Title: Self-localization of a single hole in Mott antiferromagnets

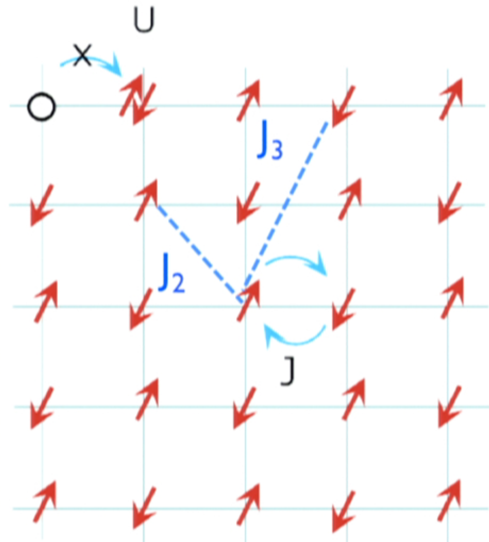
Date: Oct 19, 2012 02:30 PM

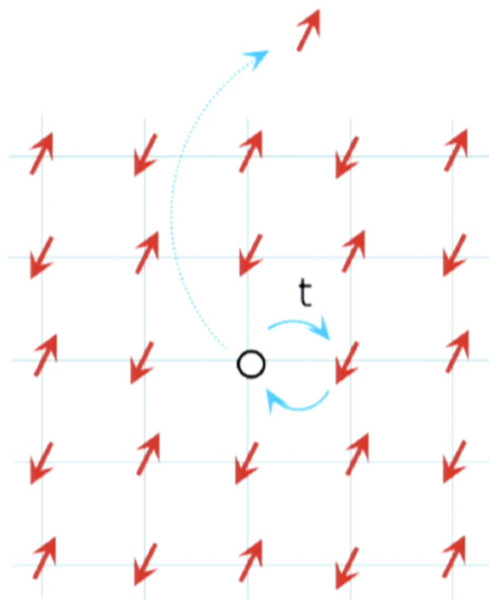
URL: http://pirsa.org/12100077

Abstract: Anderson localization - quantum suppression of carrier diffusion due to disorders - is a basic notion of modern condensed matter physics. Here I will talk about a novel localization phenomenon totally contrary to this common wisdom. Strikingly, it is purely of strong interaction origin and occurs without the assistance of disorders. Specifically, by combined numerical (density matrix renormalization group) method and analytic analysis, we show that a single hole injected in a quantum antiferromagnetic ladder is generally self-localized even though the system respects the translational symmetry. The localization length is found to monotonically decrease with the increase of leg number, indicating stronger self-localization in the two-dimensional limit. We find that a peculiar coupling between the doped charge and the quantum spin background causes quantum interference among different hole paths. The latter brings the hole's itinerant motion to a halt, a phenomenological analogy to Anderson localization. Our findings are opposite to the common belief of the quasiparticle picture for the doped hole and unveil a completely new paradigm for lightly doped Mott insulators.

frustrated Mott antiferromagnets

doped antiferromagnets

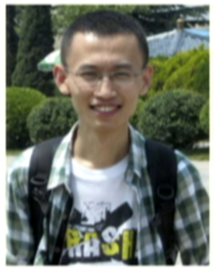




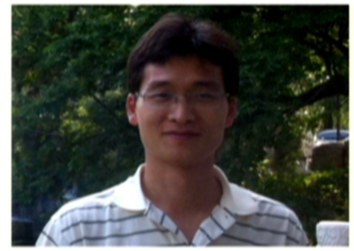
geometric frustrations

dynamic frustrations

Collaborators



Zheng Zhu (IAS, Tsinghua)



Hong-Chen Jiang (KITP/UCSB)

Yang Qi (IAS, Tsinghua)

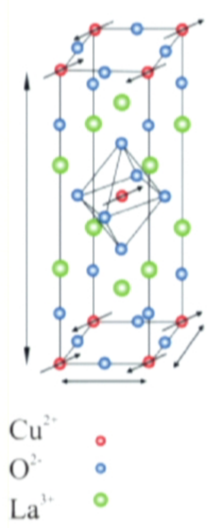
Chushun Tian (IAS, Tsinghua)

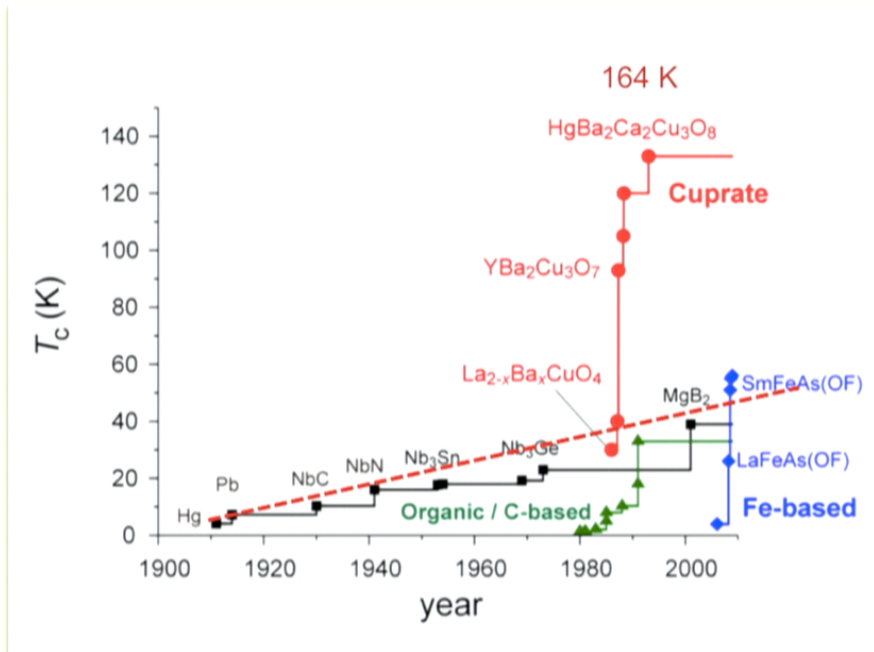
Outline

- Overview (after two decades…)
- DMRG results
- Implications

High-T_c cuprates: doped Mott insulators?

Science 1987

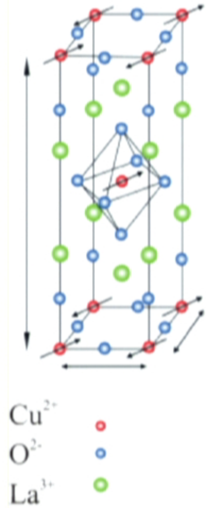




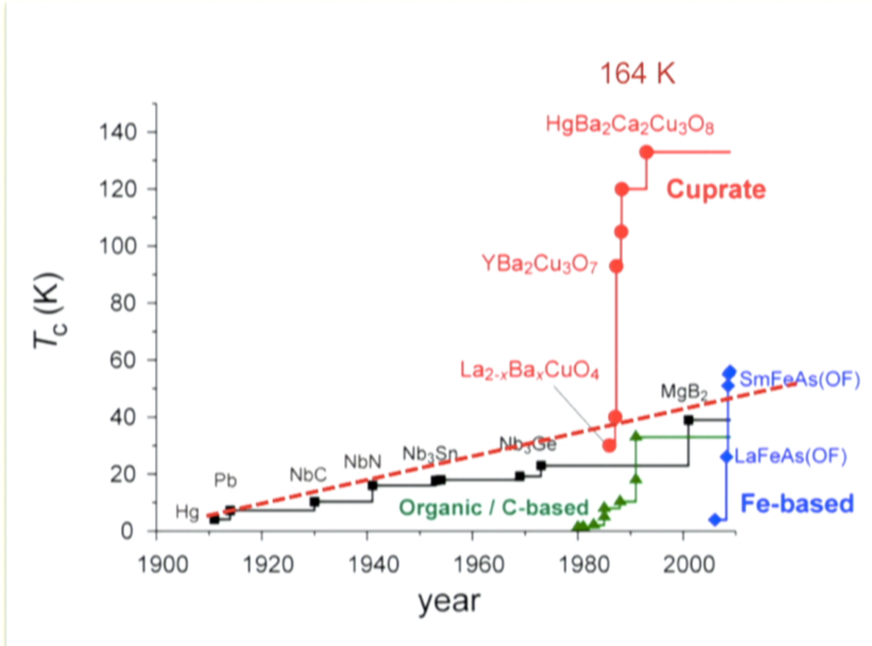
Mueller

Bednorz

High-T_c cuprates: doped Mott insulators?



Science 1987



Mueller

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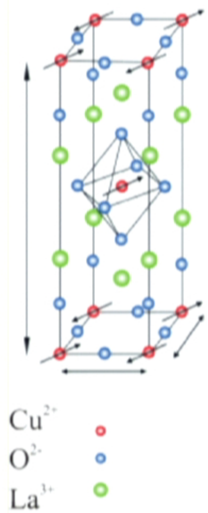

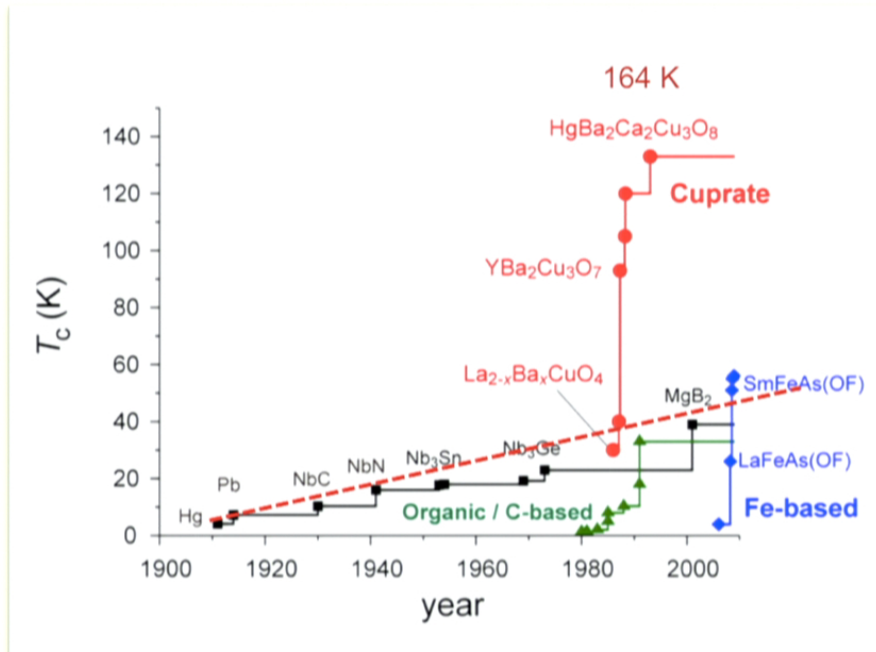

Mueller

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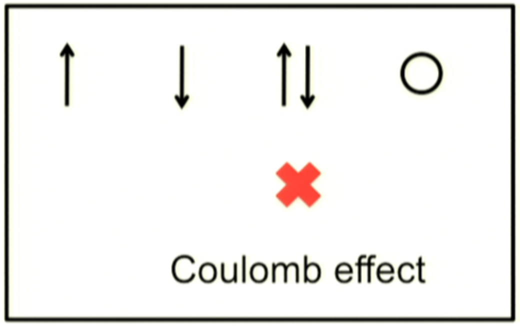
Problem of single hole doped into a Mott insulator



Half-filling:

A Mott insulator antiferromagnet

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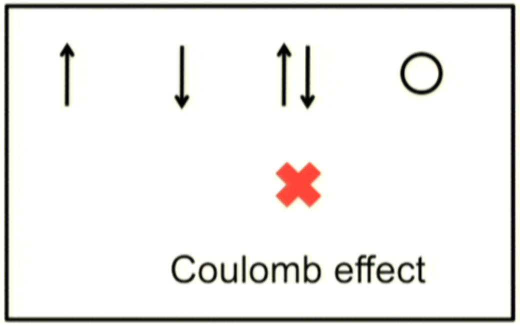
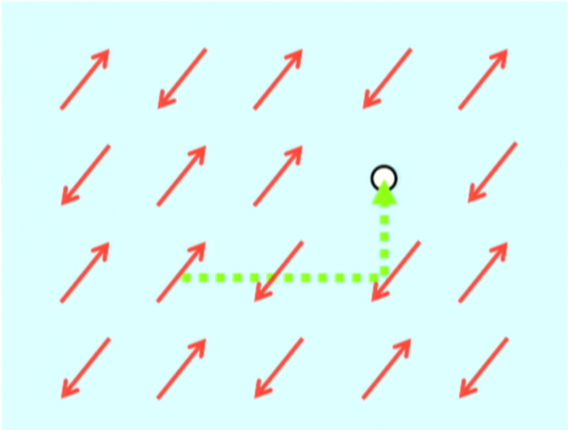
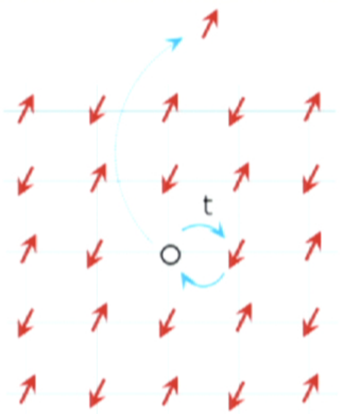


Half-filling:

A Mott insulator antiferromagnet

Question: How a single hole behaves?





Theoretical debate in one-hole problem

- Spin polaron picture (self-consistent Born approximation)

  S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein (1988);

  C. L. Kane, P. A. Lee, and N. Read (1989); ….

  → Quasiparticle

- ED result

  P.W. Leung and R. J. Gooding (1995);…

  → Quasiparticle

- P. W. Anderson's unrenormalizable phase shift argument (1990)

  → Non-quasiparticle

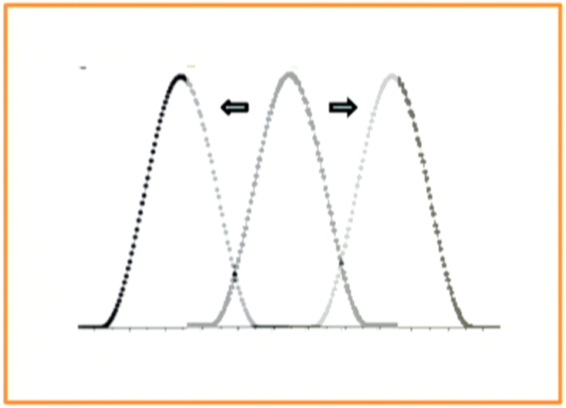
- Phase string effect *(D.N. Sheng, Y. C. Chen, ZYW (1996))*

  → Localization (non-quasiparticle) (ZYW, et al. (2001))

Quasiparticle (spin-polaron) picture

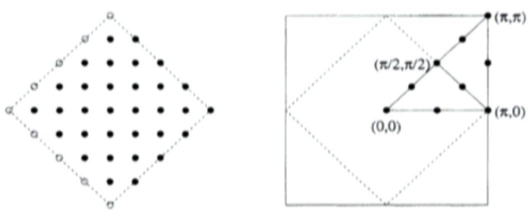
Bloch theorem holds
for a many-body system?

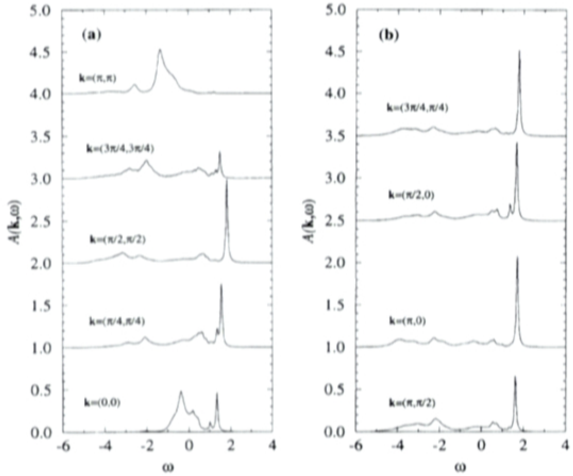
Quasiparticle (spin-polaron) picture



Bloch theorem holds for a many-body system?

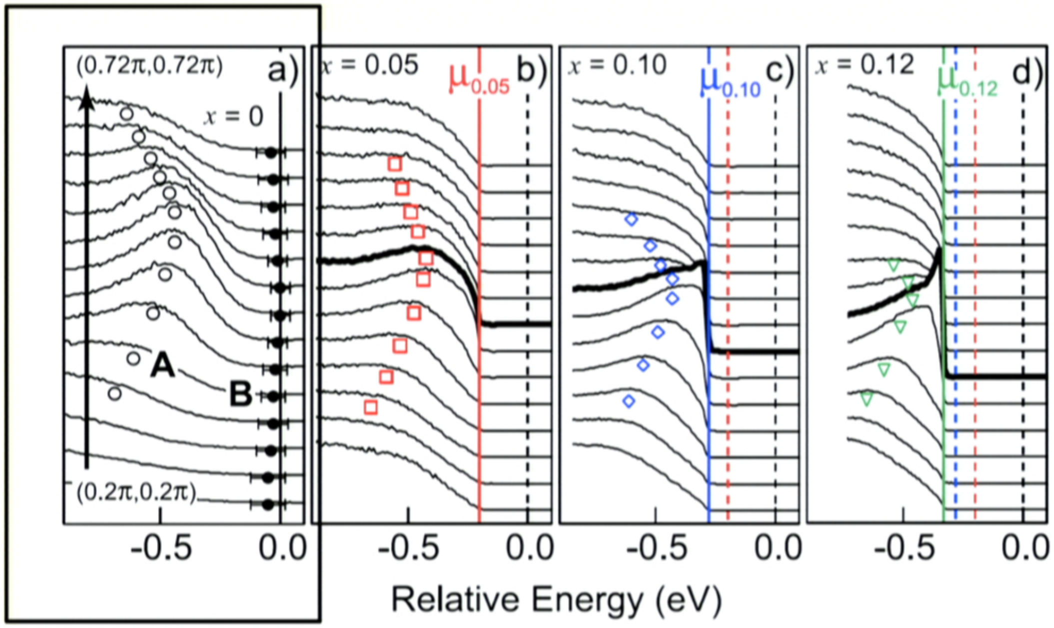
S^z (Ising)-strings can be destroyed by quantum spin flips (*C. L. Kane, P. A. Lee, and N. Read (1989)*)

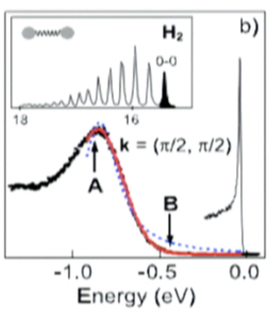




P. W. Leung and R. J. Gooding (1995)

ARPES result: A broad peak at x=0





$Ca_2CuO_2Cl_2$

K. M. Shen et al, PRL 93, 267002 (2004)

Experimental Results (STM)

Localization

C. Ye, et. al., arXiv:1201.0342v1
(Yayu Wang's group in Tsinghua)

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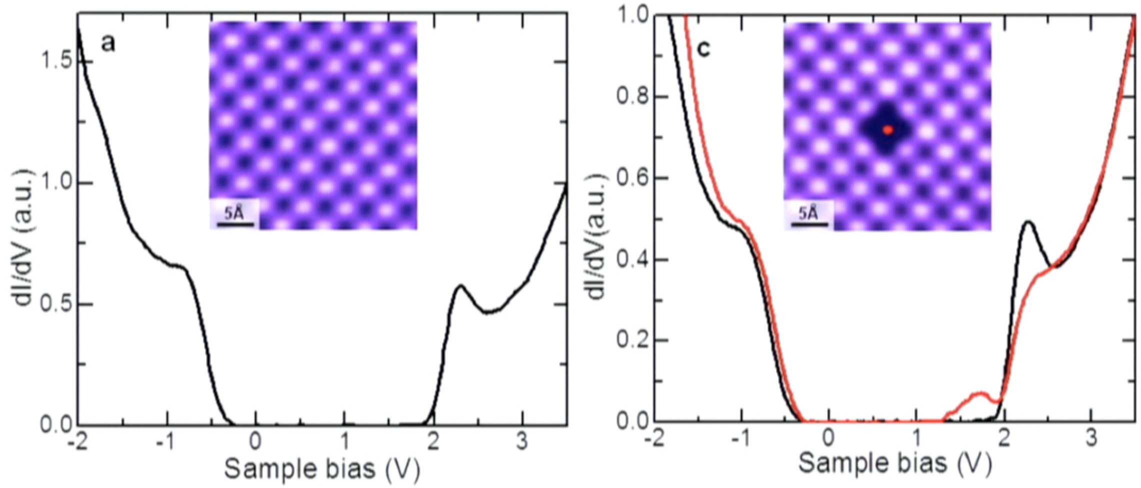

Strong localization of a single electron donated by a Cl defect

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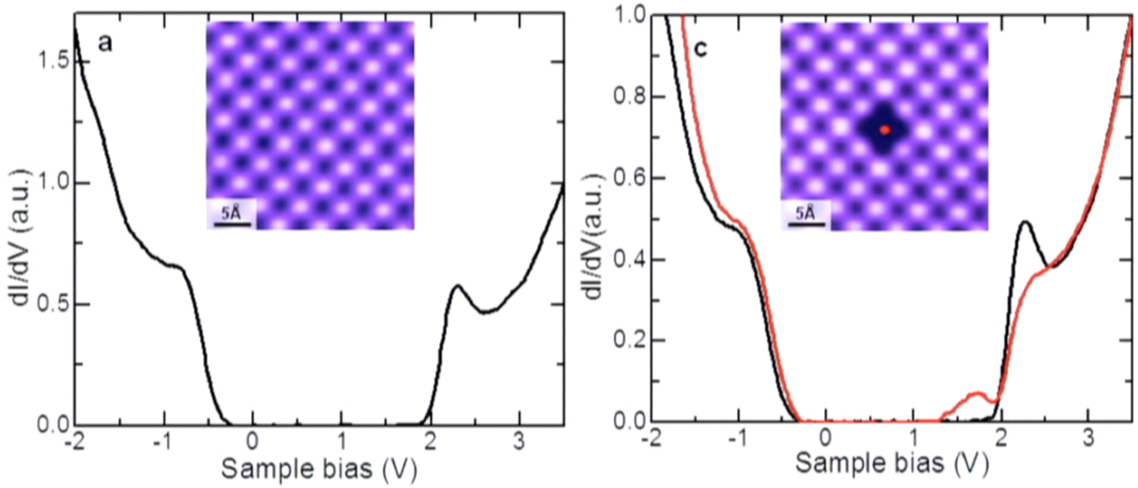


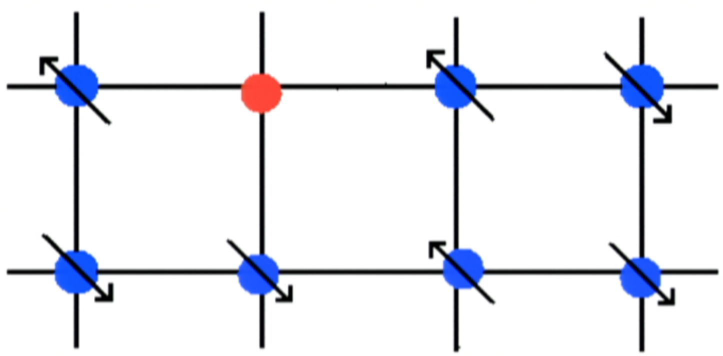
Strong localization of a single electron donated by a Cl defect

A minimal model for doped Mott insulators: t-J model

$$H = -t \sum_{\langle ij \rangle} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + J \sum_{\langle ij \rangle} \left(\mathbf{S_i} \cdot \mathbf{S_j} - \frac{1}{4} n_i n_j \right)$$

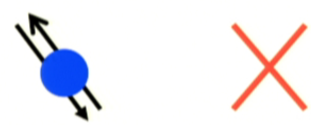
hopping

superexchange



constrained by

$$\sum_{\sigma} c_{i\sigma}^{+} c_{i\sigma} \leq 1$$



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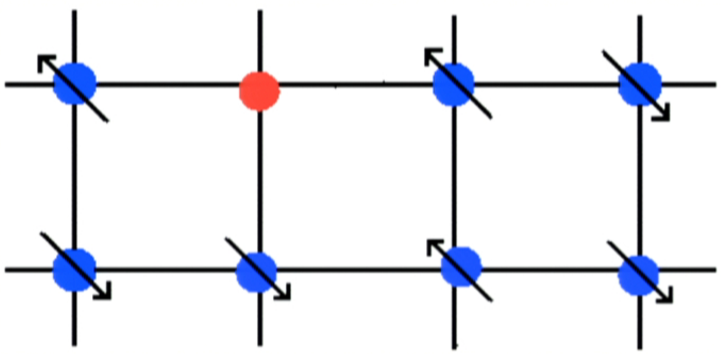
hopping **superexchange**



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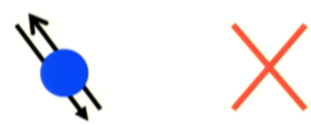


1-hole propagator: Phase string effect

$$G(j,i,E) = \langle \psi_0 | c^\dagger_{j\sigma} G(E) c_{i\sigma} | \psi_0 \rangle \propto \sum_c \tau_c W[c;E]$$

$$G(E) = \frac{1}{E - H_{t-J} - 0^+}$$

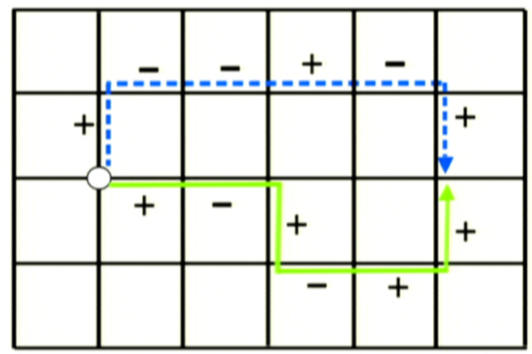
$$\tau_c \equiv (+1)\times(-1)\times(-1)\times\ldots\ldots$$
$$= (-1)^{N_h^{\downarrow}(c)}$$

$$W[c;E] = \left(\frac{t}{-E}\right)^{M_h} \left(\frac{J}{-2E}\right)^{M_{\uparrow\downarrow}+M_Q} \geq 0$$

Partition function:

$$Z = \sum_{\text{loop c}} \tau_c W(c)$$

$$W(c) = \underbrace{\frac{2t}{J}\cdot\frac{2t}{J}\cdots\frac{2t}{J}}_{M_h(C)} \sum_n \frac{(\beta J/2)^n}{n!} \delta_{M_h+M_{\uparrow\downarrow},n} \geq 0$$



D.N. Sheng, Y.C. Chen, ZYW, PRL (1996); K. Wu, ZYW, J. Zaanen, PRB (2008)

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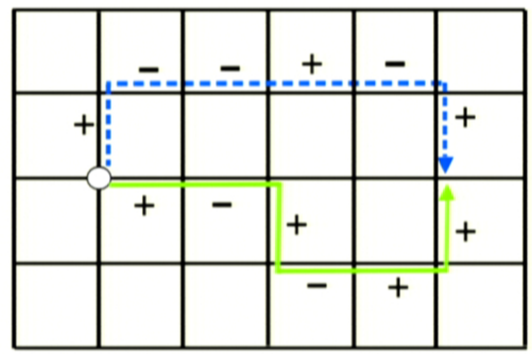

D.N. Sheng, Y.C. Chen, ZYW, PRL (1996); K. Wu, ZYW, J. Zaanen, PRB (2008)

Removing the phase string: A sign-free model

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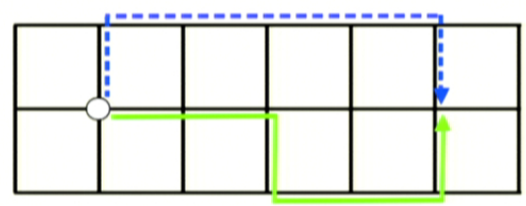
σ

$$Z = \sum_{loop\ c} \tau_c W(c)$$

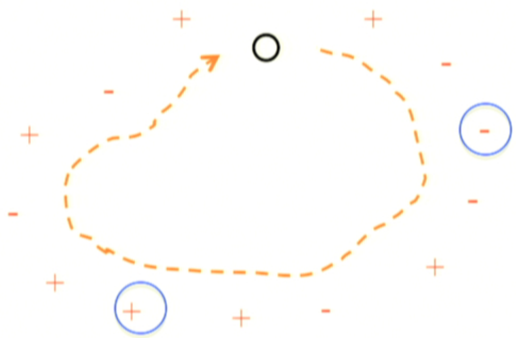


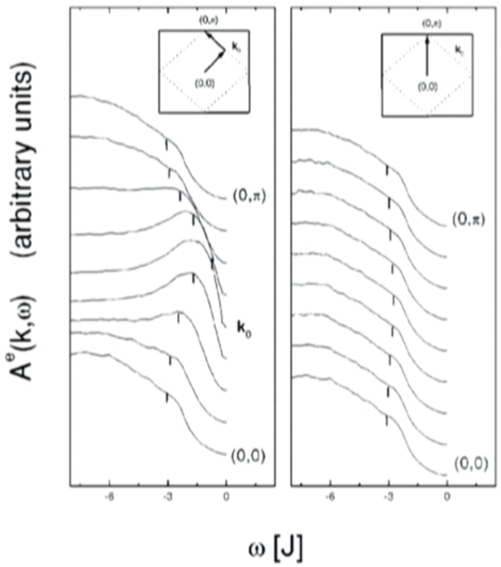
$$\tau_c \equiv 1$$

for one-hole case

$$W(c) = \underbrace{\frac{2t}{J} \cdot \frac{2t}{J} \cdots \frac{2t}{J}}_{M_h(C)} \sum_n \frac{(\beta J/2)^n}{n!} \delta_{M_h + M_{\uparrow\downarrow}, n} \geq 0$$

Prediction: self-localization of the one-hole





ZYW, V. N. Muthukumar, D.N. Sheng, C.S. Ting (2001)

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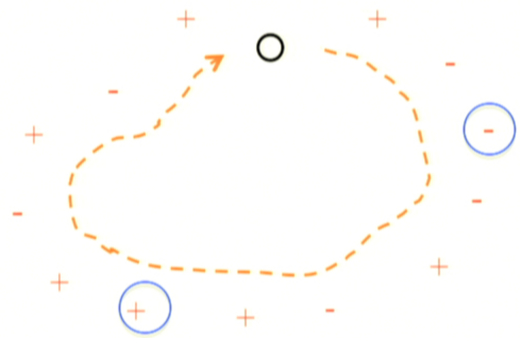
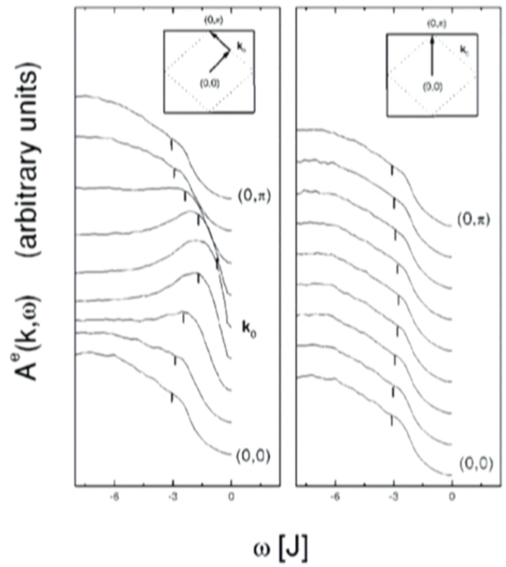


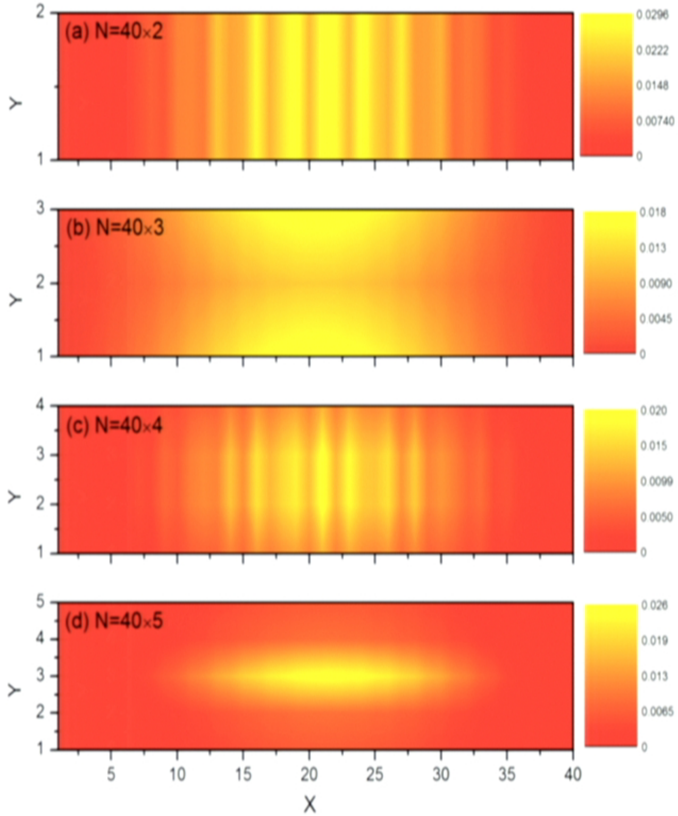
AFM state



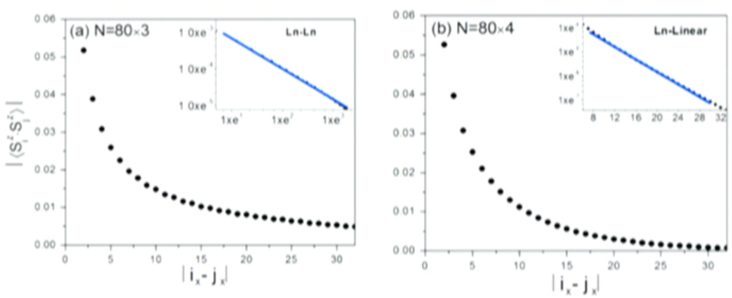
ZYW, V. N. Muthukumar, D.N. Sheng, C.S.Ting (2001)

Real space distribution of a single hole in t-J ladders

Z. Zhu, H.C. Jiang, Y. Qi, C.S.Tian, ZYW (2012)

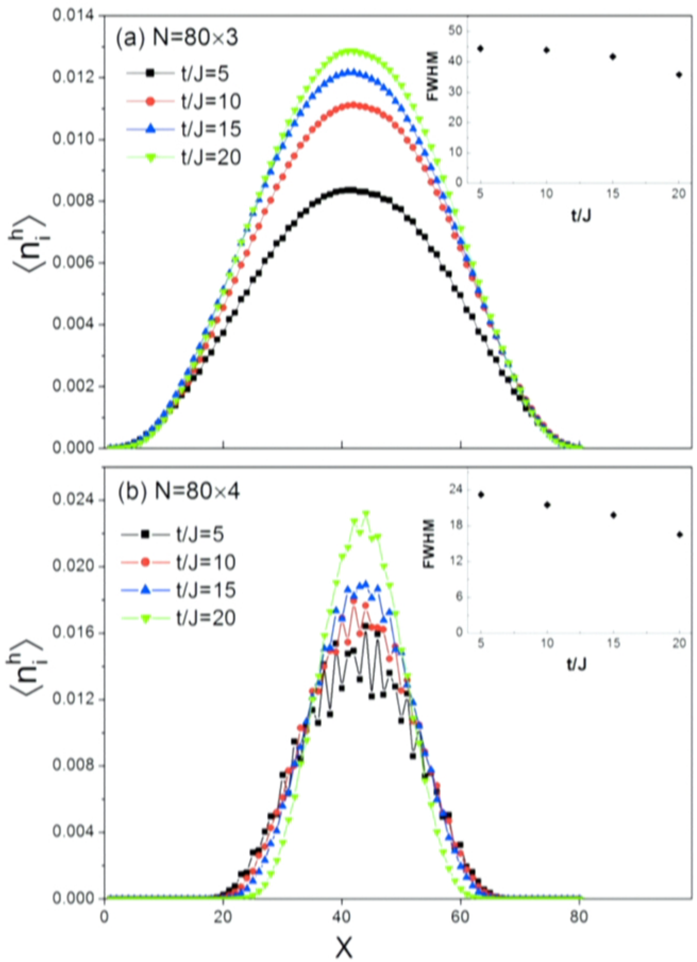


t-J ladders: $t = 3J$



odd (3-legs) even (4-legs)

Localization with the ratio t/J



$\sum_i \langle n_i^h \rangle = 1$

1. Localization length is monotonically reduced as t/J increases

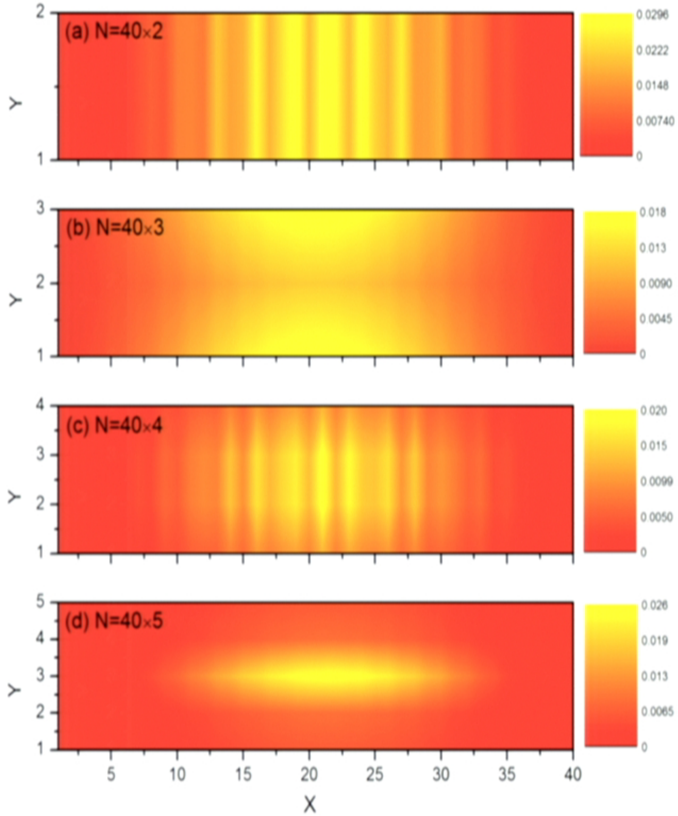
- **spin dynamics is not essential to the hole localization.**

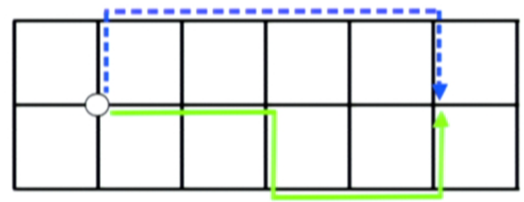
2. Oscillation for the even-leg ladders diminish as t/J increases

- **the spin-gap effect will be gradually reduced with the increase of t/J**

Real space distribution of a single hole in t-J ladders

Z. Zhu, H.C. Jiang, Y. Qi, C.S.Tian, ZYW (2012)





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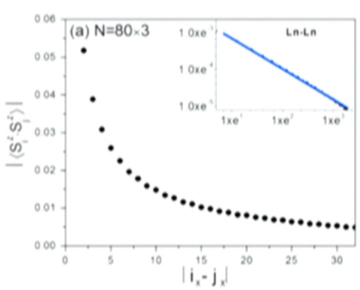
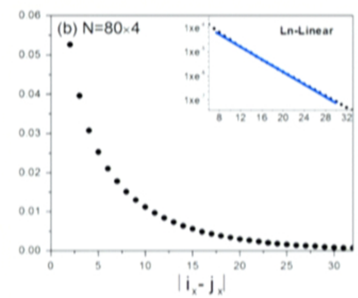
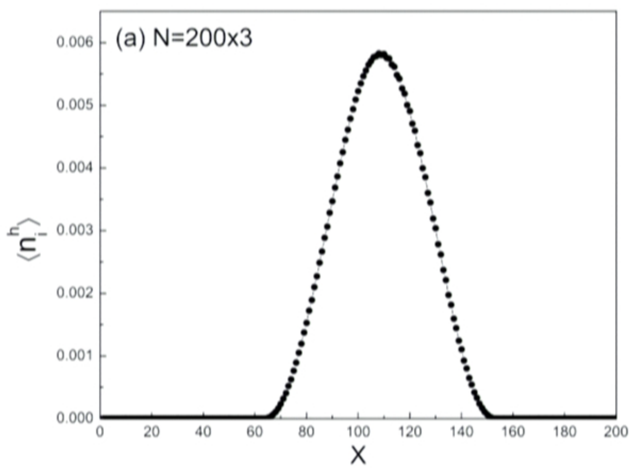


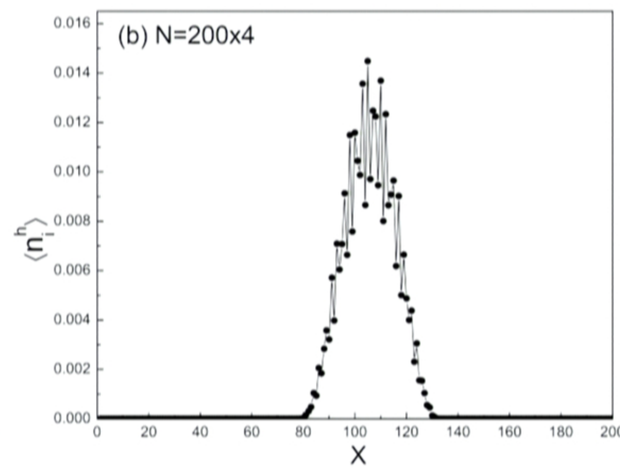
odd (3-legs)



even (4-legs)

Single-hole problem: A DMRG calculation





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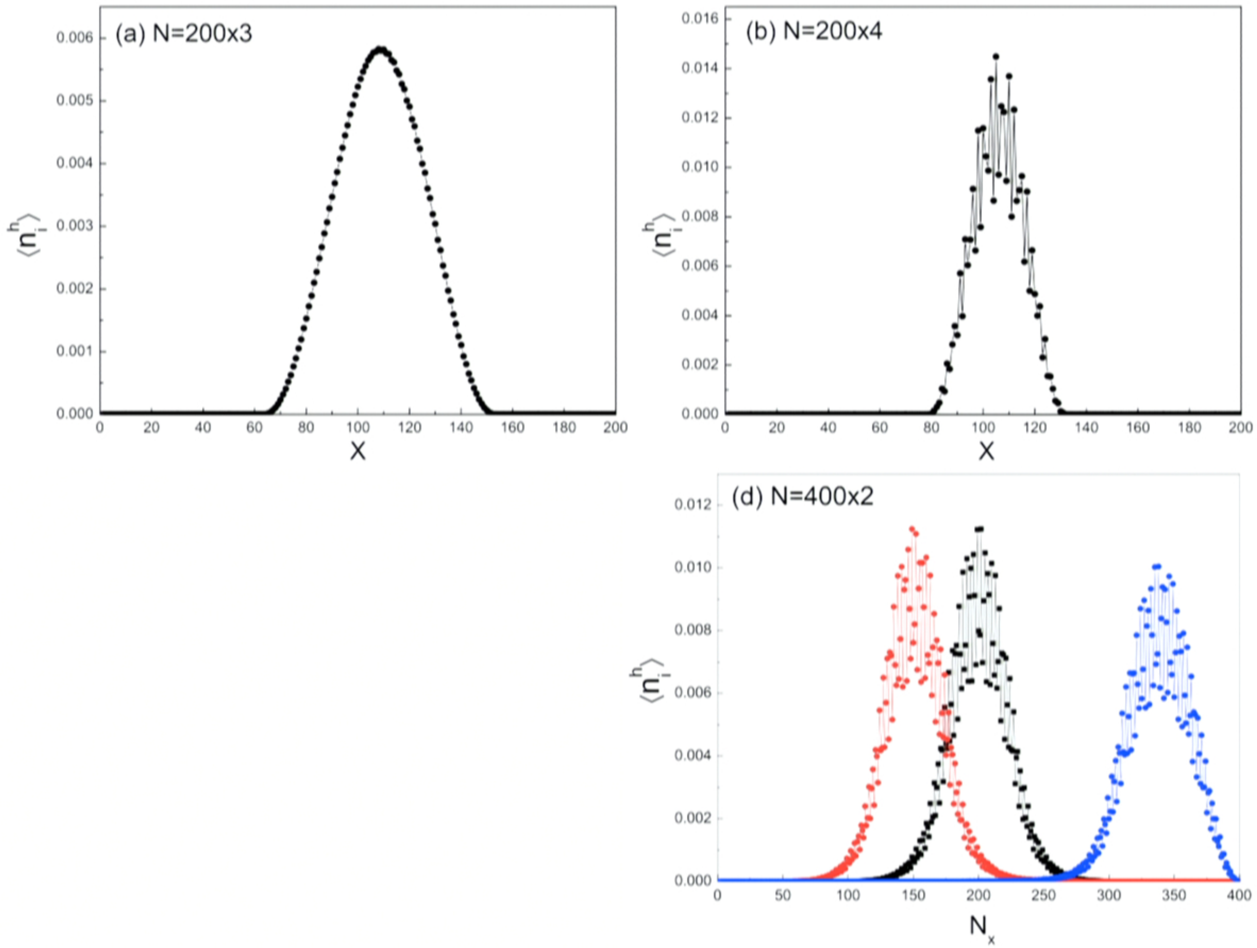

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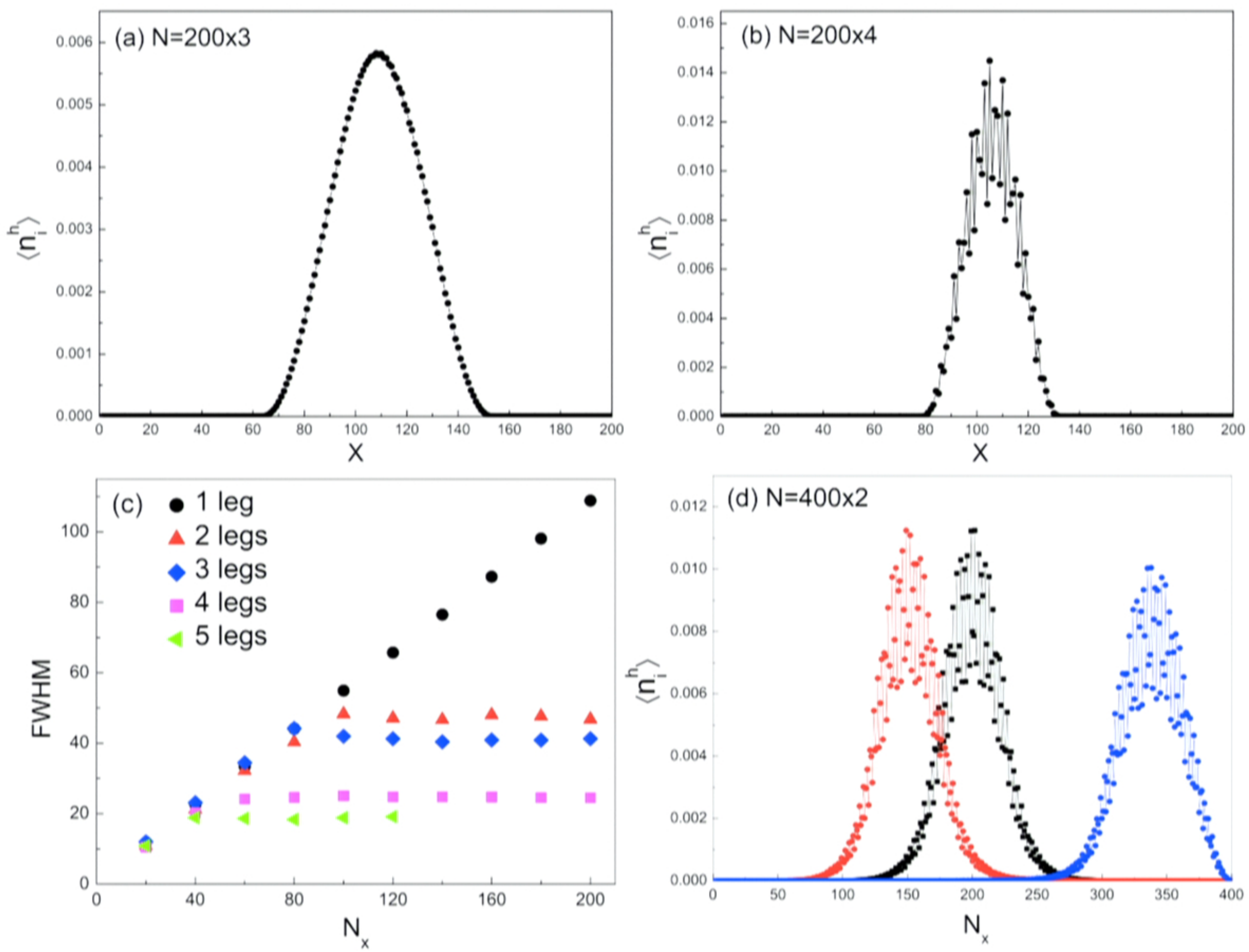
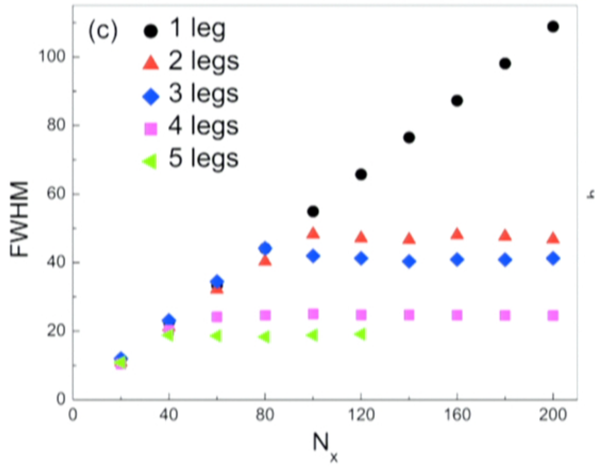
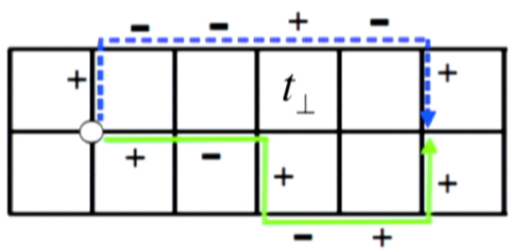


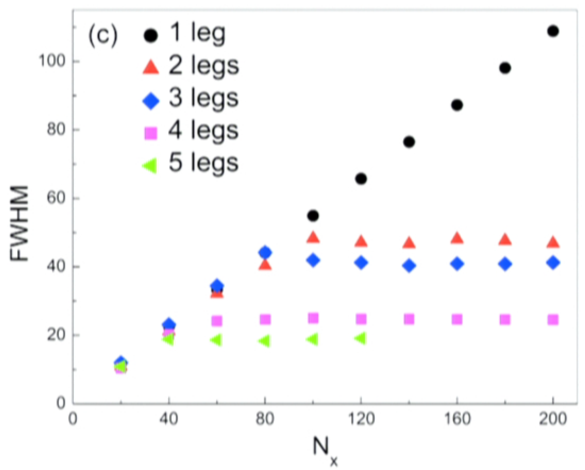
Effect of phase string effect

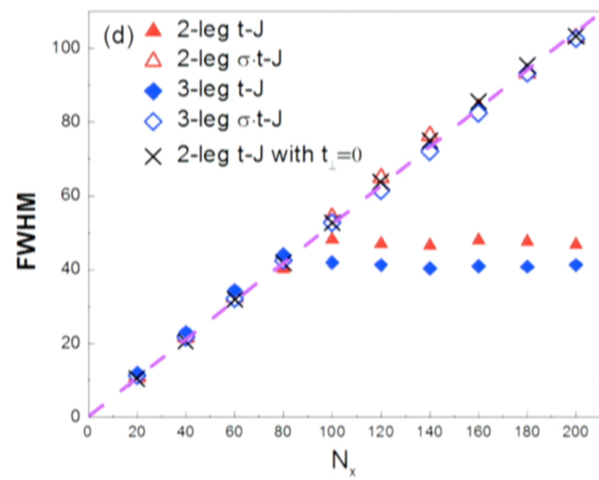


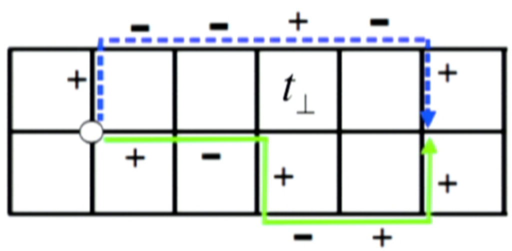


Self-localization of the hole!

Effect of phase string effect







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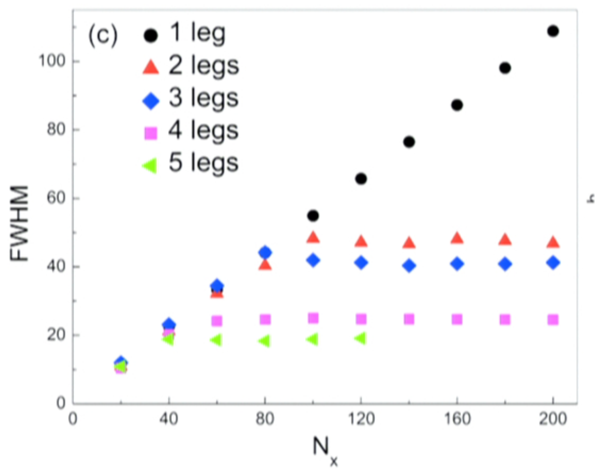
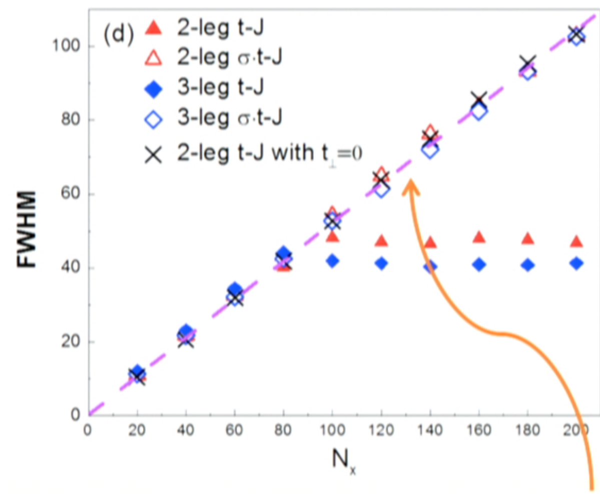
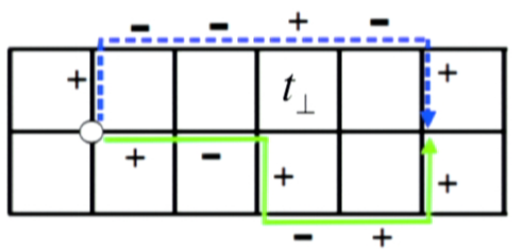
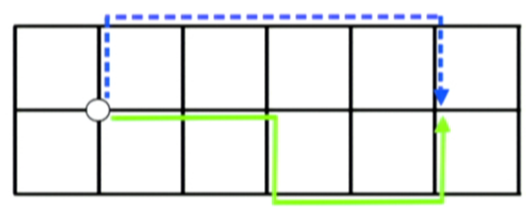




no phase string effect





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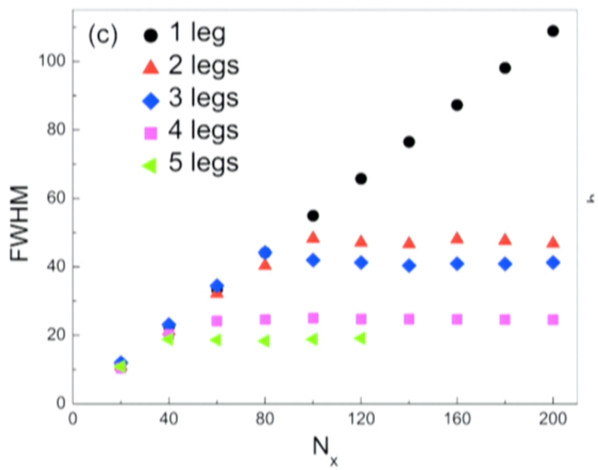
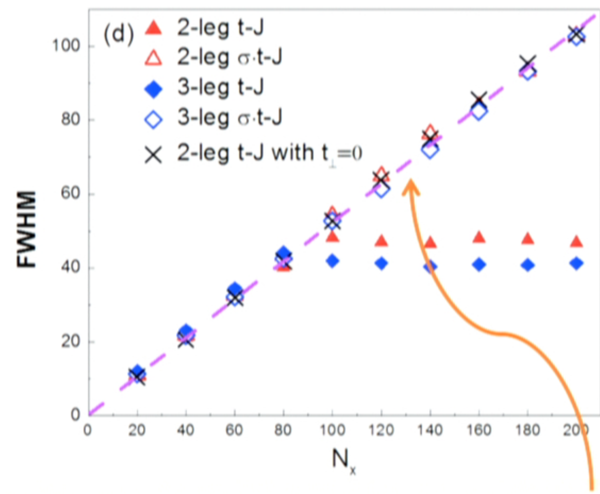
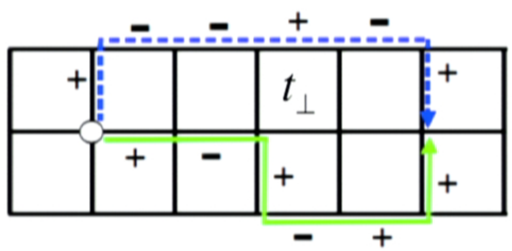
σ

Effect of phase string effect





no phase string effect



Self-localization of the hole!

$$H = -t\sum_{\langle ij\rangle}\left(c^\dagger_{i\sigma}c_{j\sigma} + h.c.\right) + J\sum_{\langle ij\rangle}\left(\mathbf{S}_i\cdot\mathbf{S}_j - \frac{1}{4}n_i n_j\right)$$

σ

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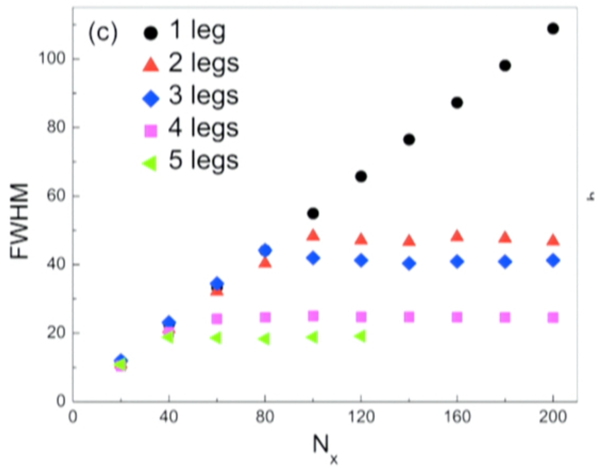
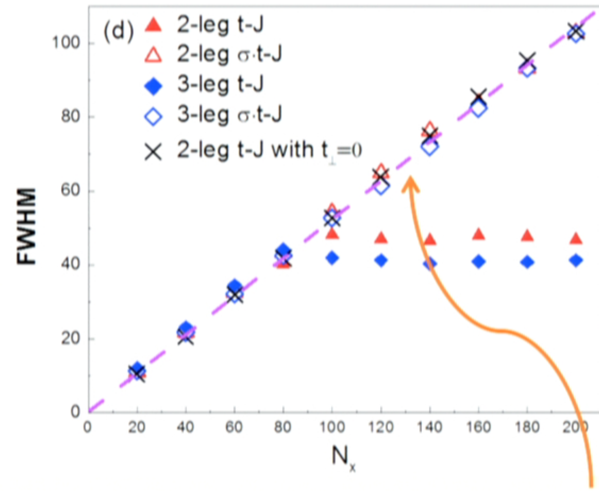
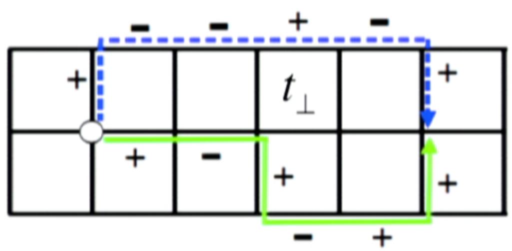




no phase string effect



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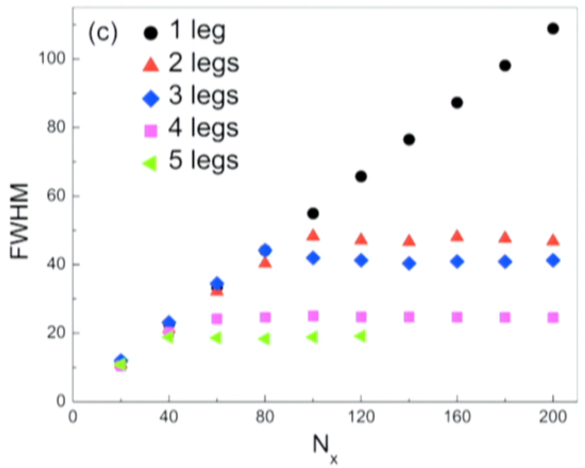

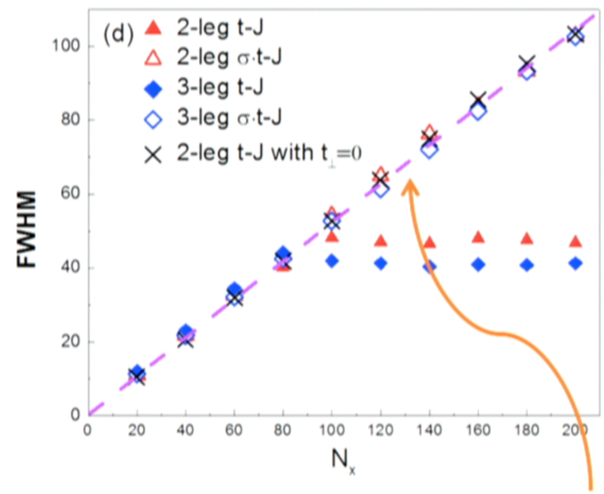

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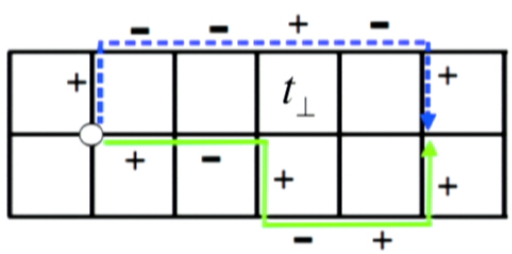

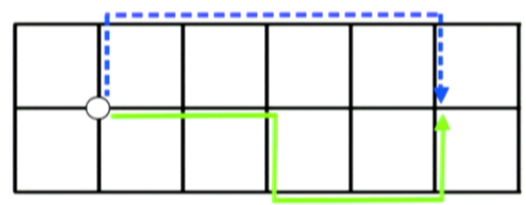

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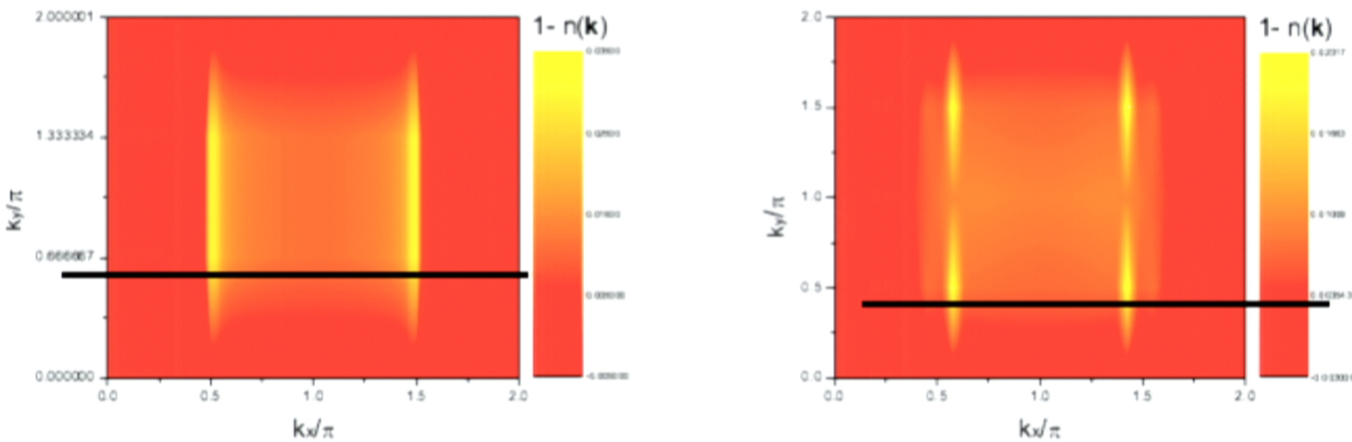
σ

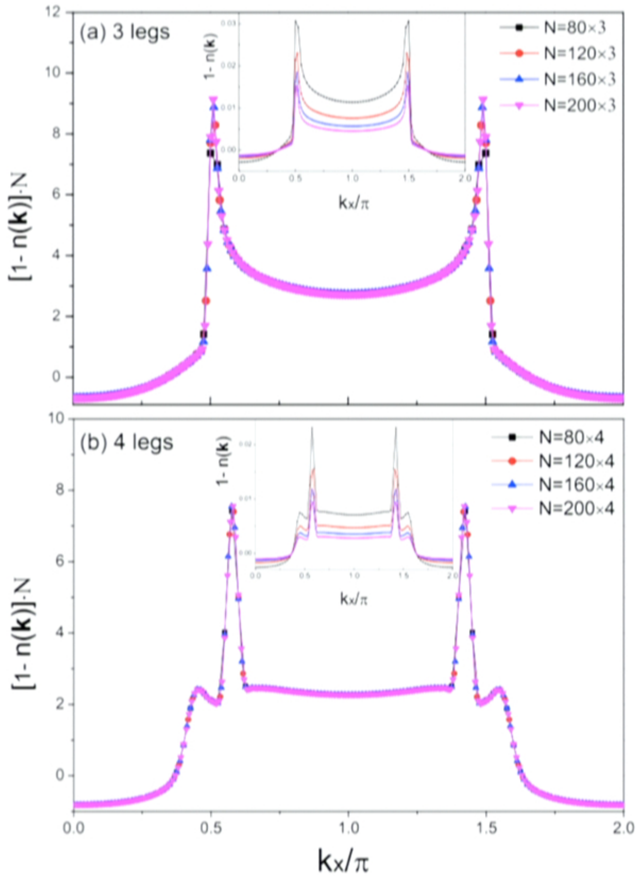
Picturing the Fermi surface



(a) N=80×3 (b) N=80×4

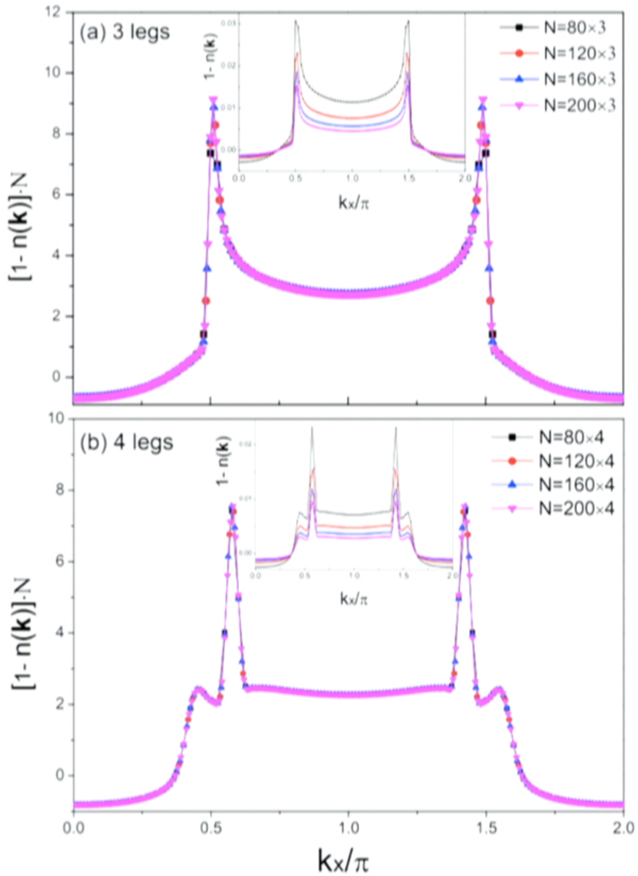
- DMRG gives n(k) of the ground state
- Jump in n(k) → Fermi surface

Vanishing quasiparticle weight



- 1-n(k) ~ 1/N
- peak height ~ 1/N
- peak width ~ const
- quasiparticle within the localized region

Vanishing quasiparticle weight



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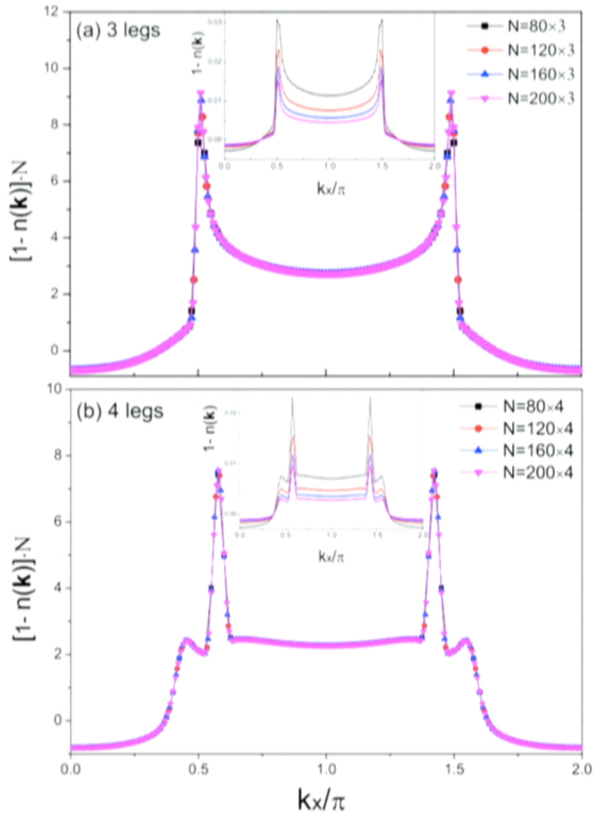
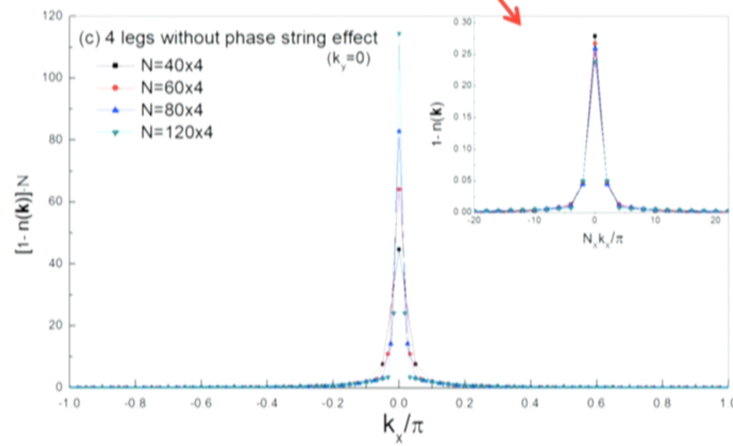
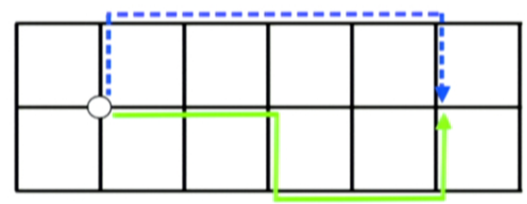
Momentum distribution



Quasiparticle picture restored!



without phase string effect



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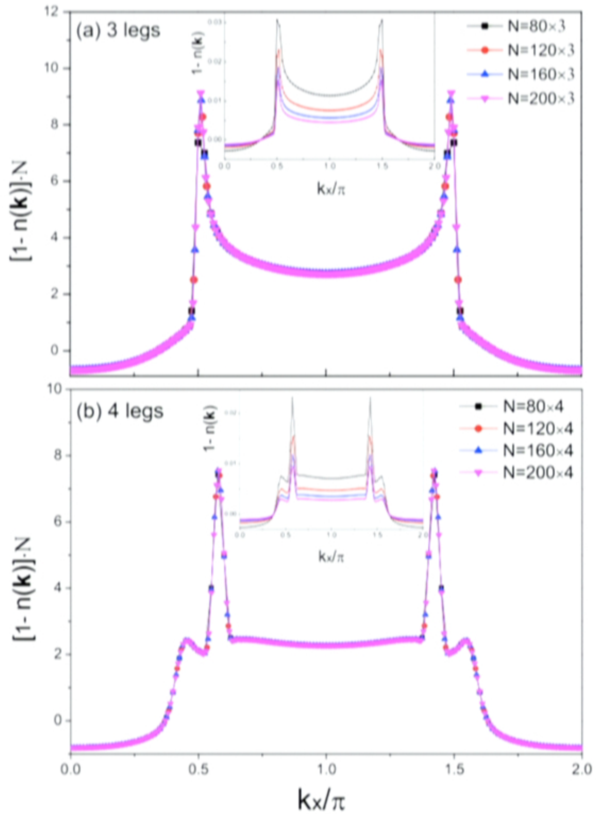

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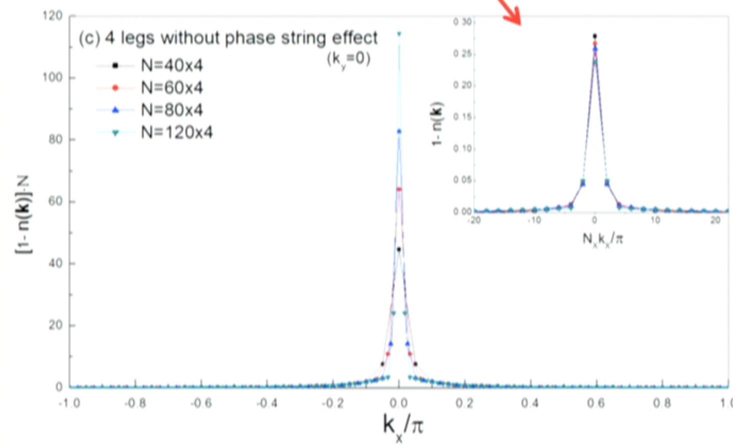

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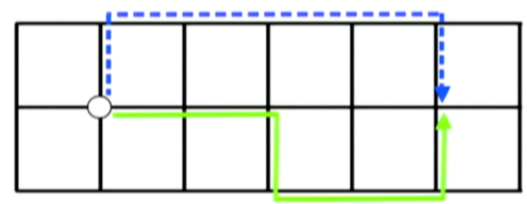

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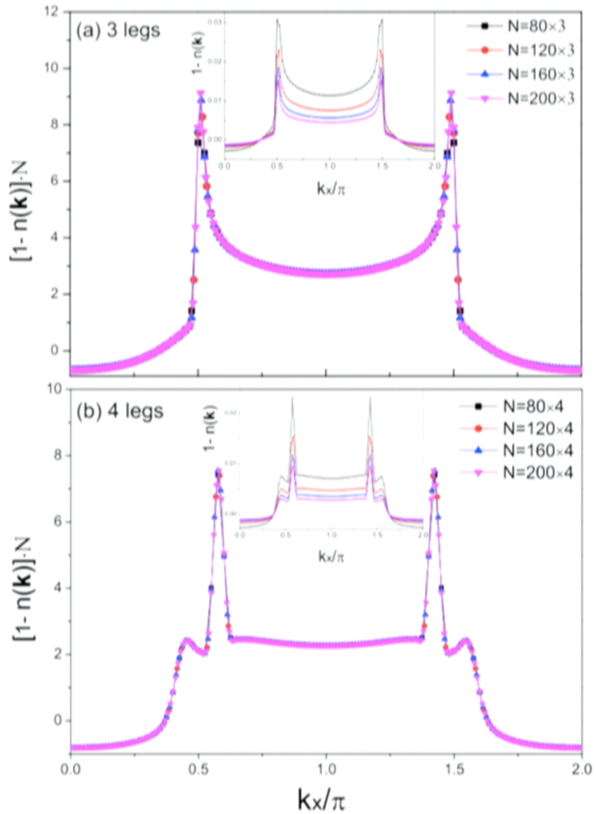

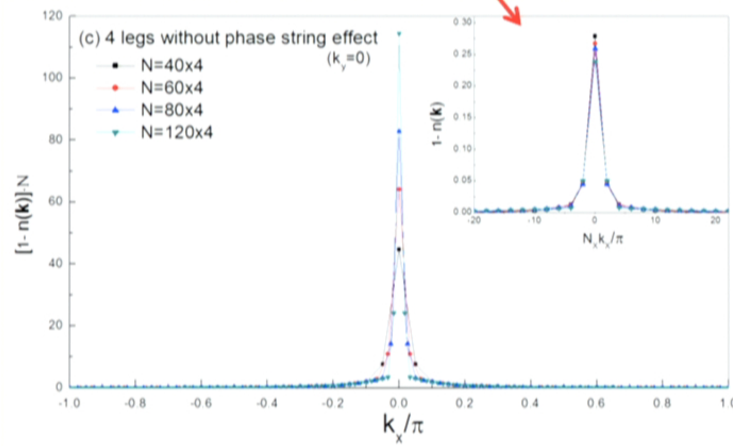

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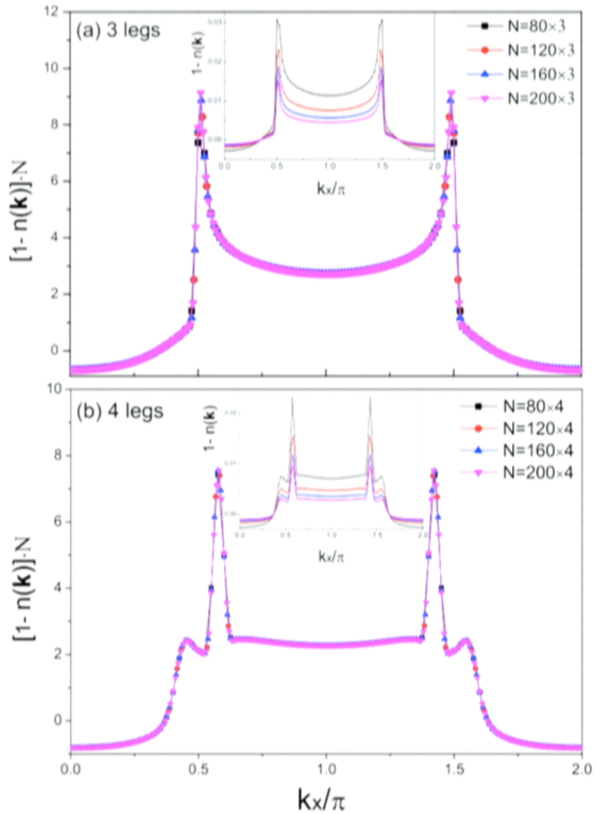

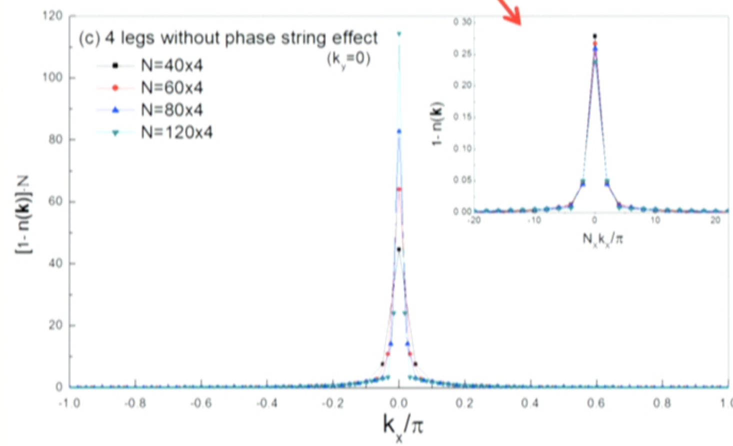

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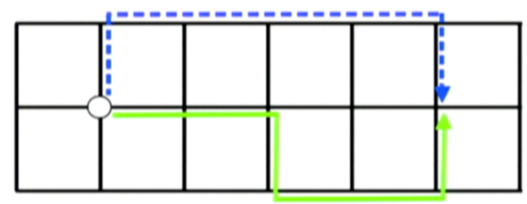

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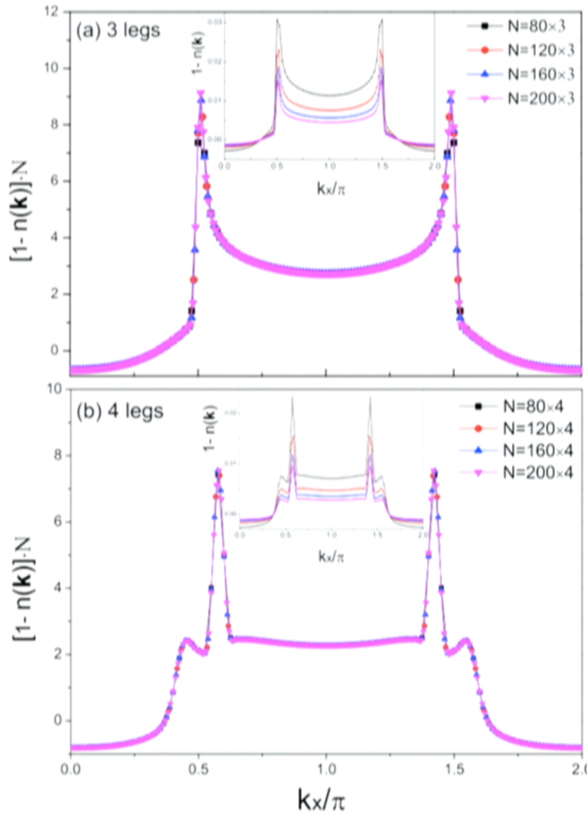

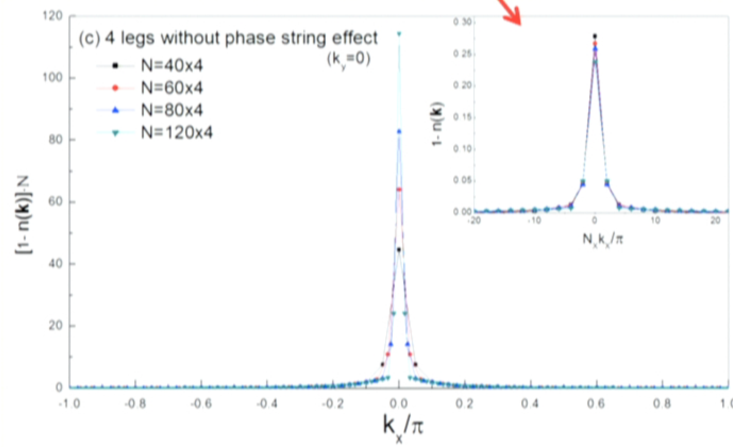

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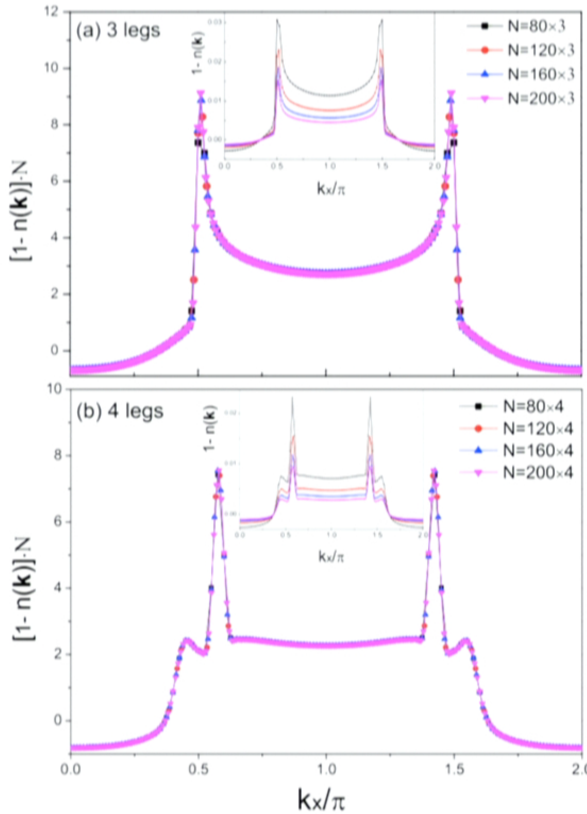

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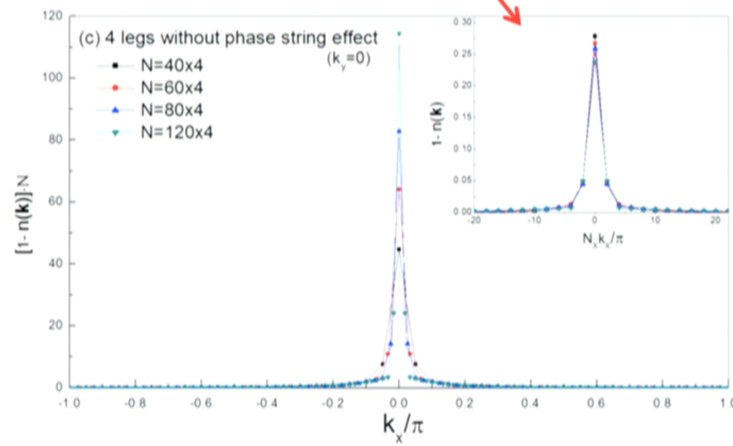
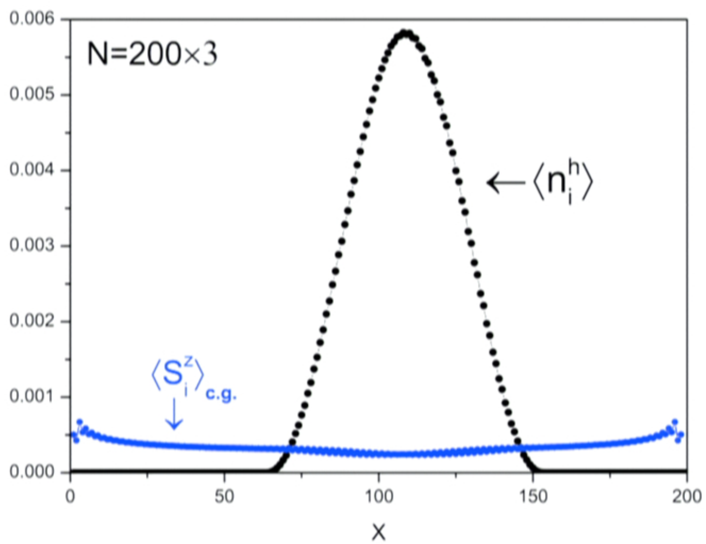
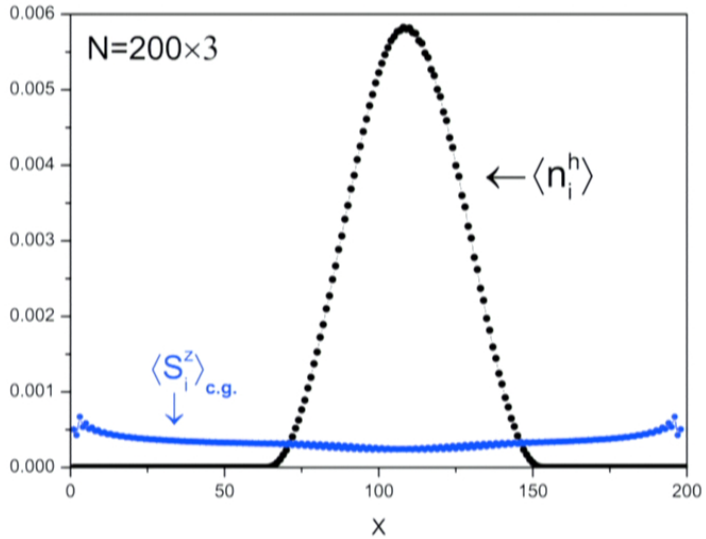


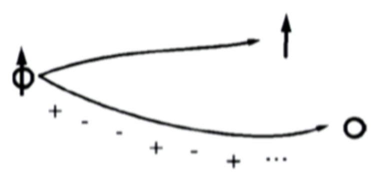
without phase string effect

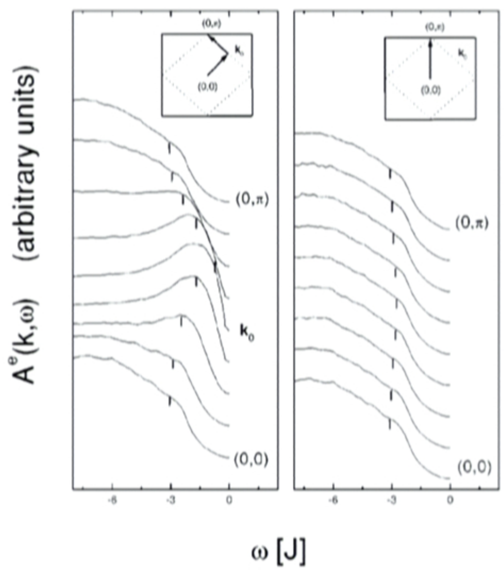
Spin-charge separation



Spin-charge separation





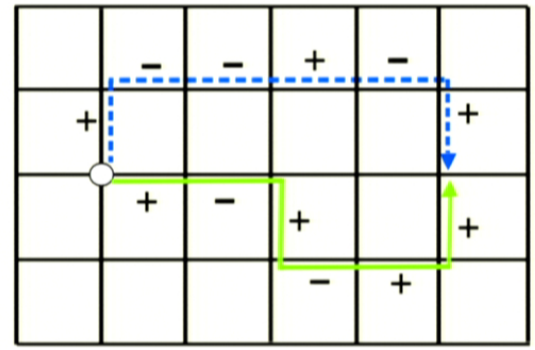


ZYW, V. N. Muthukumar, D.N. Sheng, C.S. Ting (2001)

Implications

Emergent gauge force in doped Mott insulators!

$$(+1)\times(-1)\times(-1)\times\ldots\ldots \equiv \tau_c$$



Partition function

$$Z = \sum_{loop\ c} \tau_c W(c) \qquad\qquad W(c) \geq 0$$

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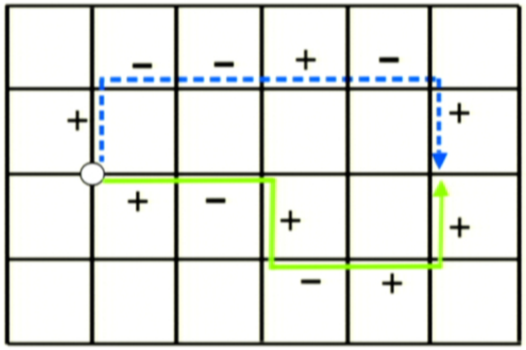

C. N. Yang (1974) , Wu and Yang (1975)

Nonintegrable phase factor:

$$P e^{i\frac{e}{\hbar c}\int_A^B A_\mu dx^\mu}$$

"An intrinsic and complete description of electromagnetism"

"Gauge symmetry dictates the form of the fundamental forces in nature"

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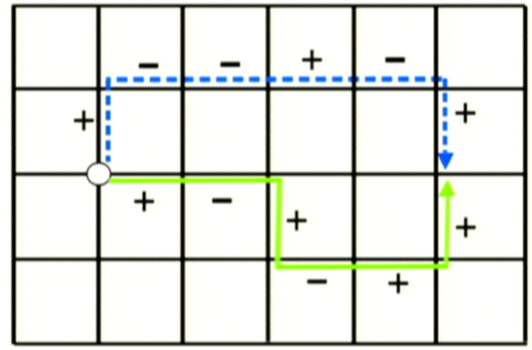

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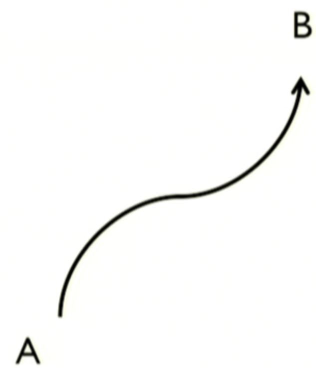

"An intrinsic and complete description of electromagnetism"

"Gauge symmetry dictates the form of the fundamental forces in nature"

At arbitrary doping, dimensions, temperature: t-J model

$$Z = \sum_c \tau_c \mathcal{Z}(c)$$

$$\tau_c = (-1)^{N_h^{\downarrow}(c)} \times (-1)^{N_h^h(c)}$$

$$\mathcal{Z}[c] = \left(\frac{2t}{J}\right)^{M_h[c]} \sum_n \frac{(\beta J/2)^n}{n!} \delta_{n, M_h + M_{\uparrow\downarrow} + M_Q}$$

$M_h(C)$ = total steps of hole hoppings

$M_{\uparrow\downarrow}(C)$ = total number of spin exchange processes

$M_Q(C)$ = total number of opposite spin encounters

$$\mathcal{Z}(c) \geq 0$$

Wu, Weng, Zaanen, PRB (2008)

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General sign struture rule:

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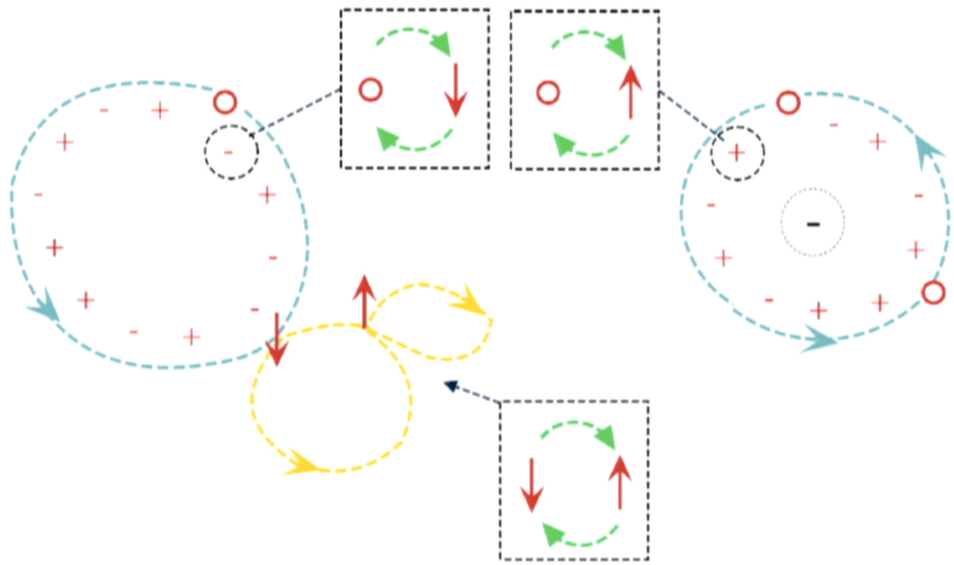
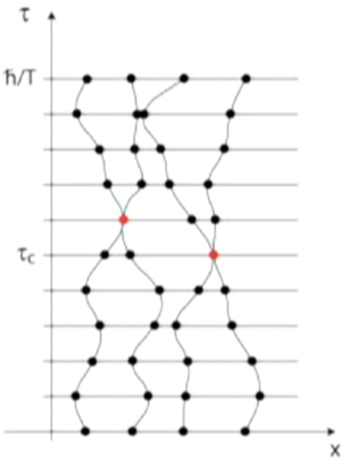


Feynman's path-integral

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}})$$
$$= \int \mathrm{d}\mathbf{R}\,\rho(\mathbf{R},\mathbf{R};\beta)$$

Fermion signs



$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N)$$

$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$
$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

t-J model:

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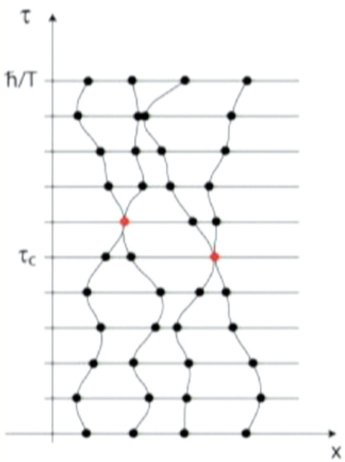

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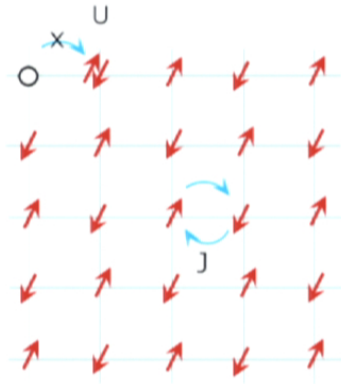
Trivial limits of phase string effect

Half-filling:



$$Z_{t-J} = \sum_c \tau_c \mathcal{Z}[c]$$

$\Uparrow$

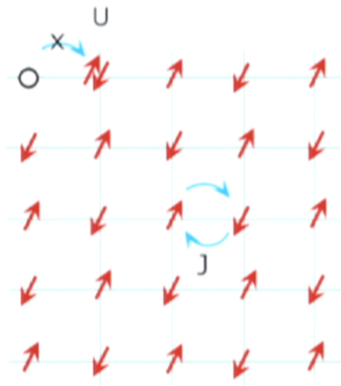
$$\tau_c = 1$$

no sign problem $N_h^{\downarrow}(c) = 0$

antiferromagnetic ground state

Trivial limits of phase string effect

Half-filling:



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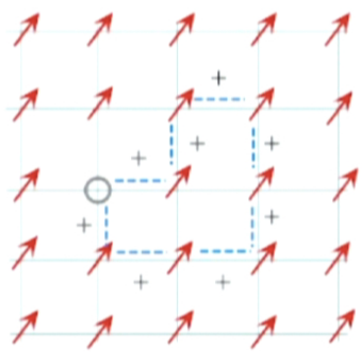
no sign problem $N_h^{\downarrow}(c) = 0$

antiferromagnetic ground state

Nagaoka state (J=0)



$$N_h^{\downarrow}(c) = 0$$

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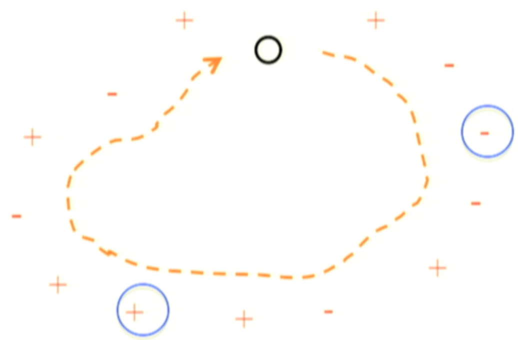
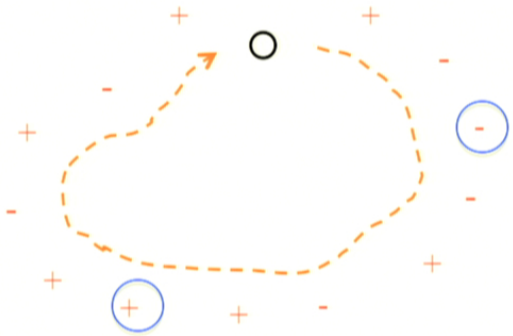
no sign problem



one hole

self-localization



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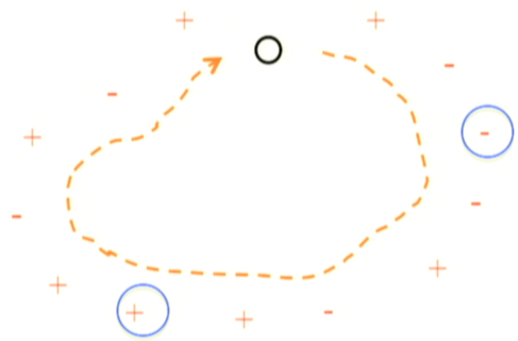

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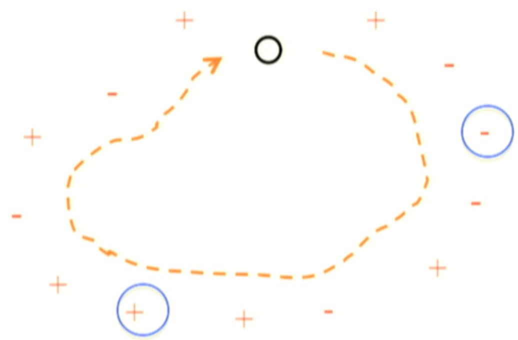

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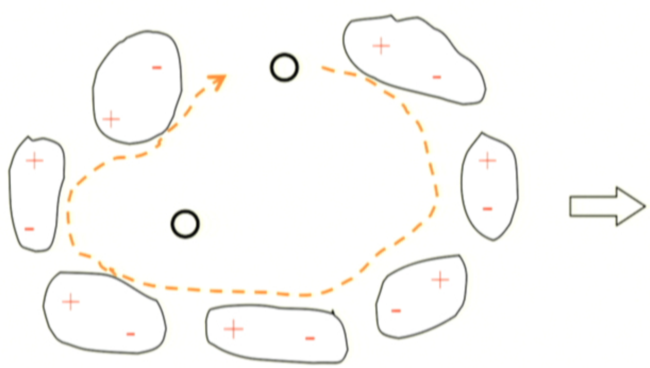
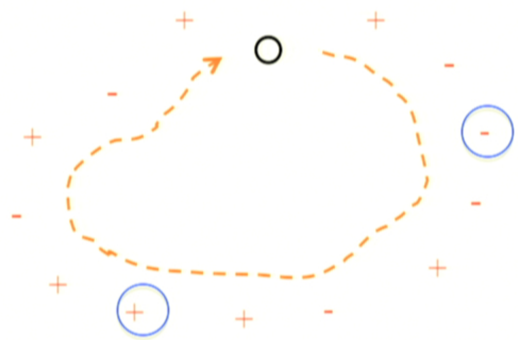


finite doping

RVB/Superconducting

minimizing the total exchange and kinetic energy



one hole

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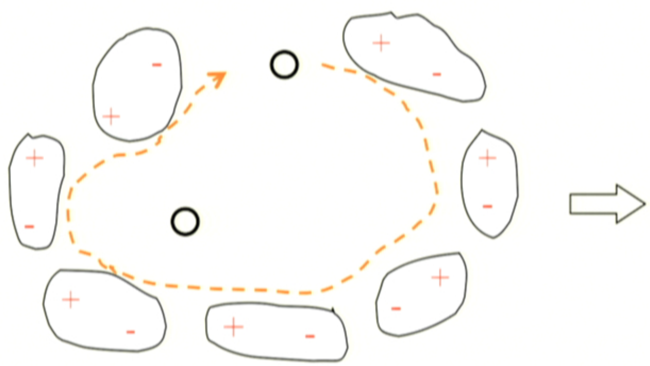

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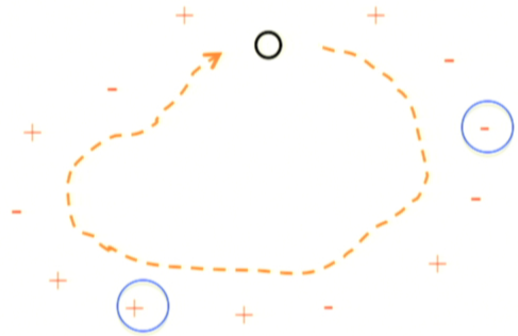

one hole

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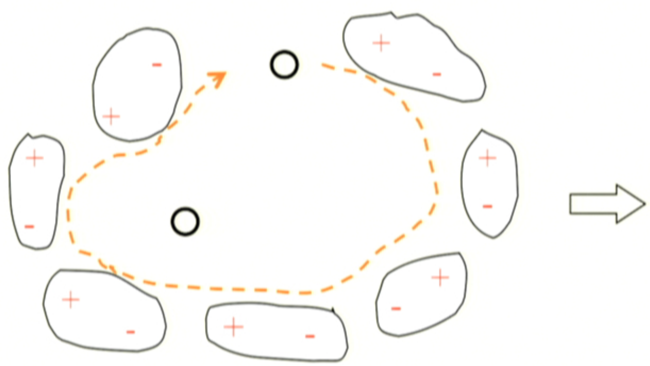

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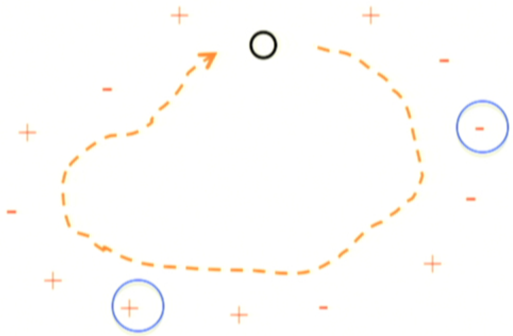

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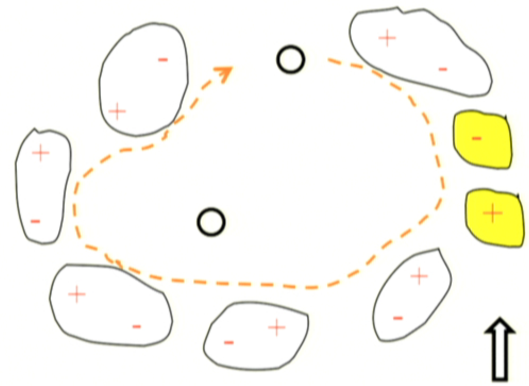
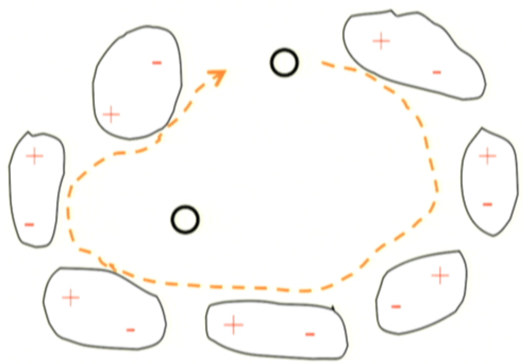


Spin-roton effect



finite doping

RVB/Superconducting

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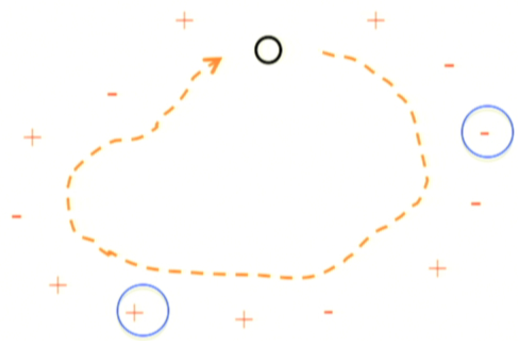
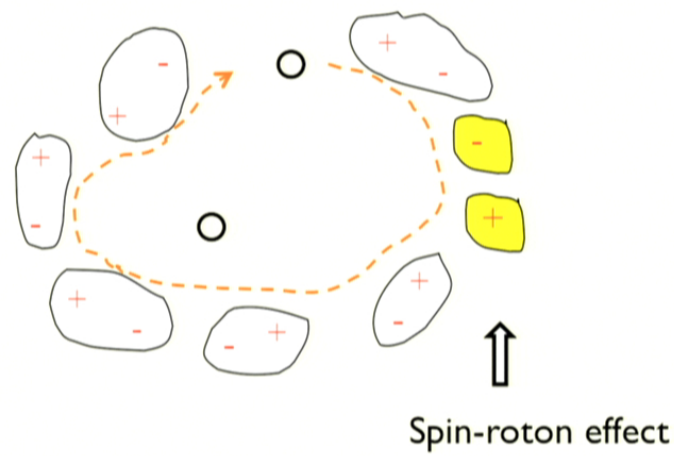
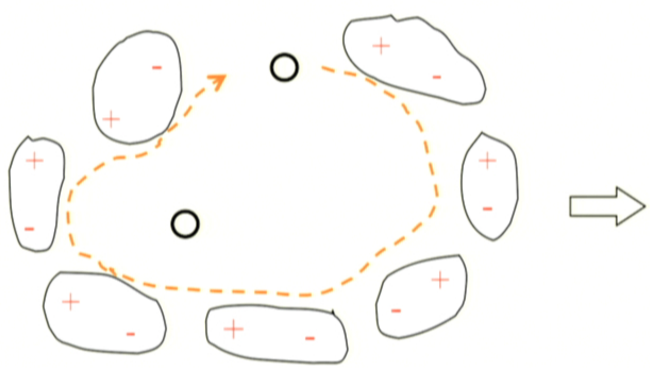
Long-range entanglement between charge and spin!



one hole

self-localization





finite doping

RVB/Superconducting

minimizing the total exchange
and kinetic energy

Perspective

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Z.Y. Weng, New J. Phys. 13 (2011) 103039

Conclusion

- Nonintegral phase factor (sign structure) dictates (1-hole) doped Mott physics:

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  $$e^{i\frac{e}{\hbar c}\int_A^B A_\mu dx^\mu} \Longrightarrow (+1)\times(-1)\times(-1)\times......$$

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- Examples:

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  2) One-hole (J=0): Nagaoka state (no sign problem)

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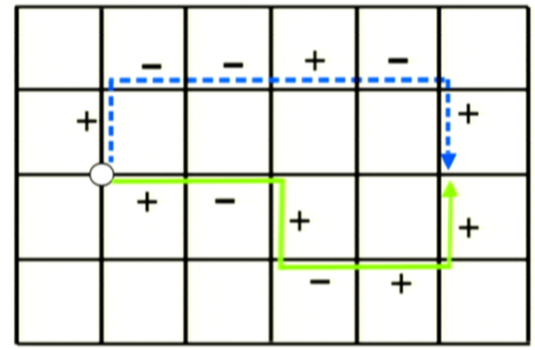
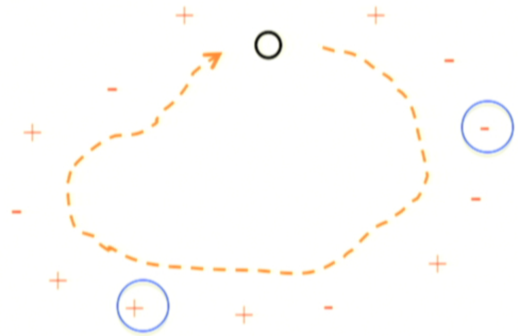
Emergent gauge force in doped Mott insulators!

$$(+1)\times(-1)\times(-1)\times\ldots\ldots\equiv\tau_c$$



Partition function

$$Z=\sum_{loop\ c}\tau_c W(c) \qquad W(c)\geq 0$$



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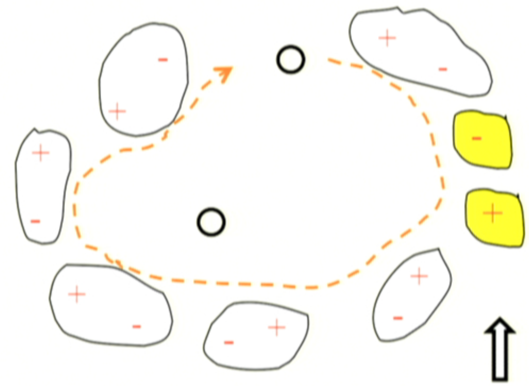

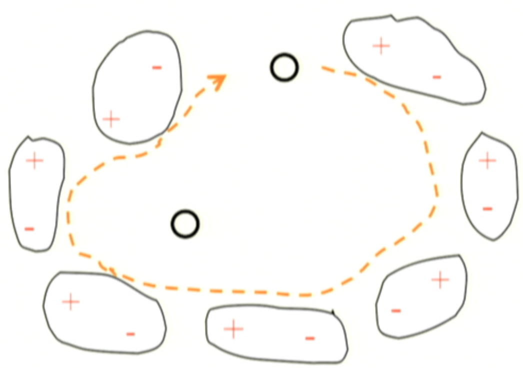

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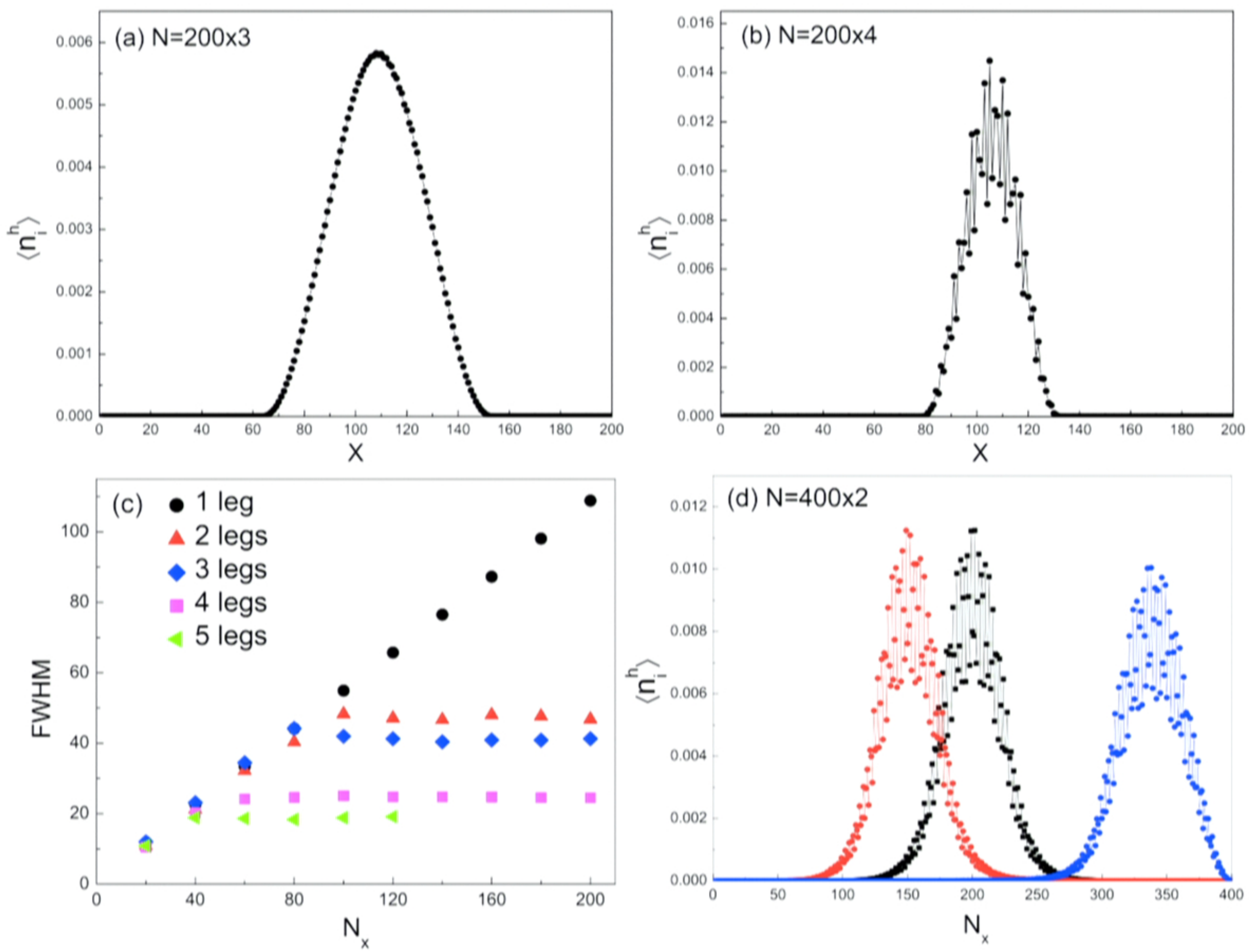
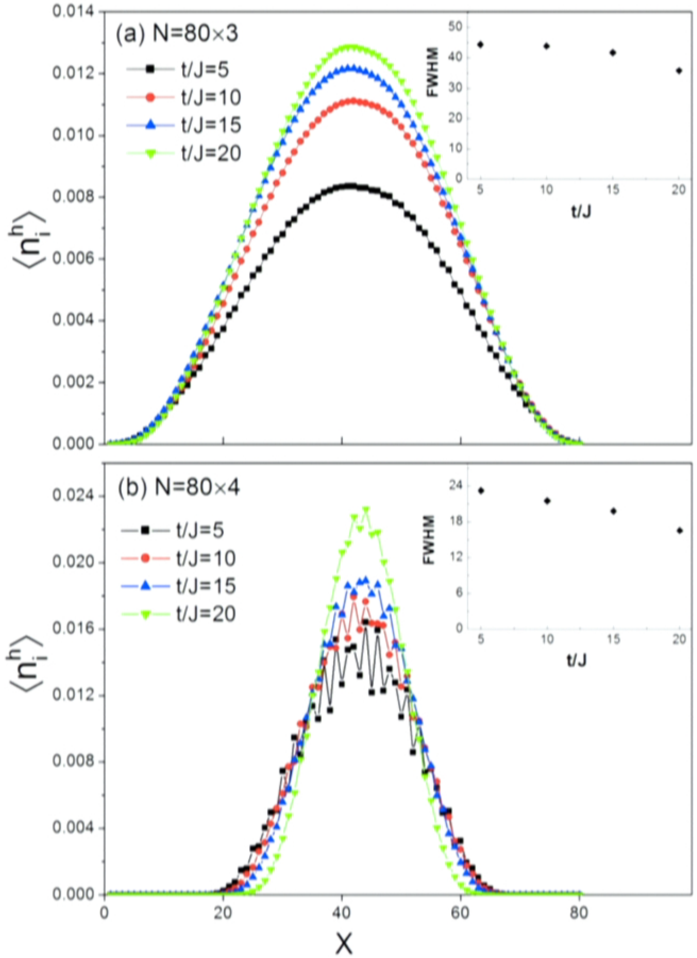
Single-hole problem: A DMRG calculation



Localization with the ratio t/J



$$\sum_i \langle n_i^h \rangle = 1$$

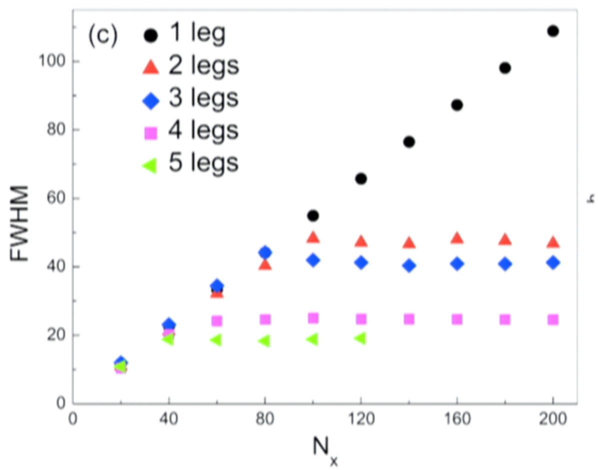
1. Localization length is monotonically reduced as t/J increases

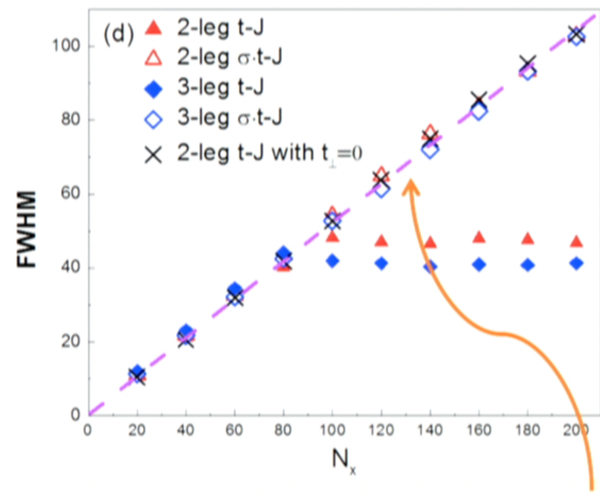
- **spin dynamics is not essential to the hole localization.**

2. Oscillation for the even-leg ladders diminish as t/J increases

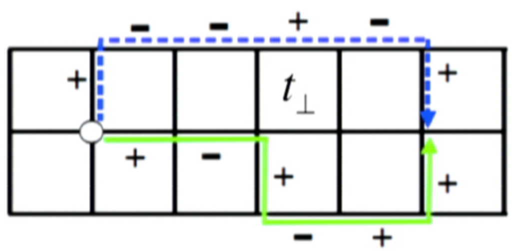
- **the spin-gap effect will be gradually reduced with the increase of t/J**

Effect of phase string effect





no phase string effect



Self-localization of the hole!

$$H = -t \sum_{\langle ij \rangle} \left(c^{\dagger}_{i\sigma} c_{j\sigma} + h.c. \right) + J \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right)$$

σ