Title: Towards an Asymptotically AdS Description of Heavy Ion Collisions

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Abstract: I will discuss recent work in simulating asymptotically anti-de Sitter spacetimes, and its relation to heavy ion collider physics. For this purpose, I intend to focus on a class of oblately deformed black hole spacetime solutions. For each of these solutions, I will map the gravitational metric in the spacetime bulk to a stress tensor one-point function of the conformal field theory defined on the spacetime boundary. During the ring-down process, wherein the deformed black hole settles down to the AdS analog of the Schwarzschild solution, I will exhibit evidence that the dual CFT stress tensor on the boundary is that of an N=4 SYM fluid, even for black holes of significant deformation well outside the perturbative regime. We will conformally map the boundary fluid onto a real-world fluid in Minkowski space, and discover a temperature profile which can be thought of as approximating that of a head-on heavy ion collision at its moment of impact. I will close with a description of recent parallel explorations.

Towards an Asymptotically AdS Description of Heavy Ion Collisions

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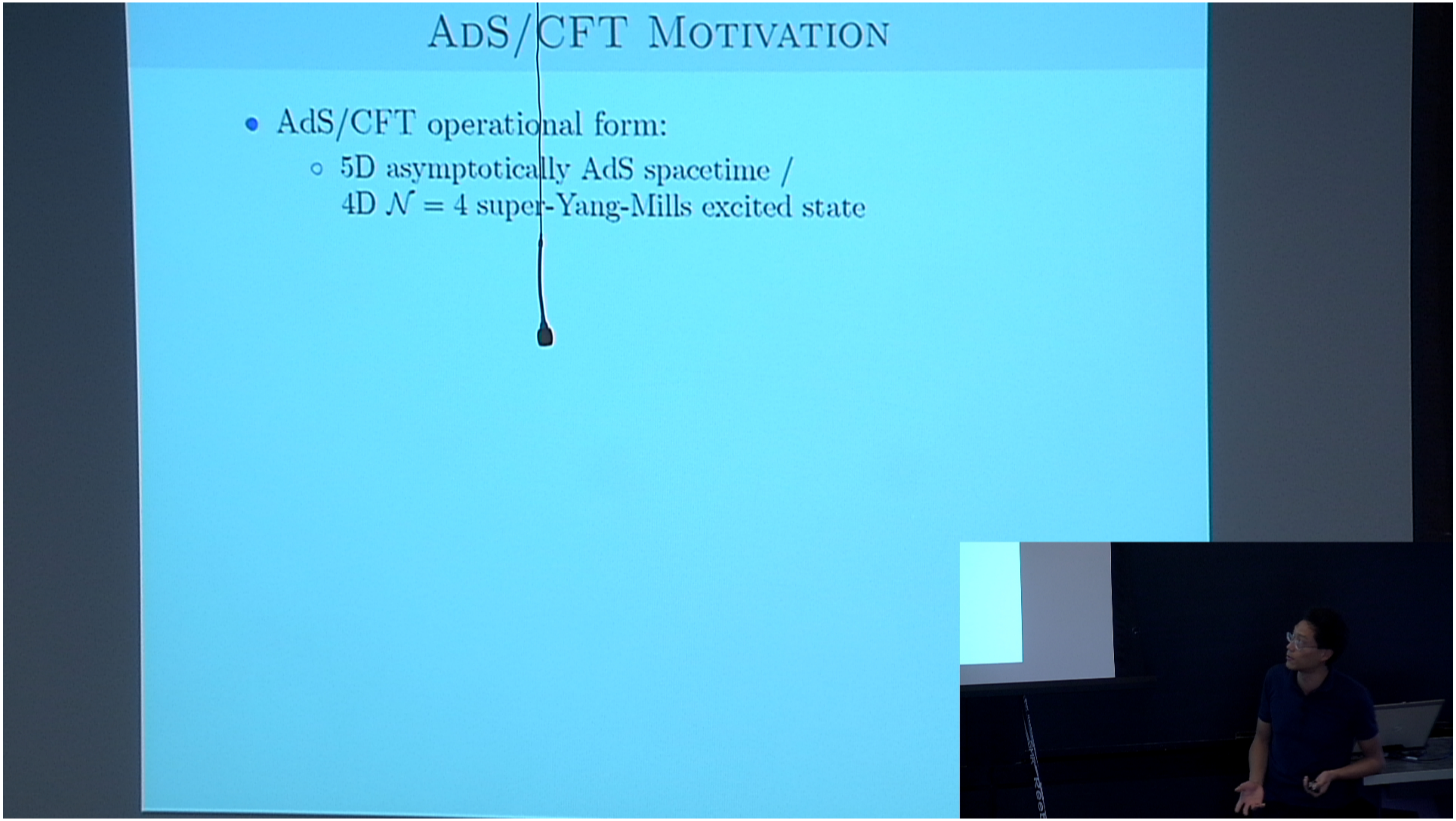
Princeton University

October 18, 2012

Outline

- AdS/CFT Motivation
- Background
- Method
- A Concrete Example
  - deformed AdS black hole, designed so that the dynamics of its dual CFT is reminiscent of a head-on heavy ion collision *(work with Frans Pretorius, Steve Gubser)*
  - boundary stress tensor consistent with an $\mathcal{N} = 4$ SYM fluid
  - minkowski flows reminiscent of quark gluon plasma flows
- Ongoing Explorations
  - Cooper-Frye particle spectrum *(work with JP Ang)*
  - BH spacetimes massive self-interacting scalar fields *(work with Frans Pretorius, Steve Gubser)*
  - non-BH spacetimes


AdS/CFT Motivation
AdS/CFT operational form:
5D asymptotically AdS spacetime /
4D $\mathcal{N} = 4$ super-Yang-Mills excited state

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- Hope is that AdS/CFT can eventually be used to make experimentally testable predictions about QCD processes
  - heavy ion physics at the Relativistic Heavy Ion Collider (RHIC), and the Large Hadron Collider (LHC)
  - a non-perturbative QCD problem: gravity description via AdS/CFT is desirable
- Previous work to relate heavy-ion collisions to gravity
  - heavy-ion collision in terms of a gravity dual[1,2]
  - field theory description of global AdS-Schwarzschild[3]
  - linearized gravity about global AdS-Schwarzschild[4]

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[3] *JHEP* **02** (1999) 010, G. Horowitz, N. Itzhaki
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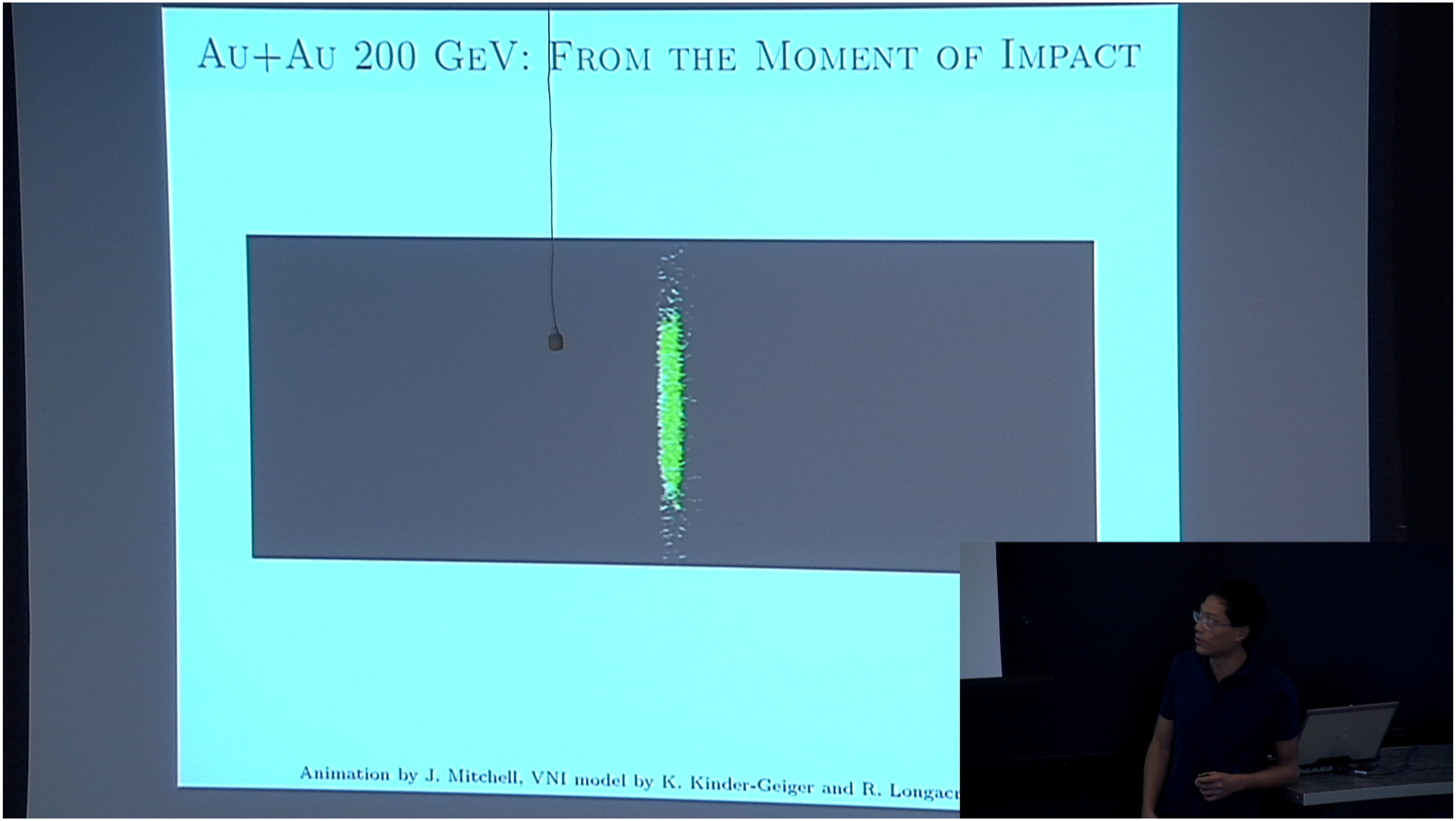
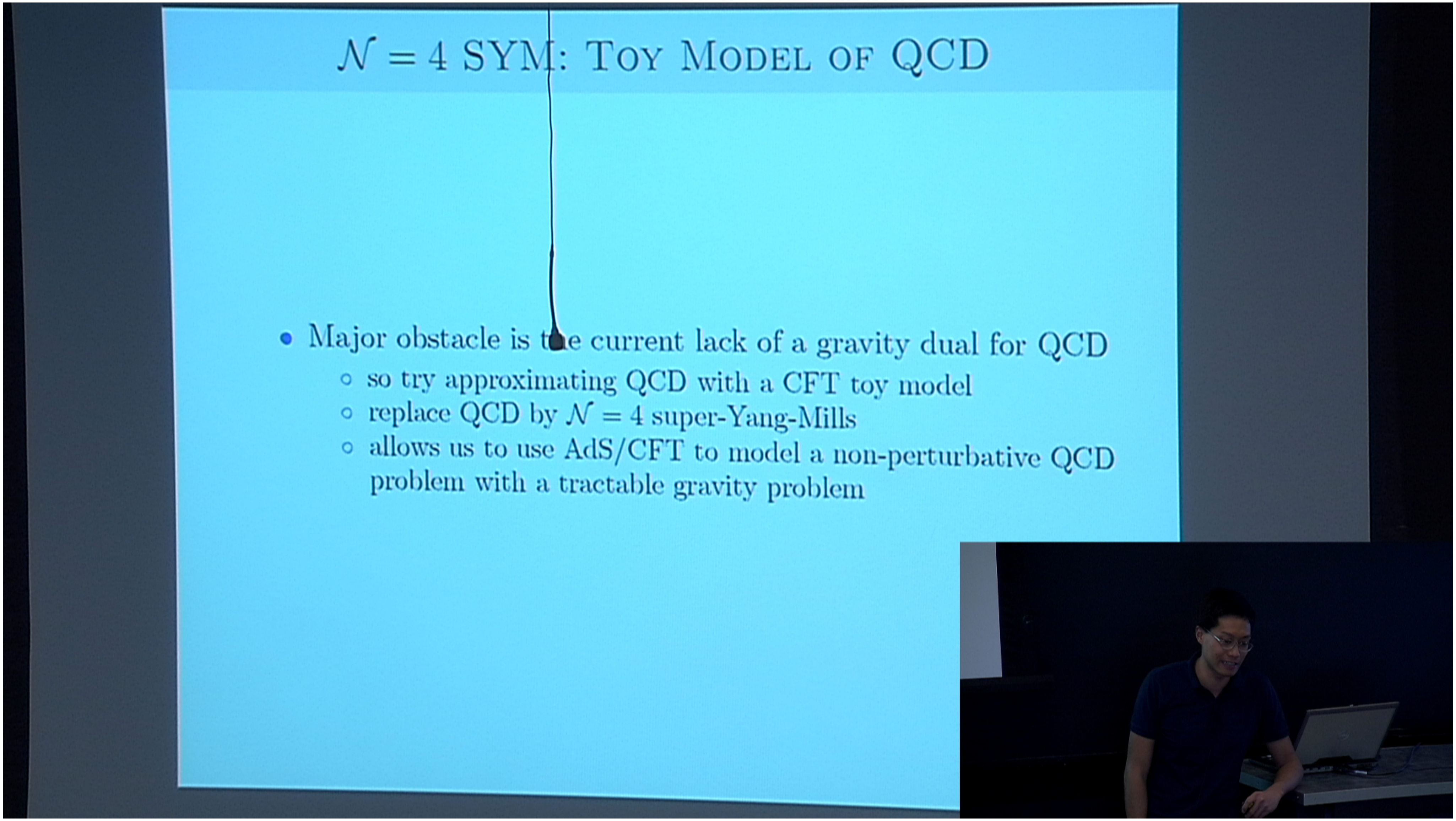
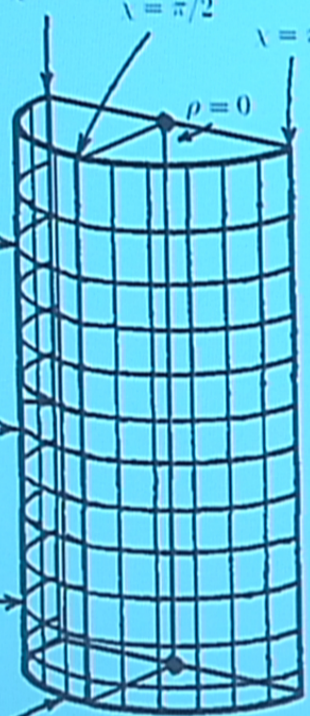

Au+Au 200 GeV: From the Moment of Impact
Animation by J. Mitchell, VNI model by K. Kinder-Geiger and R. Longacr


$\mathcal{N} = 4$ SYM: Toy Model of QCD
Major obstacle is the current lack of a gravity dual for QCD
so try approximating QCD with a CFT toy model
replace QCD by $\mathcal{N} = 4$ super-Yang-Mills
allows us to use AdS/CFT to model a non-perturbative QCD problem with a tractable gravity problem

The Anti-de Sitter Spacetime



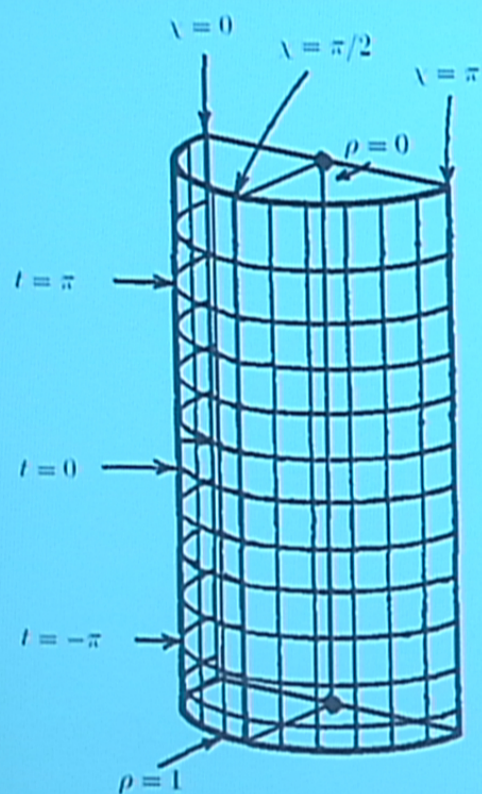
- The AdS_5 spacetime ($D = 5$, $\hat{R} < 0$)
  - $\hat{R}_{\kappa\lambda\mu\nu} = \frac{\hat{R}}{D(D-1)} (\hat{g}_{\kappa\mu}\hat{g}_{\lambda\nu} - \hat{g}_{\kappa\nu}\hat{g}_{\lambda\mu})$
  - boundary ($\rho = 1$)
    - refers to spacelike and null infinity
    - forms a timelike 4-surface
    - has $\mathbb{R} \times S^3$ topology
    - is causally connected to interior
  - SO(4,2) symmetry
  - metric in local coordinates:

$$\hat{g} = \frac{1}{(1-\rho)^2}\left(-\hat{f}(\rho)dt^2 + \frac{1}{\hat{f}(\rho)}d\rho^2 + \rho^2 d{\Omega_3}^2\right)$$

FIGURE: Conformal sketch of the AdS_5 spacetime; there is an internal 2-sphere geometry at each point of this sketch.

Here, $\hat{f}(\rho) = (1-\rho)^2 + \rho^2/L^2$.

An Asymptotically Anti-de Sitter Spacetime



- An asymptotically AdS_5 spacetime
  - ~~$R_{\kappa\lambda\mu\nu} = \frac{R}{D(D-1)}(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu})$~~
  - boundary ($\rho = 1$)
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  - metric in local coordinates:

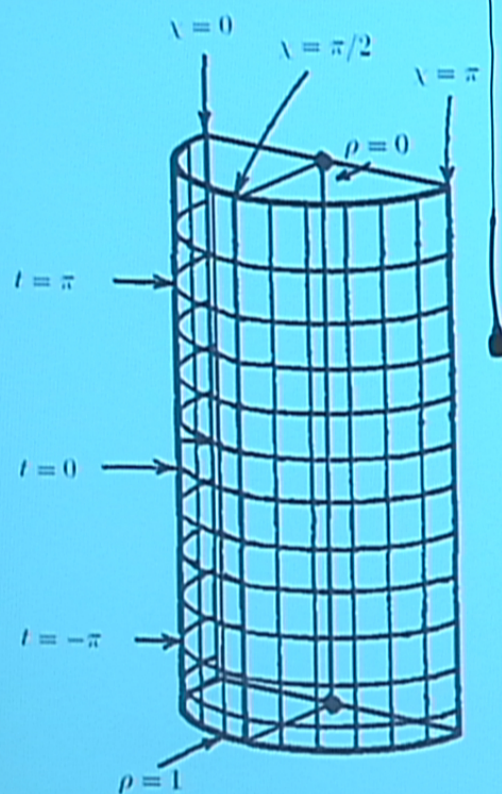
$$g = g_{ab}dx^a dx^b + \Psi^2 d\Omega_2^2$$

$$\begin{cases} g_{\rho\rho} - \hat{g}_{\rho\rho} \sim (1-\rho)^2 \\ g_{mn} - \hat{g}_{mn} \sim (1-\rho)^2 \\ g_{\rho m} - \hat{g}_{\rho m} \sim (1-\rho)^3 \end{cases} \quad \text{as} \quad \rho \to 1$$

FIGURE: Conformal sketch of an asymptotically AdS_5 spacetime with an SO(3) symmetry that acts to rotate the 2-sphere at each point.

Here, $x^a = (t, \rho, \chi)$ and $x^m = (t, \chi, \theta, \phi)$.

An Asymptotically Anti-de Sitter Spacetime



- An asymptotically AdS_5 spacetime
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  - metric in local coordinates:

$$g = g_{ab}dx^a dx^b + \Psi^2 d\Omega_2^2 = \hat{g} + h$$

$$\left\{ \begin{array}{l} g_{\rho\rho} - \hat{g}_{\rho\rho} \sim (1-\rho)^2 \\ g_{mn} - \hat{g}_{mn} \sim (1-\rho)^2 \quad \text{as} \quad \rho \to 1 \\ g_{\rho m} - \hat{g}_{\rho m} \sim (1-\rho)^3 \end{array} \right.$$

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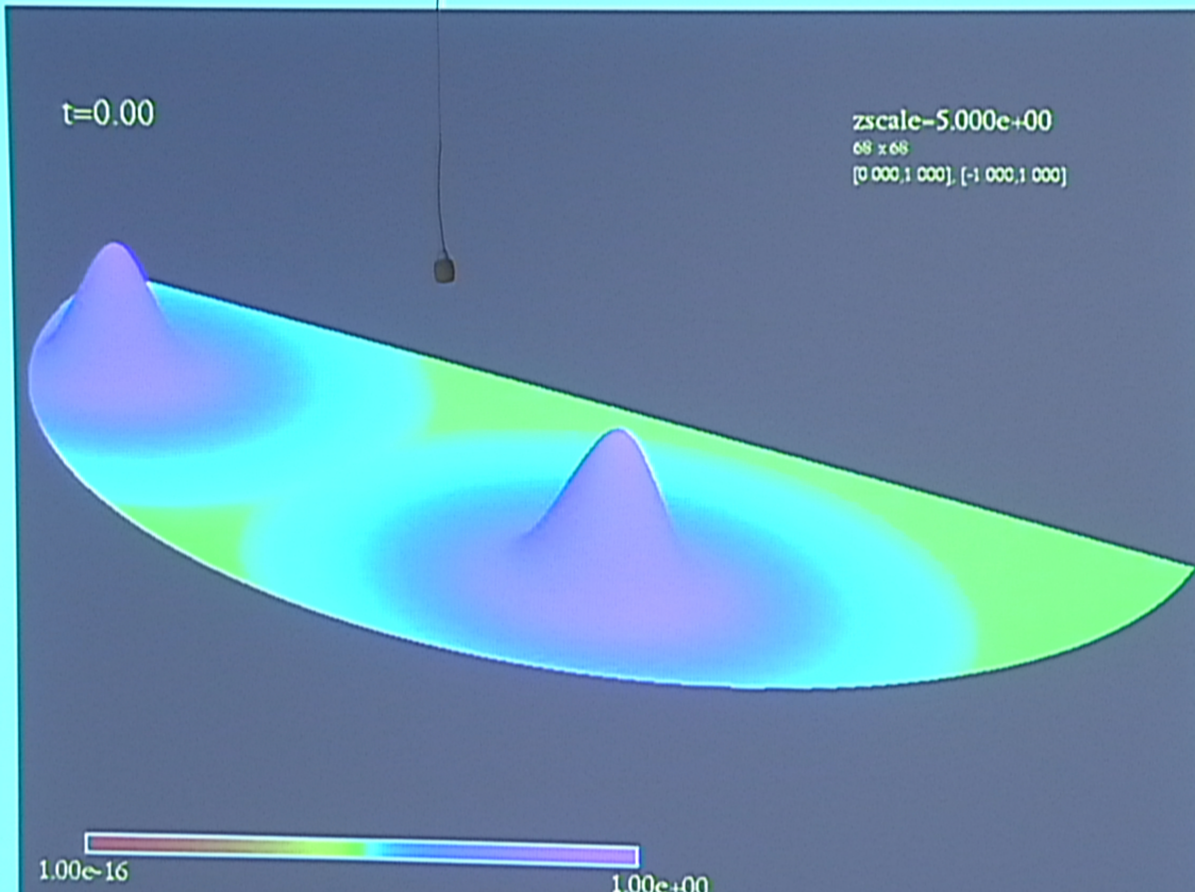
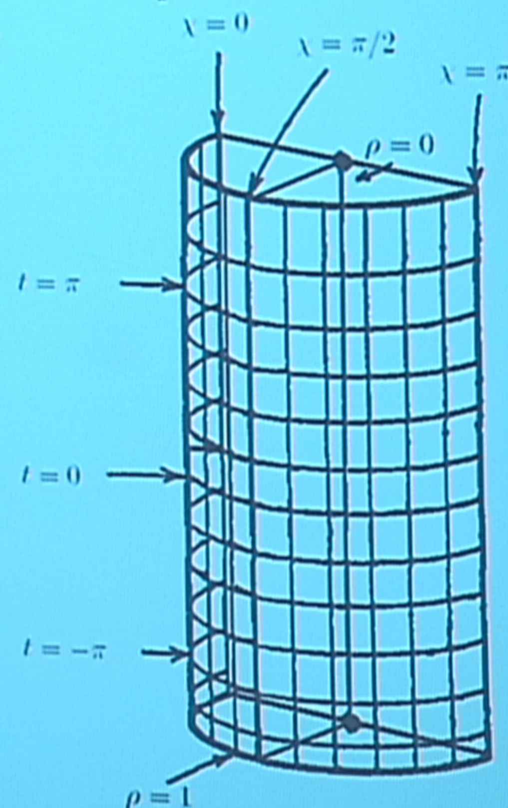
An Asymptotically AdS Black Hole Accreting Massless Scalar Lumps



Conformal Field Theory Dual

- Bulk field / CFT operator
  - e.g. metric dynamics / CFT stress tensor one-point function
    $\bar{g}_{\mu\nu}(x^m, \rho)$ $\langle T_{\mu\nu}(x^m) \rangle_{\text{CFT}}$
- Strategy: gravity $\rightarrow$ field theory
- How do bulk fields encode boundary CFT operators?



Conformal Field Theory Dual

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    $\bar{g}_{\mu\nu}(x^m, \rho)$ $\qquad$ $\langle T_{\mu\nu}(x^m) \rangle_{\mathrm{CFT}}$
- Strategy: gravity $\rightarrow$ field theory
- How do bulk fields encode boundary CFT operators?

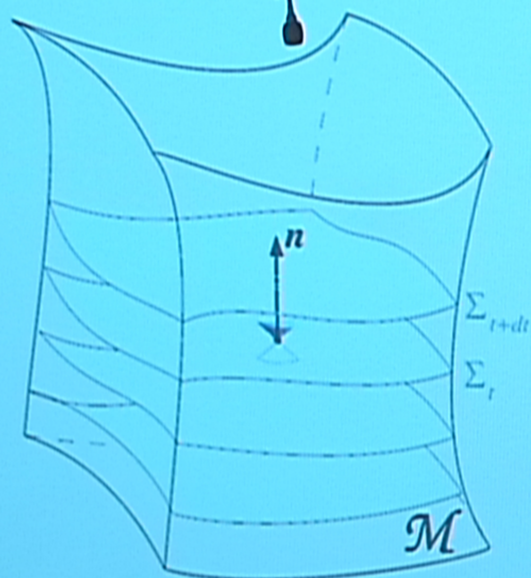
e.g.

$$\langle T_{\mu\nu} \rangle_{\mathrm{CFT}} = \lim_{\rho \to 1} \left[\frac{1}{8\pi} \left({}^{(\rho)}\Theta_{\mu\nu} - {}^{(\rho)}\Theta \Sigma_{\mu\nu} - \frac{3}{L} \Sigma_{\mu\nu} + {}^{(\rho)}G_{\mu\nu} \frac{L}{2} \right) - t_{\mu\nu} \right]$$

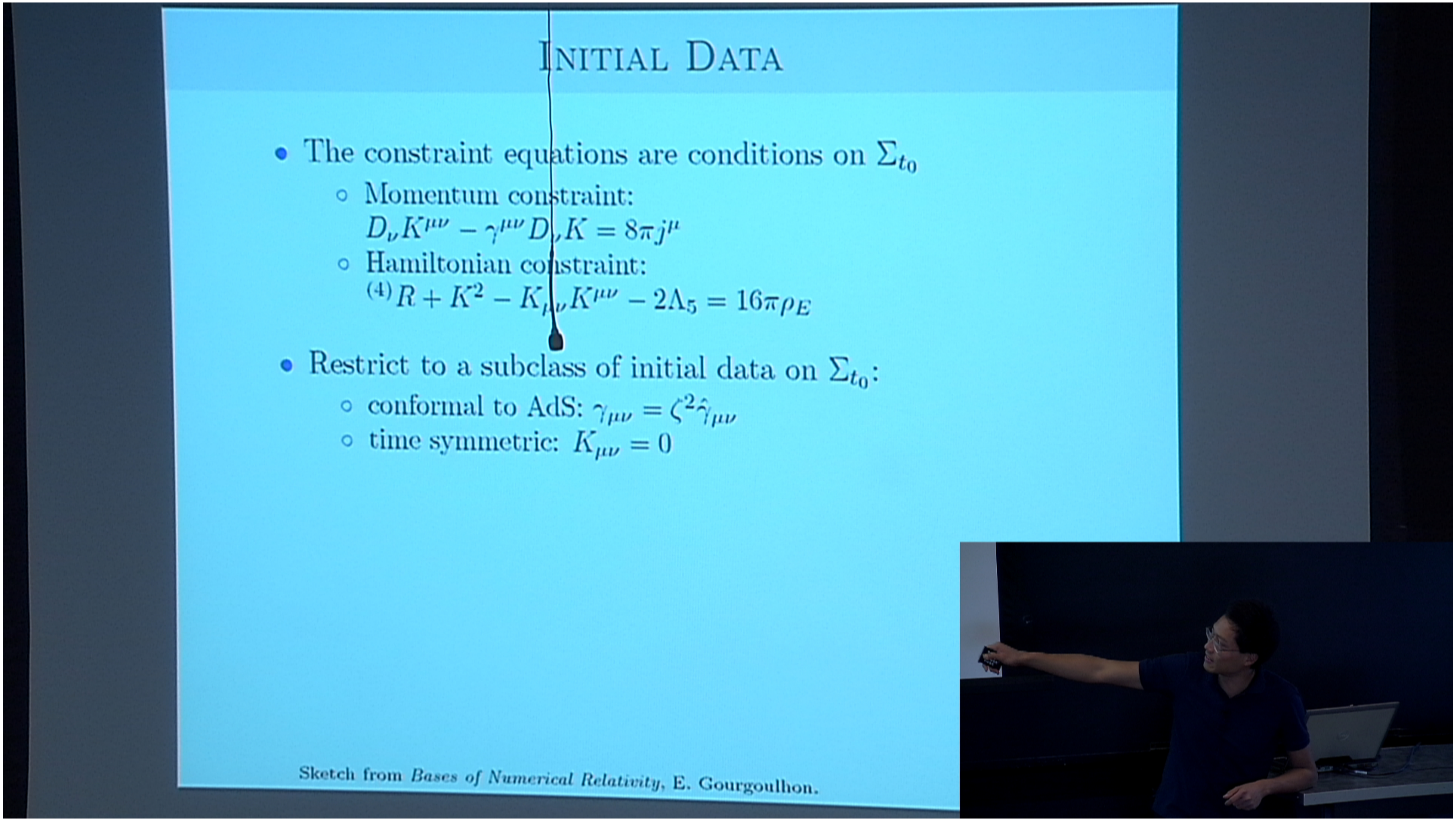
Given a $\rho = \text{const.}$ time-like hypersurface ∂M_ρ, ${}^{(\rho)}\Theta_{\mu\nu} = -\Sigma^\alpha{}_\mu \Sigma^\beta{}_\nu \nabla_{(\alpha} S_{\beta)}$ is the extrinsic curvature of ∂M_ρ, S^μ is a space-like, outward pointing unit vector normal to the surface ∂M_ρ, $\Sigma_{\mu\nu} \equiv g_{\mu\nu} - S_\mu S_\nu$ is the induced 4-metric on ∂M_ρ, ∇_α is the covariant derivative operator, and ${}^{(\rho)}G_{\mu\nu}$ is the Einstein tensor associated with $\Sigma_{\mu\nu}$. Setting $L = 1$, the non-zero components of the (non-dynamical) Casimir contribution $t_{\mu\nu}$ that we have explicitly subtracted above are $t_{tt} = 3(1-\rho)^2/(64\pi)$, $t_{\chi\chi} = (1-\rho)^2/(64\pi)$, $t_{\theta\theta} = (1-\rho)^2 \sin^2\chi/(64\pi)$, and $t_{\phi\phi} = t_{\theta\theta} \sin^2\theta$.

Initial Data

- The constraint equations are conditions on Σ_{t_0}
  - Momentum constraint:
    $D_\nu K^{\mu\nu} - \gamma^{\mu\nu} D_\nu K = 8\pi j^\mu$
  - Hamiltonian constraint:
    ${}^{(4)}R + K^2 - K_{\mu\nu}K^{\mu\nu} - 2\Lambda_5 = 16\pi\rho_E$



Sketch from *Bases of Numerical Relativity*, E. Gourgoulhon.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Restrict to a subclass of initial data on Σ_{t_0}:
conformal to AdS: $\gamma_{\mu\nu} = \zeta^2\hat{\gamma}_{\mu\nu}$
time symmetric: $K_{\mu\nu} = 0$
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- Restrict to a subclass of initial data on Σ_{t_0}:
  - conformal to AdS: $\gamma_{\mu\nu} = \zeta^2 \hat{\gamma}_{\mu\nu}$
  - time symmetric: $K_{\mu\nu} = 0$
- The momentum constraints are trivially satisfied by $j^\mu = 0$ (i.e. time derivatives of matter fields vanish on Σ_{t_0})
- Left with the Hamiltonian constraint

$$\hat{\gamma}^{\alpha\beta} \hat{D}_\alpha \hat{D}_\beta \zeta - \frac{1}{3}\Lambda_D \zeta + \frac{1}{3}\left(\Lambda_D + 8\pi\rho_E\right)\zeta^3 = 0$$

Sketch from *Bases of Numerical Relativity*, E. Gourgoulhon.

Evolution

- Symmetries of the asymptotically AdS_5 solution
  - AdS_5 compactified global coordinates $x^\mu = (t, \rho, \chi, \theta, \phi)$
  - "obvious" SO(3) symmetry acts to rotate (θ, ϕ) 2-spheres
  - we consider solutions that preserve this SO(3) symmetry

- Form of metric, for $x^a = (t, \rho, \chi)$

$$g_{\mu\nu} dx^\mu dx^\nu = \tilde{g}_{ab} dx^a dx^b + \psi^2 d\Omega_2^2$$

- 7 independent components: $g_{tt}, g_{t\rho}, g_{t\chi}, g_{\rho\rho}, g_{\rho\chi}, g_{\chi\chi}, \underbrace{g_{\theta\theta}, g_{\phi\phi}}_{\bar{g}_\psi}$

Evolution

- Express metric as $g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu}$
  - pure AdS_5 $\hat{g}_{\mu\nu}$, with deviation $h_{\mu\nu}$

- Boundary conditions at $\rho = 1$:
  - preserved under the asymptotic SO(4,2) symmetry
  - include the AdS-Schwarzschild metric as a special case
  - yield finite spacetime mass

$$\begin{aligned} h_{\rho\rho} &= f_{\rho\rho}(t,\chi,\theta,\phi)(1-\rho)^2 + \ldots \\ h_{\rho m} &= f_{\rho m}(t,\chi,\theta,\phi)(1-\rho)^3 + \ldots \\ h_{mn} &= f_{mn}(t,\chi,\theta,\phi)(1-\rho)^2 + \ldots \end{aligned}$$

$$(\text{independent components}:\ h_{tt}, h_{t\rho}, h_{t\chi}, h_{\rho\rho}, h_{\rho\chi}, h_{\chi\chi}, \underbrace{h_{\theta\theta}, h_{\phi\phi}}_{\bar{g}_{\psi}})$$

Evolution

- Express metric as $g_{\mu\nu} = \hat{g}_{\mu\nu} + (1-\rho^2)^{p_{\mu\nu}} \bar{g}_{\mu\nu}$
  - pure AdS_5 $\hat{g}_{\mu\nu}$, with deviation variable $\bar{g}_{\mu\nu}$

- Boundary conditions at $\rho = 1$:
  - preserved under the asymptotic SO(4,2) symmetry
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  - yield finite spacetime mass

$$
\begin{aligned}
\bar{g}_{\rho\rho} &= f_{\rho\rho}(t,\chi,\theta,\phi)(1-\rho) + \ldots \\
\bar{g}_{\rho m} &= f_{\rho m}(t,\chi,\theta,\phi)(1-\rho) + \ldots \\
\bar{g}_{mn} &= f_{mn}(t,\chi,\theta,\phi)(1-\rho) + \ldots
\end{aligned}
$$

$$\text{(evolved variables : } \bar{g}_{tt}, \bar{g}_{t\rho}, \bar{g}_{t\chi}, \bar{g}_{\rho\rho}, \bar{g}_{\rho\chi}, \bar{g}_{\chi\chi}, \underbrace{\bar{g}_{\theta\theta}, \bar{g}_{\phi\phi}}_{\bar{g}_{\psi}})$$

Evolution

- Gauge choice in asymptotically AdS spacetimes
  - not enough to simply demand b.c.s for $\bar{g}_{\mu\nu}$, $\bar{H}_\mu$, $\bar{\varphi}$
  - need to ensure that b.c.s are preserved by evolution

- To see this: expand in power series about $\rho = 1$

$$
\begin{aligned}
\bar{g}_{\mu\nu} &= (1-\rho)\bar{g}_{(1)\mu\nu}(t,\chi,\theta,\phi) + (1-\rho)^2\bar{g}_{(2)\mu\nu}(t,\chi,\theta,\phi) + \ldots \\
\bar{H}_\mu &= (1-\rho)\bar{H}_{(1)\mu}(t,\chi,\theta,\phi) + (1-\rho)^2\bar{H}_{(2)\mu}(t,\chi,\theta,\phi) + \ldots \\
\bar{\varphi} &= (1-\rho)\bar{\varphi}_{(1)}(t,\chi,\theta,\phi) + (1-\rho)^2\bar{\varphi}_{(2)}(t,\chi,\theta,\phi) + \ldots
\end{aligned}
$$

Evolution

- Gauge choice in asymptotically AdS spacetimes
  - not enough to simply demand b.c.s for $\bar{g}_{\mu\nu}$, $\bar{H}_\mu$, $\bar{\varphi}$
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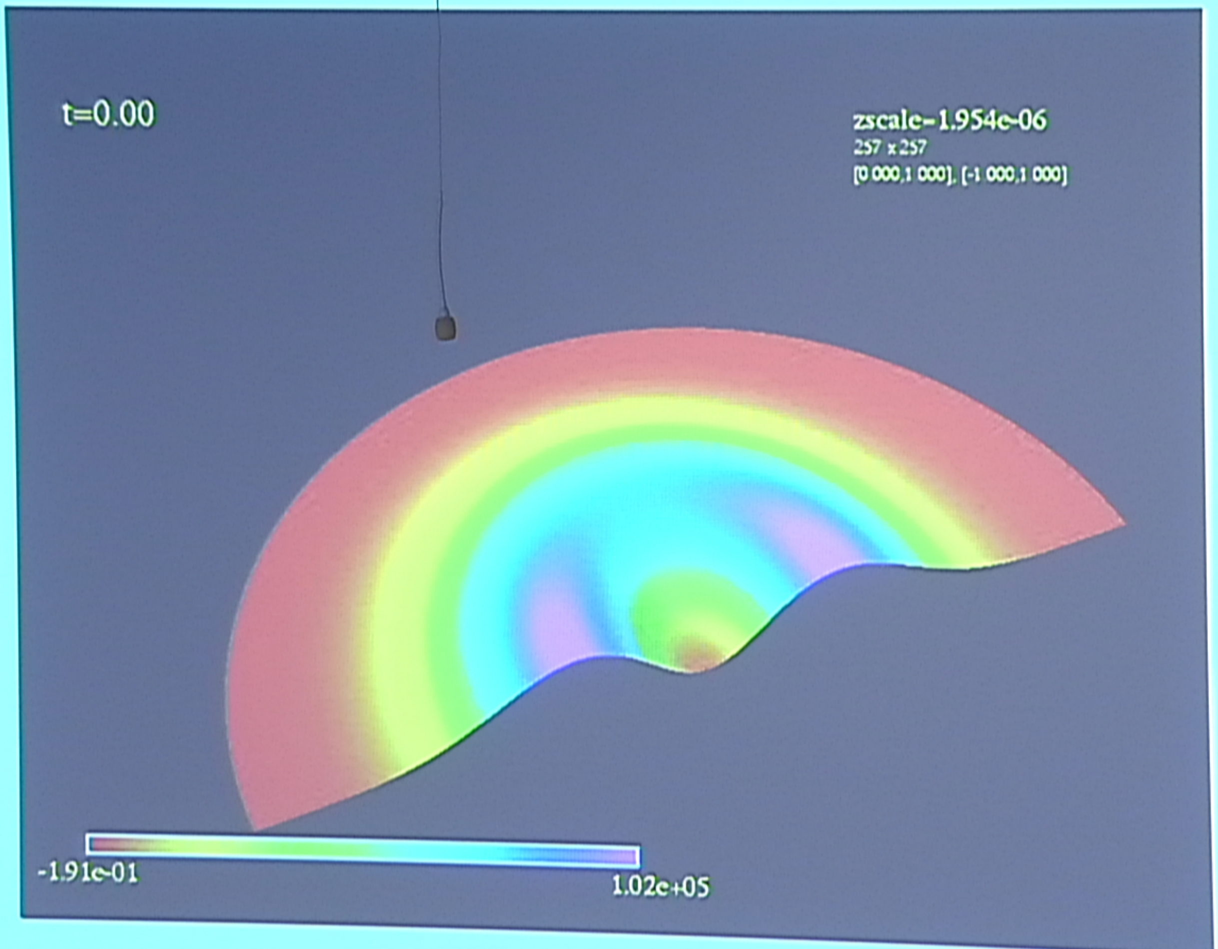
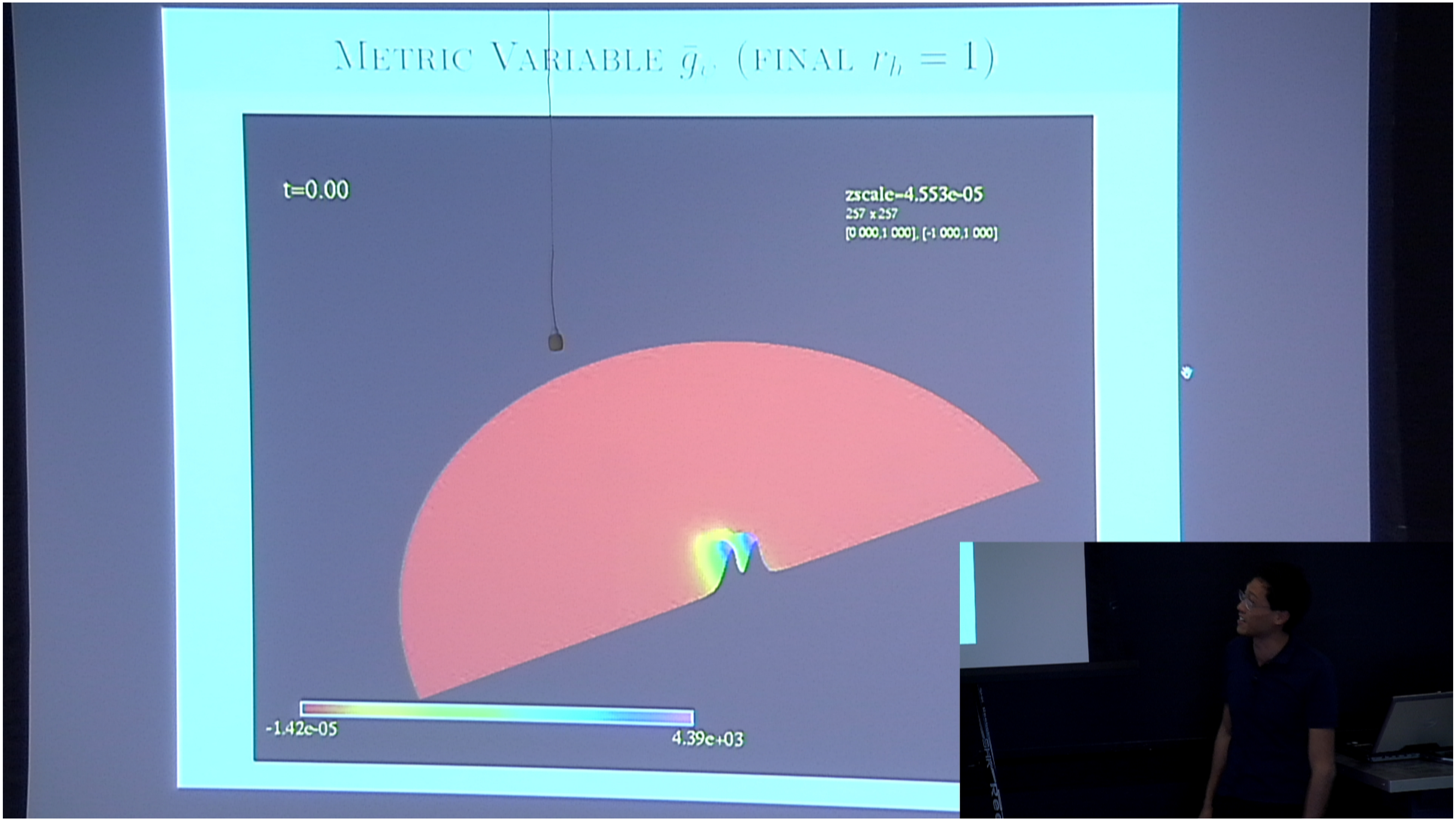
- Example: tt component of field equations near $\rho = 1$

$$\tilde{\Box}\bar{g}_{(1)tt} = (-8\bar{g}_{(1)\rho\rho} + 4\bar{H}_{(1)\rho})(1-\rho)^{-2} + \ldots$$

  - no guarantee that a given choice of $\bar{H}_\mu$ will preserve the desired fall-off
  - regularity requires a delicate cancellation between terms in the near-boundary limit
  - gauge choice: $\bar{H}_{(1)\rho} = 2\bar{g}_{(1)\rho\rho}$

Metric Variable $\bar{g}_r$ (final $r_h = 12.2$)




METRIC VARIABLE $\bar{g}_{tt}$ (FINAL $r_h = 1$)
t=0.00
zscale=4.553e-05
257 x 257
[0.000,1.000], [-1.000,1.000]
-1.42e-05
4.39e+03

Dual CFT on the Boundary

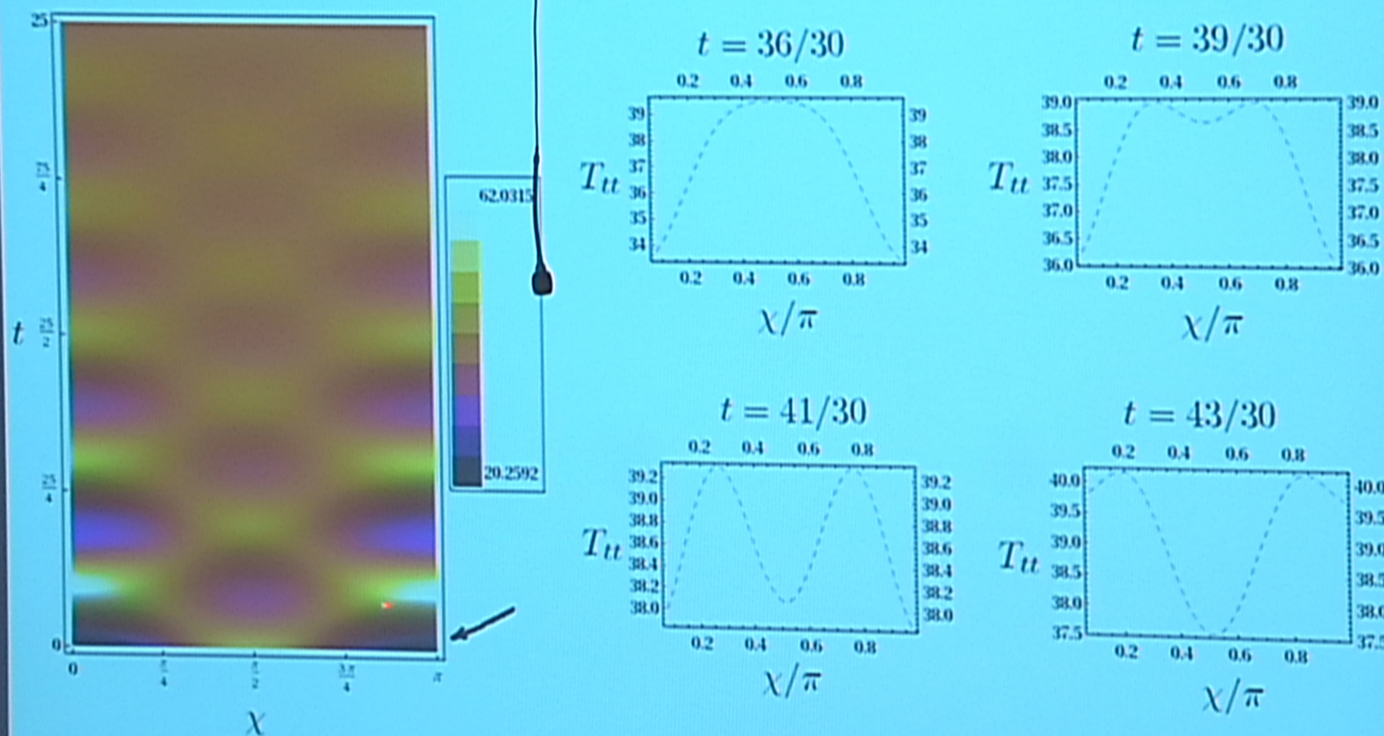


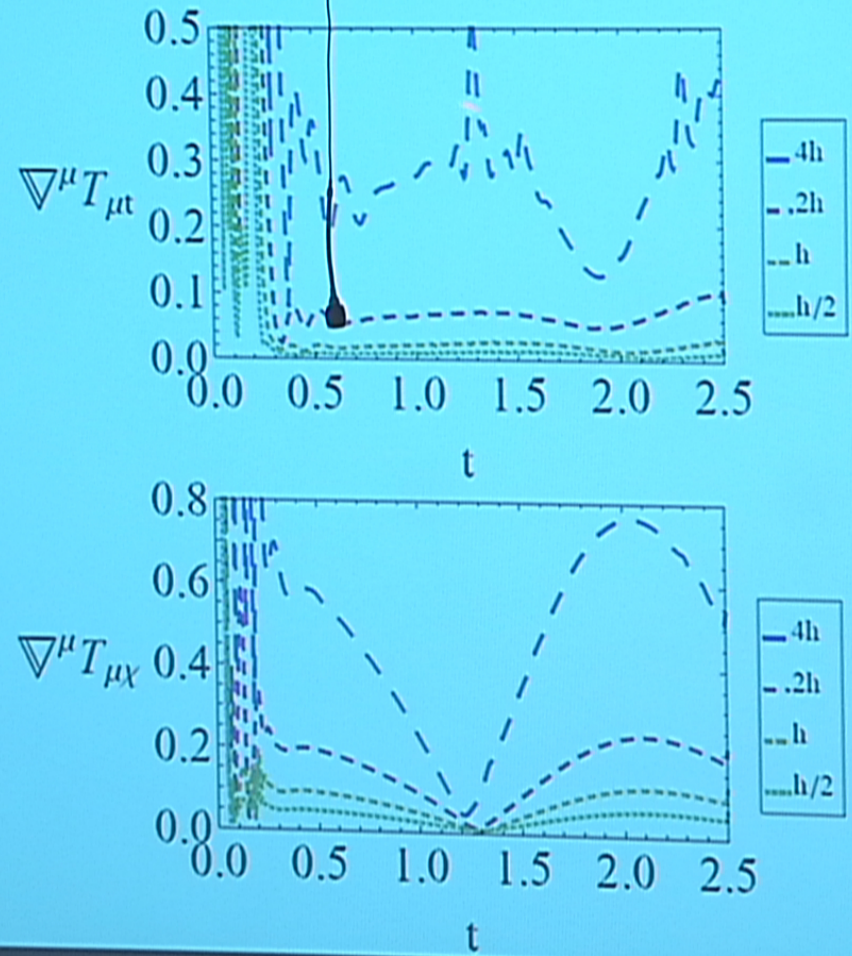
FIGURE: The energy density T_{tt} of the boundary CFT.

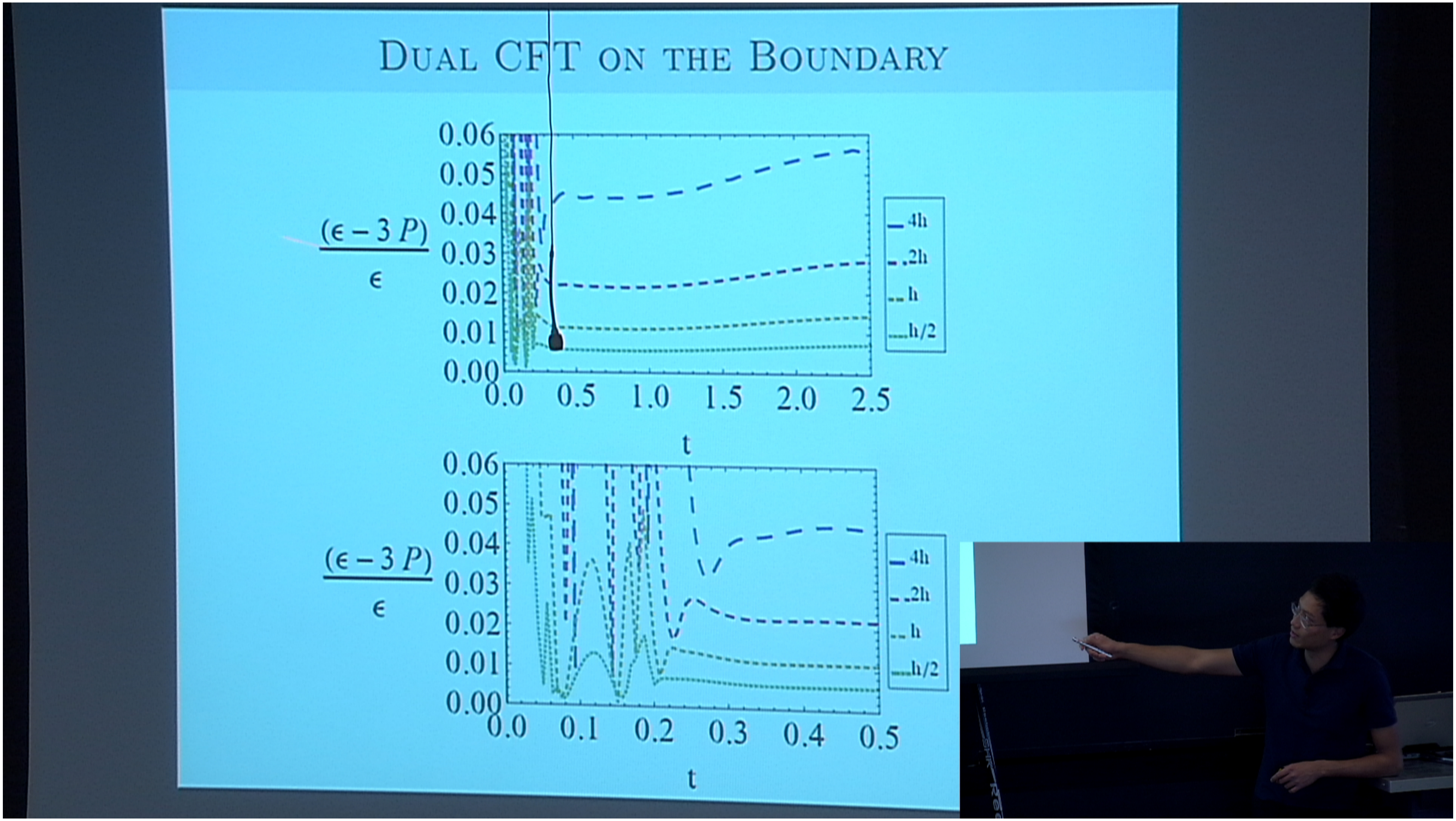
Dual CFT on the Boundary

- Relativistic hydrodynamics
  - equations of motion: $\nabla^\mu T_{\mu\nu} = 0$
  - equation of state: $\epsilon = \epsilon(P)$
- Hydrodynamic stress tensor $T_{\mu\nu}$

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P g_{\mu\nu} - 2\eta\sigma_{\mu\nu} + \Pi_{\mu\nu}$$

Dual CFT on the Boundary




Dual CFT on the Boundary
(ϵ − 3 P) / ϵ
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
t
4h
2h
h
h/2
(ϵ − 3 P) / ϵ
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.1
0.2
0.3
0.4
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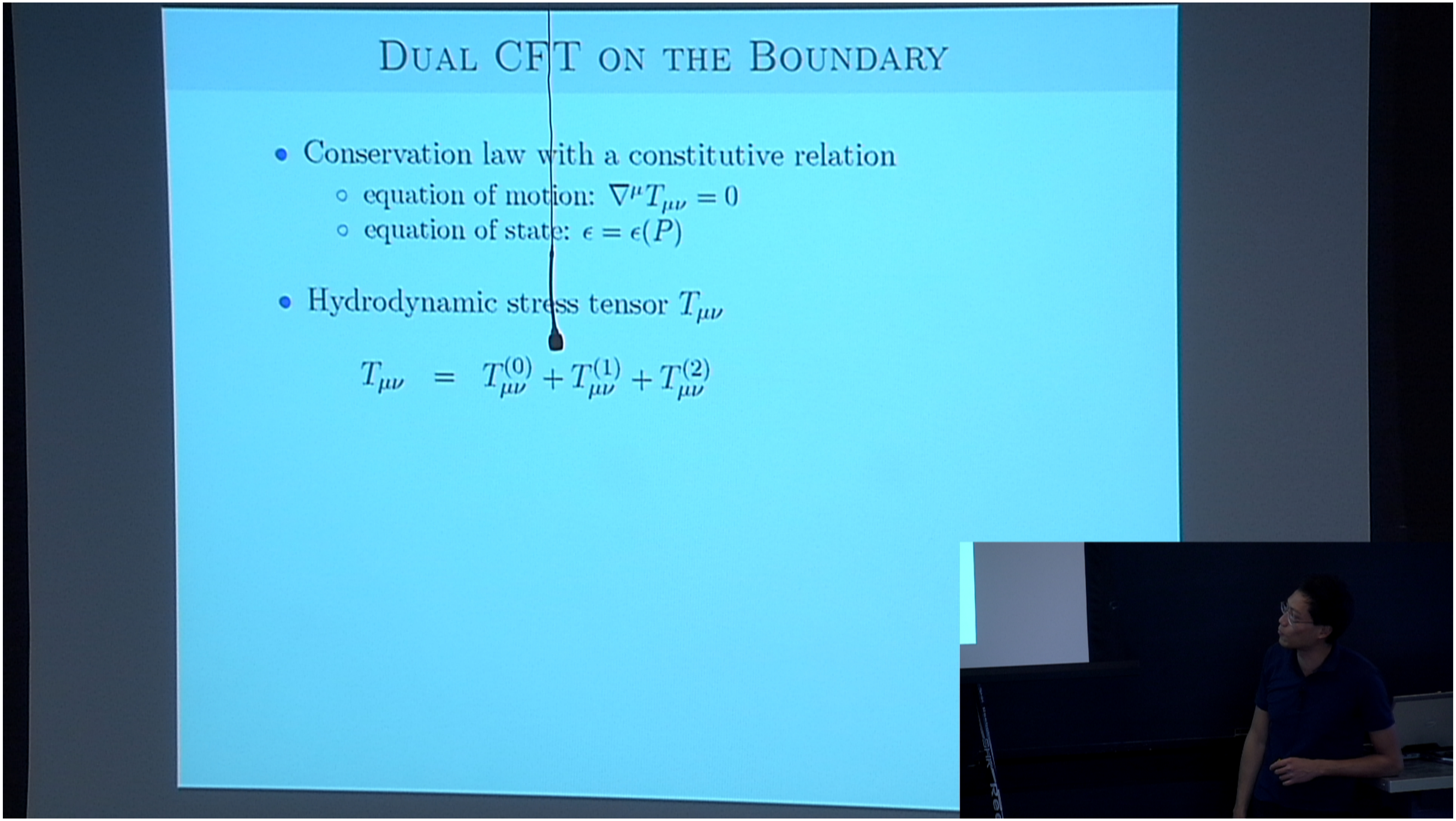
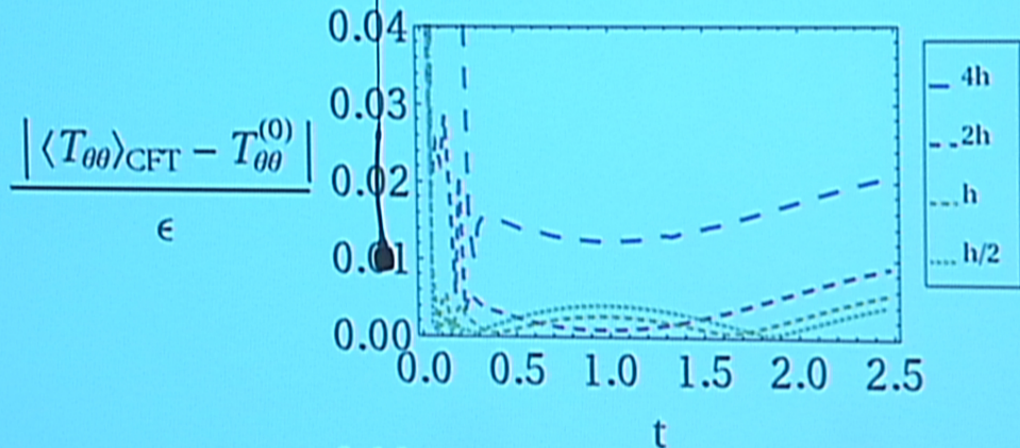
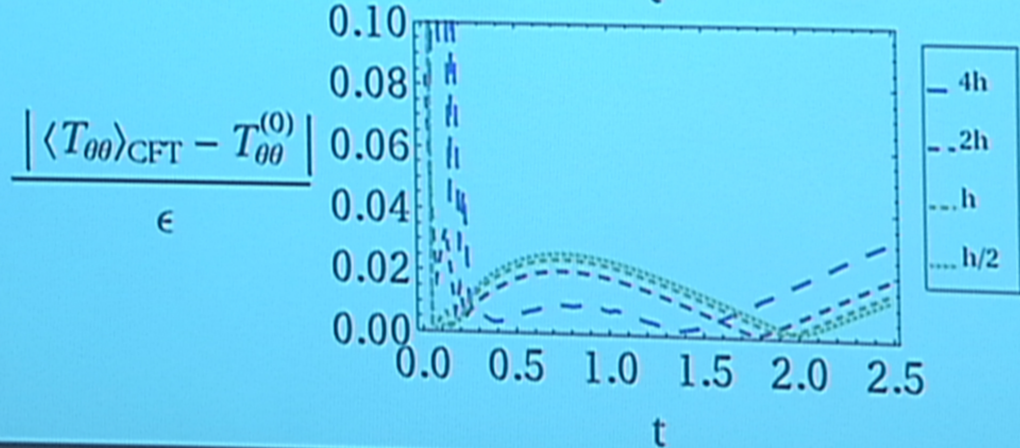

Dual CFT on the Boundary
• Conservation law with a constitutive relation
◦ equation of motion: $\nabla^\mu T_{\mu\nu} = 0$
◦ equation of state: $\epsilon = \epsilon(P)$
• Hydrodynamic stress tensor $T_{\mu\nu}$
$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu}$

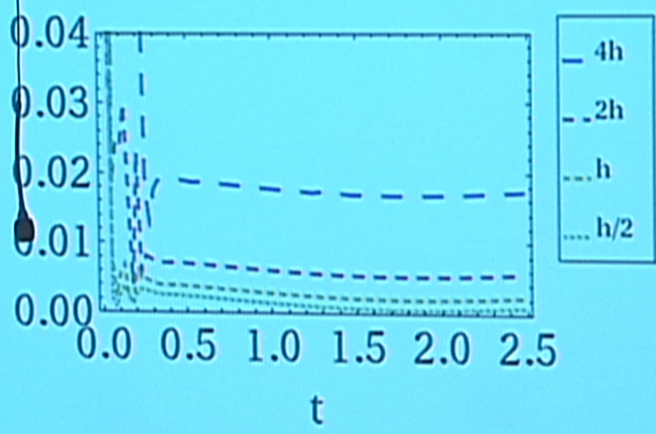
Difference Between Dual CFT and Hydrodynamics ($w_y/w_x = 4$ and $w_y/w_x = 32$)



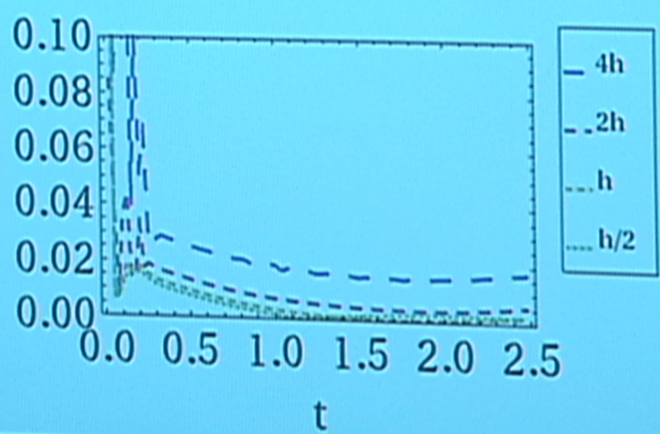


Difference Between Dual CFT and Hydrodynamics ($w_y/w_x = 4$ and $w_y/w_x = 32$)

$$\frac{\left|\langle T_{\theta\theta}\rangle_{\mathrm{CFT}} - \sum_{i=0}^{i=1} T^{(i)}_{\theta\theta}\right|}{\epsilon}$$

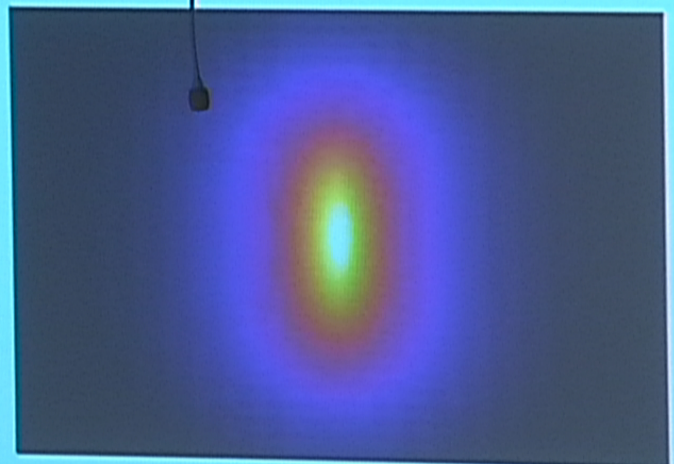


$$\frac{\left|\langle T_{\theta\theta}\rangle_{\mathrm{CFT}} - \sum_{i=0}^{i=1} T^{(i)}_{\theta\theta}\right|}{\epsilon}$$



Fluid Temperature T on an $\mathbb{R}^{3,1}$ Piece of the $\mathbb{R} \times S^3$ Boundary, for $w_y/w_x = 32$

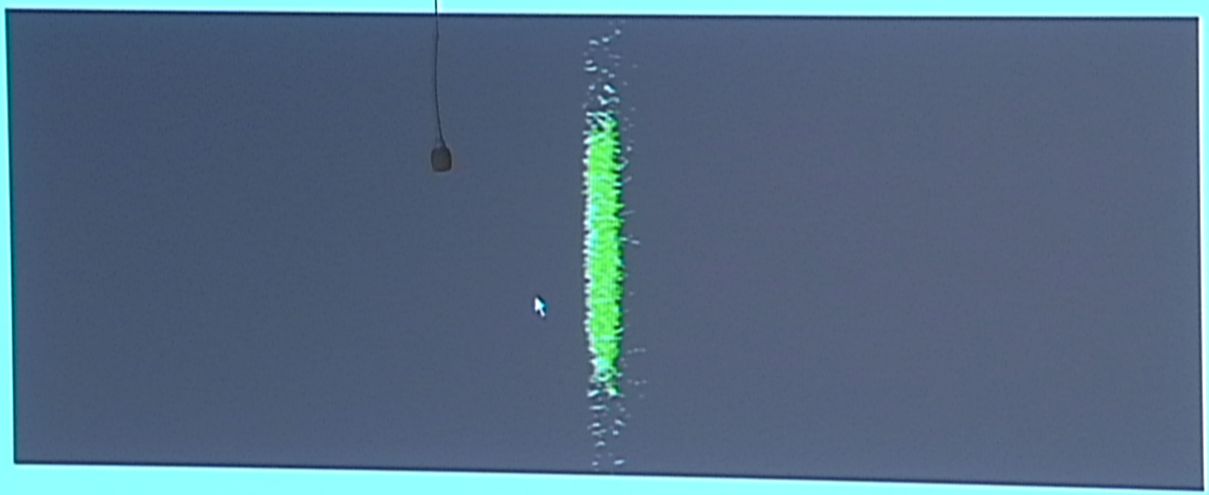
$$(t, \chi, \theta, \phi) \to (\tilde{t}, \tilde{\chi}, \tilde{\theta}, \tilde{\phi}) \to (t', x_1, x_2, x_3)$$



$\longrightarrow$ x_3

$$\begin{aligned} T &\equiv W^{-1}\epsilon^{1/4} \\ W &= \frac{1}{\cos \tilde{t}/L + \cos \tilde{\chi}} = \frac{1}{\cos(t/L) + \cos\theta \sin\chi} \end{aligned}$$

Au+Au 200 GeV: From the Moment of Impact



Animation by J. Mitchell, VNI model by K. Kinder-Geiger and R. Longacre (BNL).

Cooper-Frye Hadronization Spectrum

- Cooper-Frye prescription:
  provides snapshot of the particle momentum distribution dN/dp^3 at the hypersurface of freezout (i.e. hadronization), where the partons in the QGP fluid transition to free-streaming hadron gas behavior
- Assumptions in calculation
  - well-approximated by an isothermal hypersurface $\Sigma = \{x = (t', x_1, x_2, x_3) | T(x) = T_{freezeout}\}$
  - use values representative of a head-on Au+Au 200 GeV $\epsilon/T^4 = 11$, $L = 4.3fm = (46MeV)^{-1}$, $T_{freezeout} = 130MeV$
  - perform the computation for pions (bosons) with $\mu = 0$, $m_\pi = 140MeV$

Cooper-Frye Hadronization Spectrum

- Cooper-Frye prescription:

$$\frac{dN}{dy} = \frac{g}{(2\pi)^3} \int p_T dp_T d\phi_p \int_\Sigma p_\mu d\sigma^\mu(x) \frac{1}{\exp\left(\frac{p_\mu u^\mu(x)}{T(x)}\right) \pm 1}$$

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In the rest frame of a fluid cell with velocity $u^\mu(x)$, the $p_0 dN$ here is the energy flux of particles with momentum p_μ hadronizing through a hypersurface element $d\sigma^\mu(x)$. $T(x)$ is local temperature, $p_1 = p_T \cos\phi_p$, $p_2 = p_T \sin\phi_p$, $p_0 = \sqrt{{m_\pi}^2 + {p_T}^2}\cosh y$, $p_0 = \sqrt{{m_\pi}^2 + {p_T}^2}\sinh y$ for $p_T = \sqrt{{p_1}^2 + {p_2}^2}$.

Cooper-Frye Hadronization Spectrum

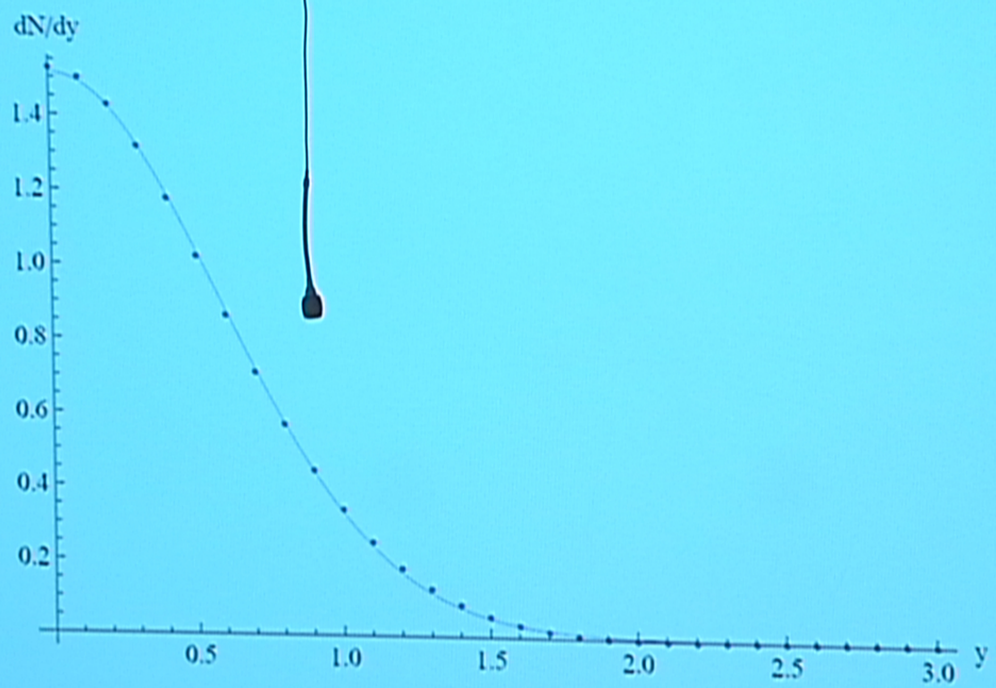

FIGURE: dN/dy vs y computed by using the Cooper-Frye prescription on the $w_y/w_x = 32$ data, compared to the Landau model $dN/dy = \frac{Ks^{1/4}}{\sqrt{2\pi \log \gamma}} \exp\left(-y^2/2\log\gamma\right)$

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Cooper-Frye Hadronization Spectrum

- The $w_y/w_x = 32$ gravity data gives a particle collision Lorentz factor of $\gamma \approx 1.4$
  - RHIC Lorentz factors $\gamma \approx 100$
  - LHC Lorentz factors $\gamma \approx 2700$

Massive Self-interacting Scalars

- Couple AdS gravity with a massive scalar field with asymptotics:
  $\varphi \sim A(1-\rho)^{\Delta} + B(1-\rho)^{4-\Delta}$, for $\Delta = 2 + \sqrt{4+m^2}$

- Choose $m = -15/4$
  - $B = 0$ Boundary Conditions

$$
\begin{aligned}
g_{mn} - \hat{g}_{mn} &= f_{mn}(t,\chi,\theta,\phi)(1-\rho)^2 + ... \\
g_{\rho m} - \hat{g}_{\rho m} &= f_{\rho m}(t,\chi,\theta,\phi)(1-\rho)^3 + ... \\
g_{\rho\rho} - \hat{g}_{\rho\rho} &= f_{\rho\rho}(t,\chi,\theta,\phi)(1-\rho)^2 + ...
\end{aligned}
$$

  - $B \neq 0$ Boundary Conditions

$$
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