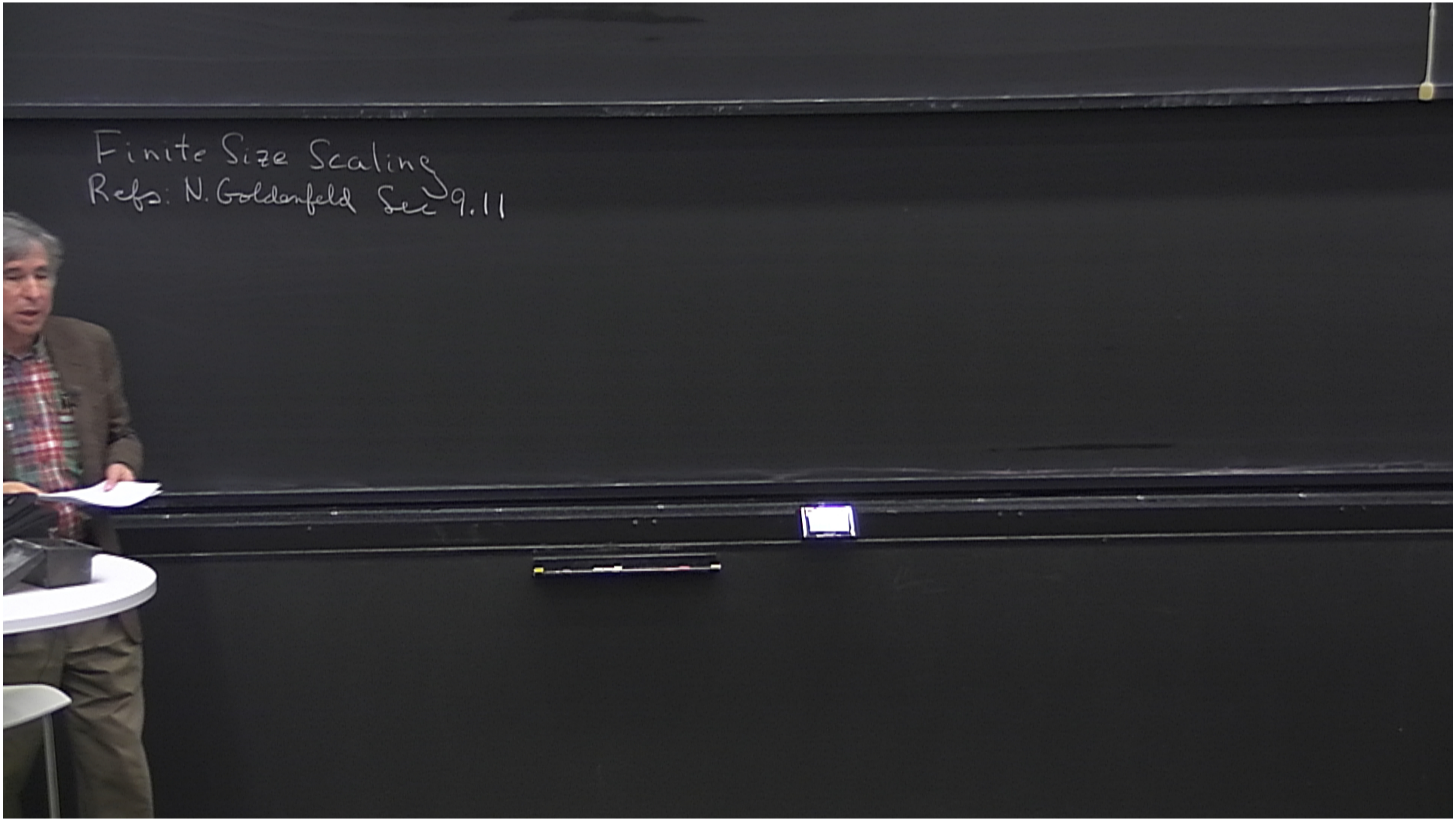
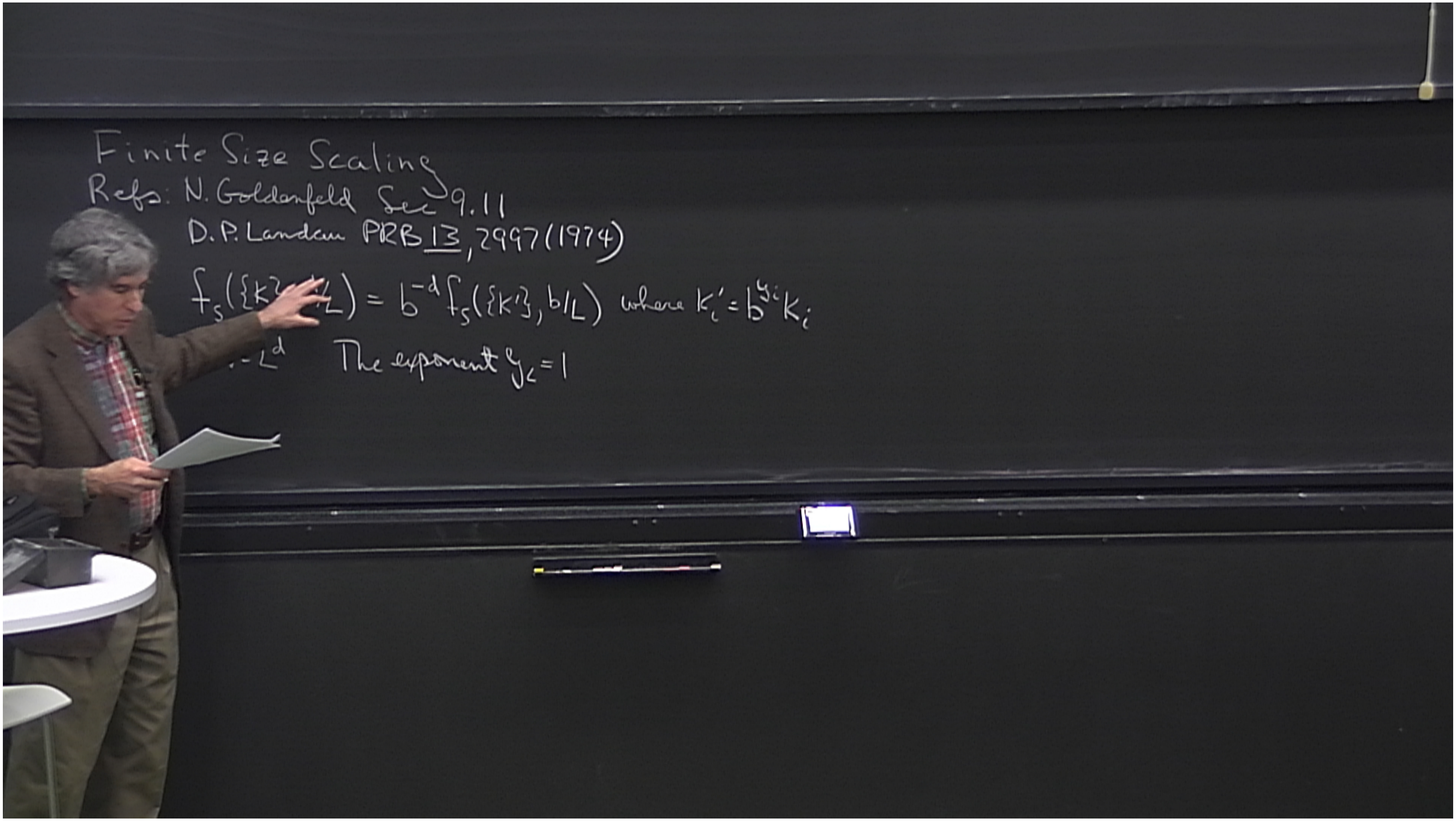
Title: Statistical Mechanics - Lecture 14

Date: Nov 29, 2012 10:30 AM

URL: http://pirsa.org/12110036

Abstract:


Finite Size Scaling
Refs: N. Goldenfeld Sec 9.11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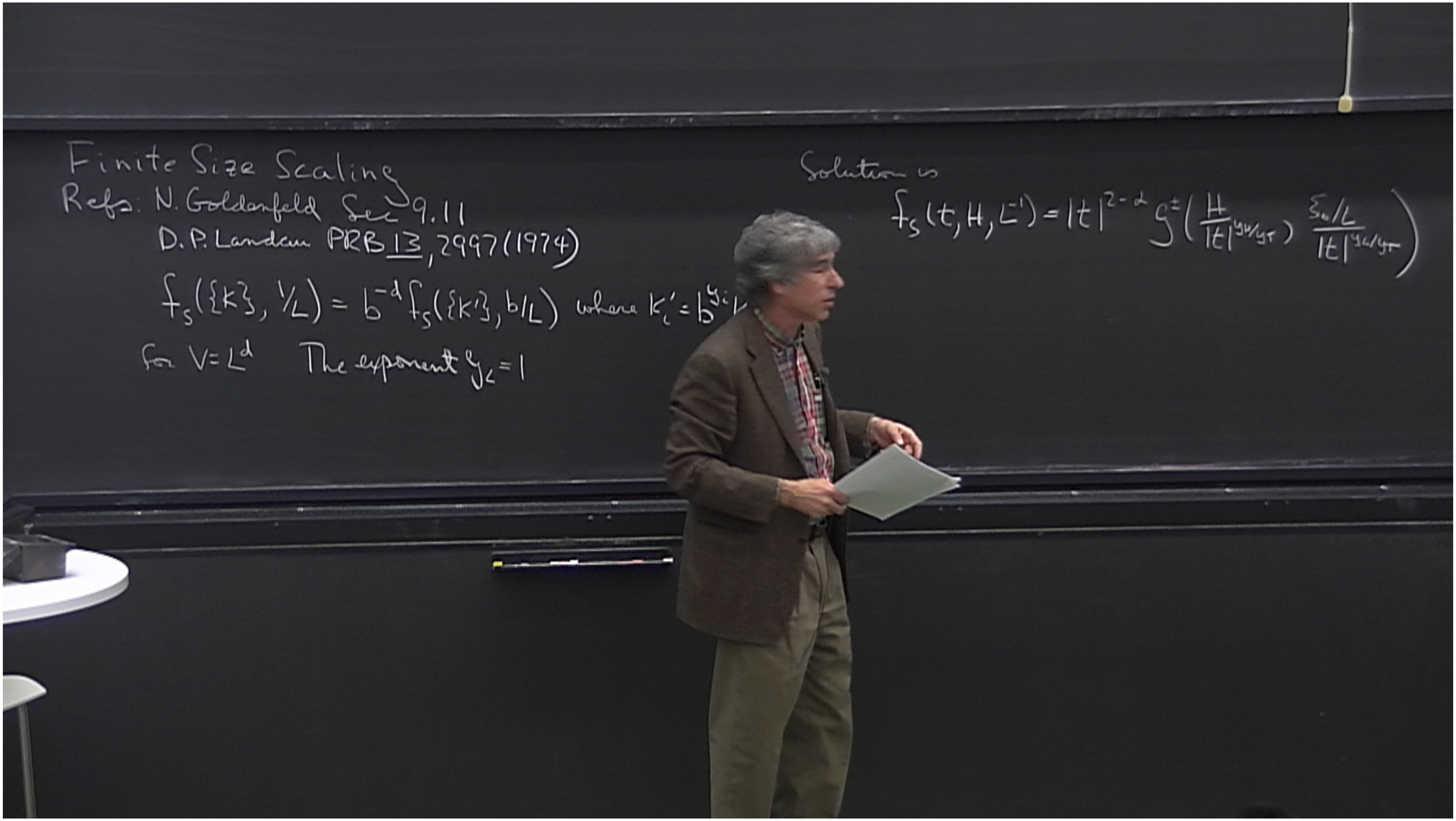
D.P. Landau PRB 13, 2997 (1974)
$f_s(\{K\}, \ldots) = b^{-d} f_s(\{K'\}, b/L)$ where $K_i' = b^{y_i} K_i$
The exponent $y_L = 1$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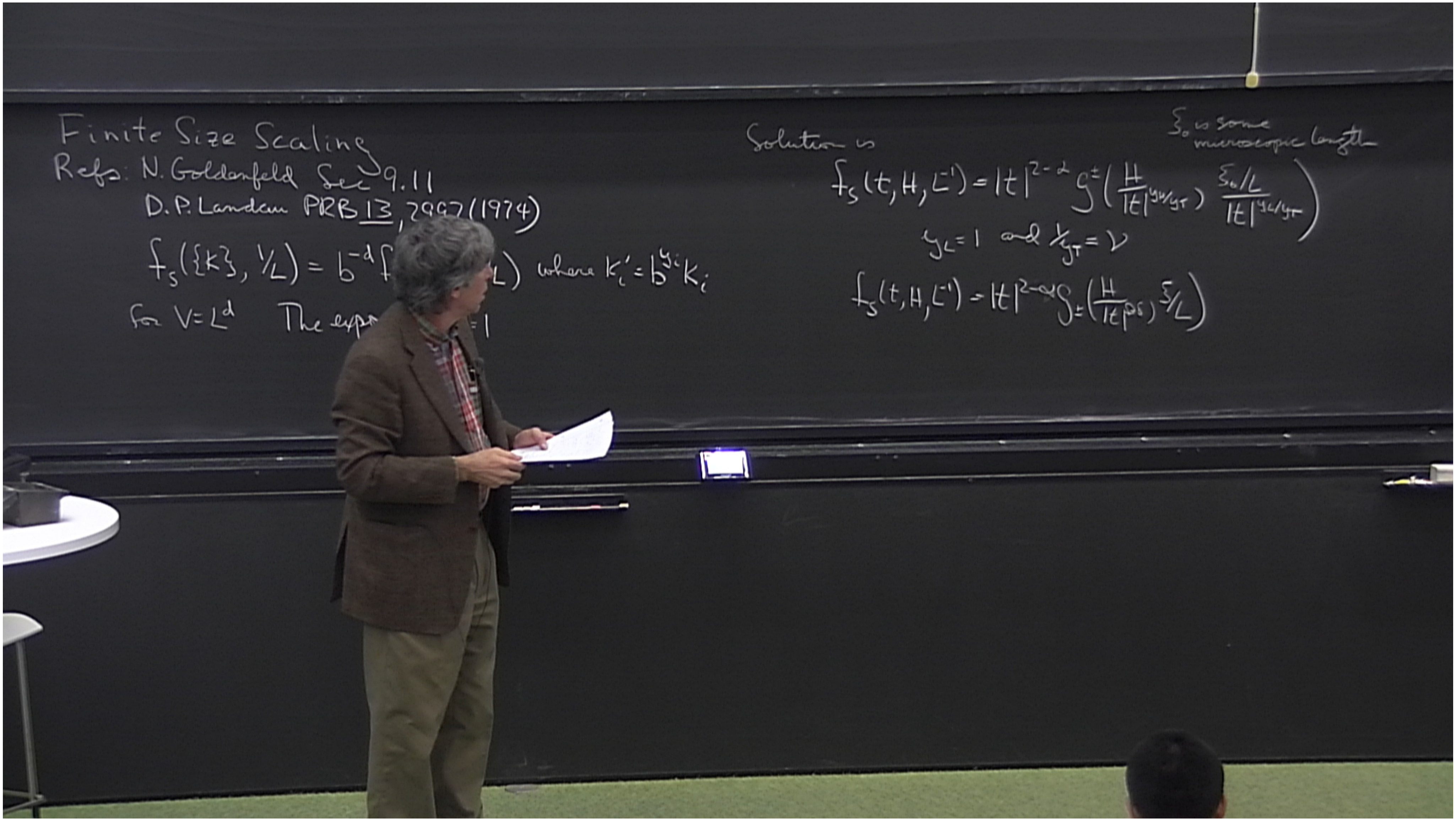
$f_s(\{K\}, 1/L) = b^{-d} f_s(\{K'\}, b/L)$ where $K_i' = b^{y_i}$
for $V = L^d$ The exponent $y_L = 1$
Solution is
$f_s(t, H, L^{-1}) = |t|^{2-d} g^{\pm}\left(\frac{H}{|t|^{y_H/y_T}}, \frac{\xi_0/L}{|t|^{y_L/y_T}}\right)$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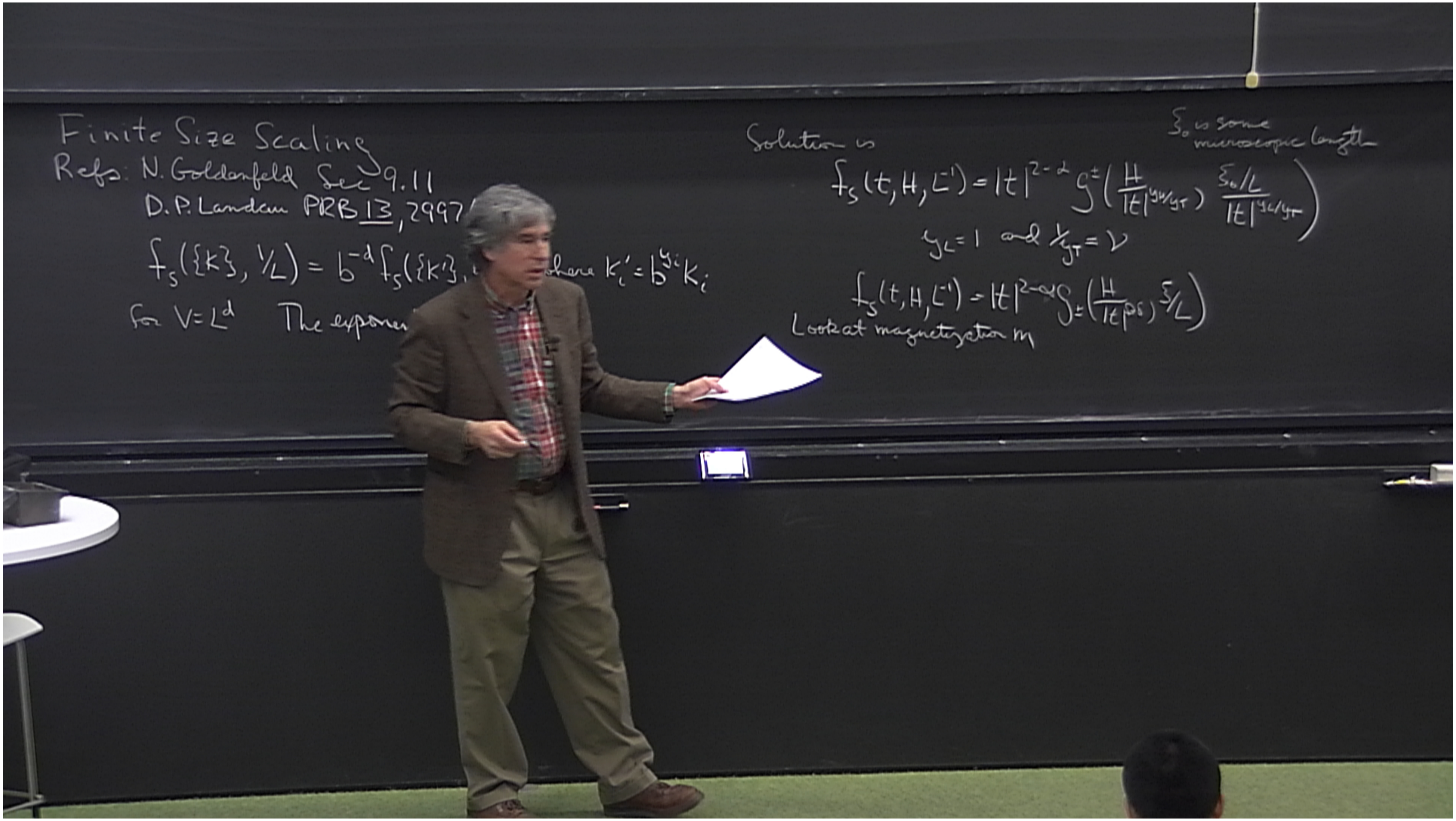
The exp
$= 1$
Solution is
ξ_0 is some microscopic length
$f_s(t, H, L^{-1}) = |t|^{2-d} g^{\pm}\left(\frac{H}{|t|^{y_H/y_T}}, \frac{\xi_0/L}{|t|^{y_L/y_T}}\right)$
$y_L = 1$ and $1/y_T = \nu$
$f_s(t, H, L^{-1}) = |t|^{2-\alpha} g_{\pm}\left(\frac{H}{|t|^{\beta\delta}}, \xi/L\right)$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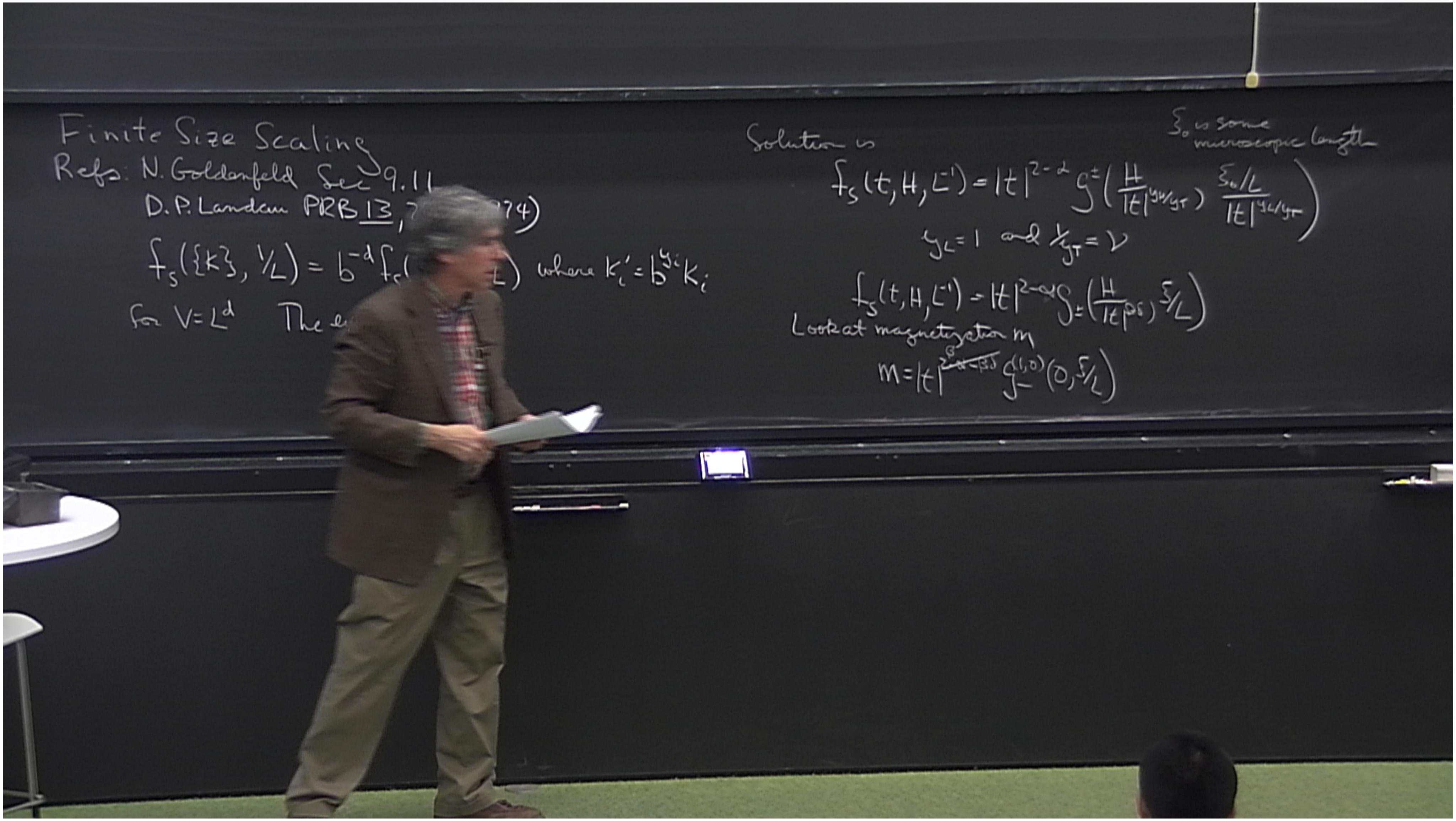
Look at magnetization m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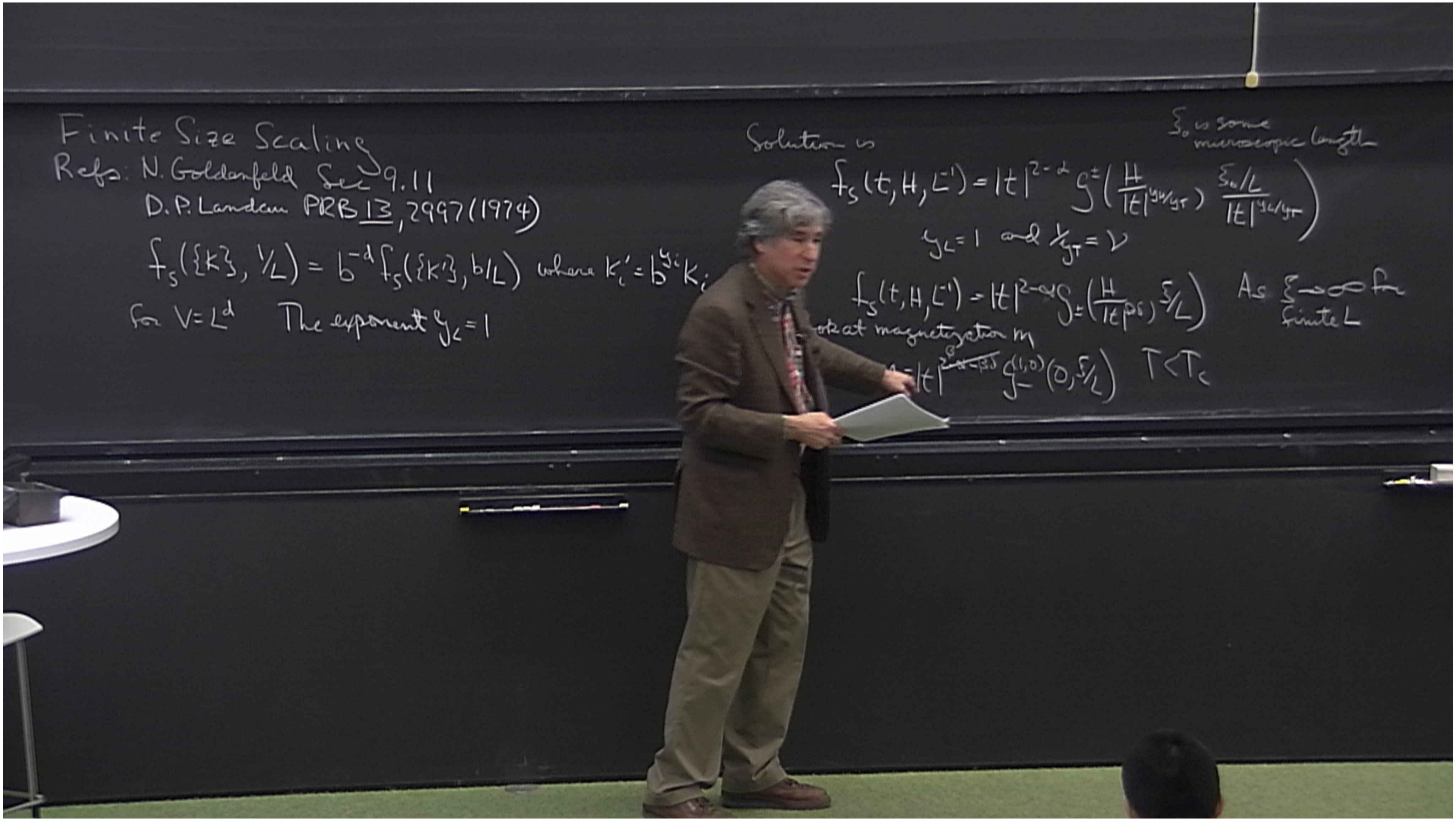
Look at magnetization m
$m = |t|^{\cancel{2-\alpha-\beta\delta}\ \beta} g_-^{(1,0)}(0, \xi/L)$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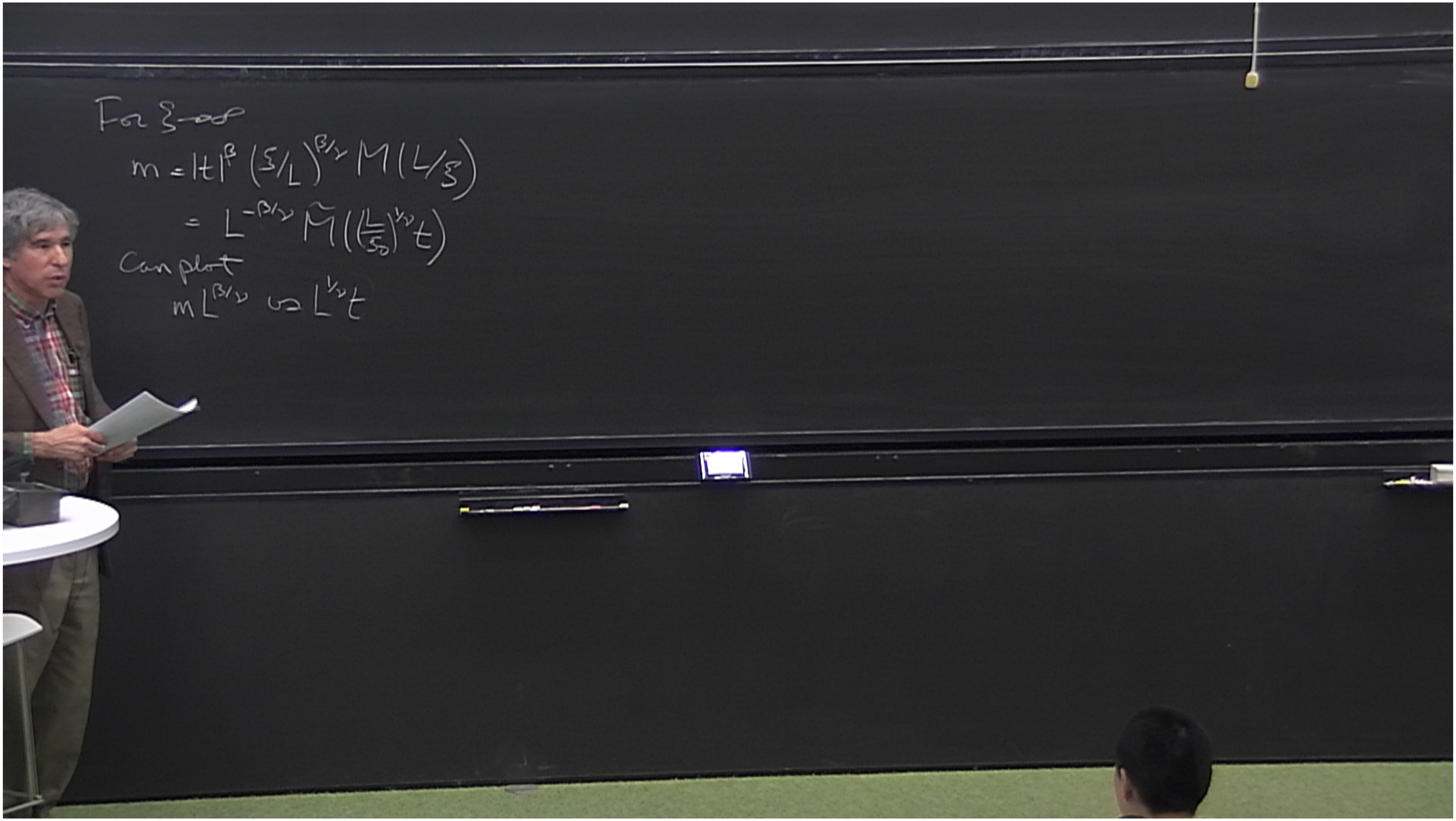
$y_L = 1$ and $1/y_T = \nu$
$f_s(t, H, L^{-1}) = |t|^{2-\alpha} g_{\pm}\left(\frac{H}{|t|^{\beta\delta}}, \xi/L\right)$
As $\xi \to \infty$ for finite L
ok at magnetization m
$= |t|^{\beta} g_-^{(1,0)}(0, \xi/L)$
$T < T_c$


For $\xi \to \infty$
$m = |t|^{\beta} (\xi/L)^{\beta/\nu} M(L/\xi)$
$= L^{-\beta/\nu} \tilde{M}\left(\left(\frac{L}{\xi_0}\right)^{1/\nu} t\right)$
Can plot
$m L^{\beta/\nu}$ vs $L^{1/\nu} t$

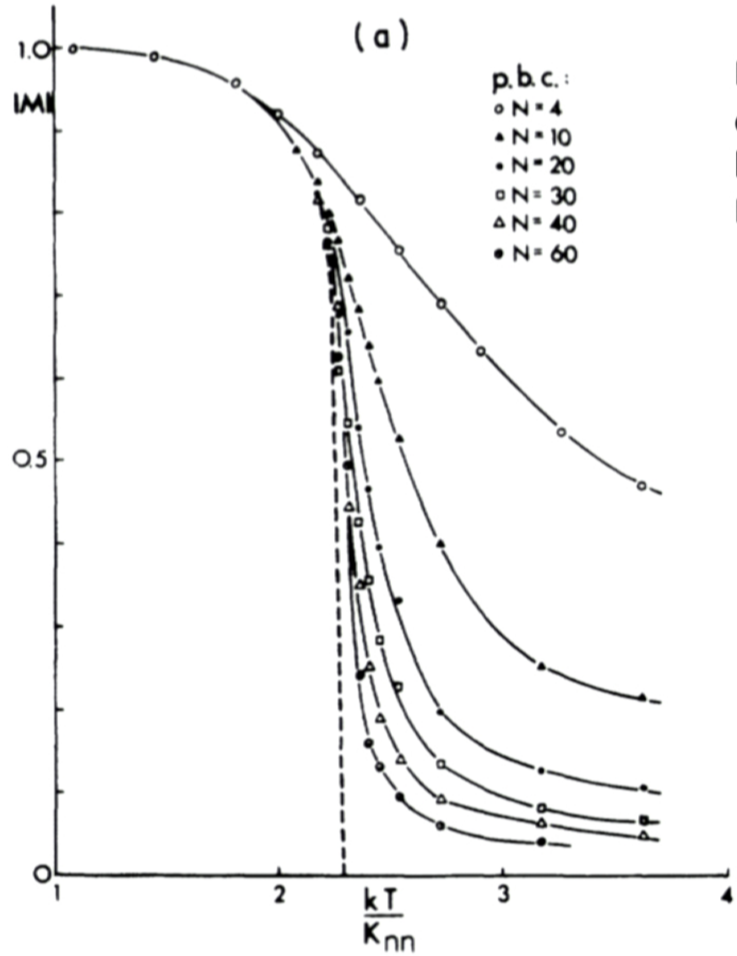
For $\xi \to \infty$

$$m = |t|^{\beta} \left(\xi/L\right)^{\beta/\nu} M\left(L/\xi\right)$$

$$= L^{-\beta/\nu} \tilde{M}\left(\left(\frac{L}{\xi_0}\right)^{1/\nu} t\right)$$

Can plot

$m L^{\beta/\nu}$ vs $L^{1/\nu} t$



Monte Carlo Data for Magnetization of 2D Ising Model with periodic boundary conditions.
D. P Landau, PRB 13, 2997 (1976).

FIG. 4. Temperature variation of the order parameter. The dashed lines represent the exact $N=\infty$ curve (Ref. 1).

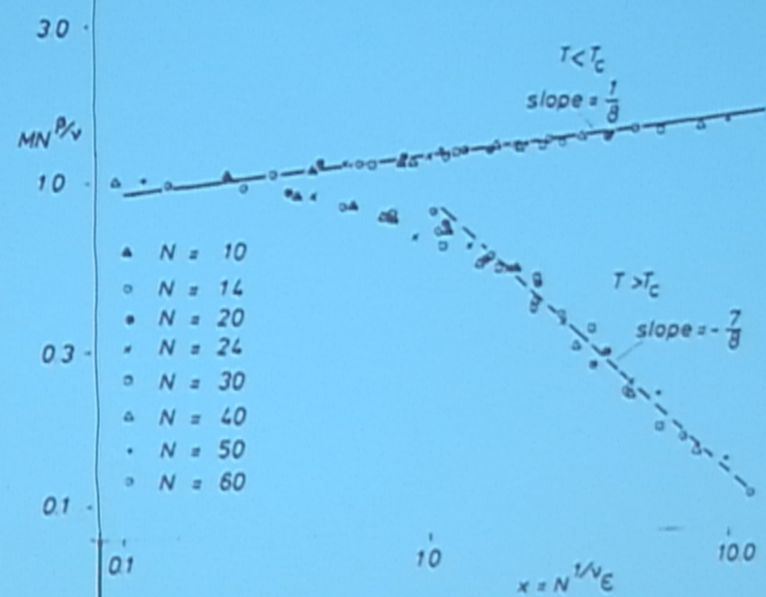


FIG. 14. Finite-size scaling plot for the order parameter for lattices with p.b.c. Data for $N=10$, ▲ ; $N=14$, ○; $N=20$, ●; $N=24$, × ; $N=30$, □ ; $N=40$, △; $N=50$, +; $N=60$, ⊙. For $T<T_c$ the solid line is $1.22\,x^{1/8}$; for $T>T_c$ the dashed line is $0.92\,x^{-7/8}$.

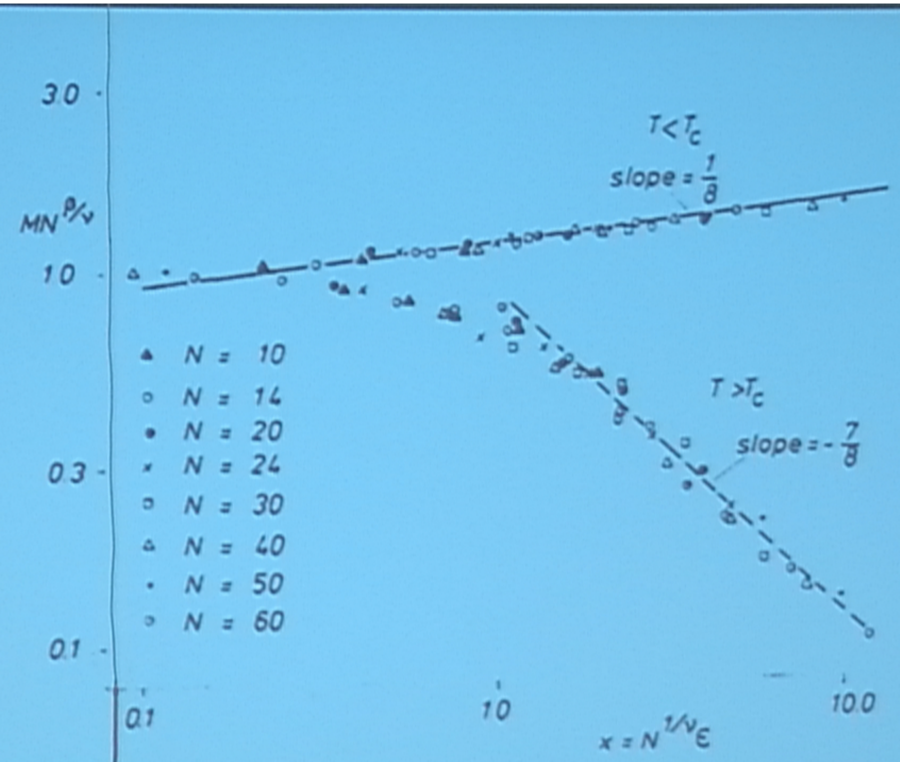
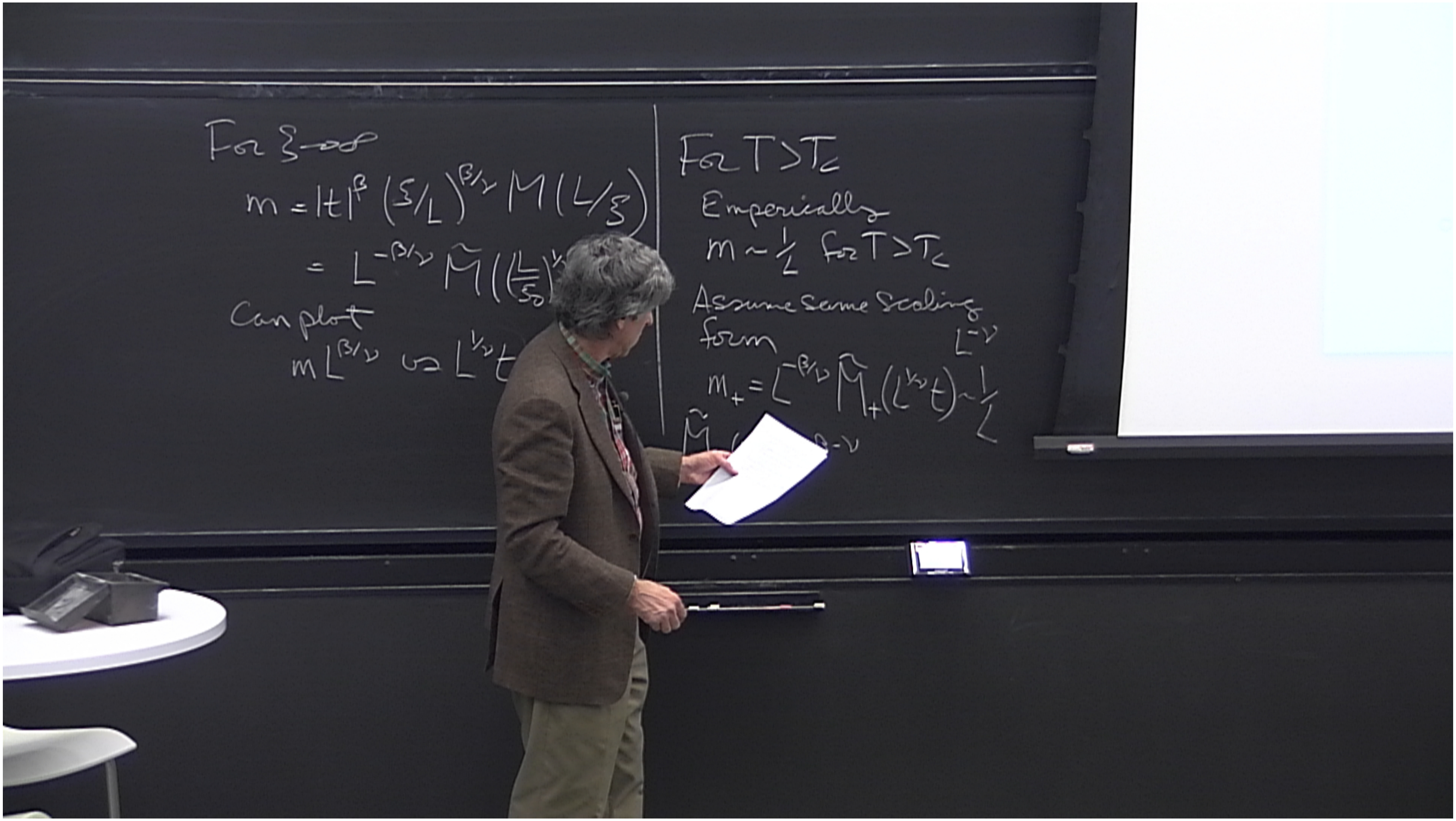


FIG. 14. Finite-size scaling plot for the order parameter for lattices with p.b.c. Data for $N=10$, ▲; $N=14$, ○; $N=20$, ●; $N=24$, ×; $N=30$, □; $N=40$, △; $N=50$, +; $N=60$, ⊙. For $T<T_c$ the solid line is $1.22\,x^{1/8}$; for $T>T_c$ the dashed line is $0.92\,x^{-7/8}$.


For $\xi \to \infty$
$m = |t|^{\beta} (\xi/L)^{\beta/\nu} M(L/\xi)$
$= L^{-\beta/\nu} \tilde{M}((L/\xi_0)^{1/\nu}$
Can plot
$m L^{\beta/\nu}$ vs $L^{1/\nu} t$
For $T > T_c$
Emperically
$m \sim 1/L$ for $T > T_c$
Assume same scaling form
$L^{-\nu}$
$m_+ = L^{-\beta/\nu} \tilde{M}_+(L^{1/\nu} t) \sim 1/L$

For $\xi \to \infty$

$$m = |t|^{\beta} \left(\xi/L\right)^{\beta/\nu} M\left(L/\xi\right)$$

$$= L^{-\beta/\nu} \tilde{M}\left(\left(\frac{L}{\xi_0}\right)^{1/\nu} t\right)$$

Can plot

$$m L^{\beta/\nu} \text{ vs } L^{1/\nu} t$$

For $T > T_c$

Emperically

$$m \sim \frac{1}{L} \text{ for } T > T_c$$

Assume same scaling form

$$L^{-\nu}$$

$$m_+ = L^{-\beta/\nu} \tilde{M}_+\left(L^{1/\nu} t\right) \sim \frac{1}{L}$$

$$\tilde{M}_+(x) \sim x^{\beta - \nu} = x^{-7/8}$$

Crossover Phenomena

Ref: N. Goldenfeld Sec 9.9

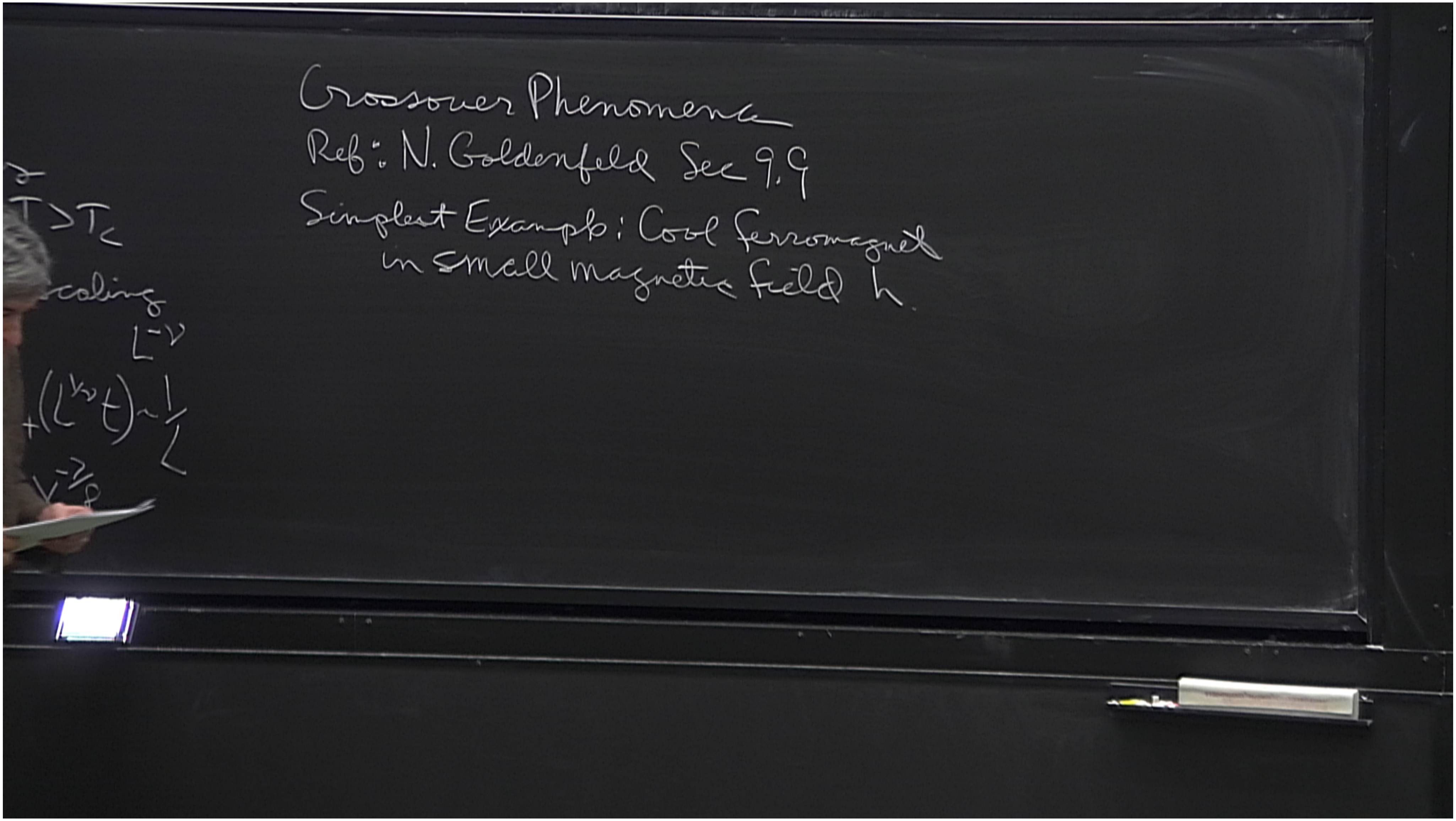
$T > T_c$

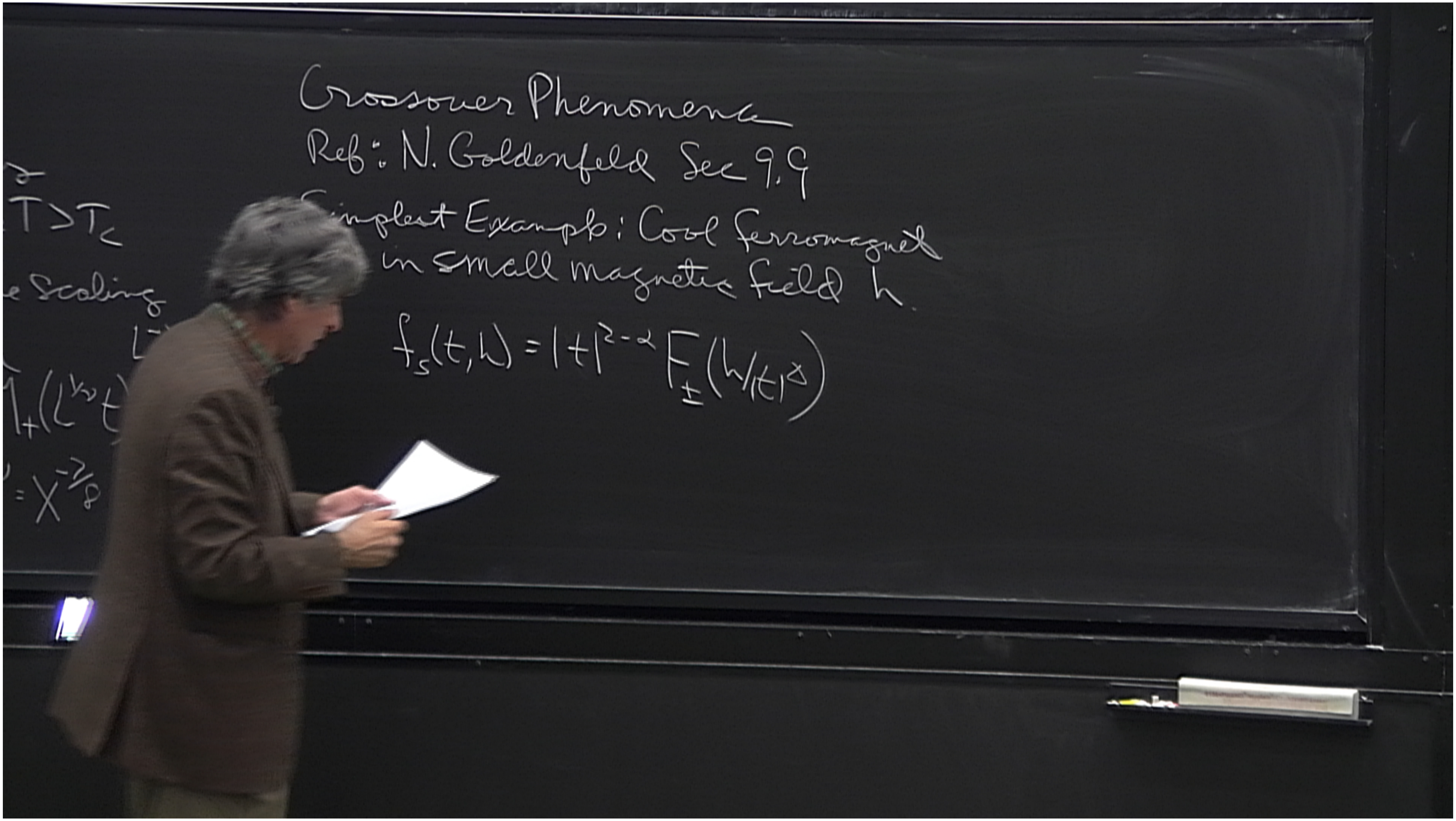
e Scaling

$L^{-\nu}$

$\eta_+(L^{1/\nu} t) \sim \frac{1}{L}$

$= X^{-7/8}$


Crossover Phenomena
Ref: N. Goldenfeld Sec 9.9
Simplest Example: Cool ferromagnet
in small magnetic field h
T > T_c
scaling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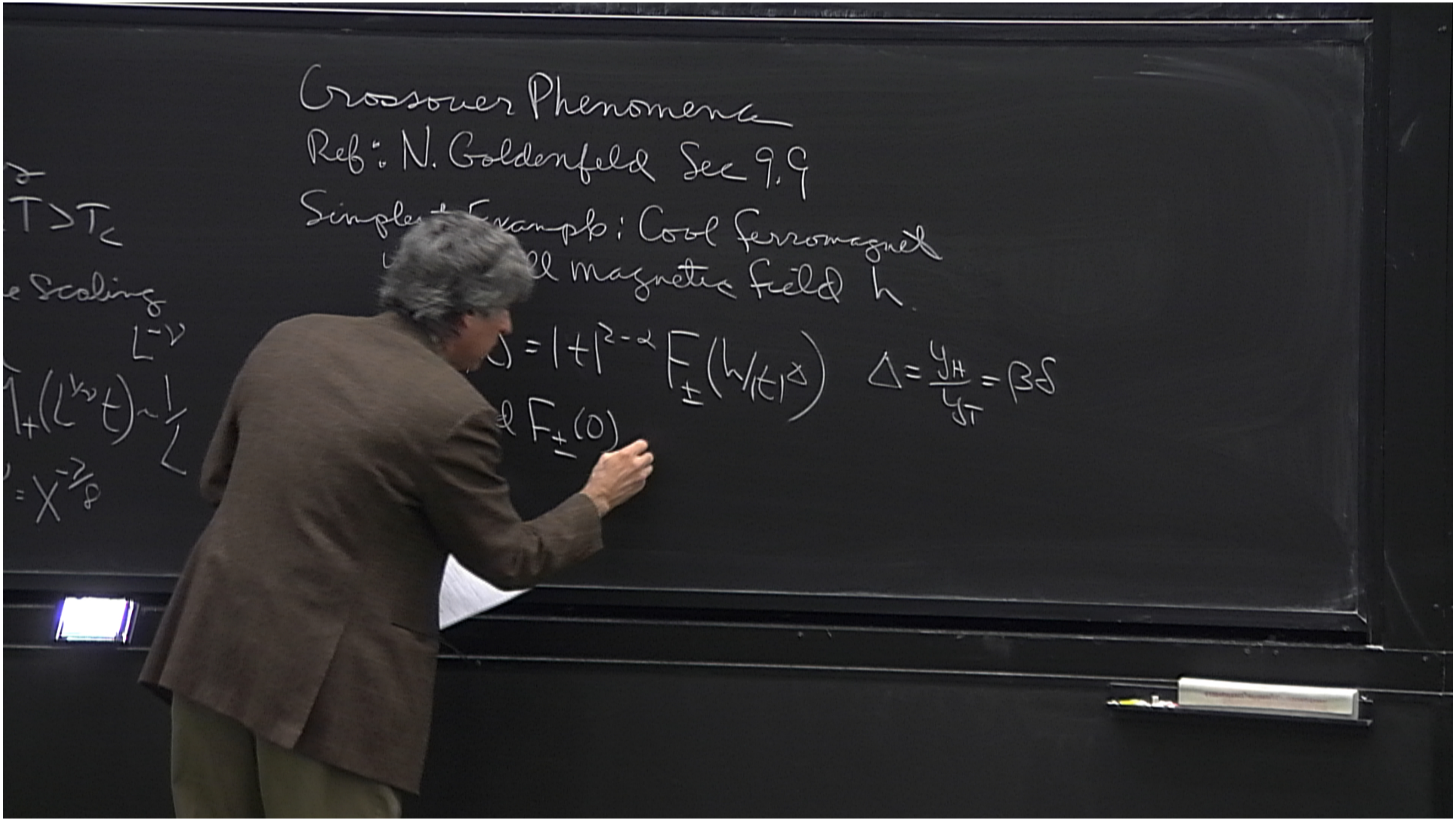
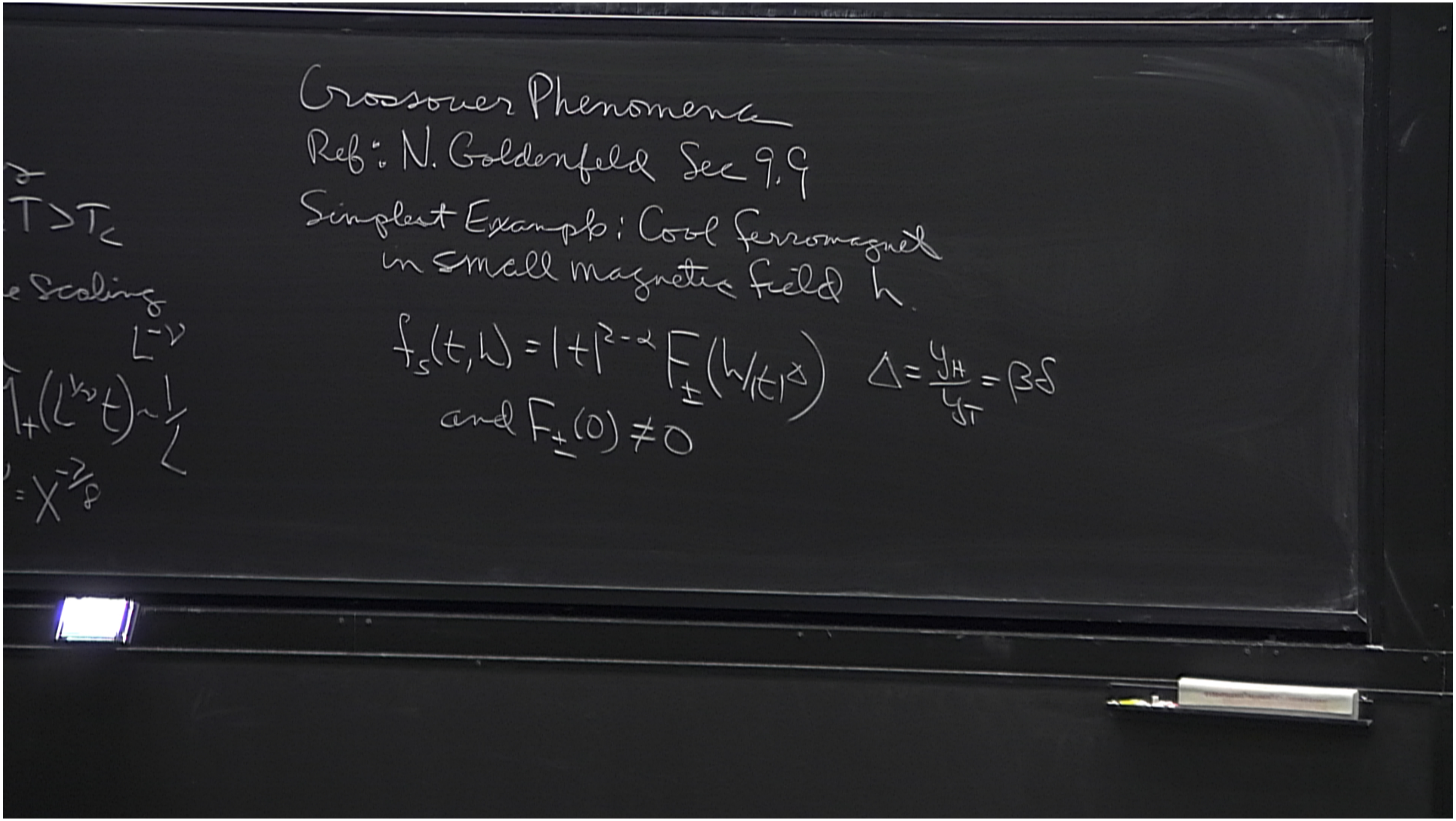
$f_s(t,h) = |t|^{2-\alpha} F_\pm(h/|t|^\Delta)$
$T > T_c$


Crossover Phenomena
Ref: N. Goldenfeld Sec 9.9
Cool ferromagnet
magnetic field h
T > T_c
Scaling
Δ = y_H / y_T = βδ


Crossover Phenomena
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Simplest Example: Cool ferromagnet
in small magnetic field h
$f_s(t,h) = |t|^{2-\alpha} F_\pm(h/|t|^\Delta)$
$\Delta = \frac{y_H}{y_T} = \beta\delta$
and $F_\pm(0) \neq 0$
$T > T_c$
Scaling
$L^{-\nu}$

Crossover Phenomena

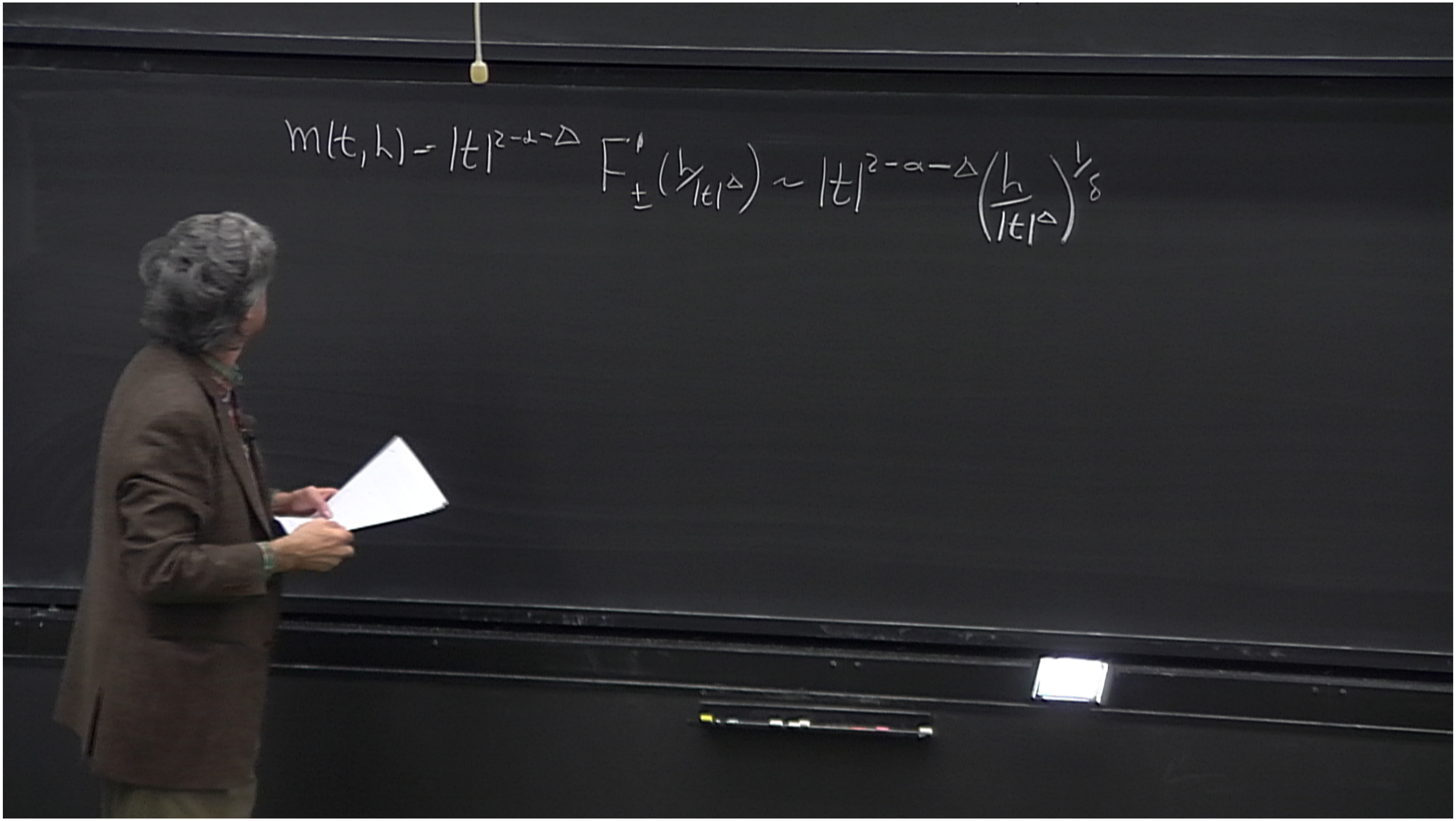
Ref: N. Goldenfeld Sec 9.9

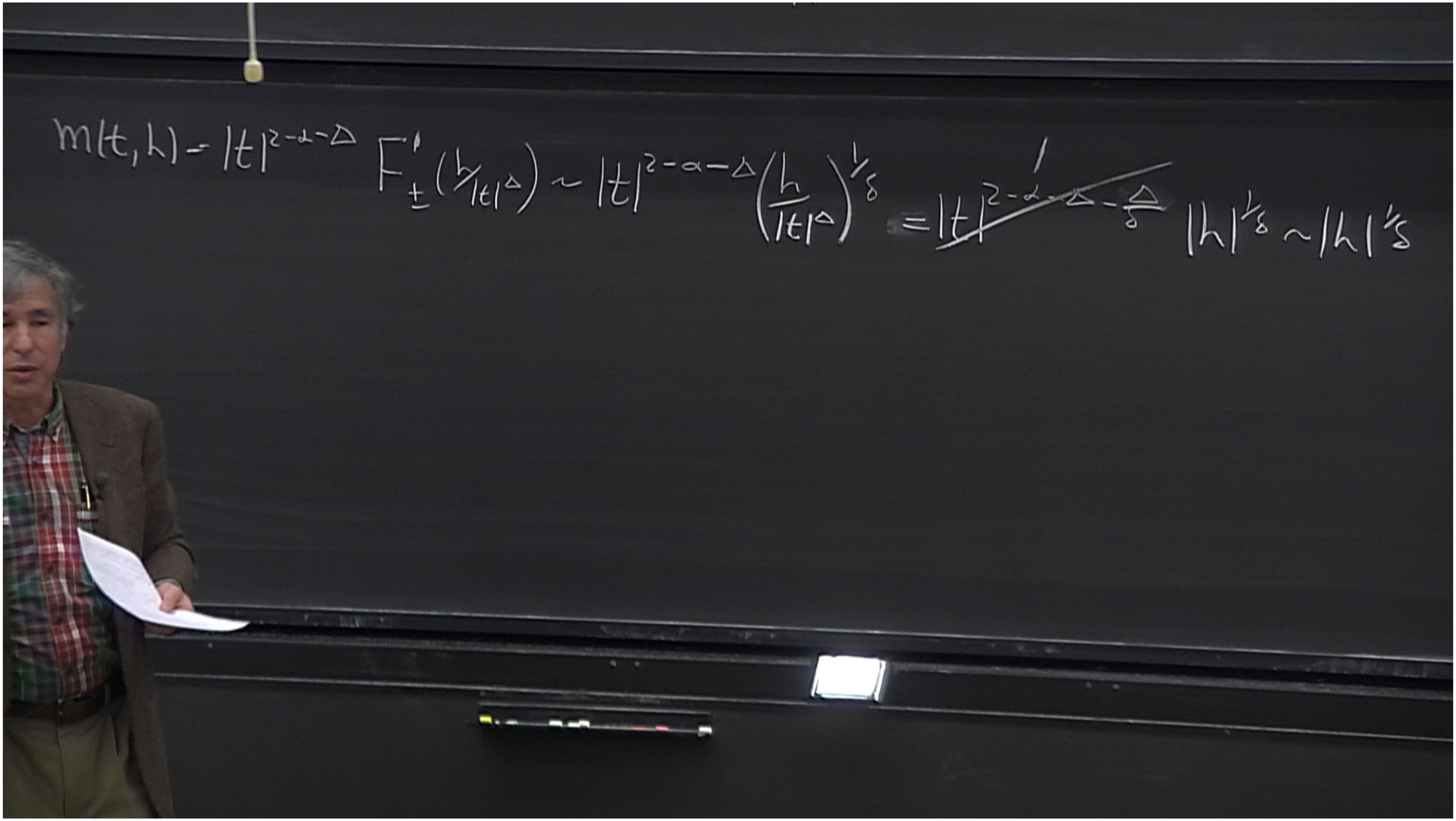
Simplest Example: Cool ferromagnet in small magnetic field h.

$$f_s(t,h) = |t|^{2-\alpha} F_{\pm}\left(h/|t|^{\Delta}\right) \qquad \Delta = \frac{y_H}{y_T} = \beta\delta$$

and $F_{\pm}(0) \neq 0$

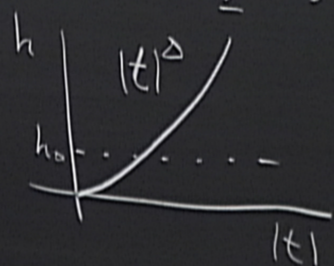
what happens for large $h/|t|^{\Delta}$?

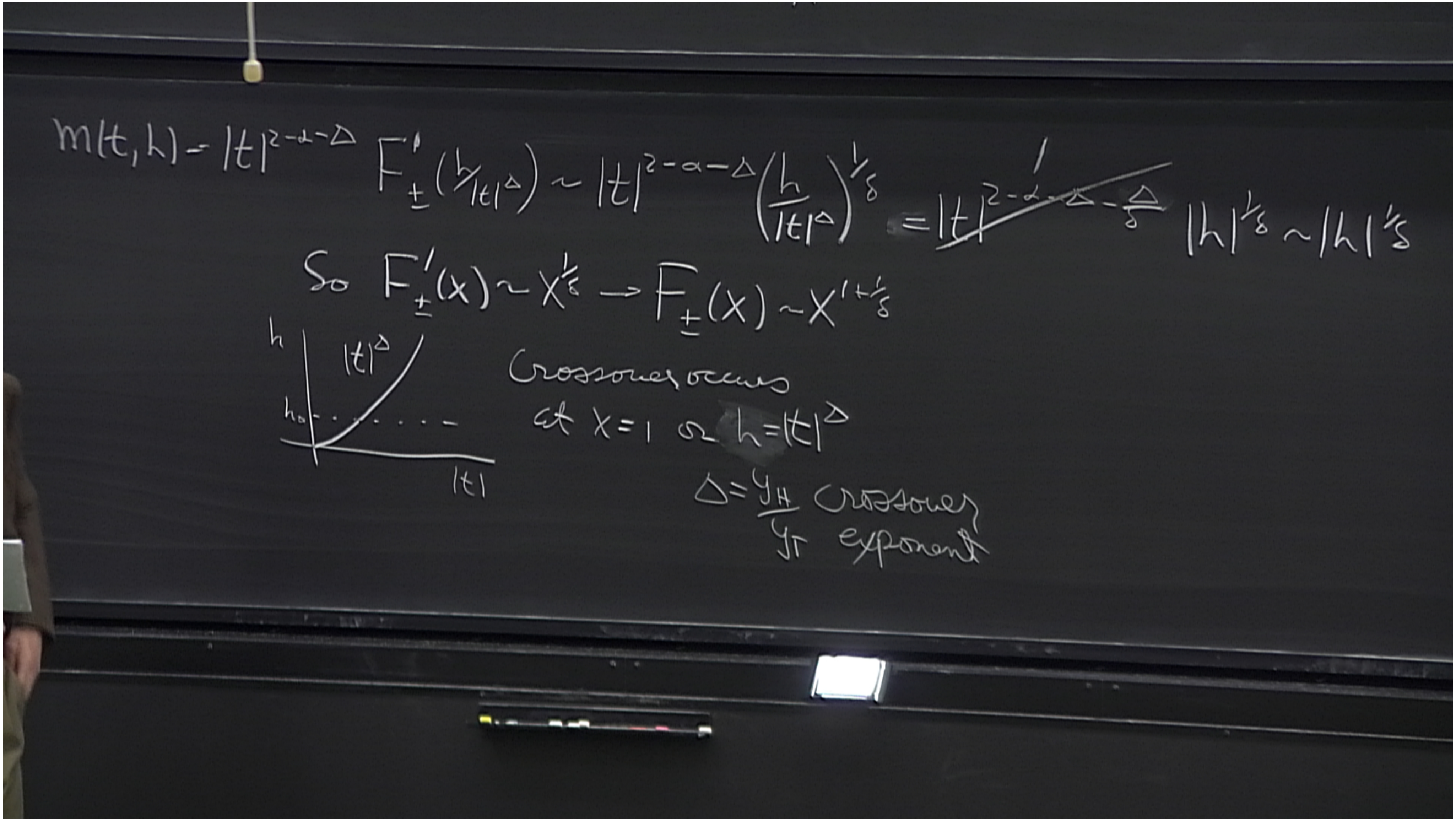

$m(t,h) \sim |t|^{2-\alpha-\Delta} F'_{\pm}(h/|t|^{\Delta}) \sim |t|^{2-\alpha-\Delta}\left(\frac{h}{|t|^{\Delta}}\right)^{1/\delta}$


$m(t,h) \sim |t|^{2-\alpha-\Delta} F'_{\pm}(h/|t|^{\Delta}) \sim |t|^{2-\alpha-\Delta}\left(\frac{h}{|t|^{\Delta}}\right)^{1/\delta} = \cancel{|t|^{2-\alpha-\Delta-\frac{\Delta}{\delta}}}^{\,1} |h|^{1/\delta} \sim |h|^{1/\delta}$

$$m(t,h) \sim |t|^{2-\alpha-\Delta} F'_{\pm}\left(h/|t|^{\Delta}\right) \sim |t|^{2-\alpha-\Delta}\left(\frac{h}{|t|^{\Delta}}\right)^{1/\delta} = \overset{1}{\cancel{|t|^{2-\alpha-\Delta-\frac{\Delta}{\delta}}}}\, |h|^{1/\delta} \sim |h|^{1/\delta}$$

So $F'_{\pm}(x) \sim x^{1/\delta} \rightarrow F_{\pm}(x) \sim x^{1+1/\delta}$

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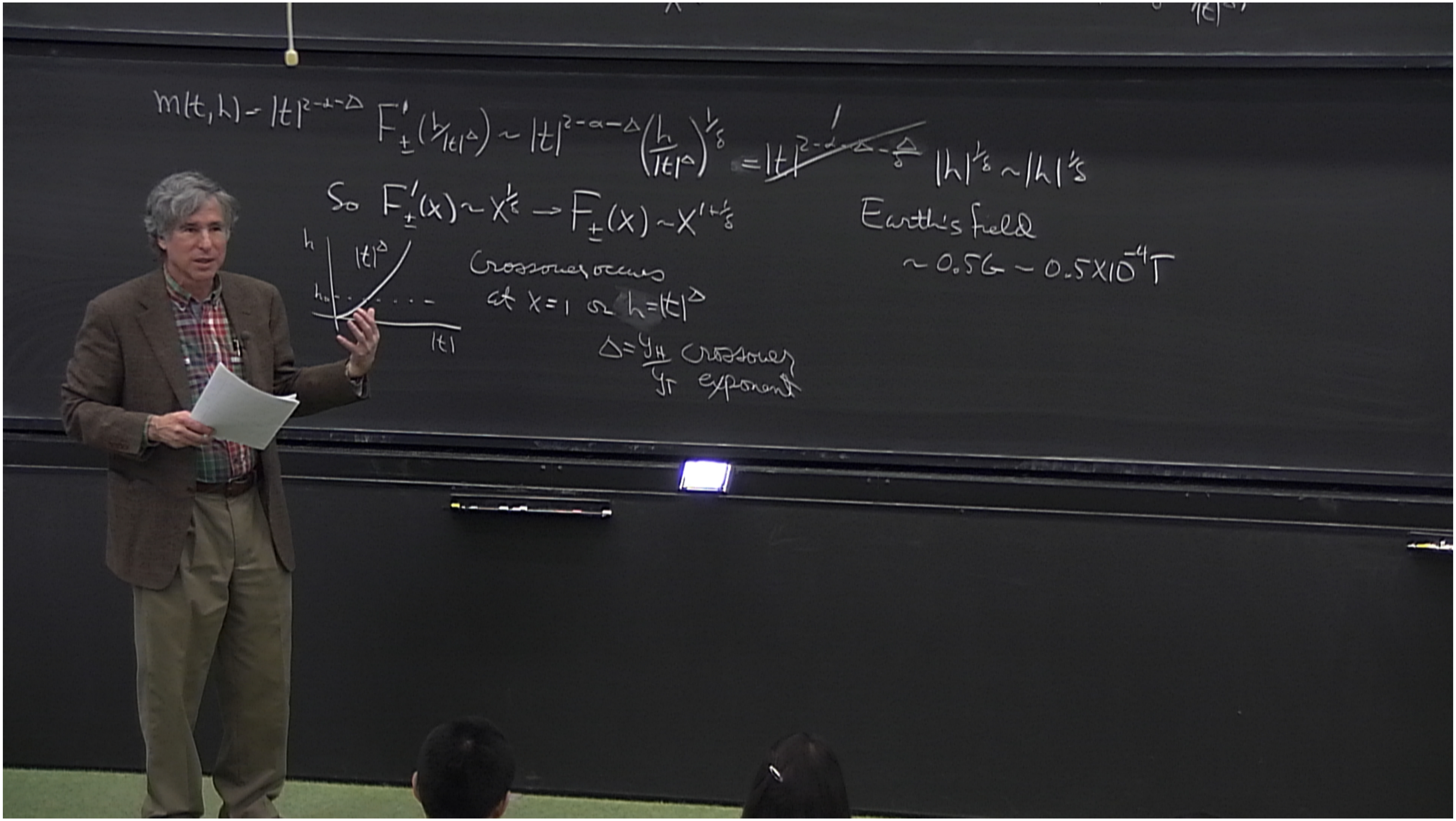
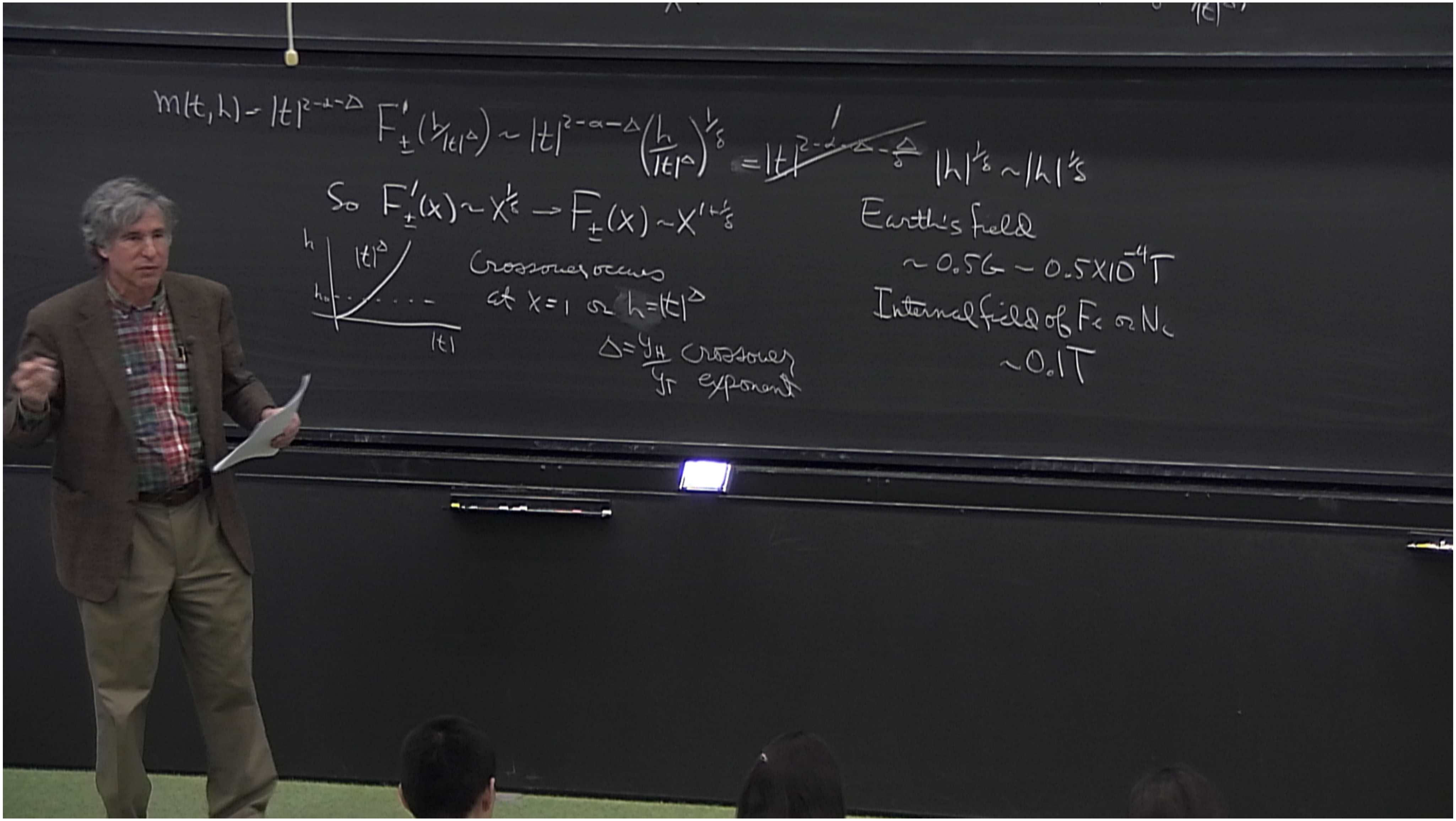
1
So $F'_\pm(x) \sim x^{1/\delta} \rightarrow F_\pm(x) \sim x^{1+1/\delta}$
h
$|t|^\Delta$
h_0
$|t|$
Crossover occurs
at $x=1$ or $h=|t|^\Delta$
$\Delta = \frac{y_H}{y_T}$ crossover exponent


m(t,h) ~ |t|^(2-α-Δ) F'±(h/|t|^Δ) ~ |t|^(2-α-Δ) (h/|t|^Δ)^(1/δ) = |t|^(2-α-Δ-Δ/δ) |h|^(1/δ) ~ |h|^(1/δ)
So F'±(x) ~ x^(1/δ) → F±(x) ~ x^(1+1/δ)
Crossover occurs at x=1 or h=|t|^Δ
Δ = yH/yT crossover exponent
Earth's field ~ 0.5G ~ 0.5×10^-4 T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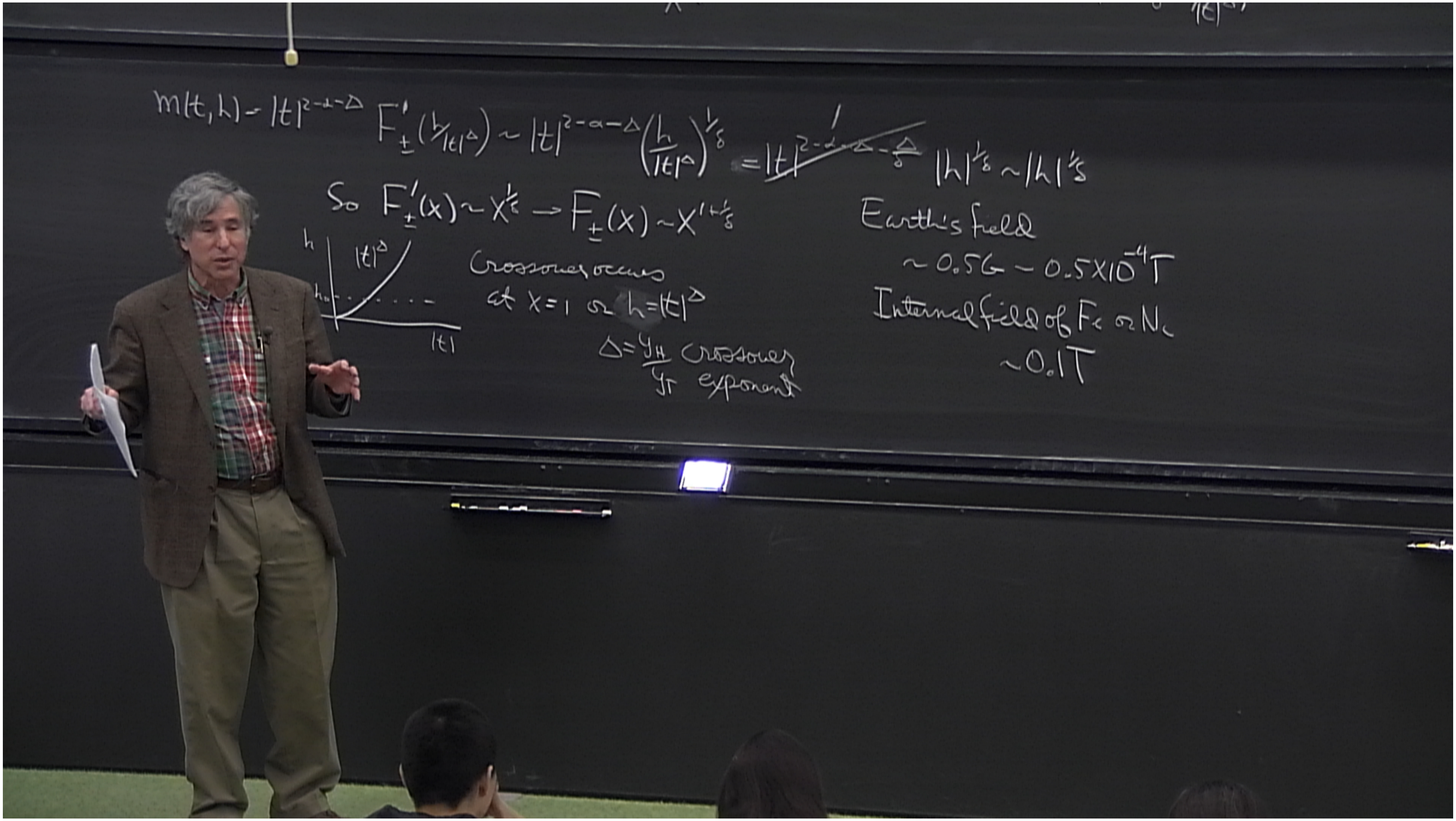
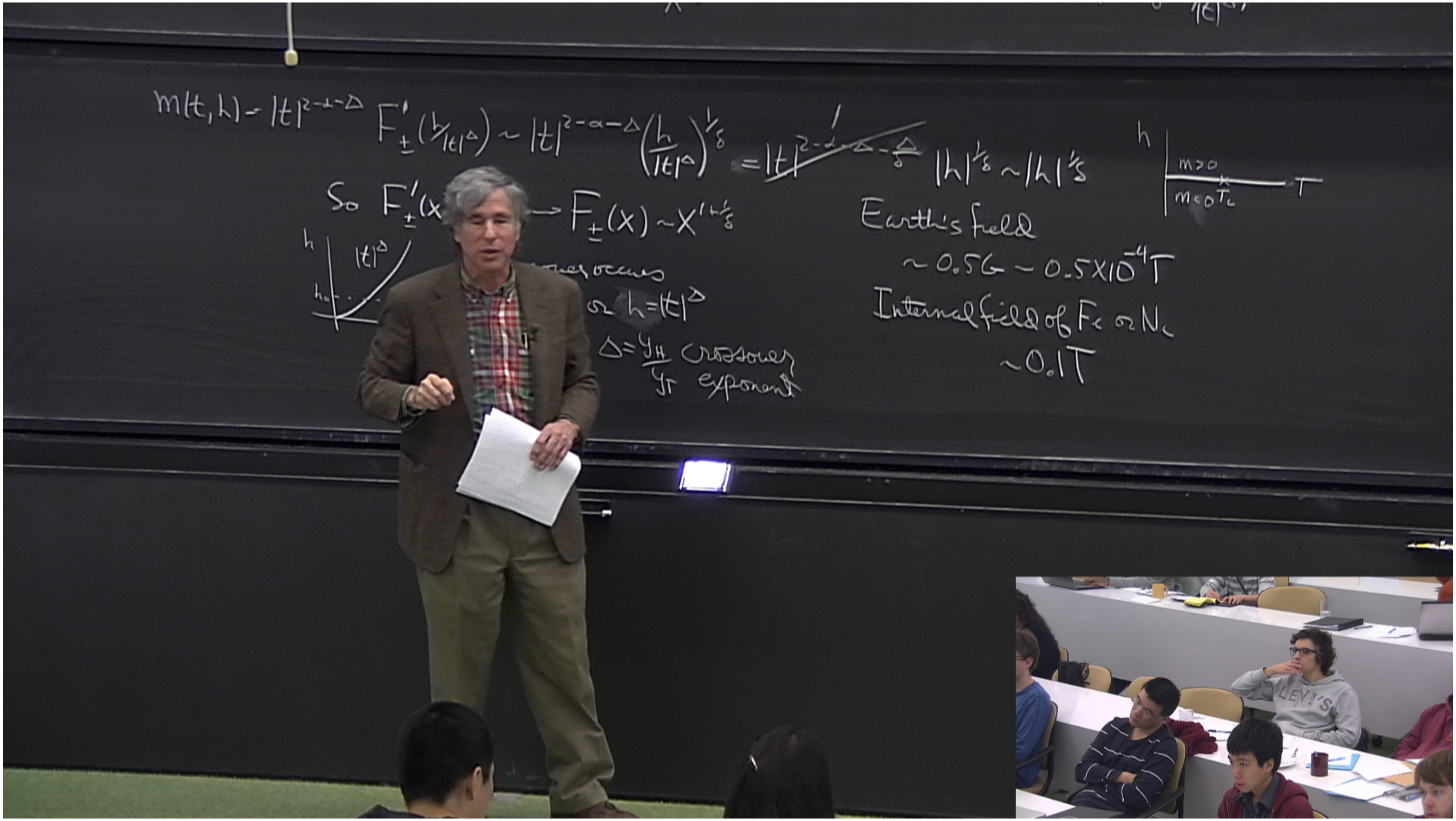
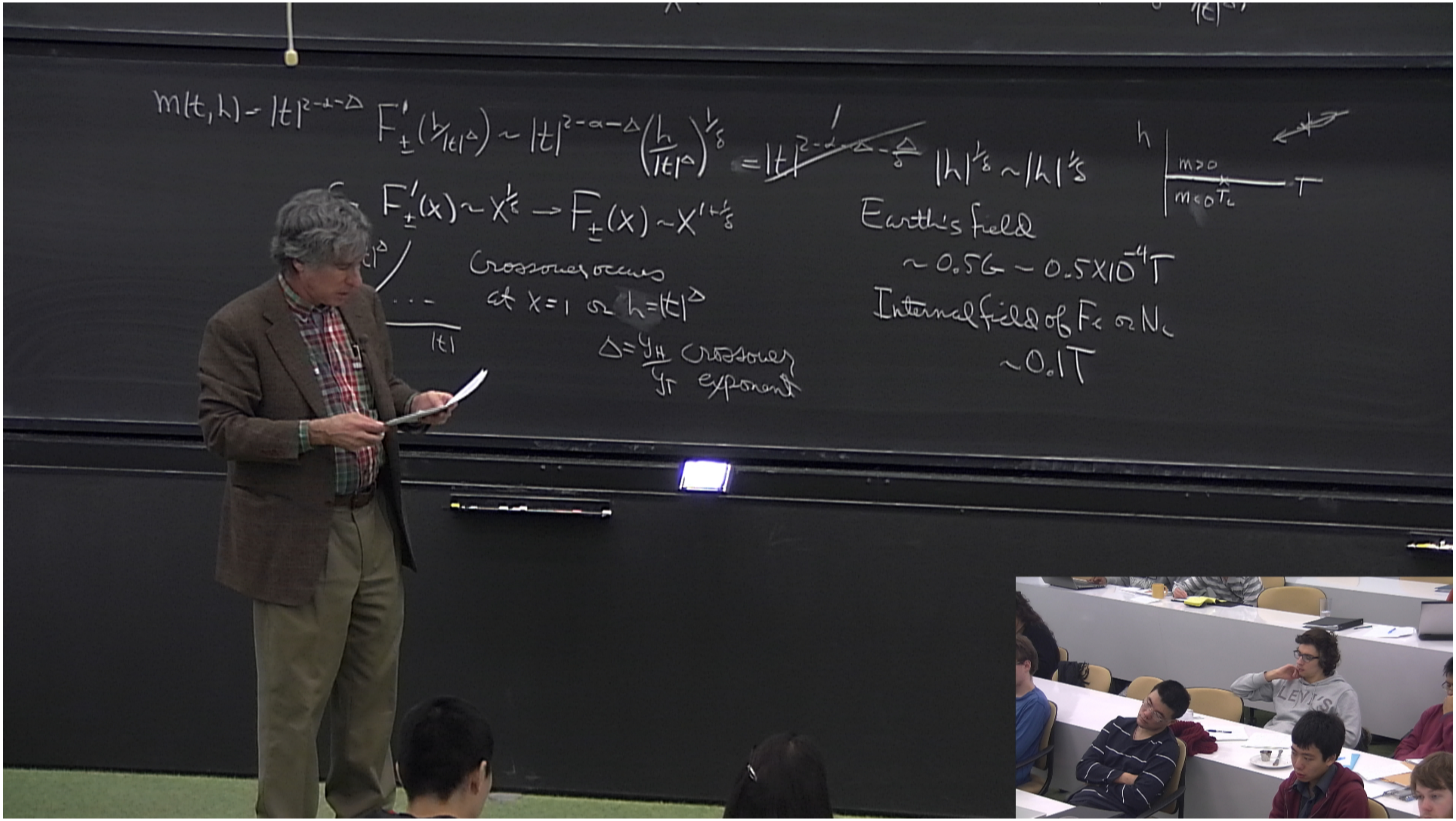
So F'±(x) ~ x^(1/δ) → F±(x) ~ x^(1+1/δ)
h
|t|^Δ
|t|
Crossover occurs at x=1 or h=|t|^Δ
Δ = yH/yT crossover exponent
Earth's field
~ 0.5G ~ 0.5×10^-4 T
Internal field of Fe or Ni
~0.1T

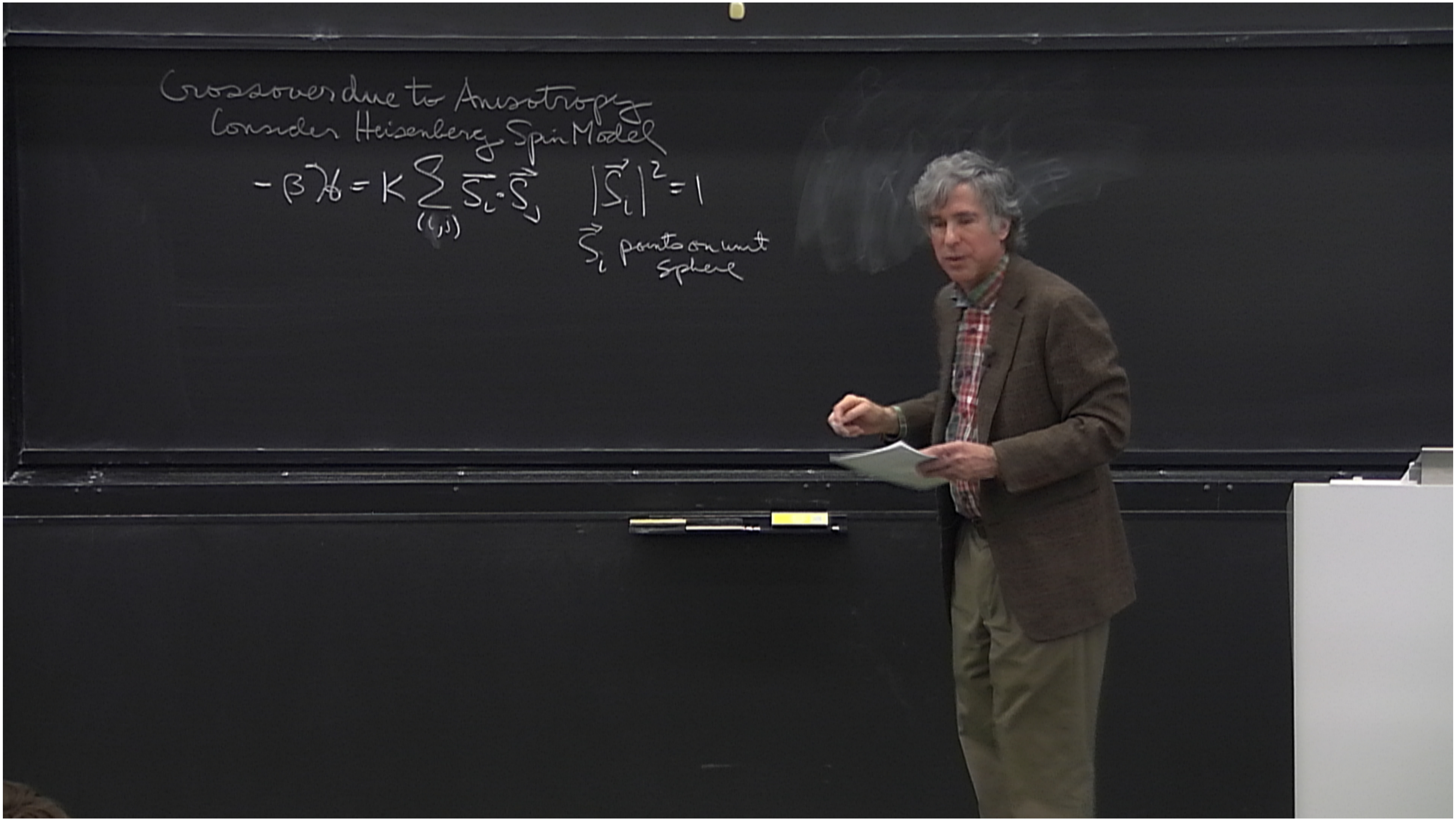


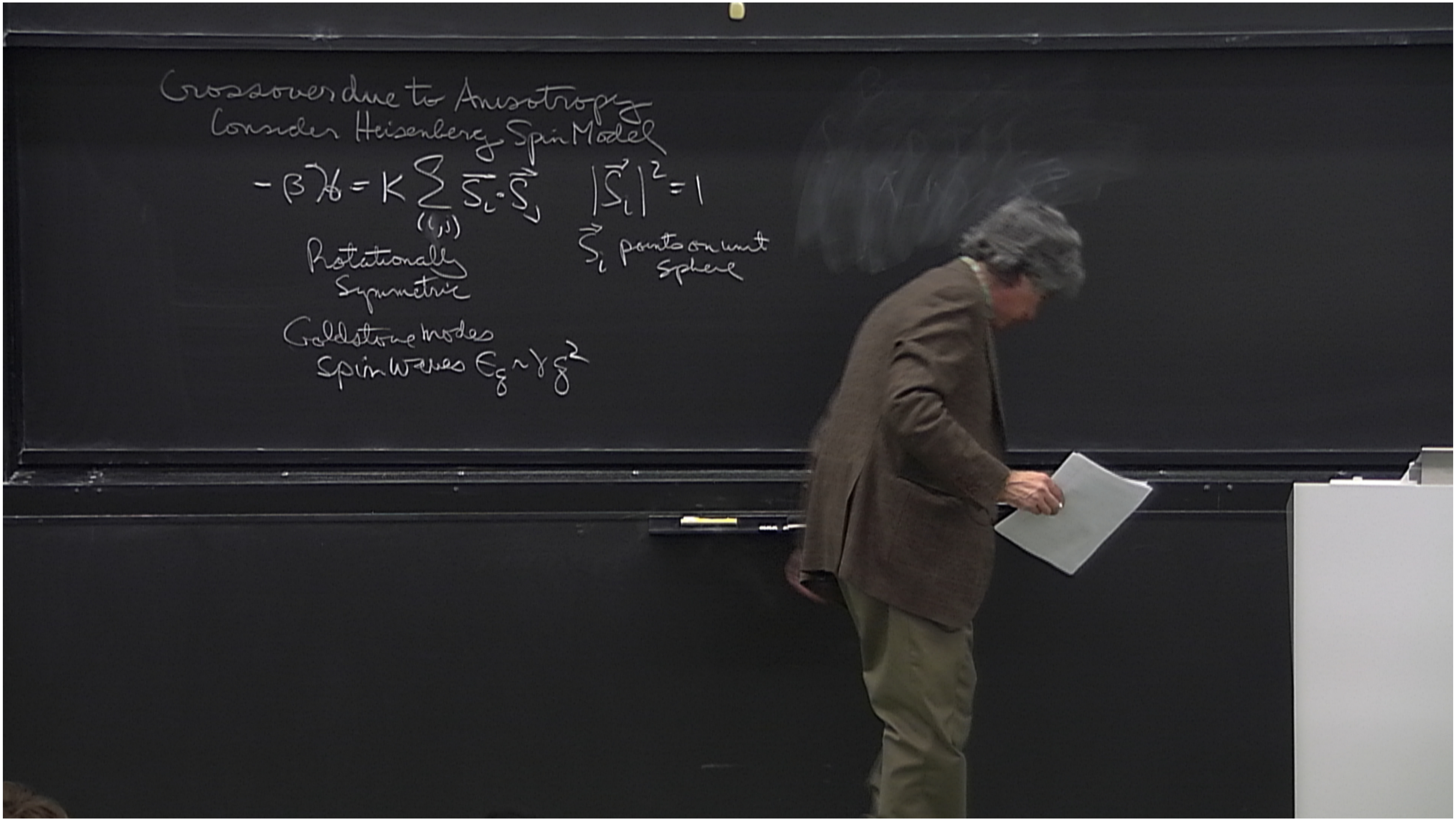
$m(t,h) \sim |t|^{2-\alpha-\Delta} F'_{\pm}(h/|t|^{\Delta}) \sim |t|^{2-\alpha-\Delta}\left(\frac{h}{|t|^{\Delta}}\right)^{1/\delta} = |t|^{2-\alpha-\Delta-\frac{\Delta}{\delta}} |h|^{1/\delta} \sim |h|^{1/\delta}$
So $F'_{\pm}(x) \sim x^{1/\delta} \to F_{\pm}(x) \sim x^{1+1/\delta}$
h
$|t|^{\Delta}$
$|t|$
Crossover occurs at $x=1$ or $h=|t|^{\Delta}$
$\Delta = \frac{y_H}{y_T}$ crossover exponent
Earth's field
$\sim 0.5G \sim 0.5\times10^{-4}T$
Internal field of Fe or Ni
$\sim 0.1T$





$m(t,h) \sim |t|^{2-\alpha-\Delta} F'_{\pm}(h/|t|^{\Delta}) \sim |t|^{2-\alpha-\Delta}\left(\frac{h}{|t|^{\Delta}}\right)^{1/\delta}$
$F'_{\pm}(x) \sim x^{1/\delta} \to F_{\pm}(x) \sim x^{1+1/\delta}$
$|h|^{1/\delta} \sim |h|^{1/\delta}$
Crossover occurs
at $x=1$ or $h=|t|^{\Delta}$
$\Delta = \frac{y_H}{y_T}$ crossover exponent
Earth's field
$\sim 0.5G \sim 0.5\times10^{-4}T$
Internal field of Fe or Ni
$\sim 0.1T$
m>0
m<0
T_c
T
h


Crossover due to Anisotropy
Consider Heisenberg Spin Model
$-\beta\mathcal{H} = K \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j$
$|\vec{S}_i|^2 = 1$
$\vec{S}_i$ points on unit sphere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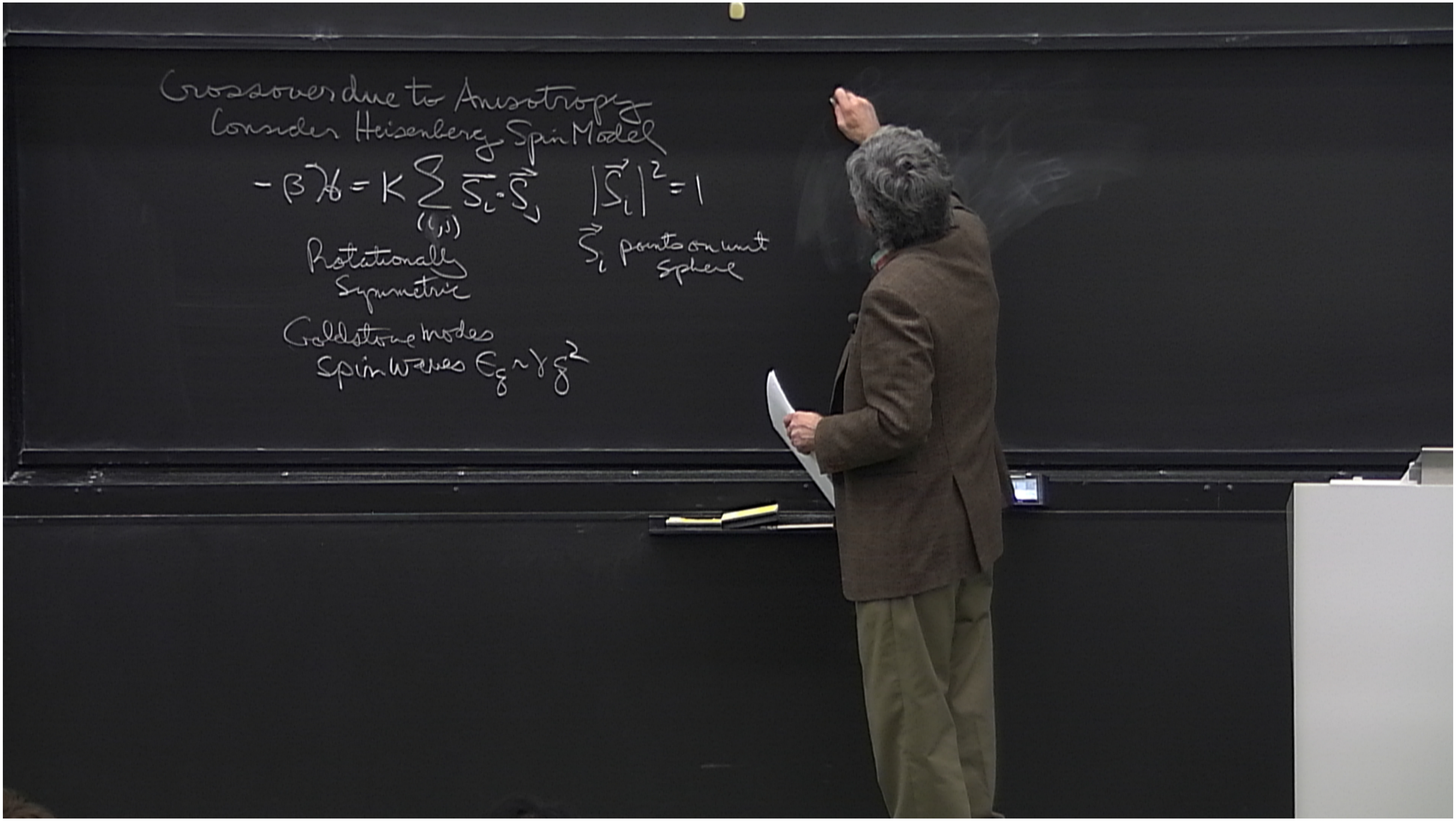
$|\vec{S}_i|^2 = 1$
Rotationally Symmetric
$\vec{S}_i$ points on unit sphere
Goldstone modes
Spin Waves $\epsilon_q \sim \gamma q^2$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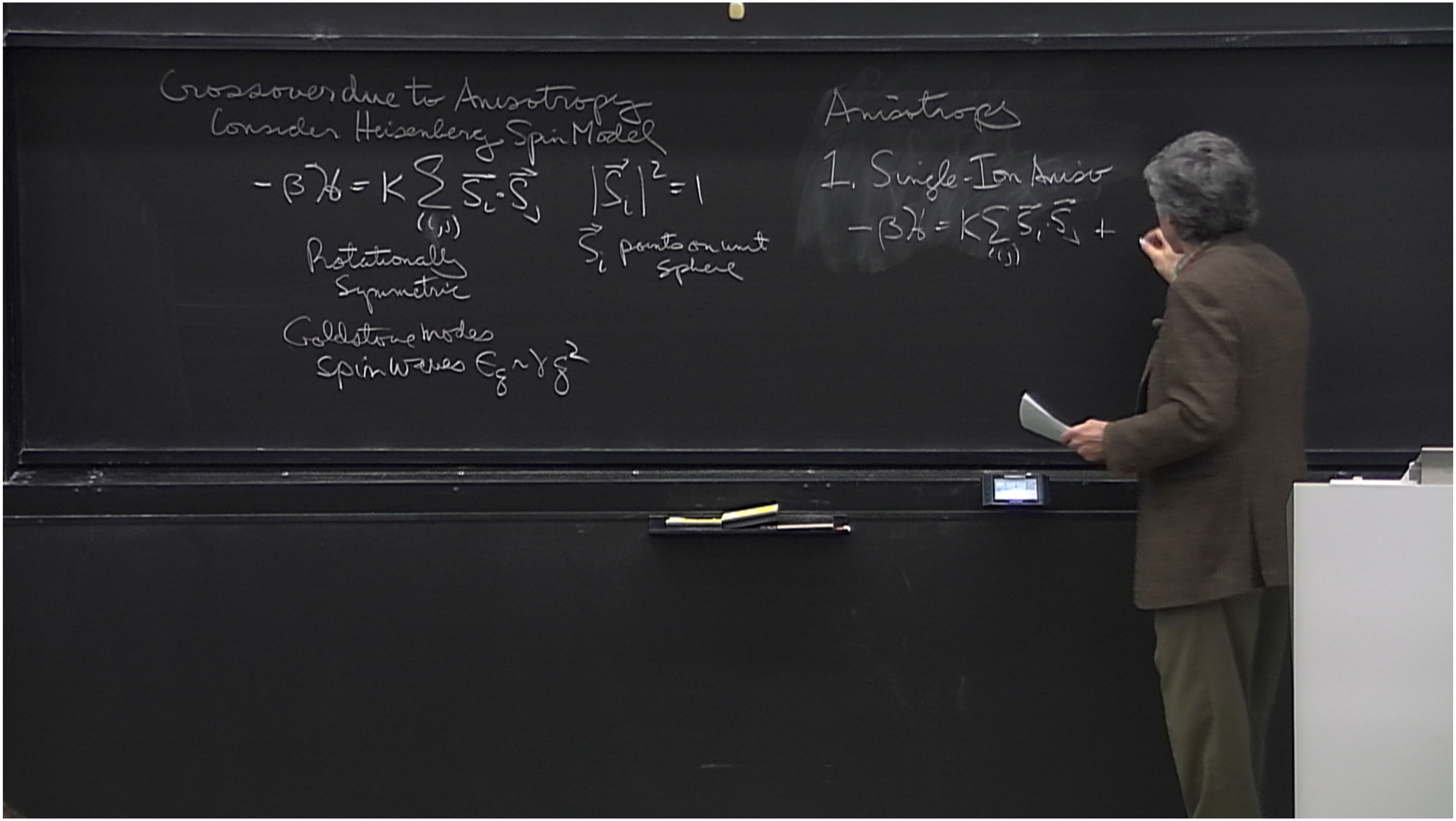
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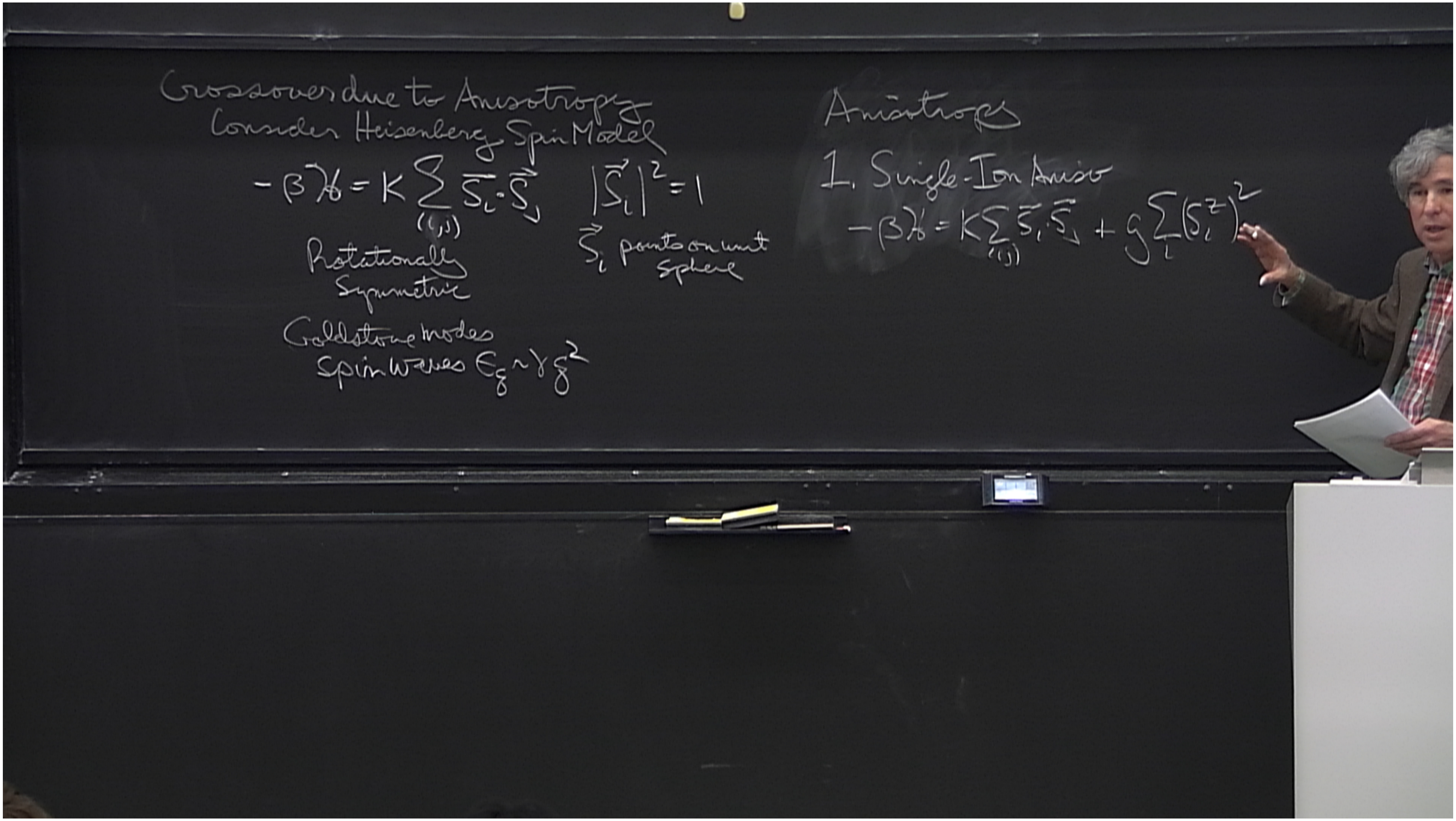
Rotationally Symmetric
$\vec{S}_i$ points on unit sphere
Goldstone modes
Spin Waves $\epsilon_q \sim \gamma q^2$
Anisotropy
1. Single-Ion Aniso
$-\beta\mathcal{H} = K \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j +$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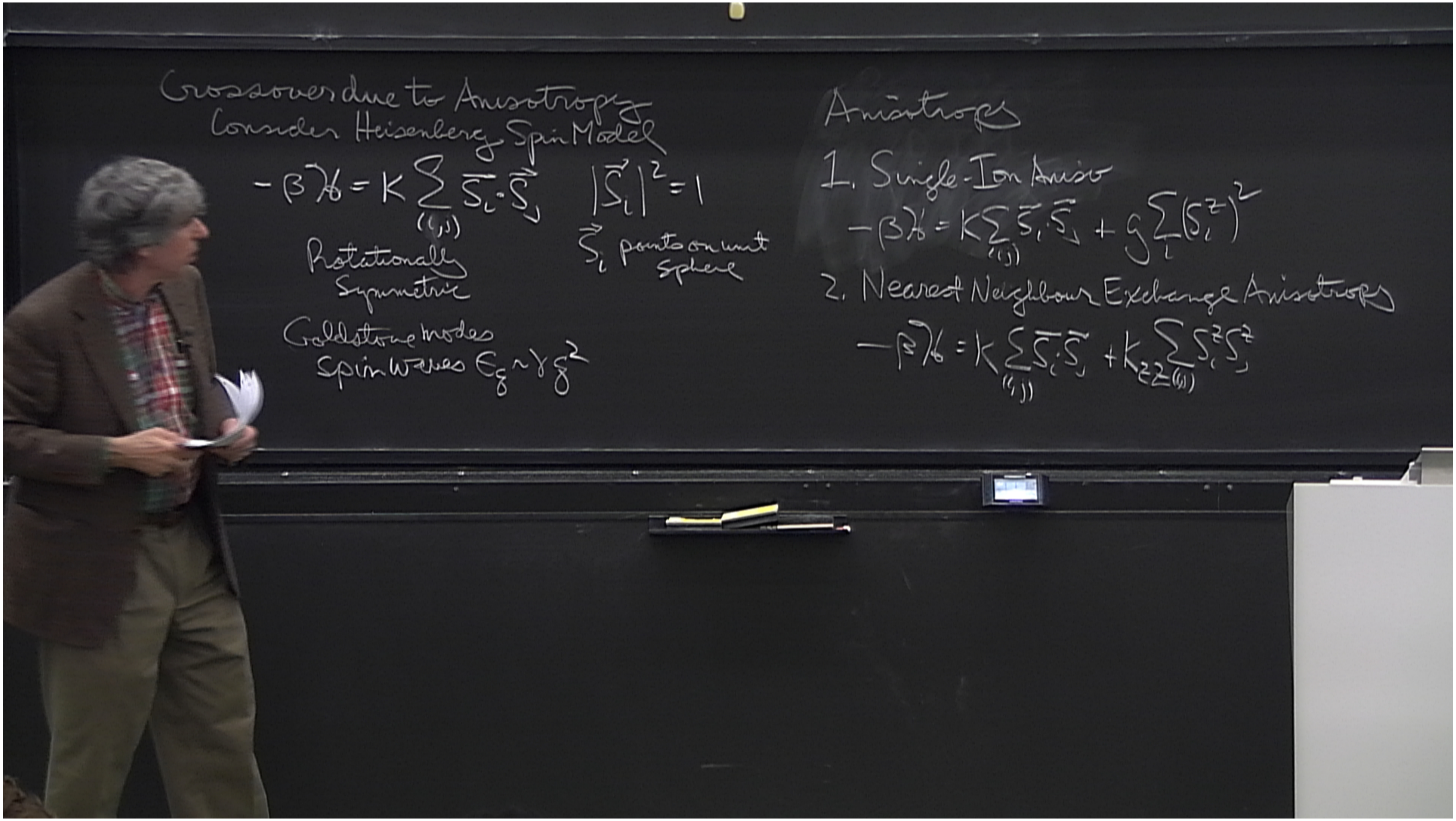
$|\vec{S}_i|^2 = 1$
Rotationally Symmetric
$\vec{S}_i$ points on unit sphere
Goldstone modes
Spin Waves $\epsilon_q \sim \gamma q^2$
Anisotropy
1. Single-Ion Aniso
$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i\cdot\vec{S}_j + g\sum_i (S_i^z)^2$


Crossover due to Anisotropy
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$|\vec{S}_i|^2 = 1$
Rotationally Symmetric
$\vec{S}_i$ points on unit sphere
Goldstone modes
Spin Waves $\epsilon_q \sim \gamma q^2$
Anisotropy
1. Single-Ion Aniso
$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i\cdot\vec{S}_j + g\sum_i (S_i^z)^2$
2. Nearest Neighbour Exchange Anisotropy
$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i\cdot\vec{S}_j + K_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z$

Crossover due to Anisotropy

Consider Heisenberg Spin Model

$$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j \qquad |\vec{S}_i|^2 = 1$$

Rotationally Symmetric

$\vec{S}_i$ points on unit sphere

Goldstone modes

Spin Waves $\epsilon_q \sim \gamma q^2$

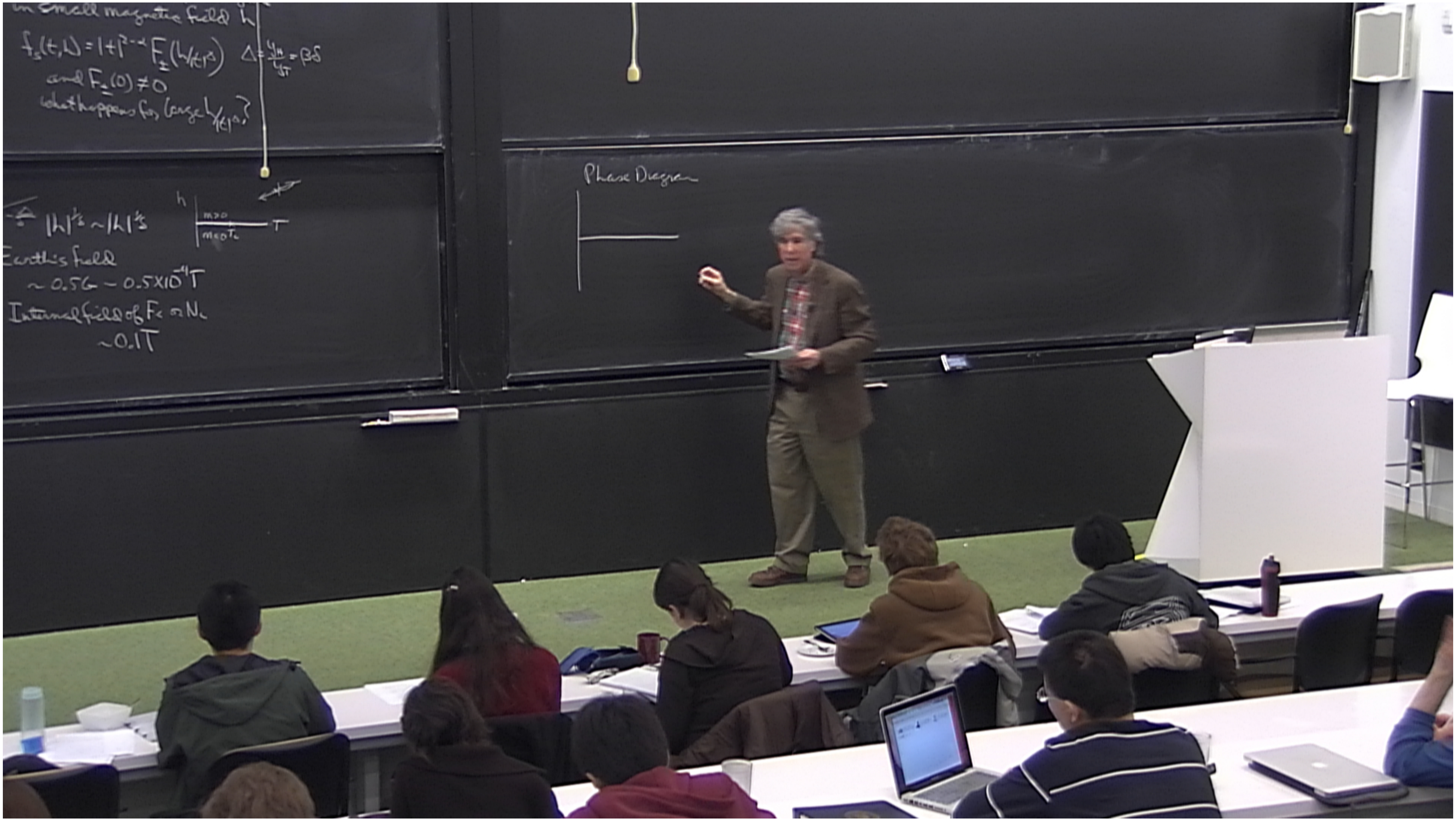
Anisotropy

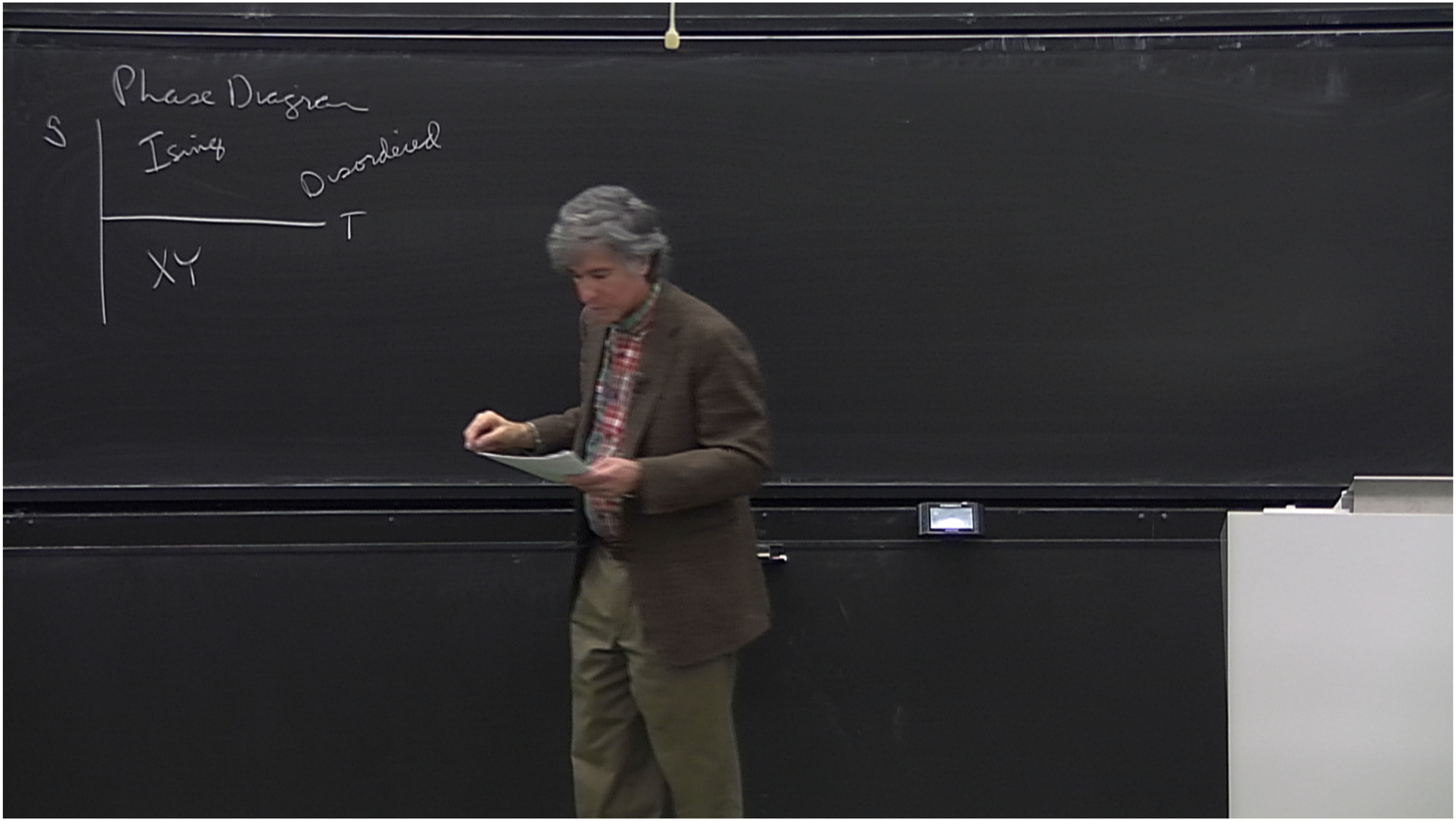
1. Single-Ion Aniso

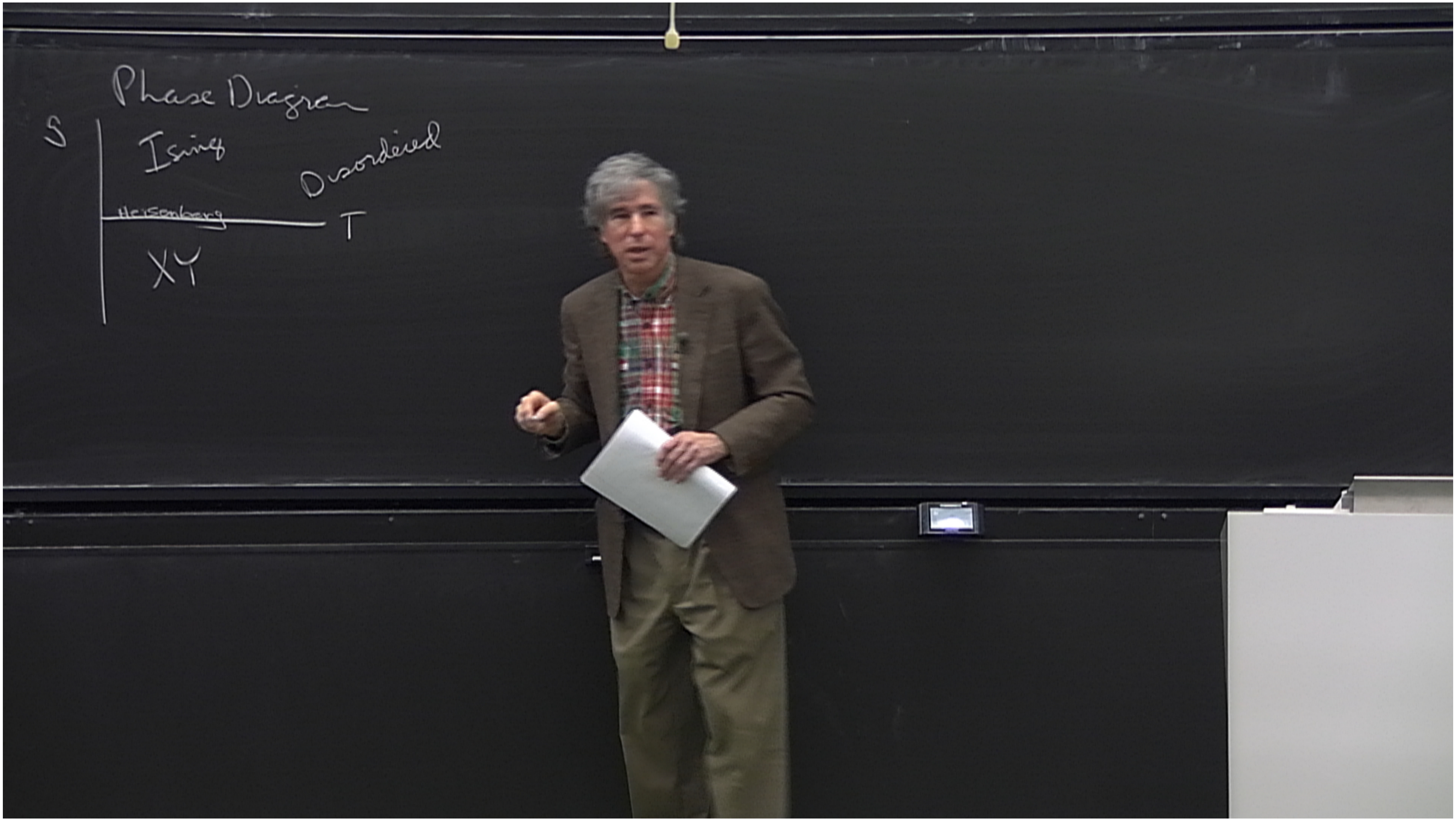
$$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + g\sum_i (S_i^z)^2$$

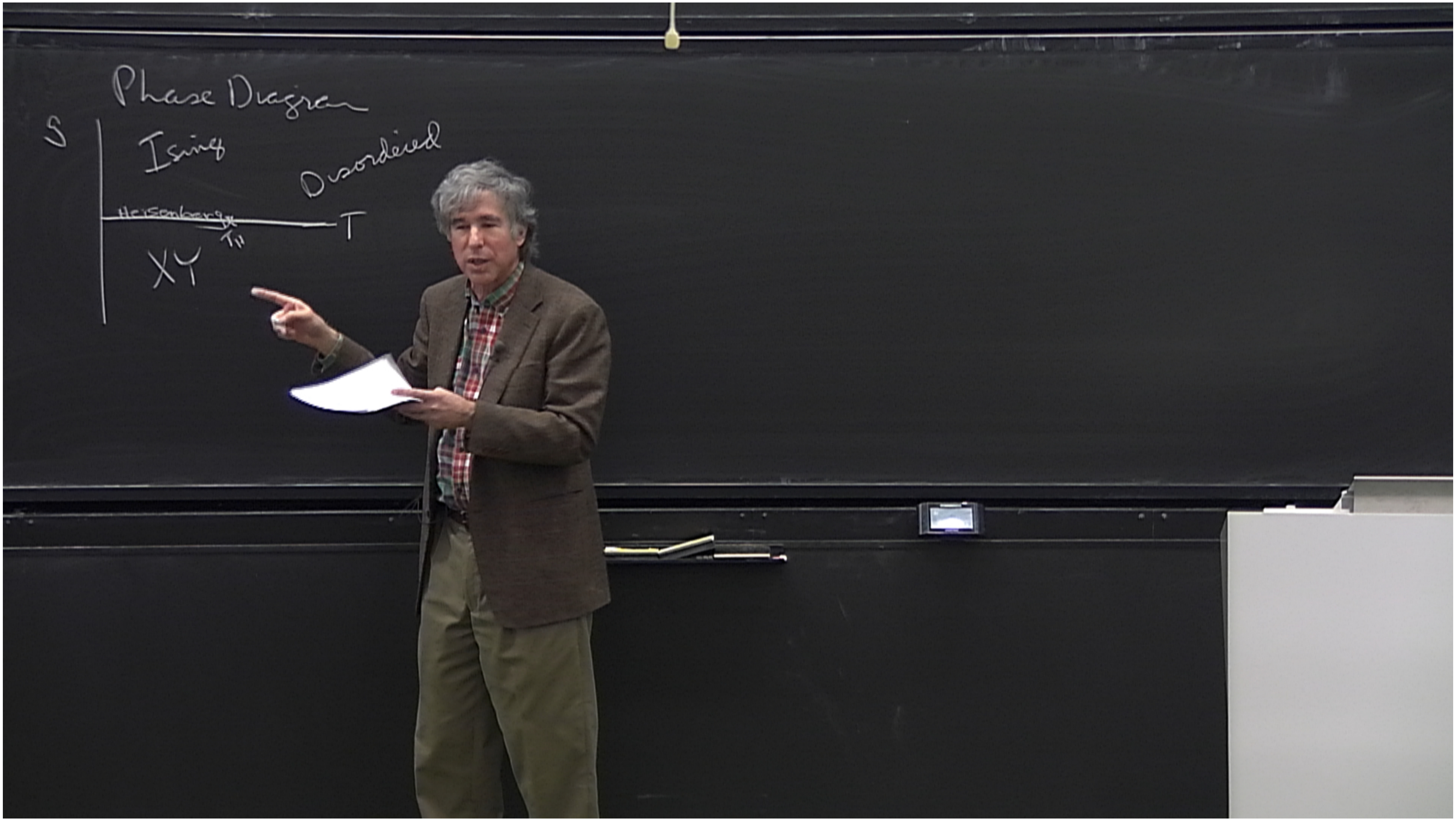
2. Nearest Neighbour Exchange Anisotropy

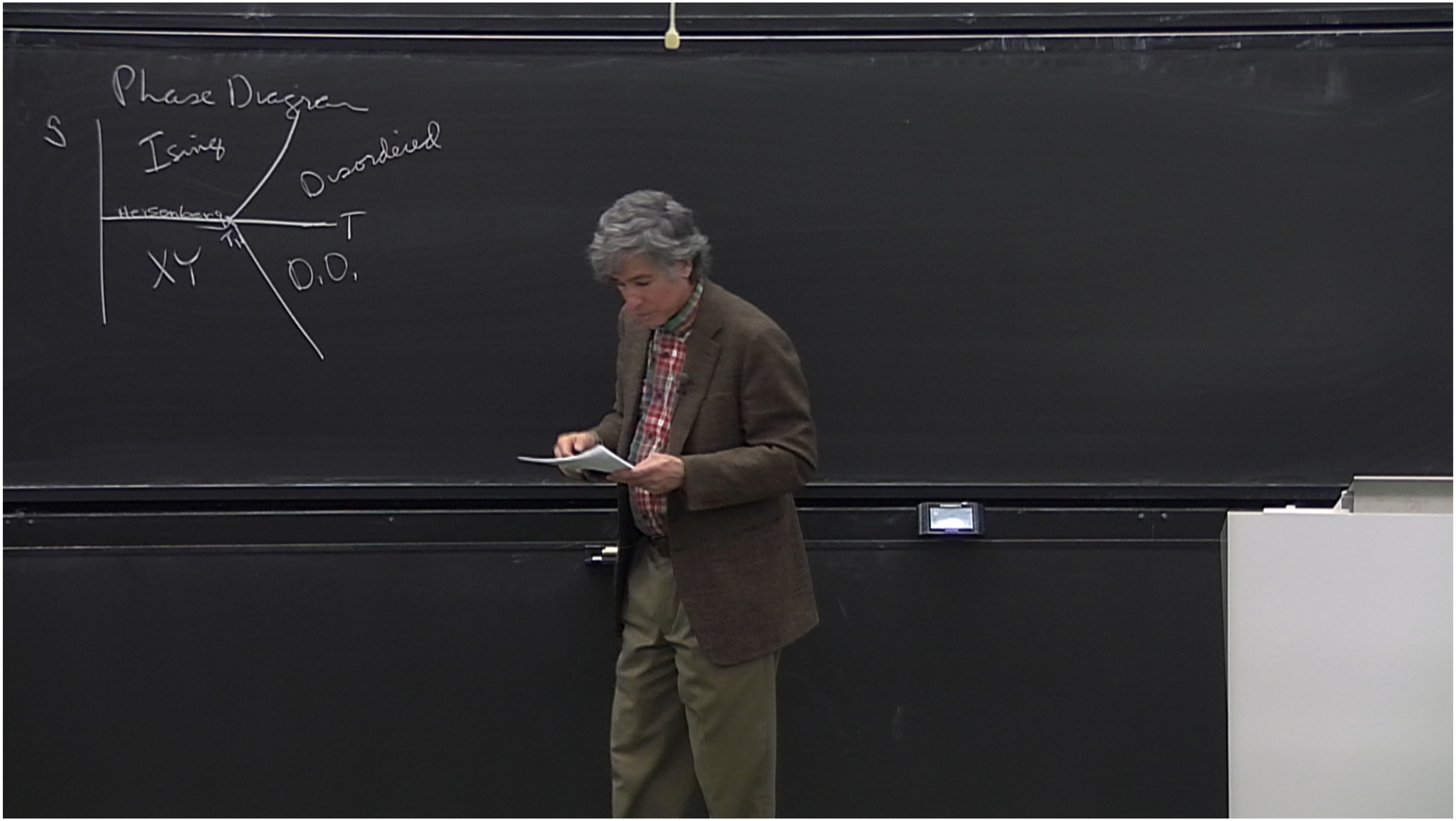
$$-\beta\mathcal{H} = K\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + K_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z$$

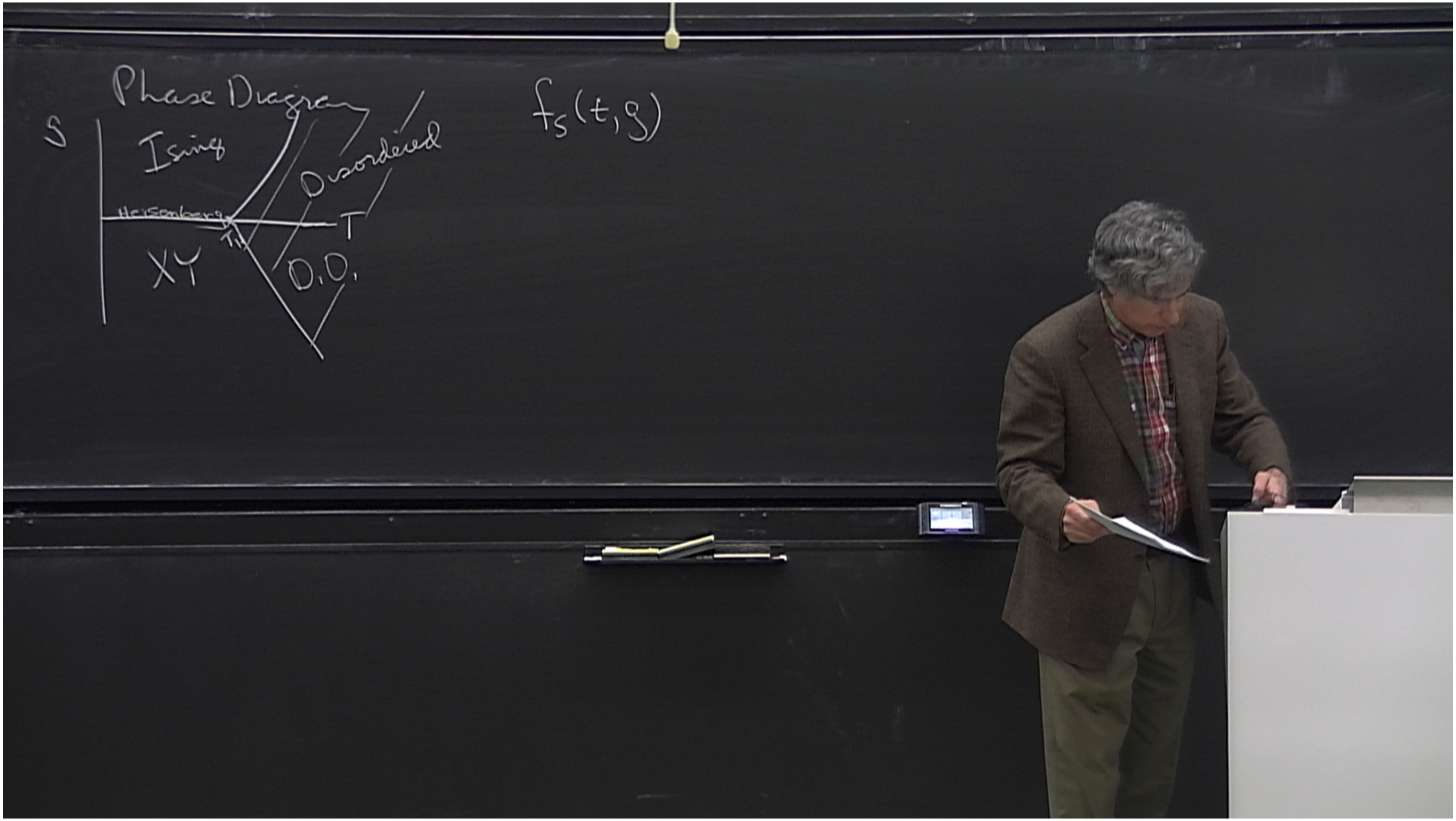

Phase Diagram
Earth's field
~ 0.5G ~ 0.5x10^-4 T
Internal field of Fe or Ni
~0.1T


Phase Diagram
S
Ising
Disordered
T
XY


Phase Diagram
S
Ising
Disordered
Heisenberg
T
XY


Phase Diagram
S
Ising
Disordered
Heisenberg
T
XY


Phase Diagram
S
Ising
Disordered
Heisenberg
T
XY
O.O.


Phase Diagram
S
Ising
Disordered
Heisenberg
T
XY
D.O.
f_S(t,g)