Title: Cosmological Constant

Date: Oct 23, 2012 05:10 PM

URL: http://pirsa.org/12100126

Abstract: Dark Matter and Dark Energy as a Possible Manifestation of a Fundamental Scale
If we take the idea of the Planck length as a fundamental (minimum) scale and if additionally we impose the Cosmological Constant ($Lambda$) as and infrared (IR) cut-off parameter. Then it is possible to demonstrate that Dark Matter effects can emerge as a consequence of an IR-UV mix effect. This opens the possibility of unifying the Dark Energy and Dark matter effects in a single approach.

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Geometric Operators in Loop Quantum Gravity with a Cosmological Constant
Loop quantum gravity is a candidate to describe the quantum gravity regime with zero cosmological constant. One of its key results is that geometric operators such as area angle volume are quantized. Not much is known when the cosmological constant is not zero. It is usually believed that to introduce this parameter in the game we need to use quantum groups. However due to the complicated algebraic structure inherent to quantum groups the geometric operators are not yet properly defined (except the area operator). I will discuss how the use of tensor operators can circumvent the difficulties and allow to construct a natural set of observables. In particular I will construct the natural geometric observables such as angle or volume and discuss some of their implications.

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Interplay Between Cosmological Constant and DSR Scale
I offer brief remarks on several ways in which the cosmological constant could provide a clue toward quantum gravity. I then focus on how DSR-relativistic theories can be made compatible with spacetime expansion (possibly cosmological-constant-governed spacetime expansion), and how this interplay could manifest itself in data.

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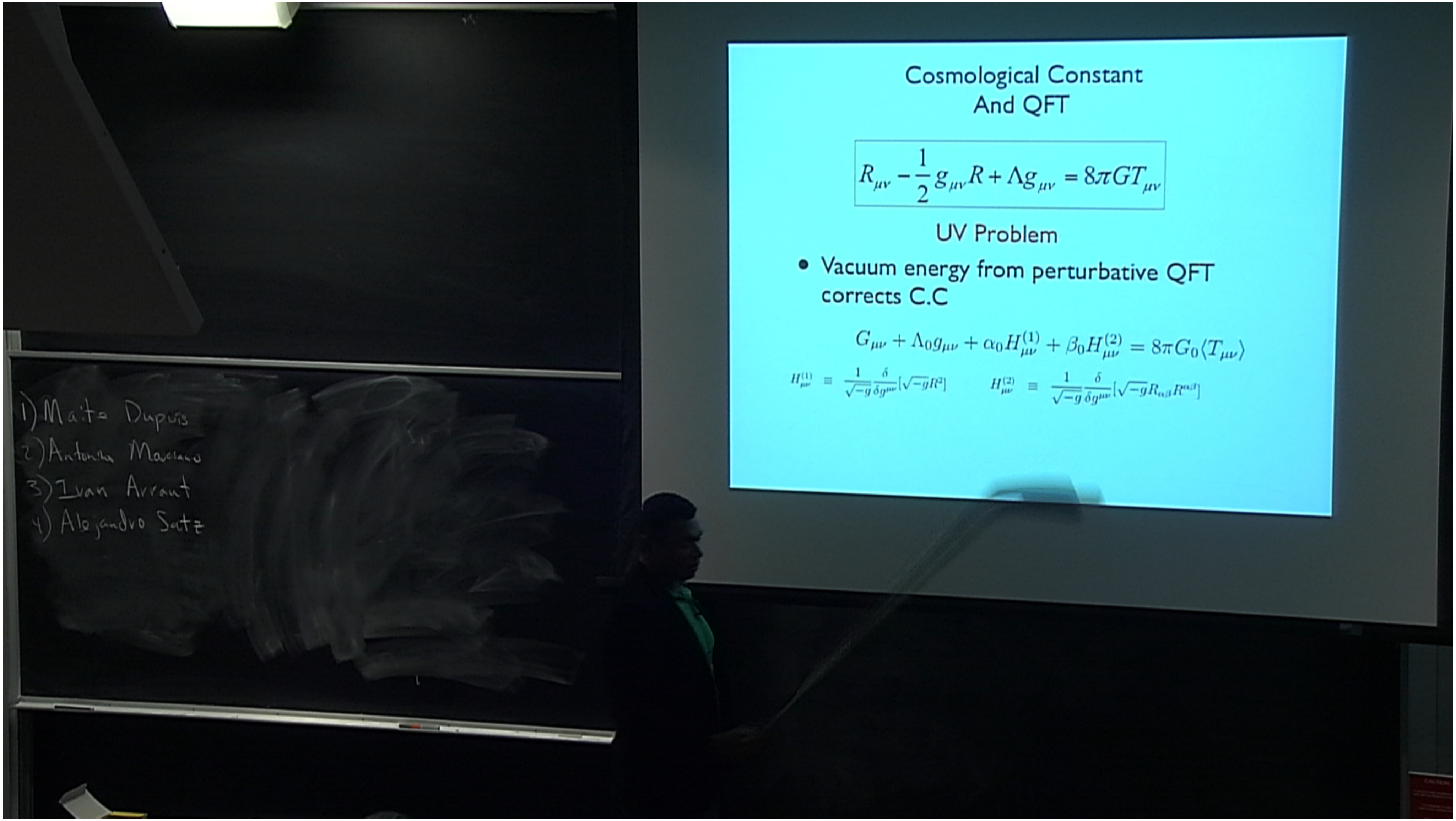
Quantum Gravity RG Flow: A Cosmological Limit Cycle
I will discuss evidence for the existence of a limit cycle in the renormalization group for quantum gravity which is visible in a minisuperspace approximation. The emergence of the limit cycle can be studied through a tuning parameter representing the number of dimensions in which fluctuations of the conformal factor are suppressed. At the critical value of the tuning parameter all RG trajectories reaching the UV fixed point have an extended semiclassical regime with a small positive cosmological constant providing a possible model for a viable cosmology without fine-tuning.


Cosmological Constant
And QFT
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
UV Problem
• Vacuum energy from perturbative QFT corrects C.C
$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \alpha_0 H^{(1)}_{\mu\nu} + \beta_0 H^{(2)}_{\mu\nu} = 8\pi G_0 \langle T_{\mu\nu} \rangle$
$H^{(1)}_{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} [\sqrt{-g} R^2]$
$H^{(2)}_{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} [\sqrt{-g} R_{\alpha\beta} R^{\alpha\beta}]$
1) Maite Dupuis
2) Antonino Marciano
3) Ivan Arraut
4) Alejandro Satz

Cosmological Constant And QFT

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

UV Problem

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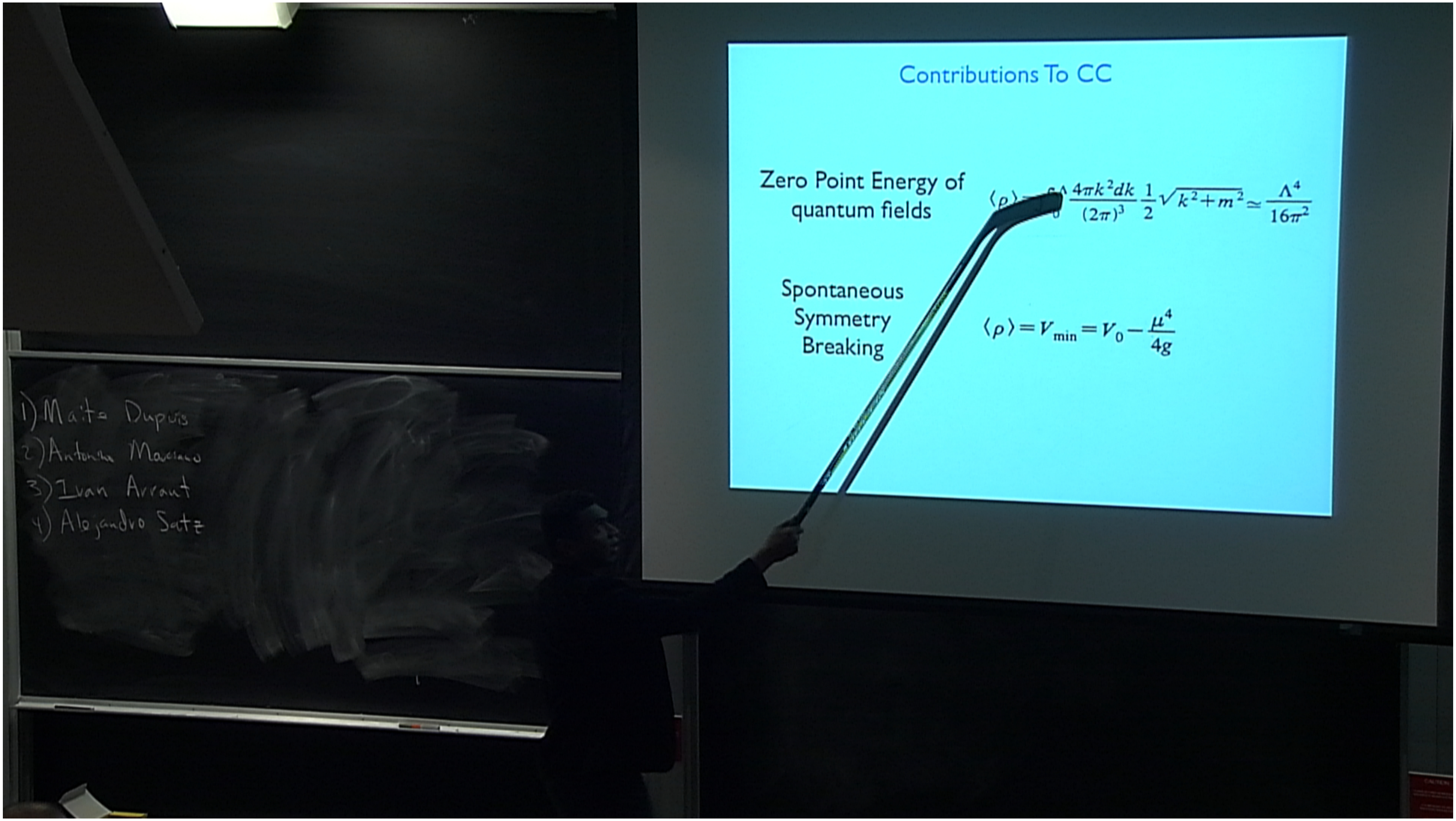
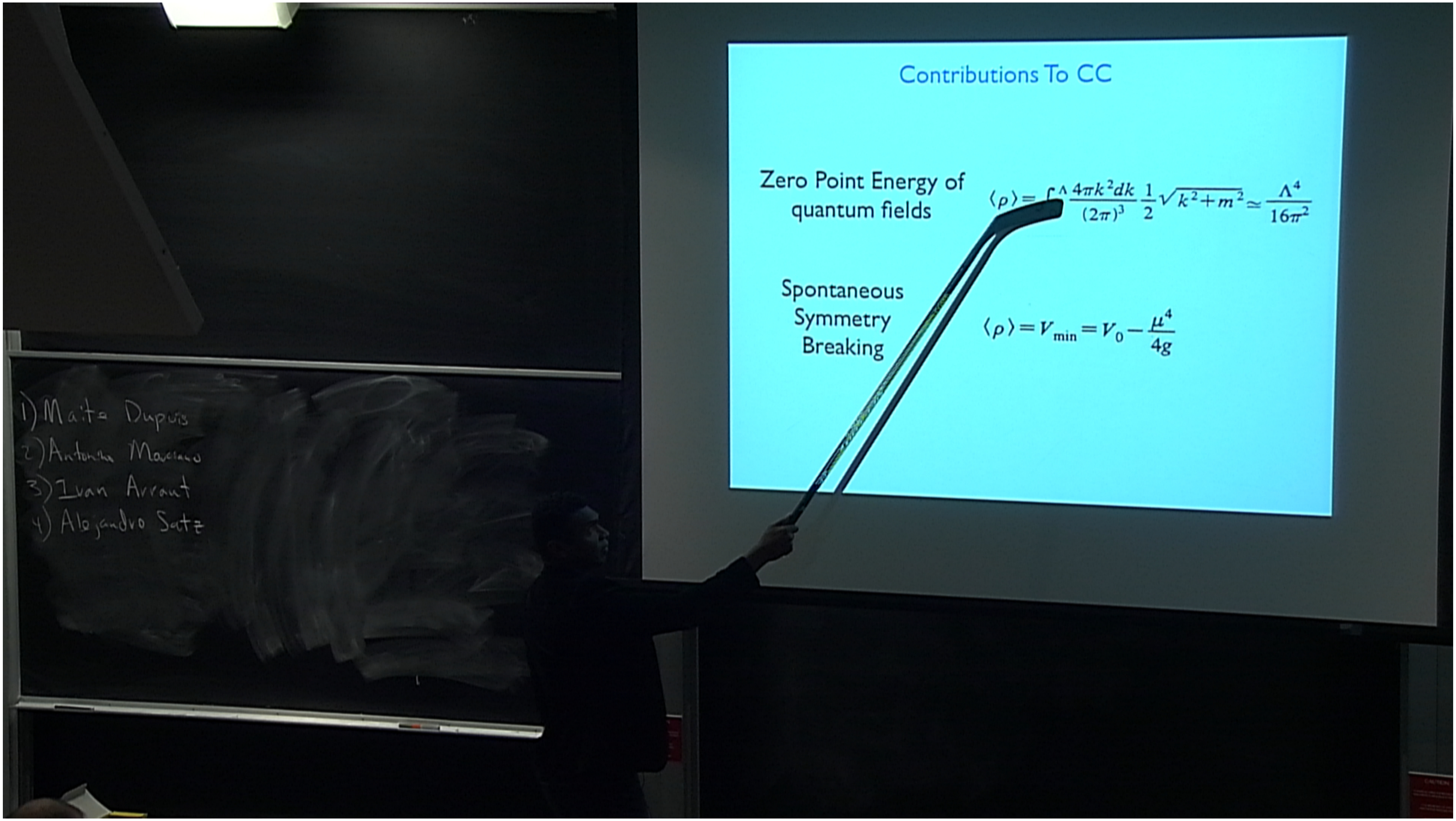

Contributions To CC
Zero Point Energy of quantum fields
$\frac{4\pi k^2 dk}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2}\simeq\frac{\Lambda^4}{16\pi^2}$
Spontaneous Symmetry Breaking
$\langle\rho\rangle = V_{\min} = V_0 - \frac{\mu^4}{4g}$
1) Maita Dupuis
3) Ivan Arrant
4) Alejandro Satz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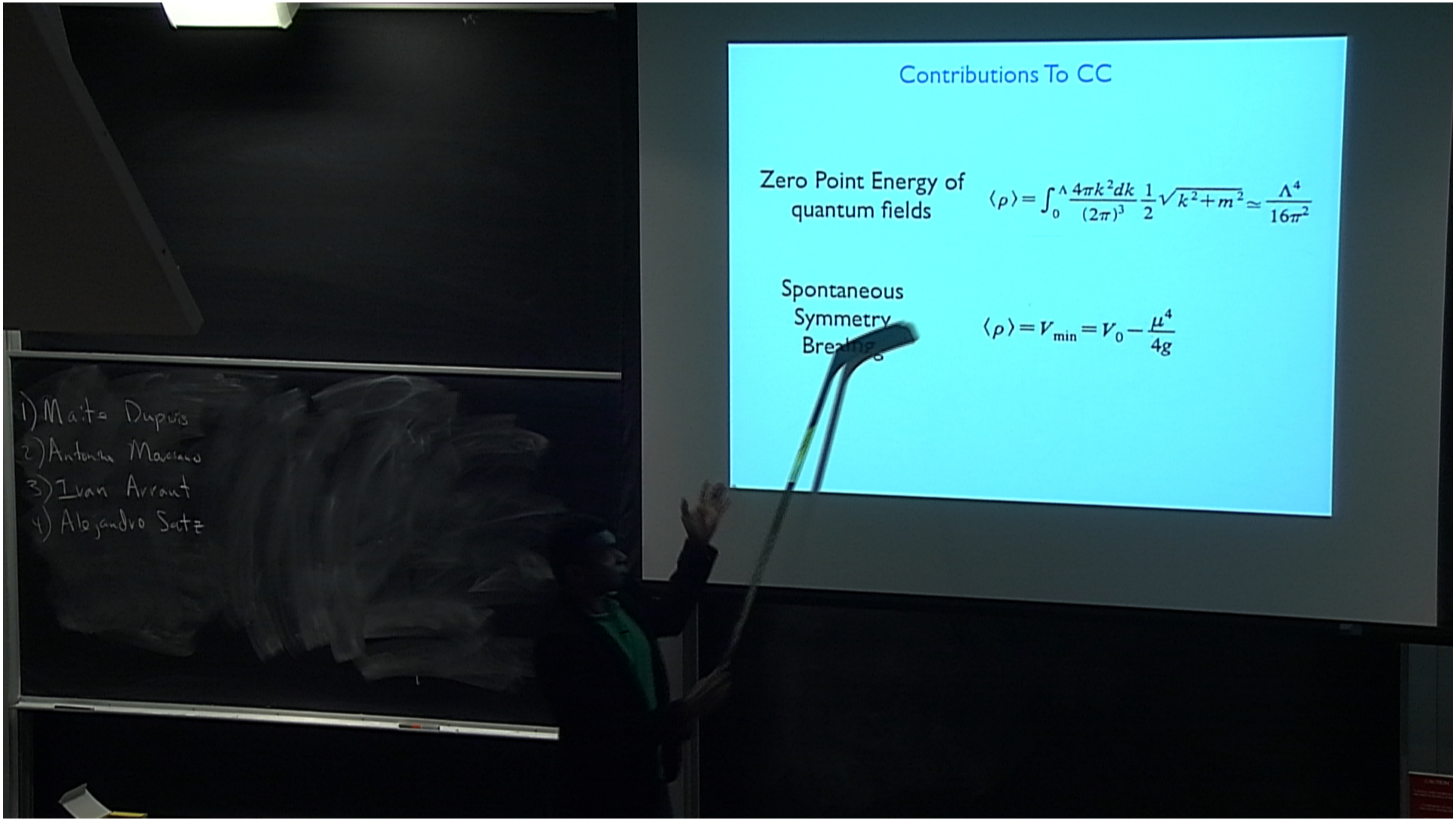
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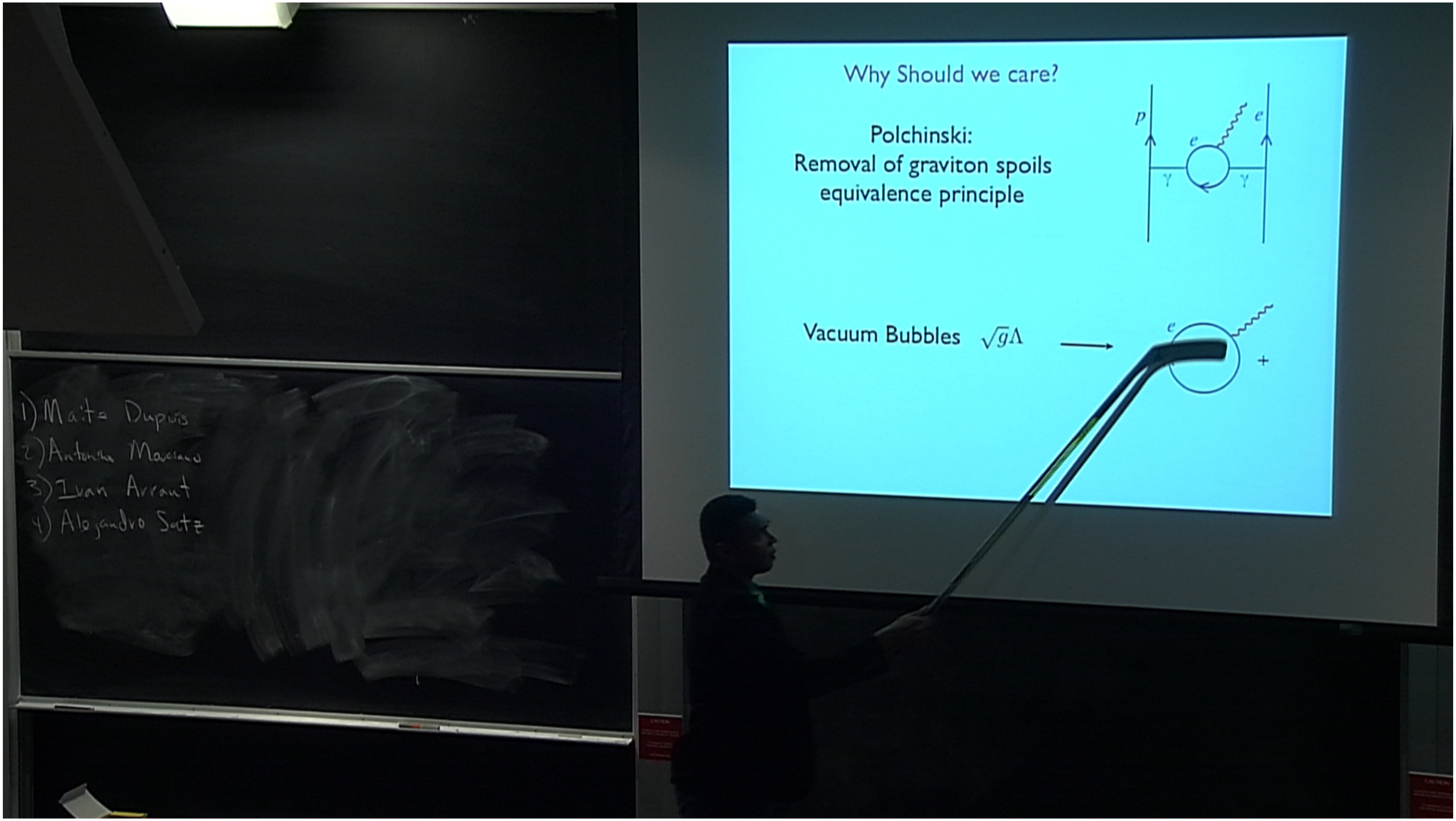
Why Should we care?
Polchinski:
Removal of graviton spoils
equivalence principle
Vacuum Bubbles $\sqrt{g}\Lambda$
1) Maite Dupuis
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Lesson

- Flat space QFT is not enough to fix the renormalized free parameters (just like we can't calculate the mass of the electron with standard model).

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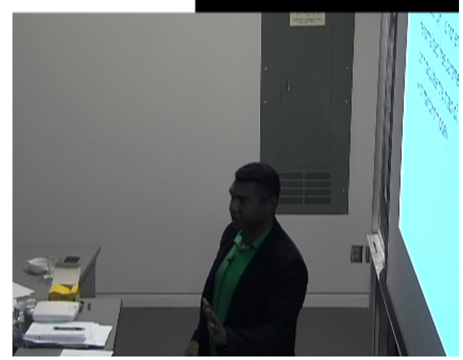

Backreaction

Quantum Gravitational Backreaction
(Tsamis, Woodard; Polyakov, Arkmedov):

Gravitational Backreaction:
Abramo, Brandenberger, Mukhanov

Graviton-Fermion Condensaton:
S.A, Moffat

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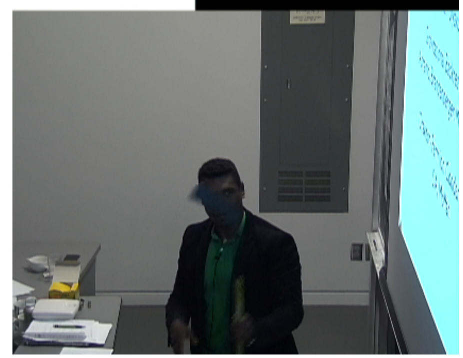


Illustration of Backreaction (Dolgov, Ford)

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \varphi \partial^\alpha \varphi - \xi R \varphi^2)$$

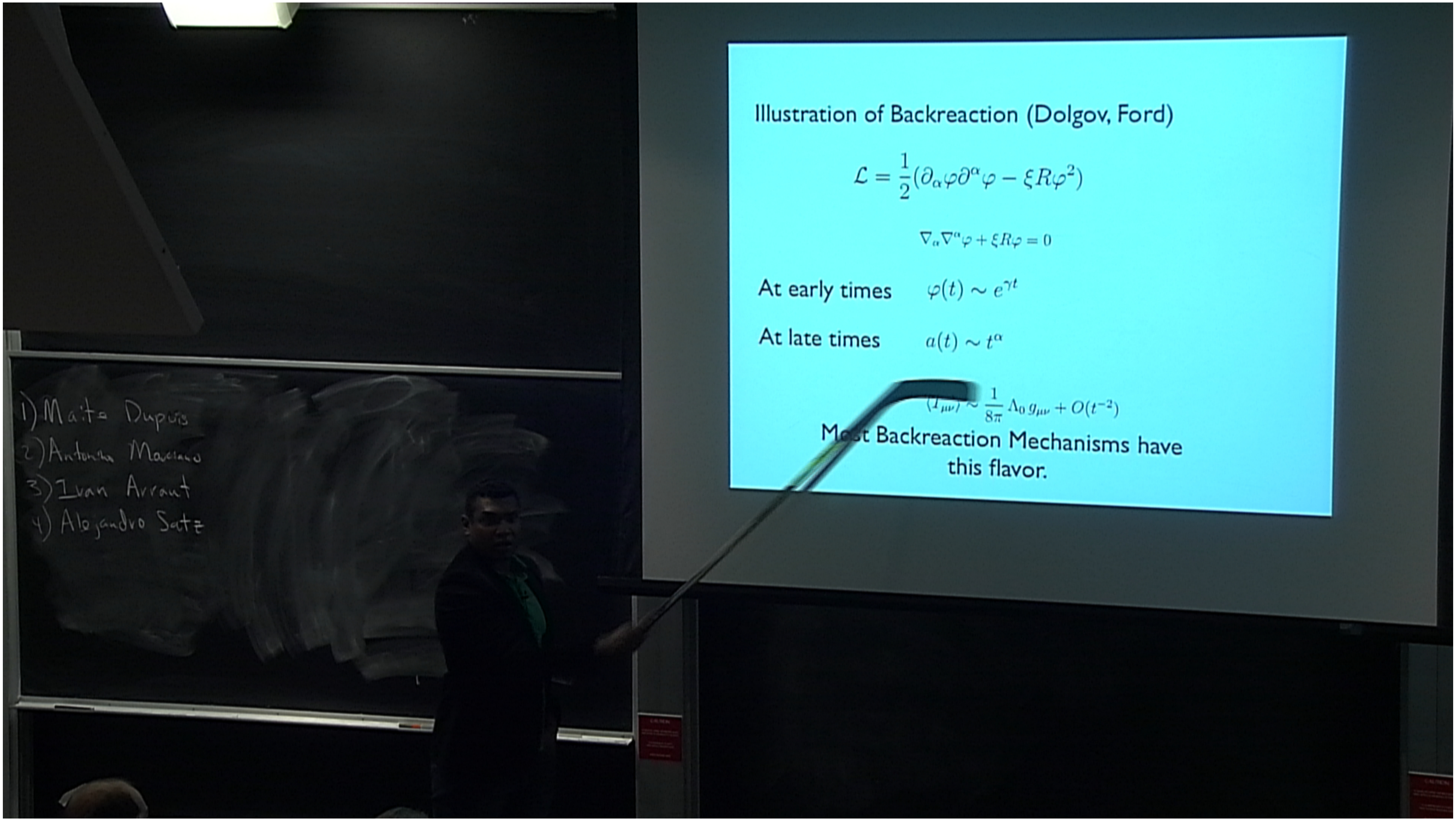
$$\nabla_\alpha \nabla^\alpha \varphi + \xi R \varphi = 0$$

At early times $\varphi(t) \sim e^{\gamma t}$

At late times $a(t) \sim t^\alpha$

$$\langle T_{\mu\nu} \rangle \sim \frac{1}{8\pi} \Lambda_0 \, g_{\mu\nu} + O(t^{-2})$$

Most Backreaction Mechanisms have this flavor.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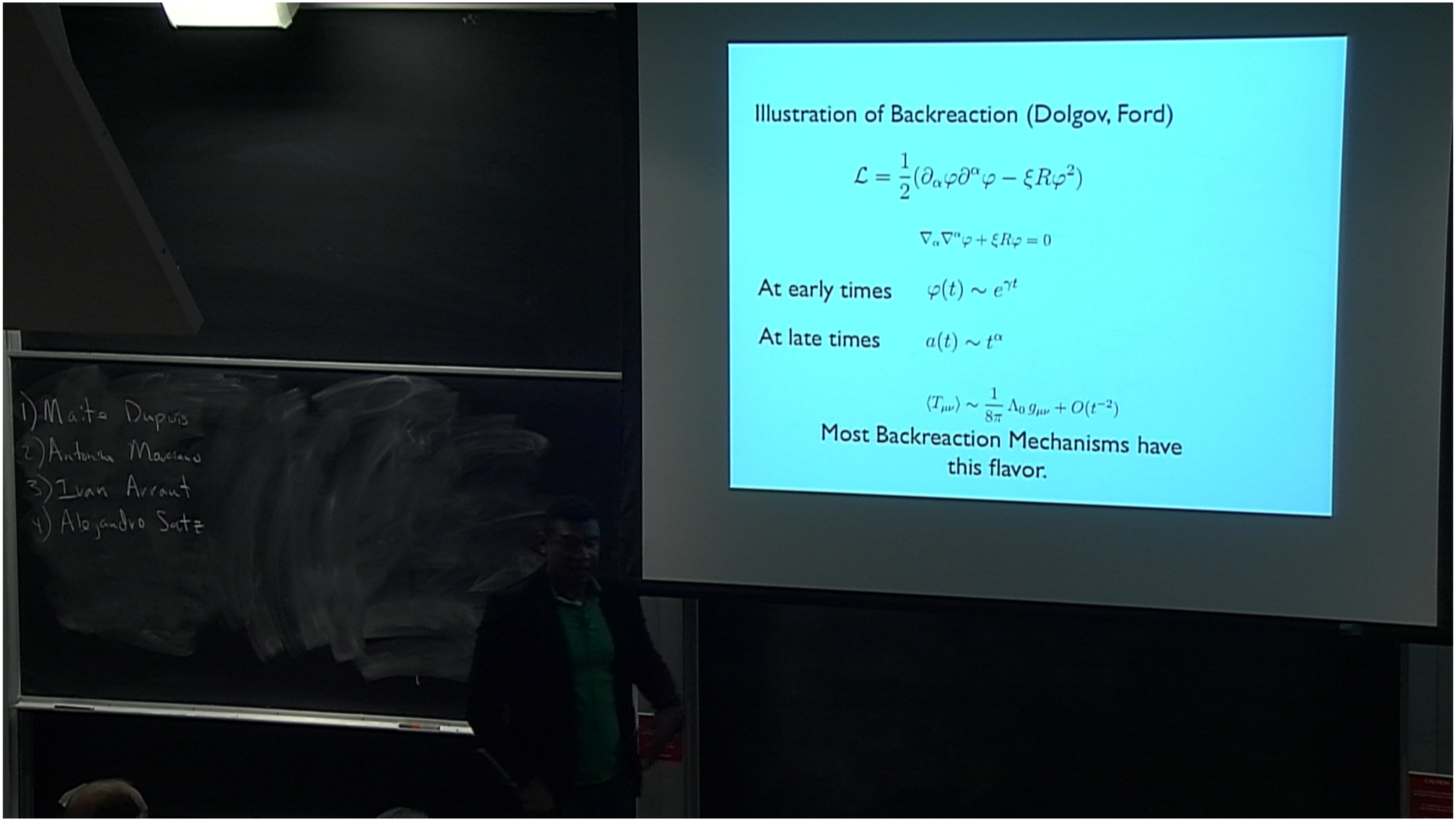
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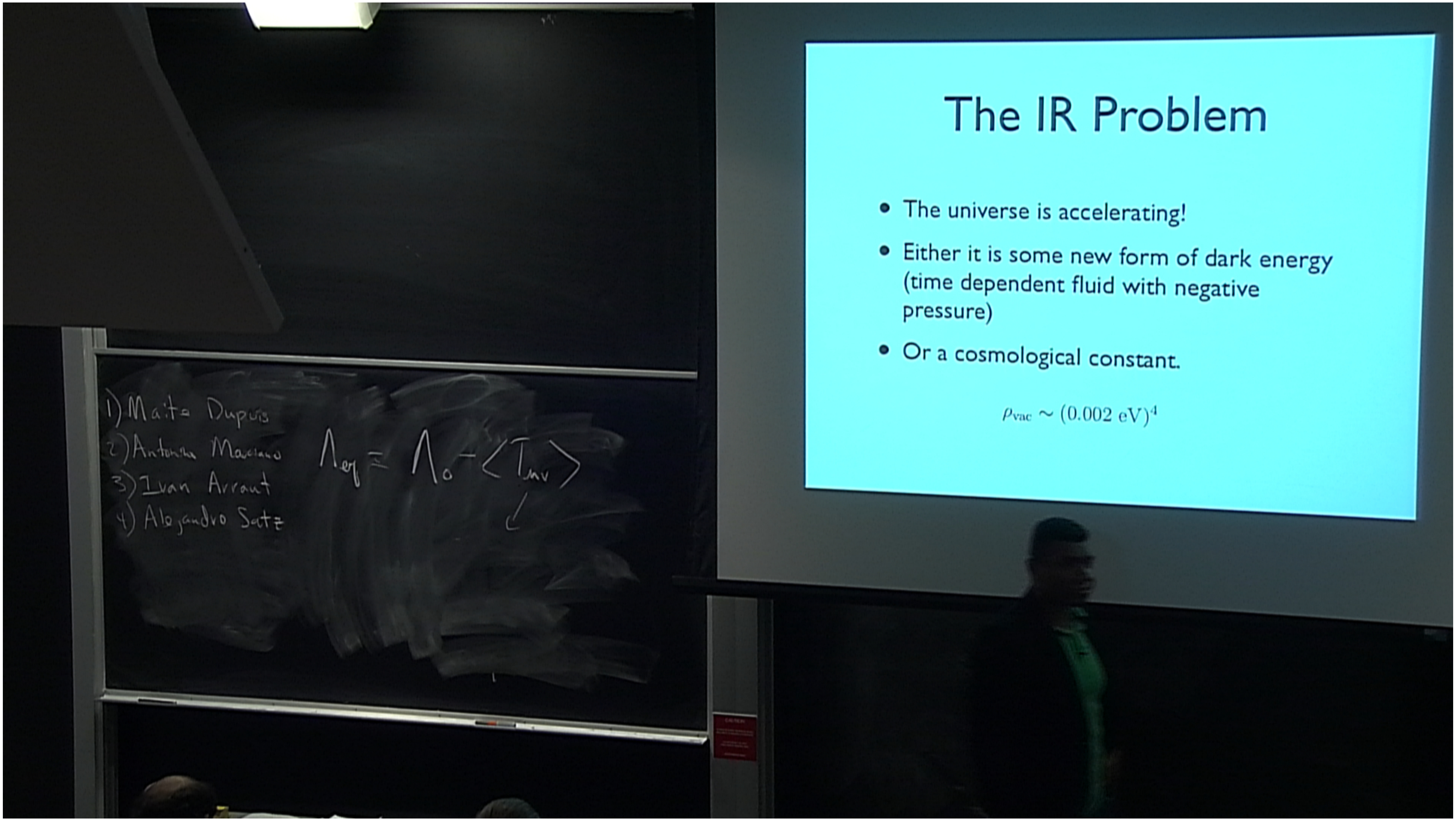
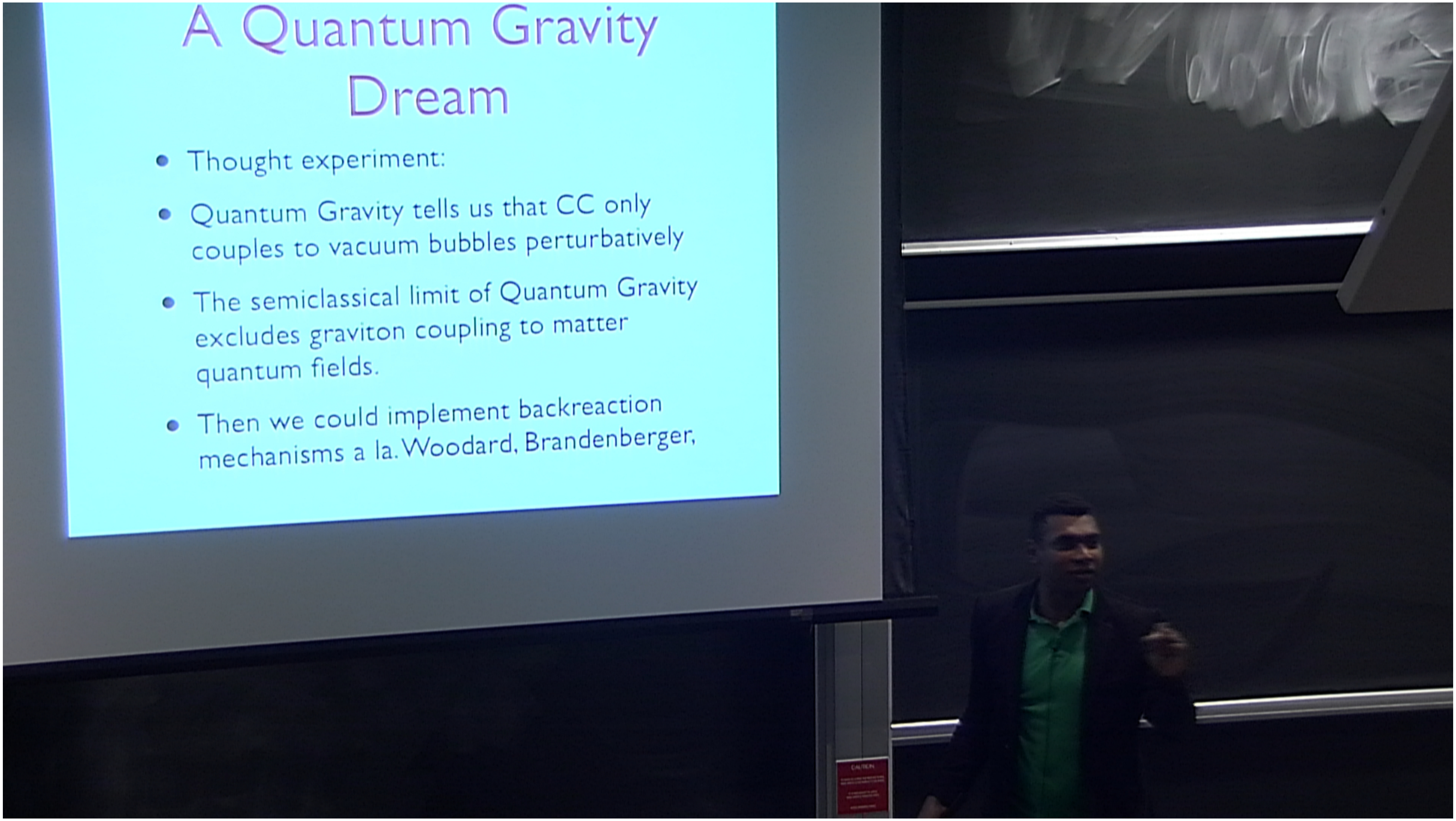
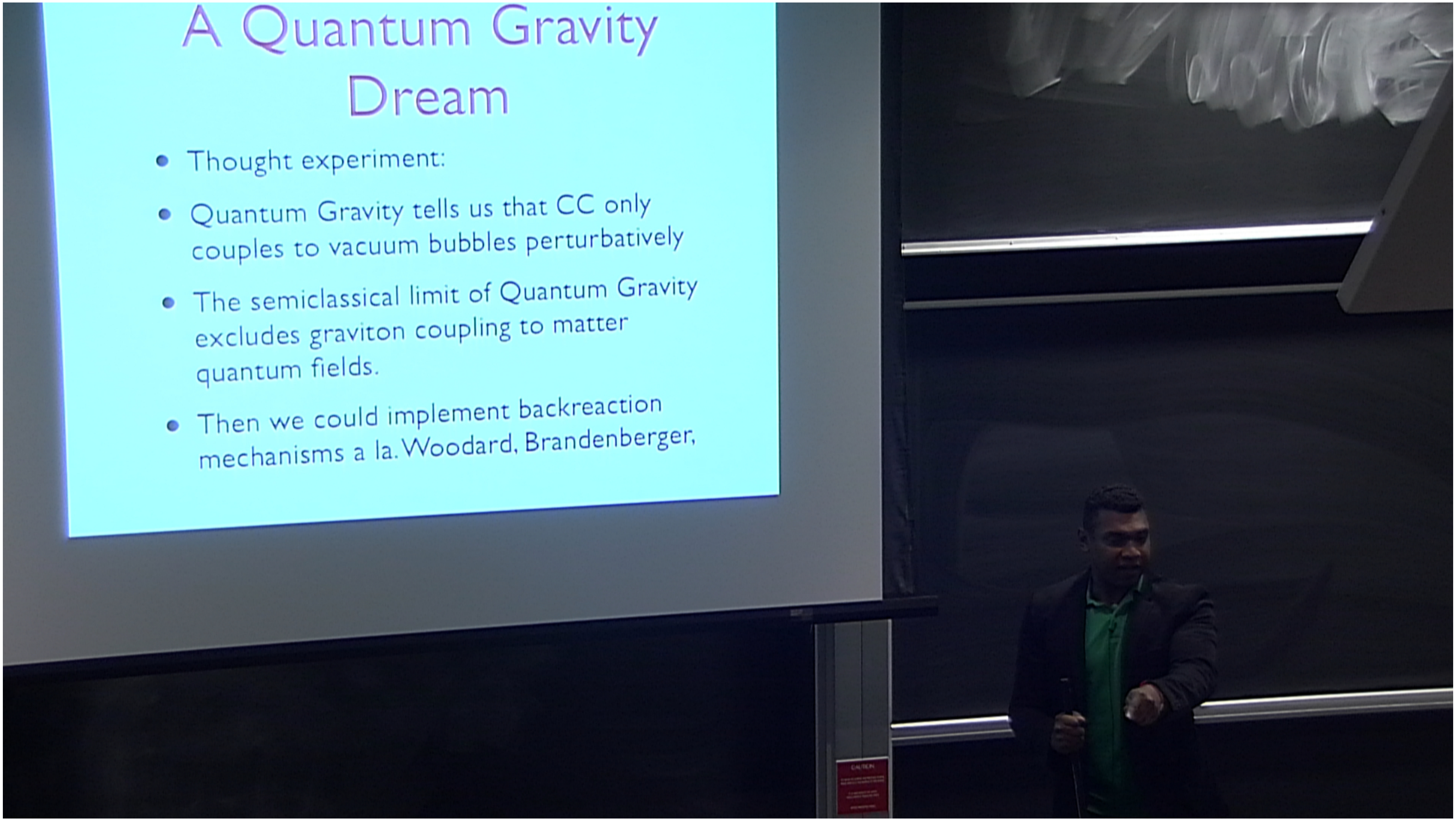

The IR Problem
The universe is accelerating!
Either it is some new form of dark energy (time dependent fluid with negative pressure)
Or a cosmological constant.
$\rho_{\rm vac} \sim (0.002 \text{ eV})^4$


A Quantum Gravity Dream
Thought experiment:
Quantum Gravity tells us that CC only couples to vacuum bubbles perturbatively
The semiclassical limit of Quantum Gravity excludes graviton coupling to matter quantum fields.
Then we could implement backreaction mechanisms a la. Woodard, Brandenberger,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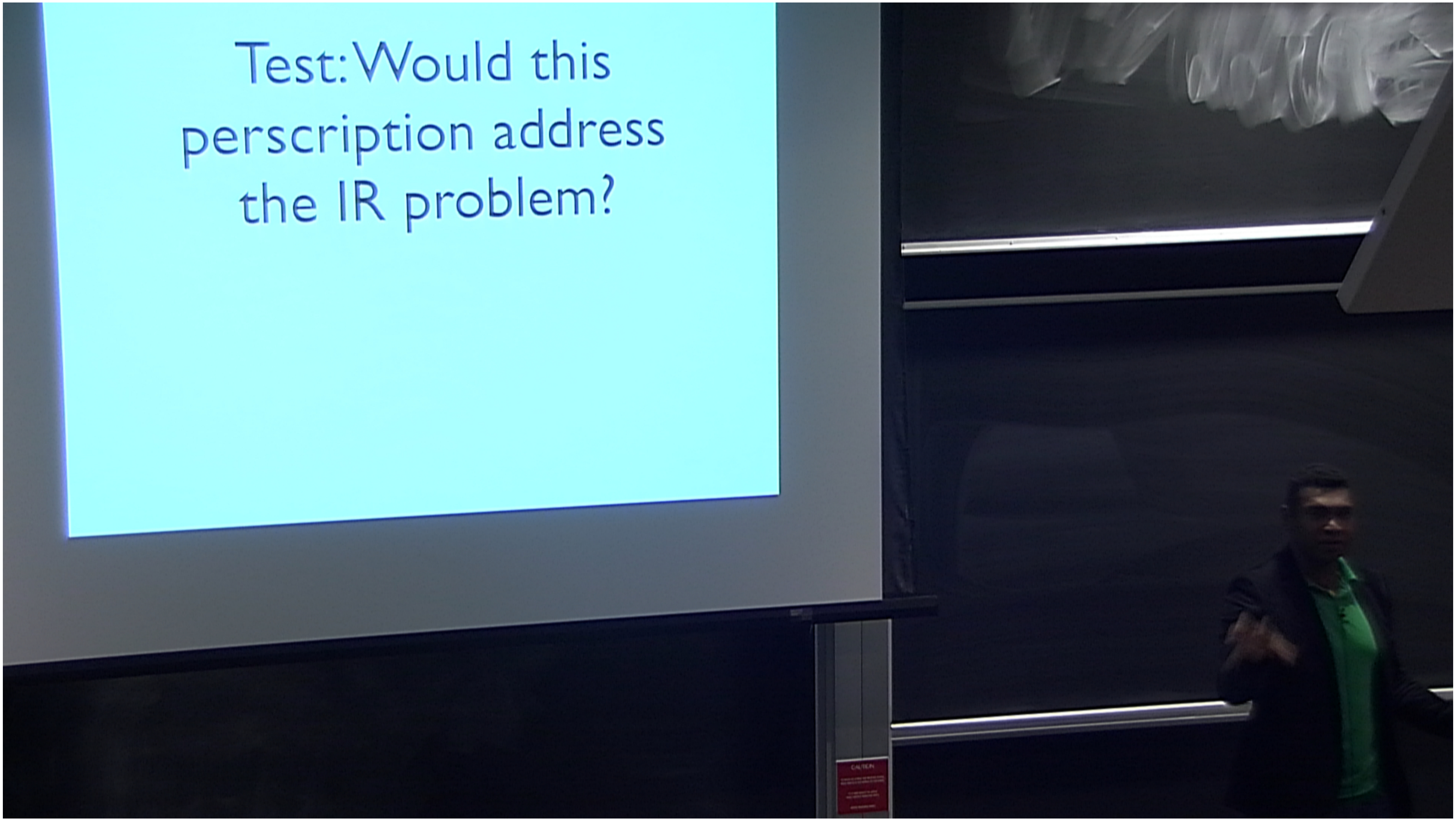
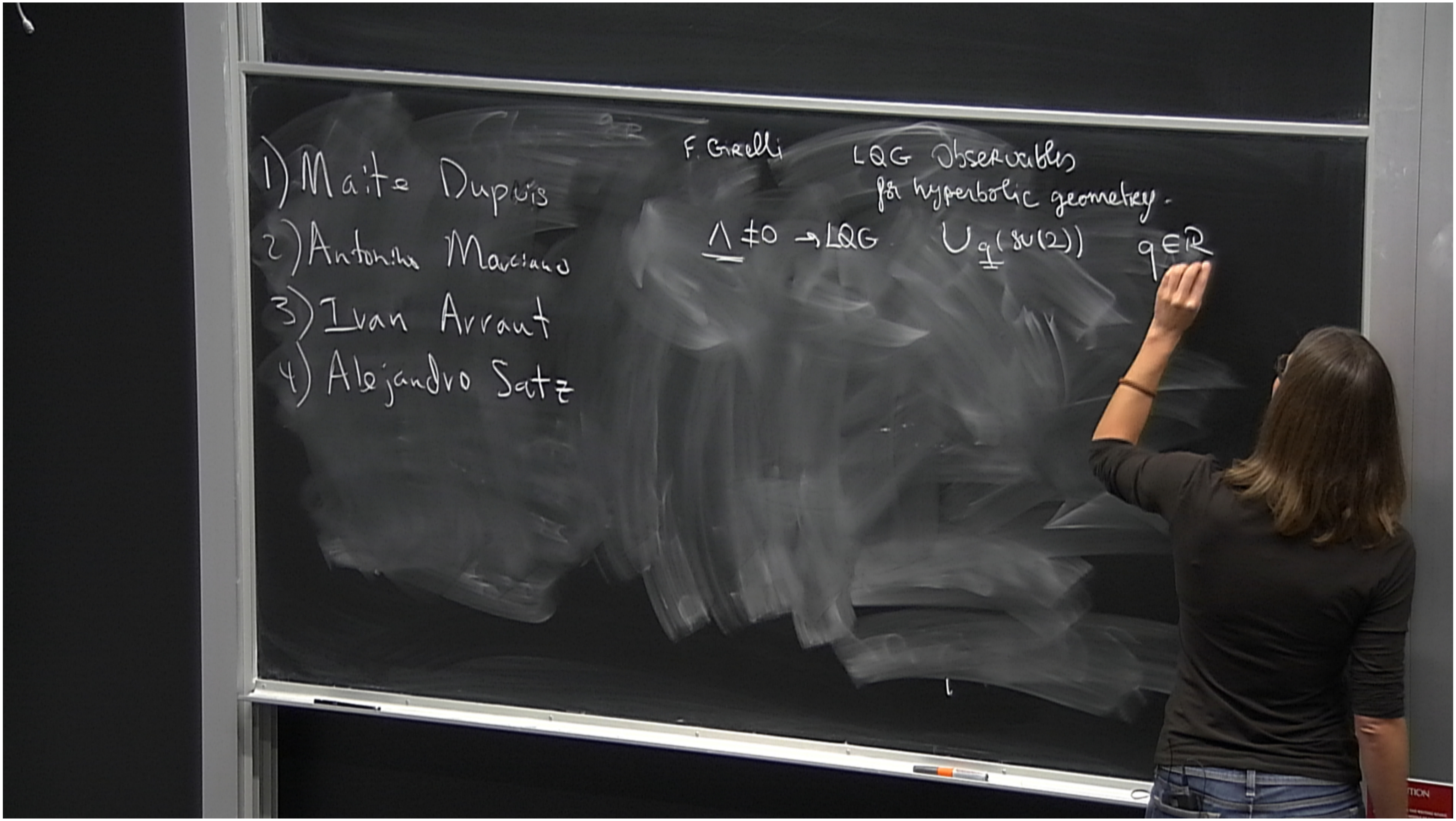

Test: Would this
perscription address
the IR problem?

Hint: Neutino Mass Oscillation

S.A th/0911.5156

$$\mu_{\rm vac} \sim m_\nu \sim \Lambda_{\rm ew}^2/M$$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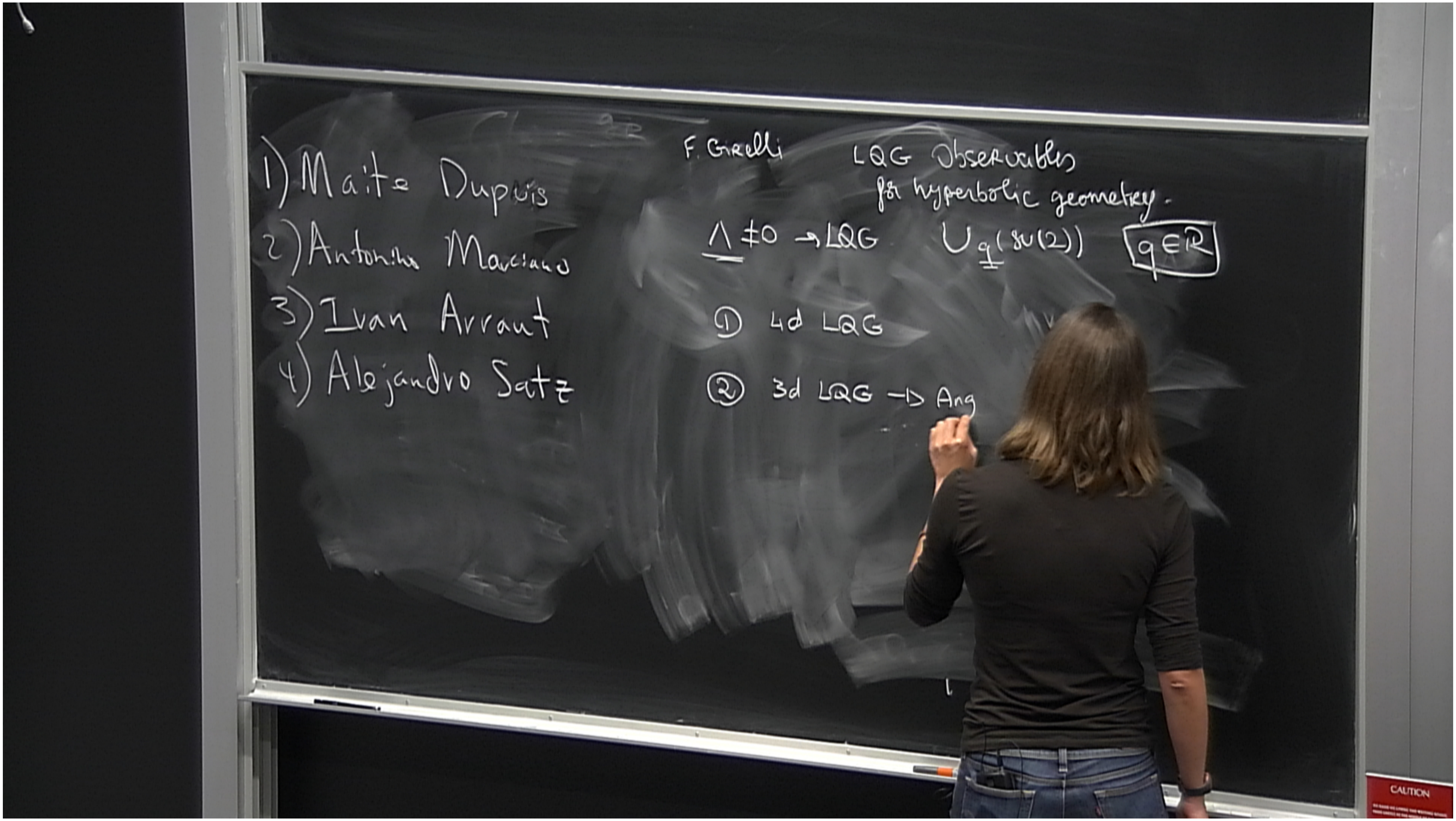
F. Girelli
LQG Observables
for hyperbolic geometry.
Λ≠0 → LQG
U_q(su(2))
q∈R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LQG Observables
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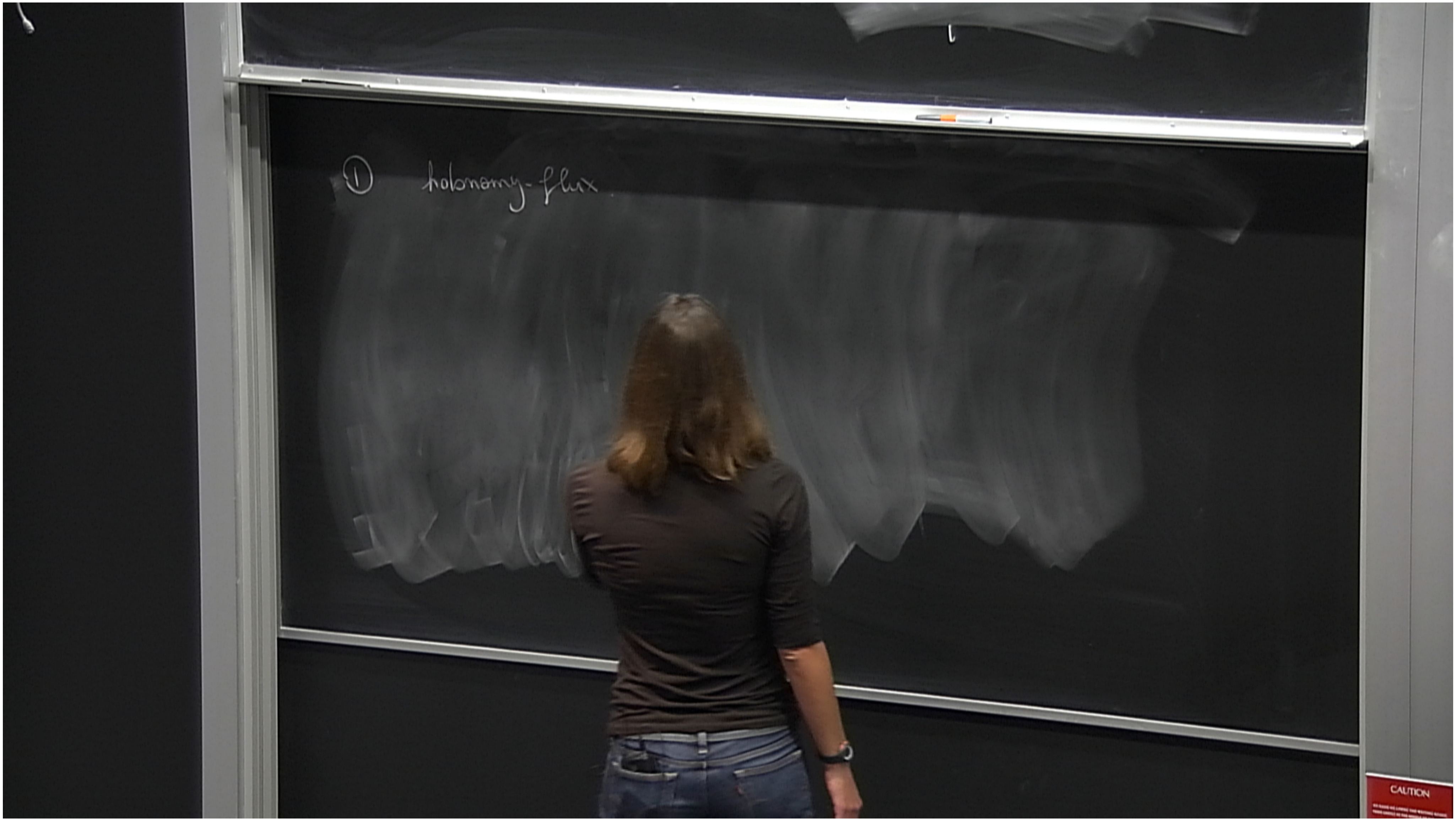
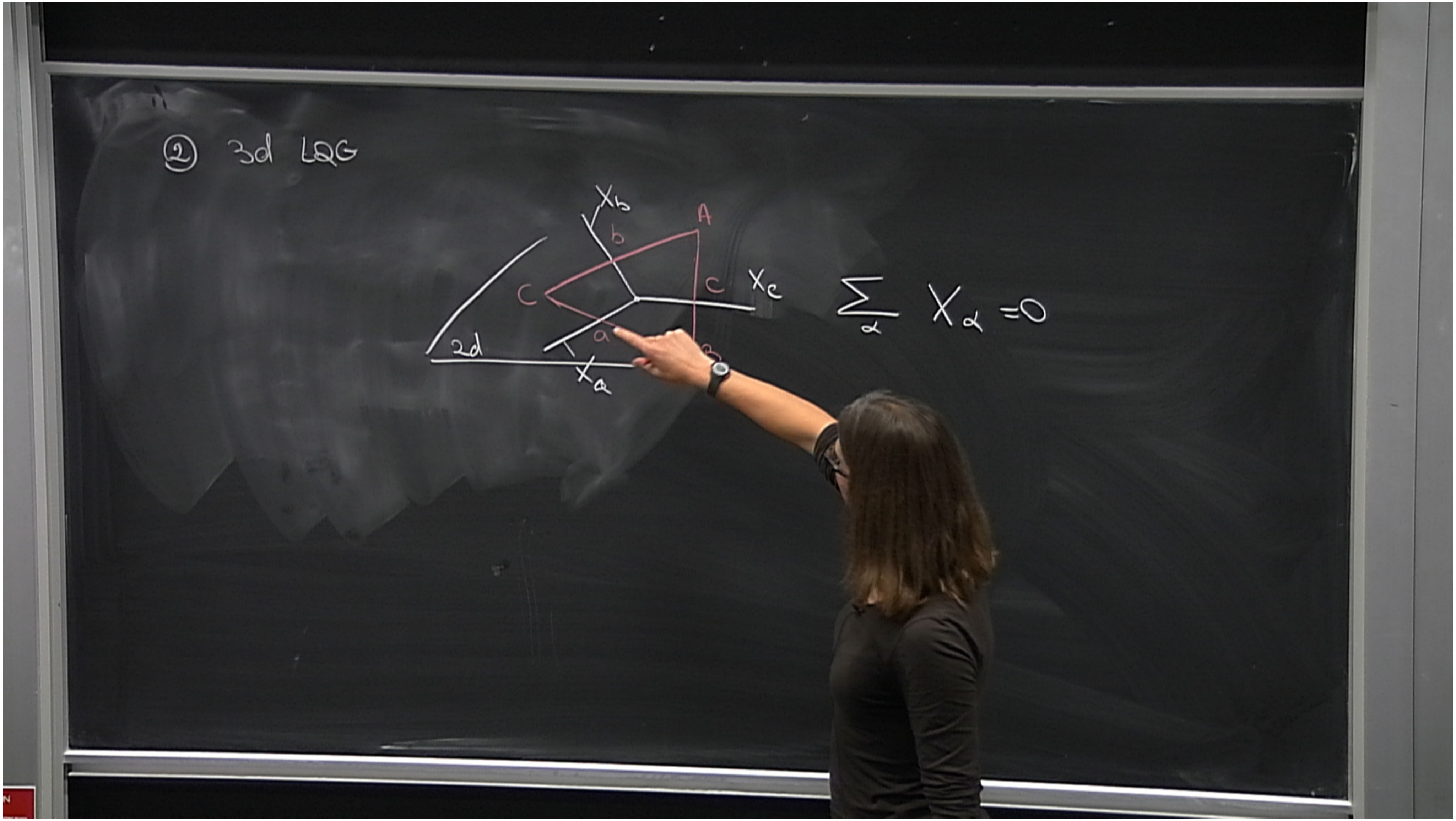
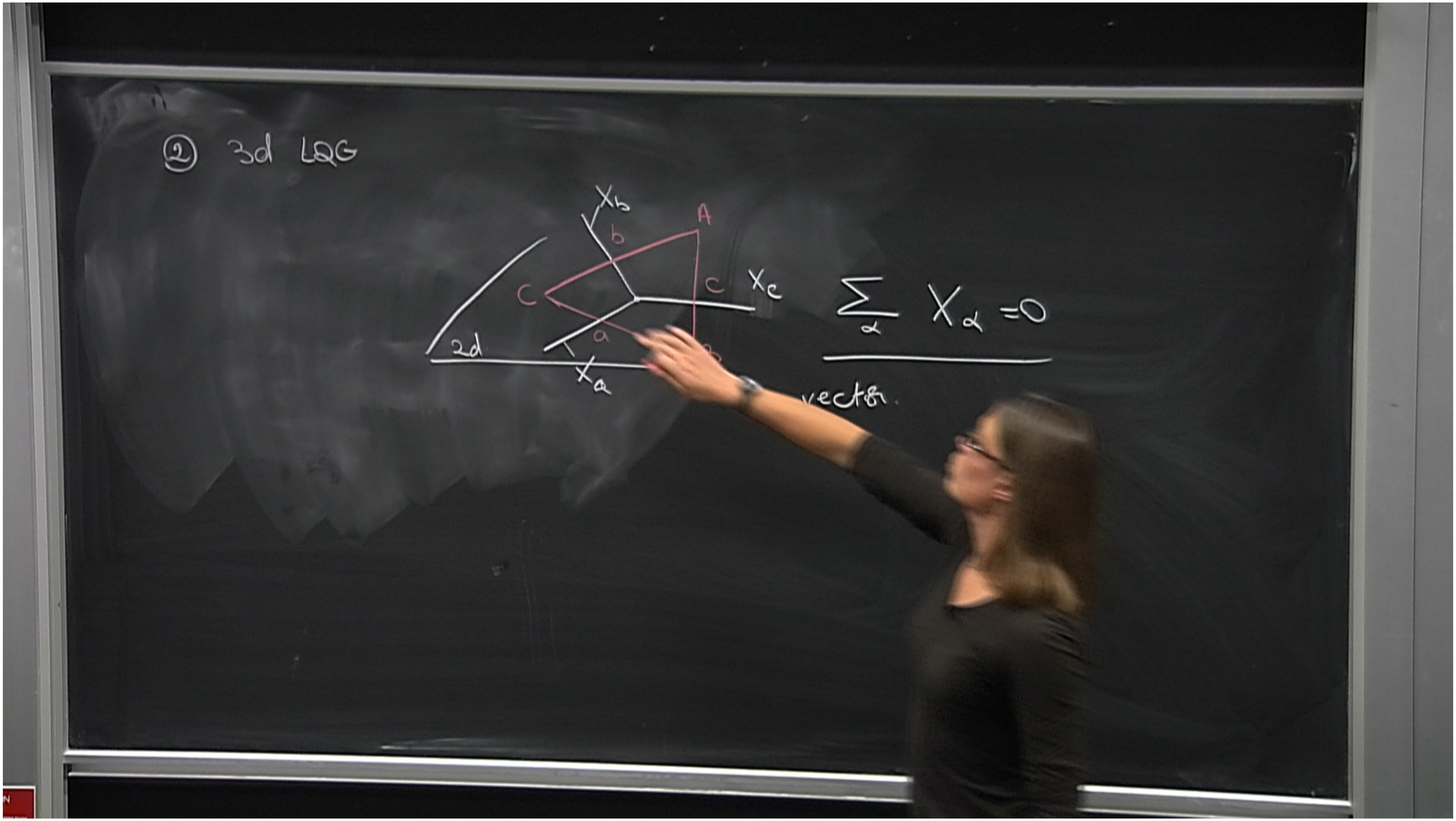
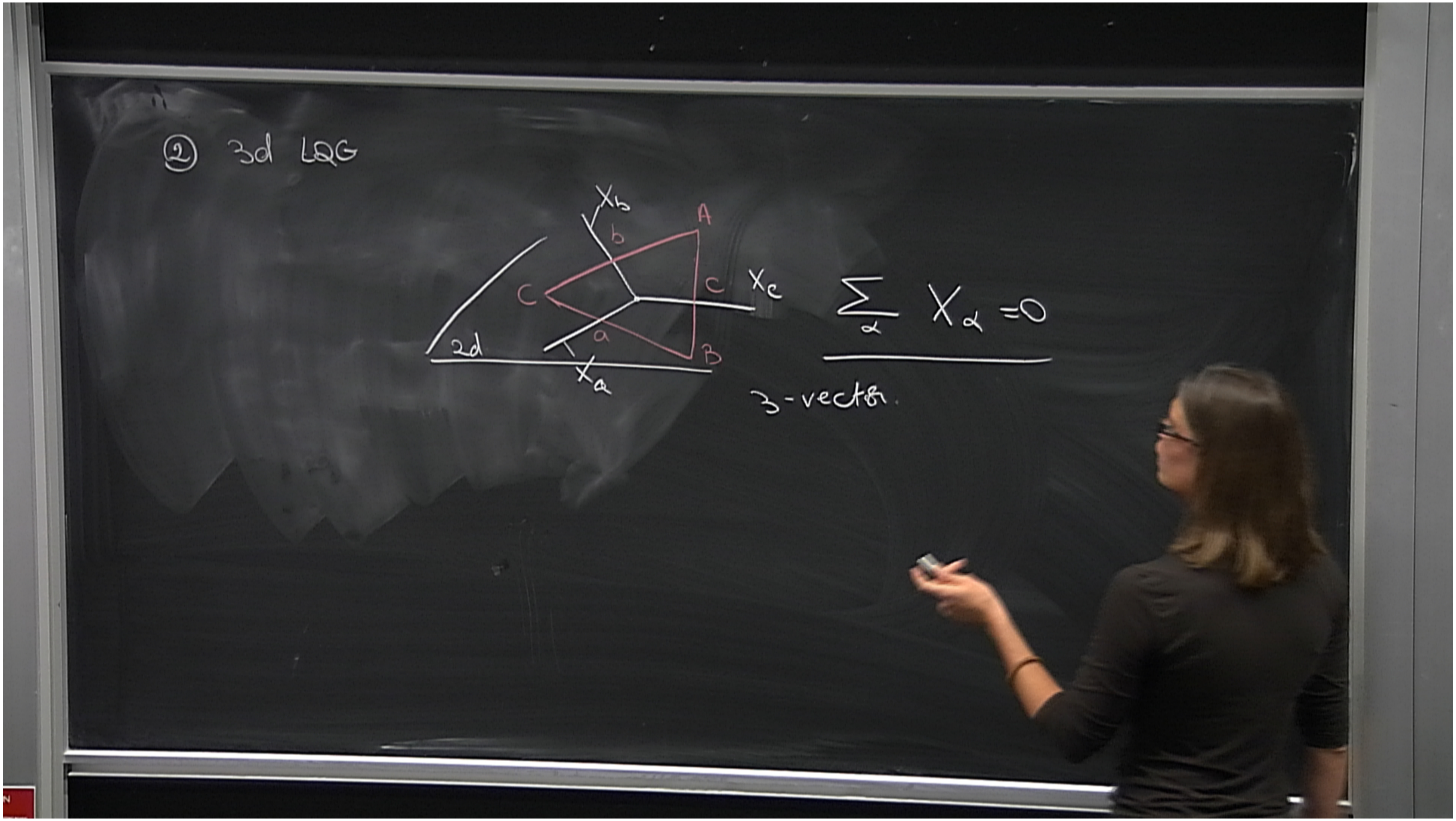
Λ ≠ 0 → LQG
U_q(su(2))
q ∈ R
① 4d LQG
② 3d LQG → Ang
CAUTION

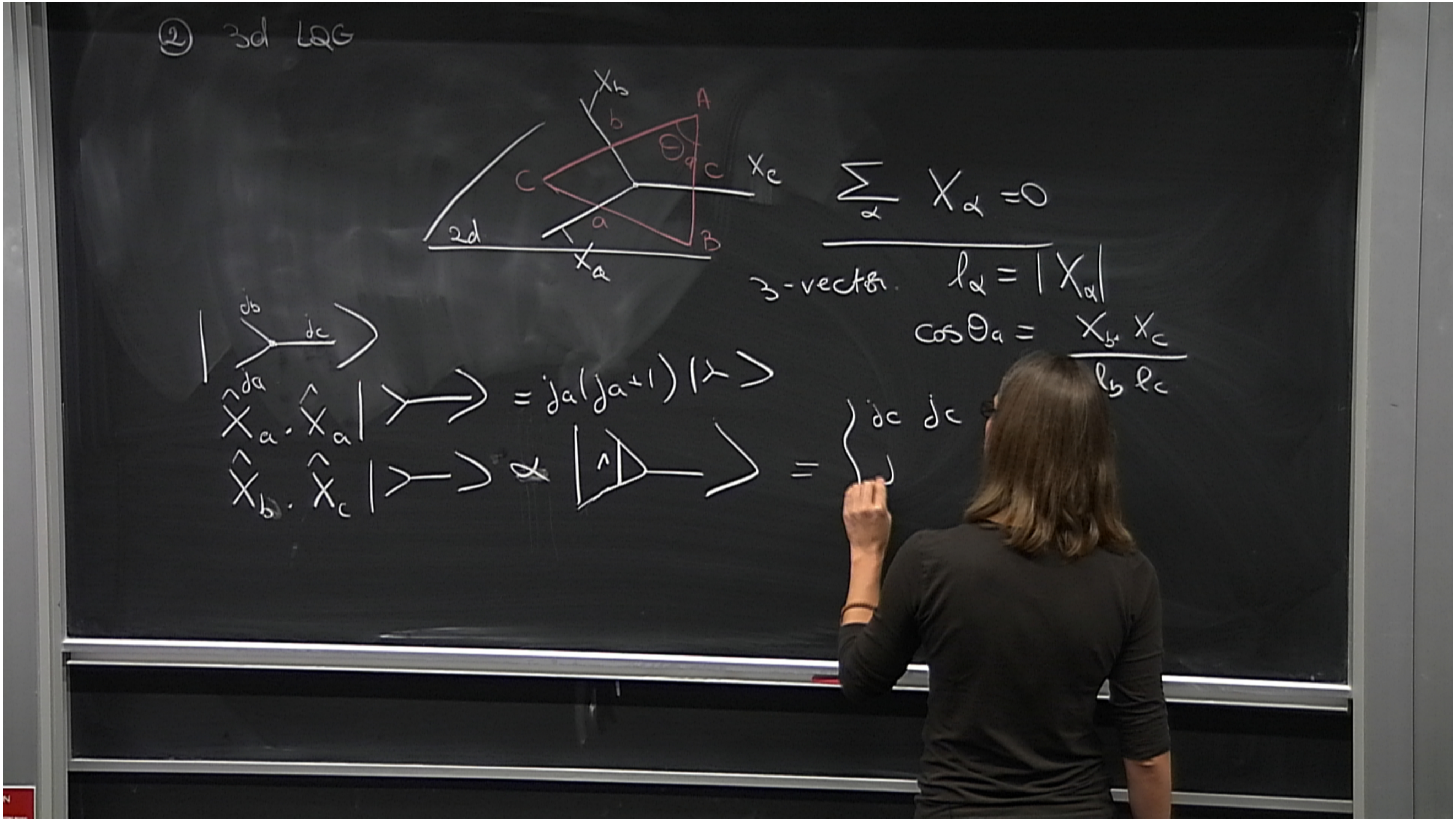

① holonomy-flux
CAUTION

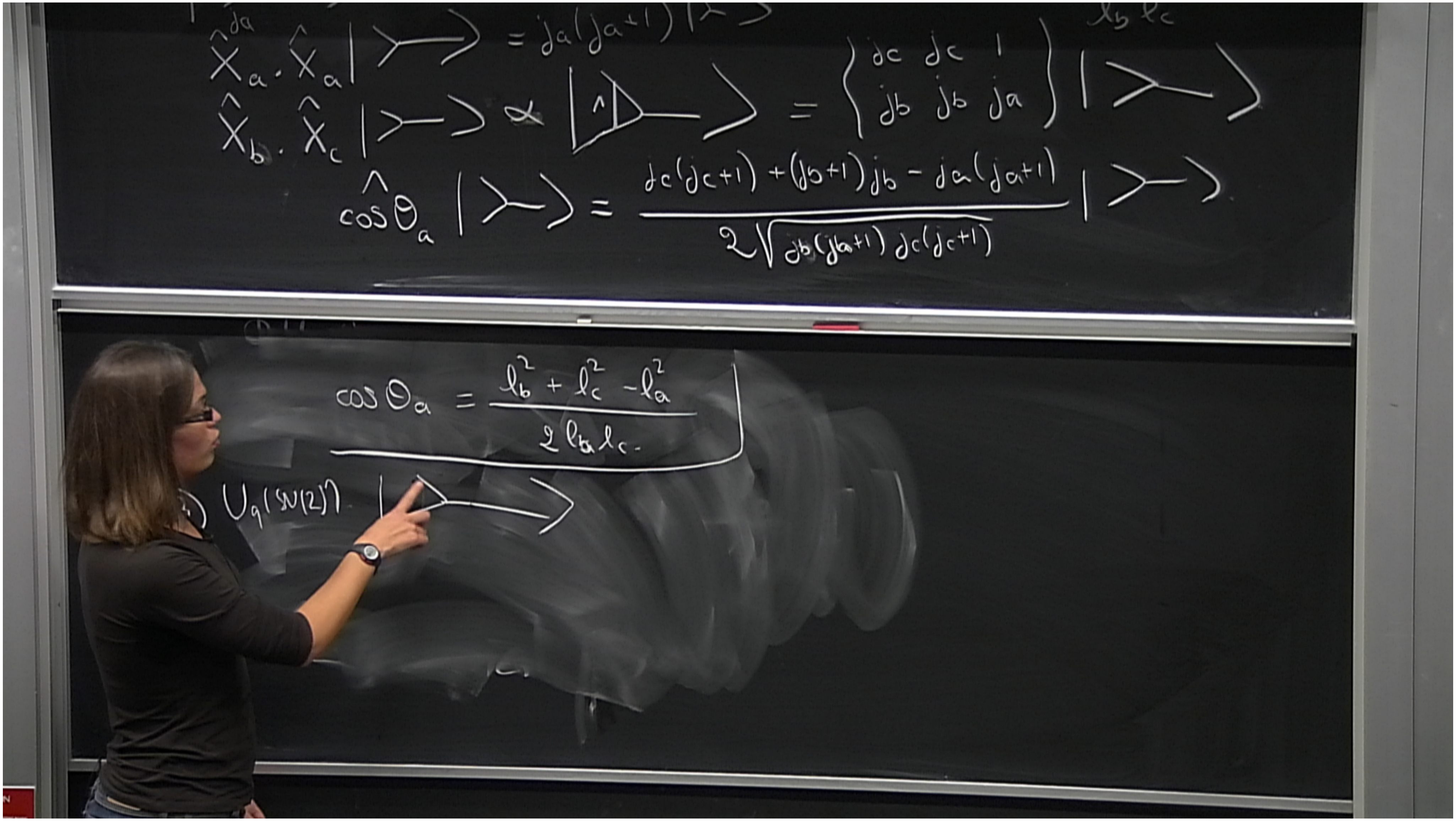


② 3d LQG
X_b
A
b
C
c
X_c
a
B
2d
X_a
$\sum_\alpha X_\alpha = 0$

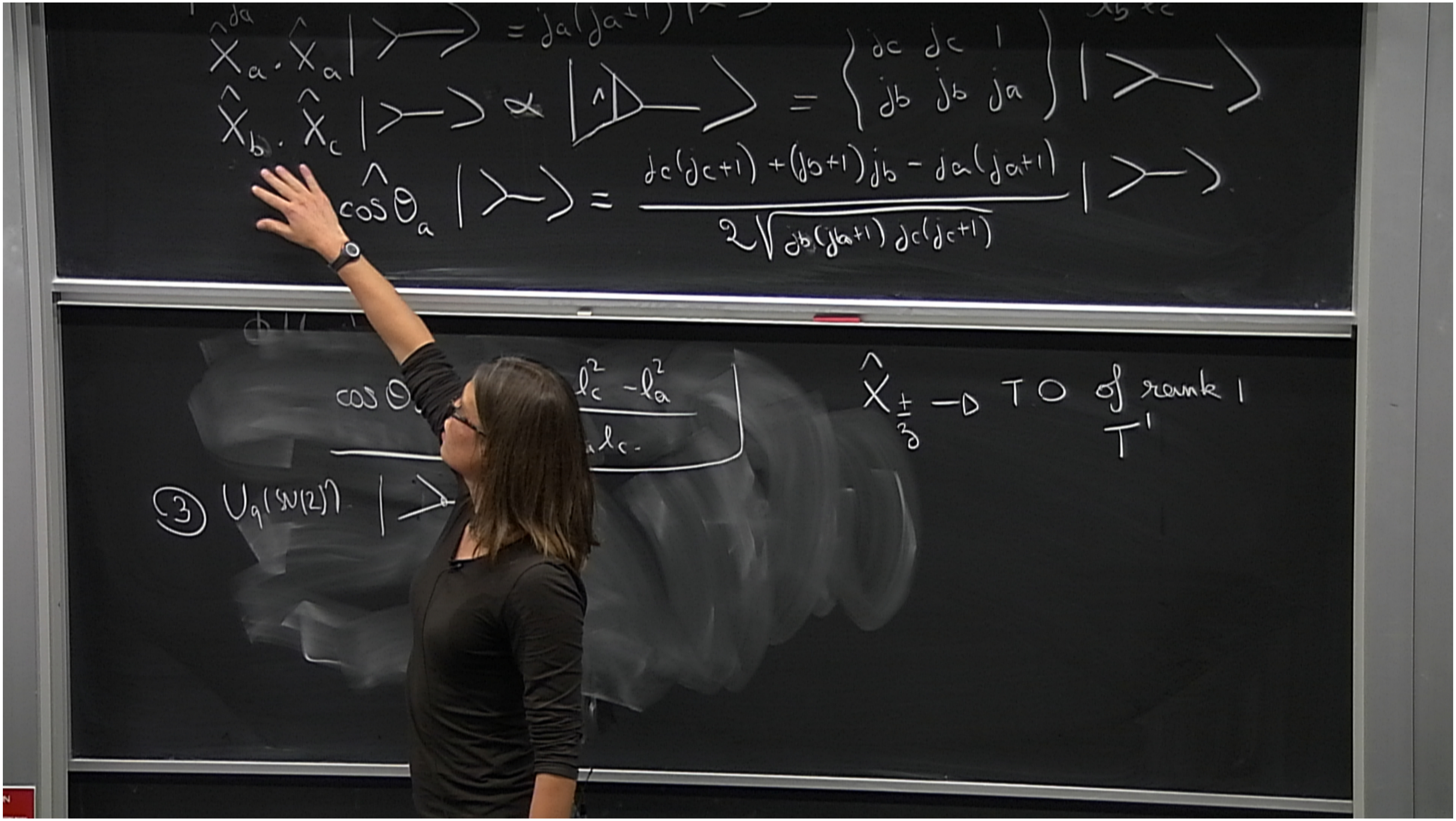

② 3d LQG
X_b
A
b
C
c
X_c
a
2d
X_a
$\sum_\alpha X_\alpha = 0$
vector

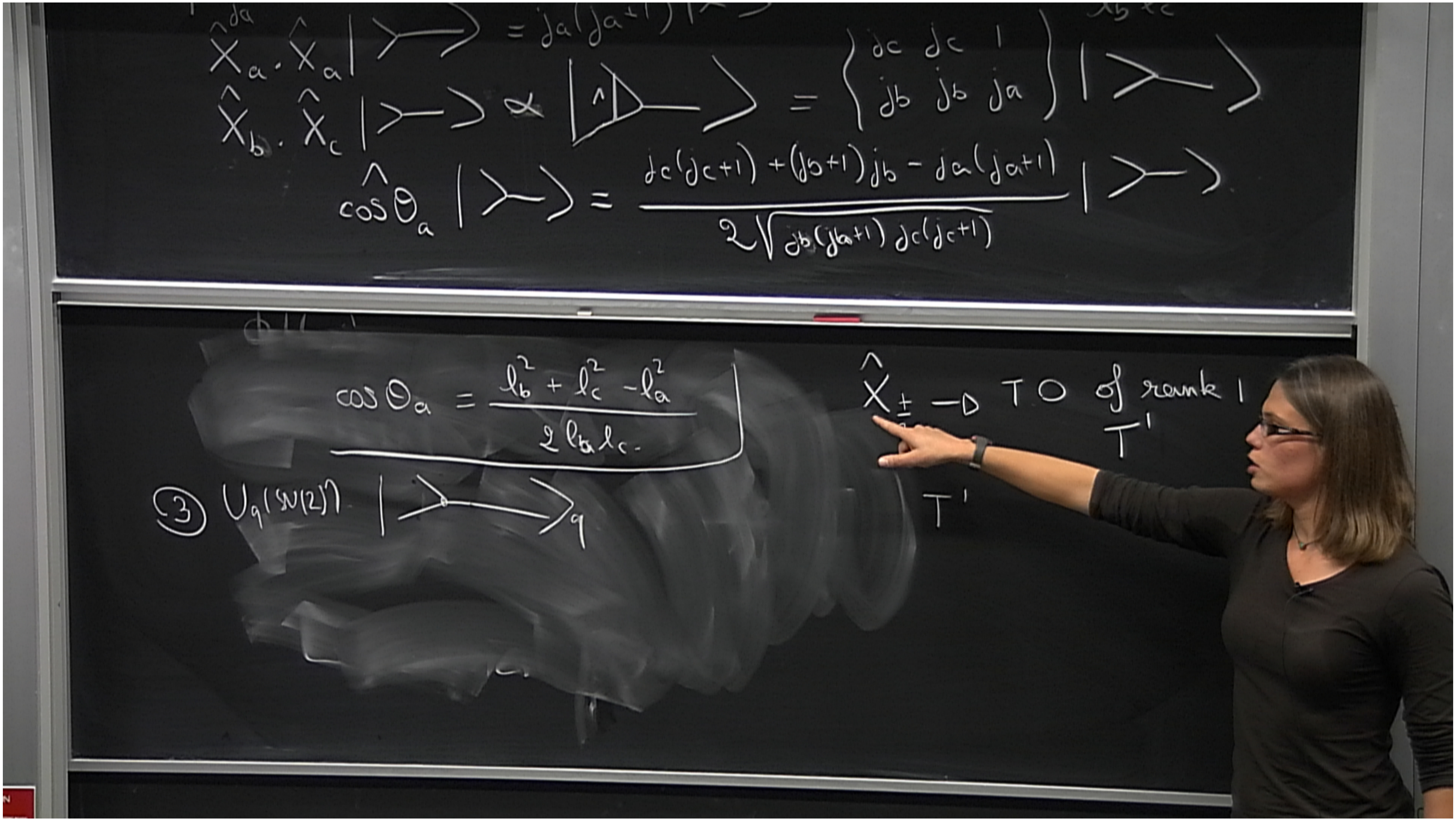

② 3d LQG
X_b
A
b
C
c
X_c
a
B
2d
X_a
$\sum_\alpha X_\alpha = 0$
3-vector

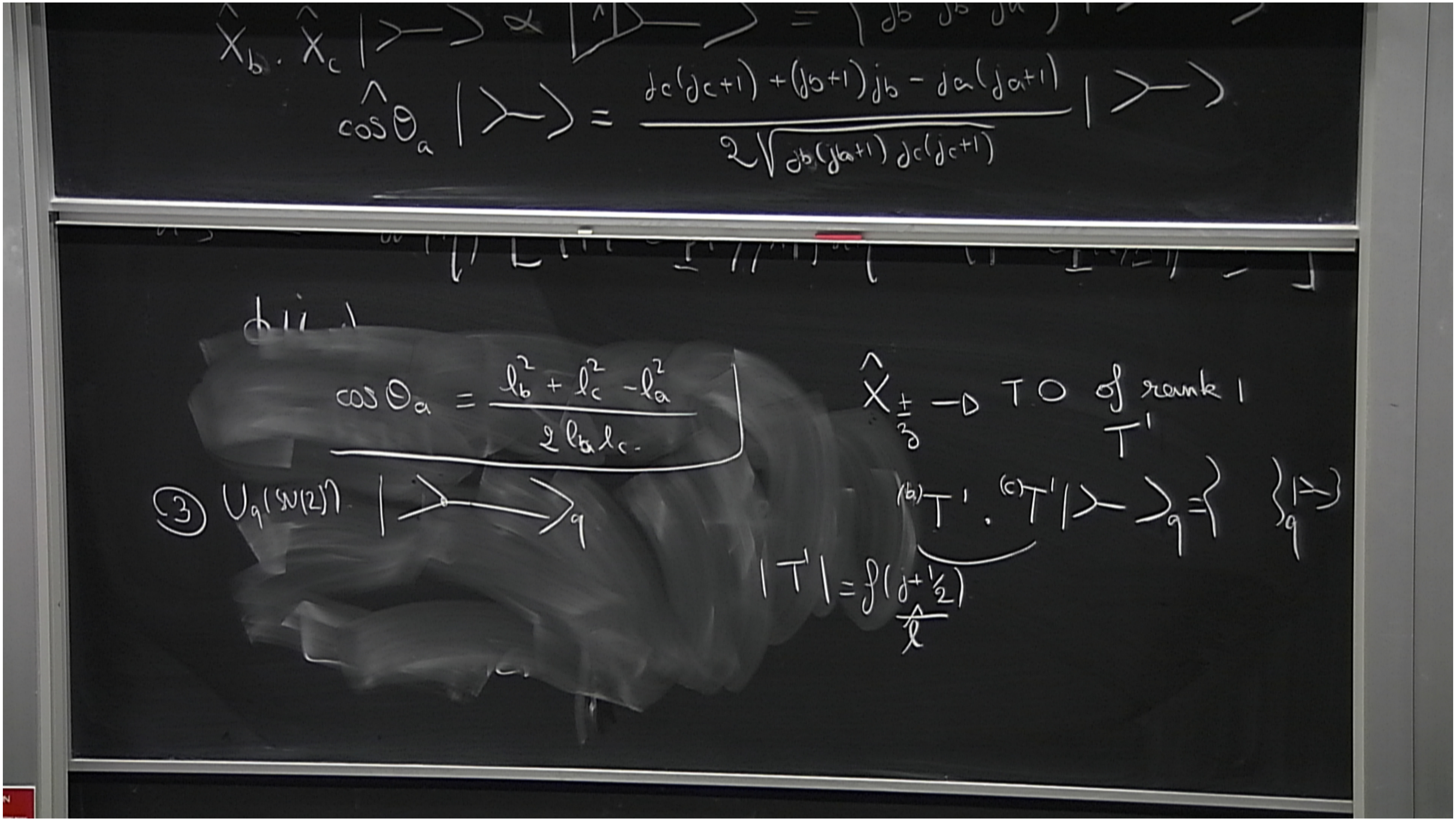

② 3d LQG
2d
A
B
C
a
b
c
Xa
Xb
Xc
Σα Xα = 0
3-vector
lα = |Xα|
cos θa = Xb·Xc / lb lc
ja
jb
jc
= ja(ja+1)

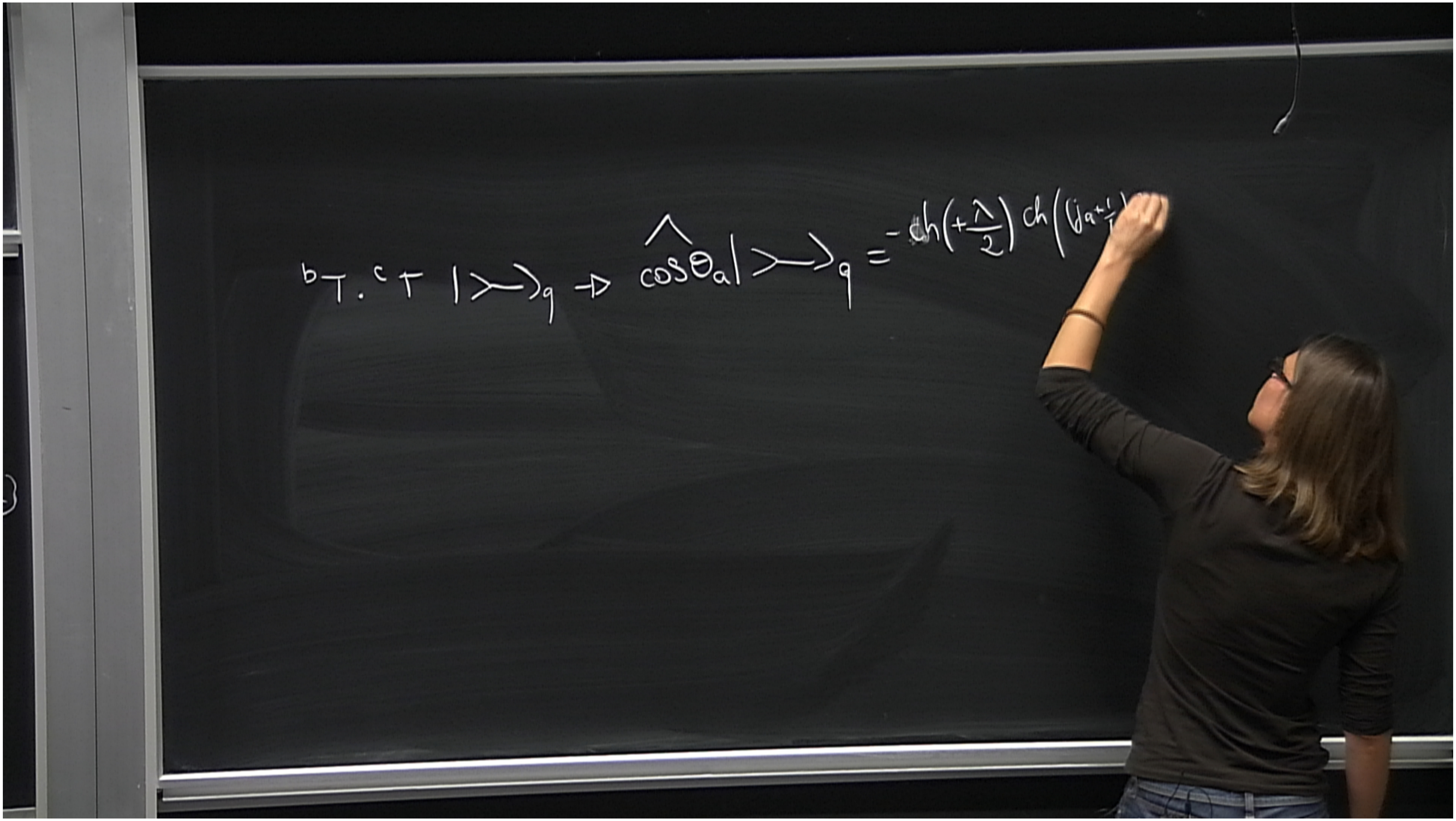


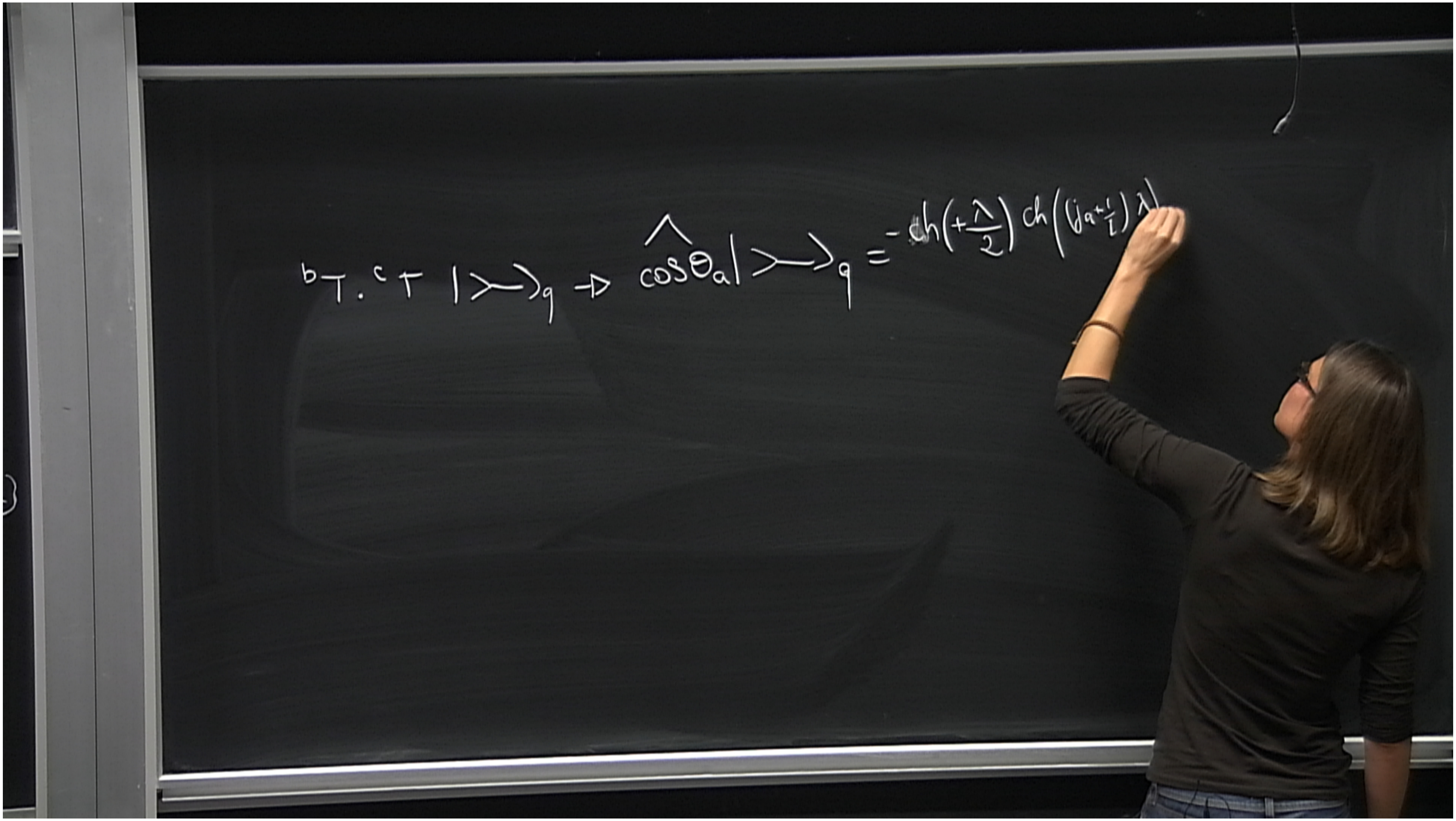
$\hat{X}_a \cdot \hat{X}_a | \succ \rangle = j_a(j_a+1) | \succ \rangle$
$\hat{X}_b \cdot \hat{X}_c | \succ \rangle \propto | \triangleright \rangle = \begin{Bmatrix} j_c & j_c & 1 \\ j_b & j_b & j_a \end{Bmatrix} | \succ \rangle$
$\hat{\cos\theta_a} | \succ \rangle = \frac{j_c(j_c+1) + (j_b+1)j_b - j_a(j_a+1)}{2\sqrt{j_b(j_b+1)\, j_c(j_c+1)}} | \succ \rangle$
$\cos\theta_a = \frac{l_b^2 + l_c^2 - l_a^2}{2 l_b l_c}$
$U_q(SU(2))?$

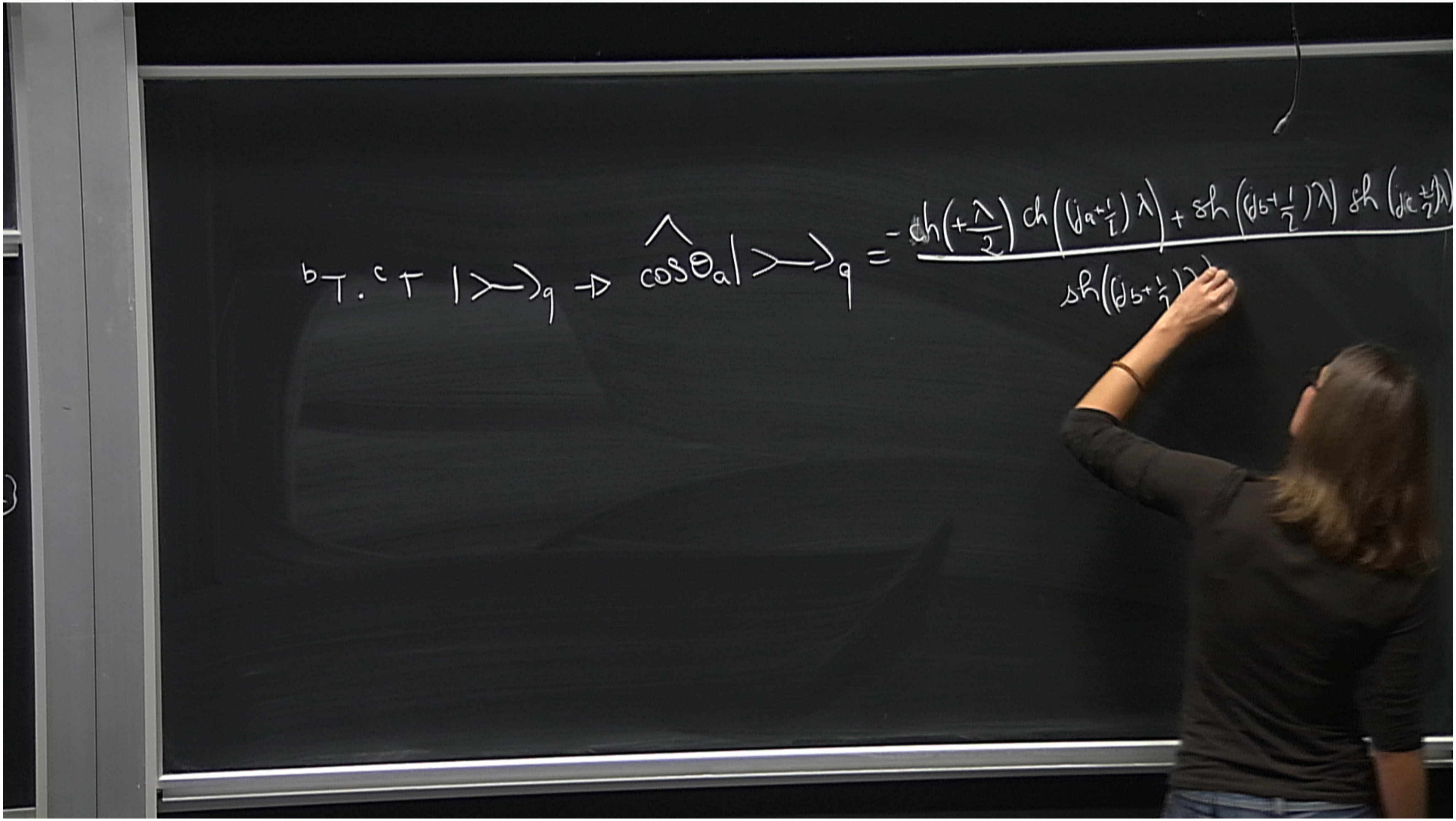

③ Uq(su(2))
TO of rank 1

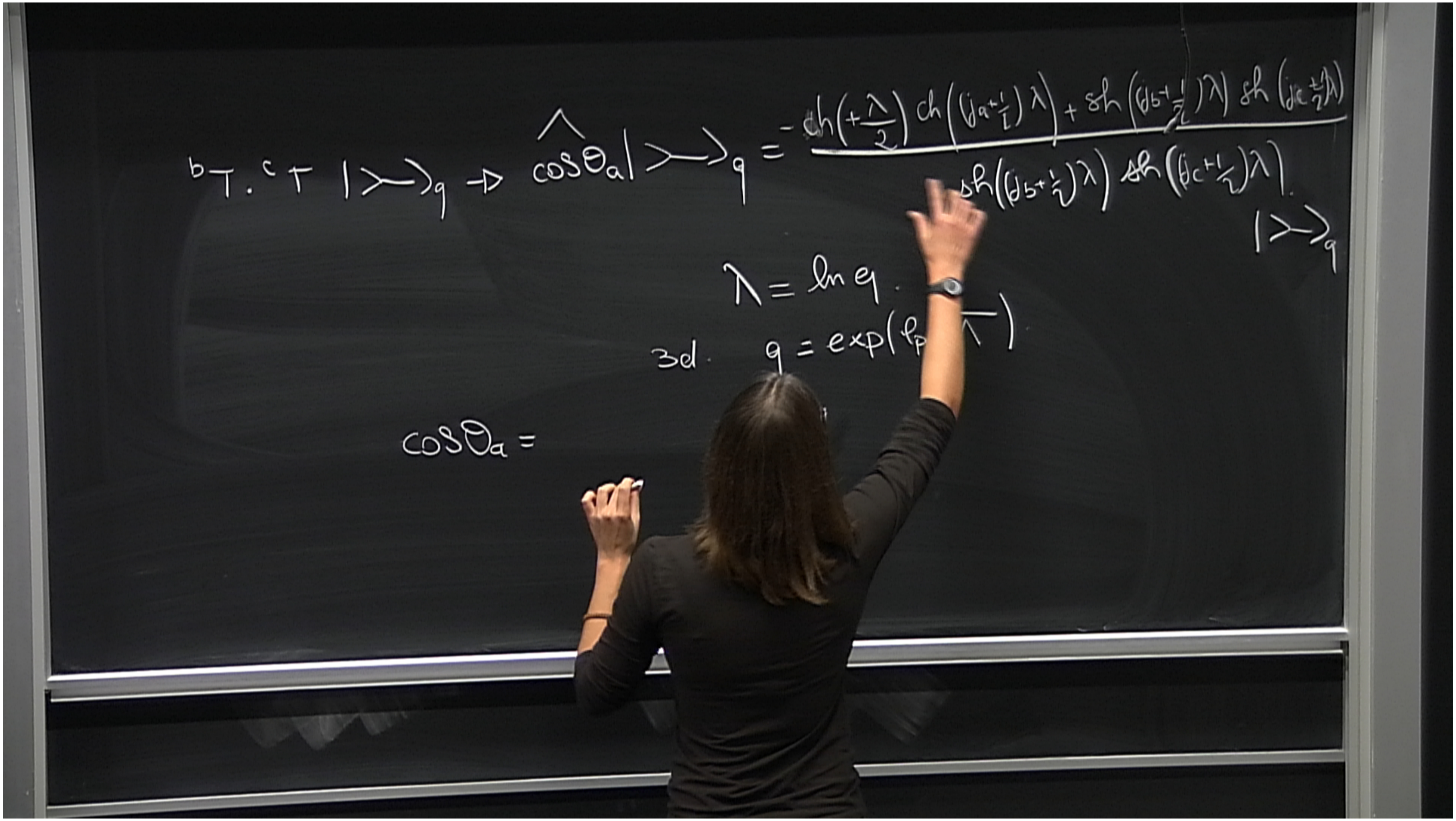

cos θa = (lb² + lc² − la²) / (2 lb lc)
③ Uq(su(2))
X̂± → TO of rank 1
T'
T'

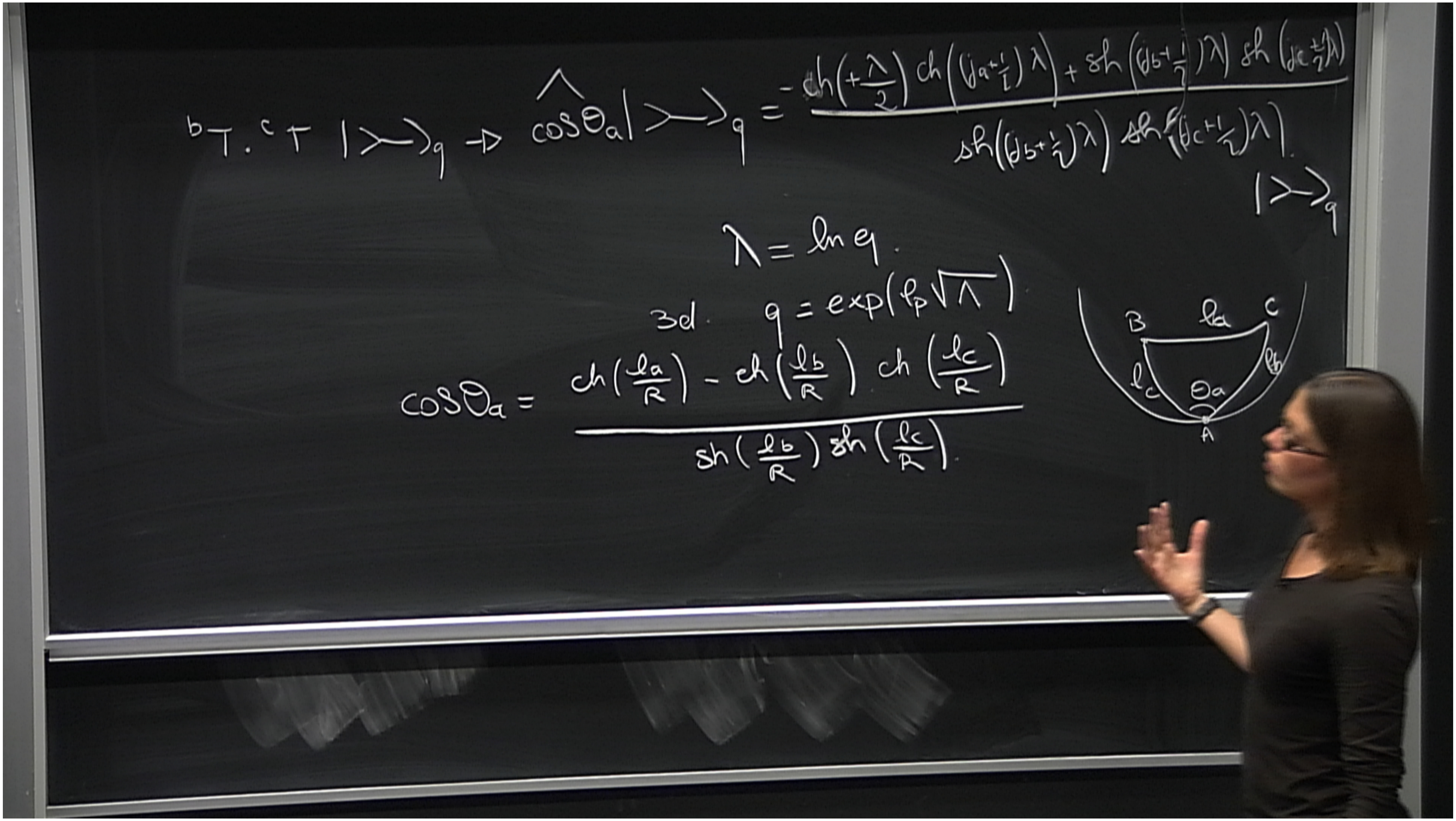

$\hat{X}_b \cdot \hat{X}_c |\rangle \propto$
$\widehat{\cos\theta_a} |\rangle = \frac{j_c(j_c+1) + (j_b+1)j_b - j_a(j_a+1)}{2\sqrt{j_b(j_b+1)\, j_c(j_c+1)}} |\rangle$
$\cos\theta_a = \frac{l_b^2 + l_c^2 - l_a^2}{2 l_b l_c}$
$\hat{X}_{\pm,3} \to$ TO of rank 1
T^1
③ $U_q(su(2))$ $|\rangle_q$
${}^{(b)}T^1 \cdot {}^{(c)}T^1 |\rangle_q = \{\ \}_q |\rangle$
$|T^1| = f(j+\tfrac{1}{2})$
$\hat{l}$

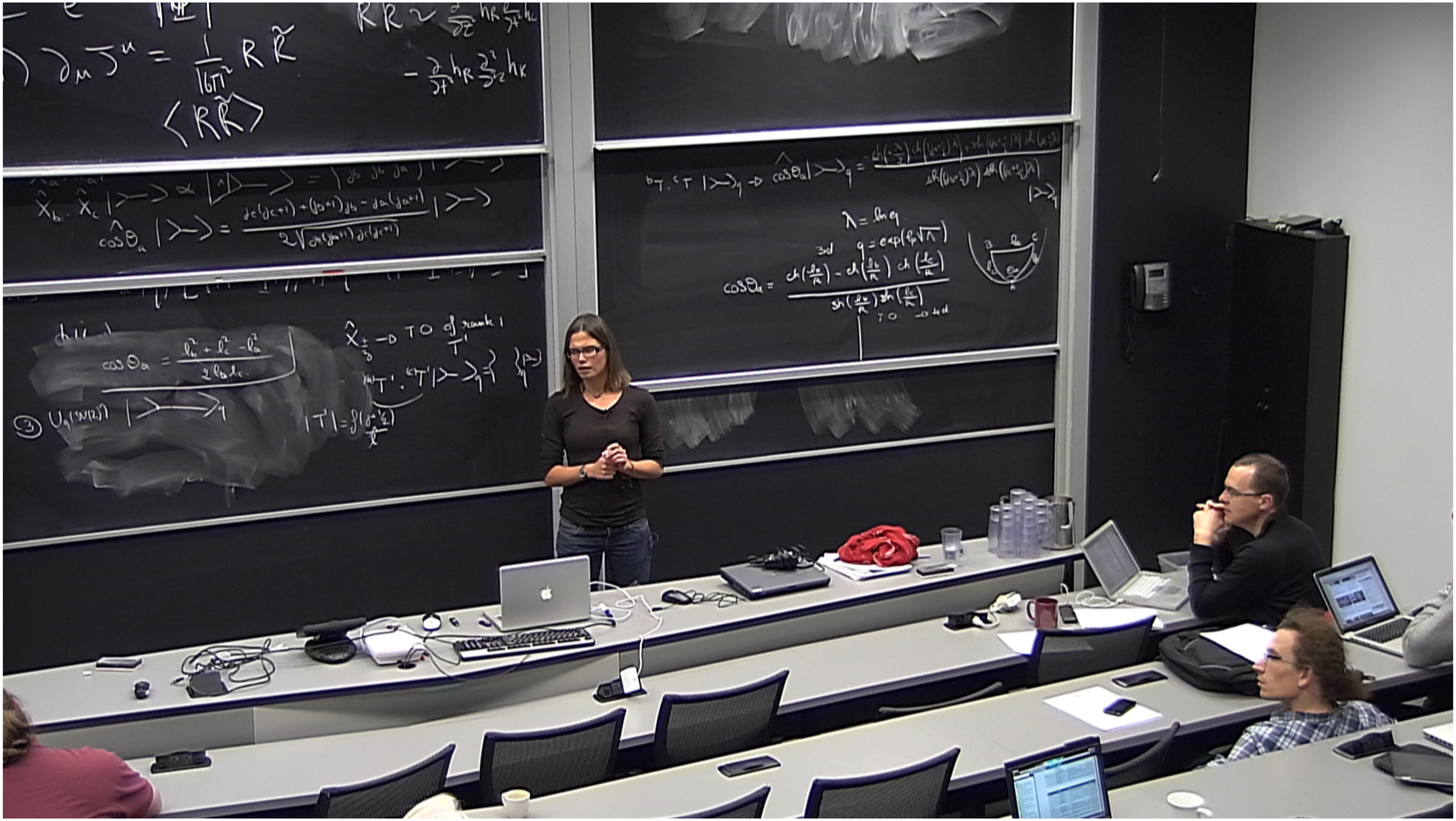



$^bT \cdot {}^cT \,|\succ\rangle_q \to \widehat{\cos\theta_a}\,|\succ\rangle_q = -\mathrm{ch}\left(+\frac{\lambda}{2}\right)\mathrm{ch}\left(\left(j_a+\frac{1}{2}\right)\lambda\right)$


$^{b}T \cdot {}^{c}T \,|\succ\rangle_q \rightarrow \widehat{\cos\theta_a}\,|\succ\rangle_q = -\frac{ch(+\frac{\lambda}{2})\, ch((j_a+\frac{1}{2})\lambda) + sh((j_b+\frac{1}{2})\lambda)\, sh((j_c+\frac{1}{2})\lambda)}{sh((j_b+\frac{1}{2})\lambda)}$


λ = ln q
3d. q = exp(
cos θa =


λ = ln q
3d   q = exp(ℓp √Λ)
cos θa = [ch(ℓa/R) − ch(ℓb/R) ch(ℓc/R)] / [sh(ℓb/R) sh(ℓc/R)]
B
C
A
ℓa
ℓb
ℓc
θa



Why we do care about H

Flat space-time:

Quantum effects and gravitational force negligible but $\sqrt{\frac{\hbar}{G_{\mathrm{Newton}}}} = m_p$

Observer independent laws | 3D QG results | 4D LQG results | 4D NC space-time

Phenomenology: burst of particles at cosmological distances

Curved space-time:

Planck-scale-deformed relativistic symmetries of a space- time with constant rate of expansion (dS)

A brief (pre)-history of H in DSR

Pre-Relative Locality:

Amelino-Camelia, IJMPD 2002 & PLB 2001; Magueijo & Smolin, PRL 2002; Kowalski-Glikman & Nowak, PLB 2002

DSR in flat space-time

Observer independent laws and (additional) invariant energy scale
Implementation in terms of quantum groups
Approximated phenomenology of GRB on flat space-time

AM, Amelino-Camelia, Bruno, Gubitosi, Mandanici & Melchiorri, arXiv:1004.1110 JCAP 1006 (2010) 030

"Quantum-groups" approach to curved space-time

Interplay between between Plansk-scale and curvature effects!
Consequences in cosmology and astrophysics using q-dS algebra
dS-slicing of FRW and application to quantum space-time

Toward Relative locality:

Hossenfelder, PRL 2010

Challenges for the description of locality in DSR theories

Relative locality:

Amelino-Camelia, Matassa, Mercati & Rosati, arXiv:1006.2126 PRL 2011
Smolin arXiv:1007.0718
Amelino-Camelia, Freidel, Kowalski-Glikman & Smolin, arXiv:1101.0931 PRD 2011

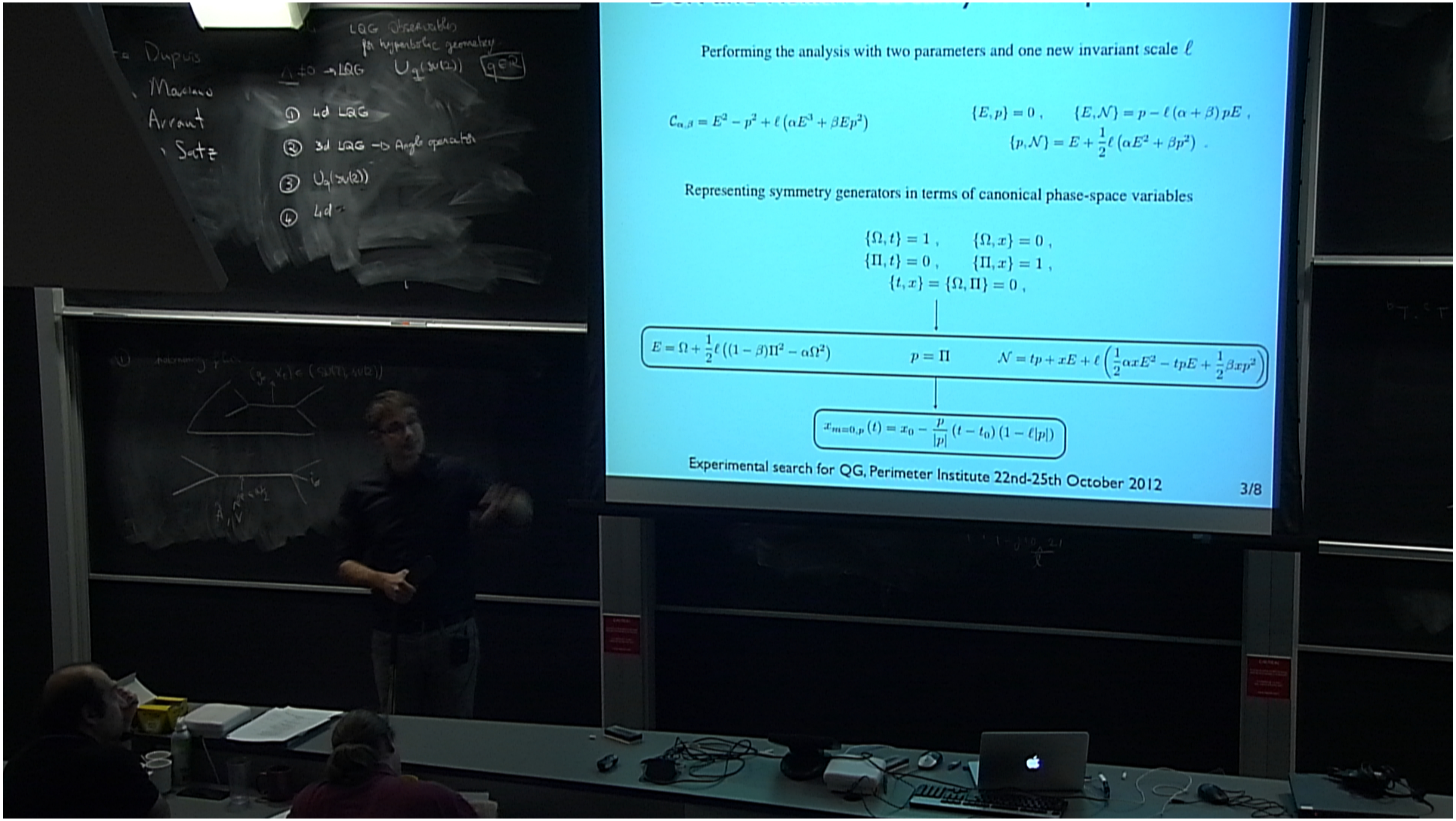
Relative locality for flat space-time

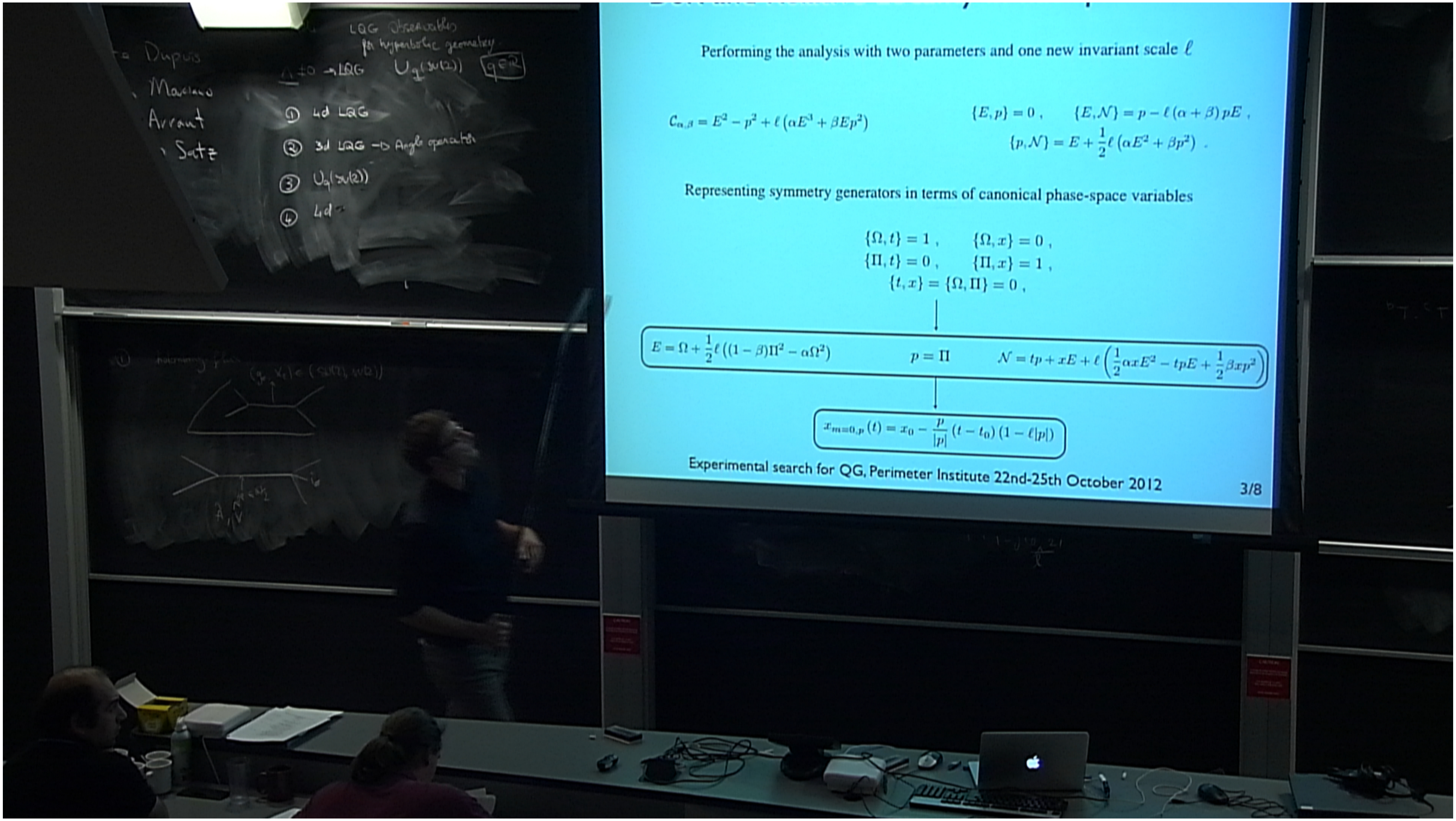
Relative locality & invariant momentum scale and curved momentum space
Relativistic theory of worldlines of particles with 2 relativistic invariants

Amelino-Camelia, AM, Matassa, Rosati, arXiv:1206.5315

Relative locality for curved space-time

Charactherization of DSR and LSB: bounds constraint sensitive at z>1
Relativistic theory of worldlines of particles with 3 relativistic invariants


Performing the analysis with two parameters and one new invariant scale ℓ
$\mathcal{C}_{\alpha,\beta} = E^2 - p^2 + \ell\,(\alpha E^3 + \beta E p^2)$
$\{E,p\} = 0\,, \quad \{E,\mathcal{N}\} = p - \ell\,(\alpha+\beta)\,pE\,,$
$\{p,\mathcal{N}\} = E + \frac{1}{2}\ell\,(\alpha E^2 + \beta p^2)\,.$
Representing symmetry generators in terms of canonical phase-space variables
$\{\Omega,t\} = 1\,, \quad \{\Omega,x\} = 0\,,$
$\{\Pi,t\} = 0\,, \quad \{\Pi,x\} = 1\,,$
$\{t,x\} = \{\Omega,\Pi\} = 0\,,$
$E = \Omega + \frac{1}{2}\ell\,((1-\beta)\Pi^2 - \alpha\Omega^2)$
$p = \Pi$
$\mathcal{N} = tp + xE + \ell\left(\frac{1}{2}\alpha x E^2 - tpE + \frac{1}{2}\beta x p^2\right)$
$x_{m=0,p}(t) = x_0 - \frac{p}{|p|}(t-t_0)(1-\ell|p|)$
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
3/8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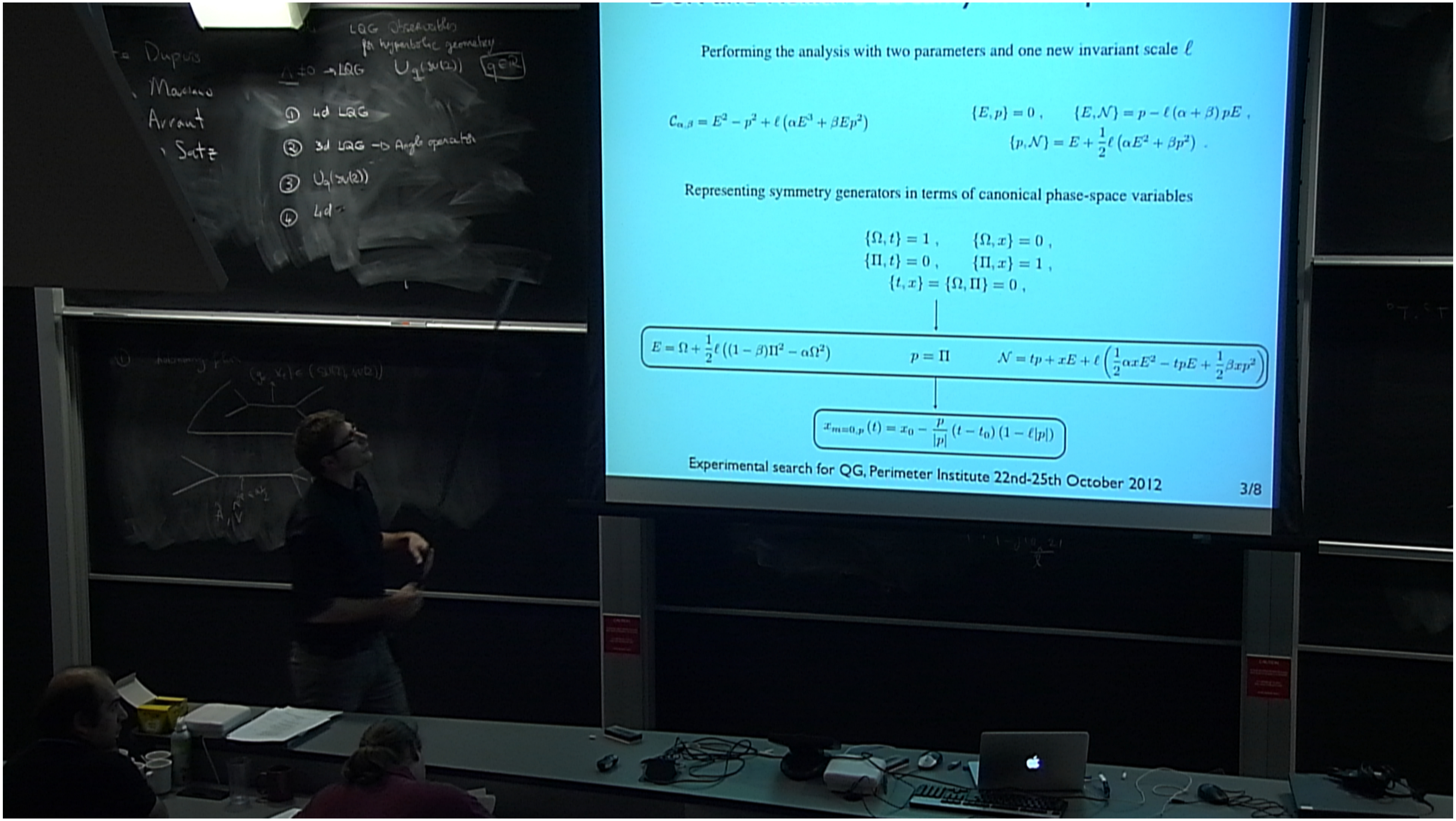
$\{\Pi,t\} = 0\,, \qquad \{\Pi,x\} = 1\,,$
$\{t,x\} = \{\Omega,\Pi\} = 0\,,$
$E = \Omega + \frac{1}{2}\ell\left((1-\beta)\Pi^2 - \alpha\Omega^2\right)$
$p = \Pi$
$\mathcal{N} = tp + xE + \ell\left(\frac{1}{2}\alpha x E^2 - tpE + \frac{1}{2}\beta x p^2\right)$
$x_{m=0,p}(t) = x_0 - \frac{p}{|p|}(t-t_0)(1-\ell|p|)$
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
3/8


Performing the analysis with two parameters and one new invariant scale ℓ
$\mathcal{C}_{\alpha,\beta} = E^2 - p^2 + \ell\,(\alpha E^3 + \beta E p^2)$
$\{E,p\} = 0\,, \quad \{E,\mathcal{N}\} = p - \ell\,(\alpha+\beta)\,pE\,,$
$\{p,\mathcal{N}\} = E + \frac{1}{2}\ell\,(\alpha E^2 + \beta p^2)\,.$
Representing symmetry generators in terms of canonical phase-space variables
$\{\Omega,t\} = 1\,, \quad \{\Omega,x\} = 0\,,$
$\{\Pi,t\} = 0\,, \quad \{\Pi,x\} = 1\,,$
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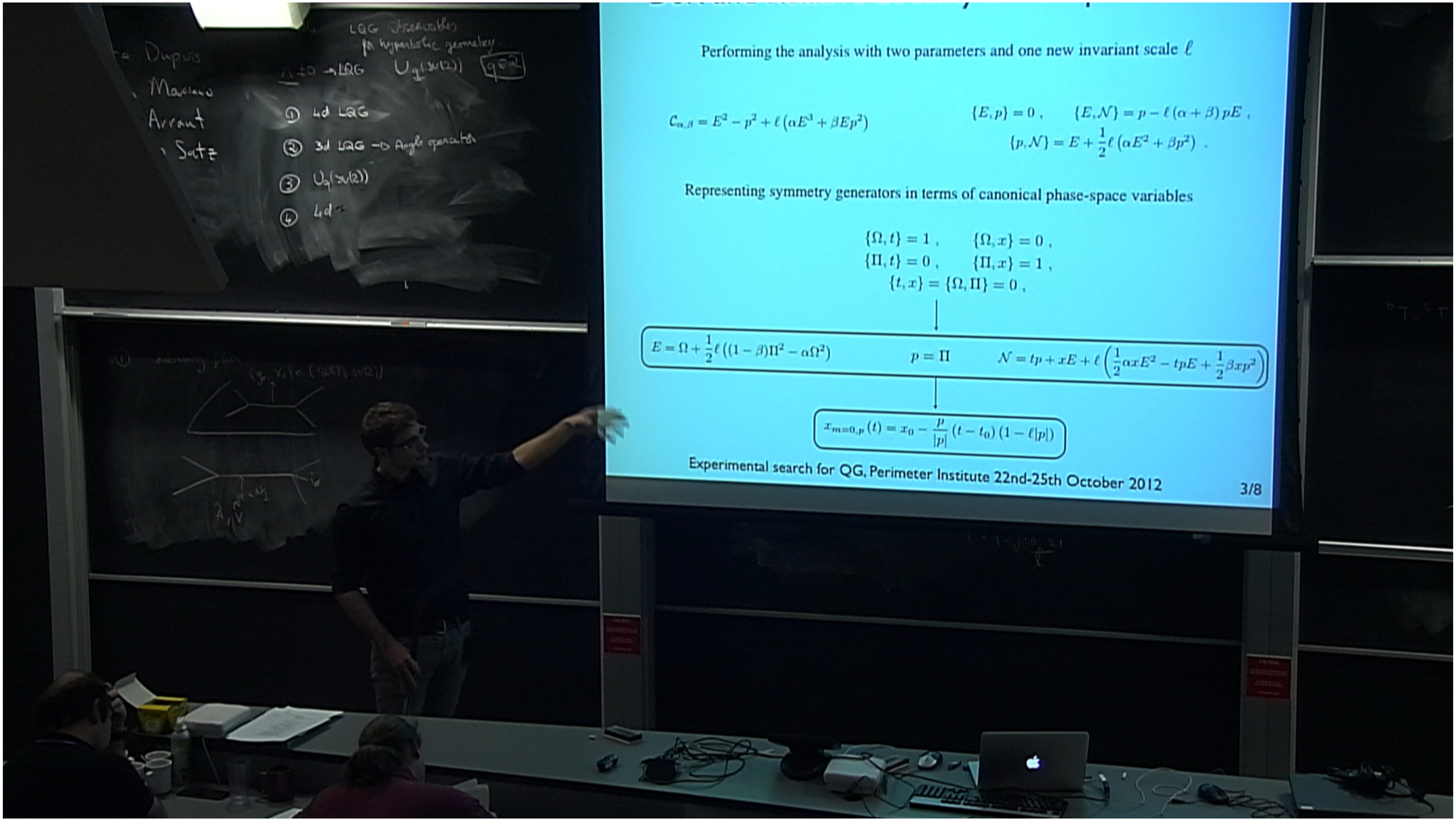
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
3/8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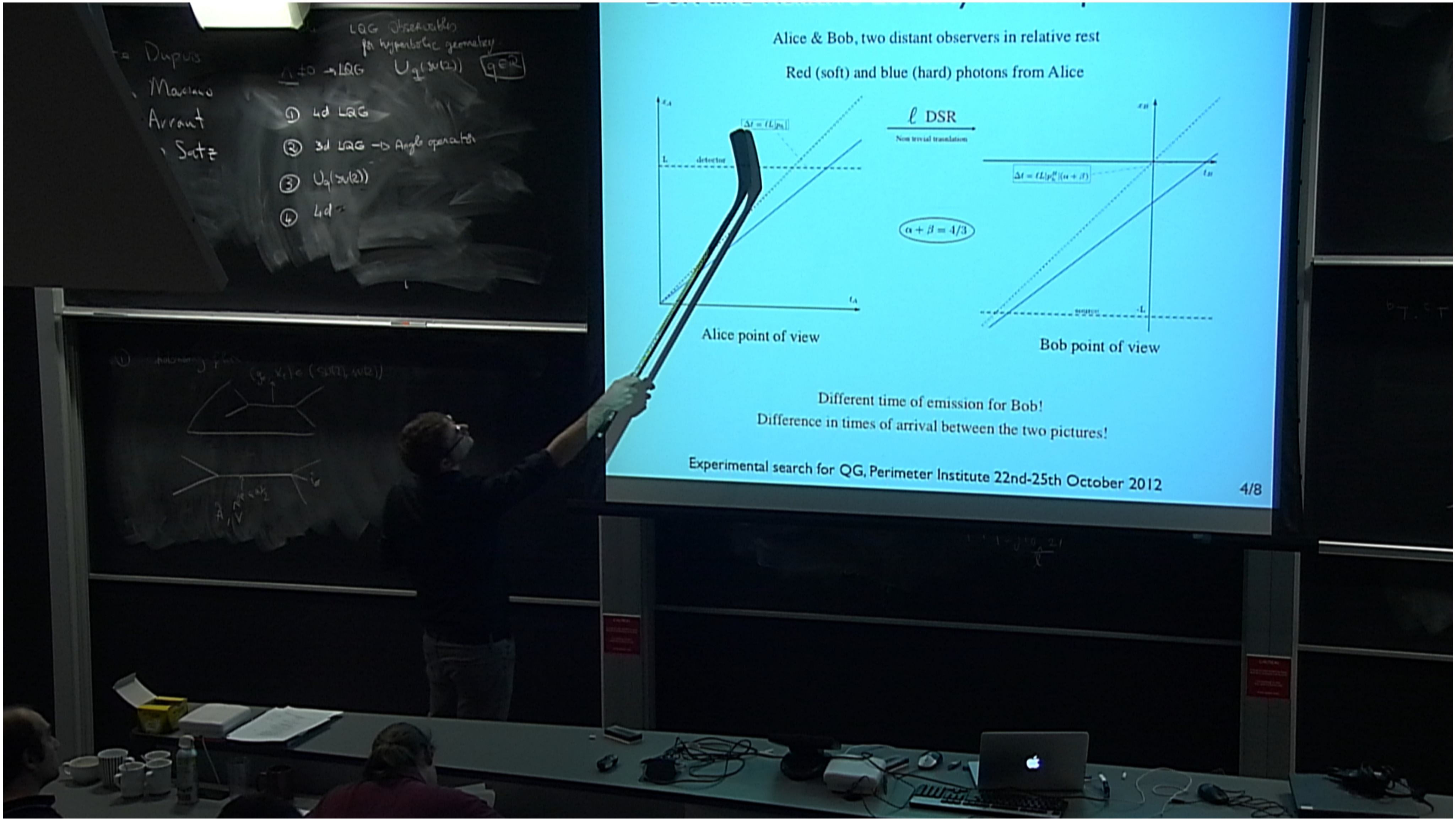
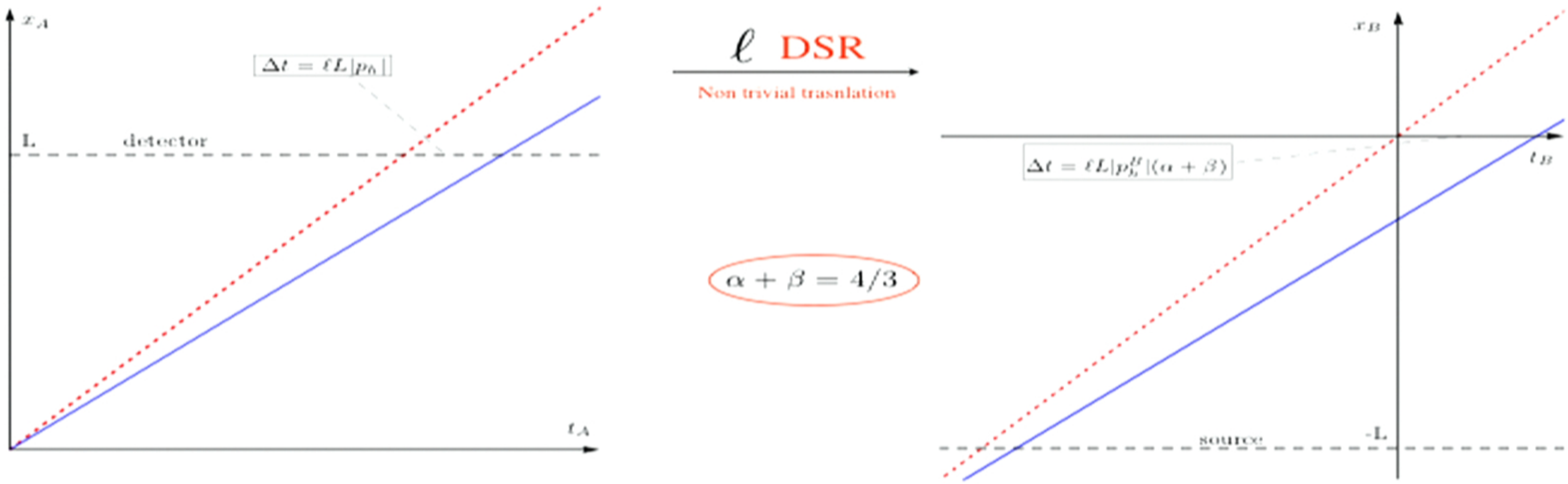
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
3/8


Alice & Bob, two distant observers in relative rest
Red (soft) and blue (hard) photons from Alice
ℓ DSR
α + β = 4/3
Alice point of view
Bob point of view
Different time of emission for Bob!
Difference in times of arrival between the two pictures!
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
4/8

DSR and Relative Locality in flat space-time II

Alice & Bob, two distant observers in relative rest

Red (soft) and blue (hard) photons from Alice



Alice point of view | Bob point of view

Different time of emission for Bob!

Difference in times of arrival between the two pictures!

DSR and Relative Locality in dS space-time I

Performing the analysis with two parameters and two new invariant scale ℓ and H

$$\mathcal{C}_{H,\alpha,\beta} = E^2 - p^2 + 2H\mathcal{N}p + \ell\left(\alpha E^3 + \beta E p^2\right)$$

$$\{E,p\} = Hp - \ell\alpha H E p\ ,$$
$$\{E,\mathcal{N}\} = p - H\mathcal{N} - \ell\alpha E(p - H\mathcal{N}) - \ell\beta E p\ ,$$
$$\{p,\mathcal{N}\} = E + \frac{1}{2}\ell\alpha E^2 + \frac{1}{2}\ell\beta p^2\ .$$

Representing symmetry generators in terms of conformal coordinates and canonical conjugated variables

$$E = -Hx\Pi + \Omega - H\eta\Omega + \frac{\ell}{2}(1-\beta)\Pi^2 + \frac{\ell}{2}H\eta\Pi^2 - \frac{\ell}{2}\alpha(\Omega - H\eta\Omega - Hx\Pi)^2$$

$$p = \Pi$$

$$\mathcal{N} = x(1-H\eta)\Omega + \left(\eta - \frac{H}{2}\eta^2 - \frac{H}{2}x^2\right)\Pi + \ell\left(\frac{1+H\eta}{2}x\Pi - (1-H\eta)\eta\Omega\right)\Pi\ .$$

Covariance of wordlined and map from Alice to Bob

$$x_{m=0,p}(\eta) = x_0 - \frac{p}{|p|}(\eta - \eta_0)(1 - \ell|p|)$$

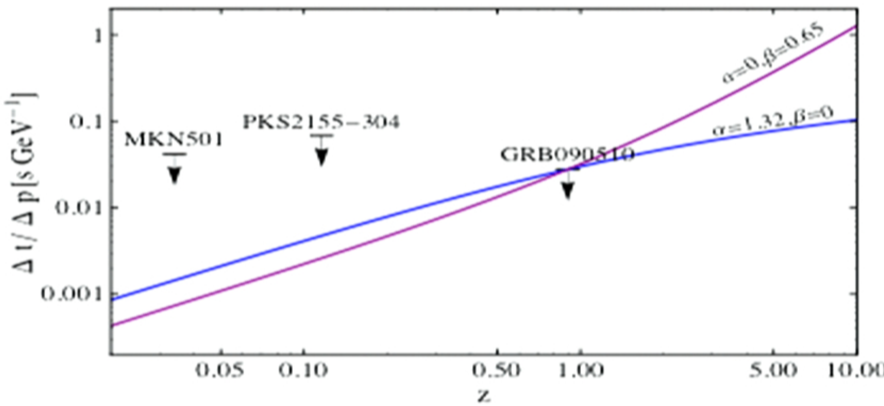
$$\mathrm{Bob} = e^{-a_x p} \triangleright e^{-a_\eta E} \triangleright \mathrm{Alice}$$
$$a_x = \frac{1 - e^{-Ha_\eta}}{H}$$

$$\eta_h^B = \frac{1-e^{Ha_\eta}}{H} + e^{Ha_\eta}\eta_h^A + \ell\,\alpha\,a_\eta e^{Ha_\eta}(1 - H\eta_h^A)\left(E_h^A + Ha_x p_h^A\right)$$

$$x_h^B = e^{Ha_\eta}(x_h^A - a_x) + \ell\beta\frac{\sinh(Ha_\eta)}{H}p_h^A - \ell\alpha a_\eta e^{Ha_\eta}H(a_x - x_h^A)\left(E_h^A + Ha_x p_h^A\right) - \ell\frac{(1-e^{-Ha_\eta})\left(1 + e^{Ha_\eta}H\eta_h^A\right)}{H}p_h^A$$

$$p_h^B = e^{-Ha_\eta}\left(p_h^A + \ell\alpha H a_\eta p_h^A\left(E_h^A + Ha_x p_h^A\right)\right)$$

Constraining models with data



Time of arrival difference

$$\Delta t = \ell|p|\left(\alpha\frac{\ln(1+z)}{H} + \beta\frac{z+\frac{z^2}{2}}{H}\right)$$

Violet line, $\alpha = 0$

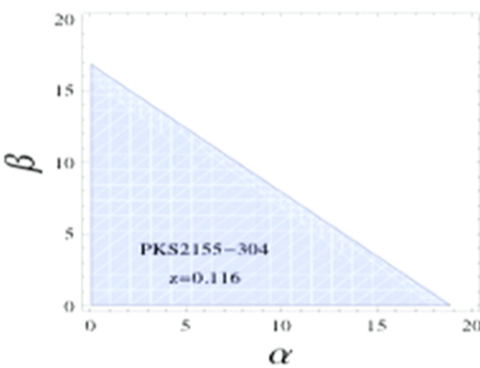
Blue line, $\beta = 0$

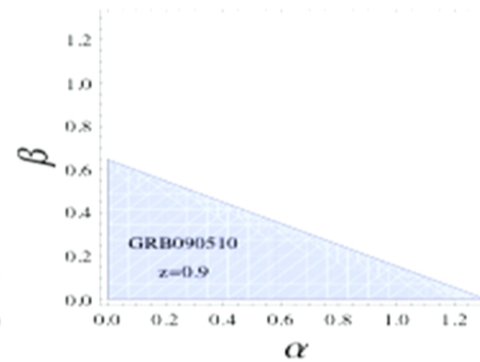
At high z, difference between Ep^2 or E^3 !

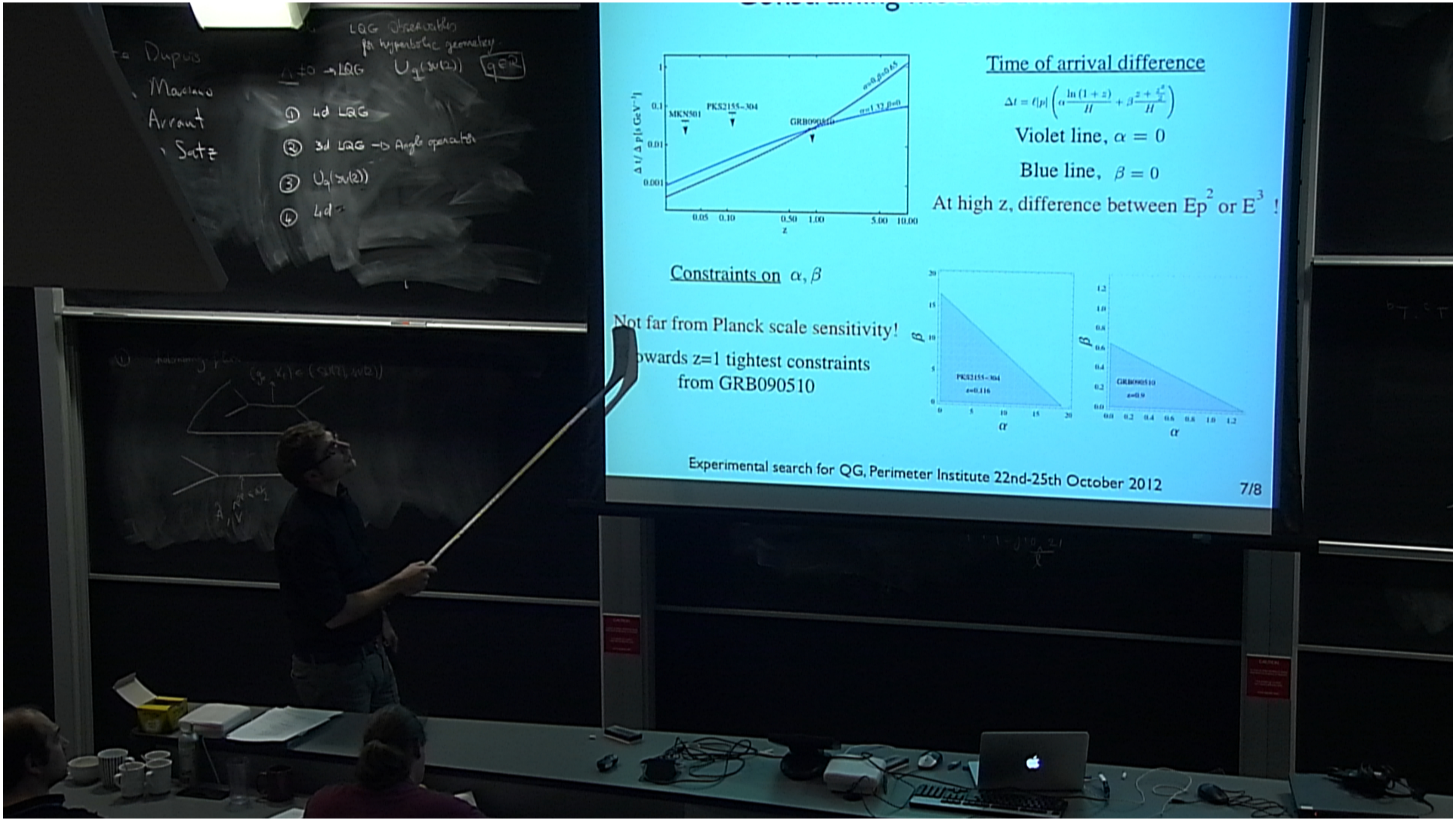
Constraints on α, β

Not far from Planck scale sensitivity!

Towards z=1 tightest constraints from GRB090510






Time of arrival difference
Violet line, $\alpha = 0$
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Constraints on α, β
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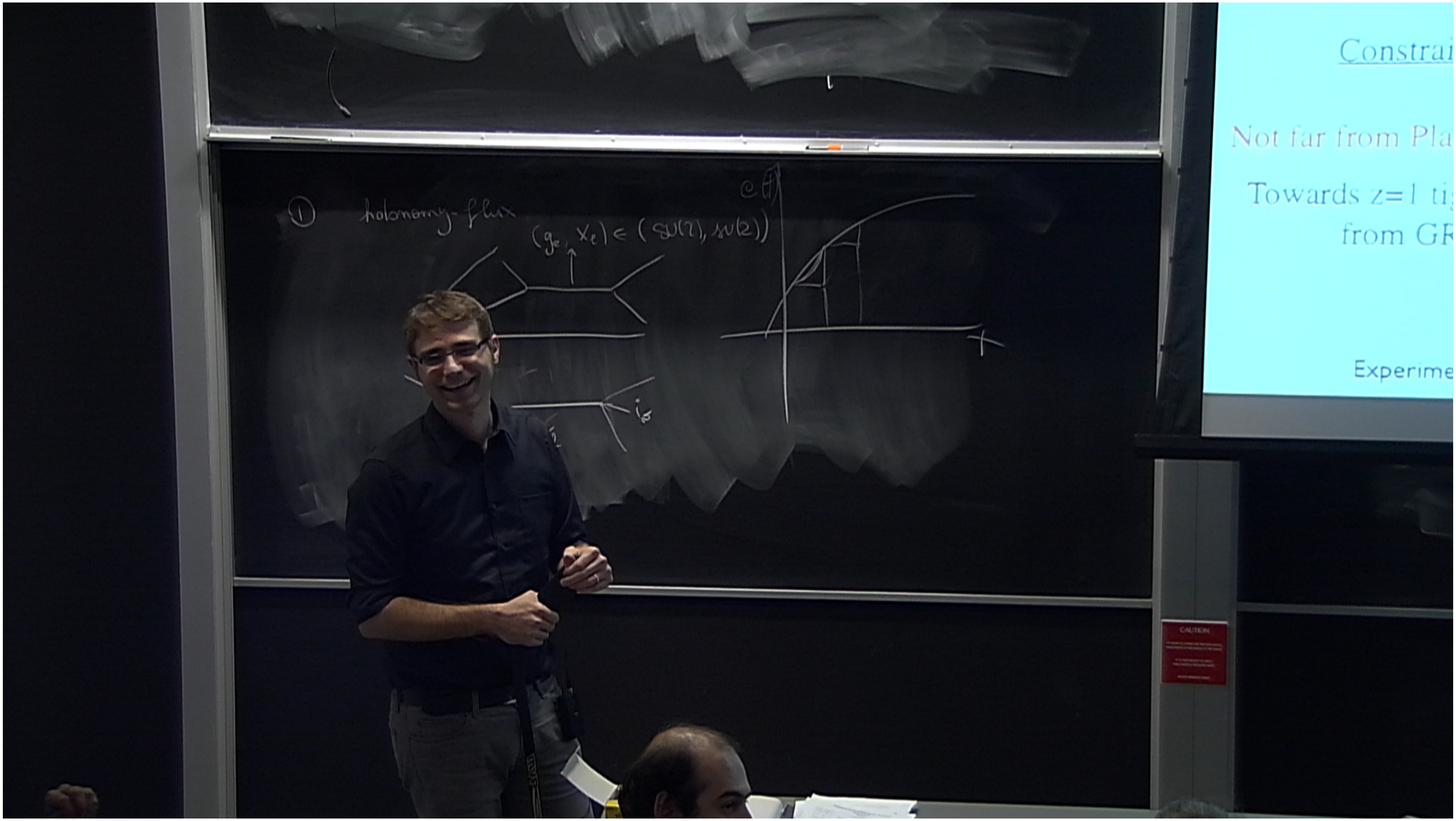
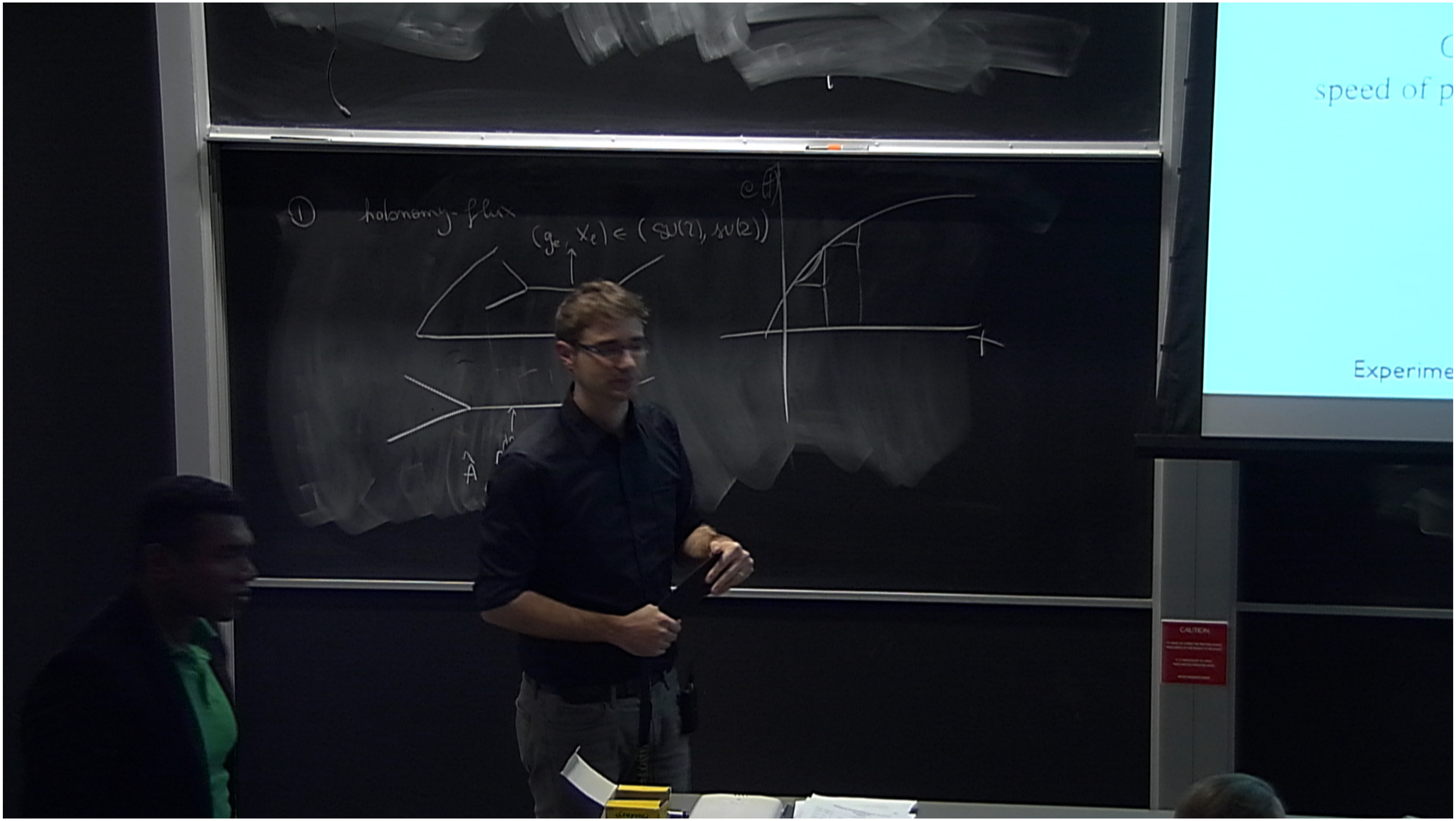
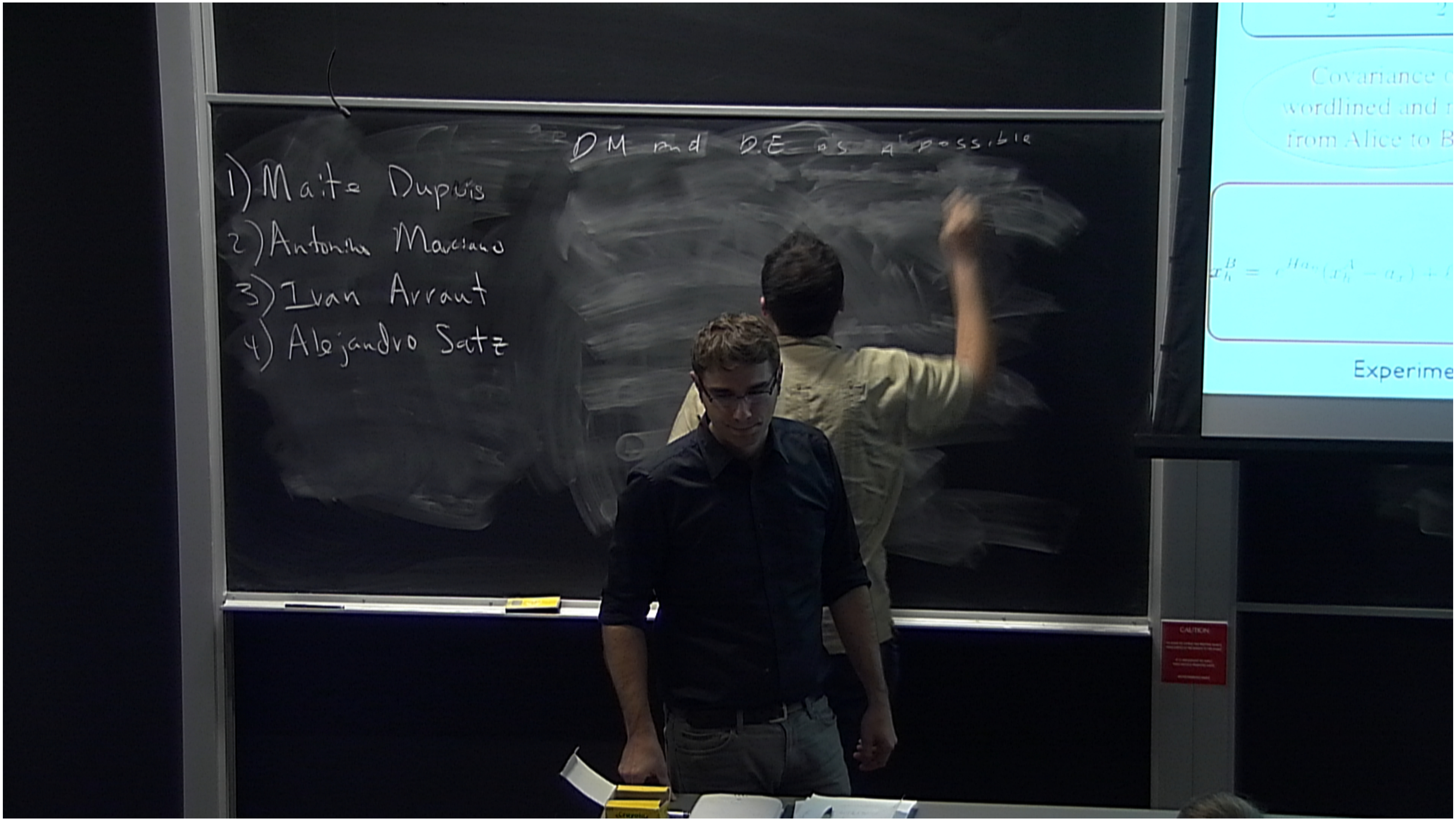
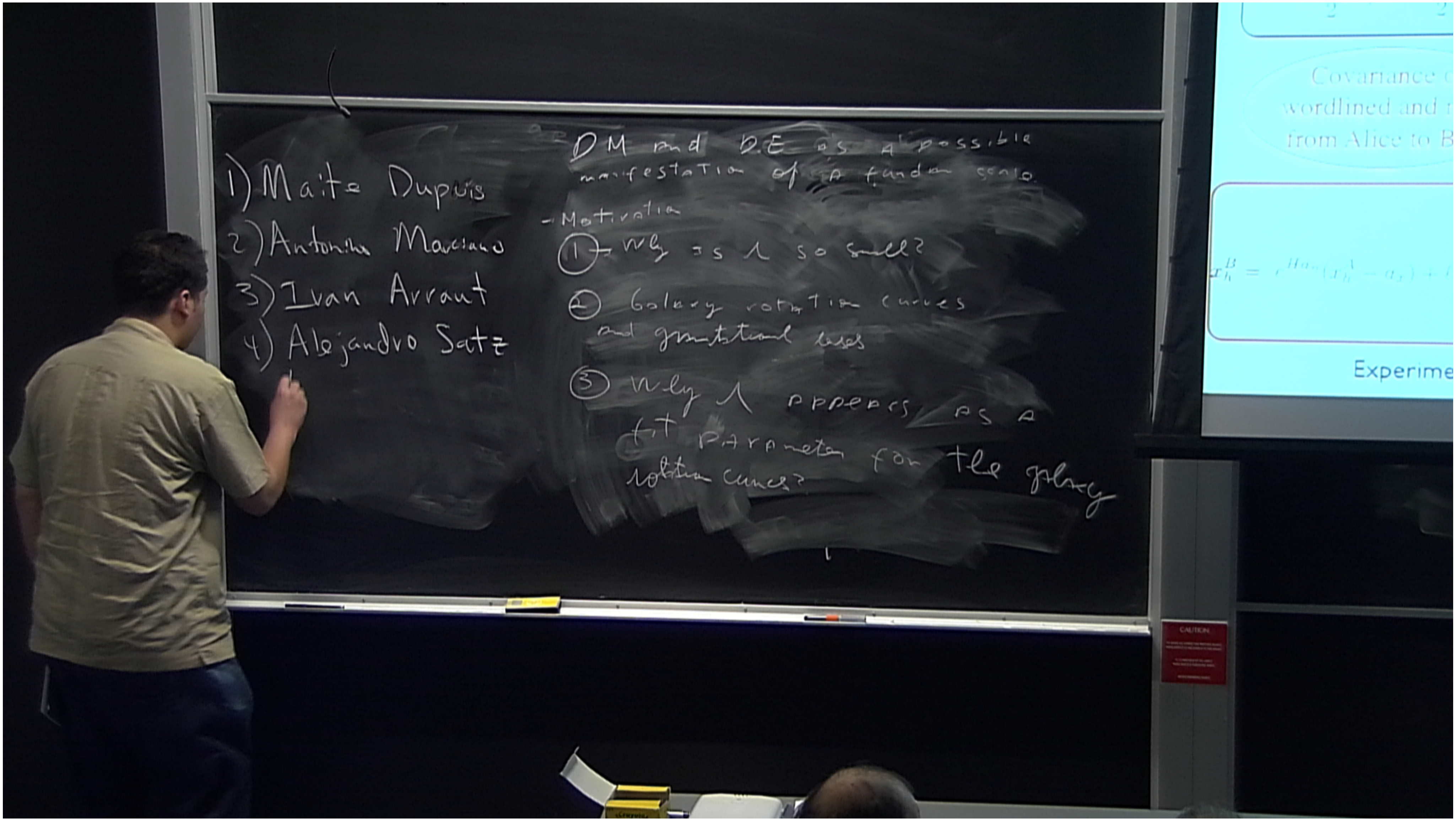
owards z=1 tightest constraints
from GRB090510
MKN501
PKS2155-304
GRB090510
Experimental search for QG, Perimeter Institute 22nd-25th October 2012
7/8
LQG Observables
for hyperbolic geometry
① 4d LQG
② 3d LQG → Angle operator
③ $U_q(su(2))$
④ 4d

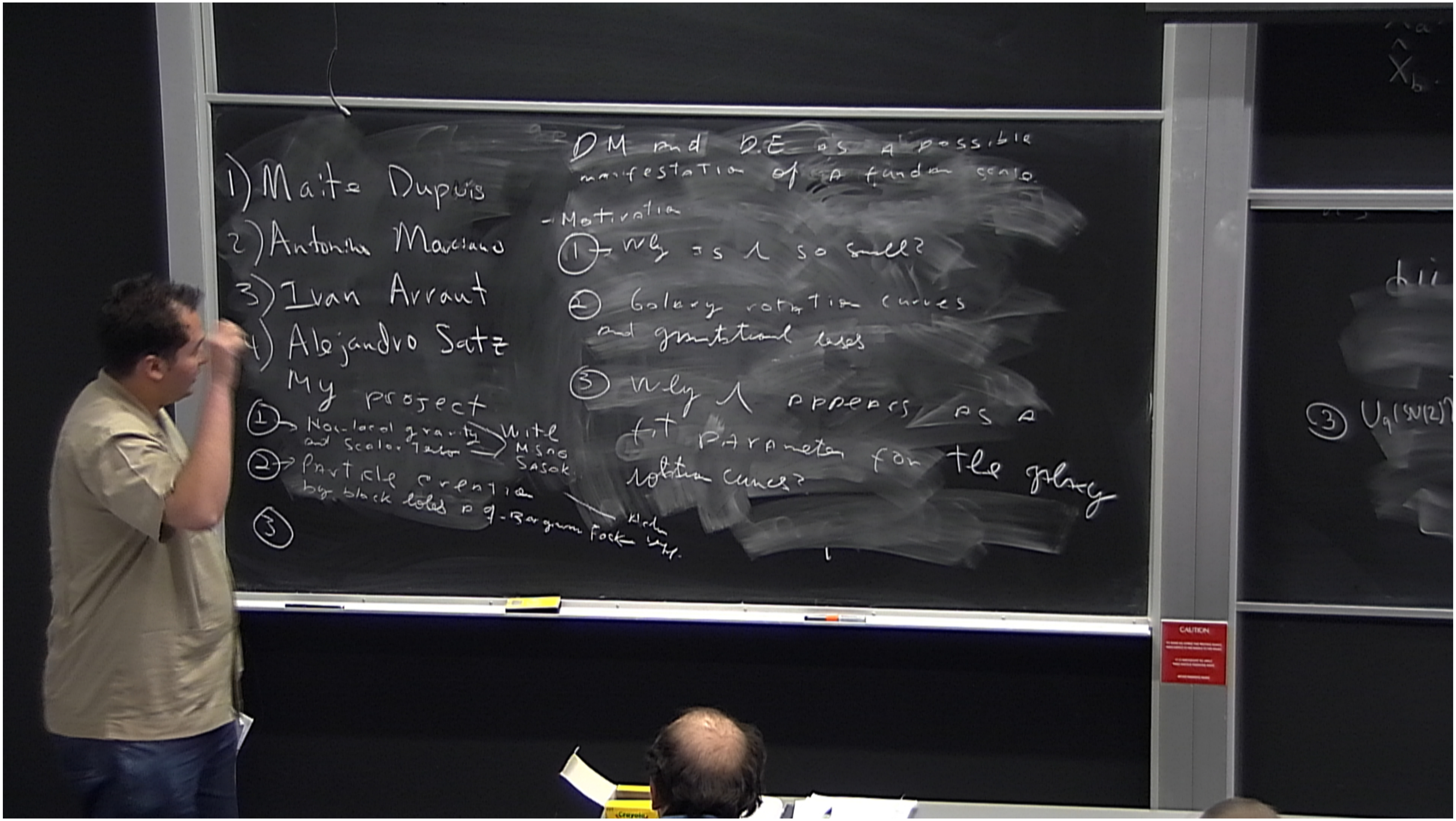

holonomy-flux
Not far from Pla
Towards z=1 ti
from GR
Experime

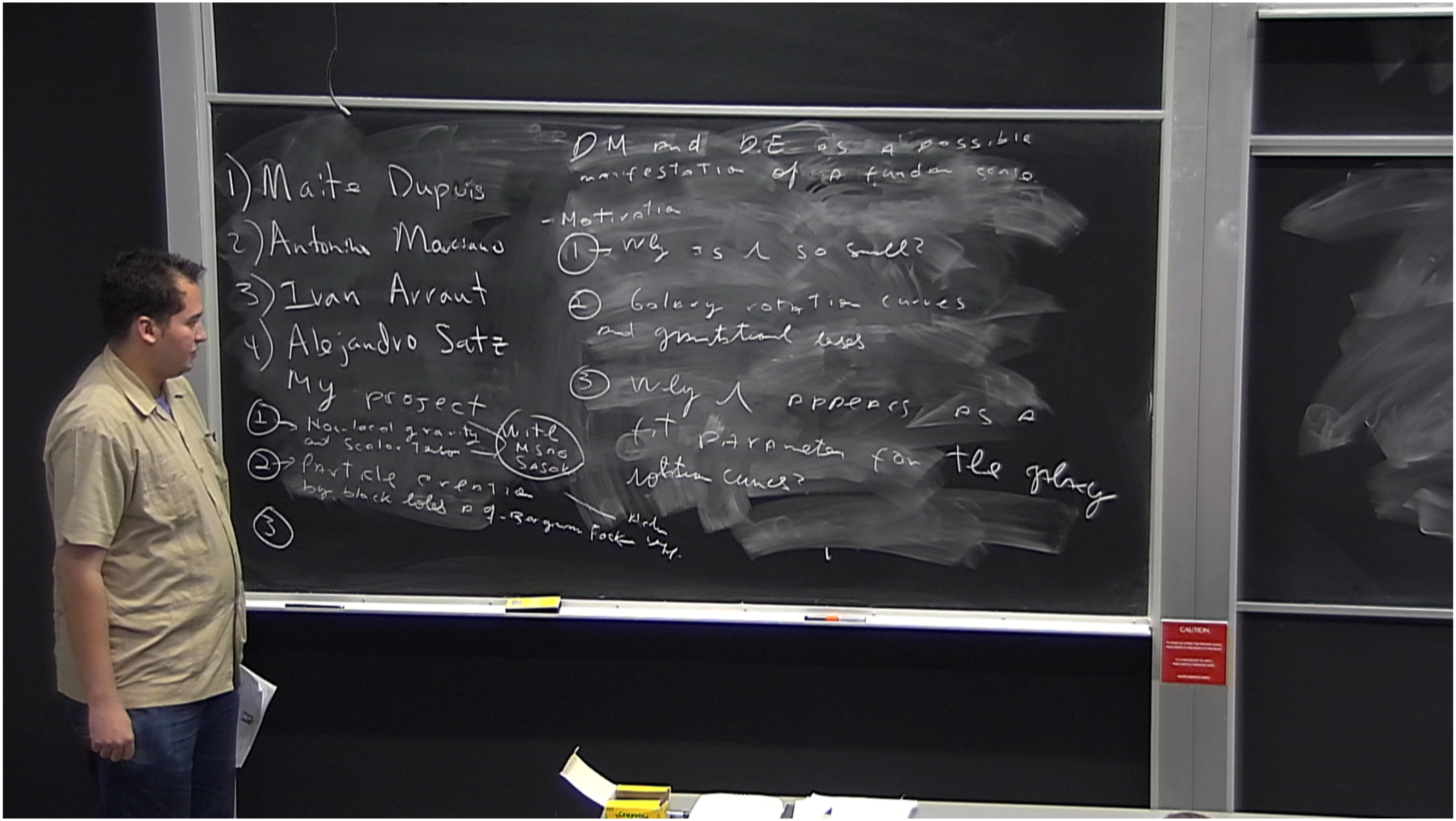

holonomy-flux
Constrai
Not far from Pla
Towards z=1 ti
from GR
Experime


① holonomy-flux
$(g_e, X_e) \in (SU(2), su(2))$
speed of p
Experime


DM and D.E as a possible
1) Maite Dupuis
2) Antonino Marciano
3) Ivan Arraut
4) Alejandro Satz


1) Maite Dupuis
2) Antonino Marciano
3) Ivan Arraut
4) Alejandro Satz
DM and D.E as a possible manifestation of a fundam scale
- Motivation
1 Why is Λ so small?
2 Galaxy rotation curves and gravitational lenses
3 Why Λ appears as a fit parameter for the galaxy rotation curves?


1) Maite Dupuis
2) Antonella Marciano
3) Ivan Arraut
4) Alejandro Satz
My project
1 Non-local gravity and Scalar-Tensor
Misao Sasaki
2 Particle creation by black holes
3
DM and D.E as a possible manifestation of a fundam scale
- Motivation
1 Why is Λ so small?
2 Galaxy rotation curves and gravitational lenses
3 Why Λ appears as a fit parameter for the galaxy rotation curves?
3 $U_q(SU(2))$


1) Maite Dupuis
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My project
Particle creation
DM and D.E as a possible
- Motivation
Why is Λ so small?
Galaxy rotation curves
Why Λ appears as a
fit parameter for the galaxy
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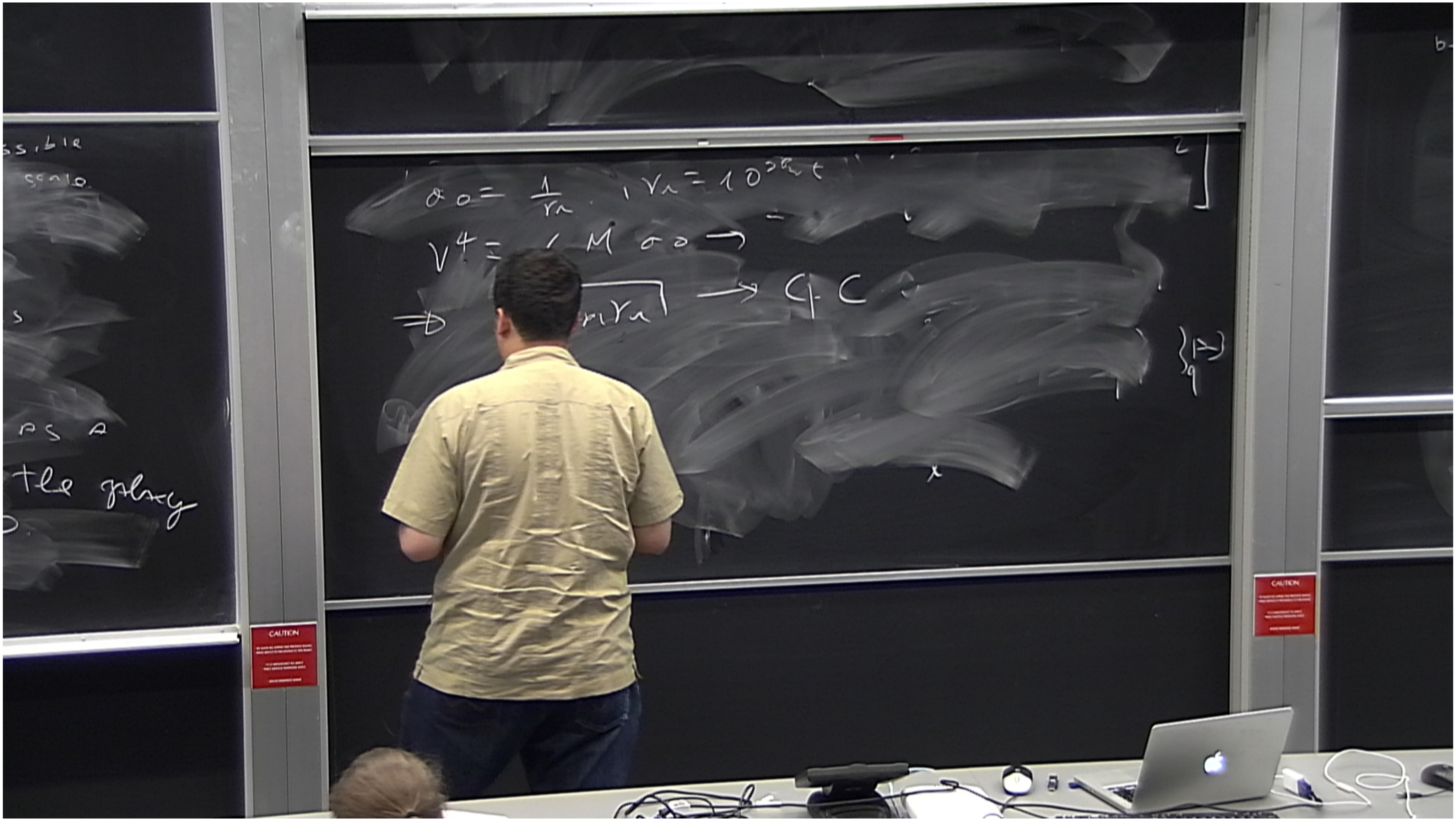
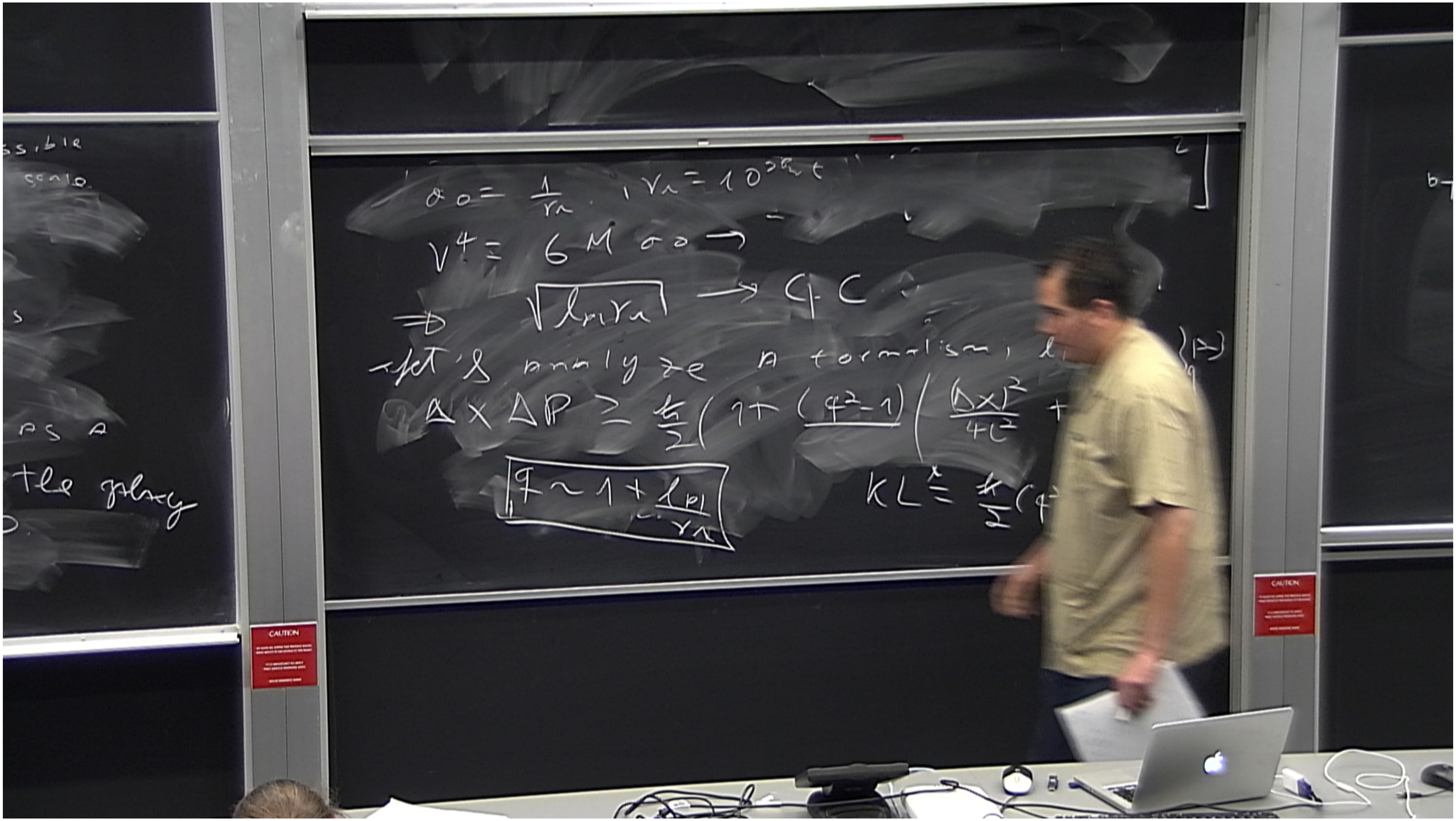
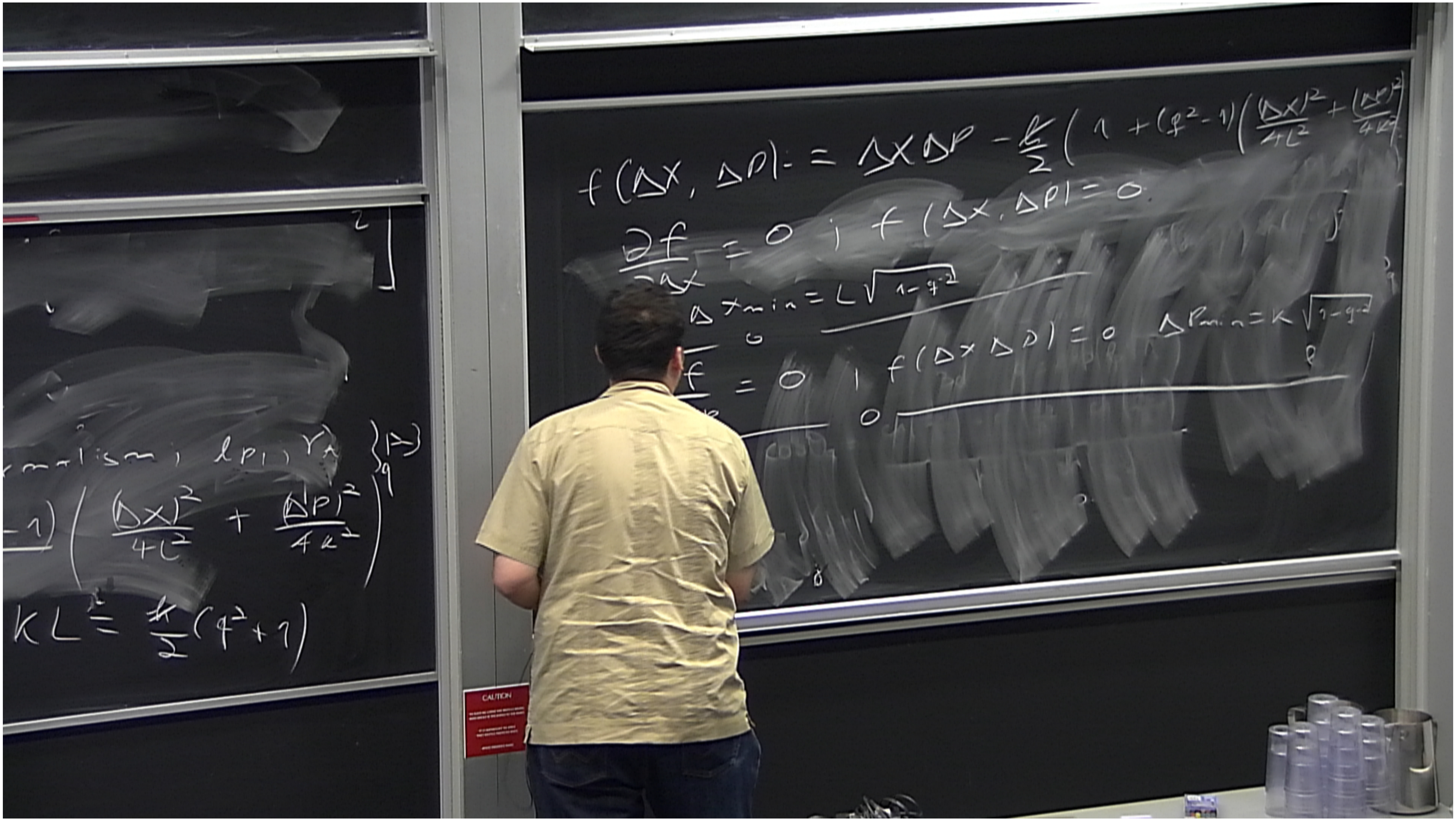
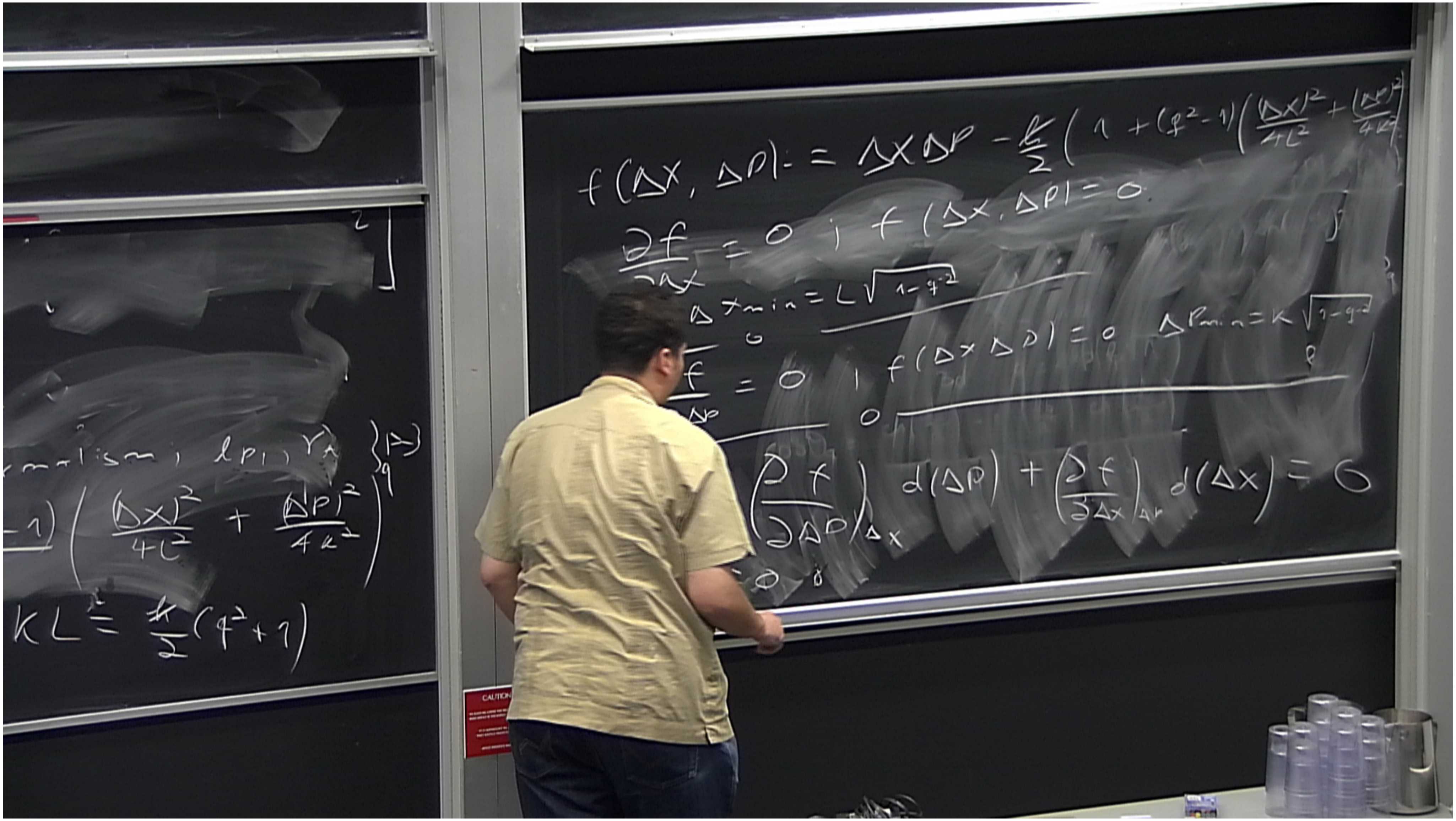
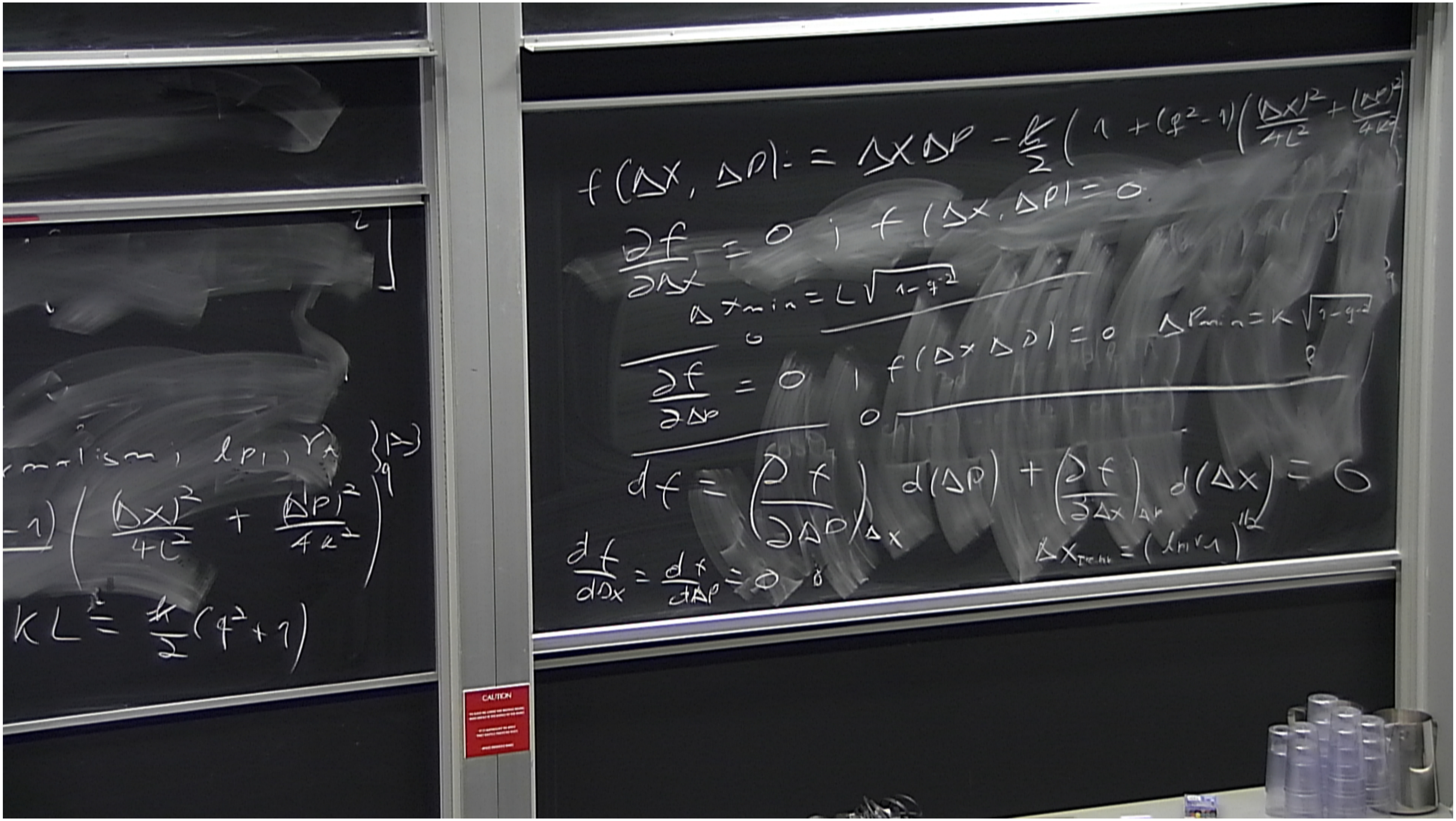


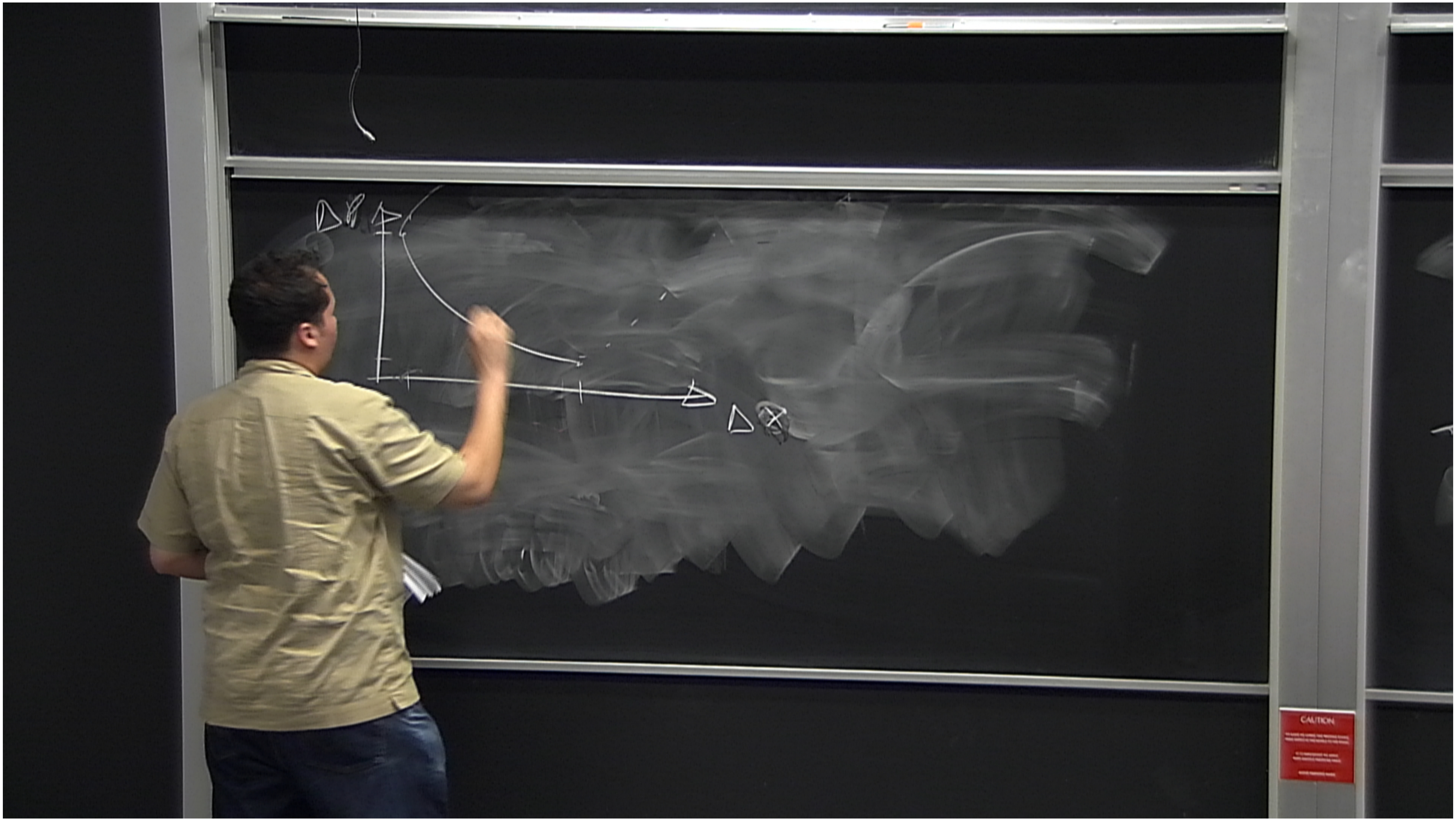


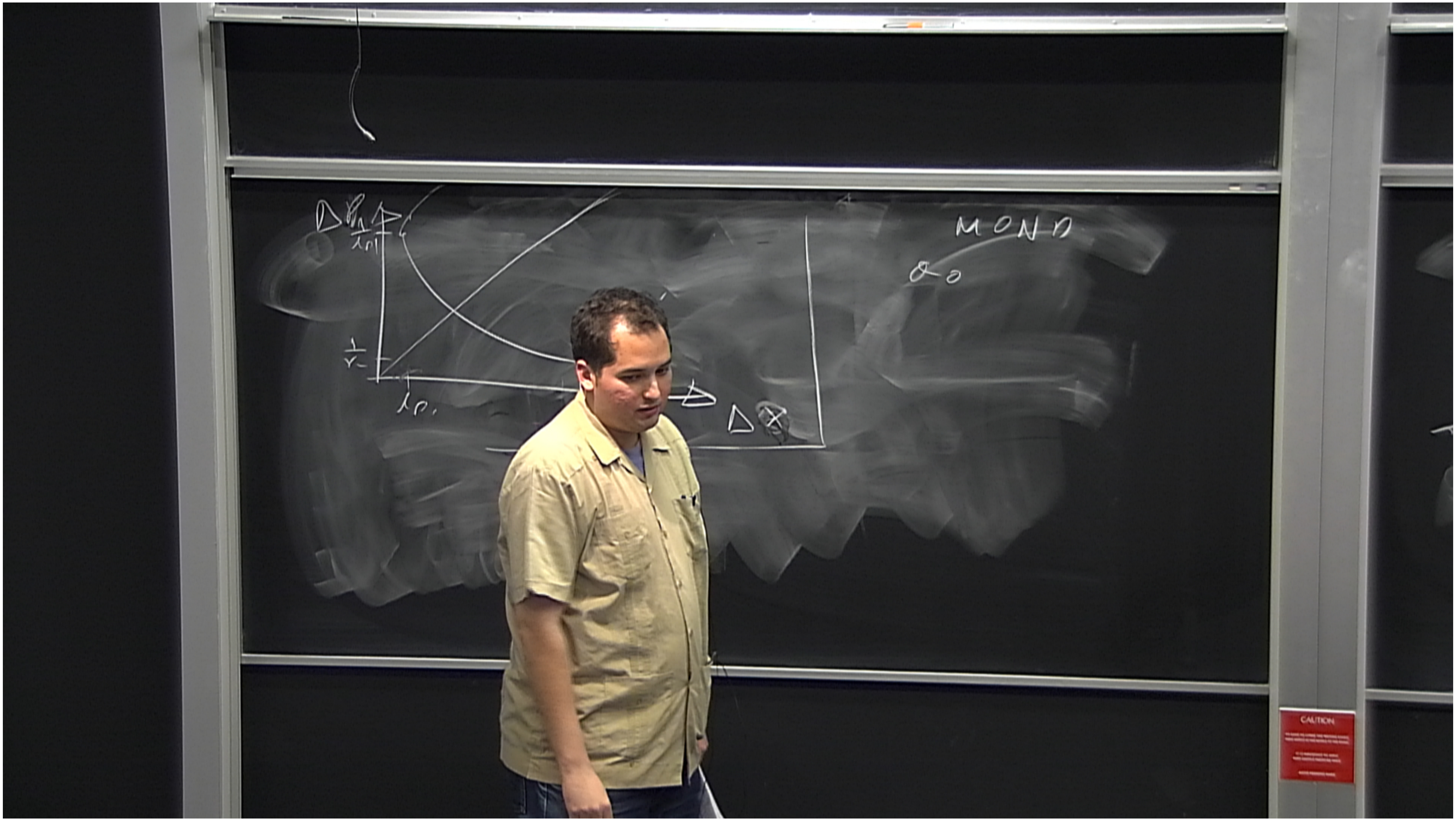
$a_0 = \frac{1}{r_h}$
$V^4 = G M a_0$
$\Rightarrow \sqrt{l_{pl} r_h}$
let's analyze a formalism
$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + (q^2-1)\frac{(\Delta x)^2}{4L^2}\right.$
$q \sim 1 + \frac{l_{pl}}{r_h}$
the galaxy
CAUTION

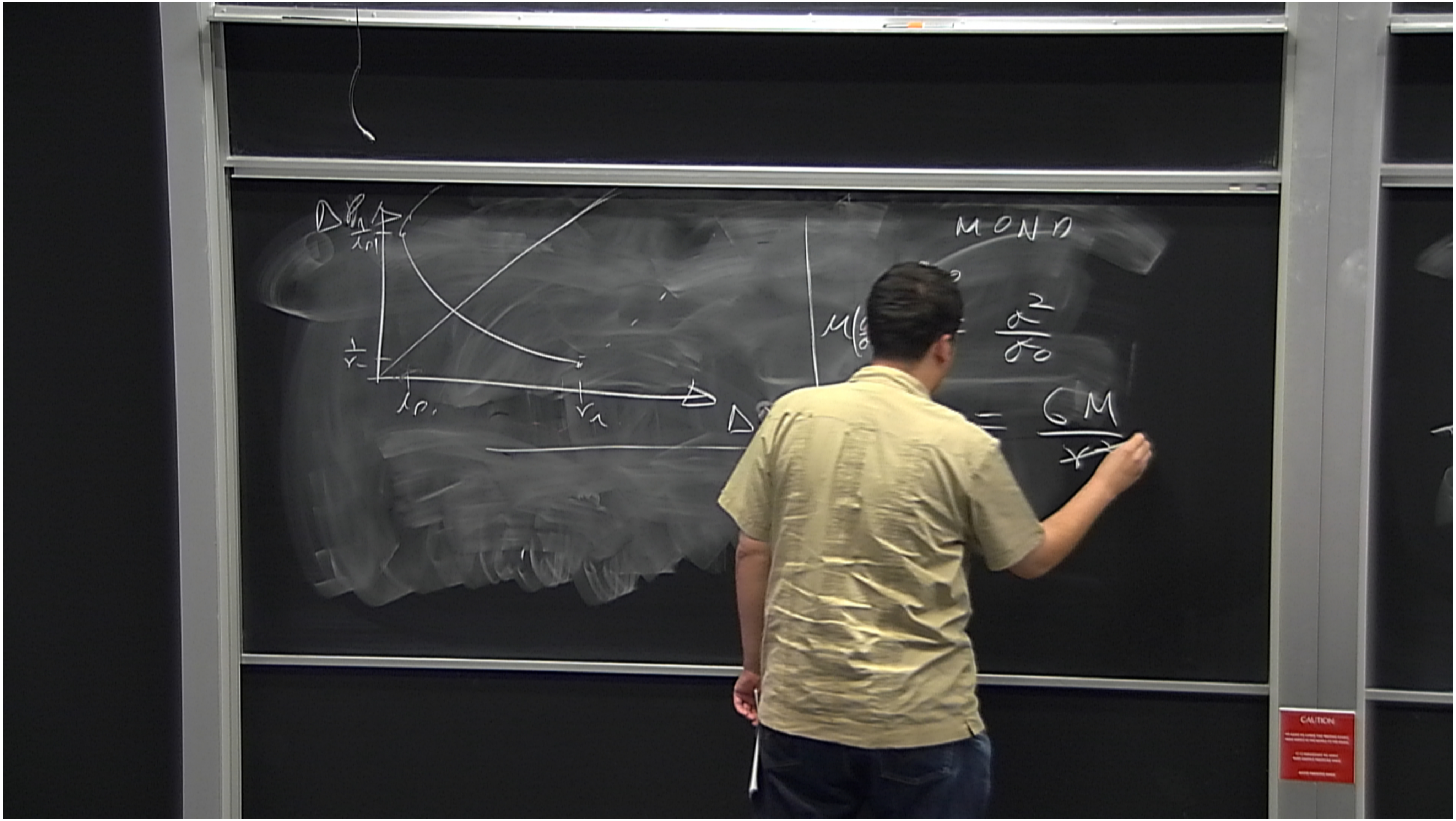


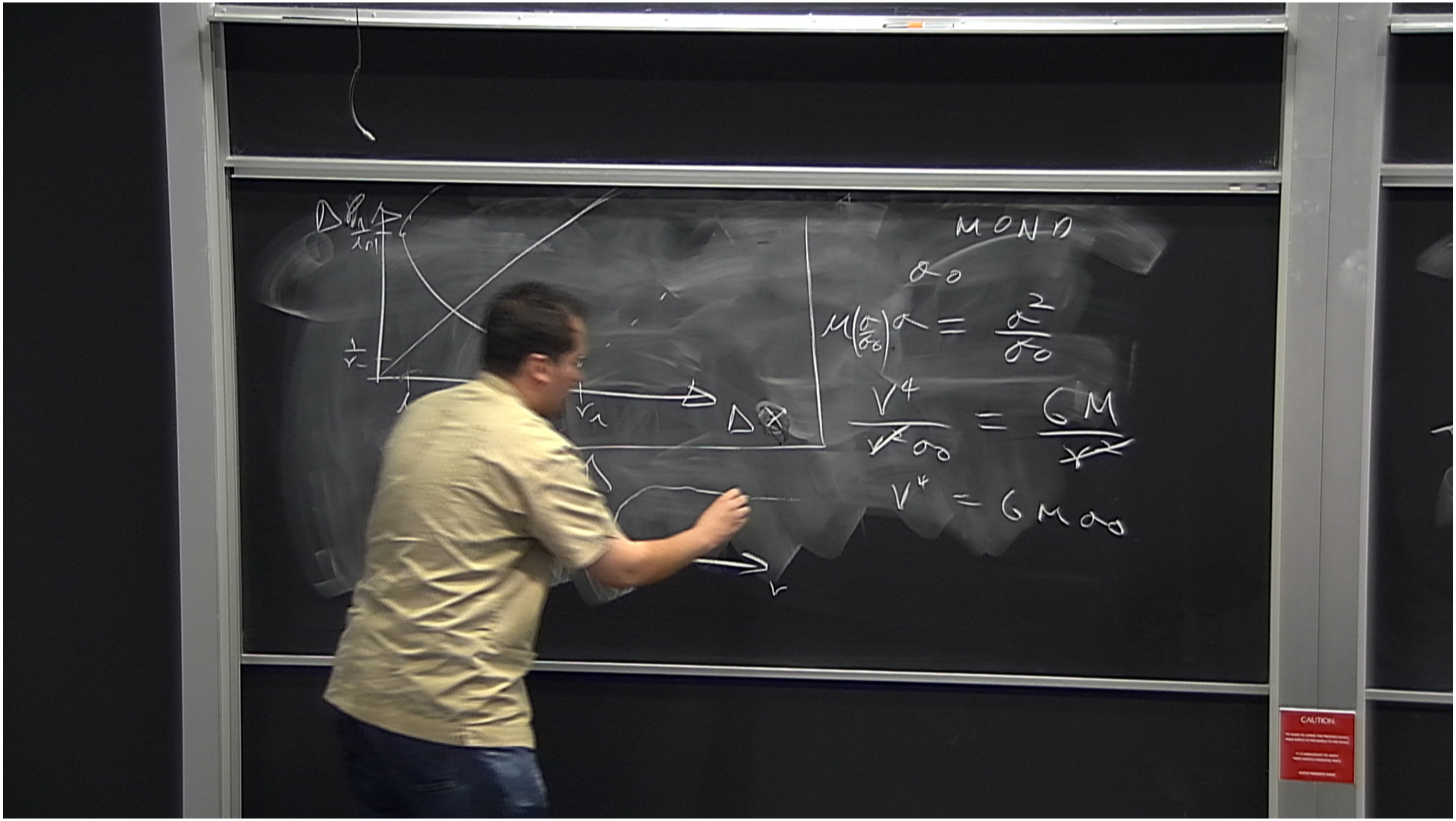


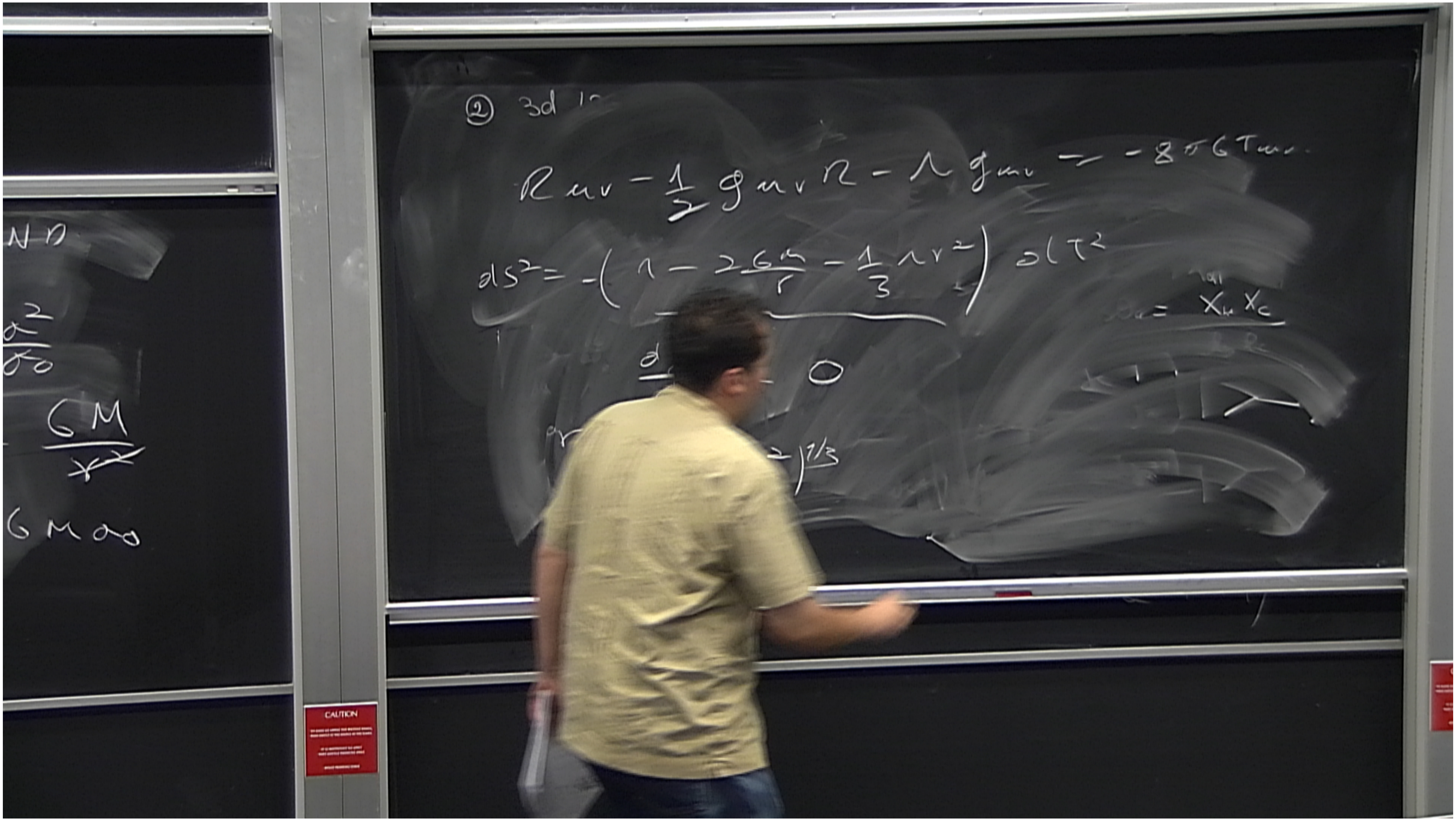

f(Δx, Δp) := ΔxΔp − ħ/2 (1 + (q² − 1)((Δx)²/4L² + (Δp)²/4K²))
f(Δx, Δp) = 0
∂f/∂Δx = 0 ; f(Δx, Δp) = 0
Δx_min = L √(1 − q²)
∂f/∂Δp = 0 ; f(Δx Δp) = 0
Δp_min = K √(1 − q²)
df = (∂f/∂Δp)_Δx d(Δp) + (∂f/∂Δx)_Δp d(Δx) = 0
df/dΔx = df/dΔp = 0
KL = ħ/2 (q² + 1)

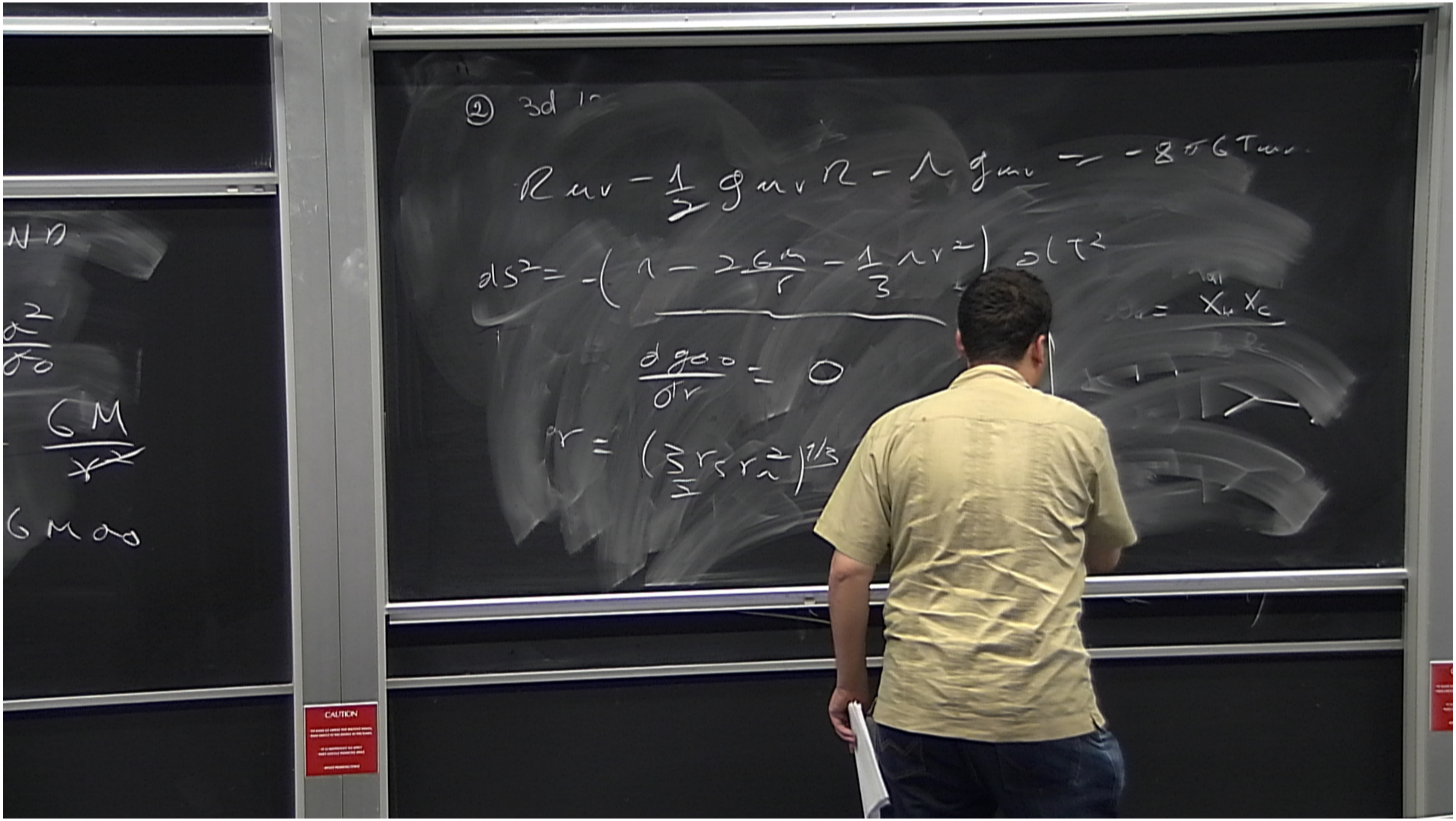



MOND
a₀


MOND


MOND
a_0
$\mu\left(\frac{a}{a_0}\right) a = \frac{a^2}{a_0}$
$\frac{V^4}{r^2 a_0} = \frac{GM}{r^2}$
$V^4 = G M a_0$


② 3d
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$
$ds^2 = -\left(1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2\right) dt^2$
$\frac{GM}{r^2}$
$G M a_0$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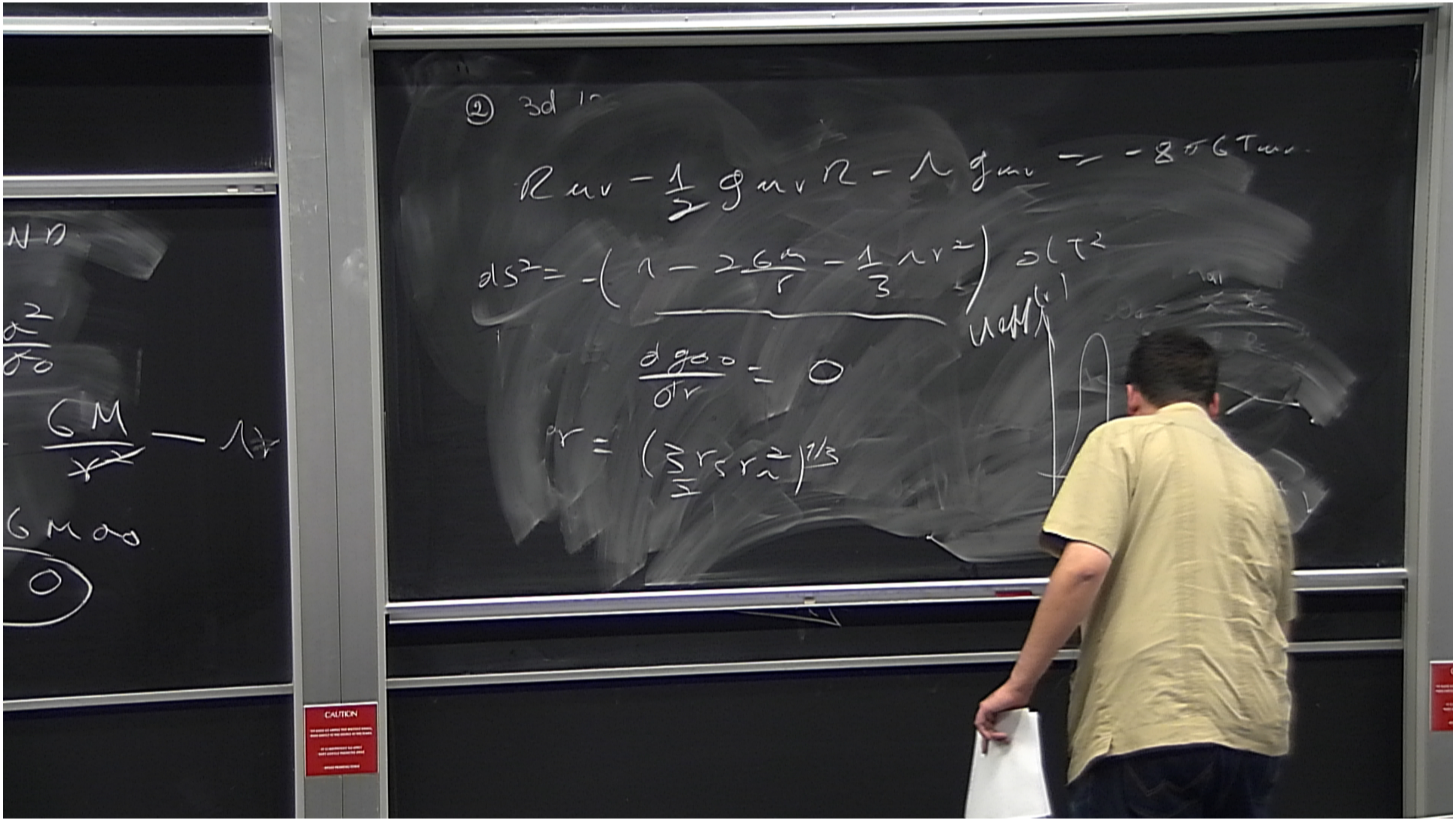
$\frac{d g_{00}}{dr} = 0$
CAUTION



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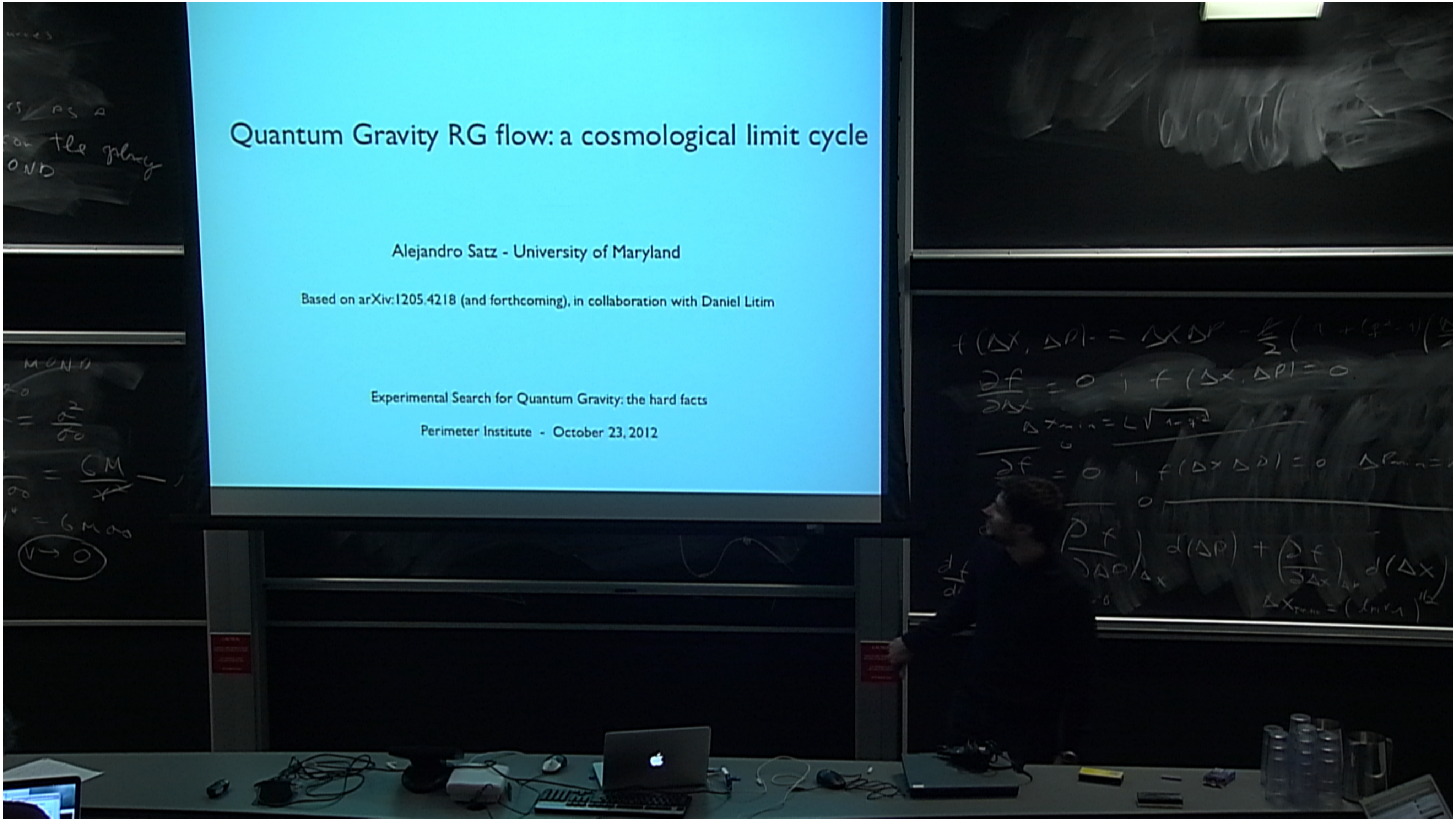
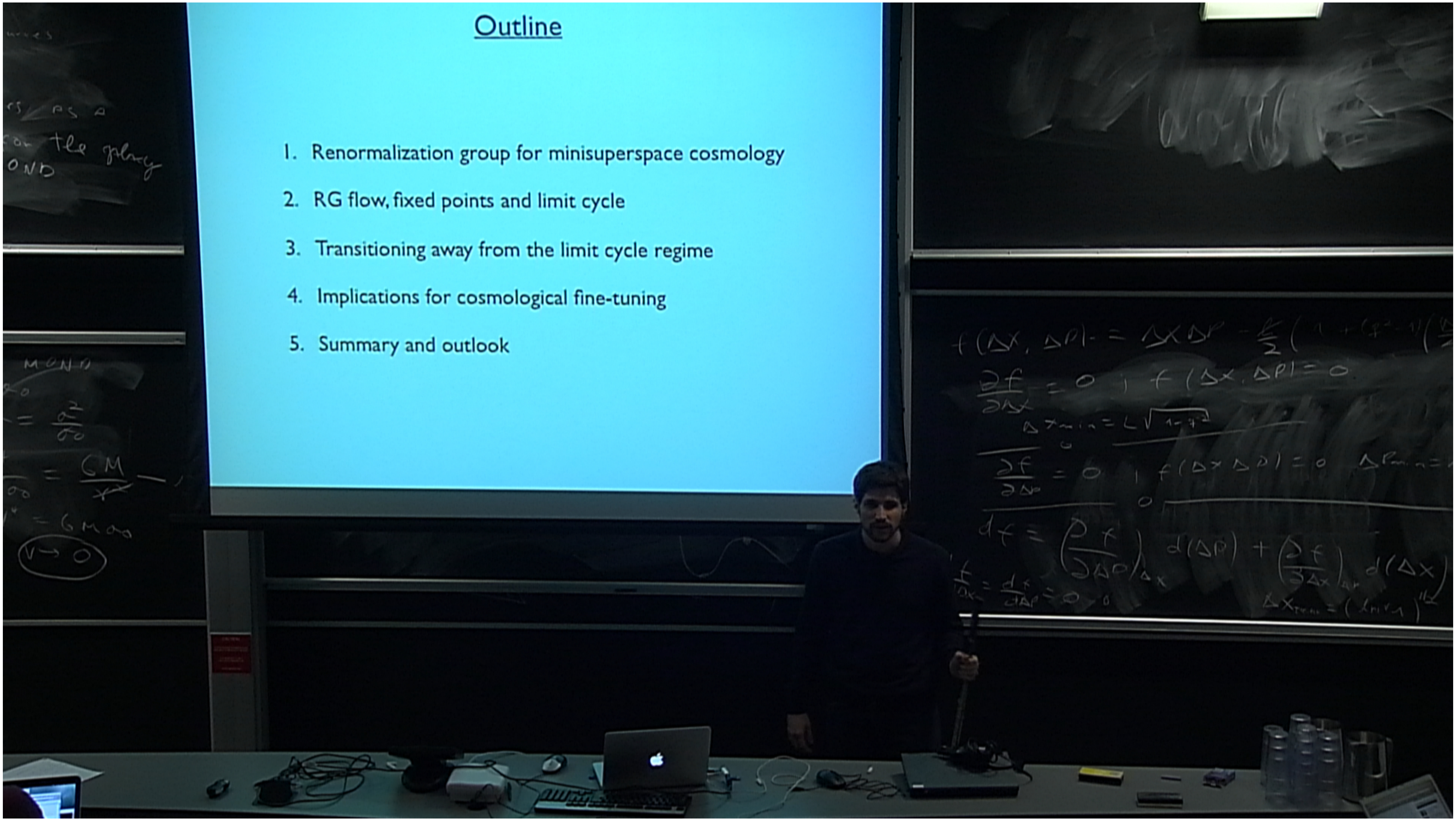

Quantum Gravity RG flow: a cosmological limit cycle
Alejandro Satz - University of Maryland
Based on arXiv:1205.4218 (and forthcoming), in collaboration with Daniel Litim
Experimental Search for Quantum Gravity: the hard facts
Perimeter Institute - October 23, 2012


Outline
1. Renormalization group for minisuperspace cosmology
2. RG flow, fixed points and limit cycle
3. Transitioning away from the limit cycle regime
4. Implications for cosmological fine-tuning
5. Summary and outlook

1. Renormalization group for minisuperspace cosmology

We consider Euclidean GR restricted to spatially flat FRW metrics

$$\mathrm{d}s^2 = a^2(t)\left[\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right] \quad , \qquad S = \int dt\, \frac{3v}{8\pi G}\left[-a'(t)^2 + \frac{\Lambda}{3}a(t)^4\right]$$

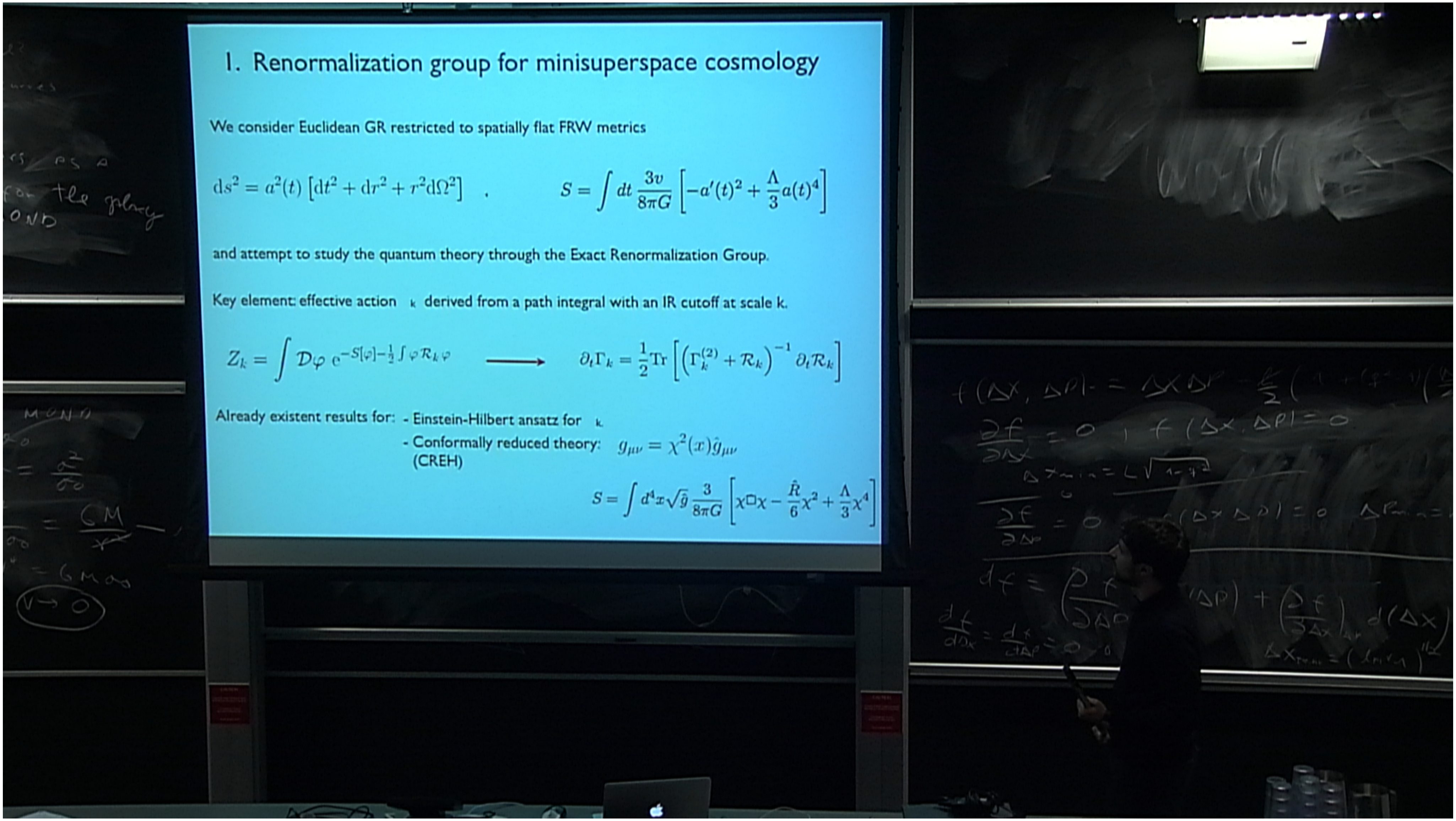
and attempt to study the quantum theory through the Exact Renormalization Group.

Key element: effective action $_k$ derived from a path integral with an IR cutoff at scale k.

$$Z_k = \int \mathcal{D}\varphi\ \mathrm{e}^{-S[\varphi]-\frac{1}{2}\int \varphi\,\mathcal{R}_k\,\varphi} \longrightarrow \partial_t\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\partial_t\mathcal{R}_k\right]$$

Already existent results for:
- Einstein-Hilbert ansatz for $_k$.
- Conformally reduced theory: $g_{\mu\nu} = \chi^2(x)\hat{g}_{\mu\nu}$ (CREH)

$$S = \int d^4x\sqrt{\hat{g}}\,\frac{3}{8\pi G}\left[\chi\Box\chi - \frac{\hat{R}}{6}\chi^2 + \frac{\Lambda}{3}\chi^4\right]$$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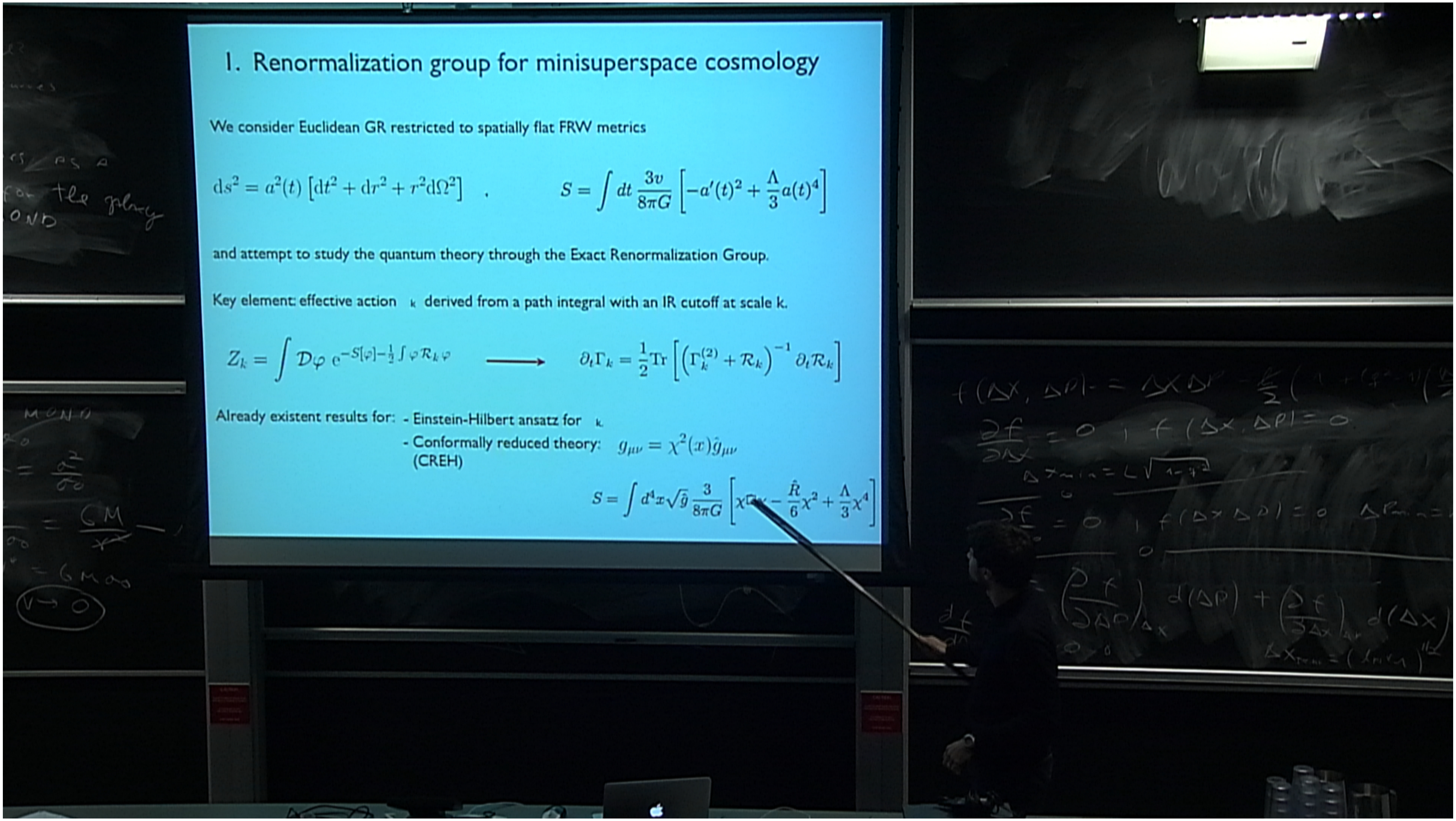
(CREH)
$S = \int d^4x\sqrt{\hat{g}}\,\frac{3}{8\pi G}\left[\chi\Box\chi - \frac{\hat{R}}{6}\chi^2 + \frac{\Lambda}{3}\chi^4\right]$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1. Renormalization group for minisuperspace cosmology (cont.)

We follow the derivation of the beta functions for G_k, $_k$ in the CREH theory, which are obtained by expanding the RG flow equation and matching terms

$$k\partial_k (G_k)^{-1} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathcal{F}(p^2, k)$$, simil. for Λ_k

Introduce a -function to suppress fluctuations in 4-n dimensions:

$$k\partial_k (G_k)^{-1} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \delta^{(4-n)} \left(\frac{p_i}{a_B k} \right) \mathcal{F}(p^2, k)$$

1. Renormalization group for minisuperspace cosmology (cont.)

We follow the derivation of the beta functions for G_k, $\;_k$ in the CREH theory, which are obtained by expanding the RG flow equation and matching terms

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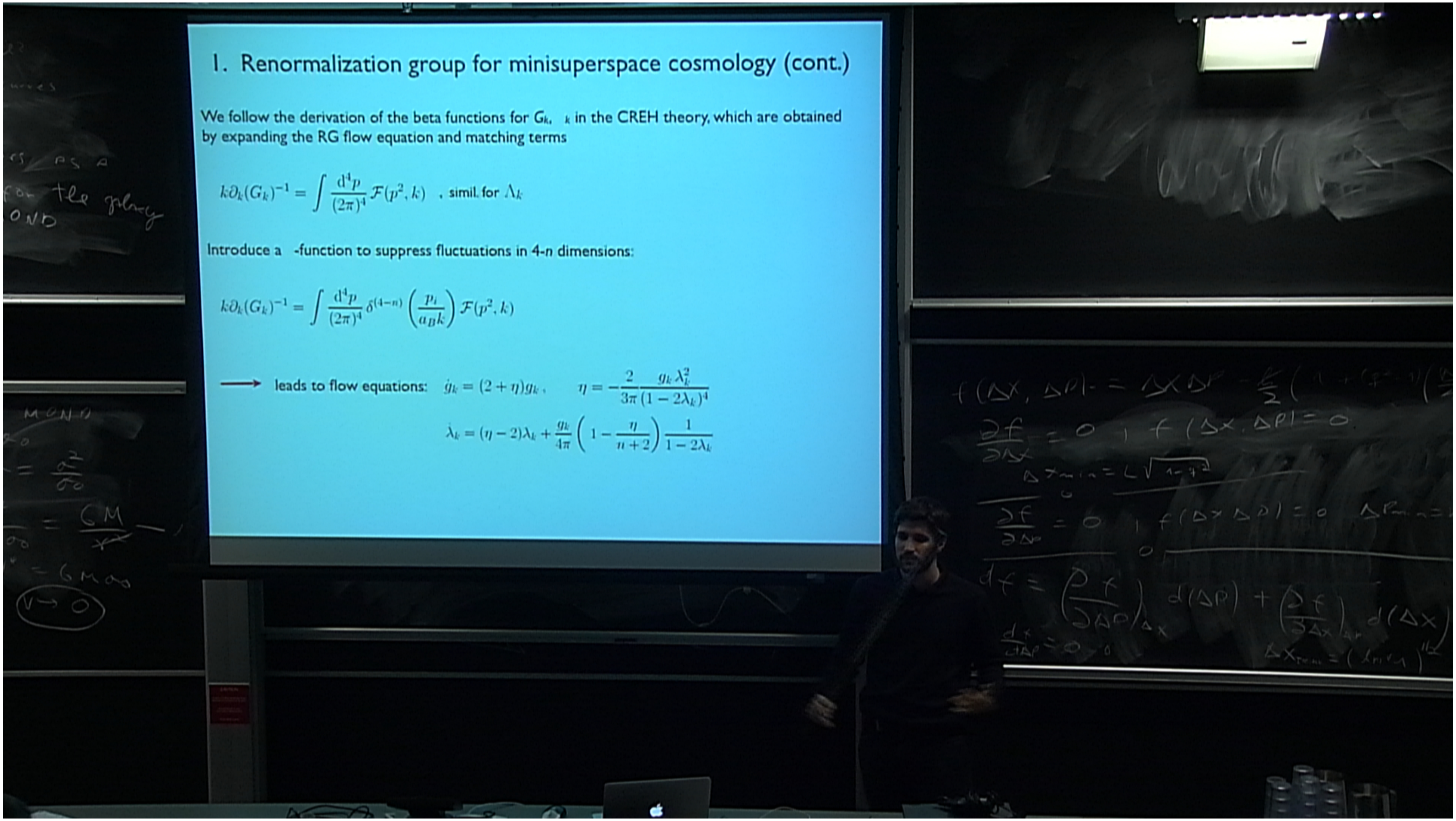
1. Renormalization group for minisuperspace cosmology (cont.)
We follow the derivation of the beta functions for G_k, k in the CREH theory, which are obtained by expanding the RG flow equation and matching terms
$k\partial_k (G_k)^{-1} = \int \frac{d^4 p}{(2\pi)^4} \mathcal{F}(p^2, k)$, simil. for Λ_k
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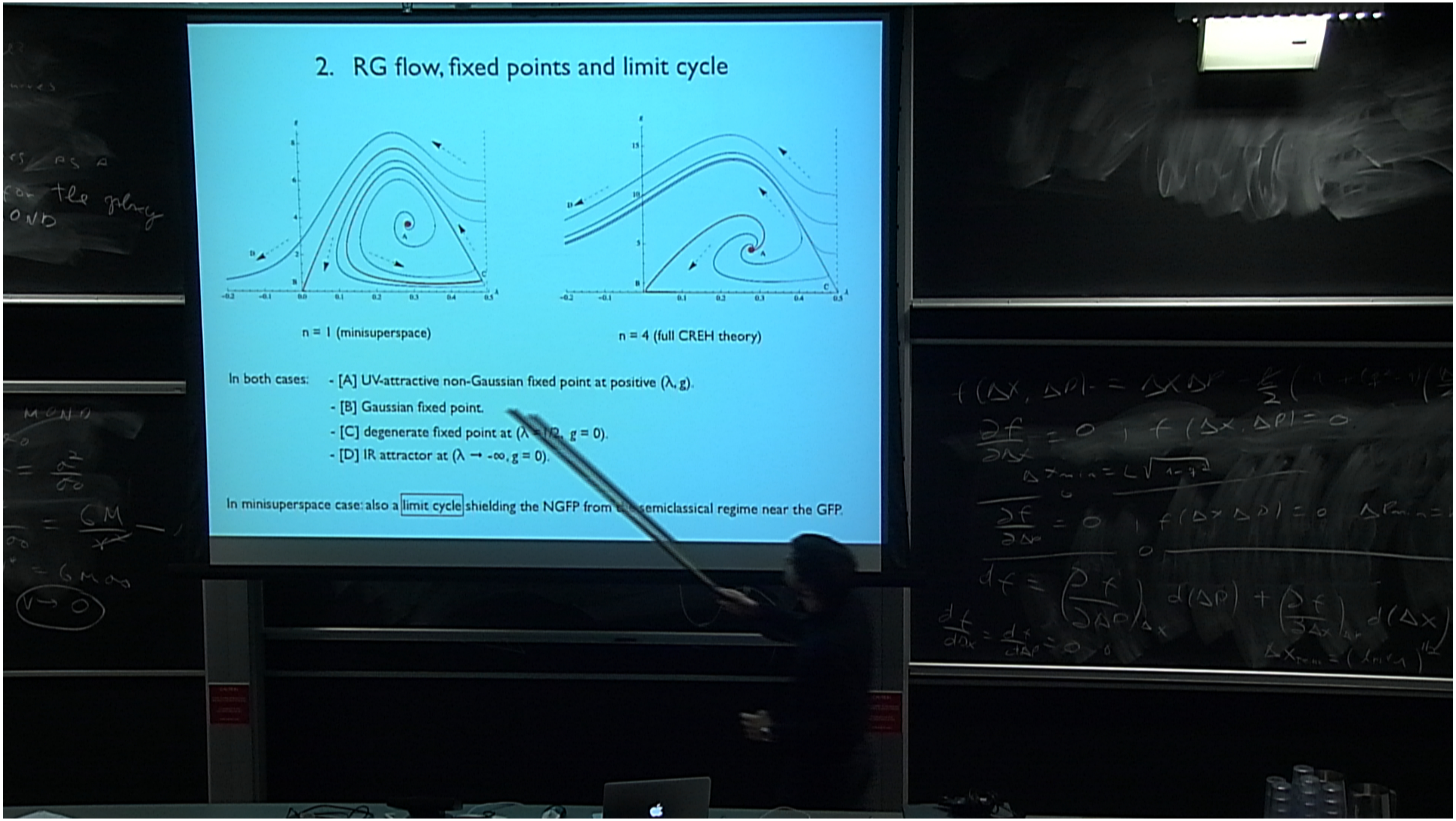
leads to flow equations: $\dot{g}_k = (2+\eta) g_k$, $\eta = -\frac{2}{3\pi} \frac{g_k \lambda_k^2}{(1-2\lambda_k)^4}$
$\dot{\lambda}_k = (\eta - 2)\lambda_k + \frac{g_k}{4\pi}\left(1 - \frac{\eta}{n+2}\right)\frac{1}{1-2\lambda_k}$


2. RG flow, fixed points and limit cycle
n = 1 (minisuperspace)
n = 4 (full CREH theory)
In both cases:
- [A] UV-attractive non-Gaussian fixed point at positive (λ, g).
- [B] Gaussian fixed point.
- [C] degenerate fixed point at (λ = 1/2, g = 0).
- [D] IR attractor at (λ → -∞, g = 0).
In minisuperspace case: also a limit cycle shielding the NGFP from the semiclassical regime near the GFP.

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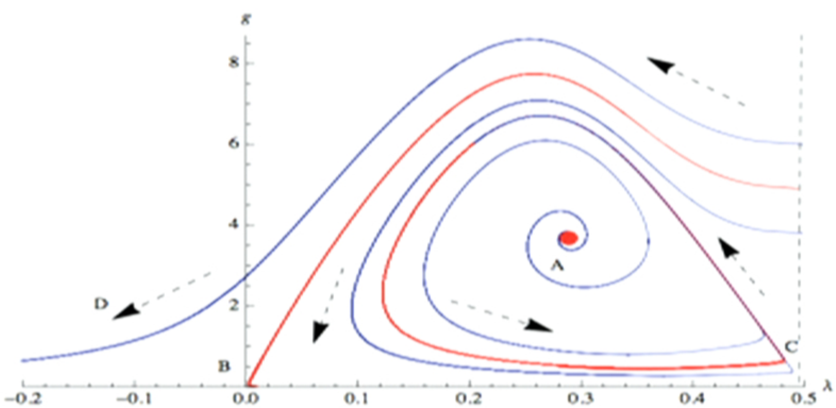
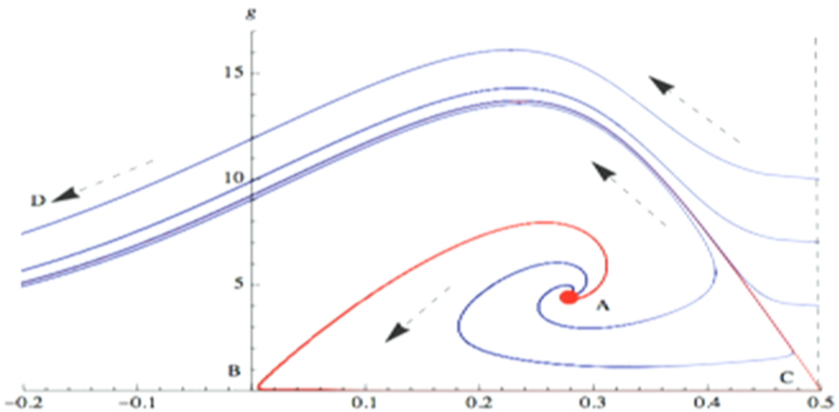


n = 1 (minisuperspace)



n = 4 (full CREH theory)

In both cases:

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In minisuperspace case: also a limit cycle shielding the NGFP from the semiclassical regime near the GFP.

4. Implications for cosmological fine-tuning

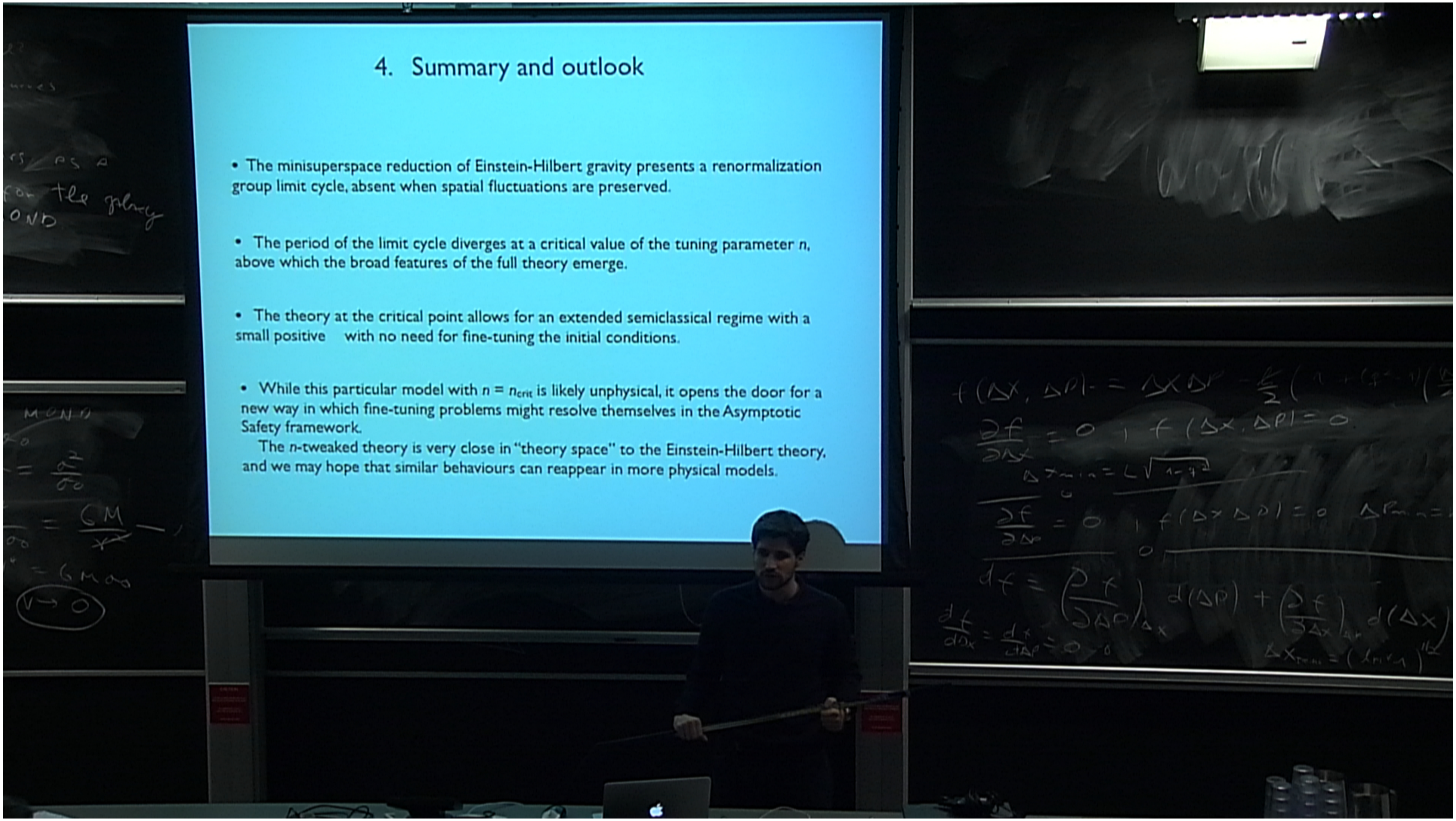
For the Asymptotic Safety research program to deliver a viable cosmology, the physical values of G_k and $_k$ must be approximately constant (and small and positive) over the wide range of scales where they are measured.

→ The RG flow trajectory realized in Nature must spend a large amount of RG "time" in the vicinity of the Gaussian fixed point, at $\lambda_k \gtrsim 0$ $g_k \gtrsim 0$.

For the usual EH and CREH truncations (and the minisuperspace too) this is not possible without fine-tuning the initial conditions of the RG flow.

For the critical value $n = n_{crit}$, all trajectories leaving the NGFP towards the IR achieve an extended semiclassical regime.

This suggest a new possible way in which the issue of the fine-tuning of the initial conditions for the flow might resolve itself.


4. Summary and outlook
• The minisuperspace reduction of Einstein-Hilbert gravity presents a renormalization group limit cycle, absent when spatial fluctuations are preserved.
• The period of the limit cycle diverges at a critical value of the tuning parameter n, above which the broad features of the full theory emerge.
• The theory at the critical point allows for an extended semiclassical regime with a small positive with no need for fine-tuning the initial conditions.
• While this particular model with n = n_crit is likely unphysical, it opens the door for a new way in which fine-tuning problems might resolve themselves in the Asymptotic Safety framework.
The n-tweaked theory is very close in "theory space" to the Einstein-Hilbert theory, and we may hope that similar behaviours can reappear in more physical models.

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