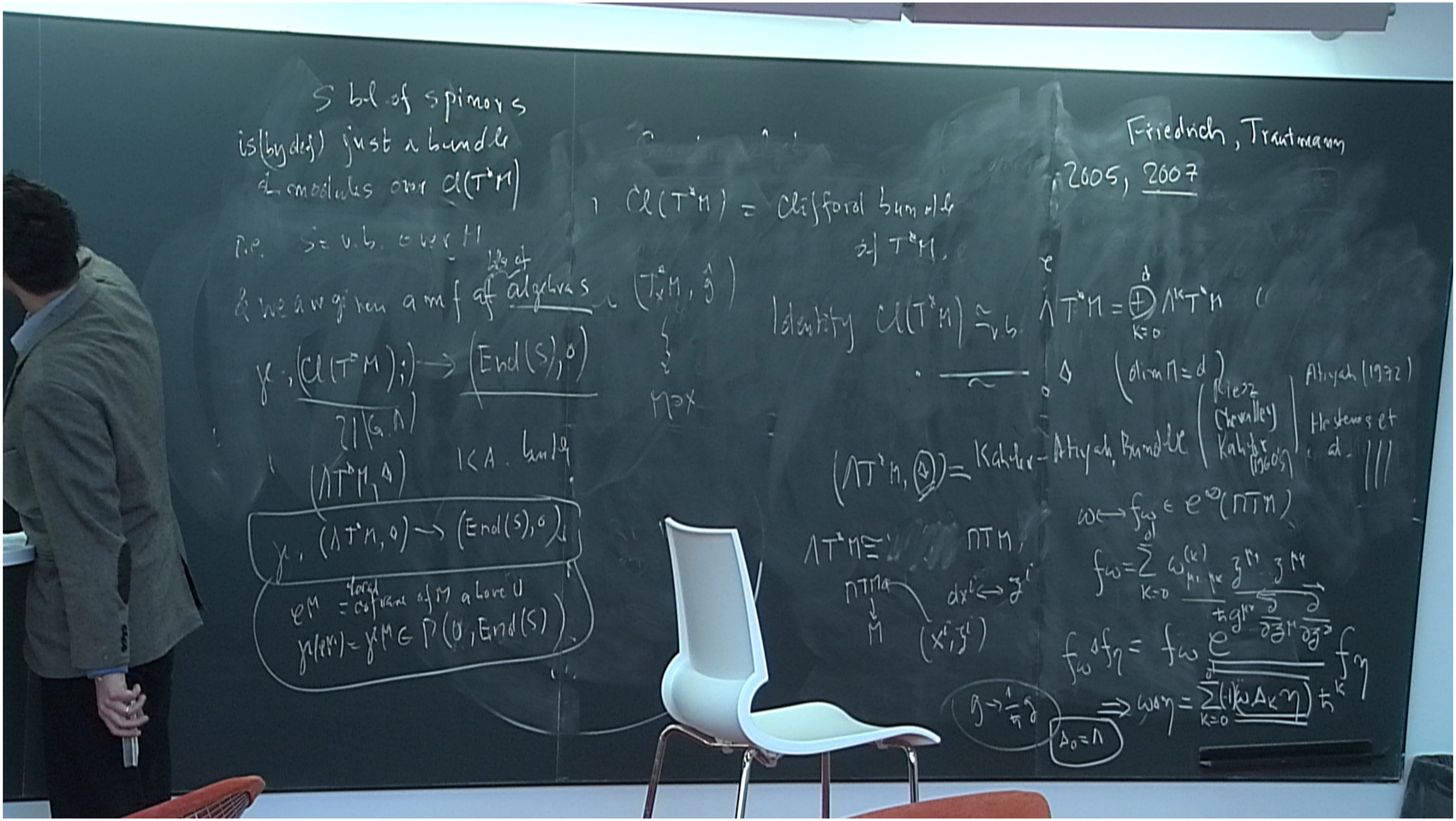

Cl(T*M) = Clifford bundle of T*M.
Cl(T*_x M, ĝ)
Identity Cl(T*M) ≃_v.s. ∧T*M = ⊕_{k=0}^{d} ∧^k T*M
(dim M = d)
Riesz
Chevalley
Kahler
(∧T*M, ◇) = Kahler-Atiyah Bundle
ω ⟷ f_ω ∈ e^∞(ΠTM)
∧T*M ≅ ΠTM
ΠTM → M
dx^i ⟷ ζ^i
(x^i, ζ^i)
f_ω = Σ_{k=0}^{d} ω^{(k)}_{μ1...μk} ζ^{μ1} ... ζ^{μk}
f_ω ◇ f_η = f_ω e^{ħ g^{μν} ∂/∂ζ^μ ∂/∂ζ^ν} f_η
g → (1/ħ) g
⟹ ω◇η = Σ_{k=0} (-1)^k (ω Δ_k η) ħ^k
Δ_0 = ∧

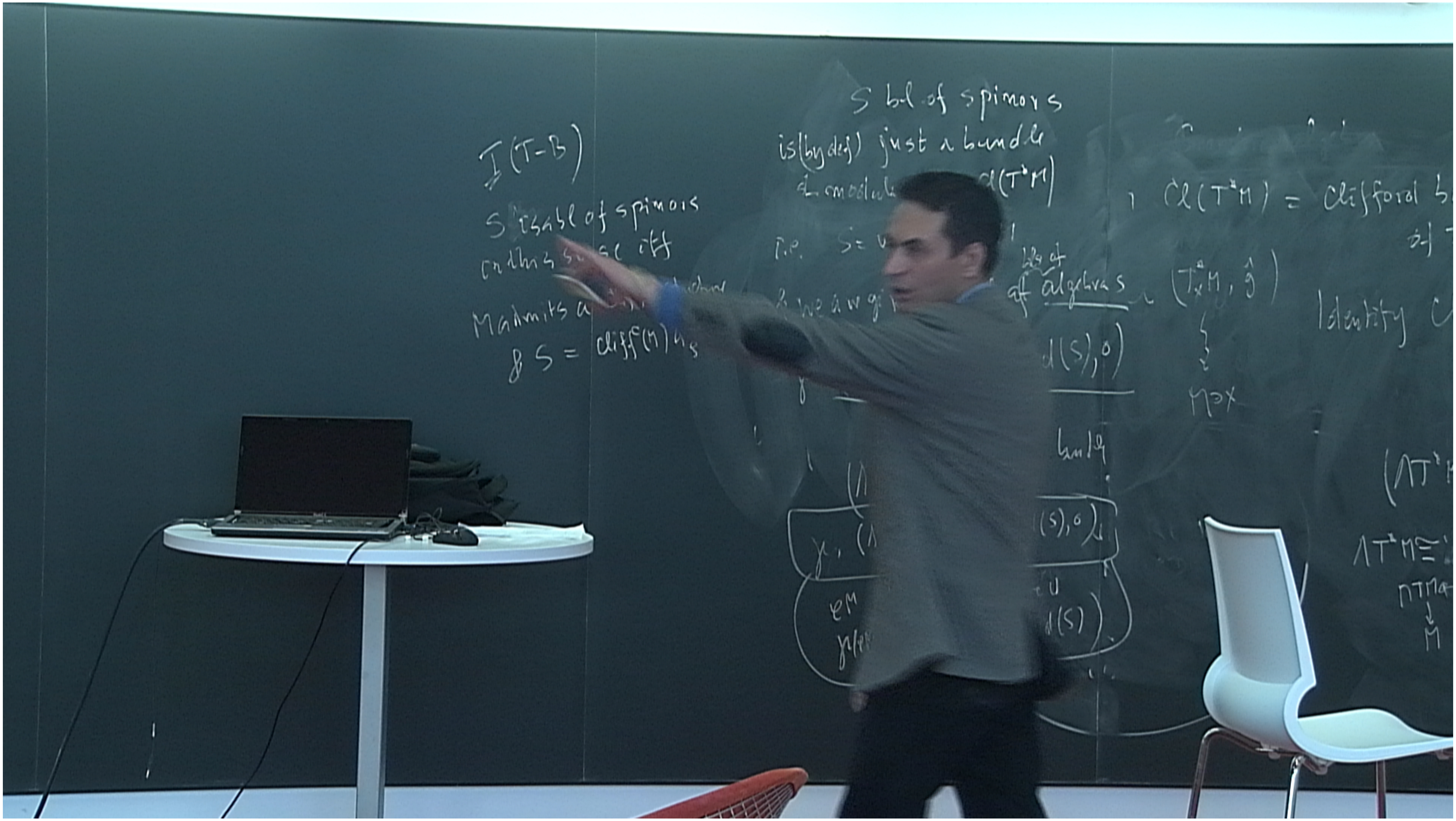

= Clifford bundle
Identity
Kahler-Atiyah Bundle
Atiyah (1972)
$\Delta_0 = \Lambda$

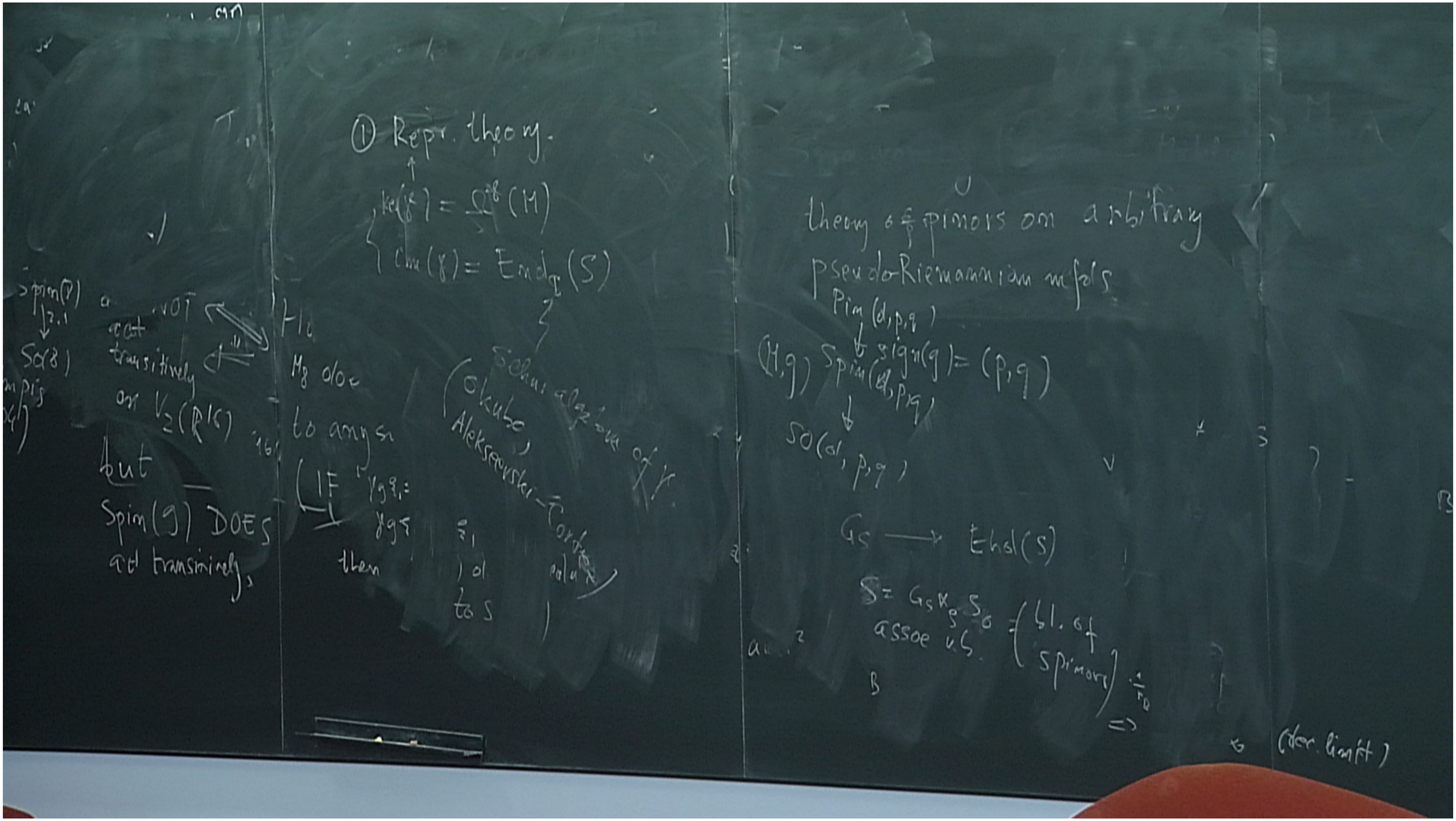

Cl(T*M) = Clifford bundle of T*M
Identity Cl(T*M) ≅ ΛT*M
ΛT*M = ⊕ Λ^k T*M
(dim M = d)
Kähler–Atiyah Bundle
Riesz
Chevalley
Kähler
Atiyah (1972)
Hestenes et al.
ΠTM
dx^i ⟷ ζ^i
ω ⟷ f_ω ∈ e^∞(ΠTM)
ω Δ_0 = Λ

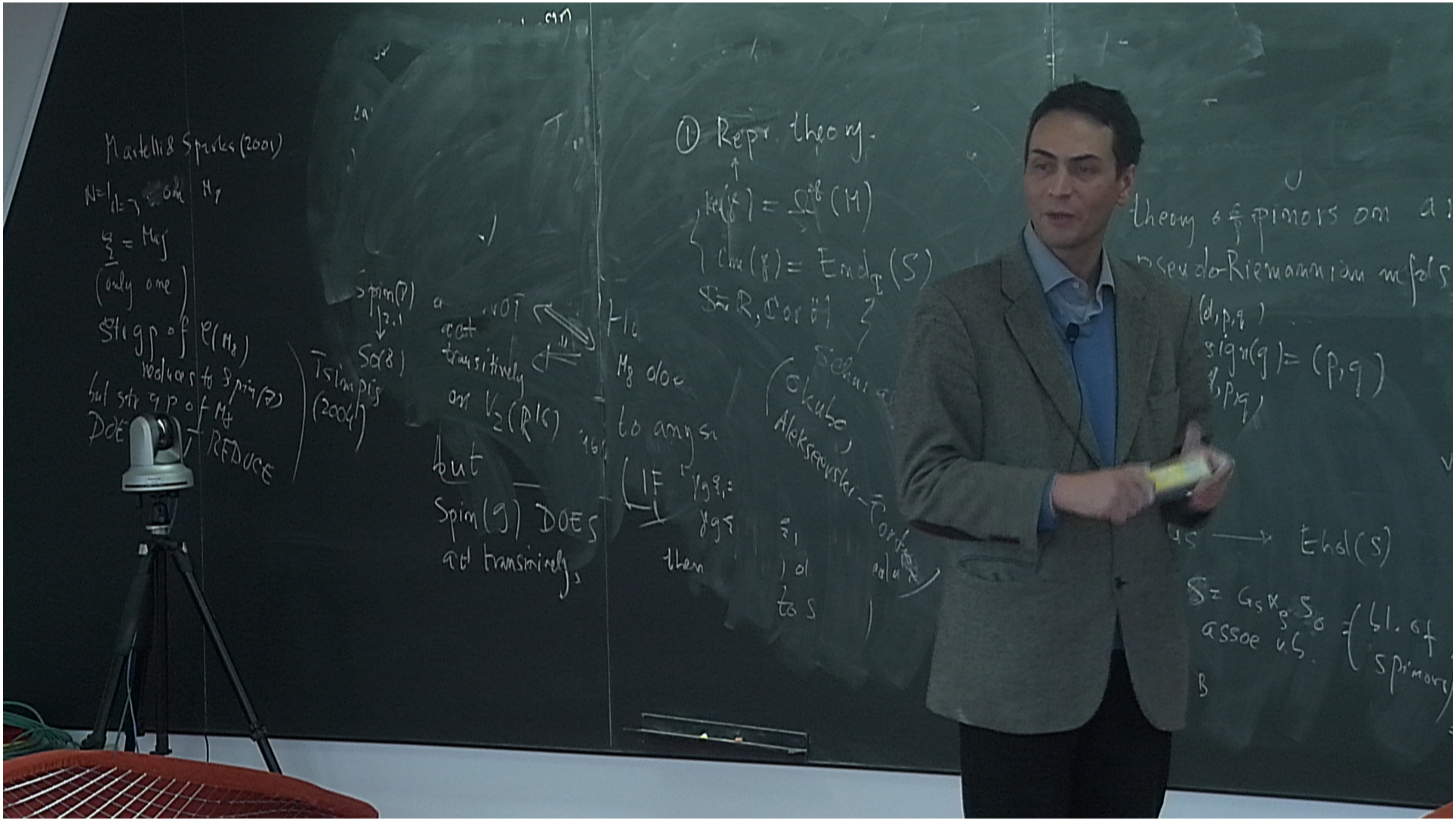

S bd of spinors
is (by def) just a bundle
of modules over Cl(T*M)
Cl(T*M) = Clifford bundle
of T*M
Identify
Friedrich, Trautmann
2007
Atiyah (1972)


S bd of spinors
is (by def) just a bundle
of algebras
Clifford bundle
Identity
Friedrich, Trautmann
2005, 2007
Kähler-Atiyah Bundle
Riesz
Chevalley
Kähler
Atiyah (1972)
Hestenes et al.


S bdl of spinors
is (by def) just a bundle
of modules over Cl(T*M)
i.e. S = v.b. over M
& we are given a mf of algebras
Cl(T*M) = Clifford bundle
of T*M.
Identity
Friedrich, Trautmann
2005, 2007
Kähler-Atiyah Bundle
Riesz
Chevalley
Kähler
1960's
Atiyah (1972)
Hestenes et al.
K.A. bundle
End(S)


S bdl of spinors
is (by def) just a bundle
Cl(T*M) = Clifford b
I(T-B)
S is a bdl of spinors
Identity


① Repr. theory.
but
Spin(9) DOES act transitively
to any s
then
to S
Okubo,
Aleksperster-Corde
theory of spinors on arbitrary
pseudoRiemannian mflds
sign(g) = (p,q)
(M,g)
Spin(d,p,q)
SO(d, p, q)
Gs ⟶ End(S)
assoc v.S.
spinors
(der. limit)


① Repr. theory
but
Spin(9) DOES
act transitively
theory of spinors on a
pseudo-Riemannian mflds
(d, p, q)
End(S)