Title: Recent progress on de Sitter S-matrix

Date: Nov 13, 2012 11:00 AM

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Abstract: The de Sitter S-matrix provides a gauge-invariant and field redefinition-invariant window into de Sitter QFTs and may provide a crucial entry in any dS/CFT dictionary. In this talk I will summarize recent progress on developing the S-matrix for theories with gauge fields and perturbative gravity. Nonrenormalization theorems, hints of supersymmetry, and perturbative stability will be discussed.

The Minkowski S-matrix

The S-matrix is an invaluable tool for QFT on Minkowski space

- gauge invariant
- invariant under field redefinitions
- admits powerful theorems which reveal structure of Minkowski QFT: Coleman-Mandula, Haag-Lopuszanksi-Sohnius, Weinberg-Witten, ...

At a more mundane level, the S-matrix:

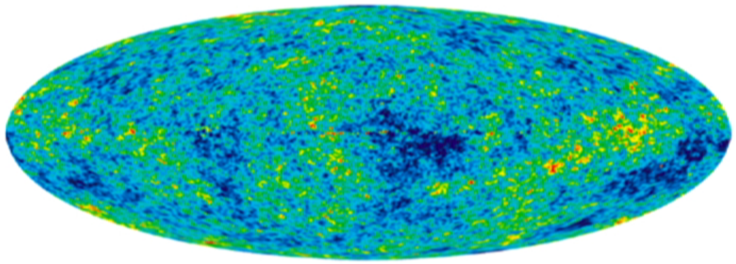
- allows clean comparison of different approaches, choices of gauge, etc.
- is useful for resolving controversies, hastening advances in knowledge

Experimentally accessible!

The S-matrix and cosmology

In cosmological setting:

- traditional scattering experiments not feasible in inflationary spacetimes
- emphasis on CMB



Lack of an S-matrix (or equivalent) has been sorely felt for decades

- remain controversies over the interpretations simple self-interacting theories on fixed backgrounds as well as the more complicated case of gravitational theories

Cosmological observables insufficient for analysing perturbative quantum gravity

- not gauge invariant to all orders
- too local; don't probe IR behavior or address stability [Tsamis, Woodard, Polyakov,...]

Bulk unitarity, dS/CFT, and the S-matrix

Concrete dS/CFT realizations involve "non-unitary" CFTs:

- Vasiliev dS_4/CFT_3 [Anninos, Hartman, Strominger, Harlow]
- dS_5/conformal gravity$_4$ [Maldacena]
- common feature: Euclidean CFT duals are *not* reflection-positive ("unitary")

Key question: how is bulk unitarity encoded in the Euclidean CFT?

- S-matrix is unitary map $S : \mathcal{H} \to \mathcal{H}$
- S is an ideal tool for studying the implications of unitarity.

More points of contact with dual theories:

- The S-matrix gives us something to compute!
- captures similar physics in a different language

bulk S-matrix $\longleftrightarrow$ asympt. behavior of bulk correlators $\longleftrightarrow$ dS/CFT

Today

In this talk we introduce the S-matrix for weakly-coupled quantum theories on global de Sitter that may be computed order-by-order in perturbation theory.

For massive fields, we can verify that the S-matrix is:

- unitary
- dS-invariant
- invariant under perturbative field redefinitions
- transforms appropriately under CPT
- reduces to the usual S-matrix in the flat-space limit

Will provide a non-trivial unitarity constraint on asymptotic behavior of renormalized bulk correlation functions.

We will offer preliminary evidence that a perturbative S-matrix exists for Einstein-Hilbert gravity.

This is not a talk on technology or techniques (I will hide essentially all computational details from you).

Outline

1. The basics
2. Pology
3. Example: tree-level scattering
4. The Optical thm, operator weights, and particle stability
5. Future directions
6. Summary

de Sitter space

Global $dS_D = \{X \in \mathbb{R}^{D,1} \mid X \cdot X = \ell^2\}$

dS isometry group is $SO(D,1)$

$$\frac{ds^2}{\ell^2} = \left[-\frac{1}{1+\eta^2}d\eta^2 + (1+\eta^2)d\Omega^2_{D-1}\right], \quad \eta \in \mathbb{R}.$$

relation to $g_{tt} = -1$ time: $\eta = \sinh(t/\ell)$

Natural "vacuum": Hartle-Hawking or Euclidean state $|\Omega\rangle$

- maximally symmetric ("dS-invariant")
- Hadamard state in free theories
- agrees with Minkowski vacuum as $\ell \to \infty$
- attractor state for local ops. in asymptotic regions [Hollands, Marolf & IM]

$$\forall\, \Psi \in \mathcal{H}_\Omega \quad \langle\phi(x)\rangle_\Psi \to \langle\phi(x)\rangle_\Omega \quad \text{as} \quad x \to \mathscr{I}^\pm$$

Asymptotic particle states

Consider a scalar field $\phi_M(x)$ with:

1. Bare mass $M^2 > 0$
2. mass gap (determined by the Källen-Lehmann weight)

Properties of initial (final) states $|\psi\rangle_{i/f}$ satisfied as $\eta \to -\infty$ $(+\infty)$:

1. normalizable: $|{}_{i/f}\langle a|b\rangle_{i/f}| < \infty$
2. definite particle content labelled by dS UIRs

$$|a\rangle_{i/f} := |n_1, n_2, \ldots, n_k\rangle_{i/f}\,, \quad n = (M^2, \vec{L})$$

3. states transform as direct products of UIRs under dS group

$$U(g)|n_1, n_2, \ldots, n_k\rangle_{i/f} = |gn_1, gn_2, \ldots, gn_k\rangle_{i/f}, \quad gn = (M^2, \vec{L}' = g\vec{L})$$

4. desire flat-space limit $\Rightarrow$ initial/final vacuua are Hartle-Hawking state $|\Omega\rangle$

Particles need not look "free" near $\mathscr{I}^{\pm}$

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Asymptotic particle states

Construction

LSZ prescription with wave packets $\psi_n(x)$

$$|n_1, n_2, \ldots, n_k\rangle_{i/f} = \lim_{\eta \to \mp\infty} a^\dagger_{n_1}(\eta) a^\dagger_{n_2}(\eta) \ldots a^\dagger_{n_k}(\eta)|\Omega\rangle,$$

$$a^\dagger_n(\eta) = -i \int d\Sigma^\nu(\vec{x}) \left[\psi_n(x) \overleftrightarrow{\nabla}_\nu \phi_M(x)\right]\Big|_\eta ,$$

Wave packets $\psi_n(x)$

Carefully chosen to ensure that ${}_{i/f}\langle a|b\rangle_{i/f}$ is free of power-law IR divergences.

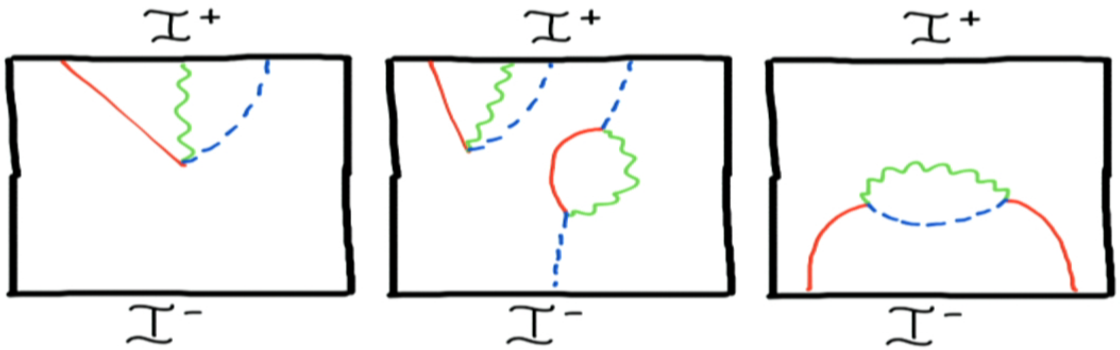
- free fields: $\psi_n(x)$ = linearized sol'ns to EOM
- interacting thys: $\psi_n(x)$ selects the "mass pole" part of $\phi_M(x)$
- for heavy fields: distinction between $\psi_n(x)$ and KG modes can generally be ignored

Captures logarithmic IR divergences expected in perturbation theory (which encode perturbative renormalization, anomalies, . . .).

Asymptotic particle states

Orthonormalization

There exist non-vanishing contributions to particle states in same basis ${}_i\langle a|b\rangle_i$.



Give to each initial particle state $|a\rangle_i$ an order $I(a)$, letting the vacuum $|\Omega\rangle$ have the lowest order. Orthonormal initial basis $\{|A\rangle_i\}$ may be constructed as follows:

$$|B\rangle_i = \frac{|b\rangle_i - \sum_{I(A)<I(b)} |A\rangle_i \, {}_i\langle A|b\rangle_i}{\left[{}_i\langle b|b\rangle_i - \sum_{I(A)<I(b)} |{}_i\langle A|b\rangle_i|^2\right]^{1/2}}, \quad I(B) = I(b).$$

The S-matrix

S-matrix:

$$S := \{ {}_f\langle A|B\rangle_i \}$$

Properties

1. The vacuum-to-vacuum amplitude is unity (use $|\Omega\rangle$ for in/out vacuum).
2. Covariance under the dS group:

$$ {}_f\langle A|B\rangle_i = {}_f\langle A|1\,|B\rangle_i = {}_f\langle A|U^{-1}(g)U(g)\,|B\rangle_i = {}_f\langle gA|gB\rangle_i \,. $$

3. Behavior under CPT: $\Theta S = S^{-1}\Theta$
4. Invariance under perturbative field-redefinitions:

$$\phi(x) \to \phi(x) + g\mathscr{O}(x), \quad |g| \ll 1.$$

5. Unitarity: $S^\dagger S = 1$ and $SS^\dagger = 1$. Equivalently, for $S = 1 + i\mathcal{T}$ have the Optical theorem

$$2\mathrm{Im}\,\mathcal{T} = \mathcal{T}^\dagger\mathcal{T}$$

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de Sitter polology

Upshot:

S-matrix essentially captures the "mass pole part" of correlation functions

- define a suitable complex mass plane
- show how correlation functions are described by poles
- not all poles are equal: mass poles are field-redefinition invariant, other poles are not

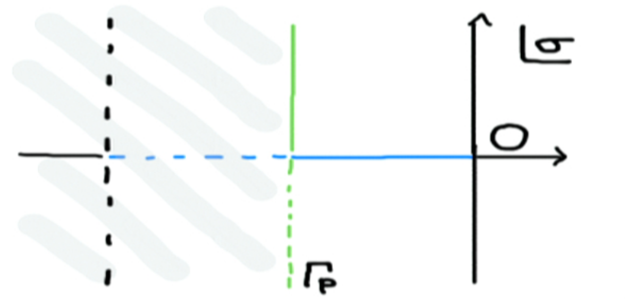
Operator weights

1-particle states form UIRs T^σ of $SO(D,1)$:

Define operator *weight*:

$$M^2(\sigma)\ell^2 = -\sigma(\sigma + D - 1),$$

$$\sigma := -\frac{(D-1)}{2} + \left[\frac{(D-1)^2}{4} - M^2\ell^2\right]^{1/2}.$$



1. principal series: (solid green, Γ_p)
$$\frac{(D-1)^2}{4} \leq M^2\ell^2, \quad \Rightarrow \quad \sigma = -\frac{(D-1)}{2} + i\rho, \;\; \rho \in \mathbb{R}, \;\; \rho \geq 0,$$

2. complementary series: (solid blue, negative real line)
$$0 < M^2\ell^2 < \frac{(D-1)^2}{4}, \quad \Rightarrow \quad \sigma \in \left(-\frac{(D-1)}{2}, 0\right),$$

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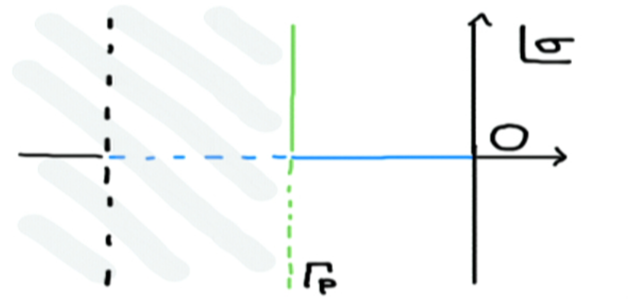

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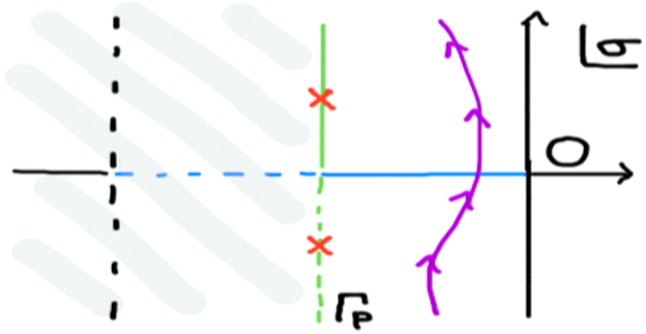
Poles in the Källen-Lehmann weight

Write as contour integral in complex σ plane: [Marolf & IM, Hollands]

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle = \int_\mu \rho(\mu) W_\mu(x_1,x_2).$$

E.g., for a free theory in the principal series:

$$\langle 0|\, \phi_\sigma(x_1)\phi_\sigma(x_2)\,|0\rangle = \int_\mu \frac{(2\mu + D - 1)}{(\mu-\sigma)(\mu+\sigma+D-1)} W_\mu(x_1,x_2) = W_\sigma(x_1,x_2)$$



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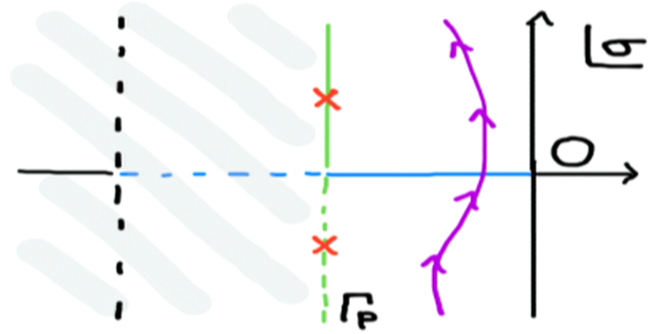

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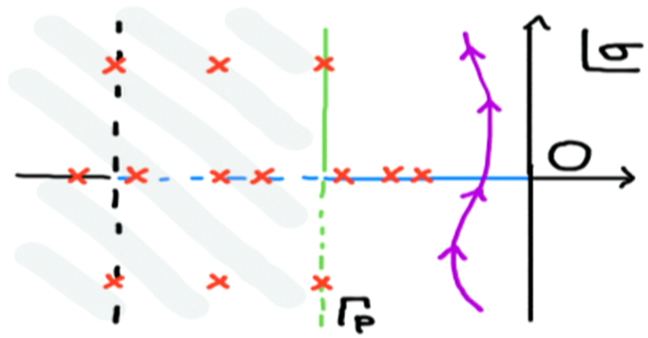
At 1-loop:

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle^{\text{1-loop}} = \int_\mu \frac{(2\mu + D - 1)\Pi(\mu)}{(\mu-\sigma)^2(\mu+\sigma+D-1)^2} W_\mu(x_1,x_2)$$



Poles in Mellin-Barnes kernels

Mellin-Barnes representations of correlation functions [Marolf & IM, Hollands]

$$\langle \phi_{\sigma_1}(x_1)\phi_{\sigma_2}(x_2)\phi_{\sigma_3}(x_3)\rangle = \int_{\mu_{12}}\int_{\mu_{23}}\int_{\mu_{31}} k(\mu_{12},\mu_{23},\mu_{31}) \left(\frac{1-Z_{12}}{2}\right)^{\mu_{12}} \left(\frac{1-Z_{23}}{2}\right)^{\mu_{23}} \left(\frac{1-Z_{31}}{2}\right)^{\mu_{31}}$$

- $Z_{ij} = Z(x_i, x_j)$ is $SO(D,1)$-invariant distance
- by deforming integration contours may obtain asymptotic expansions for various configurations
- poles in $k(\mu_{12},\mu_{23},\mu_{31})$ determine asympt. behavior $(Z_{ij})^{p_{ij}}\ldots$
- asympt. expansions depend on ratios of Z_{ij}
- S-matrix $\sim \operatorname{Res} k(\sigma_1,\sigma_2,\sigma_3)$, $\operatorname{Res} k(-(\sigma_1+D-1),\sigma_2,\sigma_3)$, ...

NB: can use this expression to prove cluster decomposition: if all x_i are taken to large separations from all y_j:

$$\langle \phi(x_1)\phi(x_2)\ldots\phi(y_1)\phi(y_2)\ldots\rangle_\Omega \to \langle \phi(x_1)\phi(x_2)\ldots\rangle_\Omega \, \langle \phi(y_1)\phi(y_2)\ldots\rangle_\Omega \, .$$

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Tree-level scattering 1

Consider simple model with $\phi_{1,2,3}(x)$, $M^2_{1,2,3}$

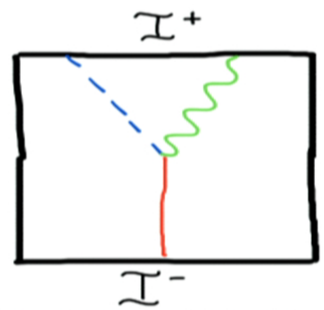
$$\mathcal{L}_{\text{int}}[\vec{\phi}] = g\phi_3\phi_2\phi_1(x)$$

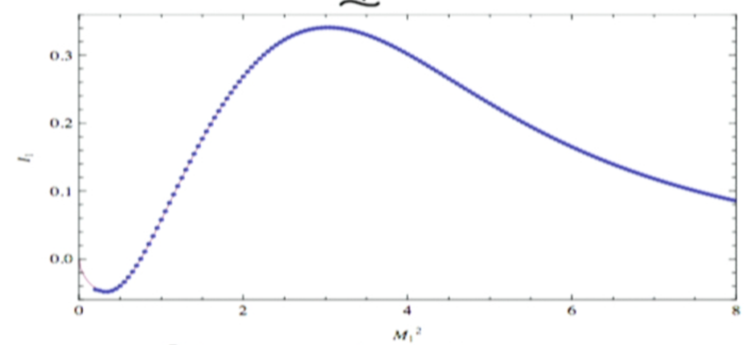
$\mathcal{O}(g)$ tree-level amplitude:

$$_f\langle n_3 n_2 | n_1 \rangle_i^{(1)} = ig \int_y u_3^* u_2^* u_1(y)$$

- non-vanishing except possibly for discrete configurations
- Im as req. by Optical theorem
- agrees with naive use of LSZ

Plot: (amplitude/ig) as a function of M_1^2 with $M_{2,3}^2 = 2,\ 1.25$ in $D = 3$.





Amplitude peaked "off-shell" at $\sigma_1 = \sigma_2 + \sigma_3$, $M^2(\sigma_2 + \sigma_3) \in \mathbb{C}$.

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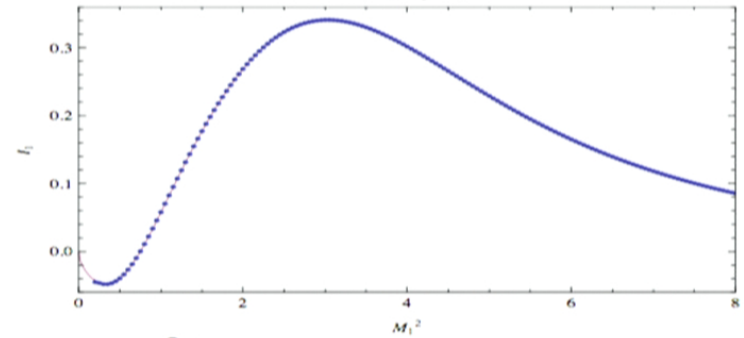

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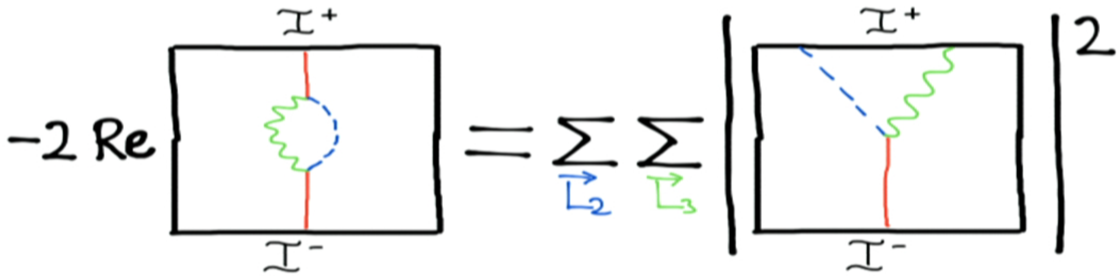
The Optical theorem

Generic operators have non-zero $1 \to$ many amplitudes

The Optical theorem

Expand $S^\dagger S = 1$ in powers of g:



Unstable particles in Minkowski space

- operators acquire complex M^2 at 1-loop ($\mathrm{Im}\, M^2 < 0$)
- complex M^2 exponentially damps correlation functions
- after 1PI summation, unstable particles not in asymptotic states

Do asympt. particle states exist at 1-loop?

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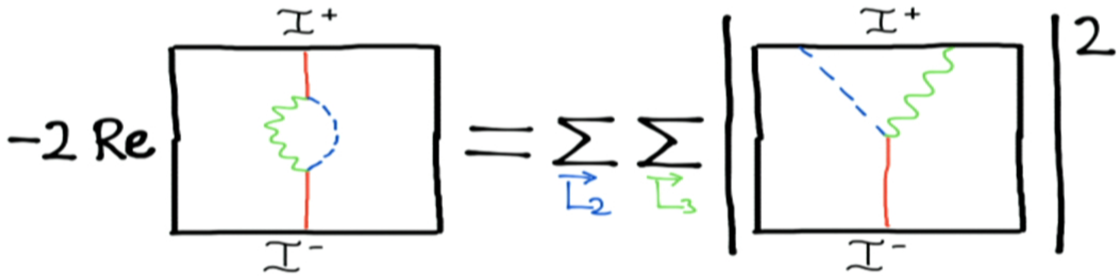

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The Optical theorem

Can relate $1 \to 1$ scattering amplitude to self-energy $\Pi(\mu)$ (or Källen-Lehmann weight) [Marolf IM 2010, Bros et. al 2006, 2008, Jatkar et. al 2011].
E.g., in our example theory at $\mathcal{O}(g^2)$

$$
\begin{aligned}
-2\mathrm{Re}\, {}_f\langle n_1|n_1\rangle_i^{(2)} &= \int_{\overline{x}}\int_x u_1^*(\overline{x})u_1(x)(\Box_x - M_1^2)(\Box_{\overline{x}} - M_1^2)\,\langle\phi_\sigma(\overline{x})\phi_\sigma(x)\rangle^{(2)} \\
&= -\left[\frac{\Pi(\sigma)-\Pi(-(\sigma+D-1))}{(2\sigma+D-1)}\right](2\log H + \text{finite}) \\
&\quad -\left[\frac{\Pi'(\sigma)+\Pi'(-(\sigma+D-1))}{(2\sigma+D-1)}\right](\text{finite}),
\end{aligned}
$$

with spacetime integrals regulated $|\overline{\eta}|,\ |\eta| < H$.
Coefficients are independent of renormalization scheme.
Optical theorem requires:

$$
\left[\frac{\Pi(\sigma)-\Pi(-(\sigma+D-1))}{(2\sigma+D-1)}\right] \leq 0
$$

How is this related to mass renormalization?

Renormalized operator weights

In perturbation theory, mass poles $\Delta_\pm$ shift (renormalized)

$$\Delta_+ := \sigma + \mathcal{O}(g^2), \quad \Delta_- := -(\sigma + D - 1) + \mathcal{O}(g^2).$$

Shift is encoded in on-mass values of the self-energy at $\mathcal{O}(g^2)$:

$$\Delta_+ = \sigma + \frac{\Pi^{(2)}(\sigma)}{(2\sigma + D - 1)}, \quad \Delta_- = -(\sigma + D - 1) - \frac{\Pi^{(2)}(-(\sigma + D - 1))}{(2\sigma + D - 1)},$$

$\Pi^{(2)}(\sigma)$, $\Pi^{(2)}(-(\sigma + D - 1))$ renormalization scheme-dependant, but combination $\Delta_+ + \Delta_-$ is independent

$$\Delta_+ + \Delta_- = -(D-1) + \frac{\Pi^{(2)}(\sigma) - \Pi^{(2)}(-(\sigma + D - 1))}{(2\sigma + D - 1)} < -(D-1).$$

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Stability of asymptotic particle states

Renormalization of weights (consequence of unitarity):

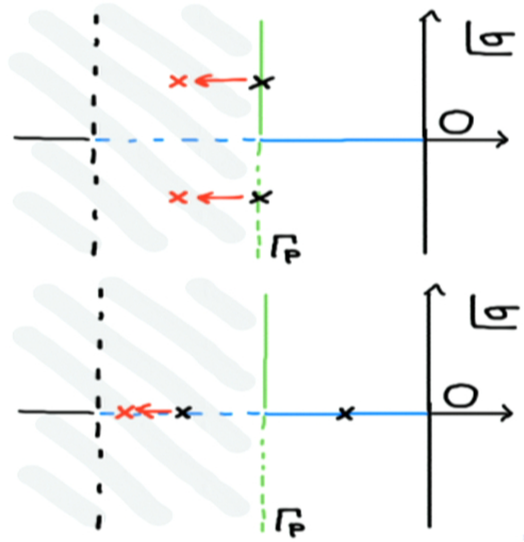
$$\Delta_+^{(2)} + \Delta_-^{(2)} = \left[\frac{\Pi^{(2)}(\sigma) - \Pi^{(2)}(-(\sigma + D - 1))}{(2\sigma + D - 1)} \right] \leq 0, \quad \Rightarrow \Delta_- + \Delta_+ \leq -(D-1).$$

Principal series fields:

- renormalized $\Delta_\pm$ do not correspond to UIRs
- renormalized masses (self-energy) have imaginary part
- "unstable" asymptotic particle states

Complementary series fields:

- renormalized Δ_+ remains in complementary series
- renormalized mass $M^2(\Delta_+)$ real
- "stable" asymptotic particle states



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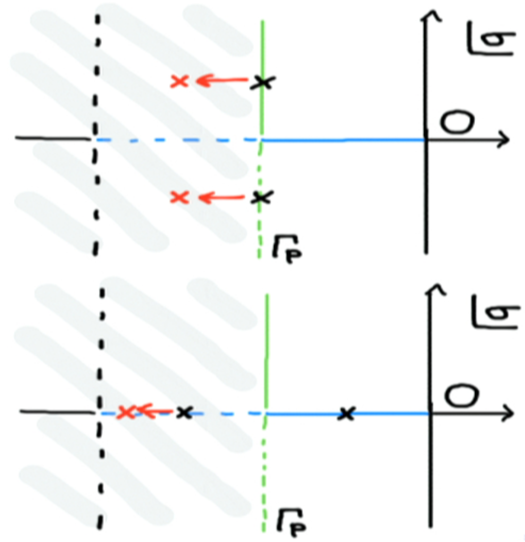

Stability of asymptotic particle states

Renormalization of weights (consequence of unitarity):

$$\Delta_+^{(2)} + \Delta_-^{(2)} = \left[\frac{\Pi^{(2)}(\sigma) - \Pi^{(2)}(-(\sigma + D - 1))}{(2\sigma + D - 1)} \right] \le 0, \quad \Rightarrow \Delta_- + \Delta_+ \le -(D-1).$$

Principal series fields:

- renormalized $\Delta_\pm$ do not correspond to UIRs
- renormalized masses (self-energy) have imaginary part
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Complementary series fields:

- renormalized Δ_+ remains in complementary series
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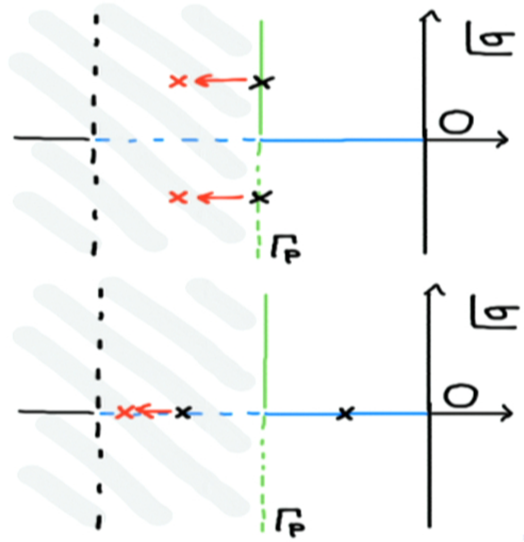

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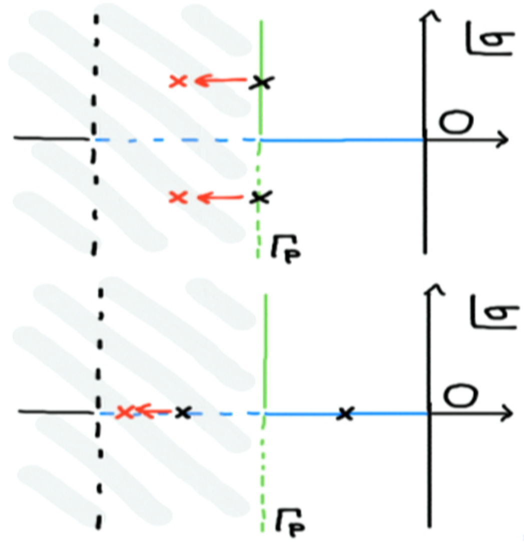

Gravitons

Stability of perturbative quantum gravity on dS an open question

- [e.g. Mottola, Tsamis, Woodard, Polyakov, ...]
- computations are difficult
- arguments for cosmological observables are *inadequate* to address stability
- what to compute?

The Tsamis-Woodard mechanism

- claim: there does not exist a dS-invariant state
- at ≥ 2 loops "IR gravitons" work to screen Λ

For a gravity S-matrix to exist:

- need dS-invariant 0-particle state Ω
- IR divergences in amplitudes should not be worse than in Minkowski

The S-matrix could provide a gauge-invariant "observable" to analyse perturbative quantum gravity in dS.

Gravitons

Algebraic approach to QFT in CST [Fewster, Hunt, Higuchi]

Makes sharp what questions may be asked, what a quantum state is

- emphasis on *-algebra of observables $\mathcal{A}(dS_D)$

$$h(f) := \int_x f^{\mu\nu}(x) h_{\mu\nu}(x), \quad f^{\mu\nu} \in C_0^\infty(dS_D), \quad \nabla_\mu f^{\mu\nu}(x) = 0$$

  locality, E.O.M., etc., may be phrased in terms of observables

- Ψ is (sufficiently regular) positive linear functional $\Psi : \mathcal{A}(dS_D) \to \mathbb{C}$

Quantization is independent of chart and gauge.

Results (easily obtained): [IM in prep, broad agreement w/ Higuchi]

- Ω exists (on any chart), admits dS-invariant Green's functions
- same state as [Miao-Tsamis-Woodard (2011)] in non-covariant gauge
- Ω is a cyclic and separating vector on any open set of dS_D
- Cosmic no-hair thm: let $B(\lambda)$ be a boost with rapidity λ

$$\forall\, \Psi \in \mathcal{H}_\Omega : \quad \left\langle B(\lambda) h(f) f(p) B^{-1}(\lambda) \right\rangle_\Psi \to \langle h(\lambda f) h(\lambda p) \rangle_\Omega \quad \text{as } |\lambda| \to \infty$$

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Exceptional configurations

For scalar theories there exist discrete configurations for which tree-level scattering amplitudes have logarithmic IR divergence

$$\sum_i \sigma_i = -(D-1) - 2n, \quad n \in \mathbb{N}_0$$

Similar IR divergence occurs for same theories in AdS:

- if combined with particular higher-derivative interactions IR divergences cancel
- these interactions precisely those that arise from dimensional reduction of SUGRA on $AdS_5 \times S_5 \to AdS_5$
- bulk fields $\leftrightarrow$ single-trace boundary operators (chiral primaries)
- chiral primary correlation functions *protected* from perturbative renormalization (independent of $g_{YM}N^2$)
- subtle consequence of SUSY in boundary theory

Precisely same choice of coefficients cancels IR divergences in dS.

Higher Spin fields

Interacting higher-spin theories exist for $\Lambda \neq 0$ [Vasiliev, Fradkin]

- contain spin-2 graviton, linearized EH gravity
- infinite tower of higher-spin fields and non-local interactions
- no Lagrangian formulation

Proposed Vasiliev dS_4/CFT_3 [Anninos, Hartman, Strominger, Harlow]

Vasiliev thy with $\Lambda > 0$ dual to free (critical) $Sp(N)$ CFT

- potentially first "microscopic" dS/CFT
- significant evidence for AdS Vasiliev/$O(N)$ CFT correspondence
- remains much to interpret in dS

The dS S-matrix for Vasiliev should be highly constrained!

- in Minkowski: S-matrix constrained to be free [Porrati, Weinberg, Weinberg-Witten]
- in AdS: CFT analysis of Maldacena & Zhiboedov shows boundary correlators have few structures
- naively, Vasiliev amplitudes correspond to exceptional configurations

Summary

The de-Sitter S-matrix

- a new tool for analysing dS QFTs
- captures gauge-, field-redefinition invariant aspects of correlator asymptotics
- elucidates implications of bulk unitarity

Key differences between Minkowski space

- asympt. states are not approximately free
- in eternal dS heavy fields are resonances
- only light fields enter into asymptotic states

Future directions

- EH gravity
- exceptional configurations/protected operators
- higher-spin theories