

Title: Statistical Mechanics - Lecture 13

Date: Nov 28, 2012 10:30 AM

URL: <http://pirsa.org/12110035>

Abstract:

Recursion Relations for $r+u$ $\epsilon=4-d$

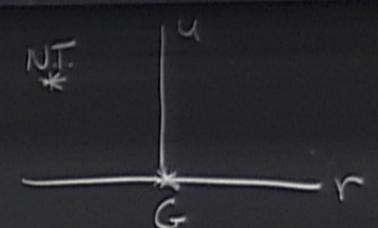
$$r_{e+1} = 4 \left[r_e + \frac{3Cu_0}{1+r_e} \right]$$

$$u_{e+1} = u_e - \frac{9Cu_0^2}{(1+r_e)^2}$$

Correct to order ϵ

Fixed Point: $r^* = 0$ (G)

$$u^* = \frac{\epsilon \ln 2}{9C}$$



Ref. K.G. Wilson + M.E. Fisher
 "Critical Exponents in 3.99 Dimensions"
 PRL 28, 240

Recursion Relations for $r+u$ $\epsilon=4-d$

$$r_{e+1} = 4 \left[r_e + \frac{3Cu_0}{1+r_e} \right]$$

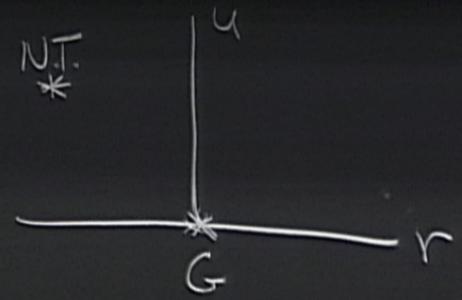
$$u_{e+1} = 2^e \left[u_e - \frac{9Cu_e^2}{(1+r_e)^2} \right]$$

2 Fixed Points

Correct to order ϵ

Gaussian F.P.: $r^* = u^* = 0$ (G)

Non-Trivial F.P.: $r^* = -\frac{4\epsilon \ln 2}{9}$, $u^* = \frac{\epsilon \ln 2}{9C}$



Ref. K.G. Wilson + M.E. Fisher

"Critical Exponents in 3.99 Dimensions"
PRL 28, 240 (1972)

Analyse flow in the $r-u$ plane

Start with G

Near G

$$r' - r^* = \left. \frac{\partial r'}{\partial r} \right|_G (r - r^*) + \left. \frac{\partial r'}{\partial u} \right|_G (u - u^*) = T_{rr} r + T_{ru} u$$

"
(1972).

Analyse flow in the $r-u$ plane

Start with G

Near G

$$r' - r^* = \left. \frac{\partial r'}{\partial r} \right|_G (r - r^*) + \left. \frac{\partial r'}{\partial u} \right|_G (u - u^*) = T_{rr} r + T_{ru} u$$

etc.

$$T_{rr} = 4 \quad T_{ru} = 12C$$

$$T_{ur} = 0 \quad T_{uu} = 2^E$$

"
(972).

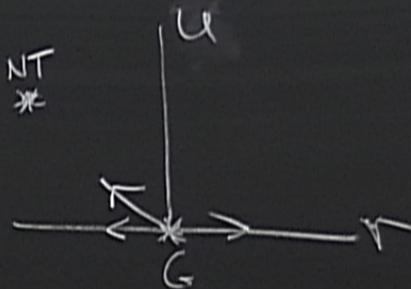
Say that $\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ then

$$T \begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} 4 & 12c \\ 0 & 2^e \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 4 \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r' \\ u' \end{pmatrix} = 4 \begin{pmatrix} r \\ u \end{pmatrix}$$

Similarly if $\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} \frac{-12cx}{4-2^e} \\ x \end{pmatrix}$

$$\text{Then } T \begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} 4 & 12c \\ 0 & 2^e \end{pmatrix} \begin{pmatrix} \frac{-12cx}{4-2^e} \\ x \end{pmatrix} = \begin{pmatrix} \frac{-48cx}{4-2^e} + 12cx \\ 2^e x \end{pmatrix} = \begin{pmatrix} \frac{-48cx + 48cx - 2^e 12cx}{4-2^e} \\ 2^e x \end{pmatrix} = 2^e \begin{pmatrix} \frac{-12cx}{4-2^e} \\ x \end{pmatrix} = 2^e \begin{pmatrix} r \\ u \end{pmatrix}$$



Linearize R.R. around N.T.

$$T = \begin{bmatrix} 4 - \frac{12Cu^*}{(1+r^*)^2} & \frac{12C}{1+r^*} \\ 2\epsilon \frac{18Cu^{*2}}{(1+r^*)^2} & 2\epsilon \left(1 - \frac{18Cu^*}{(1+r^*)^2} \right) \end{bmatrix}$$

$$u^* = \frac{\epsilon \ln 2}{4C}$$

$$\begin{aligned} & -2\epsilon \ln 2 \\ & (1 + \epsilon \ln 2)(1 - 2\epsilon \ln 2) \\ & = 1 - \epsilon \ln 2 \end{aligned}$$

$$= \begin{bmatrix} 4 - \frac{4}{3}\epsilon \ln 2 & 12C \\ 0 & 1 - \epsilon \ln 2 \end{bmatrix}$$

$$\begin{pmatrix} x + 18Cx - 2\epsilon \dots \\ \dots \end{pmatrix} = 2\epsilon \begin{pmatrix} -12Cx \\ \frac{-12Cx}{4-2\epsilon} \\ x \end{pmatrix} = 2\epsilon \begin{pmatrix} v \\ u \end{pmatrix}$$

Linearize R.R. around N.T.

$$T = \begin{bmatrix} 4 - \frac{12Cu^*}{(1+r^*)^2} & \frac{12C}{1+r^*} \\ 2\epsilon \frac{18Cu^{*2}}{(1+r^*)^2} & 2\epsilon \left(1 - \frac{18Cu^*}{(1+r^*)^2} \right) \end{bmatrix}$$

$$u^* = \frac{\epsilon \ln 2}{4C}$$

$$-2\epsilon \ln 2$$

$$(1 + \epsilon \ln 2)(1 - 2\epsilon \ln 2)$$

$$= 1 - \epsilon \ln 2$$

Eigenvalues are

$$4 - \frac{4}{3}\epsilon \ln 2 \text{ (relevant)}$$

$$1 - \epsilon \ln 2 \text{ (irrelevant)}$$

$$= \begin{bmatrix} 4 - \frac{4}{3}\epsilon \ln 2 & 12C \\ 0 & 1 - \epsilon \ln 2 \end{bmatrix}$$

$$\begin{pmatrix} -48Cx + 18\epsilon x - 2\epsilon 12Cx \\ 2\epsilon x \end{pmatrix} = 2\epsilon \begin{pmatrix} -12Cx \\ \frac{18\epsilon x}{4-2\epsilon} \end{pmatrix} = 2\epsilon \begin{pmatrix} n \\ u \end{pmatrix}$$

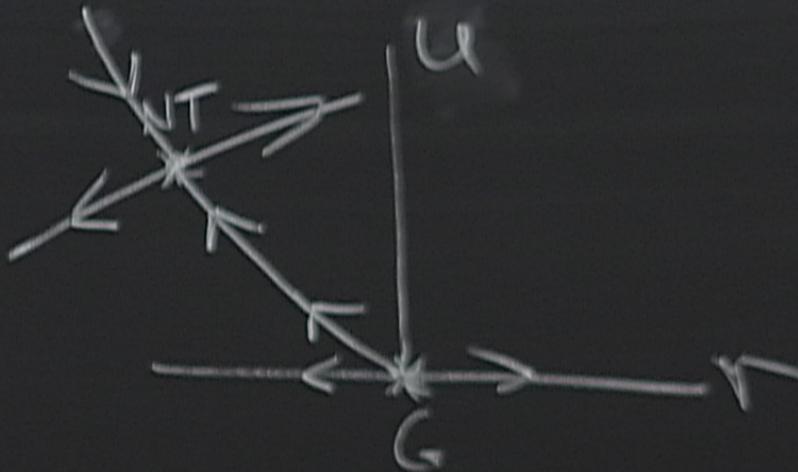
$\begin{pmatrix} x \\ 0 \end{pmatrix}$ then

$$\begin{pmatrix} 12C \\ 2G \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 4 \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$H \begin{pmatrix} v \\ 0 \end{pmatrix}$

$$\begin{pmatrix} v \\ u \end{pmatrix} = \begin{pmatrix} \frac{-12Cx}{4-2G} \\ x \end{pmatrix}$$

$$\begin{pmatrix} 12C \\ 0 \\ 2G \end{pmatrix} \begin{pmatrix} \frac{-12Cx}{4-2G} \\ x \end{pmatrix} = \begin{pmatrix} \frac{-48Cx}{4-2G} + 12Cx \\ \frac{-48Cx + 48Cx - 2G \cdot 12Cx}{4-2G} \end{pmatrix}$$



Linearize

$$T = \begin{bmatrix} 4 & - \\ & 2G \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ \end{bmatrix}$$

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ then
 $\begin{pmatrix} 12C \\ 2C \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 4 \begin{pmatrix} x \\ 0 \end{pmatrix}$
 $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x \\ 2C \end{pmatrix}$

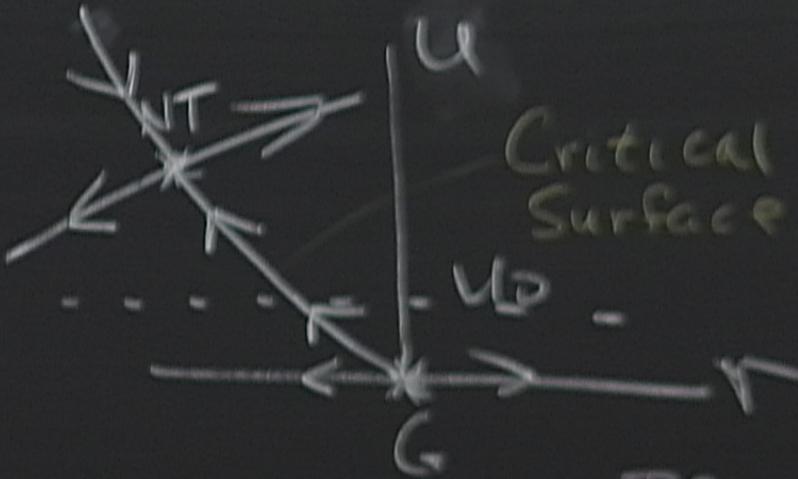
Linearize

$$T = \begin{bmatrix} 4 & - \\ & 2C \end{bmatrix}$$

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$\begin{pmatrix} \dot{x} \\ \dot{0} \end{pmatrix}$ then
 $\begin{pmatrix} \dot{x} \\ \dot{0} \end{pmatrix} = 4 \begin{pmatrix} x \\ 0 \end{pmatrix}$



Linearize R

$$T = \begin{bmatrix} 4 & -\frac{1}{2} \\ 2c & \frac{1}{2} \end{bmatrix}$$

$$\begin{pmatrix} \dot{u} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} -\frac{12cx}{4-2c} \\ x \end{pmatrix}$$

If starting Hamiltonian contains $u_0 > 0$

$$= \begin{bmatrix} 4 & -\frac{1}{2} \\ 2c & \frac{1}{2} \end{bmatrix}$$

$$\begin{pmatrix} 2c \\ 2c \end{pmatrix} \begin{pmatrix} -\frac{12cx}{4-2c} \\ x \end{pmatrix} = \begin{pmatrix} -\frac{48cx}{4-2c} + 12cx \\ -\frac{48cx + 48cx - 2c \cdot 12cx}{4-2c} \end{pmatrix}$$

Recursion Relations for $r+u \in 4-d$

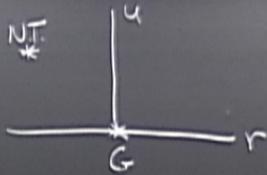
$$r_{i+1} = 4 \left[r_i + \frac{3Cu_0}{1+r_i} \right]$$

$$u_{i+1} = 2^\epsilon \left[u_i - \frac{9Cu_0^2}{(1+r_i)^2} \right] \quad \text{Correct to order } \epsilon$$

2 Fixed Points

Gaussian F.P.: $r^* = u^* = 0 \quad (G)$

Non-Trivial F.P.: $r^* = \frac{-4\epsilon \ln 2}{9}, u^* = \frac{\epsilon \ln 2}{9C}$



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Analyse flow in the $r-u$ plane

Start with G

Near G

$$r' - r^* = \frac{\partial r'}{\partial r} \Big|_G (r - r^*) + \frac{\partial r'}{\partial u} \Big|_G (u - u^*) = T_{rr} r + T_{ru} u$$

etc.

$$T_{rr} = 4 \quad T_{ru} = 12C$$

$$T_{ur} = 0 \quad T_{uu} = 2^\epsilon$$

Eigenvalues are 4 and 2^ϵ

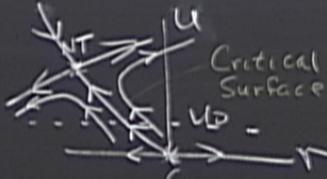
Say that $\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ then

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$$\begin{pmatrix} r' \\ u' \end{pmatrix} = 4 \begin{pmatrix} r \\ u \end{pmatrix}$$

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If starting Hamiltonian contains $u_0 > 0$

Linearize R.R. around N.T.

$$T = \begin{bmatrix} 4 - \frac{12Cu^*}{(1+r^*)^2} & \frac{12C}{1+r^*} \\ 2^\epsilon \frac{18Cu^*}{(1+r^*)^2} & 2^\epsilon \left(1 - \frac{18Cu^*}{(1+r^*)^2} \right) \end{bmatrix} \quad u^* = \frac{\epsilon \ln 2}{9C}$$

$-2\epsilon \ln 2$
 $(1 + \epsilon \ln 2)(1 - 2\epsilon \ln 2)$
 $= 1 - \epsilon \ln 2$
 Eigenvalues are
 $4 - \frac{4\epsilon \ln 2}{3}$ (relevant)
 $1 - \epsilon \ln 2$ (irrelevant)

Say that $\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ then

$$T\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} 4 & 12C \\ 0 & 2^\epsilon \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 4 \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r' \\ u' \end{pmatrix} = 4 \begin{pmatrix} r \\ u \end{pmatrix}$$

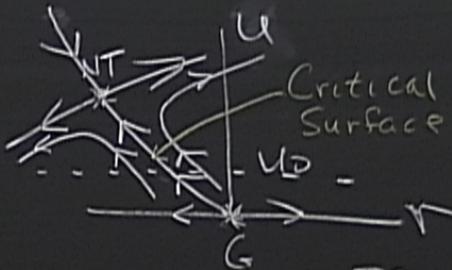
Similarly

Then $T\begin{pmatrix} r \\ u \end{pmatrix}$

$$\begin{pmatrix} -\frac{12Cx}{4-2^\epsilon} \\ x \end{pmatrix}$$

$$\begin{pmatrix} -\frac{12Cx}{4-2^\epsilon} \\ y \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{48Cx}{4-2^\epsilon} + 12Cx \\ 2^\epsilon x \end{pmatrix} = \begin{pmatrix} -\frac{48Cx + 48Cx - 2^\epsilon 12Cx}{4-2^\epsilon} \\ 2^\epsilon x \end{pmatrix} = 2^\epsilon \begin{pmatrix} -\frac{12Cx}{4-2^\epsilon} \\ x \end{pmatrix} = 2^\epsilon \begin{pmatrix} r \\ u \end{pmatrix}$$



Linearize R.R. around N.T.

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If starting Hamiltonian contains $u_0 > 0$

$$= \begin{bmatrix} 4 - \frac{4}{3}\epsilon \ln 2 & 12C \\ 0 & 1 - \epsilon \ln 2 \end{bmatrix} E$$

Exponents from the ϵ -expansion

Thermal Exponent y_T

$$\lambda_1 = b^{y_T} \quad (b=2)$$

$$= 4 - \frac{4}{3}\epsilon \ln 2 = 2^{y_T}$$

$$y_T \ln 2 = \frac{\ln 4}{2 \ln 2} + \frac{\ln(1 - \frac{\epsilon}{3} \ln 2)}{-\frac{\epsilon}{3} \ln 2}$$

$$y_T = 2 - \frac{\epsilon}{3} = \frac{1}{\nu} \rightarrow \nu = \frac{1}{2 - \frac{\epsilon}{3}} = \frac{1}{2(1 - \frac{\epsilon}{6})}$$

Exponents from the ϵ -expansion

Thermal Exponent y_T

$$\lambda_1 = b^{y_T} \quad (b=2)$$

$$= 4 - \frac{4}{3}\epsilon \ln 2 = 2^{y_T}$$

$$y_T \ln 2 = \ln 4 + \ln(1 - \frac{\epsilon}{3} \ln 2)$$

$\frac{2 \ln 2}{\quad} \quad - \frac{\epsilon}{3} \ln 2$

$$y_T = 2 - \frac{\epsilon}{3} = \frac{1}{\nu} \rightarrow \nu = \frac{1}{2 - \frac{\epsilon}{3}} = \frac{1}{2(1 - \frac{\epsilon}{6})} = \frac{1}{2} (1 + \frac{\epsilon}{6}) = \frac{1}{2} + \frac{\epsilon}{12}$$

$$\nu = \frac{1}{2} + \frac{\epsilon}{12}$$

$$d = 2 - d\nu = 2 - (4 - \epsilon)(\frac{1}{2} + \frac{\epsilon}{12}) = \frac{\epsilon}{6}$$

At this level $\mathcal{O}(\epsilon)$ $\eta = 0$ (η is $\mathcal{O}(\epsilon^2)$)

$$\gamma = \nu(2 - \eta) = 1 + \frac{\epsilon}{6}$$

Higher order results for n -component models (J. Zinn-Justin Scholarpedia)

$$\epsilon^2)) \quad \gamma = 1 + \frac{n+2}{2(n+8)} \epsilon + \frac{(n+2)(n^2+22n+52)}{4(n+8)^3} \epsilon^2 + O(\epsilon^3)$$

Hamiltonian contains $u_0 > 0$

$$\begin{pmatrix} 4 - \frac{4}{3}\epsilon \ln 2 & 12C \\ 0 & 1 - \epsilon \ln 2 \end{pmatrix}$$

Eigenvalues are

$$4 - \frac{4}{3}\epsilon \ln 2 \text{ (relevant)}$$

$$1 - \epsilon \ln 2 \text{ (irrelevant)}$$

$$\begin{pmatrix} -4PCX \\ 4-2\epsilon \end{pmatrix} + 12CX = \begin{pmatrix} -4PCX + 48CX - 2\epsilon 12CX \\ 4-2\epsilon \end{pmatrix} = 2 \begin{pmatrix} -12CX \\ 4-2\epsilon \end{pmatrix} = 2 \begin{pmatrix} n \\ u \end{pmatrix}$$

Dimension $\nu = \frac{1}{2} + \frac{\epsilon}{12}$

$$\alpha = 2 - d\nu = 2 - (4 - \epsilon)\left(\frac{1}{2} + \frac{\epsilon}{12}\right) = \frac{\epsilon}{6}$$

At this level $O(\epsilon)$ $\eta = 0$ ($\nu = 2$)

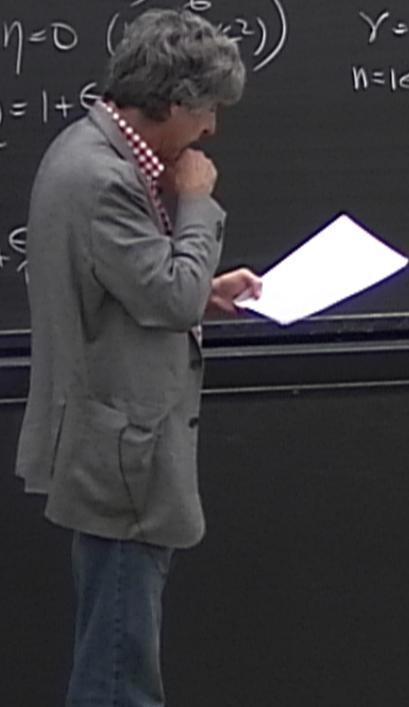
$$\gamma = \nu(2 - \eta) = 1 + \frac{\epsilon}{3} \ln 2$$

Higher order results for n-component models (J. Zinn-Justin Scholarpedia)

$$\gamma = 1 + \frac{n+2}{2(n+8)}\epsilon + \frac{(n+2)(n^2+22n-52)}{4(n+8)^3}\epsilon^2 + O(\epsilon^3)$$

$n=1 \leftarrow$ Ising Model

$$\nu = \frac{1}{2 - \frac{\epsilon}{3}} = \frac{1}{2(1 - \frac{\epsilon}{6})} = \frac{1}{2}\left(1 + \frac{\epsilon}{6}\right) = \frac{1}{2} + \frac{\epsilon}{12}$$



Higher order results for n -component models (J. Zinn-Justin Scholarpedia)

$$\epsilon^2)) \quad \gamma = 1 + \frac{n+2}{2(n+8)} \epsilon + \frac{(n+2)(n^2+22n+52)}{4(n+8)^3} \epsilon^2 + O(\epsilon^3)$$

$n=1 \leftarrow$ Ising Model
Has been calculated to order ϵ^5

Higher order results for n -component models (J. Zinn-Justin Scholarpedia)

$\gamma = \frac{\epsilon}{6}$
is $O(\epsilon^2)$)

$$\gamma(n) = 1 + \frac{n+2}{2(n+8)}\epsilon + \frac{(n+2)(n^2+22n-52)}{4(n+8)^3}\epsilon^2 + O(\epsilon^3)$$

$n=1 \leftarrow$ Ising Model

Has been calculated to order ϵ^5

$$\gamma(1) = 1.000, 1.666\dots$$

For $\epsilon=1$ ϵ^0
($d=3$)

Higher order results for n -component models (J. Zinn-Justin Scholarpedia)

$\gamma = \frac{\epsilon}{6}$
is $O(\epsilon^2)$)

$$\gamma(n) = 1 + \frac{n+2}{2(n+8)}\epsilon + \frac{(n+2)(n^2+22n-52)}{4(n+8)^3}\epsilon^2 + O(\epsilon^3)$$

$n=1 \leftarrow$ Ising Model

Has been calculated to order ϵ^5

$$\gamma(1) = 1.000, 1.666\dots, 1.2438, 1.1948, 1.3384, 0.8918$$

For $\epsilon=1$ ϵ^0 ϵ^1 ϵ^2 ϵ^3 ϵ^4 ϵ^5
($d=3$)



Higher order results for n -component models (J. Zinn-Justin Scholarpedia)

ϵ
 $\frac{6}{\epsilon}$
 $\sim O(\epsilon^2)$

$$\gamma(n) = 1 + \frac{n+2}{2(n+8)} \epsilon + \frac{(n+2)(n^2+22n+52)}{4(n+8)^3} \epsilon^2 + O(\epsilon^3)$$

$n=1 \leftarrow$ Ising Model

Has been calculated to order ϵ^5

Borel Summation

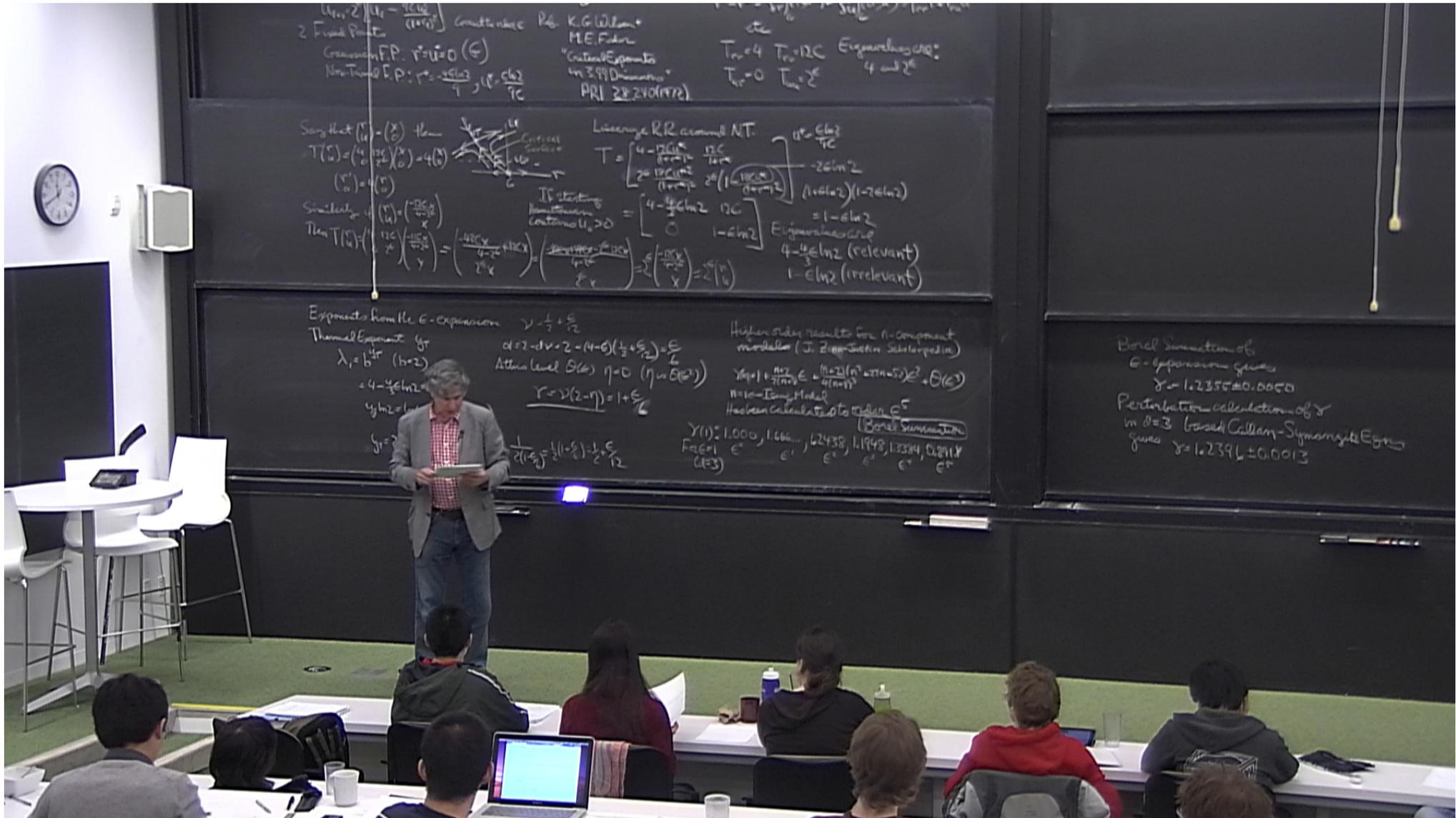
$\gamma(1) = 1.000, 1.666\dots, 1.2438, 1.1948, 1.3384, 0.8918$
 For $\epsilon=1$ $\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3, \epsilon^4, \epsilon^5$
 ($d=3$)

Borel Summation of
 ϵ -expansion gives
 $\gamma = 1.2355 \pm 0.0050$

Borel Summation of
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$$\gamma = 1.2355 \pm 0.0050$$

Perturbative calculation of γ
in $d=3$ based Callan-Symanzik Eqns
gives $\gamma = 1.2396 \pm 0.0013$



$U_{\text{eff}} = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \dots$ *Constructive* P.R. K.G. Wilson
 M.E. Fisher
 "Critical Exponents in 3.99 Dimensions"
 PRI 28 240 (1972)

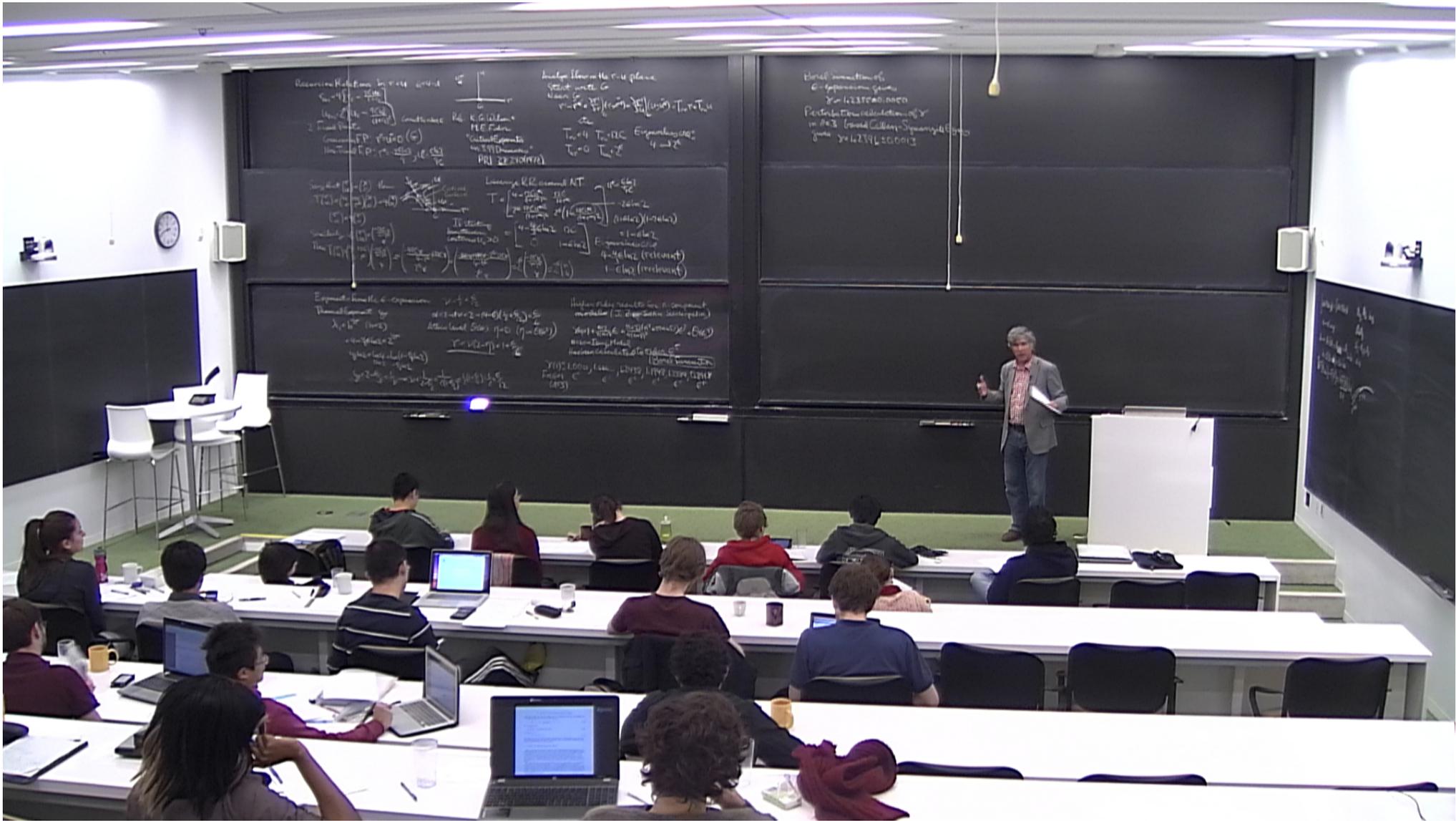
Say that $\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ then
 $T \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \begin{pmatrix} u \\ v \end{pmatrix}$
 $\begin{pmatrix} u \\ v \end{pmatrix} = 4 \begin{pmatrix} u \\ v \end{pmatrix}$
 Similarly $\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -12x \\ 2y \end{pmatrix}$
 Then $T \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -48x \\ 2y \end{pmatrix} = \begin{pmatrix} -48x \\ 2y \end{pmatrix} = 2 \begin{pmatrix} -24x \\ y \end{pmatrix} = 2 \begin{pmatrix} u \\ v \end{pmatrix}$

Lverage RR commut NT.
 $T = \begin{bmatrix} 4 - \frac{12c^2}{(1+c^2)^2} & \frac{12c}{1+c^2} \\ \frac{12c}{1+c^2} & 2 \left(1 - \frac{12c^2}{(1+c^2)^2} \right) \end{bmatrix}$
 $\begin{bmatrix} -26 \ln 2 & \\ & (1-6 \ln 2)(1-26 \ln 2) \end{bmatrix}$
 $= 1 - 6 \ln 2$
 $4 - 4 \ln 2$ (relevant)
 $1 - 6 \ln 2$ (irrelevant)

Exponents from the ϵ -expansion $\nu = \frac{1}{2} + \frac{\epsilon}{6}$
 Thermal Exponent γ_T
 $\lambda_1 = 1 - \frac{1}{2} \epsilon$ ($\nu = 2$)
 $= 4 - 4 \ln 2$
 $4 \ln 2 = 1$
 $\gamma_T = 2$
 $\frac{1}{2(1-\frac{\epsilon}{2})} = \frac{1}{2} (1 + \frac{\epsilon}{2}) = \frac{1}{2} + \frac{\epsilon}{4}$

Higher order results for 1-component model (J. Zinn-Justin, Schloerpedia)
 $\gamma_T = 1 + \frac{11\epsilon^2}{2(11\epsilon^2 - 4(11\epsilon^2 - 2))} \epsilon^2 + O(\epsilon^3)$
 $\nu = 1 - \frac{1}{2} \epsilon + \frac{1}{12} \epsilon^2 + O(\epsilon^3)$
 Has been calculated to order ϵ^5 (Borel summation)
 $\gamma_T(1) = 1.000, 1.666, 1.2438, 1.1948, 1.2334, 1.2391 \dots$
 $\nu(1) = 1.000, 1.666, 1.2438, 1.1948, 1.2334, 1.2391 \dots$

Borel Summation of ϵ -expansion gives
 $\gamma = 1.2355 \pm 0.0050$
 Perturbation calculation of γ
 in $d=3$ based Callan-Symanzik Eqns
 gives $\gamma = 1.2396 \pm 0.0013$



Correction to Scaling

Ref. F.J. Wegner PRB 5, 4529 (1972)

Recall, singular part of free energy

$$f_S(\lambda^{y_T} t, \lambda^{y_H} H) = \lambda^{\alpha} f_S(t, H)$$

implies that $f_S(t, H) = |t|^{2-\alpha} g\left(\frac{H}{|t|^{1/\nu}}\right)$ where

$2-\alpha = \frac{d}{\nu}$
 $\Delta = \frac{d}{\nu}$

Correction to Scaling

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Recall, singular part of free energy

$$f_S(\lambda^y t, \lambda^y H) = \lambda^d f_S(t, H)$$

implies that $f_S(t, H) = |t|^{2-\alpha} g\left(\frac{H}{|t|^{1/\nu}}\right)$ where

$$2-\alpha = \frac{d}{\nu}$$
$$\Delta = \frac{d-1}{\nu}$$

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Recall, singular part of free energy

$$f_S(\lambda^{y_T} t, \lambda^{y_H} H) = \lambda^{\phi} f_S(t, H)$$

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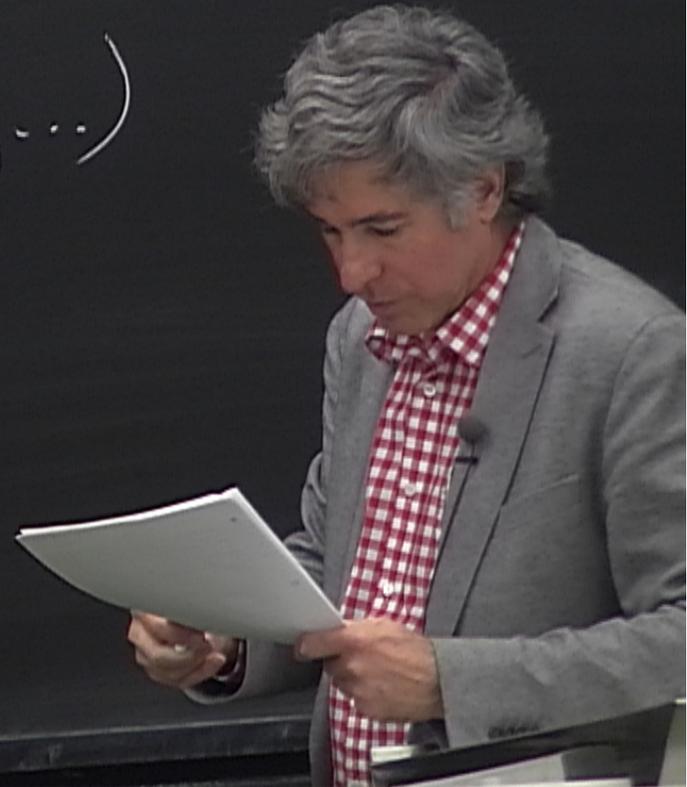
$$f_S(t, H) = |t|^{2-\alpha} g\left(\frac{H}{|t|^{1/\nu}}\right)$$

where

$$2 - \alpha = \frac{d}{5}$$
$$\Delta = \frac{H}{5^{1/5}}$$

Can be generalized to the case
of more, possibly marginal or irrelevant, fields
(coupling constants)

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of more, possibly marginal or irrelevant, fields
(coupling constants)
Generalize scaling relation
 $f_s(\lambda^{y_T} t, \lambda^{y_H} H, \lambda^{y_i} g_i, \dots) = \lambda^d f_s(t, H, g_i, \dots)$



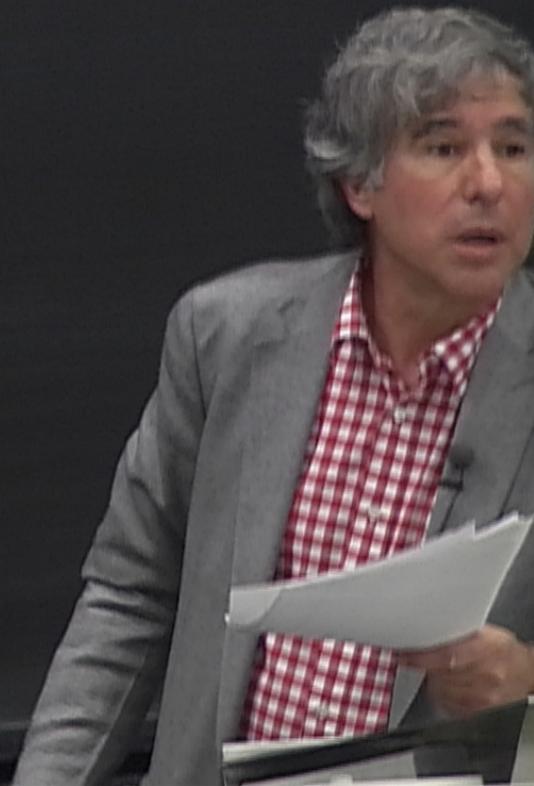
Can be generalized to the case of more, possibly marginal or irrelevant, fields (coupling constants)

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$$f_s(t, H, g_1, \dots) = |t|^{2-d} g\left(\frac{H}{|t|^\Delta}, \frac{g_1}{|t|^{\Delta_1}}, \dots\right)$$

where $\Delta = \frac{y_H}{y_T}$
 $\Delta_1 = \left(\frac{y_1}{y_T}\right)$



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$$f_s(t, H, g_i, \dots) = t^{z-2} g(H, \frac{g_i}{t^{y_i}} \dots)$$

where $\Delta = \frac{y_H}{y_T}$

$$\Delta_i = \left(\frac{y_i}{y_T} \right)$$

here
 $-\alpha = \frac{d}{y_T}$
 $\Delta = \frac{y_H}{y_T}$

An interesting irrelevant
field is $u_0 - u^*$

What is Δ_u

$$\Delta_u = \frac{y_u}{y_T} = \frac{\ln(1 - \epsilon \ln 2)}{\ln(4 - \frac{4}{3} \epsilon \ln 2)} = \Delta_u = -\frac{\epsilon}{2} + \mathcal{O}(\epsilon^2)$$

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$$\therefore \frac{u - u^*}{|t|^{-\epsilon/2}} \rightarrow 0 \text{ as } t \rightarrow 0$$

So expand

$$f_s(t, H, u_0 = u^*) = |t|^{2-\alpha} f_s\left(\frac{H}{|t|^\Delta}, 0\right) + (u_0 - u^*) |t|^{2-\alpha-\Delta} f_s^{(0,1)}\left(\frac{H}{|t|^\Delta}, 0\right) + \dots$$

$$\Delta = \frac{y_1}{y_T}$$

$$\Delta_1 = \left(\frac{y_1}{y_T} \right)^{\frac{1}{2}}$$

So expand

$$f_s(t, H, u_0 - u^*) = |t|^{2-\alpha} f_s\left(\frac{H}{|t|^\Delta}, 0\right) + (u_0 - u^*) |t|^{2-\alpha-\Delta_1} f_s^{(0,1)}\left(\frac{H}{|t|^\Delta}, 0\right) + \dots$$

$$m = \frac{\partial f}{\partial H} \rightarrow m = |t|^\beta f_s^{(1,0)}(0,0) + (u_0 - u^*) |t|^{\beta-\Delta_1} f_s^{(1,1)}(0,0) + \dots$$

+ O(ε²)

$$m = m_0 |t|^\beta (1 + a_m |t|^{-\Delta_1} + \dots)$$

ε → 0 as t → 0

$$-\Delta_1 = \frac{\epsilon}{2}$$

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t
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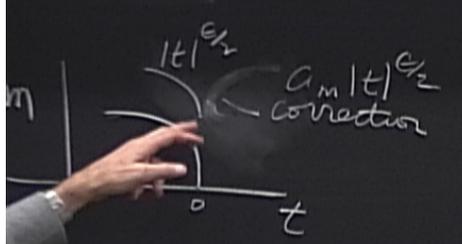
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So expand

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↑ confluent singularity

$$-\Delta_1 = \frac{\epsilon}{2}$$

$$\Delta u = -\frac{\epsilon}{2} + O(\epsilon^2)$$

$$\therefore \frac{u_0 - u^*}{|t|^{-\epsilon/2}} \rightarrow 0 \text{ as } t \rightarrow 0$$