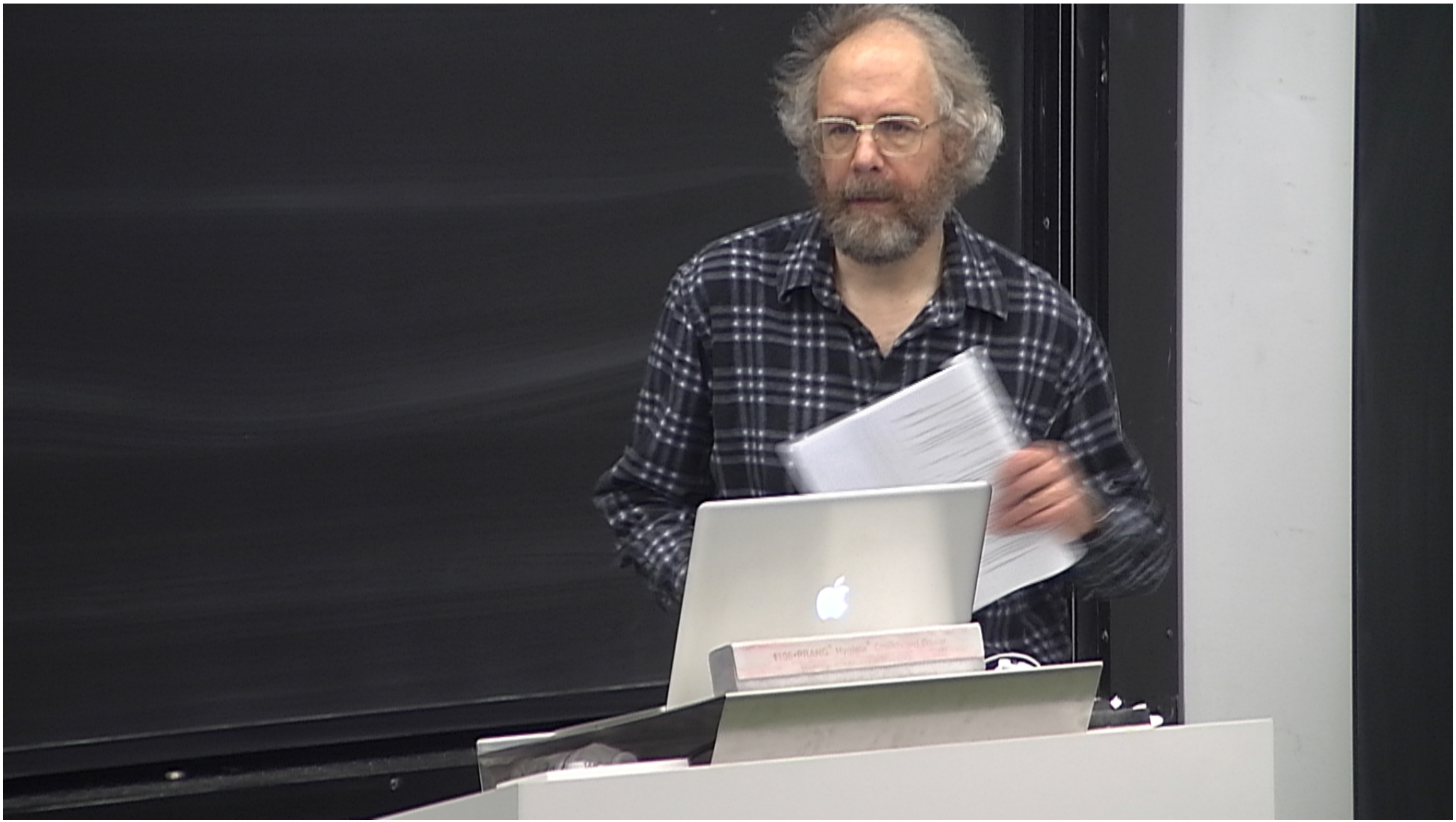
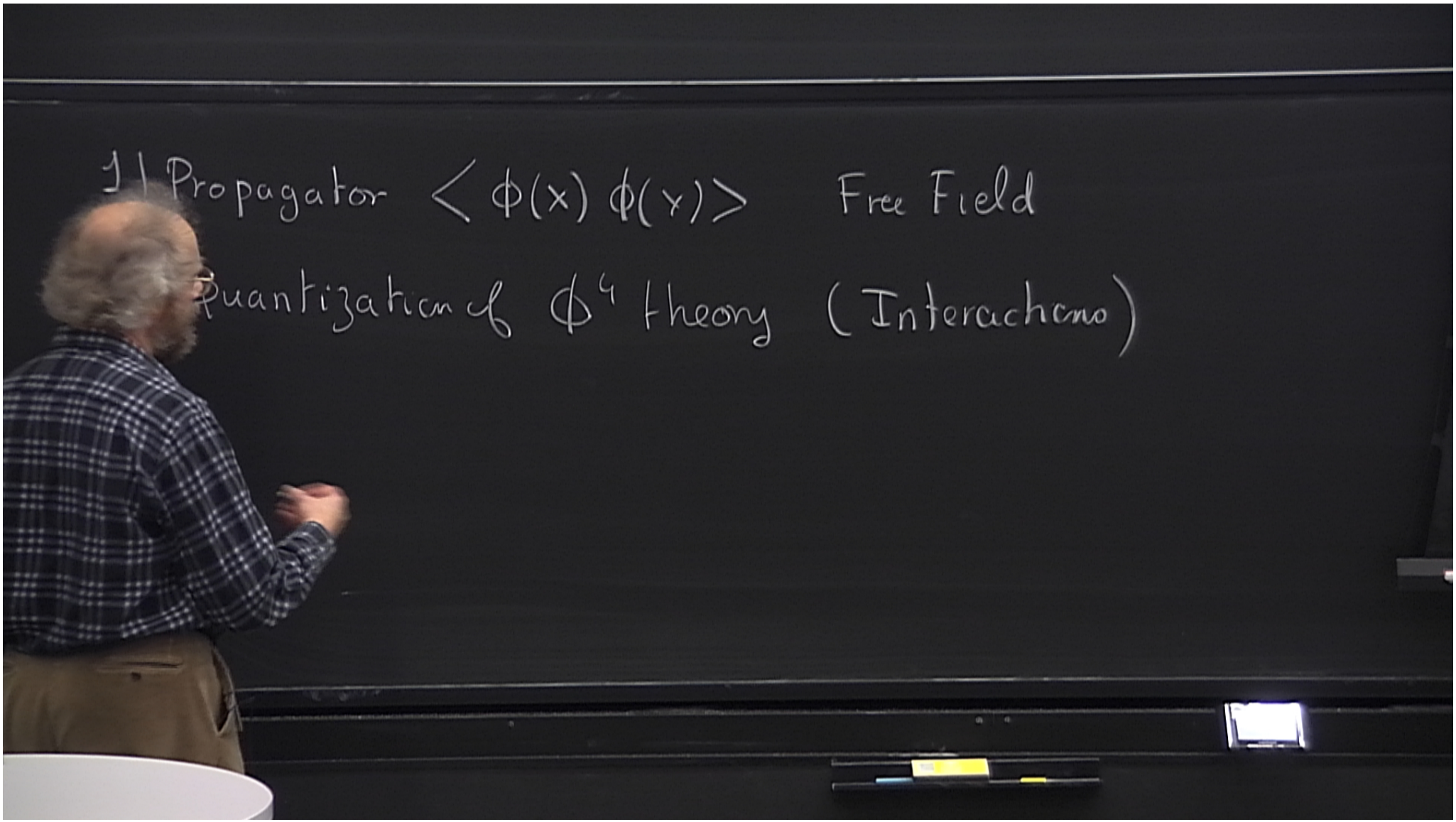
Title: Quantum Field Theory 2 - Lecture 4

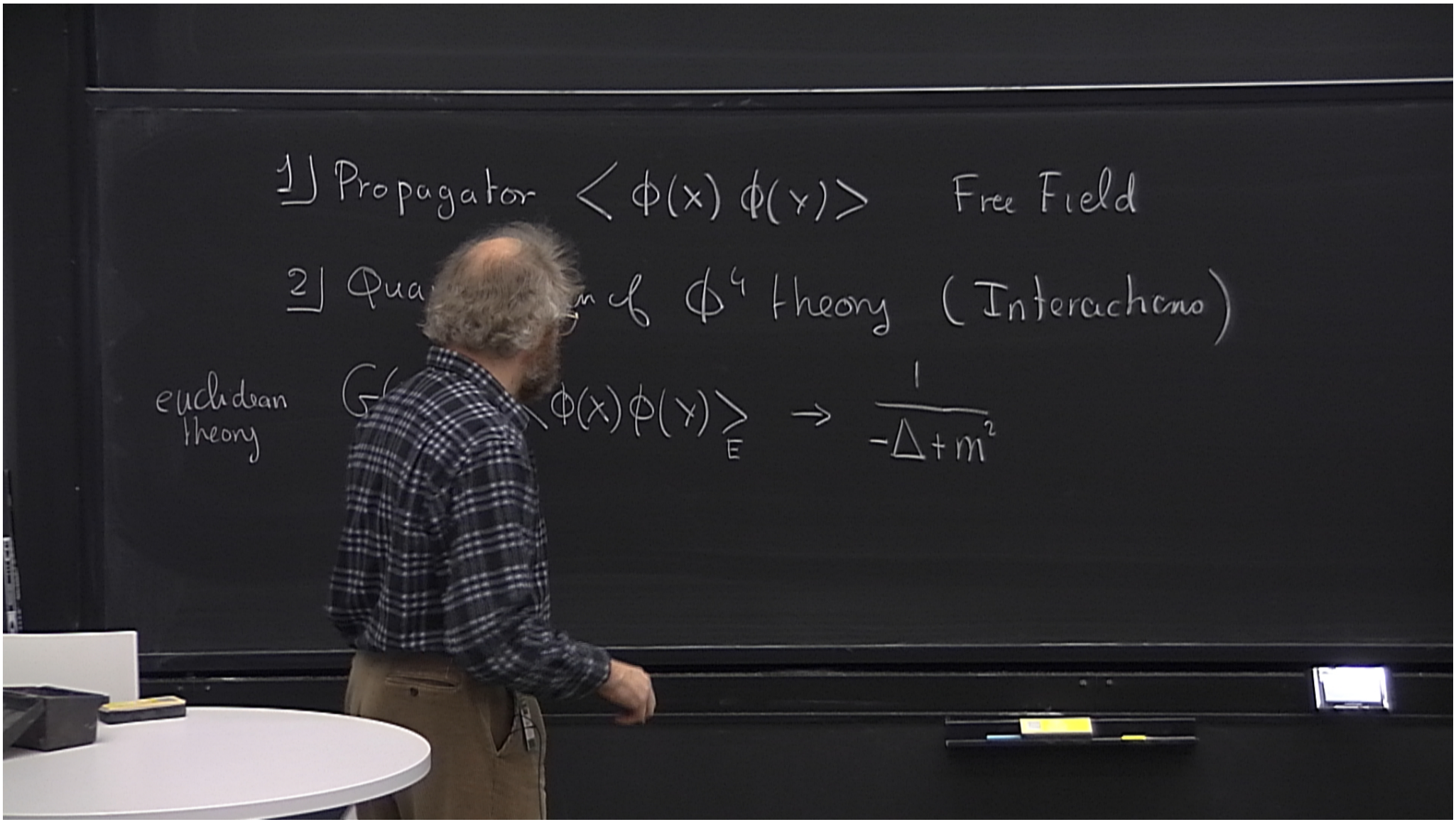
Date: Nov 15, 2012 09:00 AM

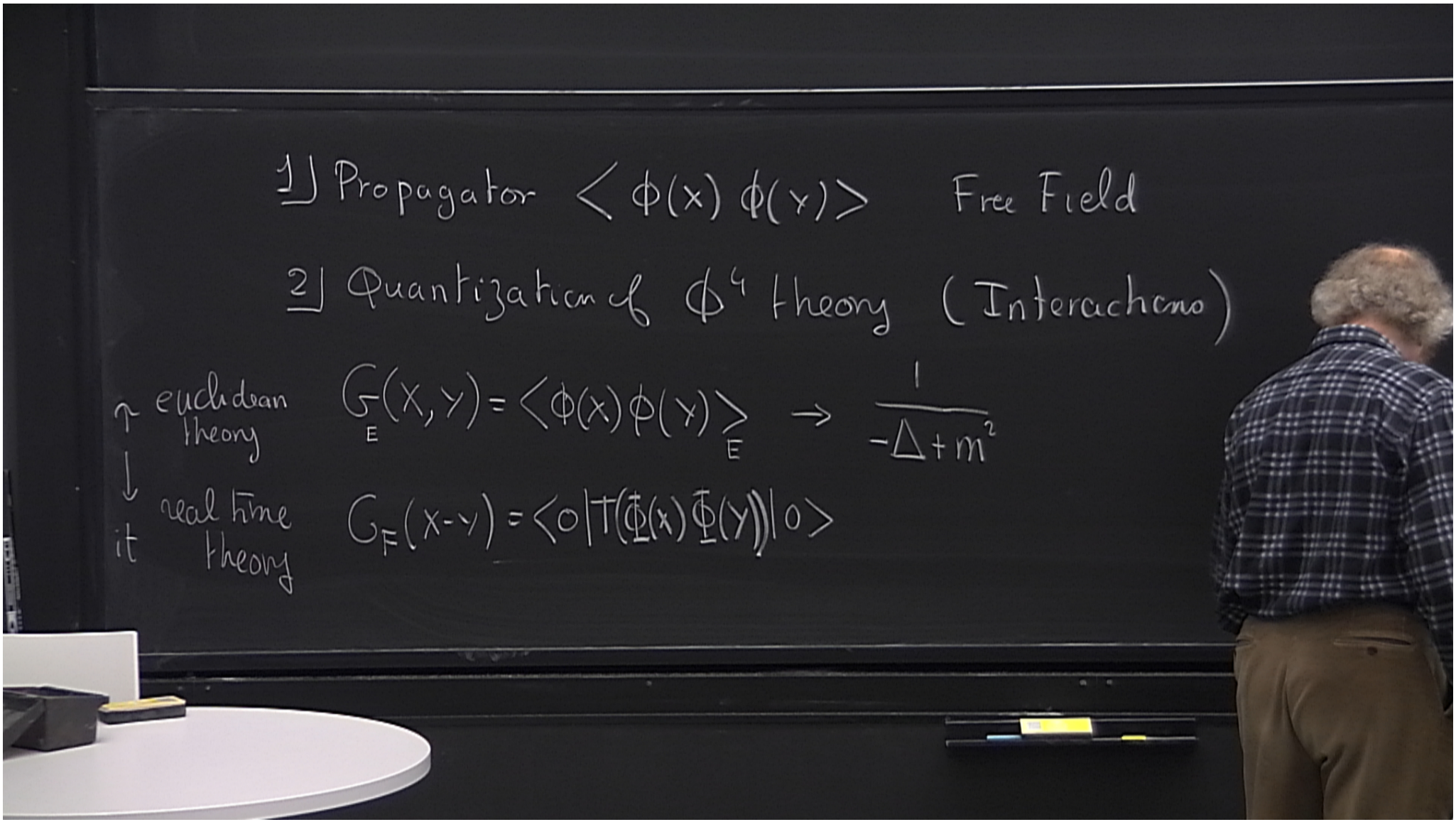
URL: http://pirsa.org/12110003

Abstract:

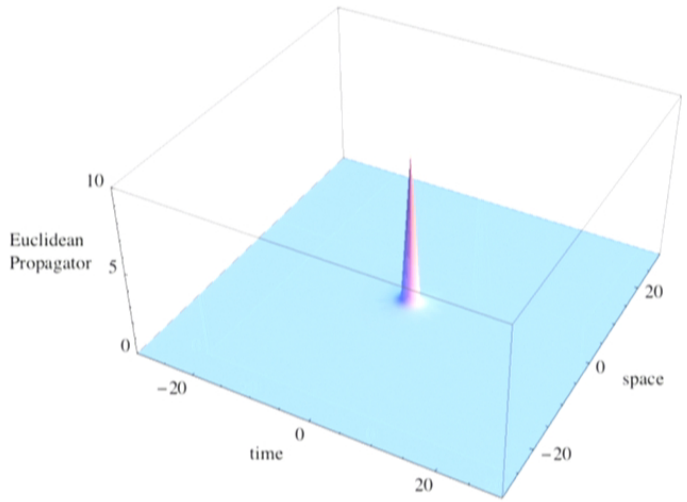


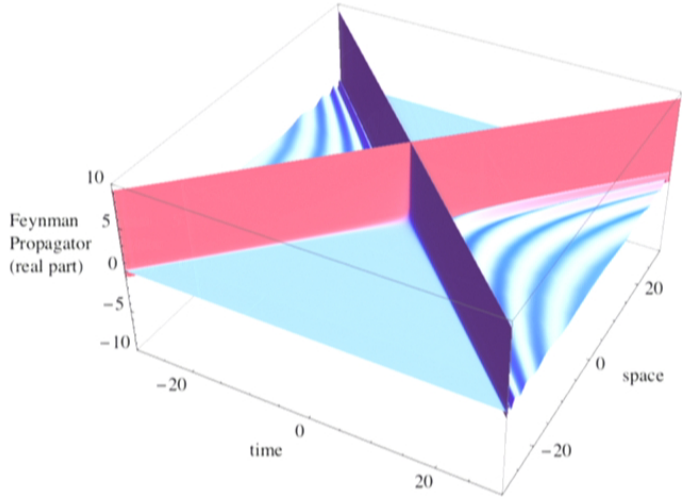

1) Propagator $\langle \phi(x)\, \phi(y) \rangle$ Free Field
Quantization of ϕ^4 theory (Interacting)

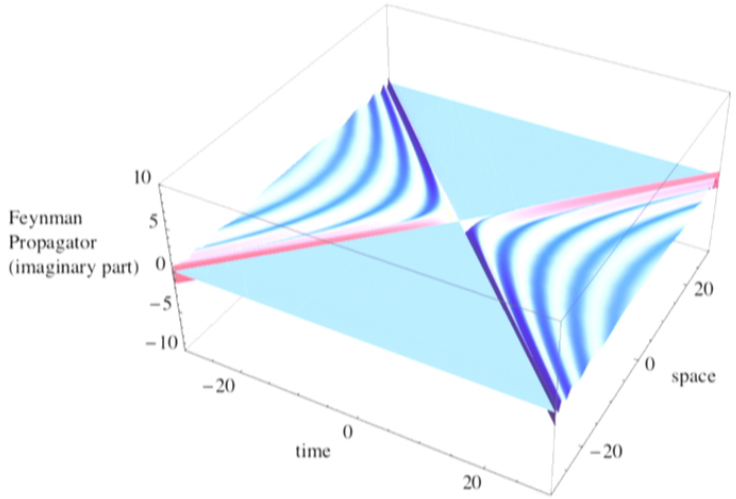

1] Propagator $\langle \phi(x)\, \phi(y) \rangle$ Free Field
2] Qua
of ϕ^4 theory (Interacting)
euclidean theory
$\phi(x)\phi(y)\rangle_E \rightarrow \frac{1}{-\Delta + m^2}$

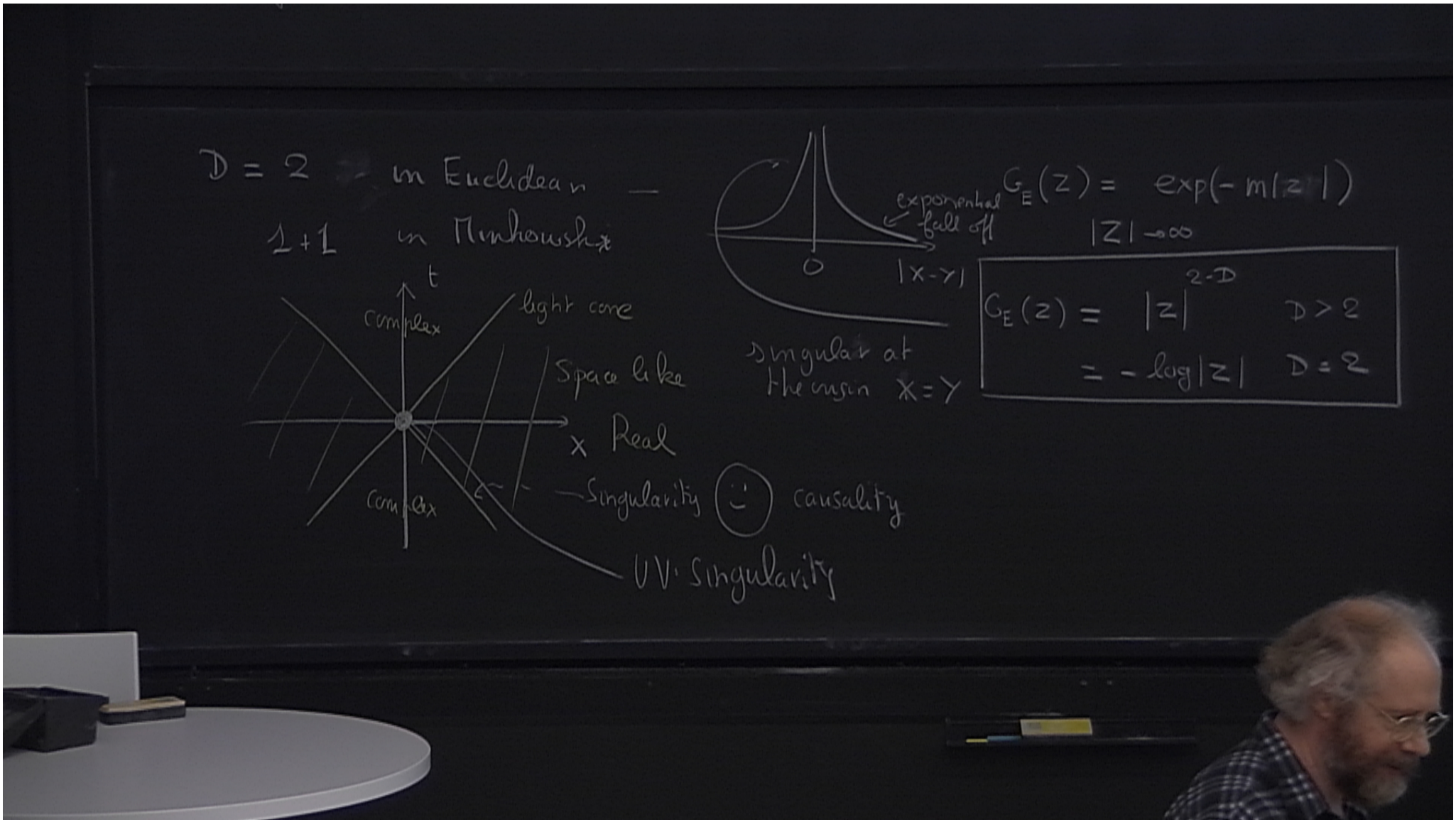

1] Propagator $\langle \phi(x)\, \phi(y) \rangle$ Free Field
2] Quantization of ϕ^4 theory (Interacting)
euclidean theory
$G_E(x,y) = \langle \phi(x)\phi(y) \rangle_E \rightarrow \frac{1}{-\Delta + m^2}$
real time theory
$G_F(x-y) = \langle 0 | T(\Phi(x)\Phi(y)) | 0 \rangle$
it

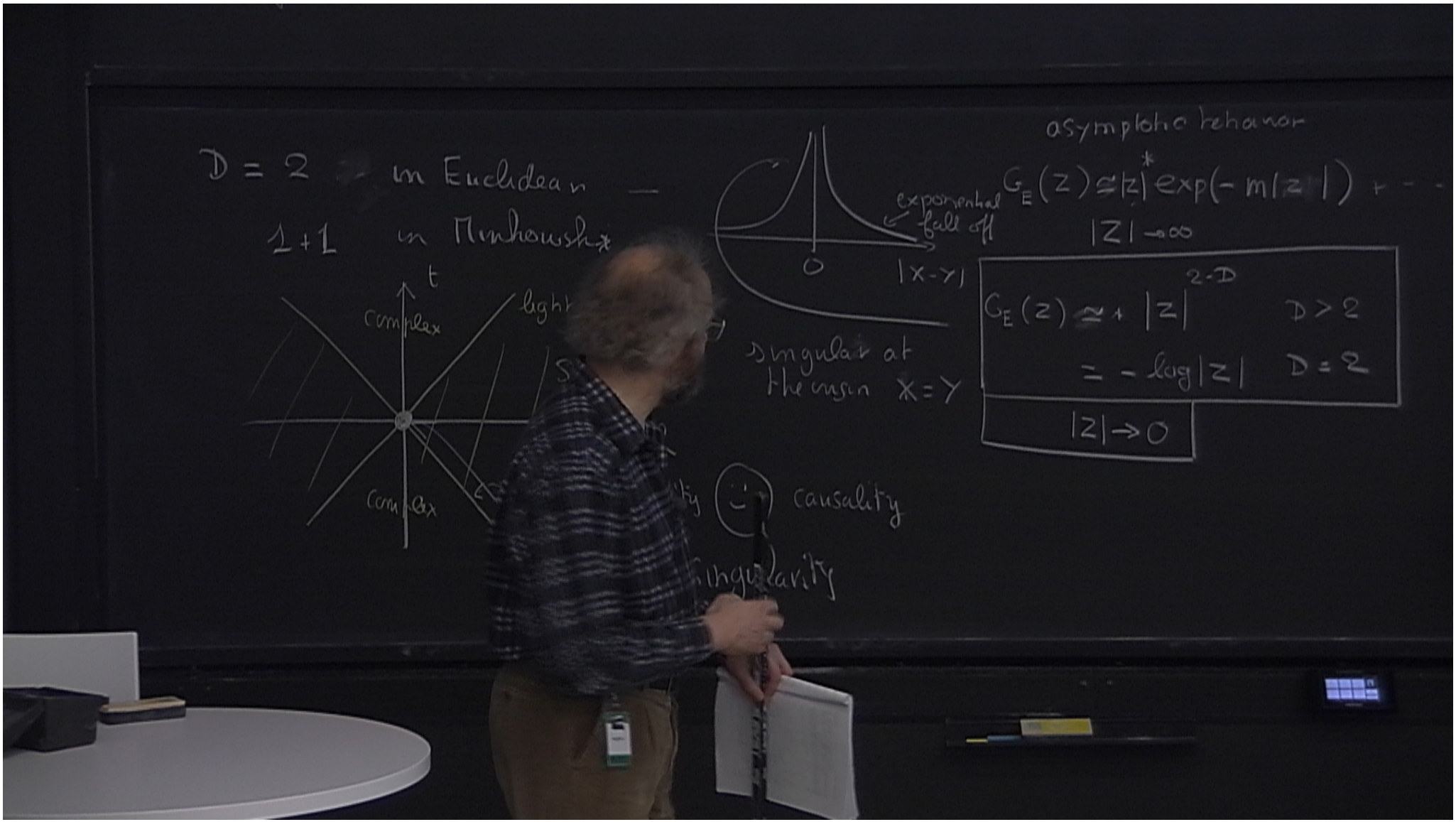
Massive Propagators in d=2

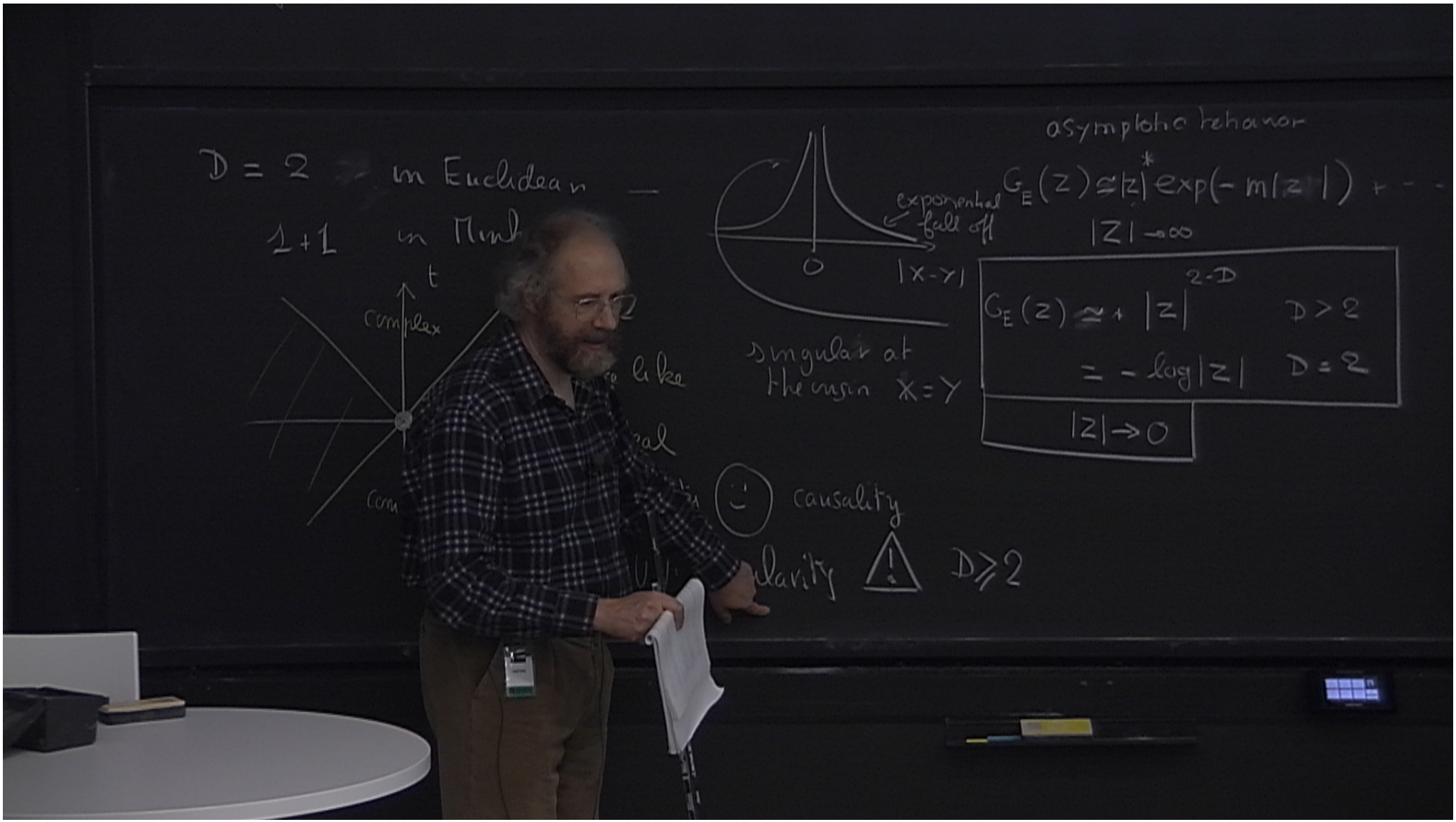


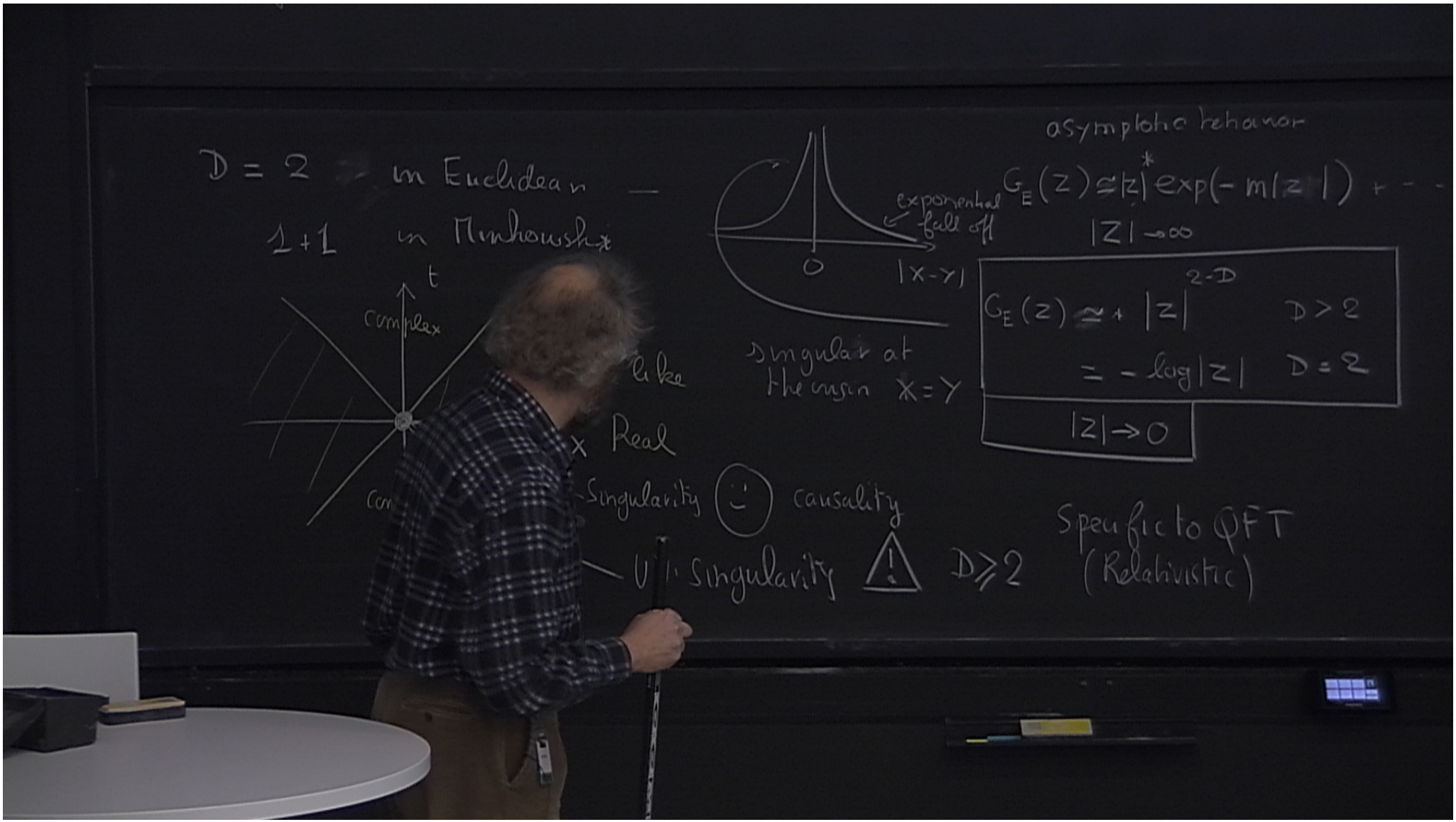





D = 2 in Euclidean
1+1 in Minkowski
t
complex
light cone
Space like
x Real
complex
Singularity
causality
UV singularity
exponential fall off
$G_E(z) = \exp(-m|z|)$
$|z| \to \infty$
0
$|x-y|$
singular at the origin $x = y$
$G_E(z) = |z|^{2-D}$ $D > 2$
$= -\log|z|$ $D = 2$


D = 2 in Euclidean
1+1 in Minkowski
t
complex
light
complex
asymptotic behavior
exponential fall off
0
|x-y|
singular at the origin x = y
|z| → 0
D > 2
D = 2
causality
singularity


D = 2 in Euclidean
1+1 in Mink
t
complex
asymptotic behavior
$G_E(z) \simeq |z|^* \exp(-m|z|) + \cdots$
exponential fall off
$|z| \to \infty$
0
$|x-y|$
$G_E(z) \simeq + |z|^{2-D}$ $D > 2$
$= -\log|z|$ $D = 2$
$|z| \to 0$
singular at the origin $x = y$
causality
$D \geq 2$


D = 2 in Euclidean
1+1 in Minkowski
t
complex
like
x Real
Singularity
causality
singularity
D ≥ 2
asymptotic behavior
$G_E(z) \simeq |z|^* \exp(-m|z|) + \cdots$
exponential fall off
$|z| \to \infty$
0
$|x-y|$
singular at the origin $x = y$
$G_E(z) \simeq + |z|^{2-D}$ $D > 2$
$= -\log|z|$ $D = 2$
$|z| \to 0$
Specific to QFT
(Relativistic)

$$\langle \phi(x)\,\phi(y) \rangle_E \nearrow \infty \qquad x \to y$$

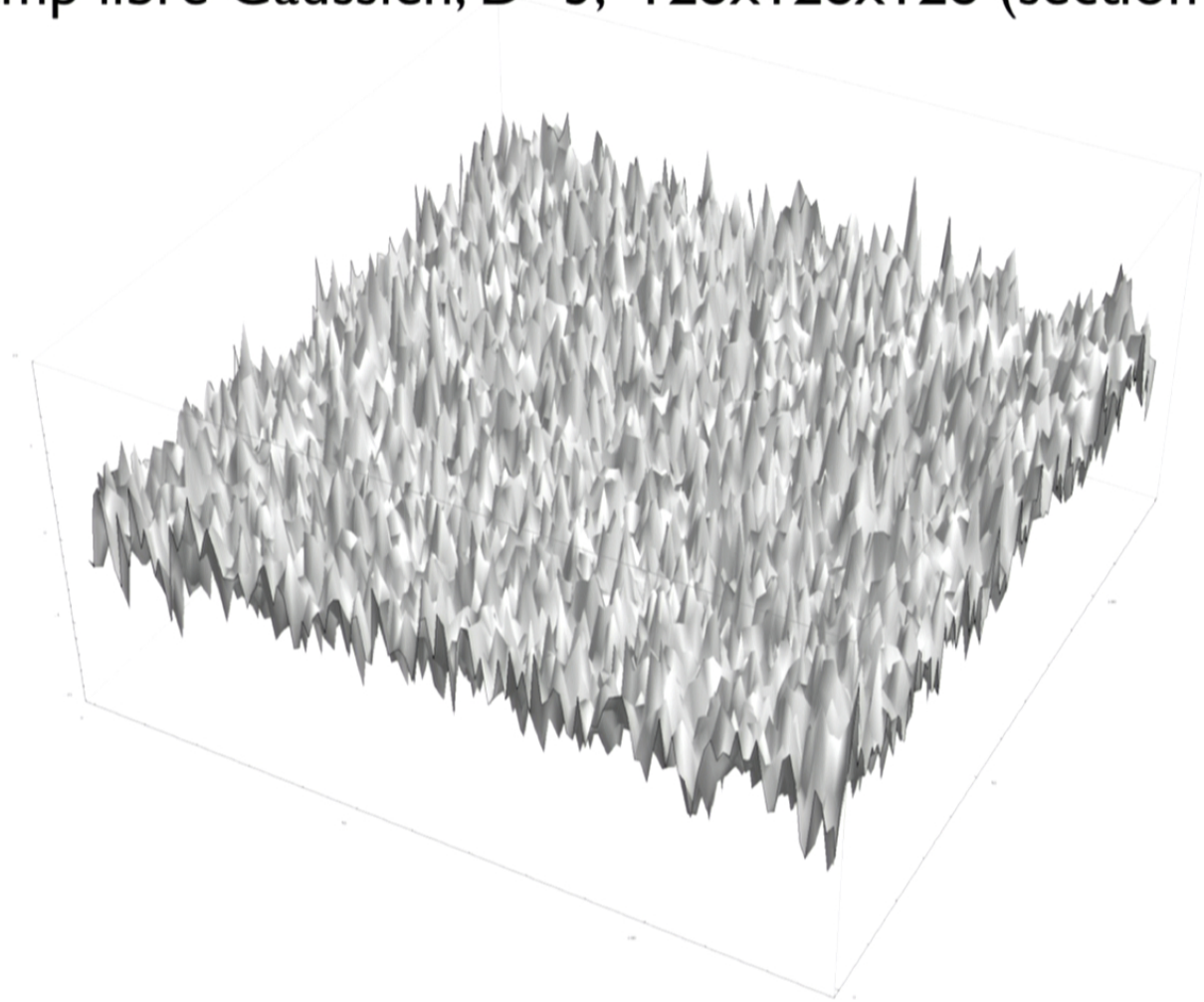
$$\langle \phi(x)^2 \rangle = \infty$$

$\phi(x)$ in the $\int D[\phi]\, e^{-S[\phi]}$

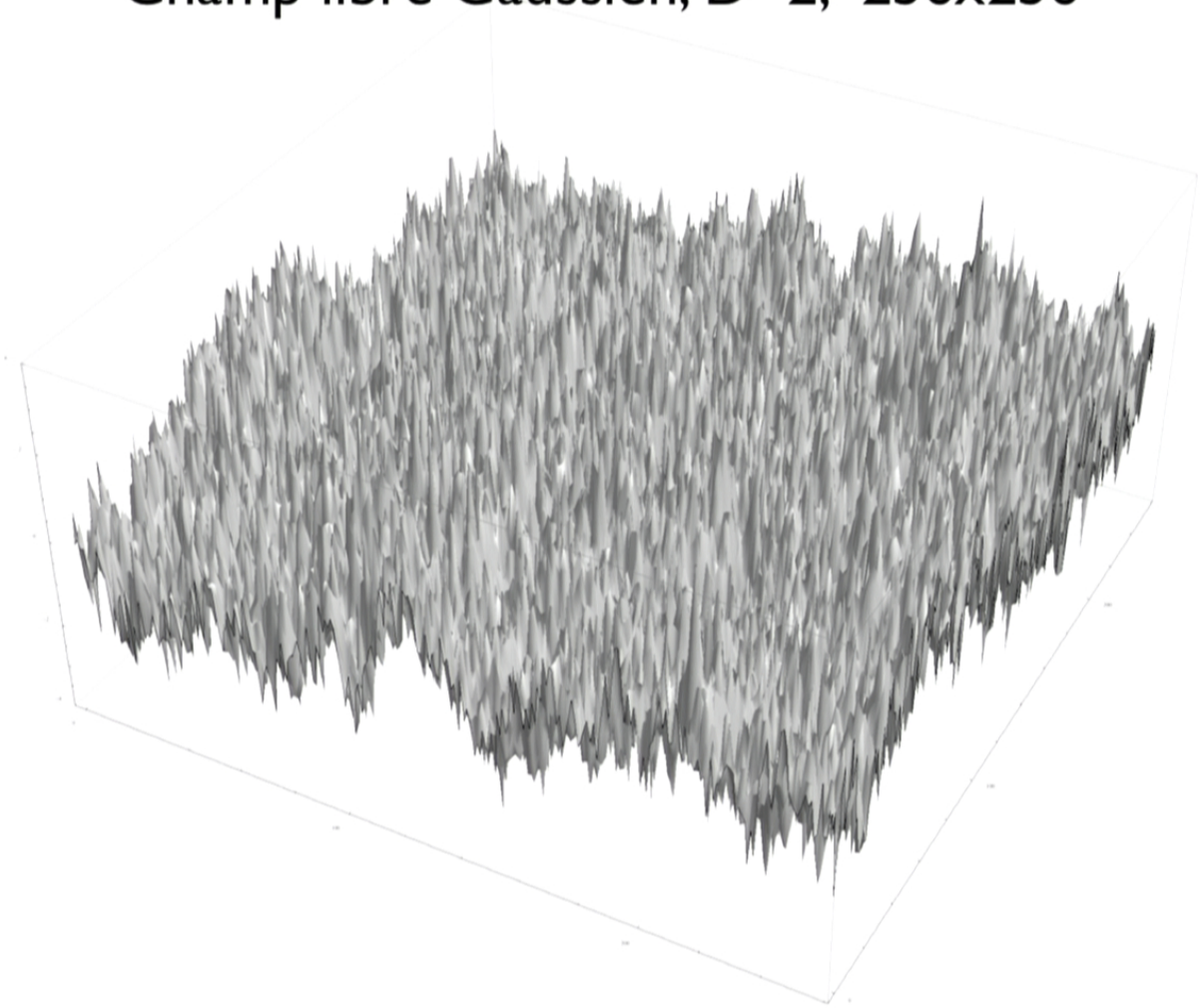
random variable

very "wild"

Champ libre Gaussien, D=3, 128x128x128 (section 2D)



Champ libre Gaussien, D=2, 256x256



$$\langle \phi(x)\,\phi(y) \rangle_E \nearrow \infty \qquad x \to y$$

$$\langle \phi(x)^2 \rangle = \infty$$

not a derivable function

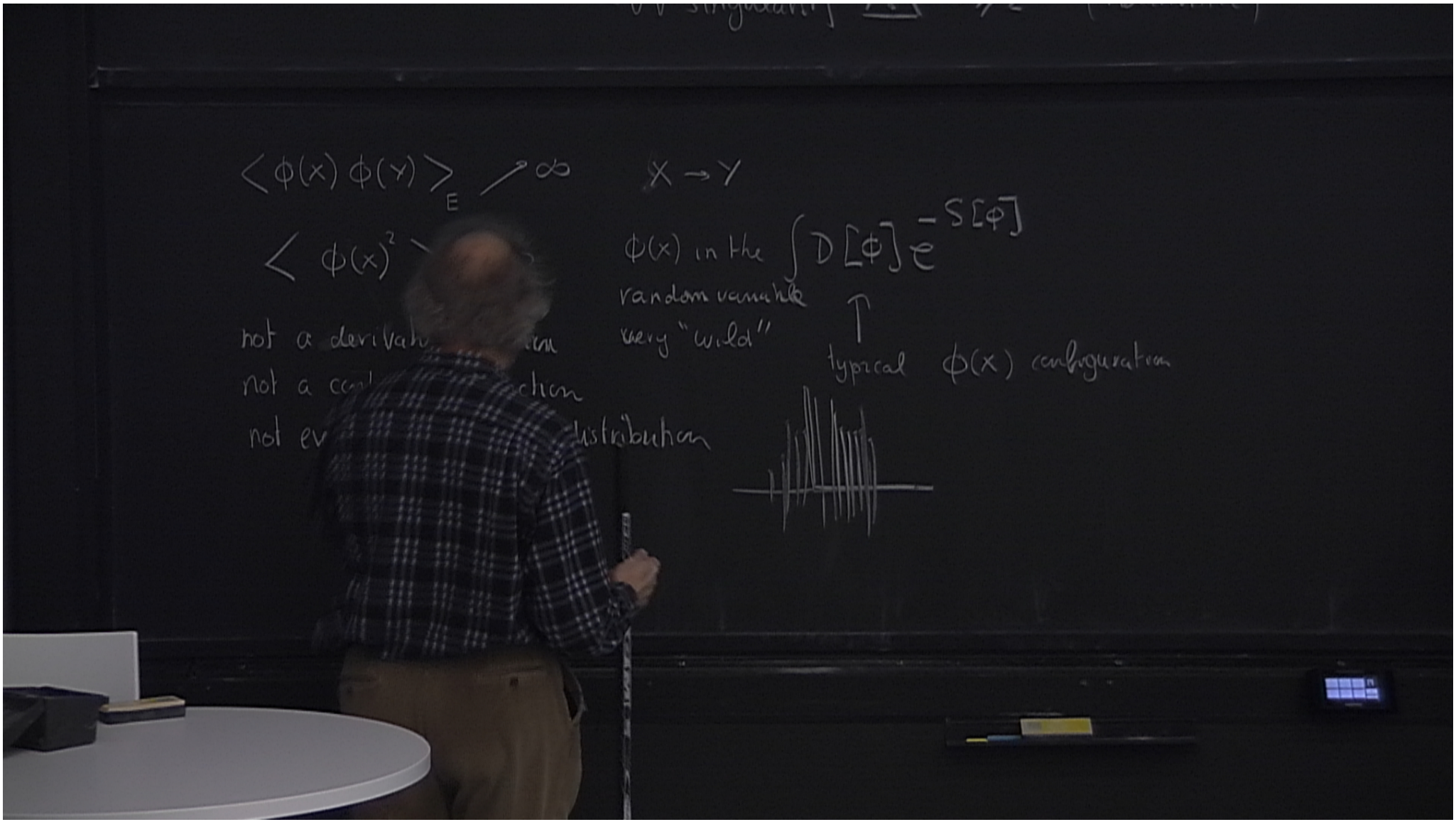
not a continuous function

$\phi(x)$ in the $\int D[\phi]\, e^{-S[\phi]}$

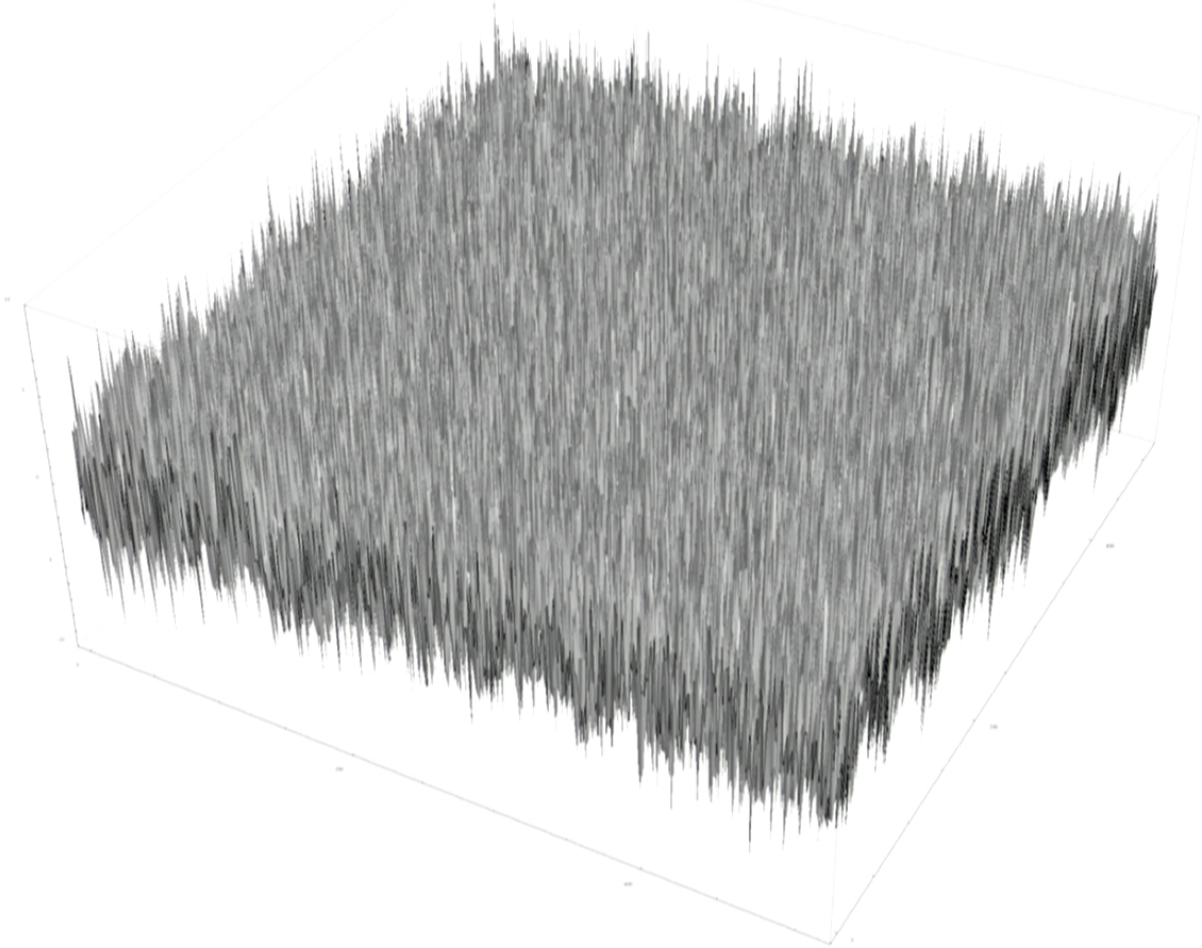
random variable

very "wild"

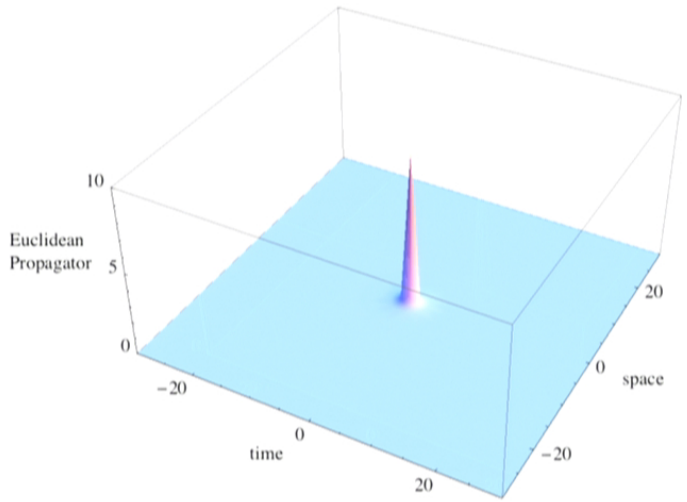
typical $\phi(x)$ configuration

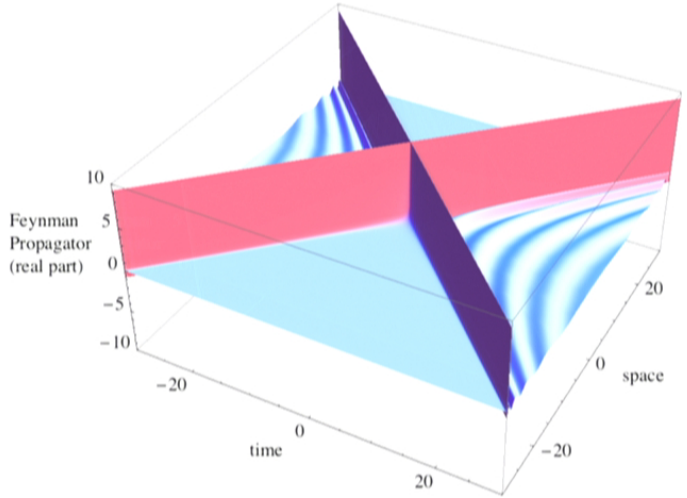

$\langle \phi(x)\,\phi(y) \rangle_E \nearrow \infty$
$x \to y$
$\langle \phi(x)^2 \rangle$
$\phi(x)$ in the $\int D[\phi]\, e^{-S[\phi]}$
random variable
very "wild"
typical $\phi(x)$ configuration
not a derivative
not a continuous function
not ev distribution

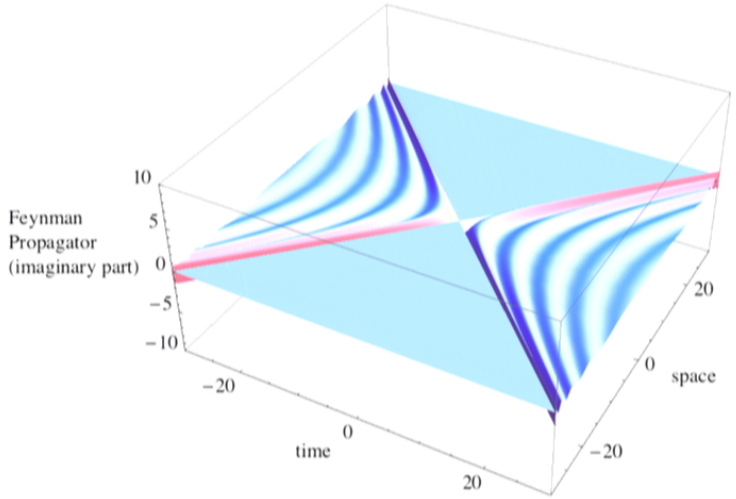
Champ libre Gaussien, D=3, 512x512x512 (section 2D)



Massive Propagators in d=2



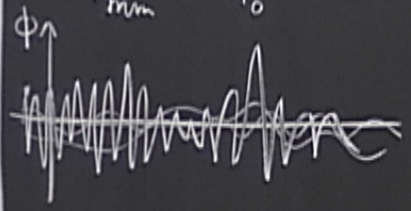




configuration

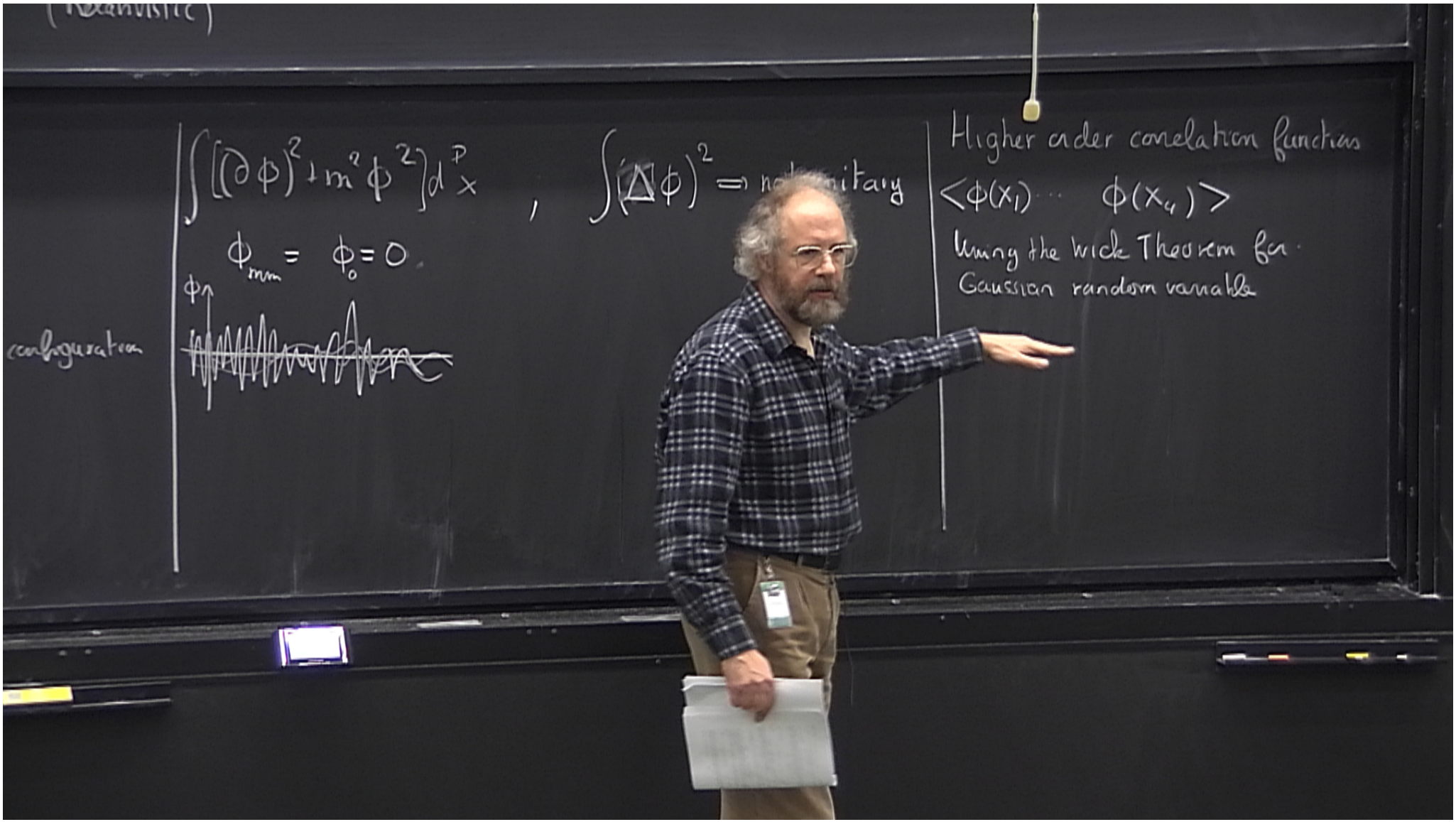
$$\int \left[(\partial \phi)^2 + m^2 \phi^2 \right] d^D x \quad , \quad \int (\Delta \phi)^2 \Rightarrow \text{not unitary}$$

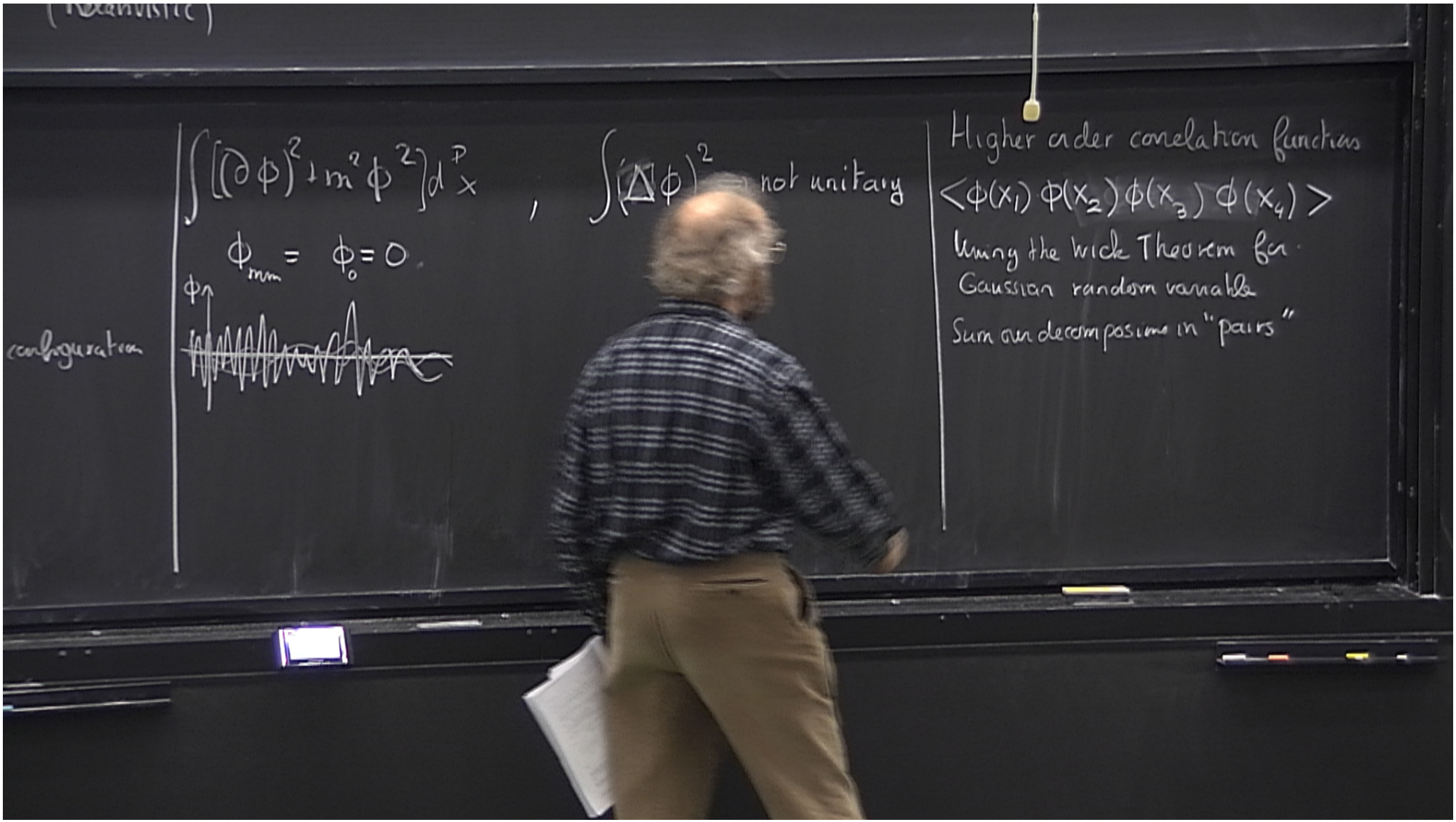
$$\phi_{min} = \phi_0 = 0 .$$

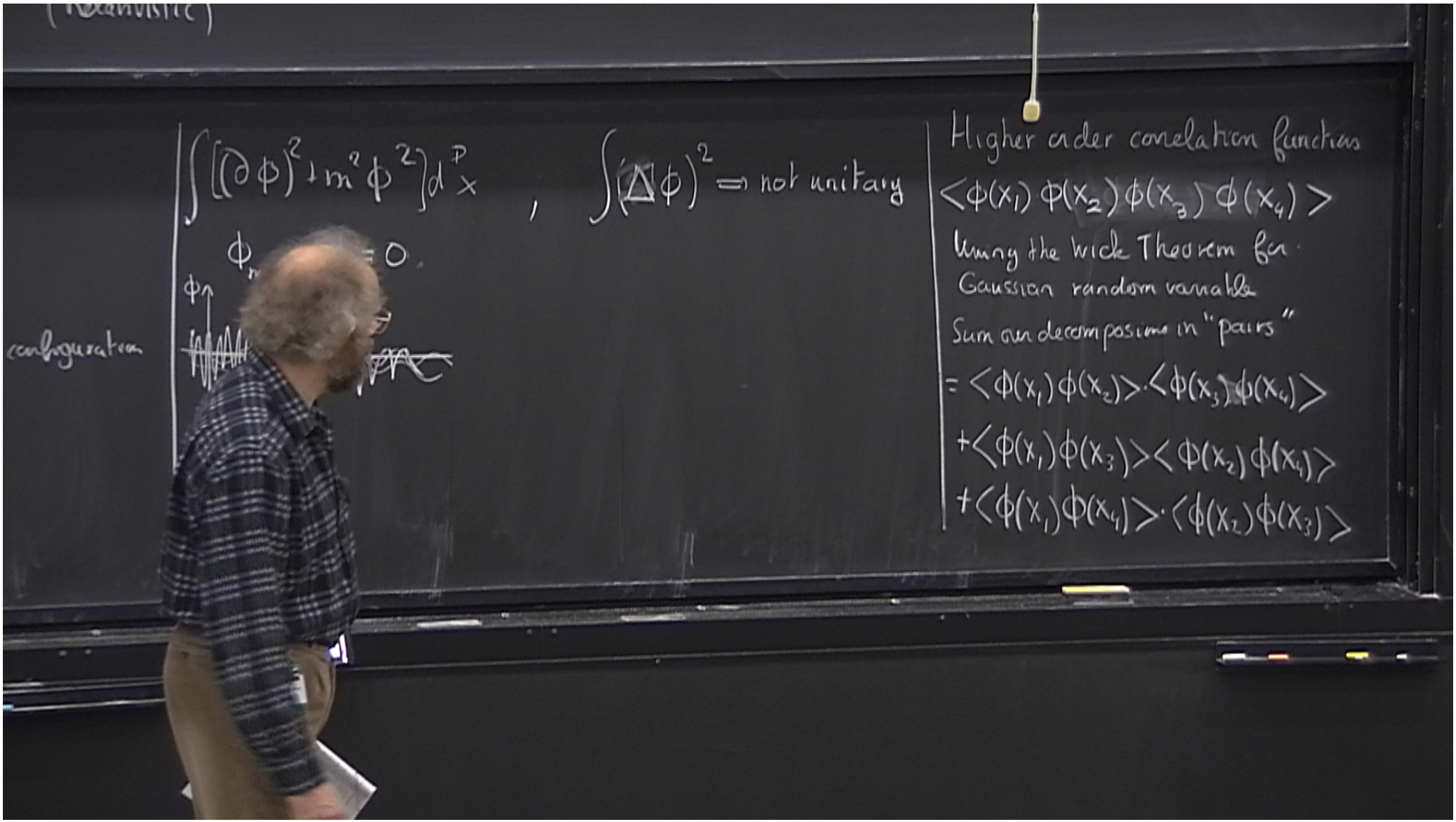


Higher order correlation functions

$$\langle \phi(x_1) \cdots \phi(x_4) \rangle$$


$\int [(\partial\phi)^2 + m^2\phi^2] d^p x$, $\int (\Delta\phi)^2 \Rightarrow$ not unitary
$\phi_{min} = \phi_0 = 0$
ϕ_n
configuration
Higher order correlation functions
$\langle \phi(x_1) \cdots \phi(x_4) \rangle$
using the Wick Theorem for
Gaussian random variable

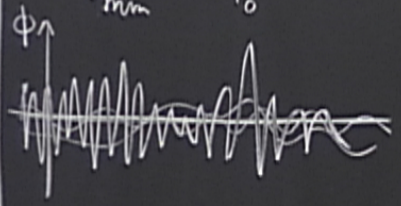

$\int [(\partial\phi)^2 + m^2\phi^2] d^p x$, $\int (\Delta\phi)^2$ not unitary
$\phi_{min} = \phi_0 = 0$
ϕ_n
configuration
Higher order correlation functions
$\langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle$
Using the Wick Theorem for
Gaussian random variable
Sum over decomposition in "pairs"


$\int [(\partial\phi)^2 + m^2\phi^2] d^p x$, $\int (\Delta\phi)^2 \Rightarrow$ not unitary
$\phi_n = 0$
configuration
Higher order correlation functions
$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$
Using the Wick Theorem for Gaussian random variable
Sum on decomposition in "pairs"
$= \langle\phi(x_1)\phi(x_2)\rangle\cdot\langle\phi(x_3)\phi(x_4)\rangle$
$+ \langle\phi(x_1)\phi(x_3)\rangle\langle\phi(x_2)\phi(x_4)\rangle$
$+ \langle\phi(x_1)\phi(x_4)\rangle\cdot\langle\phi(x_2)\phi(x_3)\rangle$

$$\int \left[(\partial\phi)^2 + m^2\phi^2 \right] d^p x \quad , \quad \int (\Delta\phi)^2 \Rightarrow \text{not unitary}$$

$$\phi_{min} = \phi_0 = 0$$

configuration



Valid For the Free field

because the action is quadratic in ϕ

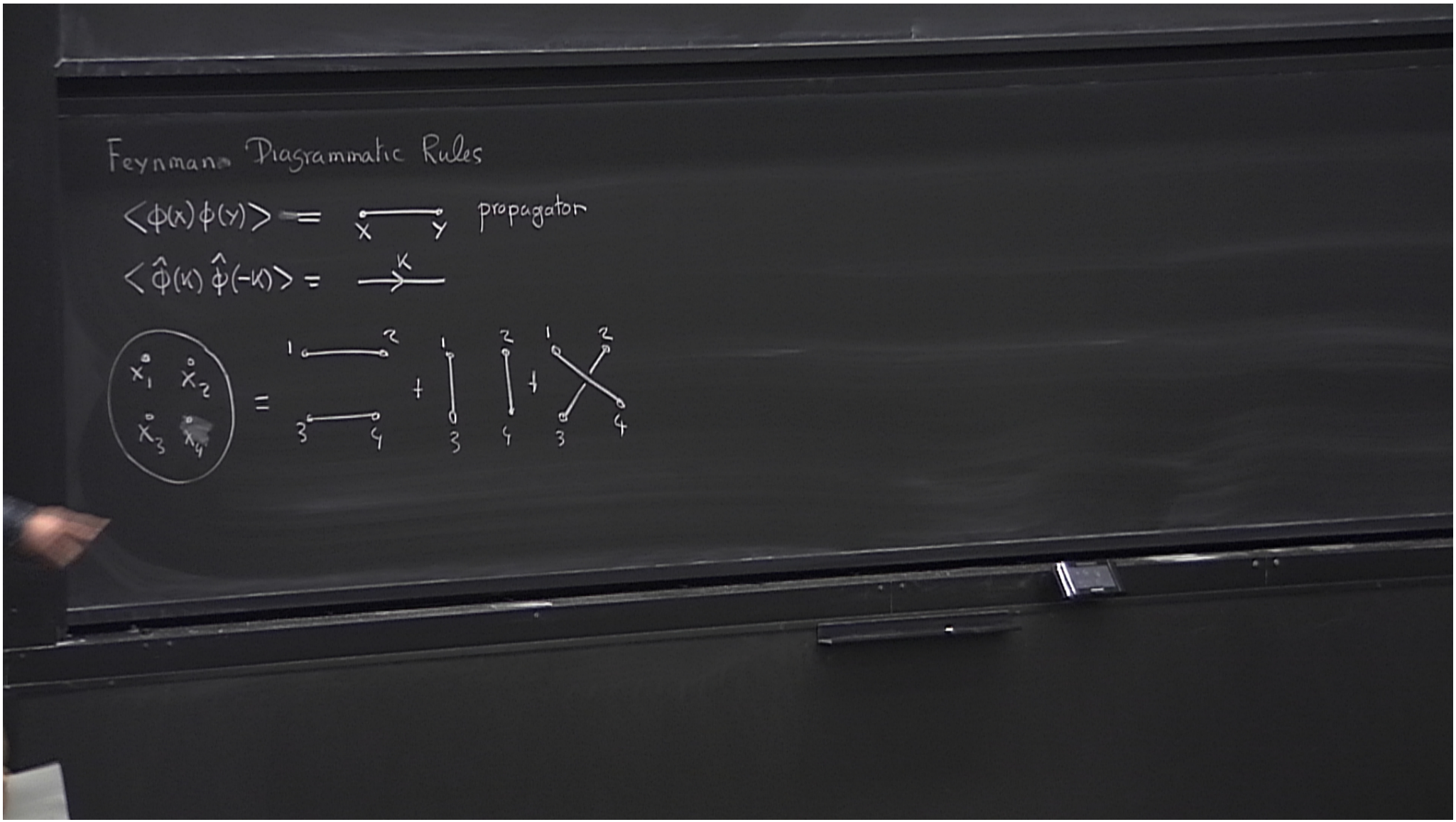
Higher order correlation functions

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4) \rangle$$

using the Wick Theorem for Gaussian random variable

Sum on decomposition in "pairs"

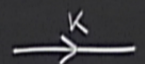
$$= \langle \phi(x_1)\phi(x_2)\rangle \cdot \langle \phi(x_3)\phi(x_4)\rangle$$
$$+ \langle \phi(x_1)\phi(x_3)\rangle \langle \phi(x_2)\phi(x_4)\rangle$$
$$+ \langle \phi(x_1)\phi(x_4)\rangle \cdot \langle \phi(x_2)\phi(x_3)\rangle$$

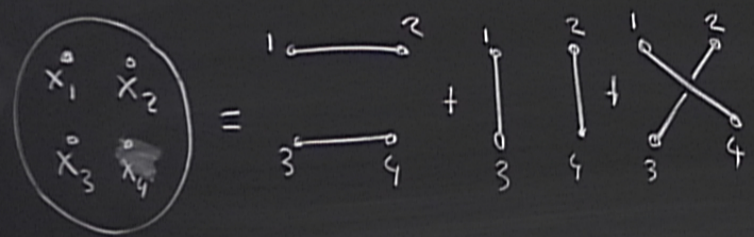

Feynman Diagrammatic Rules
$\langle \phi(x)\phi(y)\rangle =$
x
y
propagator
$\langle \hat{\phi}(k)\hat{\phi}(-k)\rangle =$
k
x_1
x_2
x_3
x_4
=
1
2
3
4
+
1
3
2
4
+
1
2
3
4

Feynman Diagrammatic Rules

$\langle \phi(x)\phi(y)\rangle =$

propagator

$\langle \hat{\phi}(k)\hat{\phi}(-k)\rangle =$ 



Interactions ϕ^4 (Euclidean case)

$$S_0[\phi] = \int \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right] d^4x \quad \text{Free Field}$$

$$S[\phi] = S_0[\phi] + g\, S_{int}[\phi]$$

$$S_{int}[\phi] = \int d^4x \, \frac{1}{4!}\phi^4(x)$$

normalization

coupling constant

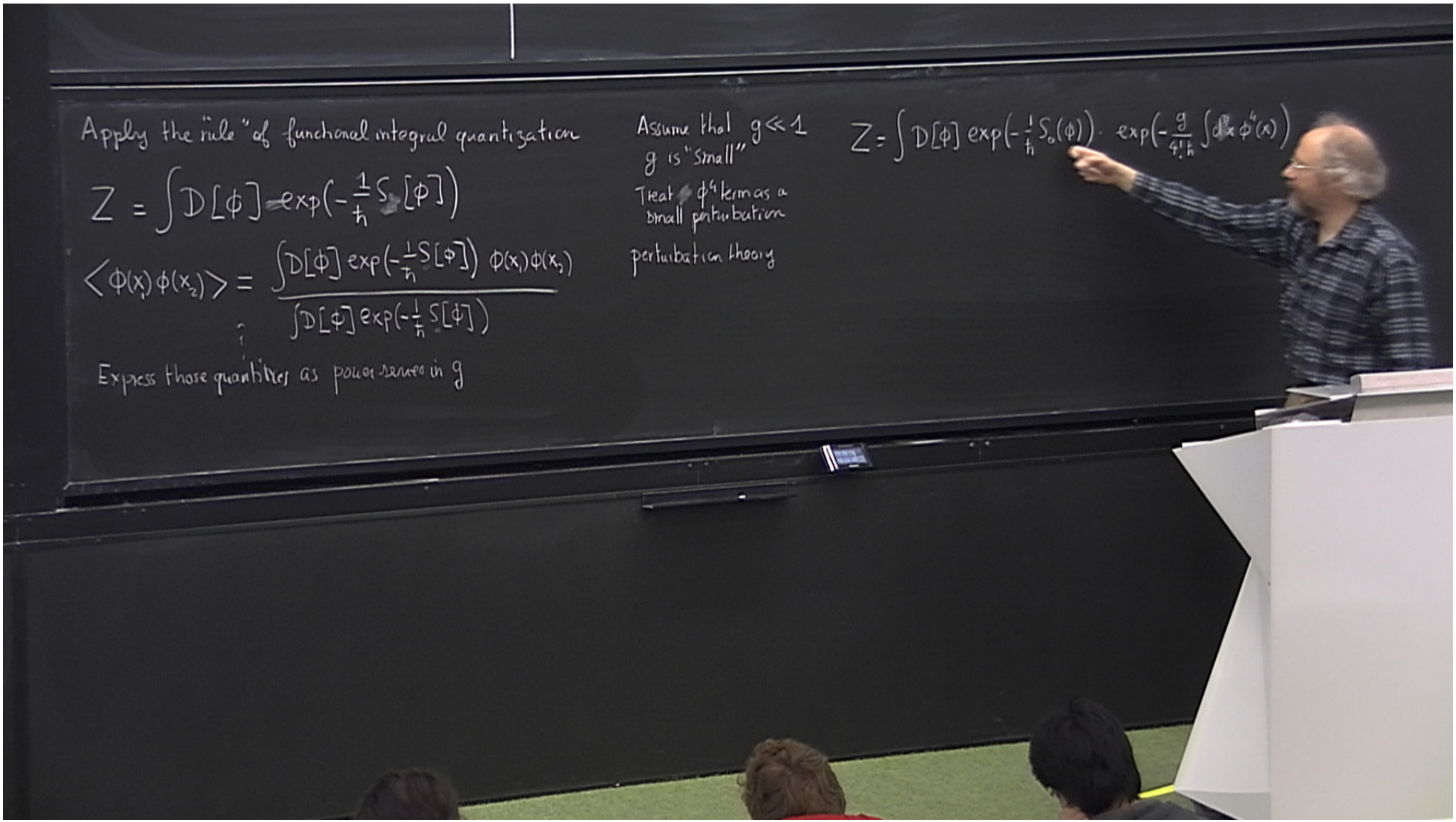
$g \geq 0$

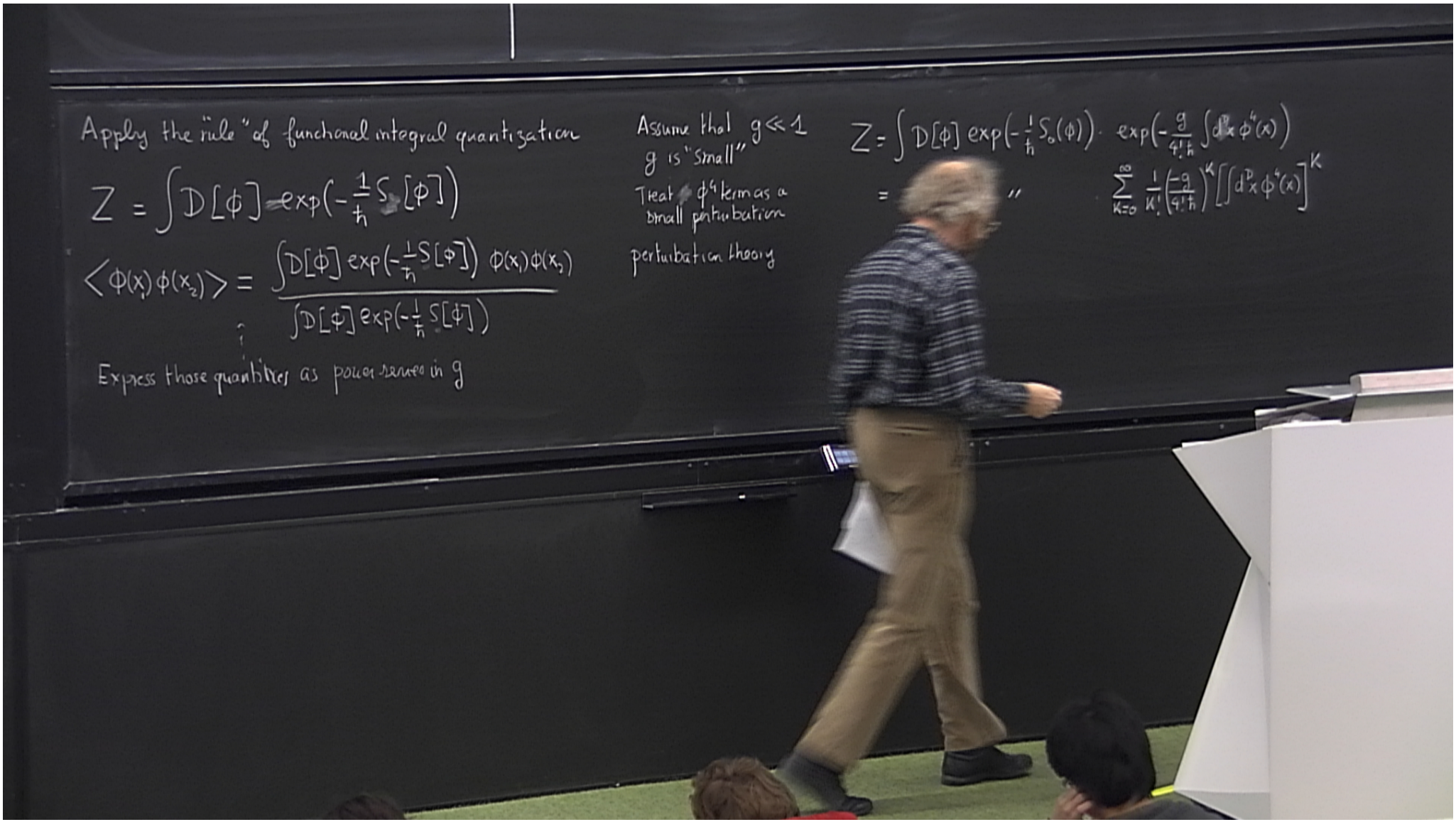
stable vacuum

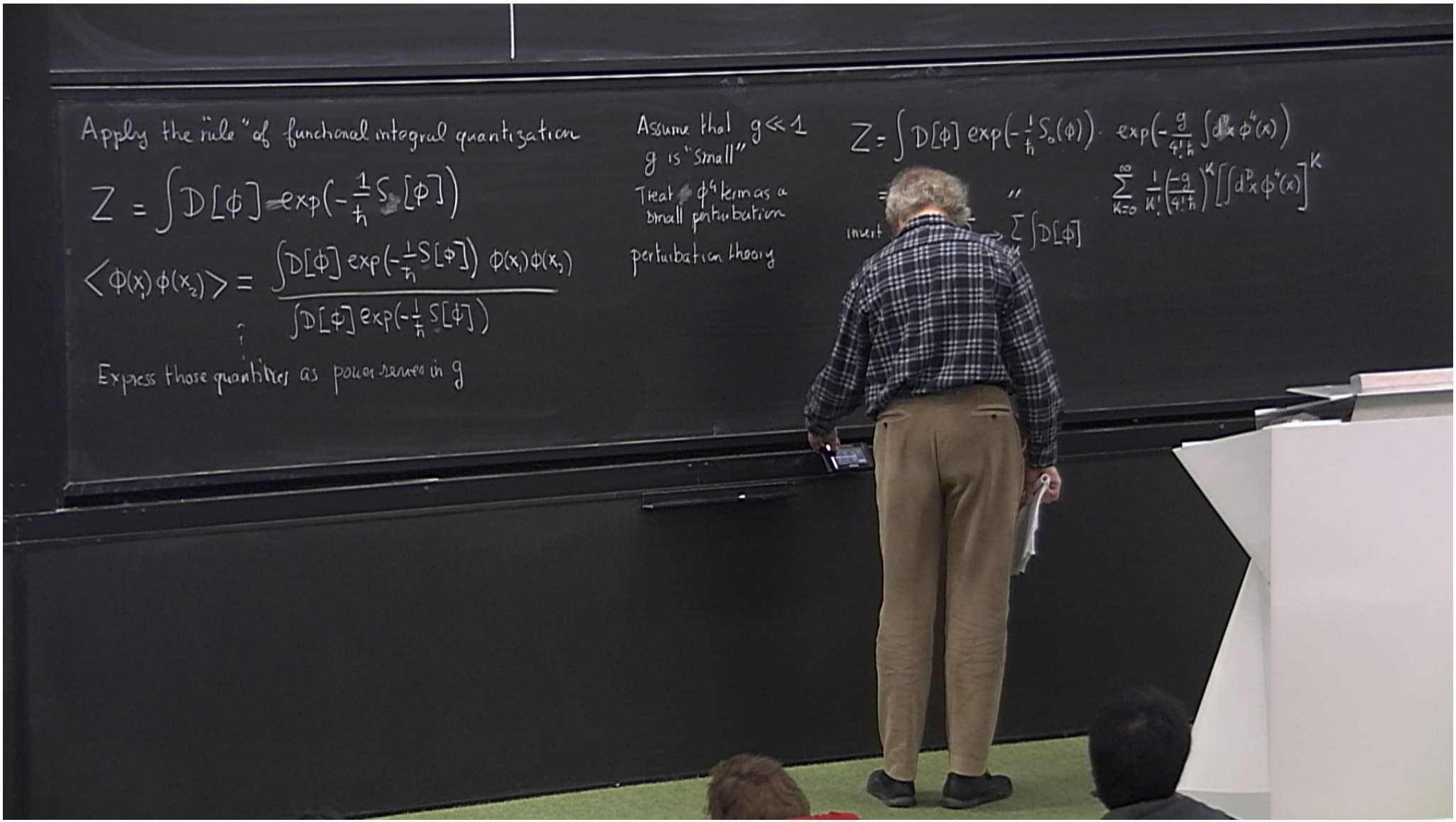
Apply the "rule" of functional integral quantization

$$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$$

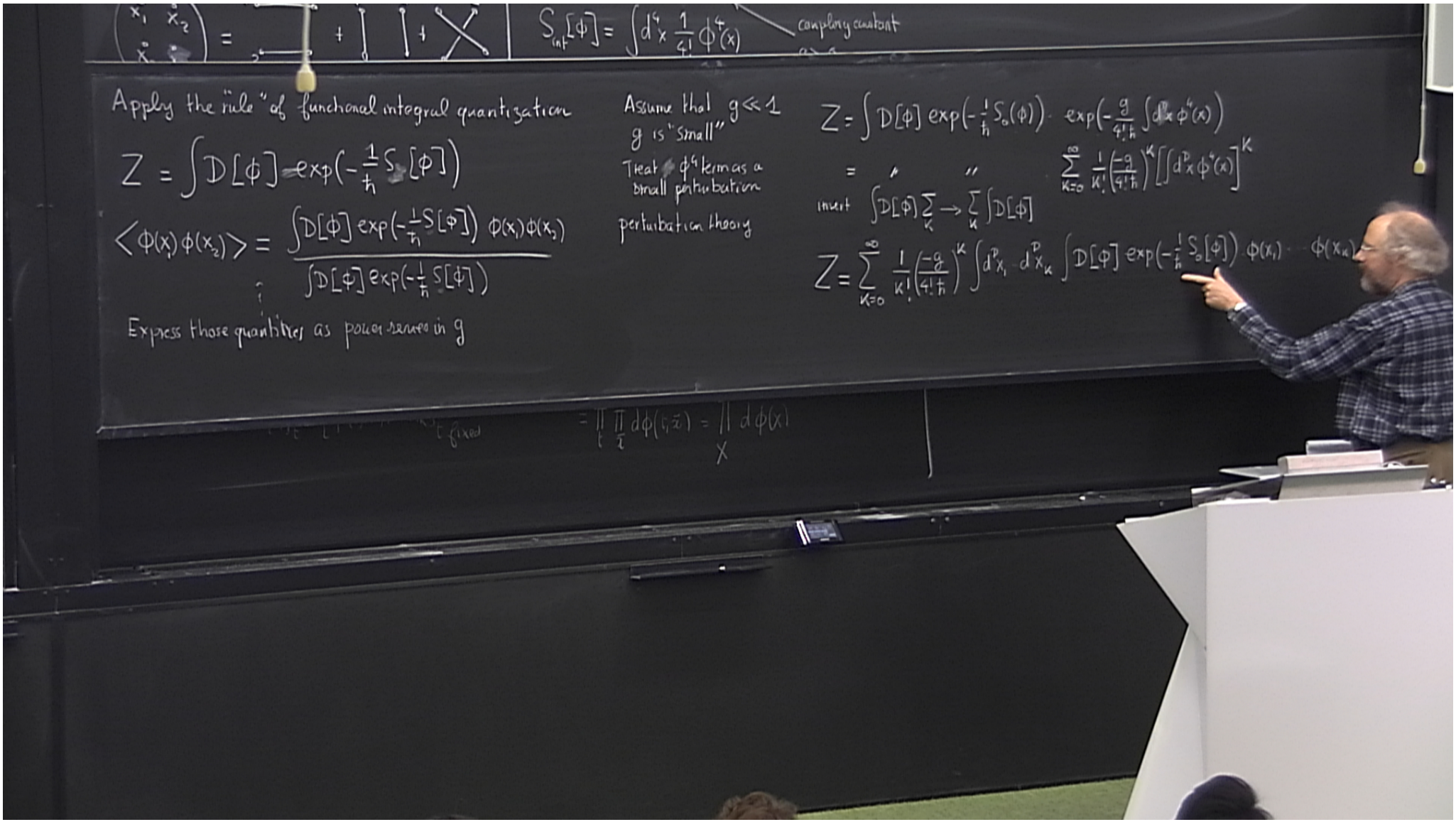
$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1)\phi(x_2)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)}$$


Apply the "rule" of functional integral quantization
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$
$\langle \phi(x_1)\phi(x_2) \rangle = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1)\phi(x_2)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)}$
?
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0(\phi)\right) \cdot \exp\left(-\frac{g}{4!\hbar} \int d^D x\, \phi^4(x)\right)$


Apply the "rule" of functional integral quantization
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1)\phi(x_2)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)}$
?
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0(\phi)\right) \cdot \exp\left(-\frac{g}{4!\hbar} \int d^D x\, \phi^4(x)\right)$
$= \sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\hbar}\right)^K \left[\int d^D x\, \phi^4(x)\right]^K$


Apply the "rule" of functional integral quantization
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1)\phi(x_2)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)}$
?
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0(\phi)\right) \cdot \exp\left(-\frac{g}{4!\hbar}\int d^D x\, \phi^4(x)\right)$
$\sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\hbar}\right)^k \left[\int d^D x\, \phi^4(x)\right]^k$
insert
$\sum_k \int D[\phi]$

Feynman Diagrammatic Rules
Interactions
Free Field
Apply the rule of functional integral quantization
Higher order correlation functions
Express these quantities as power series in g


$S_{int}[\phi] = \int d^4x \frac{1}{4!}\phi^4(x)$
coupling constant
Apply the "rule" of functional integral quantization
$Z = \int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])\, \phi(x_1)\phi(x_2)}{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])}$
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp(-\frac{1}{\hbar}S_0(\phi)) \cdot \exp(-\frac{g}{4!\hbar}\int d^Dx\, \phi^4(x))$
$= \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \left[\int d^Dx\, \phi^4(x)\right]^K$
insert $\int D[\phi] \sum_K \rightarrow \sum_K \int D[\phi]$
$Z = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \int d^Dx_1 \cdots d^Dx_K \int D[\phi] \exp(-\frac{1}{\hbar}S_0[\phi])\, \phi(x_1) \cdots \phi(x_K)$

$x_1 \quad x_2$

$x_1 \quad x_2$

$=$

$$S_{int}[\phi] = \int d^4x \, \frac{1}{4!}\phi^4(x)$$

coupling constant

Apply the "rule" of functional integral quantization

$$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$$

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar}S[\phi]\right)\phi(x_1)\phi(x_2)}{\int D[\phi]\exp\left(-\frac{1}{\hbar}S[\phi]\right)}$$

?

Express those quantities as power series in g

Assume that $g \ll 1$

g is "small"

Treat ϕ^4 term as a small perturbation

perturbation theory

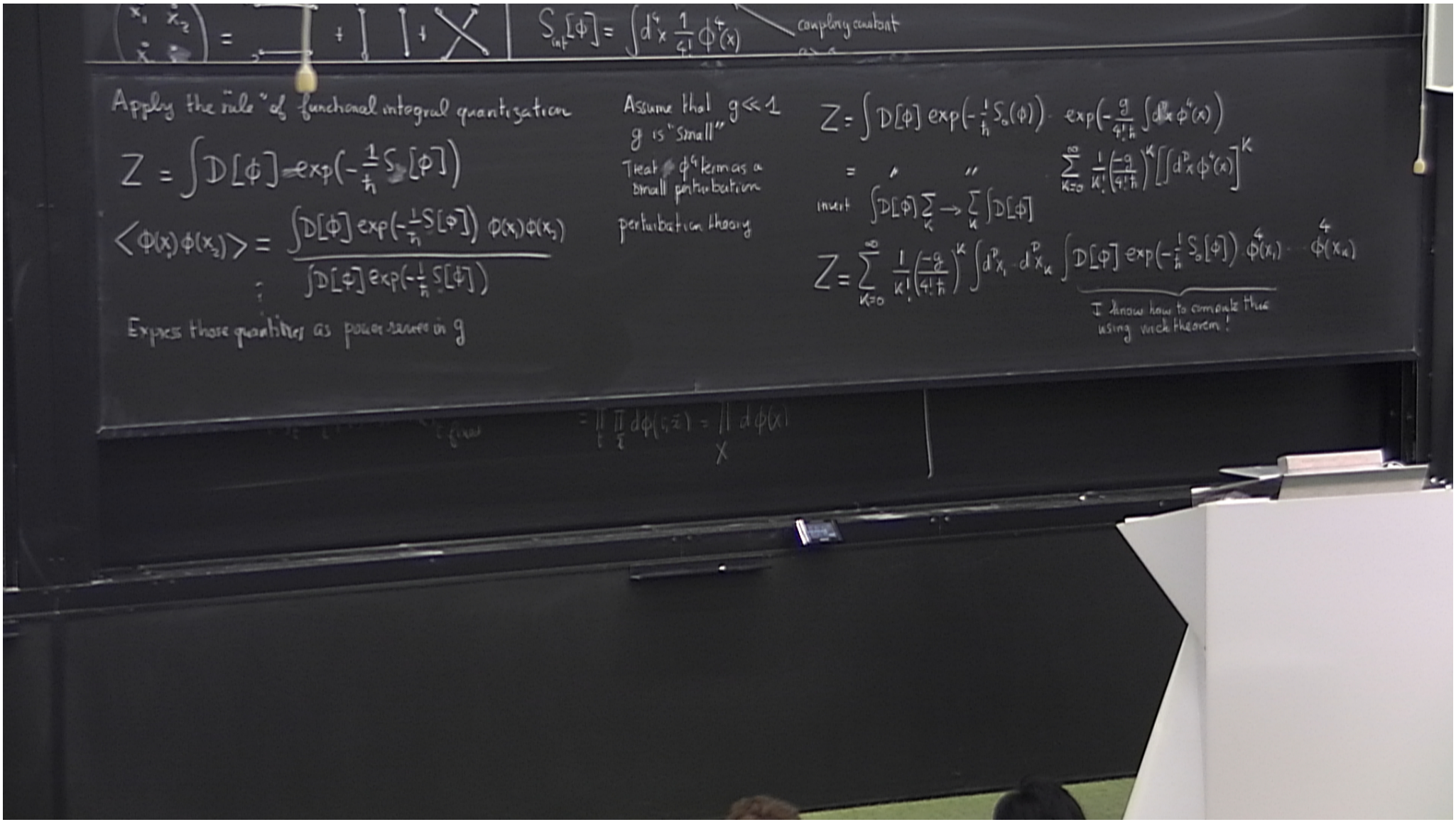
$$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar}S_0(\phi)\right) \cdot \exp\left(-\frac{g}{4!\hbar}\int d^Dx\, \phi^4(x)\right)$$

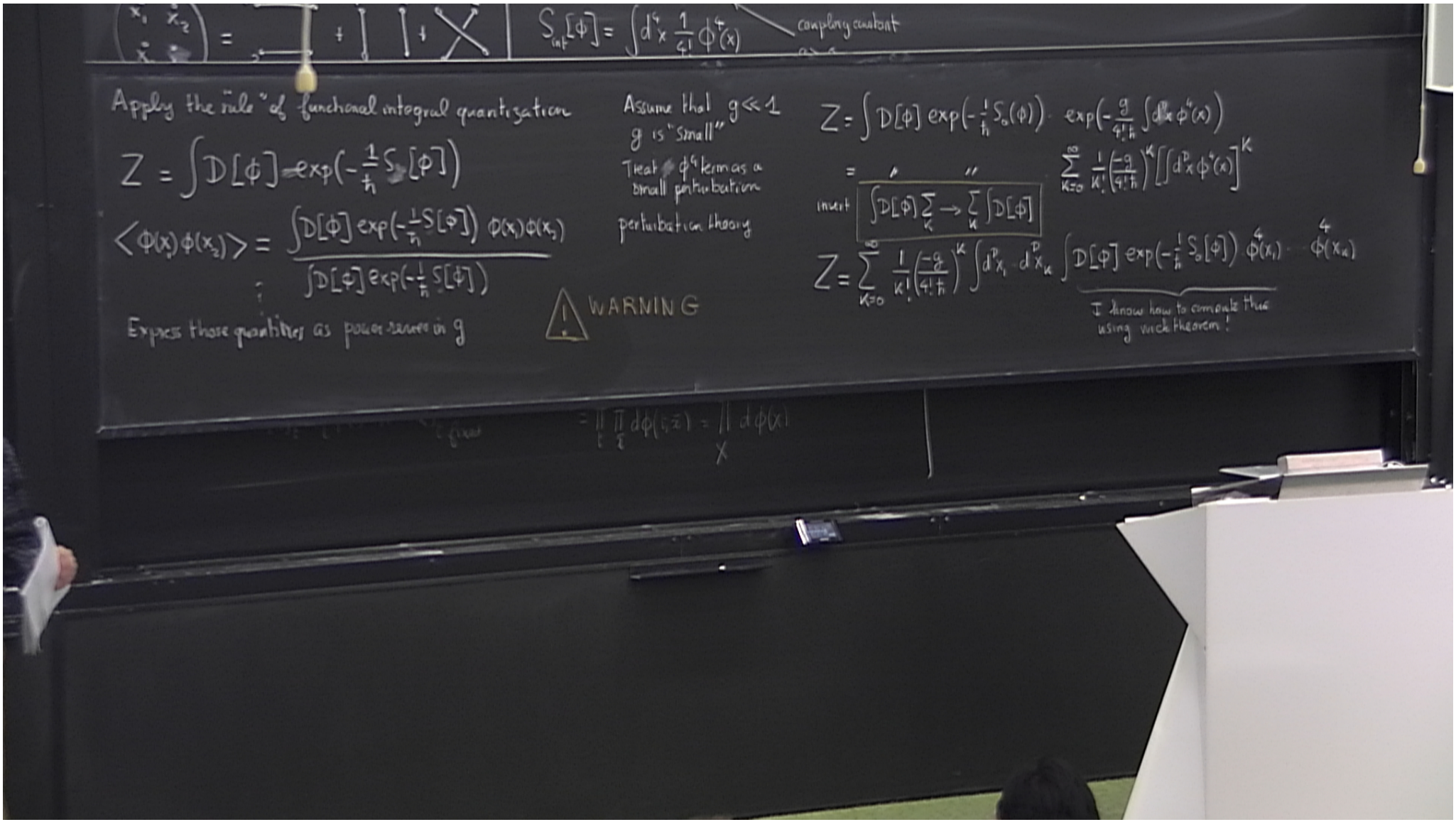
$$= \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \left[\int d^Dx\, \phi^4(x)\right]^K$$

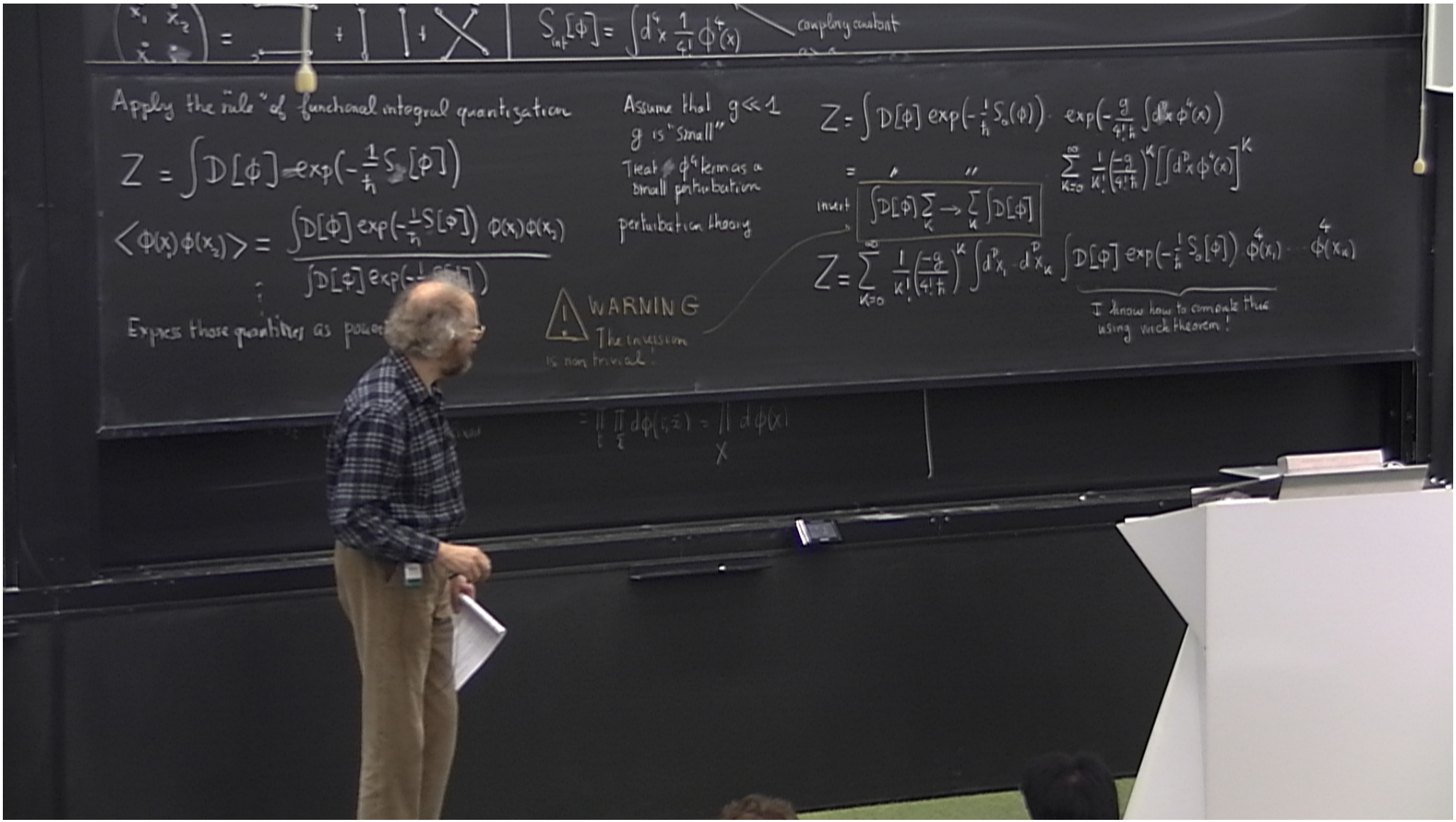
insert $\int D[\phi] \sum_K \rightarrow \sum_K \int D[\phi]$

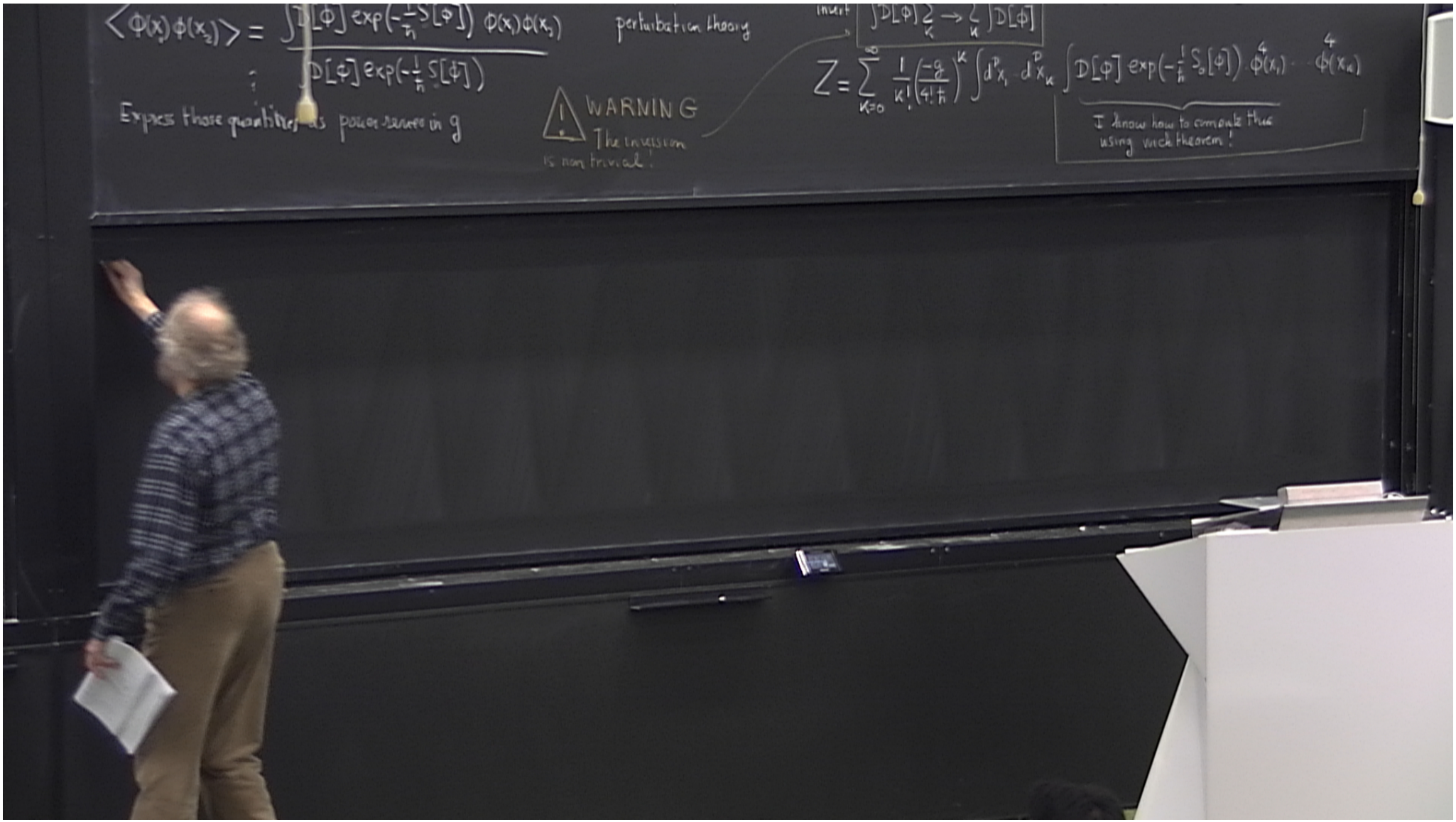
$$Z = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \int d^Dx_1 \cdots d^Dx_K \int D[\phi] \exp\left(-\frac{1}{\hbar}S_0[\phi]\right)\phi(x_1)\cdots\phi(x_K)$$

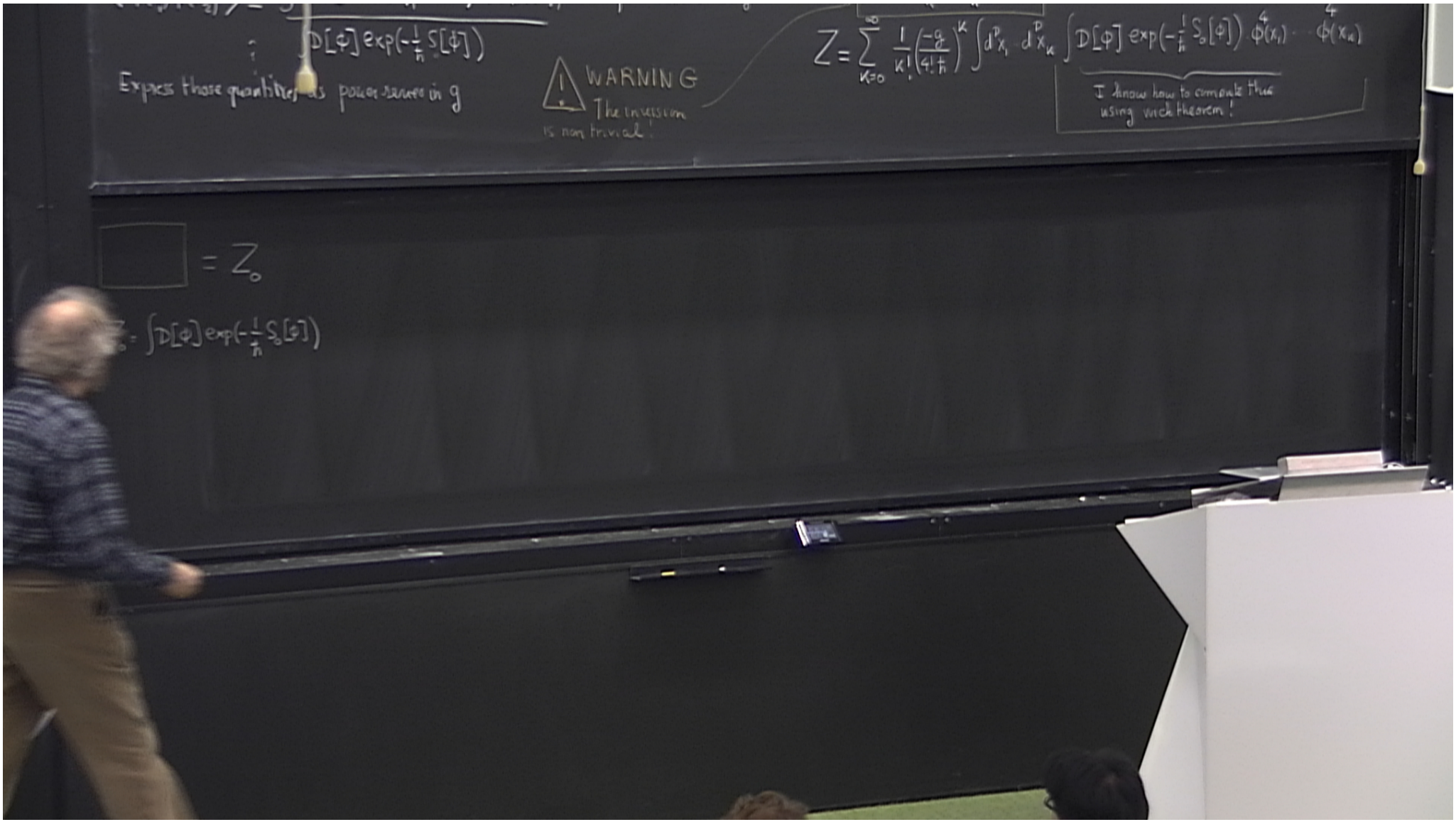
$$= \prod_t \prod_{\vec{x}} d\phi(t,\vec{x}) = \prod_x d\phi(x)$$


$S_{int}[\phi] = \int d^4x \frac{1}{4!}\phi^4(x)$
coupling constant
Apply the "rule" of functional integral quantisation
$Z = \int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])\, \phi(x_1)\phi(x_2)}{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])}$
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp(-\frac{1}{\hbar}S_0(\phi)) \cdot \exp(-\frac{g}{4!\hbar}\int d^Dx\, \phi^4(x))$
$= \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \left[\int d^Dx\, \phi^4(x)\right]^K$
invert $\int D[\phi] \sum_K \to \sum_K \int D[\phi]$
$Z = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \int d^Dx_1 \cdots d^Dx_K \int D[\phi] \exp(-\frac{1}{\hbar}S_0[\phi])\, \phi^4(x_1) \cdots \phi^4(x_K)$
I know how to compute this using wick theorem!


$S_{int}[\phi] = \int d^4x \frac{1}{4!}\phi^4(x)$
coupling constant
Apply the "rule" of functional integral quantisation
$Z = \int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])\, \phi(x_1)\phi(x_2)}{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])}$
Express those quantities as power series in g
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
WARNING
$Z = \int D[\phi] \exp(-\frac{1}{\hbar}S_0(\phi)) \cdot \exp(-\frac{g}{4!\hbar}\int d^Dx\, \phi^4(x))$
$= \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \left[\int d^Dx\, \phi^4(x)\right]^K$
insert $\boxed{\int D[\phi] \sum_K \to \sum_K \int D[\phi]}$
$Z = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \int d^Dx_1 \cdots d^Dx_K \int D[\phi] \exp(-\frac{1}{\hbar}S_0[\phi])\, \phi^4(x_1) \cdots \phi^4(x_K)$
I know how to compute this using wick theorem!


$S_{int}[\phi] = \int d^4x \frac{1}{4!}\phi^4(x)$
coupling constant
Apply the "rule" of functional integral quantisation
$Z = \int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])$
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])\, \phi(x_1)\phi(x_2)}{\int D[\phi] \exp(-\frac{1}{\hbar}S[\phi])}$
Express those quantities as power
Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
WARNING
The inversion is non trivial.
$Z = \int D[\phi] \exp(-\frac{1}{\hbar} S_0(\phi)) \cdot \exp(-\frac{g}{4!\hbar}\int d^Dx\, \phi^4(x))$
$= \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \left[\int d^Dx\, \phi^4(x)\right]^K$
invert $\boxed{\int D[\phi] \sum_K \to \sum_K \int D[\phi]}$
$Z = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{-g}{4!\hbar}\right)^K \int d^Dx_1 \cdots d^Dx_K \int D[\phi] \exp(-\frac{1}{\hbar}S_0[\phi])\, \phi^4(x_1) \cdots \phi^4(x_K)$
I know how to compute this using wick theorem!


$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])\, \phi(x_1)\phi(x_2)}{\int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])}$
perturbation theory
Express those quantities as power series in g
WARNING
The inversion is non trivial!
insert $\boxed{\int D[\phi] \sum_k \to \sum_k \int D[\phi]}$
$Z = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\hbar}\right)^k \int d^D x_1 \cdots d^D x_k \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \cdot \phi^4(x_1) \cdots \phi^4(x_k)$
I know how to compute this using wick theorem!


$\int D[\phi] \exp(-\frac{1}{\hbar} S[\phi])$
Express those quantities as power series in g
WARNING
The inversion is non trivial!
$Z = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{-g}{4!\hbar}\right)^k \int d^D x_1 \cdots d^D x_k \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \phi^4(x_1) \cdots \phi^4(x_k)$
I know how to compute this using Wick theorem!
$= Z_0$
$Z_0 = \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$

$$\int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$$

Express those quantities as power series in g

WARNING

The inversion is non trivial!

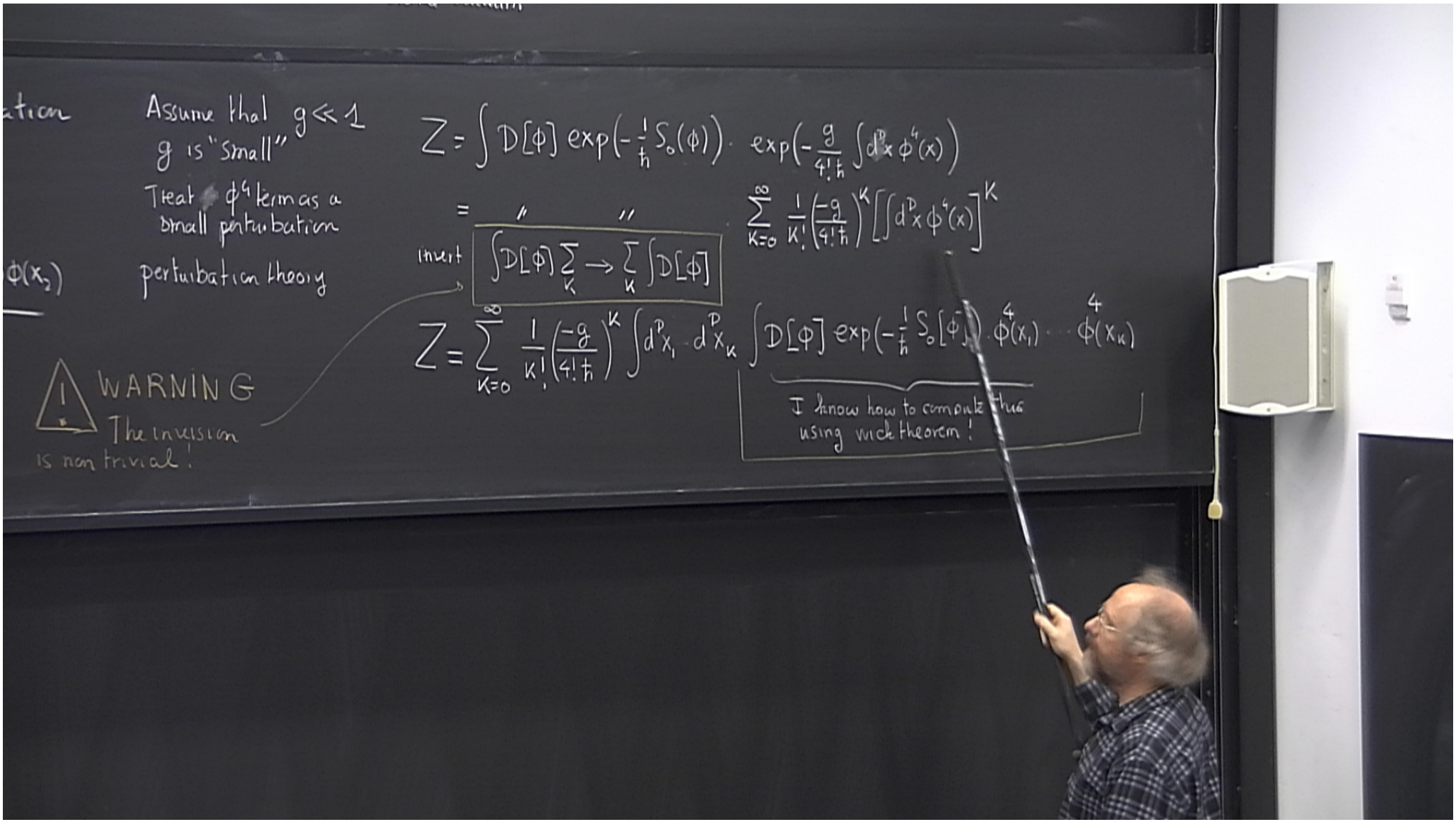
$$Z = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\,\hbar}\right)^k \int d^D x_1 \cdots d^D x_k \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right) \cdot \phi^4(x_1) \cdots \phi^4(x_k)$$

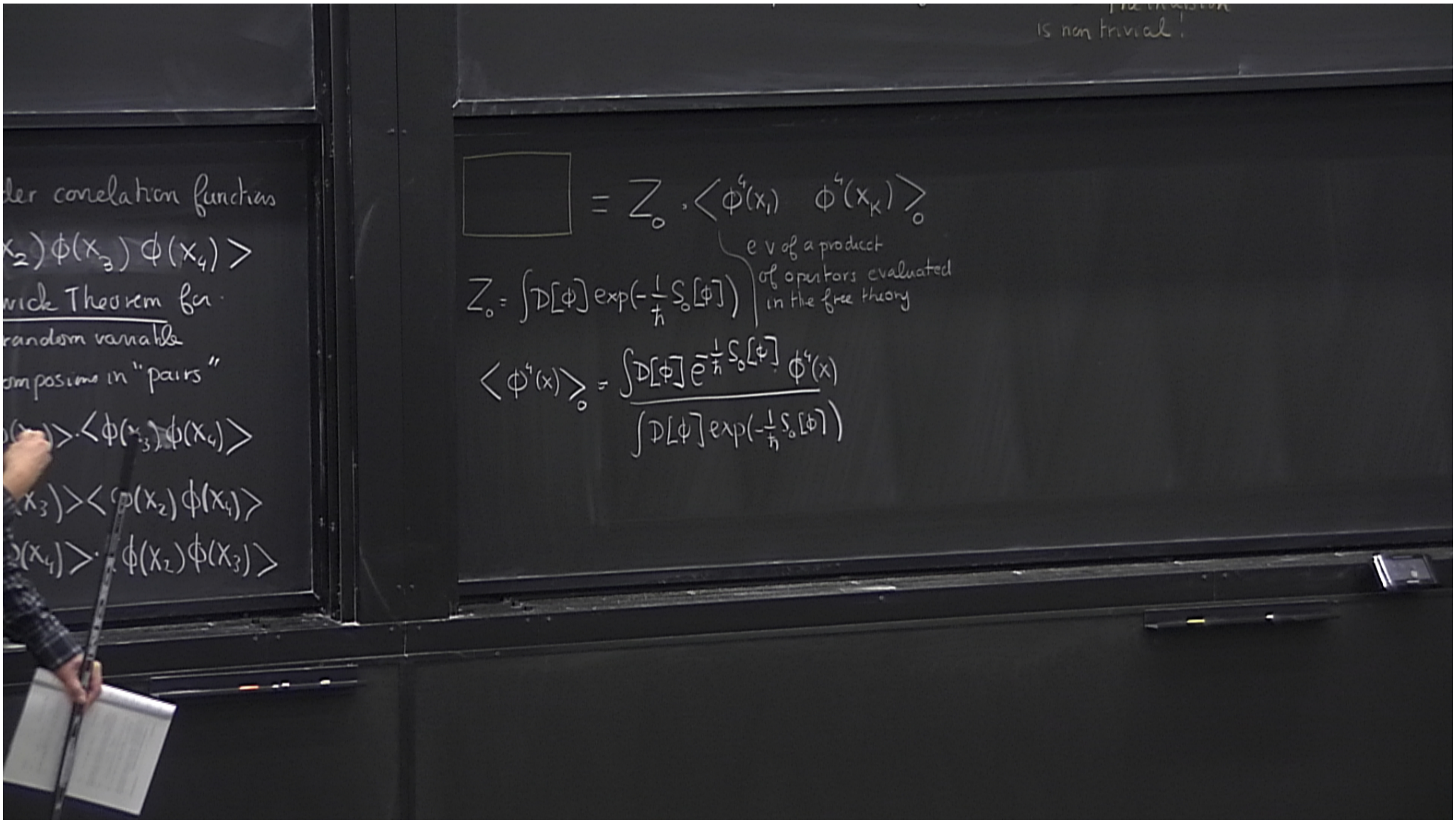
I know how to compute this using Wick theorem!

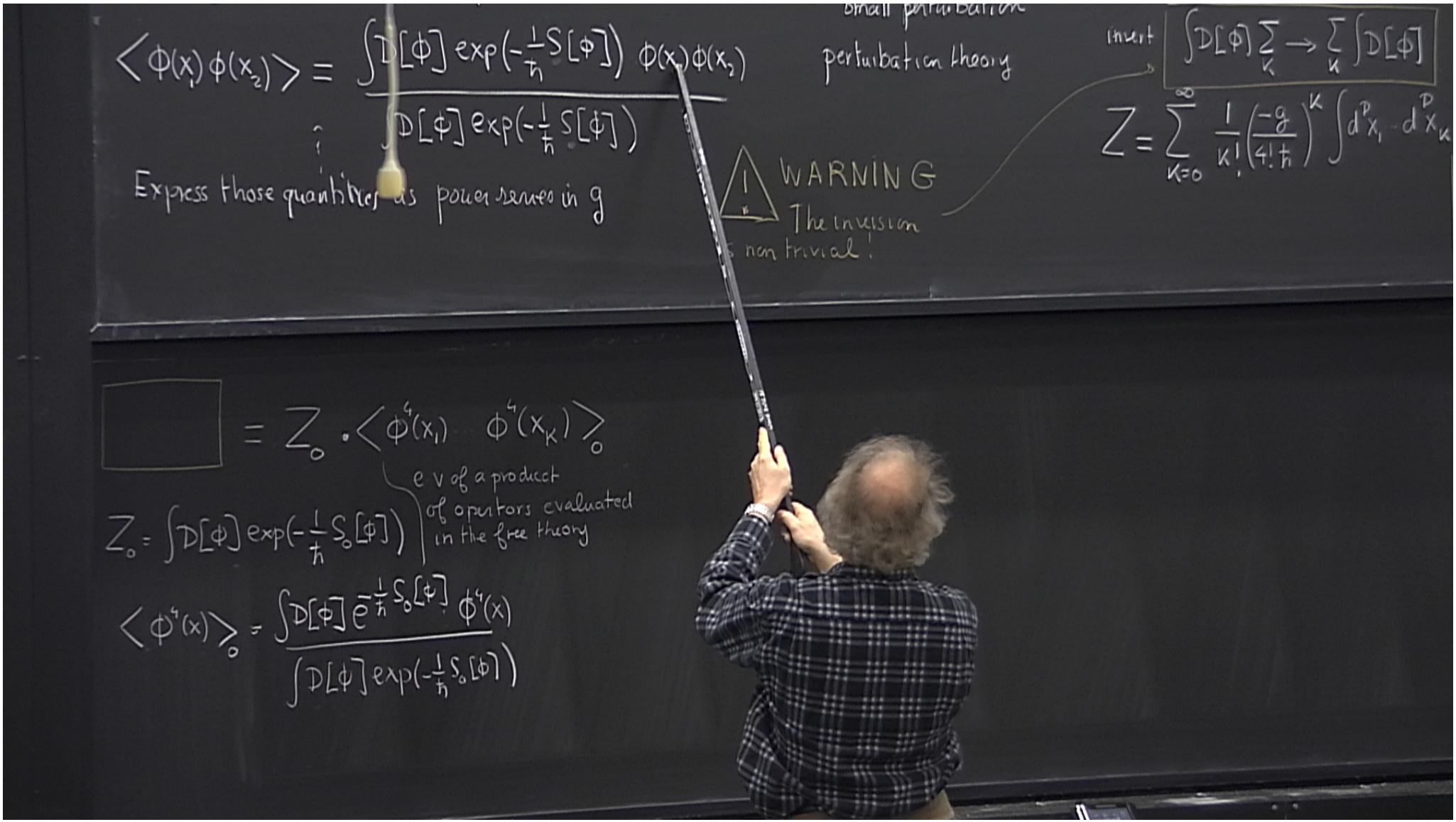
$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \cdots \phi^4(x_k) \rangle_0$$

e.v. of a product of operators evaluated in the free theory

$$Z_0 = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$


Assume that $g \ll 1$
g is "small"
Treat ϕ^4 term as a small perturbation
perturbation theory
$Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0(\phi)\right) \cdot \exp\left(-\frac{g}{4!\hbar} \int d^D x\, \phi^4(x)\right)$
$= \sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\hbar}\right)^K \left[\int d^D x\, \phi^4(x)\right]^K$
invert $\boxed{\int D[\phi] \sum_K \rightarrow \sum_K \int D[\phi]}$
$Z = \sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\hbar}\right)^K \int d^D x_1 \cdots d^D x_K \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right) \cdot \phi^4(x_1) \cdots \phi^4(x_K)$
I know how to compute this using wick theorem!
WARNING
The inversion is non trivial!


is non trivial!
ler correlation functions
$x_2)\phi(x_3)\phi(x_4)\rangle$
wick Theorem for
random variable
omposing in "pairs"
$\phi(x_3)\phi(x_4)\rangle$
$\phi(x_2)\phi(x_4)\rangle$
$\phi(x_2)\phi(x_3)\rangle$
$= Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K)\rangle_0$
e v of a product
of operators evaluated
in the free theory
$Z_0 = \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$
$\langle \phi^4(x)\rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp(-\frac{1}{\hbar}S_0[\phi])}$


$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi]\exp(-\frac{1}{\hbar}S[\phi])\,\phi(x_1)\phi(x_2)}{\int D[\phi]\exp(-\frac{1}{\hbar}S[\phi])}$
small perturbation
perturbation theory
invert $\int D[\phi]\sum_k \rightarrow \sum_k \int D[\phi]$
$Z = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\,\hbar}\right)^k \int d^D x_1 \cdots d^D x_k$
Express those quantities as power series in g
WARNING
The inversion is non trivial!
$= Z_0 \cdot \langle \phi^4(x_1) \cdots \phi^4(x_k)\rangle_0$
e v of a product of operators evaluated in the free theory
$Z_0 = \int D[\phi]\exp(-\frac{1}{\hbar}S_0[\phi])$
$\langle \phi^4(x)\rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_0[\phi]}\,\phi^4(x)}{\int D[\phi]\exp(-\frac{1}{\hbar}S_0[\phi])}$

small perturbation

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi]\exp\left(-\frac{1}{\hbar}S[\phi]\right)\phi(x_1)\phi(x_2)}{\int D[\phi]\exp\left(-\frac{1}{\hbar}S[\phi]\right)}$$

perturbation theory

Express those quantities as power series in g

WARNING

The inversion is non trivial!

invert $\boxed{\int D[\phi]\sum_k \rightarrow \sum_k \int D[\phi]}$

$$Z = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\,\hbar}\right)^k \int d^D x_1 \cdots d^D x_k \int D$$

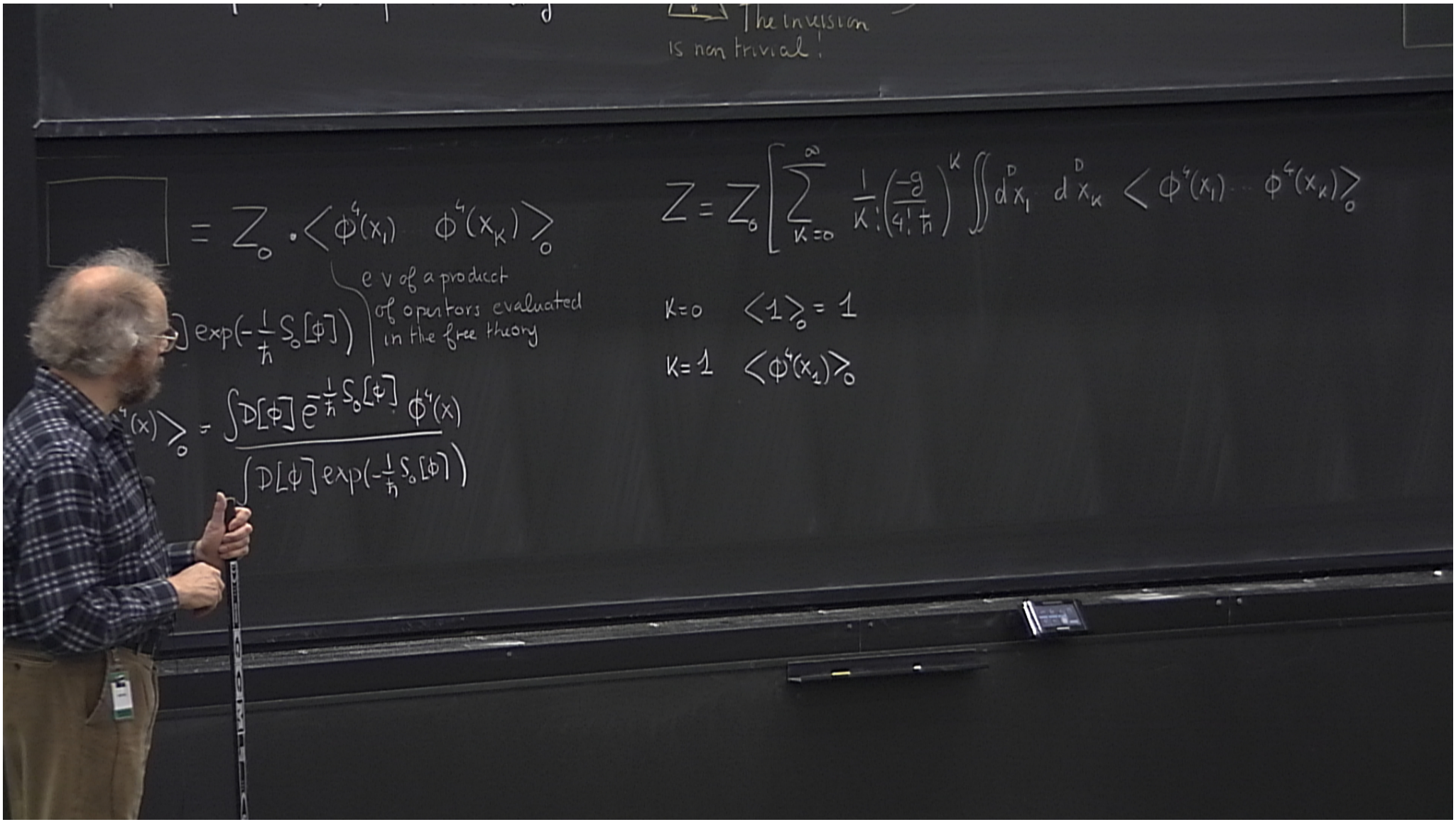
$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_k)\rangle_0$$

e v of a product of operators evaluated in the free theory

$$Z_0 = \int D[\phi]\exp\left(-\frac{1}{\hbar}S_0[\phi]\right)$$

$$\langle \phi^4(x)\rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_0[\phi]}\,\phi^4(x)}{\int D[\phi]\exp\left(-\frac{1}{\hbar}S_0[\phi]\right)}$$

$$Z = Z_0\left[\sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{4!\,\hbar}\right)^k \int\!\!\int d^D x_1 \cdots d^D x_k \langle \phi^4(x_1) \cdots \phi^4(x_k)\rangle_0\right.$$


The inversion
is non trivial!
$= Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$
e v of a product
of operators evaluated
in the free theory
$\exp(-\frac{1}{\hbar} S_0[\phi])$
$\phi^4(x) \rangle_0 = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_0[\phi]} \phi^4(x)}{\int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])}$
$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4! \hbar} \right)^K \int\int d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$
$K=0 \quad \langle 1 \rangle_0 = 1$
$K=1 \quad \langle \phi^4(x_1) \rangle_0$

The inversion is non trivial!

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

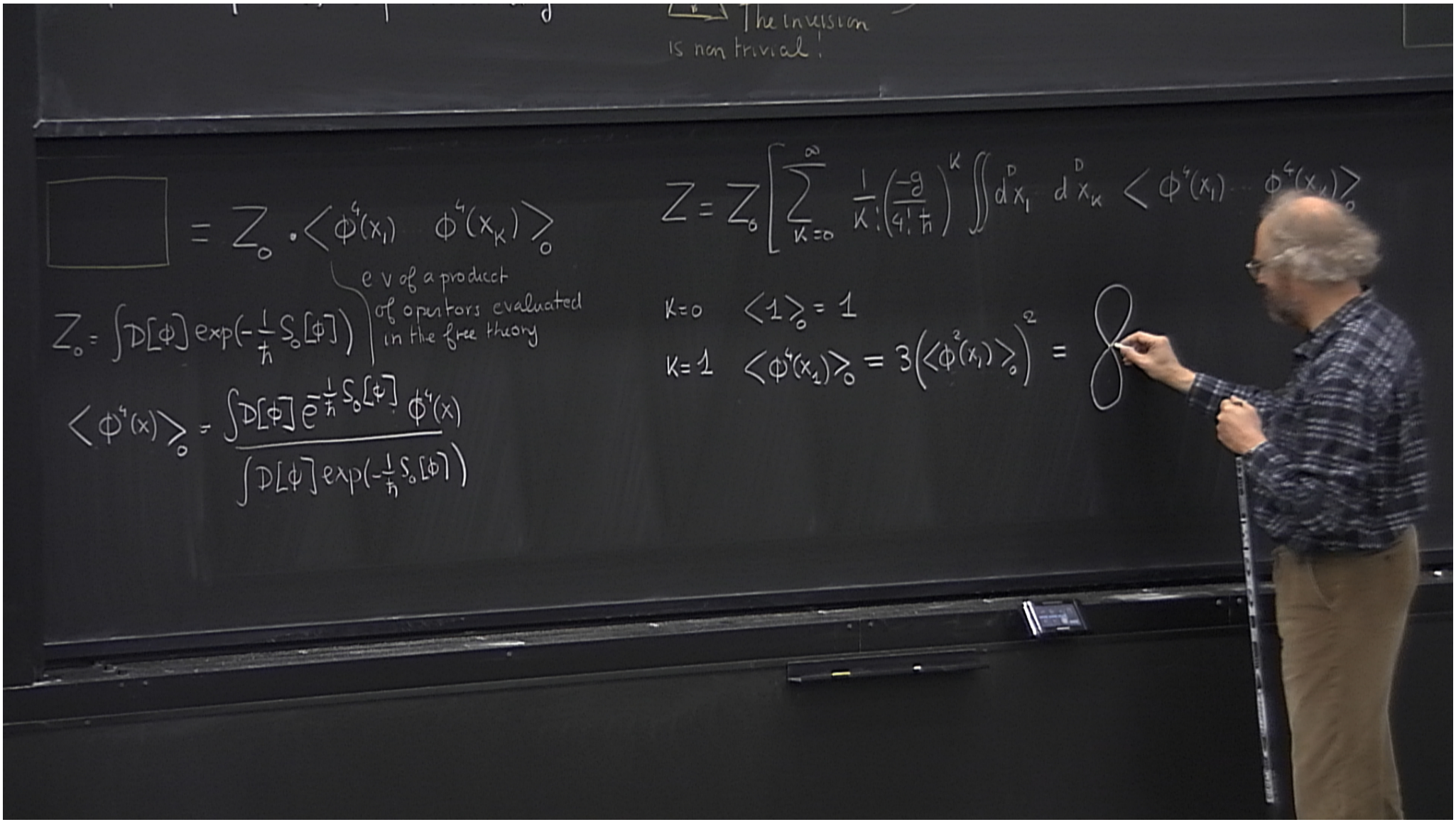
$$\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar}\right)^K \iint d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3\left(\langle \phi^2(x_1) \rangle_0\right)^2$


The inversion
is non trivial!
$= Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$
e v of a product
of operators evaluated
in the free theory
$Z_0 = \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$
$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_0[\phi]} \phi^4(x)}{\int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])}$
$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4! \hbar} \right)^K \int d^D x_1 \; d^D x_K \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$
$K=0 \quad \langle 1 \rangle_0 = 1$
$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 =$

The inversion is non trivial!

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

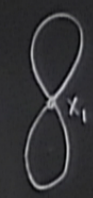
$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

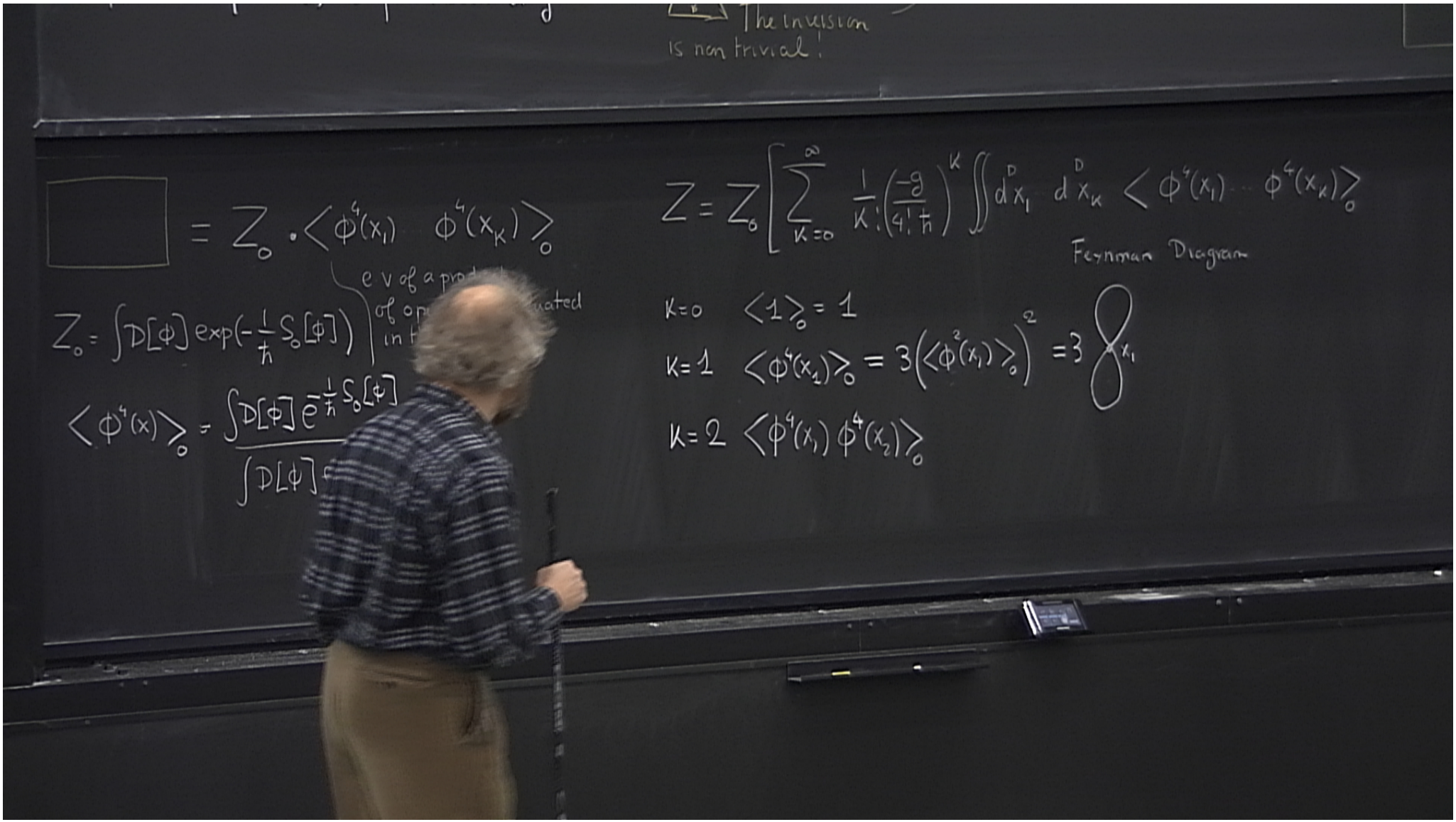
$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar} \right)^K \int\!\!\int d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3\left(\langle \phi^2(x_1) \rangle_0 \right)^2 =$ 


The inversion
is non trivial!
$= Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$
$Z_0 = \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$
$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_0[\phi]}}{\int D[\phi]}$
$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4! \hbar} \right)^K \int\int d^D x_1 \; d^D x_K \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$
Feynman Diagram
$K=0 \quad \langle 1 \rangle_0 = 1$
$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$
x_1
$K=2 \quad \langle \phi^4(x_1) \phi^4(x_2) \rangle_0$

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

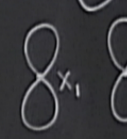
$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

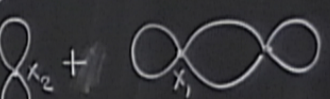
$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar}\right)^K \int d^D x_1 \cdots d^D x_K \, \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$

$K=2 \quad \langle \phi^4(x_1)\, \phi^4(x_2) \rangle_0 = g$ 

$+$ 

t fixed

$$\prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_X d\phi(X)$$

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

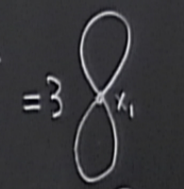
$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

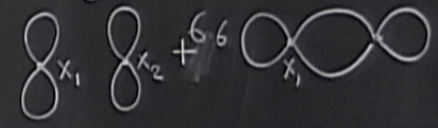
$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar}\right)^K \int d^D x_1 \cdots d^D x_K \, \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

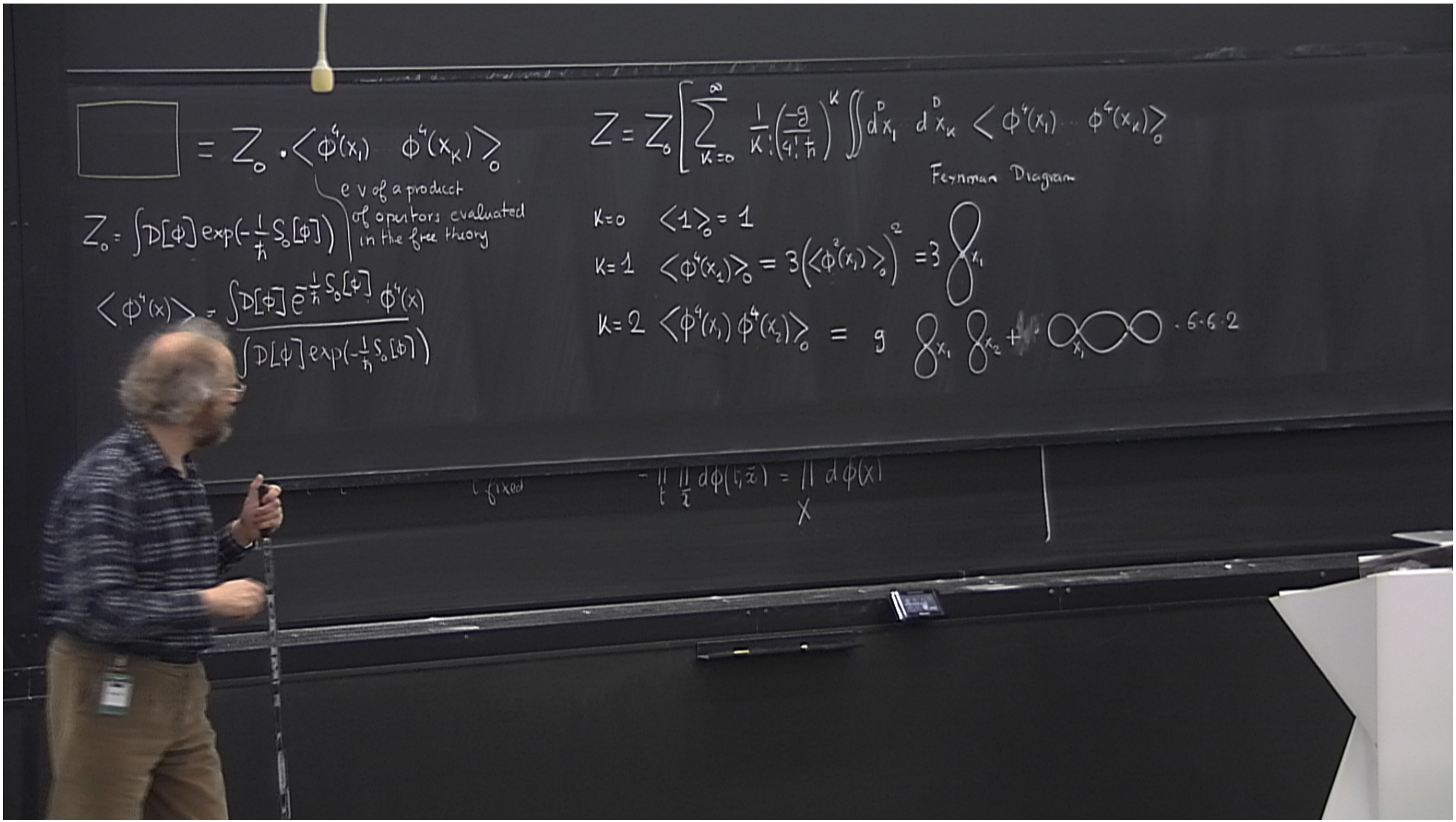
$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$ 

$K=2 \quad \langle \phi^4(x_1)\, \phi^4(x_2) \rangle_0 = g$ 

t fixed

$$\prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_X d\phi(X)$$


$= Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$
e v of a product of operators evaluated in the free theory
$Z_0 = \int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$
$\langle \phi^4(x) \rangle = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_0[\phi]} \phi^4(x)}{\int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])}$
$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4! \hbar} \right)^K \int d^D x_1 \; d^D x_K \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$
Feynman Diagram
$K=0 \quad \langle 1 \rangle_0 = 1$
$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$
x_1
$K=2 \quad \langle \phi^4(x_1) \phi^4(x_2) \rangle_0 = g$
x_1 x_2
x_1
$\cdot 6 \cdot 6 \cdot 2$
$\prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_X d\phi(X)$

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

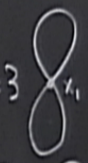
$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

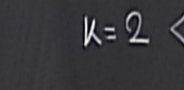
$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar}\right)^K \iint d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$ 

$K=2 \quad \langle \phi^4(x_1)\, \phi^4(x_2) \rangle_0 = \underset{3 \cdot 3}{\overset{g}{\shortparallel}}$

$+$ 

$\cdot\, \underset{72}{\overset{6 \cdot 6 \cdot 2}{\shortparallel}} +$

$\underset{24}{\overset{4!}{\shortparallel}}$

t fixed

$$\prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_x d\phi(x)$$

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

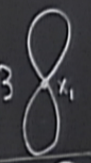
$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

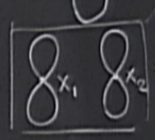
$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

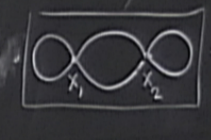
$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar} \right)^K \int\!\!\int d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

$K=0 \quad \langle 1 \rangle_0 = 1$

$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$ 

$K=2 \quad \langle \phi^4(x_1)\, \phi^4(x_2) \rangle_0 = \underset{3 \cdot 3}{\overset{9}{\shortparallel}}$ 

$+$ 

$\cdot\, 6 \cdot 6 \cdot 2 \underset{72}{\shortparallel} +$

$\underset{24}{\overset{4!}{\shortparallel}}$

t fixed $\quad \prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_x d\phi(x)$

$$\boxed{} = Z_0 \cdot \langle \phi^4(x_1) \quad \phi^4(x_K) \rangle_0$$

e v of a product of operators evaluated in the free theory

$$Z_0 = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$$

$$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)}$$

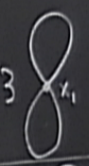
$\boxed{\text{Feynman diagrammatic rules}}$

$$Z = Z_0 \left[\sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!\,\hbar} \right)^K \iint d^D x_1 \; d^D x_K \; \langle \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0 \right.$$

Feynman Diagram

$K=0 \quad \langle 1 \rangle_0 = 1$

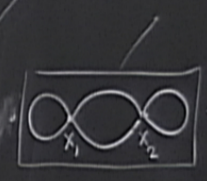
$K=1 \quad \langle \phi^4(x_1) \rangle_0 = 3 \left(\langle \phi^2(x_1) \rangle_0 \right)^2 = 3$



$K=2 \quad \langle \phi^4(x_1)\, \phi^4(x_2) \rangle_0 = \underset{3 \cdot 3}{\overset{9}{\shortparallel}}$

$[G(0)]^4$

$+$



$[G(0)]^2 [G(x_1 - x_2)]^2$

$\cdot\, 6 \cdot 6 \cdot 2 = 72$

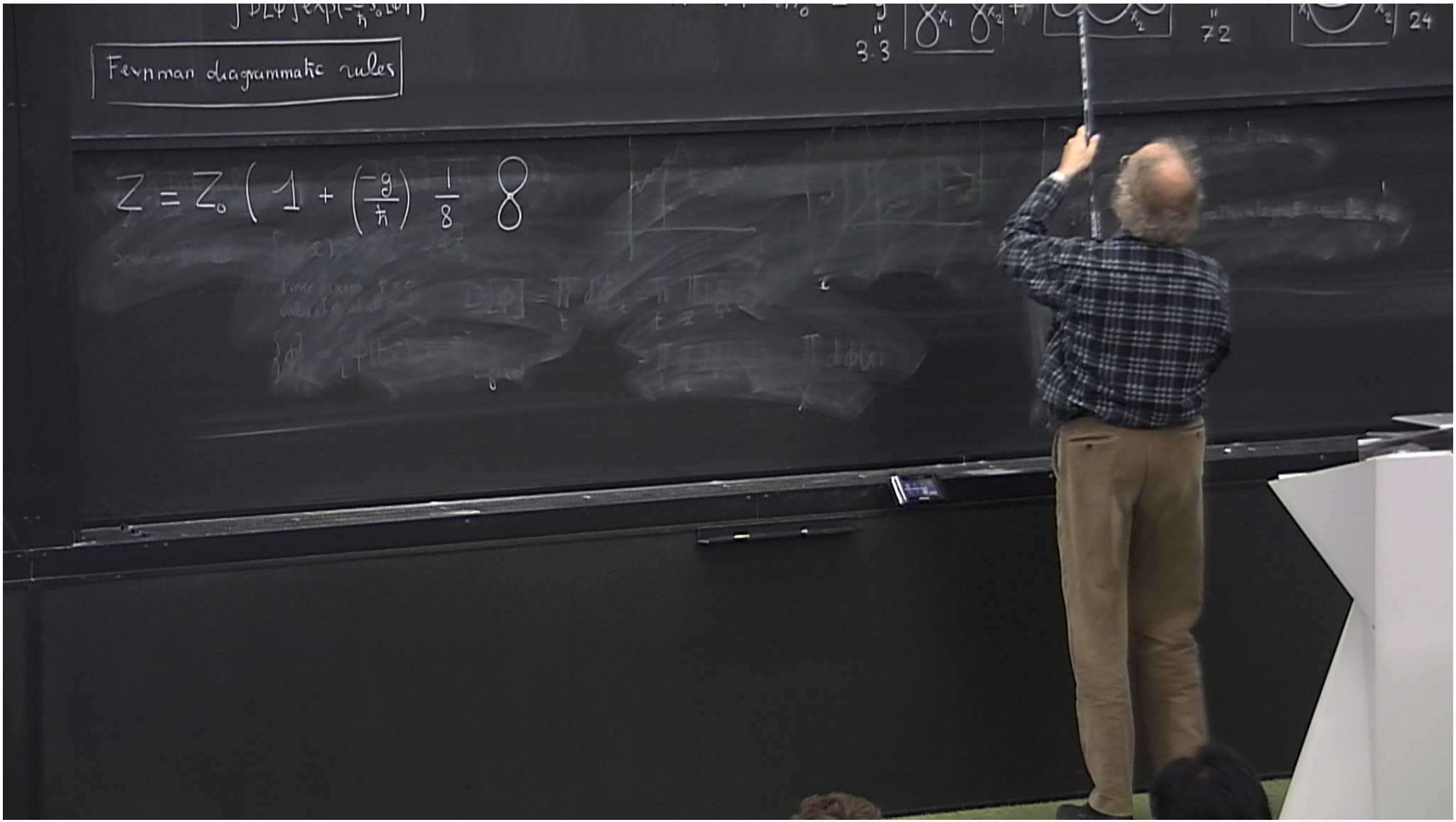
$+$

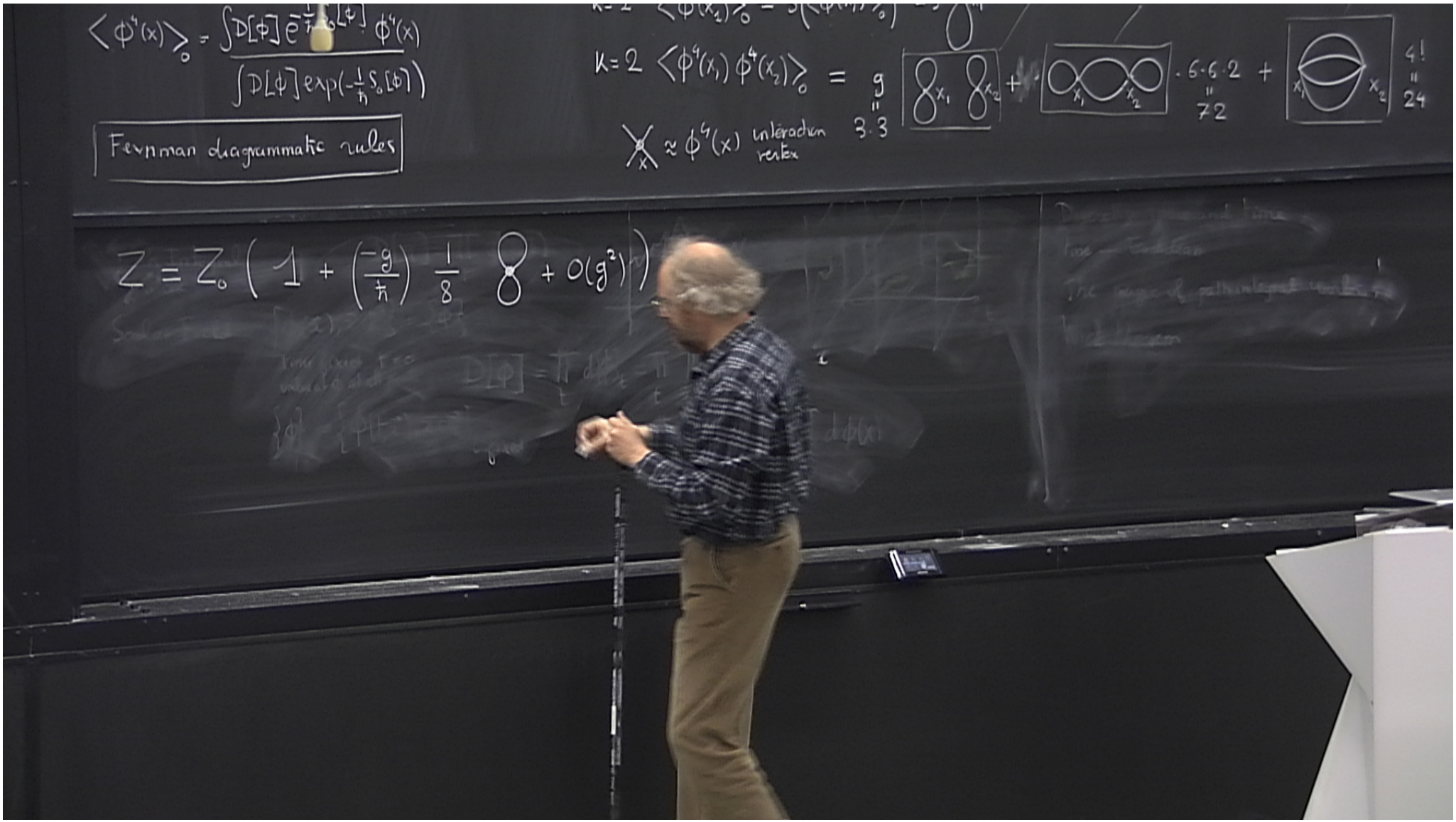
$[G(x_1 - x_2)]^4$

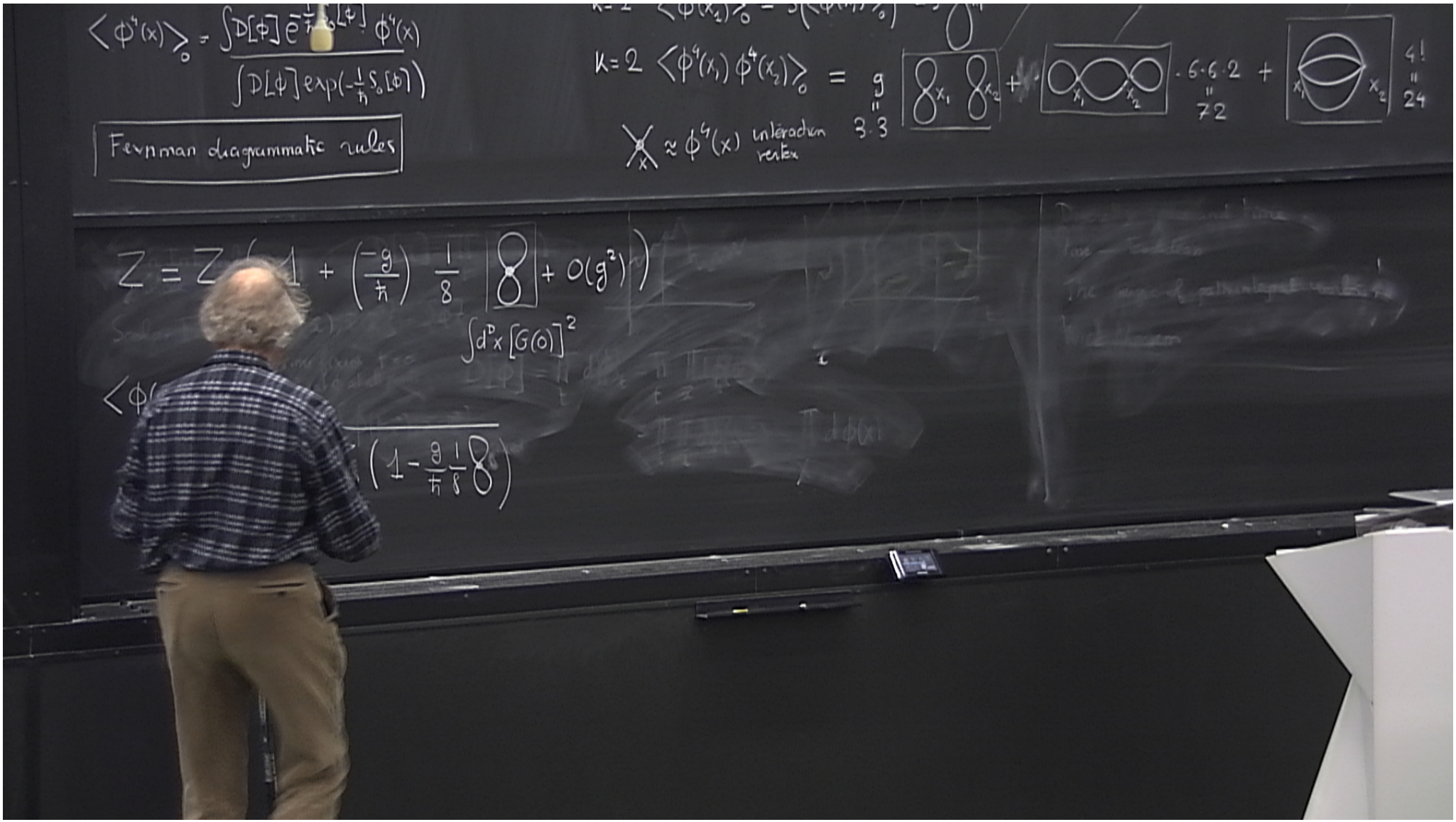
$4! = 24$

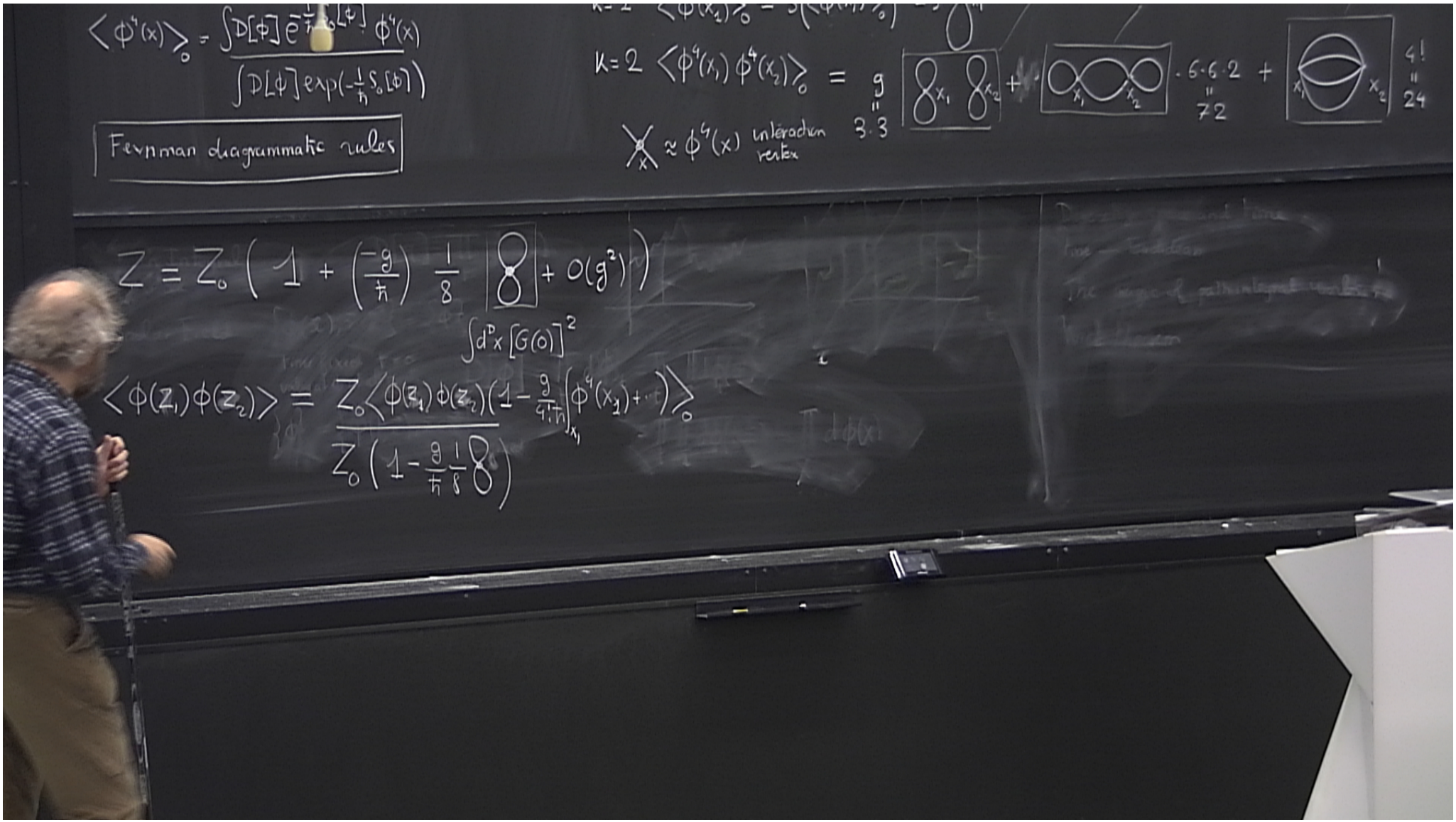
t fixed

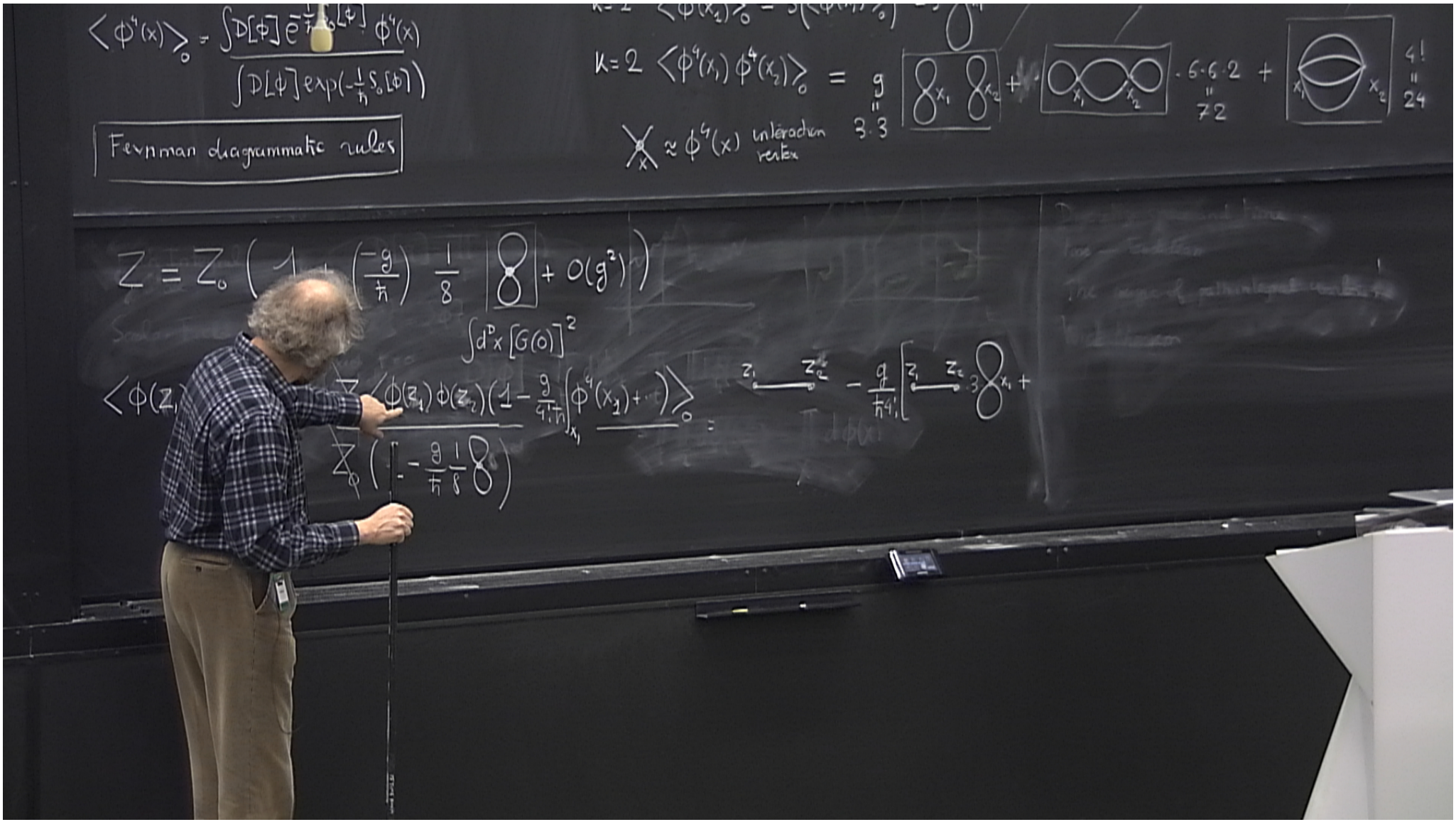
$$\prod_t \prod_{\vec{x}} d\phi(t, \vec{x}) = \prod_x d\phi(x)$$

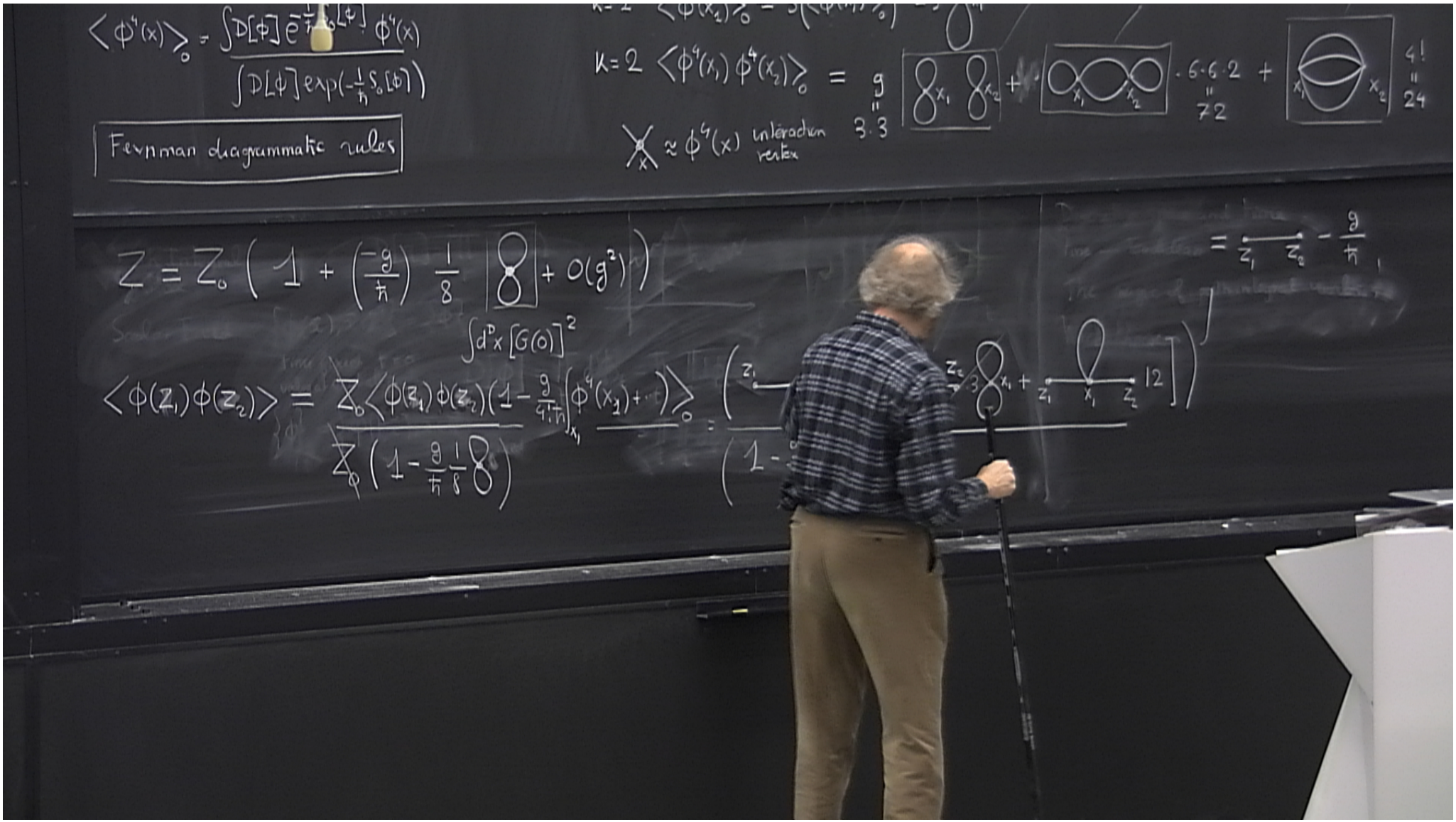

Feynman diagrammatic rules
Z = Z₀ (1 + (−g/ħ) 1/8 8
3·3
7·2
24


Feynman diagrammatic rules
interaction vertex
3·3
6·6·2
4!
24


Feynman diagrammatic rules
interaction vertex
$\int d^D x\,[G(0)]^2$

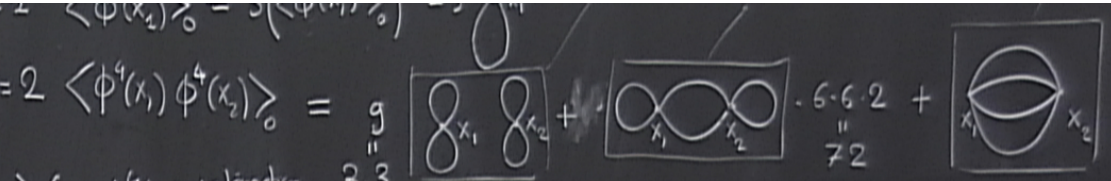

$\langle \phi^4(x) \rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_0[\phi]}\, \phi^4(x)}{\int D[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])}$
Feynman diagrammatic rules
$k=2 \quad \langle \phi^4(x_1)\phi^4(x_2) \rangle_0 = \frac{g}{3\cdot 3}$
x_1 x_2
$\cdot 6\cdot 6\cdot 2 = 72$
$4! = 24$
$x \approx \phi^4(x)$ interaction vertex
$Z = Z_0 \left(1 + \left(\frac{-g}{\hbar}\right) \frac{1}{8} + O(g^2) \right)$
$\int d^D x\, [G(0)]^2$
$\langle \phi(z_1)\phi(z_2) \rangle = \frac{Z_0 \langle \phi(z_1)\phi(z_2)(1 - \frac{g}{4!\hbar}\int_{x_1} \phi^4(x_1) + \cdots) \rangle_0}{Z_0 (1 - \frac{g}{\hbar}\frac{1}{8})}$


Feynman diagrammatic rules
interaction vertex
$Z = Z_0 \left(1 + \left(\frac{-g}{\hbar}\right) \frac{1}{8} + O(g^2)\right)$
$\int d^D x \, [G(0)]^2$
$\frac{g}{3 \cdot 3}$
$6 \cdot 6 \cdot 2 = 72$
$4! = 24$


Feynman diagrammatic rules
interaction vertex
4!
24

$$\langle \phi^4(x)\rangle_0 = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_0[\phi]}\, \phi^4(x)}{\int D[\phi]\exp\left(-\frac{1}{\hbar}S_0[\phi]\right)}$$

$\boxed{\text{Feynman diagrammatic rules}}$

$k=2$ $\langle \phi^4(x_1)\phi^4(x_2)\rangle_0 = \frac{g}{3\cdot 3}$ 

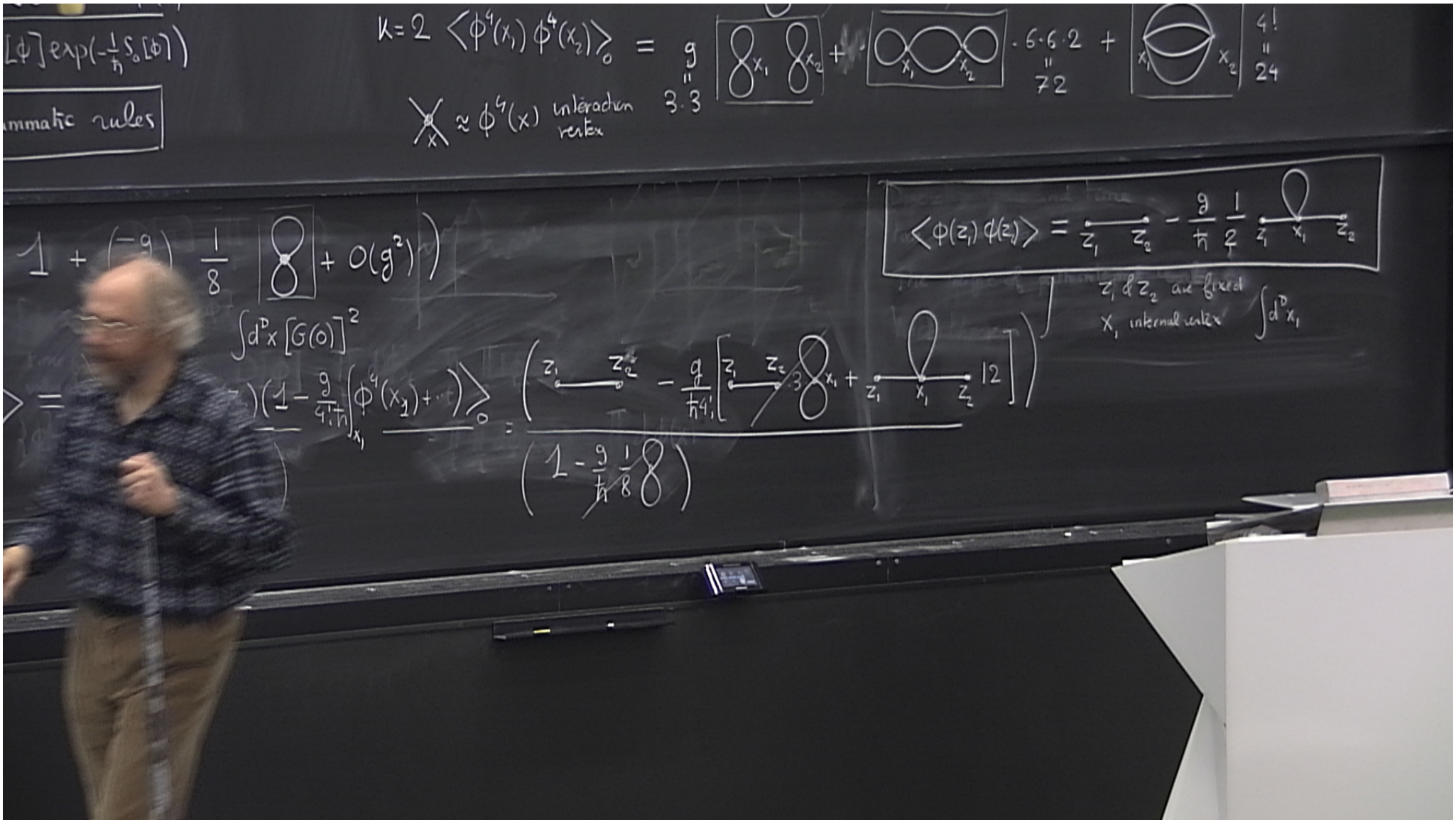
$\approx \phi^4(x)$ interaction vertex

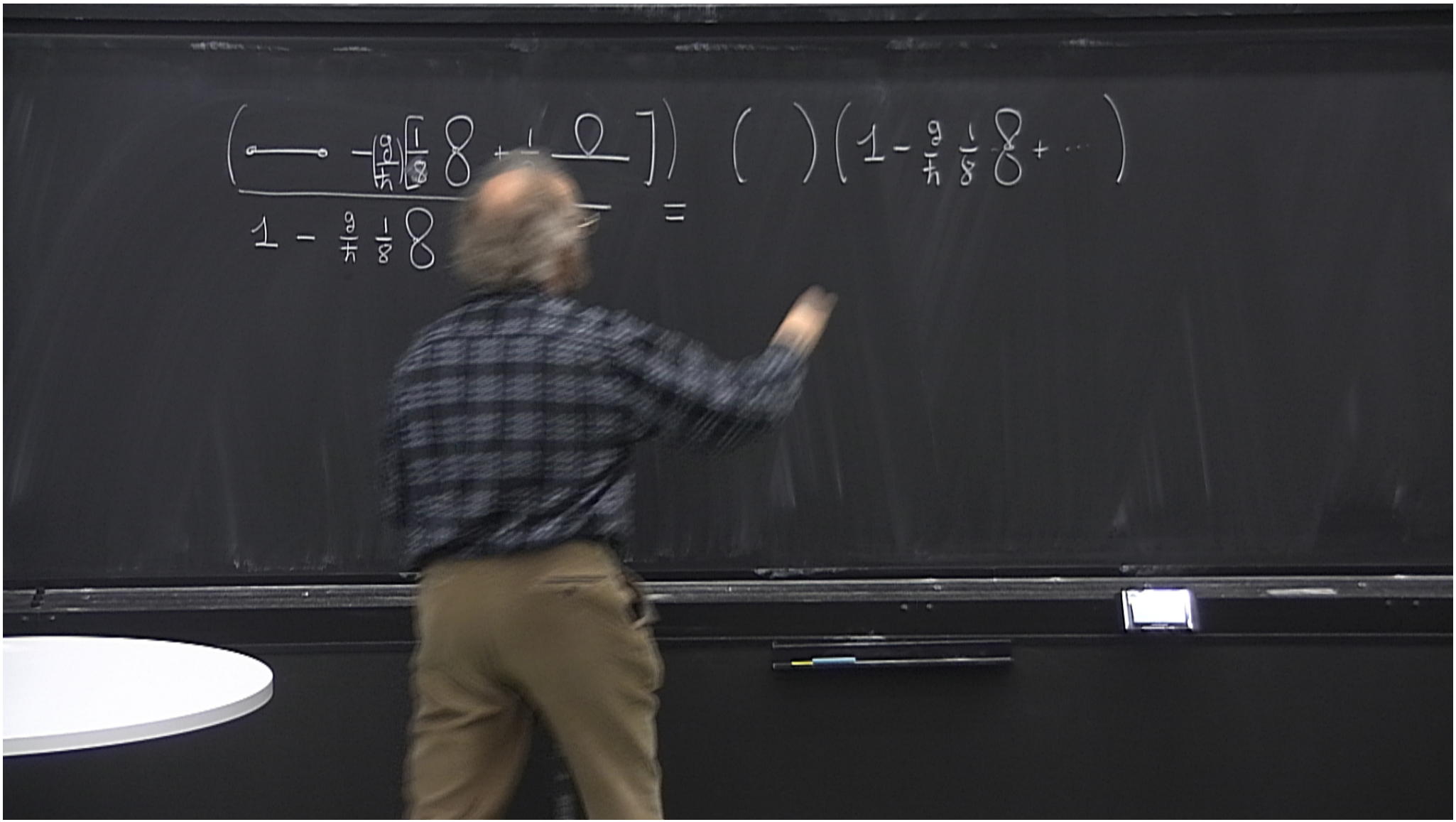
$$Z = Z_0\left(1 + \left(\frac{-g}{\hbar}\right)\frac{1}{8}\boxed{\ }\ + O(g^2)\right)$$

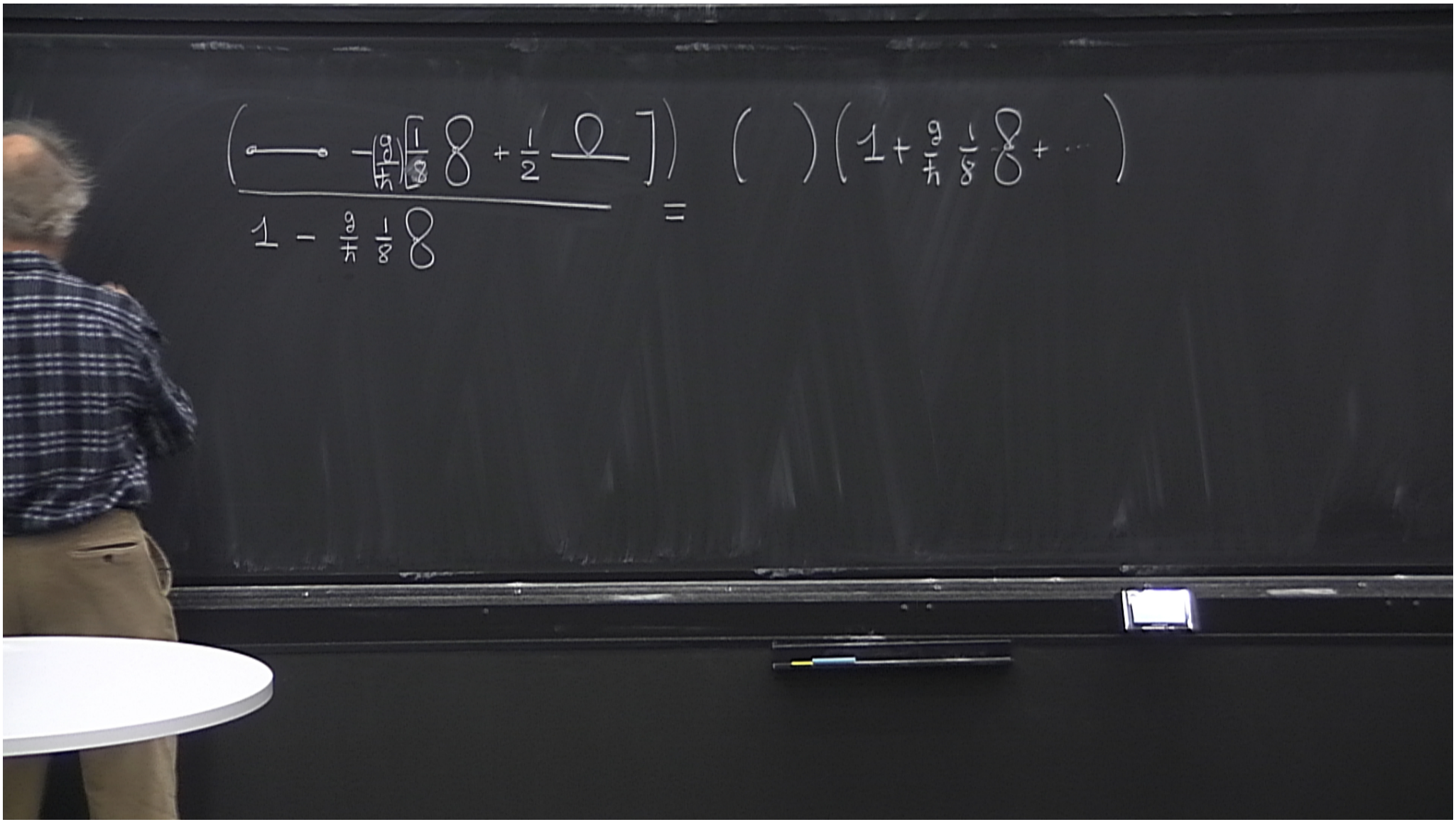
$\int d^Dx\,[G(0)]^2$

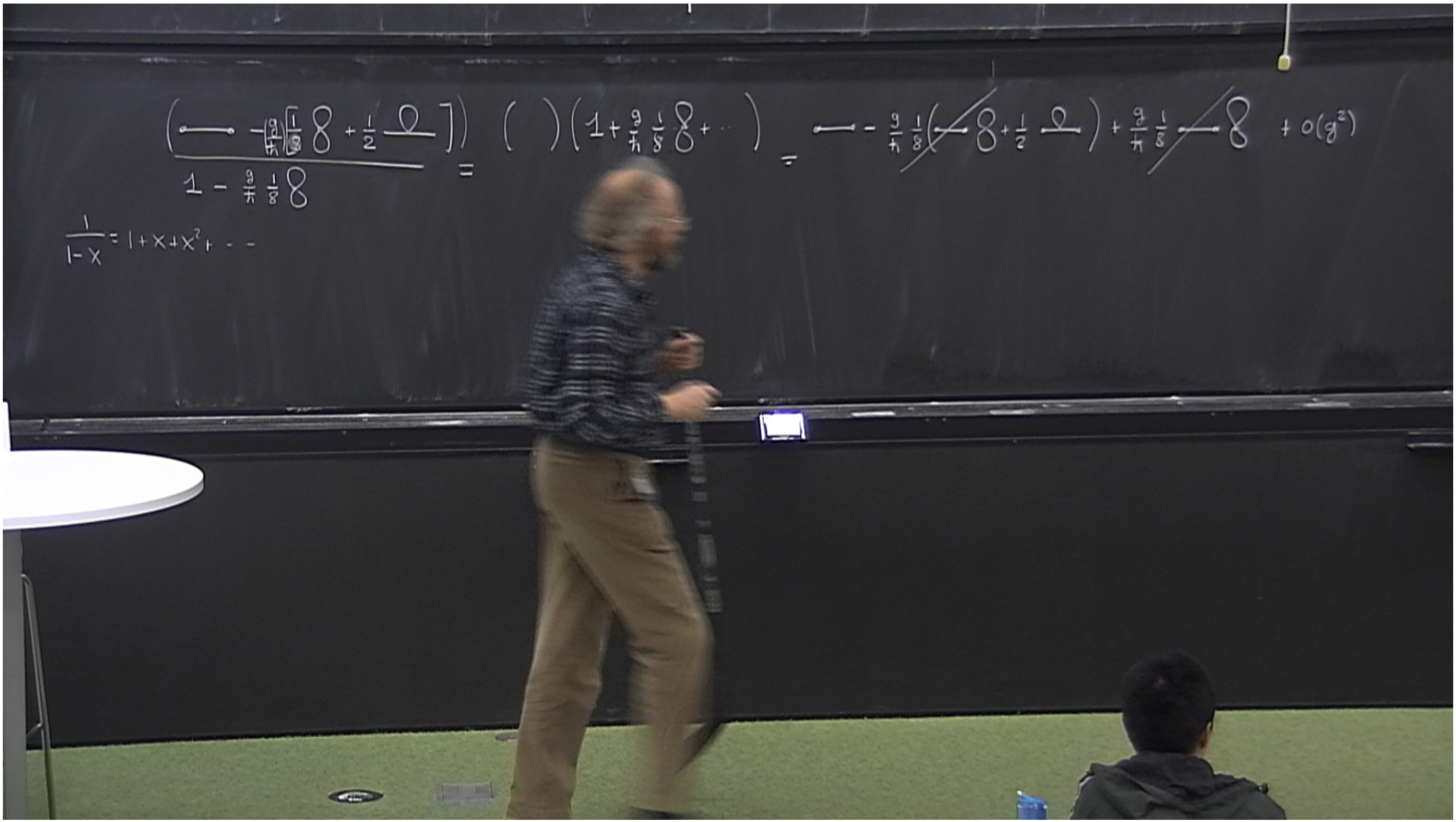
$$\langle \phi(z_1)\phi(z_2)\rangle = \frac{Z_0\langle \phi(z_1)\phi(z_2)\left(1 - \frac{g}{4!\hbar}\int_{x_1}\phi^4(x_1)+\cdots\right)\rangle_0}{Z_0\left(1-\frac{g}{\hbar}\frac{1}{8}\ \right)} = \frac{\left(z_1 \text{———} z_2 - \frac{g}{\hbar 4!}\left[z_1 \text{———} z_2\ 3\ x_1 + z_1 \text{———} x_1 \text{———} z_2\ 12\right]\right)}{\left(1-\frac{g}{\hbar}\frac{1}{8}\ \right)}$$

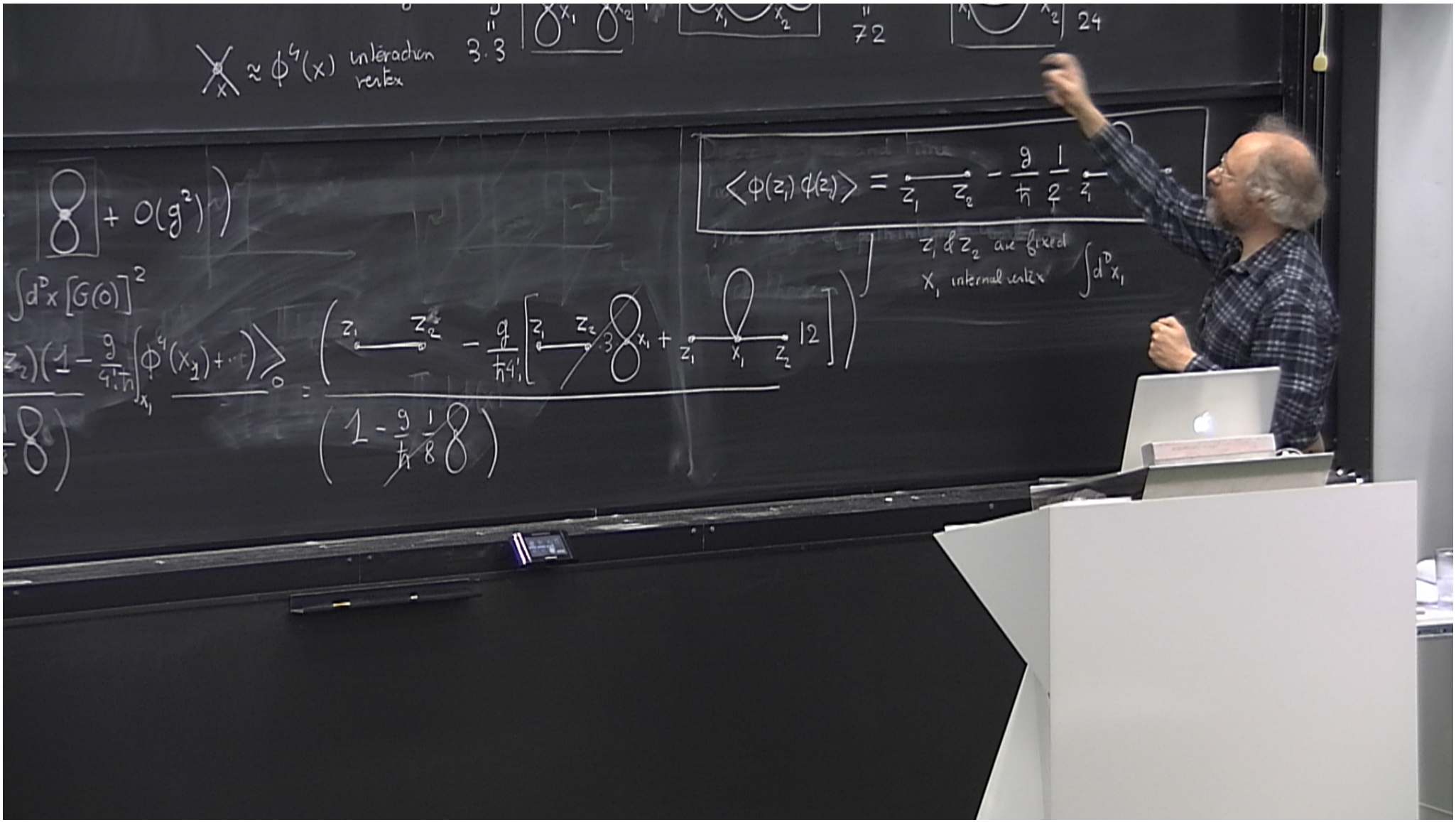
$$= z_1 \text{———} z_2 - \frac{g}{\hbar}\frac{1}{2}$$

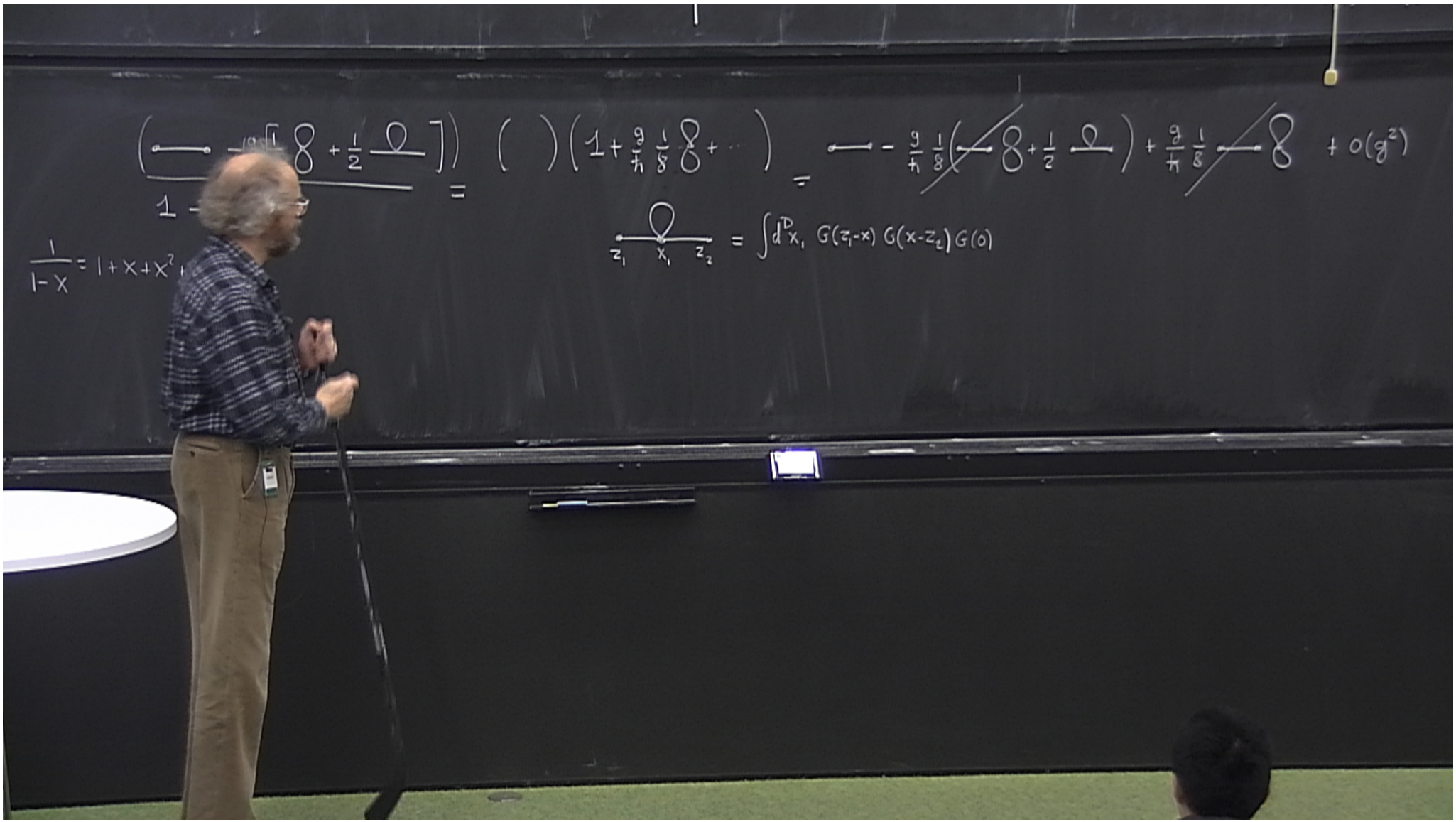

[φ] exp(-1/ħ S₀[φ])
mmatic rules
K=2 ⟨φ⁴(x₁) φ⁴(x₂)⟩₀ = g
3·3
x₁ x₂
·6·6·2 +
72
4!
24
≈ φ⁴(x) interaction vertex
1 + (-g) 1/8 + O(g²))
∫d^D x [G(0)]²
⟨φ(z₁) φ(z₂)⟩ =
z₁ & z₂ are fixed
x₁ internal vertex
∫d^D x₁
12

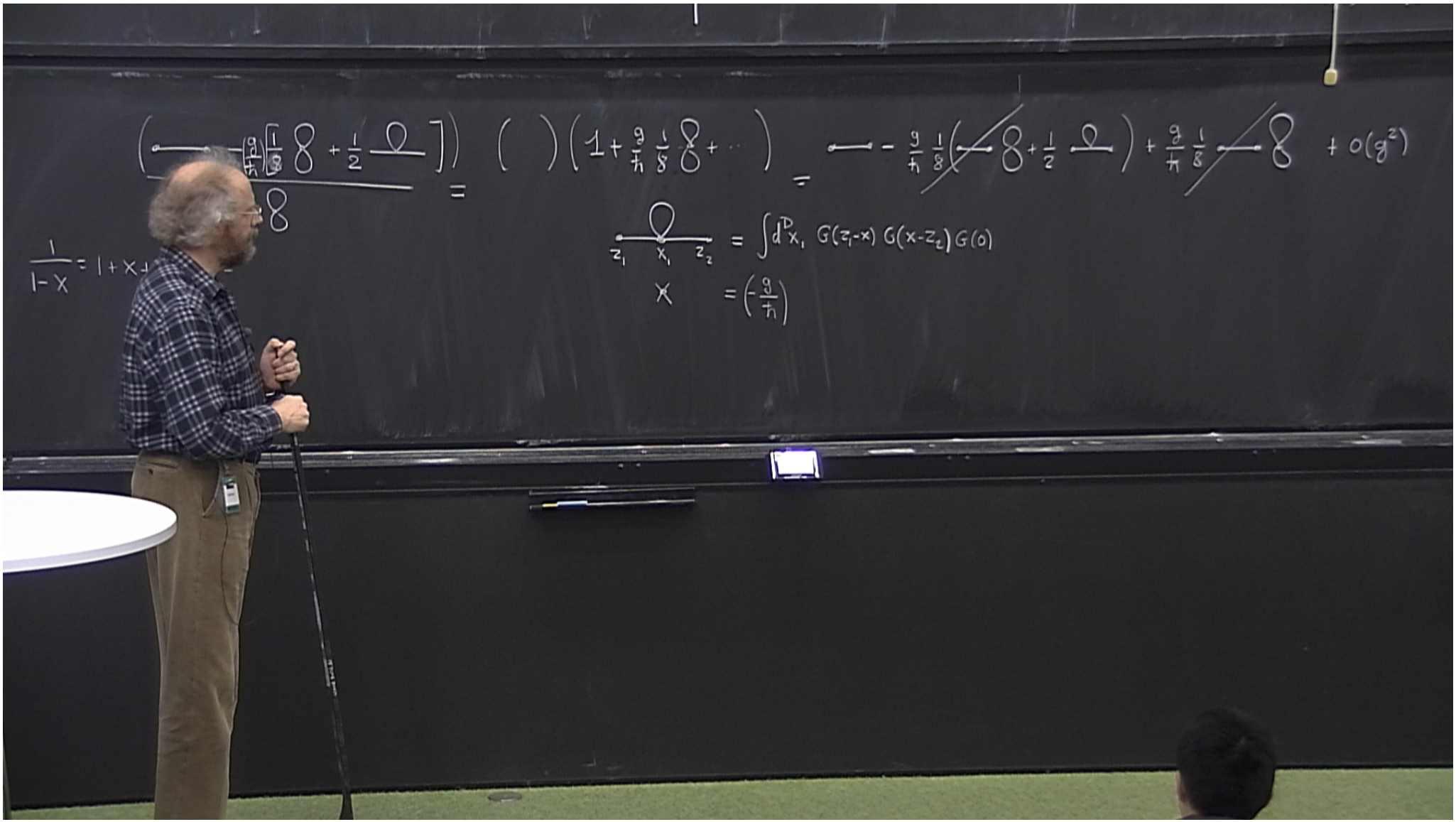



$\left(\text{—} - \left(\frac{g}{\hbar}\right)\left[\frac{1}{8}\,8 + \frac{1}{2}\,0\right]\right)$
$1 - \frac{g}{\hbar}\frac{1}{8}\,8$
$=$
$\left(\quad\right)\left(1 + \frac{g}{\hbar}\frac{1}{8}\,8 + \cdots\right)$


$\frac{1}{1-x} = 1 + x + x^2 + \cdots$


≈ φ⁴(x) interaction vertex
3·3
72
24
+ O(g²)
∫d^D x [G(0)]²
z₁ & z₂ are fixed
x₁ internal vertex
∫d^D x₁

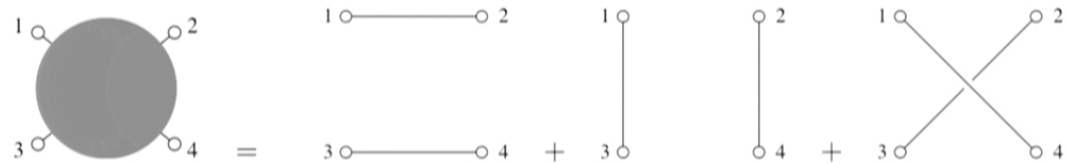

$\frac{1}{1-x} = 1 + x + x^2 +$
$z_1 \quad x_1 \quad z_2 = \int d^D x_1 \, G(z_1 - x)\, G(x - z_2)\, G(0)$
$+ O(g^2)$


z_1 x_1 z_2
$= \int d^D x_1 \, G(z_1 - x)\, G(x - z_2)\, G(0)$
$= \left(-\frac{g}{\hbar}\right)$
$\frac{1}{1-x} = 1 + x +$

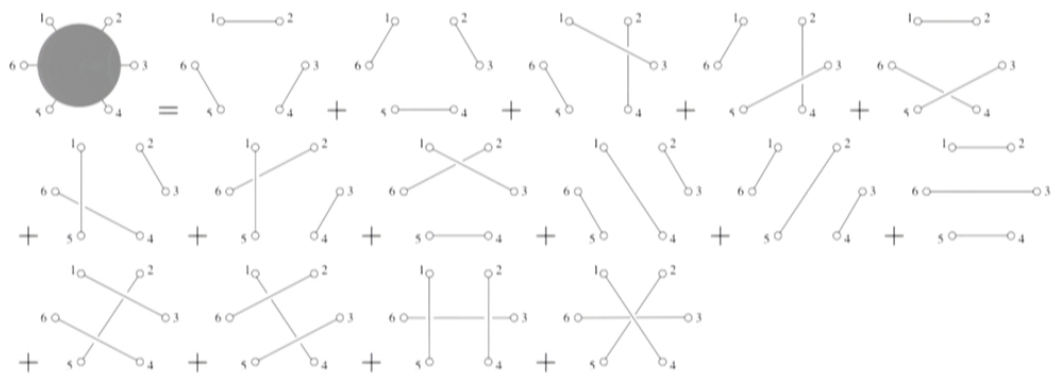
Wick theorem for the free scalar field

2 point function

4 point function



6 point function

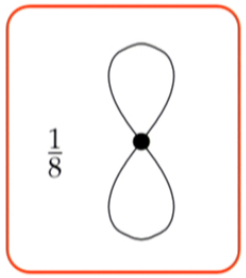


Interacting theory

vacuum diagrams

$N = 0, \; K = 1$

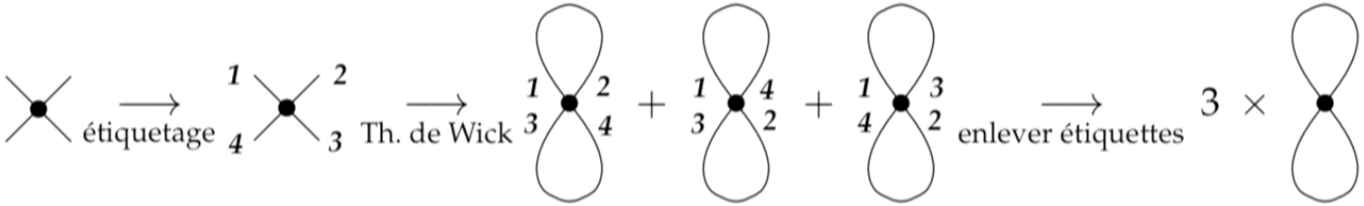
$$\frac{1}{4!}\int_{x_1} \langle \Phi^4(x_1)\rangle_0 \; =$$



$N = 0, \; K = 2$

$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2} \langle \Phi^4(x_1)\Phi^4(x_2)\rangle_0 \; =$$

Origin of the combinatorial factor



$$\frac{3}{4!} = \frac{1}{8}$$

$$\frac{\text{Number of inequivalent labellings of } G}{\text{total number of labellings}} = \frac{1}{\text{number of equivalent labellings}}$$

$$= \frac{1}{\text{order of the symmetry group of } G}$$

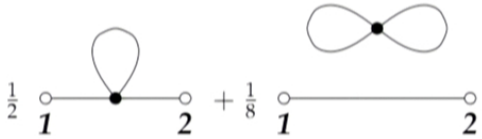
2 points diagrams

$N = 2,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\rangle_0 \ =$$

$N = 2,\ K = 1$

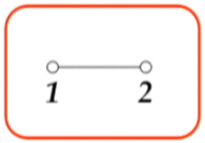
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1)\rangle_0 \ =$$



2 points diagrams

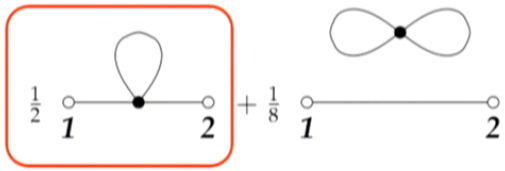
$N = 2,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\rangle_0 \ =$$



$N = 2,\ K = 1$

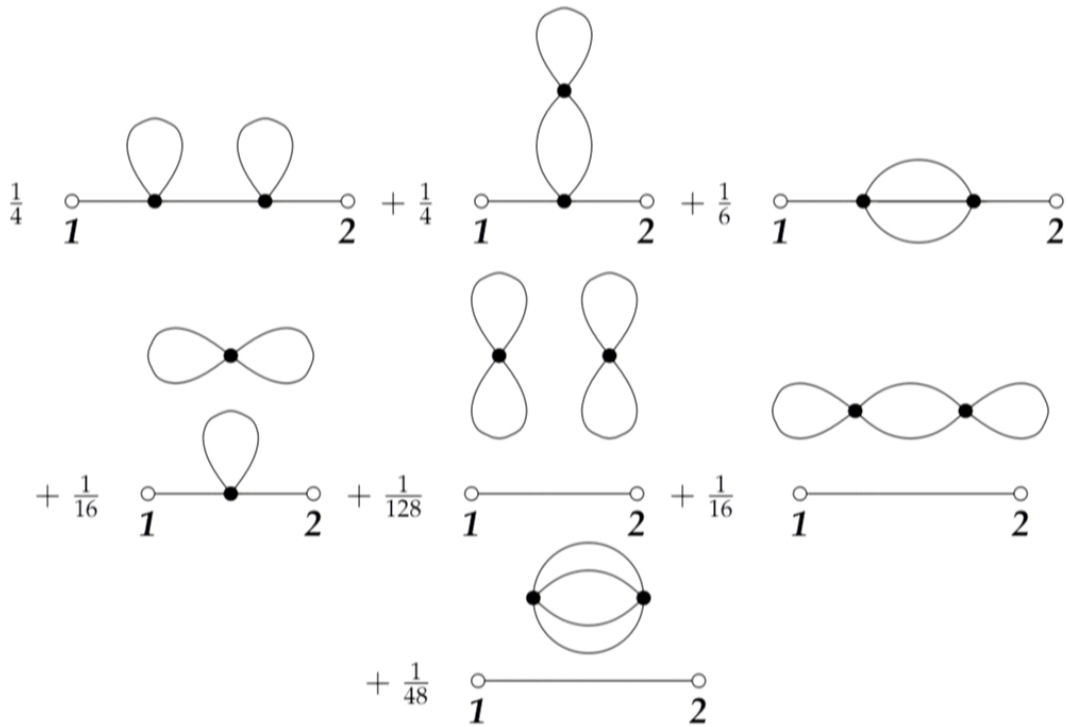
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1)\rangle_0 \ =$$



2 points diagrams (continued)

$N = 2,\ K = 2$

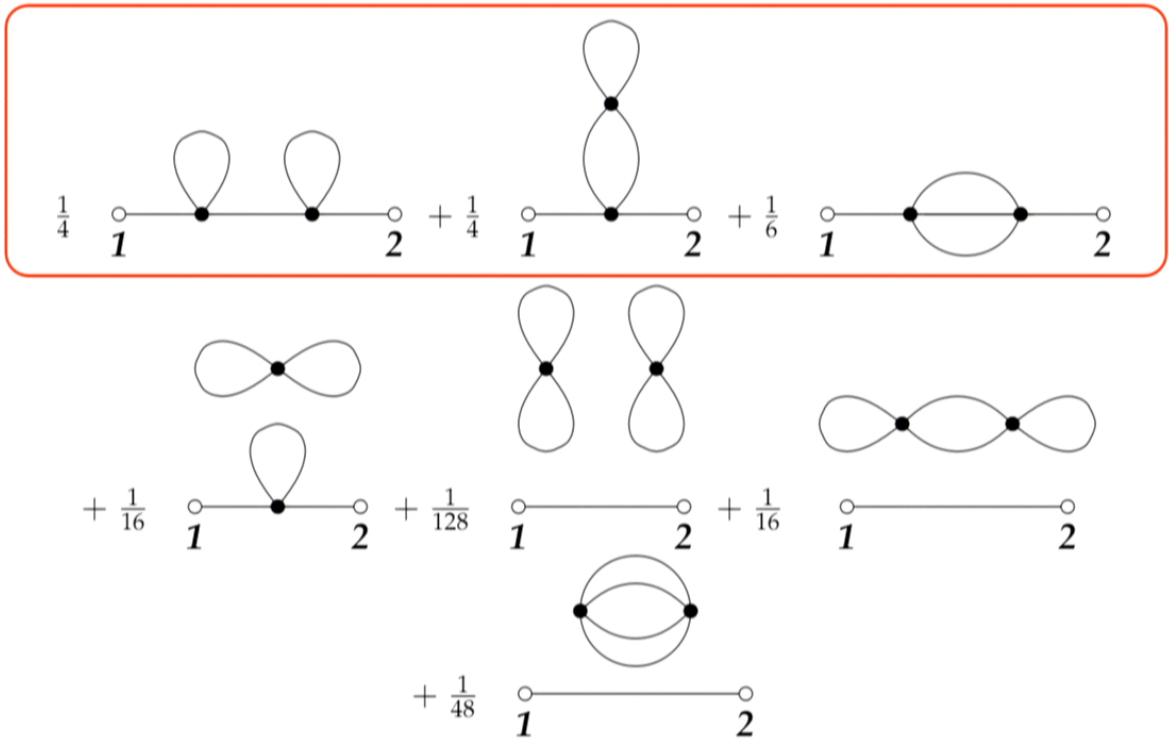
$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2}\langle\Phi(z_1)\Phi(z_2)\Phi^4(x_1)\Phi^4(x_2)\rangle_0 \;=$$



2 points diagrams (continued)

$N = 2,\ K = 2$

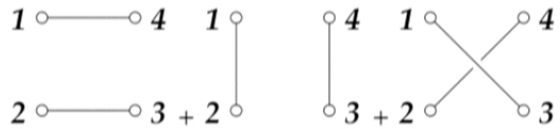
$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2}\langle\Phi(z_1)\Phi(z_2)\Phi^4(x_1)\Phi^4(x_2)\rangle_0 \;=$$



4 points diagrams

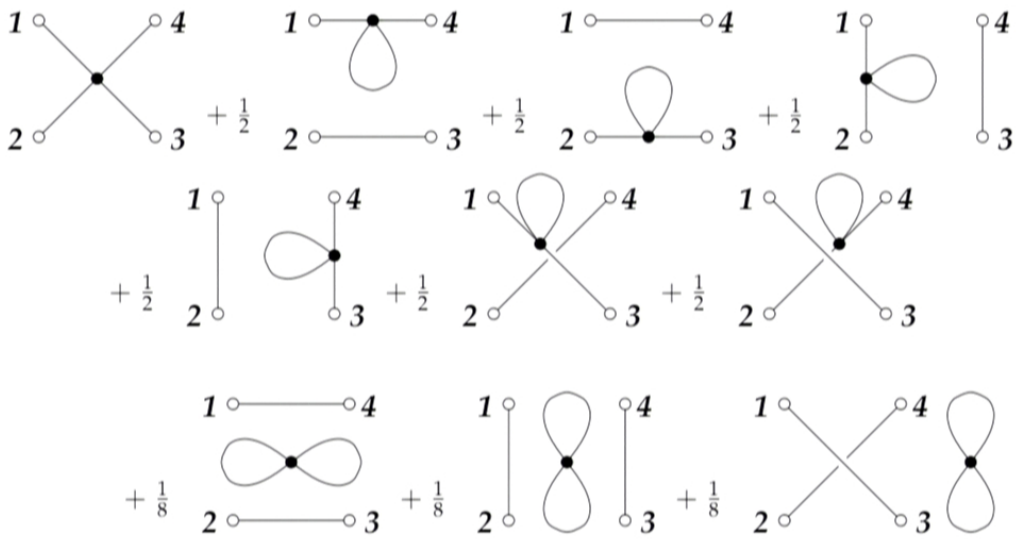
$N = 4,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\rangle_0 \ =$$

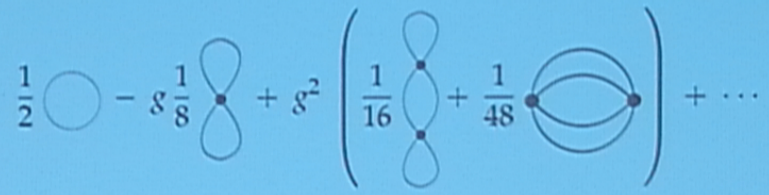


$N = 4,\ K = 1$

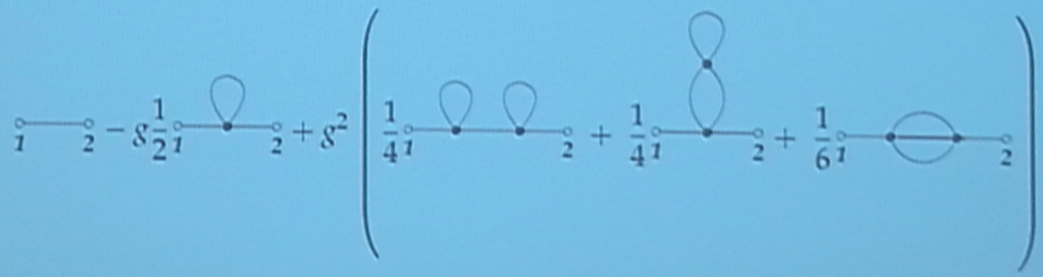
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\Phi^4(x_1)\rangle_0 \ =$$



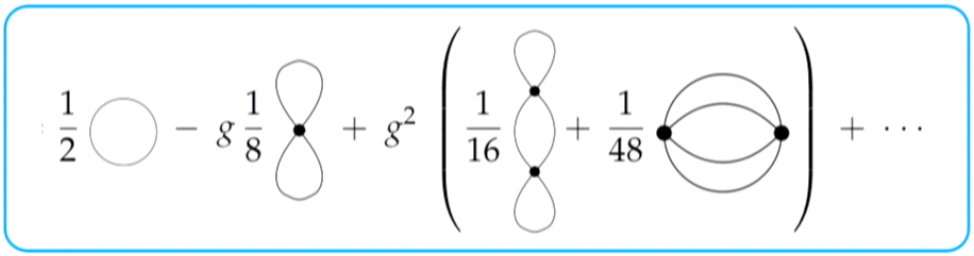
Connected vacuum diagrams



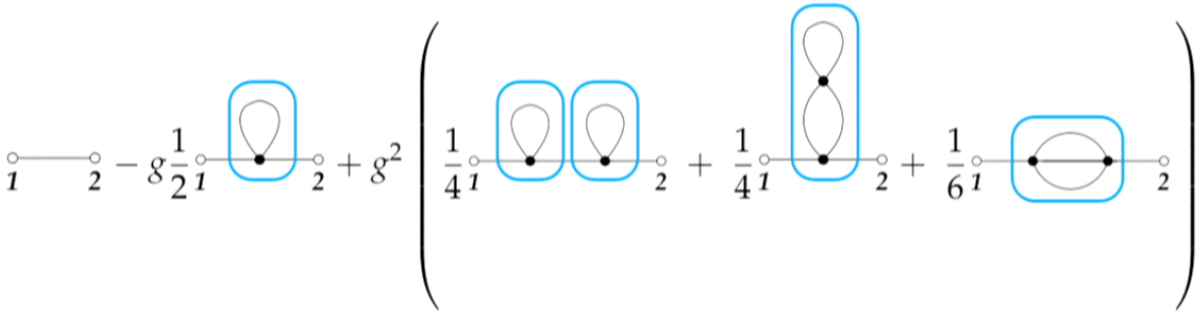
2 points function = connected 2 points function



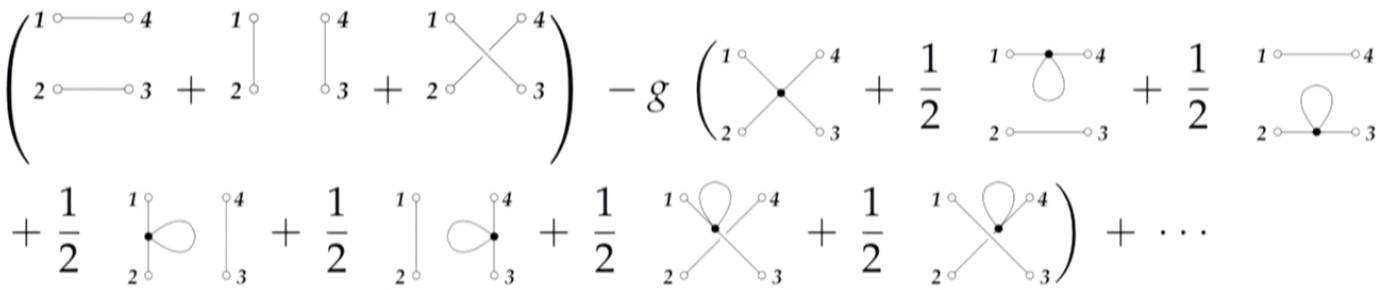
Connected vacuum diagrams



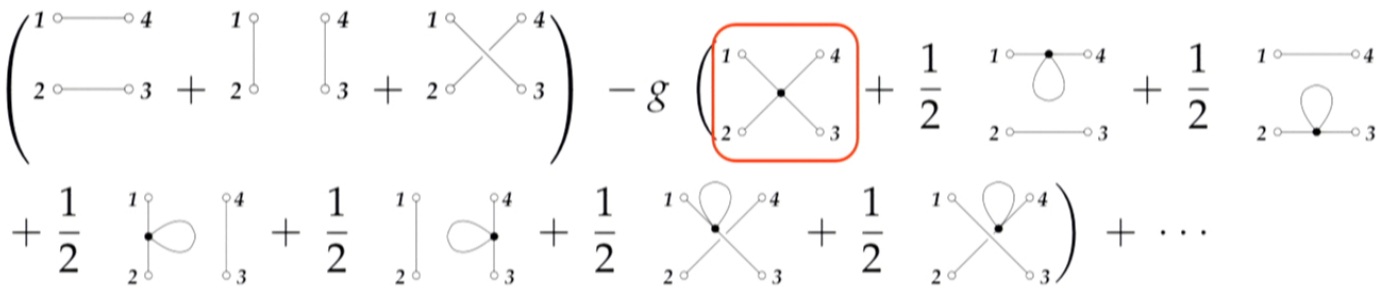
2 points function = connected 2 points function



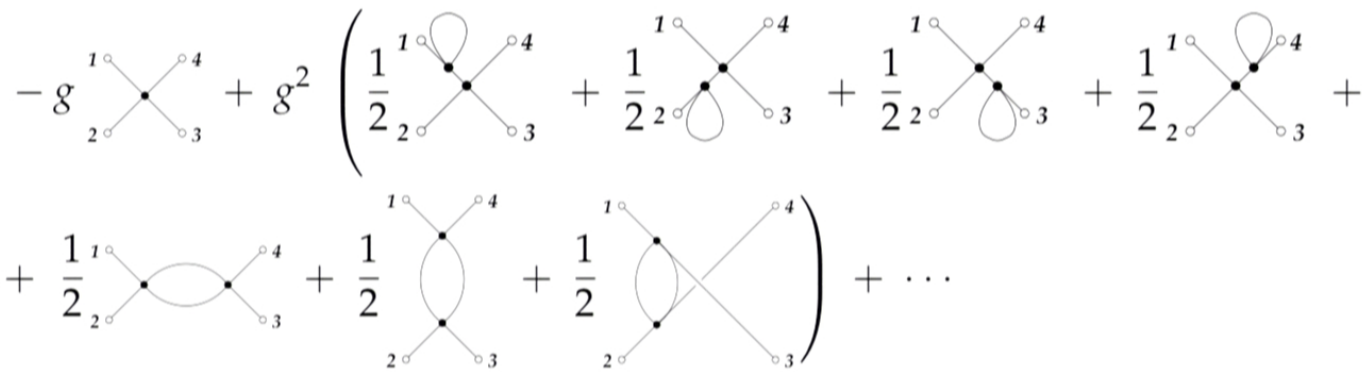
4 points function (up to order 1)



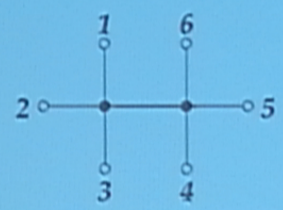
4 points function (up to order 1)



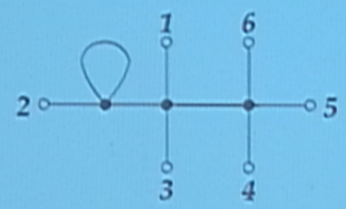
Connected 4 points function (up to order 2)

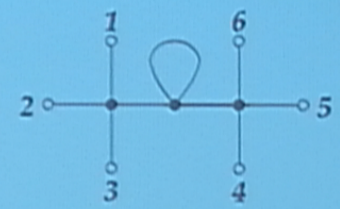


Connected 6 points function (up to order 3)

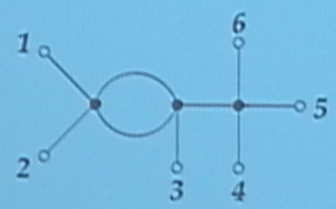
g^2 

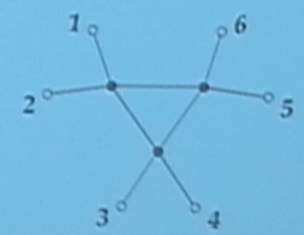
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$ $-g^3\frac{1}{2}$ 

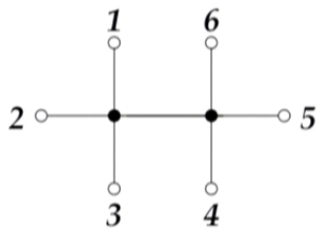
$+\cdots$

$-g^3\frac{1}{2}$ 

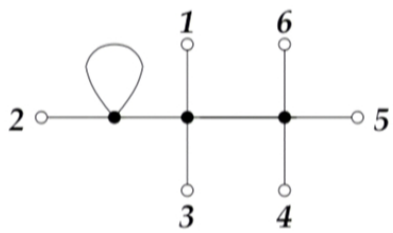
$+\cdots$ $-g^3$ 

$+\cdots$

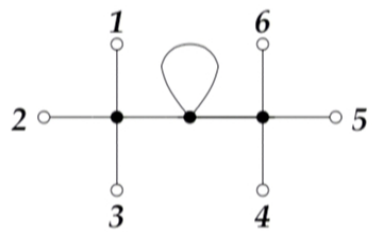
Connected 6 points function (up to order 3)

g^2 

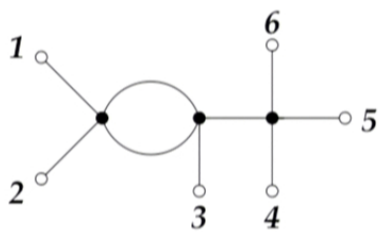
$+\cdots$

$-g^3\frac{1}{2}$ 

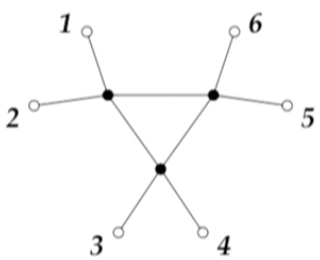
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$

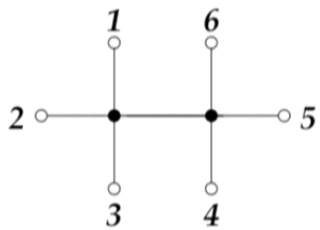
$-g^3\frac{1}{2}$ 

$+\cdots$

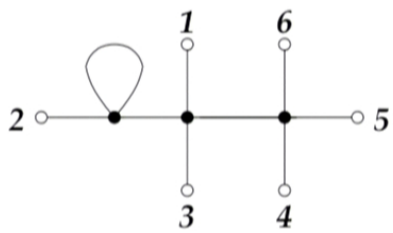
$-g^3$ 

$+\cdots$

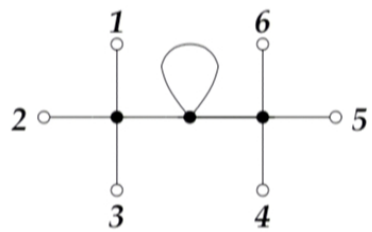
Connected 6 points function (up to order 3)

g^2 

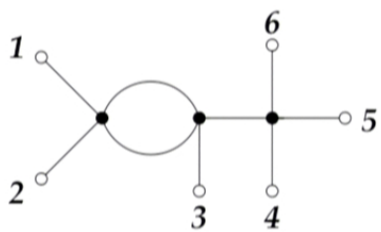
$+\cdots$

$-g^3\frac{1}{2}$ 

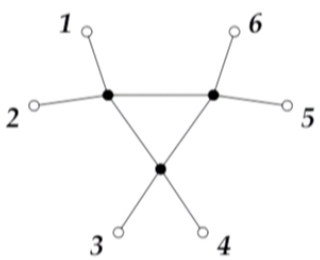
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$

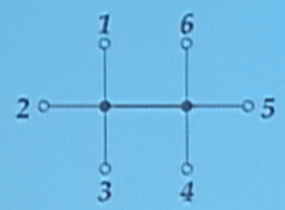
$-g^3\frac{1}{2}$ 

$+\cdots$

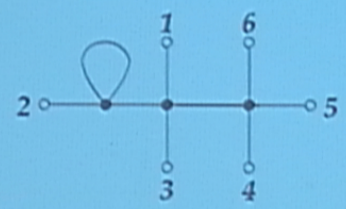
$-g^3$ 

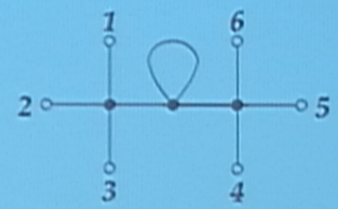
$+\cdots$

Connected 6 points function (up to order 3)

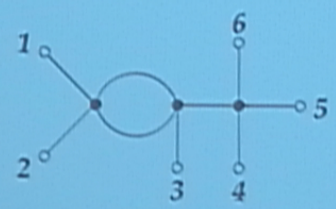
g^2 

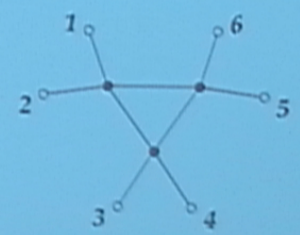
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$ $-g^3\frac{1}{2}$ 

$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$ $-g^3$ 

$+\cdots$