Title: Primordial Magnetic Fields & Non-Gaussianity

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Abstract: There are indications of a lower bound on magnetic fields in the intergalactic media. Since magnetic fields on such large scales are difficult to generate in the late universe, this might point to inflationary magnetogenesis as the origin of intergalactic magnetic fields. If the magnetic fields are generated during inflation, they are naturally correlated with the inflaton perturbations in a general class of models. This leads to a consistency relation between the power spectrum of primordial magnetic fields and the non-Gaussian three-point cross-correlation of magnetic fields with the inflaton perturbation. The size of non-Gaussianity can be expressed in a new magnetic non-linearity parameter b_NL. In the flattened shape where the non-Gaussianity is maximal, b_NL can be as large as 5000.

Primordial Magnetic Fields & Non-Gaussianity

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Based on arXiv:1210.3461 , arXiv:1207.4187 with R. K. Jain

Observations

- Micro-Gauss magnetic fields seen in galaxy clusters

➡ Galactic dynamo from small seed fields

- Initial seeds need coherence length > *10* Kpc
- and initial field strength of order 10^{-12} - 10^{-22} Gauss

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Magnetic seeds

It's problematic to produce such magnetic seeds after inflation

- The seed coherence length is smaller than horizon size at time of magnetogenesis

  (this is solar system size for electroweak phase transition)

- Process of inverse cascades can increase coherence length, but requires large amount of magnetic helicity

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More observations

- Recent claims of a lower bound on magnetic field in the intergalactic space of 10^{-15} Gauss [Neronov, Vovk 2010]

➡ Indication of inflationary magnetogenesis

- Upper bound on primordial magnetic fields of order nano-Gauss from CMB

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A little bit of history...

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

in FRW space is conformal inv. ⇒ doesn't feel expansion

➡ Electromagnetic fields are not amplified by inflation

➡ Breaking of conformal invariance needed

- Consider coupling of EM fields to other fields, which may couple to gravity in a non-conformal invariant way

➡ Production of magnetic fields? [Tuner, Widrow 1988]

Different models

- Dynamical gauge coupling

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\lambda(\phi)F_{\mu\nu}F^{\mu\nu}$$

[Ratra 1992]

- Coupling to gravity

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha_n}{4}R^n F_{\mu\nu}F^{\mu\nu}\right]$$

same as above, when Φ is the inflaton

- Axial coupling

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\lambda(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

strong constraints from NG and backreaction

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- Mass term

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m^2 A_\mu A^\mu\right]$$

  - Negative mass-squared needed for generating enough magnetic fields
  - Generating neg. mass-squared from Higgs mech. ⇒ one needs ghost scalar field with neg. kinetic energy

[Dvali et. al. 2007, Himmetoglu, Contaldi, Peloso 2009]

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Magnetogenesis in Ratra-type models

- The action of the EM field and the inflaton

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - \partial_\mu\phi\partial^\mu\phi - 2V(\phi) - \frac{1}{2}\lambda(\phi)F_{\mu\nu}F^{\mu\nu}\right)$$

- In Coulomb gauge we have ($A_0 = 0$, $\partial_i A^i = 0$)

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int d^3x\,d\tau\lambda(\phi)\left(A_i'^2 - \frac{1}{2a^2}(\partial_i A_j - \partial_j A_i)^2\right)$$

- With the magnetic field given by

$$B_i(\tau,\mathbf{x}) = \frac{1}{a}\epsilon_{ijk}\partial_j A_k(\tau,\mathbf{x})$$

- The Fourier transformed gauge field is

$$A_i(\tau, \mathbf{k}) = \int d^3x A_i(\tau, \mathbf{x}) e^{i\mathbf{x}\cdot\mathbf{k}}$$

- Quantize by defining mode expansion

$$A_i(\tau, \mathbf{k}) = \sum_{\sigma=\pm} \left[\epsilon_i^\sigma(\hat{k}) A_k(\tau) \hat{b}_{\mathbf{k}}^\sigma e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.\right]$$

with

$$[\hat{b}_{\mathbf{k}}^\sigma, \hat{b}_{\mathbf{k}'}^{\sigma'\dagger}] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \delta_{\sigma\sigma'}$$

$$\mathbf{k} \cdot \epsilon^\sigma(\hat{k}) = 0 \ , \quad \epsilon^\sigma(\hat{k}) \cdot \epsilon^{\sigma'*}(\hat{k}) = \delta_{\sigma\sigma'} \ , \quad \sum_{\sigma=\pm} \epsilon_i^\sigma(\hat{k}) \epsilon_j^{\sigma*}(\hat{k}) = \delta_{ij} - k_i k_j / k^2$$

- Defining the magnetic power spectrum

$$\left\langle B_i(\tau, \mathbf{k}) B^i(\tau, \mathbf{k}') \right\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_B(k)$$

it can be computed from

$$P_B(k) = 2 \frac{k^2}{a^4} |A_k(\tau)|^2$$

- Define pump field $S^2(\eta) = \lambda(\phi(\eta))$
- and a canonically normalized vector field $v_i = S(\tau)A_i$
- Such that the quadratic action takes the simple form

$$S_v = \frac{1}{2}\int d\tau d^3x \left[v_i'^2 - (\partial_j v_i)^2 + \frac{S''}{S}v_i^2\right]$$

- The EOM for the mode function $v_k = S(\tau)A_k$ is

$$v_k'' + \left(k^2 - \frac{S''}{S}\right)v_k = 0$$

- With $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$ the solution normalized to Bunch-Davis vacuum is

$$v_k(\tau) = \frac{\sqrt{\pi}}{2}e^{i\pi(1+n)/2}\sqrt{-\tau}H^{(1)}_{\frac{1}{2}+n}(-k\tau)$$

- which leads to

$$P_B(k) = \frac{1}{\lambda_I}\frac{\pi}{2}\frac{H^4}{k^3}\left(\frac{\tau}{\tau_I}\right)^{2n}(-k\tau)^5 H^{(1)}_{\frac{1}{2}+n}(-k\tau)H^{(2)}_{\frac{1}{2}+n}(-k\tau)$$

➡ The spectral index of the magnetic power spectrum is

$$n_B = (4 - 2n)$$

- For a scale invariant spectrum, *n*=2, backreaction remains small
- In this case, with $H \simeq 10^{14}$ GeV, a magnetic field strength of order nano-Gauss an be achieved on Mpc scales

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Strong coupling problem

- Adding the EM coupling to the SM fermions

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}\lambda(\phi)F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi\right]$$

- The physical electric coupling is

$$e_{phys} = e/\sqrt{\lambda(\phi)}$$

- Since $\sqrt{\lambda} \propto a^n$ then for $n > 0$ the electric coupling decreases by a lot during inflation, and must have been very large at the beginning

➡ QFT out of control initially [Demozzi, Mukhanov, Rubinstein 2009]

- Solutions??? Speculations [Bonvin, Caprini, Durrer 2011, Caldwell, Motta 2012, Bartolo, Matarrese, Peloso, Ricciardone 2012, and more...]

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➡NG correlation of magnetic field with inflaton field

$$\langle\zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(k_3)\rangle \qquad \zeta = \frac{H}{\dot{\phi}}\delta\phi$$

[Kamionkowski, Caldwell, Motta (2012), Jain, MSS (2012)]

(Ordinary) Non-Gaussianity

- To leading order, the perturbations are encoded in the two-point function

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_\zeta(k)$$

- A non vanishing three point function

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$$

is a signal of non-Gaussianity

- Introduce dimensionless f_{NL} :

$$f_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) \ + \ perm.$$

as a measure of non-Gaussinity

- Similarly

$$\tau_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle / P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_{14}) \ + \ perm.$$

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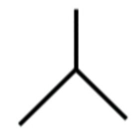
Semiclassical estimates

Squeezed limit:

- Consider



$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$ for $k_1 << k_2,\ k_3$

- The long wavelength mode will rescale the spatial coord. of the backgound of the two other modes

$$ds^2 = -dt^2 + e^{2\zeta_1} a^2(t) dx^2$$

- Taylor expanding the two point function in the background of

$$\langle \zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} = \langle \zeta_{k_2}\zeta_{k_3}\rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2}\zeta_{k_3}\rangle + \dots$$

$$\Rightarrow \quad \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1}\zeta_{k_1}\rangle\, k \frac{d}{dk} \langle \zeta_{k_2}\zeta_{k_3}\rangle$$

$$\Rightarrow \qquad \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \sim -(n_s - 1) \langle \zeta_{k_1}\zeta_{k_1}\rangle \langle \zeta_{k_2}\zeta_{k_3}\rangle$$

Maldacena consistency relation [Maldacena '02]

- This corresponds to a local f_{NL} following from a non-linear field redef.

$$\zeta(\vec{x}) = \zeta_g(\vec{x}) + \frac{3}{5} f_{NL}^{local} \zeta_g^2(\vec{x})$$

- with

$$f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim \epsilon$$

- Similarly one could consider the double squeezed limit of the four point function

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \sim (n_s - 1)^2 \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle$$

$$\tau_{NL} \sim \frac{\langle \zeta^4 \rangle}{\langle \zeta^2 \rangle^3} \sim \epsilon^2$$!!! $\times$ $k_1 << k_2,\ k_3,\ k_4$

Magnetic non-linearity parameter: b_{NL}

- Let's come back to $\langle\zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(k_3)\rangle$ and parametrize it in a similar way

➡ Introduce new magnetic non-linearity parameter: b_{NL}

- Define the cross-correlation bispectrum

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- We then define

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

$$\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k)$$

$$\langle\mathbf{B}(\mathbf{k})\cdot\mathbf{B}(\mathbf{k}')\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_B(k)$$

Local b_{NL}

- In the case where b_{NL} is momentum independent, it takes the *local* form:

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2} b_{NL}^{local} \zeta^{(G)} \mathbf{B}^{(G)}$$

- Compare with *local* f_{NL}, given by

$$\zeta = \zeta^{(G)} + \frac{3}{5} f_{NL}^{local} \left(\zeta^{(G)} \right)^2$$

Two interesting shapes

1. The squeezed limit $k_1 \ll k_2, k_3 = k$

- We obtain a new *magnetic consistency relation*

$$\langle \zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(\mathbf{k_3})\rangle = (n_B - 4)(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)P_\zeta(k_1)P_B(k)$$

with $b_{NL}^{local} = (n_B - 4)$

- Compare with *Maldacena consistency relation*

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = -(n_s - 1)(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)P_\zeta(k_1)P_\zeta(k)$$

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Two interesting shapes

2. The flattened shape $k_1/2 = k_2 = k_3$

- This is the shape where b_{NL} turns out to be maximized with

$$|b_{NL}| \sim \mathcal{O}(10^3)$$

Non-Gaussianity

- Consider

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- The effect of the long wavelength mode is to shift the background of the short wavelength modes

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$
$$= \left\langle\zeta(\tau_I,\mathbf{k}_1)\left\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\right\rangle_B\right\rangle$$

- Since the vector field only feels the background through the coupling λ, all the effect of the long wavelength mode is captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots$$

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- The effect of the long wavelength mode is to shift the background of the short wavelength modes

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$$=\left\langle\zeta(\tau_I,\mathbf{k}_1)\left\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\right\rangle_B\right\rangle$$

- Since the vector field only feels the background through the coupling λ, all the effect of the long wavelength mode is captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots$$

- Define pump field $S^2 = \lambda_0$
- and linear Gaussian canonical vector field

$$v_i = S(\tau) A_i^{(G)}$$

- Such that the quadratic action takes the simple form

$$S_v = \frac{1}{2} \int d\tau d^3x \left[v_i'^2 - (\partial_j v_i)^2 + \frac{S''}{S} v_i^2 \right]$$

- Since all the effect of the long wavelength mode is in

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta \ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots$$

➡One finds

$$\langle A_i(\tau, \mathbf{x}_2) A_j(\tau, \mathbf{x}_3)\rangle_B = \left\langle \frac{1}{\lambda_B} v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\right\rangle$$
$$\simeq \frac{1}{\lambda_0}\langle v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\rangle \quad - \frac{1}{\lambda_0^2}\frac{d\lambda}{d\ln a}\zeta_B \langle v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\rangle$$

- Using

$$d\lambda/d\ln a \;=\; \dot{\lambda}/H$$

- and

$$\lim_{k_1 \to 0} \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle$$
$$= \left\langle \zeta(\tau_I, \mathbf{k}_1) \langle A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle_B \right\rangle$$

➡One finds

$$\lim_{k_1 \to 0} \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle$$
$$\simeq -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}\langle \zeta(\tau_I, \mathbf{k}_1)\zeta(\tau_I, -\mathbf{k}_1)\rangle_0 \langle A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle_0$$

Consistency relation

- Expressing it in terms of the magnetic fields

➡ Magnetic consistency relation

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle$$
$$= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- With $\lambda(\phi(\tau)) = \lambda_I (\tau/\tau_I)^{-2n}$

➡ One has $b_{NL} = (n_B - 4)$

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The full *in-in* QFT calculation

- Perturbing the metric in the ADM formalism

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- with the metric ansatz $h_{ij} = a^2 e^{2\zeta} \left[e^{\gamma}\right]_{ij}$
- and solving for the lapse and shift

$$N = 1 + \frac{1}{H}\dot{\zeta}$$
$$N_i = \partial_i \left(-\frac{1}{H}\zeta + a^2 \epsilon\, \partial^{-2}\dot{\zeta}\right)$$

- It is easy to see that the interaction Hamiltonian

$$H_{\zeta AA} = -\frac{1}{2}\int d^3x\, a^3\, T^{\mu\nu}\delta g_{\mu\nu}$$

- becomes

$$H_{\zeta AA} = -\frac{1}{2}\int d^3x a^3 \left(\frac{1}{H}\dot{\zeta}T^{00} - \partial_i\left(-\frac{1}{H}\zeta + a^2\epsilon\partial^{-2}\dot{\zeta}\right)T^{0i} - a^2\zeta T^{ii}\right)$$

[Caldwell & Motta, 2012]

- Using the *in-in* formalism for evaluating the expectation value at some time τ_I

$$\langle\Omega|\,\mathcal{O}(\tau_I)\,|\Omega\rangle = \langle 0|\,\bar{\mathrm{T}}\left(e^{i\int_{-\infty}^{\tau_I} d\tau H_I}\right)\mathcal{O}(\tau_I)\mathrm{T}\left(e^{-i\int_{-\infty}^{\tau_I} d\tau H_I}\right)|0\rangle$$

- One obtains

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = \frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)|$$
$$\times\left[\left(\mathbf{k}_2\cdot\mathbf{k}_3+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^3}{k_2^2k_3^2}\right)k_2k_3\tilde{\mathcal{I}}_n^{(1)}+2(\mathbf{k}_2\cdot\mathbf{k}_3)^2\tilde{\mathcal{I}}_n^{(2)}\right] .$$

[Jain, MSS 2012]

$$\tilde{\mathcal{I}}_2^{(1)} = \frac{1}{(k_2k_3)^{3/2}k_t^2}\times\left[-k_1^3-2k_1^2(k_2+k_3)-2k_1(k_2^2+k_2k_3+k_3^2)-(k_2+k_3)(k_2^2+k_2k_3+k_3^2)\right]$$

$$\tilde{\mathcal{I}}_2^{(2)} = \frac{1}{(k_2k_3)^{5/2}k_t^2}\times\left[(k_1+k_2)^2(-3k_1^3-3k_1^2k_2-k_2^3)+(k_1+k_2)(-9k_1^3-6k_1^2k_2-2k_2^3)k_3\right.$$
$$+(-9k_1^3-6k_1^2k_2-2k_1k_2^2-2k_2^3)k_3^2$$
$$\left.-2(2k_1^2+k_1k_2+k_2^2)k_3^3-2(k_1+k_2)k_3^4-k_3^5+3k_1^3k_t^2(\gamma+\ln(-k_t\tau_I))\right]$$

$$k_t = k_1+k_2+k_3$$

The flattened shape

- The integral contains a growing log

➡ The effect maximal, when the coefficient of the log is maximal

➡ The correlation is maximal in flattened shape

$$k_1 = 2k_2 = 2k_3$$

- In this case

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle \simeq -96 \ln(-k_t \tau_I) P_\zeta(k_1) P_B(k_2)$$

- For the largest scales exiting the horizon about *60* e-folds before the end of inflation, so $\ln(-k_t \tau_I) \sim 60$

➡ $$\left| b_{NL}^{flat} \right| \sim 5760 \quad !!!$$

[Jain, MSS 2012]

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The squeezed limit

- In the squeezed limit, $k_1 \ll k_2, k_3 = k$, the integrals simplify significantly, and we have

$$\tilde{\mathcal{I}}_n^{(1)} = -\pi \int^{\tau_I} d\tau J_{n-1/2}(-k\tau_I) Y_{n-1/2}(-k\tau_I) \qquad \tilde{\mathcal{I}}_n^{(2)} = \tilde{\mathcal{I}}_{n+1}^{(1)}$$

- For integer values of n, one can show that

$$\tilde{\mathcal{I}}_n^{(1)} = -(n-1/2)/k^2$$

- which gives

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- which gives a local type non-linearity parameter

$$b_{NL}^{local} = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I}$$

consistent with the magnetic consistency relation [Jain, MSS 2012]

Conclusions

- If the magnetic consistency relation is violated it will rule out an important class of models for magnetogenesis
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