

Title: Quantum Field Theory - Lecture 7B

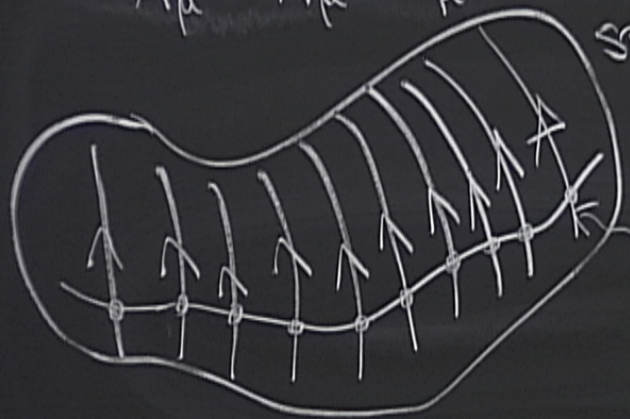
Date: Oct 17, 2012 10:30 AM

URL: <http://pirsa.org/12100068>

Abstract:

$$A_\mu \sim A_\mu + \partial_\mu \omega$$

space of all
fields



$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit



Potentials:

$$A_\mu = (\underbrace{\varphi}_{\text{electric potential}}, \underbrace{-\vec{A}}_{\text{vector potential}})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \leadsto \text{solves (2)}$$

We are left w. (1)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (\vec{E}^2 - \vec{B}^2)$$

$$\Pi_0 = \frac{\partial \mathcal{L}}{\partial (\partial_0 A_0)} \equiv 0$$

A_0 is a Lagrange multiplier

$$\frac{\delta S}{\delta A_0} = 0 - \text{constraint}$$

Redundant

$$A_\mu \rightarrow A_\mu$$

• gauge

(gradient

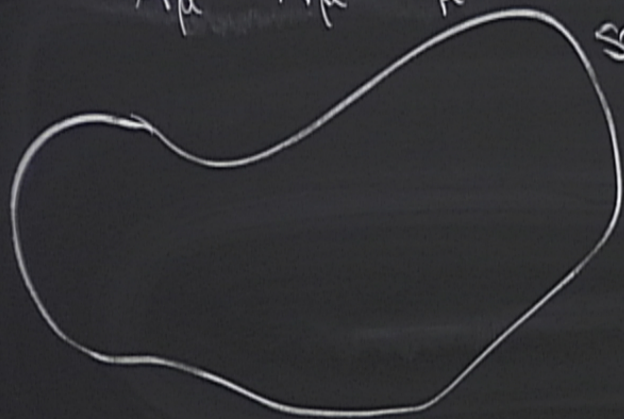
$$w = w(x)$$

$\uparrow$

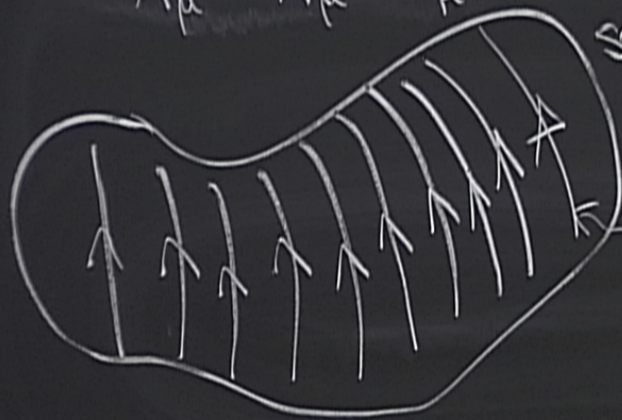
$$A_\mu \sim A_\mu + \partial_\mu \omega$$

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space



$$A_\mu \sim A_\mu + \partial_\mu \omega$$

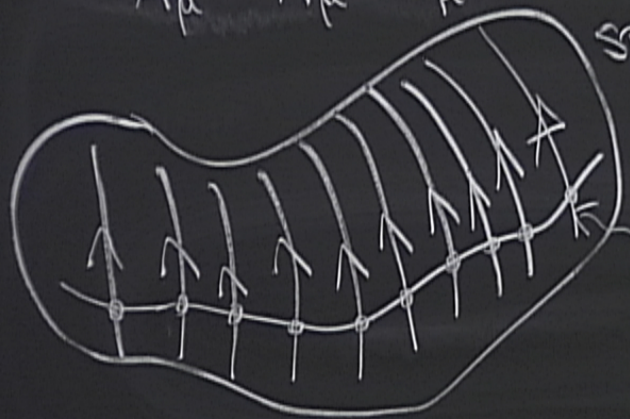


space of all
fields

$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit

$$A_\mu \sim A_\mu + \partial_\mu \omega$$

space of all
fields

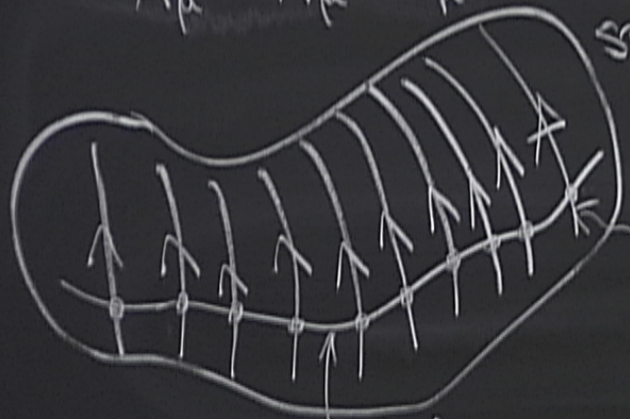


$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit



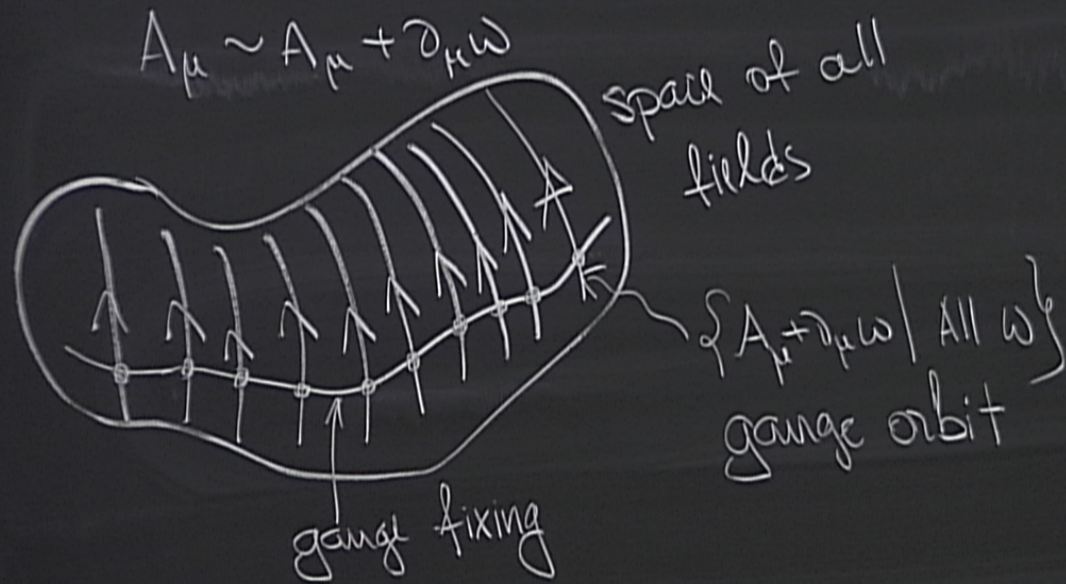
$$A_\mu \sim A_\mu + \partial_\mu \omega$$

space of all
fields



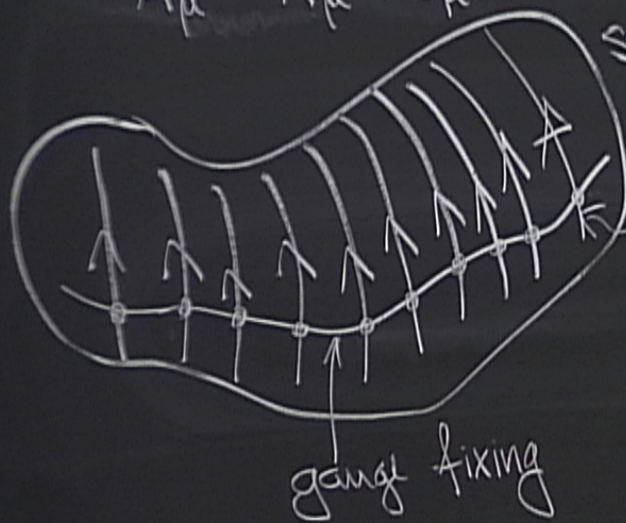
gauge fixing

$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit



$$\Phi(A_\mu, \partial_\mu A_0) = 0 : \text{gauge condition}$$

$$A_\mu \sim A_\mu + \partial_\mu \omega$$



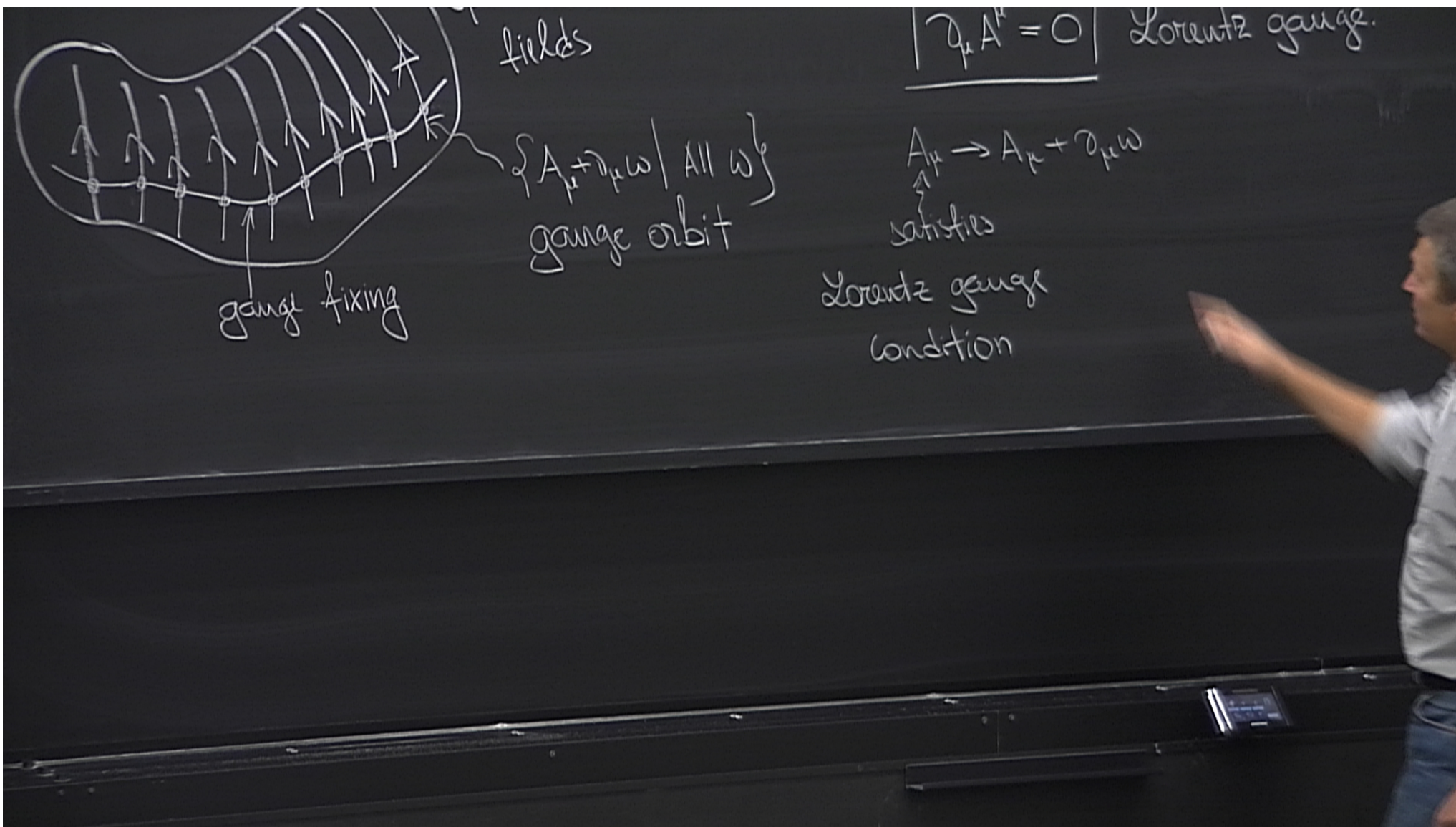
space of all fields

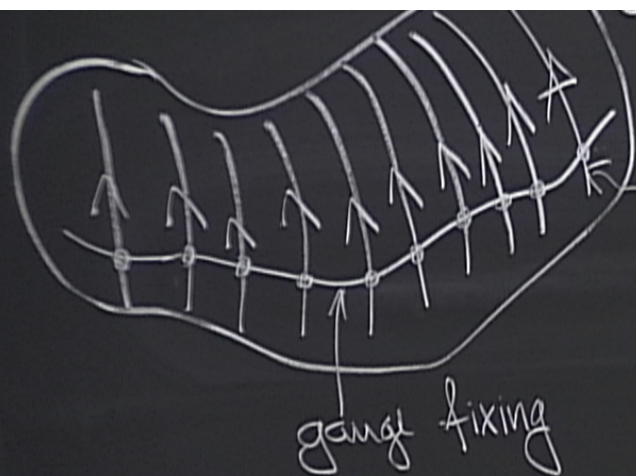
$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit

gauge fixing

$$\Phi(A_\mu, \partial_\mu A_\mu) = 0 : \text{gauge condition}$$

$$\boxed{\partial_\mu A^\mu = 0} \quad \text{Lorentz gauge.}$$





fields

$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit

gauge fixing

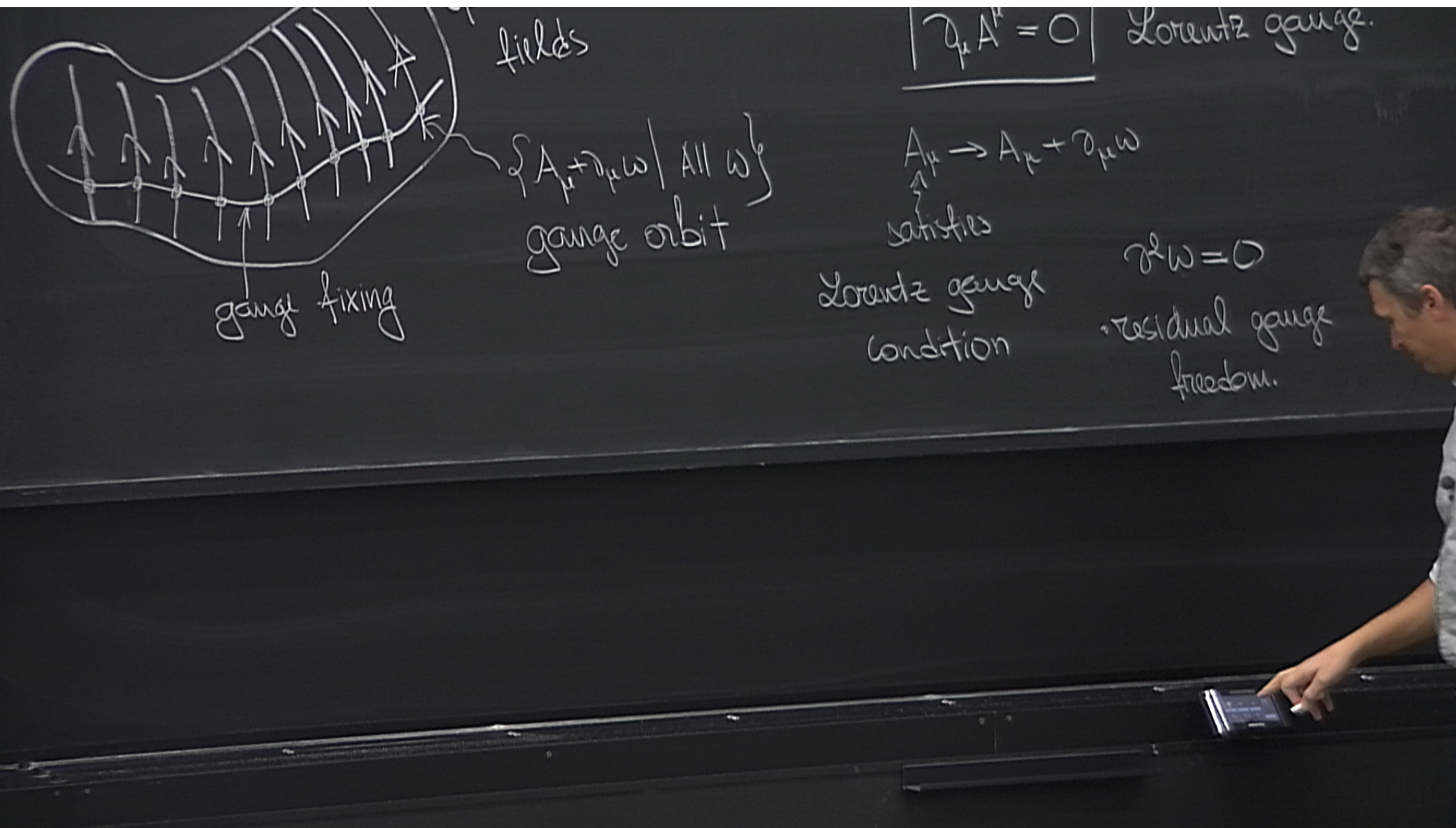
$$\underline{\partial_\mu A^\mu = 0} \quad \text{Lorentz gauge.}$$

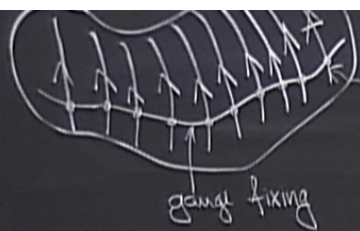
$$A_\mu \rightarrow A_\mu + \partial_\mu \omega$$

satisfies

Lorentz gauge
condition

$$\partial^2 \omega = 0$$



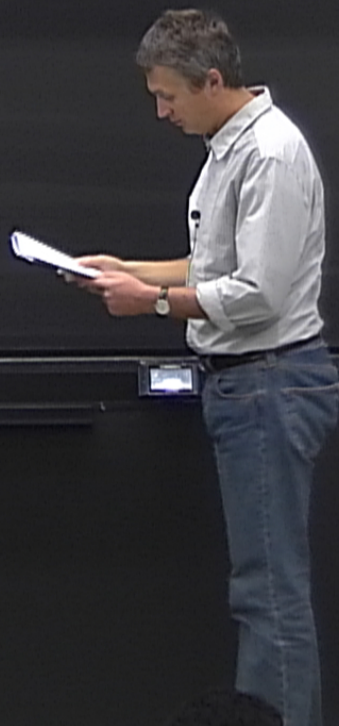


fixes

$\{A_\mu + \partial_\mu \omega \mid \text{All } \omega\}$
gauge orbit

$\partial_\mu A^\mu = 0$ Lorentz gauge.
 $A_\mu \rightarrow A_\mu + \partial_\mu \omega$
 satisfies
 Lorentz gauge condition $\partial \omega = 0$
 residual gauge freedom.

$\partial_\mu A^\mu = 0 \rightarrow$ Maxwell's equations
in Lorentz gauge



$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

$$A_\mu =$$

$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s$$

$\partial^\mu A_\mu = 0 \rightarrow$ Maxwell's equations
in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s$$

$$a_{p\mu} = \epsilon_\mu^s(p) a_s$$

$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s$$

$$a_{p\mu} = \epsilon_\mu^s(p) a_s$$

$$\mu = 0, \dots, 3$$

$$s = 1, \dots, 4$$

$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ipx} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx} \right)$$

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in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \Theta(p_0) \sqrt{2p_0} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ipx} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx} \right)$$

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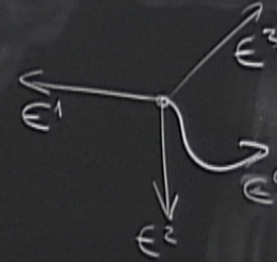
$|\vec{p}| = 2$
Imposing the gauge:

$$\partial_\mu e^{\mp i p x} = \mp i p_\mu e^{\mp i p x}$$

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$$\partial_\mu e^{\mp i p x} = \mp i p_\mu e^{\mp i p x}$$

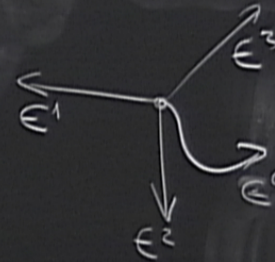
$$P_\mu \epsilon^{\mu}(p) = 0$$



Imposing the gauge:

$$\partial_\mu e^{\mp i p x} = \mp i p_\mu e^{\mp i p x}$$

$$\boxed{P_\mu \epsilon^{\mu}(p) = 0}$$



$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

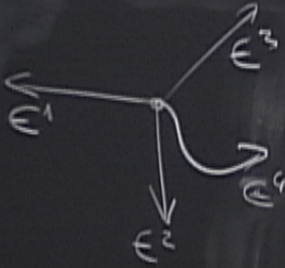
$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ipx} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx} \right)$$

$$a_{p\mu} = \epsilon_\mu^s(p) a_s$$

$$\mu = 0, \dots, 3$$

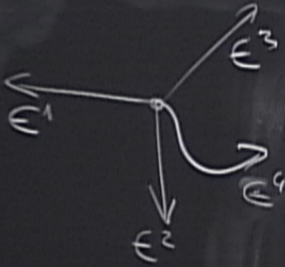
$$s = 1, \dots, 4$$

$$\underline{E}_x \quad P_t = (100\Delta) \omega$$



$$\underline{E}_x \quad p_r = (1 \ 0 \ 0 \ 1) \omega$$

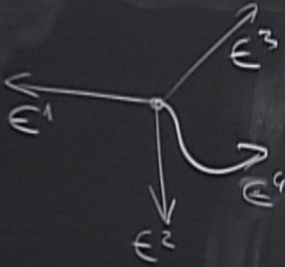
$$\epsilon^1(p) = \frac{1}{\sqrt{2}} (0$$



Ex $P_t = (1 \ 0 \ 0 \ 1) \omega$

$$\epsilon^1(p) = \frac{1}{\sqrt{2}} (0 \ 1 \ i \ 0)$$

$$\epsilon^2(p) = \epsilon^{1*} = \frac{1}{\sqrt{2}} (0 \ 1 \ -i \ 0)$$

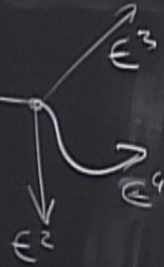


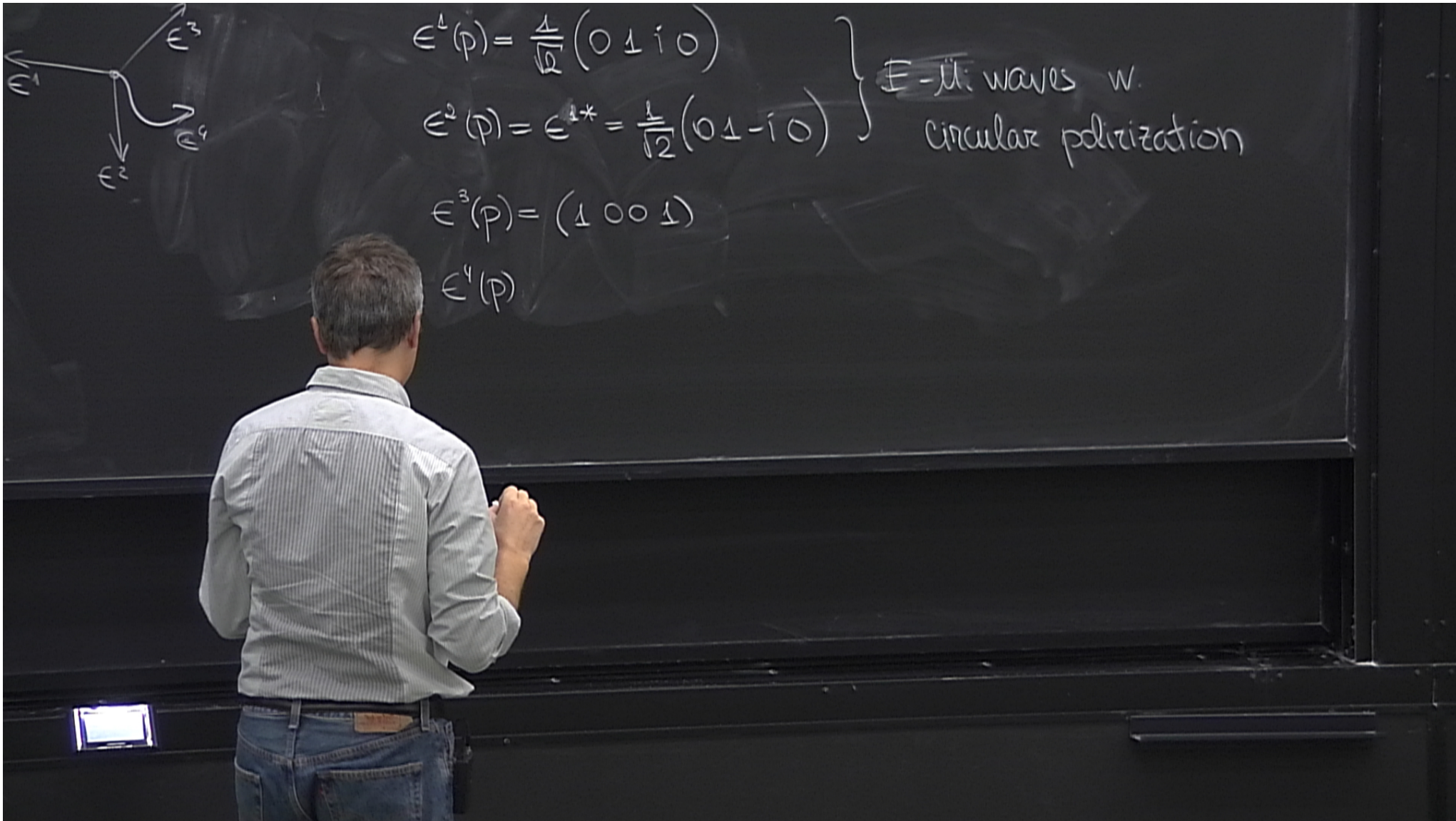
Ex $P_k = (1\ 0\ 0\ 1)\omega$

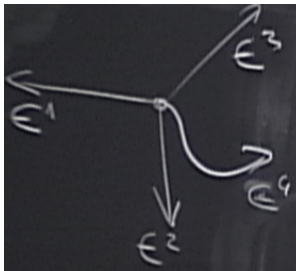
$$\epsilon^1(p) = \frac{1}{\sqrt{2}}(0\ 1\ i\ 0)$$

$$\epsilon^2(p) = \epsilon^{1*} = \frac{1}{\sqrt{2}}(0\ 1\ -i\ 0)$$

} E-M waves w.
circular polarization







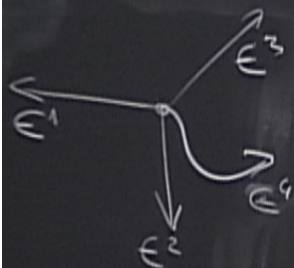
$$\epsilon^1(p) = \frac{1}{\sqrt{2}} (0 \ 1 \ i \ 0)$$

$$\epsilon^2(p) = \epsilon^{1*} = \frac{1}{\sqrt{2}} (0 \ 1 \ -i \ 0)$$

$$\epsilon^3(p) = (1 \ 0 \ 0 \ 1)$$

$$\epsilon^4(p) = (1 \ 0 \ 0 \ -1)$$

} E-M waves w.
circular polarization



$$\underline{E}_x \quad P_\mu = (1 \ 0 \ 0 \ 1) \omega$$

$$\checkmark \epsilon^1(p) = \frac{1}{\sqrt{2}} (0 \ 1 \ i \ 0)$$

$$\checkmark \epsilon^2(p) = \epsilon^{1*} = \frac{1}{\sqrt{2}} (0 \ 1 \ -i \ 0)$$

} E-M waves w.
circular polarization

$$\checkmark \epsilon_\mu^3(p) = (1 \ 0 \ 0 \ 1) = \frac{P_\mu}{\omega} \quad P_\mu P^\mu = 0$$

$$\times \epsilon_\mu^4(p) = (1 \ 0 \ 0 \ -1)$$

$$\mathcal{L} = i\bar{\psi} \frac{1+\gamma^5}{2} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi = i\bar{\psi} \gamma^\mu \left(\frac{1-\gamma^5}{2}\right)^2 \partial_\mu \psi = i\bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi$$

Ex $P_\mu = (1\ 0\ 0\ 1)\omega$

$$\sqrt{\epsilon^1(p)} = \frac{1}{\sqrt{2}}(0\ 1\ i\ 0)$$

$$\sqrt{\epsilon^2(p)} = \epsilon^{1*} = \frac{1}{\sqrt{2}}(0\ 1\ -i\ 0)$$

} E-M waves w.
circular polarization

$$\partial_\mu \omega \rightsquigarrow \sqrt{\epsilon_\mu^3(p)} = (1\ 0\ 0\ 1) = \frac{P_\mu}{\omega} \quad P_\mu P^\mu = 0$$

$$\sqrt{\epsilon_\mu^4(p)} = (1\ 0\ 0\ -1)$$

$\partial^2 A_\mu = 0 \leadsto$ Maxwell's equations
in Lorentz gauge

$$A_\mu = \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2) \theta(p_0) \sqrt{2p_0} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ipx} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx} \right)$$

$$a_{p\mu} = \epsilon_\mu^s(p) a_s$$

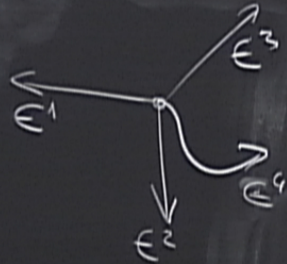
$$\mu = 0, \dots, 3$$

$$s = 1, \dots, 4$$

$$\rightarrow (x^0, -x)$$

$$\mathcal{L} = i\bar{\psi} \frac{1+\gamma^5}{2} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi = i\bar{\psi} \gamma^\mu \left(\frac{1-\gamma^5}{2}\right)^2 \partial_\mu \psi = i\bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi$$

$$\underline{E_x} \quad p_\mu = (1 \ 0 \ 0 \ 1) \omega$$



$$\checkmark E^1(p) = \frac{1}{\sqrt{2}} (0 \ 1 \ i \ 0)$$

$$\checkmark E^2(p) = E^{1*} = \frac{1}{\sqrt{2}} (0 \ 1 \ -i \ 0)$$

} E- μ waves w.
circular polarization

$$\partial_\mu \omega \rightsquigarrow \checkmark E_\mu^3(p) = (1 \ 0 \ 0 \ 1) = \frac{p_\mu}{\omega} \quad p_\mu p^\mu = 0$$

eliminated
by residual

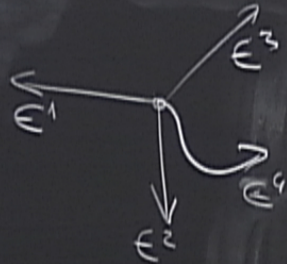
$$\times E_\mu^4(p) = (1 \ 0 \ 0 \ -1)$$

$$\Gamma_{\mu\nu} = -\Gamma_{\nu\mu}$$

$$\rightarrow (\bar{x}, -\bar{x})$$

$$\mathcal{L} = i\bar{\psi} \frac{1+\gamma^5}{2} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi = i\bar{\psi} \gamma^\mu \left(\frac{1-\gamma^5}{2}\right)^2 \partial_\mu \psi = i\bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \partial_\mu \psi$$

$$\underline{E}_x \quad p_\mu = (1 \ 0 \ 0 \ 1) \omega$$



$$\checkmark \epsilon^1(p) = \frac{1}{\sqrt{2}} (0 \ 1 \ i \ 0)$$

$$\checkmark \epsilon^2(p) = \epsilon^{1*} = \frac{1}{\sqrt{2}} (0 \ 1 \ -i \ 0)$$

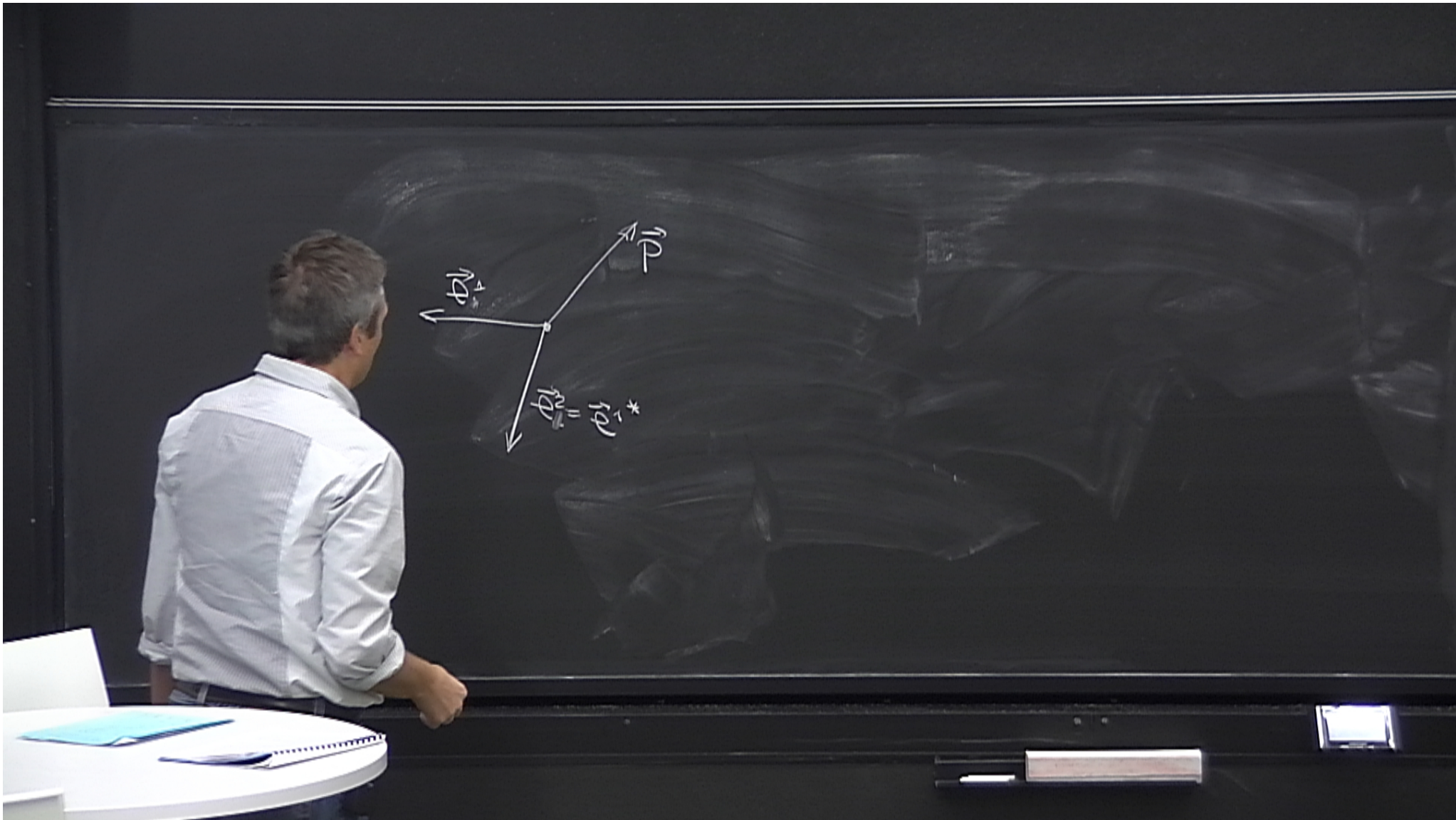
} E- $\vec{u}$: waves w.
circular polarization

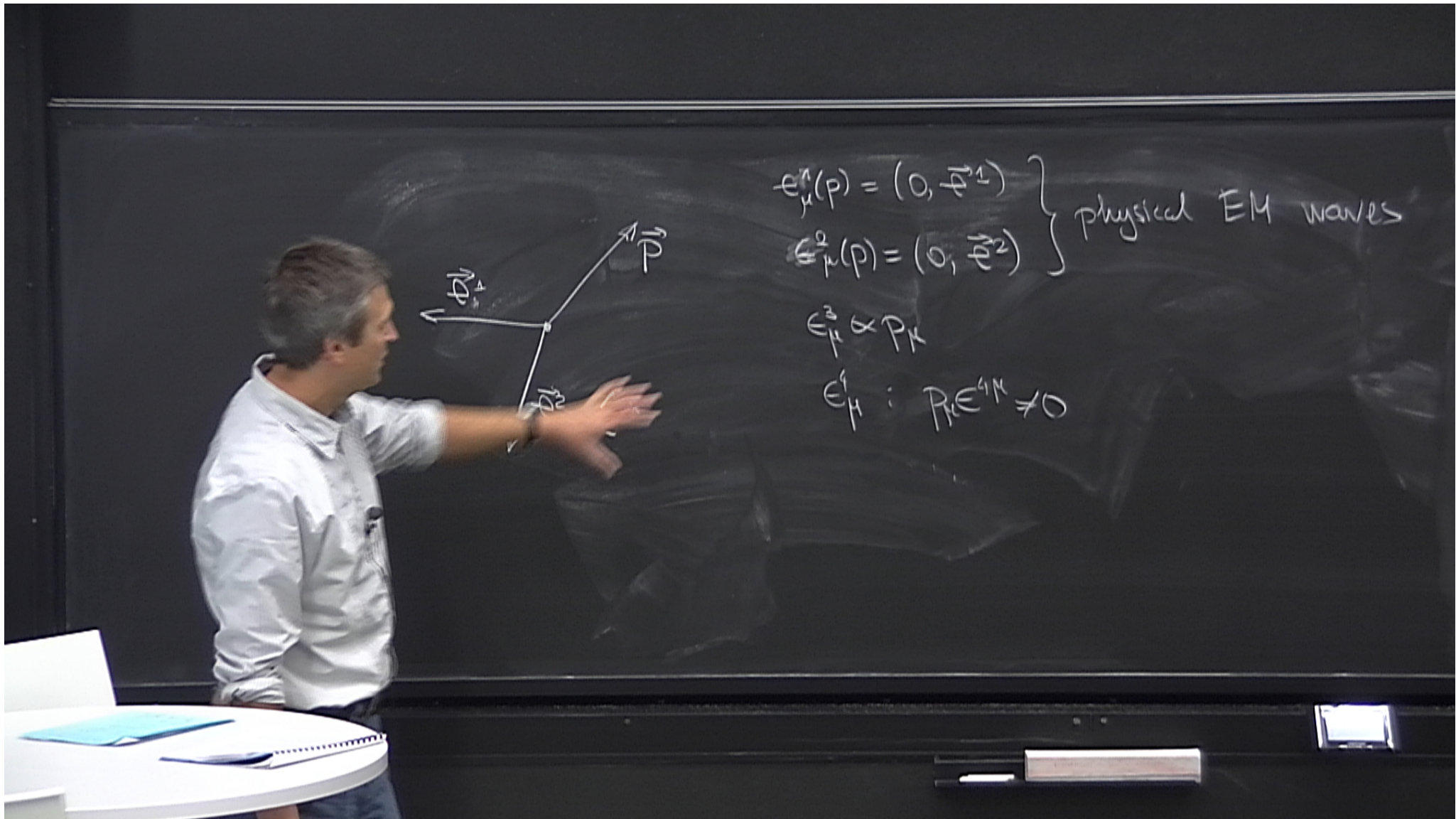
$$\partial_\mu \omega \rightsquigarrow \checkmark \epsilon_\mu^3(p) = (1 \ 0 \ 0 \ 1) = \frac{p_\mu}{\omega} \quad p_\mu p^\mu = 0$$

$$\times \epsilon_\mu^4(p) = (1 \ 0 \ 0 \ -1)$$

eliminated
by residual
gauge transformations.

$$\Gamma_{\mu\nu} = -\Gamma_{\nu\mu}$$





4 waves

Quantization:

4 waves

Quantization:

$$[a_p^s, a_q^{r\dagger}] = (2\pi)^3 \delta^{sr} \delta(p-q)$$

$$s, r = 1, 2$$

4 waves

Quantization:

$$[a_p^s, a_q^{r\dagger}] = (2\pi)^3 \delta^{sr} \delta(p-q)$$

$s, r = 1, 2$: two helicity (± 1) states of a photon

$$[a_p, a_q] = \delta_{pq} \quad [a_p, a_p^\dagger] = 1$$

$s, r = 1, 2$: two helicity (± 1) states of a photon

$\partial_\mu A^\mu = 0$ inconsistent & interpreted as an operator equation

Toy model

$$H = p_1^2 + p_2^2 + V(x_1)$$

Toy model

$$H = \frac{p_x^2}{2m} + \frac{p_z^2}{2m} + V(x_1)$$

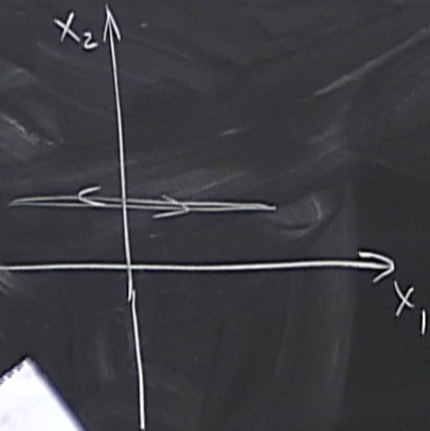
In classical mechanics

$$p_z = 0$$

x_1

Toy model

$$H = p_1^2 + p_2^2 + V(x_1)$$



In classical mechanics

$$p_2 = 0$$

In quantum mechanics:

$$[x_2, p_2] = i\hbar$$

del

$$+p_z^2 + V(x_1)$$

In classical mechanics

$$p_z = 0$$

In quantum mechanics

$[x_2, p_z] = i\hbar$ makes it impossible to set $p_z = 0$

$\rightarrow x_1$

1D model

$$H = p_1^2 + p_2^2 + V(x_1)$$

$x_2 \uparrow$

$x_1 \rightarrow$

In classical mechanics

$$p_2 = 0$$

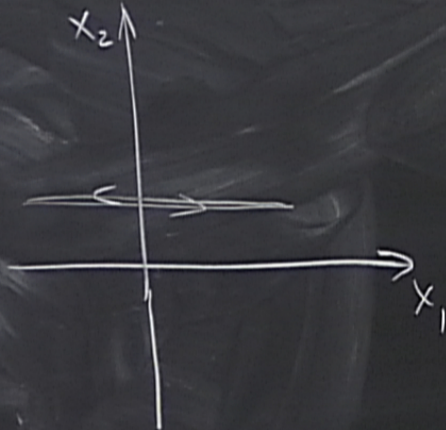
In quantum mechanics:

$[x_2, p_2] = i\hbar \rightarrow$ makes it impossible to set $p_2 = 0$

$$p_2 |\Psi\rangle = 0$$

Toy model

$$H = p_1^2 + p_2^2 + V(x_1)$$



In classical mechanics

$$p_2 = 0$$

In quantum mechanics:

$$[x_2, p_2] = i\hbar \rightarrow \text{makes it impossible}$$

$$p_2 |\Psi\rangle = 0$$



$$\Psi = \Psi(x_1)$$

ms

ms:

→ makes it impossible to set $p_z=0$

$$\partial_\mu A^\mu |phys\rangle = 0$$

ms:

ms:

→ makes it impossible to set $p_z=0$

$$\partial_\mu A^\mu |phys\rangle = 0$$