Title: Bootstrapping CFTs with the Extremal Functional Method

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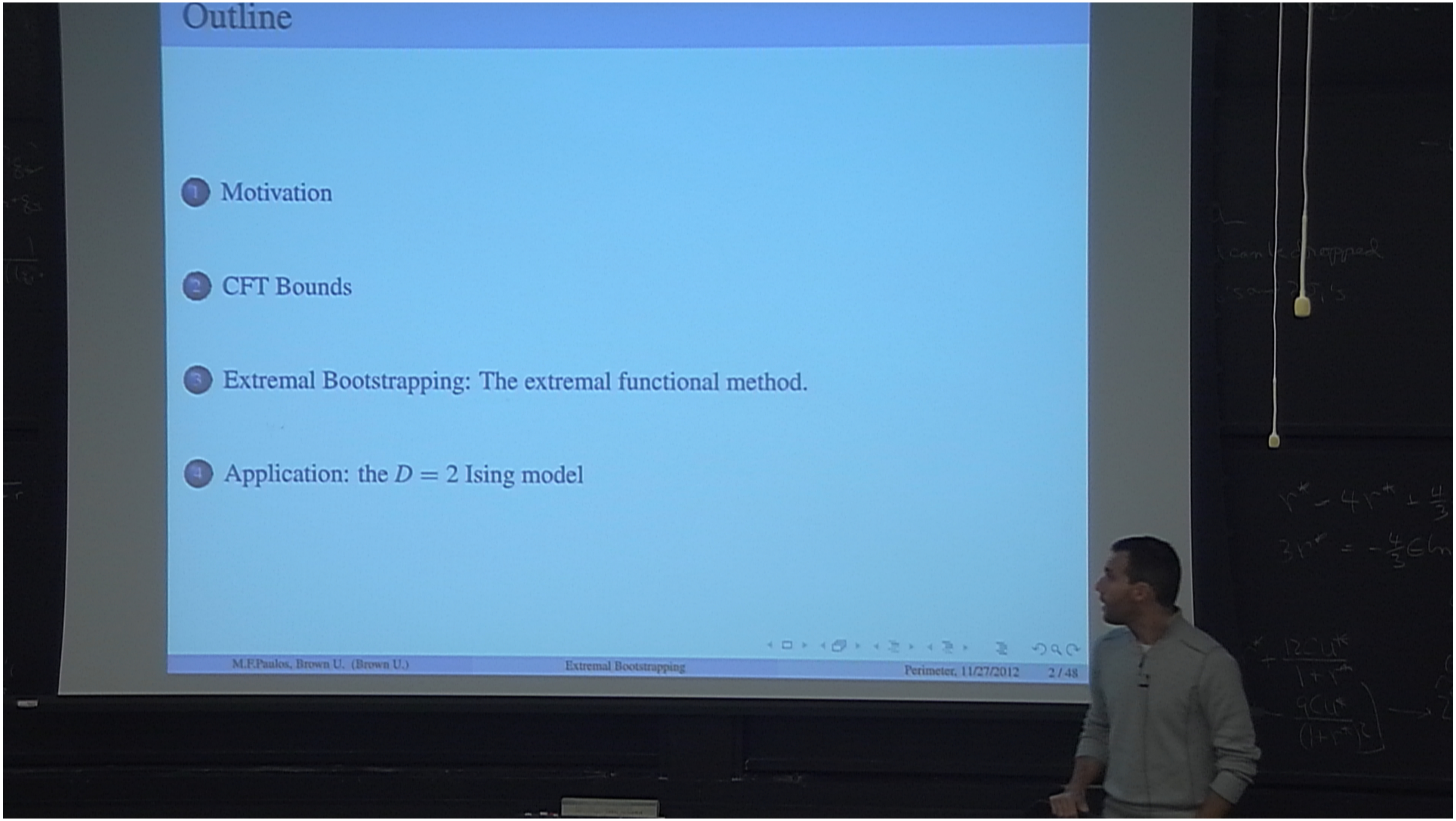
Abstract: The existence of a positive linear functional acting on the space of (differences between) conformal blocks has been shown to rule out regions in the parameter space of conformal field theories (CFTs). We argue that at the boundary of the allowed region the extremal functional contains, in principle, enough information to determine the dimensions and OPE coefficients of an infinite number of operators appearing in the correlator under analysis. Based on this idea we develop the Extremal Functional Method (EFM), a numerical procedure for deriving the spectrum and OPE coefficients of CFTs lying on the boundary (of solution space). We test the EFM by using it to rederive the low lying spectrum and OPE coefficients of the 2d Ising model based solely on the dimension of a single scalar quasi-primary -- no Virasoro algebra required. Our work serves as a benchmark for applications to more interesting, less known CFTs in the near future, such as the 3d Ising model.


Outline
1 Motivation
2 CFT Bounds
3 Extremal Bootstrapping: The extremal functional method.
4 Application: the $D = 2$ Ising model
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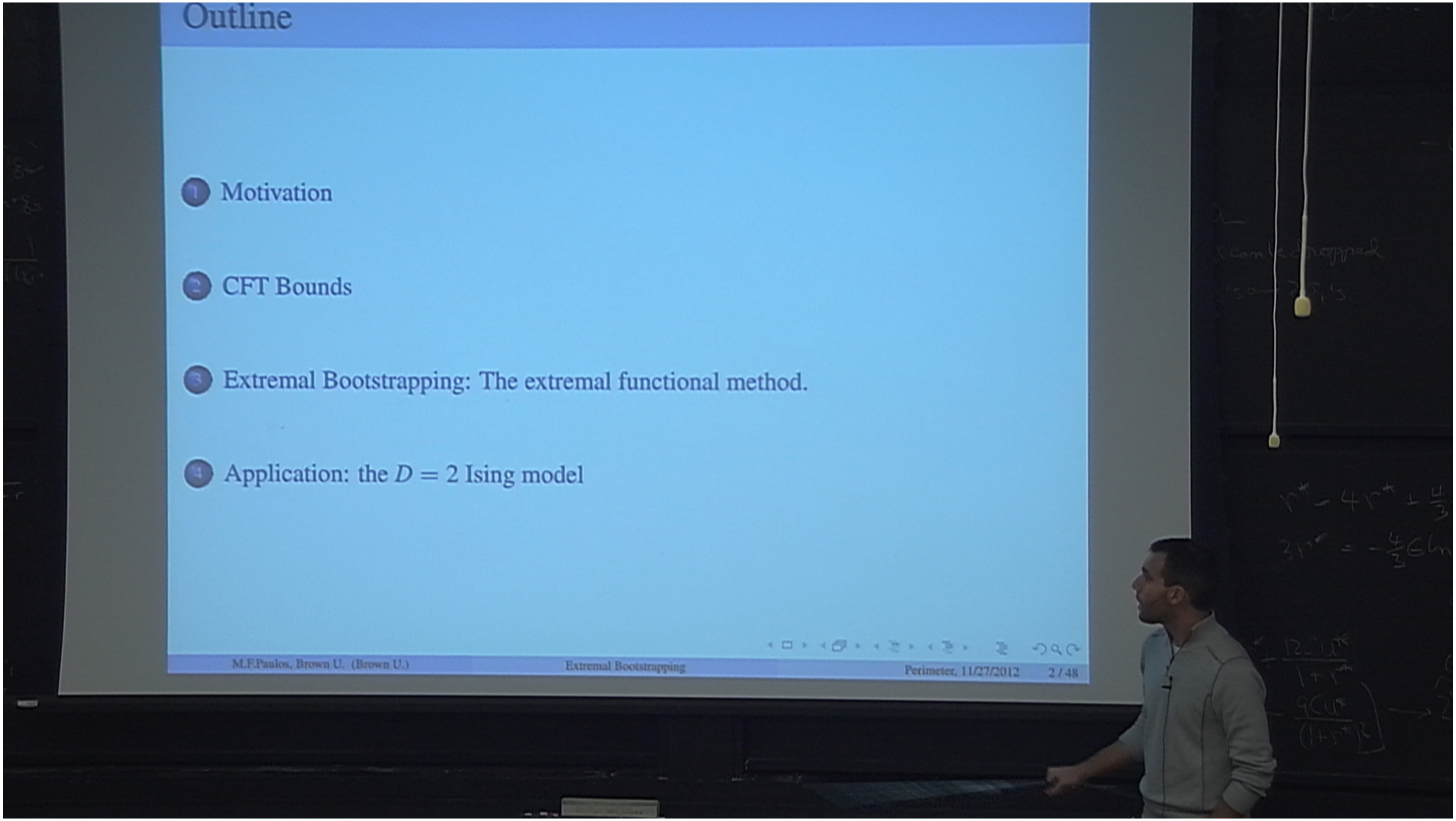
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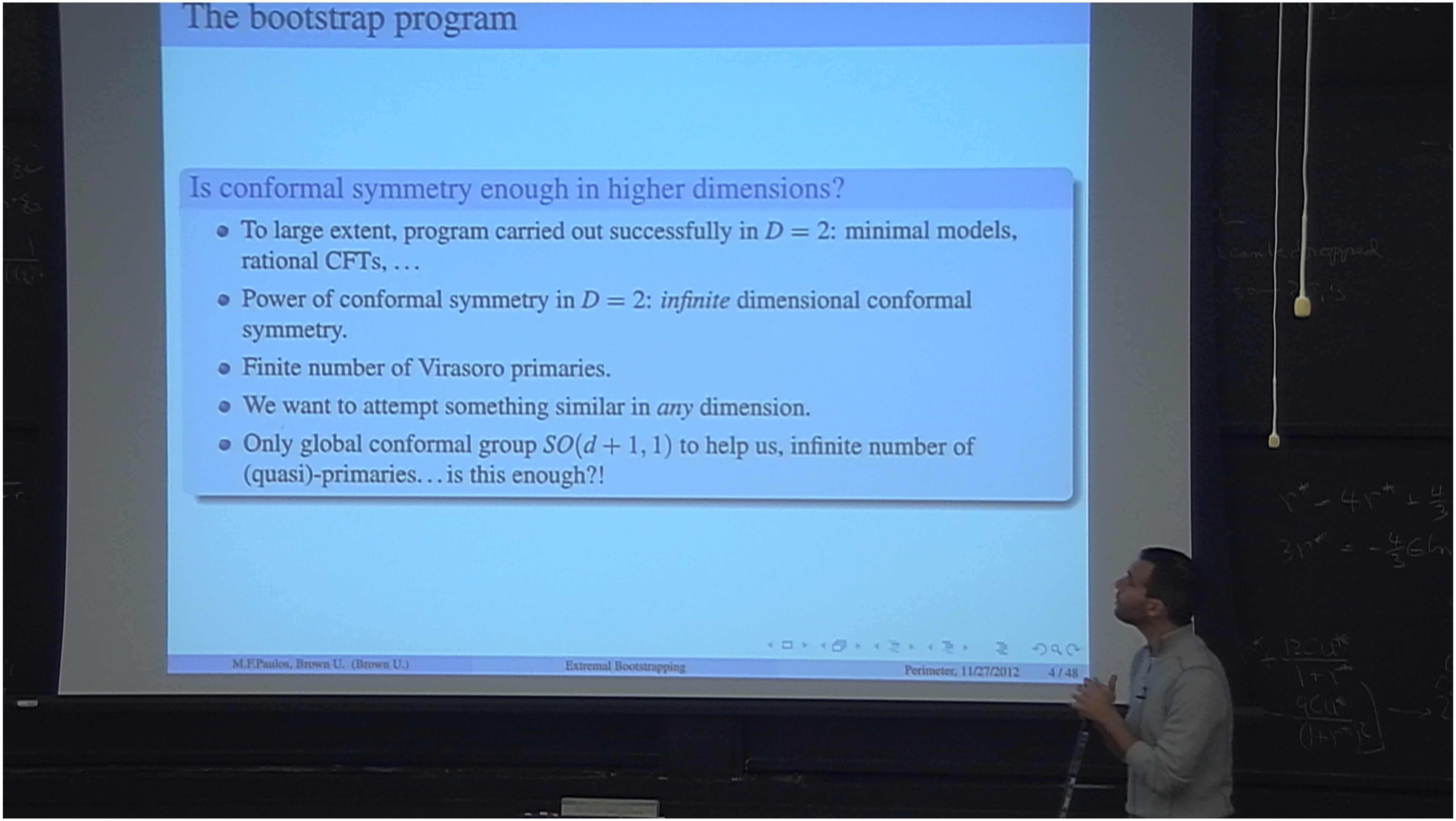
The bootstrap program
Is conformal symmetry enough in higher dimensions?
To large extent, program carried out successfully in $D = 2$: minimal models, rational CFTs, ...
Power of conformal symmetry in $D = 2$: *infinite* dimensional conformal symmetry.
Finite number of Virasoro primaries.
We want to attempt something similar in *any* dimension.
Only global conformal group $SO(d + 1, 1)$ to help us, infinite number of (quasi)-primaries. . . is this enough?!
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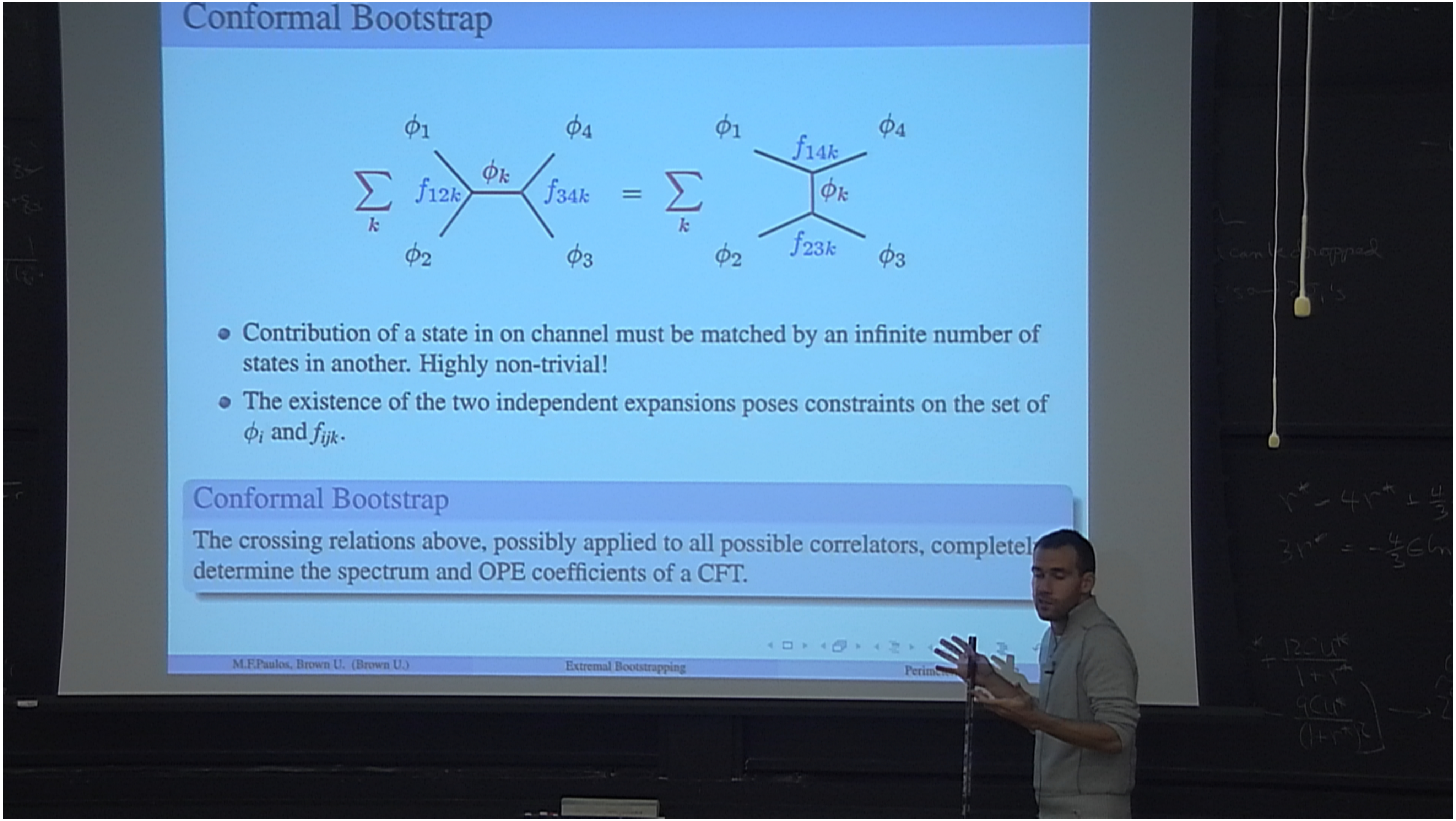
Conformal Bootstrap
$\sum_k$ f_{12k} ϕ_k f_{34k} $= \sum_k$ f_{14k} ϕ_k f_{23k}
ϕ_1 ϕ_2 ϕ_3 ϕ_4
Contribution of a state in on channel must be matched by an infinite number of states in another. Highly non-trivial!
The existence of the two independent expansions poses constraints on the set of ϕ_i and f_{ijk}.
Conformal Bootstrap
The crossing relations above, possibly applied to all possible correlators, complete
determine the spectrum and OPE coefficients of a CFT.
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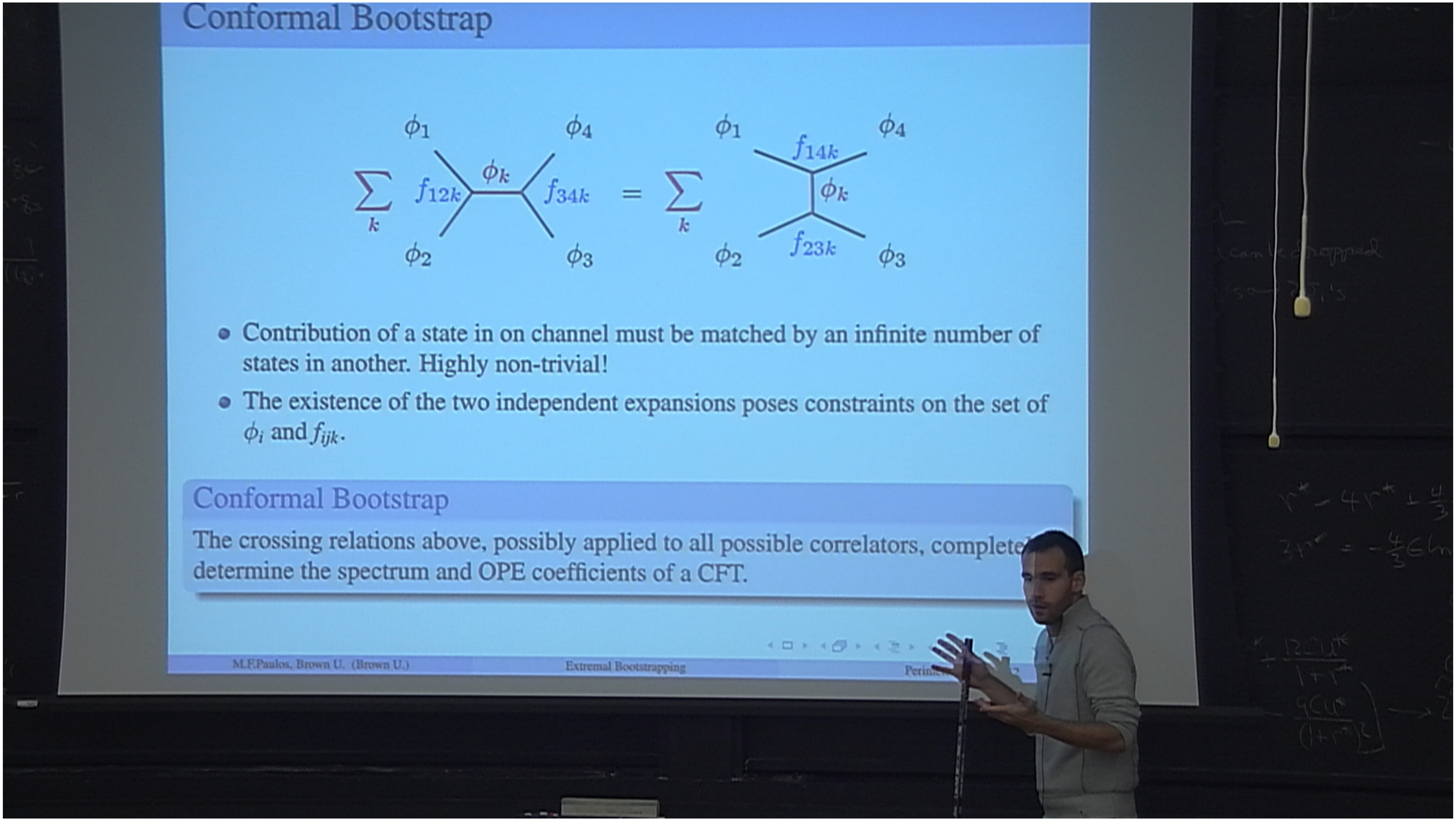
Extremal Bootstrapping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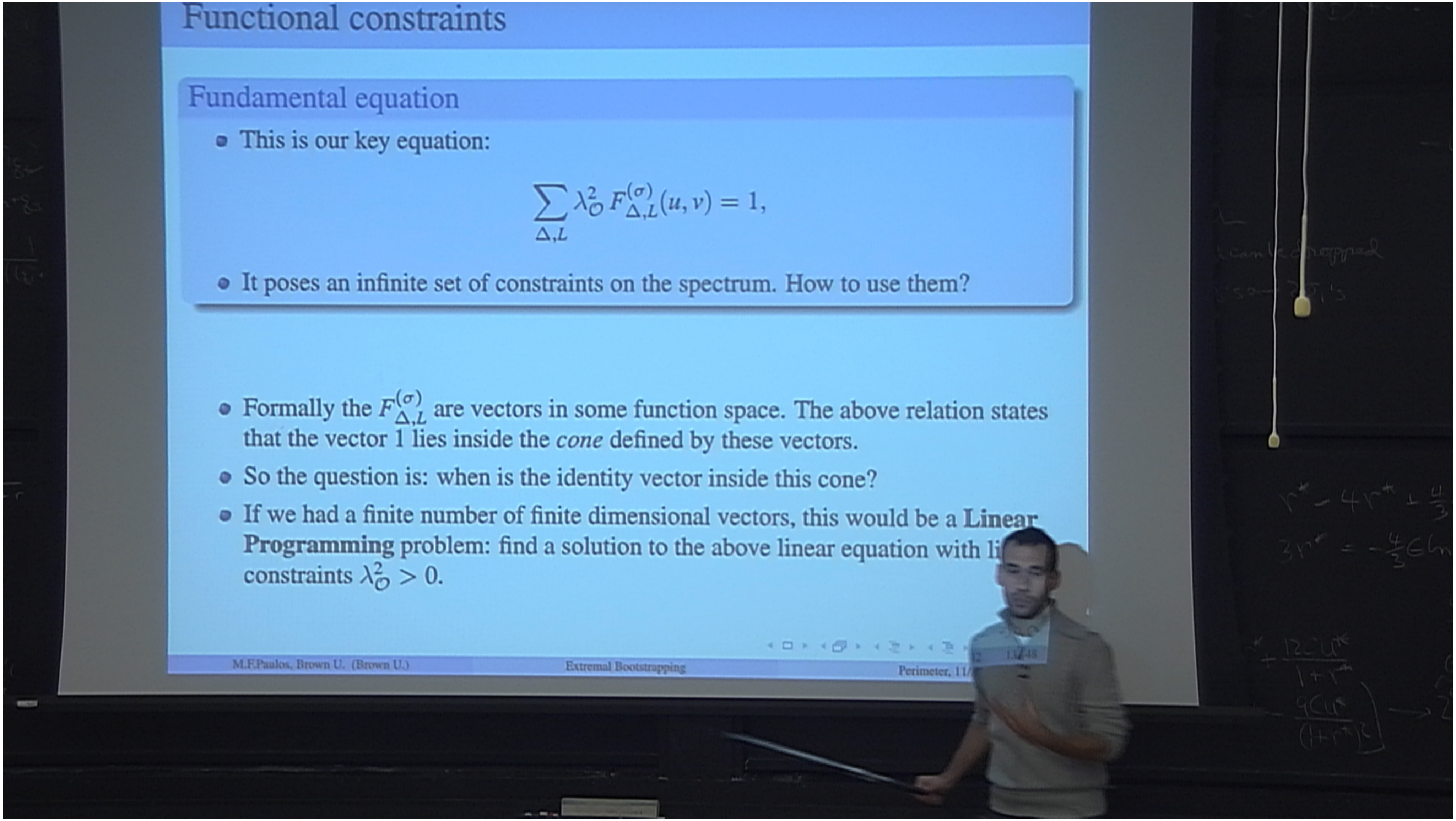
Functional constraints
Fundamental equation
This is our key equation:
$$\sum_{\Delta,L} \lambda_{\mathcal{O}}^2 F_{\Delta,L}^{(\sigma)}(u,v) = 1,$$
It poses an infinite set of constraints on the spectrum. How to use them?
Formally the $F_{\Delta,L}^{(\sigma)}$ are vectors in some function space. The above relation states that the vector 1 lies inside the cone defined by these vectors.
So the question is: when is the identity vector inside this cone?
If we had a finite number of finite dimensional vectors, this would be a Linear Programming problem: find a solution to the above linear equation with li
constraints $\lambda_{\mathcal{O}}^2 > 0$.
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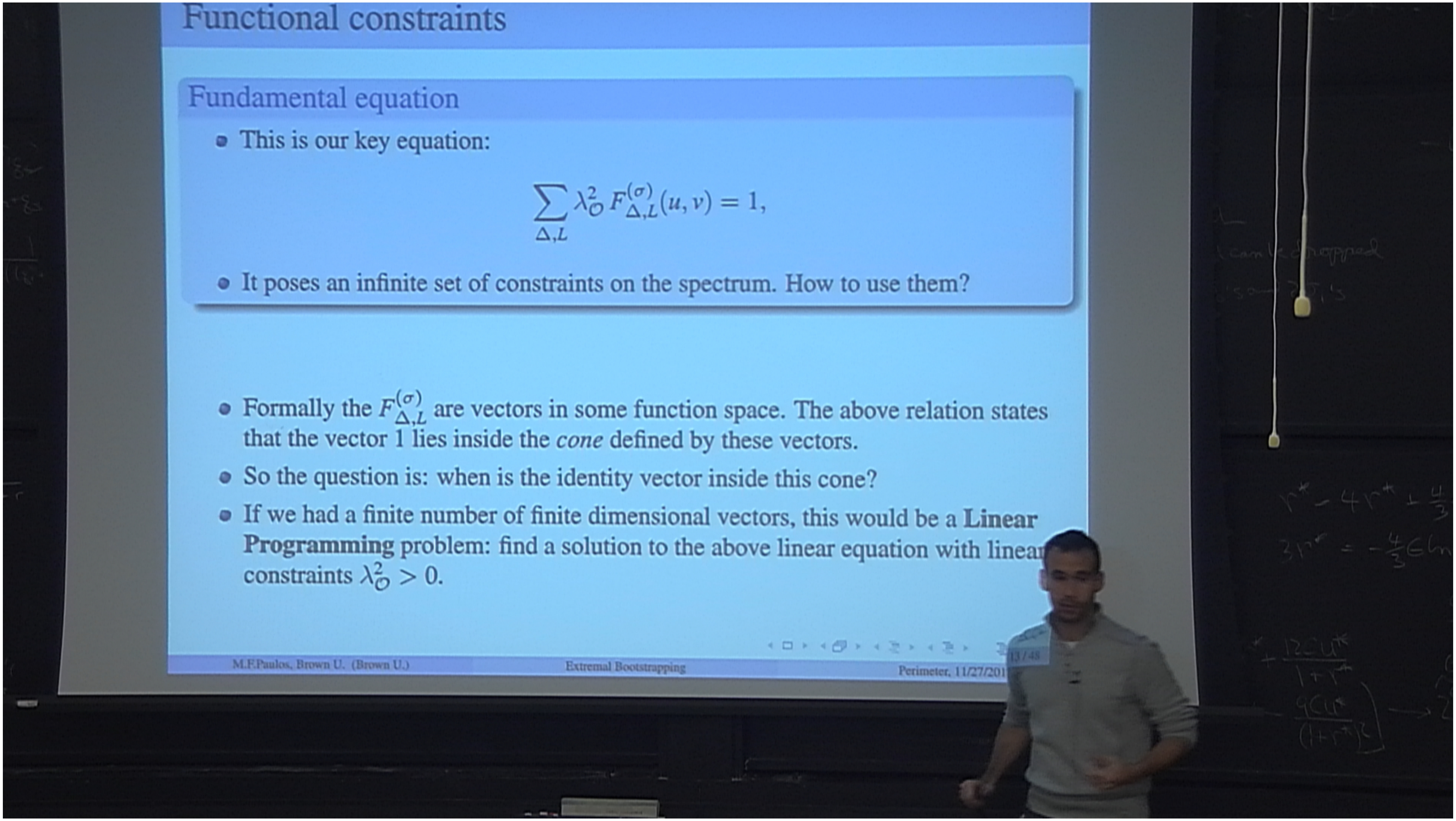
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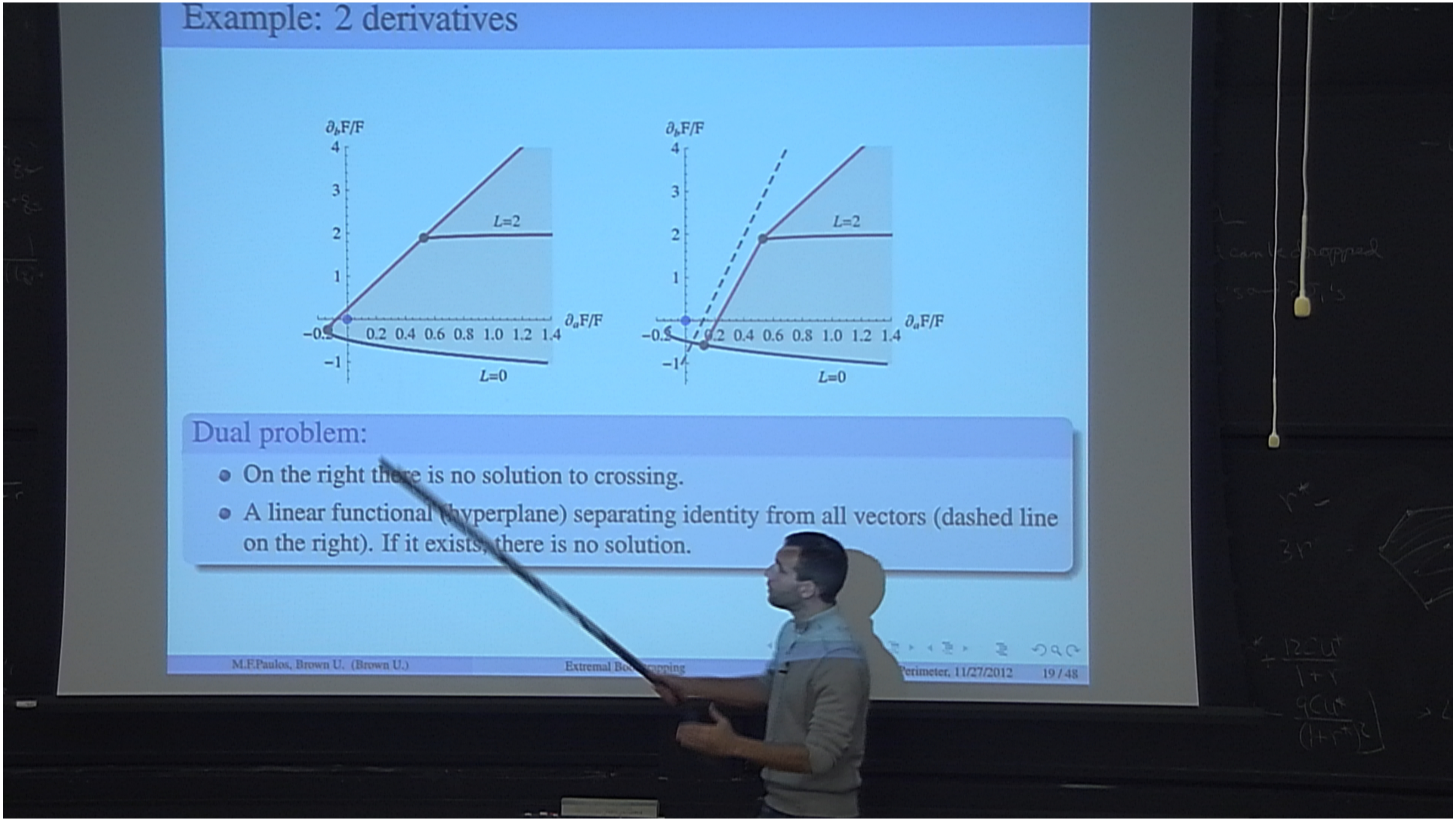
Example: 2 derivatives
$\partial_b F/F$
$\partial_a F/F$
L=2
L=0
Dual problem:
On the right there is no solution to crossing.
A linear functional (hyperplane) separating identity from all vectors (dashed line on the right). If it exists, there is no solution.
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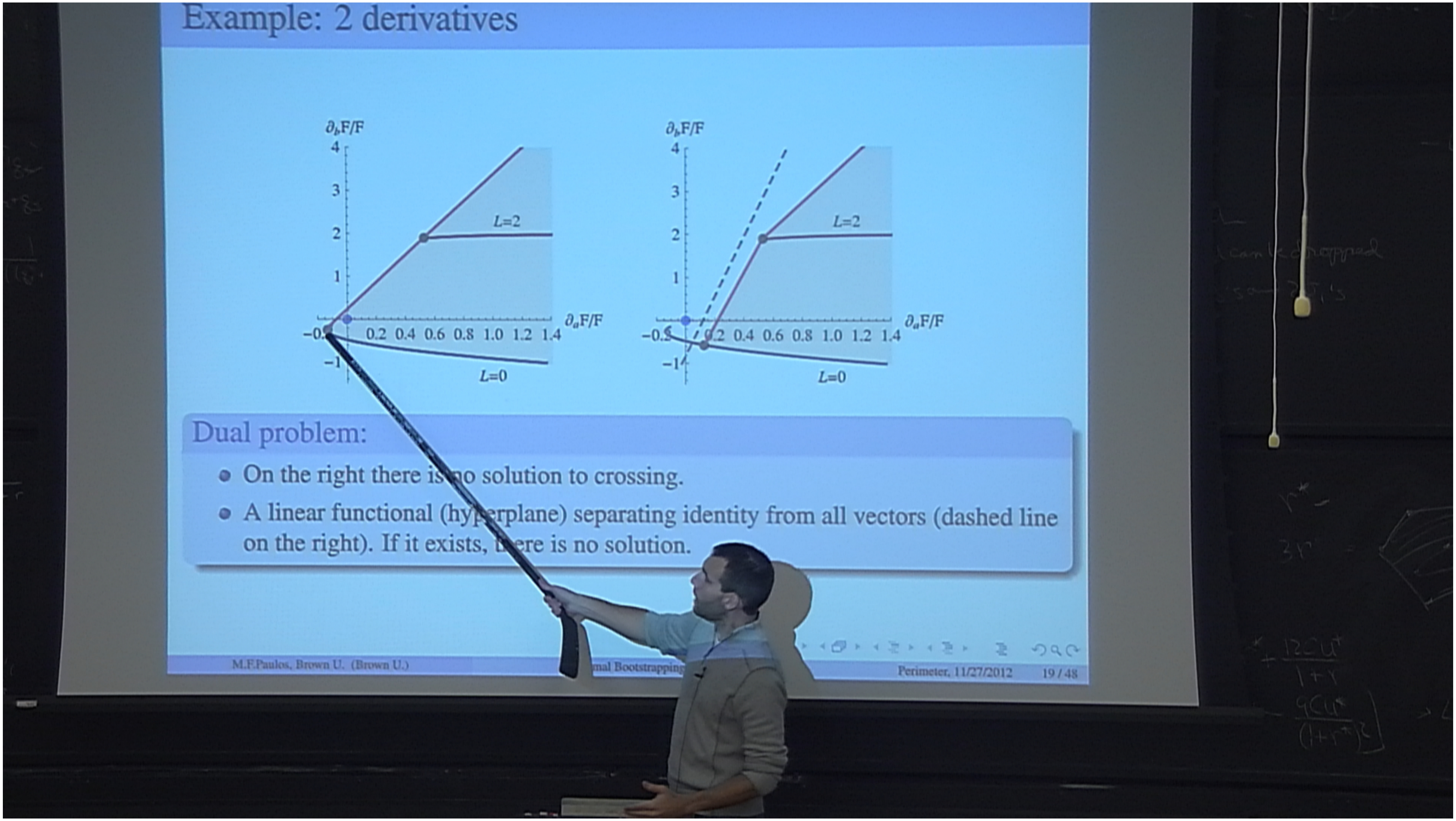
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∂bF/F
∂aF/F
L=2
L=0
0.2 0.4 0.6 0.8 1.0 1.2 1.4
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Extremal Bootstrapping

Going to Extremality

- Bounds rely only on *existence* of a linear functional. But what is the information carried by the functional (if any)?

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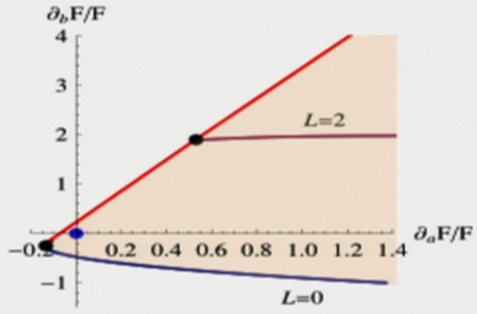
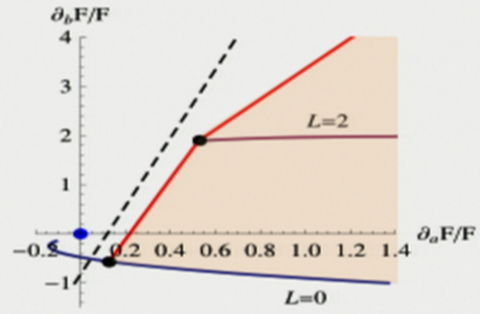


∞ solutions



0 solutions

Going to Extremality

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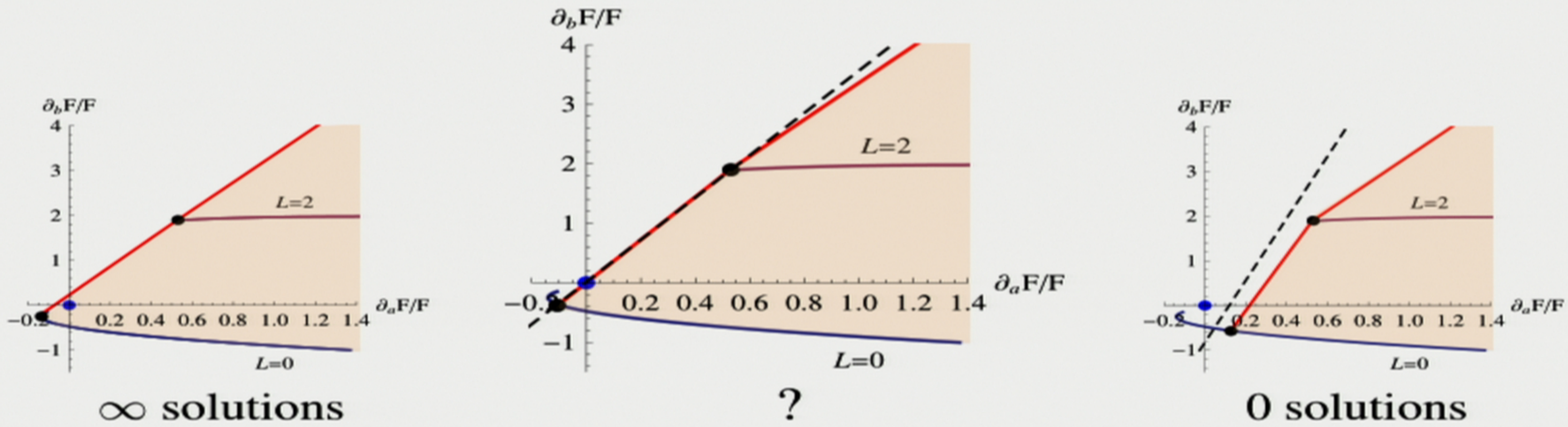
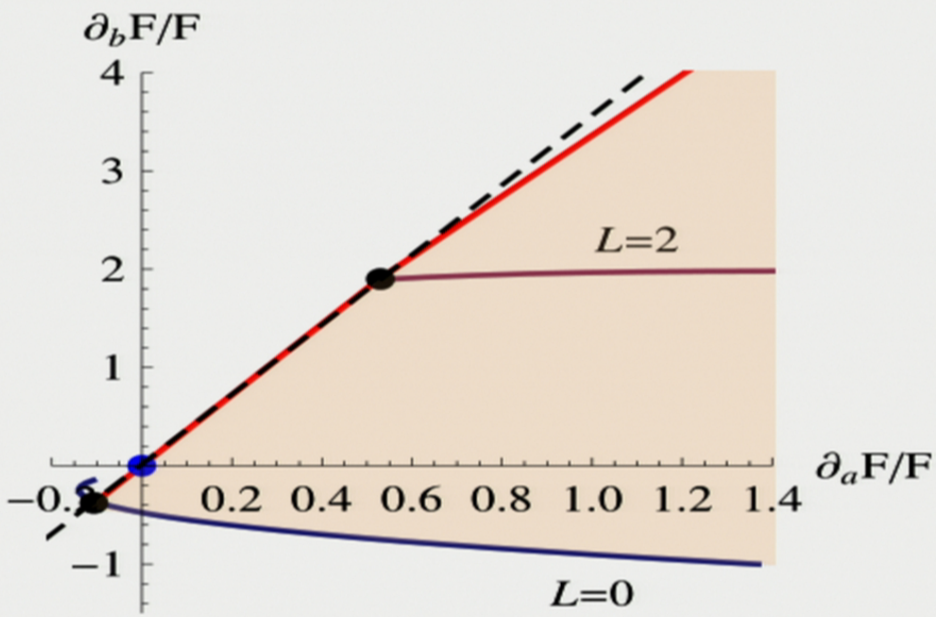


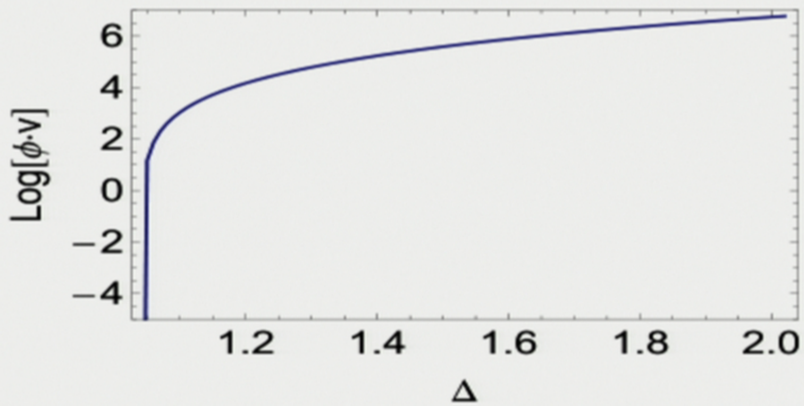
The Extremal Functional



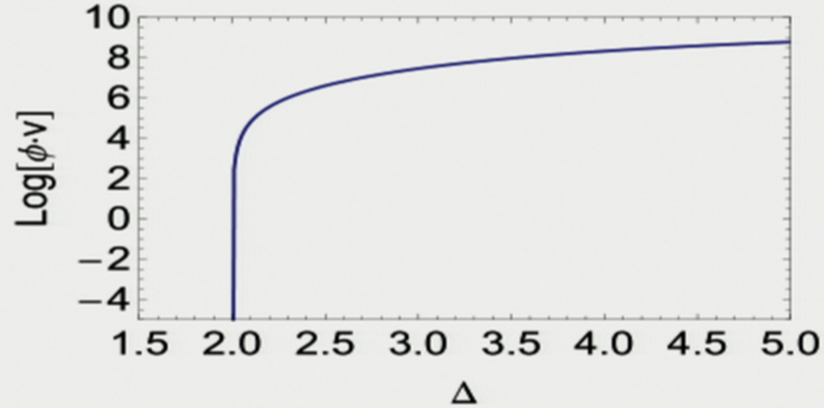
Extremal functional

- Functional obtained just above the bound curve.
- The dashed line is the **extremal functional**.

Positivity of the Extremal Functional



$L = 0$

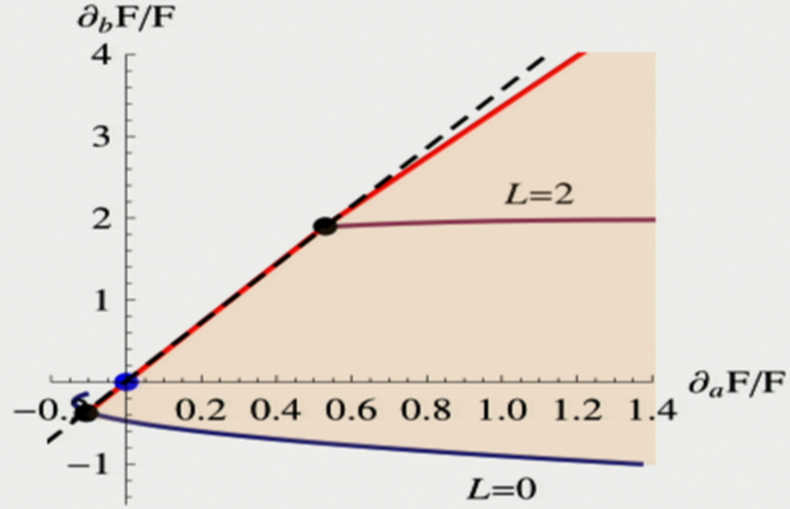


$L = 2$

- Extremal Functional is positive everywhere, except at two special vectors, its zeroes - shown as black dots before.
- Zeroes at approximately $(1.03, 0)$ (the ϵ scalar) and $(2, 2)$ (the stress-tensor!)

Solving Crossing

$$\sum_{\text{Zeroes}} \lambda^2_{\mathcal{O}_{\Delta,L}} \begin{pmatrix} F^{(\sigma)}_{\Delta,L}(\frac{1}{4},\frac{1}{4}) \\ \partial_a F^{(\sigma)}_{\Delta,L}(\frac{1}{4},\frac{1}{4}) \\ \partial_b F^{(\sigma)}_{\Delta,L}(\frac{1}{4},\frac{1}{4}) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



The Extremal Functional Method

- To find a solution to:

$$\sum_{\Delta,L} \lambda_{\mathcal{O}}^2 \, F_{\Delta,L}^{(\sigma)}(u,v) = 1,$$

1. Find the Extremal Linear Functional ϕ.

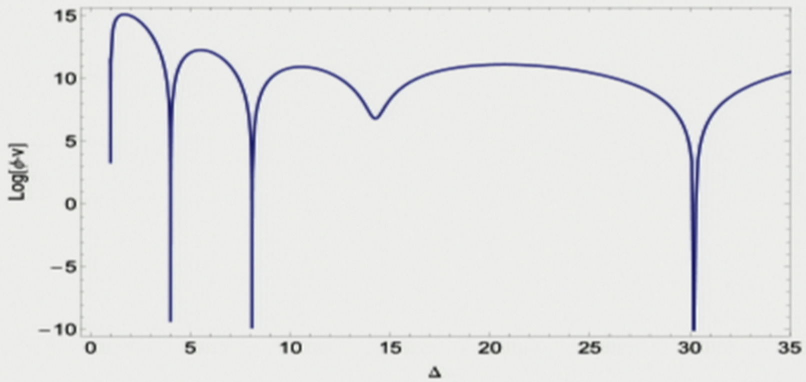
2. Determine the vectors $(F_{\Delta,L}^{(\sigma)}, \partial F_{\Delta,L}^{(\sigma)}, \ldots)$ which are *zeroes* of ϕ.

3. Solve for the linear combination of $F_{\Delta,L}^{(\sigma)}$'s which gives the identity vector. The coefficients are the square of the OPE coefficients.

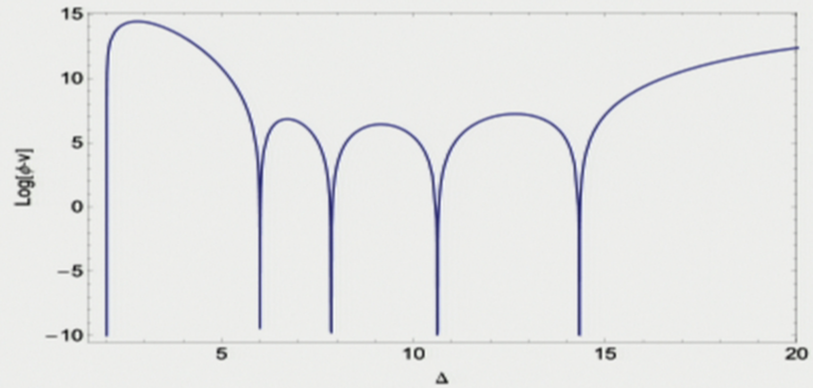
Application: the $D = 2$ Ising model

Finding the zeroes

- Results with $N = 60$ derivatives.



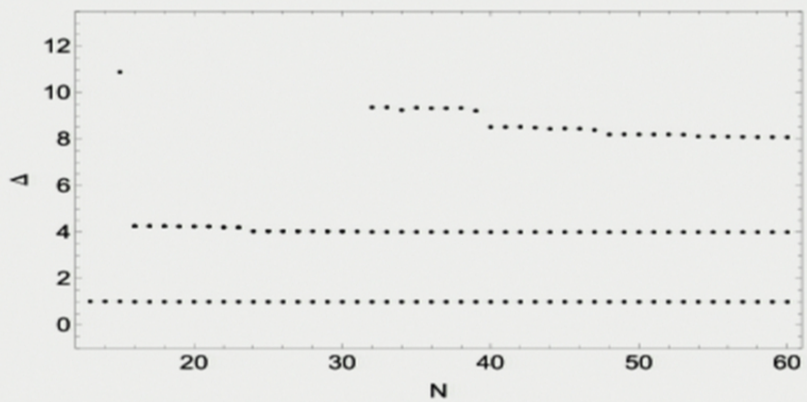
$L = 0$



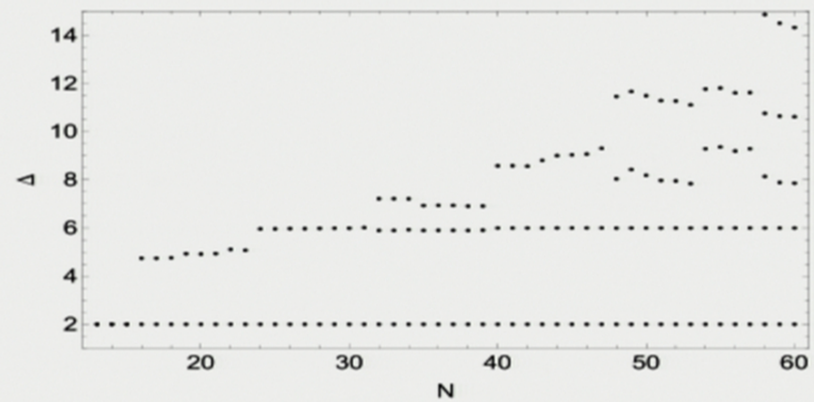
$L = 2$

Operator convergence

- Operator spectrum as N is increased.



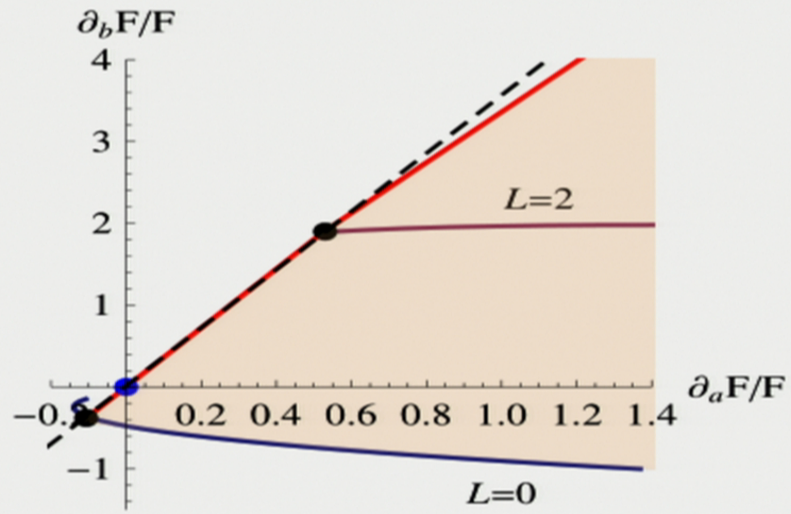
$L = 0$



$L = 2$

Finding OPE coefficients

- Polyhedron has curved directions: number of zeroes is smaller than N.
- Not possible to find unique solution due to small numerical errors.

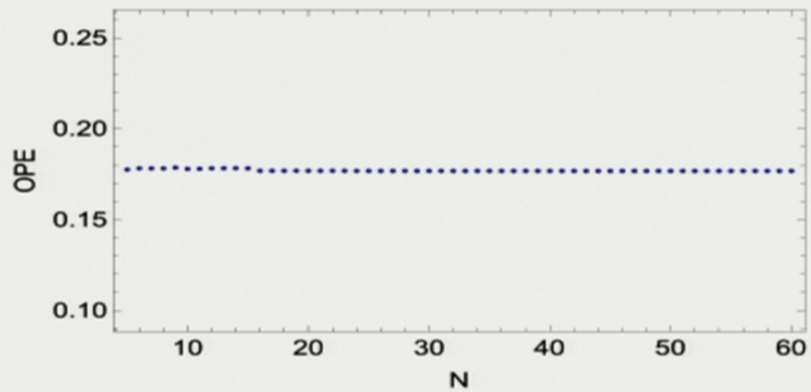


- Find "optimal solution":

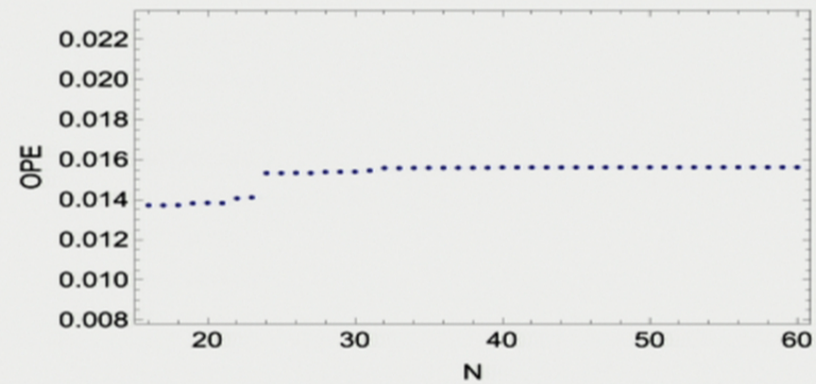
$$\text{OPE Coeffs} = \text{Min}_{\{\lambda_i\}} \left(\sum_{V_i:\ \phi\cdot V=0} \lambda_i^2 V_i - \mathbb{1} \right).$$

OPE convergence

- OPE convergence with N.



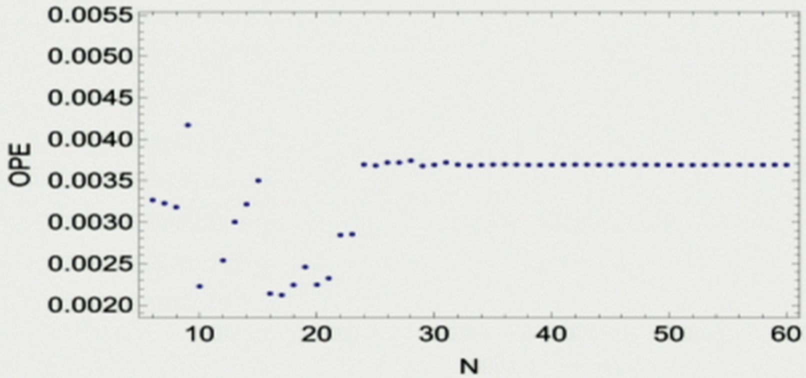
$(2, 2)$



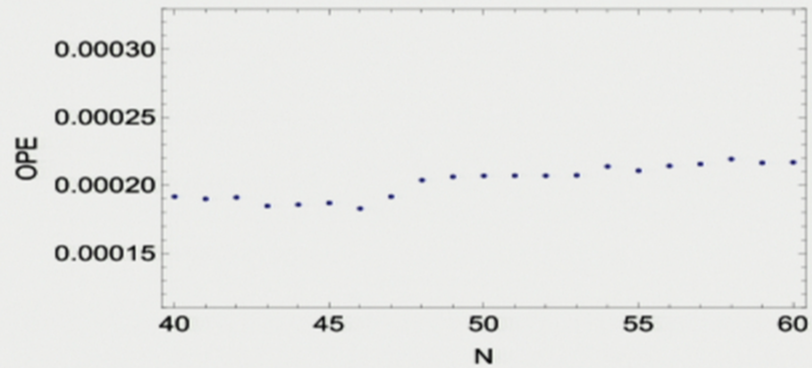
$(4, 0)$

OPE convergence

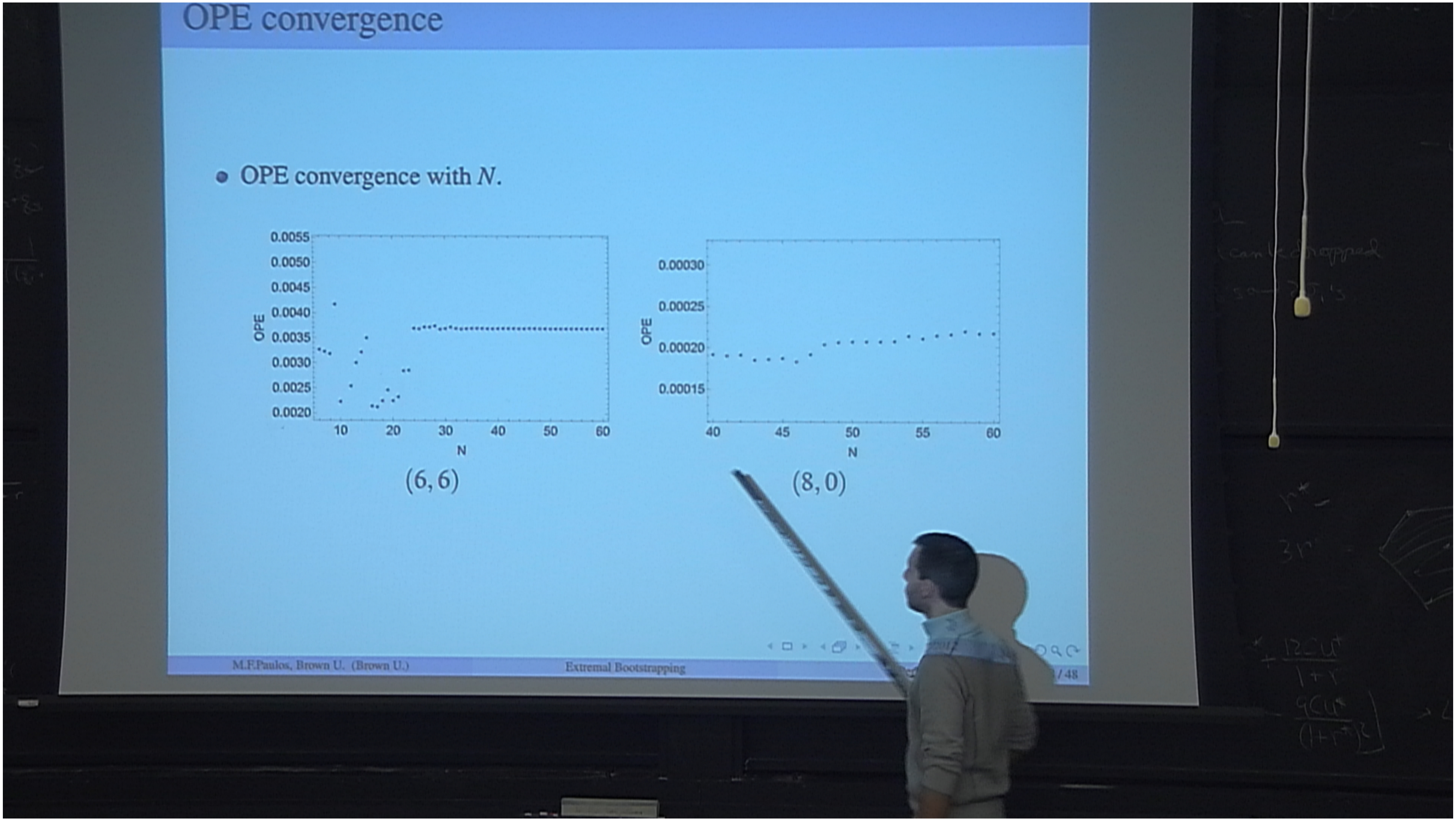
- OPE convergence with N.



$(6,6)$



$(8,0)$


OPE convergence
OPE convergence with N.
0.0055
0.0050
0.0045
0.0040
0.0035
0.0030
0.0025
0.0020
OPE
10
20
30
40
50
60
N
(6, 6)
0.00030
0.00025
0.00020
0.00015
OPE
40
45
50
55
60
N
(8, 0)
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