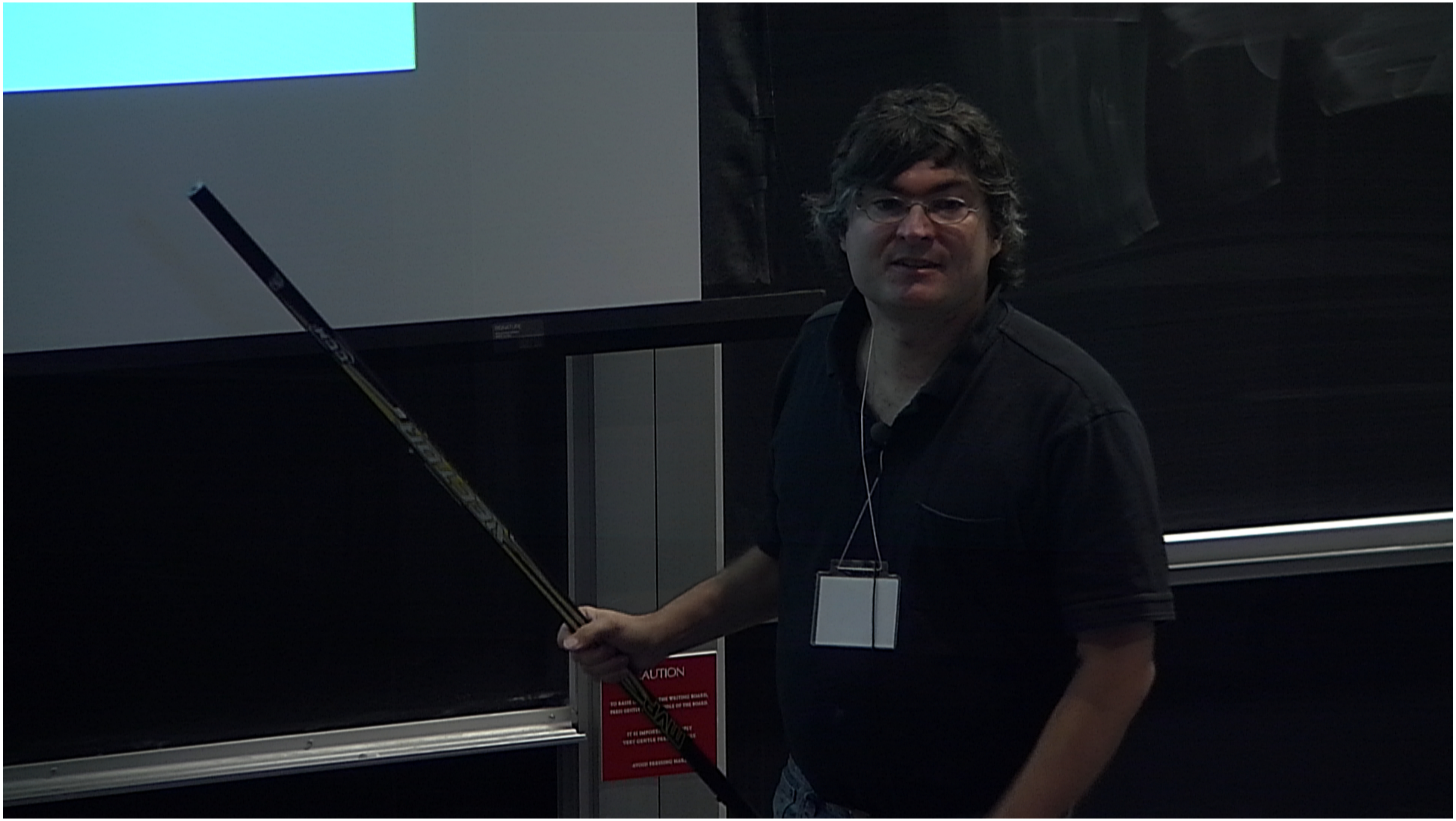
Title: Matter-wave clocks

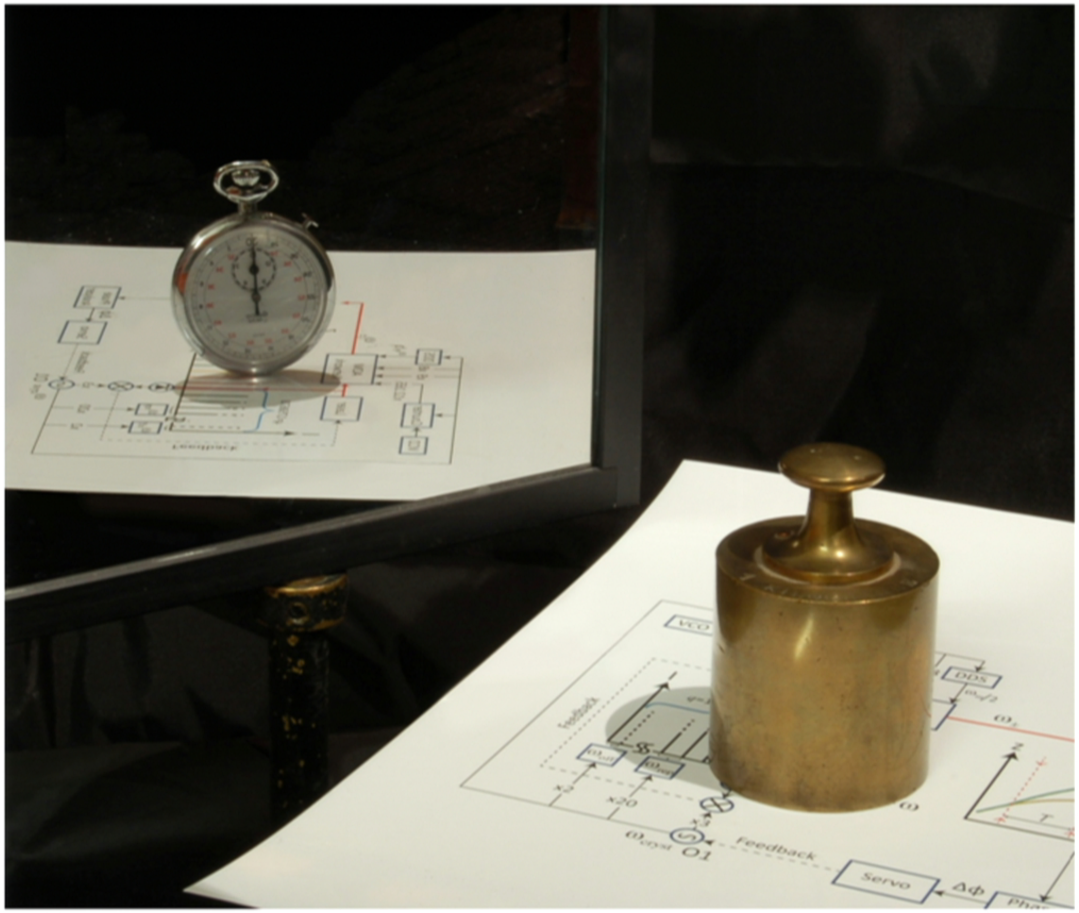
Date: Oct 22, 2012 09:15 AM

URL: http://pirsa.org/12100124

Abstract:



Matter-wave clocks



Perimeter Institute 10/23/2012

Experimental signatures for quantum gravity

Energy ↑

Gravity, Electroweak, Strong

Underlying theory (TBD): consistent, comprehensive

Gravity | Electroweak, Strong

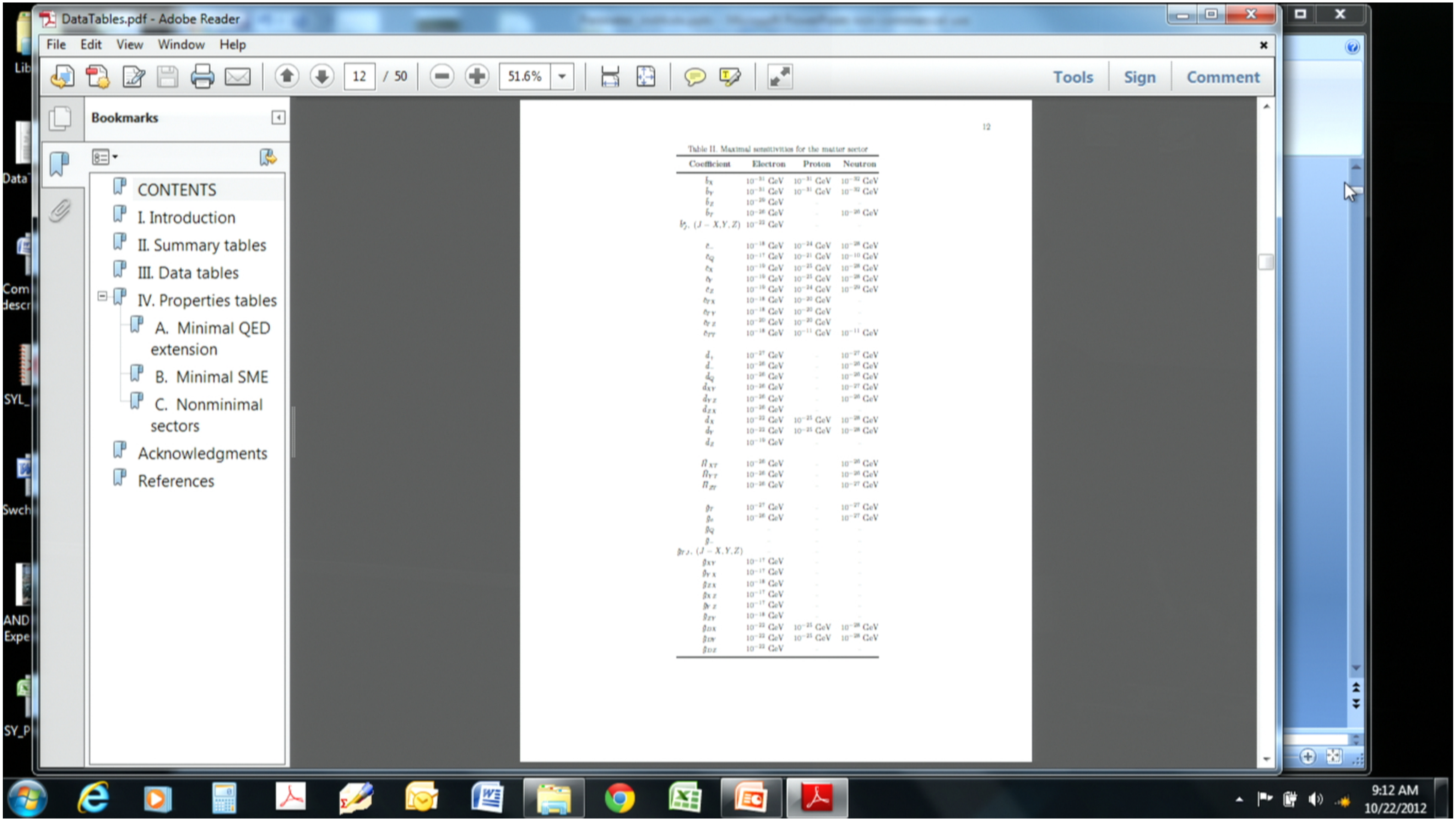
Effective theories, frameworks

E&M | Weak | Strong

- EEP violation:
  - Redshift
  - UFF
- Clock comparison
- Torsion balances
- Atom interferometers

- Lorentz violation
  - Limiting velocities
  - Anomalous dispersion
- Gamma-ray bursts
- Accelerator physics
- Clock comparison

- Decoherence,
- Noise
- 5th (6th,…) force
- Interferometers
- Nanomechanics
- 1/r law tests



12

Table II. Maximal sensitivities for the matter sector

Coefficient	Electron	Proton	Neutron
$\tilde b_X$	10^{-31} GeV	10^{-31} GeV	10^{-32} GeV
$\tilde b_Y$	10^{-31} GeV	10^{-31} GeV	10^{-32} GeV
$\tilde b_Z$	10^{-29} GeV	–	–
$\tilde b_T$	10^{-26} GeV	–	10^{-28} GeV
$\tilde b_J^*$, $(J = X, Y, Z)$	10^{-22} GeV	–	–
$\tilde c_-$	10^{-18} GeV	10^{-24} GeV	10^{-28} GeV
$\tilde c_Q$	10^{-17} GeV	10^{-21} GeV	10^{-10} GeV
$\tilde c_X$	10^{-19} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde c_Y$	10^{-19} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde c_Z$	10^{-19} GeV	10^{-24} GeV	10^{-29} GeV
$\tilde c_{TX}$	10^{-18} GeV	10^{-20} GeV	–
$\tilde c_{TY}$	10^{-18} GeV	10^{-20} GeV	–
$\tilde c_{TZ}$	10^{-20} GeV	10^{-20} GeV	–
$\tilde c_{TT}$	10^{-18} GeV	10^{-11} GeV	10^{-11} GeV
$\tilde d_+$	10^{-27} GeV	–	10^{-27} GeV
$\tilde d_-$	10^{-26} GeV	–	10^{-26} GeV
$\tilde d_Q$	10^{-26} GeV	–	10^{-26} GeV
$\tilde d_{XY}$	10^{-26} GeV	–	10^{-27} GeV
$\tilde d_{YZ}$	10^{-26} GeV	–	10^{-26} GeV
$\tilde d_{ZX}$	10^{-26} GeV	–	–
$\tilde d_X$	10^{-22} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde d_Y$	10^{-22} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde d_Z$	10^{-19} GeV	–	–
$\tilde H_{XT}$	10^{-26} GeV	–	10^{-26} GeV
$\tilde H_{YT}$	10^{-26} GeV	–	10^{-26} GeV
$\tilde H_{ZT}$	10^{-26} GeV	–	10^{-27} GeV
$\tilde g_T$	10^{-27} GeV	–	10^{-27} GeV
$\tilde g_c$	10^{-26} GeV	–	10^{-27} GeV
$\tilde g_Q$	–	–	–
$\tilde g_-$	–	–	–
$\tilde g_{TJ}$, $(J = X, Y, Z)$	–	–	–
$\tilde g_{XY}$	10^{-17} GeV	–	–
$\tilde g_{YX}$	10^{-17} GeV	–	–
$\tilde g_{ZX}$	10^{-18} GeV	–	–
$\tilde g_{XZ}$	10^{-17} GeV	–	–
$\tilde g_{YZ}$	10^{-17} GeV	–	–
$\tilde g_{ZY}$	10^{-18} GeV	–	–
$\tilde g_{DX}$	10^{-22} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde g_{DY}$	10^{-22} GeV	10^{-25} GeV	10^{-28} GeV
$\tilde g_{DZ}$	10^{-22} GeV	–	–

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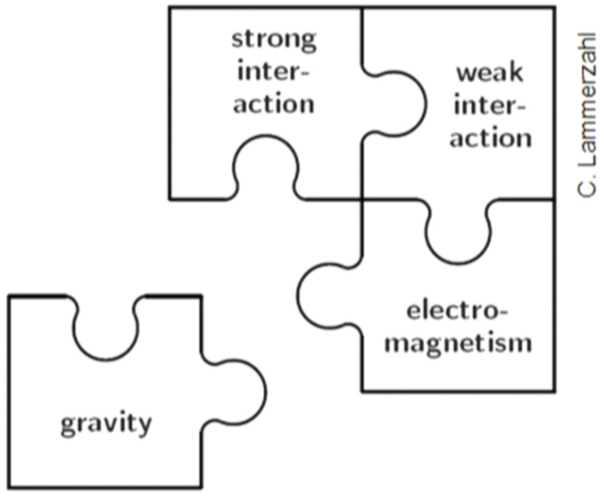
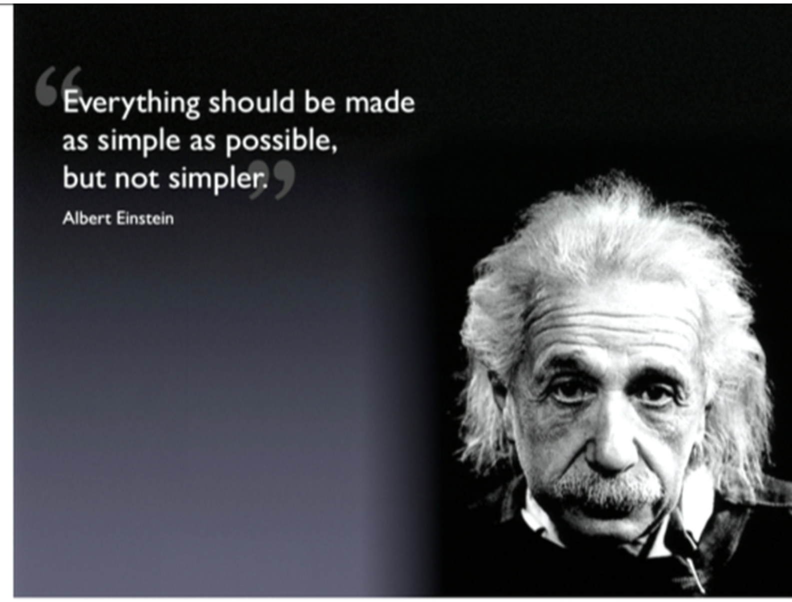




General Relativity:

- Theory of gravity, space & time used in astrophysics, cosmology, string theory,…
- Nonlinear theory: gravity causes more gravity
- Cannot be quantized
- Low-energy limit of string theory might be Lorentz-violating [V.A. Kostelecky …]

Compton oscillators + time dilation = de Broglie (1924) relations

$$h\nu_0 = m_0 c^2$$

ν_0 being measured, of course, in a system which is at rest with respect to a certain amount of energy. This hypothesis is the base of our system, and it is

Then the frequency observed by the fixed observer will be

$$\nu_1 = \nu_0 (1-\beta^2)^{1/2} = \frac{m_0 c^2}{h} (1-\beta^2)^{1/2}$$

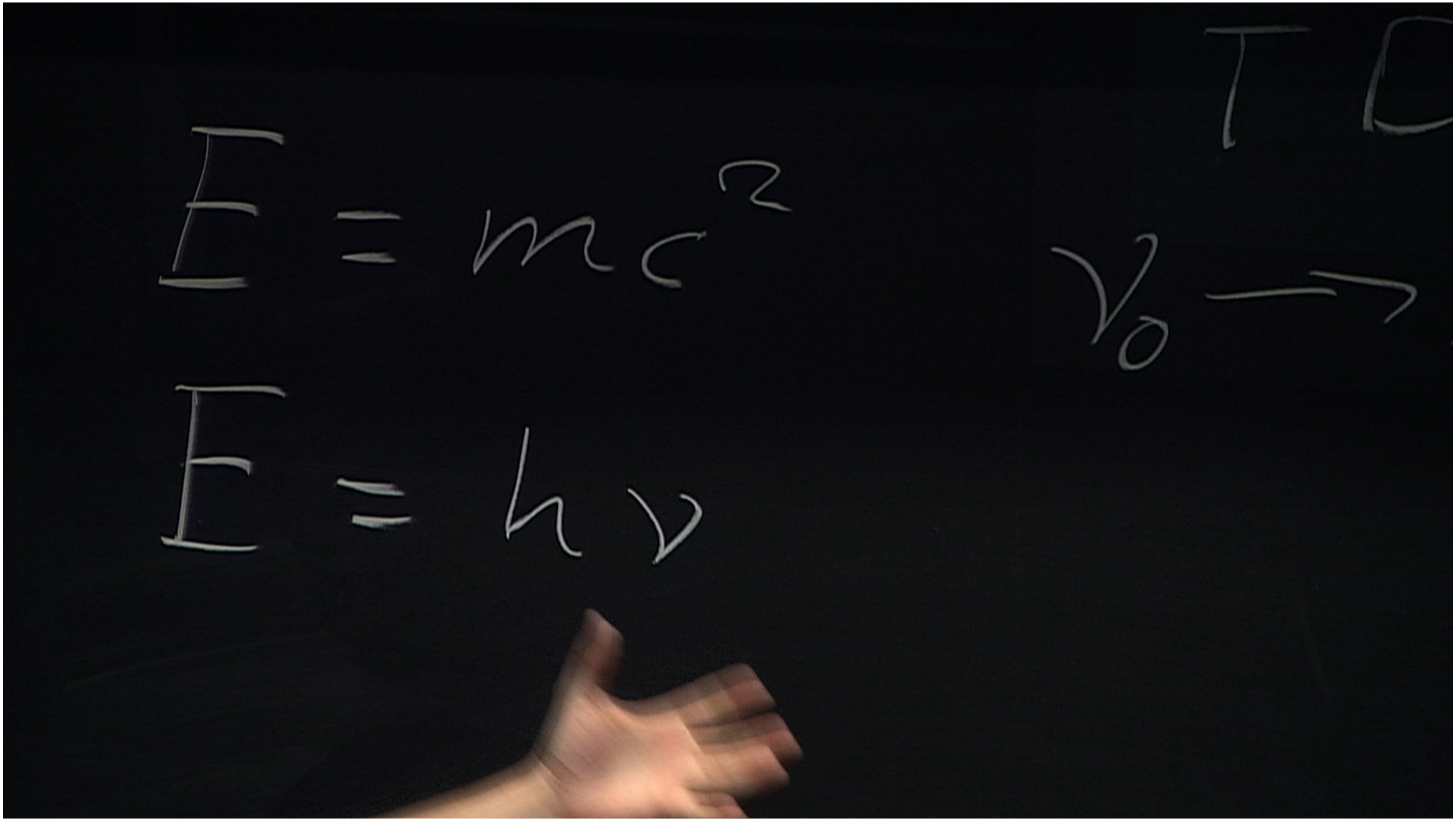
On the other hand, since the energy of the moving body is equal to $m_0c^2/(1-\beta^2)^{1/2}$ for the same observer, the corresponding frequency according to the relationship of the quantum is

$$\nu = h^{-1}[m_0 c^2/(1-\beta^2)^{1/2}].$$

The two frequencies ν_1 and ν are essentially different since the factor $(1-\beta^2)^{1/2}$ is not involved in the same way. This is a difficulty that has intrigued me for a long time; I have succeeded in eliminating it by demonstrating the following theorem that I shall call the theorem of the

Translated in Am. J. Phys. **40,** 1315 (1972)

E = mc²
E = hν
Translated in Am. J. P


$E = mc^2$
$E = h\nu$
ν_0 →

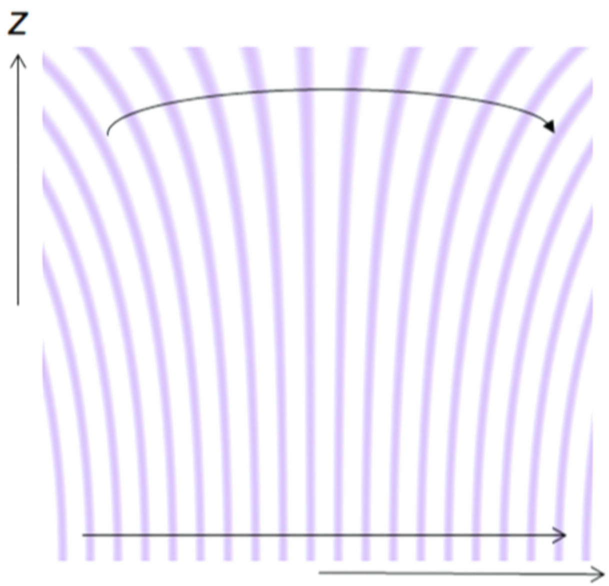
De Broglie + Gravity

Particle represents an oscillator $\omega_C = \frac{mc^2}{\hbar}.$

Time dilation $\omega_C \to \omega_C \sqrt{1 - v^2 / c^2}$

Gravitational redshift $\omega_C \to \omega_C \left(1 - \frac{\Delta U}{c^2}\right)$

Time dilation + redshift $\omega_C \to \omega_C \sqrt{- g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$



$$\omega_C \rightarrow \omega_C\left(1 - \frac{\Delta U}{c^2}\right)$$

t

e

Equivalence to Schrodinger equation

$$S = \int mc\sqrt{-g_{\mu\nu}u^{\mu}u^{\nu}}\,d\lambda$$

$$\approx \int mc^2\left(1+\frac{1}{2}h_{00}+h_{0j}\frac{v^j}{c}-\frac{1}{2}\left(\delta_{jk}-h_{jk}\right)\frac{v^j}{c}\frac{v^k}{c}\right)dt$$

$$\Psi(t+\varepsilon, x_A) = N\int d^3\xi \exp\left(i\frac{mc^2}{\hbar}\left(1+\frac{1}{2}h_{00}\right)\varepsilon\right)\exp\left(-\frac{1}{2}A_{jk}\xi^j\xi^k + B_j\xi^j\right)\Psi(t, x_A-\xi)$$

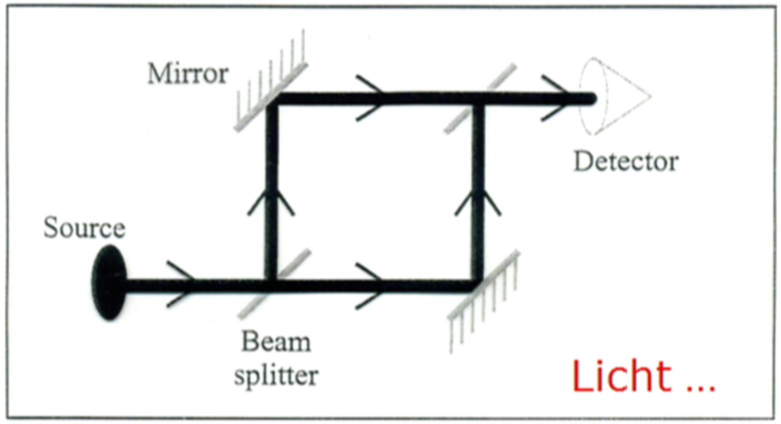
$$\int e^{-\frac{1}{2}A_{jk}\xi^j\xi^k+B_j\xi^j}d^3\xi = \frac{(2\pi)^{3/2}}{\sqrt{\det A}}e^{-\frac{1}{2}B_j(A^{-1})_{jk}B_k}$$

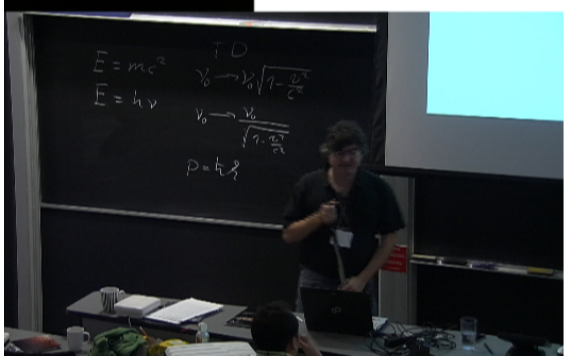
$$i\hbar\frac{d}{dt}\Psi = -mc^2\frac{1}{2}h_{00}\Psi - \frac{\hbar^2}{2m}\left(\vec{\nabla}-m\vec{h}\right)\Psi$$

Feynman, Rev. Mod. Phys. 20, 367 (1948)

Hohensee & Muller, *J. Mod. Opt.* 58, 2021 (2011)

Interferometry





TD

$$E = mc^2$$

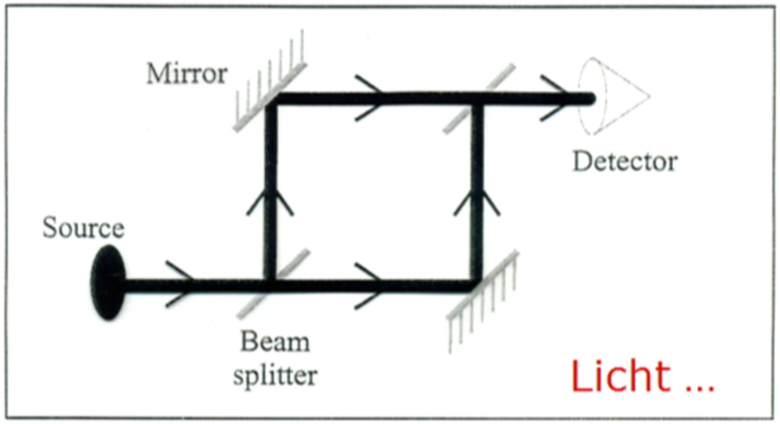
$$E = h\nu$$

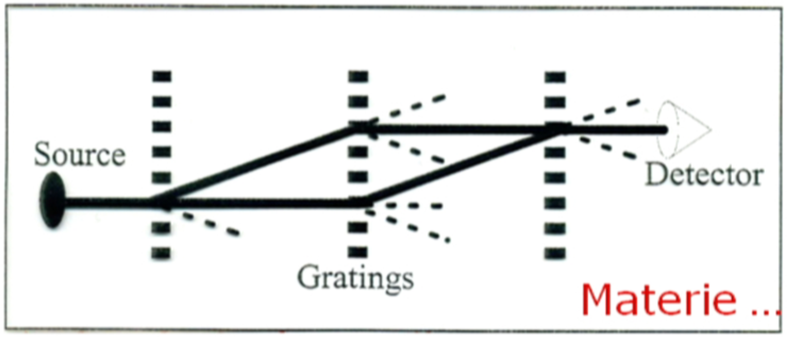
$$\nu_0 \longrightarrow \nu_0 \sqrt{1 - \frac{v^2}{c^2}}$$

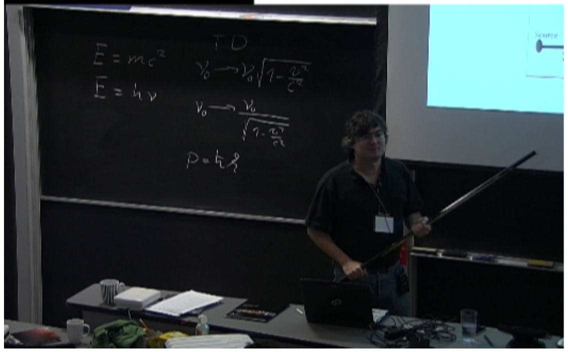
$$\nu_0 \longrightarrow \frac{\nu_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \hbar k$$

Interferometry

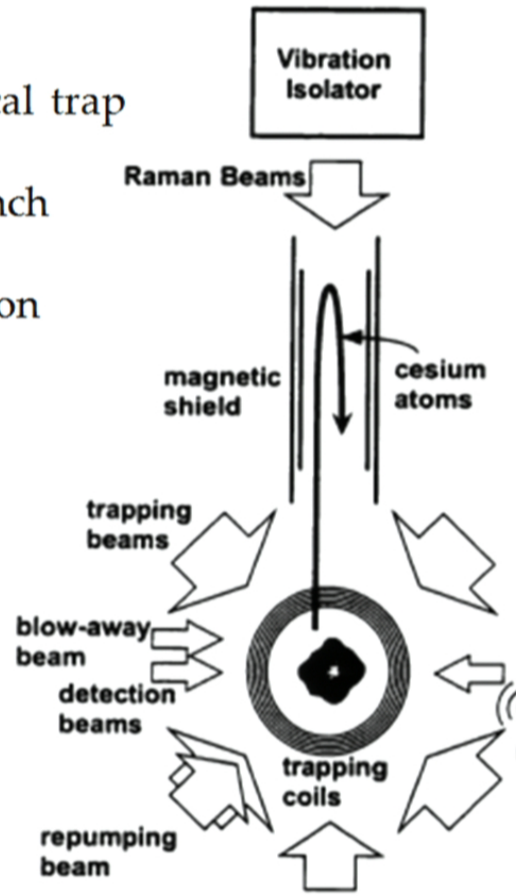


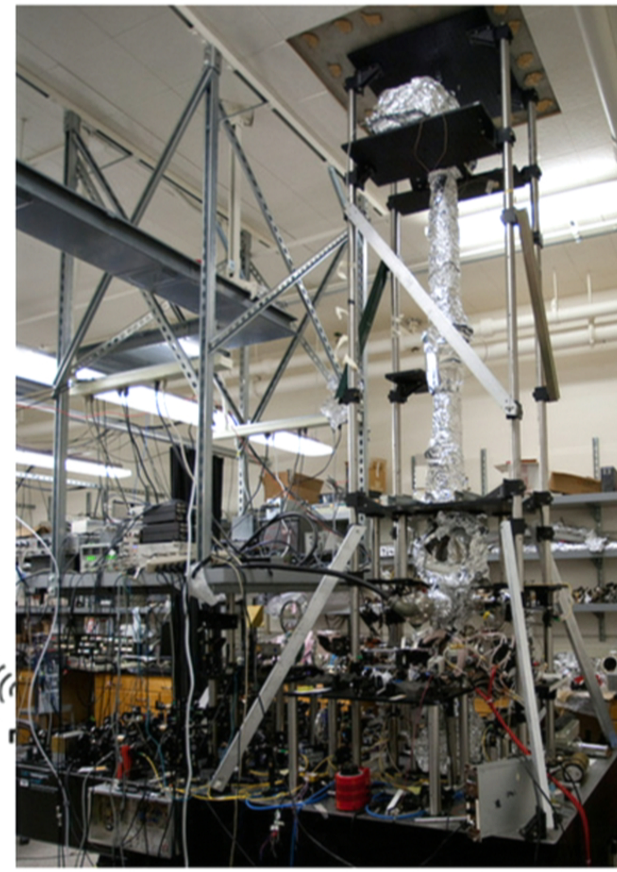


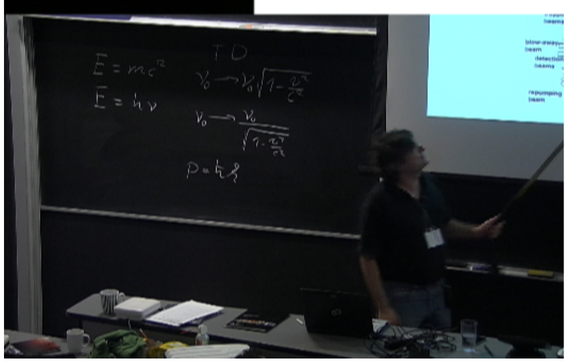


Atomic fountain

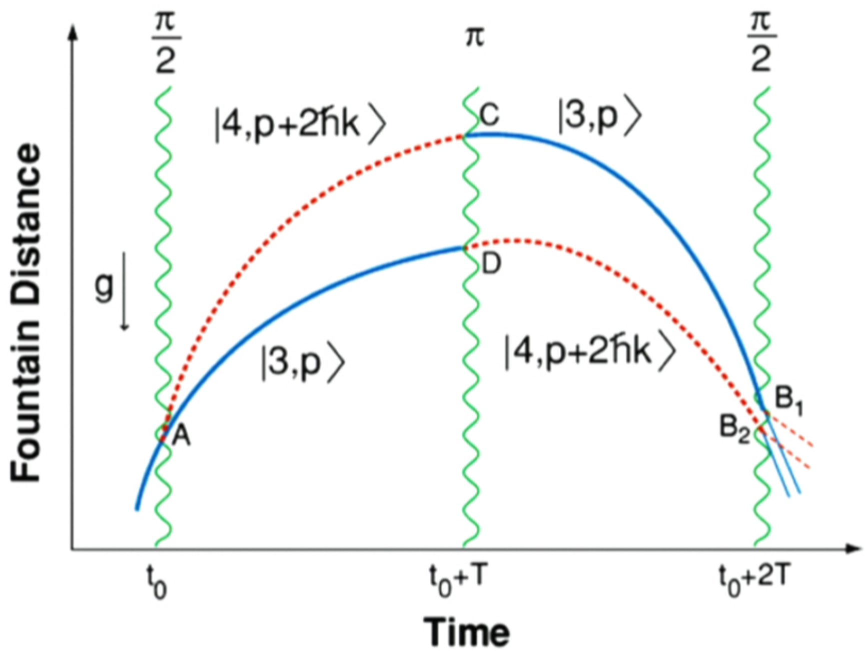
- Magneto-optical trap
- Cooling & launch
- State preparation
- Experiment





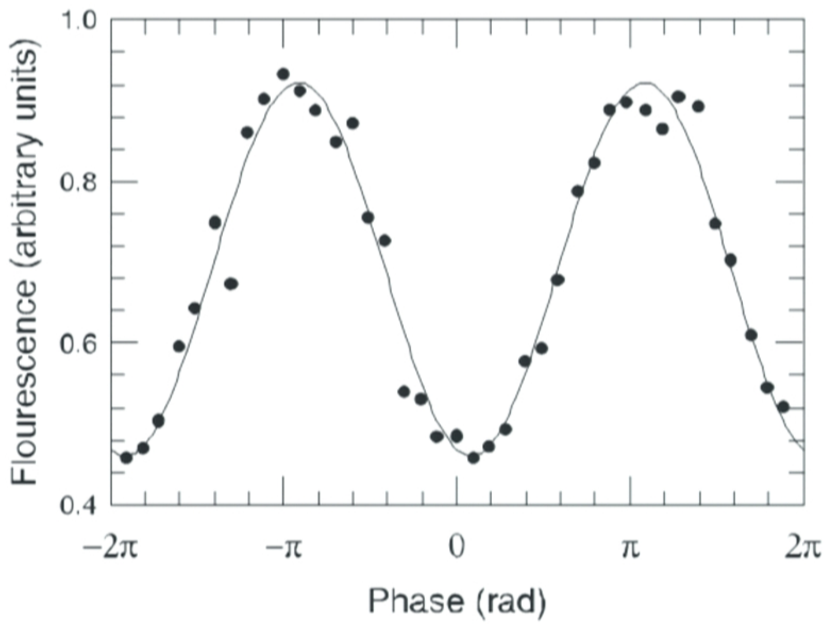


Mach-Zehnder Atom Interferometer



$$\Delta\varphi = \frac{1}{\hbar}\Delta S_{\mathrm{Cl}} + \Delta\varphi_{\mathrm{laser}} = k_{\mathrm{eff}}\, g T^2$$

Highest-precision conventional atom interferometer



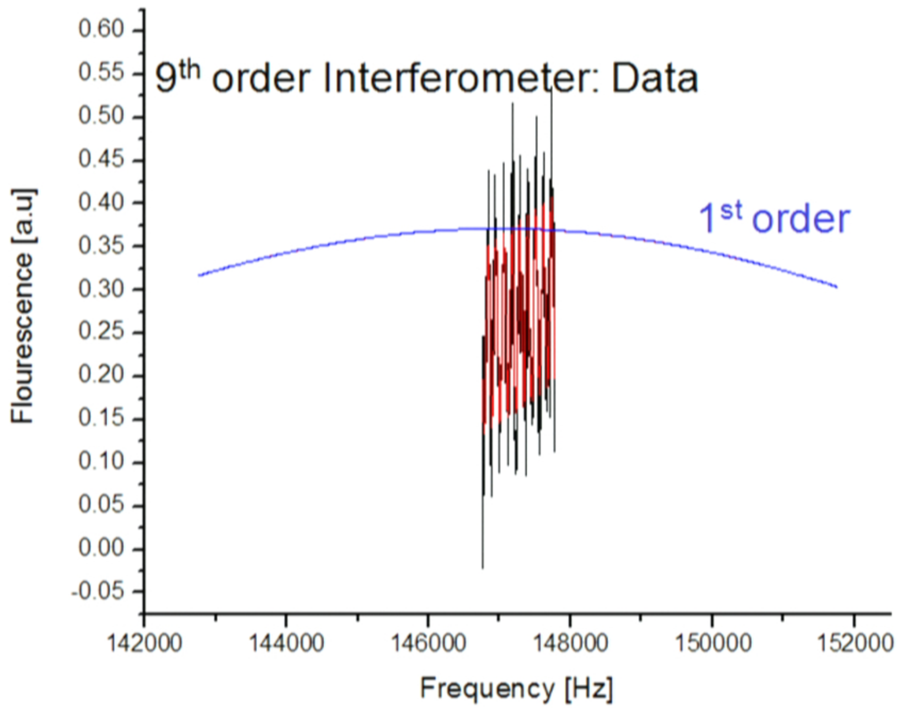
Each data point is from a single launch

Fit error 0.031rad, determines g to 1.3ng

=>11ng/sqrt(Hz)

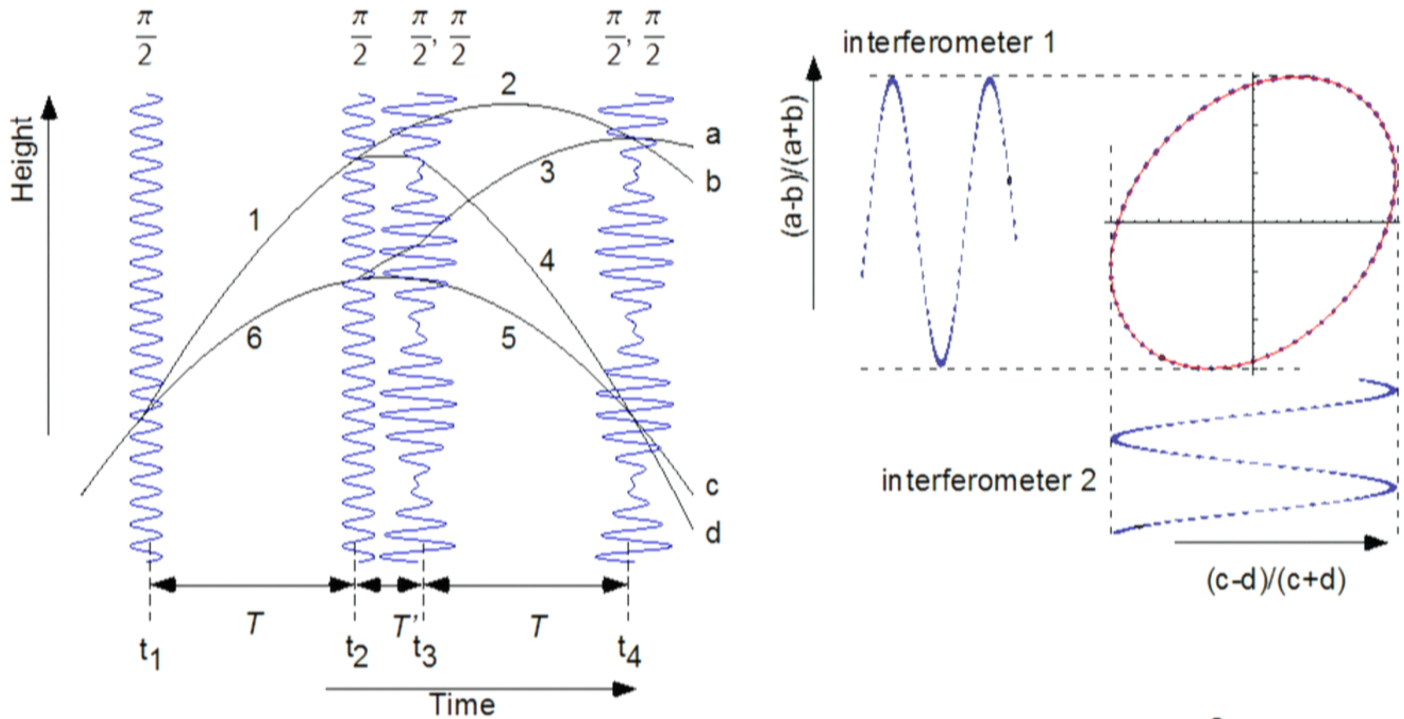
[H. M. *et al.*, PRL **100,** 031101 (2008), K.Y. Chung, PRD 2009]

Large momentum transfer



H. M. *et al.*, PRL 100 (2009): Chiow et al, PRL 2009

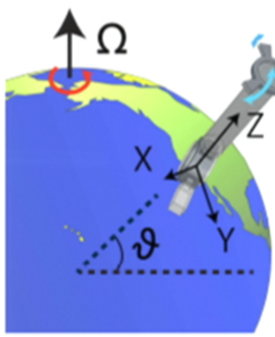
Solution: Simultaneous conjugate Interferometers

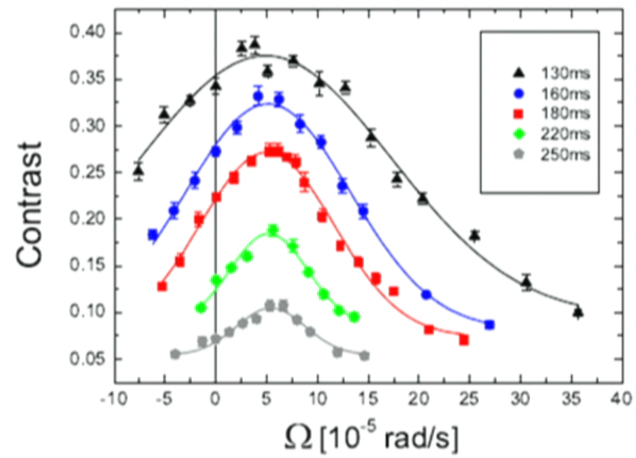


$$\Phi_1 - \Phi_2 = 4\frac{\Delta E_{kin}}{\hbar}T = 16n^2\frac{\hbar k^2}{2m}T$$

Coriolis force

$$\vec{\delta} = 4nv_r\Omega_{\oplus}T(T+T')\cos\vartheta(1,0,0).$$





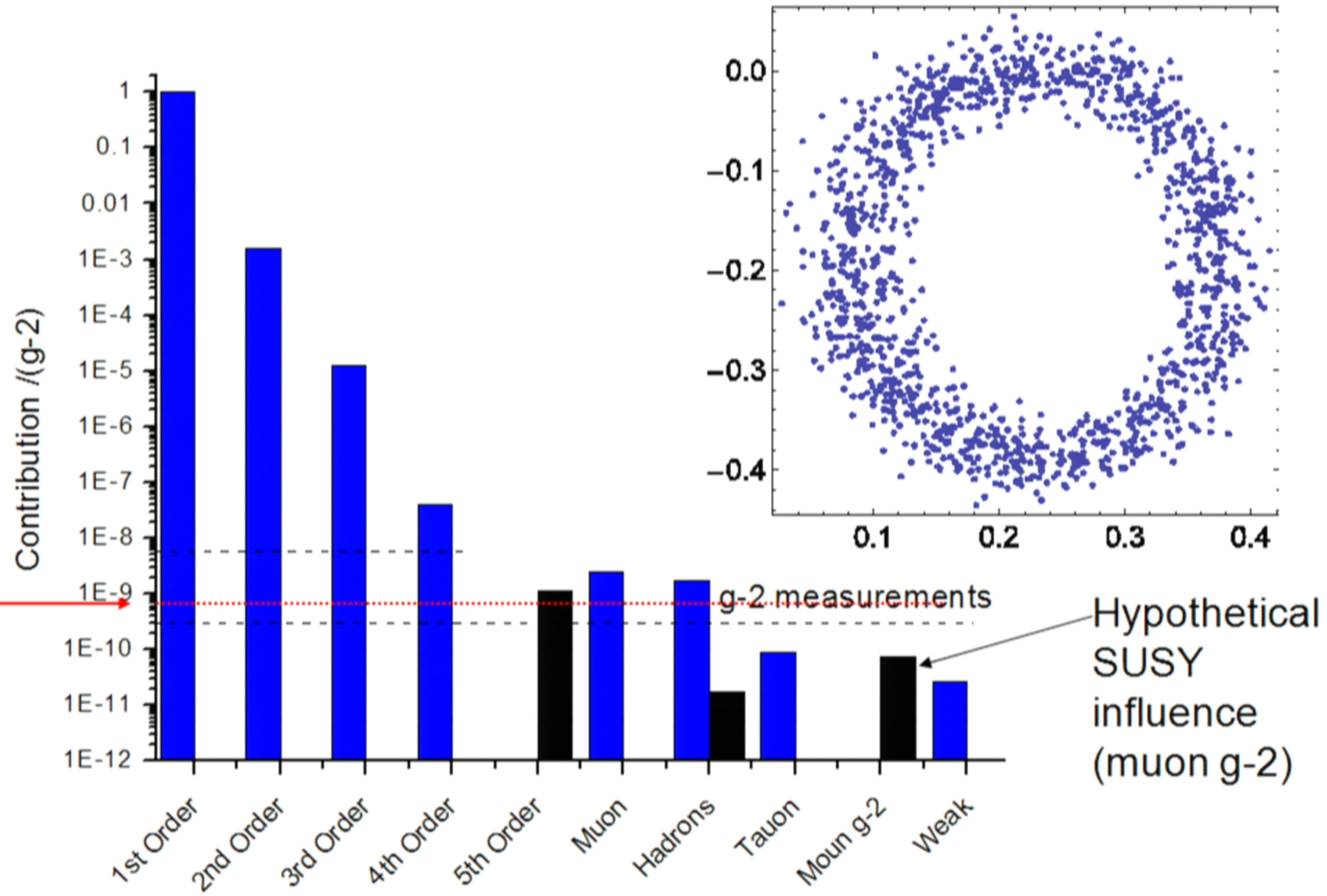
- Interferometer does not close
- Cancellation improves contrast (350%), T
- World's most sensitive atom interferometer (10 ħk, 250 ms)

Lan *et al.*, *PRL* 108, 090402 (2012)

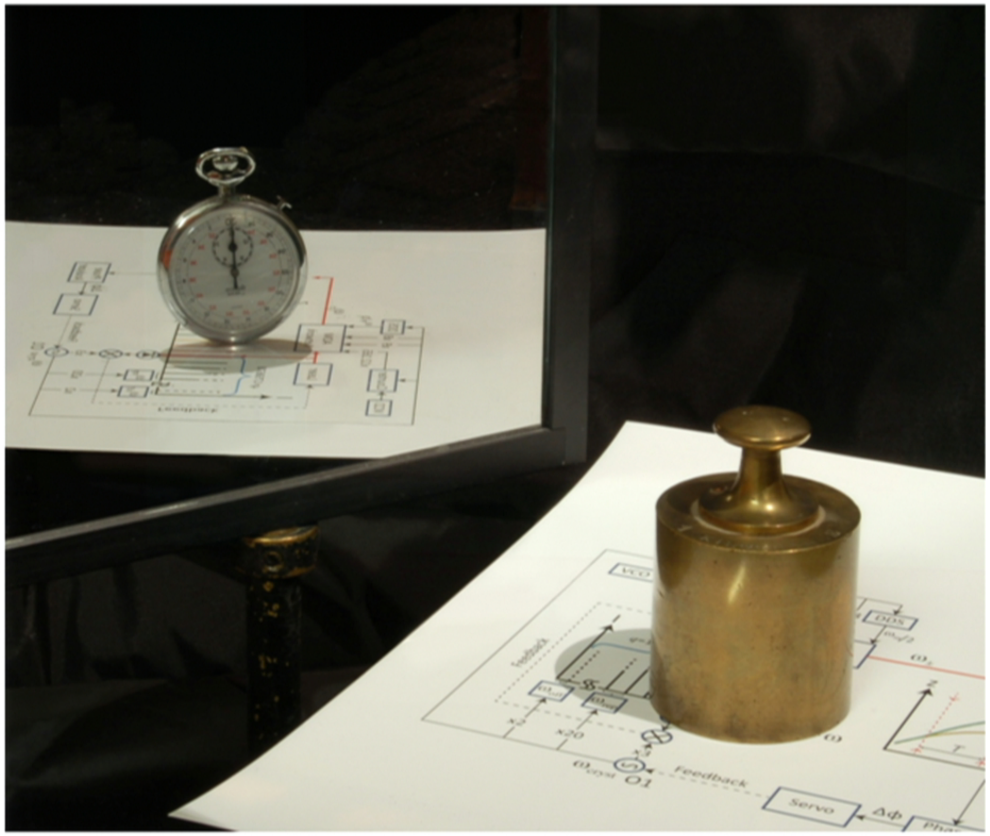
α from recoil measurements

$$\alpha^2 = 2\frac{R_\infty}{c}\frac{h}{m_{\mathrm{Cs}}}\frac{m_{Cs}}{u}\frac{u}{m_e}$$

α from electron g-2



What is Nature's elementary clock?



History

Einstein[1] 1911: solar spectral lines. Didn't work.

Einfluß der Schwerkraft auf die Ausbreitung des Lichtes. 905

müssen also die Spektrallinien des Sonnenlichtes gegenüber den entsprechenden Spektrallinien irdischer Lichtquellen etwas nach dem Rot verschoben sein, und zwar um den relativen Betrag

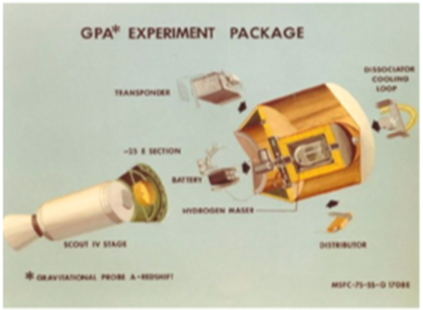
$$\frac{\nu_0 - \nu}{\nu_0} = \frac{-\Phi}{c^2} = 2 \cdot 10^{-6}.$$

Wenn die Bedingungen, unter welchen die Sonnenlinien entstehen, genau bekannt wären, wäre diese Verschiebung noch der Messung zugänglich. Da aber anderweitige Einflüsse (Druck, Temperatur) die Lage des Schwerpunktes der Spektrallinien beeinflussen, ist es schwer zu konstatieren, ob der hier

Pound & Rebka,[2] 1960: Mossbauer effect. 10^{-2}.

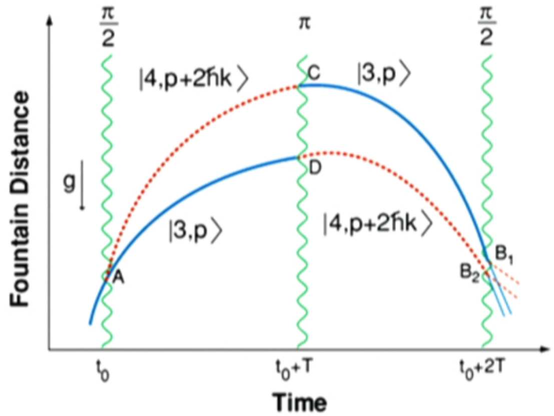
Hafele & Keating,[3] 1971: Atomic clocks in aircraft 10^{-1}.

Vessot *et al.*,[4] 1976: Atomic clocks in rocket 10^{-5}.



[1] Ann. Phys. **340,** 898 (1911). [2] PRL **4,** 337 (1960); **13,** 539 (1964); Phys. Rev. **140,** B788 (1965).
[3] Science **177,** 166 (1972): 177, 168 (1972). [4] PRL **45,** 2081 (1980).

Atom Interferometer



Redshift   Time dilation Laser

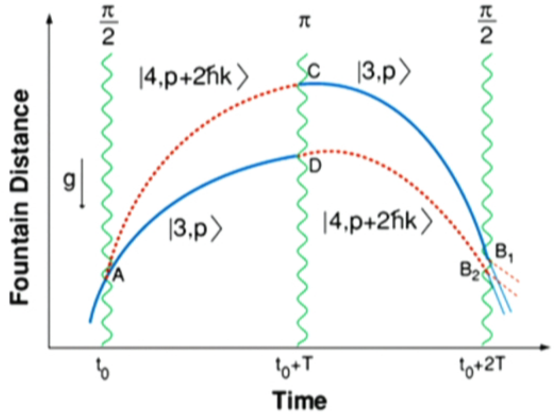
$$\Delta\varphi = \frac{mc^2}{\hbar}\int\left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{\vec{v}_s^{\,2} - \vec{v}_e^{\,2}}{2c^2}\right)dt + \sum_{i=1}^{3} k_i z(t_i)$$

=0
(non-gravitational energy conservation)

Alternative Interpretation

- If *any* energy is conserved, first terms also cancel
- then, testing UFF = testing redshift
- AI becomes indirect measurement of g
- As any redshift measurement

Atom Interferometer



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nature Vol 463|18 February 2010|doi:10.1038/nature08776

[1]Department of Physics, 366 Le Conte Hall MS 7300, University of California, Berkeley, California 94720, USA. [2]Lawrence Berkeley National Laboratory, One Cyclotron Road, Berkeley, California 94720, USA. [3]Institut für Physik, Humboldt-Universität zu Berlin, Hausvogteiplatz 5-7, 10117 Berlin, Germany. [4]US Department of Energy, 1000 Independence Avenue SW, Washington, District of Columbia 20585, USA.

A precision measurement of the gravitational redshift by the interference of matter waves

Holger Müller[1,2], Achim Peters[3] & Steven Chu[1,2,4]

One of the central predictions of metric theories of gravity, such as general relativity, is that a clock in a gravitational potential U will run more slowly by a factor of $1 + U/c^2$, where c is the velocity of light, as compared to a similar clock outside the potential[1]. This effect, known as gravitational redshift, is important to the operation of the global positioning system[2], timekeeping[3,4] and future experiments with ultra-precise, space-based clocks[5] (such as searches for variations in fundamental constants). The gravitational redshift has been measured using clocks on a tower[6], an aircraft[7] and a rocket[8], currently reaching an accuracy of 7×10^{-5}. Here we show that laboratory experiments based on quantum interference of atoms[9,10] enable a much more precise measurement, yielding an accuracy of 7×10^{-9}. Our result supports the view that gravity is a manifestation of space-time curvature, an underlying principle of general relativity that has come under scrutiny in connection with the search for a theory of quantum gravity[11]. Improving the redshift measurement is particularly important because this test has been the least accurate among the experiments that are required to support curved space-time theories[1].

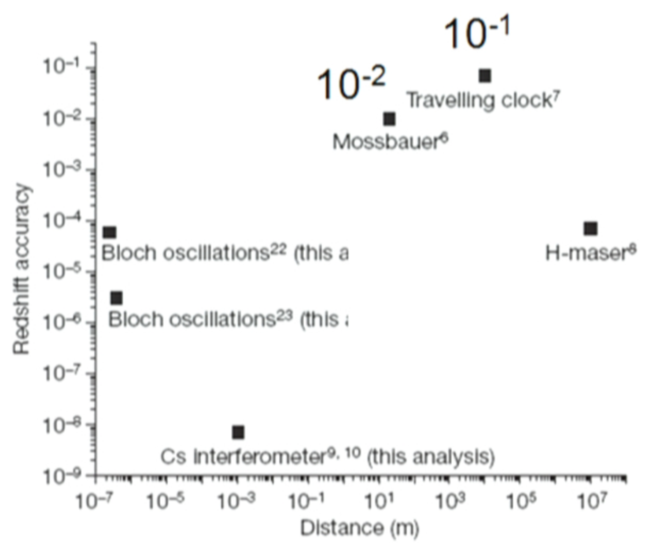

Figure 2 | Absolute determinations of the gravitational redshift. The accuracy (defined as the standard error) in β is plotted versus the relative height of the clocks.

H. M. *et al.*, *Nature* 463 (2011)

Criticism

• You are not testing GR in the same way that clocks do: If you calculate the redshift and the trajectories from one consistent model, the redshift signal goes away

• ω_C is unphysical: the atom is not a clock, but just moves in response to the gravitational force. AI measures g only.

• The atom is no real clock: it cannot be used to lock an oscillator.

Wolf *et al.*, *CQG* 28, 145017 (2011)

Standard Model Extension

The whole standard model of particle physics, plus Lorentz violation

Includes all effects of General Relativity

Energy conservation and coordinate independence, renormalizability of non-gravitational interactions…

Hohensee *et al.*, *PRL* **106,** 151102 (2011)

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Isotropic subset of SME

Effective coeff. for composite particles $(c^B)_{\mu\nu} = \sum_w \frac{N^w m^w}{m^B}(c^w)_{\mu\nu}, (a^B)_\mu = \sum_w N^w (a^w)_\mu$

Lagrangian at PNO(3)

$$S = \int m^T c \left(\sqrt{-\left(g_{\mu\nu} + 2c^T_{\mu\nu}\right)\frac{dx^\mu dx^\nu}{d\lambda d\lambda}} + \frac{1}{m^T}(a^T)_\mu \frac{dx^\mu}{d\lambda} \right) d\lambda$$

Expand to PNO(2), drop constant term, redefine particle mass $m^{\mathrm{T}} \to m^{\mathrm{T}}[1 + \frac{5}{3}(c^{\mathrm{T}})_{00}]$

$$S = \int m^T c^2 \left(\frac{\phi}{c^2}\left[1 - \frac{2}{3}(c^T)_{00} + \frac{2\alpha}{m^T}(a^T)_0\right] - \frac{v^2}{2c^2} \right) dt$$

Modifications in $g_{\mu\nu}$ common to all experiments.

Equivalence Principle and Gravitational Redshift

Michael A. Hohensee,[1,*] Steven Chu,[1,†] Achim Peters,[2] and Holger Müller[1]
[1]*Department of Physics, University of California, Berkeley, California 94720, USA*
[2]*Institut für Physik, Humboldt-Universität zu Berlin, Newtonstrasse 15, 12489 Berlin, Germany*
(Received 17 February 2011; published 11 April 2011)

We investigate leading order deviations from general relativity that violate the Einstein equivalence principle in the gravitational standard model extension. We show that redshift experiments based on matter waves and clock comparisons are equivalent to one another. Consideration of torsion balance tests, along with matter-wave, microwave, optical, and Mössbauer clock tests, yields comprehensive limits on spin-independent Einstein equivalence principle-violating standard model extension terms at the 10^{-6} level.

DOI: 10.1103/PhysRevLett.106.151102 PACS numbers: 04.80.−y, 03.30.+p, 11.30.Cp, 12.60.−i

TABLE I. Sensitivity of redshift experiments. The EEP-violation signal for each experiment is given as a linear combination of SME parameters. The observable for the Pound-Rebka Mössbauer test, e.g., is $-1.1\ \mathrm{GeV}^{-1}\alpha(\bar{a}^{n}_{\mathrm{eff}})_0 - 1.1\ \mathrm{GeV}^{-1}\alpha(\bar{a}^{e+p}_{\mathrm{eff}})_0 + (-0.34 + [-0.66])(\bar{c}^{n})_{00} + (-0.34 + [-0.006])(\bar{c}^{p})_{00} + 0.0002(\bar{c}^{e})_{00}$, with $\bar{a}^{e+p}_{\mathrm{eff}} = \bar{a}^{p}_{\mathrm{eff}} + \bar{a}^{e}_{\mathrm{eff}}$. The last column shows the measured value and 1σ uncertainty. Signals dependent on models for ξ are in square brackets. Curly brackets mark expected limits.

Method	$\alpha(\bar{a}^{n}_{\mathrm{eff}})_0$ GeV	$\alpha(\bar{a}^{e+p}_{\mathrm{eff}})_0$ GeV	$(\bar{c}^{n})_{00}$	$(\bar{c}^{p})_{00}$	$(\bar{c}^{e})_{00}$	Limit ppm
Mössbauer effect [2]	−1.072	−1.072	0.3358 − [2/3]	−0.3353 − [0.006]	0.000 182 6	1000 ± 7600
H maser on rocket [3]	−1.072	−1.072	0.3358	0.3353 − [0.67]	0.000 182 6 − [1.3]	2.5 ± 70
Cs fountain (*proj.*) [16]	−1.072	−1.072	0.3358 + [0.40]	0.34 + [0.28]	0.000 182 6 − [1.3]	{2}
Bloch oscillations [4,17]	0.1632	−0.1580	−0.051 12 − [0.0005]	0.049 40 + [0.0010]	0.000 026 90	3 ± 1
Bloch oscillations [6]	0.1492	−0.1439	−0.046 73 − [0.0006]	0.045 00 + [0.0008]	0.000 024 51	0.16 ± 0.14
Cs interferometer [4]	0.1881	−0.1835	−0.058 90 − [0.0004]	0.057 39 + [0.001]	0.000 031 26	0.007 ± 0.007
Rb interferometer [18]	0.1632	−0.1580	−0.051 12 − [0.0005]	0.049 40 + [0.001]	0.000 026 90	−0.004 ± 0.007

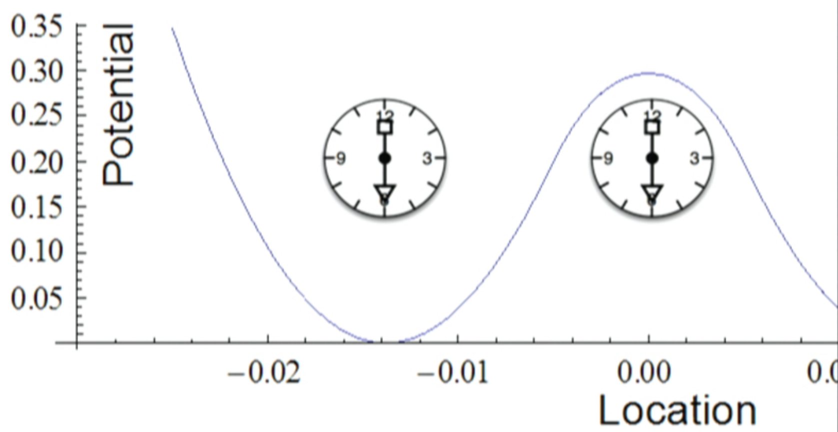
Bottom line

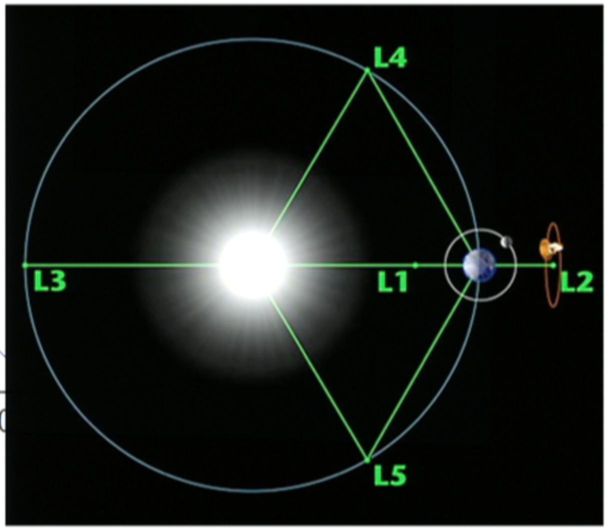
	$(a^{n}_{eff})_0$	$(a^{p}_{eff})_0 + (a^{n}_{eff})_0$	$(c^{n}_{eff})_{00}$	$(c^{p}_{eff})_{00}$	$(c^{e}_{eff})_{00}$
	GeV	GeV			
Clocks+UFF	-3±53	-1±11	-5±94	2.1±40	-1±40
	↓ 14	↓ 11	↓ 14	↓ 11	↓ 9
AI+clocks+UFF	4.3±3.7	0.8±1.0	7.6±6.7	-3.3±3.5	4.6±4.6
					↓ 3
+ future space clocks	-1.4±3.7	-0.3±1.0	-2.4±6.8	0.9±3.5	0.5±1.5

- The AI tests GR like conventional clocks
- improves overall bounds ~10-fold

Hohensee *et al.*, *PRL* **106,** 151102 (2011)

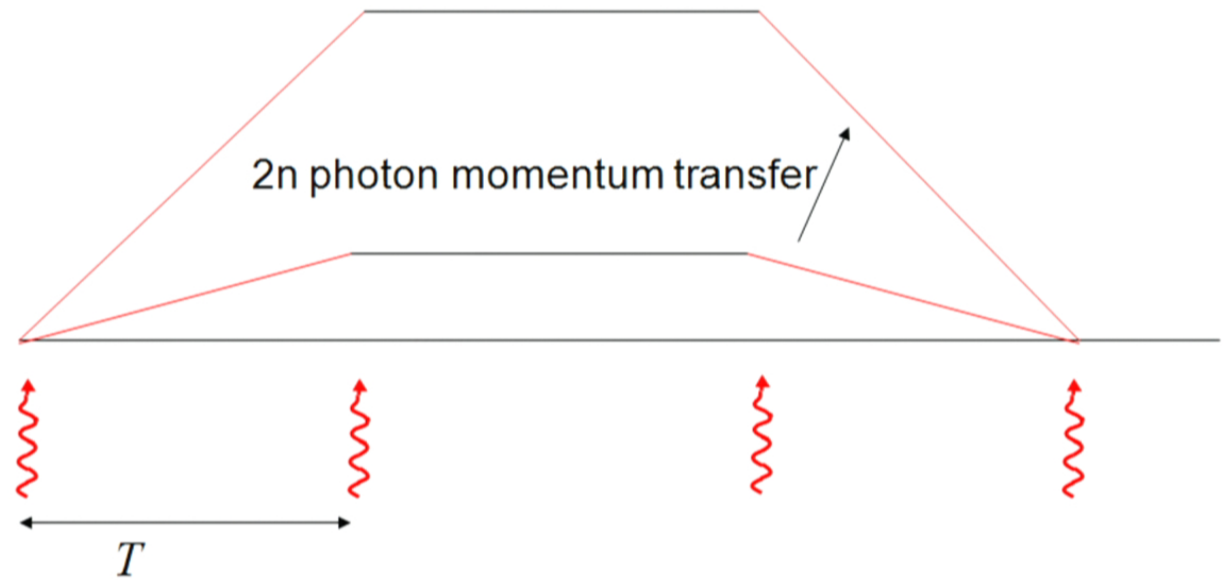
Gravity's Aharonov-Bohm effect





- Terrestrial experiment: ϱ=10 g/cm^3, R=10 m: $\Delta\nu/\nu$=5x10^{-21}
- Possible realization: Earth-moon Lagrange points

Ramsey-Borde interferometer

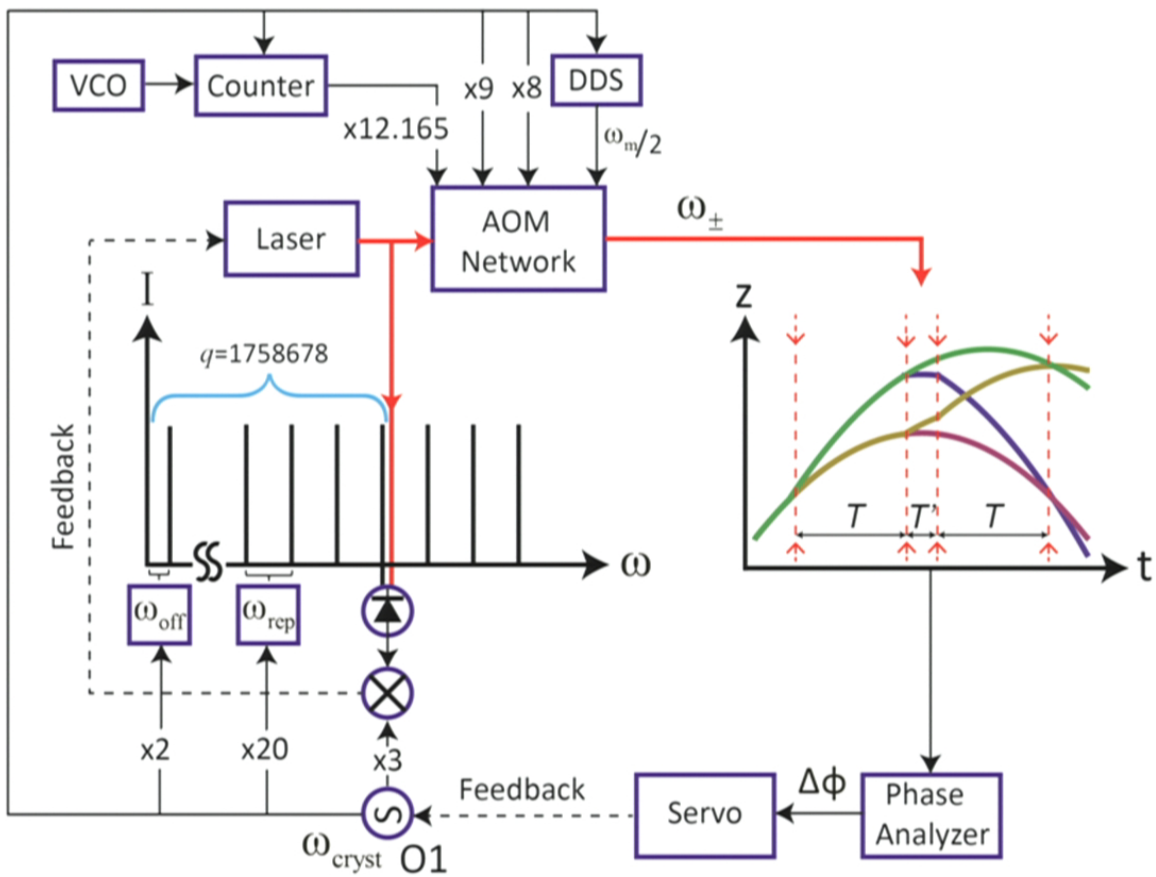


$$\Phi_1 = 8n^2\omega_r T$$

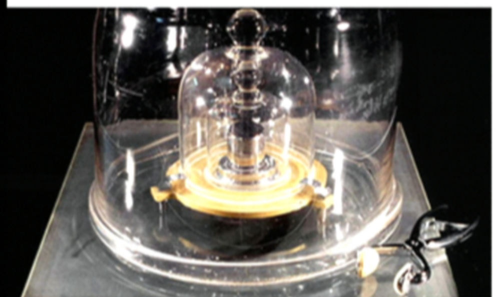
If we can set

$$\omega_r = \frac{\hbar k^2}{2m} = \frac{\omega_L^2}{\omega_C} \qquad \omega_L = N\omega_r \qquad \omega_C = N\omega_L = N^2\omega_r$$

Compton clock



Timing Weight



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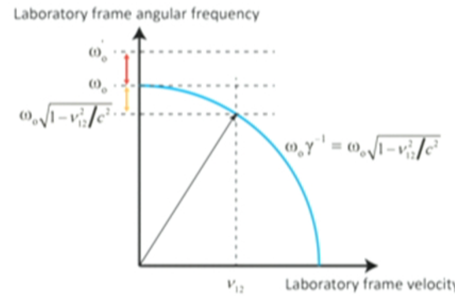

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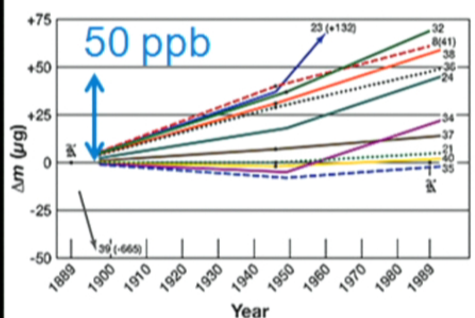


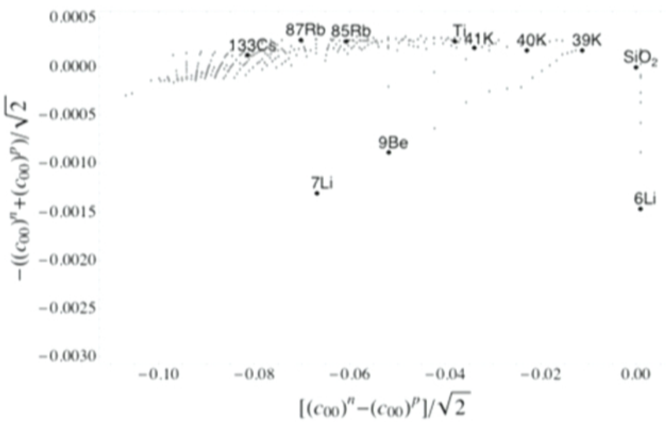
4 ppb

30 ppb

- Define *h* (GPMFC intended)
- kg: 31 ppb with present data
- AMU: 100-fold improvement compared to present Si

0.1 ppm-level limits

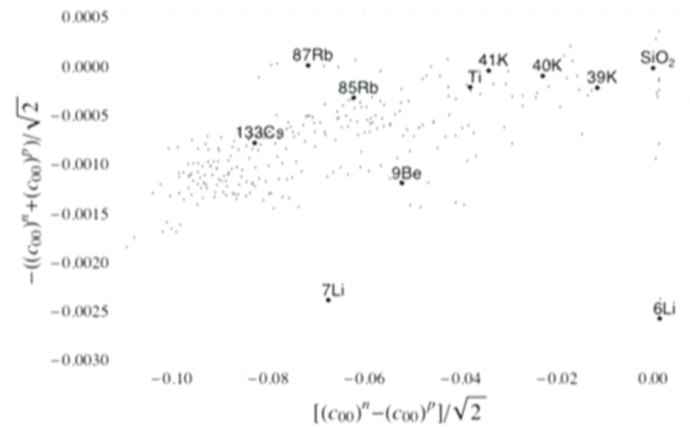
No Composition Correction



Limits in ppb	a^n	a^{p+e}	c^n	c^p	c^e
Baseline	500	250	1590	790	1740
^{87}Rb vs ^{40}K	23	10	73	32	1660
^{87}Rb vs ^{85}Rb	11	20	34	63	1660

Hohensee *et al.* PRL **106,** 151102 (2011),
- one additional torsion balance,
- natural isotopic abundance

With Composition Correction

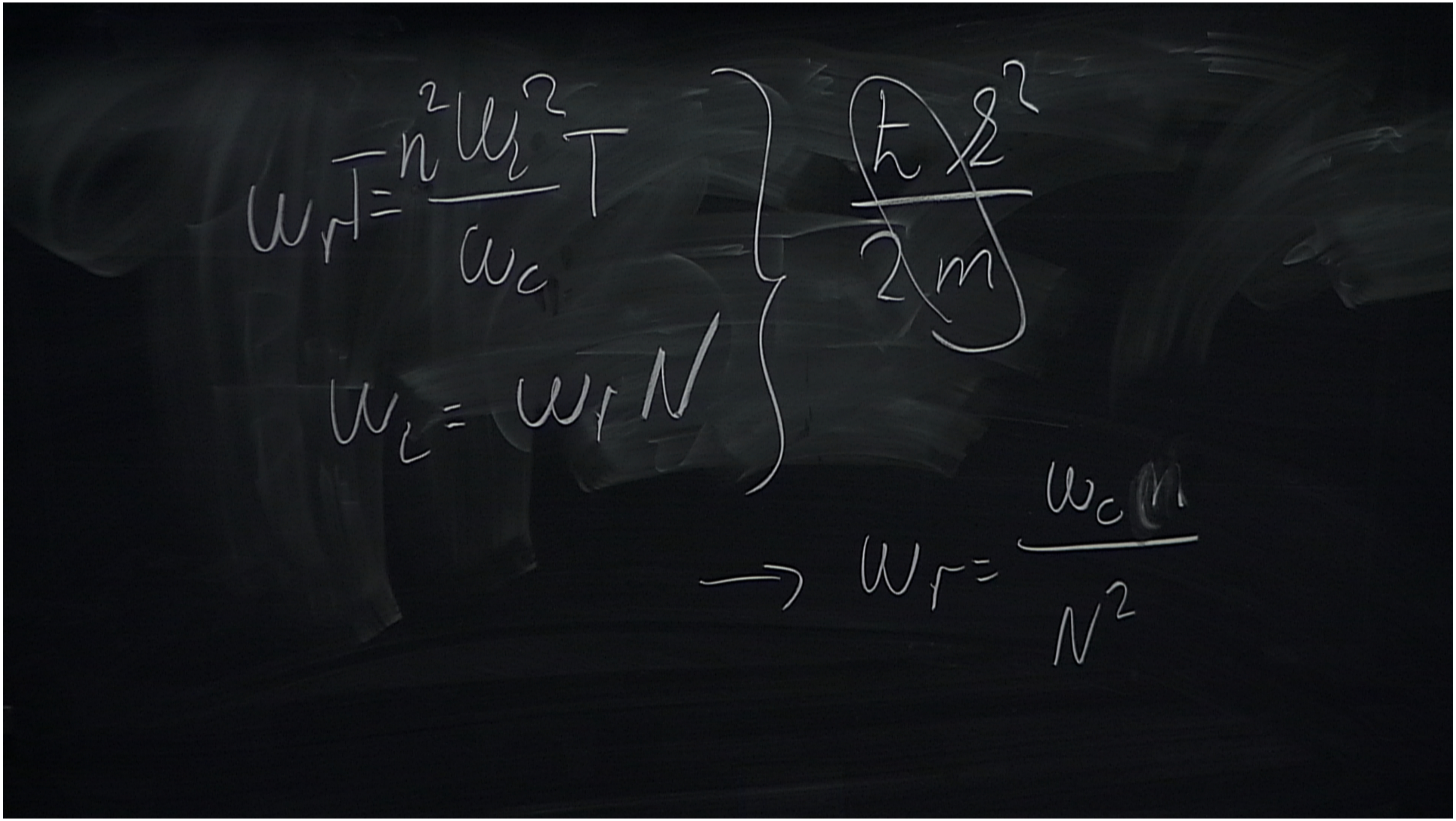


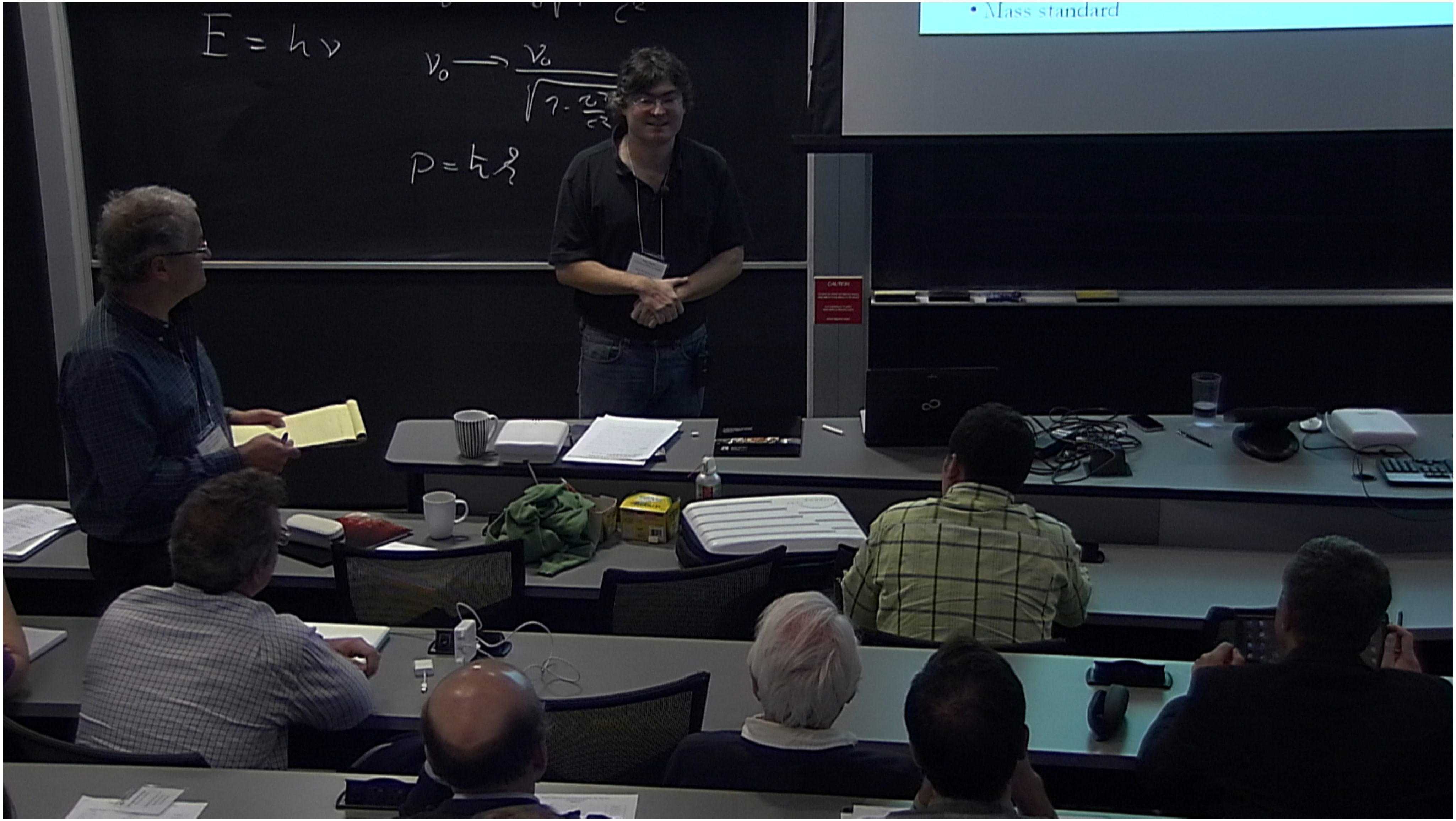
Limits in ppb	a^n	a^{p+e}	c^n	c^p	c^e
Baseline	223	95.4	1380	302	1700
^{87}Rb vs ^{40}K	0.072	0.30	0.18	0.46	1660
^{87}Rb vs ^{85}Rb	0.10	0.31	0.33	0.39	1660

Hohensee *et al.*, to be published

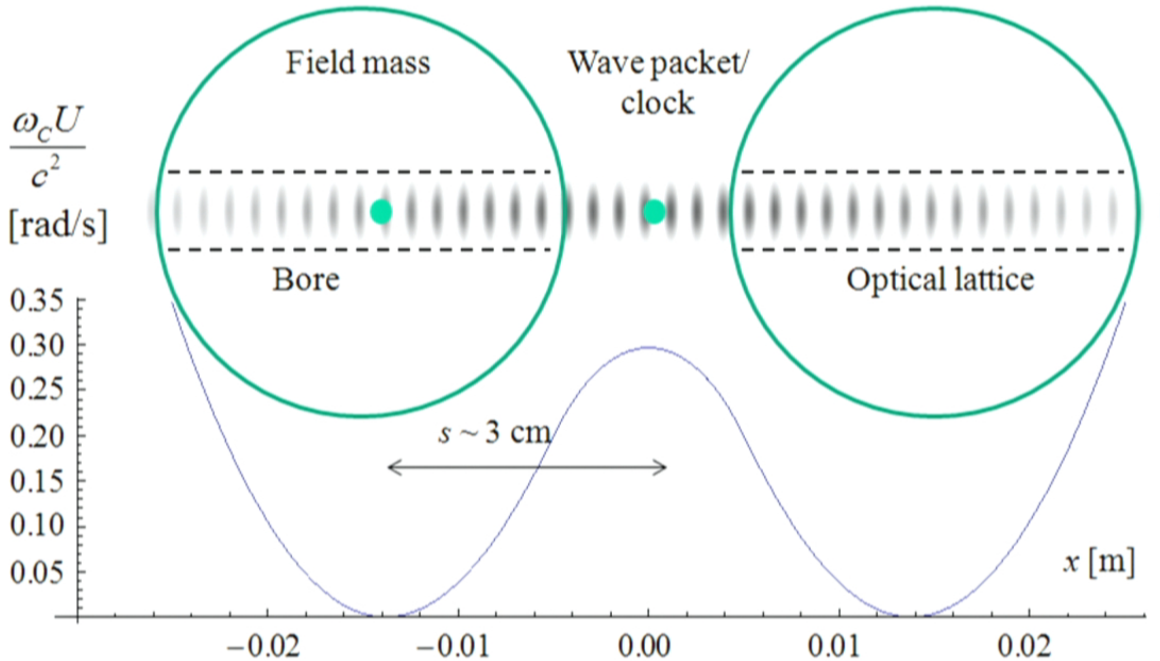
Conclusion

- LMT, Simultaneous, BBB, Coriolis compensation
- α, gravitational waves, inertial sensors
- 10^{-9} redshift test with de Broglie waves
- SME bounds for all EEP violations at 0.1 ppm level
- Gravitational AB effect
- Compton clock
- Mass standard


$\omega_r T = \frac{n^2 \omega_l^2}{\omega_c} T$
$\omega_l = \omega_r N$
$\frac{\hbar l^2}{2m}$
$\rightarrow \omega_r = \frac{\omega_c m}{N^2}$


E = hν
• Mass standard

Realization with atom-clocks



$$\varphi = \omega_C \int \frac{\Delta U}{c^2}\, dt = 0.16 \left(\frac{s}{\text{cm}}\right)^2 \left(\frac{\rho}{10\ \text{g/cm}^3}\right) \left(\frac{m}{m_{Cs}}\right) \left(\frac{T}{\text{s}}\right)$$