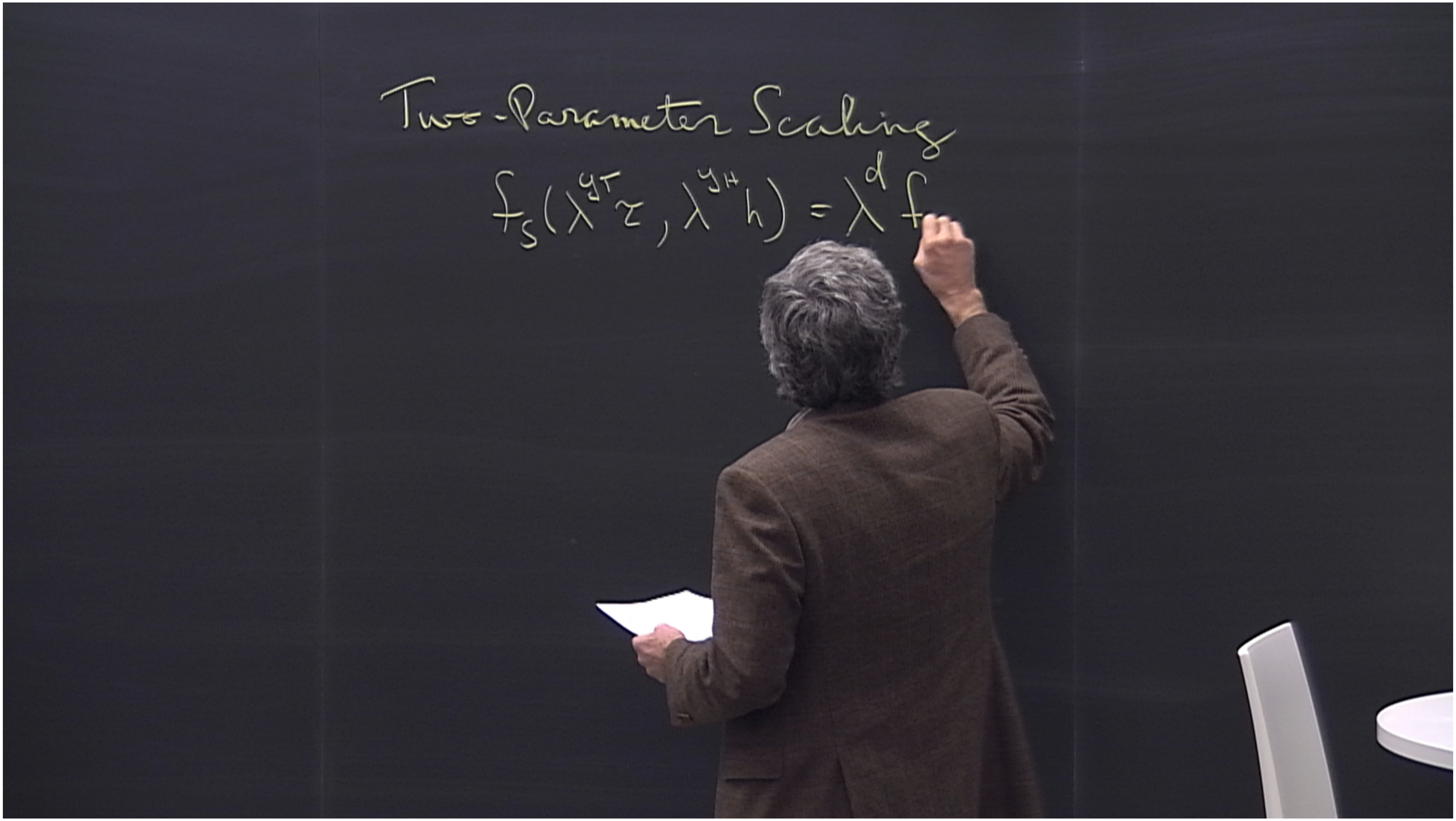
Title: Statistical Mechanics - Lecture 5

Date: Nov 16, 2012 10:30 AM

URL: http://pirsa.org/12110023

Abstract:


Two-Parameter Scaling
$f_s(\lambda^{y_T}\tau, \lambda^{y_H}h) = \lambda^d f$

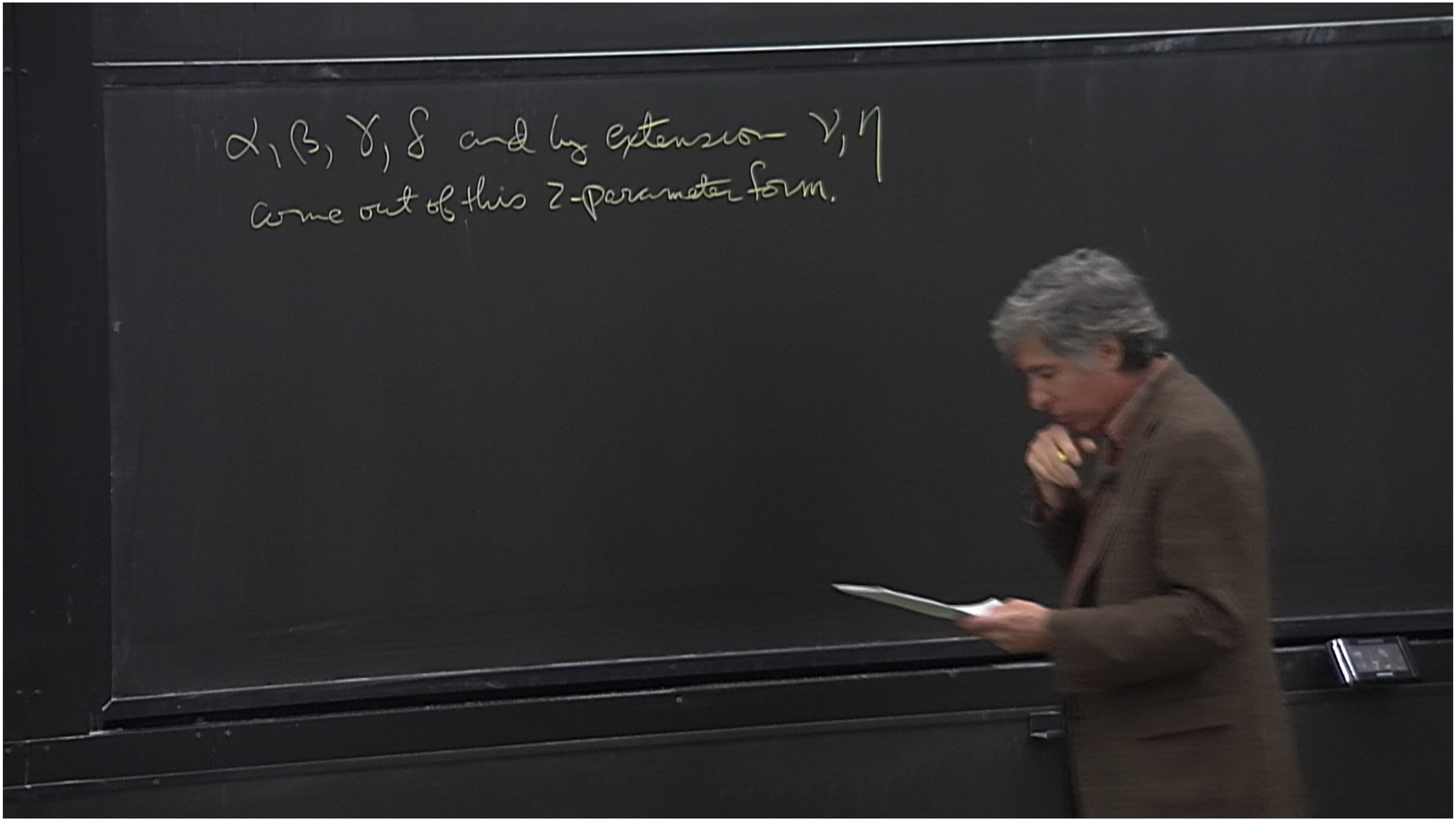
Two-Parameter Scaling

$$f_s(\lambda^{y_T}\tau, \lambda^{y_H} h) = \lambda^d f_s(\tau, h)$$

then f_s can be written as

$$f_s(\tau, h) = |\tau|^{2-\alpha}\, g\left(\frac{h}{|\tau|^{\Delta}}\right)$$

where $\Delta = \frac{y_H}{y_T}$ and $2-\alpha = \frac{d}{y_T}$


α, β, γ, δ and by extension ν, η
come out of this 2-parameter form.

$\alpha, \beta, \gamma, \delta$ and by extension ν, η

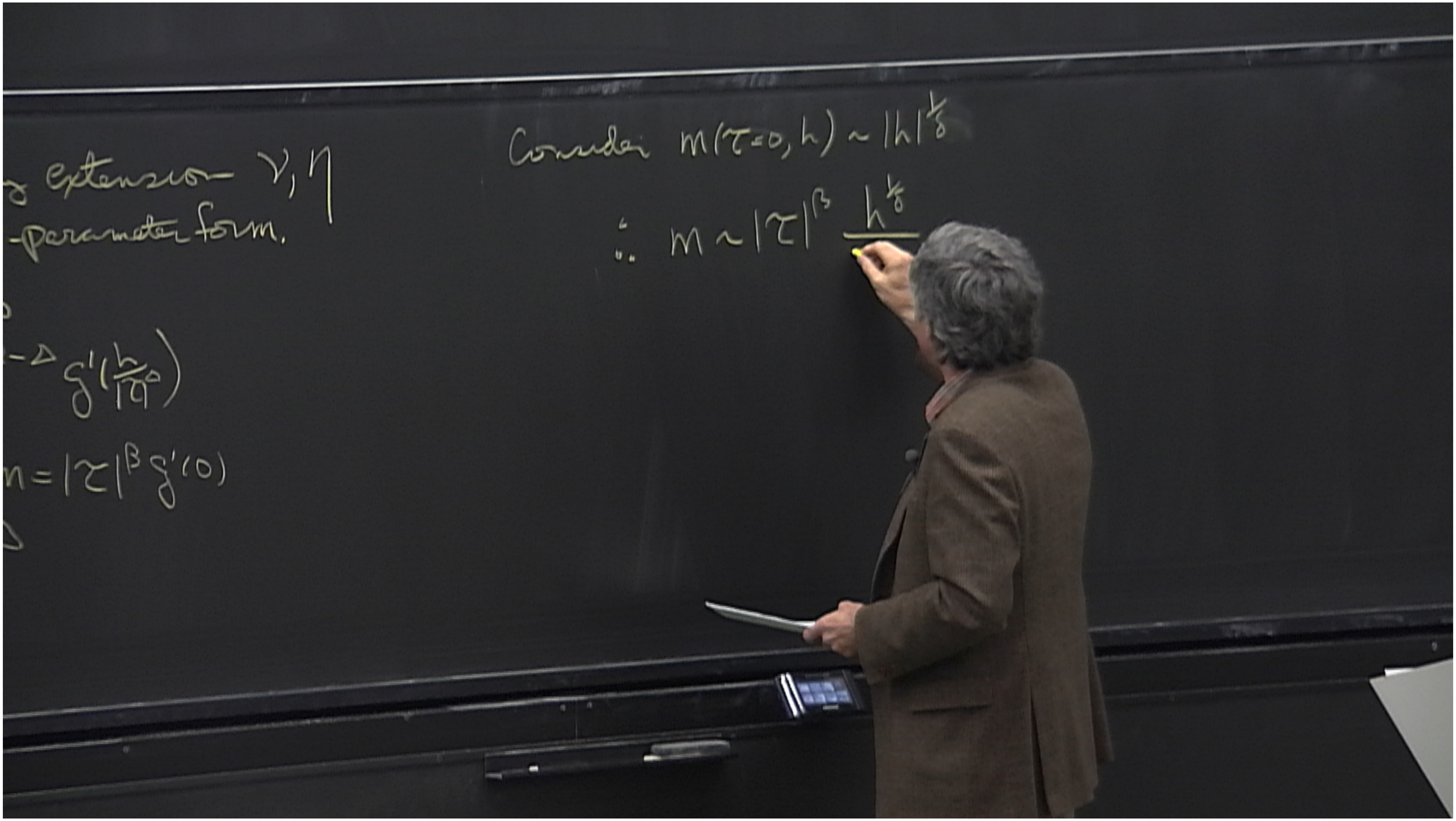
come out of this 2-parameter form.

The magnetization is

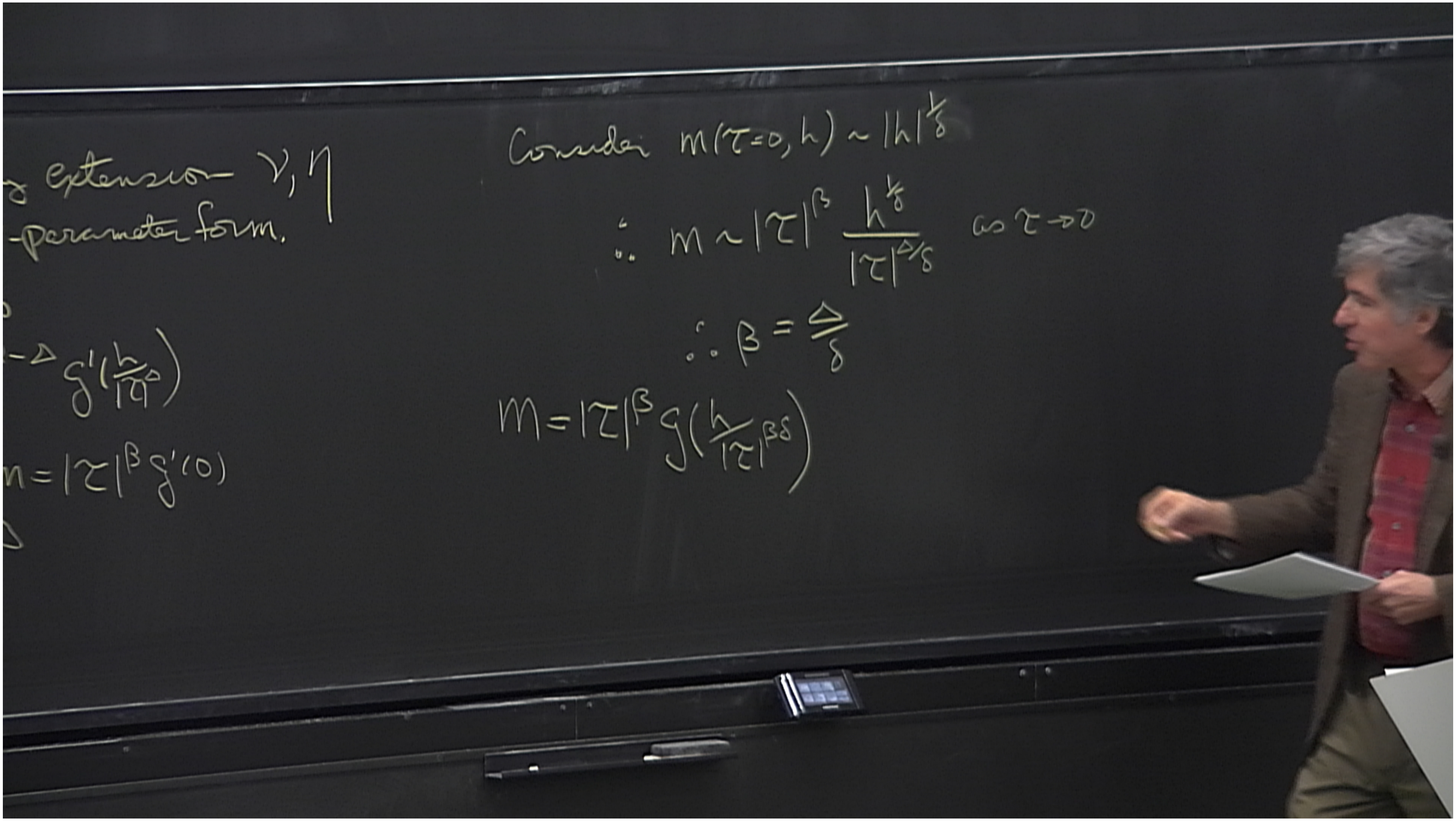
$$m = \frac{\partial f}{\partial h} = |\tau|^{2-\alpha-\Delta} g'\left(\frac{h}{|\tau|^{\Delta}}\right)$$

Consider $h = 0 \rightarrow m = |\tau|^{\beta} g'(0)$

where $\beta = 2 - \alpha - \Delta$



Consider m(τ=0, h) ~ |h|^1/δ
∴ m ~ |τ|^β h^1/δ
extension ν, η
-parameter form.
m = |τ|^β g'(0)



extension — ν, η
-parameter form.
Consider $m(\tau=0,h)\sim|h|^{1/\delta}$
$\therefore m\sim|\tau|^{\beta}\frac{h^{1/\delta}}{|\tau|^{\Delta/\delta}}$ as $\tau\to 0$
$\therefore \beta=\frac{\Delta}{\delta}$
$m=|\tau|^{\beta}g\left(\frac{h}{|\tau|^{\beta\delta}}\right)$
$m=|\tau|^{\beta}g'(0)$

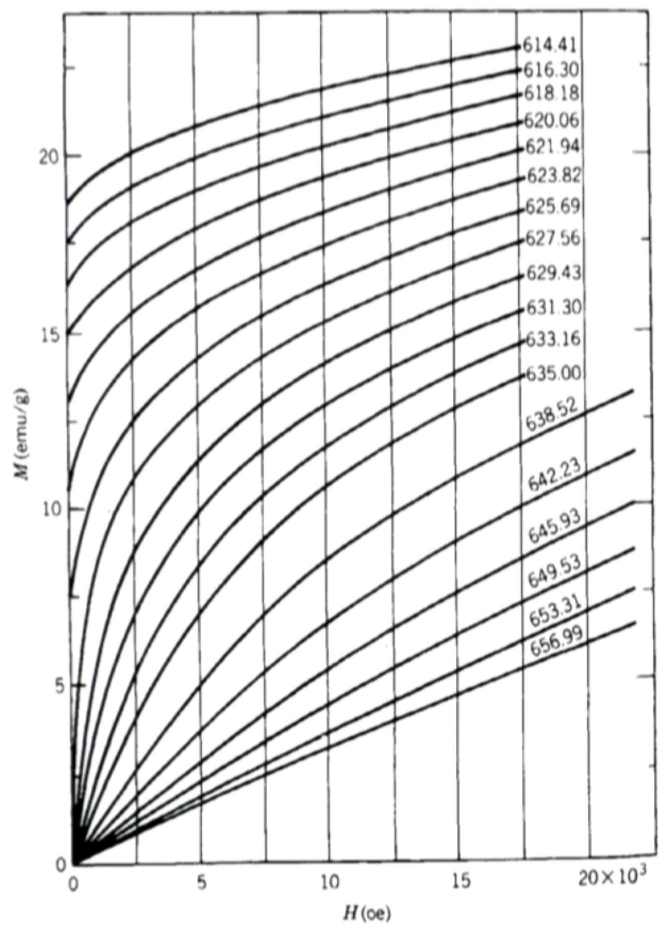
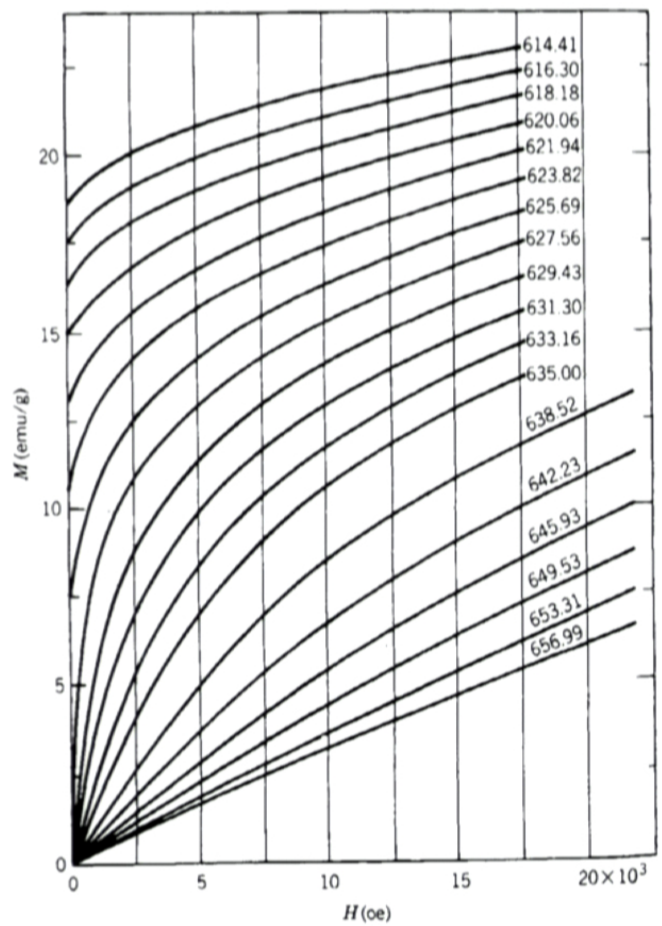


FIG. 1. Magnetization versus internal field for nickel at different temperatures, from data of Weiss and Forrer.[7]

[7] P. Weiss and R. Forrer, Ann. Phys. (Paris) **5**, 153 (1926).



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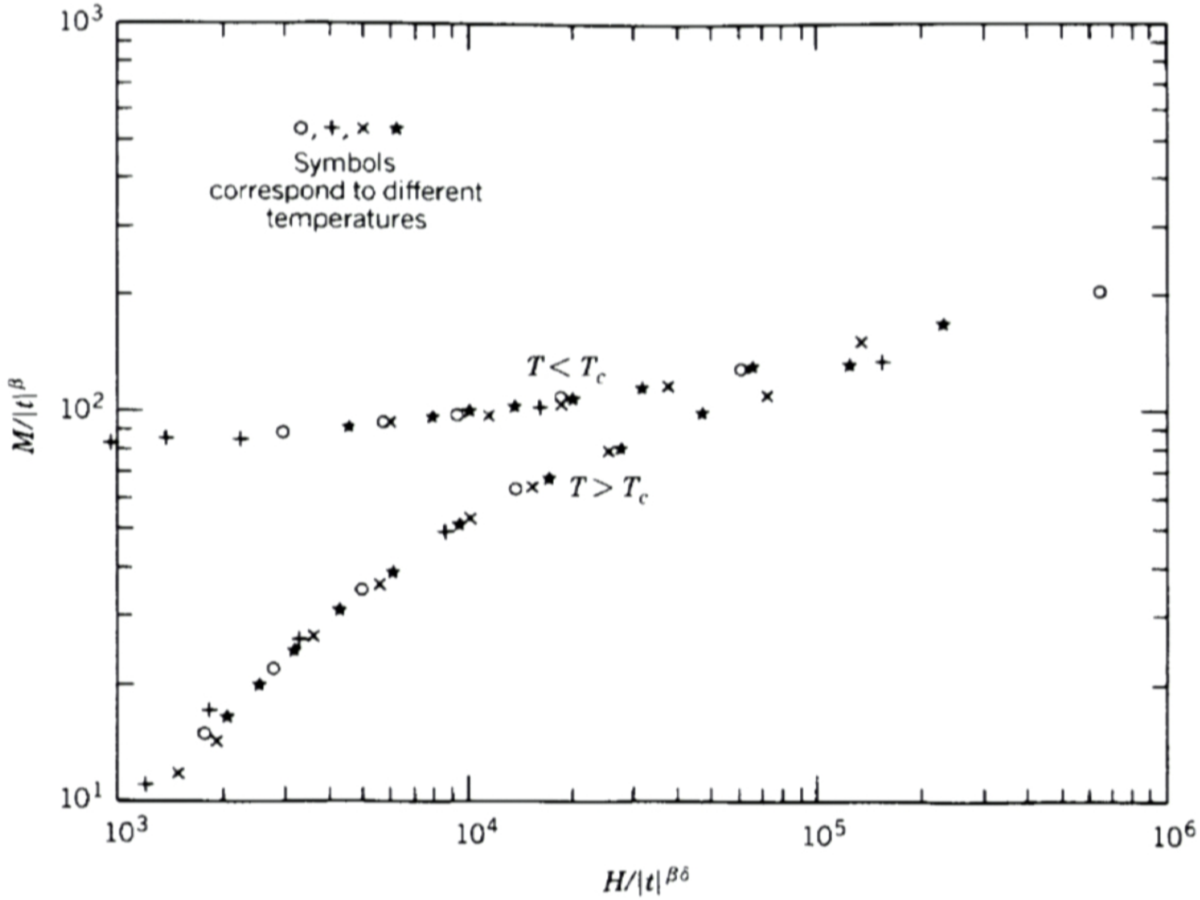
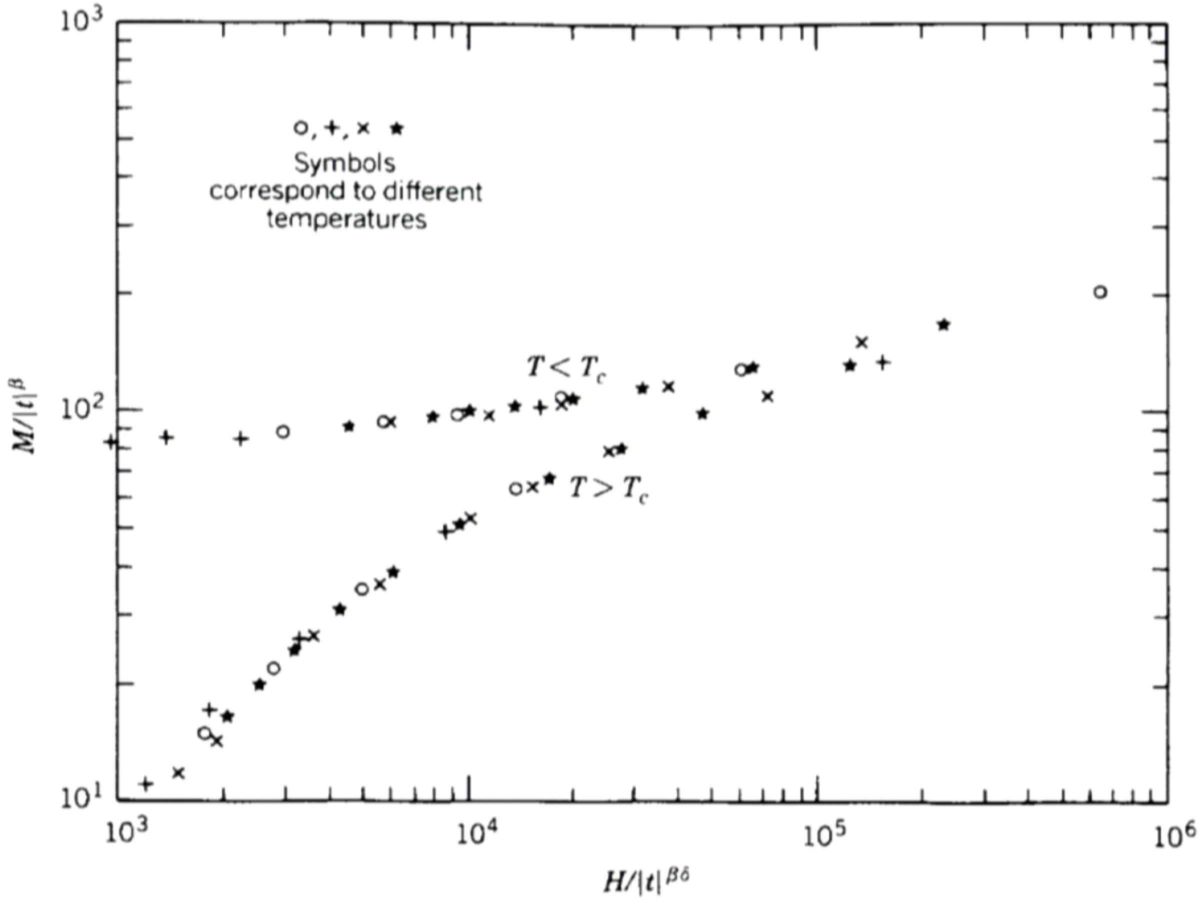


Tc = 627.2, β = 0.368, δ = 4.22



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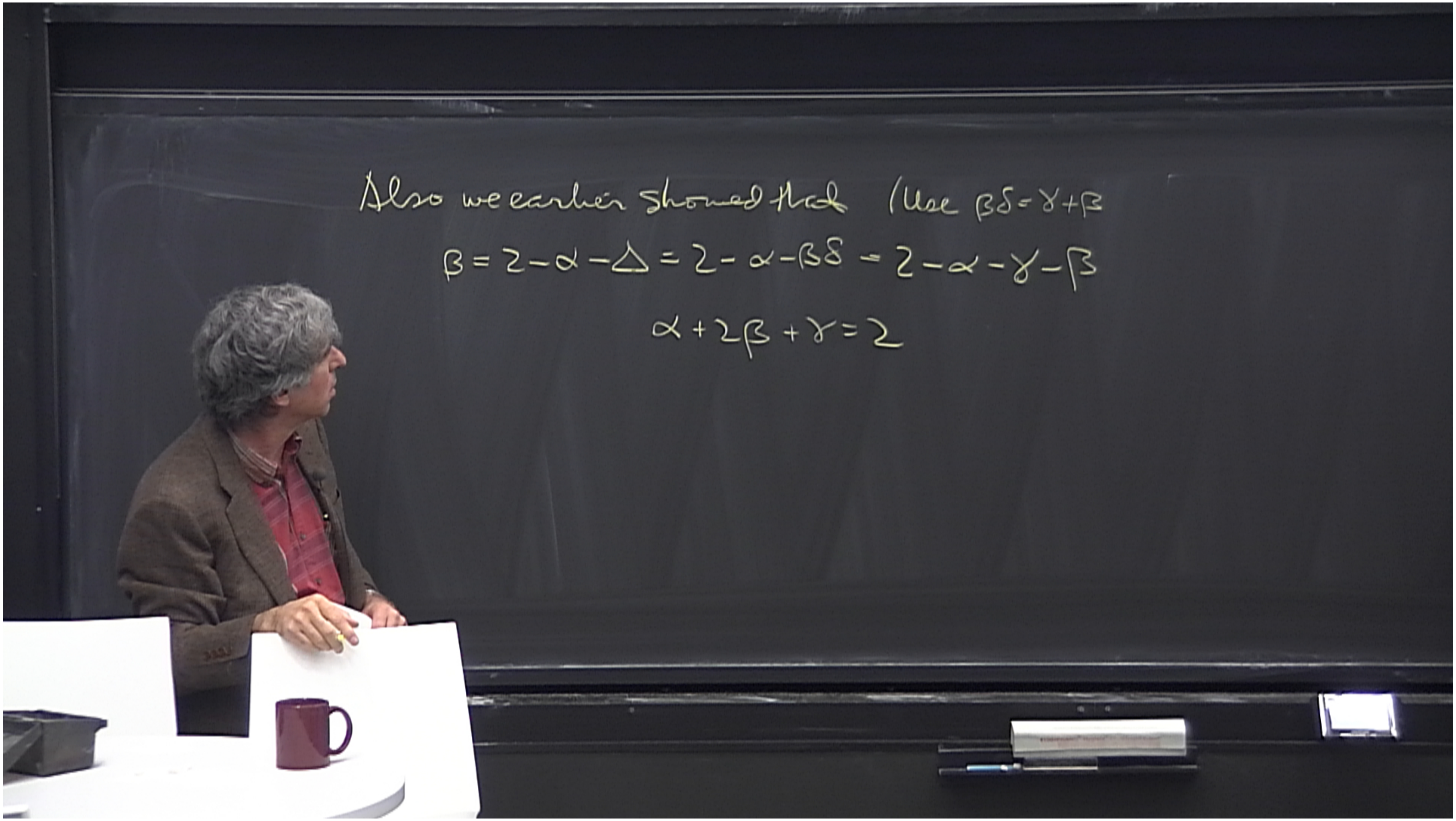
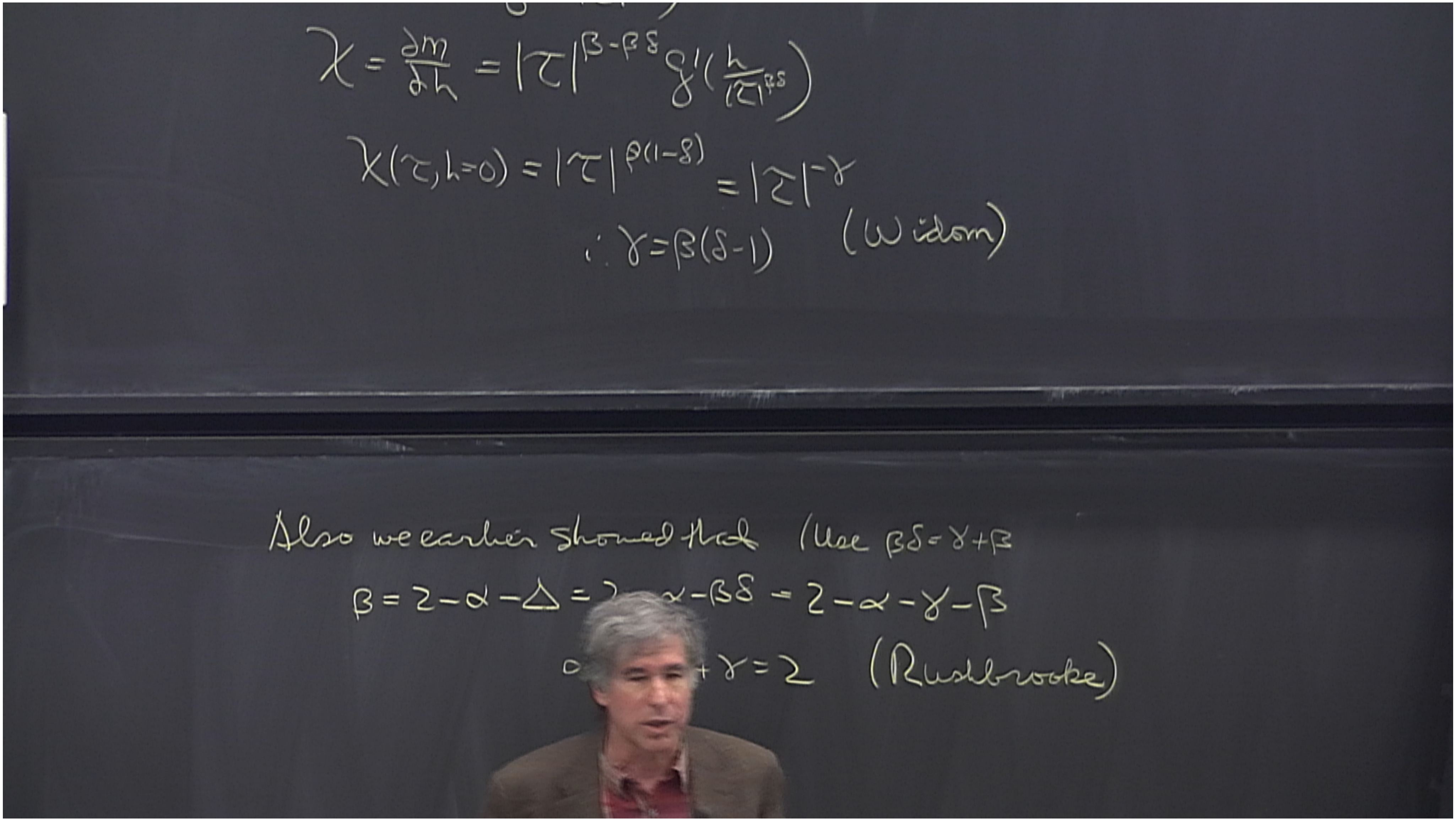

Also we earlier showed that (Use $\beta\delta = \gamma + \beta$
$\beta = 2 - \alpha - \Delta = 2 - \alpha - \beta\delta = 2 - \alpha - \gamma - \beta$
$\alpha + 2\beta + \gamma = 2$


$\chi = \frac{\partial m}{\partial h} = |\tau|^{\beta - \beta\delta} g'\left(\frac{h}{|\tau|^{\beta\delta}}\right)$
$\chi(\tau, h=0) = |\tau|^{\beta(1-\delta)} = |\tau|^{-\gamma}$
$\therefore \gamma = \beta(\delta - 1)$ (Widom)
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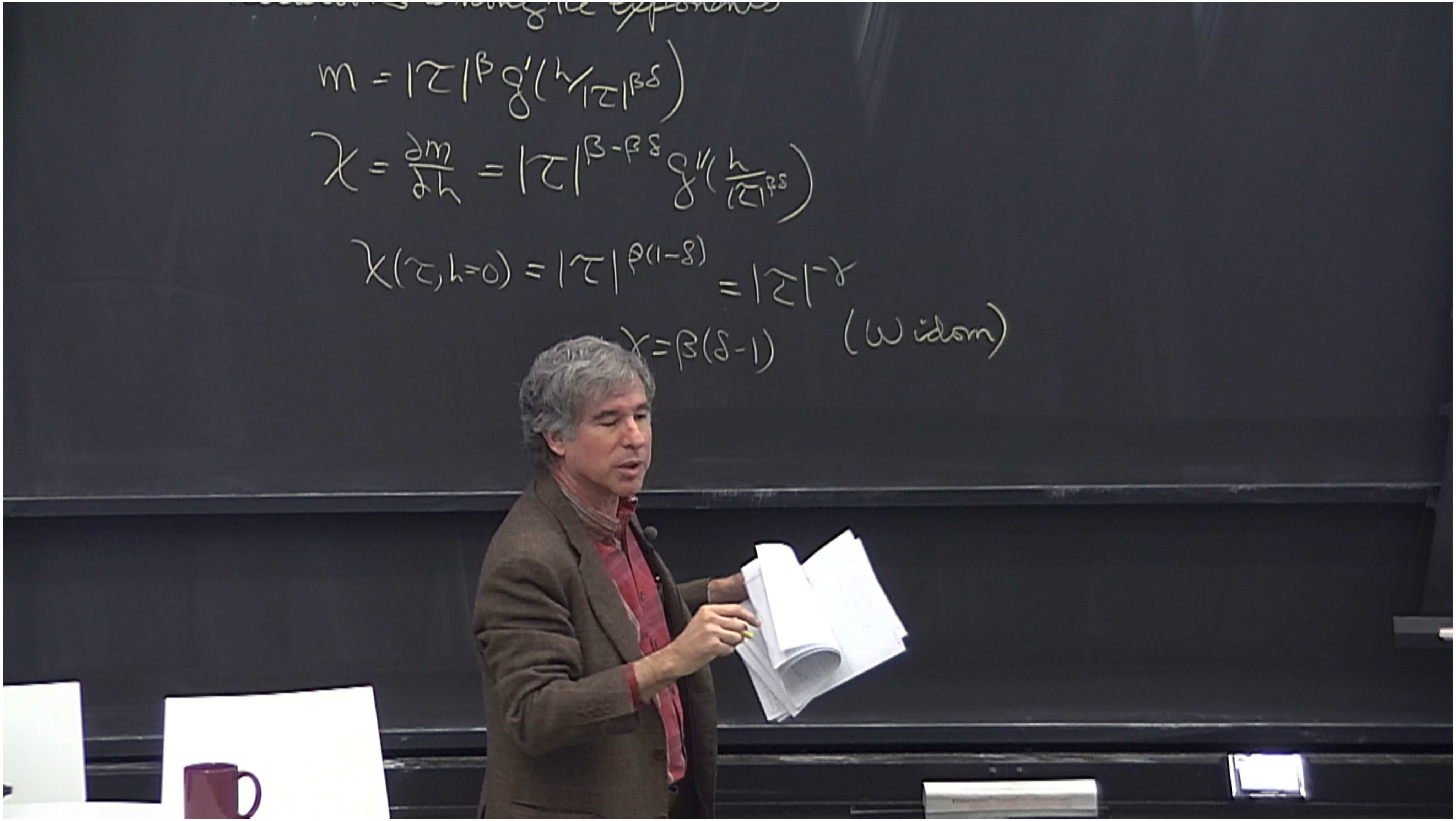
$+ \gamma = 2$ (Rushbrooke)


$m = |\tau|^{\beta} g'(h/|\tau|^{\beta\delta})$
$\chi = \frac{\partial m}{\partial h} = |\tau|^{\beta-\beta\delta} g''(\frac{h}{|\tau|^{\beta\delta}})$
$\chi(\tau, h=0) = |\tau|^{\beta(1-\delta)} = |\tau|^{-\gamma}$
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$\therefore \boxed{\gamma = \beta(\delta - 1)}$ (Widom)

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$$\beta = 2 - \alpha - \Delta = 2 - \alpha - \beta\delta = 2 - \alpha - \gamma - \beta$$

$$\boxed{\alpha + 2\beta + \gamma = 2} \quad \text{(Rushbrooke)}$$

Also recall

$$\chi \sim \xi^{2-\eta} \rightarrow \boxed{\gamma = \nu(2-\eta)} \quad \text{(Fisher)}$$

Table 16.2 Critical Exponents[a, b]

Exponent	*TH*	*EXPT*	*MFT*	*ISING2*	*ISING3*	*HEIS3*
α		0–0.14	0	0	0.12	-0.14
β		0.32–0.39	1/2	1/8	0.31	0.3
γ		1.3–1.4	1	7/4	1.25	1.4
δ		4–5	3	15	5	
ν		0.6–0.7	1/2	1	0.64	0.7
η		0.05	0	1/4	0.05	0.04
$\alpha + 2\beta + \gamma$	2	2.00 ± 0.01	2	2	2	2
$(\beta\delta - \gamma)/\beta$	1	0.93 ± 0.08	1	1	1	
$(2 - \eta)\nu/\gamma$	1	1.02 ± 0.05	1	1	1	1
$(2 - \alpha)/\nu d$	1		$4/d$	1	1	1

[a] TH, theoretical values (from scaling laws); EXPT, experimental values (from a variety of systems); MFT, mean field theory; ISINGd, Ising model in d dimension; HEIS3, classical Heisenberg model, $d = 3$.

[b] For more details and documentation see A. Z. Patashinskii and V. L. Pokrovskii, *Fluctuation Theory of Phase Transitions* (Pergamon, Oxford, 1979), Table 3, pp. 42–43.

From K. Huang, Statistical Mechanics (2nd Edition)

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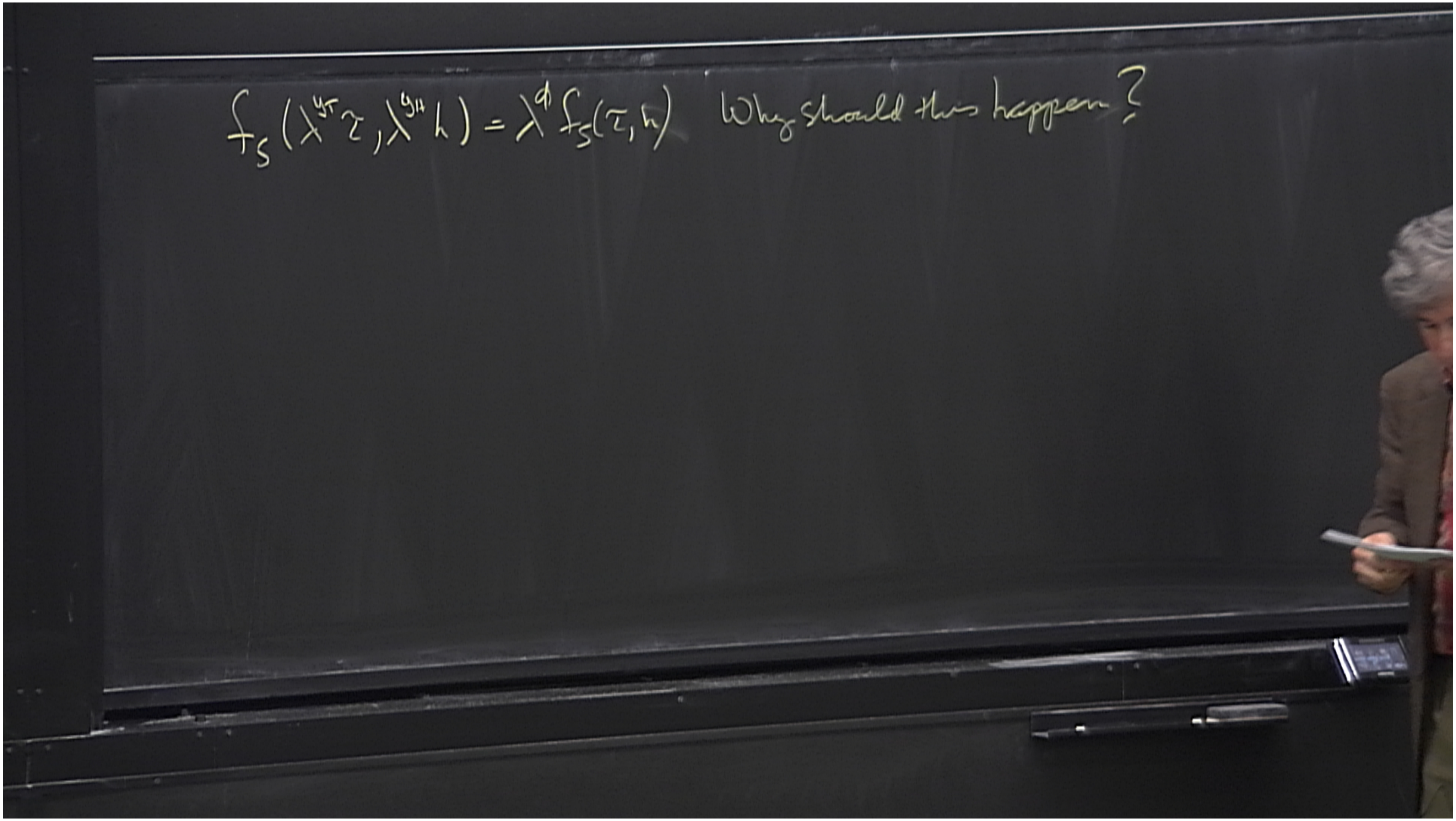
$$f_s(\lambda^{y_\tau}\tau, \lambda^{y_h} h) = \lambda^{d} f_s(\tau, h)$$ Why should this happen?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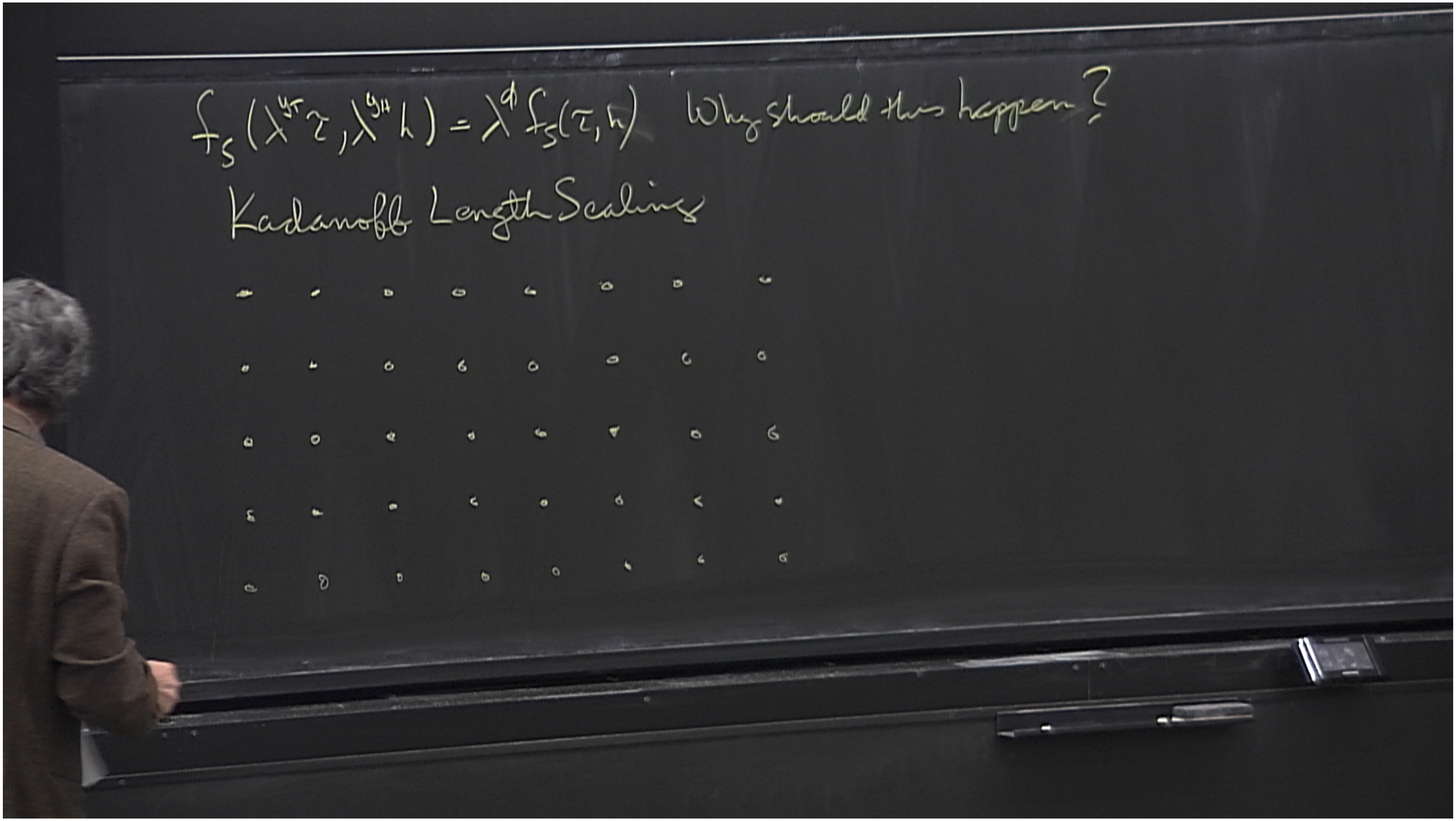
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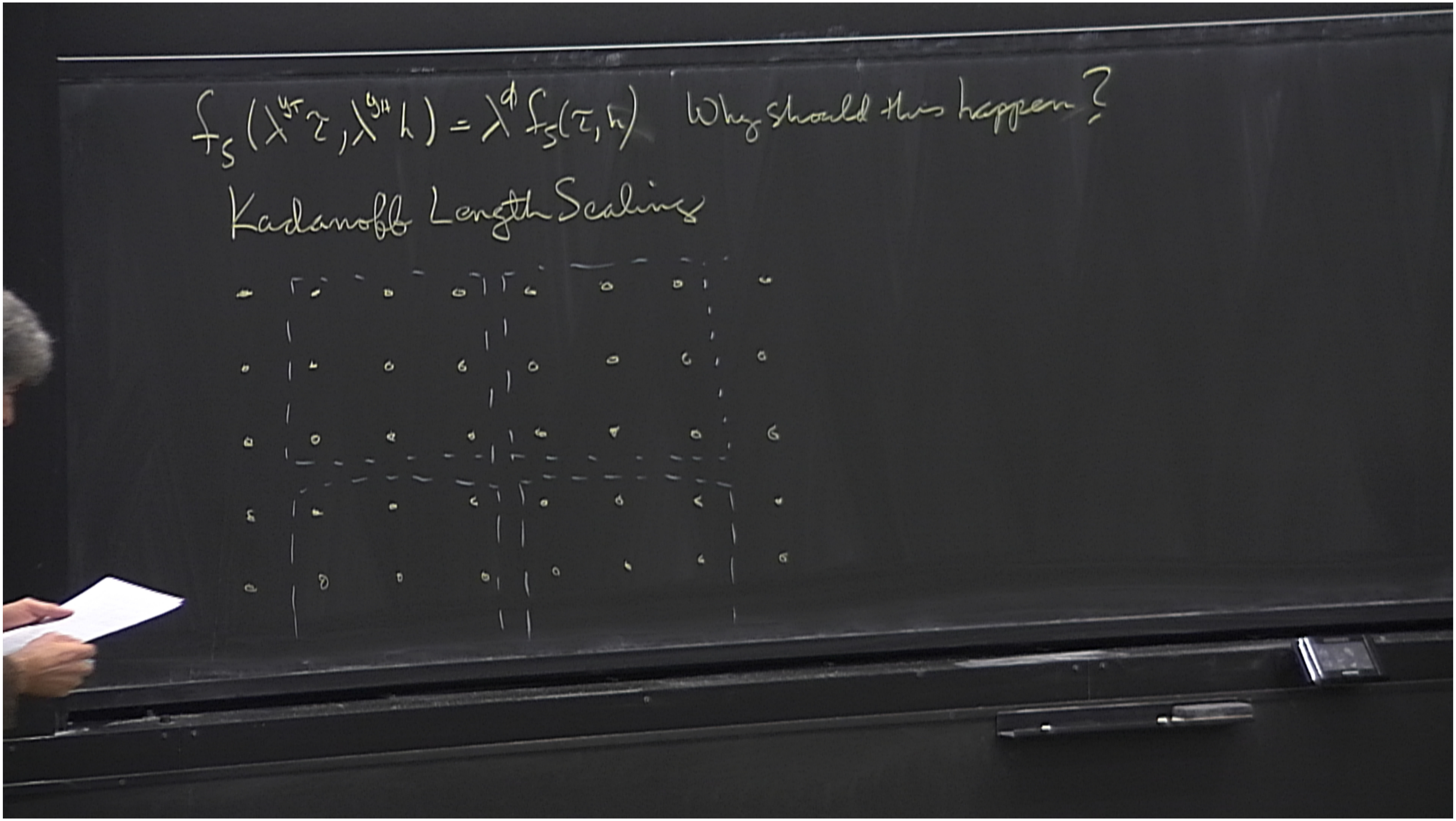
Kadanoff Length Scaling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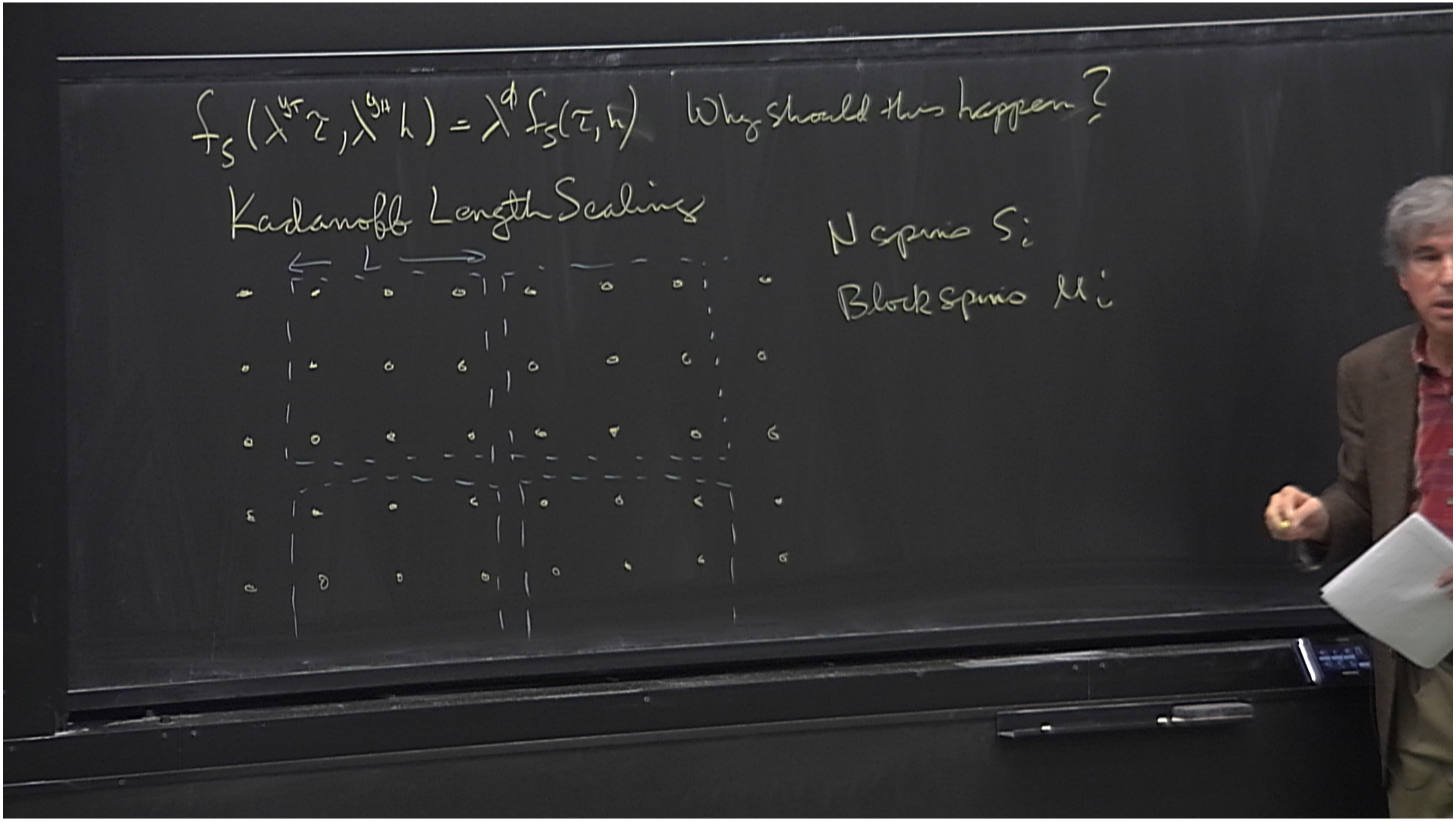
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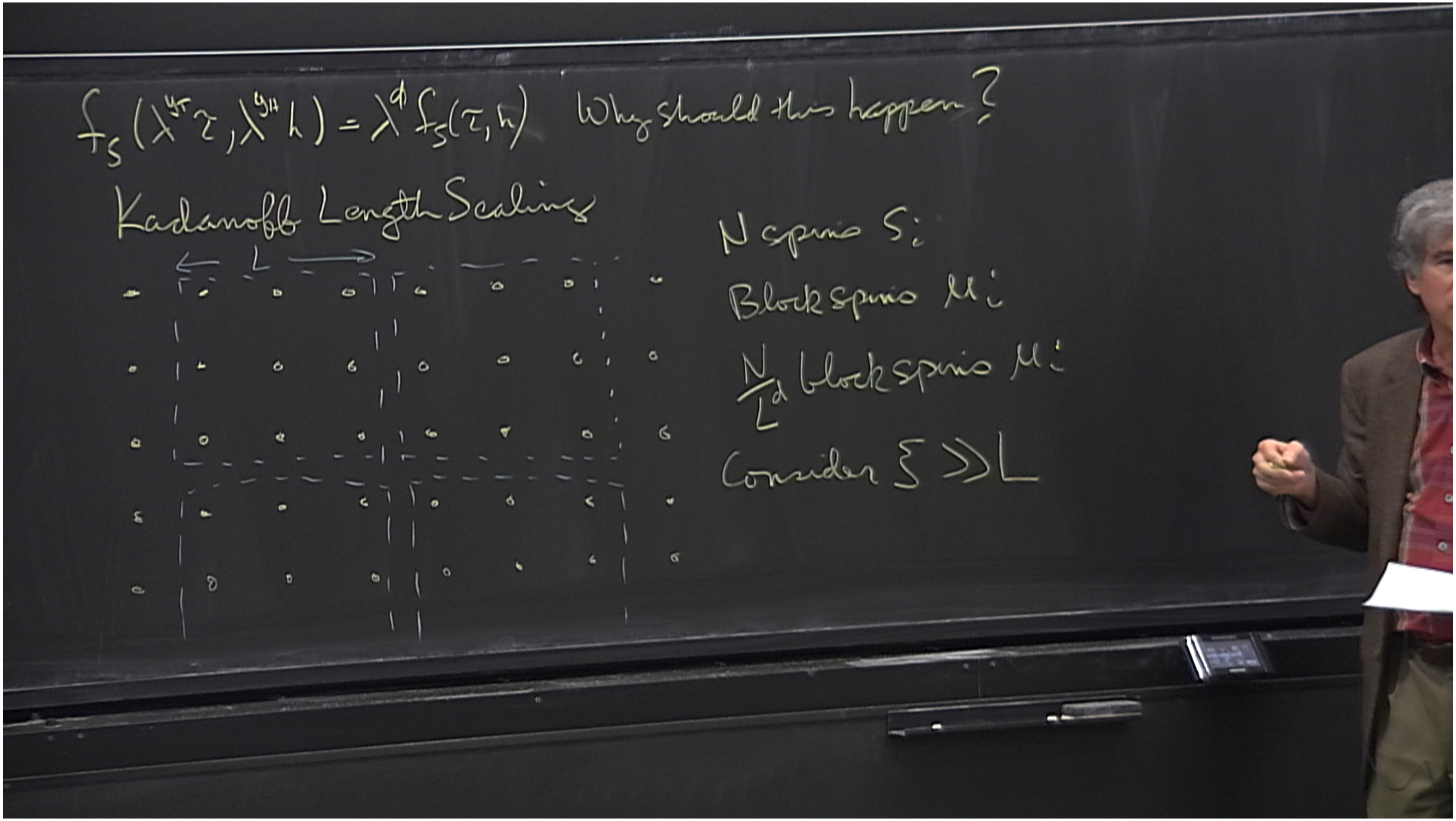
Kadanoff Length Scaling
L
N spins S_i
Block spins M_i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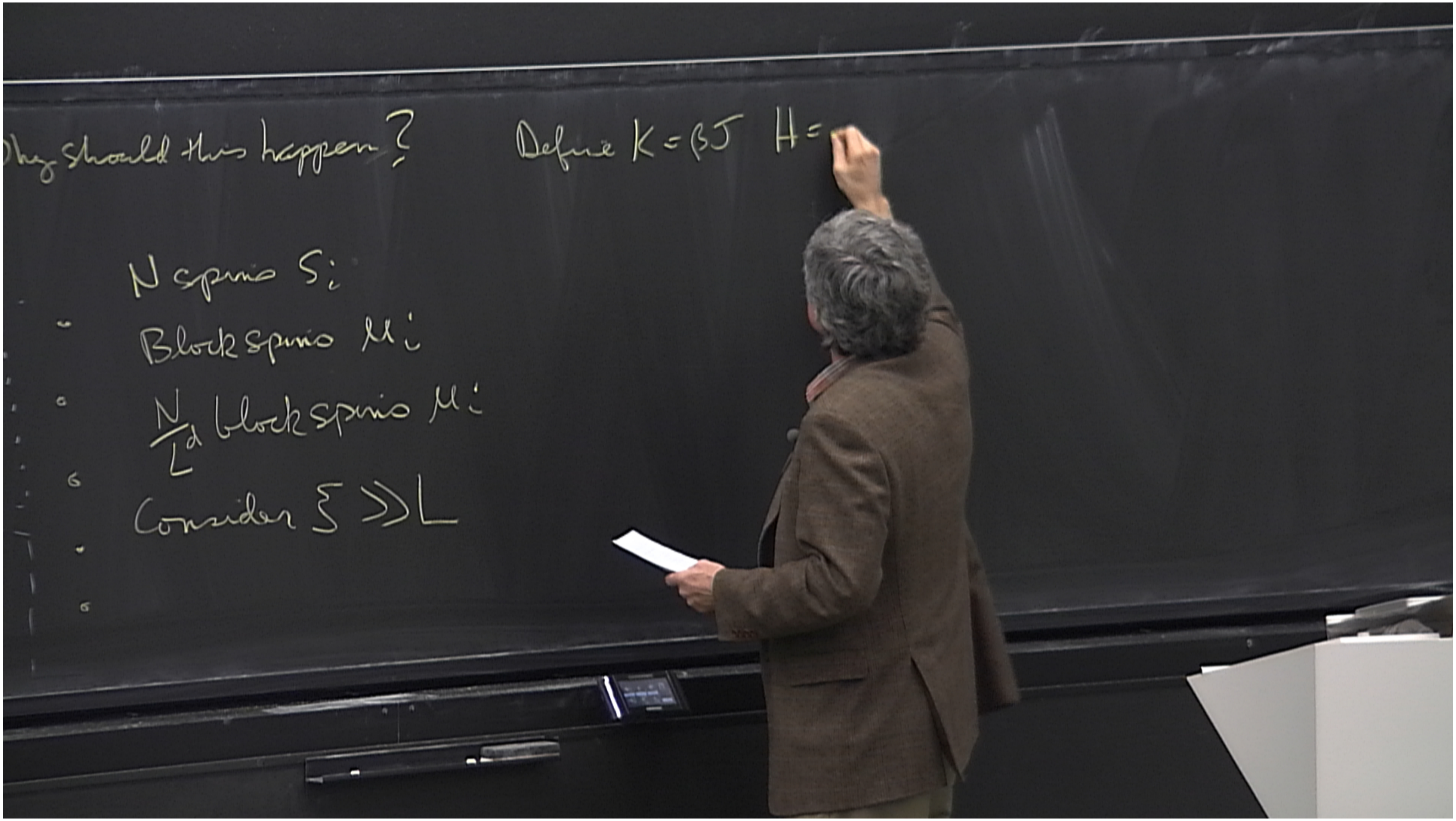
Block spins μ_i
$\frac{N}{L^d}$ block spins μ_i
Consider $\xi \gg L$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Define K = βJ H =
N spins S_i
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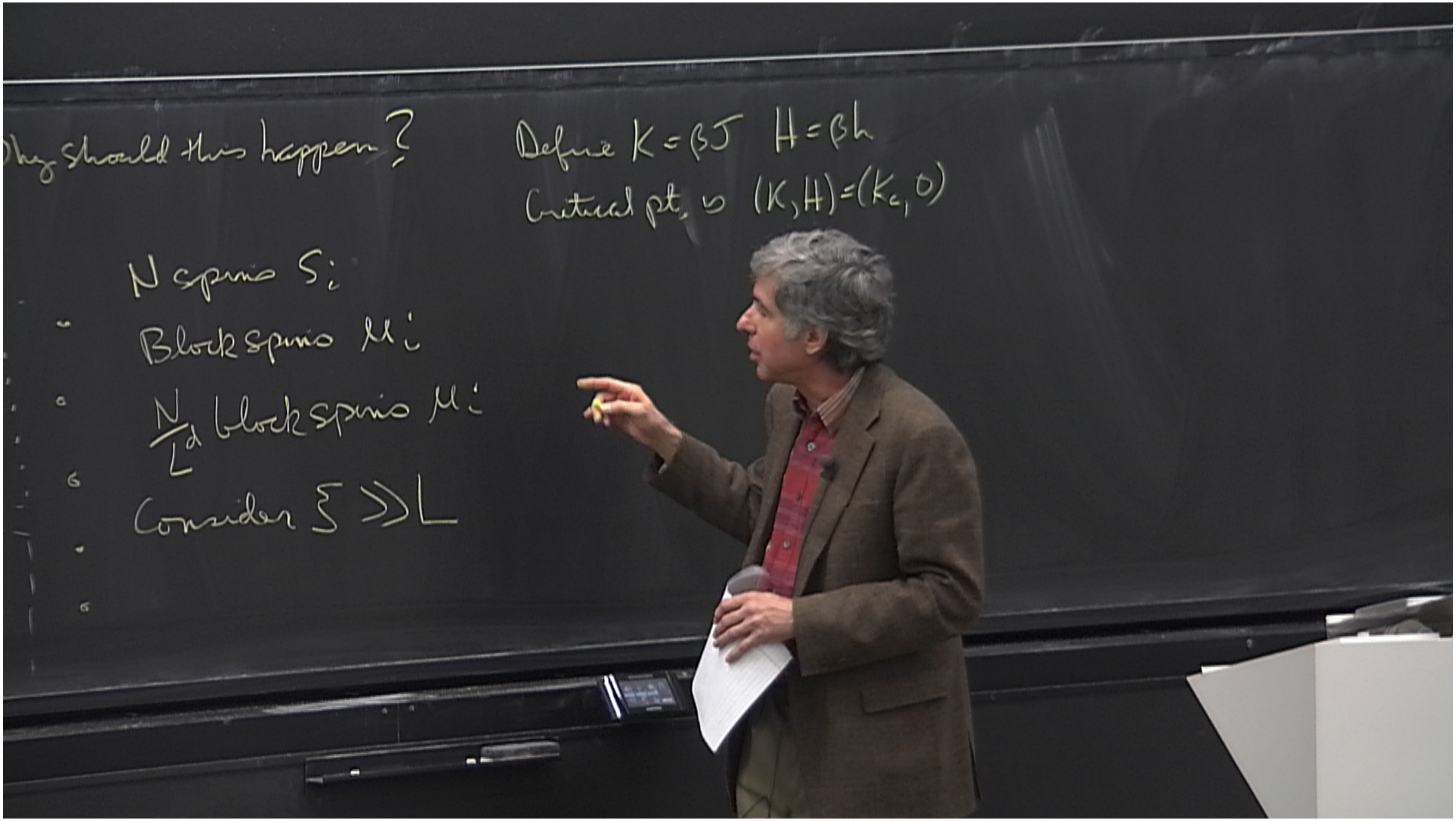
Consider ξ >> L


hy should this happen?
Define $K = \beta J$ $H = \beta h$
Critical pt. is $(K, H) = (K_c, 0)$
N spins S_i
Block spins μ_i
$\frac{N}{L^d}$ block spins μ_i
Consider $\xi \gg L$

Why should this happen?

N spins S_i

Block spins μ_i

$\frac{N}{L^d}$ block spins μ_i

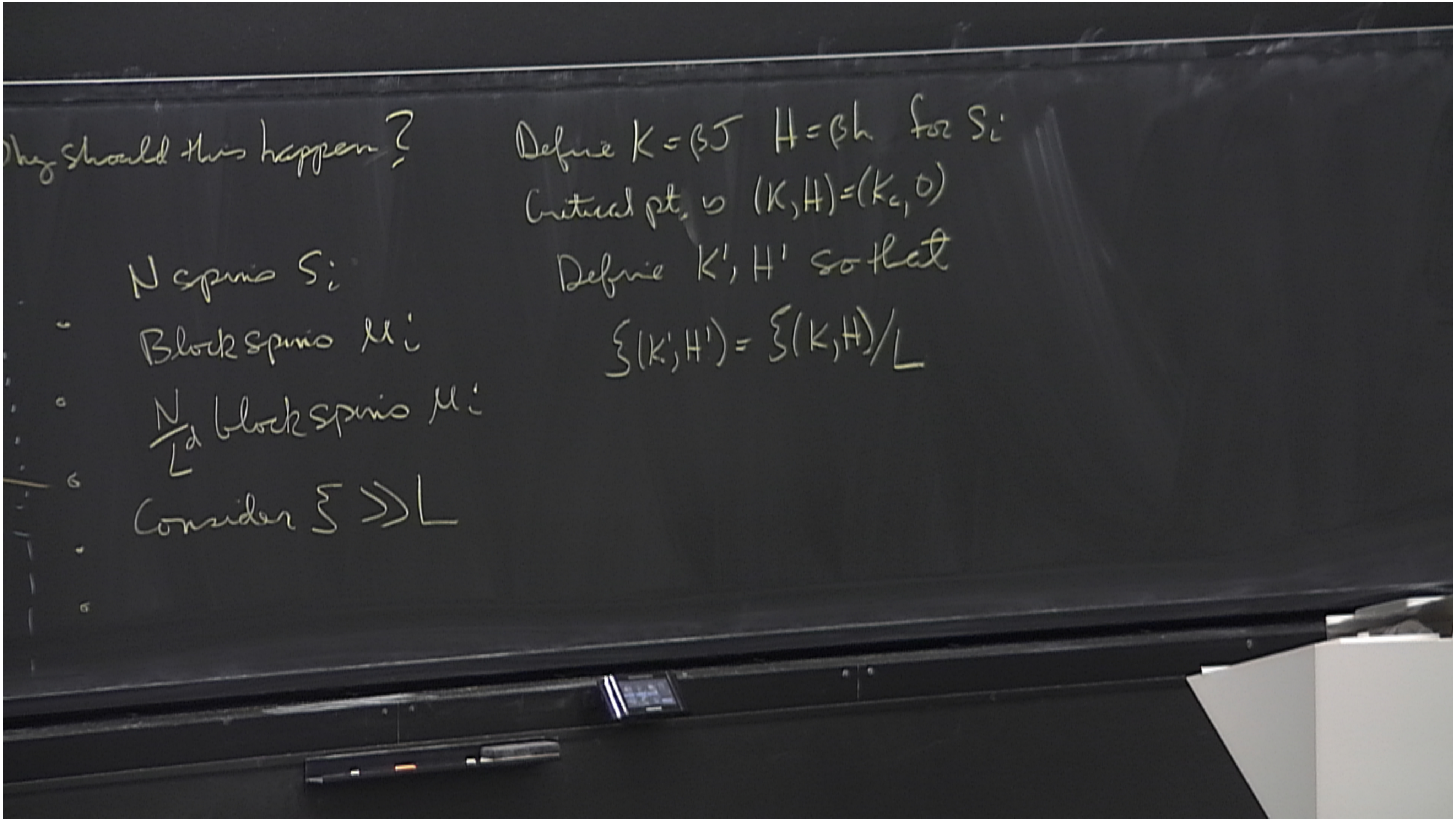
Consider $\xi \gg L$

Define $K = \beta J$ $H = \beta h$ for S_i

Critical pt. is $(K,H) = (K_c, 0)$

Define K', H' so that

$$\xi(K',H') = \xi(K,H)/L$$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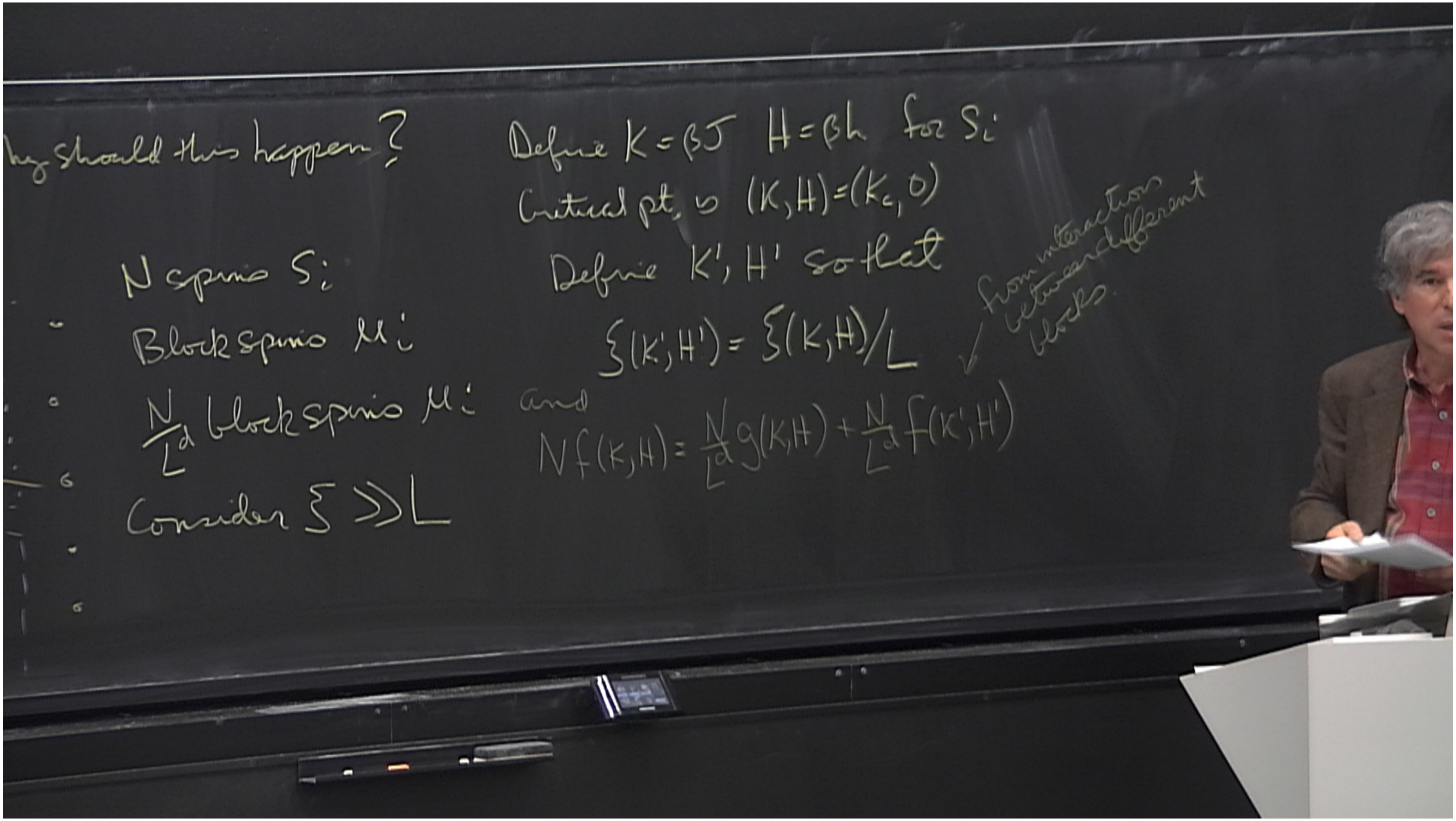
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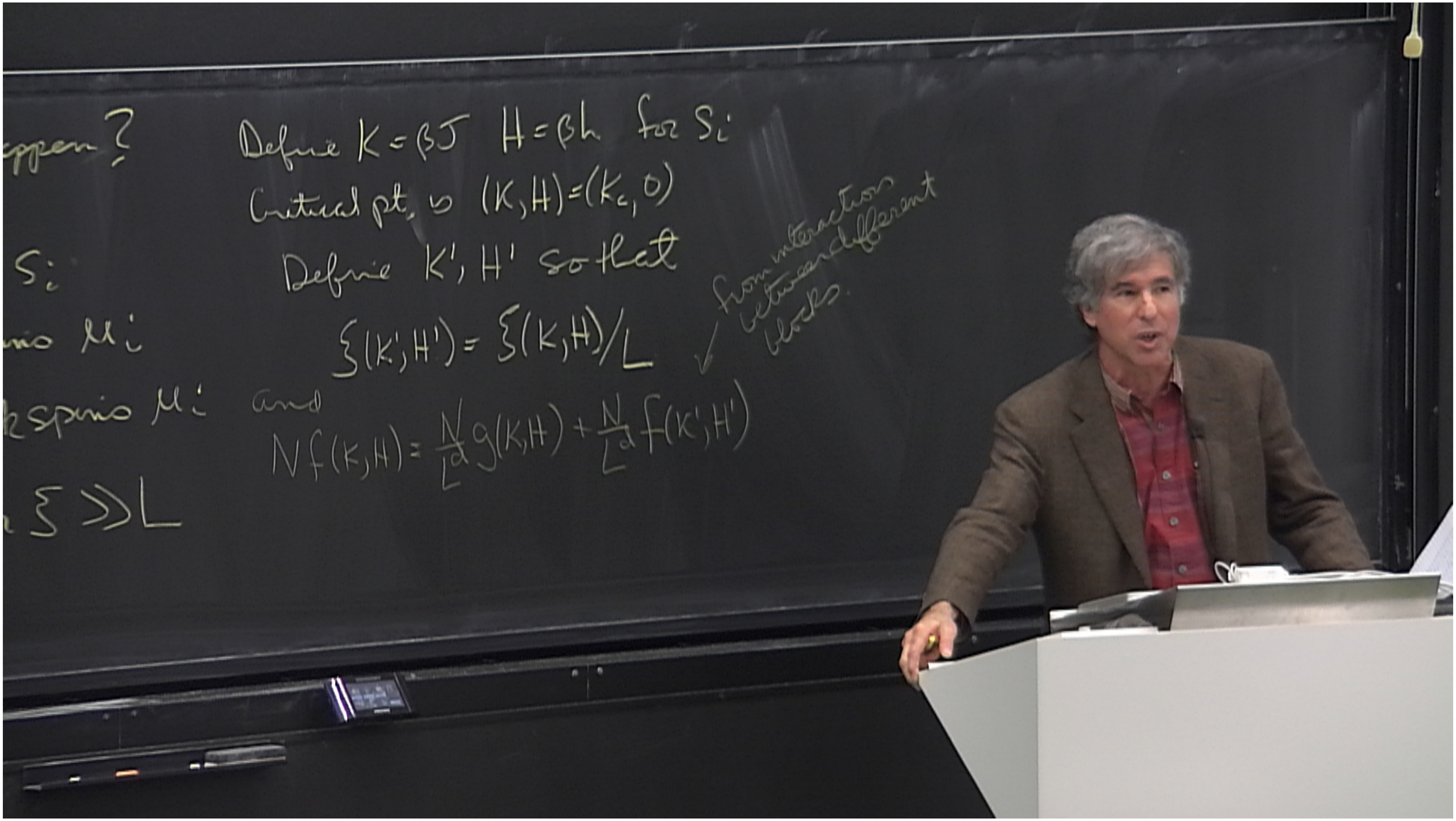
N spins S_i
Block spins μ_i
$\frac{N}{L^d}$ block spins μ_i
Consider $\xi \gg L$
Define $K=\beta J$ $H=\beta h$ for S_i
Critical pt. is $(K,H)=(K_c,0)$
Define K', H' so that
$\xi(K',H') = \xi(K,H)/L$
and
$Nf(K,H) = \frac{N}{L^d} g(K,H) + \frac{N}{L^d} f(K',H')$
from interaction between different blocks


Define K = βJ H = βh for S_i
Critical pt. is (K,H) = (K_c, 0)
Define K', H' so that
ξ(K',H') = ξ(K,H)/L
from interaction between different blocks
and
Nf(K,H) = N/L^d g(K,H) + N/L^d f(K',H')
ξ >> L

Define $t = K - K_c$

and equate singular parts

$$f_s(t', H') = L^d f_s(t, h)$$

$$\xi(t', H') = \xi(t, H)/L$$

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$$f_s(t', H') = L^d f_s(t, H)$$

$$\xi(t', H') = \xi(t, H)/L$$

Now make scaling assumption

$$t' = L^{y_T} t \qquad H' = L^{y_H} H$$

$$f_s(L^{y_T} t, L^{y_H} H) = L^d f_s(t, H)$$

Define $t = K - K_c$

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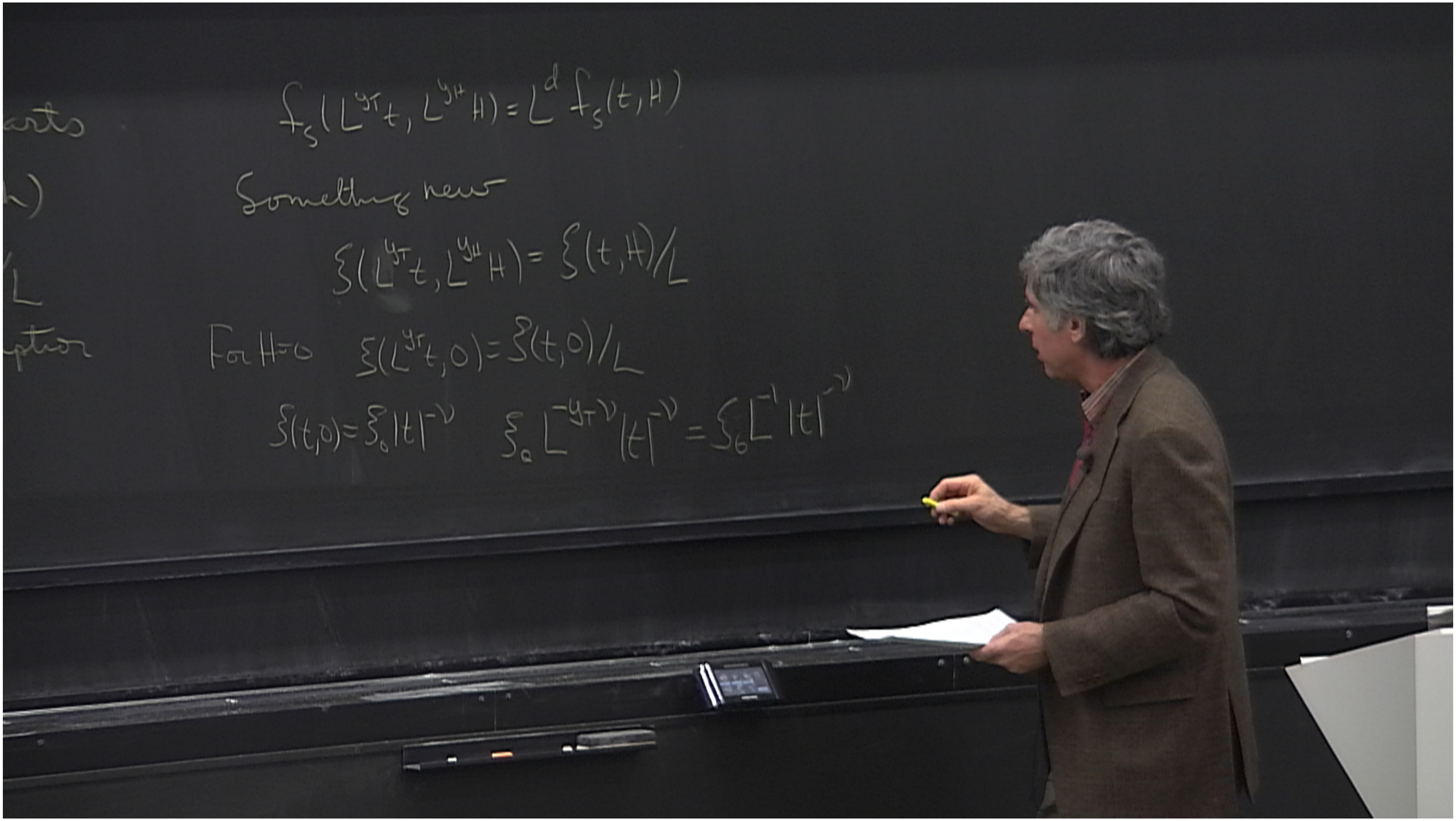
$f_s(L^{y_T} t, L^{y_H} H) = L^d f_s(t, H)$
Something new
$\xi(L^{y_T} t, L^{y_H} H) = \xi(t, H)/L$
For H=0 $\xi(L^{y_T} t, 0) = \xi(t, 0)/L$
$\xi(t,0) = \xi_0 |t|^{-\nu}$ $\xi_0 L^{-y_T \nu} |t|^{-\nu} = \xi_0 L^{-1} |t|^{-\nu}$

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$$f_s(L^{y_T} t, L^{y_H} H) = L^d f_s(t, H)$$

$$\nu = \frac{1}{y_T} = \frac{2-\alpha}{d}$$

Something new

$$2 - \alpha = d\nu$$

"hyperscaling" due to Josephson

$$\xi(L^{y_T} t, L^{y_H} H) = \xi(t, H)/L$$

$$y_T \nu = 1$$

For $H=0$ $\quad \xi(L^{y_T} t, 0) = \xi(t, 0)/L$

$$\xi(t,0) = \xi_0 |t|^{-\nu} \qquad \xi_0 L^{-y_T \nu} \cancel{|t|^{-\nu}} = \xi_0 L^{-1} \cancel{|t|^{-\nu}}$$