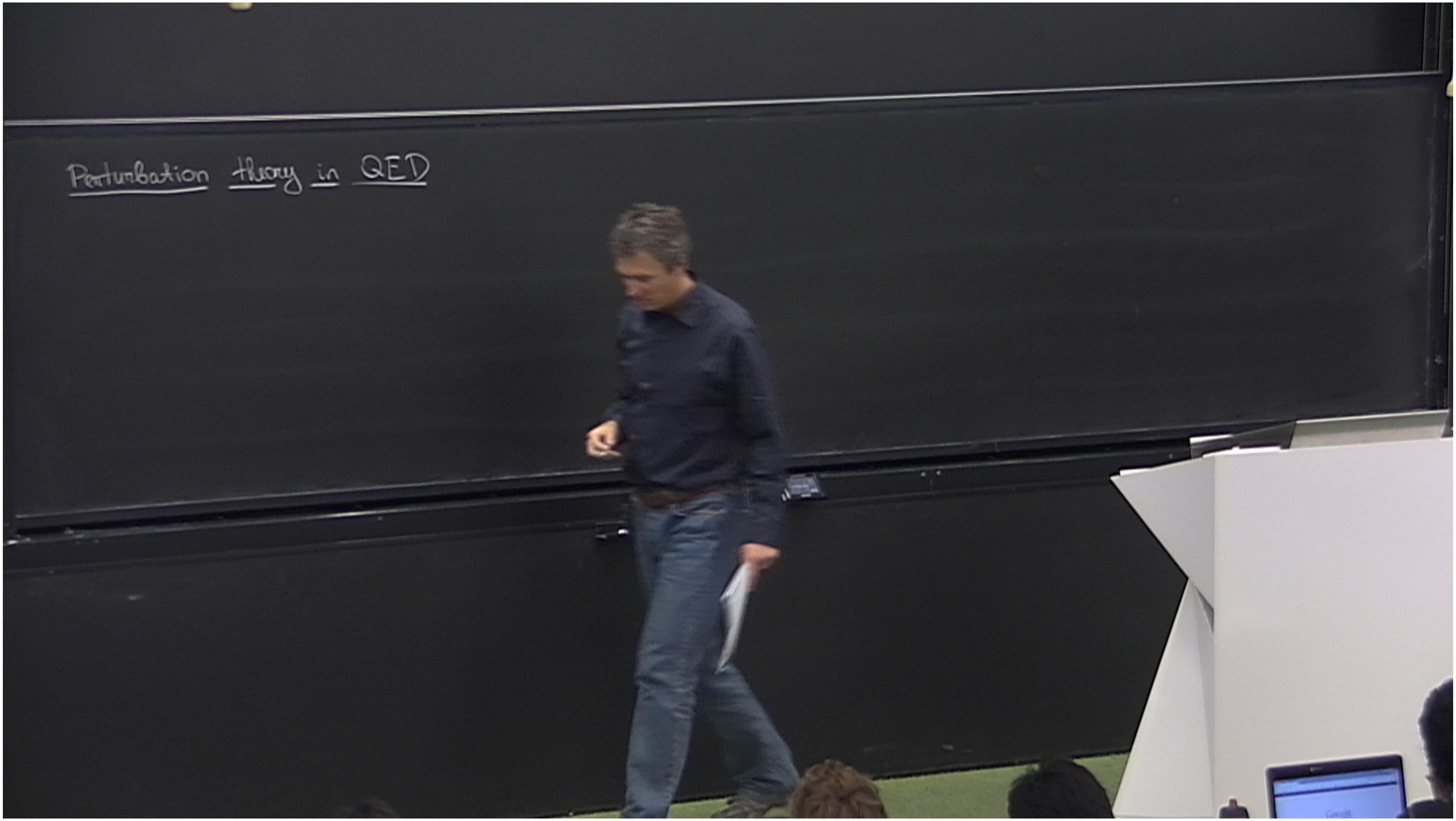
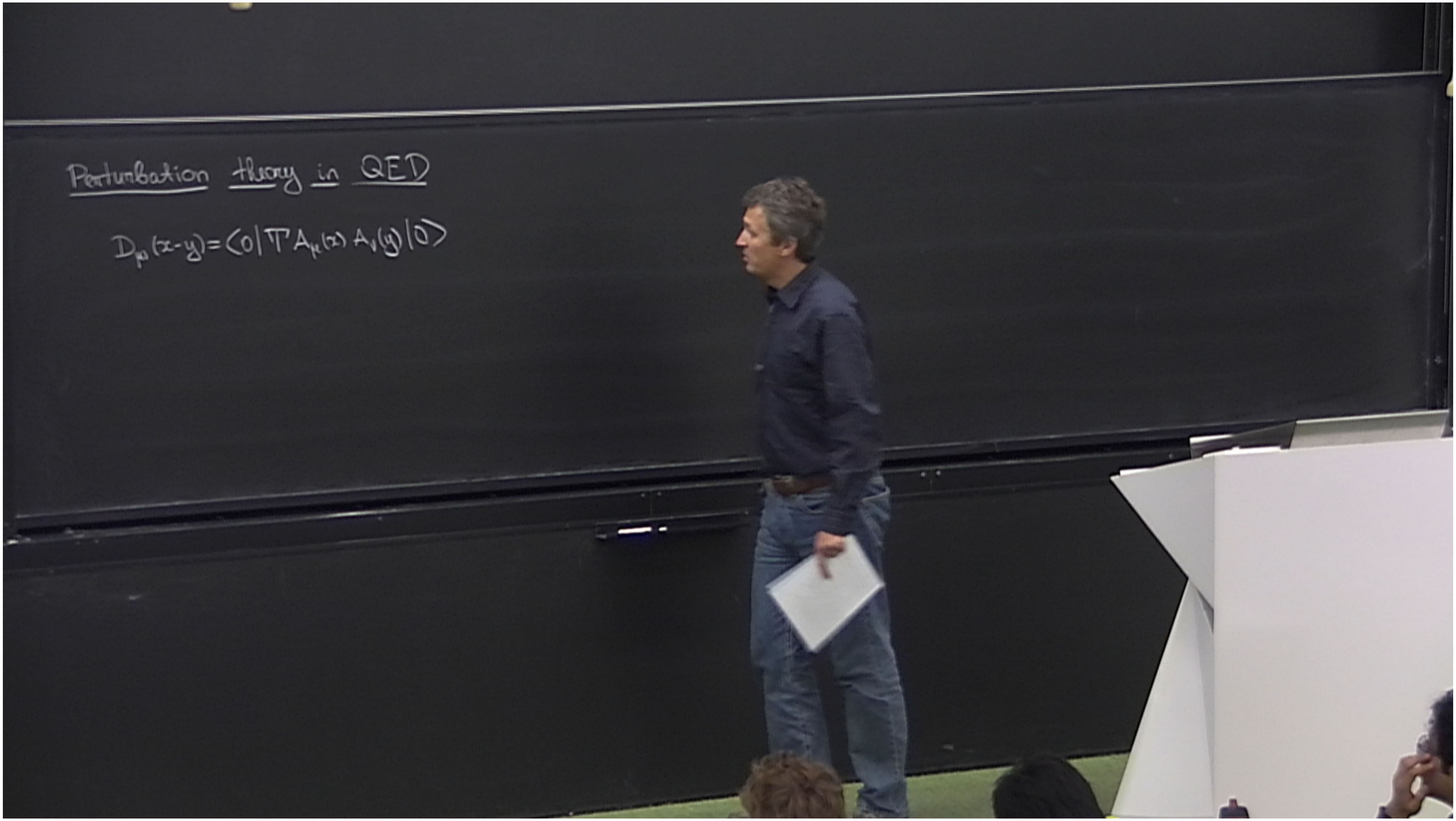
Title: Quantum Field Theory - Lecture 11B

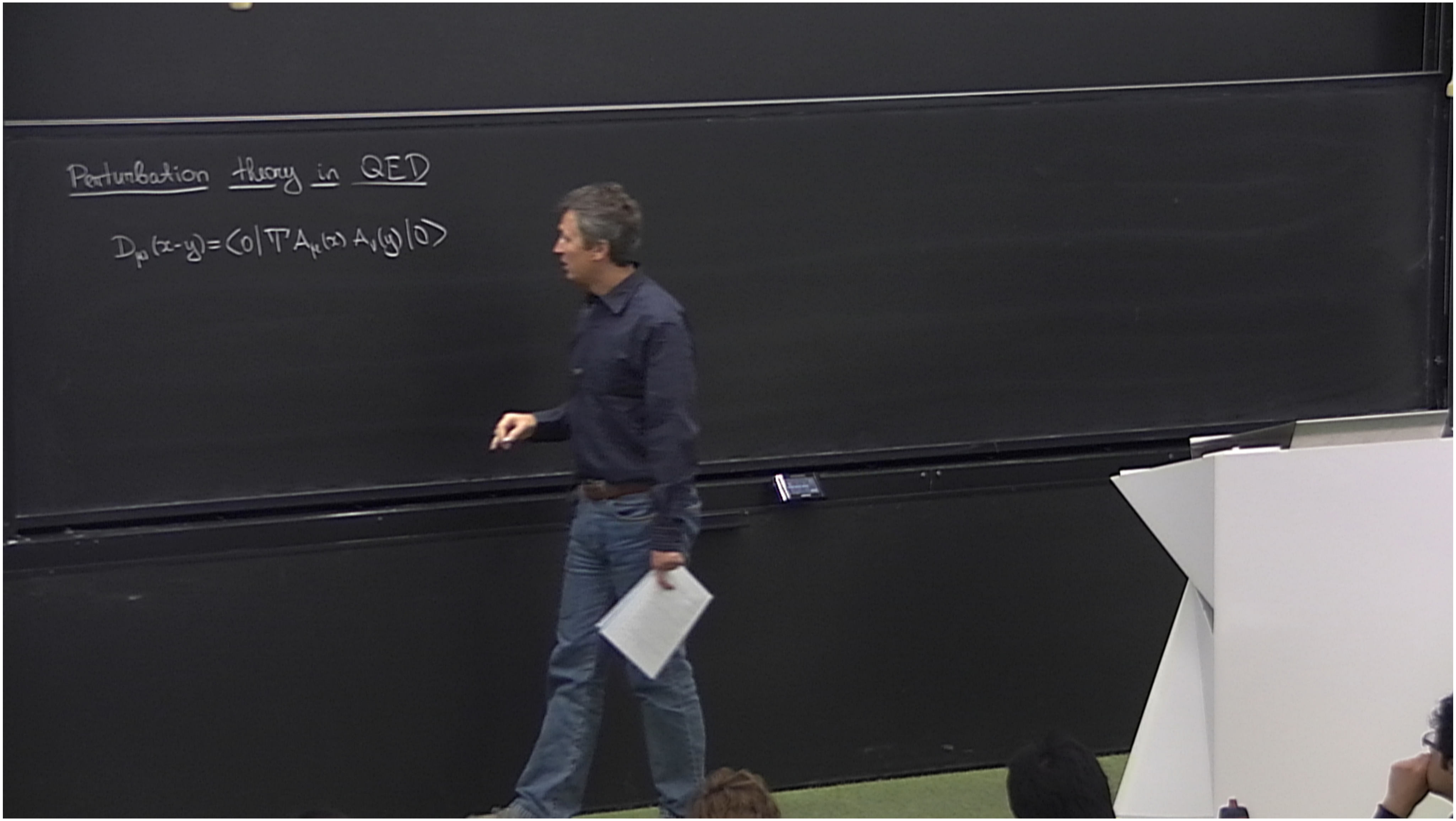
Date: Oct 23, 2012 10:30 AM

URL: http://pirsa.org/12100072

Abstract:


Perturbation theory in QED


Perturbation theory in QED
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$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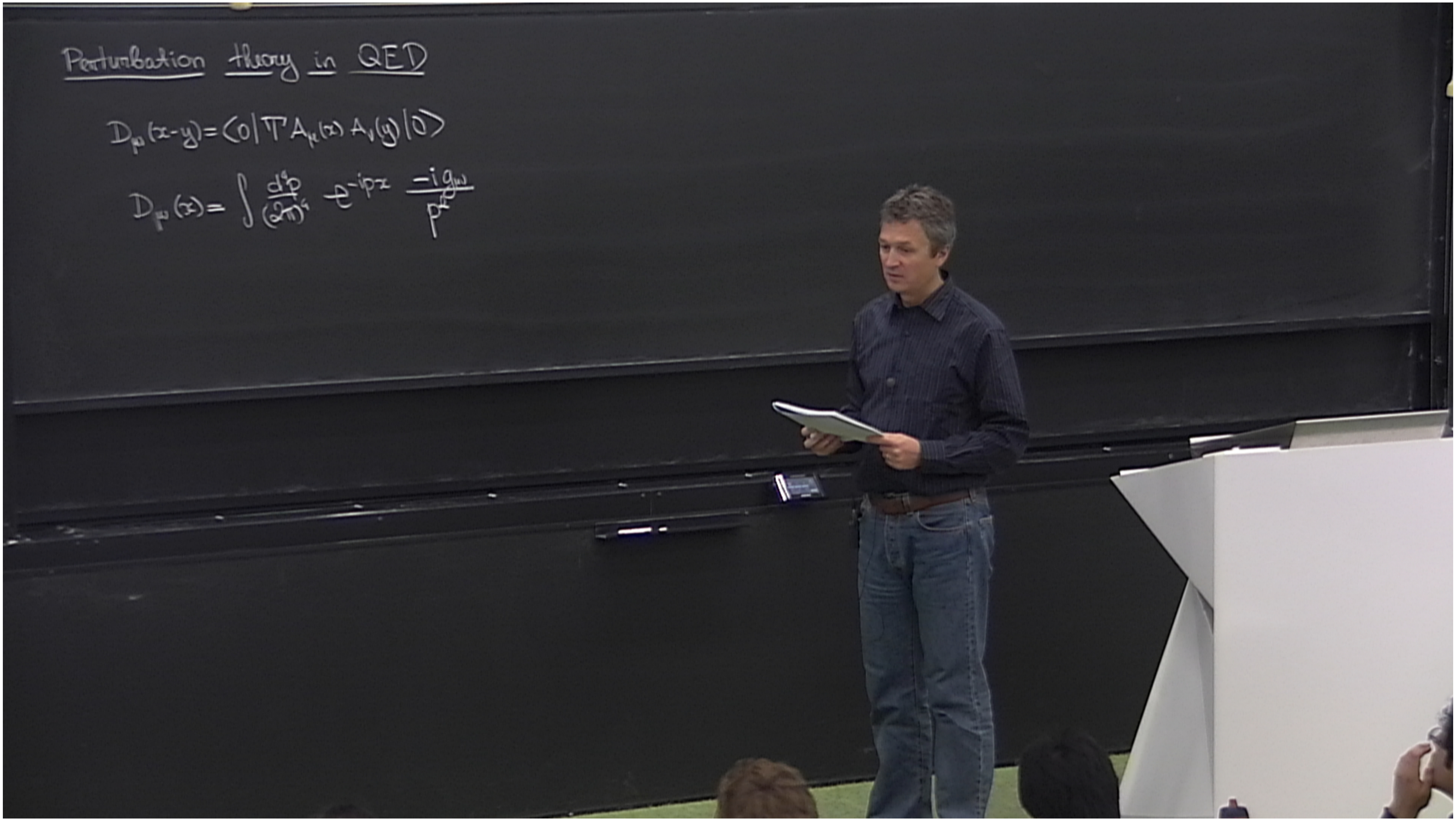
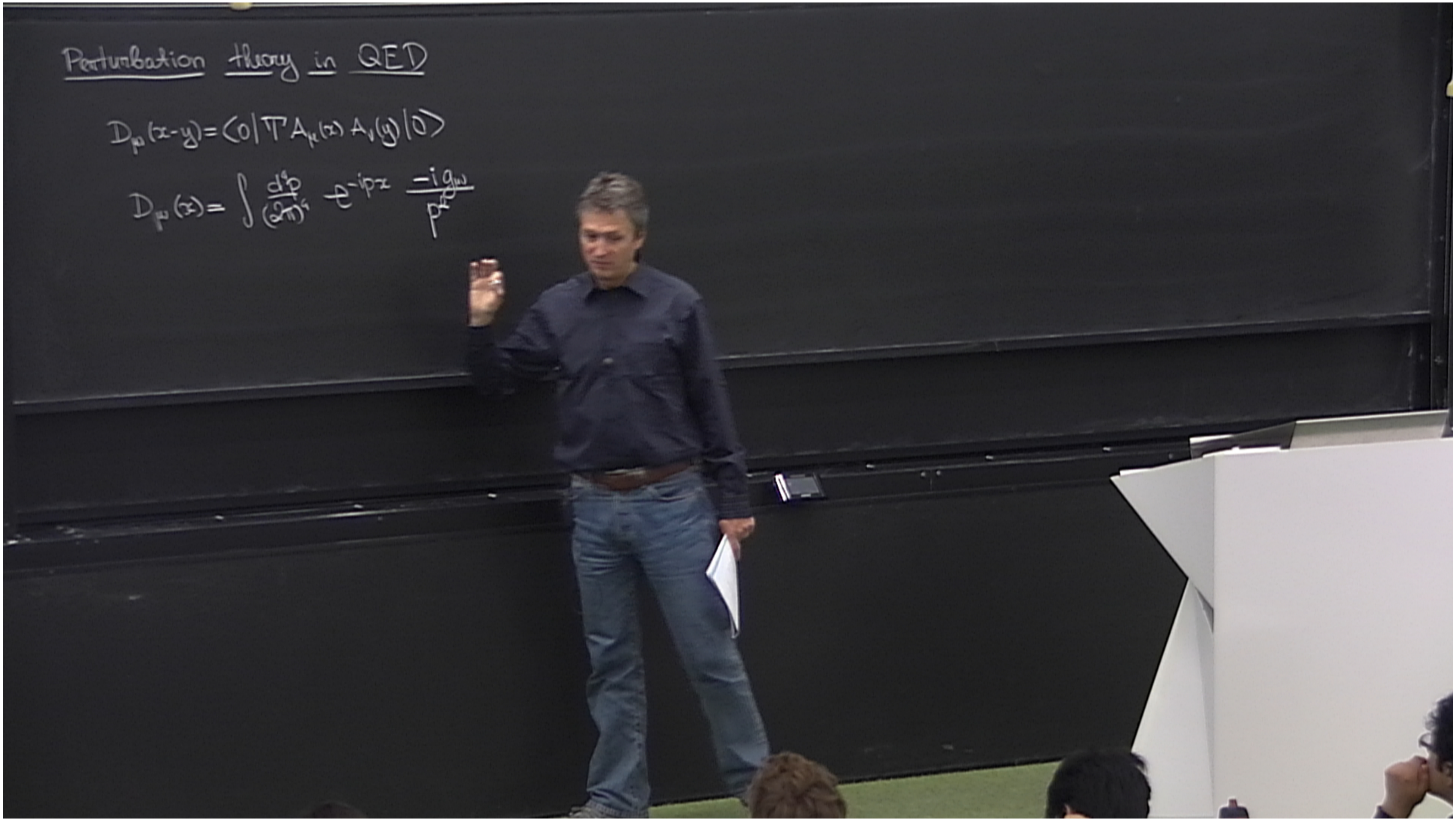

Perturbation theory in QED
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$
$D_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{-ig_{\mu\nu}}{p^2}$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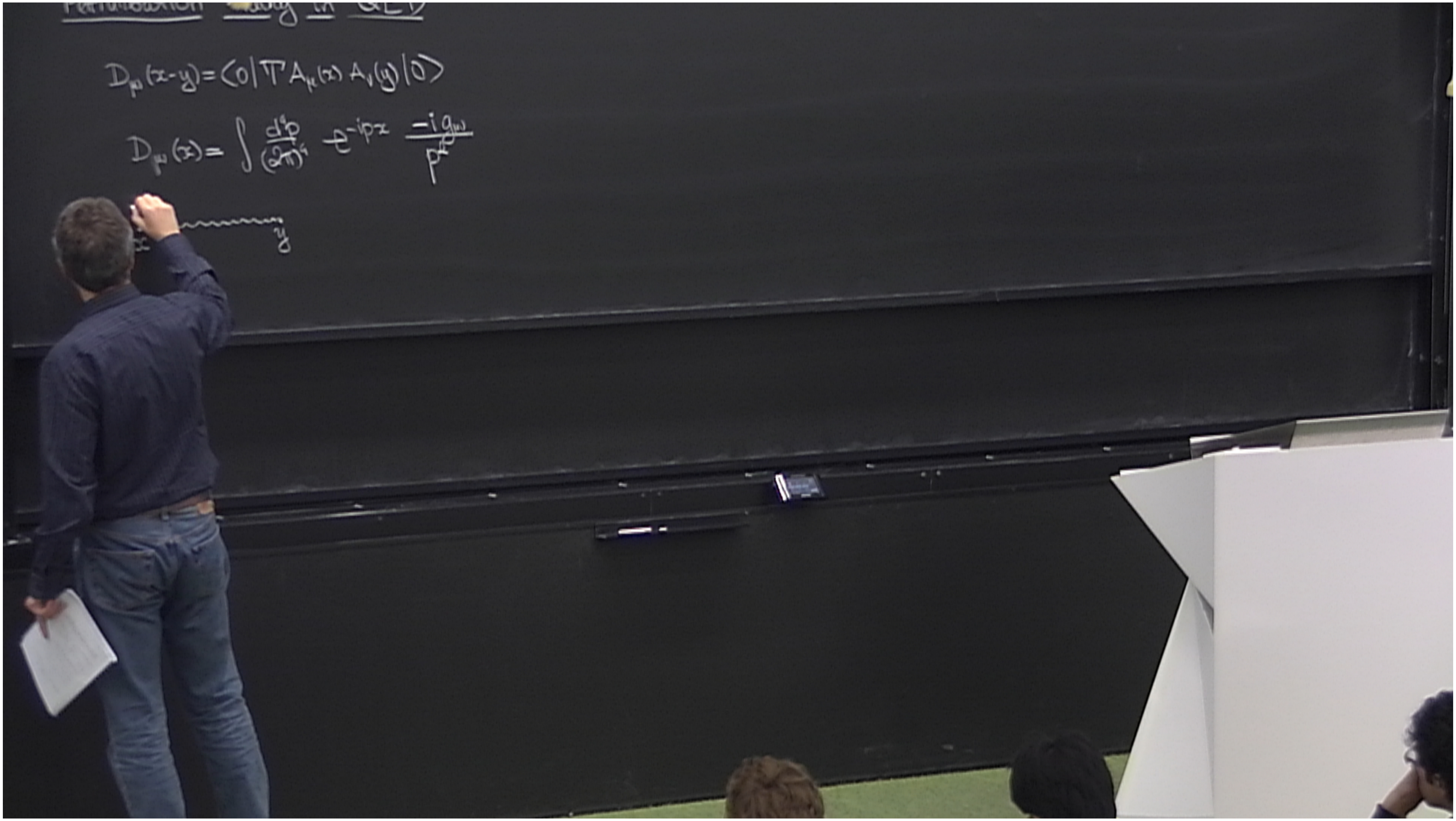
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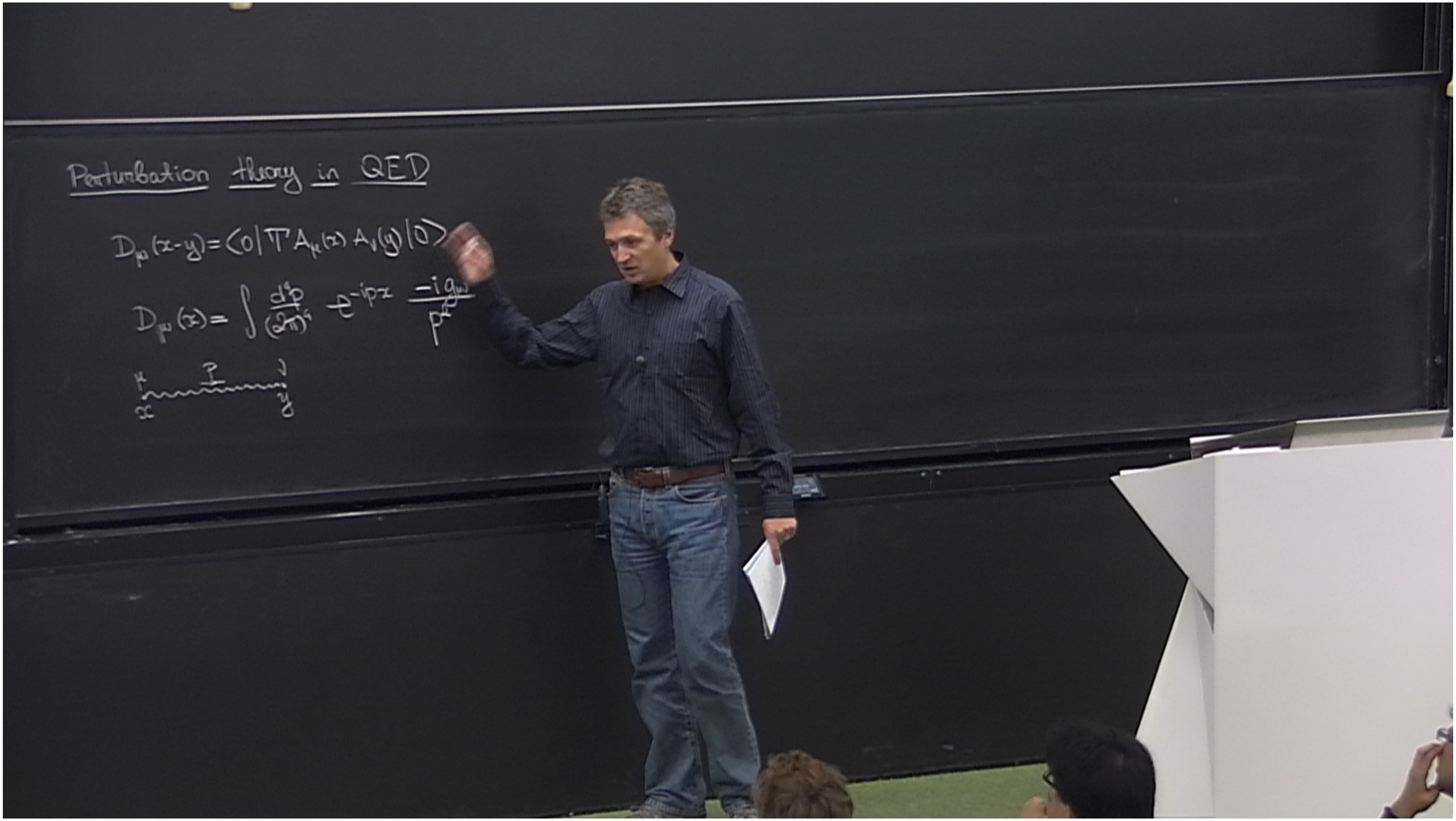
x
y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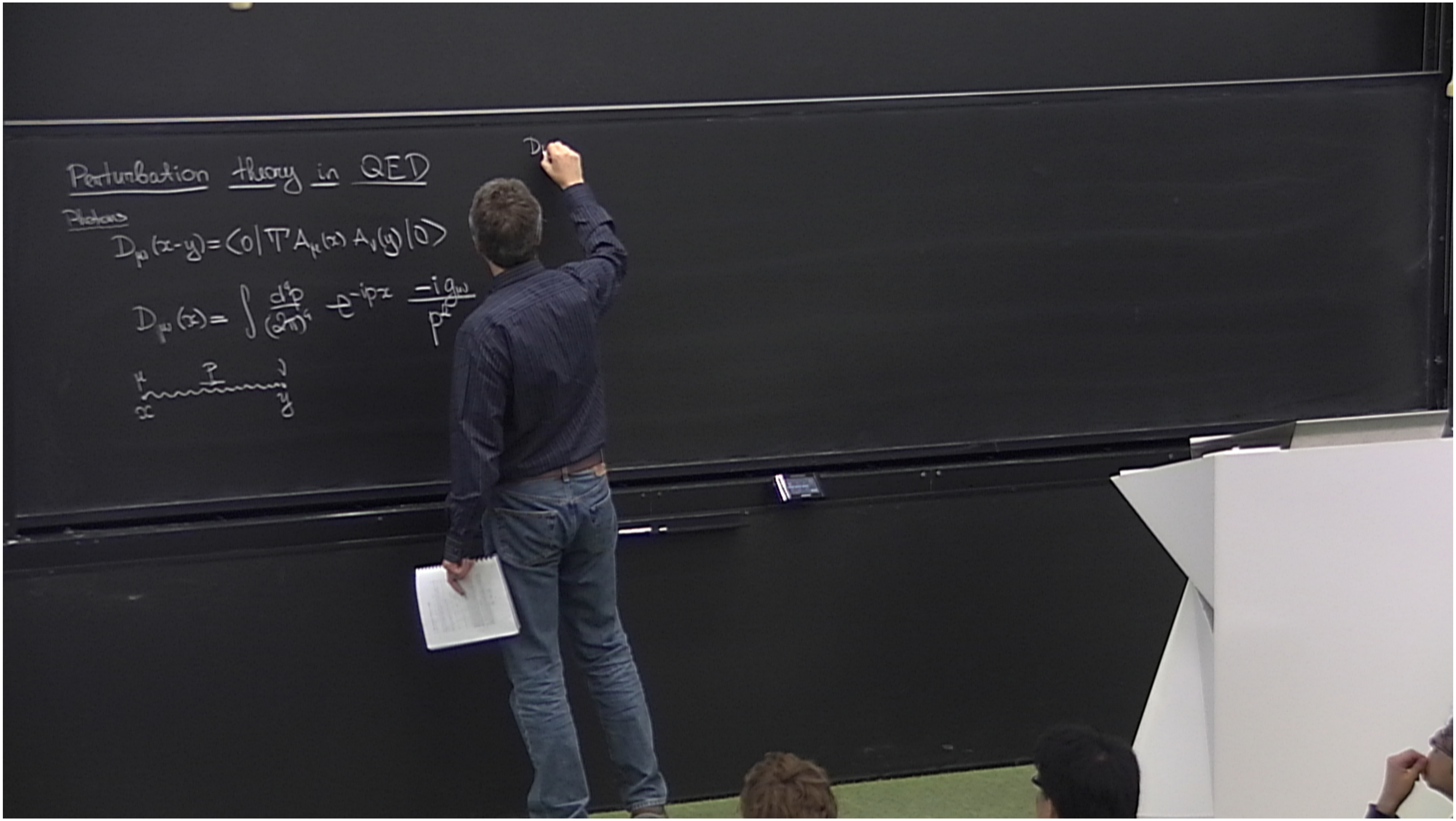
μ p ν
x y


Perturbation theory in QED
Photons
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$
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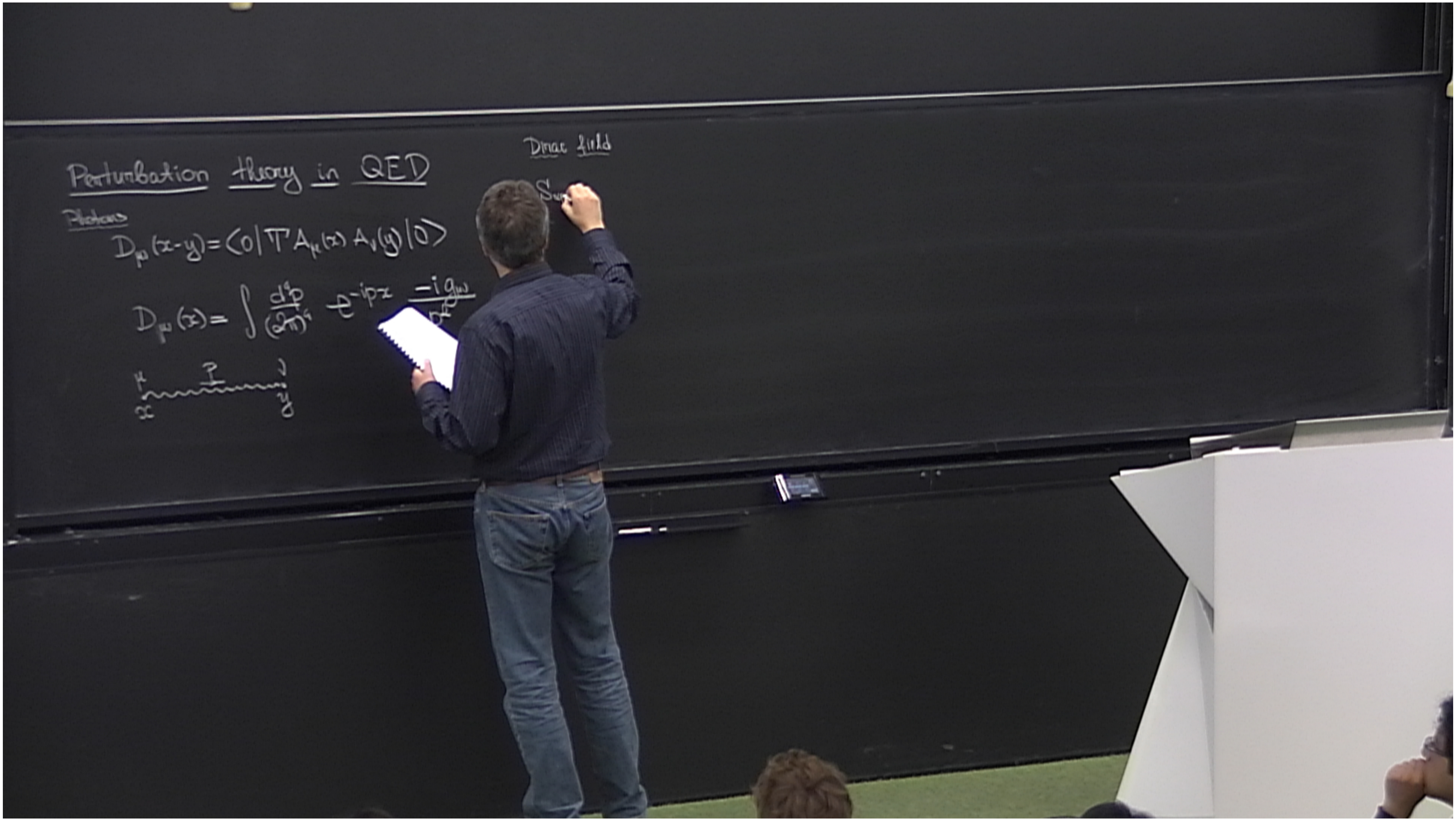
μ p ν
x y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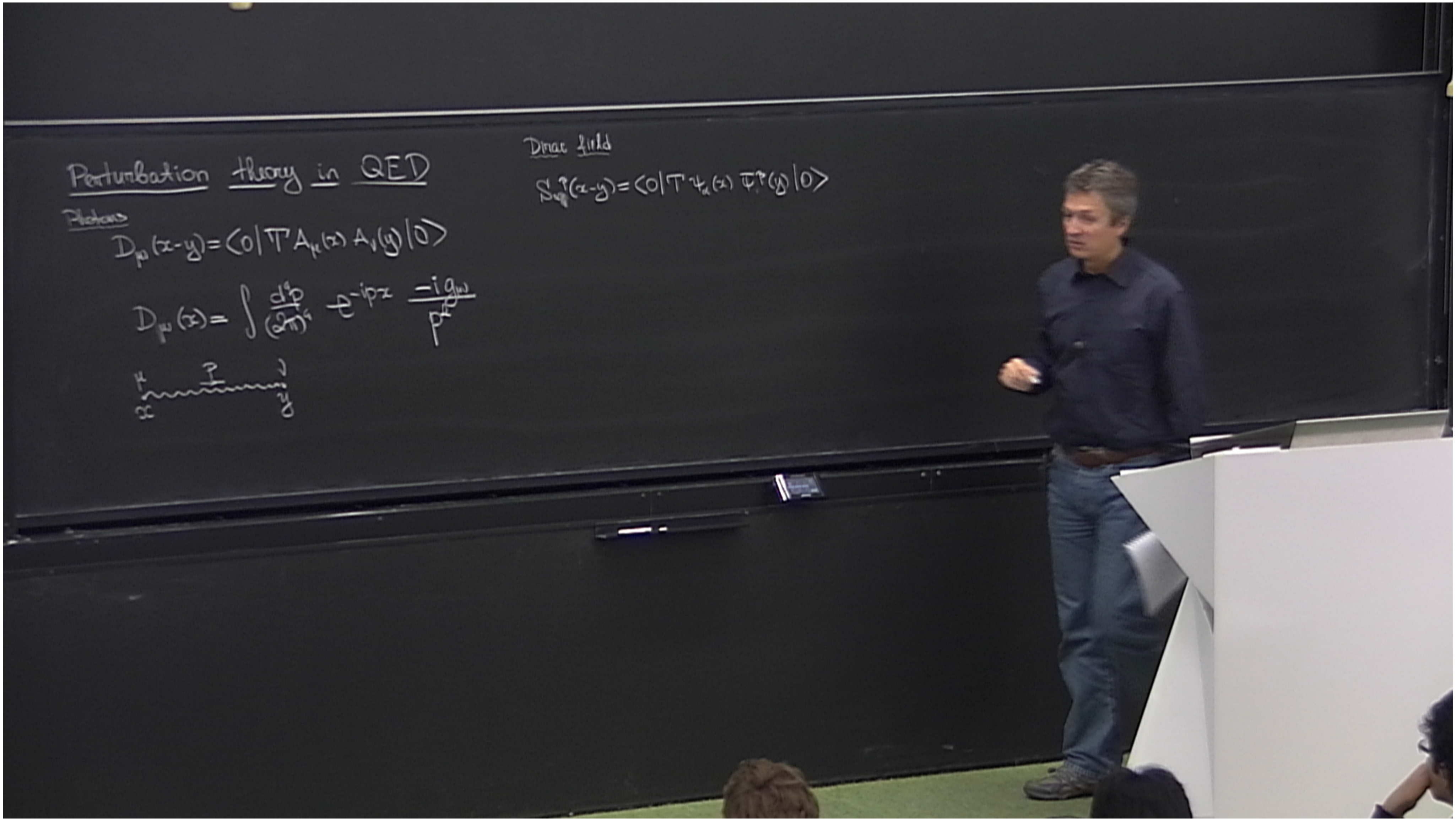
μ p ν
x y
Dirac field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Photons
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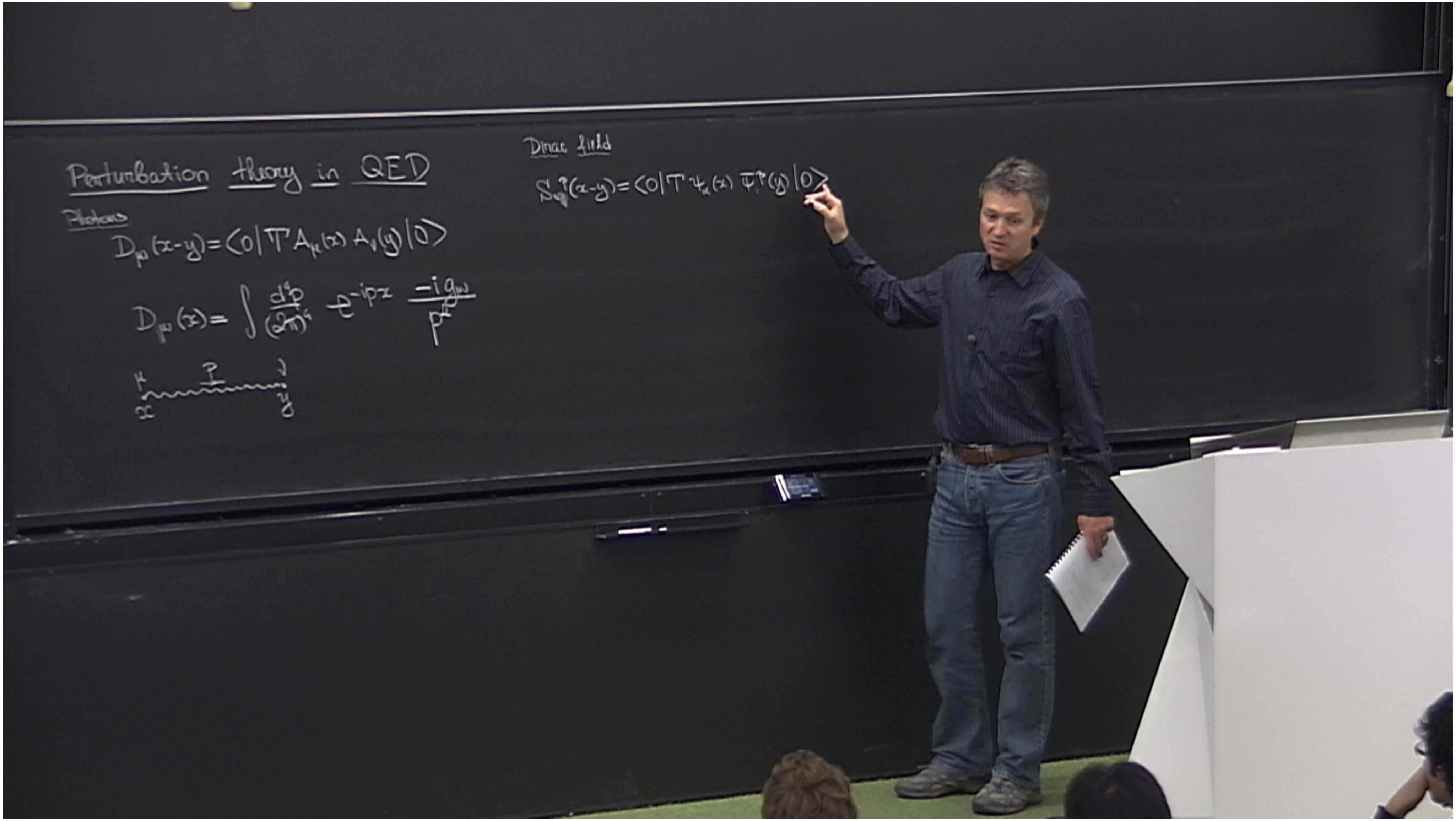
$D_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{-i g_{\mu\nu}}{p^2}$
μ p ν
x y
Dirac field
$S_{\alpha\beta}(x-y) = \langle 0|T \psi_\alpha(x) \bar{\psi}_\beta(y)|0\rangle$



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Photons
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Dirac field
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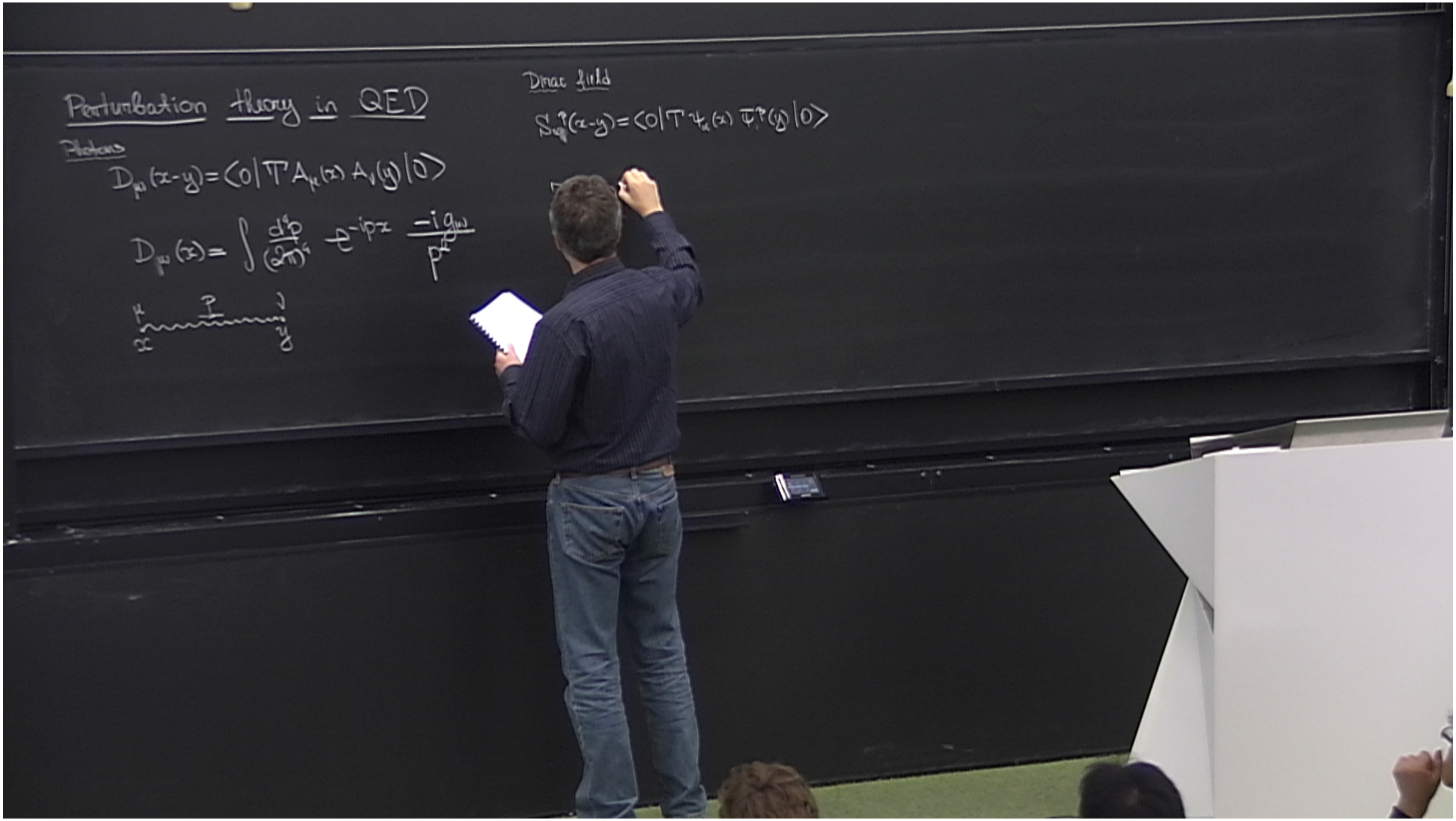

Perturbation theory in QED
Photons
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$
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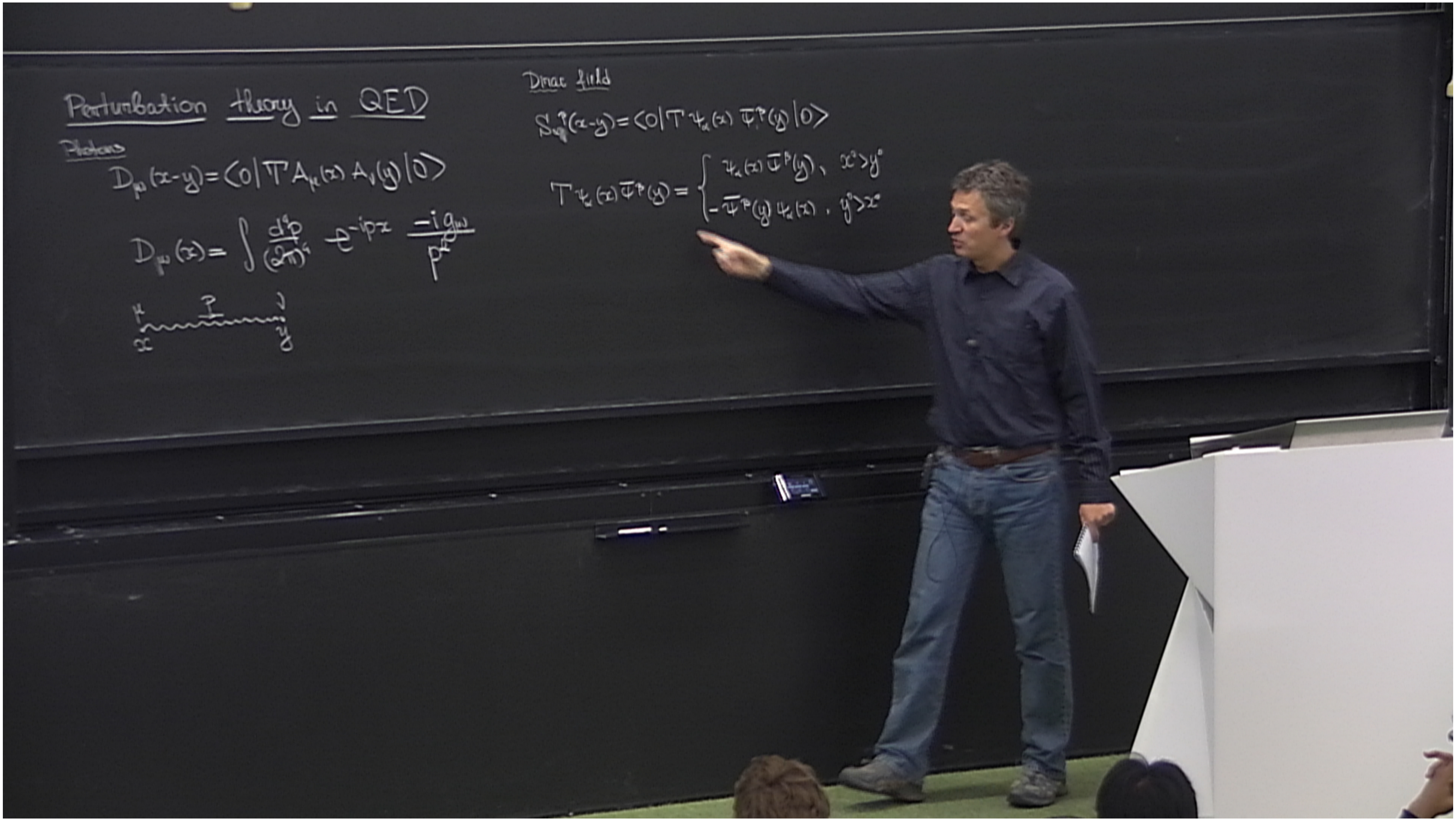
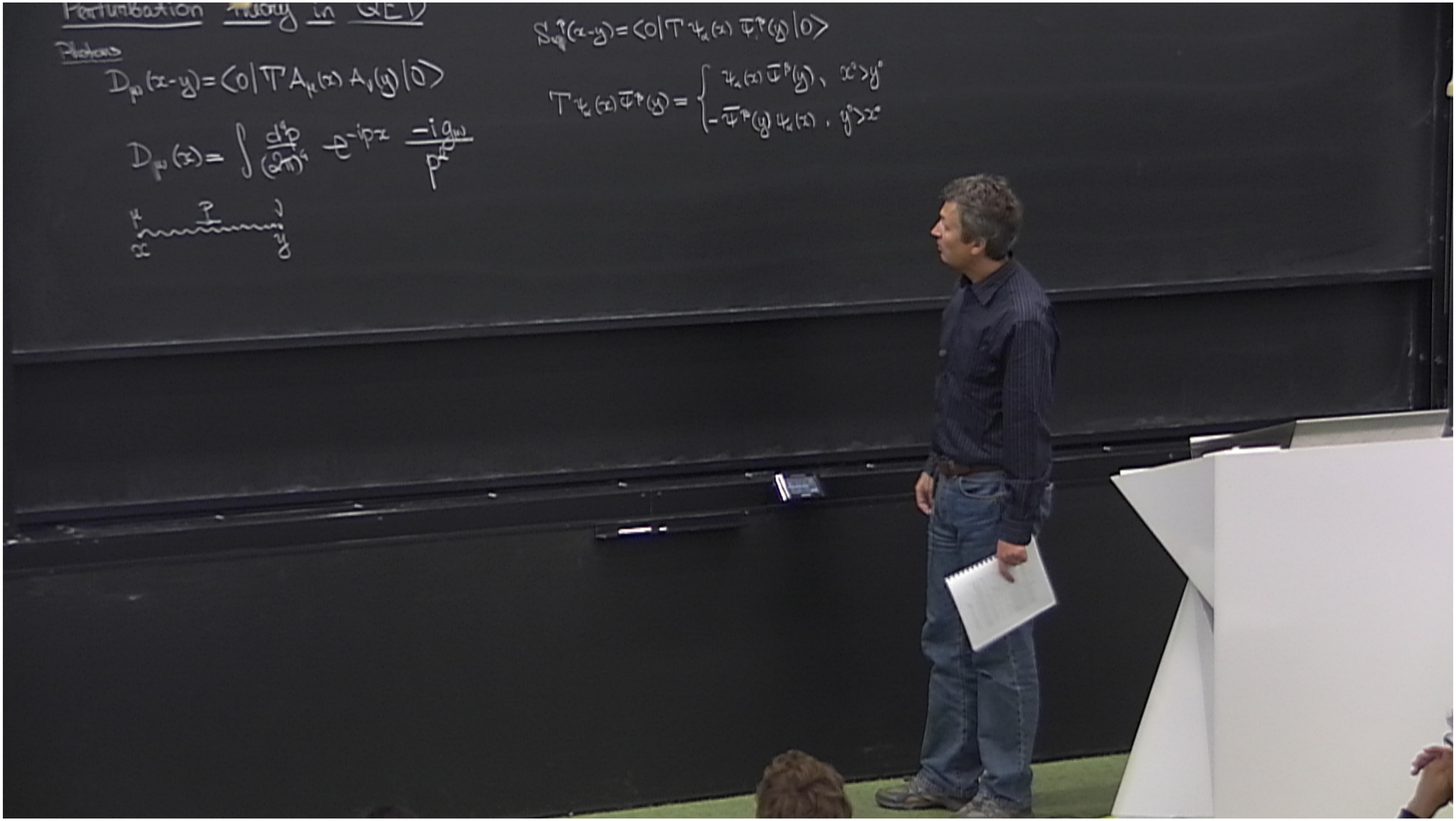
μ p ν
x y
Dirac field
$S_{\alpha\beta}(x-y) = \langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(y) | 0 \rangle$


Perturbation theory in QED
Photons
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$
$D_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{-i g_{\mu\nu}}{p^2}$
μ p ν
x y
Dirac field
$S_{\alpha}^{\beta}(x-y) = \langle 0 | T \psi_\alpha(x) \bar{\psi}^\beta(y) | 0 \rangle$
$T \psi_\alpha(x) \bar{\psi}^\beta(y) = \begin{cases} \psi_\alpha(x) \bar{\psi}^\beta(y), & x^0 > y^0 \\ -\bar{\psi}^\beta(y) \psi_\alpha(x), & y^0 > x^0 \end{cases}$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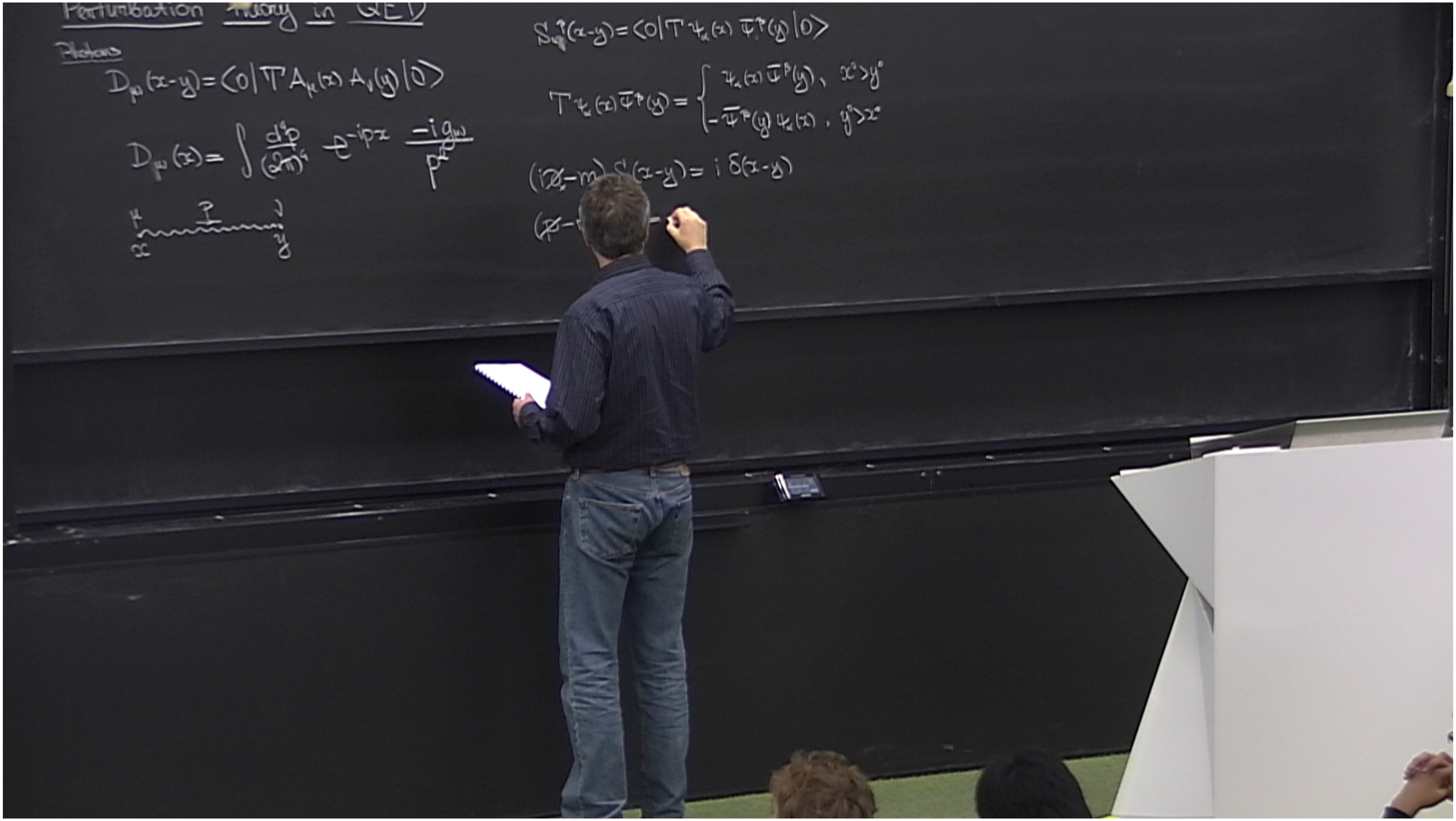
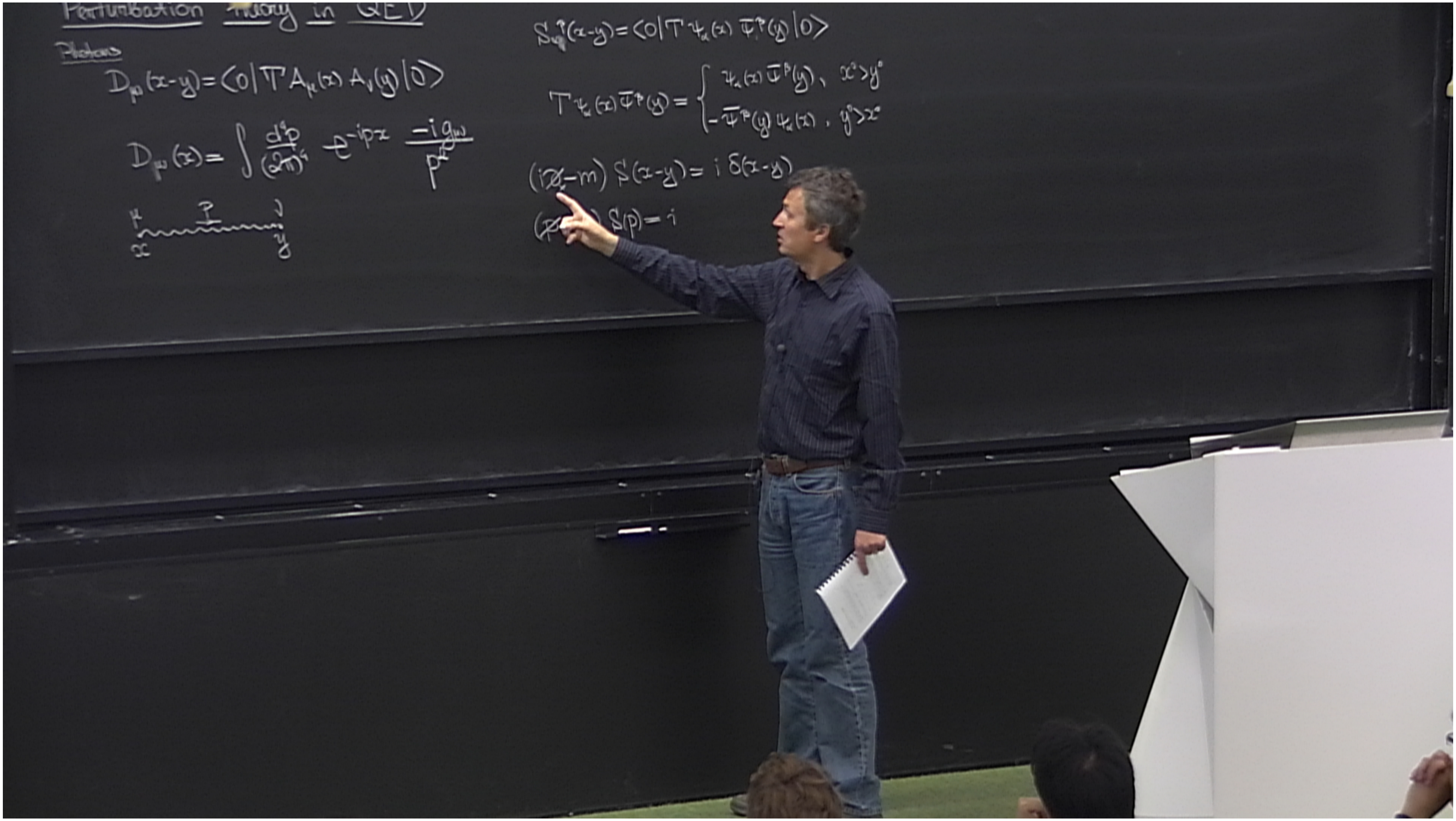


Perturbation Theory in QED
Photons
$D_{\mu\nu}(x-y) = \langle 0|T A_\mu(x) A_\nu(y)|0\rangle$
$D_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{-ig_{\mu\nu}}{p^2}$
μ p ν
x y
$S_{\alpha\beta}(x-y) = \langle 0|T \psi_\alpha(x) \bar{\psi}^\beta(y)|0\rangle$
$T \psi_\alpha(x) \bar{\psi}^\beta(y) = \begin{cases} \psi_\alpha(x) \bar{\psi}^\beta(y), & x^0 > y^0 \\ -\bar{\psi}^\beta(y) \psi_\alpha(x), & y^0 > x^0 \end{cases}$
$(i\partial\!\!\!/_x - m) S(x-y) = i\,\delta(x-y)$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Photons
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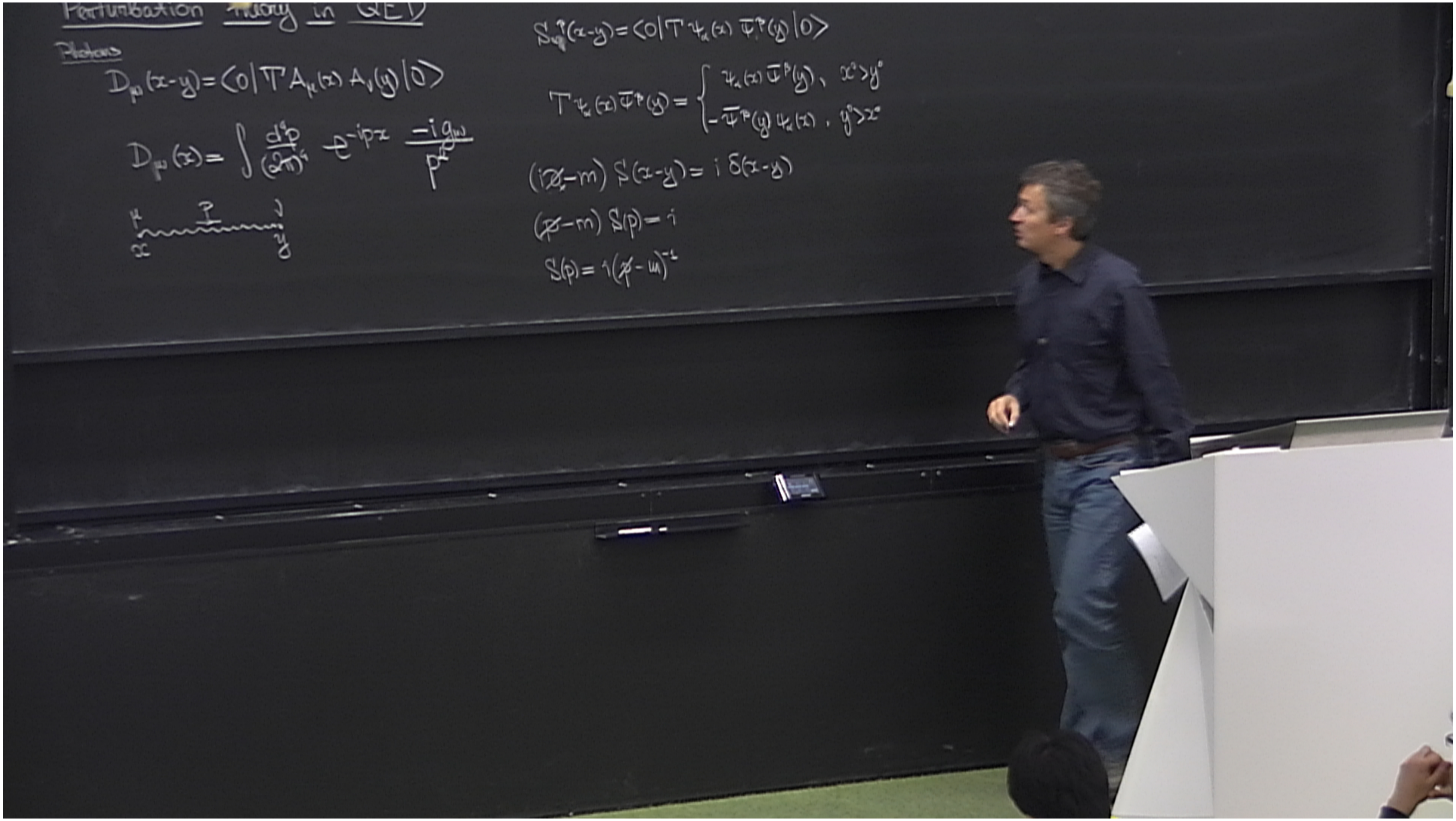
$(i\partial\!\!\!/ - m) S(x-y) = i\,\delta(x-y)$
$S(p) = i$


Perturbation Theory in QED
Photons
$D_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$
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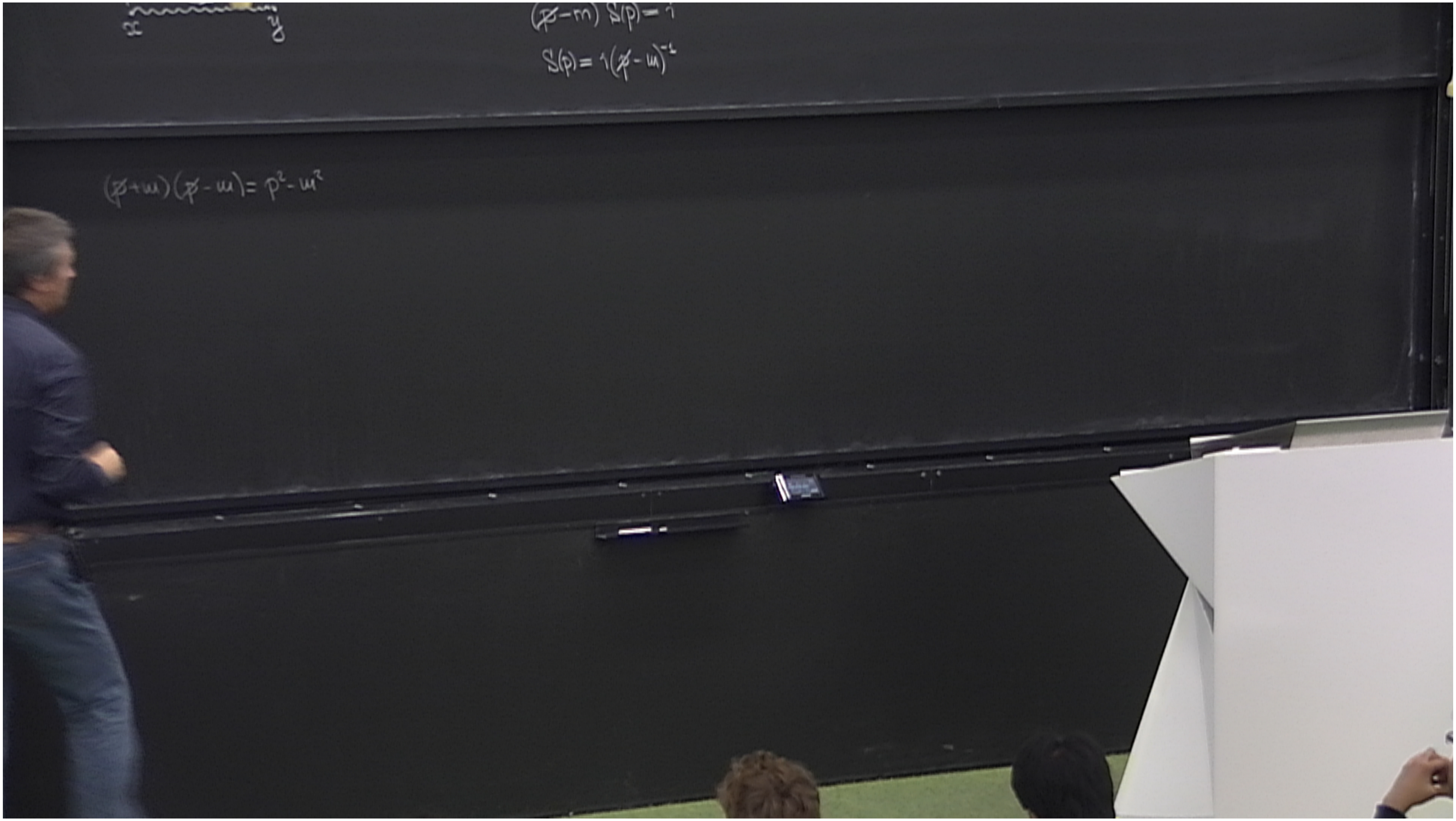
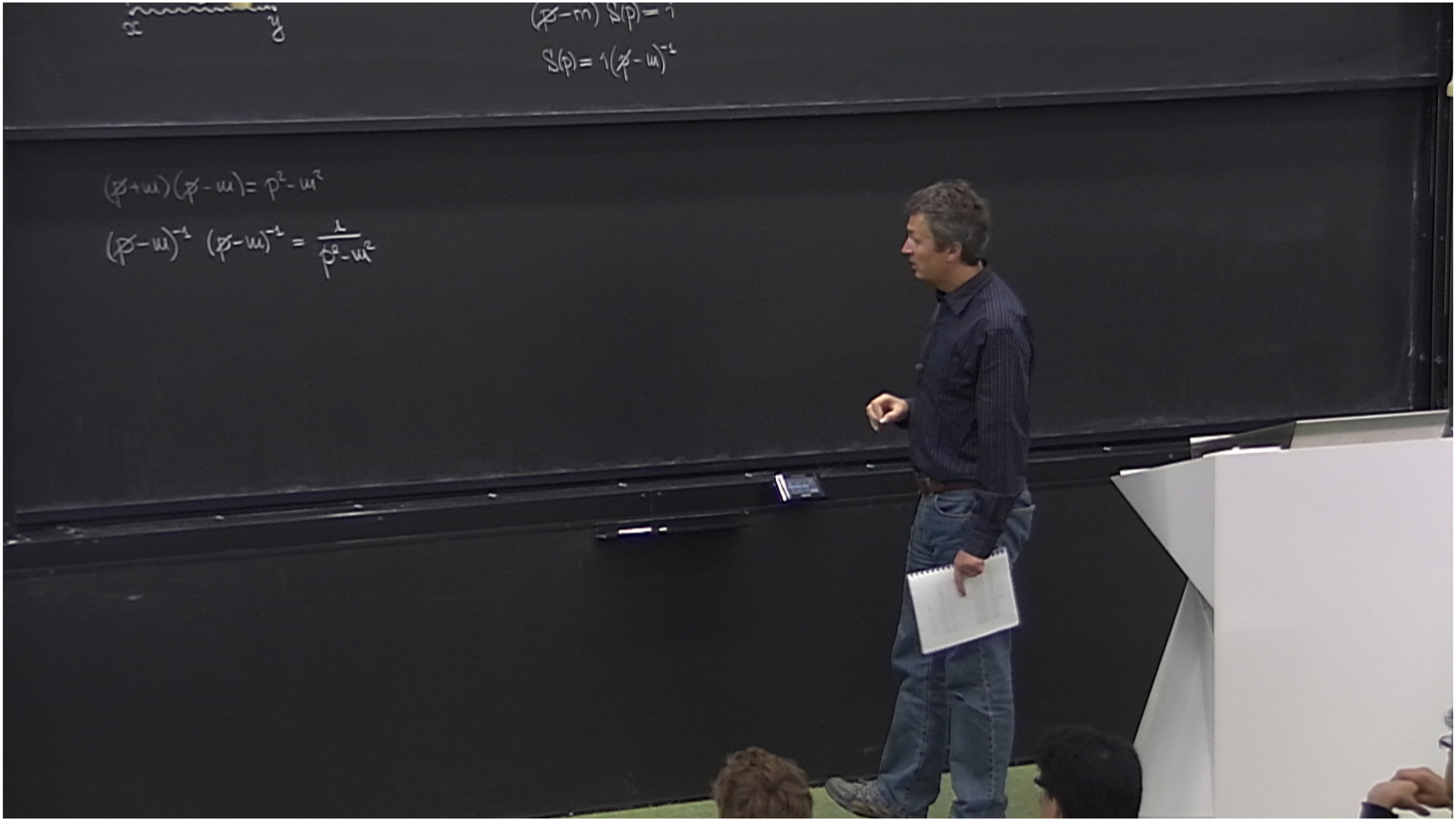
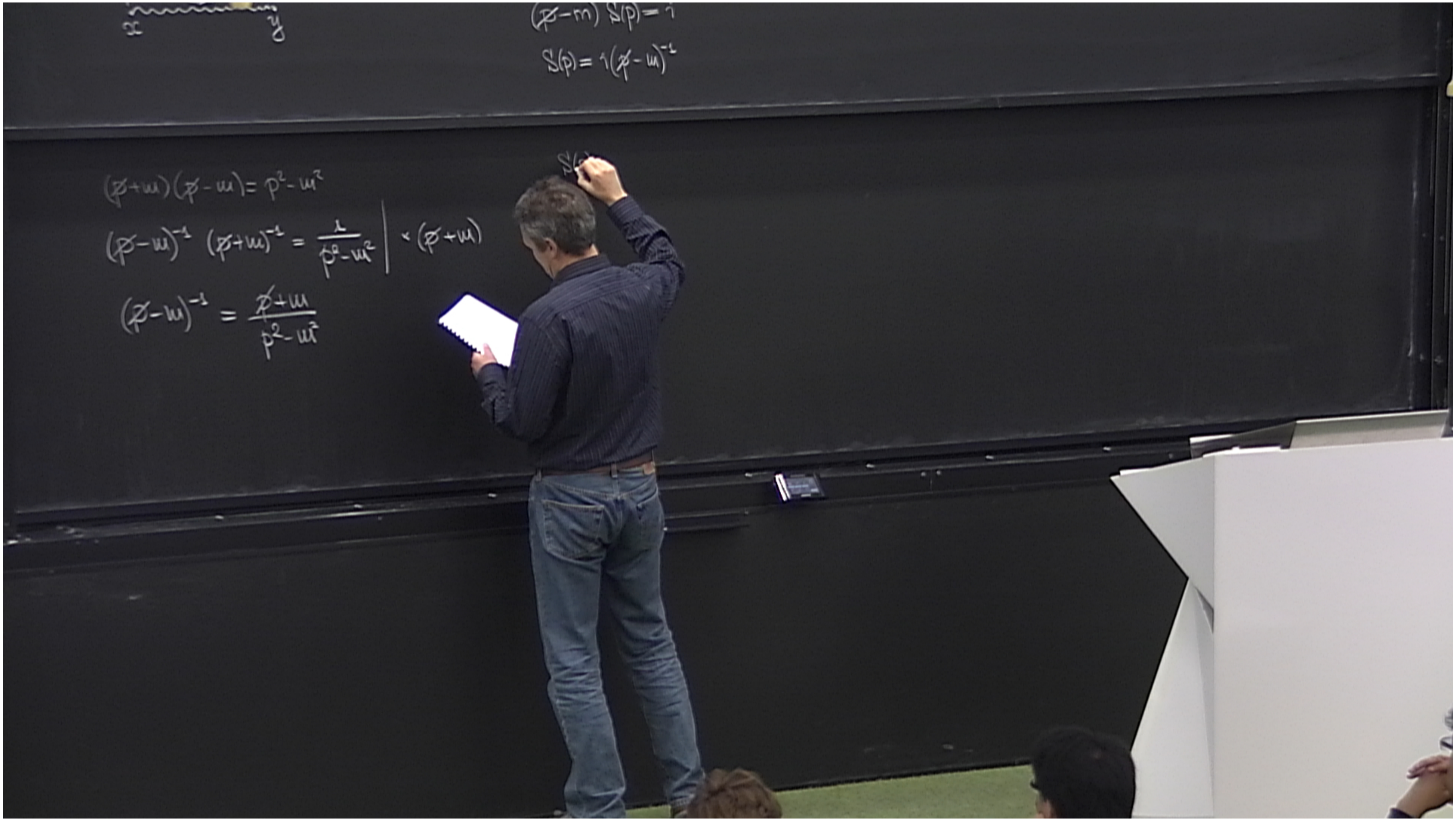
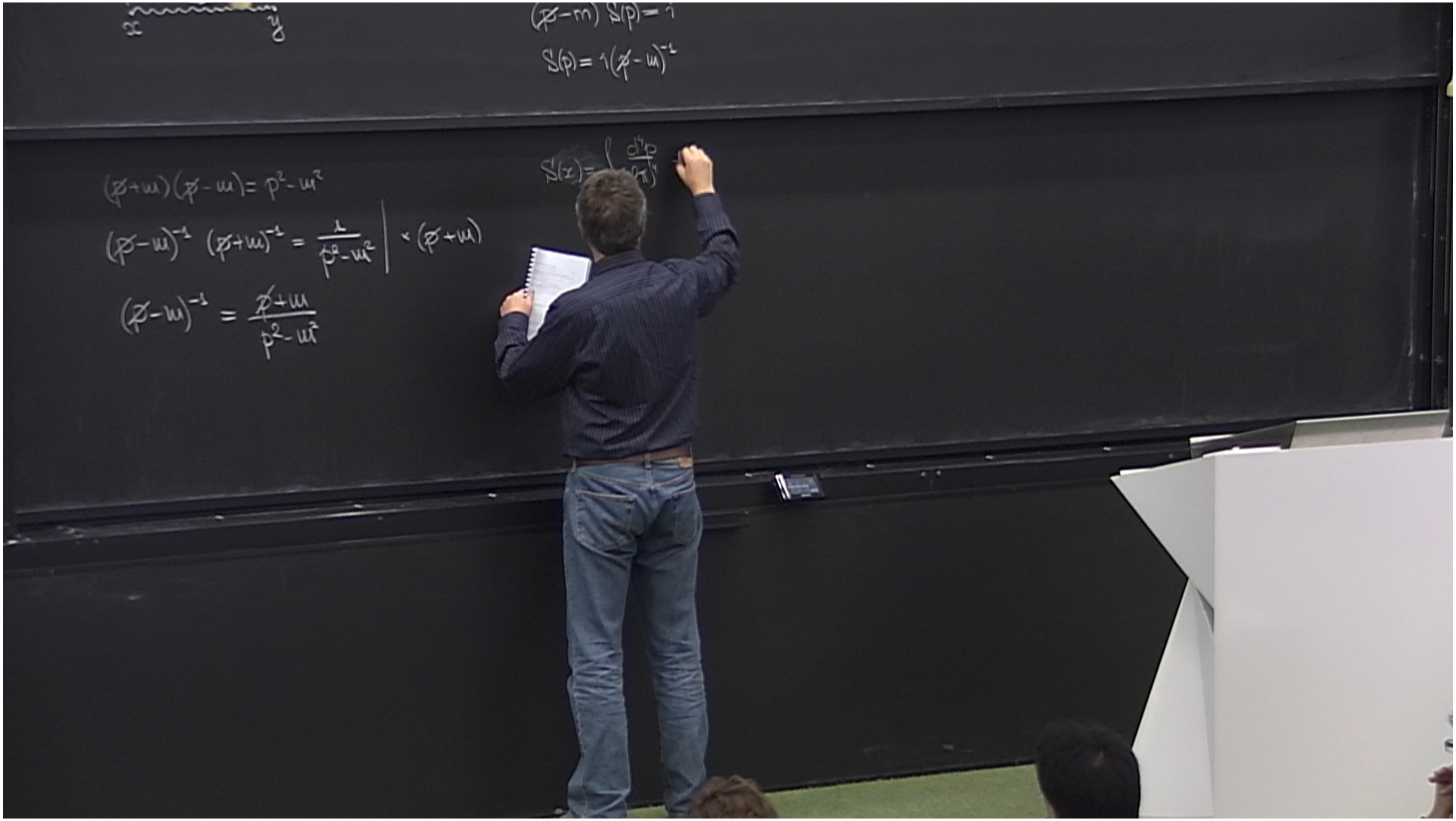
μ p ν
x y
$S_\alpha^\beta(x-y) = \langle 0 | T \psi_\alpha(x) \bar\Psi^\beta(y) | 0 \rangle$
$T \psi_\alpha(x) \bar\Psi^\beta(y) = \begin{cases} \psi_\alpha(x) \bar\Psi^\beta(y), & x^0 > y^0 \\ -\bar\Psi^\beta(y) \psi_\alpha(x), & y^0 > x^0 \end{cases}$
$(i\partial\!\!\!/_x - m) S(x-y) = i\,\delta(x-y)$
$(p\!\!\!/ - m) S(p) = i$
$S(p) = i(p\!\!\!/ - m)^{-1}$

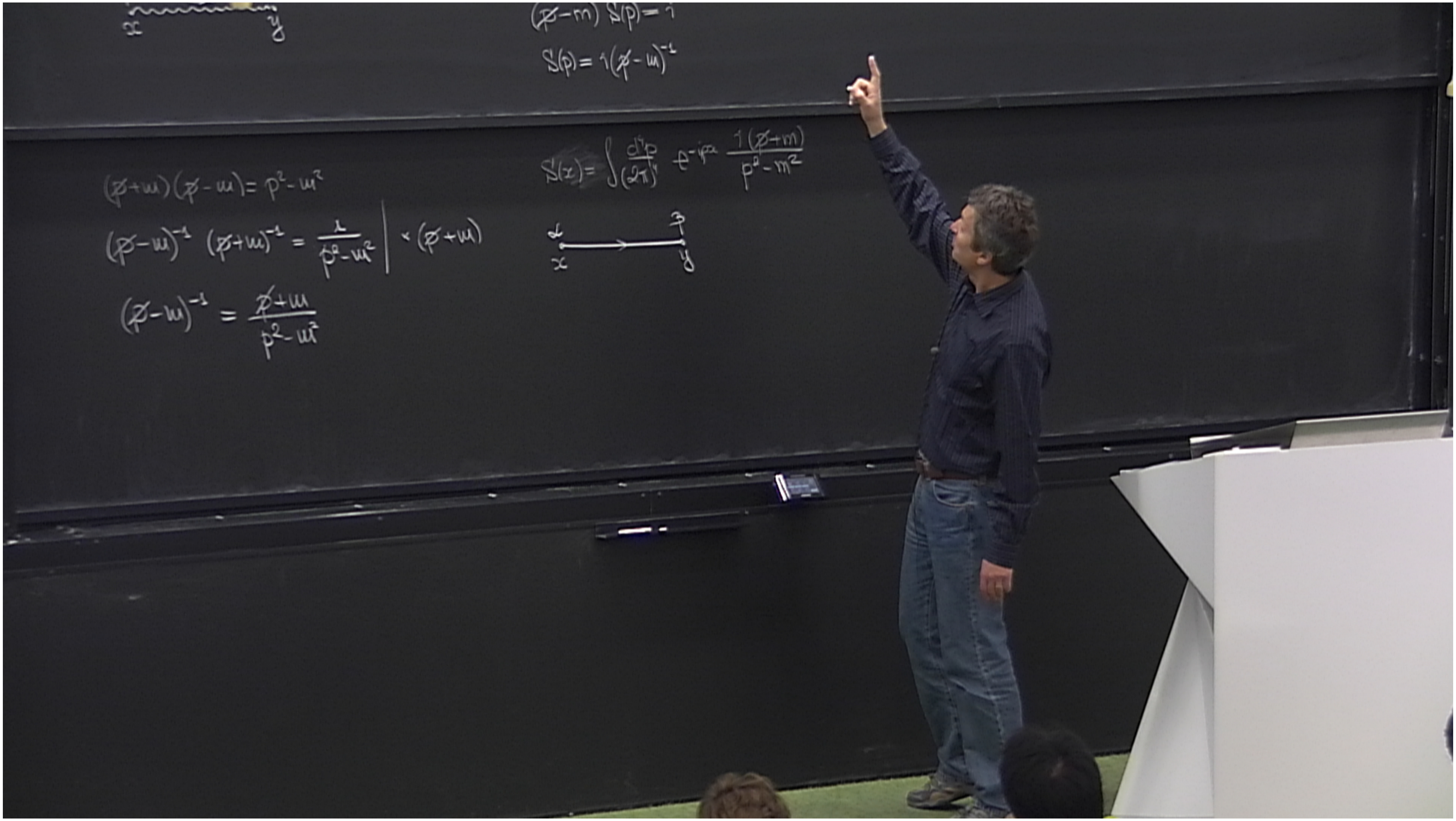

x
y
$(\not{p}-m)\, S(p) = i$
$S(p) = i(\not{p}-m)^{-1}$
$(\not{p}+m)(\not{p}-m) = p^2 - m^2$

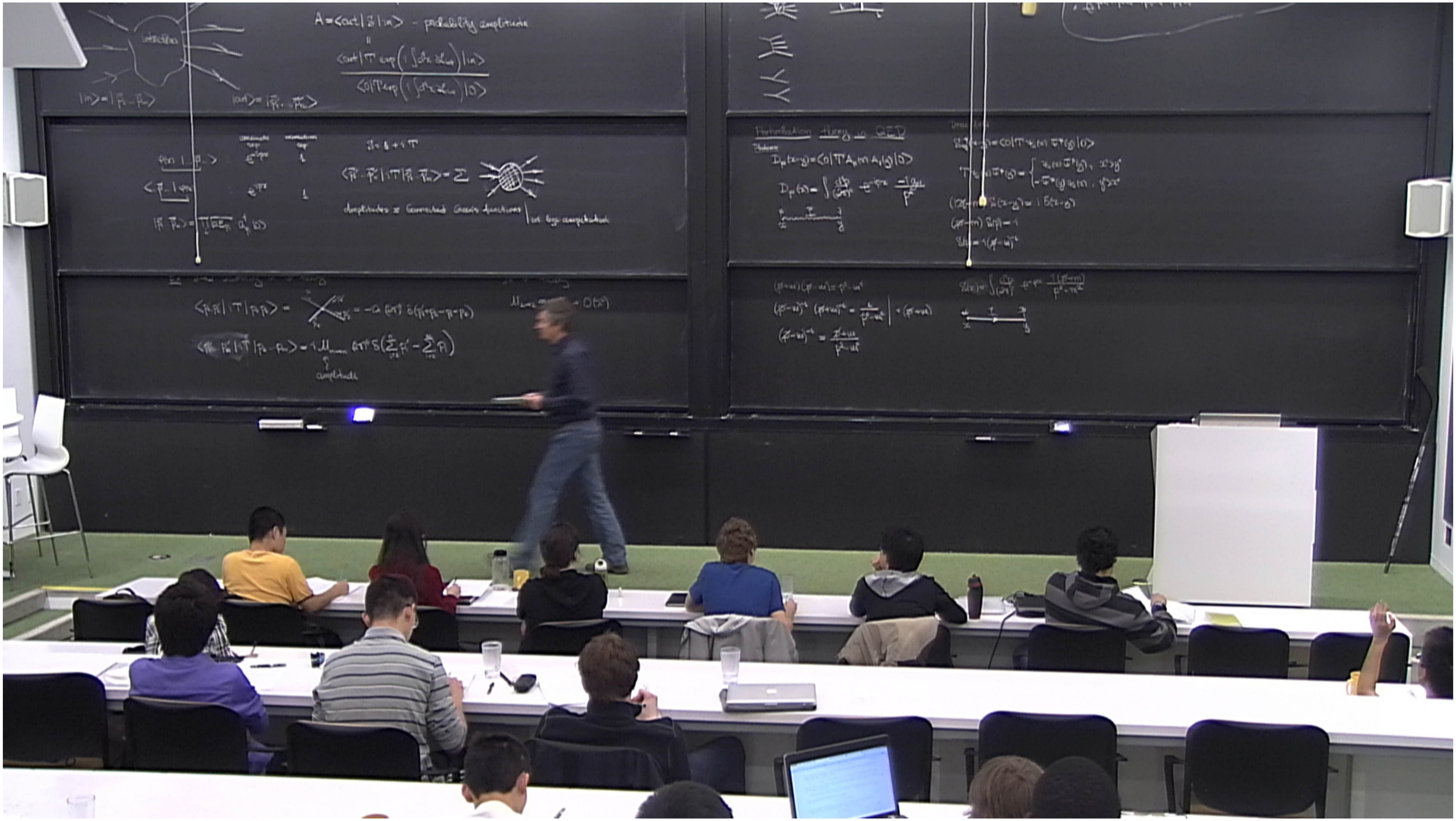

x
y
$(\not p - m)\, S(p) = i$
$S(p) = i(\not p - m)^{-1}$
$(\not p + m)(\not p - m) = p^2 - m^2$
$(\not p - m)^{-1}\,(\not p - m)^{-1} = \frac{1}{p^2 - m^2}$

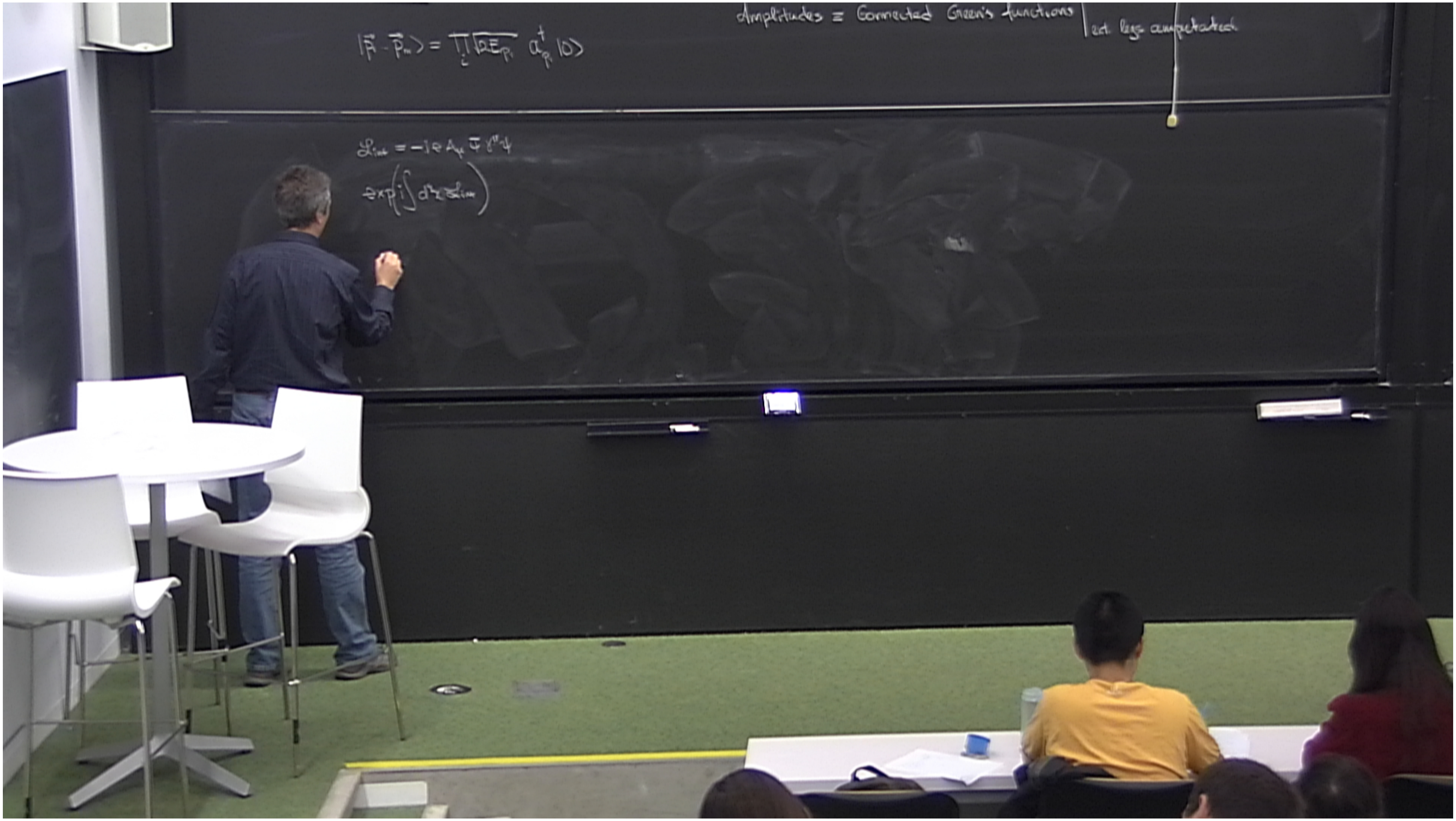


x y
$(\not{p}-m)S(p)=i$
$S(p)=i(\not{p}-m)^{-1}$
$(\not{p}+m)(\not{p}-m)=p^2-m^2$
$(\not{p}-m)^{-1}(\not{p}+m)^{-1}=\frac{1}{p^2-m^2}$ $\times(\not{p}+m)$
$(\not{p}-m)^{-1}=\frac{\not{p}+m}{p^2-m^2}$

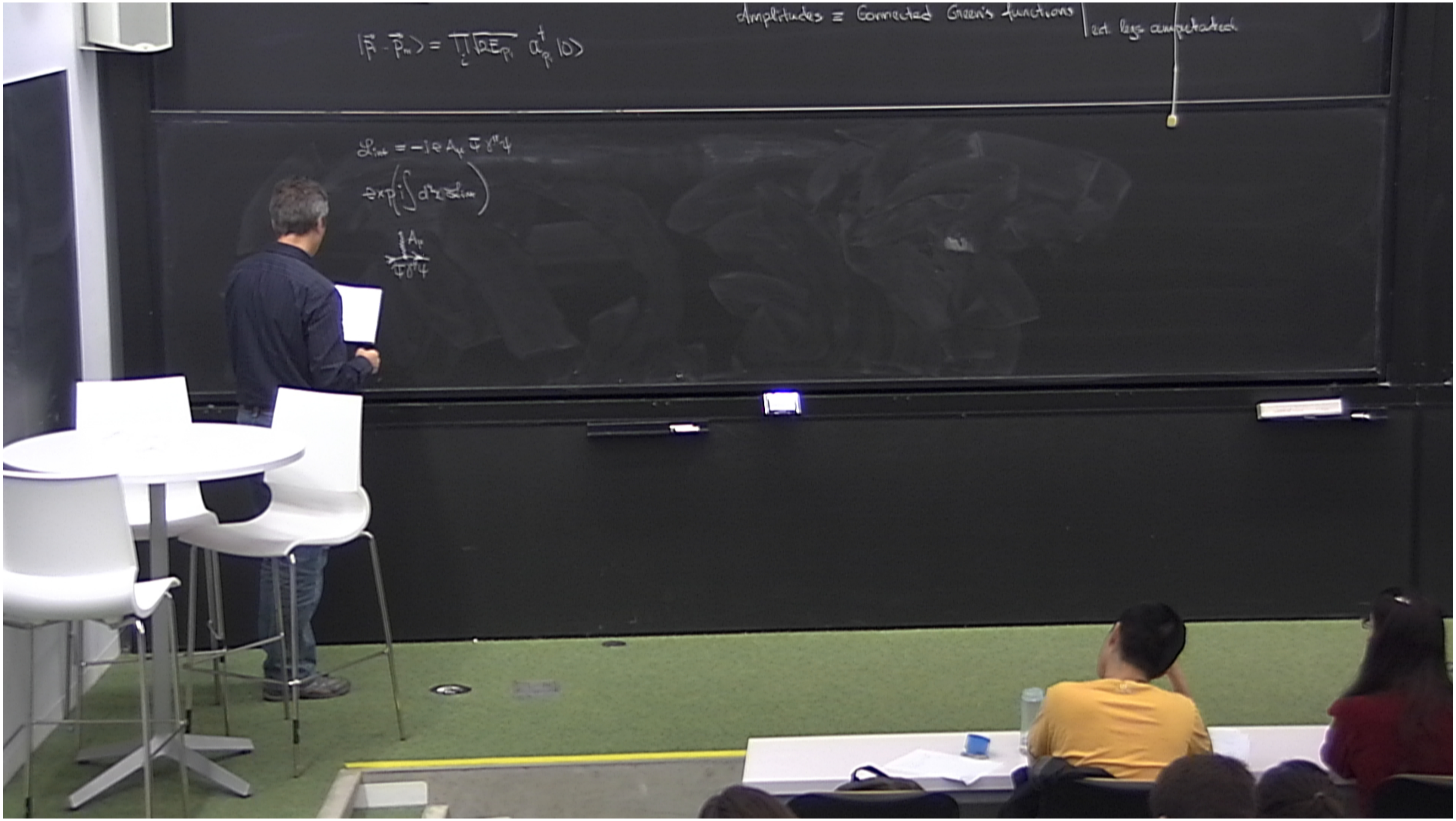

x y
$(\not{p}-m)\,S(p)=i$
$S(p)=i(\not{p}-m)^{-1}$
$(\not{p}+m)(\not{p}-m)=p^2-m^2$
$(\not{p}-m)^{-1}\,(\not{p}+m)^{-1}=\frac{1}{p^2-m^2}\ \Big|\ \times(\not{p}+m)$
$(\not{p}-m)^{-1}=\frac{\not{p}+m}{p^2-m^2}$
$S(x)=\int\frac{d^4p}{(2\pi)^4}$

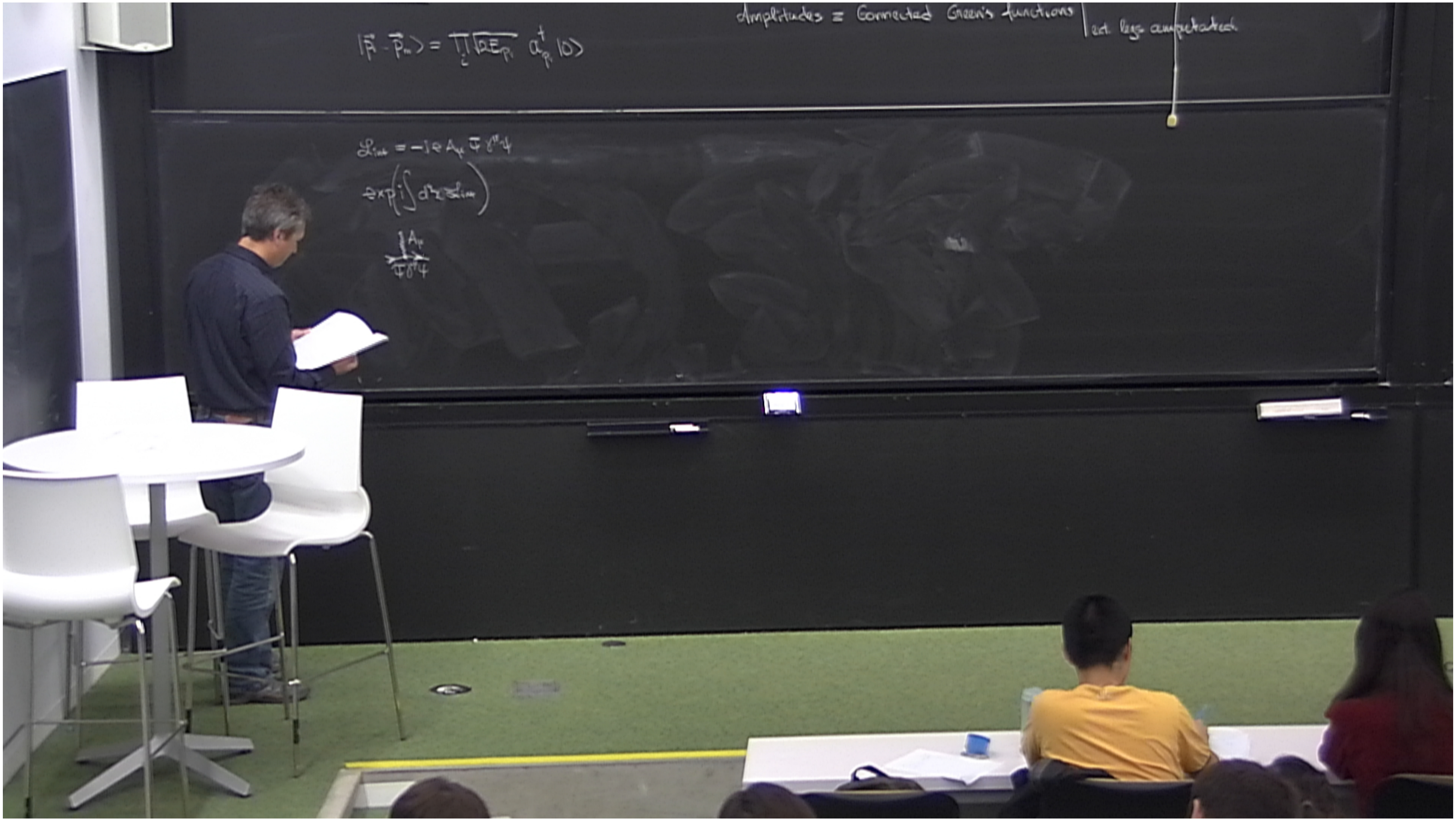

$(\not{p} - m)\, S(p) = i$
$S(p) = i(\not{p} - m)^{-1}$
$(\not{p} + m)(\not{p} - m) = p^2 - m^2$
$(\not{p} - m)^{-1} (\not{p} + m)^{-1} = \frac{1}{p^2 - m^2} \Big| \times (\not{p} + m)$
$(\not{p} - m)^{-1} = \frac{\not{p} + m}{p^2 - m^2}$
$S(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{i(\not{p} + m)}{p^2 - m^2}$
α
β
x
y



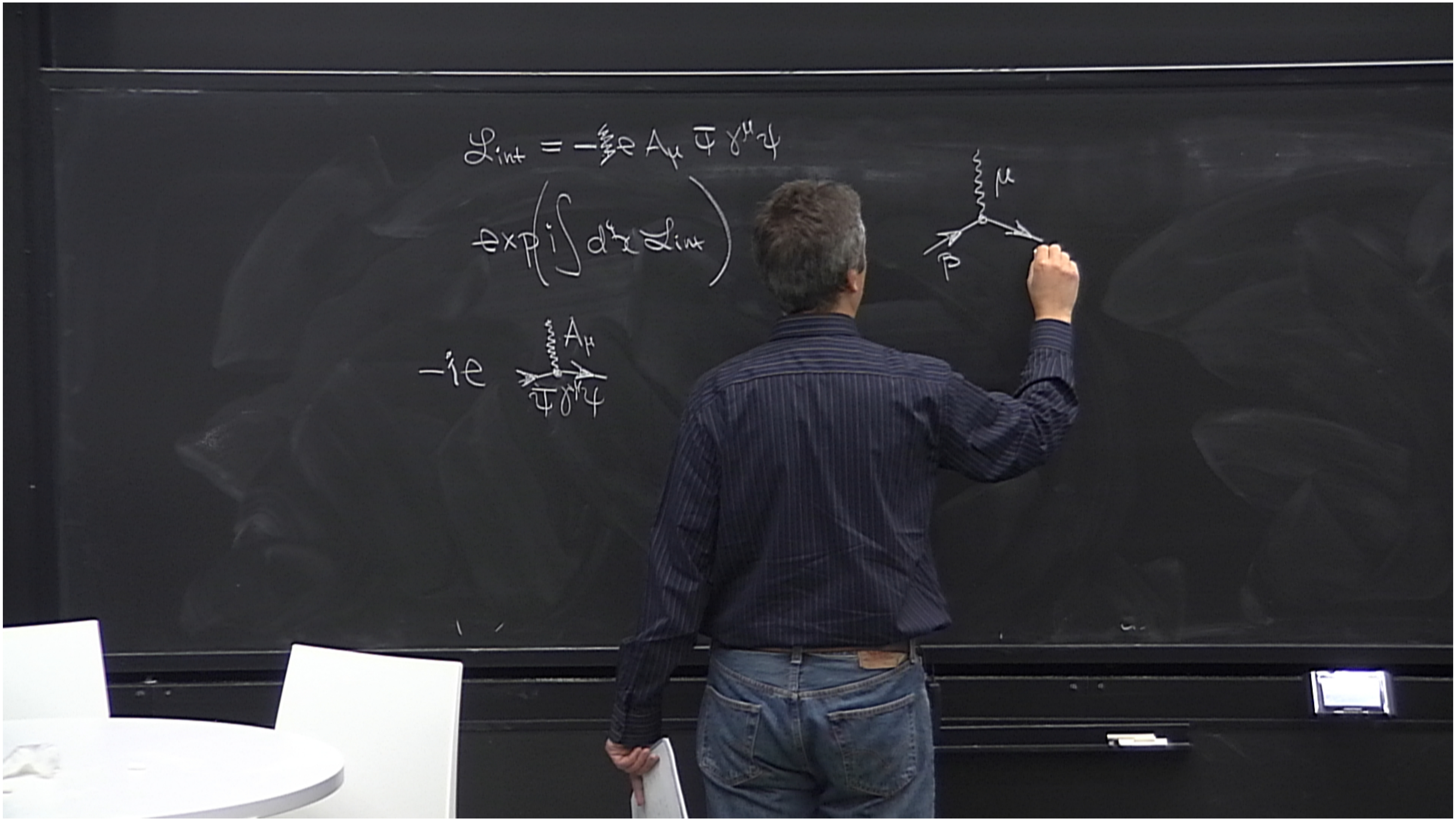


Amplitudes ≡ Connected Green's functions | ext. legs amputated
$|\vec{p}_1 \cdots \vec{p}_n\rangle = \prod_i \sqrt{2E_{p_i}}\, a^\dagger_{p_i} |0\rangle$
$\mathcal{L}_{int} = -ie A_\mu \bar{\psi} \gamma^\mu \psi$
$\exp\left(i \int d^4x\, \mathcal{L}_{int}\right)$

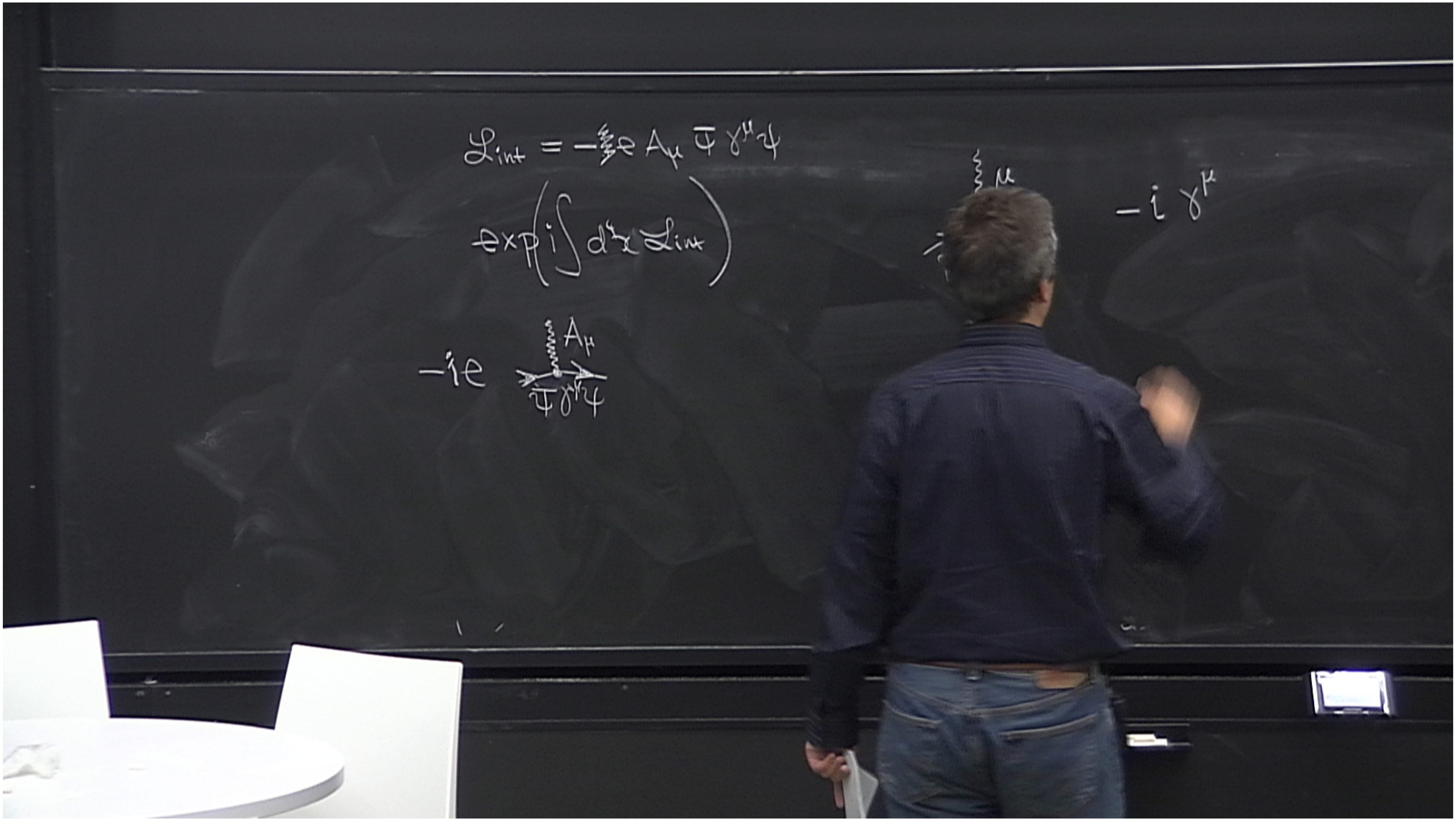

Amplitudes ≡ Connected Green's functions | ext. legs amputated
|p1 ... pn> = ∏i √2Epi a†pi |0>
Lint = −ie Aμ ψ̄ γμ ψ
exp(i ∫ d4x Lint)
Aμ
ψ̄ γμ ψ



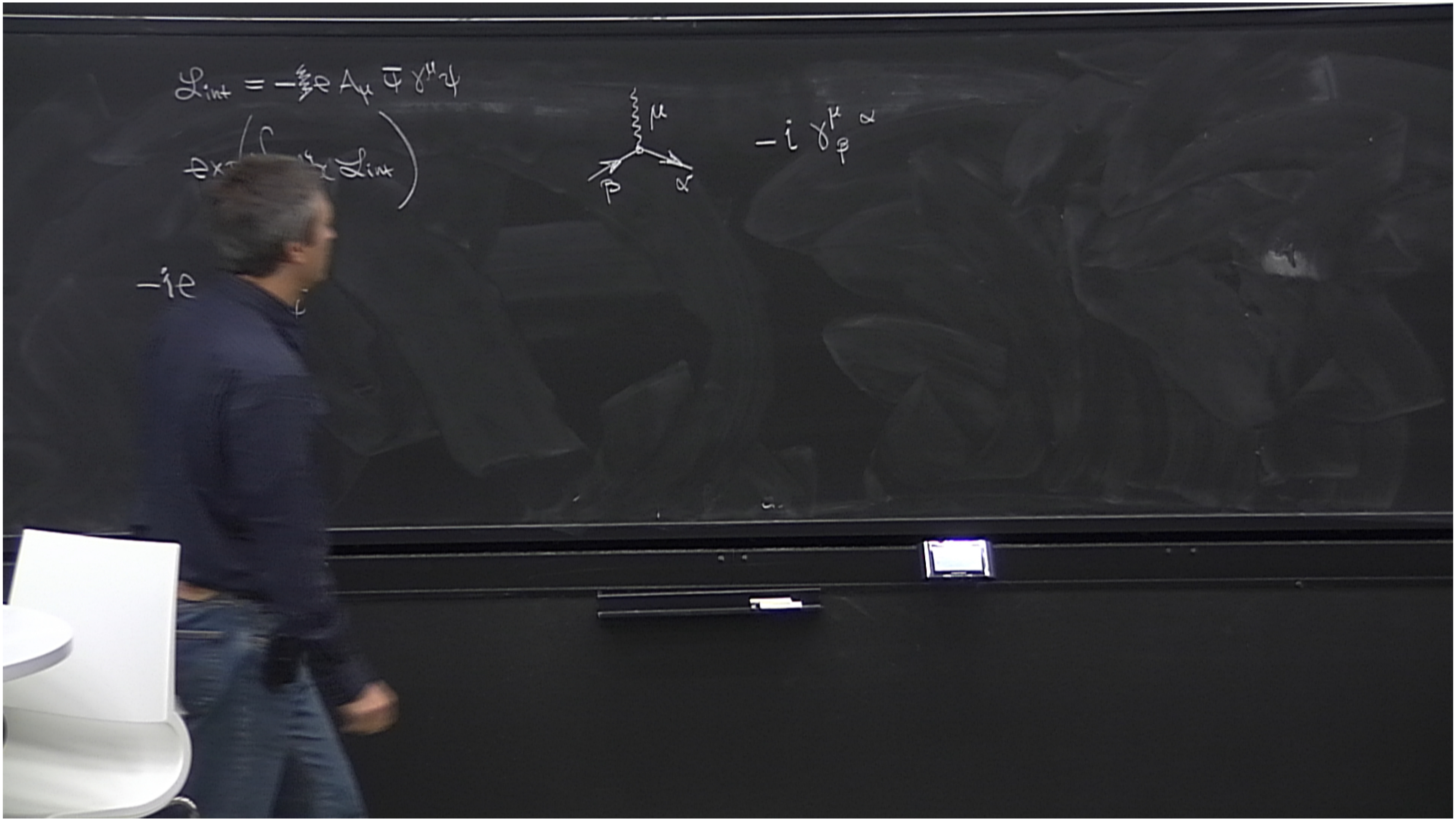
Amplitudes = Connected Green's functions
ext. legs amputated



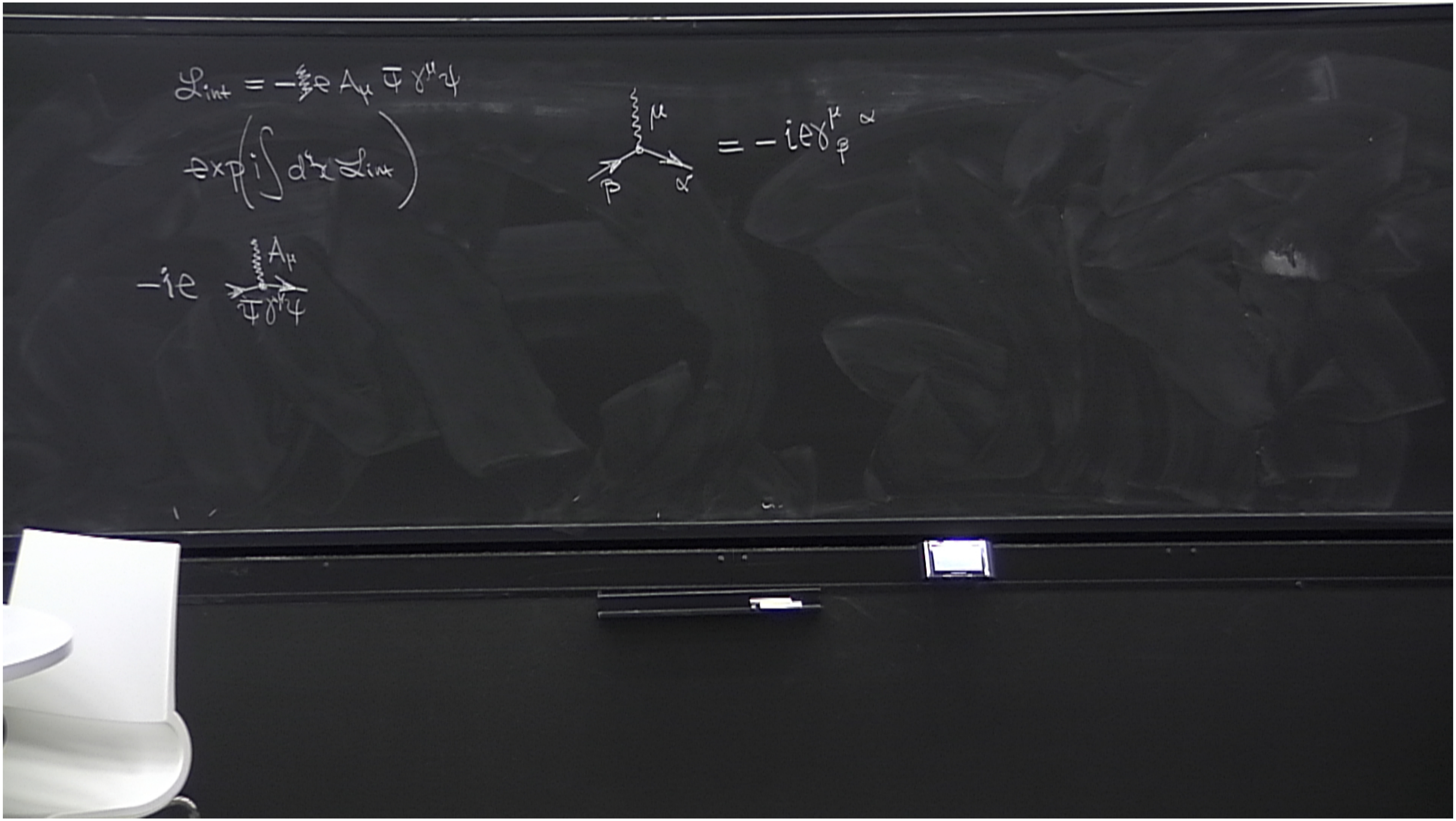
$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \psi$
$\exp\left(i \int d^4x \, \mathcal{L}_{int}\right)$
$-ie$
A_μ
$\bar{\Psi} \gamma^\mu \psi$
μ
p

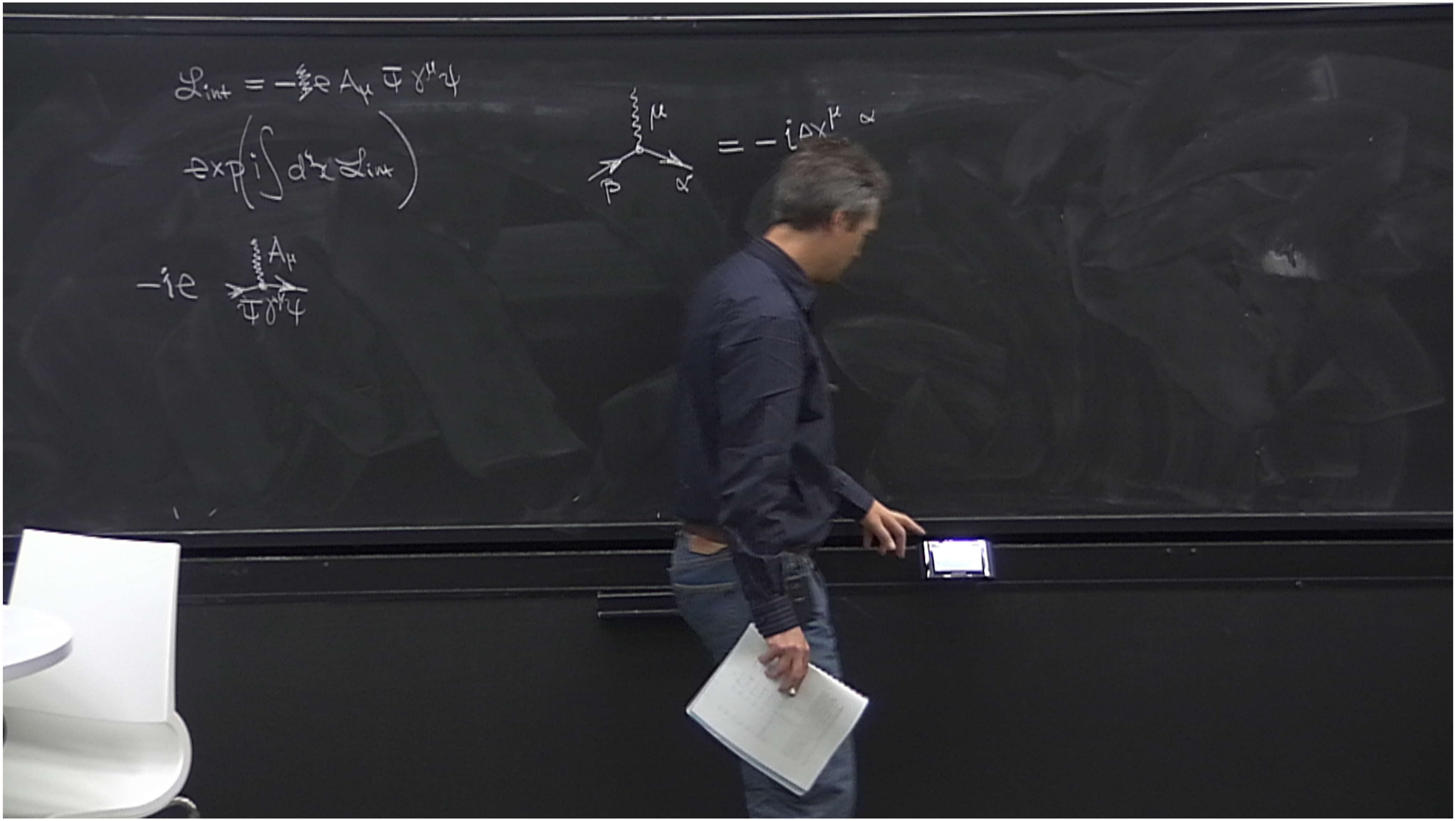


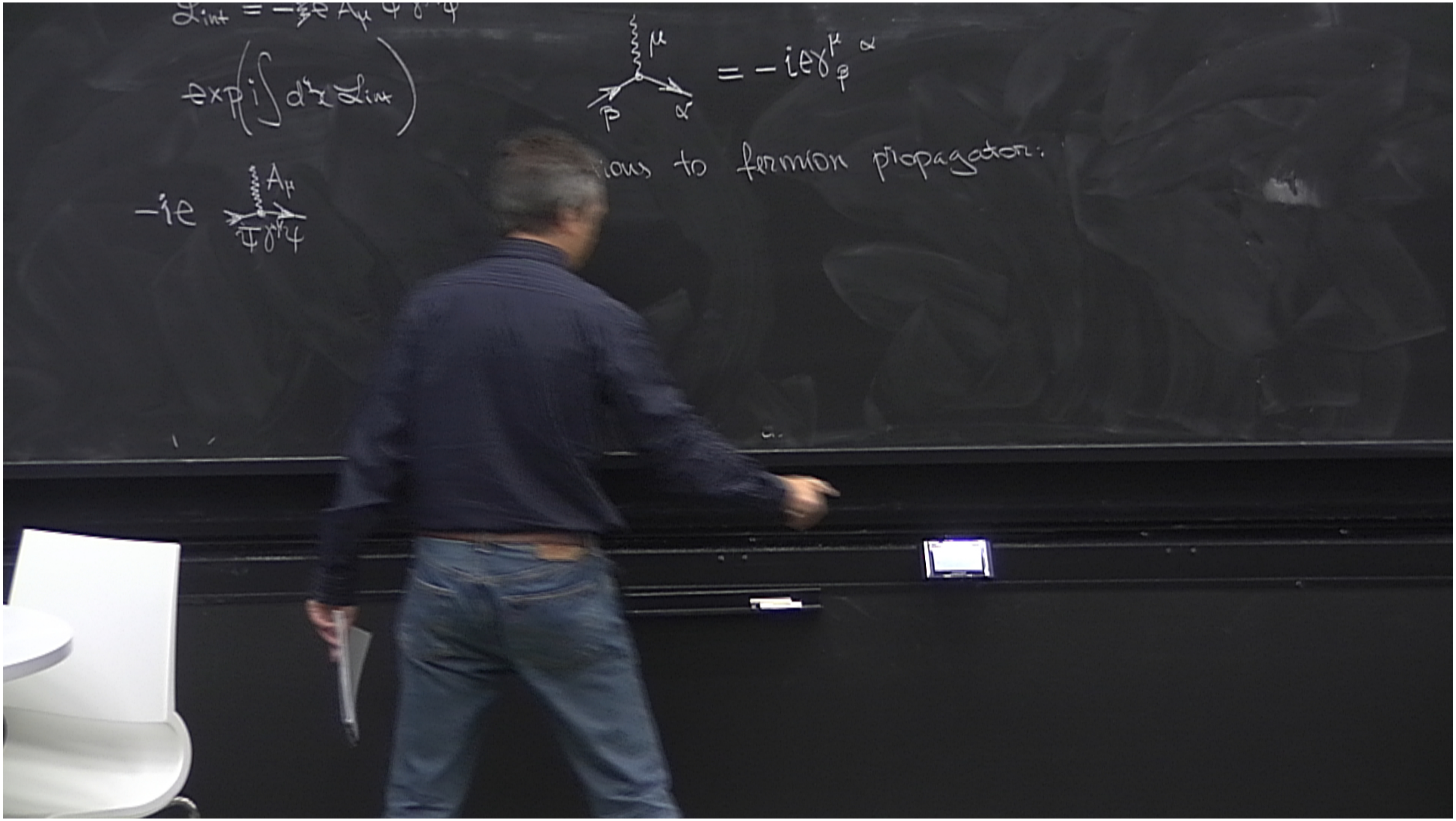
$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \psi$
$\exp\left(i \int d^4x \, \mathcal{L}_{int}\right)$
A_μ
$-ie$
$\bar{\Psi} \gamma^\mu \psi$
μ
$-i \gamma^\mu$

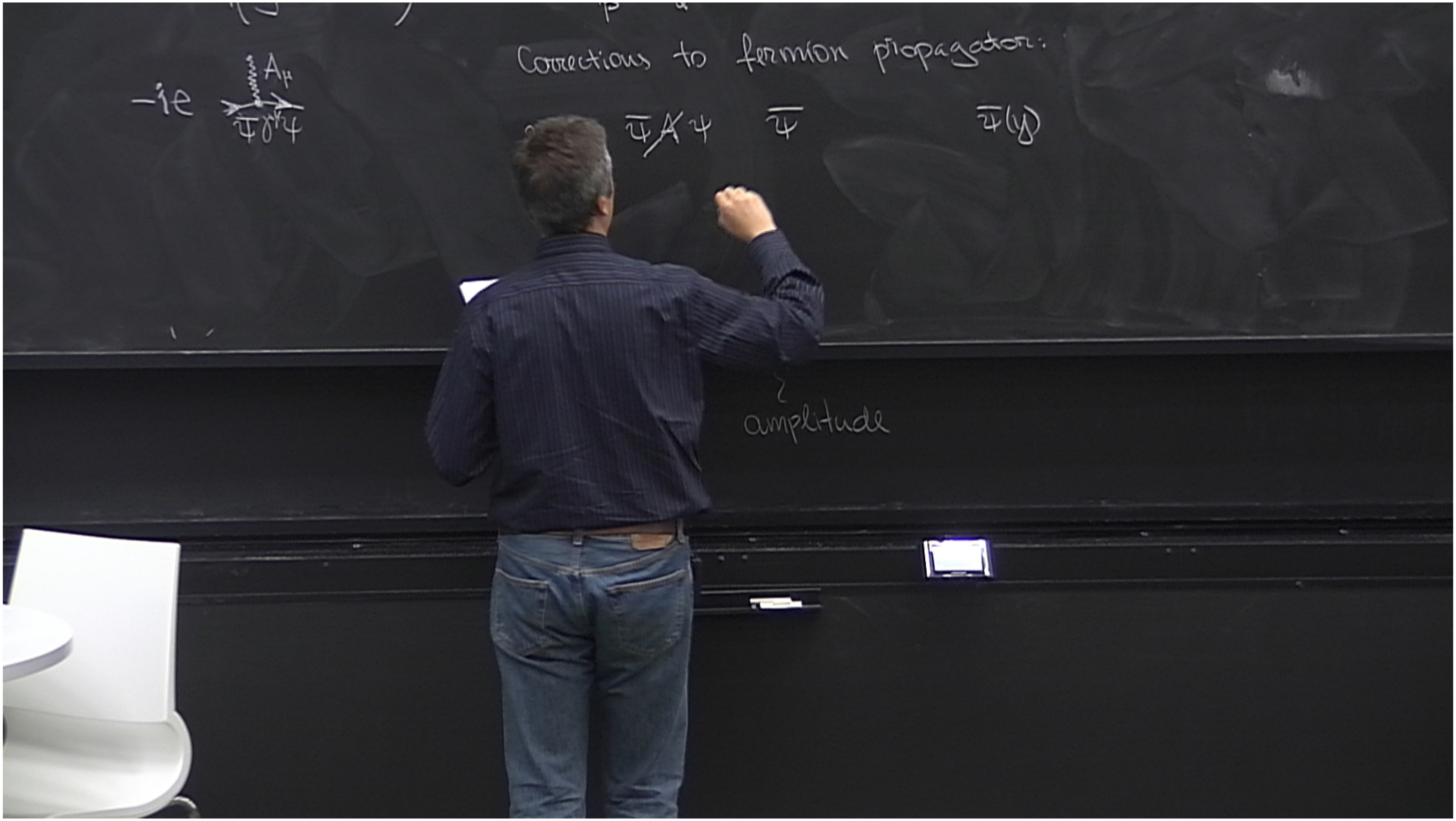


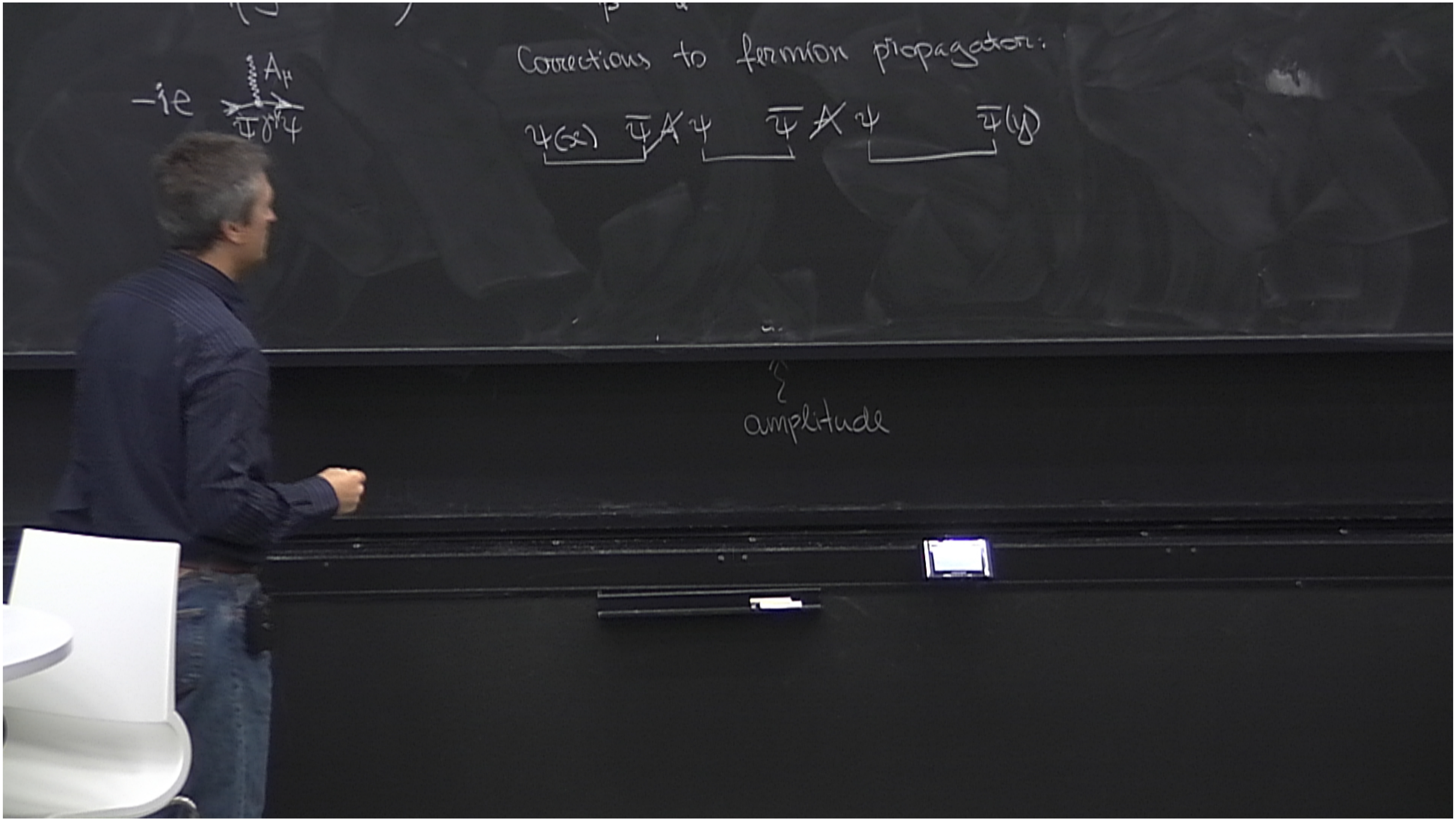
$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \Psi$
$-i\, \gamma^\mu{}_\beta{}^\alpha$
μ
β
α
$-ie$

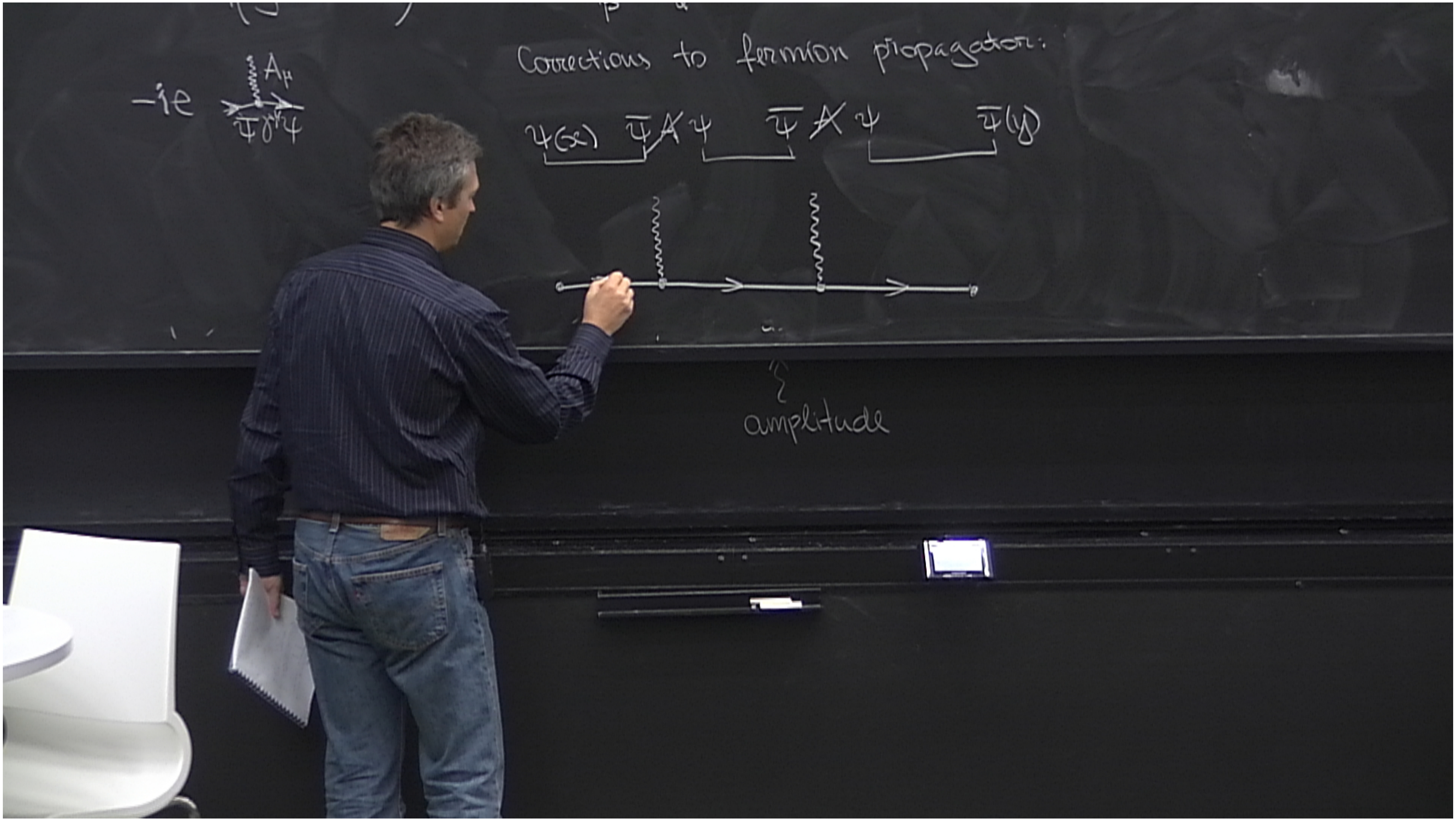

$\mathcal{L}_{int} = -e A_\mu \bar\Psi \gamma^\mu \Psi$
$\exp\left(i\int d^4x \mathcal{L}_{int}\right)$
$= -ie\gamma^\mu{}_\beta{}^\alpha$
$-ie$
A_μ
$\bar\Psi \gamma^\mu \Psi$

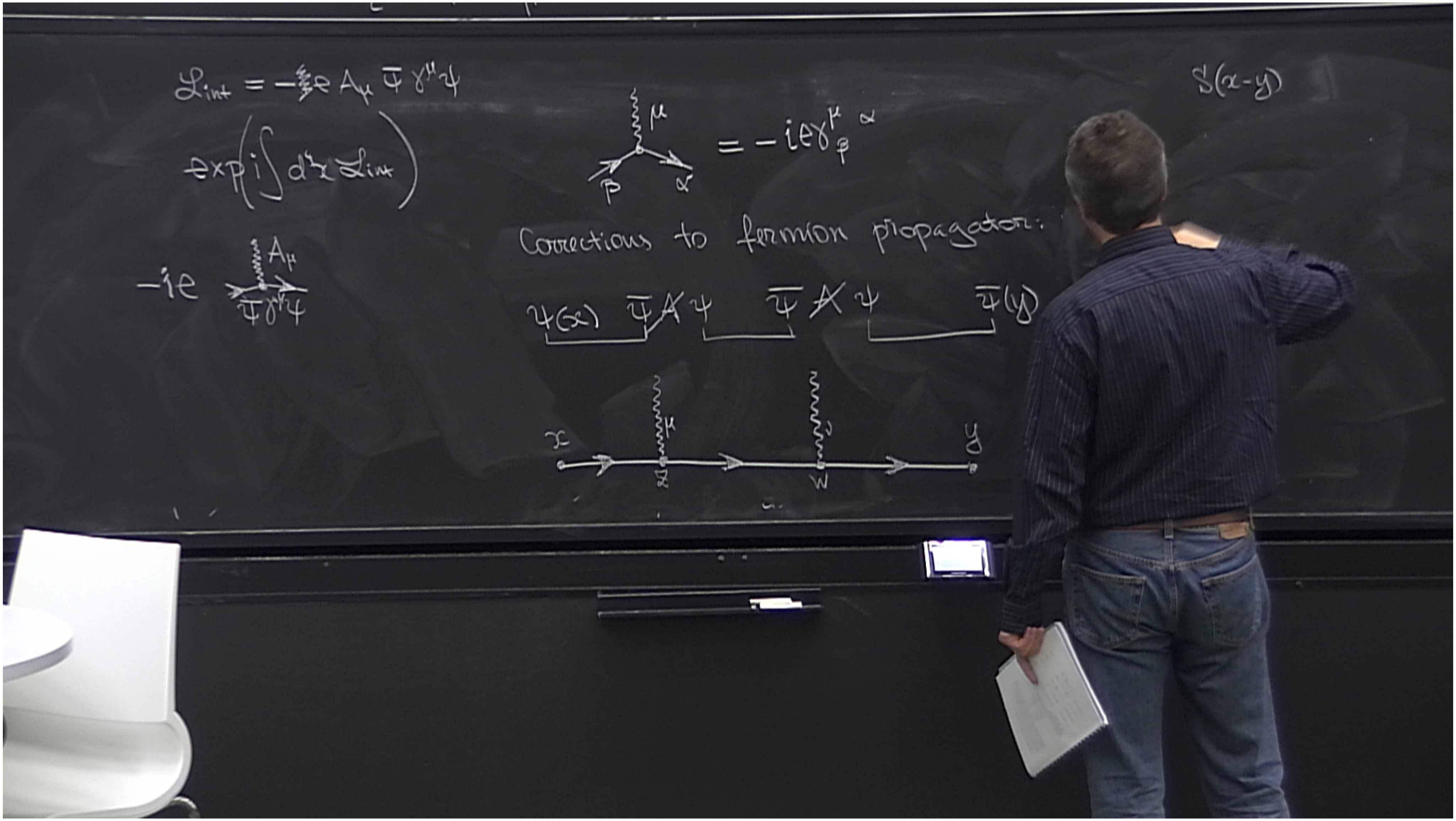

$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \psi$
$\exp\left(i \int d^4x \, \mathcal{L}_{int}\right)$
$-ie$
A_μ
$\bar{\Psi} \gamma^\mu \psi$
μ
β
α

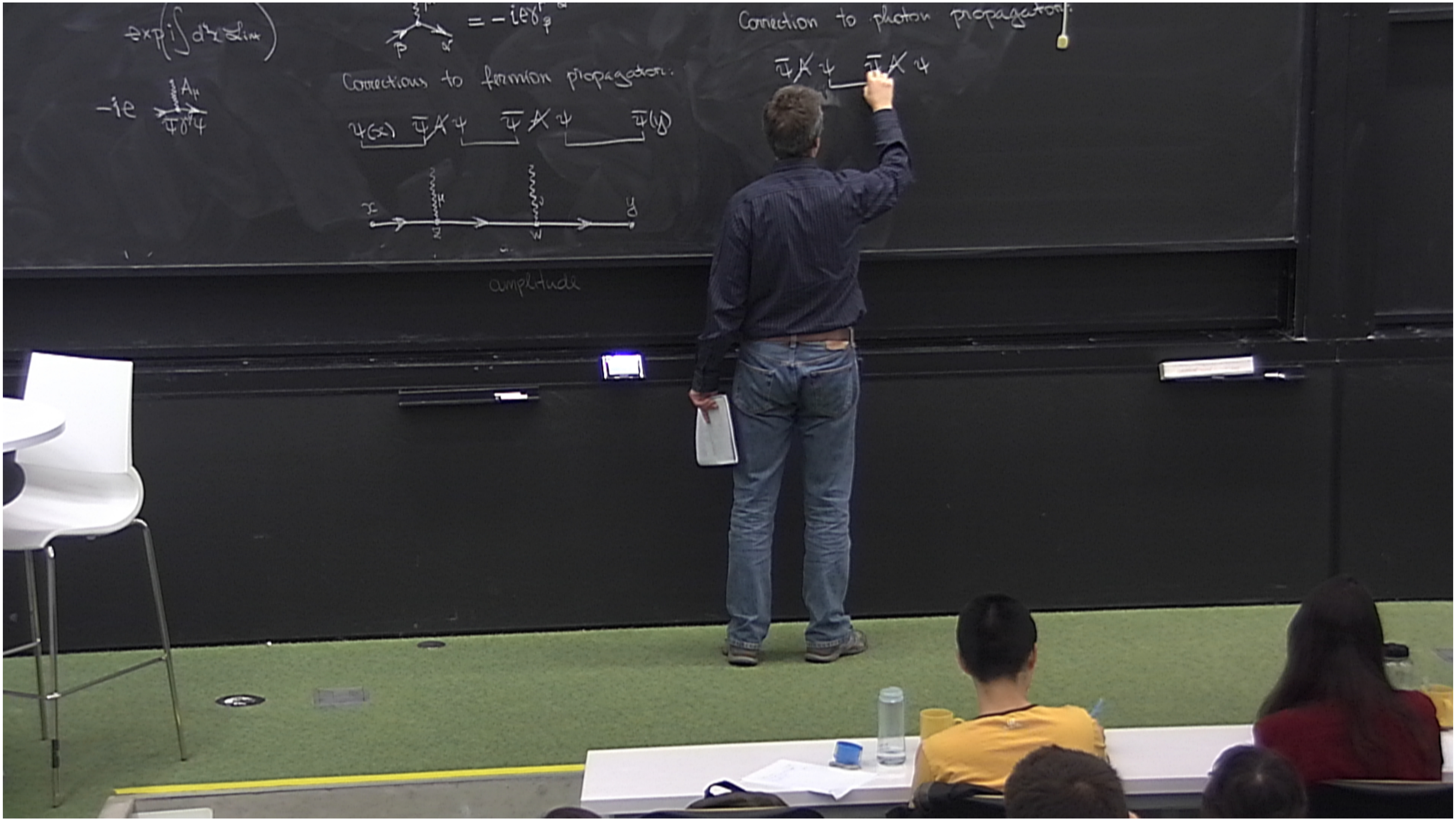

exp(i∫d⁴x ℒint)
= −ieγ^μ_β^α
ions to fermion propagator:
−ie
Aμ

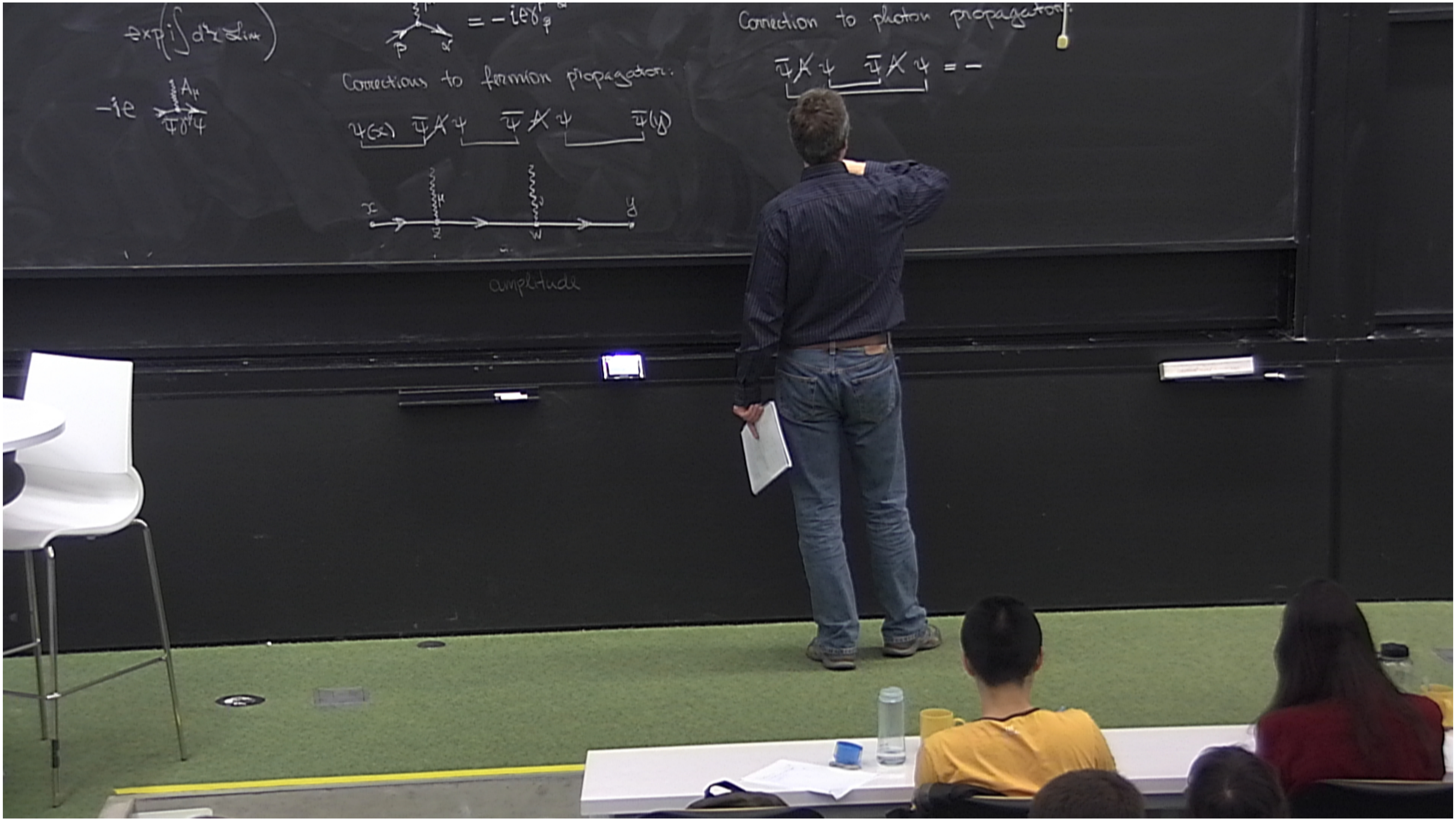

Corrections to fermion propagator:
−ie
amplitude


Corrections to fermion propagator:
$-ie$
A_μ
$\bar{\Psi}\gamma^\mu\Psi$
$\Psi(x)\ \bar{\Psi}\not{A}\Psi\ \ \bar{\Psi}\not{A}\Psi\ \ \bar{\Psi}(y)$
amplitude

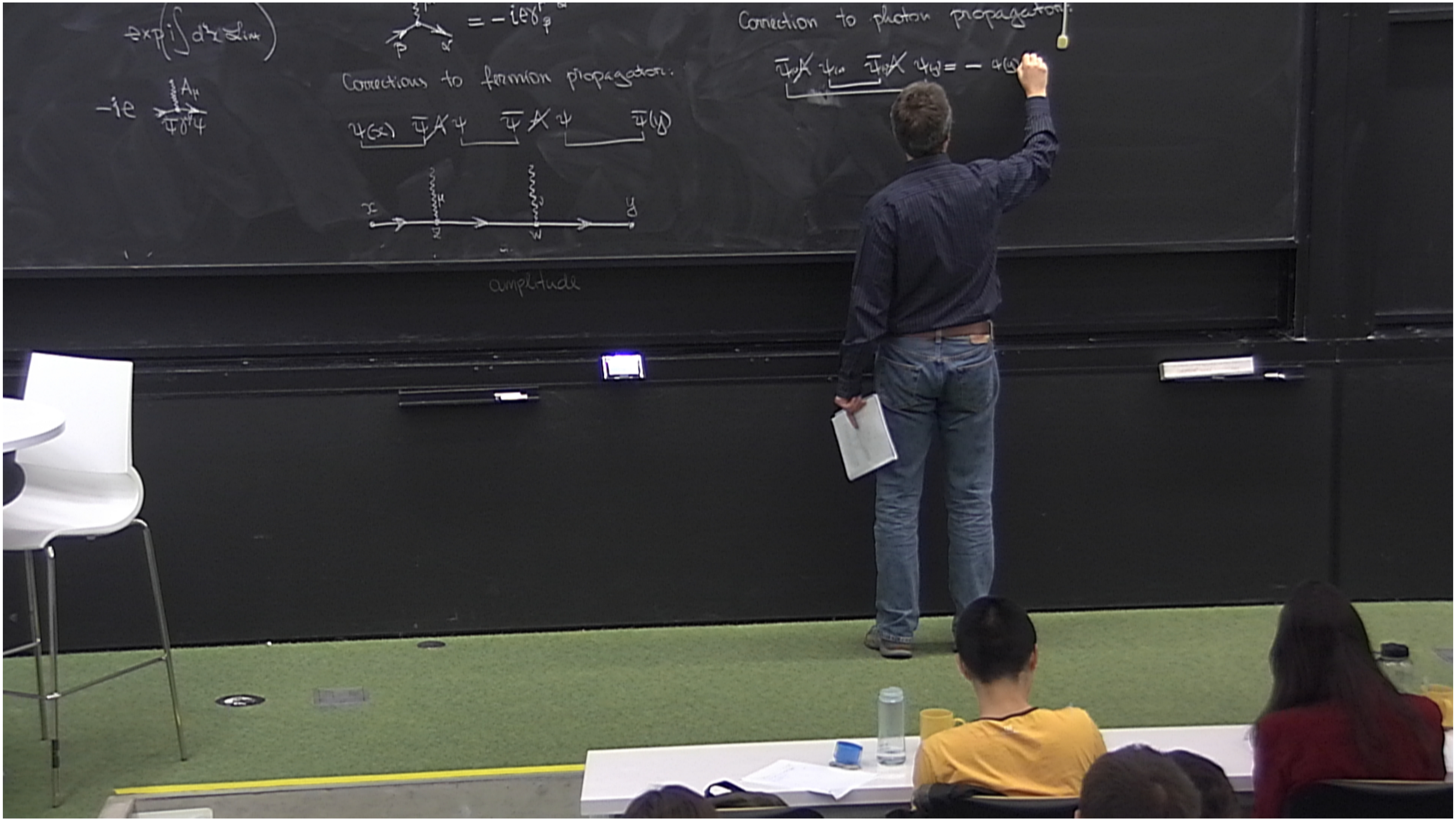

Corrections to fermion propagator:
$-ie$
A_μ
$\bar\Psi \gamma^\mu \Psi$
$\Psi(x) \; \bar\Psi \not{A} \Psi \; \bar\Psi \not{A} \Psi \; \bar\Psi(y)$
amplitude


$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \psi$
$\exp\left(i\int d^4x \mathcal{L}_{int}\right)$
$-ie$
A_μ
$\bar{\Psi}\gamma^\mu\psi$
$= -ie\gamma^\mu_\beta{}^\alpha$
$S(x-y)$
Corrections to fermion propagator:
$\psi(x)\ \bar{\Psi} \cancel{A} \psi\ \bar{\Psi} \cancel{A} \psi\ \bar{\Psi}(y)$

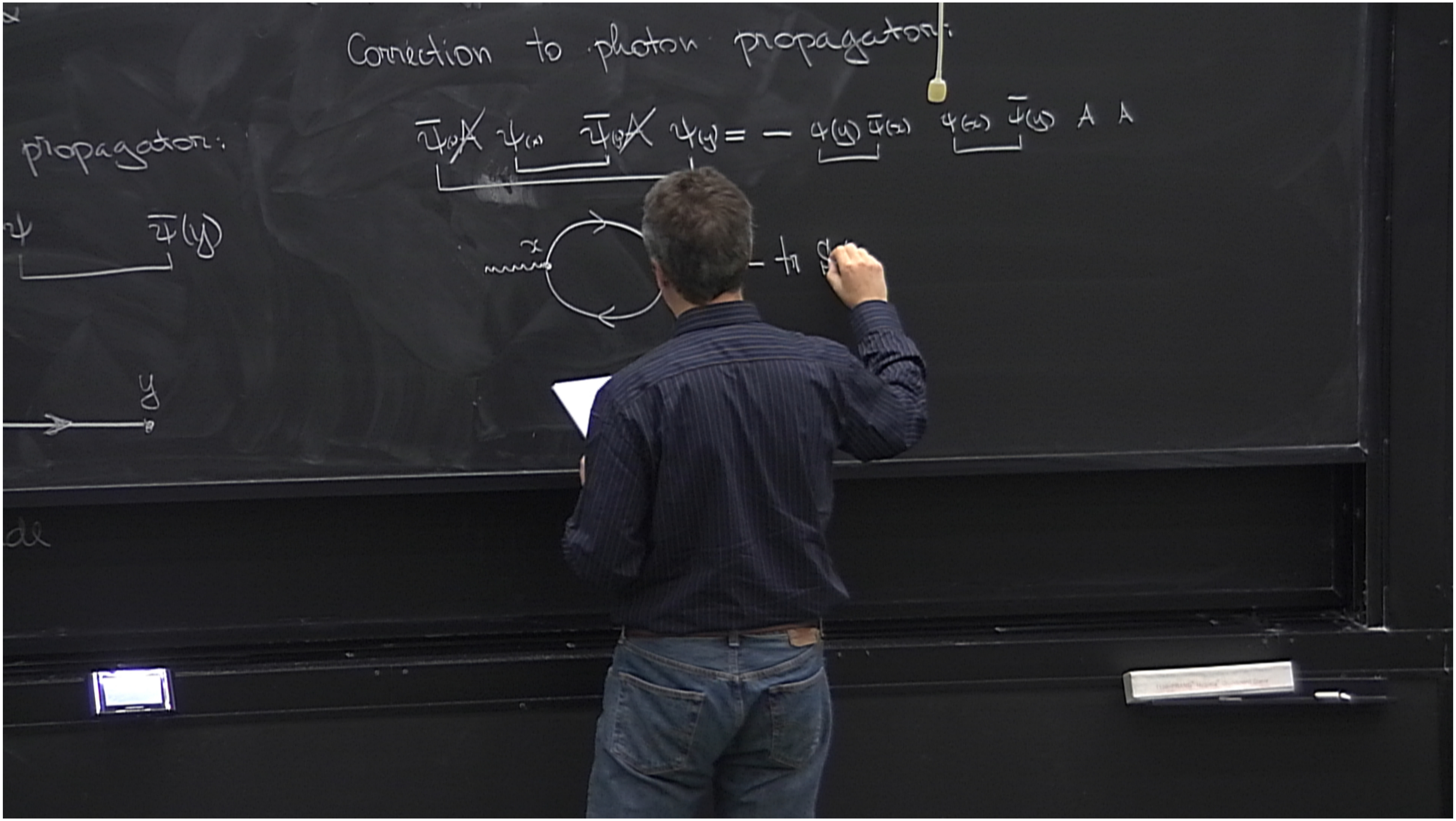

Corrections to fermion propagator:
Correction to photon propagator:
amplitude

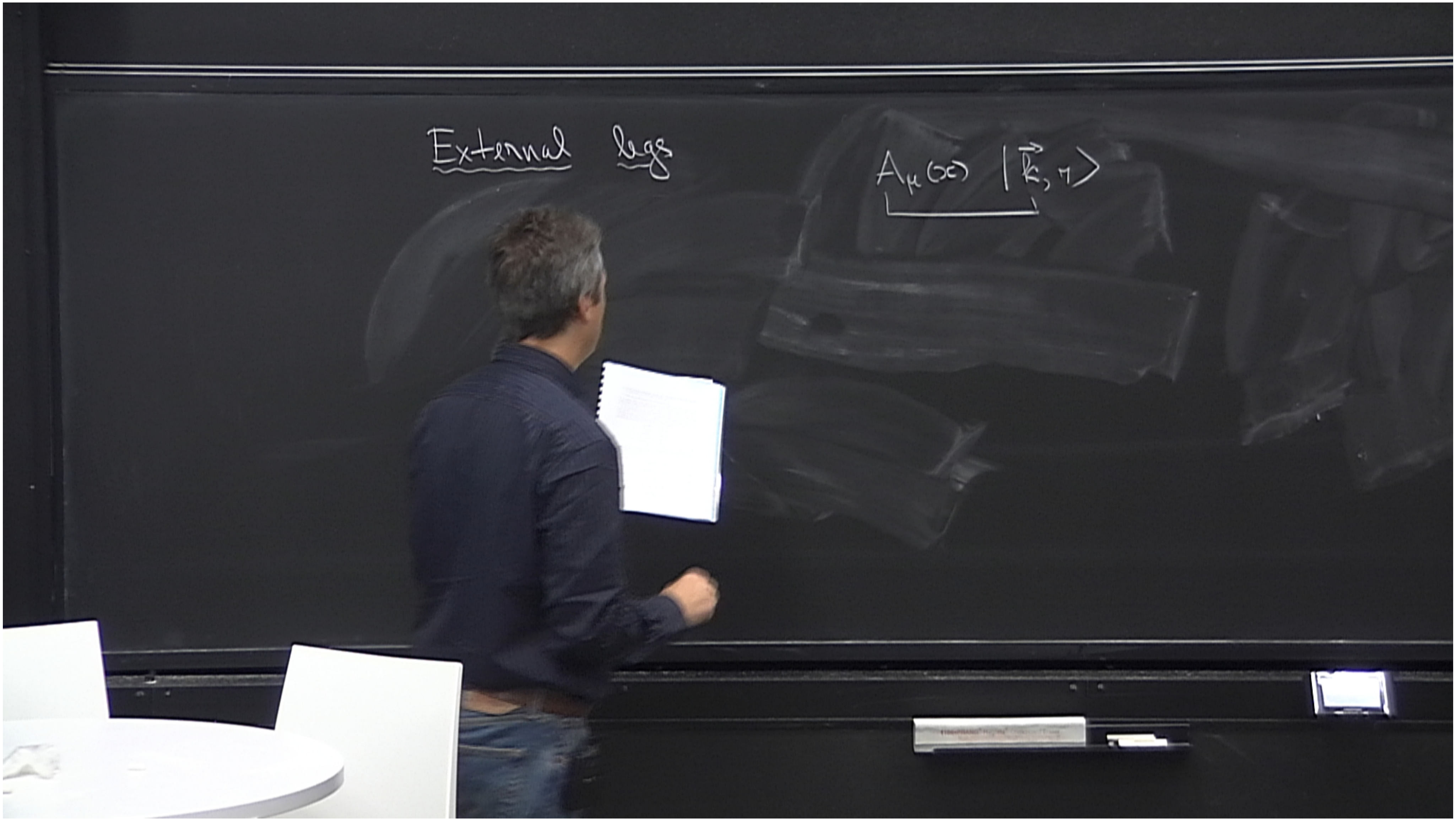


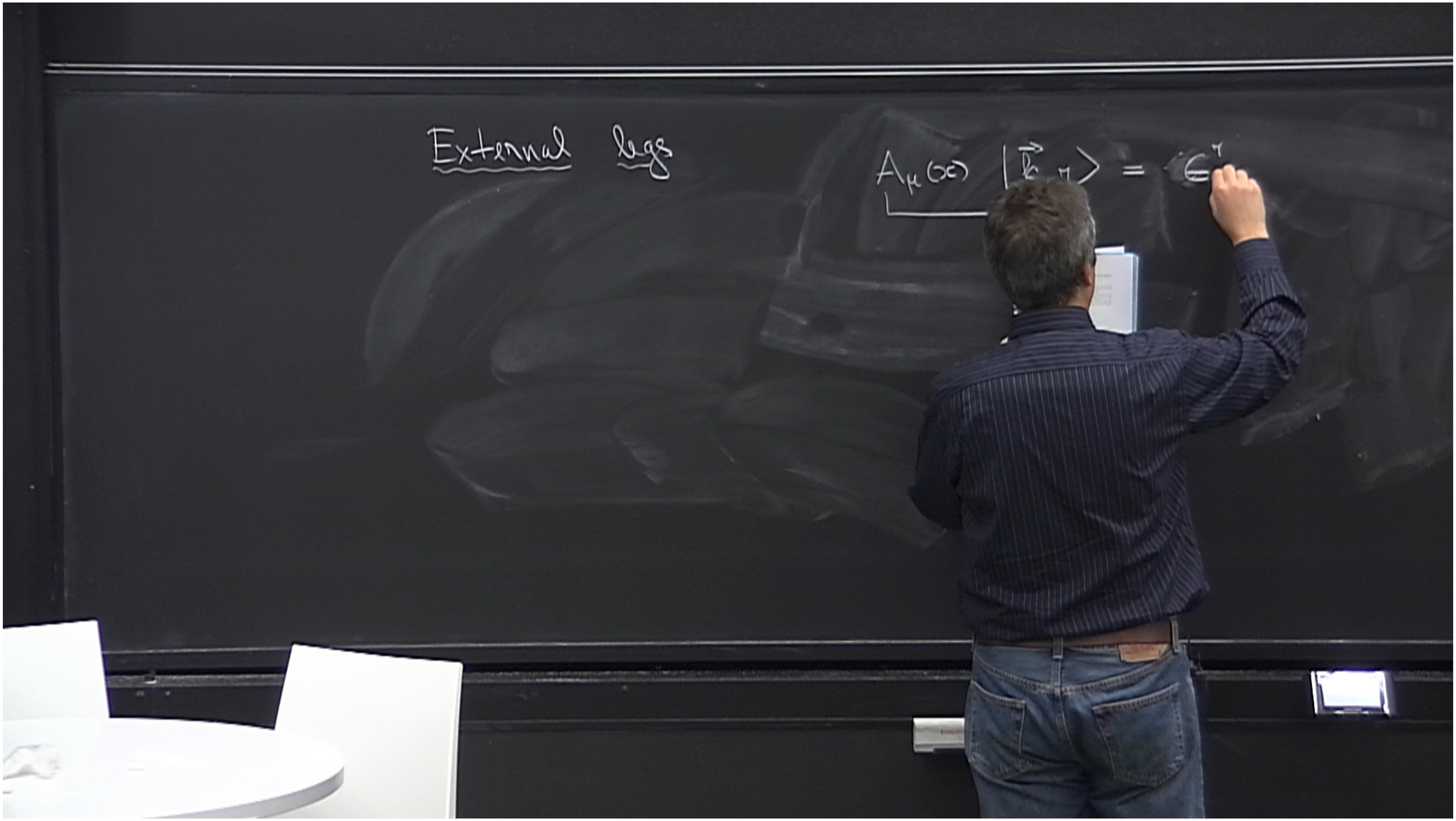
Corrections to fermion propagator:
Correction to photon propagator:
amplitude

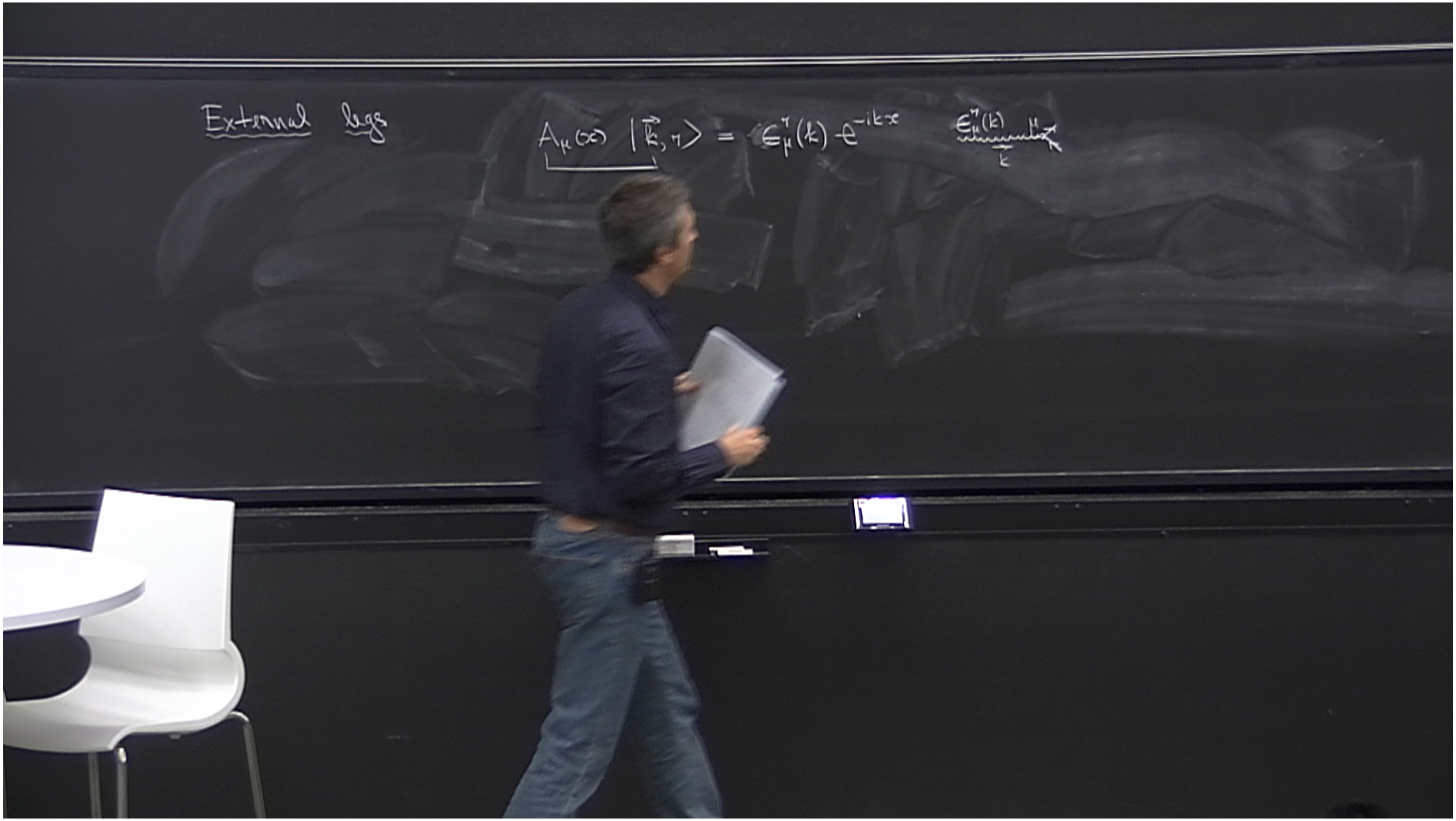


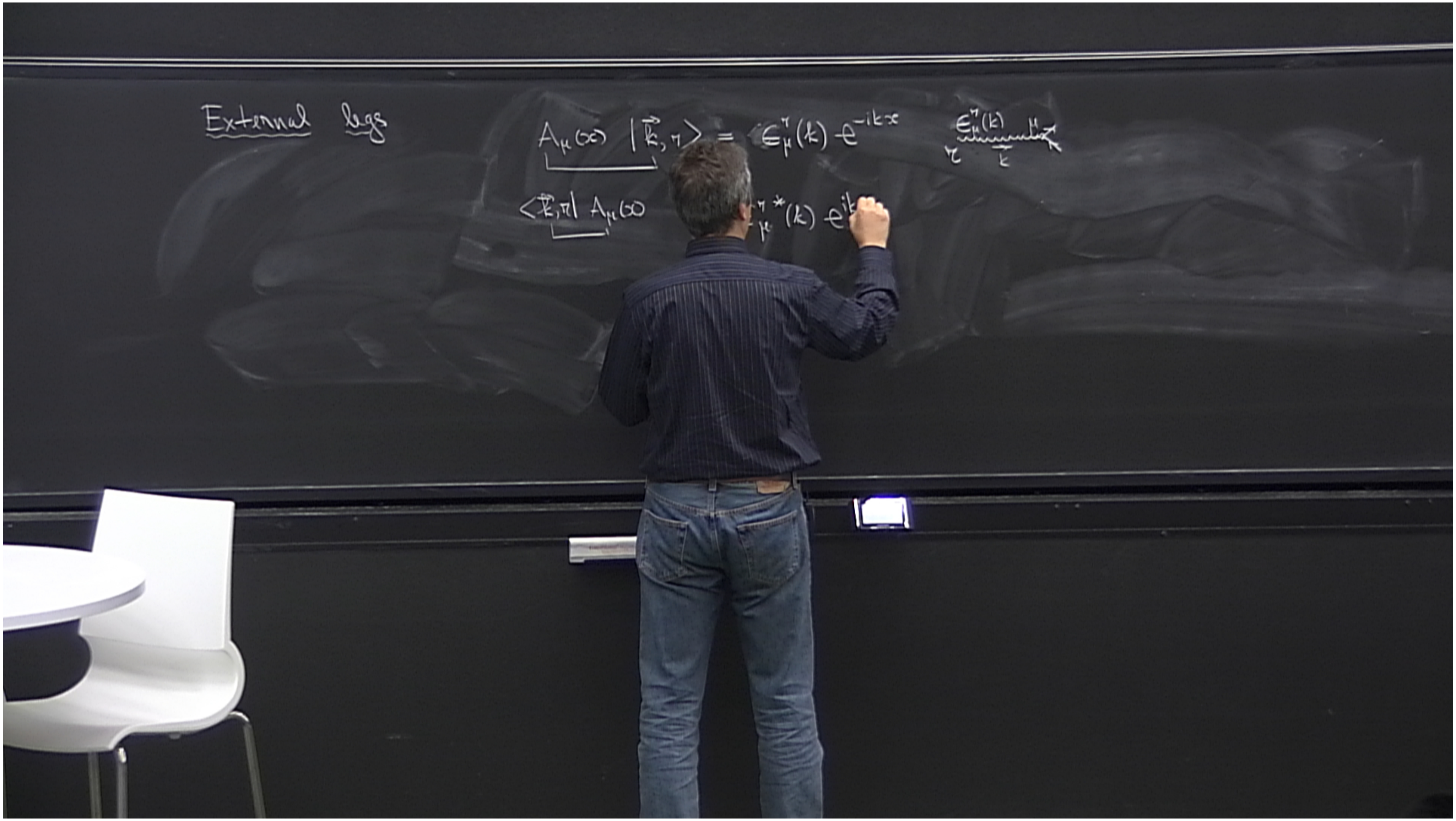
Correction to photon propagator.
Corrections to fermion propagator.
amplitude

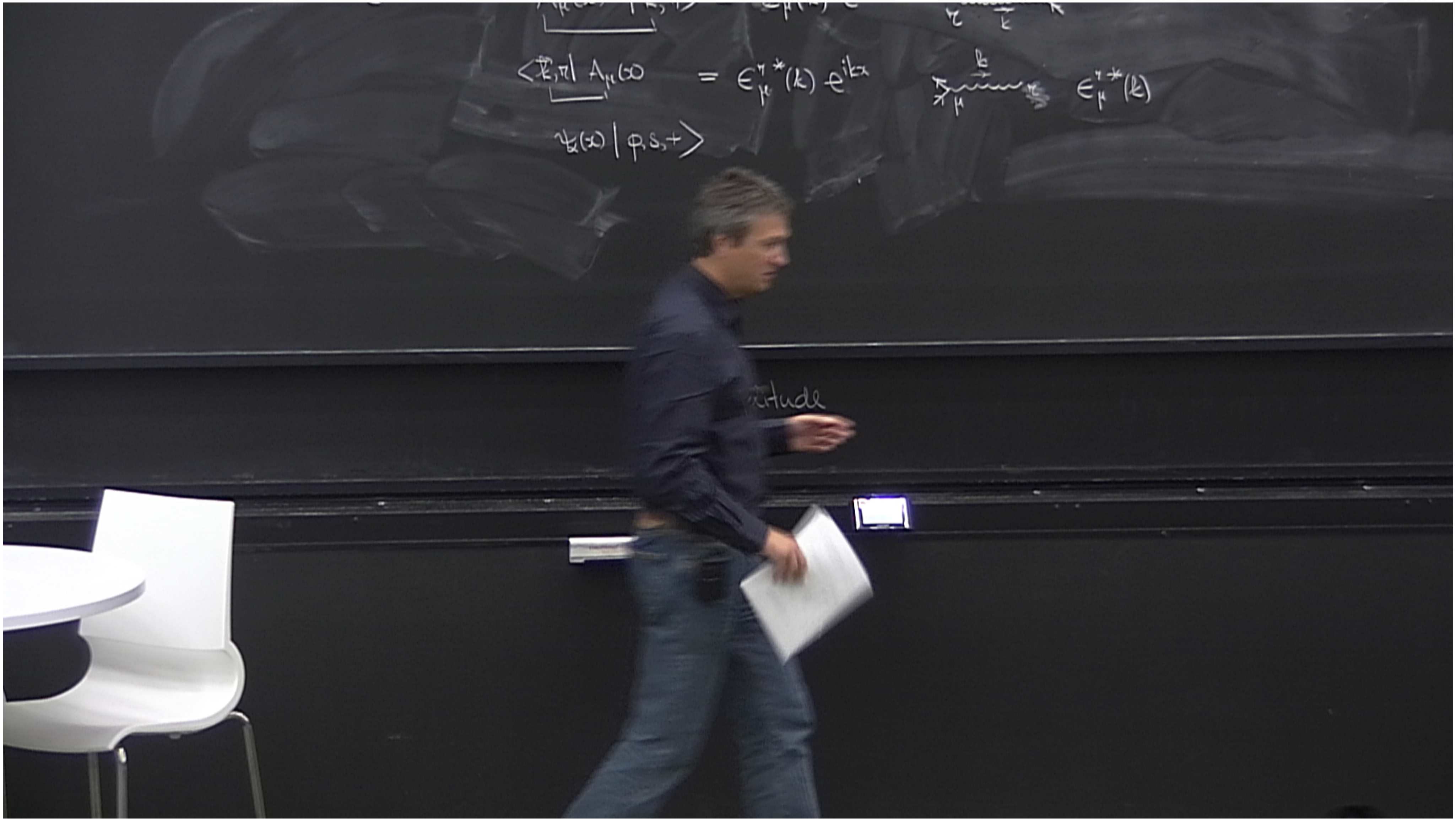

Correction to photon propagator:
propagator:

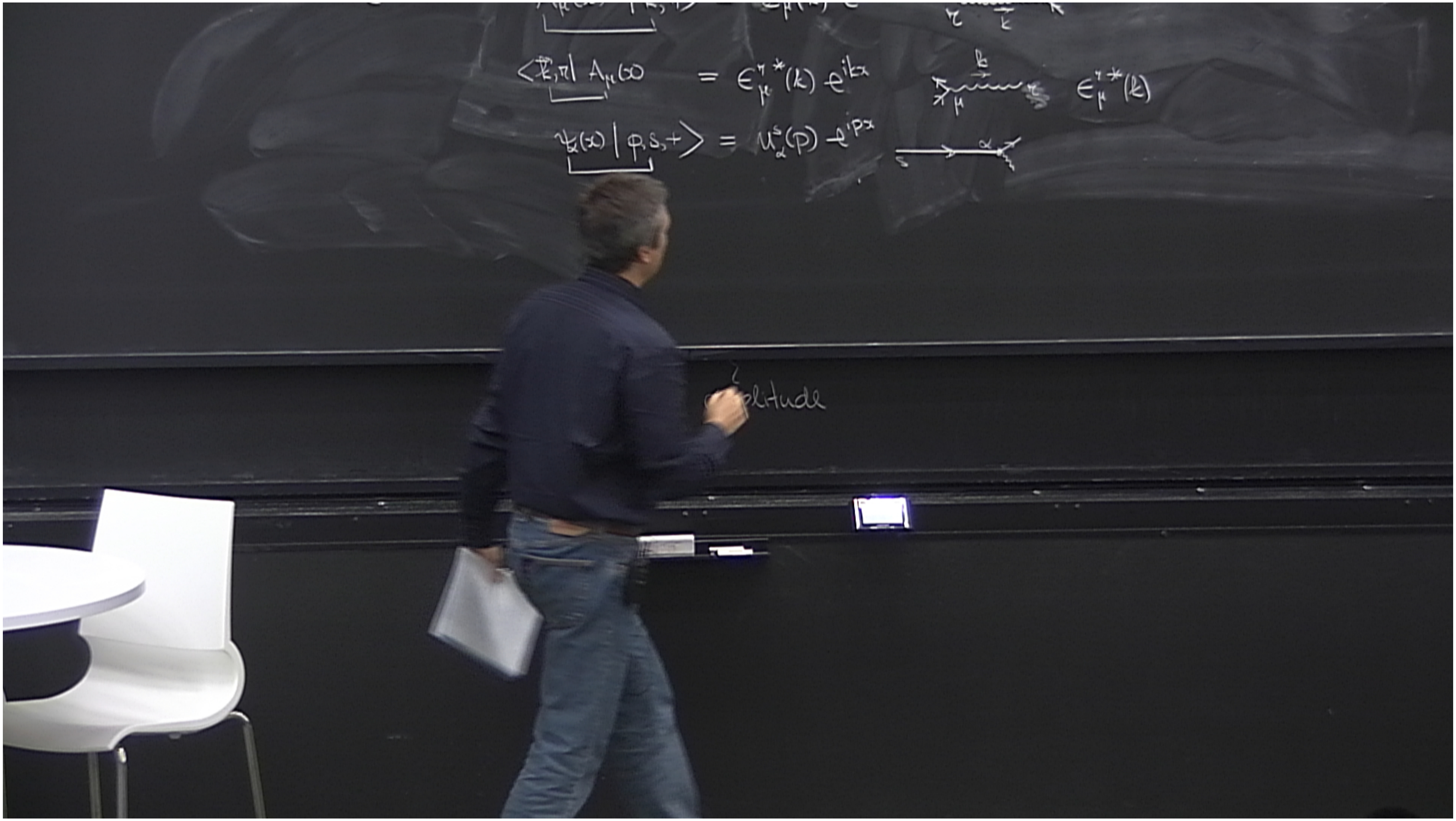

External legs
$A_\mu(x) \; |\vec{k}, r\rangle$

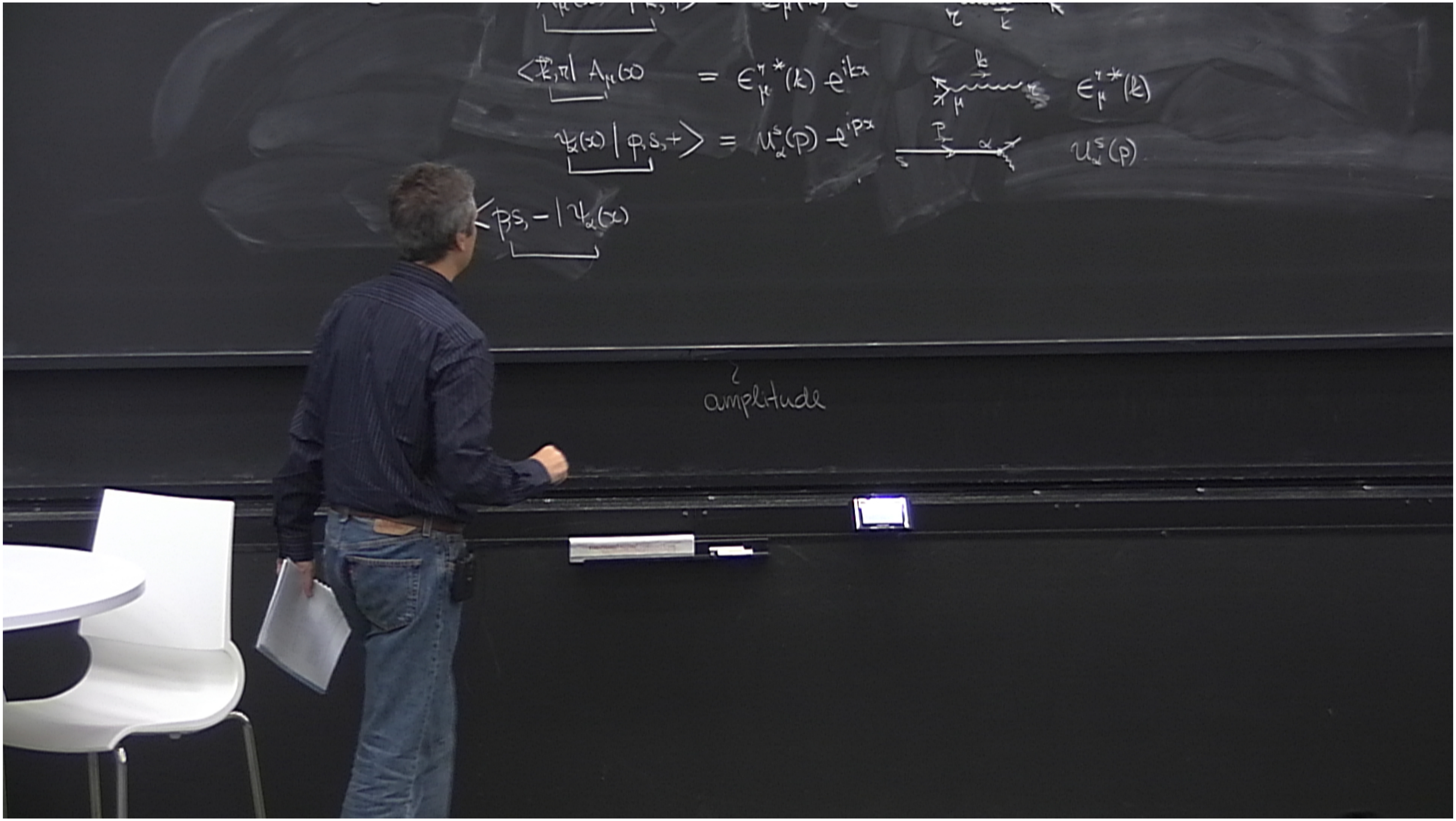

External legs
$A_\mu(x)\,|\vec{k}, r\rangle = \epsilon^r$

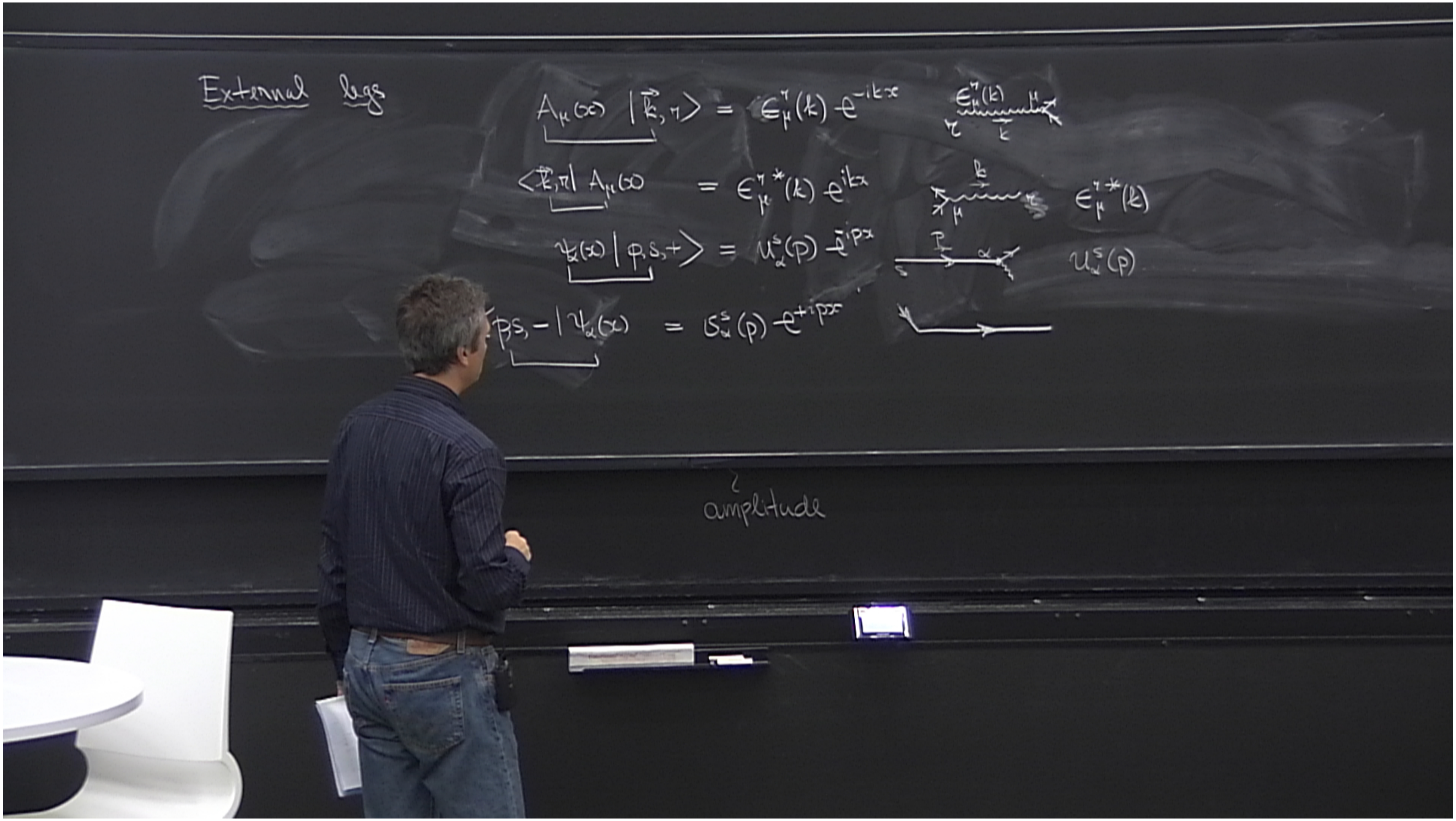

External legs
$A_\mu(x) \, |\vec{k}, \eta\rangle = \epsilon^\eta_\mu(k) \, e^{-ikx}$
$\epsilon^\eta_\mu(k)$
k


External legs


$\langle \vec{k},\pi | A_\mu(x) = \epsilon_\mu^{\pi *}(k)\, e^{ikx}$
$\epsilon_\mu^{\pi *}(k)$
$\psi_\alpha(x) | \varphi, s, + \rangle$


⟨k,r| A_μ(x) = ε_μ^{r*}(k) e^{ikx}
ε_μ^{r*}(k)
ψ_α(x) | p,s,+⟩ = u_α^s(p) e^{-ipx}
plitude


$\langle k, r | A_\mu(x) = \epsilon^{r*}_\mu(k)\, e^{ikx}$
$\epsilon^{r*}_\mu(k)$
$\psi_\alpha(x) | p, s, + \rangle = u^s_\alpha(p)\, e^{-ipx}$
$u^s_\alpha(p)$
$\langle p, s, - | \psi_\alpha(x)$
amplitude


External legs
$A_\mu(x)\,|\vec{k},\eta\rangle = \epsilon^\eta_\mu(k)\,e^{-ikx}$
$\epsilon^\eta_\mu(k)$
$\langle\vec{k},\eta|\,A_\mu(x) = \epsilon^{\eta\,*}_\mu(k)\,e^{ikx}$
$\epsilon^{\eta\,*}_\mu(k)$
$\psi_\alpha(x)\,|p,s,+\rangle = u^s_\alpha(p)\,e^{-ipx}$
$u^s_\alpha(p)$
$\langle p,s,-|\,\bar{\psi}_\alpha(x) = \bar{v}^s_\alpha(p)\,e^{+ipx}$
amplitude

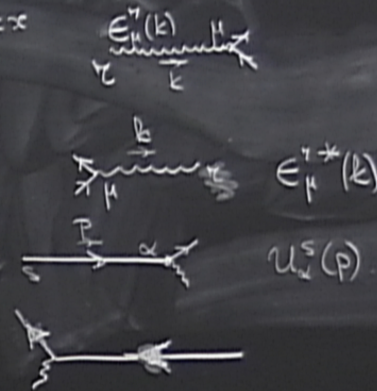
External legs

$$\underbrace{A_\mu(x)\,|\vec{k}, r\rangle} = \epsilon^r_\mu(k)\, e^{-ikx}$$

$$\underbrace{\langle \vec{k}, r|\, A_\mu(x)} = \epsilon^{r\,*}_\mu(k)\, e^{ikx}$$

$$\underbrace{\psi_\alpha(x)\,|p, s, +\rangle} = u^s_\alpha(p)\, e^{-ipx}$$

$$\underbrace{\langle p s, -|\, \psi_\alpha(x)} = v^s_\alpha(p)\, e^{+ipx}$$



amplitude

$$\underbrace{A_\mu(x)\,|\vec{k}},r\rangle = \epsilon^r_\mu(k)\,e^{-ikx}$$

$$\langle \underbrace{\vec{k},r|\,A_\mu}(x) = \epsilon^{r\,*}_\mu(k)\,e^{ikx} \qquad \epsilon^{r\,*}_\mu(k)$$

$$\underbrace{\psi_\alpha(x)\,|\,p},s,+\rangle = u^s_\alpha(p)\,e^{-ipx} \qquad u^s_\alpha(p)$$

$$\langle \underbrace{p,s,-\,|\,\psi_\alpha}(x) = v^s_\alpha(p)\,e^{+ipx} \qquad v^s_\alpha(p)$$

$$\underbrace{\bar\psi^\beta(x)\,|\,p},s,-\rangle = \bar v^{s\beta}(p)$$

$$\langle \underbrace{p,s,+\,|\,\bar\psi^\beta}(x) = \bar u^{s\beta}(p)$$