Title: Condensed Matter Seminar

Date: Oct 16, 2012 03:00 PM

URL: http://pirsa.org/12100078

Abstract:

Valence Bonds Crystal Ground states

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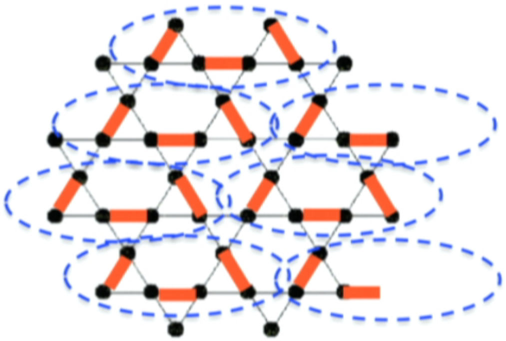
Columnar dimer state

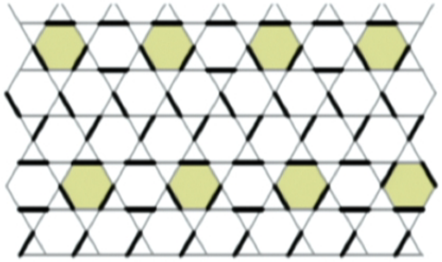
Budnik AA, 2004



soft to local rotations

Striped hexagons

Nikolic, Senthil

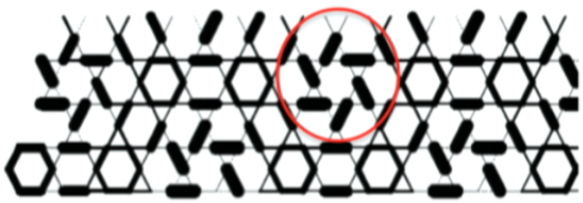


"Honeycomb VBC"

36 site cell

Marston,Zeng
Singh,Huse
Evenbly,Vidal

"pinwheel" p6



Spin liquid states

DMRG: Yan, Huse, White

Gapped Spin Liquid

n_{svd} = 8000

triplet gap > 0
No lattice symmetry breaking
exponentially decaying corr.

$$e_0^{GSL} = -0.4385$$

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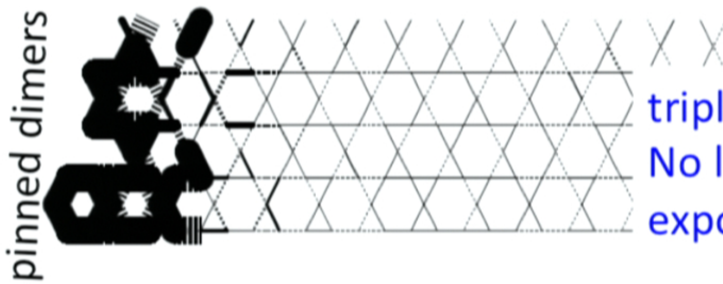

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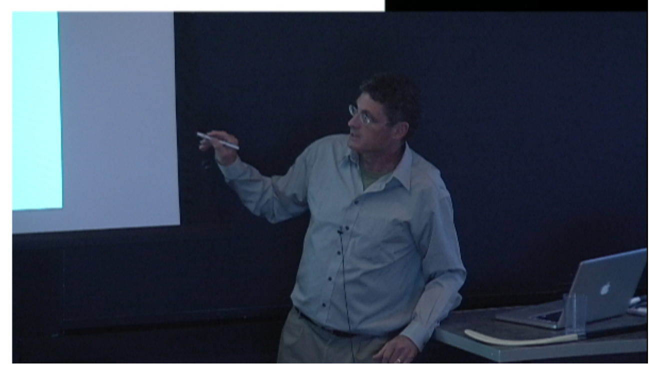
Z2 entangllement entropy (Jiang, Wang, Balents)



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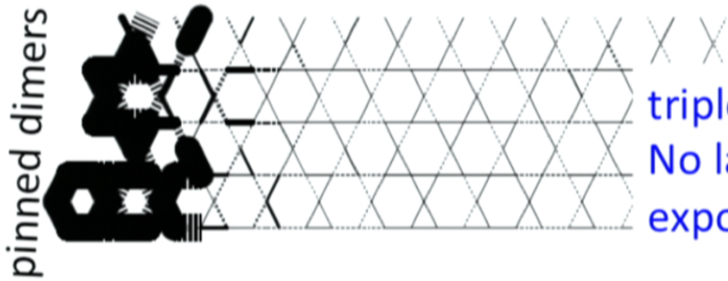

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VMC: Iqbal, Becca, Sorella, Poilblanc Algebraic Spin Liquid

$$|\Psi_{p-\mathrm{LS}}\rangle = \left(1 + \sum_{k=1}^{p} \alpha_k \hat{\mathcal{H}}^k\right)|\Psi_{\mathrm{VMC}}\rangle$$

Triplet gap = 0
power law spin correlations

$$e_0^{ASL} = -0.4365$$

Spin liquid states

DMRG: Yan, Huse, White   Gapped Spin Liquid

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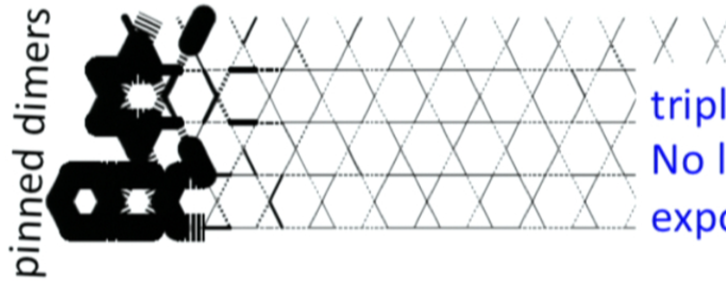

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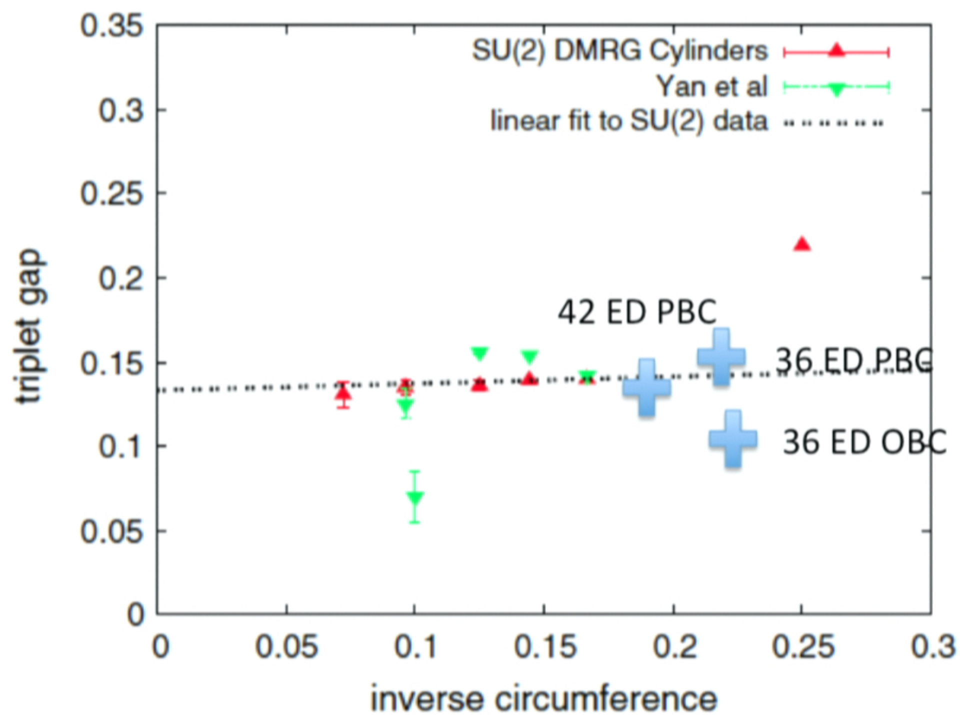
power law spin correlations

$$e_0^{ASL} = -0.4365$$

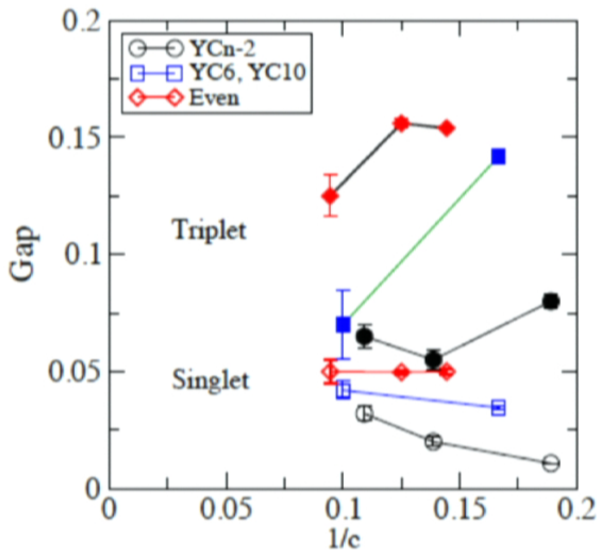
Only 0.46% difference in energy --
qualitative difference in long range correlations

Triplet gap

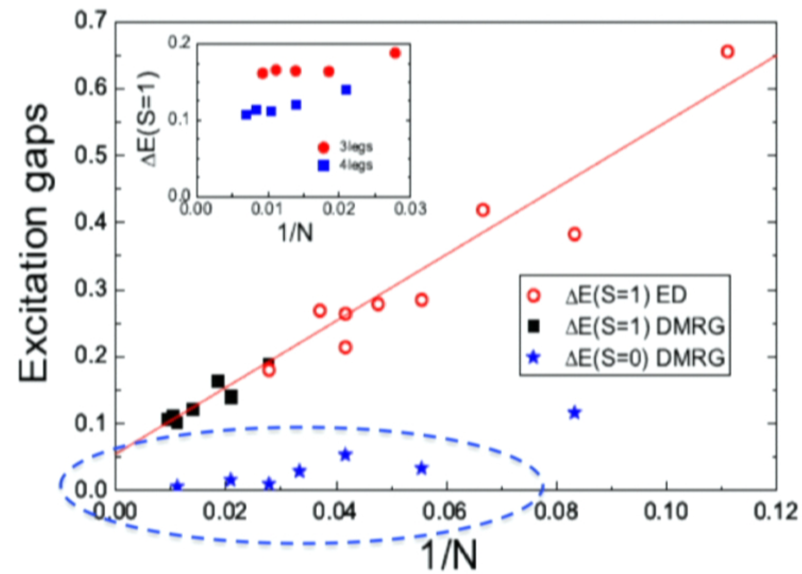


Stefan Depenbrock,[1,*] Ian P. McCulloch,[2] and Ulrich Schollwöck[1]

Singlet gap



Yan, Huse, White



Jiang, Weng, Sheng

Does the singlet gap stay finite in the thermodynamic limit ?

Contractor Renormalization (CORE)

Using ED to compute the effective Hamiltonian in the singlets sector for larger lattices

General Method: C.Morningstar, M.Weinstein, PRD (1996).

Square lattice Hubbard Model: E. Altman and A. A, PRB (2002).

Checkerboard+Pyrochlore: Berg, Altman, AA PRL (2003)

Kagome (first attempt): Budnik, A.A. PRL (2004);

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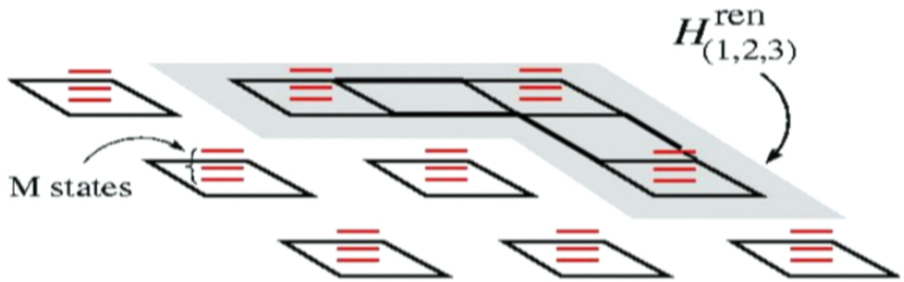
Exact Diagonalizations (ED) of connected clusters



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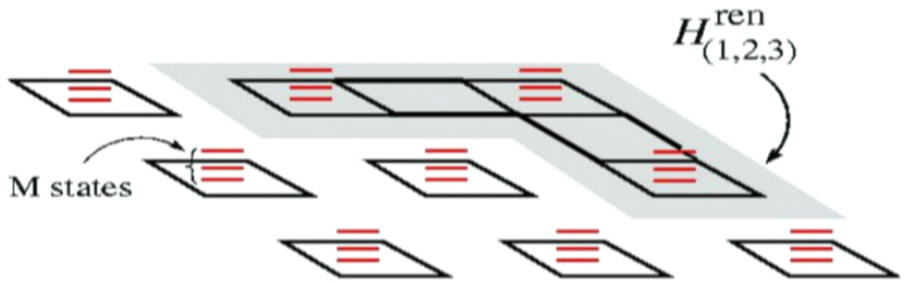

Find low spectrum of H on the connected cluster.

Project the wavefunctions onto the reduced Hilbert space

$$\varepsilon_n, |\psi_n\rangle \Rightarrow |\tilde{\Psi}_n\rangle \quad |\tilde{\Psi}_n\rangle = \frac{1}{Z_n}\left[P|\psi_n\rangle - \sum_{n'<n} |\tilde{\Psi}_{n'}\rangle\langle\tilde{\Psi}_{n'}|\psi_n\rangle\right]$$

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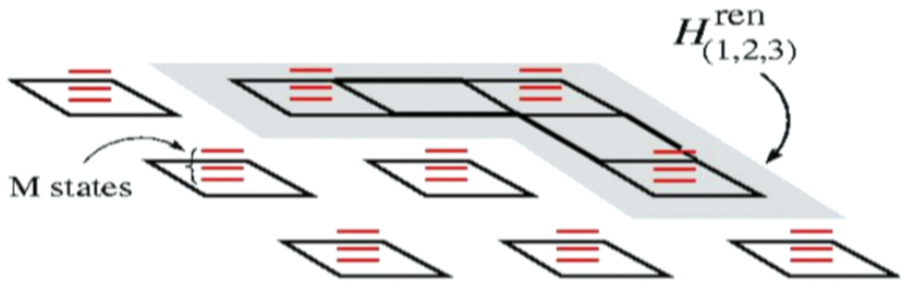

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Construct renormalized cluster hamiltonians:

$$H^{ren}_{(1,\ldots,N)} \equiv \sum_{n=1}^{M^N} \varepsilon_n |\tilde{\psi}_n\rangle\langle\tilde{\psi}_n|$$

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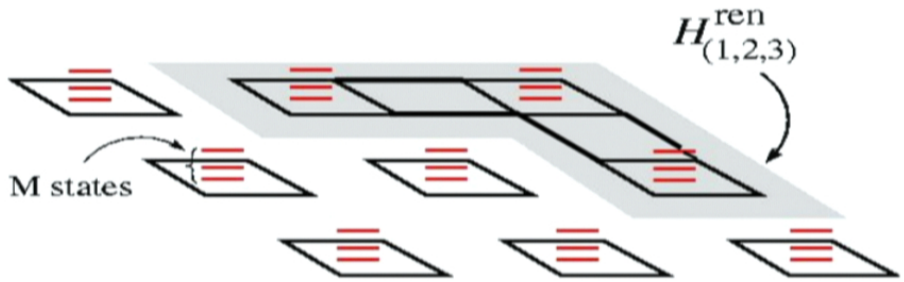

Find low spectrum of H on the connected cluster.

Project the wavefunctions onto the reduced Hilbert space

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Old perturbative RG

CORE effective hamiltonian

1. Effective Interactions $h^{(n)}_{1,2\ldots n} = \mathcal{H}^{\mathsf{ren}}_{1,2\ldots n} - \sum_{all\ sub\ clusters} h_{\{1\},\{2\},\ldots,\{n-1\}}$

For fast convergence: sum minimally embedding shapes (e.g. rectangles)

E. Altman's thesis.

2. Expansion in interaction range

$$H_{eff} = \sum_i h_i + \sum_{\langle ij \rangle} h_{ij} + \sum_{\langle ijk \rangle} h_{ijk} + \sum_{ijk..} h_{ijk...} \ldots$$

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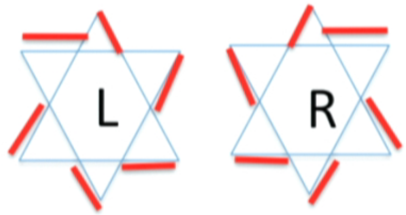
Heff may be solved by
ED, Variationally, DMRG, or CORE iteration

Star of (Magen) David Blocking

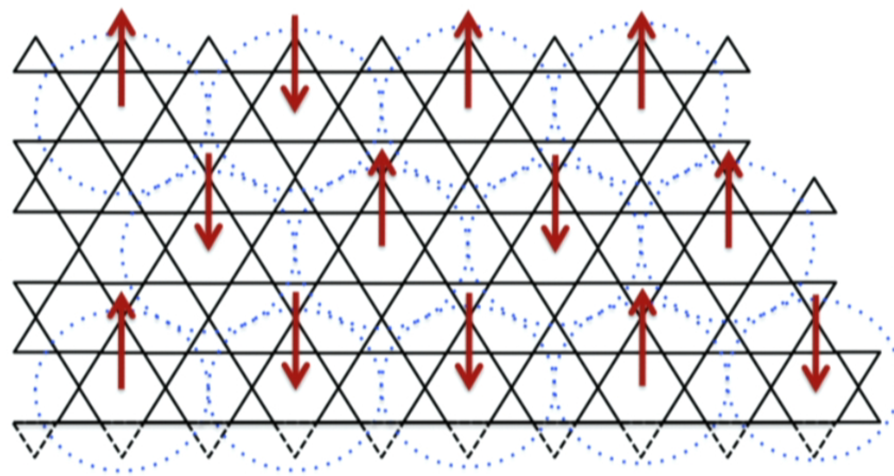


$$|\uparrow\rangle = \frac{1}{\sqrt{2+1/16}}(|L\rangle + |R\rangle)$$

$$|\downarrow\rangle = \frac{1}{\sqrt{2-1/16}}(|L\rangle - |R\rangle)$$

Ground state doublet

Pseudospin "Ising" basis



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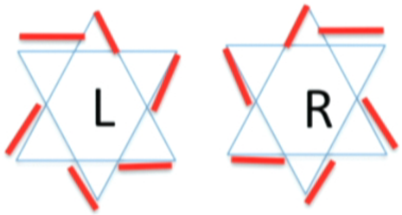

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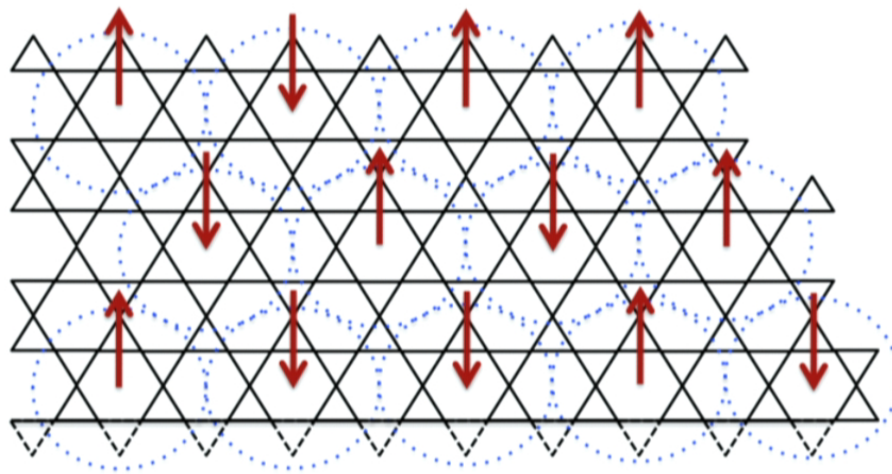
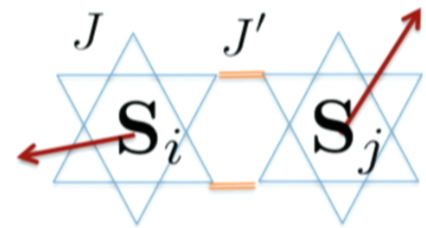


Reduced Hilbert space size: $2^{N/12} << 2^N$

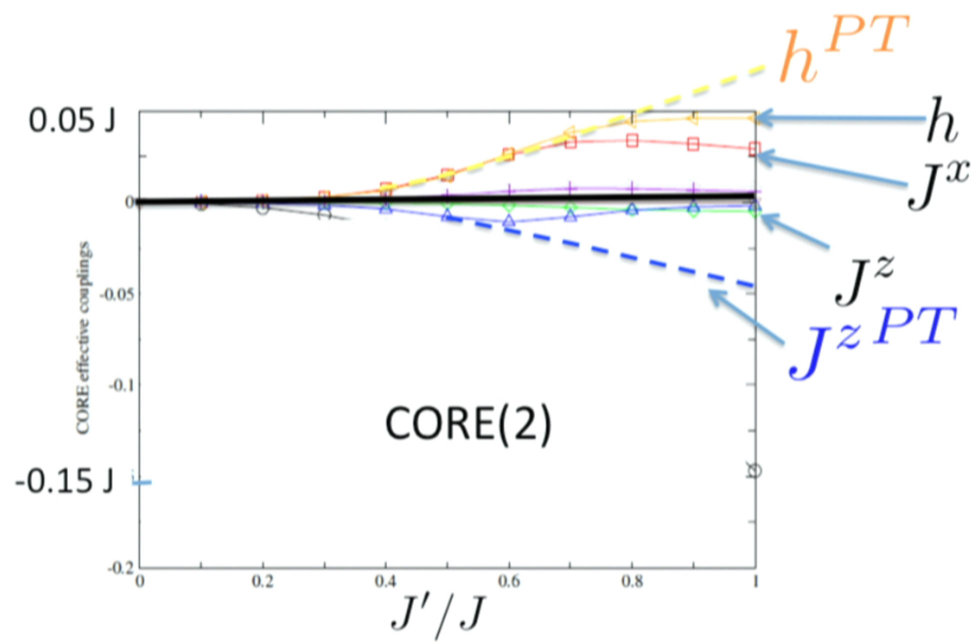
CORE(2) versus Perturbation Theory

Perturbation Theory (PT)

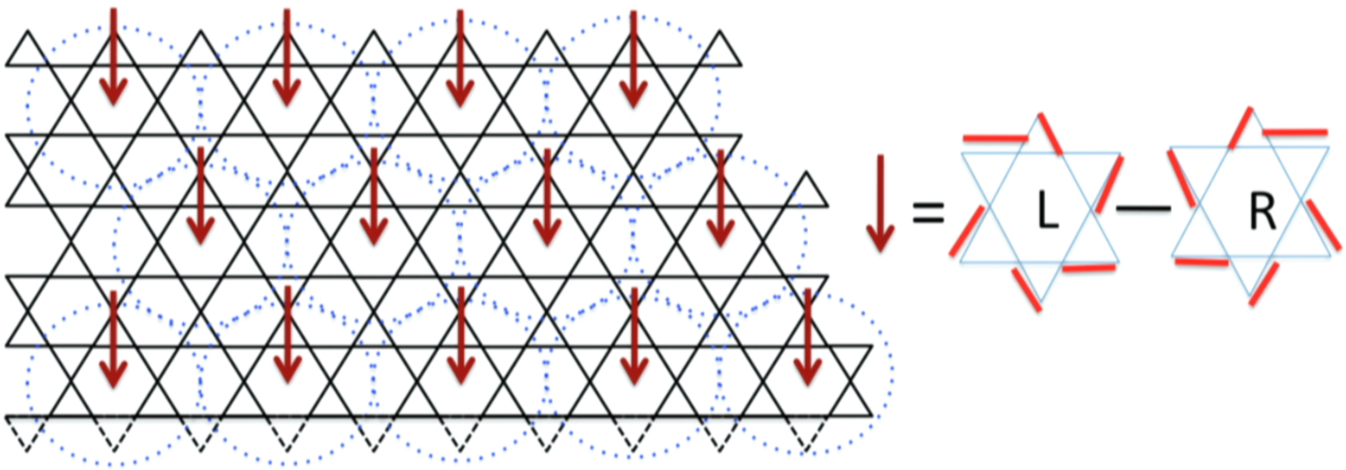
Syromyatnikov, Maleyev, PRB (2002)



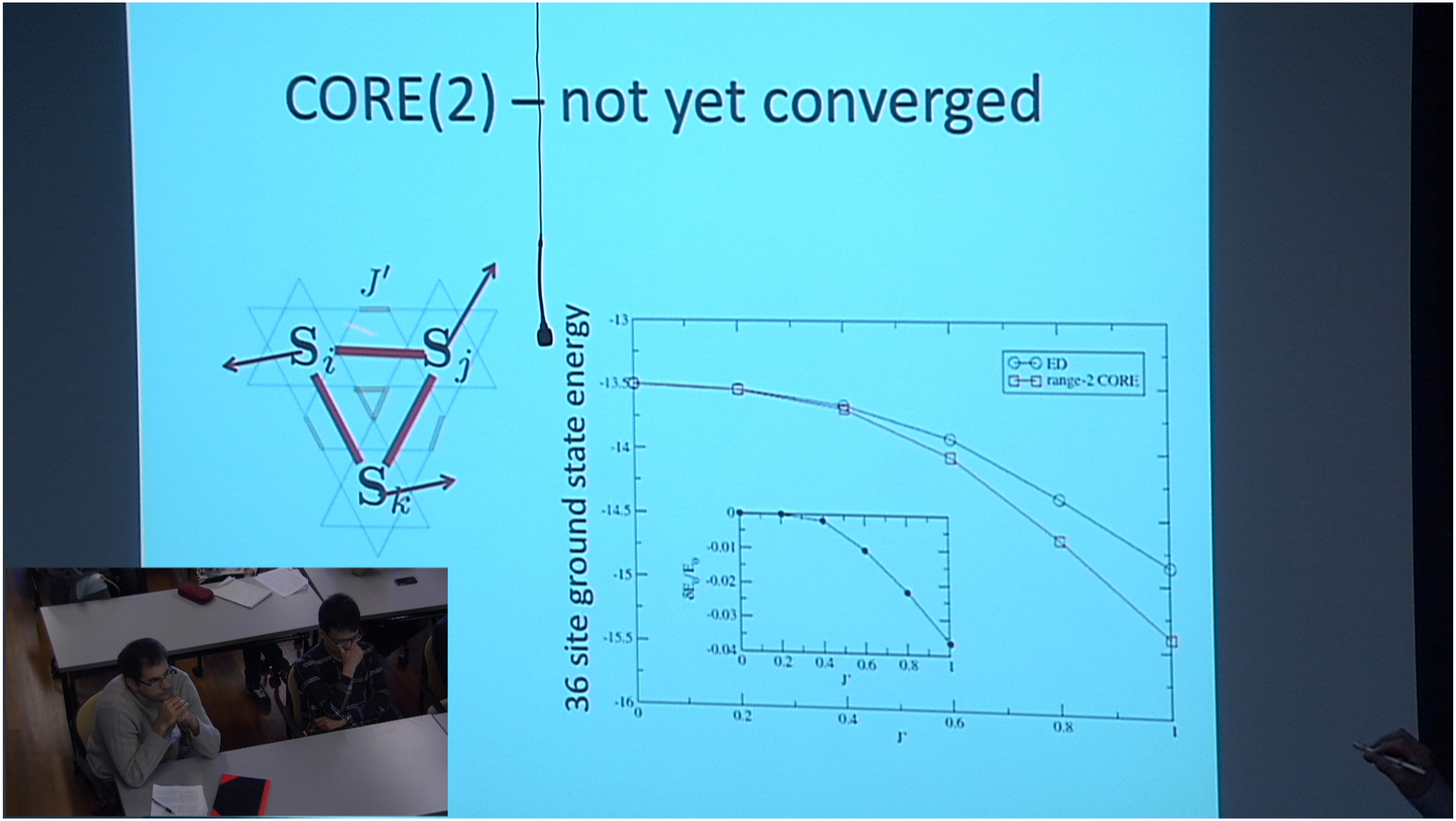
$$H^{CORE(2)} = E_0 + h(S_i^z + S_j^z) + J^x S_i^x S_j^x + J^y S_i^y S_j^y + J^z S_i^z S_j^z$$



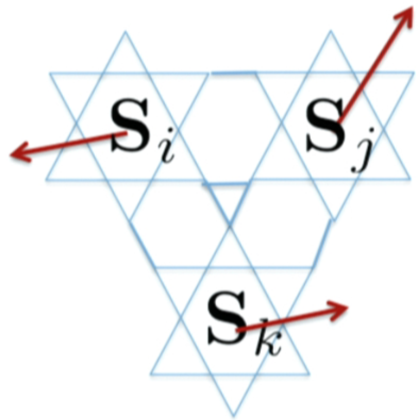
Groundstate of CORE(2)

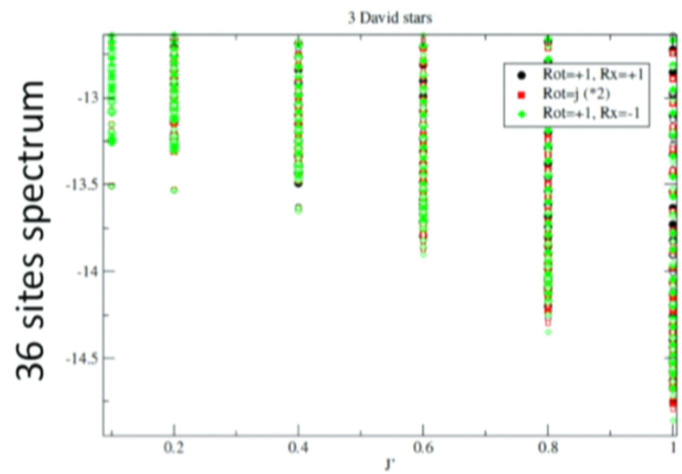


1. Product of ***antisymmetric*** singlets
2. Broken Lattice translational symmetry (12 site unit cell)
3. Point group symmetry p6m (triangular lattice)


CORE(2) – not yet converged
J'
S i
S j
S k
36 site ground state energy
ED
range-2 CORE
-13
-13.5
-14
-14.5
-15
-15.5
-16
0
0.2
0.4
0.6
0.8
1
J'
δE0/E0
-0.01
-0.02
-0.03
-0.04

Range 3 computation





Sylvain Capponi:

Lanczos: 36 sites OBC, with point group symmetries

$750*10^6$ configurations. Toulouse CALMIP supercomputer

Ravi Chandra:

Lanczos-SVD algorithm*: 2 x 18 sites, 200 SVD states

Desktop PC with 15GB memory; relative precision 10^{-4}

*Weinstein, AA, Ravi Chandra PRE 84 (2011)

CORE(3) Interaction Parameters

$$H^{CORE(3)} = Nc_0 + \sum_i h\sigma_i^z + \sum_{\langle ij\rangle,\alpha} J_\alpha \sigma_i^\alpha \sigma_j^\alpha$$
$$+ \sum_{\langle ijk\rangle_\Delta,\alpha} J_{z\alpha\alpha}\sigma_i^z \sigma_j^\alpha \sigma_k^\alpha$$

J_2	$J_2 = 0$ (Kagomé)	$J_2 = +0.1$	$J_2 =$ -0.1
c_0	-5.24629	-5.17068	-5.48631
h	-0.0692243	0.0593234	-0.362797
J_x	-0.00909679	-0.0154208	0.00112276
J_y	-0.0118789	0.00183224	-0.0176992
J_z	0.0210562	0.00368622	0.0201406
J_{zxx}	-0.02792	-0.0196486	-0.0282832
J_{zyy}	0.00455018	-0.00474948	0.00452468
J_{zzz}	0.000660095	-0.00141008	0.0104951

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Truncation Error CORE(3)

Comparing ground state energy of CORE(3) to DMRG on large lattices

number of stars	$E_0^{CORE_3}$	E_0^{DMRG}	Error
2×2	-0.418452	-0.417213	-0.001239
2×3	-0.423953	-0.422336	- 0.001617
3×4	-0.431150	-0.428046	-0.003104
3×5	-0.432688	-0.429191	-0.003497

Ranges > 3 effective interactions are very small!
→ CORE(3) converges well.

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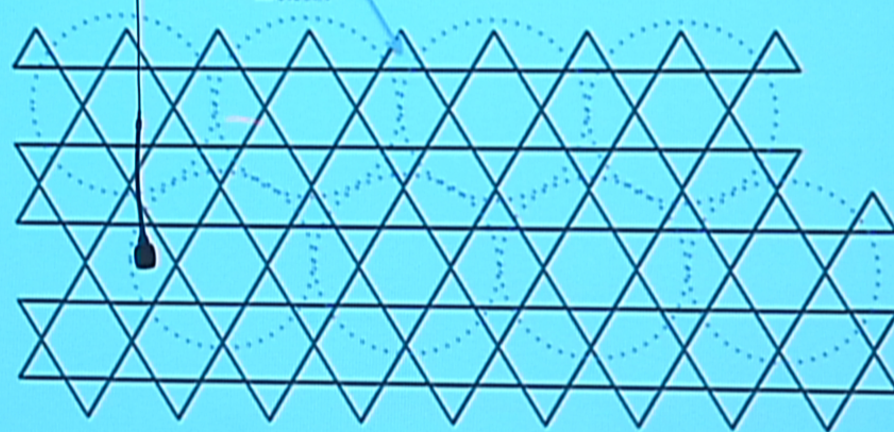
Is the GS a Spin Liquid?

$$H^\delta = \frac{1}{2}\sum_{\Delta}(\mathbf{S}_\Delta^2 - 9/4) + \delta\frac{1}{2}\sum_{\Delta_{inter}}(\mathbf{S}_\Delta^2 - 9/4)$$



$$E_{\Delta_{inter}} = \frac{dE_0}{d\delta}$$

$$E_\Delta = -0.687J$$

$$E_{\Delta_{inter}} = -0.665J$$

< 3% modulation

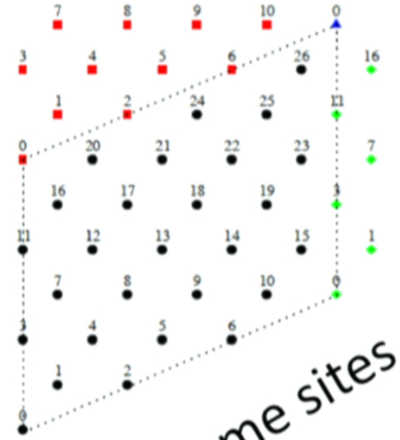
consistent with truncation error

Singlets spectra of large lattices

3x4 MD's

# A1	# A2
1 -64.37144334047481208927	1 -64.36732936063638987889
2 -63.96754639332078085090	2 -63.86789783887156346509
3 -63.79832541628604047901	3 -63.84784614830449811507
4 -63.75933331555023642068	4 -63.70772139593811544955
5 -63.62067587174262683902	5 -63.57801300230658370083
6 -63.51830836371206601143	6 -63.51410861653796757764

CORE(3) on 27 MD's



324 Kagome sites

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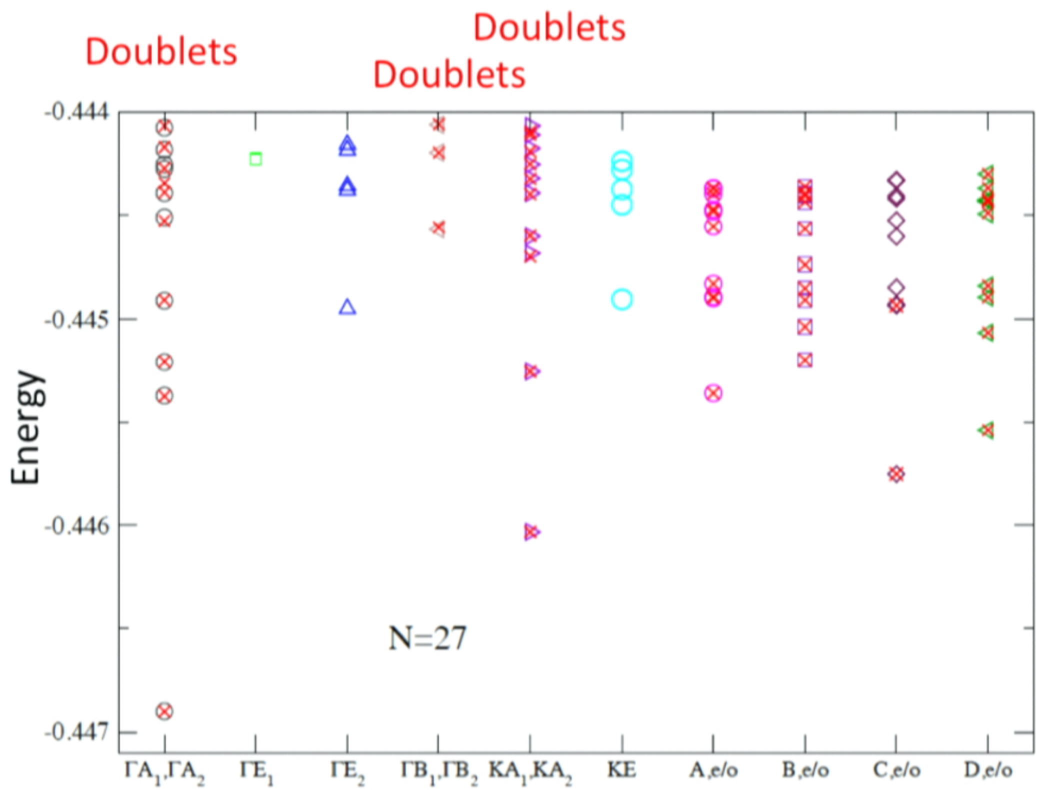

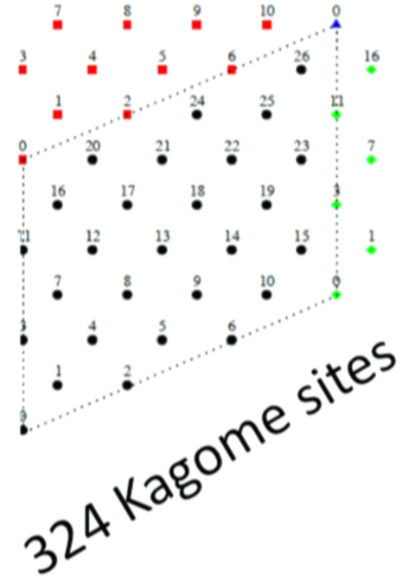

Mean field theory-HVBC

$$E^{FM} = Nc_0h\sum_i \cos\theta + \sum_{\langle ij\rangle} J_x \sin\theta_i \sin\theta_j + J_z \cos\theta_i \cos\theta_j$$

$$+ \sum_{\langle ijk\rangle} J_{zxx} \cos\theta_i \sin\theta_j \sin\theta_k + 2J_{zzz} \cos\theta_i \cos\theta_j \cos\theta_k$$

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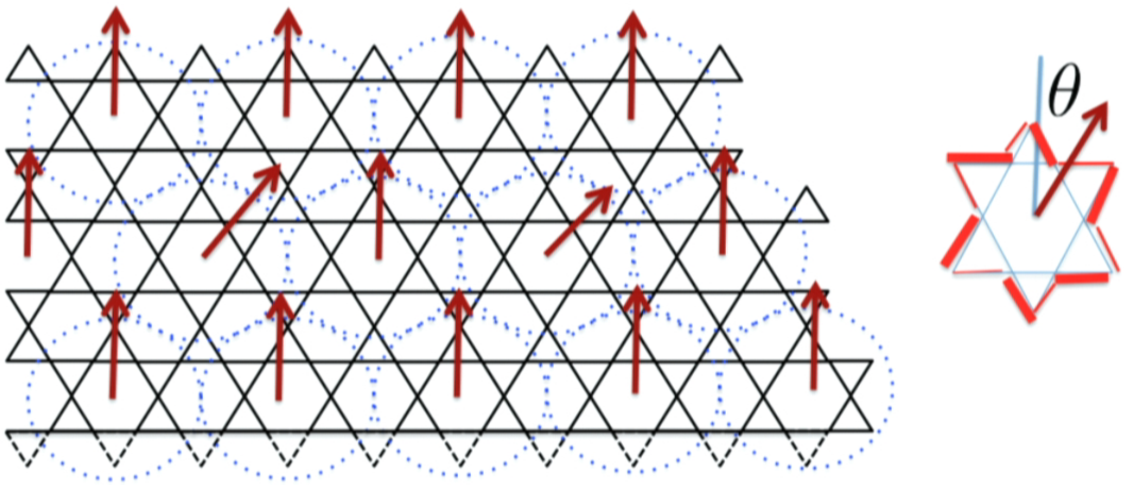
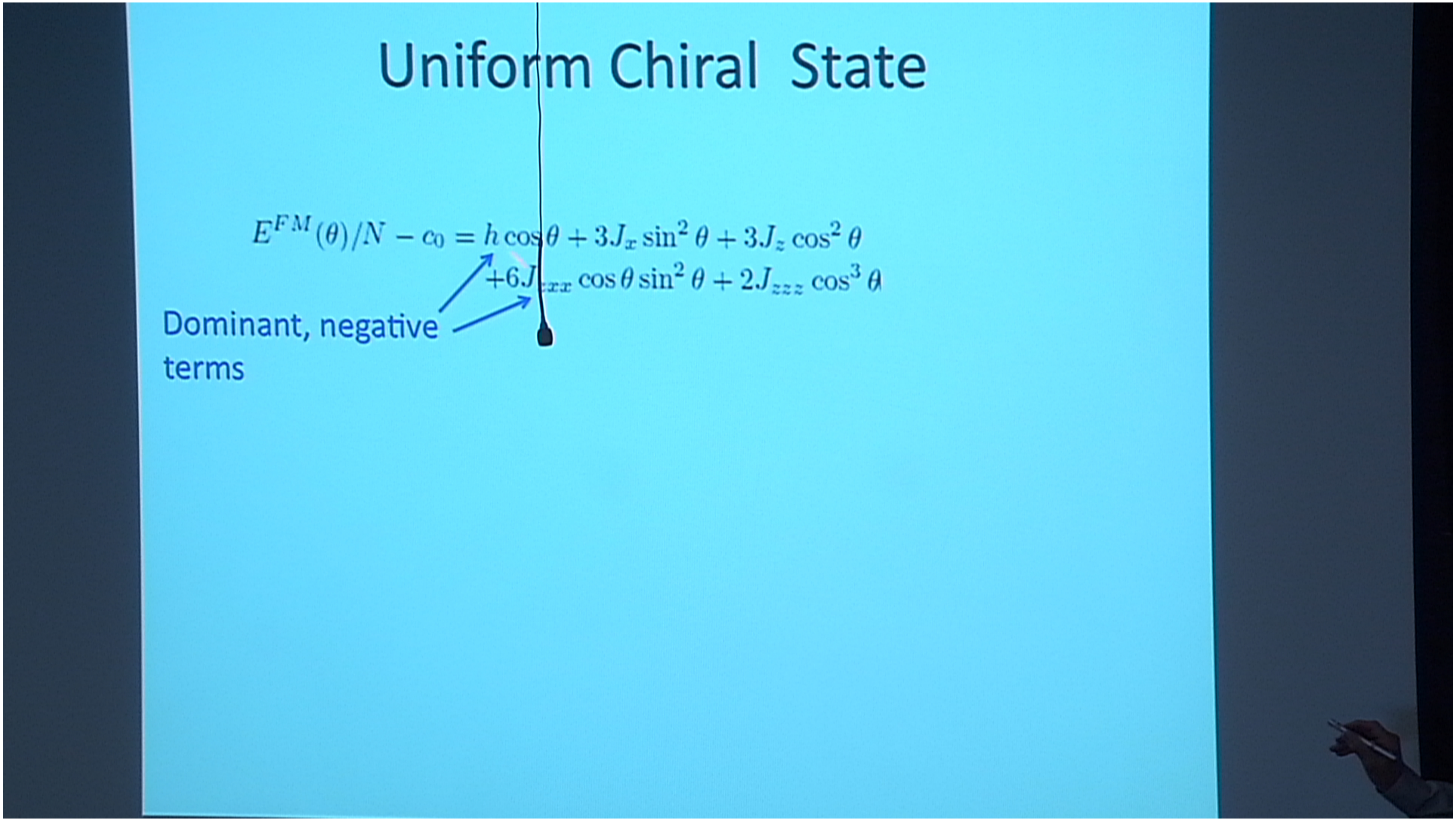
$$+ \sum_{\langle ijk\rangle} J_{zxx} \cos\theta_i \sin\theta_j \sin\theta_k + 2J_{zzz} \cos\theta_i \cos\theta_j \cos\theta_k \quad (7)$$



Uniform Chiral State

$$E^{FM}(\theta)/N - c_0 = h\cos\theta + 3J_x \sin^2\theta + 3J_z \cos^2\theta$$
$$+6J_{zxx}\cos\theta\sin^2\theta + 2J_{zzz}\cos^3\theta$$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Dominant, negative terms

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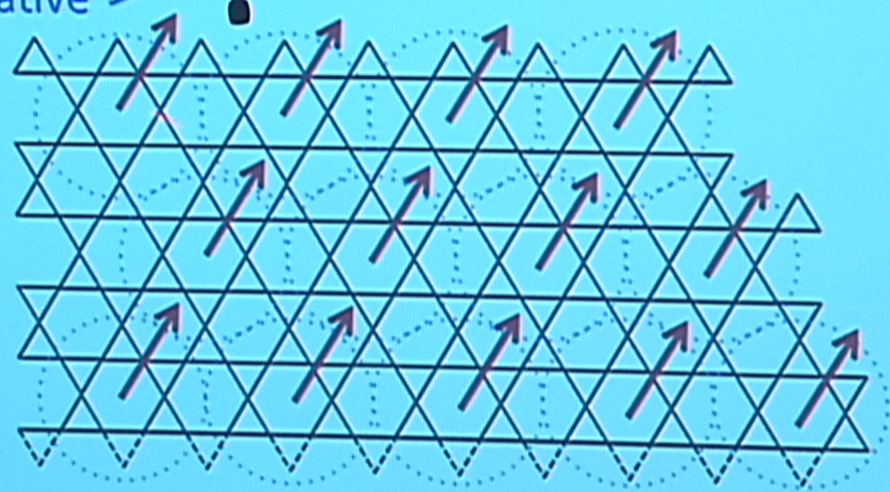

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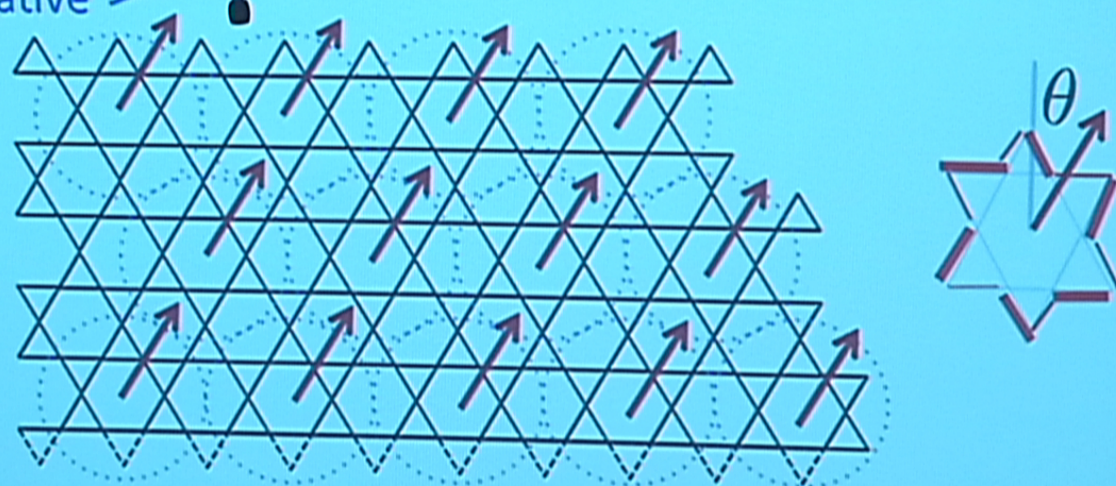
$$+6J_{zxx}\cos\theta\sin^2\theta + 2J_{zzz}\cos^3\theta$$

Dominant, negative terms



consistent with p6 (2D chiral) Spin Liquid

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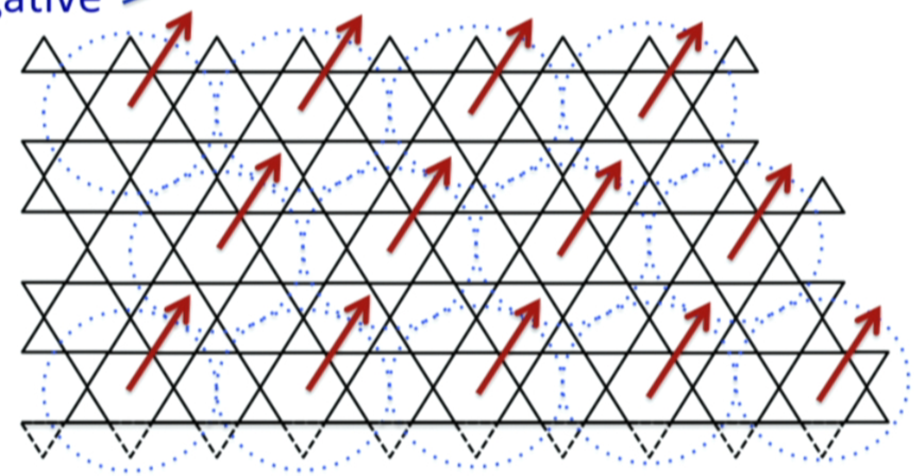

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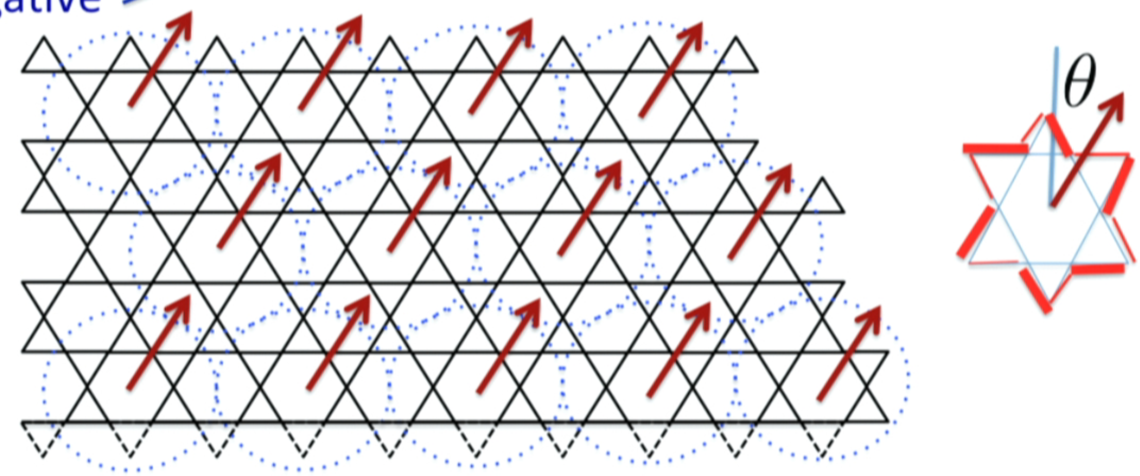

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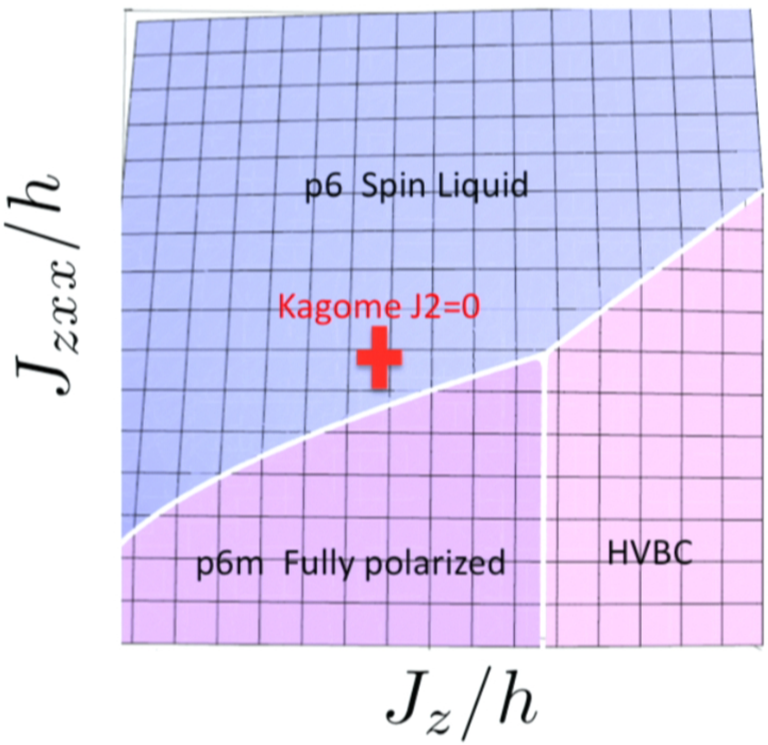
Projected Phase diagram

$$H = h\sum_i S_i^z + J^z \sum_{\langle ij\rangle} S_i^z S_j^z + J^{zxx}\sum_{ijk} S_i^z S_j^x S_k^x$$



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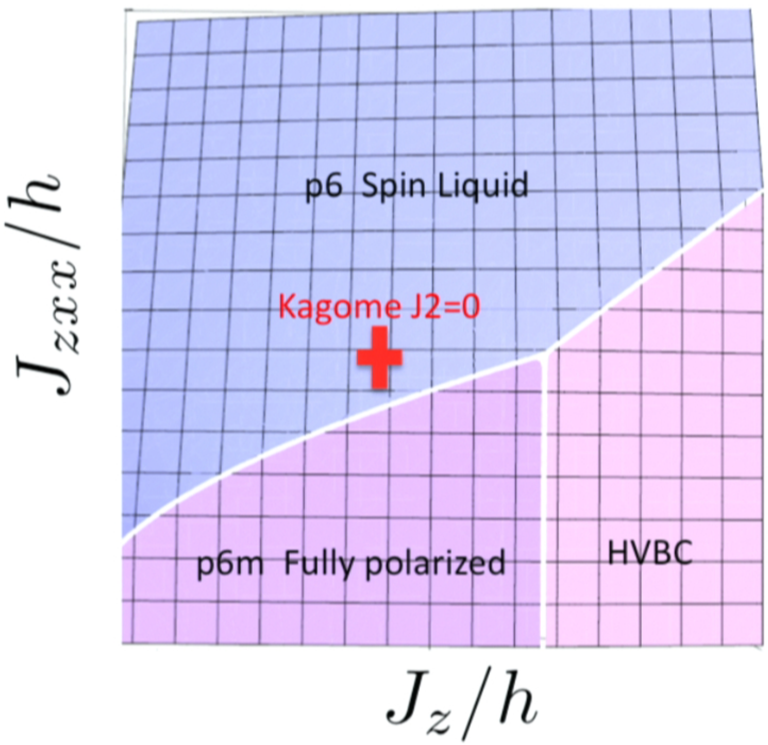

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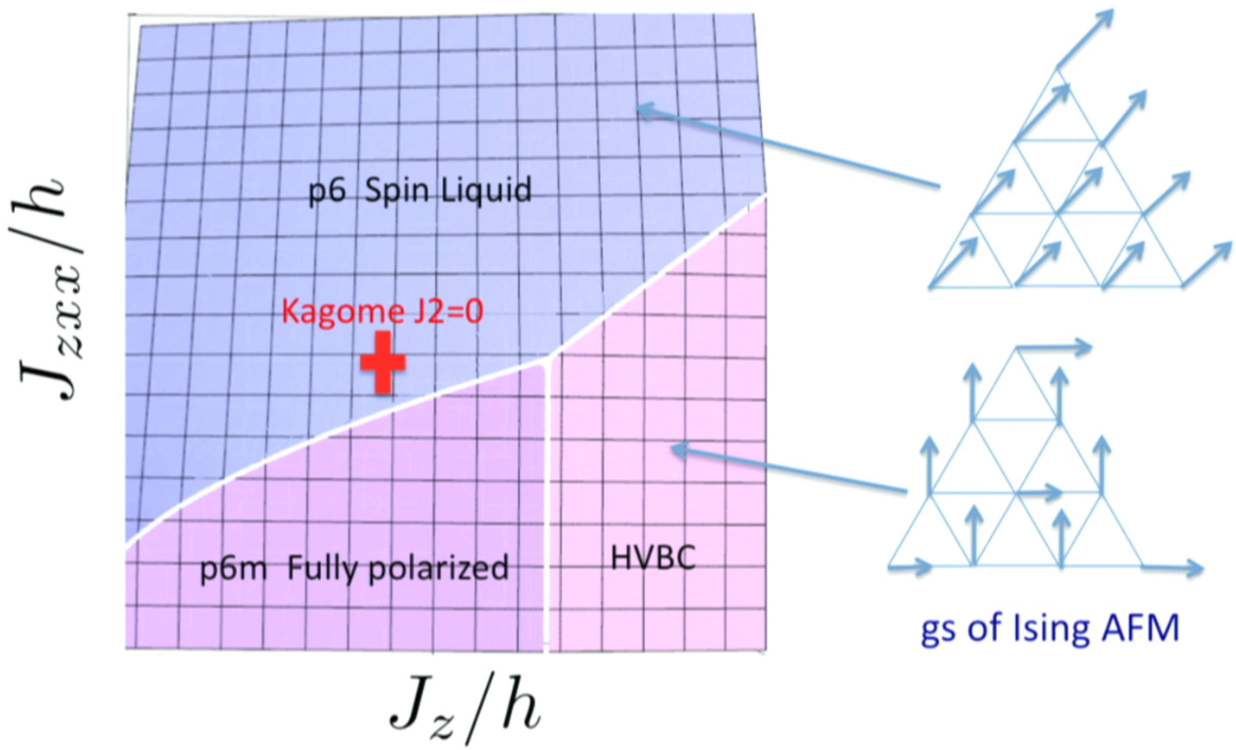
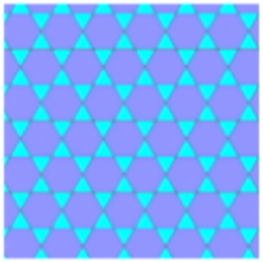
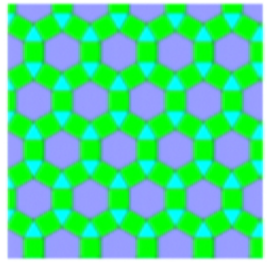


Wall paper groups (6-fold rotations)

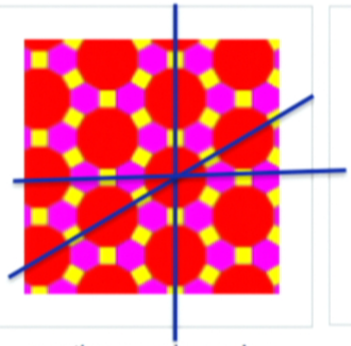
p6m has reflections in three distinct directions.



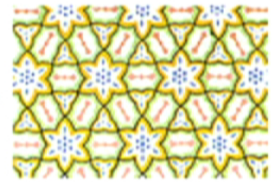
one of the 8 semi-regular tessellations



another semi-regular tessellation



another semi-regular tessellation



Persian glazed tile

Examples of group *p6* has ***no*** reflections or glide reflections.

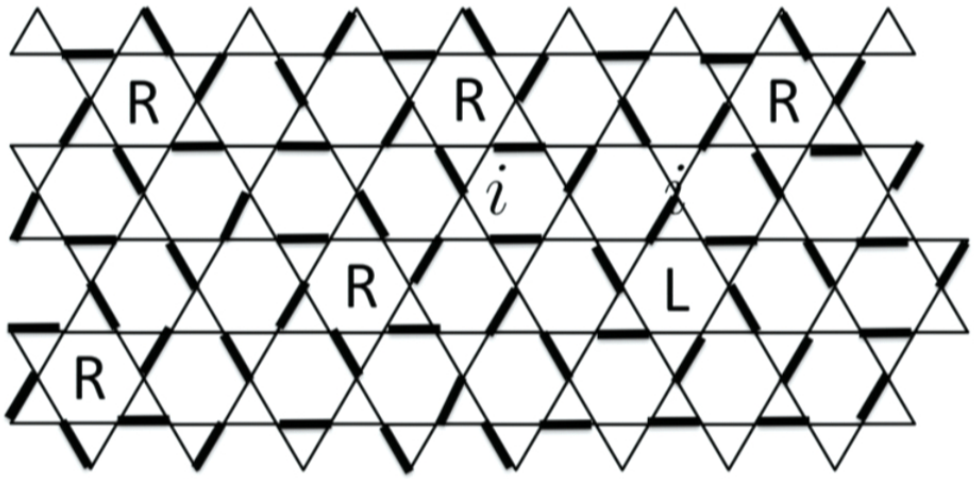
p6

Computer generated

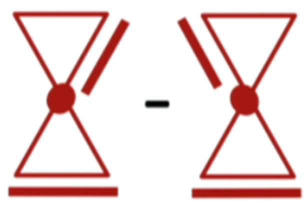
Wall panelling, the Alhambra, Spain

Persian ornament

Chiral Order Parameter



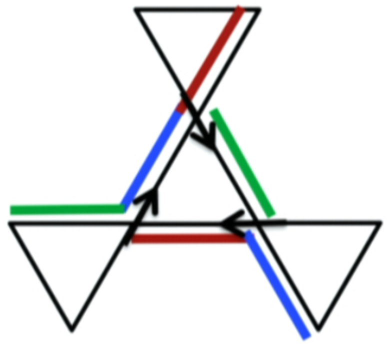
Chiral Dimer Configuration

$\mathcal{C}_i =$ 

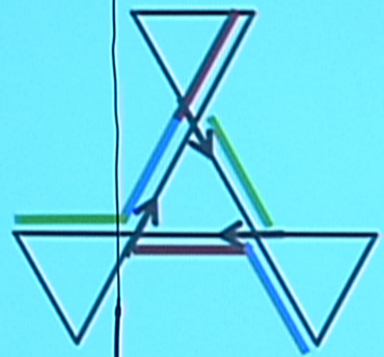
$$\mathcal{C}_i = \sum_{\mathbf{d}} \left(\mathcal{S}_{\mathbf{d}} \mathcal{S}_{\boldsymbol{\eta}^r(\mathbf{d})} - \mathcal{S}_{\mathbf{d}} \mathcal{S}_{\boldsymbol{\eta}^l(\mathbf{d})} \right)$$

$$m_c = \lim_{N \to \infty} \frac{1}{N} \sum_i \langle \mathcal{C}_i \rangle$$

2-Dimer Chiral Order Parameter



$$\delta\rho_d > 0$$
$$\langle \mathcal{C} \rangle > 0$$



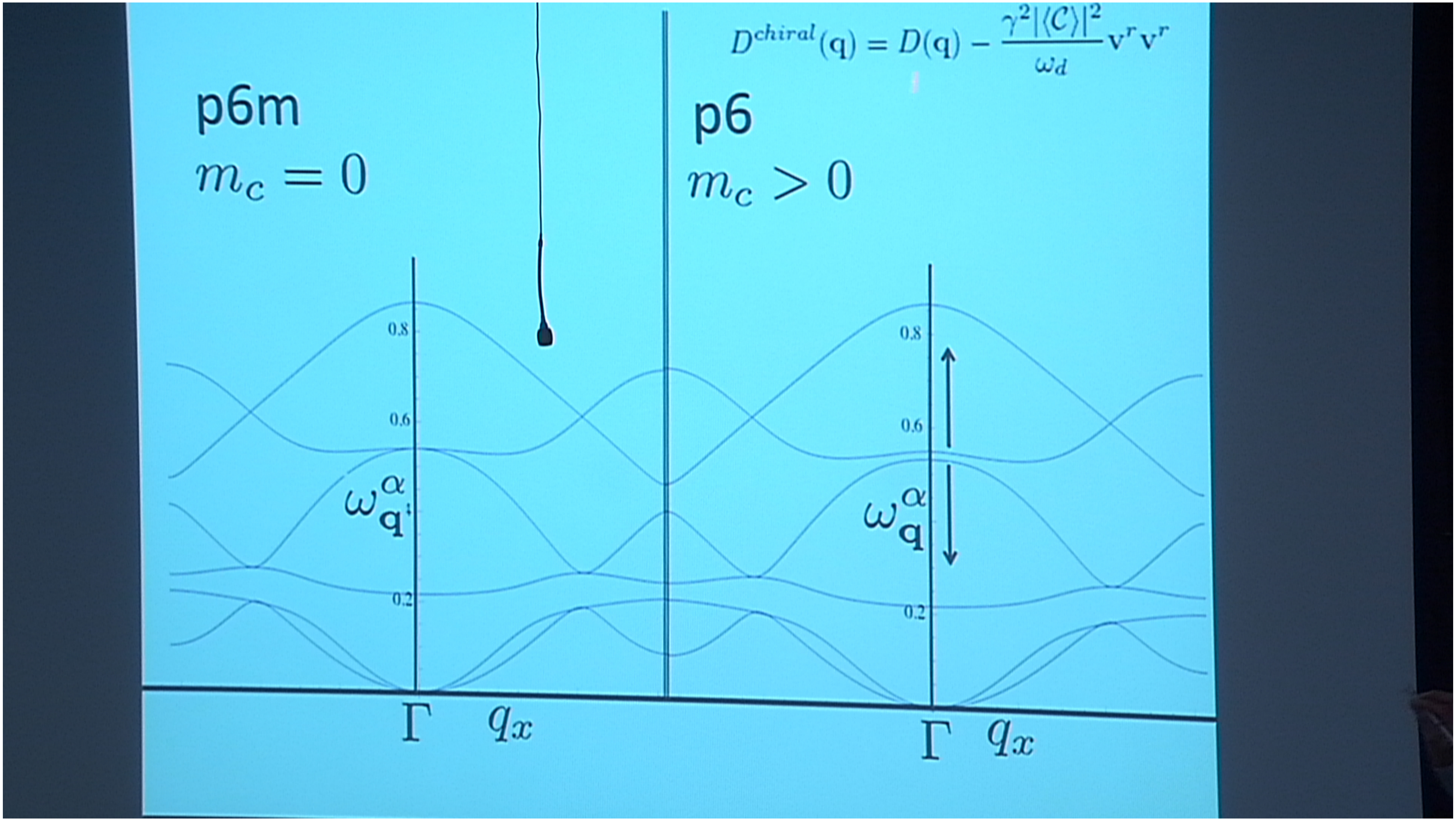
$$\delta\rho_d > 0$$

$$\langle\mathcal{C}\rangle > 0$$

$$\mathcal{L}_d = \frac{1}{2}\int d^2x \frac{1}{2}(\partial_t^2 - \omega_d^2)\delta\rho_d^2(\mathbf{x})$$

$$\mathcal{L}_{d-ion} = -\gamma|\langle\mathcal{C}\rangle| \int d^2x \delta\rho_d^2(\mathbf{x})\mathbf{v}^r \cdot \mathbf{u}(\mathbf{x})$$

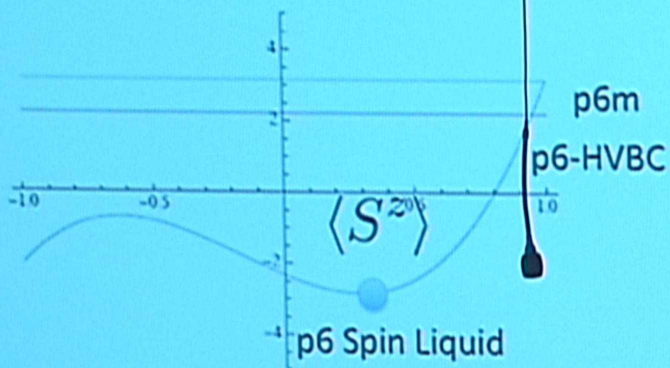
$$\mathbf{v}^r = \frac{1}{\sqrt{3}}(\cos(\frac{\pi}{3}), \sin(\frac{\pi}{3}), -1, 0, \cos(\frac{\pi}{3}), -\sin(\frac{\pi}{3})$$



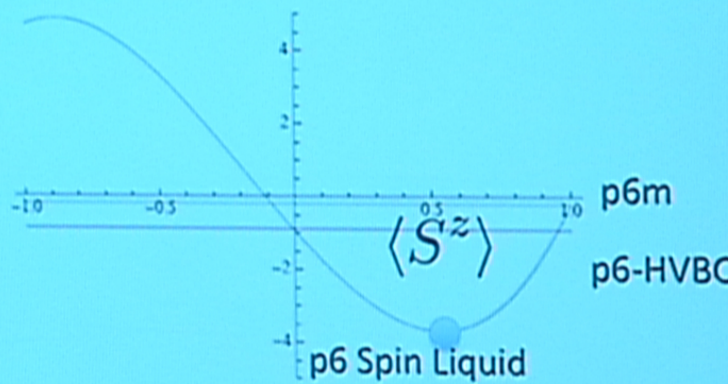
$D^{chiral}(\mathbf{q}) = D(\mathbf{q}) - \frac{\gamma^2|\langle \mathcal{C} \rangle|^2}{\omega_d} \mathbf{v}^r \mathbf{v}^r$
p6m
$m_c = 0$
p6
$m_c > 0$
0.8
0.6
0.2
$\omega_{\mathbf{q}}^{\alpha}$
Γ
q_x

Varying J2 – work in progress

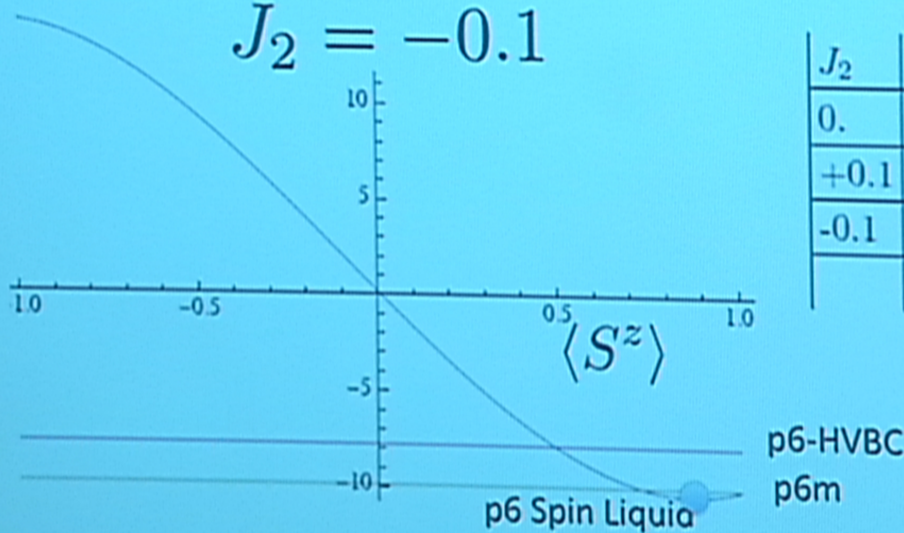
$J_2 = 0.1$



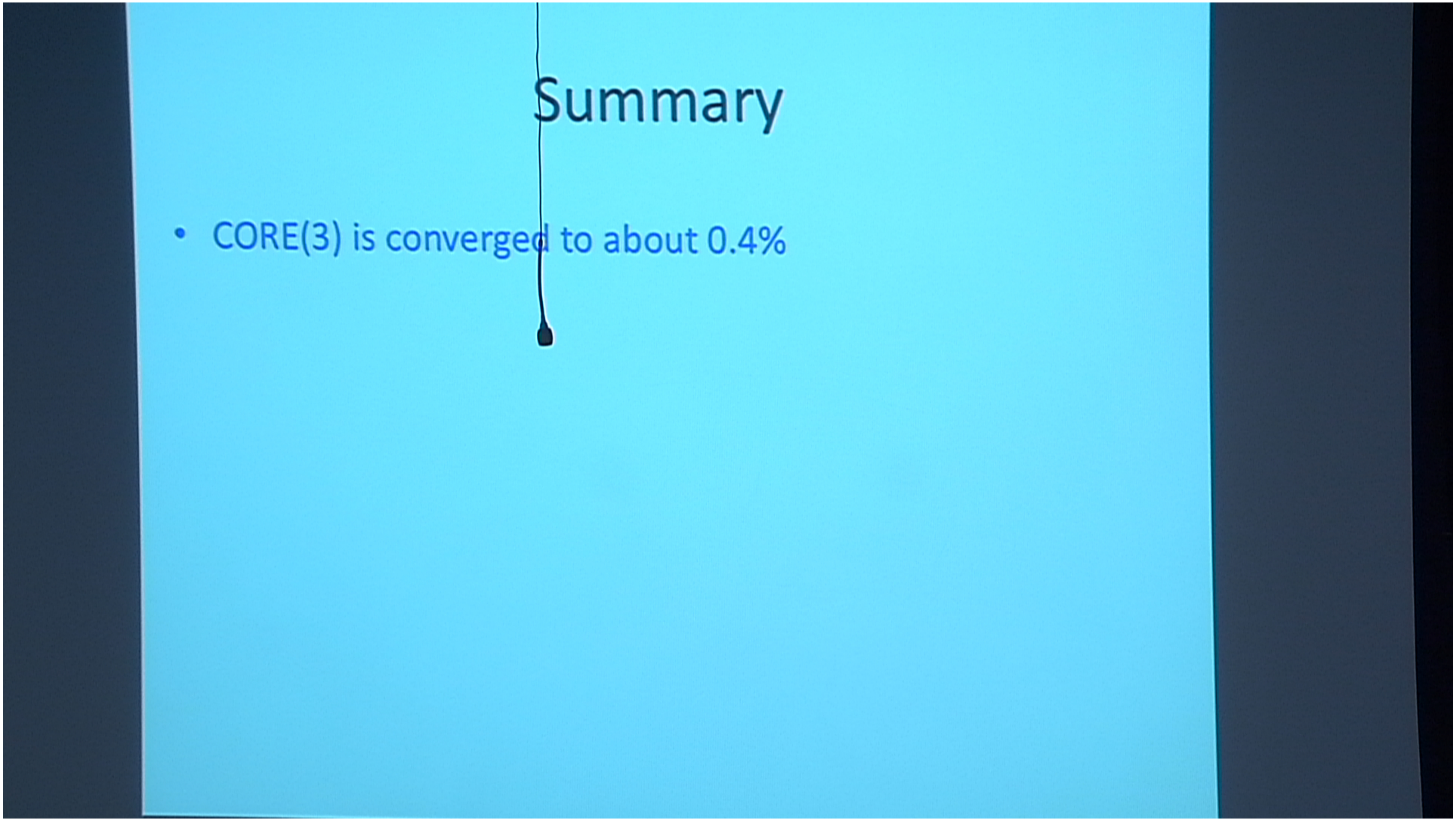
$J_2 = 0.$



$J_2 = -0.1$



J_2	M_z^{MF}	M_z^{ED}	M_x^{MF}
0.	0.26473		0.42417
+0.1	0.13904	+0.14756	0.48028
-0.1	0.43511	0.4999	0.246325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- *CORE is proving useful...*