Title: Anomaly-induced transport and thermodynamics

Date: Nov 13, 2012 02:00 PM

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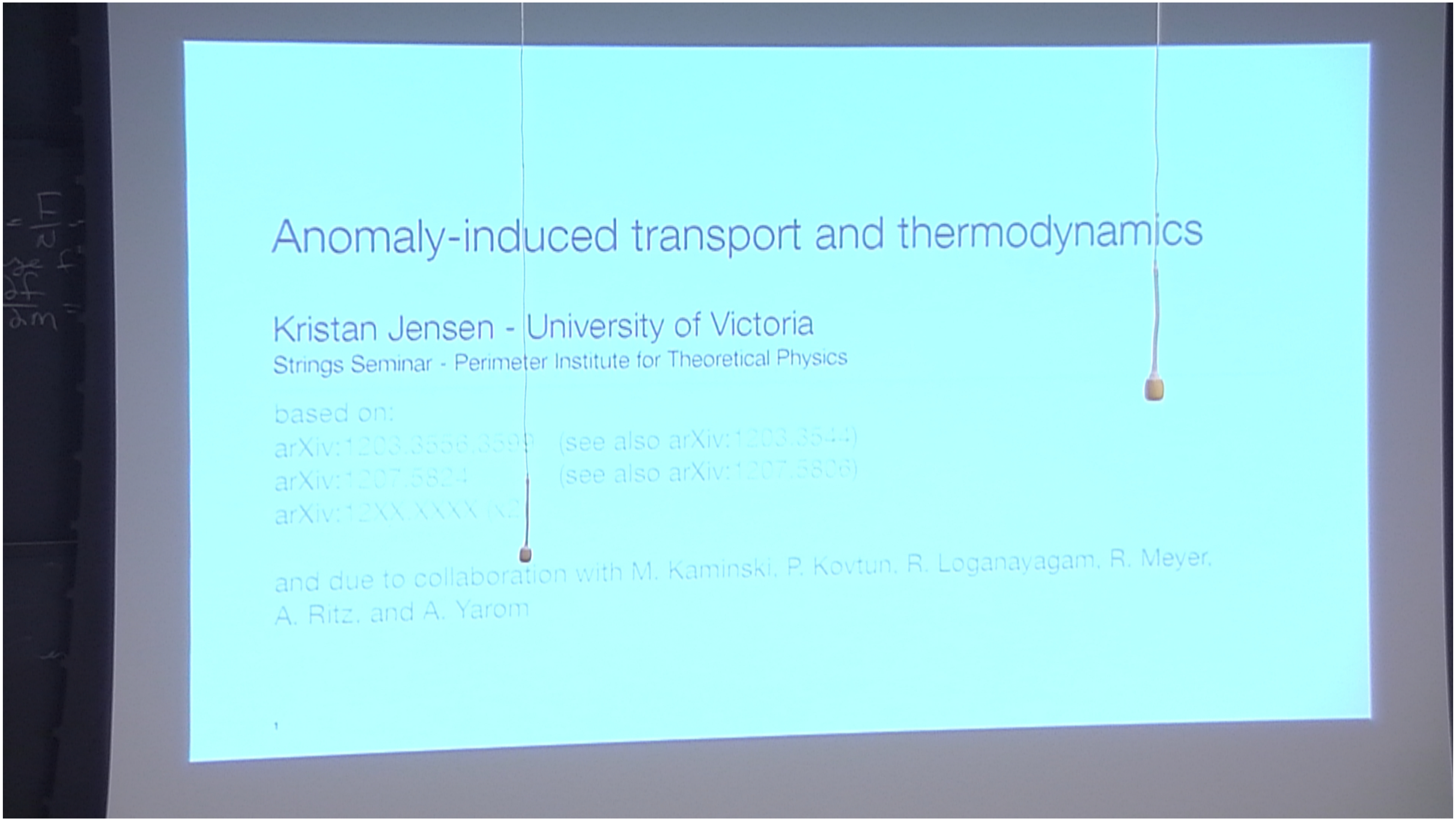
Abstract: I will discuss recent progress in the study of anomaly-induced transport, focusing on the chiral vortical effect in 3+1 dimensions. Most of my discussion will be framed in light of a larger story, namely progress in making exact statements about finite-temperature quantum field theory, for which the chiral magnetic and vortical effects are instructive prototypes.


Anomaly-induced transport and thermodynamics
Kristan Jensen - University of Victoria
Strings Seminar - Perimeter Institute for Theoretical Physics
based on:
arXiv:1203.3556,3599 (see also arXiv:1203.3544)
arXiv:1207.5824 (see also arXiv:1207.5806)
arXiv:12XX.XXXX (x2
and due to collaboration with M. Kaminski, P. Kovtun, R. Loganayagam, R. Meyer,
A. Ritz, and A. Yarom
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The context

1. Chiral magnetic effect (CME), chiral vortical effect (CVE) in 4d.
   - Experimentally accessible, and interesting to theorists

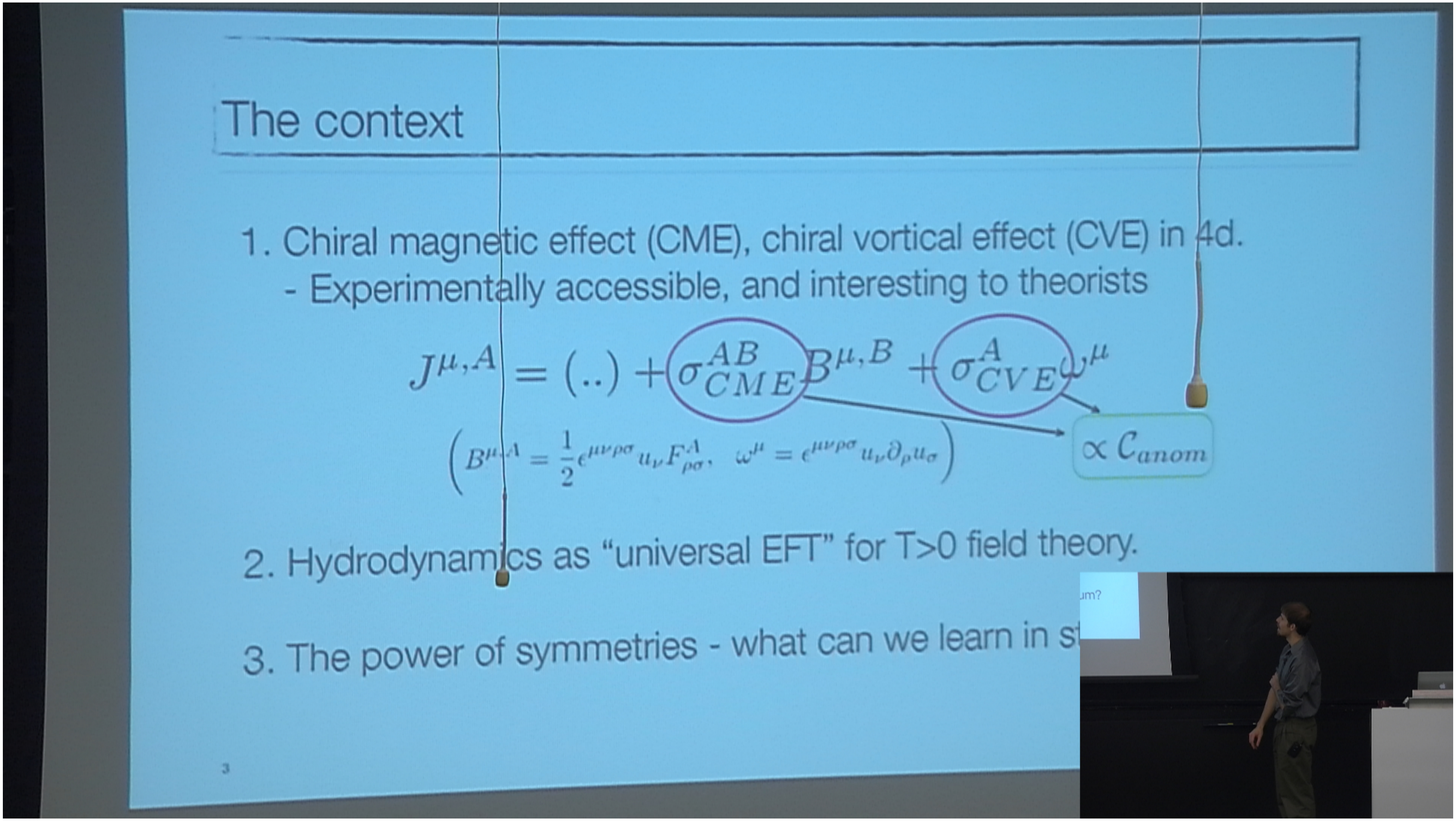
$$J^{\mu,A} = (..) + \sigma^{AB}_{CME} B^{\mu,B} + \sigma^{A}_{CVE} \omega^{\mu}$$

$$\left(B^{\mu,A} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu F^A_{\rho\sigma}, \quad \omega^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma \right)$$

2. Hydrodynamics as “universal EFT” for T>0 field theory.

3. The power of symmetries - what can we learn in static equilibrium?

2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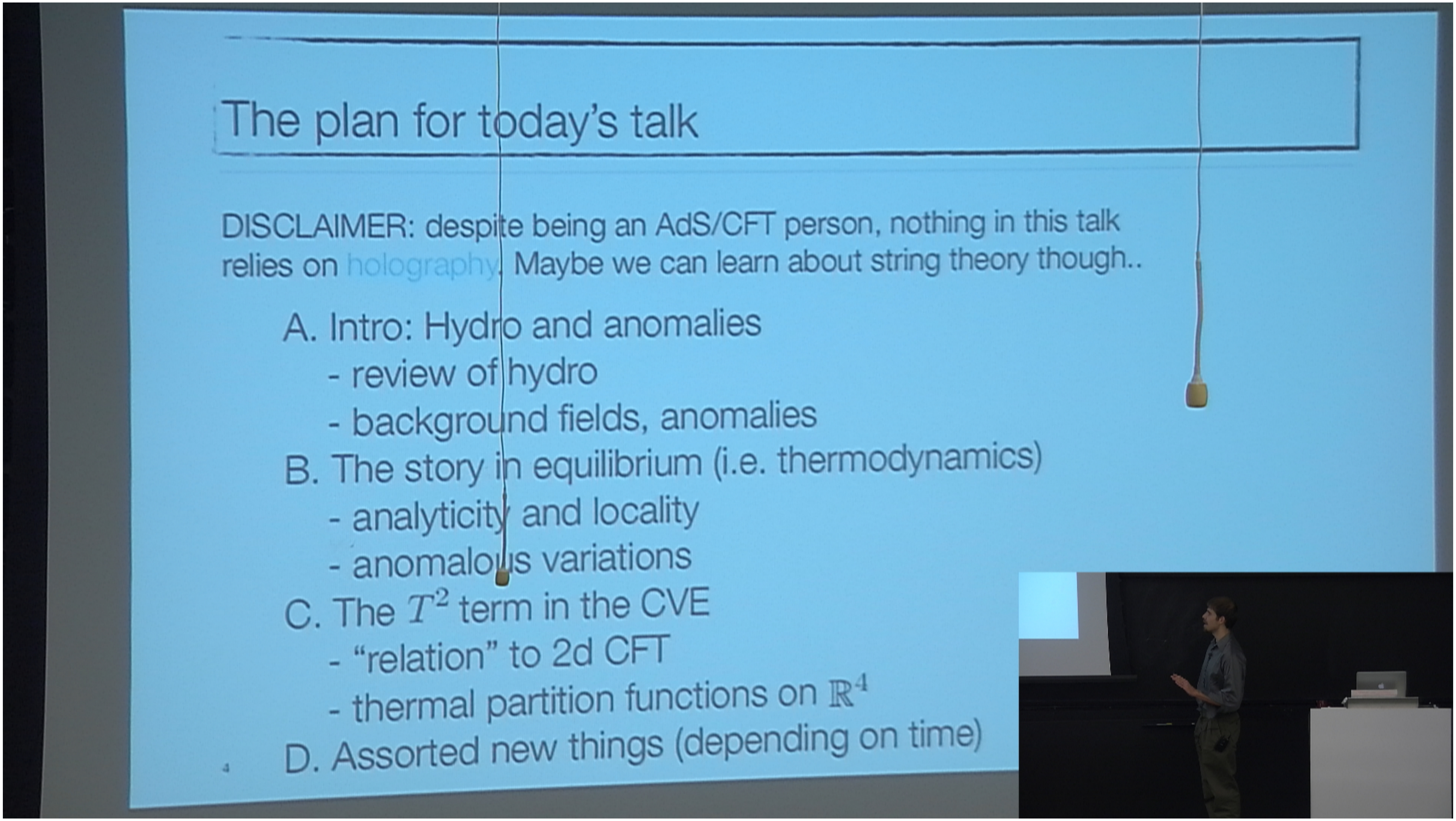
The plan for today's talk
DISCLAIMER: despite being an AdS/CFT person, nothing in this talk relies on holography. Maybe we can learn about string theory though..
A. Intro: Hydro and anomalies
- review of hydro
- background fields, anomalies
B. The story in equilibrium (i.e. thermodynamics)
- analyticity and locality
- anomalous variations
C. The T^2 term in the CVE
- "relation" to 2d CFT
- thermal partition functions on $\mathbb{R}^4$
D. Assorted new things (depending on time)
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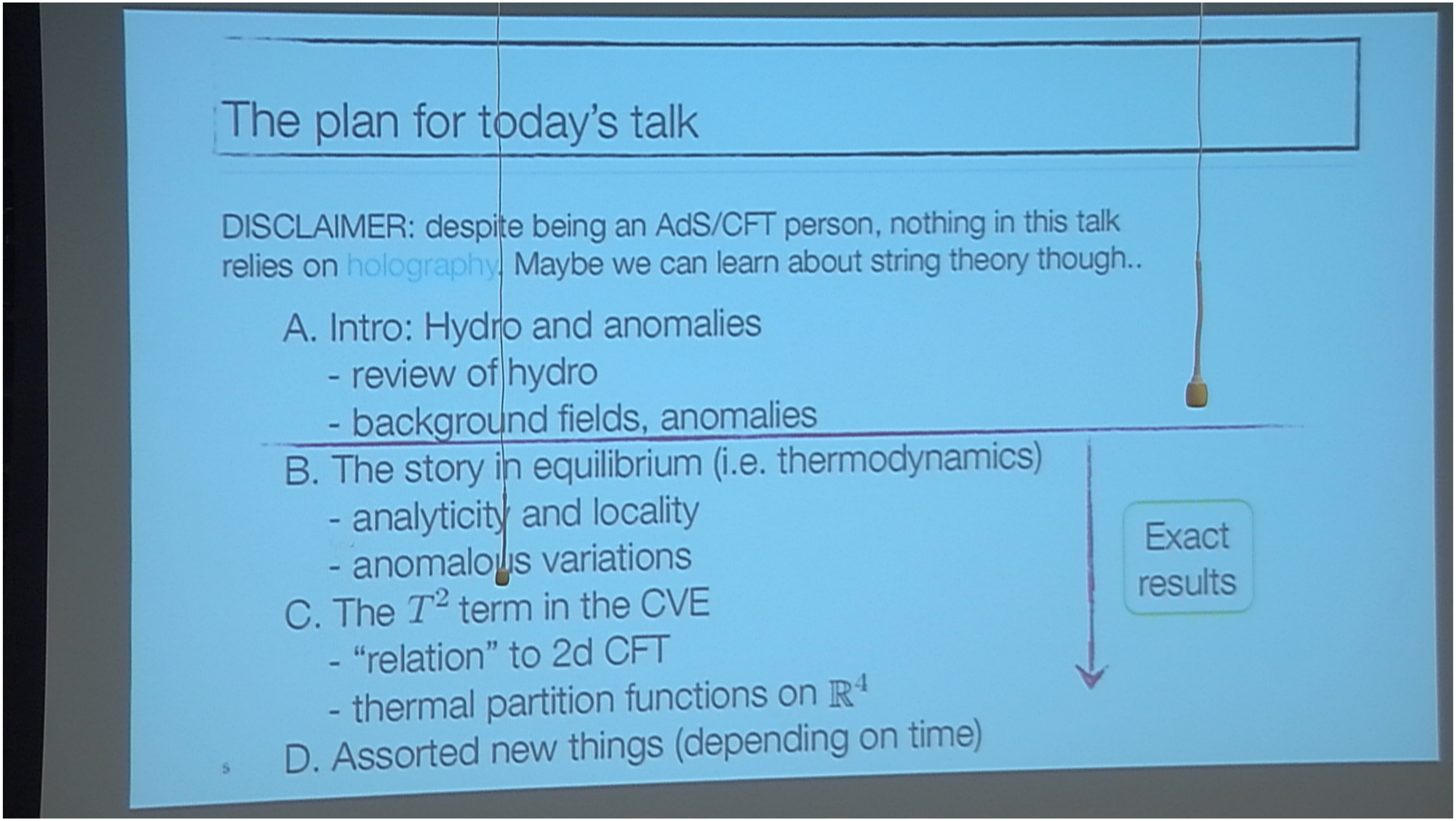
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Exact results
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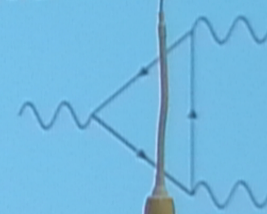
The actors in today's story

Lead actor: 4d, 2d* relativistic field theories with assorted global symmetries ($U(1)^n$, coordinate invariance).



Tragic flaw: global symmetries are anomalous.

BUT: tragic flaws make for revelatory insight *and* excellent theatre.

Character development: flaws revealed under suitable circumstances. Couple (anomalous) symmetry currents to background fields ($A_\mu, g_{\mu\nu}$).

* We could look at odd-d theories too. But no anomalies, so less drama.

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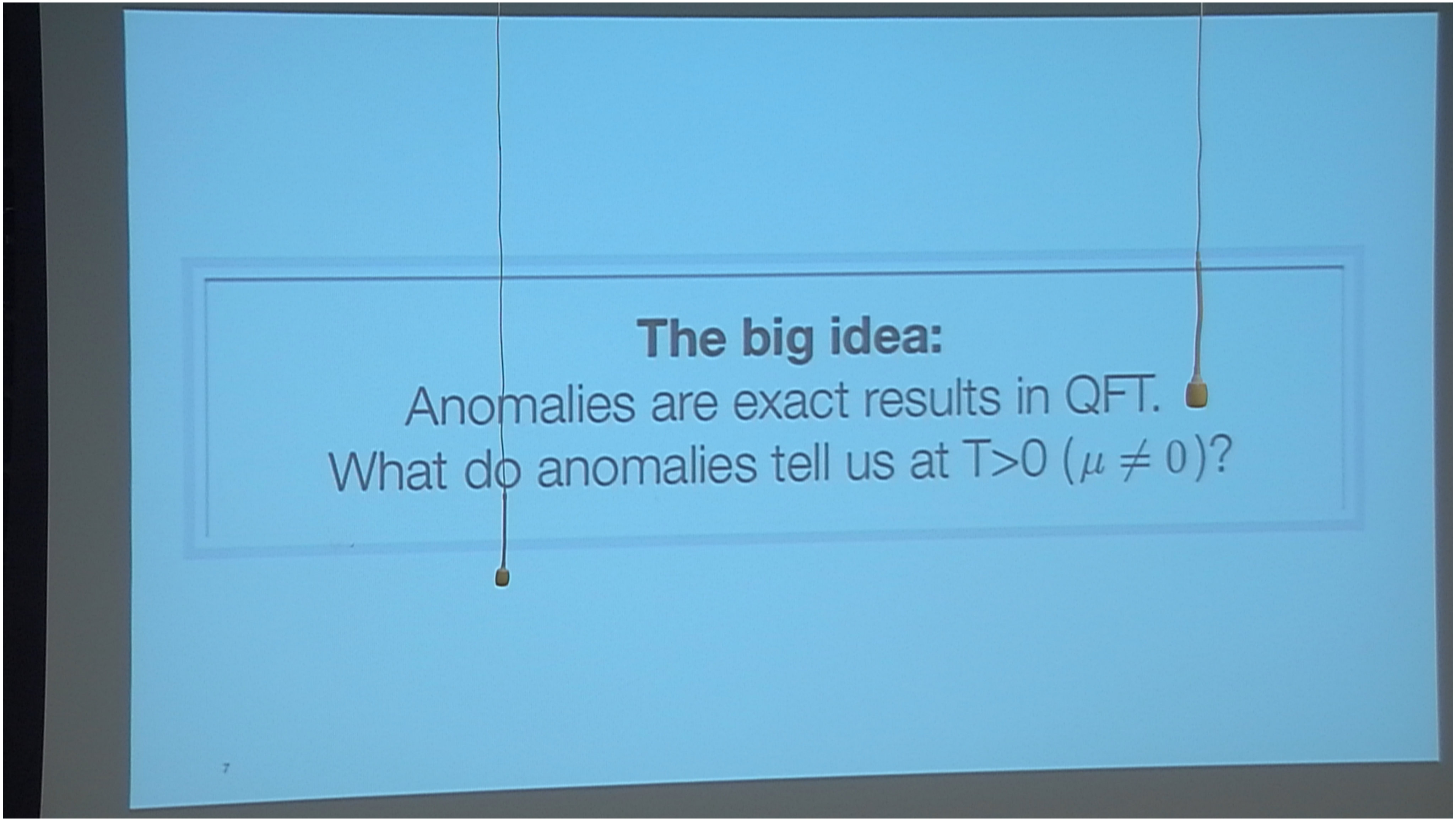
The big idea:
Anomalies are exact results in QFT.
What do anomalies tell us at T>0 ($\mu \neq 0$)?
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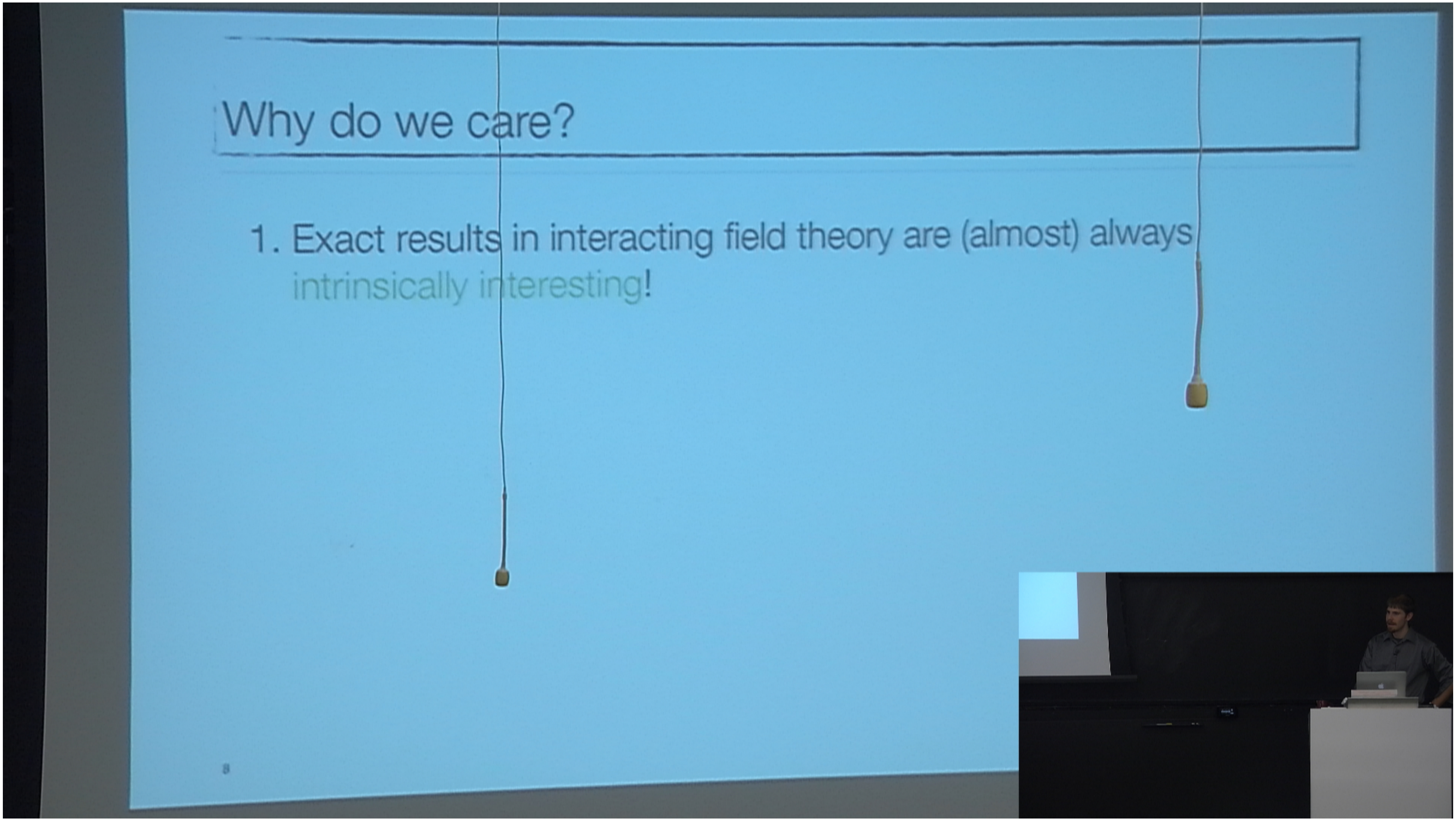
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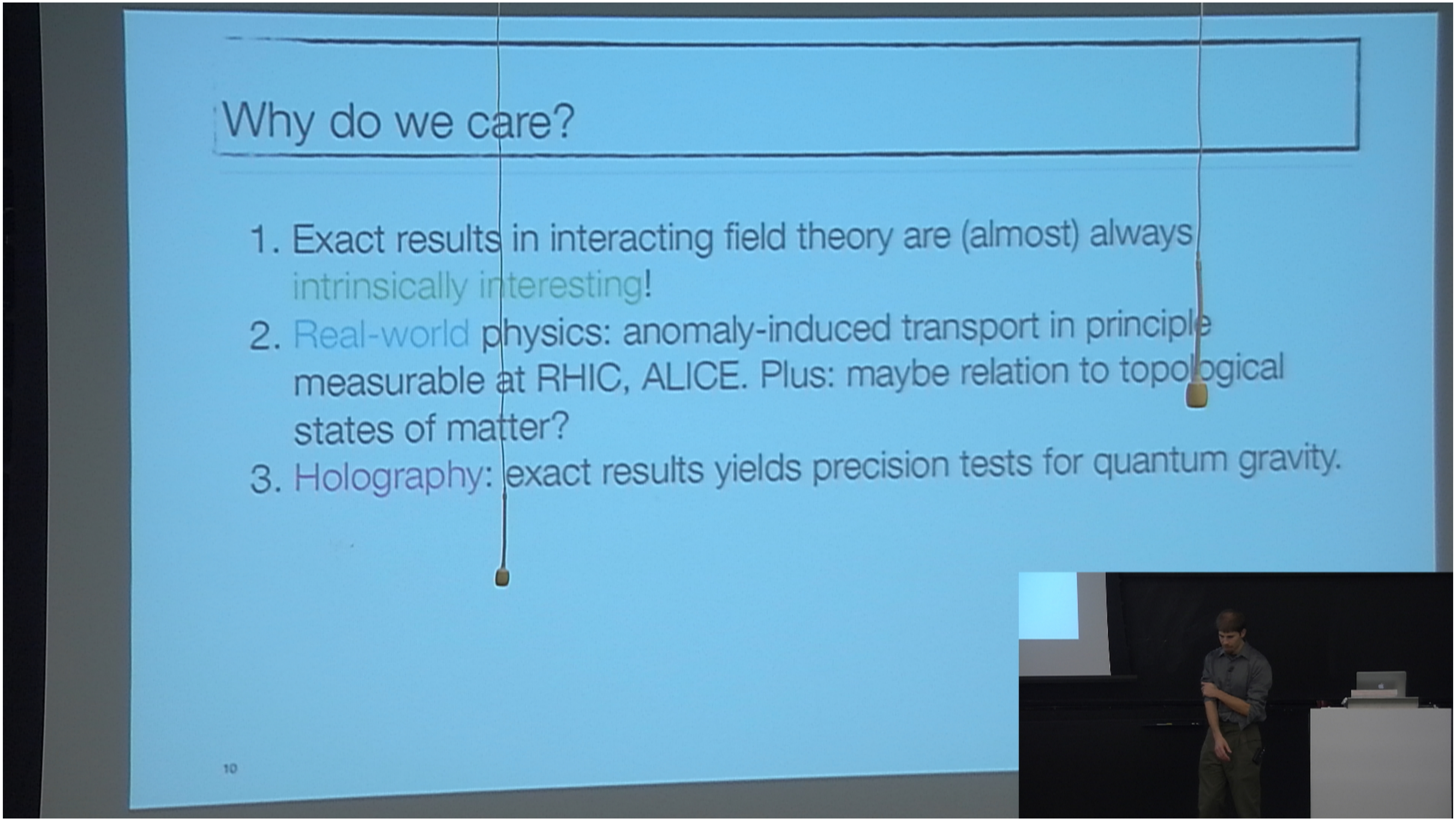
8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3. Holography: exact results yields precision tests for quantum gravity.
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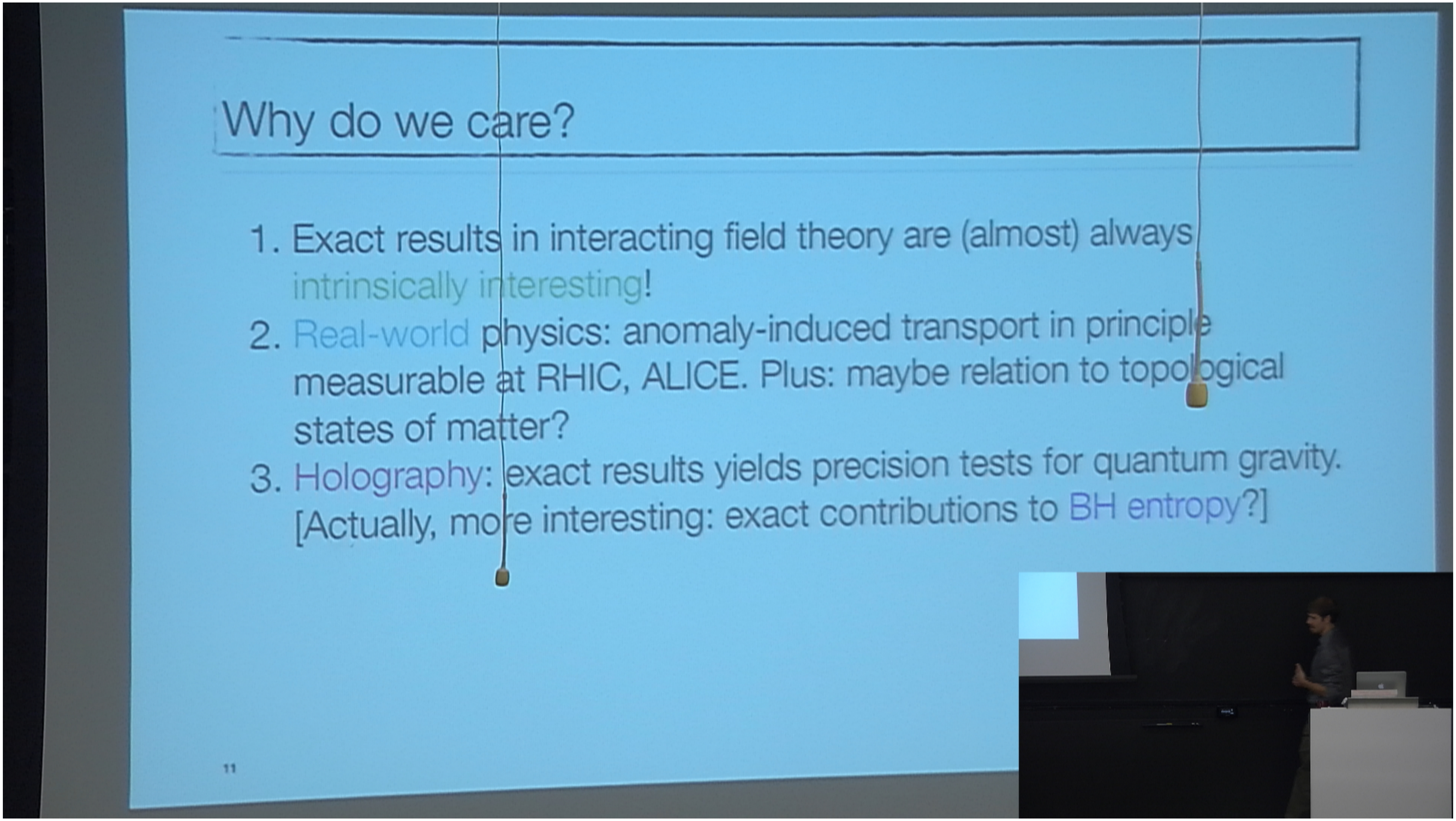
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3. Holography: exact results yields precision tests for quantum gravity. [Actually, more interesting: exact contributions to BH entropy?]
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First things first: anomalies and symmetry currents

Two primary definitions of currents when there are anomalies:

1. Consistent currents. $$J^\mu = \frac{\delta W}{\delta A_\mu}, \quad T^{\mu\nu} = 2\frac{\delta W}{\delta g_{\mu\nu}}$$ → generating functional

NOTE: consistent currents are not gauge-covariant. For instance,

$$(\delta A_\mu)\delta_\Lambda J^\mu = \delta_\Lambda \delta_A W = \delta_A \delta_\Lambda W = -\int d^d x\, \Lambda\, \delta_A(\nabla_\mu J^\mu)$$

2. Covariant currents. $$J^\mu_{cov} = \frac{\delta W}{\delta A_\mu} + P^\mu_{BZ}, \quad T^{\mu\nu}_{cov} = 2\frac{\delta W}{\delta g_{\mu\nu}} + P^{\mu\nu}_{BZ}$$

where P_{BZ} is a local functional of background fields.

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First things first: anomalies and Ward identities

Potential anomalies must satisfy Wess-Zumino consistency condition:
➔ solution allows complete classification (anomaly polynomial).

In two dimensions, gauge and gravitational anomalies:

$$\nabla_\mu J^\mu_{cov} = c_s \epsilon^{\mu\nu} F_{\mu\nu}, \qquad \nabla_\nu T^{\mu\nu}_{cov} = F^\mu{}_\nu J^\nu_{cov} + c_g \epsilon^{\mu\nu} \nabla_\nu R$$

In four dimensions, gauge and mixed gauge-gravitational anomalies:

$$\nabla_\mu J^\mu_{cov} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \left[3 c_A F_{\mu\nu} F_{\rho\sigma} + c_m R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\sigma} \right],$$

$$\nabla_\nu T^{\mu\nu}_{cov} = F^\mu{}_\nu J^\nu_{cov} + \frac{c_m}{2} \nabla_\nu \left[\epsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} \right]$$

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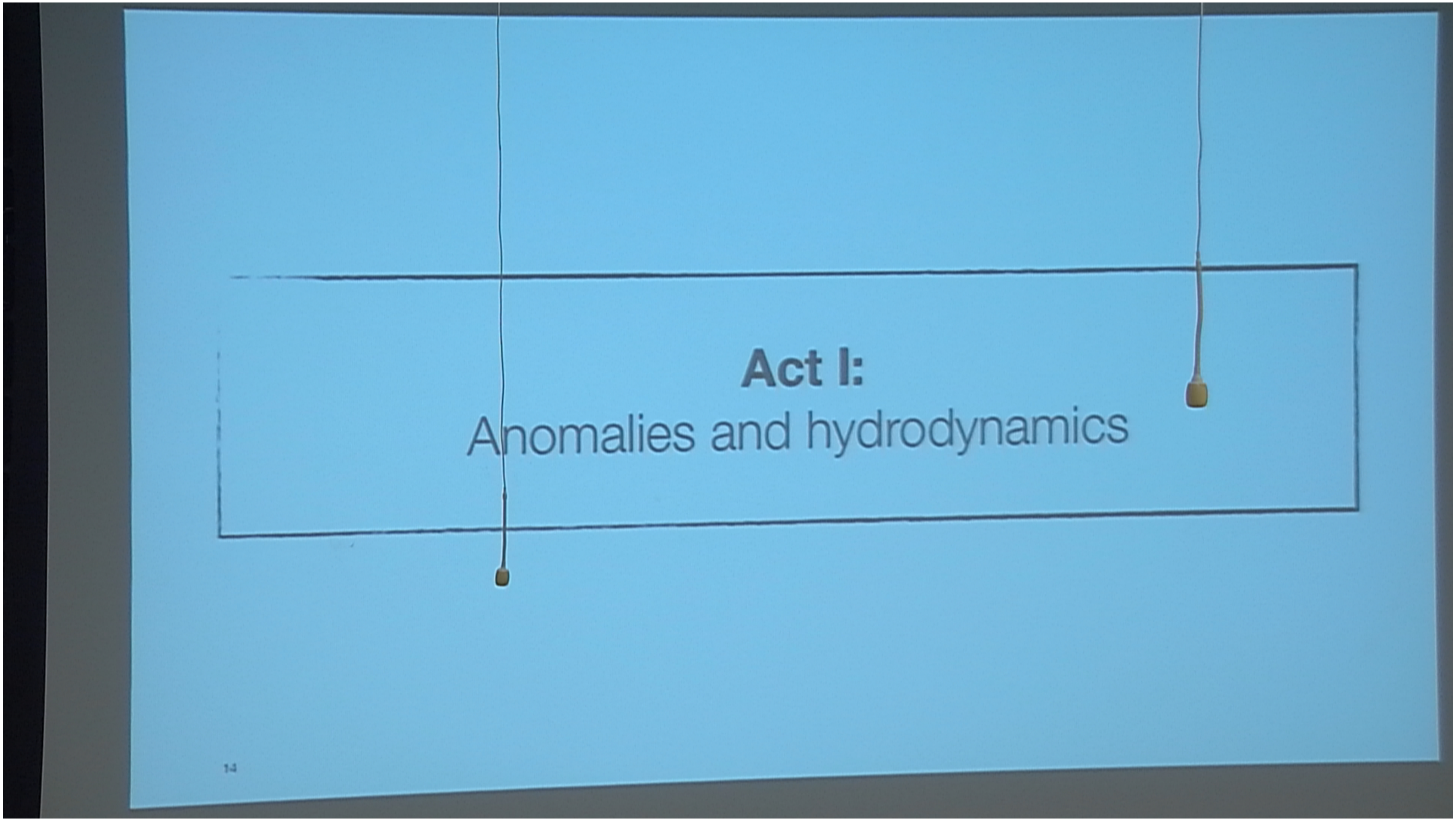
Act I:
Anomalies and hydrodynamics
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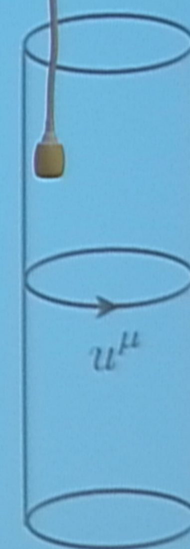
Primer to hydrodynamics (Landau, Lifshitz in nutshell)

Start with translationally-invariant equilibrium state in flat space at T>0:

➔ States parametrized by $(T, \mu, u^\mu), u^2 = -1$

(In Euclideanized theory $T = 1/L,\ \mu = \ln P_A/L$)



$$J^\mu = \rho u^\mu,$$

$$SO(1, d-1) \to SO(d-1) \Rightarrow \quad T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu},$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Big idea #1: study small fluctuations around equilibrium by promoting $(T, \mu, u^\mu) \to (T(x), \mu(x), u^\mu(x))$

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Primer to hydrodynamics - II

Big Idea #2: express J^μ, $T^{\mu\nu}$ in derivative expansion [Bhattacharyya, et al].
Allow arbitrary coefficients in front of all tensor structures, e.g.

$$T^{\mu\nu} = (..) + \zeta(\nabla \cdot u)\Delta^{\mu\nu} + ..$$

ζ → Bulk viscosity

Big idea #3: determine $(T(x), \mu(x), u^\mu(x))$ by imposing Ward identities:

$$\nabla_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu,$$

$$\nabla_\mu J^\mu = 0$$

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Primer to hydrodynamics - II

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Primer to hydrodynamics - III

Big idea #4: use field redefinitions (e.g. $T \to T + a(T,\mu)(\nabla \cdot u)$), along with solution to conservation eq'ns to eliminate coefficients in constitutive relations.

➔ Known as change of hydro frame.

NOTE: almost same language as involved in constructing EFT!

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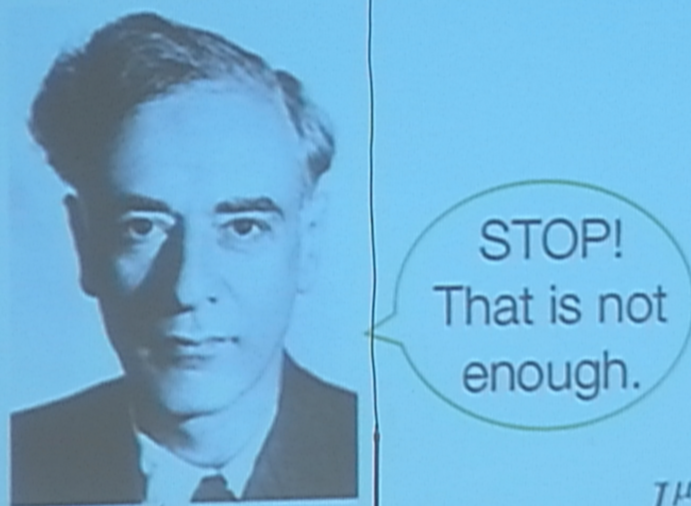
Argument *ad* Landau



Should also demand local version of second law: $\exists\, s^\mu$ so that $\nabla_\mu s^\mu \geq 0$

➔ "entropy current"

In first-order hydro, this tells you

$$J^\mu = \rho u^\mu + \sigma E^\mu - T\Delta^{\mu\nu}\partial_\mu \frac{\mu}{T},$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (P - \zeta(\nabla\cdot u))\Delta^{\mu\nu} - \eta\sigma^{\mu\nu},$$

$$\eta > 0,\ \zeta > 0,\ \sigma > 0$$

Now bring in the anomalies

Now: what happens with gauge anomalies in 4d? [Son, Surowka]

- modified Ward identities: $\nabla_\mu J^\mu_{cov} = \frac{3c_A}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$
- broken parity means more transport coefficients: $J^\mu_{cov} = (..) + \xi_B B^\mu + \xi_\omega \omega^\mu$

$$R = \frac{\rho}{\epsilon + P}$$

Solution for entropy current fixes ξ_B, ξ_ω

- which end up dissipationless (can be understood from P,T [Kharzeev, Yee])

$$\xi_B = -6c_A\mu\left(1 - \frac{R\mu}{2}\right) + f_1 T\left(1 - R\mu\right) - f_2 R T^2,$$

$$\xi_\omega = -3c_A\mu^2\left(1 - \frac{2R\mu}{3}\right) + f_1\mu T\left(1 - R\mu\right) + f_2 T^2(1 - 2R\mu) + f_3 T^3$$

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CPT [later]

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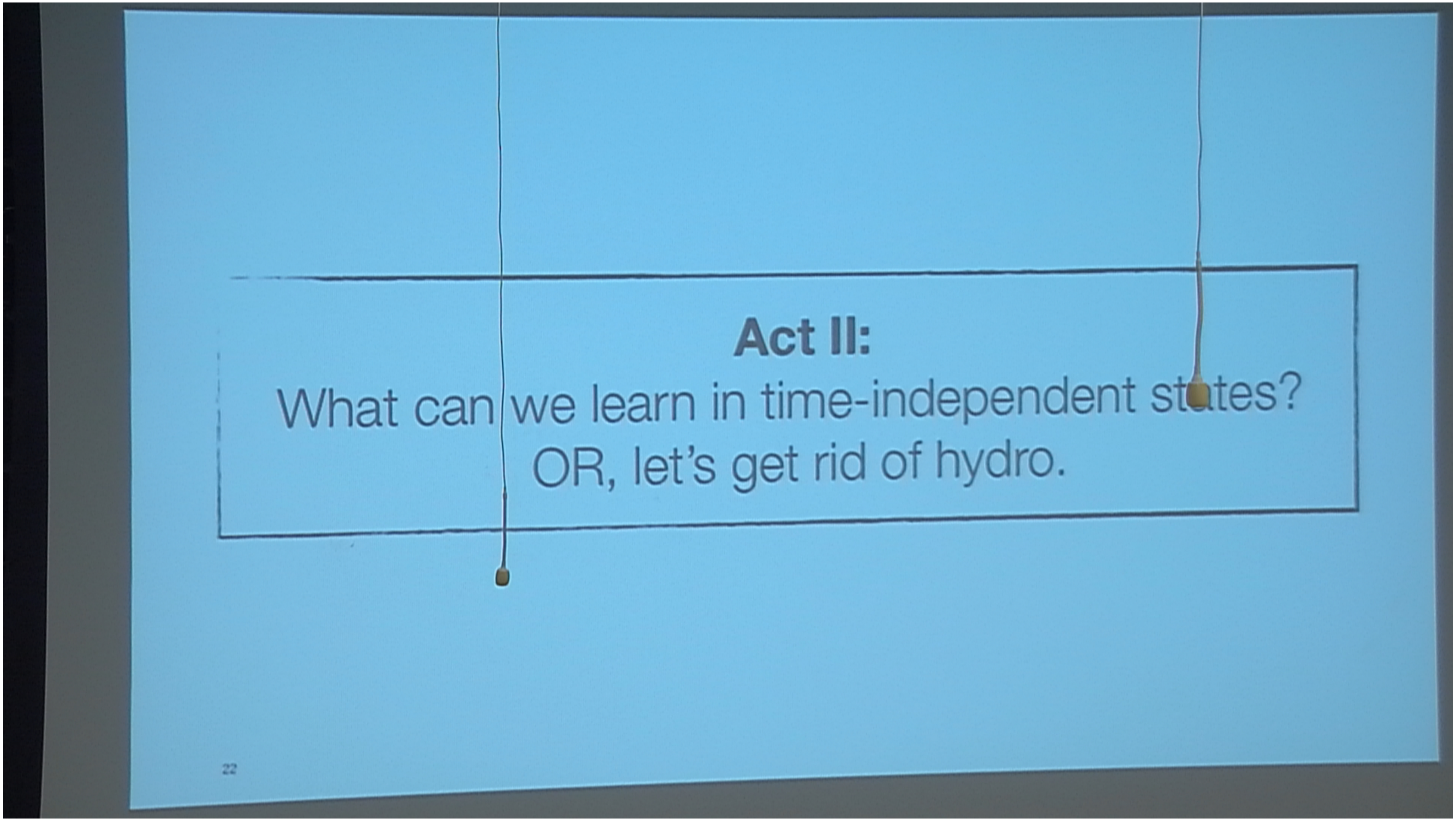
Act II:
What can we learn in time-independent states?
OR, let's get rid of hydro.
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Zero-frequency correlators and analyticity

Two useful facts about $\omega = 0$ correlators:

1. All real-time functions (ra, raa, rra, &c) are $\propto$ Euclidean correlators [Evans].
2. Most theories are have <..> ~ $O(1) + O(k) + O(k^2) + \dots$

Manifestation of screening. Holds except for special cases:
superfluid phase, U(1) gauge field, critical points

Suppose we take {<..>} and Taylor expand to $O(k^n)$. Call these $\{\langle .. \rangle_n\}$

- Fourier transform to position space: get **local** $\{\langle .. \rangle_n\}$
- Imagine constructing $W_n = \int dt d^{d-1}x \, \langle J^\mu(x) \rangle A_\mu(x) + \dots$

23 result will be local functional of background fields!

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The Equilibrium generating functional

Summary: there exists a local functional of time-independent $A_\mu, g_{\mu\nu}$ which reproduces low-momentum static correlators [Jensen, et al] [Banerjee, et al].

Next, parametrize possible W_n by classifying local scalars to $\mathcal{O}(\partial^n)$

Time-independence amounts to timelike vector field v with $\mathfrak{L}_v = 0$

- can think of geometry as U(1) fibration over spatial slice
- write down all observables local in space (including those which are non-local in Euclidean time)

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The Equilibrium generating functional - II

What sort of terms do we get? Parity-preserving ones are..

$$O(\partial^0): T(x) = \frac{1}{L(x)},\ \mu(x) = \frac{1}{L(x)} \ln P_A(x),\ u^\mu = \frac{v^\mu}{\sqrt{-v^2}},$$

$$O(\partial^1): E_\mu = F_{\mu\nu}u^\mu,\ a^\mu = u^\nu \nabla_\nu u^\mu,\ \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\rho}\Delta^{\nu\sigma}(\nabla_\rho u_\sigma - \nabla_\sigma u_\rho)$$

So, to first order in derivatives, we have for P-preserving theory:

$$W_1 = \int dt d^{d-1}x \sqrt{-g} P(T,\mu) \longrightarrow$$

Compute $T^{\mu\nu}$, J^μ; get ideal hydro*.

*Look at second order in derivatives, or first-order 2+1d with P-violation. Matches equilibrium hydro!

[Jensen, et al] [Bhattacharya, et al]

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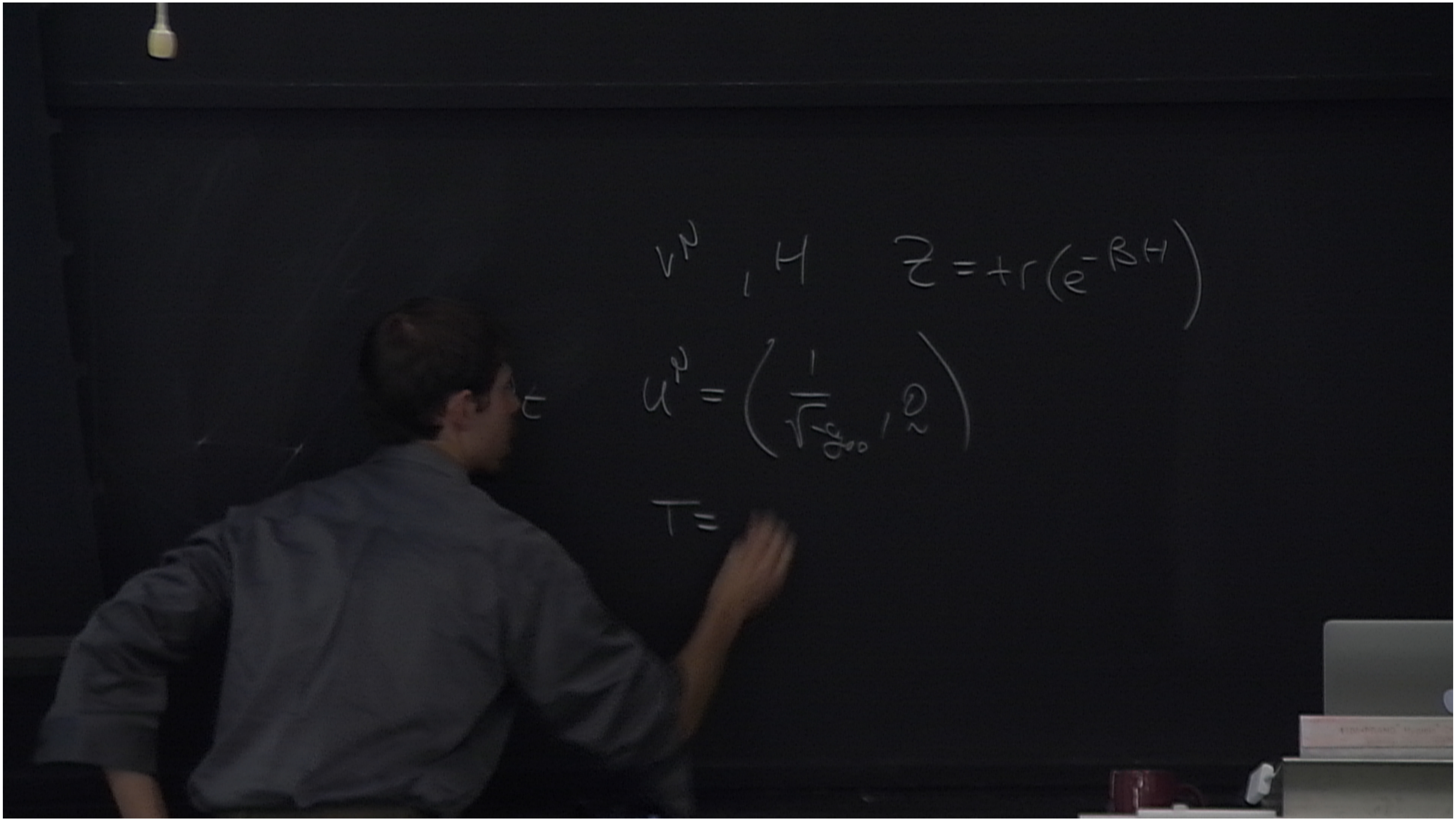
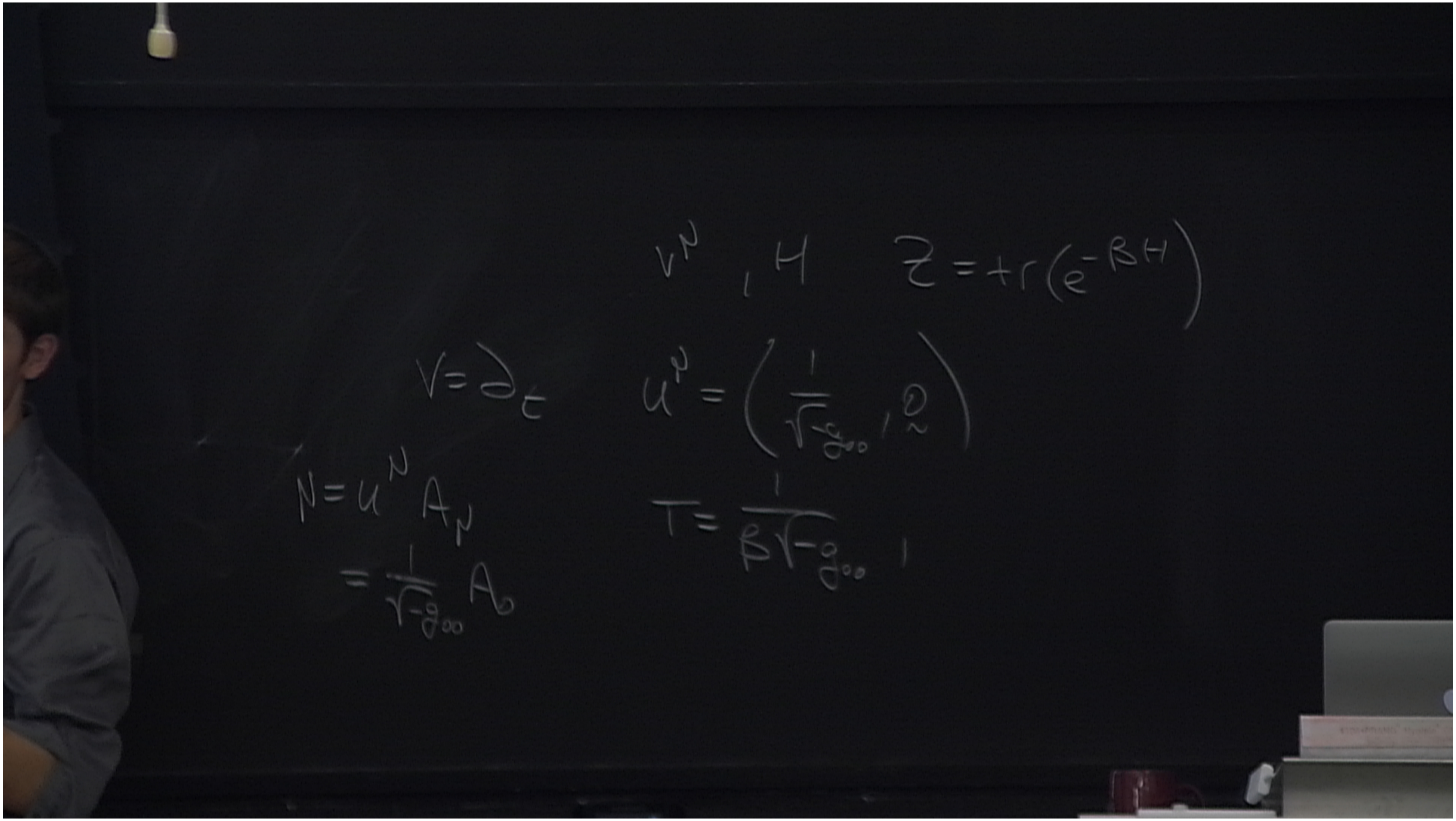

V^N , H   Z = tr(e^{-βH})
u^ν = (1/√(-g_00) , 0)
T =


v^ν, H $Z = tr(e^{-\beta H})$
$v = \partial_t$
$u^\nu = \left(\frac{1}{\sqrt{-g_{00}}}, 0\right)$
$T = \frac{1}{\beta\sqrt{-g_{00}}}$
$\mu = u^\nu A_\nu$
$= \frac{1}{\sqrt{-g_{00}}} A_0$

The Equilibrium generating functional - III

What's the **physics** of this construction?

- The procedure manifests all of the various Ward identities and properties of Euclidean field theory: [KJ]

1. Analyticity of zero-frequency correlators at k=0.
2. Local gauge/coordinate invariance.
3. "Global" Ward identities (due to thermal circle).
4. Zero-frequency reciprocity (bosonic variational derivatives commute)

$$\langle ..\mathcal{O}_1(k_1)..\mathcal{O}_2(k_2)..\rangle = \langle ..\mathcal{O}_2(k_2)..\mathcal{O}_1(k_1)..\rangle$$

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Now bring back the anomalies

Specialize to 4d theory with U(1) anomaly and so P-violation.

- In order to get: $\nabla_\mu J^\mu_{cov} = \frac{3c_A}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$
- We need: $\delta_\Lambda W = -\frac{c_A}{4}\int d^4x\,\Lambda\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$

We can get this, along with an extra CPT-preserving term by

$$W_1 = \int d^4x\sqrt{-g}\left(P(T,\mu) + \tilde{c}_{4d}\beta^{-1}T(B^0 + \mu\omega^0)\right),$$

$$W_A = -2c_A\int d^4x\sqrt{-g}\mu A_\mu\left(B^\mu + \frac{\mu}{2}\omega^\mu\right)$$

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Must be constant

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Match to hydrodynamics

Next, compute $J^{\mu}_{cov}, T^{\mu\nu}_{cov}$; match to Landau frame hydro..

And get the *exact* same result which follows from entropy current (with $\tilde{c}_{4d} \propto f_2$).

Physics observations:

1. Chiral conductivities are consequence of consistent thermodynamics with background fields.
2. Anomalous part of W is local, in contrast with Lorentz covariant case.
3. We're a good chunk toward the goal of replacing hydro. with EFT.
4. However, we still have an unfixed coefficient: $\tilde{c}_{4d}$

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Act III:

Fixing $\tilde{c}_{4d}$ in terms of the mixed anomaly

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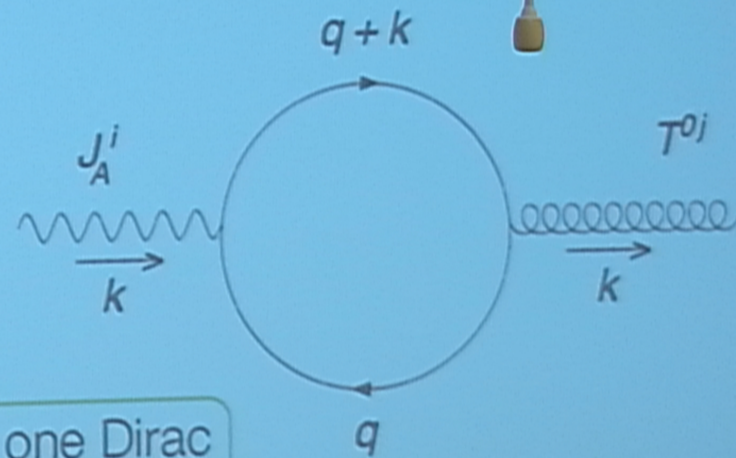
Some backstory: calculations at weak, strong coupling

Kubo formulae for chiral conductivities: [Amado, Landsteiner, Pena-Benitez]

$$\langle J^i(k) T^{0j}(-k) \rangle = i\epsilon^{ijk} k_k (3c_A \mu^2 - \tilde{c}_{4d} T^2) + O(k^2)$$

- Calculation for free Dirac fermion:

[Landsteiner, Megias, Pena-Benitez] [Vilenkin]



1-loop diagram gives

$$\tilde{c}_{4d} = \frac{1}{12} = -8\pi^2 c_m$$

For one Dirac fermion

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Some backstory: calculations at weak, strong coupling

At strong coupling, use AdS/CFT! [Landsteiner, Megias, Melgar, Pena-Benitez]

- Mixed anomaly described in bulk as mixed Chern-Simons term

$$S_{CS} \propto c_m \int A \wedge \mathrm{tr}(R \wedge R)$$

- Calculation of $\langle J^i(k) T^{0j}(-k) \rangle$ from linear response around black brane gives..

$$\tilde{c}_{4d} = -8\pi^2 c_m$$

→ Identical to weak-coupling. Conjecture: identity!

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New tools required

Two *a priori* reasons why previous methods (including the entropy current) will never give you $\tilde{c}_{4d} = -8\pi^2 c_m$:

1. Mixing orders in the derivative expansion. Mixed anomaly corresponds to $W_A \sim O(\partial^3)$, while $\tilde{c}_{4d} \sim O(\partial)$.

2. The transcendental factor of $8\pi^2$: the entropy current or constraints from Ward identities give algebraic relations between coefficients.

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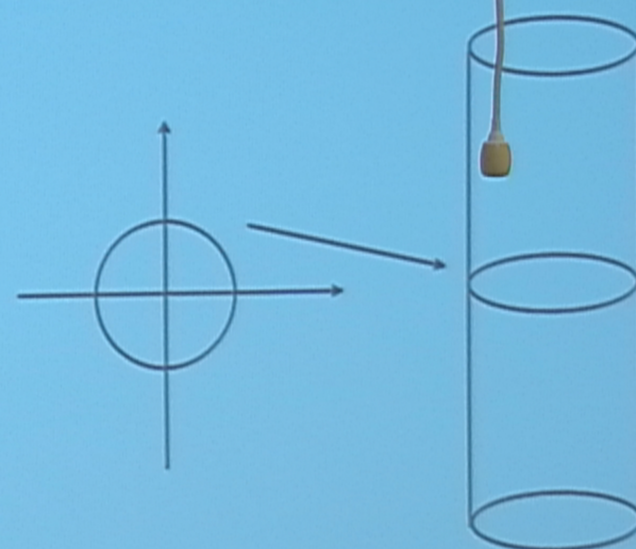
But all is not lost: Cardy formula

But we know a classic example in 2d CFT that behaves in just this way!

Pressure of 2d CFT related to central charge as [Affleck], [Bloete, Cardy, Nightingale]

$$P = 4\pi^2 c_w T^2, \; T^\mu_\mu = c_w R$$

Arises because vacuum state is conformal to thermal state, pressure comes from inhomogeneous Weyl transformation.



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Recovering the Cardy formula

We'd like to use the generating functional to get the same result. How?

Big idea: the generating functional computes $T^{\mu\nu}$, &c on manifolds with U(1) isometry and non-trivial cycle.. like $\mathbb{R}^{2,*}$, where we take the Euclidean time to be the angular variable.

$$g = r^2 d\tau^2 + r^2 + h$$

$$\Rightarrow u^\mu \partial_\mu = \frac{i}{r}\partial_\tau, \; T(r) = \frac{1}{2\pi r}$$

$$(\nabla T^{-1})^2 = 4\pi^2$$

derivative expansion breaks down!

Two potential issues:

1. Breakdown of derivative expansion (can get away with it if we're careful).
2. Assumes $\mathcal{Z}_{\mathbb{R}^{2,*}} = \mathcal{Z}_{\mathbb{R}^2}$ (usually true).

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Recovering the Cardy formula - II

For 2d CFT, conformal symmetry constrains W to be:

$$W = \int d^2x\sqrt{-g}\,[p_0 T^2 + \tilde{c}_{2d}\beta^{-1}T\epsilon^{0\mu}u_\mu + c_w a_\mu a^\mu - c_g u_\alpha u^\beta \epsilon^{\mu\nu}\partial_\nu \Gamma^\alpha_{\beta\nu}]$$

Pressure — Analogue of $\tilde{c}_{4d}$. — Conformal anomaly — Gravitational anomaly

Note: don't have to worry about breakdown, since this is it.

$$T^{\mu\nu}_{cov} = \left(\frac{p_0}{4\pi^2} - c_w\right) r^{-2}(g^{\mu\nu} + u^\mu u^\nu) + \left(\frac{\tilde{c}_{2d}}{4\pi^2} + 2c_g\right) r^{-2} u^{(\mu}\epsilon^{\nu)\rho}u_\rho$$

$\Rightarrow$

$$p_0 = 4\pi^2 c_w,$$
$$\tilde{c}_{2d} = -8\pi^2 c_g$$

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2d theories without conformal symmetry

When we lose CFT, infinite number of terms in W (asymptotic series).

Now what?

2d theories without conformal symmetry

When we lose CFT, infinite number of terms in W (asymptotic series).

Now what?

Observe: no diffeo. invariant scalars linear in $h_{\tau r}(r)$ around flat $\mathbb{R}^{2,*}$,
EXCEPT term with $\tilde{c}_{2d}$ (and grav. anomaly)

Implies that flat space $T^{\tau r}_{cov}$ *only* comes from $\tilde{c}_{2d}$, c_g : gives

$$T^{\tau r}_{cov} = \frac{i(\tilde{c}_{2d} + 8\pi^2 c_g)}{4\pi^2 r^3} \Rightarrow \boxed{\tilde{c}_{2d} = -8\pi^2 c_g}$$

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One slide for the 4d result

One can calculate more directly, but the simplest way is:

Put 4d theory on $\mathcal{M}_2 \times \mathbb{S}^2$, with cycle inside $\mathcal{M}_2$ and flux on $\mathbb{S}^2$

1. 4d mixed gauge-gravitational anomaly --> 2d gravitational anomaly

$$c_g = \Phi_B c_m$$

2. Term with $\tilde{c}_{4d}$ reduces to term with $\tilde{c}_{2d}$ (reduction of CS term)

$$\tilde{c}_{2d} = \Phi_B \tilde{c}_{4d}$$

$$\tilde{c}_{2d} = -8\pi^2 c_g \Rightarrow \tilde{c}_{4d} = -8\pi^2 c_m$$

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Act IV:

The future

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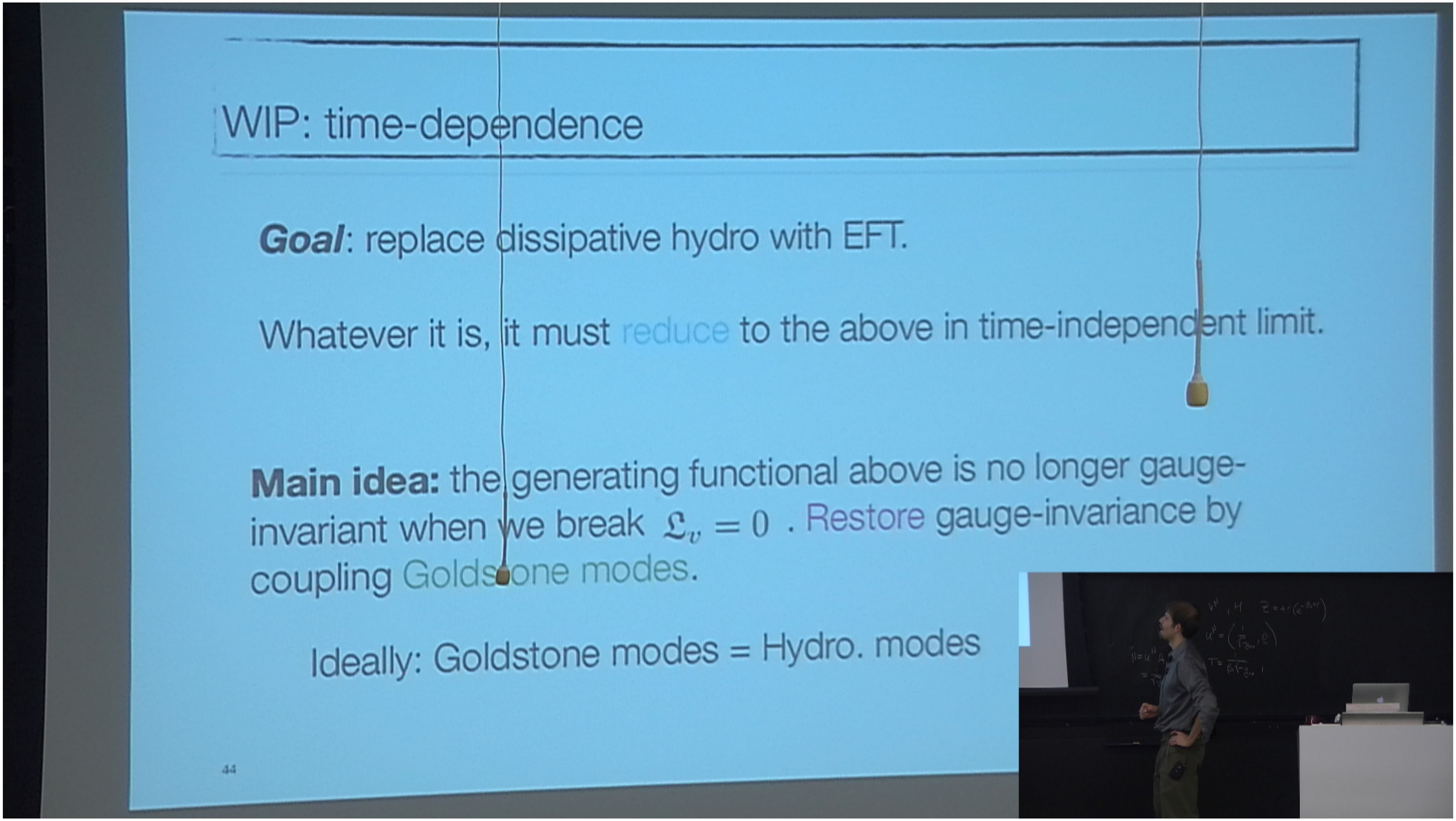
WIP: time-dependence

Goal: replace dissipative hydro with EFT.

Whatever it is, it must reduce to the above in time-independent limit.

Main idea: the generating functional above is no longer gauge-invariant when we break $\mathfrak{L}_v = 0$. Restore gauge-invariance by coupling Goldstone modes.

Ideally: Goldstone modes = Hydro. modes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WIP: perturbative corrections to $\tilde{c}_{4d}$ (T^2 term in CVE)

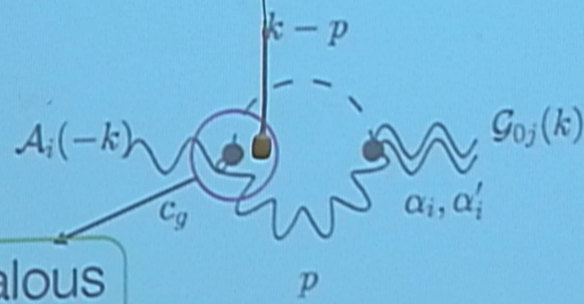
When there are AVV anomalies with **dynamical** U(1)s, get corrections:

$$\tilde{c}_{4d} = -8\pi^2 c_m + O(e^2) + O(e^2|e|) + ..$$

[Golkar, Son], [Hou, Hie, Ren]

IR correction, sensitive to screening

[KJ, Kovtun, Ritz]



Anomalous vertex

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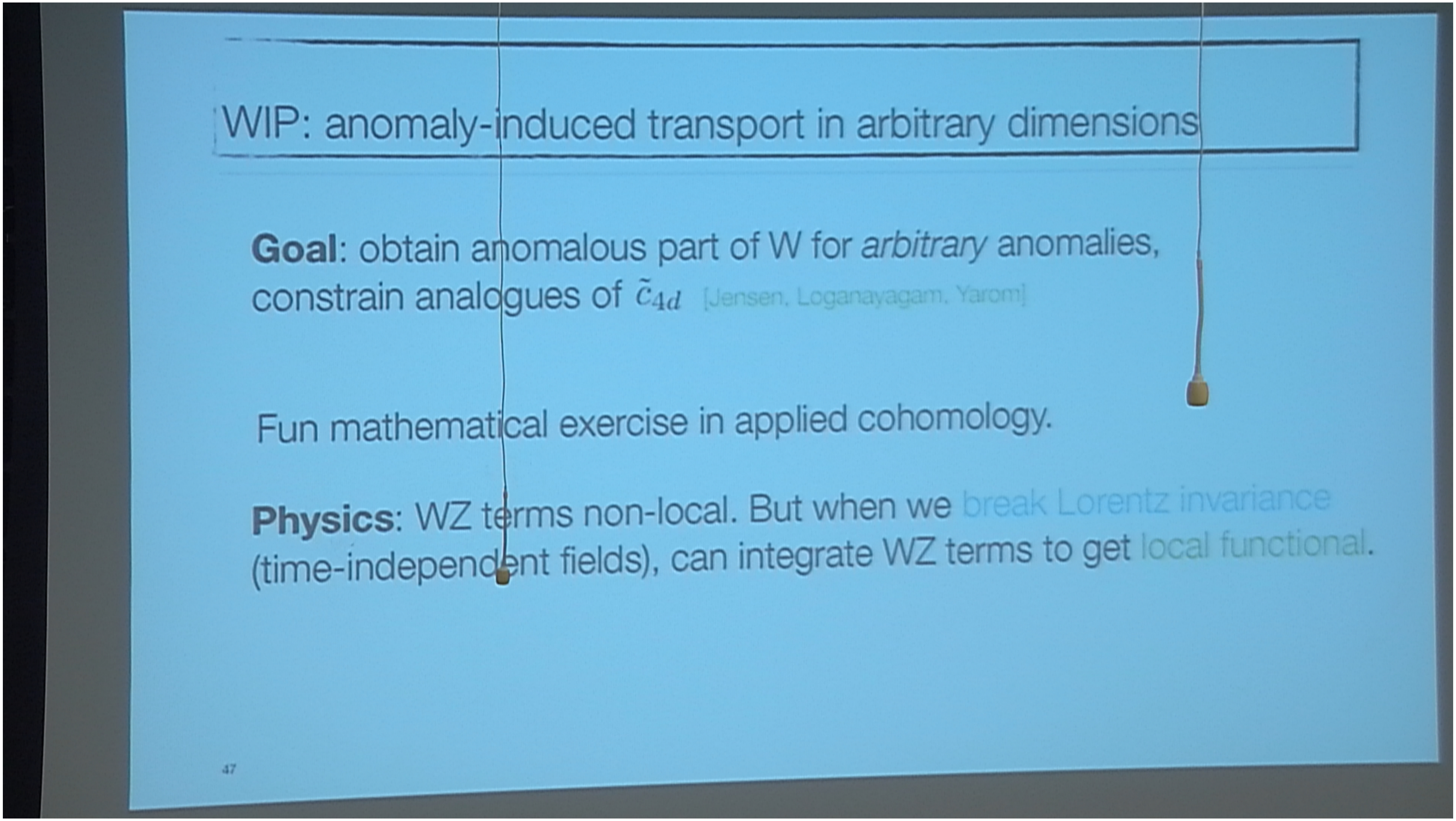
WIP: anomaly-induced transport in arbitrary dimensions
Goal: obtain anomalous part of W for arbitrary anomalies,
constrain analogues of $\tilde{c}_{4d}$ [Jensen, Loganayagam, Yarom]
Fun mathematical exercise in applied cohomology.
Physics: WZ terms non-local. But when we break Lorentz invariance
(time-independent fields), can integrate WZ terms to get local functional.
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Conclusions

1. Lots of exact results in thermal field theory (no SUSY, CFT).
2. Anomalies give local W's in time-independent equilibrium.
3. Replacing hydro with field theory gives **the** low-energy EFT for T>0 field theory (probably).
4. Lots of work to do!

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