Title: Kicking Chameleons: Early Universe Challenges for Chameleon Gravity

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Abstract: Chameleon gravity is
a scalar-tensor theory that mimics general relativity in the Solar System. The scalar degree of freedom is hidden in high-density environments because the effective mass of the chameleon scalar depends on the trace of the stress-energy tensor. In the early Universe, when the trace of the stress-energy tensor is nearly zero, the chameleon is very light and Hubble friction prevents it from reaching its potential minimum. Whenever a particle species becomes non-relativistic, however, the trace of the stress-energy tensor is temporarily nonzero, and the chameleon begins to roll. I will show that these "kicks" to the chameleon field have catastrophic consequences for chameleon gravity. The velocity imparted to the chameleon is sufficiently large that the chameleon's mass changes rapidly as it slides past its potential minimum. This nonadiabatic process shatters the chameleon field by generating extremely high-energy perturbations, casting doubt on chameleon gravity's viability as an alternative to general relativity.

Kicking Chameleons:

Early Universe Challenges for Chameleon Gravity

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PI Cosmology Seminar
October 25, 2012

Overview: A Chameleon Catastrophe

Part I: Chameleon Cosmology Crash Course

What is chameleon gravity?
What is the chameleon's initial state?
What are the "kicks" and why are they important?

Part II: Classically Kicking Chameleons

How do chameleons respond to kicks?
How much do the chameleons move?
How fast do the chameleons move?

Part III: Quantum Chameleon Kicks

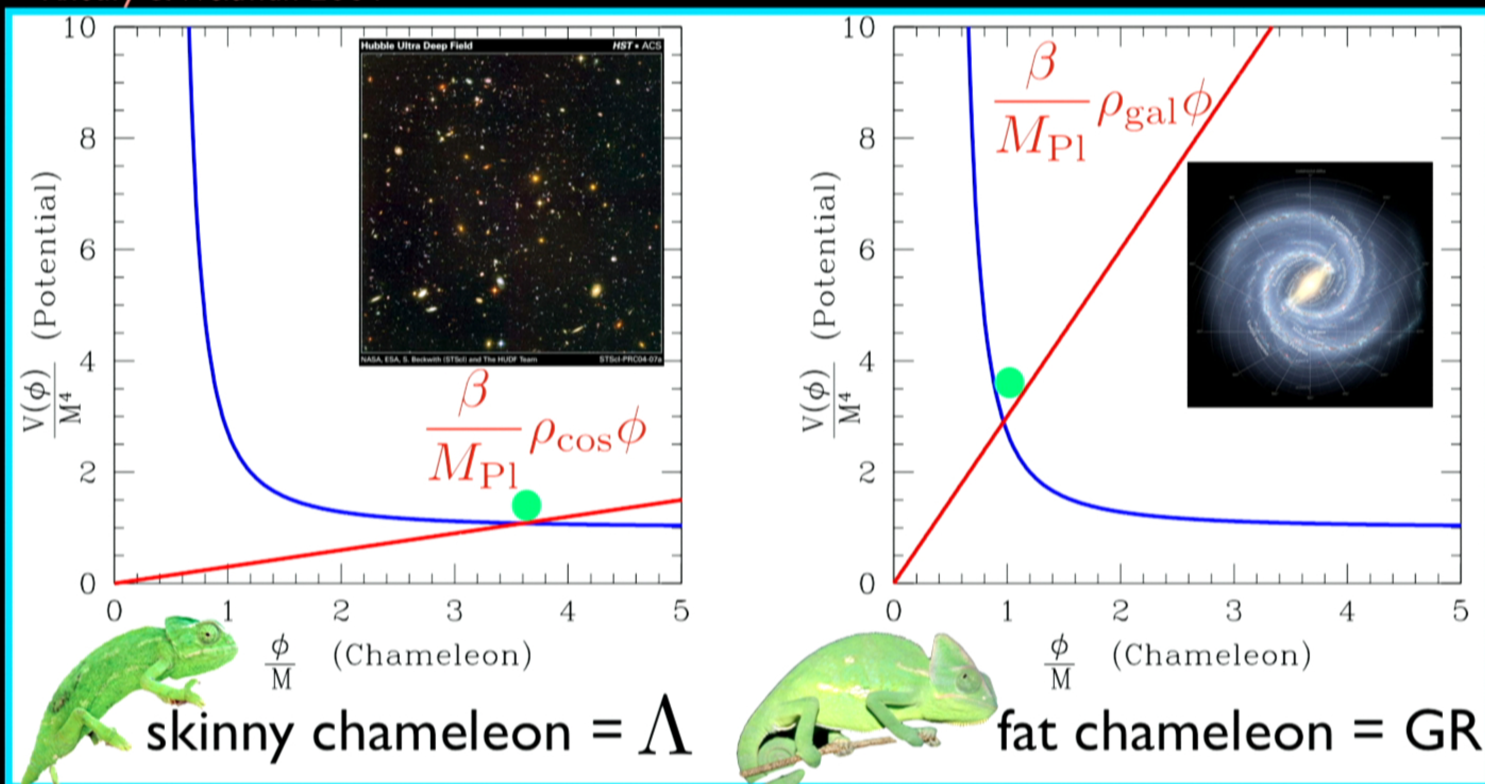
Why do rapid mass changes generate perturbations?
What perturbations result from the kicks?
Why is the chameleon in trouble?

Chameleon Gravity

Scalar-Tensor Gravity: we must hide the scalar!

Chameleon Gravity: scalar's mass depends on environment

Khoury & Weltman 2004



Chameleon Gravity: Nuts and Bolts

Chameleon gravity: a screened scalar-tensor theory *Khoury & Weltman 2004*

$$S = \int d^4x\sqrt{-g_*}\left[\frac{M_{\rm Pl}^2}{2}R_* - \frac{1}{2}(\nabla_*\phi)^2 - V(\phi)\right] + S_m\left[\tilde{g}_{\mu\nu}, \psi_m\right]$$

Einstein frame: standard GR + scalar field (chameleon field)

Matter couples to different metric (Jordan Frame)

$$\tilde{g}_{\mu\nu} = e^{2\beta\phi/M_{\rm Pl}}g^*_{\mu\nu}$$

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Densities in both frames:

$$\tilde{T}^\mu{}_\nu \equiv \mathrm{diag}\left[-\tilde{\rho}, \tilde{p}, \tilde{p}, \tilde{p}\right]$$

$$T_*{}^\mu{}_\nu \equiv \mathrm{diag}\left[-\rho_*, p_*, p_*, p_*\right]$$

$$T_*{}^\mu{}_\nu = \left(e^{4\beta\phi/M_{\rm Pl}}\right)\tilde{T}^\mu{}_\nu$$

Assume FRW in both frames:

$$\tilde{a} = e^{\beta\phi/M_{\rm Pl}} a_* \qquad d\tilde{t} = e^{\beta\phi/M_{\rm Pl}} dt_*$$

scale factor — *proper time*

Key parameter: the chameleon coupling constant β

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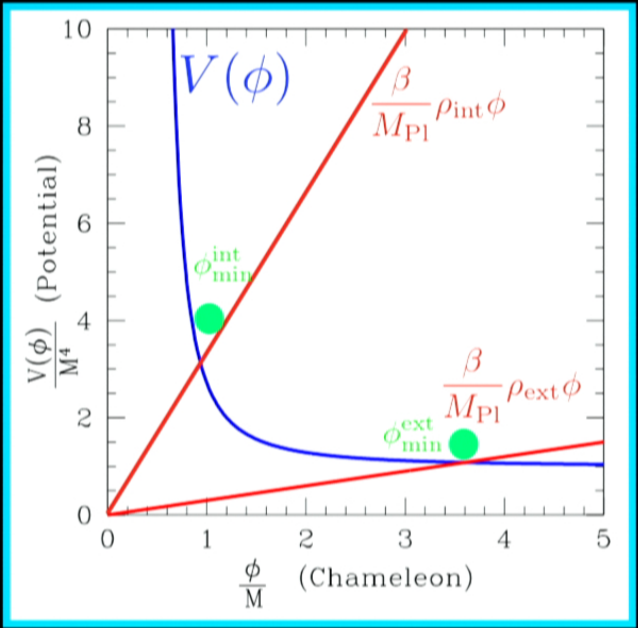
The Effective Potential

Vary action w.r.t. Einstein metric: $G_{\mu\nu} = 8\pi G\left(T^*_{\mu\nu} + T^\phi_{\mu\nu}\right)$

Vary action w.r.t. chameleon field: $\left(\tilde{g}_{\mu\nu} = e^{2\beta\phi/M_{\rm Pl}} g^*_{\mu\nu}\right)$

$$\ddot{\phi} + 3H_*\dot{\phi} = -\left[\frac{dV}{d\phi} + \frac{\beta}{M_{\rm Pl}}(\rho_* - 3p_*)\right]$$

derivative of effective potential



Thin shell mechanism: *Khoury & Weltman 2004*

$$\frac{\phi^{\rm ext}_{\rm min} - \phi^{\rm int}_{\rm min}}{M_{\rm Pl}} \lesssim \beta\frac{GM_s}{R_s}$$

Inside an massive body, $\phi \simeq \phi^{\rm int}_{\rm min}$ and the scalar force outside the massive body is suppressed because

$$m_{\rm int} = \sqrt{V''_{\rm eff}(\phi^{\rm int}_{\rm min})} \gg R_s$$

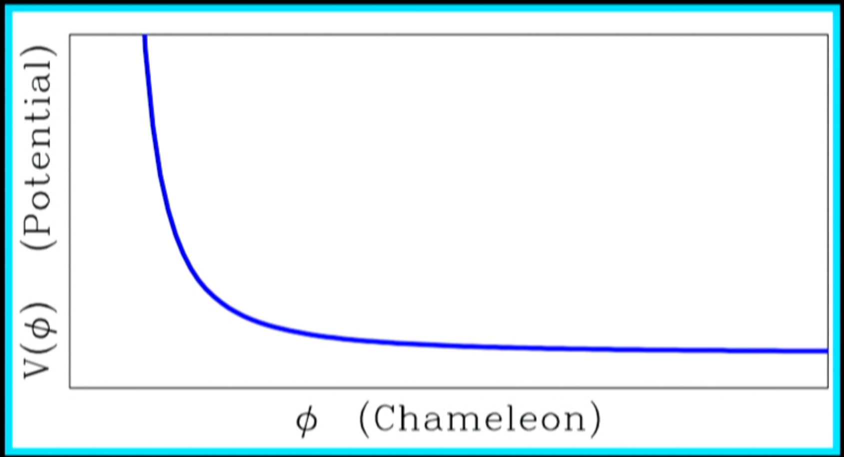
Chameleon Cosmology

Fiducial Chameleon Potential: *Brax et al. 2004*

$$V(\phi) = M^4 \exp\left[\left(\frac{M}{\phi}\right)^n\right] \overset{\phi \gg M}{\simeq} M^4\left[1 + \left(\frac{M}{\phi}\right)^n\right]$$

Evade Solar System gravity tests and provide dark energy:

$$M \simeq 0.001\,\mathrm{eV} \simeq (\rho_{\mathrm{de}})^{1/4}$$



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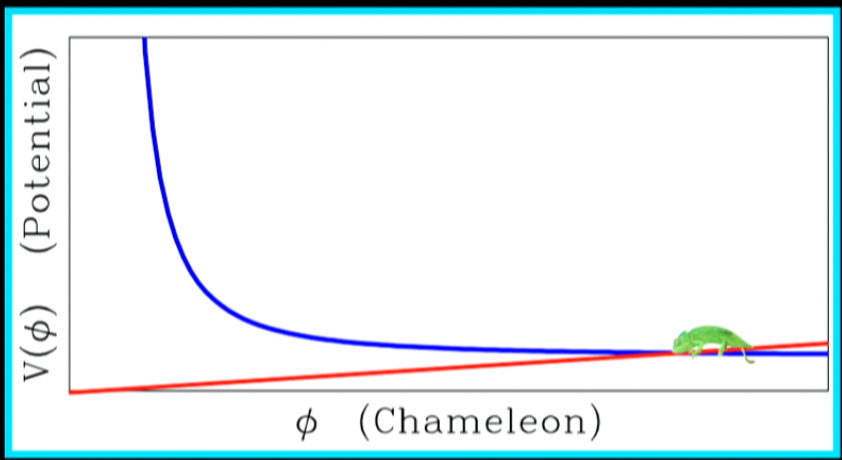
Where is the chameleon now?



$$\rho_{\mathrm{mat},0} = 0.3\rho_{\mathrm{crit},0}$$

$$\phi_{\mathrm{min}} = 5.9 \times 10^9 M \ll M_{\mathrm{Pl}}$$

$\phi_{\mathrm{min}} \ll M_{\mathrm{Pl}}$ **always!**

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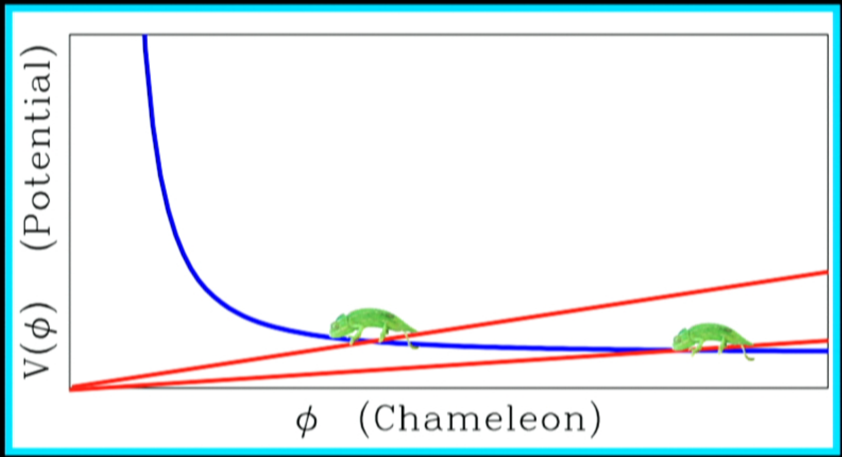

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$$\rho_{\mathrm{gal}} = 0.6\,\mathrm{GeV/cm^3}$$

$$\phi_{\mathrm{min}} = 8.3 \times 10^7 M$$

Chameleon Initial Conditions

During inflation: $\rho - 3p \simeq 4\rho_{\rm infl}$ pins chameleon $\phi \ll M$ *Brax et al. 2004*

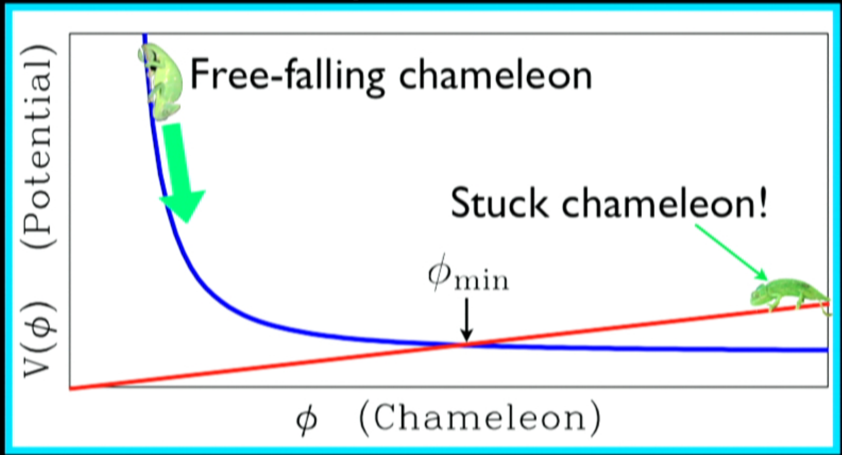
After reheating: $\rho - 3p \simeq 0$ the chameleon quickly slides down its bare potential and rolls to $\phi \gg \phi_{\rm min}$

For $\phi \gg \phi_{\rm min}$

$$\ddot{\phi} + 3H_*\dot{\phi} = -\frac{\beta}{M_{\rm Pl}}(\rho_* - 3p_*) \implies \Delta\phi \simeq \frac{\dot{\phi}_i}{H_i} = M_{\rm Pl}\sqrt{6\Omega_{\dot{\phi},i}}$$

Chameleon rolls out to $\phi_{\rm min} \ll \phi \lesssim M_{\rm Pl}$

Hubble friction prevents the chameleon from rolling back to $\phi_{\rm min}$



Unsticking the Chameleon

Particles in thermal equilibrium:

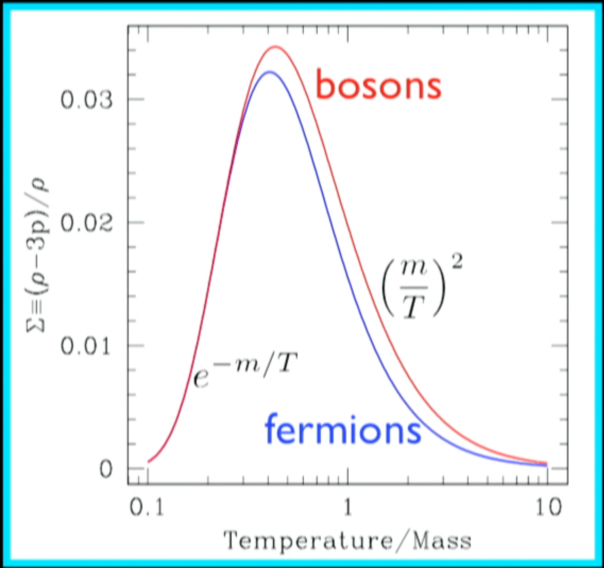
Damour & Nordtvedt 1993
Damour & Polyakov 1994
Brax et al. 2004
Coc et al. 2006, 2009

Jordan-frame density

$$\tilde{\rho} = \frac{g}{2\pi^2}\int_m^\infty \frac{E^2(E^2-m^2)^{1/2}}{e^{E/T}\pm 1}\,dE$$

Jordan-frame pressure

$$\tilde{p} = \frac{g}{6\pi^2}\int_m^\infty \frac{(E^2-m^2)^{3/2}}{e^{E/T}\pm 1}\,dE = \frac{\tilde{\rho}}{3}\left[1+\mathcal{O}\left(\frac{m^2}{T^2}\right)\right]$$



Define the kick function:

$$\Sigma(T_J) \equiv \frac{\tilde{\rho}_R - 3\tilde{p}_R}{\tilde{\rho}_R} = \frac{\rho_{*R} - 3p_{*R}}{\rho_{*R}}$$

$$\ddot{\phi} + 3H_*\dot{\phi} = -\left[\frac{dV}{d\phi} + \frac{\beta}{M_{\rm Pl}}\rho_{*R}\Sigma\right]$$

Every time a mass-threshold is crossed, the chameleon gets kicked!

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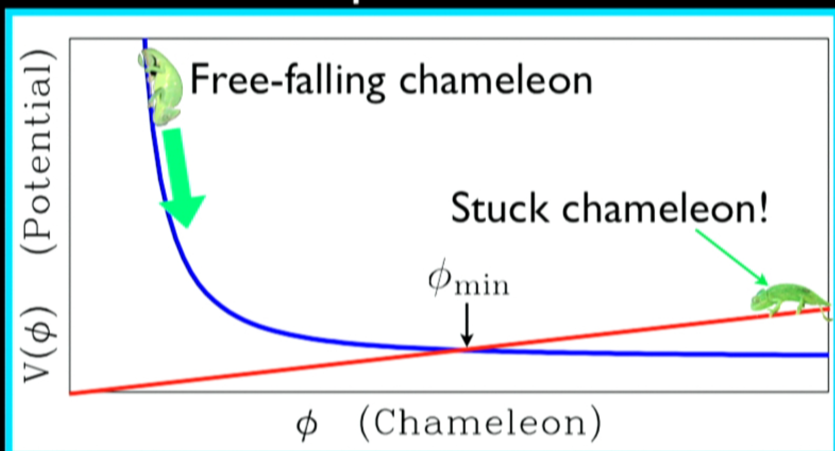
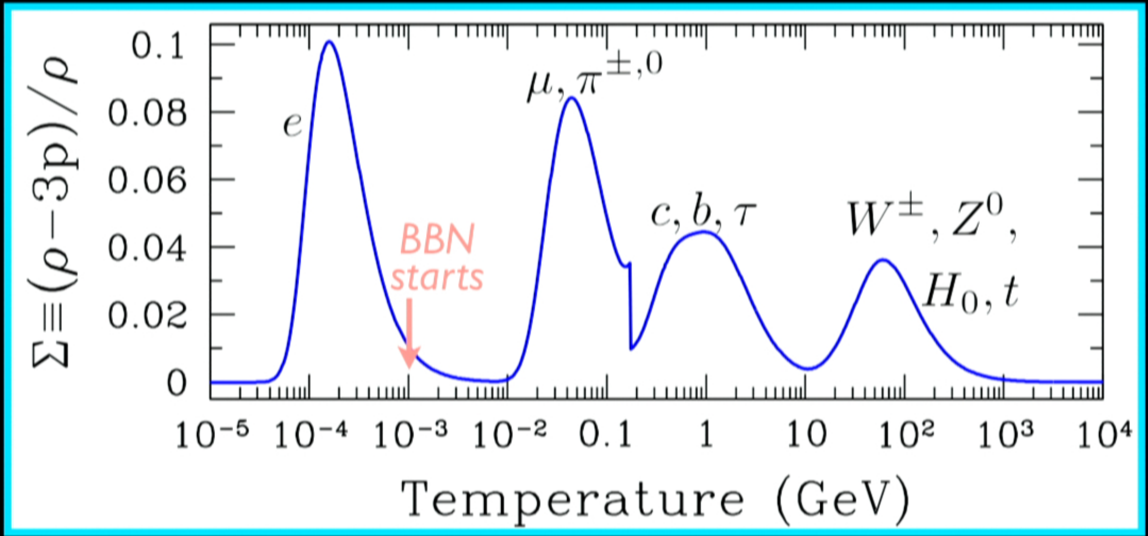


Kicks from the Standard Model

Every particle in the Standard Model (and beyond) kicks the chameleon.

- there are 4 distinct "combo-kicks" with increasing amplitude
- there is a kick during BBN between n,p freeze-out and helium production
- kicks dominate over dark matter: $\rho_{*R}\Sigma \gg \rho_{*M}$ for $T_J \gtrsim 0.024\,\mathrm{MeV}$
- during the kicks, $\phi_{\min} \lesssim M$

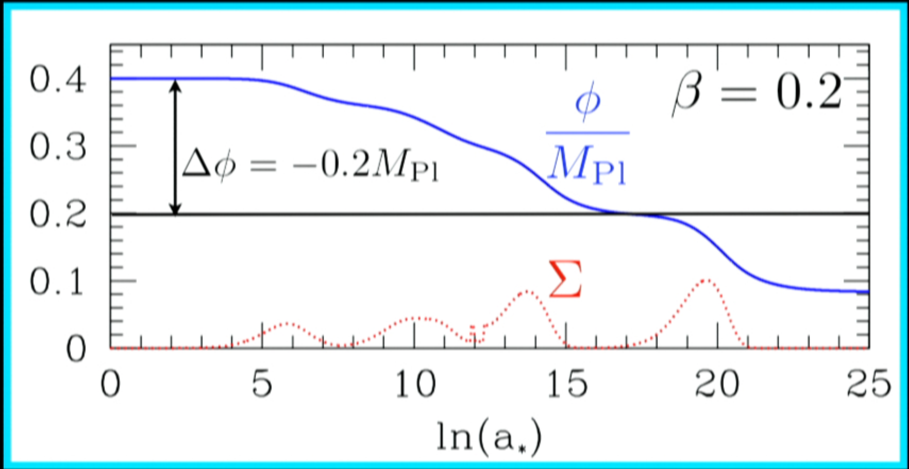


The Old Story

The kicks save the chameleon: $\Delta\phi \simeq -\beta M_{\rm Pl}$ prior to BBN. *Brax et al. 2004*

- treat kicks individually and assume that $|\beta\Delta\phi| \ll M_{\rm Pl} \Leftrightarrow \beta^2 \ll 1$
- BBN requirement ($\phi_{\rm BBN} \lesssim (0.1/\beta) M_{\rm Pl}$) is satisfied for

$$\phi_i \lesssim \left(\beta + \frac{0.1}{\beta}\right) M_{\rm Pl}$$



For a wide range of initial conditions, the chameleon reaches the minimum of its effective potential and happily lives there for the rest of its days.

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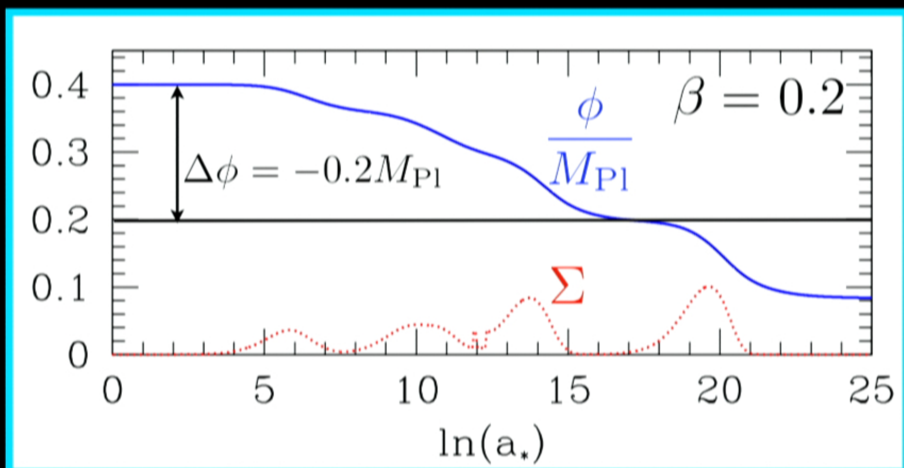

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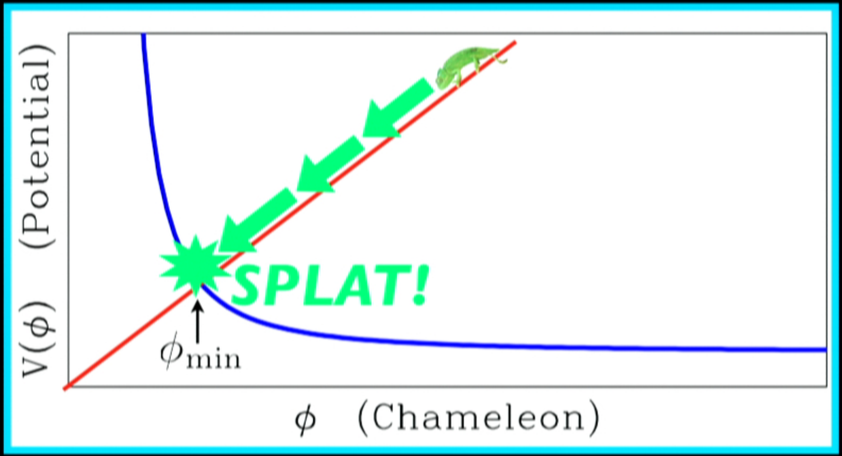
No happily ever after!

The standard chameleon story misses several important features:

1. Ignoring the feedback of $\Delta\phi$ on T_J severely underestimates chameleon motion for $\beta \gtrsim 1.8$.
2. Nearly all chameleons reach $\phi_{\rm min}$ with a large velocity and climb up their bare potentials.
3. The classical picture is incomplete because the rebound is violent enough to excite quantum perturbations.



Part II: Classical Kicks

What is the chameleon's velocity when it reaches $\phi_{\rm min}$?

Part III: Quantum Kicks

What happens to the chameleon when it rebounds?

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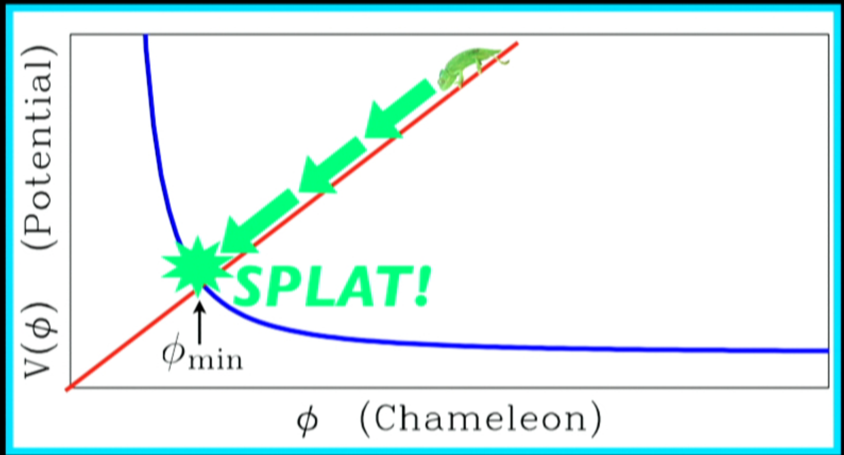

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The Equation of Motion Revisited

$$\ddot{\phi} + 3H_*\dot{\phi} = -\left[\frac{dV}{d\phi} + \frac{\beta}{M_{\rm Pl}}\rho_{*R}\left(\Sigma + \frac{\rho_{*M}}{\rho_{*R}}\right)\right]$$

- change variables: $p = \ln(a_*)$ $\quad \varphi \equiv \phi/M_{\rm Pl}$ $\quad \varphi'(p) = \sqrt{6\Omega_{\dot{\phi}}}$
- assume $\phi \gg \phi_{\rm min}$ and neglect the bare potential
- recall that $(\rho_{*M}/\rho_{*R}) \ll \Sigma \lesssim 0.1$ and keep only first order in Σ
- use Friedmann eqn. in Einstein frame

$$\varphi'' + \varphi'\left[1 - \frac{(\varphi')^2}{6}\right] = -3\beta\left[1 - \frac{(\varphi')^2}{6}\right]\Sigma(T_J)$$

Jordan-frame temperature: $g_{*S}(T_J)\tilde{a}^3T_J^3 = \text{constant}$

$$T_J\left[\frac{g_{*S}(T_J)}{g_{*S}(T_{J,i})}\right]^{1/3} = T_{J,i}\frac{\tilde{a}_i}{\tilde{a}} = \frac{T_{J,i}}{a_*}e^{\beta(\varphi_i - \varphi)}$$

compute and invert numerically

old story: $e^{-\beta\Delta\varphi} \simeq 1$

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The Surfing Solution

Keeping the full expression for T_J reveals a new solution!

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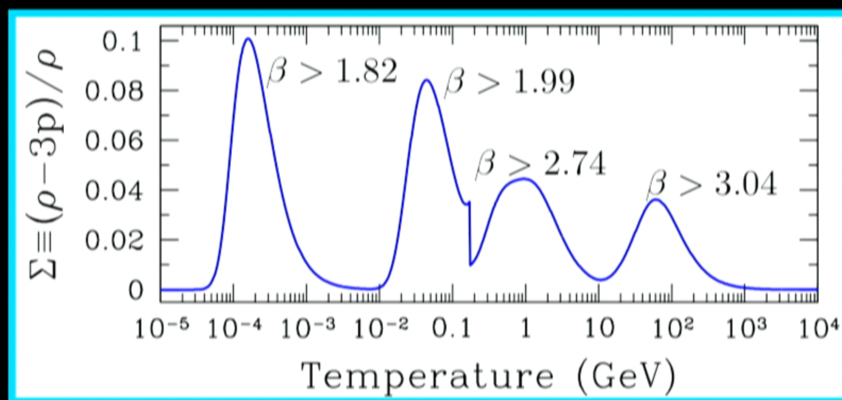
The surfing solution only exists if

$$\beta \geq \sqrt{\frac{1}{3\Sigma_{max}}}$$



The Surfing Solution

Keeping the full expression for T_J reveals a new solution!

$$\varphi = \frac{-p+\lambda}{\beta} \Rightarrow T_J \left[\frac{g_{*S}(T_J)}{g_{*S}(T_{J,i})}\right]^{1/3} = T_{J,i} e^{\beta(\varphi_i - \varphi) - p} = T_{J,i} e^{\beta\lambda}$$

$p = \ln(a_*)$

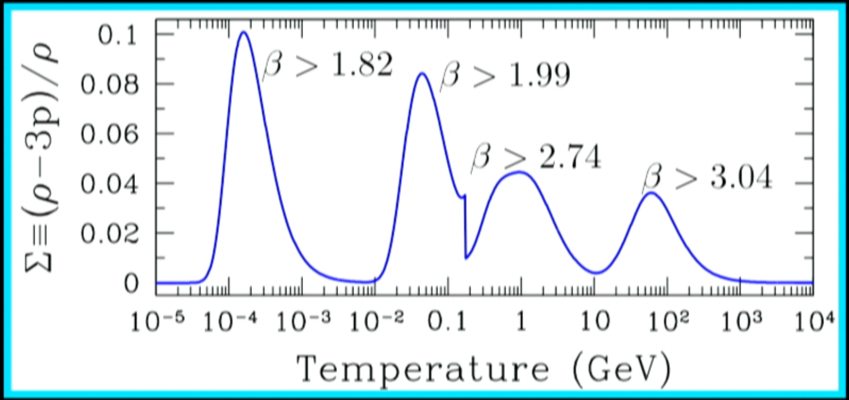
The temperature is constant in the Jordan frame!

$\varphi'(p) = -\frac{1}{\beta}$ solves $\varphi'' + \varphi'\left[1 - \frac{(\varphi')^2}{6}\right] = -3\beta\left[1 - \frac{(\varphi')^2}{6}\right]\Sigma$

provided that $\beta = \sqrt{\frac{1}{3\Sigma(T_J)}}$ for some value of T_J.

The surfing solution only exists if

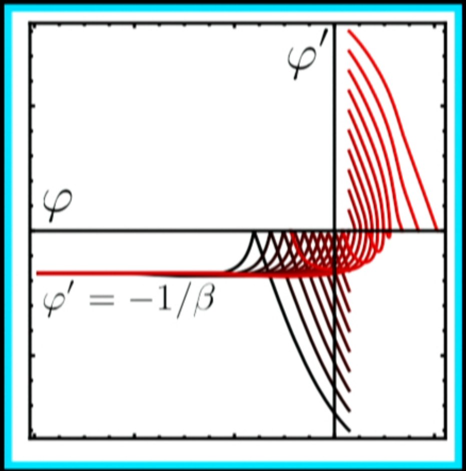
$$\beta \geq \sqrt{\frac{1}{3\Sigma_{\max}}}$$

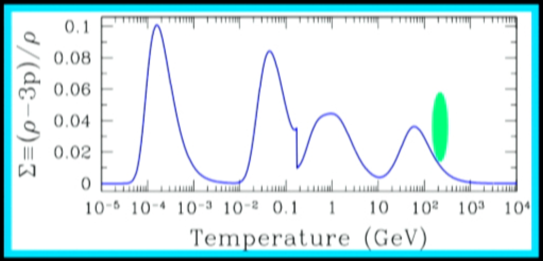


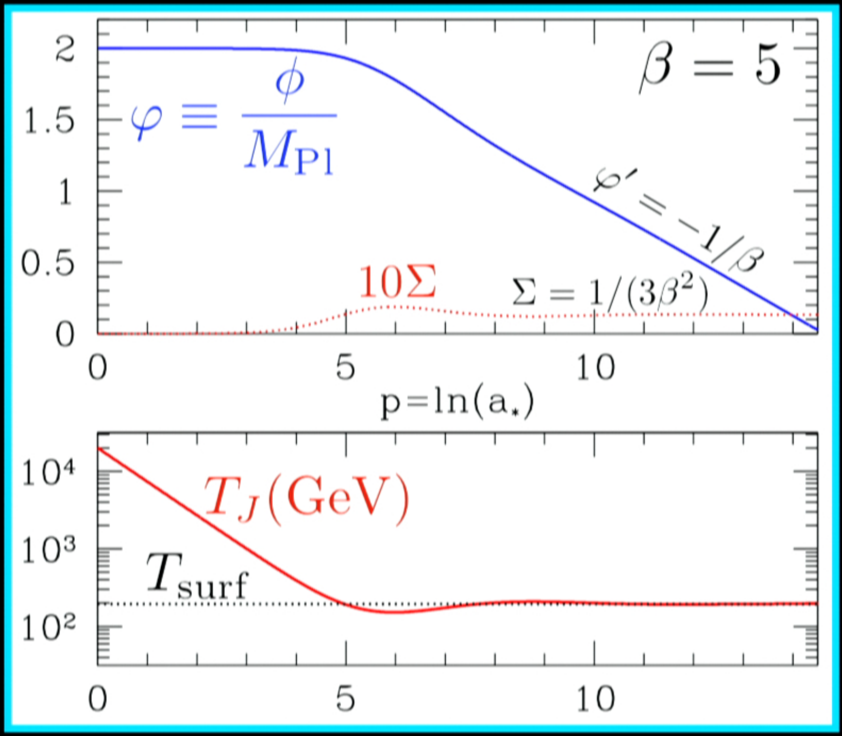
Surfing Chameleons

Chameleons that can surf, do surf!

- valid for any $\phi_i \gg \phi_{\rm min}$ and $\Omega_{\dot\phi} \lesssim 0.5$
- solution holds until $\phi \simeq \phi_{\rm min} \lesssim M$







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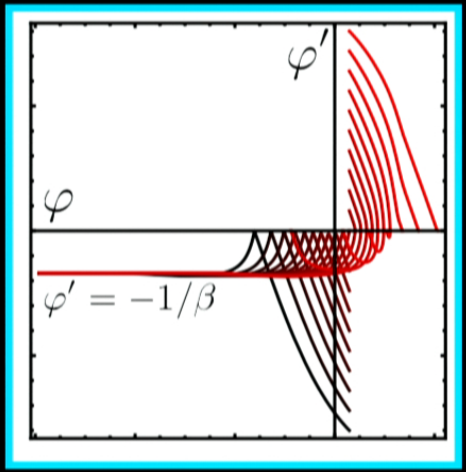
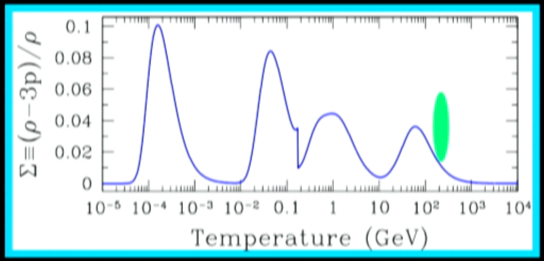
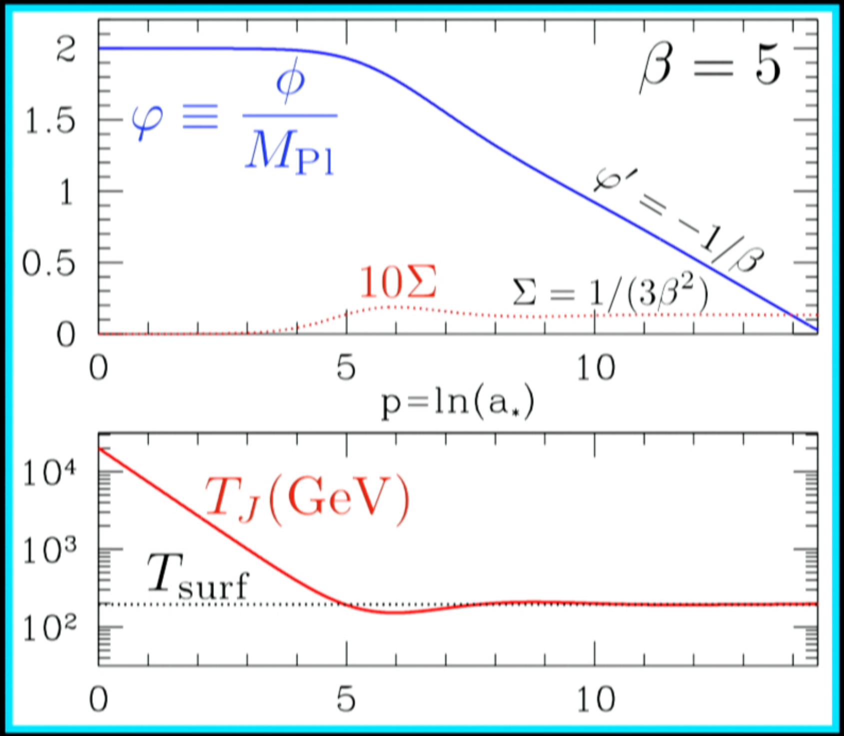
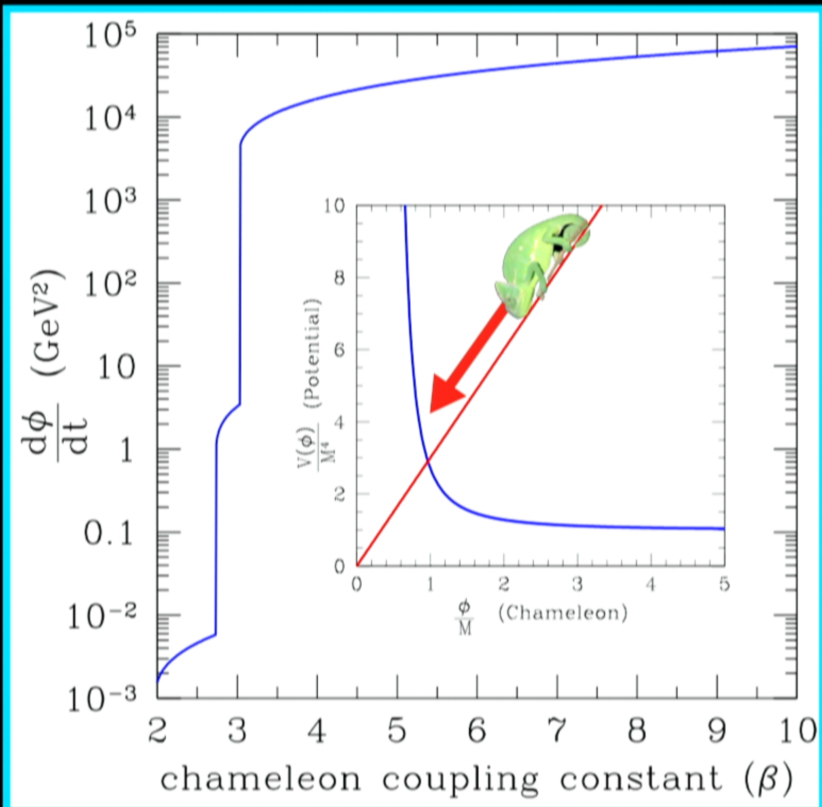






Surfing Velocity

For every value of $\beta > 1.82$, the surf solution has $\Sigma(T_{\rm surf}) = \dfrac{1}{3\beta^2}$



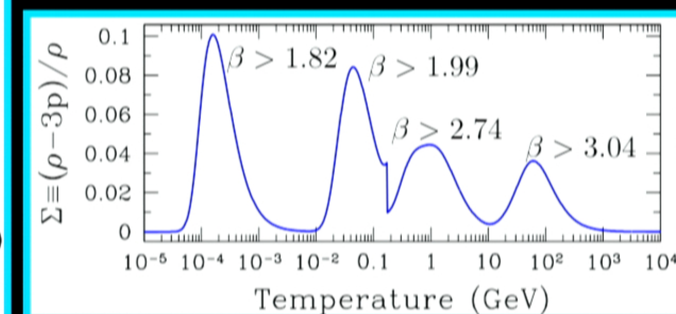
$$\dot\phi = H_*\phi'(p) = -\frac{H_* M_{\rm Pl}}{\beta}$$

$$\dot\phi = \sqrt{\frac{2\rho_{*R}}{6\beta^2 - 1}}$$

At the end of the surf

$$\phi \ll M_{\rm Pl}$$

$$\rho_{*R} \simeq \tilde\rho = \frac{\pi^2}{30} g_*(T_{\rm surf}) T^4_{\rm surf}$$



What if the chameleon can't surf?

Return to chameleon equation of motion for $\phi \gg \phi_{\rm min}$:

$$\frac{1}{a^3}\frac{d}{dt}\left(a_*^3\dot{\phi}\right) = -\frac{\beta}{M_{\rm Pl}}\rho_{*R}\Sigma(T_J)$$

Integrate twice: $$\frac{\Delta\phi}{M_{\rm Pl}} = -3\beta\int_1^{e^p}\frac{dx}{x^2}\int_1^x \Sigma(T_J[\phi(a_*,a_*])da_*$$

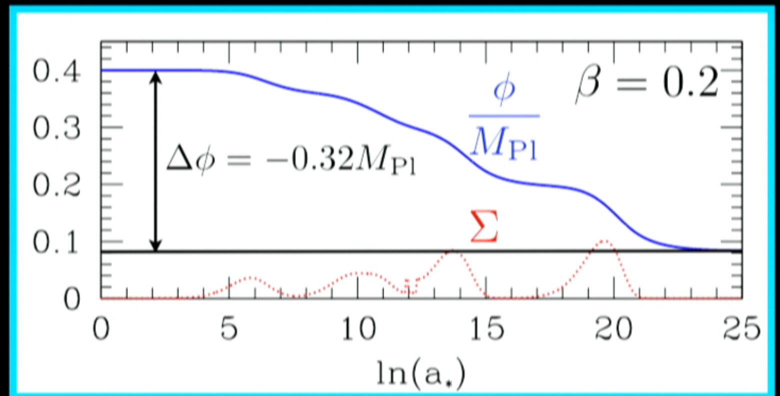
But we can't use that because T_J depends on chameleon's motion:

$$T_J\left[\frac{g_{*S}(T_J)}{g_{*S}(T_{J,i})}\right]^{1/3} = \frac{T_{J,i}}{a_*}e^{\beta(\phi_i-\phi)/M_{\rm Pl}} \rightarrow 1$$

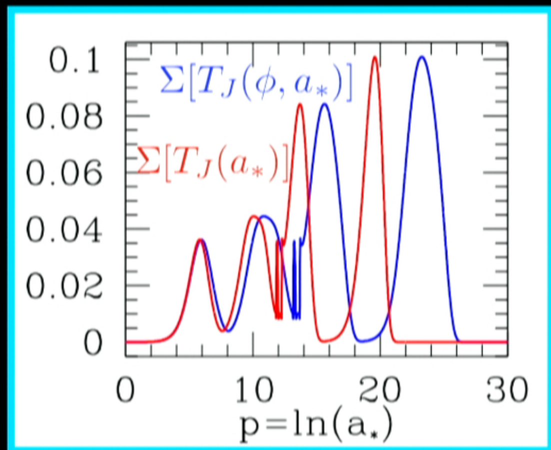
If $\beta|\Delta\phi| \ll M_{\rm Pl}$,

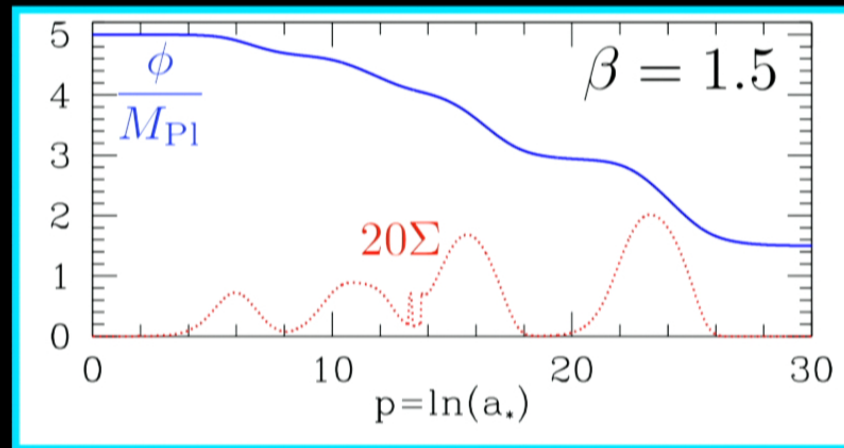
$$\Delta\phi \simeq -1.58\beta M_{\rm Pl}$$

- works well for $\beta < 0.7$
- underestimates $\Delta\phi$ for larger β



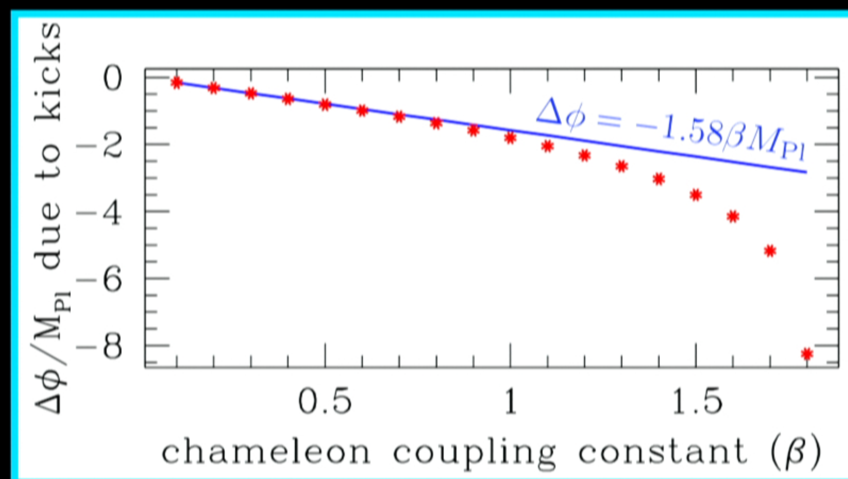
What if the chameleon can't surf?





For larger β values:

- motion of ϕ affects T_J
- slows Jordan-frame cooling
- extends duration of kicks
- $|\Delta\phi| > 1.58\beta M_{\rm Pl}$
- the surfer is the limit



Impact is difficult to avoid!

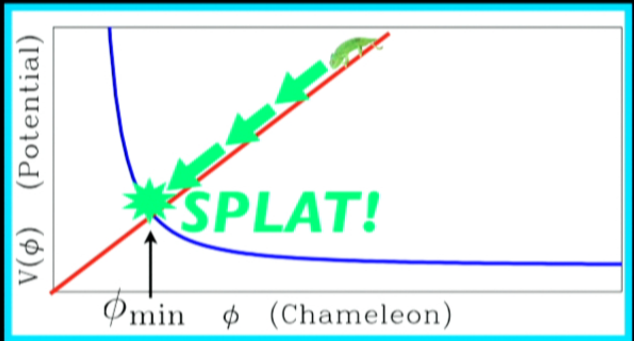
The kicks move the chameleon toward the minimum of its effective potential, but does the chameleon always reach it?

- first 3 combo kicks give $\Delta\phi \gtrsim -\beta M_{\rm Pl}$ prior to BBN
- last kick gives $\Delta\phi \gtrsim -0.56\beta M_{\rm Pl}$ during BBN
- to avoid messing with BBN, $\phi_{\rm BBN} \lesssim (0.1/\beta)M_{\rm Pl}$
- for $\beta > 0.42,\ \phi_{\rm BBN} \leq 0.56 M_{\rm Pl}$: the last kick takes $\phi < \phi_{\rm min}$
- for smaller β values, avoiding impact requires

$$(\Delta + 0.56)\beta < \frac{\phi_i}{M_{\rm Pl}} < \Delta + \frac{0.1}{\beta}$$

with $\Delta \simeq 1$ for the standard model.

Only weakly coupled $(\beta < 0.42)$ chameleons can avoid impact, and the initial condition must be finely tuned based on the entire particle content of the Universe!



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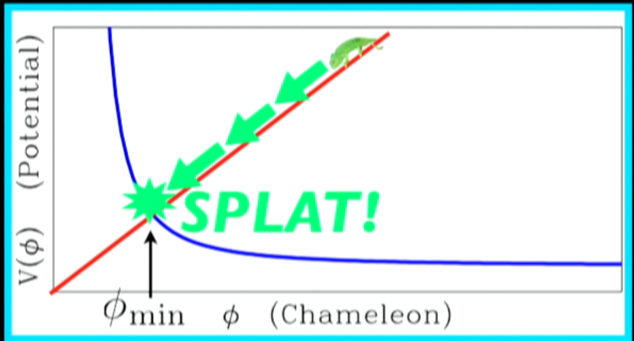
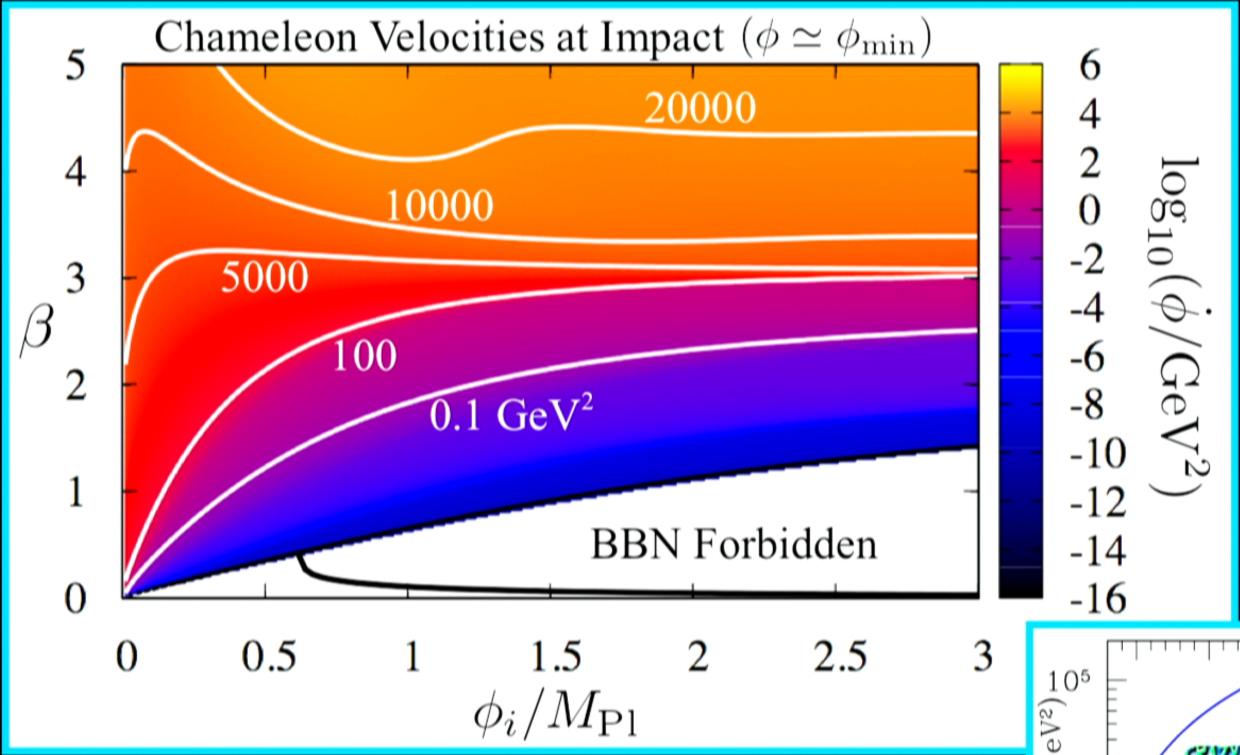
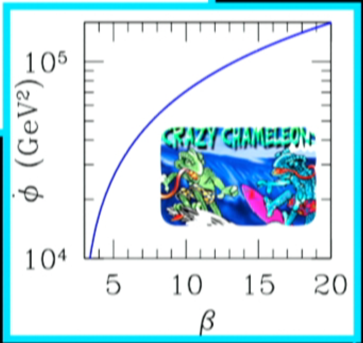


Fast-Moving Chameleons





- nearly all chameleons are kicked to $\phi \simeq \phi_{\rm min}$
- at $\phi \simeq \phi_{\rm min}$, $\dot\phi \sim {\rm GeV}^2 \gg M^2 \sim 10^{-24}{\rm GeV}^2$
- adding particles will increase the chameleon velocity!

A Classical Impact

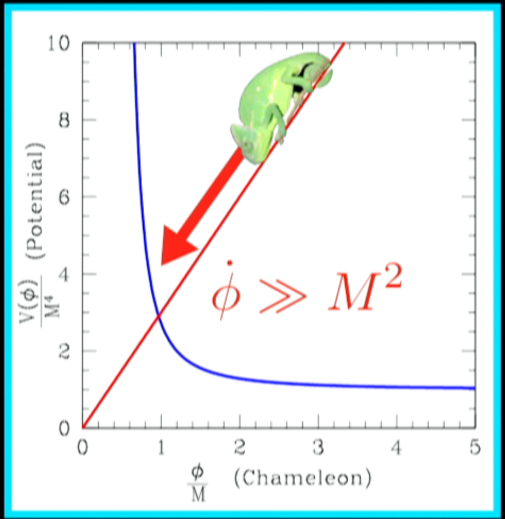
Now that $\phi \simeq \phi_{\rm min}$, we need to consider the chameleon potential:

$$V(\phi) = M^4 \exp\left[\left(\frac{M}{\phi}\right)^2\right] \quad \text{with} \quad M = 0.001\,\text{eV}$$

During the kicks, $0.13M \lesssim \phi_{\rm min} \lesssim 0.62M$, but the chameleon doesn't stop there - it's moving too fast!

The chameleon rolls up its potential until $V(\phi_b) = \dot{\phi}^2/2$

$$0.085M \lesssim \left(\phi_b = M\left[\ln\left(\frac{\dot{\phi}^2}{2M^4}\right)\right]^{-1/2}\right) \lesssim 0.11M$$



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- short time scale: $H_* \Delta t \lesssim M/\phi'(p) \lesssim \beta M/M_{\rm Pl}$
- Hubble friction + kicks: $\Delta\dot\phi \simeq (M/M_{\rm Pl})\dot\phi$
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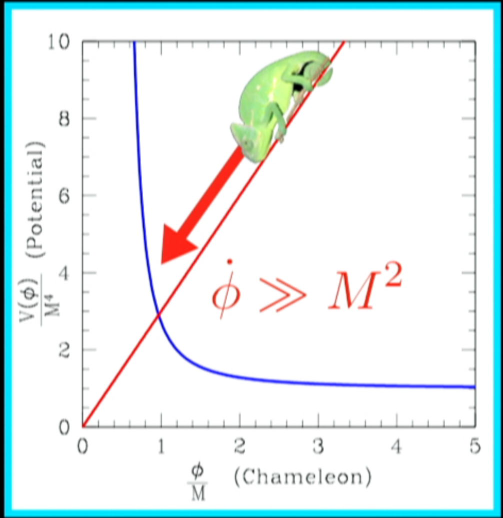

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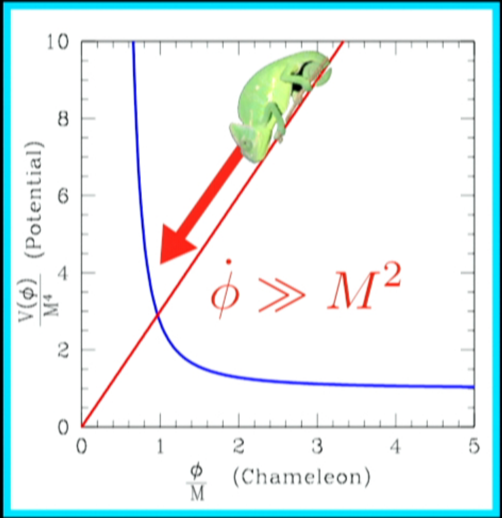

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During the kicks, $0.12M \lesssim \phi_{\rm min} \lesssim 0.62M$, but the chameleon doesn't stop t… o fast!

The chamel… until $V(\phi_b) = \dot{\phi}^2/2$

$$0.085M \lesssim \left(\phi_b \ldots\right) \lesssim 0.11M$$

Classically, the impact is a reflection!

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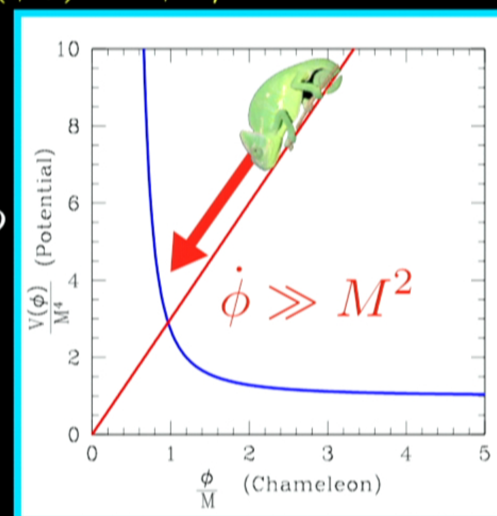

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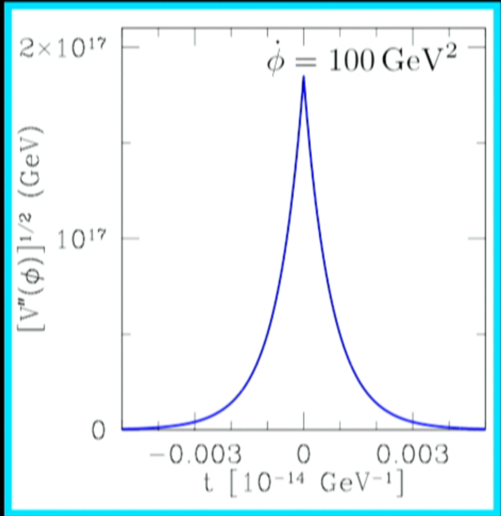
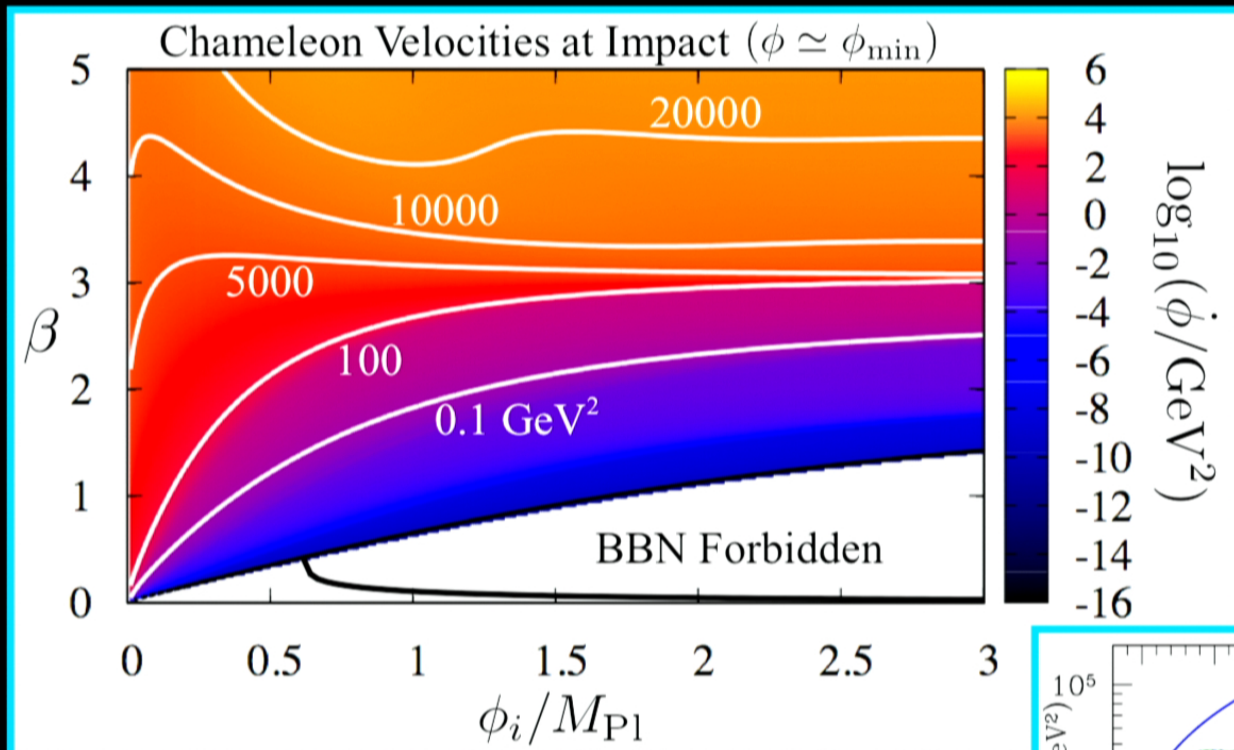
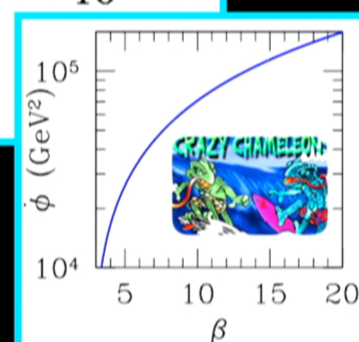


Part III
Quantum Chameleon Kicks

Fast-Moving Chameleons





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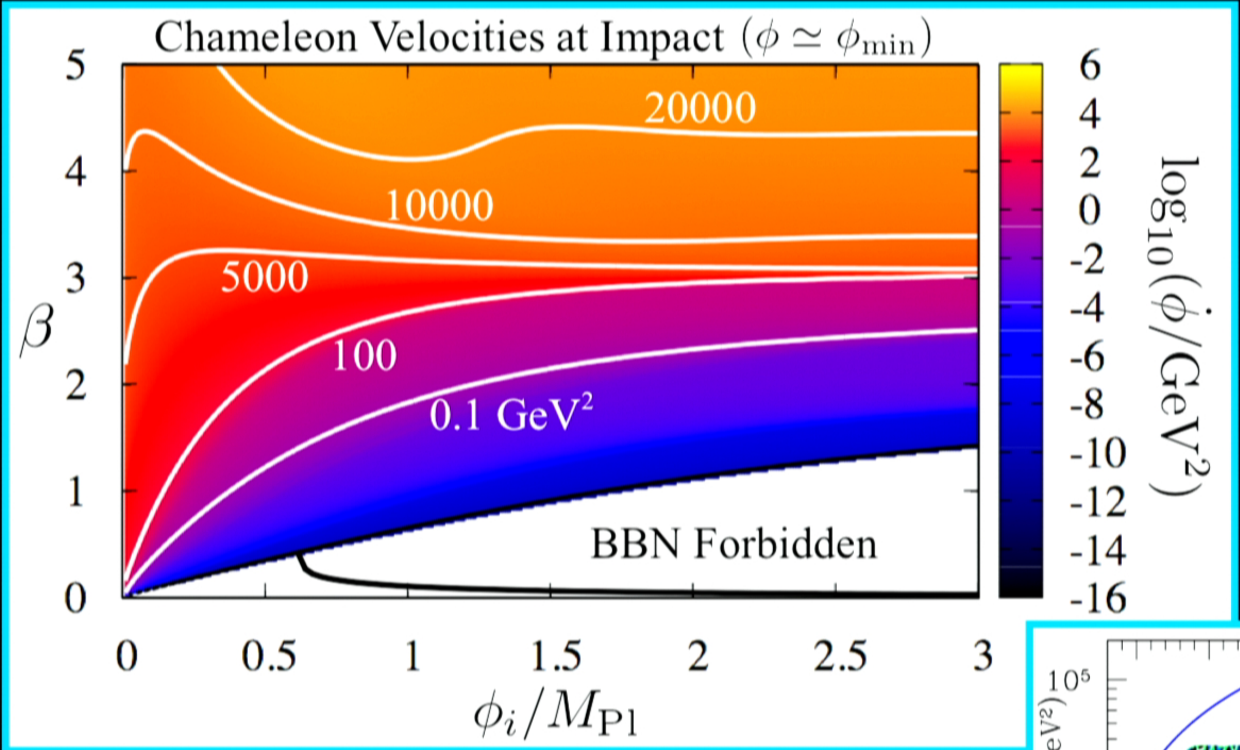

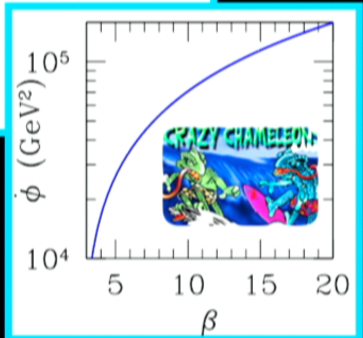

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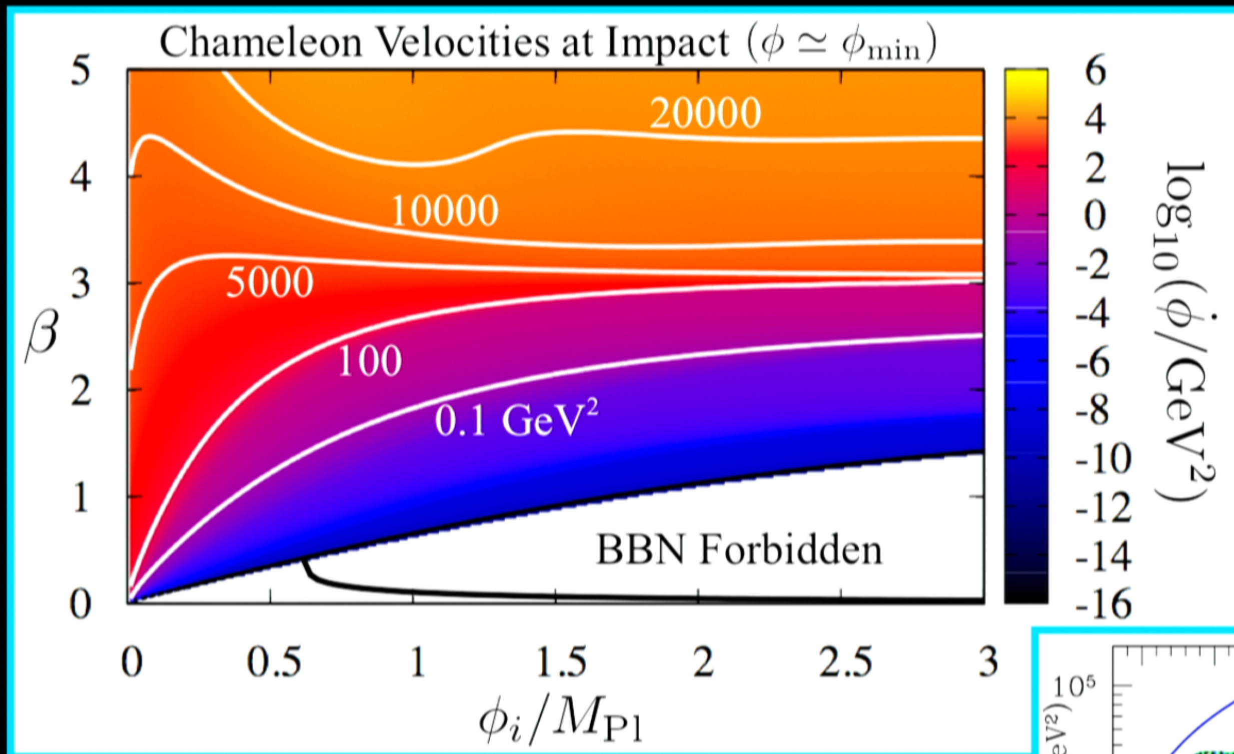

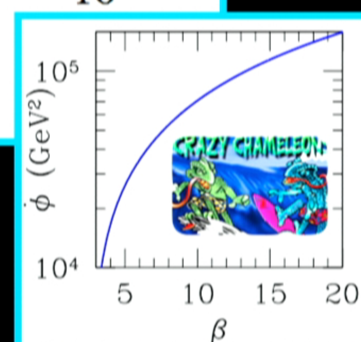

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Quantum Particle Production

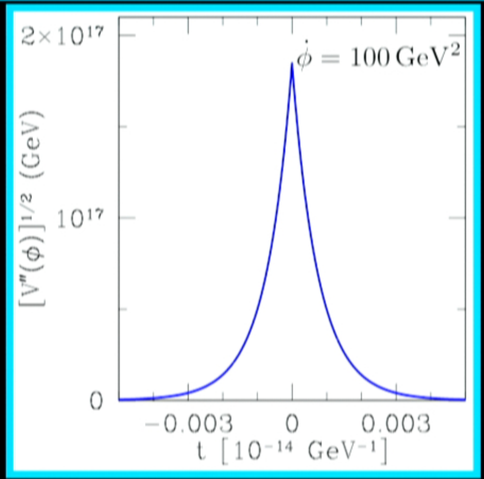
Rapid changes in $V''(\phi)$ excite perturbations!

$$\ddot{\phi} + 3H_*\dot{\phi} - \frac{\nabla^2}{a^2}\phi + V'(\phi) = 0$$

$$\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$$

$$\delta\phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[\hat{a}_{\vec{k}}\phi_k(t)e^{i\vec{k}\cdot\vec{x}} + \hat{a}^\dagger_{\vec{k}}\phi^*_k(t)e^{-i\vec{k}\cdot\vec{x}}\right]$$

$$\ddot{\phi}_k + \omega_k^2(t)\phi_k = 0 \qquad \omega_k^2 \equiv k^2 + V''(\bar{\phi})$$



Quantum Particle Production

$$\phi_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega_k(t)}} e^{-i\int^t \omega_k(t')dt'} + \frac{\beta_k(t)}{\sqrt{2\omega_k(t)}} e^{+i\int^t \omega_k(t')dt'}$$

solves $\ddot{\phi}_k + \omega_k^2(t)\phi_k = 0$ provided that

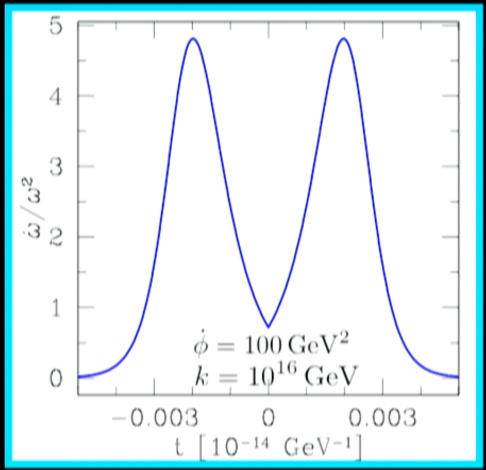
$$\dot{\alpha}_k = \frac{\dot{\omega}_k}{2\omega_k} e^{2i\int^t \omega_k(t')dt'} \beta_k$$

$$\omega_k^2 \equiv k^2 + V''(\bar{\phi})$$

$$\dot{\beta}_k = \frac{\dot{\omega}_k}{2\omega_k} e^{-2i\int^t \omega_k(t')dt'} \alpha_k$$

We get particle production ($|\beta_k|^2 \gtrsim 1$) when

$$\frac{|\dot{\omega}_k|}{\omega_k^2} = \frac{|V'''(\phi)\dot{\phi}|}{2\omega_k^3} \gtrsim 1$$



Chameleon particles: first estimate

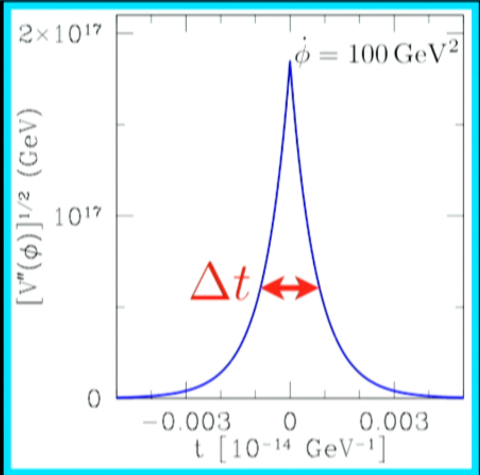
Let's treat the spike in $V''(\phi)$ as a δ - function:

$$\ddot{\phi}_k + \left[k^2 + \Lambda\delta(t - t_*)\right]\phi_k = 0$$

If we start with no perturbations, then

$$\beta_k(t > t_*) = i\frac{\Lambda}{2k}e^{-2ikt_*}$$

After the bounce: $n_k = \dfrac{\Lambda^2}{4k^2}$ $\quad E_k = \dfrac{\Lambda^2}{8\pi^2}k^2$



Wait, perturbations are excited at infinitely high wavenumbers?

No, modes with $k \gg 1/\Delta t$ are not excited: $\dfrac{|\dot{\omega}_k|}{\omega_k^2} \ll 1$ for $k \gg \dfrac{1}{\Delta t}$

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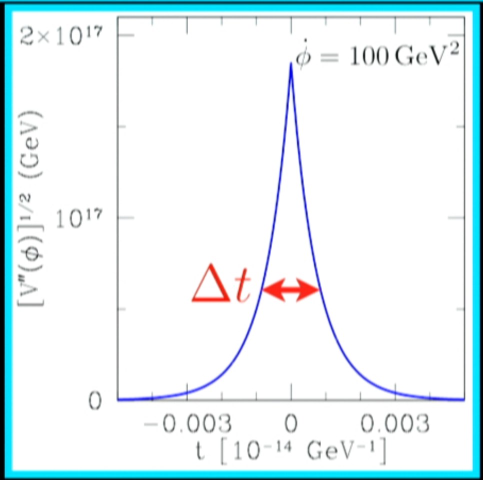

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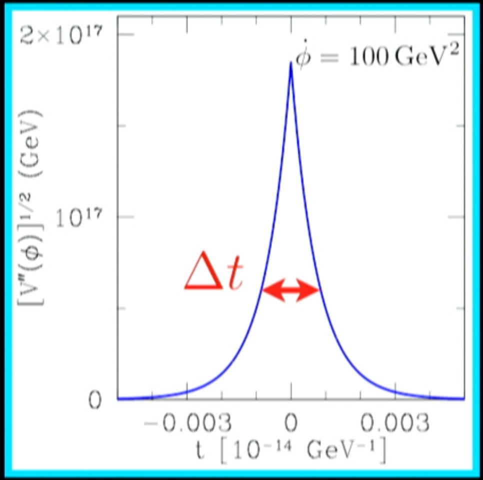
After the bounce: $n_k = \frac{\Lambda^2}{4k^2} \quad E_k = \frac{\Lambda^2}{8\pi^2}k^2$

up to $k \lesssim k_{\mathrm{peak}} = 1/\Delta t$

For our potential, $k_{\mathrm{peak}} \simeq \frac{1}{2}\frac{\dot{\phi}_i}{M}\ln^{3/2}\left[\frac{\dot{\phi}_i^2}{2M^4}\right]$

$$\Lambda = \int_{t_*-\Delta t}^{t_*+\Delta t} V''(\phi)\,dt \simeq \frac{2}{\dot{\phi}_i}V'(\phi_b) = 4k_{\mathrm{peak}}$$



How much energy in perturbations?

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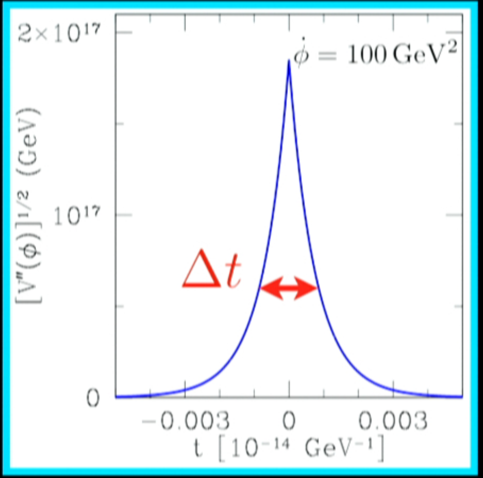

After the bounce: $n_k = \dfrac{\Lambda^2}{4k^2}$ $\quad E_k = \dfrac{\Lambda^2}{8\pi^2}k^2$

up to $k \lesssim k_{\mathrm{peak}} = 1/\Delta t$

For our potential, $k_{\mathrm{peak}} \simeq \dfrac{1}{2}\dfrac{\dot{\phi}_i}{M}\ln^{3/2}\left[\dfrac{\dot{\phi}_i^2}{2M^4}\right]$

$$\Lambda = \int_{t_*-\Delta t}^{t_*+\Delta t} V''(\phi)\,dt \simeq \frac{2}{\dot{\phi}_i}V'(\phi_b) = 4k_{\mathrm{peak}}$$

How much energy in perturbations? WAY TOO MUCH!!

$$\frac{E_{k,\mathrm{peak}}}{E_i} = \frac{1}{16\pi^2}\left(\frac{\dot{\phi}_i}{M^2}\right)^2 \ln^6\left[\frac{\dot{\phi}_i^2}{2M^4}\right]$$

Adding in Backreaction

Since the energy in perturbations is significant, we must revisit the chameleon equation of motion: $(\partial_t^2 - \nabla^2)\phi + V'(\phi) = 0$

$$\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$$ *split field into background and perturbation*

$$(\partial_t^2 - \nabla^2)(\bar{\phi} + \delta\phi) + V'(\bar{\phi}) + \sum_{n=1}^{\infty} \frac{1}{n!} V^{(n+1)}(\bar{\phi})\delta\phi^n = 0$$

Take spatial average:

drop higher order backreaction

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + \frac{1}{2} V'''(\bar{\phi})\langle\delta\phi^2\rangle + \sum_{n=4}^{\infty} \frac{1}{n!} V^{(n+1)}(\bar{\phi})\langle\delta\phi^n\rangle = 0$$

Linear perturbations with first-order backreaction:

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + \frac{1}{2} V'''(\bar{\phi})\langle\delta\phi^2\rangle = 0$$ *background equation with backreaction*

$$\ddot{\phi}_k + \left[k^2 + V''(\bar{\phi})\right]\phi_k = 0$$ *linearized perturbation equations*

$$\langle\delta\phi^2\rangle = \int \frac{d^3k}{(2\pi)^3}\left(|\phi_k|^2 - \frac{1}{2\omega_k}\right)$$ $\omega_k^2 \equiv k^2 + V''(\bar{\phi})$

Hello Computer

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This is a closed system, so we can solve it numerically.

- initial conditions: $\bar\phi = 2M,\ \dot{\bar\phi} = \dot\phi_i,\ n_k = 0\ \forall\ k$
- solve for a number of k values with $k_{\rm IR} \leq k \leq k_{\rm max}$
- choose $k_{\rm max} \gg k_{\rm peak}$ -- these modes aren't excited
- results depend on $k_{\rm IR}$: the longest wavelength perturbation that is treated linearly. Neglecting its interactions with other modes introduces errors, so chose $k_{\rm IR} \lesssim 0.1k_{\rm peak}$

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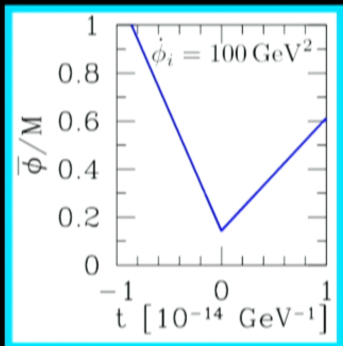
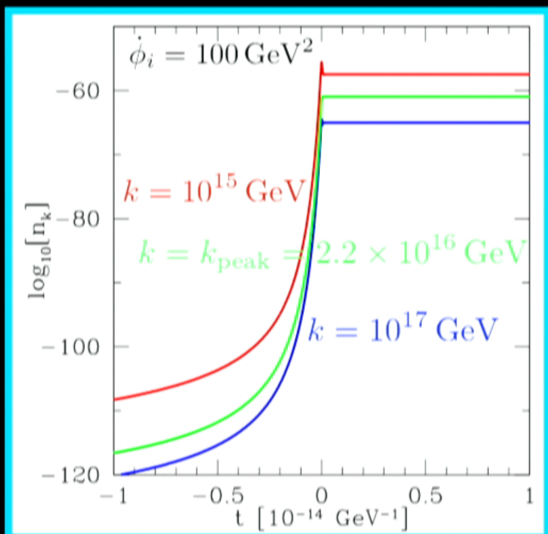
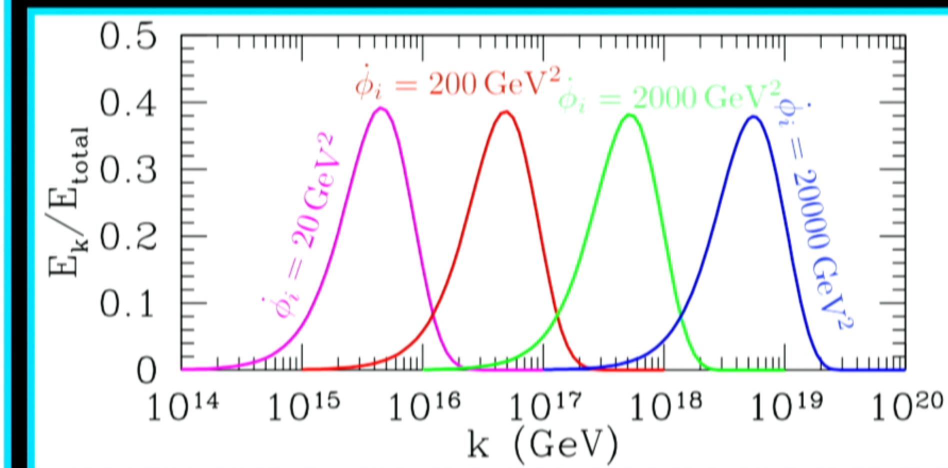
Numerical Results

The numerical results confirm our expectations.



- The chameleon bounces off its bare potential.
- Perturbations are generated during the bounce, taking energy away from the background evolution.
- The perturbation energy spectrum is peaked; most of the energy is in modes with $k_{\rm peak} \simeq (\Delta t)^{-1}$.
- The occupation numbers remain small ($n_k \ll 1$).





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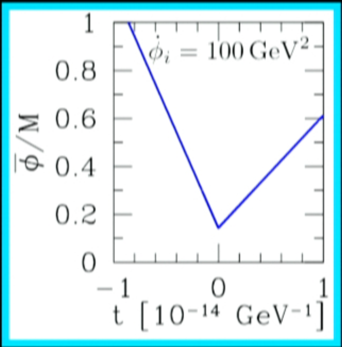

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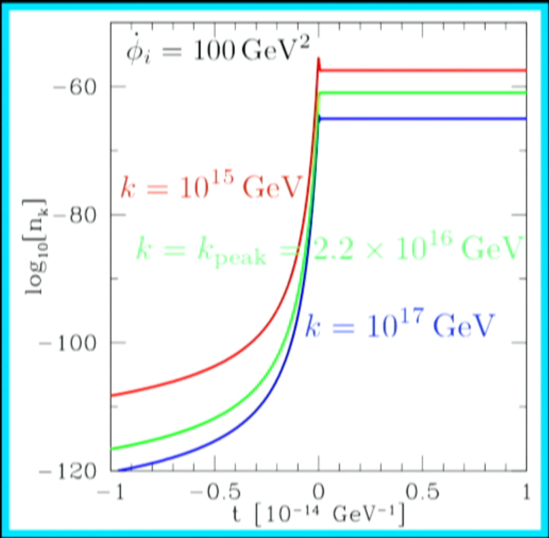

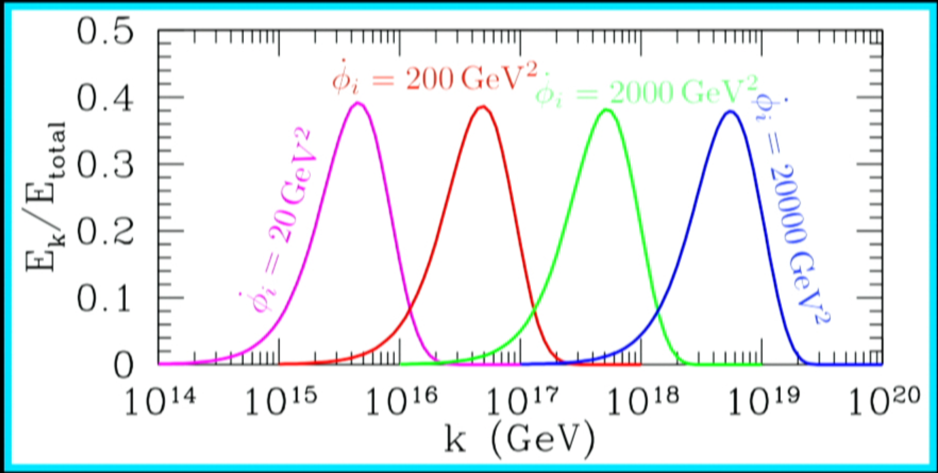

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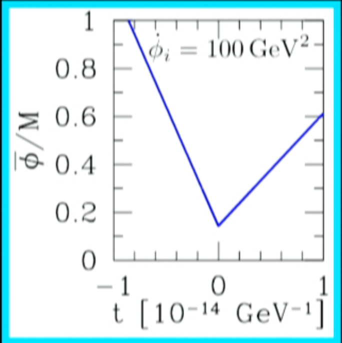

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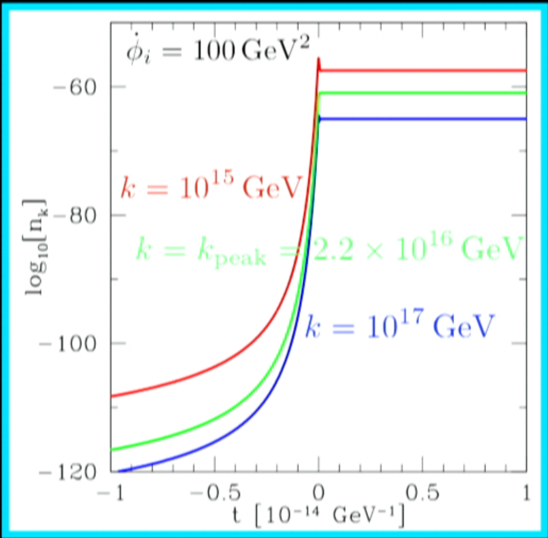

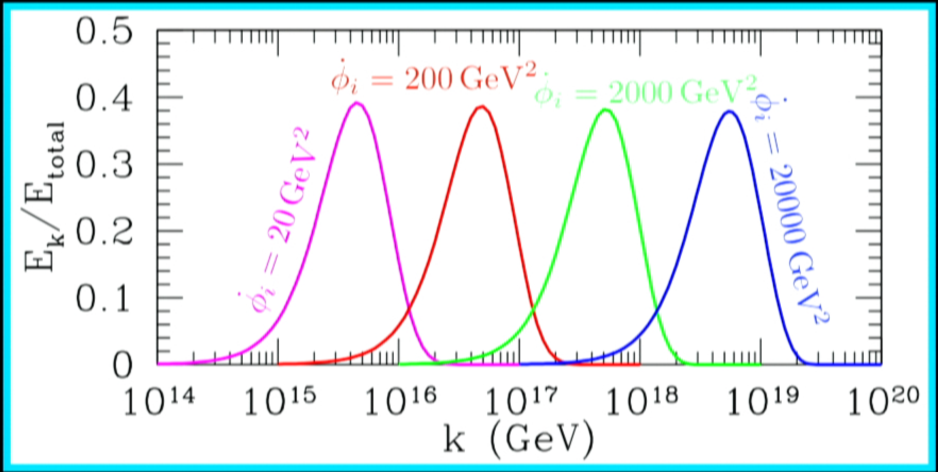

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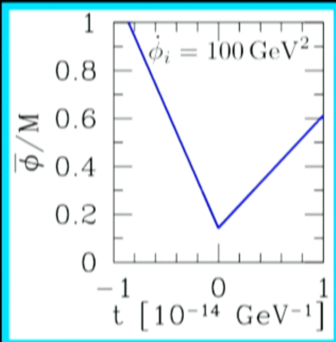

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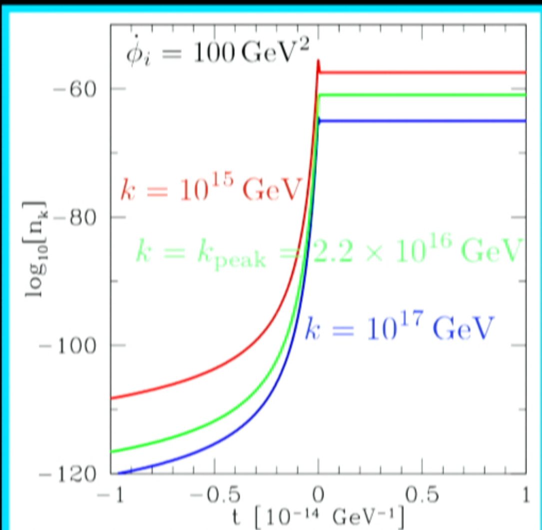
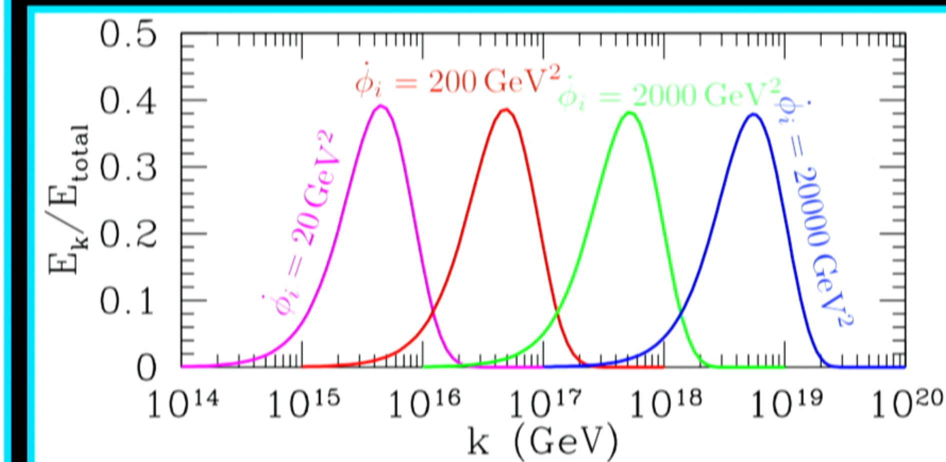
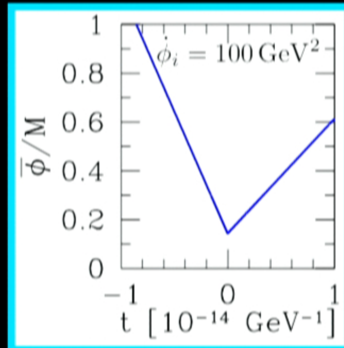


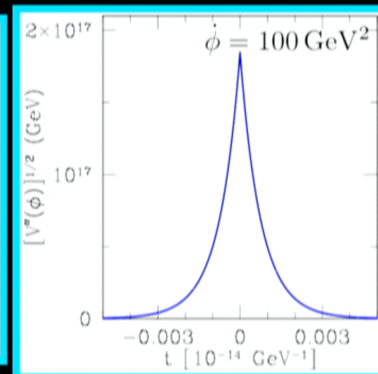


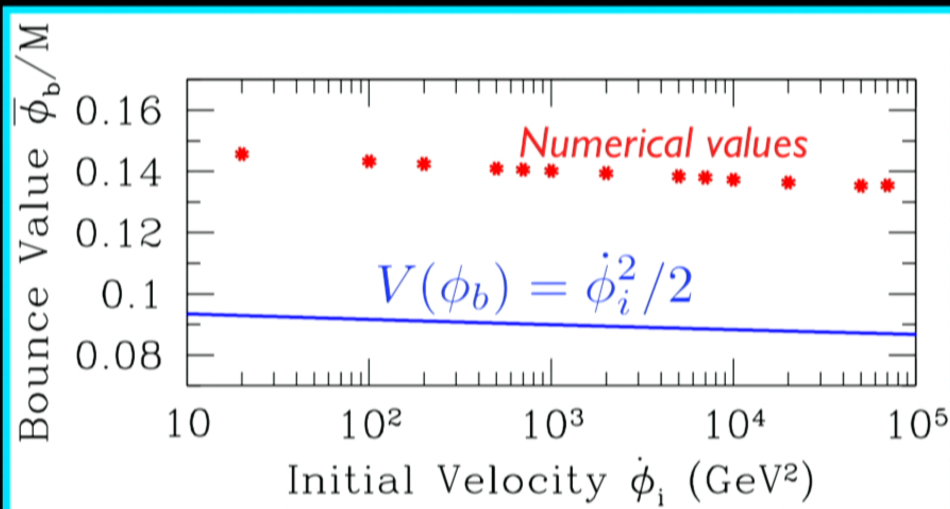
Numerical Surprises

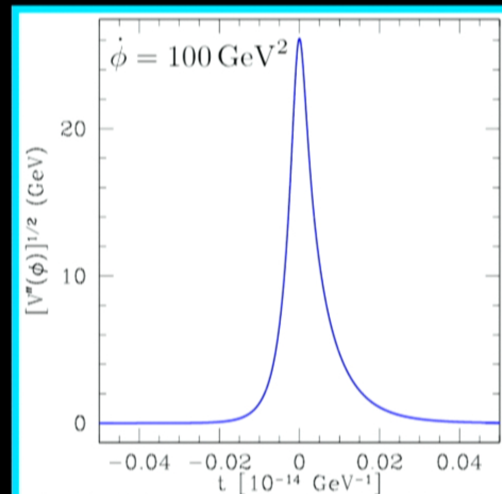
The numerical results confirm our expectations…
except when they don't!

- The chameleon turns around sooner than expected: $V(\phi_b) \ll \dot{\phi}_i^2/2$
- The effective mass $\sqrt{V''}$ is much smaller than expected, with $\sqrt{V''} \ll (\Delta t)^{-1}$









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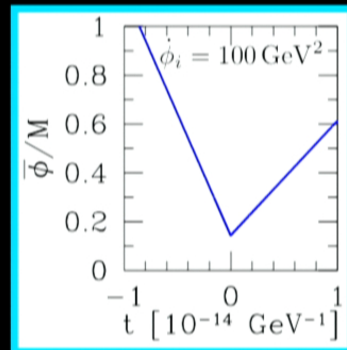
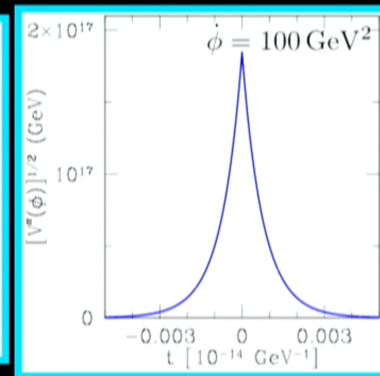
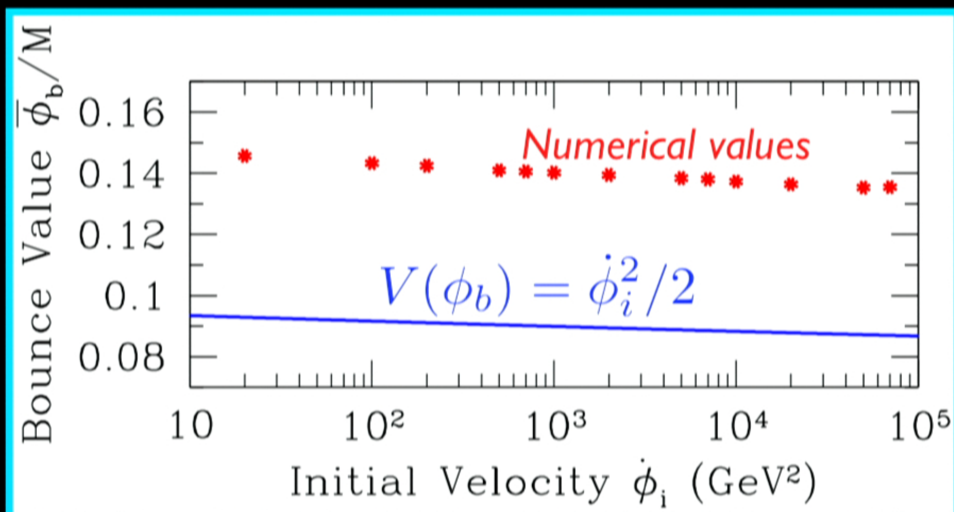
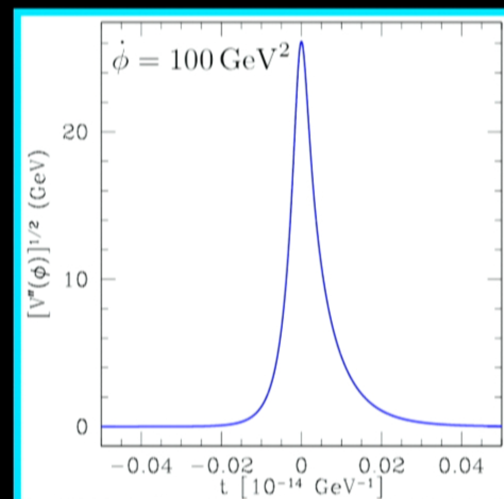




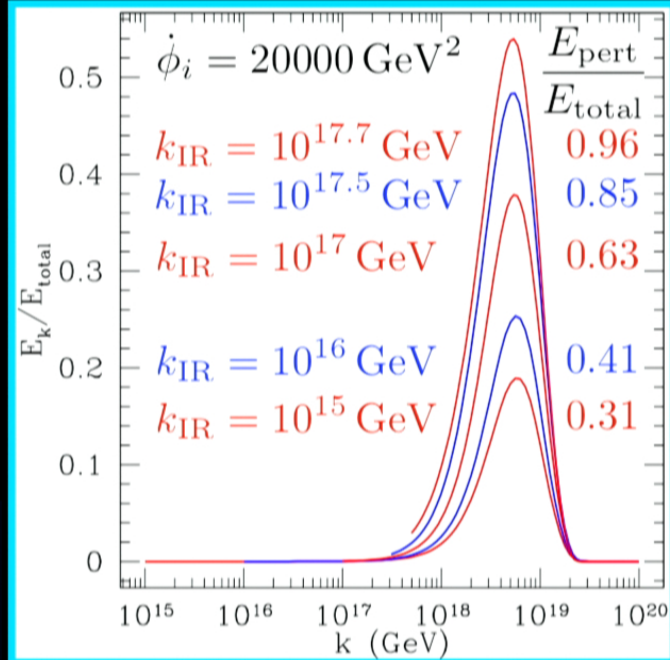


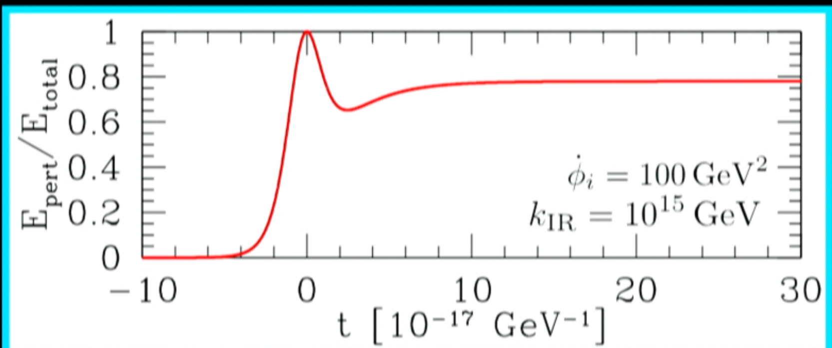


More Numerical Surprises

The numerical results confirm our expectations...
except when they don't!

- At the bounce, all of the chameleon's energy is in perturbations.
- Shortly after the bounce, the perturbations return some of this energy to the background evolution.
- The amount of energy returned depends on the minimum wavenumber.





Back to the Backreaction

Studying the backreaction of the perturbations on the chameleon background provides insight into these numerical surprises.

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + \frac{1}{2}V'''(\bar{\phi})\langle\delta\phi^2\rangle = 0$$

background equation with backreaction

negligible

$$\langle\delta\phi^2\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left(|\beta_k|^2 + \mathrm{Re}\left[\alpha_k\beta_k^* e^{-2i\int^t \omega_k(t')dt'}\right]\right)$$

Bogoliubov expansion

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We know that the occupation numbers are small: $|\beta_k|^2 \ll 1$ & $\alpha_k \simeq 1$

$$\dot{\beta}_k = \frac{\dot{\omega}_k}{2\omega_k}e^{-2i\int^t \omega_k(t')dt'}\alpha_k \Longrightarrow \beta_k(t) = \int_0^t \frac{\dot{\omega}_k(t')}{2\omega_k(t')}e^{-2i\int^{t'} \omega_k(t'')dt''}dt'$$

"perturbative" particle production

$$\dot{\omega}_k = \frac{V'''\dot{\phi}}{2\omega_k}$$

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(negligible: $|\beta_k|^2$)

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recall that we impose IR cut-off

This is the same nonlocal "dissipative" correction derived using in-in formalism by Boyanovsky, de Vega, Holman, Lee & Singh (1994).

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We now have a new equation of motion for the spatially averaged chameleon field: $\ddot{\bar{\phi}} + V'(\bar{\phi}) + D(t) = 0$

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) - \frac{V'''[\bar{\phi}(t)]}{16\pi^2}\int_{0}^{t} V'''[\bar{\phi}(t')]\dot{\bar{\phi}}(t')\mathrm{Ci}\left[2k_{\mathrm{IR}}(t-t')\right]dt' = 0$$

$t_{\min} \simeq t_b - M/\dot{\phi}$ $\quad \mathrm{Ci}(x) \equiv -\int_x^\infty \frac{\cos y}{y}dy \simeq \gamma_E + \ln(x)$ for $x \ll 1$

- the "dissipation" term *D(t)* is nonlocal; it has memory
- *D(t)* is strongly peaked near the bounce
- before the bounce, $k_{\mathrm{IR}}(t-t') \ll 1$ and *D(t)* acts like a friction term; it has the same sign as $\dot{\bar{\phi}}$ and it slows the chameleon down.
- but unlike friction, *D(t)* does not decrease as the chameleon slows down. *D(t)* is more like a potential, and it can turn the chameleon around!
- for a time after the bounce, *D(t)* is negative even though $\dot{\bar{\phi}} > 0$; like a potential, *D(t)* returns **some** energy to the rebounding chameleon.

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A new potential from perturbations

With some manipulation, we can see that the perturbation backreaction acts like a new potential as $\phi \to \phi_{\rm min}$.

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + D(t) = 0$$

$k_{\rm IR}(t - t') \ll 1$

$$D(t) = -\frac{V'''[\bar{\phi}(t)]}{16\pi^2} \int_{t_{\rm min}}^{t} \left[\frac{d}{dt'} V''[\bar{\phi}(t')]\right] \{\gamma_E + \ln[2k_{\rm IR}(t - t')]\} dt'$$

Integrate by parts, and approx. $\int_{t_{\rm min}}^{t} \frac{V''[\bar{\phi}(t')]}{t - t'} dt' \simeq V''[\bar{\phi}(t)] \int_{t_{\rm min}}^{t} \frac{dt'}{t - t'}$

$$D(t) \simeq -\frac{V'''[\bar{\phi}(t)]}{16\pi^2} \{V''[\bar{\phi}(t)] - V''[\bar{\phi}(t_{\rm min})]\} \{\gamma_E + \ln[2k_{\rm IR}(t - t_{\rm min})]\}$$

small — nearly constant

$$D(t) \equiv V_D'(\bar{\phi}) = \kappa V'''(\bar{\phi}) V''(\bar{\phi})$$

$$V_D(\phi) = \frac{\kappa}{2} \left[V''(\bar{\phi})\right]^2$$

$0.02 \lesssim \kappa \lesssim 0.05$

calibrate using numerical results

For $\phi \lesssim M$, $V_D(\phi)$ dominates over the chameleon's bare potential!

New Models for a New Potential

$V_D(\phi) = \frac{\kappa}{2}\left[V''\left(\bar{\phi}\right)\right]^2$ controls the chameleon's motion.

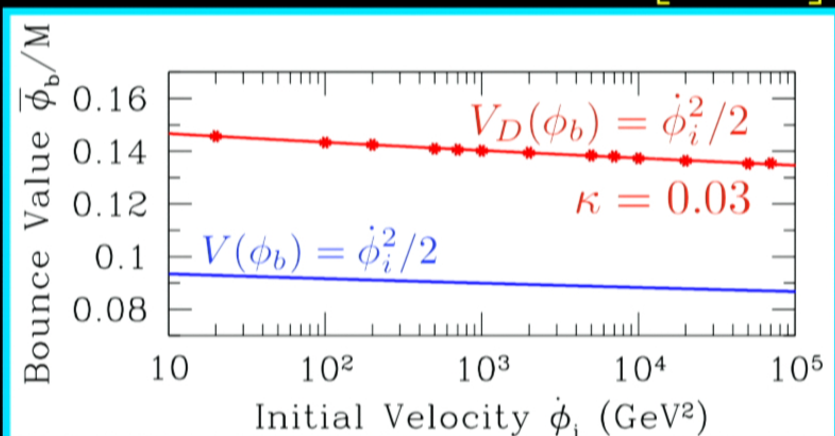
Predict when the chameleon bounces: $V_D(\phi_b) = \dot{\phi}_i^2/2$

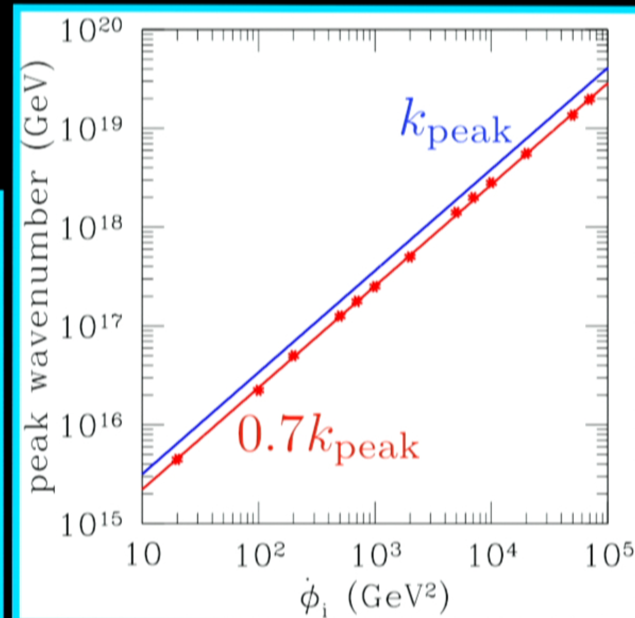
Predict the peak wavenumber in the perturbation energy spectrum:

$$\Delta t = 2\sqrt{\frac{2V_D''(\phi_b)}{V_D'(\phi_b)V_D'''(\phi_b)}} = \frac{2\sqrt{2}}{\sqrt{V_D''(\phi_b)}}$$

$$k_{\rm peak} = (\Delta t)^{-1}$$

$$k_{\rm peak} = \frac{\dot{\phi}}{M}\left(\frac{M}{\phi_b}\right)^3 \simeq 0.25\frac{\dot{\phi}}{M}\ln^{3/2}\left[\frac{\dot{\phi}_i^2}{16\kappa M^4}\right]$$





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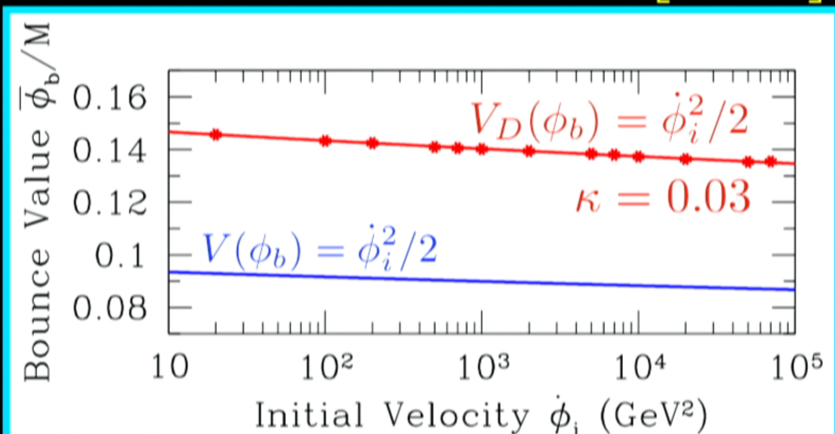
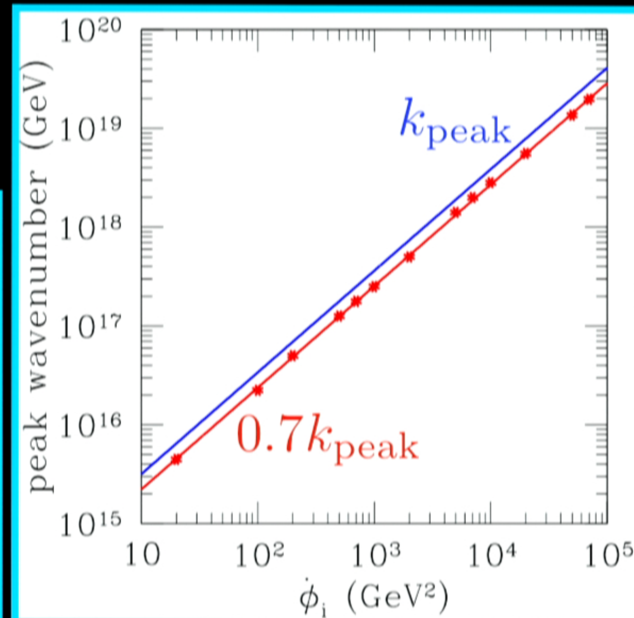
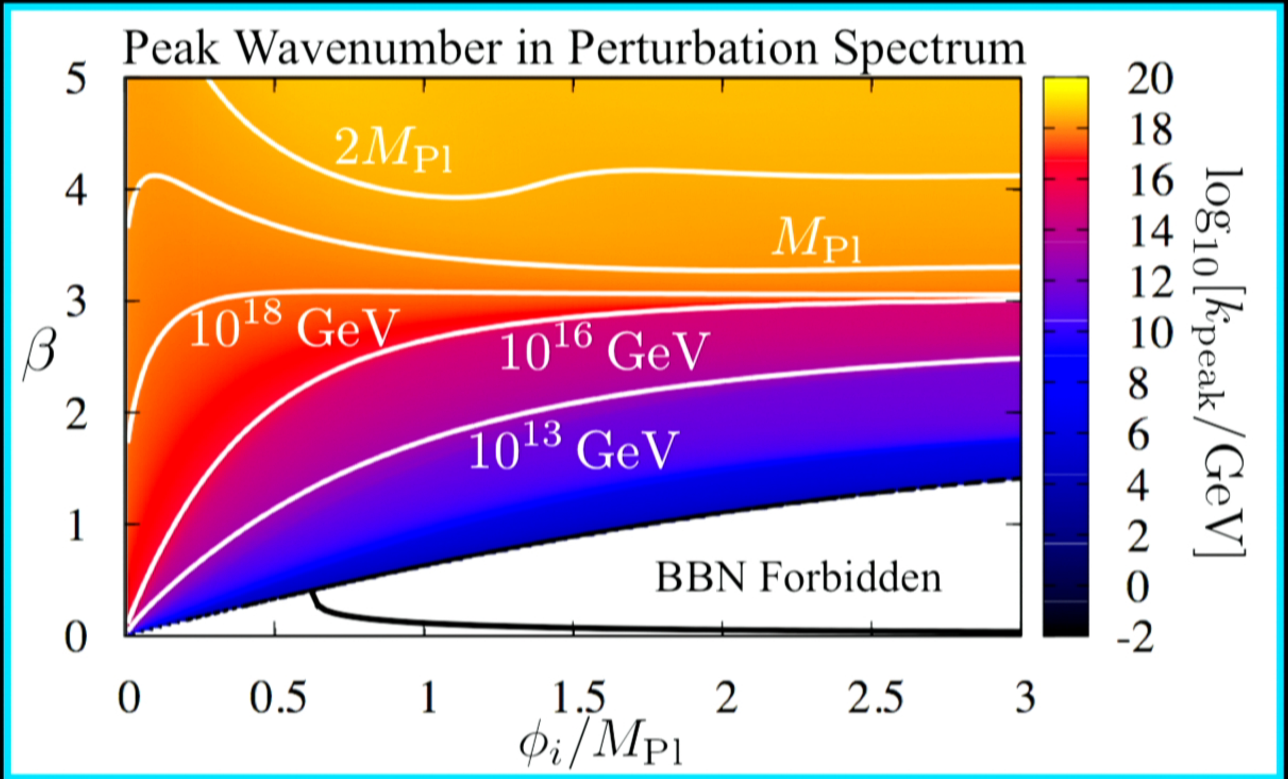




High-Energy Chameleons



$$k_{\rm peak} = 0.7\frac{\dot{\phi}}{M}\left(\frac{M}{\phi_b}\right)^3 \simeq 0.18\frac{\dot{\phi}}{M}\ln^{3/2}\left[\frac{2\dot{\phi}_i^2}{M^4}\right]$$

Summary: A Chameleon Catastrophe

AE, Barnaby, Burrage, Huang 1211.xxxx

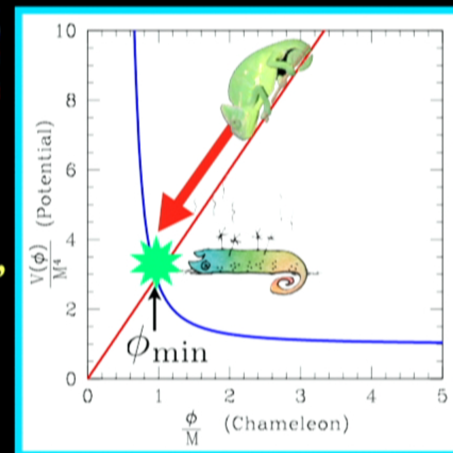
What happens when you kick a chameleon?

It hits its bare potential at a fatal velocity, and then it shatters into pieces!

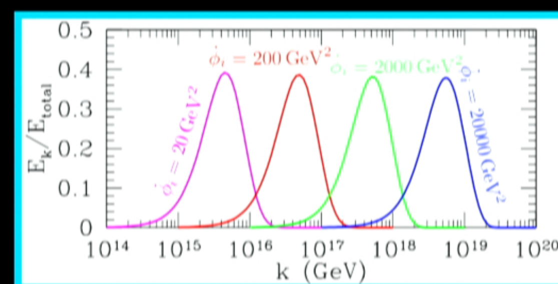
The chameleon's interaction with standard model particles hurtles it toward the minimum of its effective potential.

- Chameleons with $\beta > 1.8$ surf toward $\phi_{\rm min}$
- $\beta < 0.42$ and a finely tuned ϕ_i is needed to avoid impact
- At impact, $\dot{\phi} \gtrsim \mathrm{GeV}^2$ and $\Omega_{\dot{\phi}} \lesssim 1/(6\beta^2)$

Because $\dot{\phi} \gg M^2$, the rebound is highly nonadiabatic, and perturbations are excited.



- Most (maybe all?) the chameleon's energy goes into perturbations.
- The perturbations have wavenumbers $k \gtrsim 10^{13}$ GeV
- The perturbations interact with themselves and with matter: the final state is unknown.
- Chameleons demonstrate how the presence of an extreme hierarchy of scales can challenge a theory's stability. Are there other examples?



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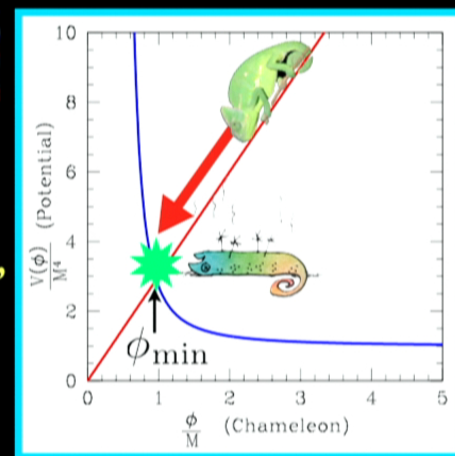

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