Title: 3 point functions in the AdS4/CFT3 correspondence

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Abstract: I will present recent developments in the computation of three point functions in the AdS4/CFT3 correspondence. More specifically I will consider two different computations for three point functions of operators belonging to the SU(2)XSU(2) sector of ABJM. I will discuss first the generalization of the determinant representation, found by Foda for the three-point functions of the SU(2) sector of N = 4 SYM, to the ABJM theory and

secondly semiclassical computations in the case where two operators are heavy and one is light and BPS, comparing the results obtained in the gauge theory side using a coherent state description of the heavy operators with its string theory counterpart calculated holographically.

Niels Bohr Institutet

Agnese Bissi

3-point functions in the AdS_4/CFT_3 correspondence

Niels Bohr Institute
Niels Bohr International Academy

based on

hep-th:1211.1359

with C.Kristjansen, A.Martirosyan and M.Orselli

Perimeter Institute – November 20, 2012

Outline

Motivation

- Building blocks in conformal field theories are 2-point and 3-point functions of local gauge invariant operators.

$$\langle \mathcal{O}_i(x)\,\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

The spectral problem is solved with integrability:
Find $\Delta = \Delta(\lambda, N)$ by diagonalizing the dilatation operator ✔

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\rangle = \frac{c_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_i+\Delta_k-\Delta_j}}$$

- In principle using the OPE all the higher point correlation functions are known:

$$\mathcal{O}_\alpha(x_1)\mathcal{O}_\beta(x_2) \sim \sum_\gamma \frac{c_{\alpha\beta\gamma}}{|\,x_1-x_2\,|^{\Delta_\alpha+\Delta_\beta-\Delta_\gamma}}\mathcal{O}_\gamma(x_2)$$

One slide review of the AdS_5/CFT_4 correspondence

AdS

- Type IIB string theory on $AdS_5 \times S^5$ with metric in Poincaré coordinates given by $ds^2 = R^2 \frac{dz^2 + dx_\mu^2}{z^2} + R^2 d\Omega_5^2$
- $g_s = \frac{4\pi\lambda}{N}$, $\frac{R^2}{\alpha'} = \sqrt{\lambda}$
- Single string states of energy Δ

CFT

- $\mathcal{N} = 4$ SYM in 4 dimension with gauge group $SU(N)$ containing 1 A_μ, 4 λ_i and 6 ϕ_i transforming in the adjoint rep
- $\lambda = g_{YM}^2 N$
- Single trace operators of conformal dimension Δ

Holographic prescription

- Correlation function using AdS/CFT:

$$Z_{bulk}\left[\phi\left(\vec{x},z\right)|_{z=0}=\phi_0\left(\vec{x}\right)\right]=\langle e^{\int d^4x\,\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{\text{field theory}}$$

  $\phi_0\left(\vec{x}\right)$ is an arbitrary function specifying the boundary values of the bulk field ϕ.

- Taking derivatives with respect to ϕ_0 and setting it to zero we obtain the correlation functions of the operator.
- Changes in the boundary conditions of AdS correspond to changes in the Lagrangian of the field theory. Infinitesimal changes in the boundary condition correspond to the insertion of an operator.

Short trip into integrability I

- Final goal: Find anomalous dimensions of the operators
- Group the 6 scalars into 3 complex scalars: $X = \phi_1 + i\phi_2$, $Y = \phi_3 + i\phi_4$, $Z = \phi_5 + i\phi_6$
- Restrict to the SU(2) sector $X = \phi_1 + i\phi_2$, $Z = \phi_5 + i\phi_6$
- Gauge invariant operators take the form $\mathcal{O} = \mathrm{Tr}\,(ZXXX\ldots Z)$
- Correspondence between operators and a configurations of an SU(2) spin chain
- Express the dilatation operator in terms of operators acting on a spin chain: $D = \mathbb{I}L + \frac{\lambda}{8\pi^2}\hat{H}$, $\hat{H} = \sum_{l=1}^{L}(\mathbb{I}_{l,l+1} - \mathbb{P}_{l,l+1})$
- The operators are the eigenvectors and the anomalous dimension is the eigenvalue of $\hat{H}$.

[Minahan and Zarembo, 2002]

Short trip into integrability II

Next step: diagonalize the hamiltonian $\hat{H}$

There are two (or more) well established different methods to solve this problem: coordinate and algebraic Bethe Ansatz.

[Faddeev and Takhtajan, 1988]

Very roughly the idea is:

- recast the problem as $\sum_{l=1}^{L} \mathbb{P}_{l,l+1} = -i \frac{d}{du} \log T(u)|_{u=0}$ where u is the spectral parameter and T is the transfer matrix
- the problem now is to find eigenvectors and eigenfunctions of $T(u)$
- using Yang Baxter algebra for $T(u)$ it is possible to extract creation operators which, acting on a reference state (a state with all spin up or all down), generates all the possible states
- it turns out that in order for these states to be eigenstates of $T(u)$ the spectral parameter u should satisfy Bethe equations

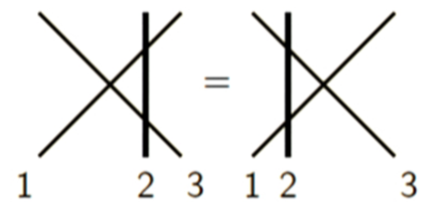
Short trip into integrability III

Some key points in this procedure:

- Auxiliary space: additional vector space, isomorphic to the physical vector space. Note that $T(u)$ acts on the physical space, because it is defined as a trace on the auxiliary space of the monodromy matrix. A nice way of thinking to the auxiliary space is as a probe spin space.
- The R-matrix: it is the building block. It acts on the tensor product of the physical space and of the auxiliary space and it obeys unitarity, crossing symmetry and Yang-Baxter equation.
- $[T(u_1), T(u_2)] = 0$ $\to$ $T(u)$ is the generating function of all conserved charges.

Short trip into integrability IV

- The R matrix is $R(u) = u\mathbb{I} + i\mathbb{P}$ and $R_{aj}(u) : \mathcal{A} \times \mathcal{P} \to \mathcal{A} \times \mathcal{P}$ where $\mathcal{A}$ is the auxiliary space and $\mathcal{P}$ the physical space.
- It satisfies Yang Baxter equations



- Monodromy matrix

$$M_a(u) = R_{a1}(u) \dots R_{aL}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

- Transfer matrix $T(u) = \mathrm{Tr}_a M_a(u)$

Operators

- For our purposes we can classify single trace operators of $\mathcal{N} = 4$ SYM in two groups:
  HEAVY: when the anomalous dimension scales as $\sqrt{\lambda}$ at strong coupling, they are dual to classical string states;
  LIGHT: when the conformal dimension scales as 1. These are BPS operators, so the scaling is the same also at weak coupling. They are dual to supergravity modes.

3 point functions in AdS_5/CFT_4

- It is known how to compute $\langle \mathtt{LLL} \rangle$ both at weak and strong coupling.
  [Kristjansen,Plefka, Semenoff and Staudacher, 2002] [Freedman, Mathur, Matusis and Rastelli, 1998]
  [Constable, Freedman, Headrick and Minwalla, 2002] [Arutyunov and Frolov, 2000]
  [Chu, Khoze and Travaglini, 2002]
  [Beisert,Kristjansen,Plefka, Semenoff and Staudacher, 2002]
  [Roiban and Volovich, 2004]
  [Okuyama and Tseng, 2004]
  [Alday, David, Gava and Narain, 2005]
- $\langle \mathtt{HHH} \rangle$ is more involved mainly because at weak coupling we need to Wick contract at least 1 out of 3 operators with 2 of them, the combinatorial problem becomes much more involved. At strong coupling, in general it is needed to solve string EOM with the topology of a sphere with 3 punctures (with some asymptotic properties).
  [Janik and Wereszczynski, 2011]
  [Kazama and Komatsu, 2011]
  [Buchbinder and Tseytlin, 2011]
- Are there simpler cases?
- Do we have more efficient ways to analyze the problem?

Semiclassical method: $\langle \text{HHL} \rangle$

[Zarembo, 2010], [Costa, Monteiro, Santos and Zoakos,2010]

$$\langle \mathcal{O}_I(x) \rangle_{\mathcal{W}} = \frac{\langle \mathcal{W}\, \mathcal{O}_I(x) \rangle}{\langle \mathcal{W} \rangle}$$

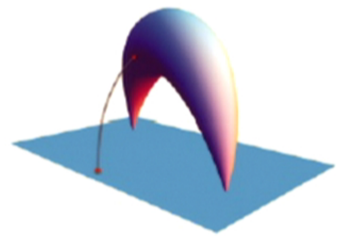


Figure from Zarembo's paper

- $\mathcal{W} = \bar{\mathcal{O}}_J(x_1)\mathcal{O}_k(x_2)$: non local operator dual to classical string
- $\mathcal{O}_I(x)$: local operator dual to a sugra mode

$$\langle \mathcal{O}_I(y) \rangle_{\mathcal{W}} = \lim_{\varepsilon \to 0} \frac{\pi}{\varepsilon^{\Delta_I}} \sqrt{\frac{2}{\Delta_I - 1}} \left\langle \phi_I(y, \varepsilon)\, \frac{1}{Z_{\rm str}} \int \mathcal{D}\mathbb{X}\; \mathrm{e}^{-S_{\rm str}[\mathbb{X}]} \right\rangle_{\rm bulk}$$

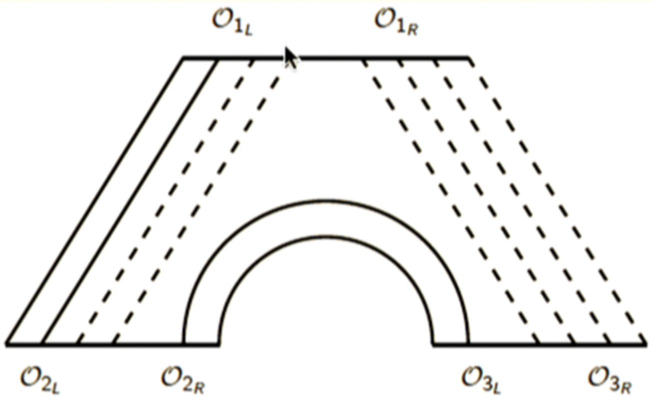
$$S_{\rm str} = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \sqrt{h} h^{\mathbf{ab}} \partial_{\mathbf{a}} \mathbb{X}^M \partial_{\mathbf{b}} \mathbb{X}^N G_{MN} + \ldots$$

$$G_{MN} = g_{MN} + \gamma_{MN}$$

γ_{MN} is the disturbance created by the local operator insertion

[Zarembo, 2010]

Three point functions as scalar products/1



1. Map the operators to closed spin chain states, $\mathcal{O}_i \to |\mathcal{O}_i\rangle$
2. Break the spin chains into 2 open subspin chains $|\mathcal{O}_i\rangle \to |\mathcal{O}_i\rangle_l$ and $|\mathcal{O}_i\rangle_r$
3. Write the initial spin chain as tensor product of the two open subspin chains $|\mathcal{O}_i\rangle = \sum_a |\mathcal{O}_{i_a}\rangle_l \otimes |\mathcal{O}_{i_a}\rangle_r$
4. In order to mimic the Wick contraction operation it is needed to flip one of the two subspin chains from a ket to a bra and then evaluate the scalar products of the appropriate states $C_{123} \sim \sum_{a,b,c} {}_r\langle\mathcal{O}_{3_c}|\mathcal{O}_{1_a}\rangle_l \, {}_r\langle\mathcal{O}_{1_a}|\mathcal{O}_{2_b}\rangle_l \, {}_r\langle\mathcal{O}_{2_b}|\mathcal{O}_{3_c}\rangle_l$

[Escobedo, Gromov, Sever and Vieira, 2011]

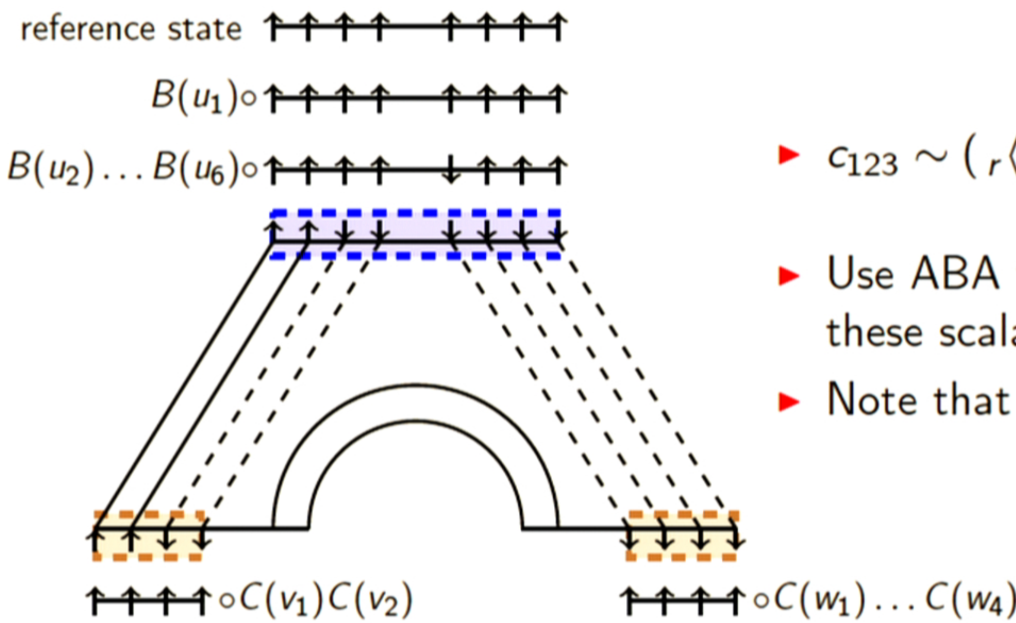
Three point functions as scalar products/2

Operator	Vacuum		Excitation	
$\mathcal{O}_1$	$L_1 - N_1$	Z	N_1	X
$\mathcal{O}_2$	$L_2 - N_2$	$\bar{Z}$	N_2	$\bar{X}$
$\mathcal{O}_3$	$L_3 - N_3$	Z	N_3	$\bar{X}$

- In order to have a well defined planar 3 point-function: $N_1 = N_2 + N_3$.
- The scalar product ${}_r\langle \mathcal{O}_2 | \mathcal{O}_3 \rangle_l$ is trivial and one sum disappears.
- It remains only one sum!

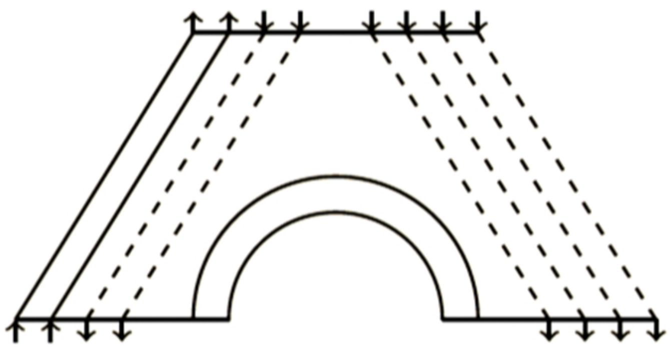
[Escobedo, Gromov, Sever and Vieira, 2011]

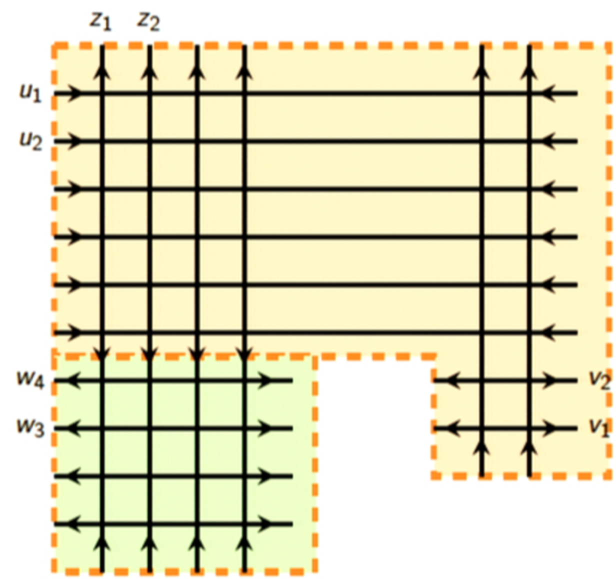
How to evaluate scalar products



- $c_{123} \sim \left({}_r\langle \mathcal{O}_3| \otimes {}_l\langle \mathcal{O}_2| \right) |\mathcal{O}_1\rangle$ [Foda, 2011]
- Use ABA techniques to express these scalar products
- Note that $\mathcal{O}_1$ is not cut

Six vertex vertex model





- The weight of the vertices is the entry of the R-matrix
- The orange part is the Slavnov scalar product and the green one is the partition function
- $C_{123} \sim Z_{N_3}(w_i) S[N_1, N_2](u_i, v_i)$

[Foda, 2011]
[Foda and Wheeler, 2012]

Extend the procedure to AdS_4/CFT_3

WHAT WE DID:

We calculate planar, tree level, non extremal three point functions of operators belonging to the SU(2)×SU(2) sector of ABJM:

- generalize the determinant representation to the case of ABJM using integrability and six vertex model techniques.
- semiclassical computation for 2 heavy operators and 1 light

VS
(in the Frolov-Tseytlin limit)

gauge theory computation in the coherent state approximation.

Six vertex model III

DOMAIN WALL PARTITION FUNCTION

$$Z_{2J}(\{u\}_{2J},\{z\}_{2J}) = \langle \downarrow_{z_{2J}} | \prod_{i=1}^{2J} \mathcal{B}(u_i,\{z_{2J}\}) | \uparrow_{z_{2J}} \rangle$$

SLAVNOV SCALAR PRODUCT

$$S[\{u\}_{N_1},\{v\}_{N_2},\{z\}_J] =$$

$$= \langle \downarrow_{z_{N_3,J}} | \prod_{i=1}^{N_2} \mathcal{C}(u_i,\{z\}_J) \prod_{j=1}^{N_1} \mathcal{B}(v_j,\{z\}_J) | \uparrow_{z_J} \rangle$$

where

$$\langle \downarrow_{z_{N_3,J}} | = \langle \downarrow_{z_1} | \otimes \cdots \otimes \langle \downarrow_{z_{N_3}} | \otimes \langle \uparrow_{z_{N_3+1}} | \otimes \cdots \otimes \langle \uparrow_{z_J} |,$$

GAUDIN NORM

$$\mathcal{N}(\{u\}) = S[\{u\}_N,\{u\}_N, i/2]$$

The AdS_4/CFT_3 correspondence [Aharony, Bergman, Jafferis and Maldacena, 2008]

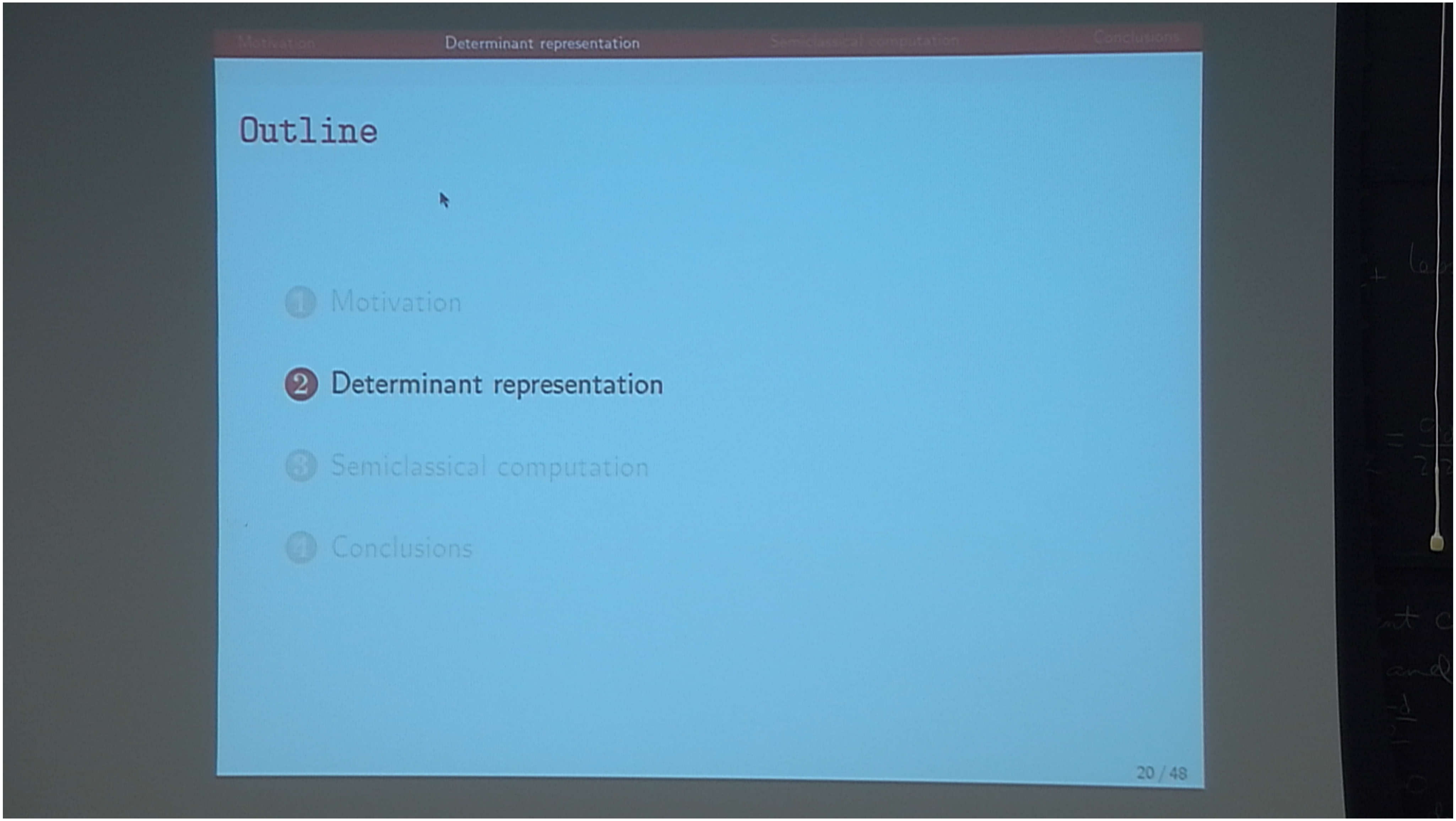
AdS

- Type IIA string theory on $AdS_4 \times \mathbb{C}P^3$ with metric $ds^2 = R^2 \left[\frac{1}{4} ds^2_{AdS_4} + ds^2_{\mathbb{C}P^3}\right]$
- $g_s = \left(\frac{2^5 \pi^2 N}{k^5}\right)^{1/4}, \frac{R^2}{l_s^2} = 4\pi\sqrt{2\lambda}$, $R = R_{\mathbb{C}P^3} = 2R_{AdS}$
- This is true ONLY when k, N are both large, for generic k the dual theory is M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$.

CFT

- $\mathcal{N} = 6$ superconformal Chern-Simons theory in 3 dimension with gauge group $U(N)_k \times U(N)_{-k}$ where k is the Chern-Simons level (ABJM theory)
- $\lambda = \frac{N}{k}$
- Bosonic field content: 2 complex scalars transforming in the $N \times \bar{N}$ and 2 complex scalars in $\bar{N} \times N$

Note another big difference wrt the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence that will be important later: the amount of supersymmetry. The former has 36 supercharges while the latter 24.


Motivation
Determinant representation
Semiclassical computation
Conclusions
Outline
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2 Determinant representation
3 Semiclassical computation
4 Conclusions
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Operators in ABJM [Aharony, Bergman, Jafferis and Maldacena, 2008]

- Gauge invariant scalar operators are

$$\mathcal{O} = C^{b_1 b_2 \cdots b_n}_{a_1 a_2 \cdots a_n} \mathrm{tr}(\mathcal{Z}^{a_1} \bar{\mathcal{Z}}_{b_1} \cdots \mathcal{Z}^{a_n} \bar{\mathcal{Z}}_{b_n})$$

  where

$$\mathcal{Z}^a = (Z_1, Z_2, \bar{W}_1, \bar{W}_2) \ , \quad \bar{\mathcal{Z}}_a = (\bar{Z}_1, \bar{Z}_2, W_1, W_2)$$

  are the multiplets of the $SU(4)$ R-symmetry.
- $\mathcal{Z}^a$ transforms in the fundamental rep and $\bar{\mathcal{Z}}_a$ in the antifundamental rep.
- Z_1, Z_2 transform in the $N \times \bar{N}$ representation of $U(N) \times U(N)$ and W_1, W_2 in the $\bar{N} \times N$ representation.
- The conformal dimension of all the scalars is $\Delta = \frac{1}{2}$ and the bare dimension of the operator is n.

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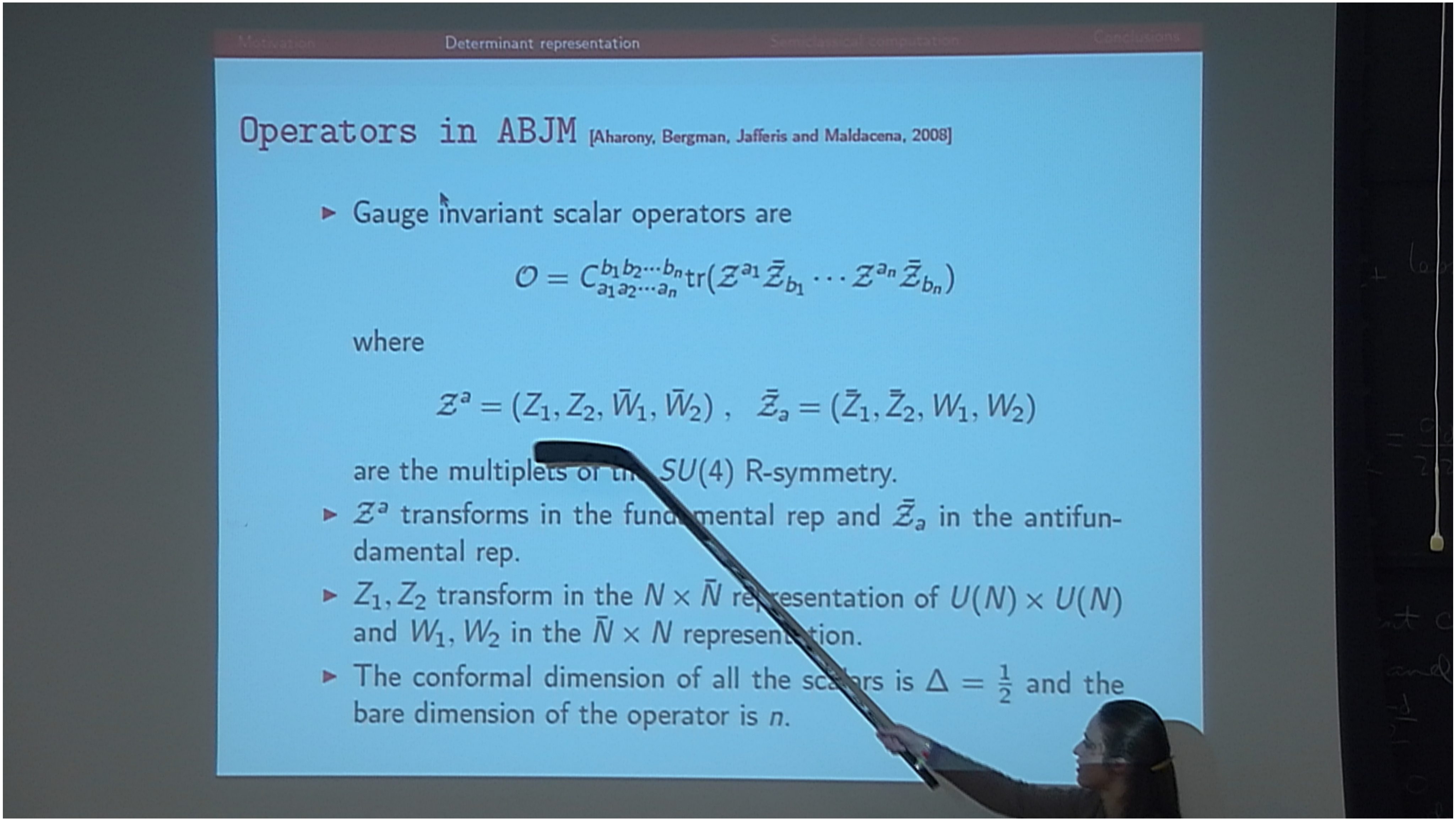
Determinant representation
Operators in ABJM [Aharony, Bergman, Jafferis and Maldacena, 2008]
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$$\mathcal{Z}^a = (Z_1, Z_2, \bar{W}_1, \bar{W}_2) \, , \quad \bar{\mathcal{Z}}_a = (\bar{Z}_1, \bar{Z}_2, W_1, W_2)$$

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$SU(2) \times SU(2)$ sector

- $SU(2) \times SU(2)$ sector is obtained by considering operators made out of 2 scalars among $\mathcal{Z}^a$ and 2 scalars among $\bar{\mathcal{Z}}_a$ transforming in two separate $SU(2)$ subgroups of $SU(4)$.
- Consider the scalars $Z_{1,2}$ and $W_{1,2}$, the operators are of the form

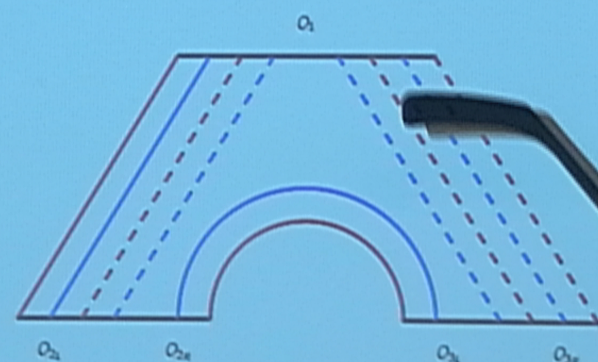
$$\mathcal{O} = C^{j_1 j_2 \cdots j_J}_{i_1 i_2 \cdots i_J} \mathrm{tr}(Z_{i_1} W_{j_1} \cdots Z_{i_J} W_{j_J}).$$

- The 2 loop dilatation operator becomes the Hamiltonian of two decoupled ferromagnetic $XXX_{1/2}$ Heisenberg spin chains, one living at the even sites and the other one living at odd sites. The two chains being related only through the momentum constraint.

[Minahan and Zarembo, 2008]

Operators

Operator	Vacuum odd	Excitation odd	Vacuum even	Excitation even
$\mathcal{O}_1$	$(J-J_1)\ \ Z_1$	$J_1\ \ Z_2$	$(J-J_2)\ \ W_1$	$J_2\ \ W_2$
$\mathcal{O}_2$	$(J_1+j_2)\ \ \bar{Z}_2$	$(J-J_1-j_1)\ \ \bar{Z}_1$	$(J_2+j_2)\ \ \bar{W}_2$	$(J-J_2-j_1)\ \ \bar{W}_1$
$\mathcal{O}_3$	$j_2\ \ W_2$	$j_1\ \ Z_1$	$j_2\ \ Z_2$	$j_1\ \ W_1$



Integrability in $SU(4)$ [Minahan and Zarembo, 2008]

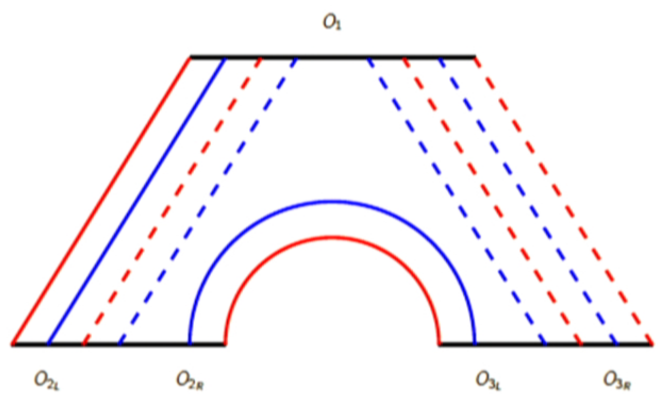
- The dilatation operator of ABJM theory acts as a Hamiltonian for this spin chain and is conjectured to be integrable.
- Introduce the R-matrix, a monodromy matrix and a transfer matrix.
- For the alternating $SU(4)$ spin chain $\rightarrow$ 4 R-matrices

$$
\begin{aligned}
R_{ab} &: V_a \otimes V_b \longrightarrow V_a \otimes V_b, & R_{ab}(u_o) &= u_o\, I_a \otimes I_b + \eta P_{ab},\\
R_{\bar{a}\bar{b}} &: V_{\bar{a}} \otimes V_{\bar{b}} \longrightarrow V_{\bar{a}} \otimes V_{\bar{b}}, & R_{\bar{a}\bar{b}}(u_e) &= u_e\, I_{\bar{a}} \otimes I_{\bar{b}} + \eta P_{\bar{a}\bar{b}},\\
R_{a\bar{b}} &: V_a \otimes V_{\bar{b}} \longrightarrow V_a \otimes V_{\bar{b}}, & R_{a\bar{b}}(u_o) &= u_o\, I_a \otimes I_{\bar{b}} + K_{a\bar{b}},\\
R_{\bar{a}b} &: V_{\bar{a}} \otimes V_b \longrightarrow V_{\bar{a}} \otimes V_b, & R_{\bar{a}b}(u_e) &= u_e\, I_{\bar{a}} \otimes I_b + K_{\bar{a}b}
\end{aligned}
$$

- V_a ,$V_{\bar{a}}$ are the vector spaces of the fundamental and anti-fundamental representation.
- I = identity operator, P=permutation operator, K=$SU(4)$ trace. u_e and u_o are spectral parameters and η is the shift.

Operators

Operator	Vacuum odd	Excitation odd	Vacuum even	Excitation even
$\mathcal{O}_1$	$(J - J_1)\ \ Z_1$	$J_1\ \ Z_2$	$(J - J_2)\ \ W_1$	$J_2\ \ W_2$
$\mathcal{O}_2$	$(J_1 + j_2)\ \ \bar{Z}_2$	$(J - J_1 - j_1)\ \ \bar{Z}_1$	$(J_2 + j_2)\ \ \bar{W}_2$	$(J - J_2 - j_1)\ \ \bar{W}_1$
$\mathcal{O}_3$	$j_2\ \ W_2$	$j_1\ \ \bar{Z}_1$	$j_2\ \ Z_2$	$j_1\ \ \bar{W}_1$



Integrability in $SU(2) \times SU(2)$ I

- In the $SU(2) \times SU(2)$ sector the trace operator K does not contribute and

$$R_{ab}(u_o, z_o) = [u_o - z_o] \begin{pmatrix} \frac{[u_o - z_o + \eta]}{[u_o - z_o]} & 0 & 0 & 0 \\ 0 & 1 & \frac{[\eta]}{[u_o - z_o]} & 0 \\ 0 & \frac{[\eta]}{[u_o - z_o]} & 1 & 0 \\ 0 & 0 & 0 & \frac{[u_o - z_o + \eta]}{[u_o - z_o]} \end{pmatrix}_{ab} \equiv [u_o - z_o]\,\mathcal{R}_{ab}$$

$$R_{\bar{a}\bar{b}}(u_e, z_e) = [u_e - z_e] \begin{pmatrix} \frac{[u_e - z_e + \eta]}{[u_e - z_e]} & 0 & 0 & 0 \\ 0 & 1 & \frac{[\eta]}{[u_e - z_e]} & 0 \\ 0 & \frac{[\eta]}{[u_e - z_e]} & 1 & 0 \\ 0 & 0 & 0 & \frac{[u_e - z_e + \eta]}{[u_e - z_e]} \end{pmatrix}_{\bar{a}\bar{b}} \equiv [u_e - z_e]\,\mathcal{R}_{\bar{a}\bar{b}}$$

$$\begin{aligned} R_{a\bar{b}}(u_o, z_e) &= [u_o - z_e]\, I \\ R_{\bar{a}b}(u_e, z_o) &= [u_e - z_o]\, I \end{aligned}$$

- $R_{ab}(u_o, z_o)$ and $R_{\bar{a}\bar{b}}(u_e, z_e)$ each are R-matrix of an $SU(2)$ spin chain.

Integrability in $SU(2) \times SU(2)$ I

- In the $SU(2) \times SU(2)$ sector the trace operator K does not contribute and

$$R_{ab}(u_o, z_o) = [u_o - z_o] \begin{pmatrix} \frac{[u_o - z_o + \eta]}{[u_o - z_o]} & 0 & 0 & 0 \\ 0 & 1 & \frac{[\eta]}{[u_o - z_o]} & 0 \\ 0 & \frac{[\eta]}{[u_o - z_o]} & 1 & 0 \\ 0 & 0 & 0 & \frac{[u_o - z_o + \eta]}{[u_o - z_o]} \end{pmatrix}_{ab} \equiv [u_o - z_o]\, \mathcal{R}_{ab}$$

$$R_{\bar{a}\bar{b}}(u_e, z_e) = [u_e - z_e] \begin{pmatrix} \frac{[u_e - z_e + \eta]}{[u_e - z_e]} & 0 & 0 & 0 \\ 0 & 1 & \frac{[\eta]}{[u_e - z_e]} & 0 \\ 0 & \frac{[\eta]}{[u_e - z_e]} & 1 & 0 \\ 0 & 0 & 0 & \frac{[u_e - z_e + \eta]}{[u_e - z_e]} \end{pmatrix}_{\bar{a}\bar{b}} \equiv [u_e - z_e]\, \mathcal{R}_{\bar{a}\bar{b}}$$

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- $R_{ab}(u_o, z_o)$ and $R_{\bar{a}\bar{b}}(u_e, z_e)$ each are R-matrix of an $SU(2)$ spin chain.

Integrability in $SU(2) \times SU(2)$ II

- The monodromy matrices are

$$M_a(u_{a_o}, \{z_o, z_e\}_J) = \left(\prod_{i=1}^{J} [u_{a_o} - z_{i_o}][u_{a_o} - z_{i_e}]\right) \mathcal{R}_{a1}(u_{a_o}, z_{1_o}) \ldots \mathcal{R}_{aJ}(u_{a_o}, z_{J_o}),$$

$$M_{\bar{a}}(u_{a_e}, \{z_o, z_e\}_J) = \left(\prod_{i=1}^{J} [u_{a_e} - z_{i_o}][u_{a_e} - z_{i_e}]\right) \mathcal{R}_{\bar{a}\bar{1}}(u_{a_e}, z_{1_e}) \ldots \mathcal{R}_{\bar{a}\bar{J}}(u_{a_e}, z_{J_e})$$

- The monodromy matrix can be written in this useful form

$$M_a(u_{a_o}, \{z_o, z_e\}_J) = \begin{pmatrix} A_o(u_{a_o}, \{z_o, z_e\}_J) & B_o(u_{a_o}, \{z_o, z_e\}_J) \\ C_o(u_{a_o}, \{z_o, z_e\}_J) & D_o(u_{a_o}, \{z_o, z_e\}_J) \end{pmatrix}_a$$

and a similar expression for $M_{\bar{a}}(u_{a_e}, \{z_o, z_e\}_J)$, where B is the spin flipping operator. Note that we have 2 different flipping operators, one acting on odd and one on even sites.

Consequences

- 4 R-matrices for the $SU(4)$ spin chain

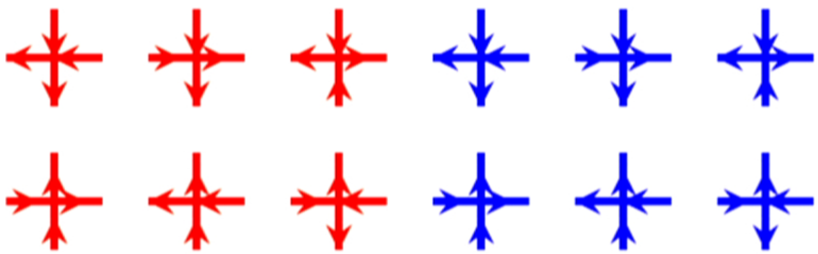
$$SU(2) \times SU(2)$$

- 2 R-matrices trivialize
- 2 are the R-matrices of 2 independent $SU(2)$ spin chains
- The lowering operators B_e and B_o become the usual $SU(2)$ spin flipping operators for even and odd sites.

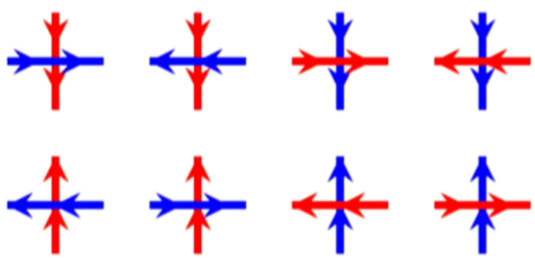
- In order to obtain an eigenstate both sets of rapidities $\{u_o\}$ and $\{u_e\}$ have to satisfy the $SU(2)$ Bethe equations
- The only connection between the two sets of rapidities $\{u_o\}$ and $\{u_e\}$ is momentum constraint (the total momentum of all excitations should vanish)

Six vertex model I

- From the R matrix it is possible to assign a weight to each vertex



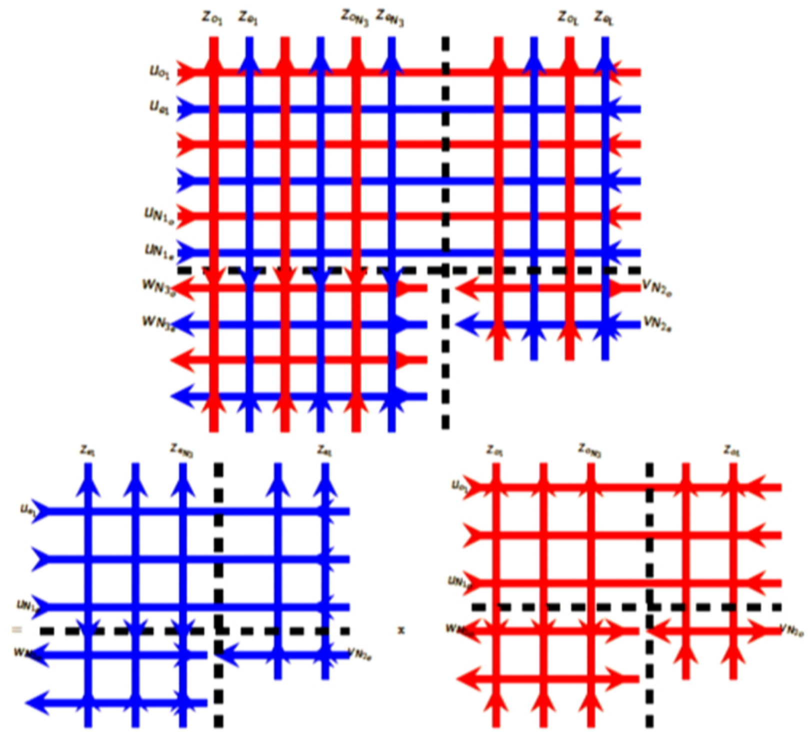
- The first 2 lines refer to $R_{ab}(u_o, z_o)$ and $R_{\overline{a}\overline{b}}(u_e, z_e)$.



- The weights of the blue-red vertices is 1 .

Six vertex model II

The full 6 vertex model is equivalent to the product of two 6 vertex models. This is a direct consequence of the fact that the mixed vertices have unit weight.



Six vertex model III

DOMAIN WALL PARTITION FUNCTION

$$Z_{2J}(\{u\}_{2J},\{z\}_{2J}) = \langle \downarrow_{z_{2J}} | \prod_{i=1}^{2J} \mathcal{B}(u_i,\{z_{2J}\}) | \uparrow_{z_{2J}} \rangle$$

SLAVNOV SCALAR PRODUCT

$$S[\{u\}_{N_1},\{v\}_{N_2},\{z\}_J] =$$

$$= \langle \downarrow_{z_{N_3,J}} | \prod_{i=1}^{N_2} \mathcal{C}(u_i,\{z\}_J) \prod_{j=1}^{N_1} \mathcal{B}(v_j,\{z\}_J) | \uparrow_{z_J} \rangle$$

where

$$\langle \downarrow_{z_{N_3,J}} | = \langle \downarrow_{z_1} | \otimes \cdots \otimes \langle \downarrow_{z_{N_3}} | \otimes \langle \uparrow_{z_{N_3+1}} | \otimes \cdots \otimes \langle \uparrow_{z_J} |,$$

GAUDIN NORM

$$\mathcal{N}(\{u\}) = S[\{u\}_N,\{u\}_N, i/2]$$

3-point function

$$\boxed{C_{123}} = \mathcal{N}_{123}\left({}_r\langle\mathcal{O}_3| \otimes {}_l\langle\mathcal{O}_2|\right)|\mathcal{O}_1\rangle$$
$$= \mathcal{N}_{123} Z_{j_1}(\{w_o\})\, S[J, J_1, J - J_1 - j_1](\{u_o\}, \{v_o\}) \times$$
$$Z_{j_1}(\{w_e\})\, S[J, J_2, J - J_2 - j_1](\{u_e\}, \{v_e\})$$

where

$$\boxed{\mathcal{N}_{123}} = \frac{\sqrt{J(j_1 + j_2)(J + j_2 - j_1)}}{\sqrt{\mathcal{N}_{1o}\mathcal{N}_{1e}\mathcal{N}_{2o}\mathcal{N}_{2e}\mathcal{N}_{3o}\mathcal{N}_{3e}}}$$

RESULT

The result is (up to the normalization) a product of two $\mathcal{N} = 4$ SYM correlation function, reflecting the properties of the spin chains. The normalization takes into account the cyclicity condition of the trace (momentum constraint).

3-point function

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$$Z_{j_1}(\{w_e\})\, S[J, J_2, J - J_2 - j_1](\{u_e\},\{v_e\})$$

where

$$\boxed{\mathcal{N}_{123}} = \frac{\sqrt{J(j_1+j_2)(J+j_2-j_1)}}{\sqrt{\mathcal{N}_{1o}\mathcal{N}_{1e}\mathcal{N}_{2o}\mathcal{N}_{2e}\mathcal{N}_{3o}\mathcal{N}_{3e}}}$$

RESULT

The result is (up to the normalization) a product of two $\mathcal{N} = 4$ SYM correlation function, reflecting the properties of the spin chains. The normalization takes into account the cyclicity condition of the trace (momentum constraint).

<HHL>

- $\mathcal{O}_1$ and $\mathcal{O}_2$ are much longer than $\mathcal{O}_3$
- To simplify we choose $\boxed{j_1 = j_2 = j}$ and $\boxed{J_1 = J_2}$ and our operators become

$\mathcal{O}_1$	$(J - J_1)\ \ Z_1$	$J_1\ \ Z_2$	$(J - J_1)\ \ W_1$	$J_1\ \ W_2$
$\mathcal{O}_2$	$(J_1 + j)\ \ \bar{Z}_2$	$(J - J_1 - j)\ \ \bar{Z}_1$	$(J_1 + j)\ \ \bar{W}_2$	$(J - J_1 - j)\ \ \bar{W}_1$
$\mathcal{O}_3$	$j\ \ W_2$	$j\ \ \bar{Z}_1$	$j\ \ Z_2$	$j\ \ \bar{W}_1$

  in the limit $\boxed{1 \ll j \ll J_1, J}$
- we calculate

$$\boxed{r = \frac{C^{\bullet\bullet\circ}}{C^{\circ\circ\circ}}}$$

1. gauge theory computation $\rightarrow$ coherent state approach
2. string theory computation $\rightarrow$ semiclassical computation

Gauge computation: coherent states

$\mathcal{O}_1$ and $\mathcal{O}_2$ are represented by coherent states

$$\mathcal{O}_1 = \ldots(\mathbf{u}_o^{(2k-1)} \cdot \mathbf{Z})(\mathbf{u}_e^{(2k)} \cdot \mathbf{W})(\mathbf{u}_o^{(2k+1)} \cdot \mathbf{Z})(\mathbf{u}_e^{(2k+2)} \cdot \mathbf{W})\ldots$$

$$\mathcal{O}_2 = \ldots(\bar{\mathbf{v}}_o^{(2k-1)} \cdot \bar{\mathbf{Z}})(\bar{\mathbf{v}}_e^{(2k)} \cdot \bar{\mathbf{W}})(\bar{\mathbf{v}}_o^{(2k+1)} \cdot \bar{\mathbf{Z}})(\bar{\mathbf{v}}_e^{(2k+2)} \cdot \bar{\mathbf{W}})\ldots$$

where

- $\mathbf{Z} = (Z_1, Z_2)$, $\mathbf{W} = (W_1, W_2)$, $\bar{\mathbf{Z}} = (\bar{Z}_1, \bar{Z}_2)$ and $\bar{\mathbf{W}} = (\bar{W}_1, \bar{W}_2)$
- The vectors $\mathbf{u}_o = (u_o^1, u_o^2)$ and $\mathbf{u}_e = (u_e^1, u_e^2)$ belong to $\mathbb{C}^2$ and are unit normalized
- $\mathcal{O}_1$ and $\mathcal{O}_2$ are eigenstates of the two loop dilatation operator $\rightarrow \mathbf{u}_o^{(p)} \equiv \mathbf{u}_o(\pi p/J)$ must be periodic in p with period $2J$ and fulfill the equations of motion of the Landau-Lifshitz sigma model.

Gauge computation: light operator

$\mathcal{O}_3$ is a BPS operator

$$\boxed{\mathcal{O}_3 = \mathcal{N}_3 \mathrm{tr}((Z_1 W_1)^j (\bar{W}_2 \bar{Z}_2)^j) + \text{irrelevant terms}}$$

where $\mathcal{N}_3 = \frac{(j!)^2}{\sqrt{(2j)!(2j-1)!}}$.

Note:

Differently from the $\mathcal{N} = 4$ case, 3 point functions of BPS operators are NOT protected, they depend on the coupling $\lambda = \frac{N}{k}$.

Gauge computation: $C^{\bullet\bullet\circ}$

There are two contributions to be taken into account

1. contractions involving $\mathcal{O}_3$ given by

$$\prod_{m=k}^{k+j-1} u_o^1\left(\frac{(2m-1)\pi}{J}\right) u_e^1\left(\frac{2m\pi}{J}\right) \bar{v}_o^2\left(\frac{(2m-1)\pi}{J}\right) \bar{v}_e^2\left(\frac{2m\pi}{J}\right)$$

   This notation means that in $\mathcal{O}_1$ as well as in $\mathcal{O}_2$ the fields at the sites $2k-1, 2k, \ldots, 2k+2j-2$ are contracted with $\mathcal{O}_3$.

2. contractions between $\mathcal{O}_1$ and $\mathcal{O}_2$

$$B = \prod_{m=1}^{J} (\mathbf{u}_o^{(2m-1)} \cdot \bar{\mathbf{v}}_o^{(2m-1)}) (\mathbf{u}_e^{(2m)} \cdot \bar{\mathbf{v}}_e^{(2m)})$$

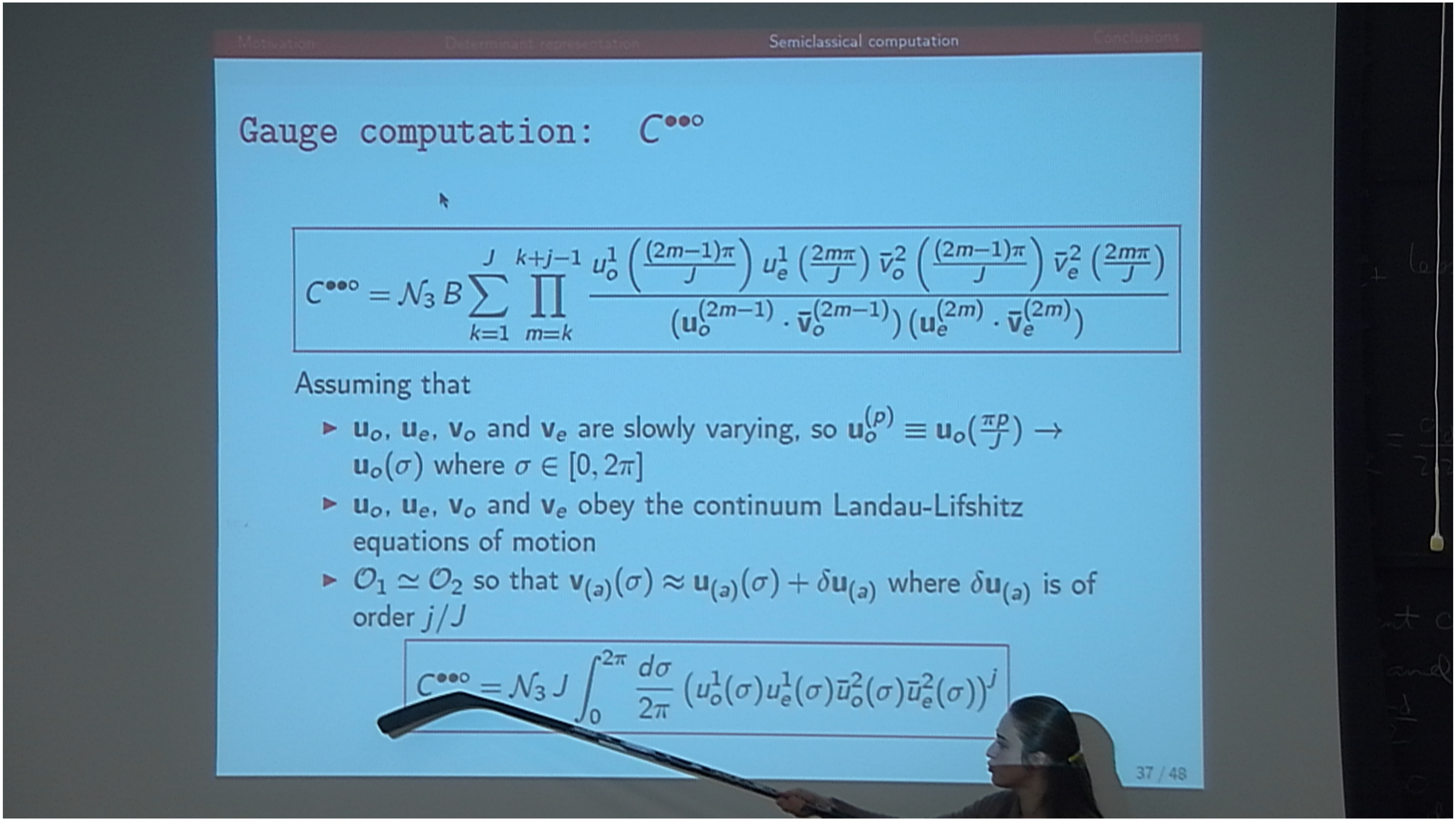
Gauge computation: $C^{\bullet\bullet\circ}$

$$C^{\bullet\bullet\circ} = \mathcal{N}_3\, B \sum_{k=1}^{J} \prod_{m=k}^{k+j-1} \frac{u_o^1\left(\frac{(2m-1)\pi}{J}\right) u_e^1\left(\frac{2m\pi}{J}\right) \bar{v}_o^2\left(\frac{(2m-1)\pi}{J}\right) \bar{v}_e^2\left(\frac{2m\pi}{J}\right)}{(\mathbf{u}_o^{(2m-1)} \cdot \bar{\mathbf{v}}_o^{(2m-1)})\,(\mathbf{u}_e^{(2m)} \cdot \bar{\mathbf{v}}_e^{(2m)})}$$

Assuming that

- $\mathbf{u}_o$, $\mathbf{u}_e$, $\mathbf{v}_o$ and $\mathbf{v}_e$ are slowly varying, so $\mathbf{u}_o^{(p)} \equiv \mathbf{u}_o(\frac{\pi p}{J}) \to \mathbf{u}_o(\sigma)$ where $\sigma \in [0, 2\pi]$
- $\mathbf{u}_o$, $\mathbf{u}_e$, $\mathbf{v}_o$ and $\mathbf{v}_e$ obey the continuum Landau-Lifshitz equations of motion
- $\mathcal{O}_1 \simeq \mathcal{O}_2$ so that $\mathbf{v}_{(a)}(\sigma) \approx \mathbf{u}_{(a)}(\sigma) + \delta\mathbf{u}_{(a)}$ where $\delta\mathbf{u}_{(a)}$ is of order j/J

$$C^{\bullet\bullet\circ} = \mathcal{N}_3\, J \int_0^{2\pi} \frac{d\sigma}{2\pi} \left(u_o^1(\sigma) u_e^1(\sigma) \bar{u}_o^2(\sigma) \bar{u}_e^2(\sigma)\right)^j$$


Motivation
Determinant representation
Semiclassical computation
Conclusions
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Gauge computation: $C^{\circ\circ\circ}$

$C^{\circ\circ\circ}$ is the 3 point function of 3 chiral primaries with the same charges as the operators we have before.

- There is a procedure for $\mathcal{N} = 4$ due to [Kostov, 2011] to take the limit of the general result in a determinant form to obtain the result for 3 chiral primaries. Our result, up to a normalization constant, is two copies of the $\mathcal{N} = 4$ that in the limit gives

$$C^{\circ\circ\circ} = J\sqrt{2j}\frac{(J-J_1+j)!J_1!((J-j)!)^2 j!^2}{(J!)^2(J-J_1)!(J_1-j)!(2j)!} \xrightarrow{J,J_1\to\infty} \mathcal{N}_3 J s^j$$

  where $s = \left(\frac{J_1(J-J_1)}{J^2}\right)$.

- We also checked this result with a perturbative prescription of [Hirano, Kristjansen and Young, 2012]

String computation: $C^{\bullet\bullet\circ}$

Compute the holographic dual to the correlator computed on the gauge theory side.

The main steps are:

1. specify the background metric
2. fluctuation computation (insertion of the light operator)
3. evaluate the fluctuations on the classical string solution (2 point function of the heavy operators)

String computation: background metric

- The metric of type IIA string theory on $AdS_4 \times \mathbb{C}P^3$ in Poincaré coordinates is $ds^2 = R^2\left[\frac{1}{4} ds^2_{AdS_4} + ds^2_{\mathbb{C}P^3}\right]$
- $$ds^2_{AdS_4} = \frac{dz^2 + d\mathbf{x}_\mu^2}{z^2}$$, $\mathbf{x}_\mu = (x_1, x_2, x_3)$
- to zoom in to the $SU(2)\times SU(2)$ sector of type IIA string theory on $AdS_4 \times \mathbb{C}P^3$ we need to start from M-theory on $AdS_4 \times S^7$, use a suitable set of coordinates and after reduce to 10 dim type IIA background. The explicit form of the $ds^2_{\mathbb{C}P^3}$ is

$$ds^2_{\mathbb{C}P^3} = \left[\frac{1}{8} d\Omega_2^2 + \frac{1}{8} d\Omega_2'^2 + (d\delta + \omega)^2\right]$$

where $\omega = \frac{1}{4}(\sin\theta_1 d\varphi_1 + \sin\theta_2 d\varphi_2)$, $\delta = \frac{1}{4}(\phi_1 + \phi_2 - \phi_3 - \phi_4)$
$\varphi_1 = \phi_1 - \phi_2$, $\varphi_2 = \phi_4 - \phi_3$

[Grignani, Harmark and Orselli, 2009]

- (θ_i, φ_i), $i = 1, 2$, parametrize 2 two-spheres corresponding to the two $SU(2)$ sectors.

String computation: Frolov-Tseytlin limit

- introduce a parametrization $\mathbf{U}_{e,o}(\sigma,\tau) = e^{i\tau/\kappa}\mathbf{u}_{e,o}(\sigma,\tau)$ with the conditions $\bar{\mathbf{u}}_e \cdot \mathbf{u}_e = 1$ and $\bar{\mathbf{u}}_o \cdot \mathbf{u}_o = 1$
- In order to compare with the gauge theory result we take the Frolov-Tseytlin limit which is

$$\kappa \to 0 \,, \quad \frac{1}{\kappa}\partial_\tau \mathbf{u}_{e,o} \quad \text{fixed} \,, \quad \partial_\sigma \mathbf{u}_{e,o} \quad \text{fixed}$$

$\kappa_{FT} = \frac{1}{\kappa} \to \infty$ [Frolov and Tseytlin, 2003]
[Grignani, Harmark and Orselli, 2009]

- $\mathbf{u}_{e,o}$ are solutions of the Landau Lifshitz equations of motion satisfying the Virasoro condition $\bar{\mathbf{u}}_e \cdot \partial_\sigma \mathbf{u}_e + \bar{\mathbf{u}}_o \cdot \partial_\sigma \mathbf{u}_o = 0$. Note that it is shown that by taking the $SU(2) \times SU(2)$ sigma model limit one obtains 2 Landau Lifshitz added together related only through momentum constraint

[Grignani, Harmark and Orselli, 2009]

$$C^{\bullet\bullet\circ} = J\frac{\lambda^{\frac{1}{4}}2^{\frac{3}{4}}}{\sqrt{\pi}}\sqrt{4j+1}\int_0^{2\pi}\frac{d\sigma}{2\pi}(u_e^1\bar{u}_e^2u_o^1\bar{u}_o^2)^j$$

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String computation: $C^{\circ\circ\circ}$

Again we need $C^{\circ\circ\circ}$ to properly normalize the three point function.

- Using the result of [Hirano, Kristjansen and Young, 2012] we obtain

$$C^{\circ\circ\circ} = \frac{\lambda^{\frac{1}{4}} 2^{-\frac{1}{4}}}{\sqrt{\pi}} \sqrt{4j+1} \frac{(2J+1)(J-j)!}{(J+j)!} \frac{(J-J_1+j)!}{(J-J_1)!} \frac{J_1!}{(J_1-j)!}$$

- in the limit $J, J_1 \to \infty$ with $J - J_1$ large

$$\boxed{C^{\circ\circ\circ} = \frac{\lambda^{\frac{1}{4}} 2^{\frac{3}{4}}}{\sqrt{\pi}} J s^j \sqrt{4j+1}}$$

- Note: the λ dependence is different from the gauge theory side computation, as expected because this object is not protected.

Comparison

In the limit $J, J_1 \to \infty$ with $J - J_1$ large we compare the results obtained at strong coupling and at weak coupling and they agree:

$$r_{\lambda \gg 1} = \left.\frac{C^{\bullet\bullet\circ}}{C^{\circ\circ\circ}}\right|_{\lambda \gg 1} = \left.\frac{C^{\bullet\bullet\circ}}{C^{\circ\circ\circ}}\right|_{\lambda \ll 1} = r_{\lambda \ll 1}$$

Note: the same agreement has been found in the AdS_5/CFT_4 case.

[Escobedo, Gromov, Sever and Vieira, 2011]

Conclusions

Computation of non extremal, planar, three point functions in the context of the AdS_4/CFT_3 correspondence and, more specifically, in the $SU(2) \times SU(2)$ sector of the theories :

- determinant expression from the gauge theory side in terms of known quantities in the six vertex model language, this approach is general meaning that can be applied to operators with any length and number of impurities
- semiclassical computation both from the gauge and string theory side of a 3 point function of 2 heavy and 1 light operators.

Outlook

- Reproduce the semiclassical result from the more general gauge theory expression. This would be extremely interesting for analytical predictions
- Compute correlators at higher loops and higher sector.