Title: Lorentz symmetry: Broken, intact or deformed?

Date: Oct 24, 2012  03:30 PM

URL: http://pirsa.org/12100108

Abstract: I will discuss whether higher energy Lorentz violation should be considered a natural expectation in theories of quantum gravity with a preferred frame. If spacetime is a causal set then Lorentz symmetry is unbroken. Quantum superpositions of the speed of light can exist in superpositions. The simplest case one can look at is a superposition of flat spaces which differ only in the value of the speed of light. I will lay out how such superpositions can be incorporated into quantum field theory, and discuss the fate of Lorentz-invariance in this scenario. In my talk I will briefly introduce the idea of relative locality, being a particular regime of quantum gravity characterized by negligible Planck length and finite Planck mass. Then I will discuss possible scenarios concerning the fate of Lorentz symmetry in this regime.

Horava-Lifshitz gravity models contain higher order operators suppressed by a characteristic scale, which is required to be parametrically smaller than the Planck scale. We show that recomputed synchrotron radiation constraints from the Crab nebula suffice to exclude the possibility that this scale is of the same order of magnitude as the Lorentz breaking scale in the matter sector. This highlights the need for a mechanism that suppresses the percolation of Lorentz violation in the matter sector and is effective for higher order operators as well. Breaking Lorentz invariance: the Universe loves it! I show how the local Lorentz and / or diffeomorphism invariances may be broken by a varying speed of light, softly or harshly, depending on taste. Regardless of the fundamental implications of such dramas, these symmetry breakings may be of great practical use in cosmology. They may solve the horizon and flatness problems. A near scale-invariant spectrum of fluctuation may arise, even without inflation. Distinct observational imprints may be left.

Is there hope to see quantum gravity effects if the underlying theory is strictly respecting of Lorentz invariance? I will discuss a novel class of possibilities, suggested by analogy with some simple solid state physics, including one that has lead to an actual experiment, which has placed the first relevant constraints on these kind of effects.
We have made progress, and that progress has raised both the possibility that Li may not be exact and better ways we can test the symmetry.

Why would you want to study such a thing? I may have thought there was an aether, but haven’t we made progress this century?
20+ years of convergence of “possible” and “can”

<table>
<thead>
<tr>
<th>Possible</th>
<th>Can</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog models (1981)</td>
<td>mSME (Colladay/Kostelecky 1994)</td>
</tr>
<tr>
<td>String LIV (1989)</td>
<td>GRB time delays (GAC, et. al. 1998)</td>
</tr>
<tr>
<td>Loop LIV (1999)</td>
<td>Advances in interferometry, atomic clocks, etc. (1990’s onwards)</td>
</tr>
<tr>
<td>Non-commutative geometry (1999)</td>
<td>UHE astrophysics (2000’s onwards)</td>
</tr>
<tr>
<td>DSR, κ-Poincare, Relative locality (2000)</td>
<td>Neutrino physics (1990’s onwards)</td>
</tr>
<tr>
<td>Horava-Lifshitz gravity (2009)</td>
<td></td>
</tr>
</tbody>
</table>

Thousands upon thousands of papers and scientists

Most explored area of QG “pheno” by far
20+ years of convergence of “possible” and “can”

<table>
<thead>
<tr>
<th>Possible</th>
<th>Can</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog models (1981)</td>
<td>mSME (Colladay/Kostelecky 1994)</td>
</tr>
<tr>
<td>String LIV (1989)</td>
<td>GRB time delays (GAC, et. al. 1998)</td>
</tr>
<tr>
<td>Loop LIV (1999)</td>
<td>Advances in interferometry, atomic clocks, etc. (1990’s onwards)</td>
</tr>
<tr>
<td>Non-commutative geometry (1999)</td>
<td>UHE astrophysics (2000’s onwards)</td>
</tr>
<tr>
<td>DSR, κ-Poincare, Relative locality (2000)</td>
<td>Neutrino physics (1990’s onwards)</td>
</tr>
<tr>
<td>Horava-Lifshitz gravity (2009)</td>
<td></td>
</tr>
</tbody>
</table>

Thousands upon thousands of papers and scientists

Most explored area of QG “pheno” by far
### 20+ years of convergence of “possible” and “can”

<table>
<thead>
<tr>
<th>Possible</th>
<th>Can</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog models (1981)</td>
<td>mSME (Colladay/Kostelecky 1994)</td>
</tr>
<tr>
<td>String LIV (1989)</td>
<td>GRB time delays (GAC, et. al. 1998)</td>
</tr>
<tr>
<td>Loop LIV (1999)</td>
<td>Advances in interferometry, atomic clocks, etc. (1990’s onwards)</td>
</tr>
<tr>
<td>Non-commutative geometry (1999)</td>
<td>UHE astrophysics (2000’s onwards)</td>
</tr>
<tr>
<td>DSR, κ-Poincare, Relative locality (2000)</td>
<td>Neutrino physics (1990’s onwards)</td>
</tr>
<tr>
<td>Horava-Lifshitz gravity (2009)</td>
<td></td>
</tr>
</tbody>
</table>

Thousands upon thousands of papers and scientists

Most explored area of QG “pheno” by far
Two main approaches

Lorentz symmetry modifications only
- Keep as much of regular physics as you can – modify ONLY Lorentz symmetry
- Mostly leads to effective field theory approaches with broken Lorentz symmetry

Lorentz symmetry modifications plus
- Lorentz modifications are a consequence of a more drastic re-interpreting of fundamental laws/picture of spacetime and particle behavior
- Leads to deformed Lorentz symmetry, metric superpositions, etc.

I don’t actually believe these arrows apply as much anymore (although I confess I used to)
Lorentz modifications only

Pick a field theory, figure out all the operators, forget why you might have written them down in the first place, and go to town constraining them. Example, rotationally invariant QED

<table>
<thead>
<tr>
<th>Dim</th>
<th>CPT Odd</th>
<th>CPT Even</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-E_{Pl}b_{\mu}\overline{\psi}\gamma^{\mu}\psi$</td>
<td>$\times$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{2}icu_{\mu}u_{\nu}\overline{\psi}\gamma^{\mu}\gamma^{\nu}\psi$</td>
<td>$\frac{1}{2}idu_{\mu}u_{\nu}\overline{\psi}\gamma^{\mu}\gamma^{\nu}\psi$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{1}{E_{Pl}}(\eta_{L}P_{L} + \eta_{R}P_{R})\overline{\psi}(u \cdot D)^{2}\psi$</td>
<td>$-\frac{1}{E_{Pl}}\overline{\psi}(u \cdot D)^{2}(\alpha_{L}^{(5)}P_{L} + \alpha_{R}^{(5)}P_{R})\psi$</td>
</tr>
<tr>
<td>6</td>
<td>$? \left(\frac{1}{E_{Pl}}\overline{\psi}(u \cdot D)^{3}(u \cdot \gamma)(\alpha_{L}^{(6)}P_{L} + \alpha_{R}^{(6)}P_{R})\psi</td>
<td></td>
</tr>
</tbody>
</table><p>ight)$ | $-\frac{1}{E_{Pl}}\overline{\psi}(u \cdot D)^{3}(u \cdot \gamma)(\alpha_{L}^{(6)}P_{L} + \alpha_{R}^{(6)}P_{R})\psi$ |</p>

You can be very successful with this, but are you doing anything significant for quantum gravity?
Lorentz modifications only

Pick a field theory, figure out all the operators, forget why you might have written them down in the first place, and go to town constraining them. Example, rotationally invariant QED

<table>
<thead>
<tr>
<th>Dim</th>
<th>CPT Odd</th>
<th>CPT Even</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$-E_p b u_{\mu} \bar{\psi} \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \psi$</td>
<td>$\times$</td>
</tr>
<tr>
<td>4</td>
<td>$\times$</td>
<td>$\frac{1}{2} i c u_{\mu} u_{\nu} \bar{\psi} \gamma^\mu D^\nu \psi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{2} i d u_{\mu} u_{\nu} \bar{\psi} \gamma^\mu D^\nu \psi$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{1}{E_p} \bar{\psi} (\eta_L P_L + \eta_R P_R) \psi (u \cdot D)^2 \psi$</td>
<td>$-\frac{1}{E_p} \bar{\psi} (u \cdot D)^2 (\alpha_L^{(5)} P_L + \alpha_R^{(5)} P_R) \psi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-\frac{i}{E_p} \bar{\psi} (u \cdot D)^3 (u \cdot \gamma) (\alpha_L^{(6)} P_L + \alpha_R^{(6)} P_R) \psi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-\frac{i}{E_p} \bar{\psi} (u \cdot D)^3 (u \cdot \gamma) (\bar{\alpha}_L^{(6)} P_L + \bar{\alpha}_R^{(6)} P_R) \psi$</td>
</tr>
<tr>
<td>6</td>
<td>$? \quad \times \quad ?$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

**Photon**

<table>
<thead>
<tr>
<th>Dim</th>
<th>CPT Odd</th>
<th>CPT Even</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>4</td>
<td>$\times$</td>
<td>$-\frac{1}{4} (k_F) u_{\kappa} \eta_{\lambda \mu} u_{\nu} F^{\kappa \lambda} F^{\mu \nu}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{5}{E_p} u^\mu F_{\mu \nu} (u \cdot \partial) u_{\alpha} \bar{F}^{\alpha \nu}$</td>
<td>$\times$</td>
</tr>
<tr>
<td>6</td>
<td>$? \quad \times \quad ?$</td>
<td>$-\frac{1}{2E_p} F_{\gamma}^{(6)} F^{\mu \nu} u_{\mu} u_{\sigma} (u \cdot \partial)^2 F_{\sigma \nu}$</td>
</tr>
</tbody>
</table>

You can be very successful with this, but are you doing anything significant for quantum gravity?
Lorentz modifications only

First issue – you are not really doing quantum gravity...you are simply doing field theory with new matter content.

Say I found a new vector field with a vev or a scalar field with derivative vev. Yay, I win a Nobel prize.

But, even though I've broken Lorentz symmetry in vacuum – have I simply found new physics/dark energy, etc?

No necessary link between LV in EFT and QG.
Lorentz modifications only

First issue – you are not really doing quantum gravity…you are simply doing field theory with new matter content.

Say I found a new vector field with a vev or a scalar field with derivative vev. Yay, I win a Nobel prize.

But, even though I’ve broken Lorentz symmetry in vacuum – have I simply found new physics/dark energy, etc?

No necessary link between LV in EFT and QG.
2nd issue is naturalness. So L1 isn't there...well, why is it *almost* there?

The problem: Imagine you wanted to naively suppress LV but still have it. You could put in a higher mass dimension operator and suppress it by some large scale $M$. But...

Loop effects will generically generate large lower dimension operators as well so your LV is effectively unsuppressed.

What about a custodial mechanism (like SUSY) or other cutoff to loop contributions at a lower scale $m$?

$$\text{Dim 3,4 ops } \sim \frac{m}{M} \text{ or } \frac{m^2}{M^2}$$

Tightest constraint on dim 4 ops $\sim 10^{-28}$

$m<100$ TeV if $M$ is Planck mass
That works, but you had to introduce another mechanism at low energies to make things work and have hierarchy issues. Additionally, the other mechanism is at an energy that theoretically can be explored.

It’s actually Lorentz modifications plus

For experimentalists, if you see a model with not only LIV but also a fairly natural mechanism to suppress it, you may want to pay attention.

My take: it is reasonable certainly to test in EFT, but I just wish there was more focus on testing specific sectors of LIV theories that accomplish something (c.f. Horava-Lifshitz)
Lorentz modifications plus

Maybe Lorentz invariance modifications are just collateral damage…

Postulate a fundamental change in how we view spacetime, locality, etc. Lorentz symmetry modifications, if they exist, are simply part of a larger picture (3 talks on these topics)

This approach is actually direct quantum gravity phenomenology

In general, these approaches produce effects that do not fit nicely within EFT, i.e.

\[ \omega^2 = k^2 \pm \frac{k^3}{M} \]

for all photons

which is exactly why

a) They are harder to constrain
b) They are harder to figure out appropriate observables, make consistent etc.
c) More varied, more time consuming to get concrete predictions for observable phenomena
**Lorentz modifications alone**

- Easy and natural ideas well checked and excluded, experimentalists and theorists beware
- Is it QG or new classical physics?
- A little bit of a runaway train without a conductor, in that people can spend their lives getting better constraints, but must be careful the constraints mean anything other than saying “I know our theory works down to this precision”?
- Proposed LIV should both solve a problem and provide a protection mechanism

**Lorentz modifications plus**

- Non-Lorentz part is part of the picture (many other parts besides just Lorentz symmetry testing)
- Ideas harder to turn into many testable predictions
- Often qualitatively new ideas for experimentalists to measure, but less well defined
Breaking Lorentz invariance: the Universe loves it!

João Magueijo
2012
Imperial College, London
Varying \( c \) theories

[JM, Rept. Prog. Phys. 66, 2025]

- Covariant and Lorentz invariant
  [Moffat, Magueijo, etc, etc]

- Bimetric theories
  [Moffat, Clayton, Drummond, etc, etc]

- Preferred frame
  [Albrecht, Magueijo, Barrow, etc, etc]

- Deformed dispersion relations
  [Amelino-Camelia, Mavromatos, Magueijo & Smolin, etc, etc]
Dirac and Manci on their honeymoon, Brighton, January 1937
“Look what happens to people when they get married”
(Niels Bohr)
A non-inflationary solution to the horizon problem
A non-inflationary solution to the horizon problem
But who cares about the horizon problem... Here's the real problem:
The zero-th order “holy grail” of cosmology:

\[ k^3 |\zeta(k)|^2 = A^2 \left( \frac{k}{k_c} \right)^{n_s - 1} \]

- Near scale-invariance
  \[ n_s \sim 1 \]
- Amplitude
  \[ A \sim 10^{-5} \]
Bimetric theories

A metric for gravity (Einstein frame):

\[ g_{\mu\nu} \xrightarrow{\text{gravity}} S = \int dx^4 \sqrt{-g} R \]

A metric for matter (matter frame):

\[ g_{\mu\nu} \xrightarrow{\text{matter}} S_m = \int dx^4 \sqrt{-g} L(g_{\mu\nu}, \Psi, \text{etc}) \]
This is a rather conservative thing to do...

- If the two metrics are conformal, we have a varying-G (Brans-Dicke) theory
  \[ \hat{g}_{\mu\nu} = e^\phi g_{\mu\nu} \]

- If they are disformal we have a VSL theory
  \[ \hat{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi \]

- The speed of light differs from the speed of gravity (larger if $B > 0$, with $+---$)
The minimal bimetric VSL theory

\[ \hat{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi \quad B = B(\phi) = \text{const} \]

\[ S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-\hat{g}} R[\hat{g}_{\mu\nu}] + \int d^4x \sqrt{-\hat{g}} \mathcal{L}_m[\hat{g}_{\mu\nu}, \Phi_{\text{Matter}}] + S_\phi \]

\[ S_\phi = ??? \]
What sort of fluctuations come out of these theories?

- If we project onto the Einstein frame, we end up with the same formalism usually used for inflation, but…
- including a varying speed of sound.
- This is the so-called K-essence inflation (an inflaton with non-quadratic kinetic terms).
The tools of (K-essence) varying speed of sound:

\[ \mathcal{L} = K(X) - V(\phi) \]

\[ X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \]

\[ p = K - V \]
\[ \rho = 2XK,X - K + V \]

\[ c_s^2 = \frac{K,X}{K,X + 2XK,X,X} \]

Check formulae with inflation, cuscaton, etc…
How to compute fluctuations:

\[ \zeta = \frac{\nu}{z} \]

\[ z = \frac{a}{c_s} \]

\[ \nu'' + \left[ c_s^2 k^2 - \frac{z''}{z} \right] \nu = 0 \]
How to compute fluctuations:

\[ \zeta = \frac{v}{z} \quad \text{and} \quad z = \frac{a}{c_s} \]

\[ v'' + \left[ c_s^2 k^2 - \frac{z''}{z} \right] v = 0 \]
How to compute fluctuations:

\[ \zeta = \frac{v}{z} \]

\[ z = \frac{a}{c_s} \]

\[ v'' + \left[ c_s^2 k^2 - \frac{z''}{z} \right] v = 0 \]
Why the horizon problem leads to a real problem:

- If \( c_s = \text{const} \)
- If \( 1 + 3w > 0 \) (with \( w = \frac{p}{\rho} \))

\[
v'' + \left[ c_s^2 k^2 \frac{z''}{z} \right] v = 0
\]

Dominates at late times

\[
z = \frac{a}{c_s}
\]
How inflation solves the problem:

- With $1 + 3w < 0$, $\eta < 0$

$$v'' + \left[ c_s^2 k^2 - \frac{\dot{z}''}{z} \right] v = 0$$

- Dominates earlier
- Dominates later

- But why do we get scale invariance?

$$\propto \frac{1}{\eta^2}$$
How a varying speed of light solves the problem:

- With $1 + 3\omega > 0$ but $c_s \propto \eta^\beta$
  with $\beta < -1$ we still get:

$$v'' + \left[ c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

Dominates earlier

$\alpha \propto \frac{1}{\eta^2}$

Dominates later
A remarkable result (!!!!!!!!!!!!!!)

- For ALL equations of state

\[ c_s \propto \rho \implies n_s = 1 \]

This scaling seems to be uniquely associated with scale invariance.
(For experts only; cf. k-essence)

- This can be understood:

\[ k^3 \zeta^2 \sim \frac{(5 + 3w)^2}{1 + w} \frac{\rho}{M^4_{Pl} c_s} \]
Where does the amplitude come from?

- Obviously the variations in $c$ must be cut off at low energies:

$$c_s = c \left( 1 + \frac{\rho}{\rho_*} \right)$$

- The cut-off scale fixes the amplitude:

$$k^3 \zeta^2 \sim \frac{(5 + 3w)^2}{1 + w} \frac{\rho_*}{M_{Pl}^4} \sim 10^{-10}$$
The minimal bimetric VSL theory

\[ \hat{g}_{\mu \nu} = g_{\mu \nu} + B \partial_\mu \phi \partial_\nu \phi \]

\[ B = B(\phi) = \text{const} \]

\[ S = \frac{M_{Pl}^2}{2} \int d^4 x \sqrt{-g} R[g_{\mu \nu}] + \int d^4 x \sqrt{-\hat{g}} L_m[\hat{g}_{\mu \nu}, \Phi_M] + S_\phi \]

\[ S_\phi = ??? \]
Something truly cool...

\[ S_{\phi}^1 = \int d^4 x \sqrt{-\hat{g}} (-2\hat{\Lambda}) \]

\[ \hat{g} = g (1 + 2B X) \quad X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \]

\[ S_{\phi}^1 = \int d^4 x \sqrt{-g} \sqrt{1 - 2BX (-2\hat{\Lambda})} \]
Apply to (anti)DBI to find that...

\[ c_s = c \left( 1 + \frac{\rho}{\rho_*} \right) \]
So our remarkable result is even more remarkable

- Not only is it possible to identify a universal varying speed of sound law associated with scale invariance…
- but this law can be realized by an anti-DBI model (in the Einstein frame), which…
- turns out to be the minimal dynamics associated with a bimetric VSL
What we did with bimetric VSL can also be done with DSR

- Deformed dispersion relations can give a frequency dependent speed of light

\[ E^2 - g^2 p^2 = m^2 \]

- The speed of sound would then also vary in time, by proxy, via expansion:

\[ \omega = kg(\lambda k/a) \]

\[ c = \frac{d\omega}{dk} = (\gamma + 1) \frac{\omega}{k} \propto \left( \frac{\lambda k}{a} \right)^\gamma \]
What we did with bimetric VSL can also be done with DSR

- Deformed dispersion relations can give a frequency dependent speed of light
  \[ E^2 - g^2 p^2 = m^2 \]
- The speed of sound would then also vary in time, by proxy, via expansion:

\[ \omega = k g \left( \lambda k / a \right) \]
\[ c = \frac{d\omega}{dk} = (\gamma + 1) \frac{\omega}{k} \propto \left( \frac{\lambda k}{a} \right)^{\gamma} \]
Also in this context scale-invariance is associated with an universal law

\[ v'' + \left[ \omega^2 - \frac{z''}{z} \right] v = 0 \]

\[ \omega^2 - k^2(1 + (\lambda k)^2)^2 = m^2 \]

\[ \lambda \sim 10^5 L_{Pl} \]

Cf. Horava-Lifschitz.
Beyond the zeroth order holy grail

\[ f_{NL} = 30 \frac{A_{k_1=k_2=k_3}}{K^3} \]

\[ k_1 = k_2 = k_3 = K/3 \]

Standard inflation

\[ f_{NL} \sim \epsilon \sim 0.1 \]

VSL

\[ f_{NL} \sim 1 > 0 \]

DBI inflation

\[ f_{NL} \sim -100 \]
Is this then another "theory of anything"? No.
Breaking Lorentz invariance is good for you… if you’re a cosmologist

- An alternative to inflation for solving the cosmological problems
- Observationally distinct from inflation
Breaking Lorentz invariance is good for you… if you’re a cosmologist

- An alternative to inflation for solving the cosmological problems
- Observational evidence against inflation
Observational constraints on scale hierarchy in Horava-Lifshitz gravity

Stefano Liberati
SISSA - INFN
Trieste, Italy

Experimental Search for Quantum Gravity:
the hard facts
October 22-25, 2012

Talk @
PERIMENTAL INSTITUTE FOR THEORETICAL PHYSICS

Work mainly done in collaboration with:
L. Maccione and T. Sotiriou
Hořava-Lifshitz Gravity

Basic idea: modify the graviton propagator in the UV by adding to the action terms containing higher order spatial derivatives of the metric, but refrain from adding higher order time derivatives in order to preserve unitarity.

Power counting renormalizability requires that the action includes terms with at least 6 spatial derivatives in 4 dimensions.

All lower order operators compatible with the symmetry of the theory are expected to be generated by radiative corrections

$$S_{HL} = \frac{M_{Pl}^2}{2} \int dt d^3 x N \sqrt{h} \left( L_2 + \frac{1}{M_*^2} L_4 + \frac{1}{M_*^4} L_6 \right),$$

where $h$ is the determinant of the induced metric $h_{ij}$ on the spacelike hypersurfaces, and $L_2 = K_{ij} K^{ij} - \lambda K^2 + \xi (3) R + \eta a_i a^i$ with $K$ is the trace of the extrinsic curvature. $K_{ij}$, $(3) R$ is the Ricci scalar of $h_{ij}$. $N$ is the lapse function, and $a_i = \partial_i \ln N$.

$L_4$ and $L_6$ denote a collection of 4th and 6th order operators respectively and $M_*$ is the scale that suppresses these operators.

These Infrared (IR) Lorentz violations are controlled by three dimensionless parameters that take the values $\lambda=1$, $\xi=1$, $\eta=0$ in General Relativity (GR).

Unfortunately $L_4$ and $L_6$ contain a very large number of operators ($\sim 10^3$) and so have been proposed several restrictions to the theory to limit them. In particular projectability: $N=N(t)$ | Detailed balance.

There is still debate about these constraints, we shall not deal with them here and our conclusions are general and does not hinge on the exact form of $L_4$ and $L_6$. 

Constraints on Hořava-Lifshitz Gravity

How much can be $M_*$?

It is indeed bounded from below and above

$M_{\text{obs}} < M_* < 10^{16} \text{ GeV} \quad M_{\text{obs}} \approx \text{few meV} \quad (\text{from sub mm tests})$

Due to the reduced symmetry with respect to GR, the theory propagates an extra scalar mode. If one chooses to restore diffeomorphism invariance, then this mode manifests as a foliation-defining scalar field.

In order to avoid scalar mode strong coupling in $L_4$ which would jeopardize of power counting renormalizability. In projectable version hopeless (strong coupling at very low energies), in non projectable version constraints from Solar System no-observation of preferred frame effects.

However LIV cannot be confined to gravity!

· Higher order operators will always induce lower order ones by radiative corrections!
  [Collins et al. PRL93 (2004), Iengo, Russo, Serone 2009]
· So in general even starting with a Lorentz invariant matter sector at tree level one expects that matter LIV operators will be generated via graviton radiative corrections
· Let us assume that some protective mechanism can be envisaged to protect the lowest order operators (universal coefficient of $p^2$ in MDR $c=1$), i.e Horava gravity IR viable.
· Then the symmetries of the LIV operators in Hořava-Lifshitz action naturally leads to the expectation for matter MDR (we assume no LIV at three level in matter and that CPT,$\mathbb{P}$ even nature of LIV in gravity sector is maintained in the LIV terms induced in matter)

$E^2 = m^2 + p^2 + \eta \frac{p^4}{M_{\text{LV}}^2} + O \left( \frac{p^6}{M_{\text{LV}}^4} \right).$

Now: Is $M_{\text{LIV}} \sim M_*$

or

$M_{\text{LIV}} \gg M_*$?
LIV in the EFT matter sector: current constraints

We already know that: if $M < 10^{16}$ then one cannot have $M_{\text{LIV}} \ll M$. In fact...

$$E_{\gamma}^2 = k^2 + \xi_{\pm}^{(n)} \frac{k^n}{M_{\text{pl}}^{n-2}} \quad \text{photons}$$

$$E_{\text{matter}}^2 = m^2 + p^2 + \eta_{\pm}^{(n)} \frac{p^n}{M_{\text{pl}}^{n-2}} \quad \text{leptons/hadrons},$$

where, in EFT, $\xi^{(n)} \equiv \xi_{\pm}^{(n)} = (-)^n \xi_{\pm}^{(n)}$ and $\eta^{(n)} \equiv \eta_{\pm}^{(n)} = (-)^n \eta_{\pm}^{(n)}$.

<table>
<thead>
<tr>
<th>Order</th>
<th>photon</th>
<th>$e^-/e^+$</th>
<th>Hadrons</th>
<th>Neutrinos$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=2</td>
<td>N.A.</td>
<td>$O(10^{-13})$</td>
<td>$O(10^{-27})$</td>
<td>$O(10^{-8})$</td>
</tr>
<tr>
<td>n=3</td>
<td>$O(10^{-14})$ (GRB)</td>
<td>$O(10^{-16})$ (CR)</td>
<td>$O(10^{-14})$ (CR)</td>
<td>$O(30)$</td>
</tr>
<tr>
<td>n=4</td>
<td>$O(10^{-8})$ (CR)</td>
<td>$O(10^{-8})$ (CR)</td>
<td>$O(10^{-6})$ (CR)</td>
<td>$O(10^{-4})^*$ (CR)</td>
</tr>
</tbody>
</table>

GRB = gamma rays burst, CR = cosmic rays
$a$ From neutrino oscillations we have constraints on the difference of LIV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. $^*$ Expected constraint from future experiments.

However... $n=4$ constraints could be weakened due to current uncertainties on UHECR nature still allowing $M_{\text{LIV}} \ll M$.

Can we exclude this without using UHECR?
**Synchrotron radiation**

LI synchrotron critical frequency:

$$\omega_c^L = \frac{3 \, eB \gamma^2}{2 \, m}$$

- $e$ - electron charge
- $m$ - electron mass
- $B$ - magnetic field

However in order to get a real constraint one needs a detailed re-derivation of the synchrotron effect with LV based on EFT.

This leads to a modified formula for the peak frequency:

$$\omega_c^{LV} = \frac{3 \, eB}{2 \, E} \gamma^3$$

While the rate of energy loss differs from the LV one only nearby the VC threshold...

$$\eta < 0 \quad E^2 = m^2 + p^2 + \eta \frac{p^n}{M_{LV}^{n-2}} \quad \eta > 0$$

$\gamma$ is a bounded function of $E$. There is now a maximum achievable synchrotron frequency $\omega_{max}$ for ALL electrons!

$$\omega_{c,\text{max}}^{max(n)} = \frac{3eB}{2m} \left(1 - \frac{4}{3n}\right)^{3/2} \left(\frac{4 \left(M_{LV}/m\right)^{n-2}}{-\eta(n-1)(3n-4)}\right)^{2/n}$$

So one gets a constraints from asking $\omega_{max} \geq (\omega_{max})_{observed}$

$\gamma$ diverges as $p_{th}$ is approached. This is unphysical as also the energy loss rates diverges in this limit, however signifies a rapid decay of the electron energy and a violent phase of synchrotron radiation.

What is the best studied synching object?
The Crab Nebula – Remnant of a SuperNova explosion

exploded in 1054 A.D.
distance ~1.9 kpc from Earth
pulsar wind powered nebula
most powerful object in the sky
spectrum spans 21 decades in frequency, from radio to ~80 TeV
leptonic origin of the radiation
electrons accelerated to > PeV
theoretical model understood only roughly at radio frequencies quite well
enough at >keV energies.
(Kennel & Coroniti, 1984)

E < 1 GeV → synchrotron
E > 1 GeV → IC scattering
The EM spectrum of the Crab nebula

Interesting part:
old EGRET data,
also new AGILE
and FERMI results!

Crab nebula (and other SNR) well explained by synchrotron self-Compton (SSC) model
1. Electrons are accelerated to very high energies at pulsar
2. High energy electrons emit synchrotron radiation
3. High energy electrons undergo inverse Compton (mainly with synchrotron ambient photons)

Re-compute the full Crab spectrum without Lorentz Invariance.

- Fix most of the free parameters (magnetic field strength, electron energy density...) from low frequency observations (well defined procedure, see later)
- Check that LV modifications enter only in the high energy part of the spectrum
- Compare with experimental points and make constraints (chi-square analysis).
Results

Crab Nebula spectrum for the LI case (blue, solid curve), for the LV case $n=4$, with $M_{LV} = 10^{16}$ GeV and $n>0$ (red, dashed curve), and for the case with same parameters but $n<0$ (magenta, dot-dashed curve). While, as discussed, the $n<0$ case would lead to premature fall off of the synchrotron spectrum, we see here that for $n>0$ there is a sudden surge of emission at high frequencies, followed by a dramatic drop due to the onset of vacuum Čerenkov emission at the characteristic threshold energy $E_{\text{th}} = [(nM_{LV})^{1/2}/\gamma]^{1/2}$.

Dependence of the reduced $\chi^2$ on $M_{LV}$.

By considering the offset from the minimum of the reduced $\chi^2$ we set exclusion limits at 90%, 95% and 99% Confidence Level (CL).

Mass scales $M_{LV} = 2 \times 10^{16}$ GeV are excluded at 95% CL.

The window for $M_{LV} \sim M_*$ is closed.
Conclusions

- We placed constraints on LV in the matter sector in HL models by exploiting the broad band spectrum of the Crab Nebula.

- We obtain $M_{LV} \approx 2 \times 10^{16}$ GeV, assuming CPT and $P$ invariance to be preserved in the matter sector.

- Hence our current constraints appear incompatible with the possibility that $M_{LV} \sim M_Z$.

- Therefore a mechanism, suppressing the percolation of LV in the matter sector, must be present in HL models, and such mechanism should not only protect lower order operators.
Ways out?

- The condition $M_c < 10^{16}$ GeV was a consequence of the need to protect perturbative renormalizability by assuring that the mass scale of the Horava scalar mode $M_{sc} > M_c$ plus the observational constraints on $L_2$ that generically imply $M_{sc} < 10^{16}$ GeV.

- This can be avoided by suitable fine tuning of the $L_2$ parameters $\lambda, \xi, \eta$ hence allowing much higher $M_{sc}$. Problem: Finely tuned solution.

- Alternatively, one could resort to breaking of P invariance and allowing for a strong hierarchy between the two scales $M_{L/2}$ and $M_{L/2}$ now suppressing LIV in electrons of opposite helicity. Problem: Finely tuned solution.

- A much more appealing option is offered by the mechanism proposed by Pospelov & Shang (arXiv.org/1010.5249v2) of "gravitational confinement": no LIV is present at the tree level in the matter sector (so to avoid the need for additional custodial symmetries) and that $M_o > M_{Pl}$.

- In this case radiative corrections will percolate LV operators from the gravity sector to the matter ones but the gravitational coupling $G = M^{-2}$ will do so by introducing strong suppression factors of the order $(M_u/M_{Pl})^2$. It has been shown that dimension 4 operators of matter can be efficiently screened from LV this way in HL models if at least $M_c < 10^{16}$ GeV and this is expected to be the case for higher order operators as well.
sector to the
magnetic suppression
matter can be
this is expected

Order: João
Stefano
Niraysh
Sabine
Vincen
Rafael
Daniel

\( \frac{M^2}{N_f^2} \)
Experimental Search for Quantum Gravity: the hard facts
Perimeter Institute, Oct. 24, 2012

IS AETHER TECHNICALLY NATURAL?

Niayesh Afshordi

UNIVERSITY OF WATERLOO
COSMOLOGIST’S
QUANTUM GRAVITY PROBLEMS

- Finiteness/Renormalizability → Big Bang/Black Hole Singularities/Initial Conditions

- Old Cosmological Constant Problem (Pauli 1920’s)
  - $|\rho_{\text{vac}}| \geq 10^{33} \text{ kg/m}^3$ (Standard Model of Particle Physics)

- New Cosmological Constant Problem: Dark Energy
  - $\rho_{\text{vac}} = (7.1 \pm 0.9) \times 10^{-27} \text{ kg/m}^3$

- Coincidence Problem
  - $\rho_{\text{vac}} \approx 2.7 \rho_{\text{m,o}}$

Perimeter Institute, Oct. 24, 2012
COSMOLOGICAL CONSTANT PROBLEM

- Einstein Equation:
  \[ (\text{Planck Mass})^2 \times \text{Einstein Curvature} = \text{Energy-Momentum} \]

**Anthropic Solution:** We live in a rare but habitable part of Cosmos*

*Perimeter Institute, Oct. 24, 2012*
Eternal inflation predicts that time will end

Raphael Bousso\textsuperscript{a,b,c}, Ben Freivogel\textsuperscript{d}, Stefan Leichenauer\textsuperscript{a,b} and Vladimir Rosenhaus\textsuperscript{a,b}

\textsuperscript{a} Center for Theoretical Physics and Department of Physics
University of California, Berkeley, CA 94720-7300, U.S.A.
\textsuperscript{b} Lawrence Berkeley National Laboratory, Berkeley, CA 94720-8162, U.S.A.
\textsuperscript{c} Institute for the Physics and Mathematics of the Universe
University of Tokyo, 5-1-5 Kashiwa-no-Ha, Kashiwa City, Chiba 277-8568, Japan
\textsuperscript{d} Center for Theoretical Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

\textbf{ABSTRACT:} Present treatments of eternal inflation regulate infinities by imposing a geometric cutoff. We point out that some matter systems reach the cutoff in finite time. This implies a nonzero probability for a novel type of catastrophe. According to the most successful measure proposals, our galaxy is likely to encounter the cutoff within the next 5 billion years.
Eternal inflation predicts that time will end

Raphael Bousso\textsuperscript{a,b,c}, Ben Freivogel\textsuperscript{d}, Stefan Leichenauer\textsuperscript{a,b} and Vladimir Rosenhaus\textsuperscript{a,b}

\textsuperscript{a} Center for Theoretical Physics and Department of Physics
University of California, Berkeley, CA 94720-7300, U.S.A.
\textsuperscript{b} Lawrence Berkeley National Laboratory, Berkeley, CA 94720-8162, U.S.A.
\textsuperscript{c} Institute for the Physics and Mathematics of the Universe
University of Tokyo, 5-1-5 Kashiwa-no-Ha, Kashiwa City, Chiba 277-8568, Japan
\textsuperscript{d} Center for Theoretical Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

\textbf{ABSTRACT:} Present treatments of eternal inflation regulate infinities by imposing a geometric cutoff. We point out that some matter systems reach the cutoff in finite time. This implies a nonzero probability for a novel type of catastrophe. According to the most successful measure proposals, our galaxy is likely to encounter the cutoff within the next 5 billion years.
WHICH PRINCIPLES TO KEEP?

- Lorentz symmetry
- Locality
- Action principle
- Unitarity

- Falsifiability
- Predictivity

Perimeter Institute, Oct. 24, 2012
COSMOLOGICAL CONSTANT PROBLEM

- Einstein Equation:

\[ (\text{Planck Mass})^2 \times \text{Einstein Curvature} = \text{Energy-Momentum} \]

MODIFY GRAVITY!!

- Spacetime curvature
  \[ \sim (10^{-3} \text{eV})^4 \]

- Standard model
  \[ \sim (100 \text{ GeV})^4 + \text{excitations} \]

Perimeter Institute, Oct. 24, 2012
AETHER SCORE CARD

✧ **Triumphs**

✧ Renormalizability of UV divergences (*a la Hořava*)

✧ Horizon problem, scale-invariant cosmic initial conditions (*a la Magueijo, Mukohyama*)

✧ Cosmological Constant Problem (*Prescod-Weinstein et al.*, Kamiab & NA, Aslanbeigi et al.)

✧ Black Hole entropy (*Saravani et al.*, in prep.)

✧ **Tribulations**

✧ Why is high energy physics Lorentz-invariant? *(with Maxim Pospelov?)*

Perimeter Institute, Oct. 24, 2012
THREE ARGUMENTS FOR WHY PARTICLE PHYSICS HAS LORENTZ SYMMETRY

1. Strong coupling of Aether beyond -meV
2. Quantum Anomaly of Aether
3. 2nd cosmological constant problem

Perimeter Institute, Oct. 24, 2012
1ST ARGUMENT: STRONG COUPLING

❖ Coupling with a dynamical aether produces a vertex that can become strongly coupled

\[ \mathcal{L}_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \epsilon (\partial^\mu \tau \partial_\mu \phi)^2. \]

❖ Aether energy-momentum is bounded by cosmology

\[ \rho^{(0)} + p^{(0)} < 3 \Omega_\Lambda |1 + w| M_p^2 H^2 < (1.7 \text{ meV})^4 \]

❖ In lieu of fine-tuning, weak-coupling and cosmological bound imply:

\[ \epsilon = |1 - c_\phi| \lesssim \left( \frac{E}{5 \text{ meV}} \right)^{-4}, \text{ for } E \gtrsim 5 \text{ meV.} \]

Perimeter Institute, Oct. 24, 2012
2ND ARGUMENT: QUANTUM ANOMALY

- In quantum aether theories, the path integral measure should be invariant under time-reparametrization in the preferred frame
  \[ \mathcal{L}_\tau = \frac{M^2}{c_\psi^3} \left[ c_\psi^{-2} \dot{u}^\mu \dot{u}_\mu - (\nabla_\mu u^\mu)^2 \right], \quad u^\mu \equiv \frac{\partial^\mu \tau}{\sqrt{\partial^\alpha \tau \partial_\alpha \tau}}. \]

- Only covariant measure:
  \[ D_\tau \mathcal{M}[g_{\mu \nu}, \tau] = \prod_x \frac{d\tau_x}{\sqrt{|\partial^\mu \tau \partial_\mu \tau|}}, \]

- Anomaly:
  \[ S_q[g_{\mu \nu}, \tau] = S_{cl}[g_{\mu \nu}, \tau] + \frac{i \Lambda_\psi^4}{2c_\psi^2} \int d^4 x \sqrt{-g} \ln |\partial^\mu \tau \partial_\mu \tau|, \]

- IR Instability: Pushing the IR scale below Hubble:
  \[ E \gtrsim \frac{c_\psi (1 + c_\psi^2)^{1/2} \Lambda_\psi^2}{M} \quad \lambda_\psi^4 \lesssim \frac{H^2 M^2}{c_\psi^2 (1 + c_\psi^2)} \lesssim \frac{(1 \text{ meV})^4}{c_\psi + c_\psi^{-1}} \lesssim (1 \text{ meV})^4 \]

Perimeter Institute, Oct. 24, 2012
**2ND ARGUMENT: QUANTUM ANOMALY**

- In quantum aether theories, the path integral measure should be invariant under time-reparametrization in the preferred frame. 
  \[ \mathcal{L}_\tau = \frac{M^2}{c_\psi^3} \left[ c_\psi^{-2} \dot{u}^\nu \dot{u}_\mu - (\nabla_\mu u^\nu)^2 \right], \quad u^\mu = \frac{\partial^\mu \tau}{\sqrt{\partial_\alpha \tau \partial_\alpha \tau}}. \]

- Only covariant measure: 
  \[ D_\tau \mathcal{M}[g_{\mu\nu}, \tau] = \prod_x \frac{d\tau_x}{\sqrt{|\partial_\mu \tau \partial_\mu \tau|}}, \]

- Anomaly: 
  \[ S_q[g_{\mu\nu}, \tau] = S_{cl}[g_{\mu\nu}, \tau] + \frac{i\Lambda_\psi^4}{2c_\psi^3} \int d^4x \sqrt{-g} \ln |\partial_\mu \tau \partial_\mu \tau|, \]

- IR Instability: 
  
  \[ E \gtrsim \frac{c_\psi (1 + c_\psi^2)^{1/2} \Lambda_\psi^2}{M} \]

  \[ \Lambda_\psi^4 \lesssim \frac{H^2 M^2}{c_\psi^2 (1 + c_\psi^2)} \lesssim \frac{(1 \text{ meV})^4}{c_\psi + c_\psi^{-1}} \lesssim (1 \text{ meV})^4 \]

*Perimeter Institute, Oct. 24, 2012*
3rd Argument: 2nd CC Problem

- If Lorentz is violated, matter sees an induced metric: 
  \[ \tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\epsilon \partial_{\mu} \tau \partial_{\nu} \tau, \]

- The vacuum stress tensor: 
  \[ \langle T_{\mu\nu} \rangle \sim \Lambda_{\phi}^4 \tilde{g}_{\mu\nu} = \Lambda_{\phi}^4 (g_{\mu\nu} + 2\epsilon \partial_{\mu} \tau \partial_{\nu} \tau), \]

- Cosmological bound implies
  \[ \rho + p = 2\Lambda_{\phi}^4 \epsilon < 3\Omega_{\Lambda} |1 + w| M_p^2 H^2 < (1.7 \text{ meV})^4, \]
  \[ \Rightarrow \epsilon = |1 - c_{\phi}| \lesssim \left( \frac{\Lambda_{\phi}}{1.4 \text{ meV}} \right)^{-4} < \left( \frac{E}{1.4 \text{ meV}} \right)^{-4}. \]
CONCLUSIONS

diamond Reviving *Aether* might address many fundamental puzzles that have haunted physics and cosmology for the past century

Perimeter Institute, Oct. 24, 2012
CONCLUSIONS

- Reviving *Aether* might address many fundamental puzzles that have haunted physics and cosmology for the past century

- *Conjecture*: Lorentz violation is “confined” to < few meV, up to the scale that gravity is UV-completed (decoupling via strong coupling, e.g., QCD, Vainshtein mechanism)
Quantum superpositions of the speed of light


Sabine Hossenfelder

NORDITA STOCKHOLM
Motivation

- Why haven’t we heard from aliens?
- Possibly because we haven’t figured out what communication method they use.
- A superluminal one would clearly be the mode of choice.
Lorentz invariance violation and deformation

Presently two ways for superluminal information exchange:

1. **Breaking of Lorentz-invariance**
   A preferred frame relative to which speeds can be arbitrarily large. No problem with causality or locality because everything is relative to one distinguished frame. Problem: quantum fields that couple to the preferred frame introduce higher order operators to the standard model which are extremely tightly constrained. Also unclear why Lorentz invariance holds to such high precision.

2. **Deformation of Lorentz-invariance**
   The speed of light is energy-dependent yet observer-independent and can, for high energies, in principle be arbitrarily large. Problem: This model suffers from several severe problems, notoriously locality is severely messed up.
A new look at an old problem

- QG: thinks of the metric as operator valued.
- Consider the simplest case: Flat space. Only difference between eigenstates is value of speed of light.
- Putting the speed of light into the metric is not necessary, but makes notation and interpretation easier.
- One could call this quantum non-gravity, as there is no gravity involved here.
- Note that this is in contrast to the limit $\hbar, G \to 0$ ($m_{\text{Pl}}$ finite) that has been the motivation for deformations of Lorentz-invariance.
Quantum superpositions of the speed of light

- The metric is an operator $\hat{g}$ acting on the wavefunction describing the background and fields. It has eigenvalues $\eta(c)$ that are $\eta$ to different values of $c$.

$$\hat{g}\eta(c) = \eta(c)\eta(c), \quad \eta(c) = \int dc \, \alpha(c)\eta(c).$$

- We will not assume that the background metric it is in one particular eigenstate to one specific speed of light, but instead allow superpositions of different eigenstates.

- Each subspace has its own Lorentz-group, depending on $c$, under which everything is Lorentz-invariant.

$$\hat{g}'\eta'(c) = U(\Lambda)\hat{g}\eta(c) = \Lambda^T\eta(c)\Lambda(c)\eta'(c) = \eta(c)\eta'(c).$$

- These representations of the Lorentz-group are called ‘equivalent’ to each other. They are usually not considered because the parameter $c$ is assumed to be fixed by experiment.
Quantum superpositions of the speed of light

- The metric is an operator $\hat{g}$ acting on the wavefunction describing the background and fields. It has eigenvalues $\eta(c)$ that are $\eta$ to different values of $c$.

$$\hat{g}|\eta(c)\rangle = \eta(c)|\eta(c)\rangle, \quad |g\rangle = \int dc \, \alpha(c)|\eta(c)\rangle,$$

- We will not assume that the background metric it is in one particular eigenstate to one specific speed of light, but instead allow superpositions of different eigenstates.

- Each subspace has its own Lorentz-group, depending on $c$, under which everything is Lorentz-invariant.

$$\hat{g}'|\eta'(c)\rangle = U(\Lambda)|g|\eta(c)\rangle = \Lambda(c)\Lambda(c')|\eta'(c)\rangle = \eta(c)|\eta'(c)\rangle.$$

- These representations of the Lorentz-group are called ‘equivalent’ to each other. They are usually not considered because the parameter $c$ is assumed to be fixed by experiment.
Quantum superpositions of the speed of light

- The metric is an operator \( \hat{g} \) acting on the wavefunction describing the background and fields. It has eigenvalues \( \eta(c) \) that are \( \eta \) to different values of \( c \).

\[
\hat{g} |\eta(c)\rangle = \eta(c)|\eta(c)\rangle, \quad |g\rangle = \int dc \alpha(c)|\eta(c)\rangle,
\]

- We will not assume that the background metric is in one particular eigenstate to one specific speed of light, but instead allow superpositions of different eigenstates.

- Each subspace has its own Lorentz-group, depending on \( c \), under which everything is Lorentz-invariant.

\[
\hat{g'}|\eta'(c)\rangle = U(\Lambda_\ell)\hat{g}|\eta(c)\rangle = \Lambda^T(c)\eta(c)\Lambda(c)|\eta'(c)\rangle = \eta(c)|\eta'(c)\rangle.
\]

- These representations of the Lorentz-group are called ‘equivalent’ to each other. They are usually not considered because the parameter \( c \) is assumed to be fixed by experiment.
Equations of motion

In the equations of motion the metric is replaced with the operator.

\[ \hat{\Box} = \partial_\mu \partial_\nu \hat{g}^{\mu\nu} . \]

It is then

\[ \hat{\Box} |\Phi\rangle = \sum_c \int d^3 p \, \alpha(c) \partial_\mu \partial_\nu \hat{g}^{\mu\nu} |\eta(c)\rangle |\tilde{p}, c\rangle \]
\[ = \sum_c \int d^3 p \, \alpha(c) \partial_\mu \partial_\nu \eta^{\mu\nu}_c |\eta(c)\rangle |\tilde{p}, c\rangle . \]

This expansion will fulfill the Klein-Gordon equation when

\[ |\tilde{p}, c\rangle =: \nu_{\tilde{p}, c}(x) = \alpha e^{-i(E t - \tilde{p} \cdot \tilde{x})} \text{ with } \delta(E - pc) , \]

where \( p = |\tilde{p}|. \)

Every momentum now corresponds to a superposition of different energies, depending on the value of \( c \).
The measurement

- So far, everything is entirely Lorentz-invariant with the subspaces each having their own property.
- We will assume, based on our experience, that detectors (macroscopic objects inducing decoherence) are elements of the same and the usual eigenspace.
- We will assume that the measurement breaks the extended Lorentz-symmetry. With the measurement, we find the particle in one particular eigenstate in the restframe of the detector (the restframe inducing decoherence.
- After measurement, all observers agree on the outcome.
- There are no problems with causality because there is a preferred frame: The frame of the measurement. This frame however is not a fundamental one that appears fundamentally in the equations of motion.
Closed Curves

Fig. 1. Closed curves. Top left: Without arrow of time. Top right: With inconsistent arrows of time, generating the possibility for grandfather paradox. Bottom: With consistent arrow of time. Closed curves could be constructed with only 3 straight lines. We have included a fourth to show that taking into account the finite amount of time necessary to process information does not remove the problem.
Outlook: Interactions and Coupling

Interaction vertices can mix eigenstates of different $c$. If our background spacetime is not in an exact eigenstate, this would show up in particle interactions. This can, at the very least, be used to put bounds on the model (the parameters quantifying the mixing of eigenspaces).

Interaction term takes the form

$$L_{\text{int}} = eM\bar{\psi} \gamma^\nu A_\nu \psi,$$

with the transition matrix

$$\langle \eta_{(c)} | M | \eta_{(c')} \rangle = M_{cc'},$$

and $M = M^\dagger$. $M$ is not necessarily diagonal because the $c$-subspace of the fermions does not need to be the same as that of the gauge field.
Summary

- Superluminal information exchange is possible
  - Without breaking Lorentz invariance
  - Without being in conflict with locality
  - Without being in conflict with causality
- This modification of special relativity may contain interesting phenomenology.
RELATIVE LOCALITY AND LORENTZ SYMMETRY

Experimental Search for Quantum Gravity: the hard facts
PI, October 24, 2012
Relative locality

- Relative locality is a setup to discuss deformations of a relativistic particle dynamics.
- From the space-time perspective it relaxes the absolute status of locality; from the momentum space perspective it allows for arbitrariness of its geometry (and the two are interconnected).
- The 2+1 gravity coupled to particles is a theory of this kind; it is hoped that also in 3+1 spacetime dimension there exists a RL sector related to no gravity limit of QG. If so, RL may provide an interesting QG phenomenology model.
Relative locality – generalities

- It is assumed that the momentum space is equipped with the following structures:

  a. The origin corresponding to zero (four) momentum;
  b. The metric $g^{\mu\nu}$ (tetrad $e_a^{\mu}$) that govern the free particle action;
  c. The connection $\Gamma_{\mu}^{\nu\rho}$ that governs interactions between particles (conservation rules).

- Physics of particles kinematics and contact interactions is expressed in terms of these geometric quantities.
Relative locality – generalities

- It is assumed that the momentum space is equipped with the following structures:
  
  a. The origin corresponding to zero (four) momentum;
  b. The metric $g^{\mu\nu}$ (tetrad $e_a{}^{\mu}$) that govern the free particle action;
  c. The connection $\Gamma_{\mu}{}^{\nu\rho}$ that governs interactions between particles (conservation rules).

- Physics of particles kinematics and contact interactions is expressed in terms of these geometric quantities.
Relative locality – free action

- The free action consists of a kinematic term and dispersion relation, both governed by metric/tetrad.
- The mass^2 is defined to be a geodesic distance between the momentum space origin and the point $P$, with coordinates $p_{\mu}$, $m^2 = D(0,P)$.
- Then the free particle action takes the form

$$S = -\int_0^1 d\tau \ x^a \ e^\mu_a (p) \dot{p}_\mu + \lambda \left( D(0,P) - m^2 \right)$$

$x^\mu$
Relative locality – free action

- The free action consists of a kinematic term and dispersion relation, both governed by metric/tetrad.
- The mass$^2$ is defined to be a geodesic distance between the momentum space origin and the point $P$, with coordinates $p_\mu$, $m^2 = D(0,P)$.
- Then the free particle action takes the form

$$S = -\int_0^1 \! d\tau \, x^a e^\mu_a(p) \dot{p}_\mu + \lambda \left( D(0,P) - m^2 \right)$$

$$x^\mu$$
Relative locality – free action

- The free action consists of a kinematic term and dispersion relation, both governed by metric/tetrad.
- The mass$^2$ is defined to be a geodesic distance between the momentum space origin and the point $P$, with coordinates $p_\mu$, $m^2 = D(0, P)$.
- Then the free particle action takes the form

$$S = -\int_0^1 d\tau \, x^a \, e^\mu_a(p) \, \dot{p}_\mu + \lambda \left( D(0, P) - m^2 \right)$$

$$x^\mu$$
Relative Locality – interactions

- There is a contact interaction of particles in the vertices, where (modified) momentum conservation law is imposed.
- The interaction is described by an additional term in the action

\[ S^{int} = z^\mu K_\mu (p^{(1)}, p^{(2)}, \ldots) \]

The form of \( K_\mu = (p_1 \oplus p_2 + \ldots) \mu \) is in one to one correspondence with the connection on the momentum space manifold.
Relative Locality – interactions

- There is a contact interaction of particles in the vertices, where (modified) momentum conservation law is imposed.
- The interaction is described by an additional term in the action

\[ S^{\text{int}} = z^\mu K_\mu (p^{(1)}_\mu, p^{(2)}_\mu, \ldots) \]

The form of \( K_\mu = (p_1 \oplus p_2 + \ldots)_\mu \) is in one to one correspondence with the connection on the momentum space manifold.

\[ K \equiv (p \oplus r) \otimes q = 0 \]
Lorentz symmetry – generalities

- Depending on the geometry of momentum space, the Lorentz symmetry may:
  
  a. Be an exact symmetry group of the resulting theory;
  b. Not be a symmetry of the resulting theory (LIV) case;
  c. Become (Hopf) deformed;
  d. ???
Exact Lorentz symmetry

- It is always possible to make the free action Lorentz-invariant:

1. Assume that rotational invariance is classical, so that \( D(0, P) \) is a scalar under rotation, i.e.,

\[
D(0, P) = D(0, P)(p_0, \vec{p}^2)
\]

2. Then there always exist three boost generators \( N_i \) with the following properties

\[
[N_i, N_j] = -\epsilon_{ijk} M_k
\]

\[
N_i \triangleright D(0, P) = 0
\]
Exact Lorentz symmetry

- One can check that the free action is invariant under

\[ \delta_\xi p_\mu = \xi^i N_i \triangleright p_\mu = \xi^i F_{(i)\mu}(p) \]

\[ \delta_\xi x^\mu = -\xi^i x^\nu F_{(i)\nu}^{\phantom{(i)\nu}} \]

- Actually, one could find momentum space coordinates \(\pi_\mu\), in which the free action is classical

\[ S = -\int_0^1 d\tau \ x^\mu \dot{\pi}_\mu + \lambda \left( \pi^2 - m^2 \right) \]

- Then the Lorentz symmetry fate rests entirely on the form of the interaction term.
Lorentz symmetry – generalities

- Depending on the geometry of momentum space, the Lorentz symmetry may:
  a. Be an exact symmetry group of the resulting theory;
  b. Not be a symmetry of the resulting theory (LIV) case;
  c. Become (Hopf) deformed;
  d. ???
Exact Lorentz symmetry

- One can check that the free action is invariant under

\[ \delta \xi p_{\mu} = \xi N_i \nabla p_{\mu} = \xi i F_{(i)\mu}(p) \]

\[ \delta \xi x^\mu = -\xi x^\nu F_{(i)\nu}^\mu \]

- Actually, one could find momentum space coordinates \( \pi_{\mu} \), in which the free action is classical

\[ S = -\int_0^1 d\tau x^\mu \pi_{\mu} + \lambda \left( \pi^2 - m^2 \right) \]

- Then the Lorentz symmetry fate rests entirely on the form of the interaction term.
Exact Lorentz symmetry: example

- For example consider a standard free action and the conservation law (in a tri-valent vertex) of the form

\[ 0 = \sum_{n=1}^{3} p^{(n)}_{\mu} + \frac{1}{\kappa} \epsilon_{mnp} p^{(m)}_{\mu} p^{(n)}_{\nu} p^{(p)}_{\rho} \]

- In this case the connection is a Lorentz tensor.
Exact Lorentz symmetry

- One can check that the free action is invariant under

\[ \delta_{\xi} p_\mu = \xi^i N_i \triangleright p_\mu = \xi^i F_{(i)|\mu}(p) \]

\[ \delta_{\xi} x^\mu = -\xi^i x^\nu F^i_{(i)\nu}^\mu \]

- Actually, one could find momentum space coordinates \( \pi_\mu \), in which the free action is classical

\[ S = -\int_0^1 d\tau \; x^\mu \dot{\pi}_\mu + \lambda \left( \pi^2 - m^2 \right) \]

- Then the Lorentz symmetry fate rests entirely on the form of the interaction term.
Co-product type of symmetry

- Before the Leibnizian structure of the Lorentz symmetry action has been implicitly assumed, i.e.,

\[
\xi^i N_i \triangleright (p^{(1)} \oplus p^{(2)})_\mu = \\
(\xi^i N_i \triangleright p^{(1)})_\mu + (p^{(1)} \oplus \xi^i N_i \triangleright p^{(2)})_\mu
\]

- But this may be generalized if the operation \( \oplus \) is sufficiently nontrivial (e.g., if momenta are functions on the group, as in 2+1 gravity case.)
Co-product type of symmetry

- Using the 2+1 dim. experience one generalizes the Leibnitzian symmetry setup to

\[ \xi^i N_i \triangleright (p^{(1)} \oplus p^{(2)})_\mu = \]

\[ (\xi^i N_i \triangleright p^{(1)} \oplus p^{(2)})_\mu + (p^{(1)} \oplus \tilde{\xi}^i (p^{(1)}) N_i \triangleright p^{(2)})_\mu \]

- This is directly related to the co-product structure of the corresponding Hopf algebra.

- Some consistency conditions guaranteeing that

\[ \xi^i N_i \triangleright (p^{(1)} \oplus p^{(2)} \oplus \ldots)_\mu = 0 \quad \text{if} \quad (p^{(1)} \oplus p^{(2)} \oplus \ldots)_\mu = 0 \]

- must be satisfied (in particular associativity).

Conclusion

- In RL Lorentz symmetry might be an exact symmetry in spite of the fact that the kinematics is deformed;
- it may be not present at all and become a low energy artifact;
- It may become deformed in the Hopf-algebra manner (for associative momentum composition law)
- If the composition law is not associative it is not clear if any symmetry can be present.
If spacetime is a causal set
• then
Lorentz symmetry is unbroken

Rafael D. Sorkin

Perimeter Institute
How (kinematically) does a causal set $C$ give rise to a spacetime $M$?

cf. Dionigi Benincasa’s talk
What does this slogan mean?

Macroscopic causal order reflects microscopic order of causet, and macroscopic 4-volume reflects number of elements: \( N = V \)

As Dionigi explained, we say that \( M \approx C \) when we can find points in \( M \) representing the elements of \( C \) and distributed with unit density

Causetters have always identified such a *faithful embedding* with a Poisson

(Why? Return to this …)
What does this slogan mean?

Macroscopic causal order reflects microscopic order of causet, and macroscopic 4-volume reflects number of elements: \( N = V \)

As Dionigi explained, we say that \( M \approx C \) when we can find points in \( M \) representing the elements of \( C \) and distributed with unit density.

Causetters have always identified such a faithful embedding with a Poisson

(Why? Return to this ...)
Macroscopic causal order reflects microscopic order of causet, and
macroscopic 4-volume reflects number of elements: \( N = V \)

As Dionigi explained, we say that \( M \approx C \) when we can find points in \( M \)
representing the elements of \( C \) and distributed with unit density

Causetters have always identified such a faithful embedding with a Poisson

(Why? Return to this . . .)

If we make this identification then Lorentz symmetry follows
A theorem on Poisson processes

Let $\Omega = \text{space of all sprinklings of } \mathbb{M}^d$ \hfill ($\Omega = \text{“sample space”}$)

Poisson process induces a measure $\mu$ on $\Omega$

Let $f$ be a rule for deducting a direction from a sprinkling $f : \Omega \to H = \text{unit vector}$

Require $f$ to be equivariant ($f \circ \Lambda = \Lambda \circ f$, $\Lambda \in \text{Lorentz}$)

Assume that $f$ is measurable (hardly an assumption)

**Theorem** \hfill *No such $f$ exists*

(not even on a partial domain of positive measure)
Let $\Omega = \text{space of all sprinklings of } \mathbb{M}^d$ (\(\Omega = \text{“sample space”}\))

Poisson process induces a measure $\mu$ on $\Omega$

Let $f$ be a rule for deducing a direction from a sprinkling $f : \Omega \to H = \text{unit vectors}$

Require $f$ to be equivariant ($f \circ \Lambda = \Lambda \circ f$, $\Lambda \in \text{Lorentz}$)

Assume that $f$ is measurable (hardly an assumption)

**Theorem** \(\text{No such } f \text{ exists}\)

(not even on a partial domain of positive measure)

So with probability 1, a sprinkling cannot determine a frame
Poisson process induces a measure $\mu$ on $\Omega$

Let $f$ be a rule for deducing a direction from a sprinkling $f : \Omega \to H =$ unit vectors

Require $f$ to be equivariant ($f \circ \Lambda = \Lambda \circ f$, $\Lambda \in \text{Lorentz}$)

Assume that $f$ is measurable (hardly an assumption)

**Theorem**  
*No such $f$ exists*

(not even on a partial domain of positive measure)

*So with probability 1, a sprinkling cannot determine a frame*
Require $f$ to be *equivariant* ($f \circ \Lambda = \Lambda \circ f$, $\Lambda \in \text{Lorentz}$)

Assume that $f$ is measurable (hardly an assumption)

**Theorem**  *No such $f$ exists*

(not even on a partial domain of positive measure)

So with probability 1, a sprinkling cannot determine a frame

proof rests on $\infty$ Haar measure of Lorentz group
proof rests on $\infty$ Haar measure of Lorentz group
Red squares are in same coordinate location (Boost is an active transformation.)
Red squares are in same coordinate location.
(Boost is an active transformation.)
But why Poisson sprinkling?

How can one judge whether points are evenly distributed in $M$?

They are if any order-interval in $M$ of volume $V$ contains $N \approx V$ embedded points.

(This seems an unbiased criterion, interval is nearest analog of Euclidean sphere.)

A regular lattice fails this test!  

Poisson sprinkling passes the test and it seems to be more-or-less unique in doing so.
But why Poisson sprinkling?

How can one judge whether points are evenly distributed in $M$?

They are if any order-interval in $M$ of volume $V$ contains $N \approx V$ embedded points.

(This seems an unbiased criterion, interval is nearest analog of Euclidean sphere)

A regular lattice fails this test!

Poisson sprinkling passes the test and it seems to be more-or-less unique in doing so.
This seems an unbiased criterion, interval is nearest analog of Euclidean sphere.

A regular lattice fails this test!  

Poisson sprinkling passes the test and it seems to be more-or-less unique in doing this.
CONJECTURE 1. No point process realizes $N = V$ better than Poisson in the sense that it would yield for all $V$ and all order-intervals of volume $V$

$$\langle (N - V)^2 \rangle \leq CV$$

for $C < 1$

CONJECTURE 2. If a point process obeys (16) with $C = 1$, then it

"almost coincides with Poisson in almost all order-intervals"
CONJECTURE 1. No point process realizes $N = V$ better than Poisson in the sense that it would yield for all $V$ and all order-intervals of volume $V$

$$\langle (N - V)^2 \rangle \leq CV$$

for $C < 1$

CONJECTURE 2. If a point process obeys (1) with $C = 1$, then it

"almost coincides with Poisson in almost all order-intervals"
sense that it would yield for all $V$ and all order-intervals of volume $V$

$$\langle (N - V)^2 \rangle \leq CV$$

for $C < 1$

**CONJECTURE 2.** If a point process obeys (†) with $C = 1$, then it

"almost coincides with Poisson in almost all order-intervals"

A related conjecture of interest, possibly easier to prove, takes symmetry as an input rather than an output:

**CONJECTURE 3.** Every Poincaré invariant point-process in $\mathbb{M}^d$ is
"almost coincides with Poisson in almost all order-intervals"

A related conjecture of interest, possibly easier to prove, takes symmetry as an input rather than an output:

**CONJECTURE 3.** Every Poincaré invariant point-process in $\mathbb{M}^d$ is a convex combination of Poisson processes
Consequences:

\[ N = V \implies \text{Poisson} \implies \text{Lorentz} \]

\[ \implies \text{We must seek QG phenomenology that respects (local) Lorentz sym} \]

“Bad”: “no hope” for some popular models

good: phenomenological models highly constrained

Some examples …
• swerves (Lorentz-invariant diffusion in momentum space) (cosmic rays??)
• diffusion and drift of photon energy
• diffusion and drift of photon polarization
• "extinction" and scattering of light etc.
• Δ fluctuations (phenomological models need completion!)
• high frequency transparency (?)

A very general consequence is **nonlocality** (cf. Dionigi's talk):

\[
\text{discreteness } + \text{ Lorentz invariance } + \text{ locality } \implies \text{ contradiction}
\]

Should we hope Poisson is *not quite* unique?
• “extinction” and scattering of light etc.

• ω fluctuations (phenomological models need completion!)

• high frequency transparency (?)

A very general consequence is nonlocality (cf. Dionigi’s talk):

\[ \text{discreteness} + \text{Lorentz invariance} + \text{locality} \implies \text{contradiction} \]

Should we hope Poisson is not quite unique?
• "extinction" and scattering of light etc.

• A fluctuations (phenomological models need completion!)

• high frequency transparency (?)

A very general consequence is **nonlocality** (cf. Dionigi's talk):

\[
\text{discreteness} + \text{Lorentz invariance} + \text{locality} \implies \text{contradiction}
\]

Should we hope Poisson is *not quite* unique?
- "extinction" and scattering of light etc.
- A fluctutations *(phenomological models need completion!)*
- high frequency transparency (?)

A very general consequence is **nonlocality** (cf. Dionigi's talk):

\[
\text{discreteness} + \text{Lorentz invariance} + \text{locality} \implies \text{contradiction}
\]

Should we hope Poisson is *not quite* unique?
Plan:

1) The point of view.
2) The Proposal.
3) The experiment.
4) Analysis of the results.
1) QG is often considered as tied to Lorentz Invariance Violation
Radiative processes could bring to low energies phenomena the
information about the rest frame favored by the high energy effects.
The usual lessons from of QFT indicate that all terms compatible with
the symmetries will be generated by radiative corrections. Their
suppression controlled by dimensional analysis. Generically not by
\(E/M_p\).

The lesson: A space-time granularity tied to a preferential rest frame
seems incompatible with QFT together with even relatively low
precision experiments unless there is a strong fine running (the
“Naturalness problem”).

Take the view that if there is some space-time granularity it is not
related to a preferential rest frame.
1) QG is often considered as tied to Lorentz Invariance Violation. Radiative processes could bring to low energies phenomena the information about the rest frame favored by the high energy effects. The usual lessons from of QFT indicate that all terms compatible with the symmetries will be generated by radiative corrections. Their suppression controlled by dimensional analysis. Generically not by $E/M_p$.

The lesson: A space-time granularity tied to a preferential rest frame seems incompatible with QFT together with even relatively low precision experiments unless there is a strong fine running (the “Naturalness problem”).

Take the view that if there is some space-time granularity it is not related to a preferential rest frame.
1) QG is often considered as tied to Lorentz Invariance Violation. Radiative processes could bring to low energies phenomena the information about the rest frame favored by the high energy effects. The usual lessons from of QFT indicate that all terms compatible with the symmetries will be generated by radiative corrections. Their suppression controlled by dimensional analysis. Generically not by $E/M_p$.

The lesson: A space-time granularity tied to a preferential rest frame seems incompatible with QFT together with even relatively low precision experiments unless there is a strong fine running (the “Naturalness problem”).

Take the view that if there is some space-time granularity it is not related to a preferential rest frame.
The Nature of Quantum space-time:

The basic point of view is that space-time is emergent.... E.E. are similar to Navier-Stokes (N-S) equations for a fluid (i.e. hydrodynamical analogy).

Gravity as described by GR, would be, according to this view, just some averaging of underlying substrate represents the gravitational DOF at the quantum level.

The fundamental description of the gravity DOF does not rely on a metric tensor or closely connected variables.

The hydrodynamical description is appropriate (for the fluid) at length scales $\gg$ the mean free path of the particles, and time scales $\gg$ their mean free time.

We will assume that the metric description is approximately valid for curvatures $\ll \frac{1}{L_{\text{Planck}}^2}$.
Alternative Approach Towards QG Phenomenology:
How could this granularity become manifest, if at all?
The fundamental structure is unknown (no workable Quantum Theory of Gravitation) - We must rely on symmetry arguments and analogies:
Solid State Physics: Consider a crystal with fundamental cubic symmetry.
Say, we do not know the fundamental symmetry, and will try detecting such structure using a macroscopic crystal with cubic form.
Suppose we are looking for some experimental signature of incompatibility with the macro cubic symmetry: We will not see it, and will conclude that, if there is any discrete structure, it must be cubic.

Next: Use a Non Cubic Crystal and look for signs of discrepancies between micro and macro symmetries (X Rays would show cubic symmetry even for a spherical crystal).
By analogy to the solid state situation, in our case, we need space-times that differ from Minkowski in the macro; The departure from that is characterized by Riemman.
The idea is to search for nontrivial couplings of Riemman with matter, which would become manifest “at the quantum level”.

In fact, up to now, there is no experimental test of the interaction of quantum matter with gravitation (Curvature) !!

We look for exotic couplings of matter and curvature. We try to make an “educated guess” while looking for observability: Ricci looks like self coupling ( \( R_{\mu\nu}(x) \sim T^\text{matt}_{\mu\nu}(x) \)). We need to focus on Weyl. Bulk matter in the lab is made of fermions (also photons).

Things of the form \( \mathcal{L} = W_{abcd} \bar{\psi}^\gamma_{a} \gamma^b \gamma^c \gamma^d \psi \) do not work.

We’ll look for something of the form: \( \mathcal{L} = c \Xi^{ab} \bar{\psi} \gamma^{[a} \gamma_{b]} \psi \).

Consider Weyl as a mapping from the space of two forms \( S \) to itself. This space comprises a 6 dimensional vector space endowed with a pseudo-riemannian metric:

\[
G_{abcd} = \frac{1}{2} (g_{ae}g_{db} - g_{ad}g_{eb}).
\]
The idea is to search for nontrivial couplings of Riemman with matter, which would become manifest “at the quantum level”.

In fact, up to now, there is no experimental test of the interaction of quantum matter with gravitation (Curvature) !!

We look for exotic couplings of matter and curvature. We try to make an “educated guess” while looking for observability: Ricci looks like self coupling ($R_{\mu \nu}(x) \sim T^{\text{man}}_{\mu \nu}(x)$). We need to focus on Weyl. Bulk matter in the lab is made of fermions (also photons).

Things of the form $\mathcal{L} = W_{abcd} \bar{\psi} \gamma^a \gamma^b \gamma^c \gamma^d \psi$ do not work.

We’ll look for something of the form: $\mathcal{L} = e \Xi_{ab}[\bar{\psi} \gamma^a \gamma^b] \psi$.

Consider Weyl as a mapping from the space of two forms $S$ to itself. This space comprises a 6 dimensional vector space endowed with a pseudo-riemannian metric:

$$G_{abcd} = \frac{1}{2} (g_{ac}g_{db} - g_{ad}g_{cb}).$$
The mapping provided by Weyl is not, in general, self adjoint, but we can separate it into its self-adjoint parts.

We construct two self-adjoint maps out of the Weyl tensor

\[
(W_+)_\,^a_b{}^c_d = \frac{1}{2} \left( W_{ab}^{\;cd} + W_{ab}^{\dagger \;cd} \right), \\
(W_-)_\,^a_b{}^c_d = \frac{1}{4} \epsilon_{ab}^{\;ef} \left( W_{ef}^{\;cd} - W_{ef}^{\dagger \;cd} \right),
\]

(1)

where \( W_{ab}^{\dagger \;cd} \) stands for the adjoint (with respect to \( G_{abcd} \)) of \( W_{ab}^{\;cd} \) and \( \epsilon_{abcd} \) is the natural space-time volume 4-form.

We can now look for the eigenvectors and eigenvalues of these operators. The eigenvectors are two forms.

They, together with the corresponding eigenvalues, contain information about Weyl in a canonical and invariant form.
Constructing the Interaction

The tensor field $\Xi_{ab} \ldots$ can be constructed out of the eigenvectors and eigenvalues of the self-adjoint maps, and such is our proposal. Let $\lambda^{(\pm,l)}$ and $X^{(\pm,l)}_{ab} \in \mathcal{S}$ be such that

\[
(W_{\pm})_{ab}^{\phantom{ab}cd} X^{(\pm,l)}_{cd} = \lambda^{(\pm,l)} X^{(\pm,l)}_{ab},
\]

\[
\epsilon^{abcd} X^{(\pm,l)}_{ab} X^{(\pm,l)}_{cd} = 0,
\]

\[
G^{abcd} X^{(\pm,l)}_{ab} X^{(\pm,l)}_{cd} = -1.
\]

This fixes the $X$'s uniquely, up to a sign, (unless there are further degeneracies!).

Define $\bar{X}^{(\pm,l)}_{ab} = \epsilon_{ab}^{\phantom{ab}cd} X^{(\pm,l)}_{cd}$ (which is degenerate with $X^{(\pm,l)}_{cd}$).

Note: The role of the space-time orientation, possibility of P, T violating effects!
Constructing the Interaction
The tensor field $\Xi_{ab}$ ... can be constructed out of the eigenvectors and eigenvalues of the self-adjoint maps, and such is our proposal.
Let $\lambda^{(\pm,l)}$ and $X^{(\pm,l)}_{ab} \in S$ be such that

\[
(W_{\pm})_{ab}^{\quad cd} X^{(\pm,l)}_{cd} = \lambda^{(\pm,l)} X^{(\pm,l)}_{ab},
\]
\[
\epsilon^{abcd} X^{(\pm,l)}_{ab} X^{(\pm,l)}_{cd} = 0,
\]
\[
G^{abcd} X^{(\pm,l)}_{ab} X^{(\pm,l)}_{cd} = -1.
\]

This fixes the $X$'s uniquely, up to a sign, (unless there are further degeneracies!).
Define $X^{(\pm,l)}_{ab} = \epsilon_{ab}^{\quad cd} X^{(\pm,l)}_{cd}$ (which is degenerate with $X^{(\pm,l)}_{cd}$).
Note: The role of the space-time orientation, possibility of P, T violating effects!
Use it to construct the following Lagrangian terms for Fermions of the form:

\[ \mathcal{L} = e \Xi_{ab} \bar{\psi} \gamma^{[a} \gamma^{b]} \psi, \]

Looks like a SME-term but with the constant tensor field \( \rightarrow \) an object locally determined by the underlying curvature structure.

Note: regarding only standard model fields is a dimension 3 operator! No large radiative corrections (as long as graviton loops are not involved).

The object that eliminates the sign ambiguity is:

\[ \Xi_{ab} = \sum_{\alpha, \beta = \pm} \sum_{l,m=1}^{3} M^{(\alpha, \beta, l, m)} G^{efgh} X_{ef}^{(\alpha, l)} X_{gh}^{(\beta, m)} g^{cd} X_{[a}^{(\alpha, l)} X_{b]}^{(\beta, m)} \]

plus similar terms obtained by replacing \( G^{efgh} \) by \( \epsilon^{efgh} \) and \( X_{ef}^{(\alpha, l)} \) by \( X_{ef}^{(\alpha, l)} \).
where the coefficients

\[ M^{(\alpha, \beta, l, m)} = \xi^{(\alpha, \beta, l, m)} |\lambda^{(\alpha, l)}|^{1/4} |\lambda^{(\beta, m)}|^{1/4} \]

and similar ones called \( N^{(\alpha, \beta, l, m)}, \tilde{M}^{(\alpha, \beta, l, m)}, \tilde{N}^{(\alpha, \beta, l, m)} \) for the other combinations. The free dimensionless parameters of the model are: \( \xi^{(\alpha, \beta, l, m)}, c^{(\alpha, l)}, \) and \( d^{(\alpha, l)} \), and the corresponding terms for the other combinations.

They are subject to the restriction:

\[ c^{(\alpha, l)}, d^{(\alpha, l)} > -1/2, \]

so that they vanish for flat space-time (and there are no divergences).
where the coefficients

\[ M^{(\alpha, \beta, l, m)} = \xi^{(\alpha, \beta, l, m)} |\lambda^{(\alpha, l)}|^{1/4} |\lambda^{(\beta, m)}|^{1/4} \]

and similar ones called \( N^{(\alpha, \beta, l, m)} \), \( \tilde{M}^{(\alpha, \beta, l, m)} \), \( \tilde{N}^{(\alpha, \beta, l, m)} \) for the other combinations. The free dimensionless parameters of the model are: \( \xi^{(\alpha, \beta, l, m)} \), \( c^{(\alpha, l)} \), and \( d^{(\alpha, l)} \), and the corresponding terms for the other combinations.

They are subject to the restriction:

\[ c^{(\alpha, l)}, d^{(\alpha, l)} > -1/2, \]

so that they vanish for flat space-time (and there are no divergences).
where the coefficients

\[ M^{(\alpha, \beta, l, m)} = \xi^{(\alpha, \beta, l, m)} |\lambda^{(\alpha, l)}|^{1/4} |\lambda^{(\beta, m)}|^{1/4} \]

and similar ones called \( N^{(\alpha, \beta, l, m)} \), \( \tilde{M}^{(\alpha, \beta, l, m)} \) and \( \tilde{N}^{(\alpha, \beta, l, m)} \) for the other combinations. The free dimensionless parameters of the model are: \( \xi^{(\alpha, \beta, l, m)} \), \( c^{(\alpha, l)} \), and \( d^{(\alpha, l)} \), and the corresponding terms for the other combinations.

They are subject to the restriction:

\[ c^{(\alpha, l)}, d^{(\alpha, l)} > -1/2, \]

so that they vanish for flat space-time (and there are no divergences).
This corresponds to a low energy effective Hamiltonian for an electron:

\[
\mathcal{H}^e = \sum_{l,m=1}^{3} \Delta \xi^{(l,m)} |\alpha^{(l)}|^{1/4} |\beta^{(m)}|^{1/4} \left( \frac{|\alpha^{(l)}|^{1/2}}{M_p} \right) c^{(+,l)} \left( \frac{|\beta^{(m)}|^{1/2}}{M_p} \right) c^{(-,m)}
\]

\[
\left[ \vec{a}^{(l)} \cdot \vec{b}^{(m)} \right] \left[ \vec{a}^{(l)} \times \vec{b}^{(m)} \right] \cdot \vec{\sigma},
\]

where \( \Delta \xi^{(l,m)} = \xi^{(+,-,l,m)} - \xi^{(-,+,-,m,l)} \) is a free parameter of the model.

Consider relevant experiments. Important feature is the dependence on polarization. This makes the experiments hard. Furthermore, the effects look like electric and magnetic fields. It becomes very hard!!

One also needs good gravitational tidal forces.

Possibilities:
- Cold Neutrons
- Neutrino Astrophysics
- High precision experiments of the Fifth force type.
This corresponds to a low energy effective Hamiltonian for an electron:

\[ \mathcal{H}^e = \sum_{l,m=1}^{3} \Delta \xi^{(l,m)} |\alpha^{(l)}|^{1/4} |\beta^{(m)}|^{1/4} \left( \frac{|\alpha^{(l)}|^{1/2}}{M_P} \right)^{c^{(+,l)}} \left( \frac{|\beta^{(m)}|^{1/2}}{M_P} \right)^{c^{(-,m)}} \]

\[ \left[ \vec{a}^{(l)} \cdot \vec{b}^{(m)} \right] \left[ \vec{a}^{(l)} \times \vec{b}^{(m)} \right] \cdot \vec{\sigma}, \]

where \( \Delta \xi^{(l,m)} = \xi^{(+,-,l,m)} - \xi^{(-,+;m,l)} \) is a free parameter of the model.

Consider relevant experiments. Important feature is the dependence on polarization. This makes the experiments hard. Furthermore, the effects look like electric and magnetic fields. It becomes very hard!!

One also needs good gravitational tidal forces.

Possibilities:
Cold Neutrons, Neutrino Astrophysics, High precision experiments of the Fifth force type.
A devoted Experiment has been done (EoT-Wash) [Class. Quantum Grav. 28 145011 (2011)] using torsion balances and polarized matter, with no magnetization!
Their setup measures the torque:

\[ T_z = N \sum_{l,m=1}^{3} \Delta \xi^{(l,m)} |\alpha^{(l)}|^{1/4} |\beta^{(m)}|^{1/4} \left( \frac{|\alpha^{(l)}|^{1/2}}{M_P} \right)^{c^{(+,l)}} \times \left( \frac{|\beta^{(m)}|^{1/2}}{M_P} \right)^{c^{(-,m)}} \left( f_{y}^{(l,m)} \cos \phi - f_{x}^{(l,m)} \sin \phi \right), \]
These lead to bounds:

\[
9.19 \left| \Delta \xi^{(2,2)} \right| \left( \frac{|\alpha^{(2)}|^{1/2}}{M_P} \right)^{c^{(+,2)}} \left( \frac{|\beta^{(2)}|^{1/2}}{M_P} \right)^{c^{(-,2)}} < 3.7 \\
7.94 \left| \Delta \xi^{(2,3)} \right| \left( \frac{|\alpha^{(2)}|^{1/2}}{M_P} \right)^{c^{(+,2)}} \left( \frac{|\beta^{(3)}|^{1/2}}{M_P} \right)^{c^{(-,3)}} < 3.7 \\
8.17 \left| \Delta \xi^{(3,2)} \right| \left( \frac{|\alpha^{(3)}|^{1/2}}{M_P} \right)^{c^{(+,3)}} \left( \frac{|\beta^{(2)}|^{1/2}}{M_P} \right)^{c^{(-,2)}} < 3.7 \\
7.06 \left| \Delta \xi^{(3,3)} \right| \left( \frac{|\alpha^{(3)}|^{1/2}}{M_P} \right)^{c^{(+,3)}} \left( \frac{|\beta^{(3)}|^{1/2}}{M_P} \right)^{c^{(-,3)}} < 3.7.
\]
With similar results for the over parameters.
Use it to construct the following Lagrangian terms for Fermions of the form:

\[ \mathcal{L} = e \Xi_{ab} \bar{\psi} \gamma^{[a} \gamma^{b]} \psi. \]

Looks like a SME-term but with the constant tensor field \( \Xi \) an object locally determined by the underlying curvature structure.

Note: regarding only standard model fields is a dimension 3 operator! No large radiative corrections (as long as graviton loops are not involved).

The object that eliminates the sign ambiguity is:

\[ \Xi_{ab} = \sum_{\alpha, \beta = \pm} \sum_{l,m=1}^{3} N^{(\alpha, \beta, l, m)} G^{efgh}_{\alpha \beta} X^{(\alpha, l)}_{ef} X^{(\beta, m)}_{gh} g^{cd} X^{(\alpha, l)}_{c[a} X^{(\beta, m)}_{b]d}. \]

Similar terms obtained by replacing \( G^{efgh} \) by \( \epsilon^{efgh} \) and \( X^{(\alpha, l)}_{ef} \) by...
Constructing the Interaction

The tensor field $\Xi_{ab} \ldots$ can be constructed out of the eigenvectors and eigenvalues of the self-adjoint maps, and such is our proposal. Let $\lambda^{(\pm,l)}$ and $X_{ab}^{(\pm,l)} \in \mathcal{S}$ be such that

\[
(W_{\pm})_{ab}^{cd} X_{cd}^{(\pm,l)} = \lambda^{(\pm,l)} X_{ab}^{(\pm,l)},
\]
\[
\epsilon^{abcd} X_{ab}^{(\pm,l)} X_{cd}^{(\pm,l)} = 0,
\]
\[
G^{abcd} X_{ab}^{(\pm,l)} X_{cd}^{(\pm,l)} = -1.
\]

This fixes the $X$'s uniquely, up to a sign, (unless there are further degeneracies!)

Define $\tilde{X}_{ab}^{(\pm,l)} = \epsilon_{ab}^{\quad cd} X_{cd}^{(\pm,l)}$ (which is degenerate with $X_{cd}^{(\pm,l)}$)

Note: The role of the space-time orientation, possibility of $P, T$ violating effects!
Use it to construct the following Lagrangian terms for Fermions of the form:

\[ \mathcal{L} = e \Xi_{ab} \bar{\psi} \gamma^{[a} \gamma^{b]} \psi, \]

Looks like a SME-term but with the constant tensor field \( \rightarrow \) an object locally determined by the underlying curvature structure.

Note: regarding only standard model fields is a dimension 3 operator!

No large radiative corrections (as long as graviton loops are not involved).

The object that eliminates the sign ambiguity is:

\[ \Xi_{ab} = \sum_{\alpha, \beta = \pm} \sum_{l, m = 1}^{3} M^{(\alpha, \beta, l, m)} G_{ef}^{fgh} X_{c[a}^{(\alpha, l)} X_{gh}^{(\beta, m)} g^{cd} X_{c[a}^{(\alpha, l)} X_{b]d}^{(\beta, m)} \]

Similar terms obtained by replacing \( G_{ef}^{fgh} \) by \( \epsilon_{ef}^{fgh} \) and \( X_{c[a}^{(\alpha, l)} \) by...