Title: AdS(3)/CFT(2) correspondence and integrability

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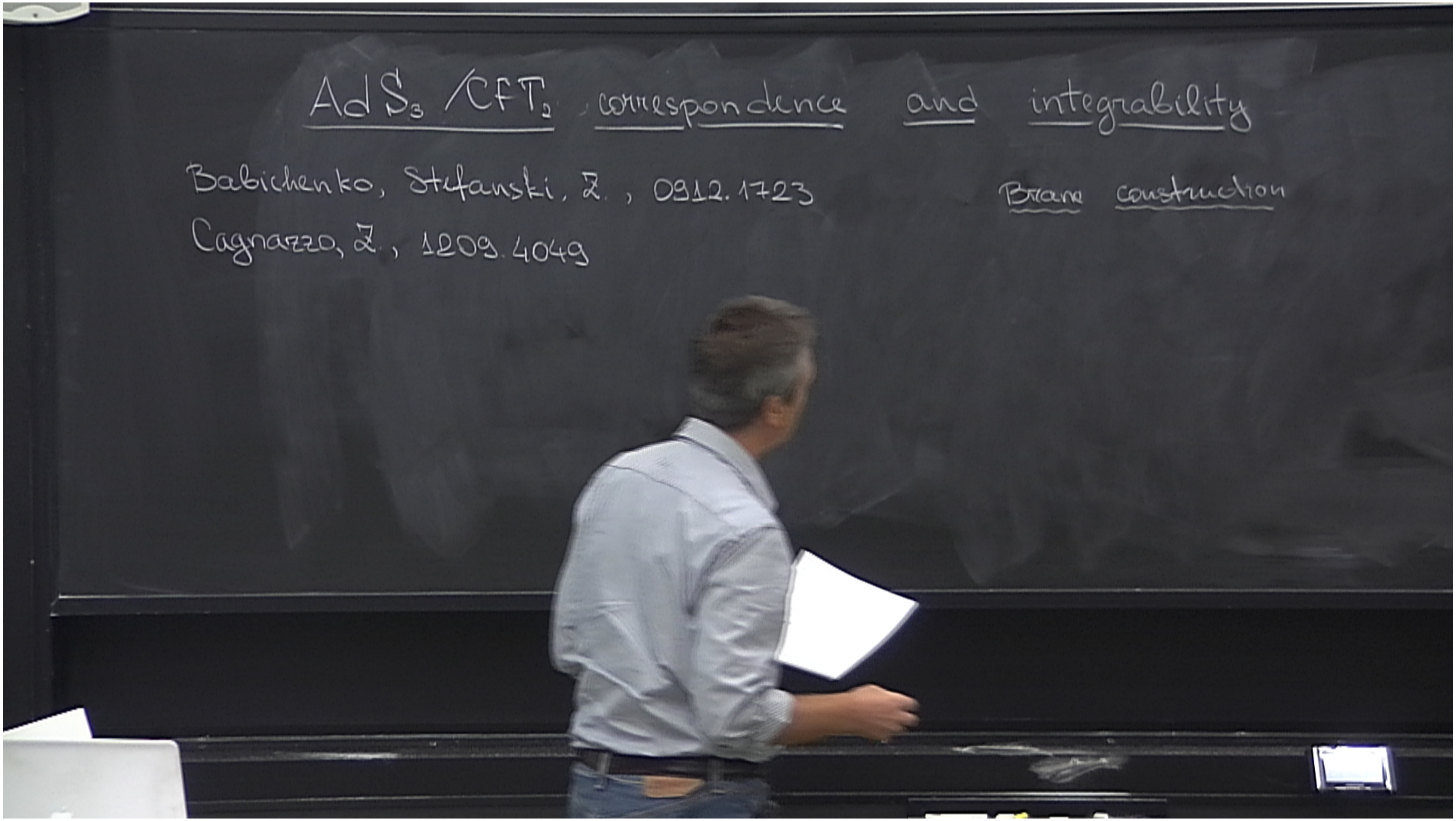
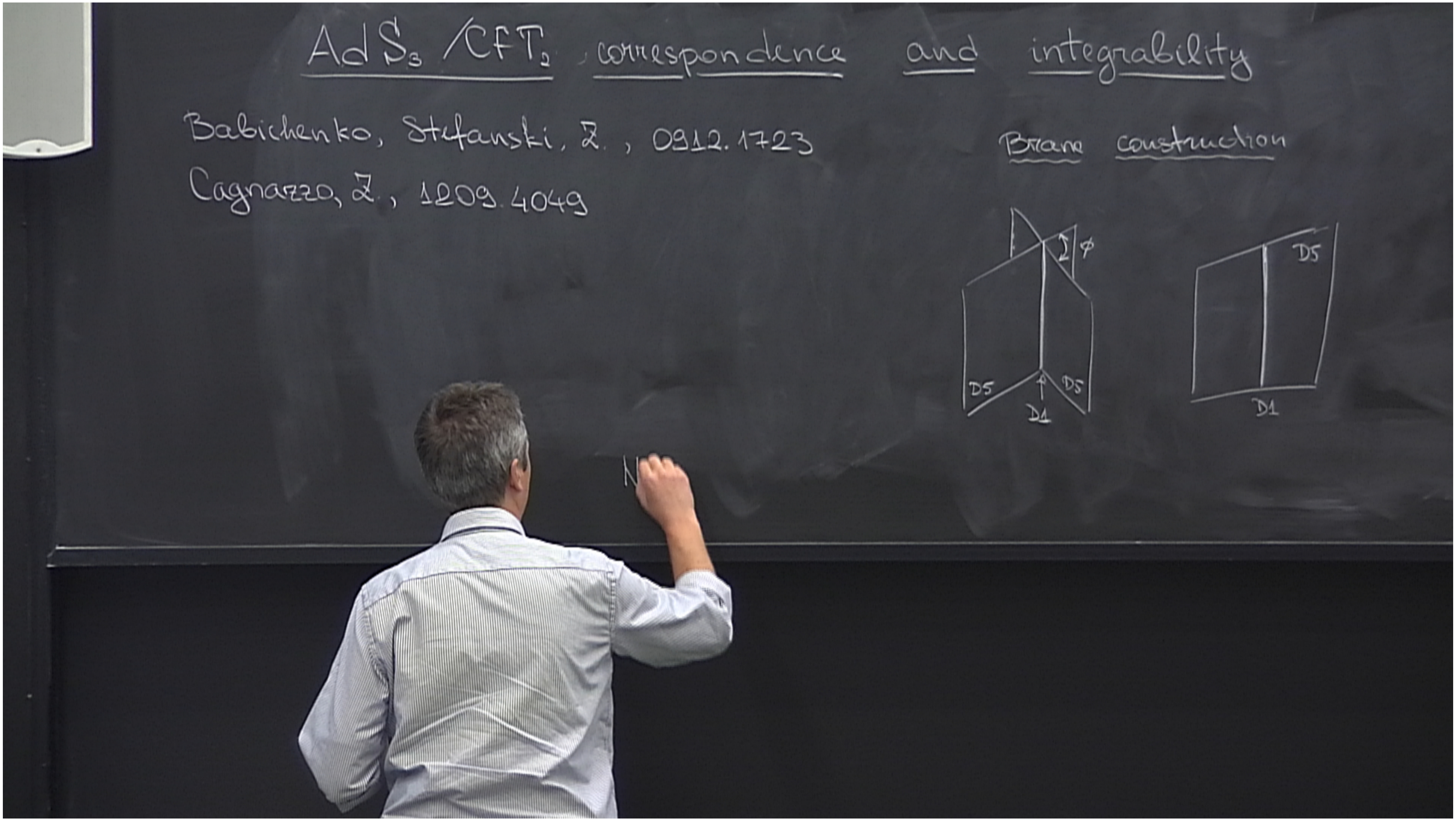
Abstract: Integrability has been successfully used to compute the non-perturbative spectrum, Wilson loops and scattering amplitudes in the AdS/CFT correspondence. Most of these results apply to N=4, D=4 SYM / strings on AdS(5)xS(5). Strings on AdS(3)xM, where M is either

S(3)xT(4) or S(3)xS(3)xS(1), are also integrable and potentially solvable by the same methods (Bethe ansatz, Y-system, TBA etc). An interesting aspect of string theory on AdS(3) is a large number of parameters that preserve integrability. One parameters, in particular, interpolates between the integrable RR AdS(3) background and conformal NSNS background described by a non-compact WZW model. Another interesting aspect of string theory on AdS(3) is emergence of target-space Viraroso symmetry. I will review how integrability arises in the AdS(3)/CFT(2) correspondence, and will describe what is (not) know about the Bethe ansatz solution of the relevant sigma-models.


AdS3 /CFT2 correspondence and integrability
Babichenko, Stefanski, Z., 0912.1723
Cagnazzo, Z., 1209.4049
Brane construction


AdS3 /CFT2 correspondence and integrability
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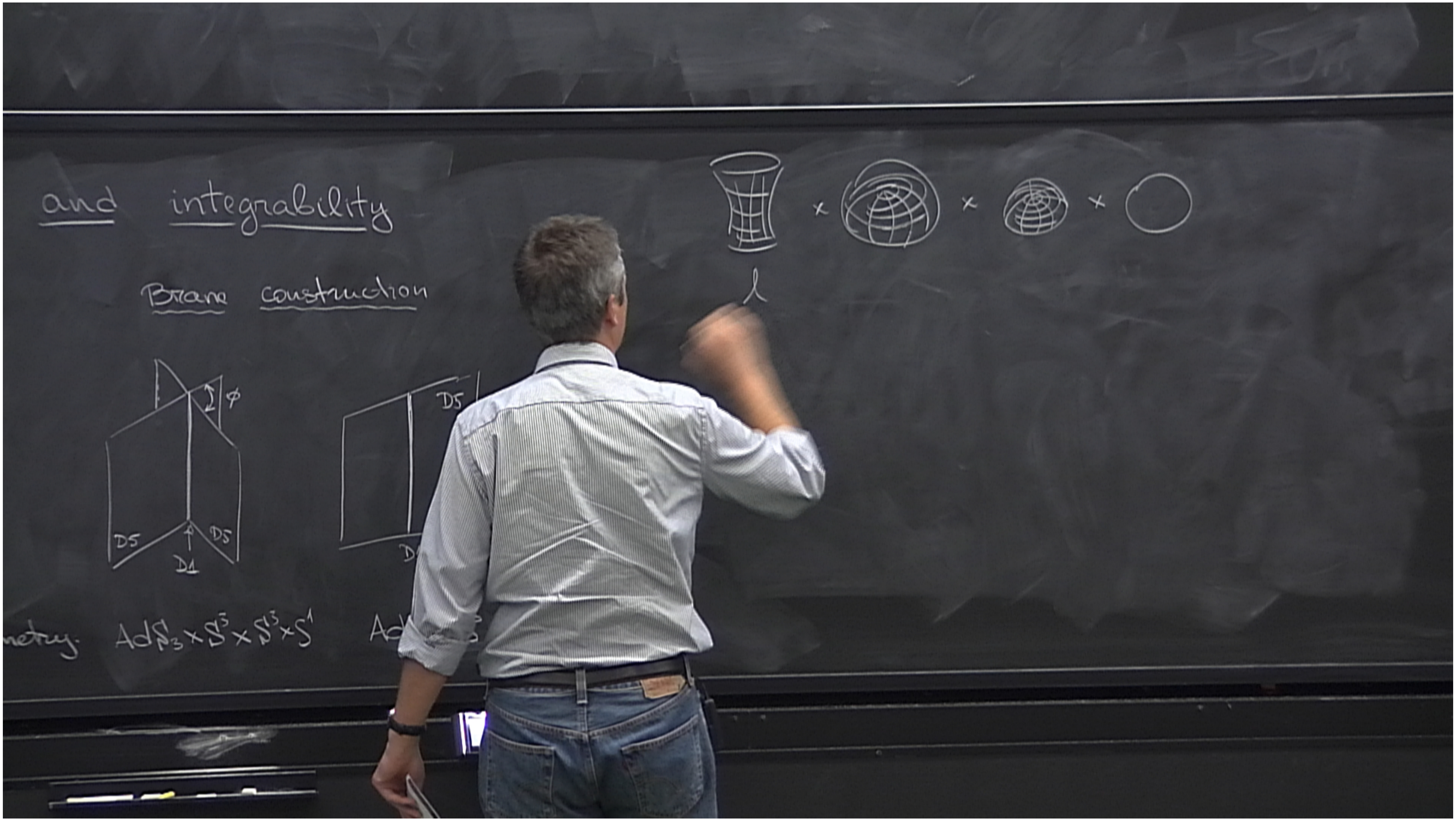
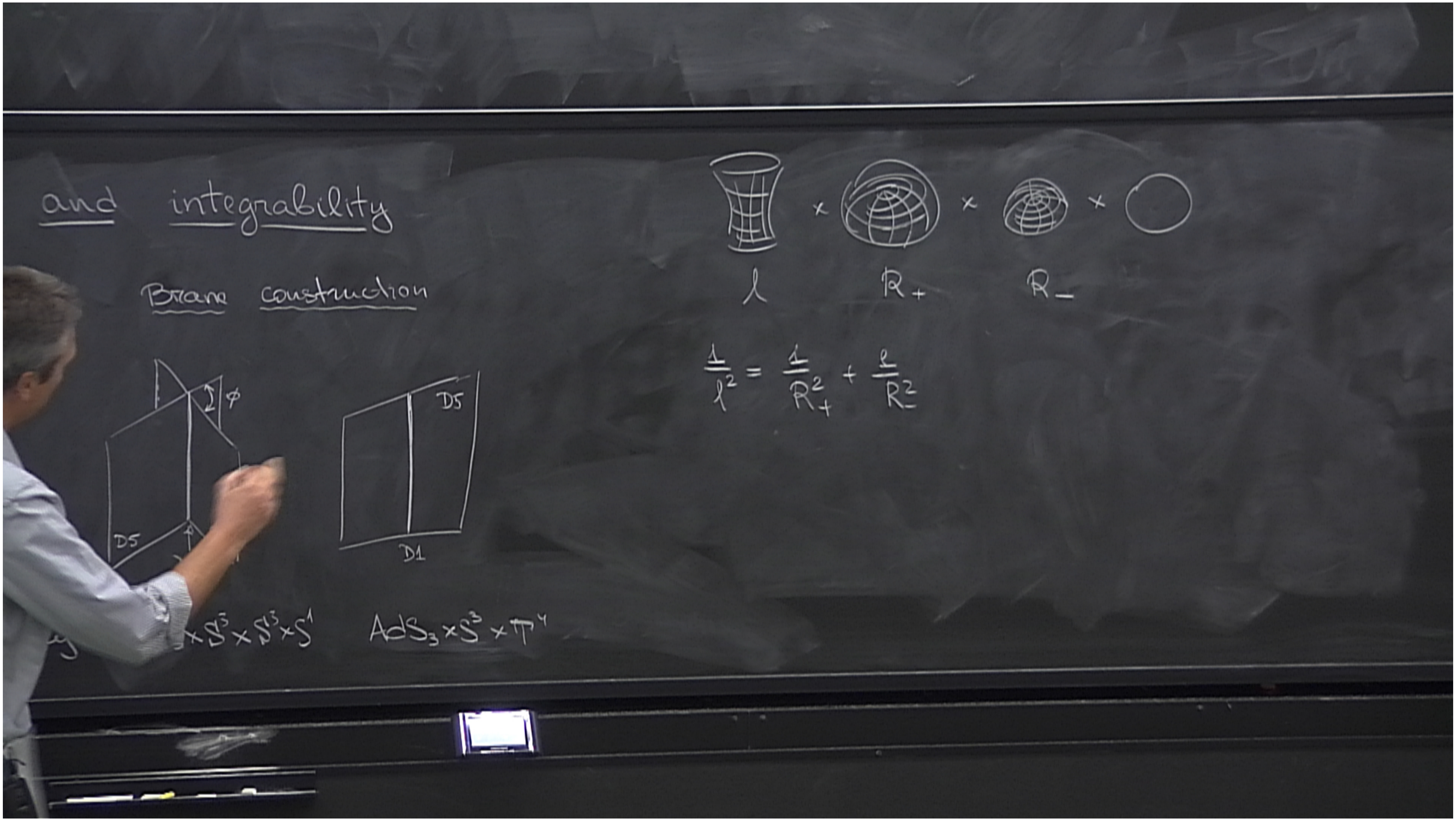
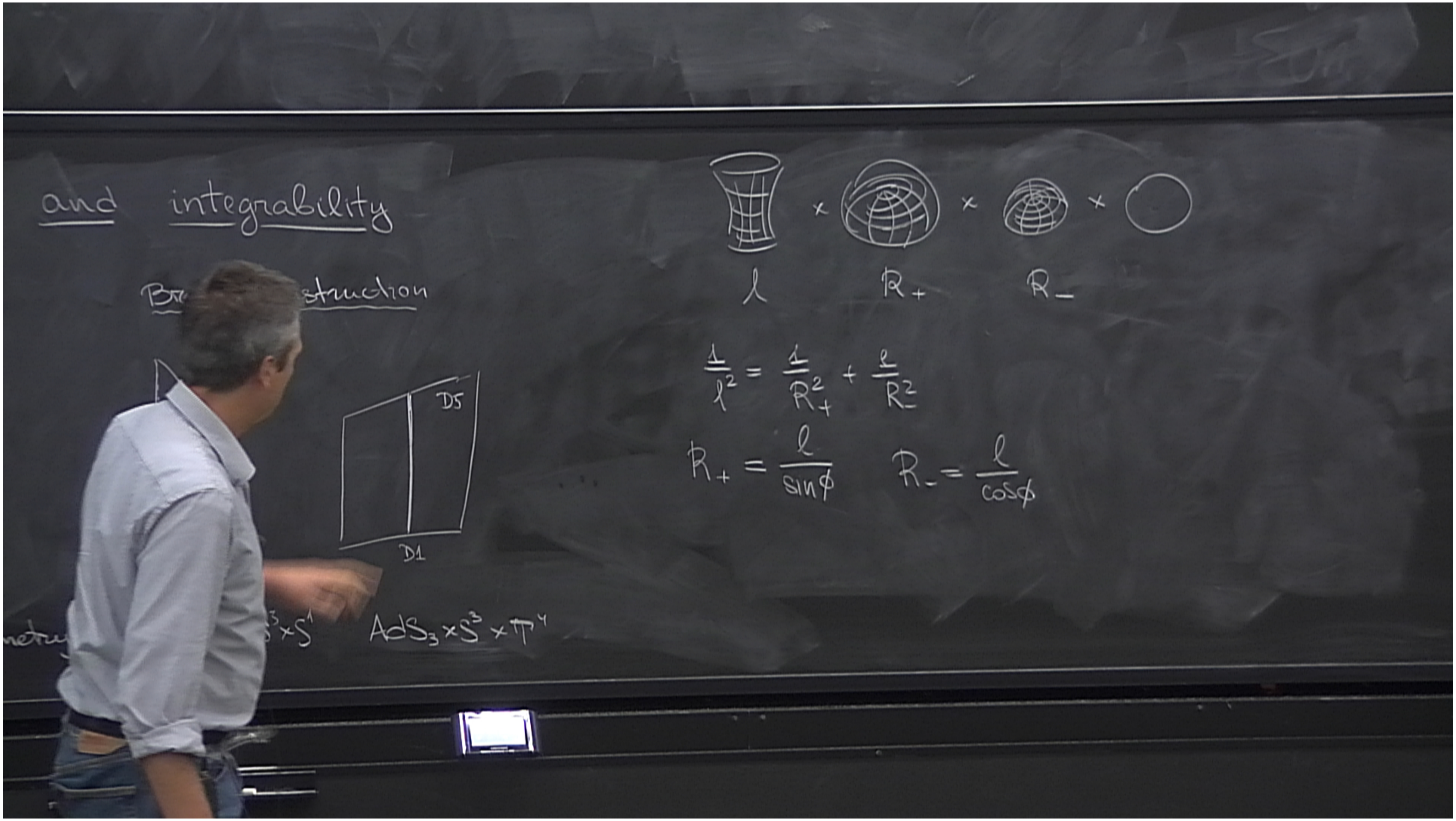
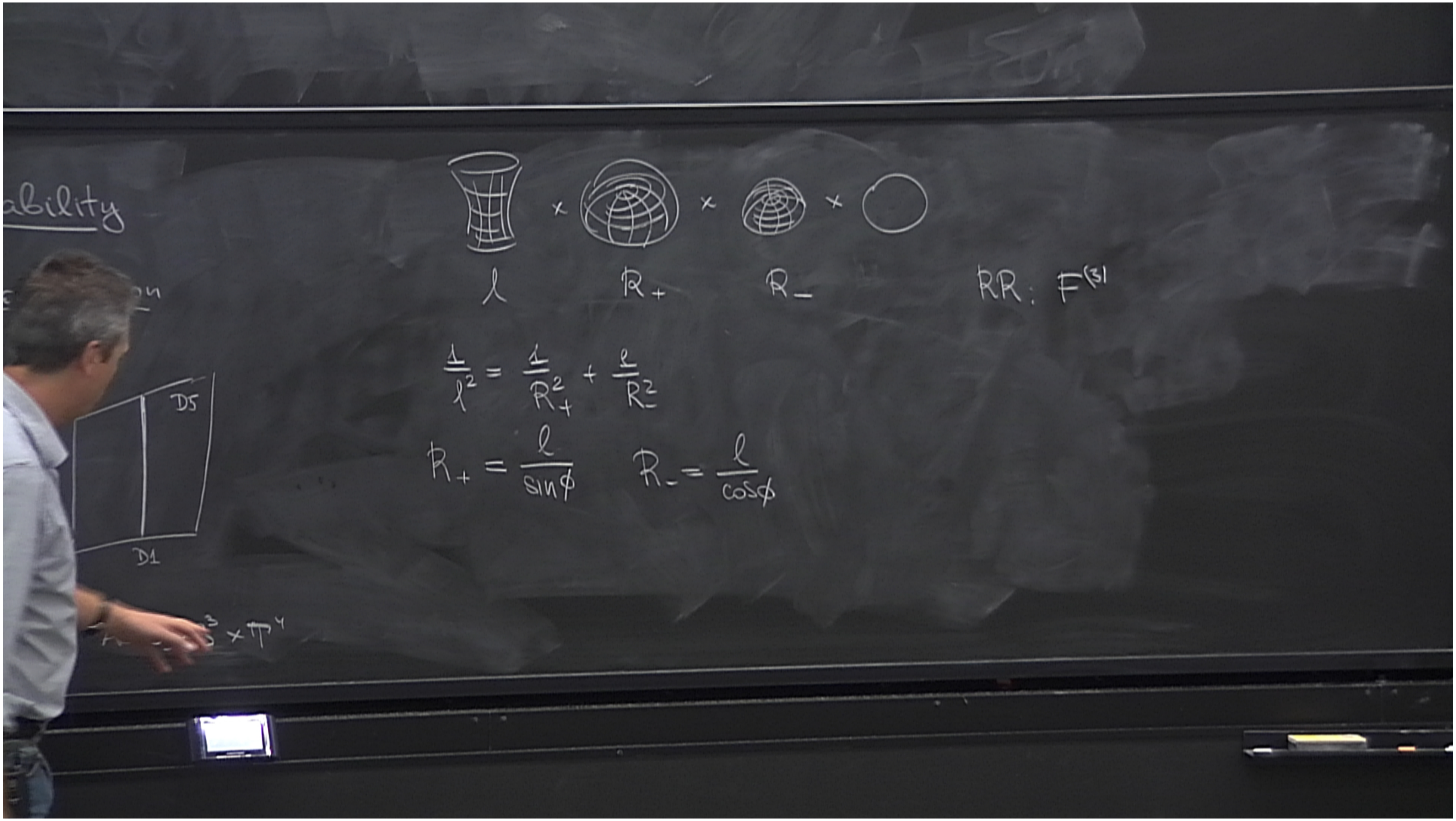
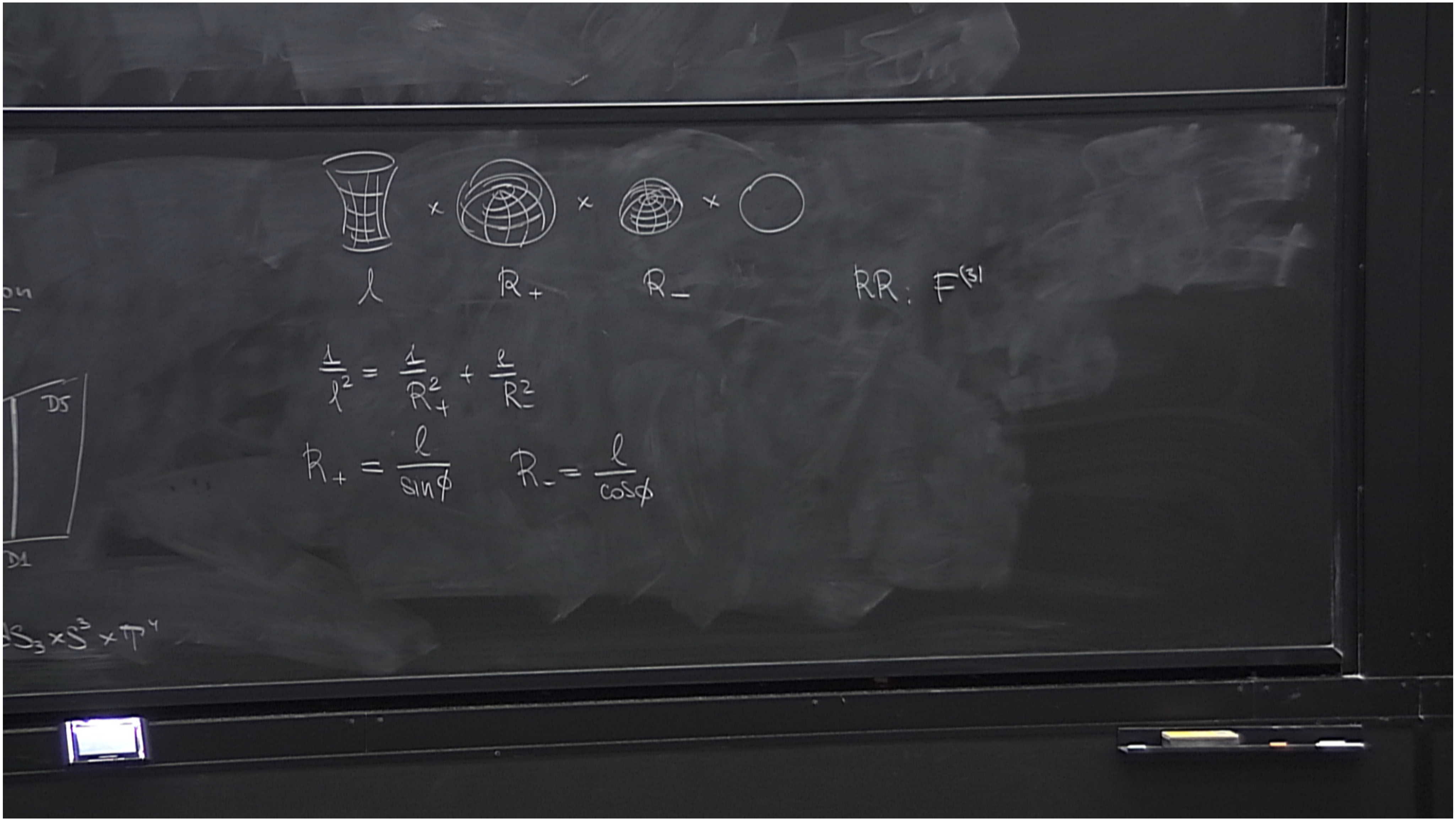
Brane construction
Σ
φ
D5
D5
D1
D5
D1

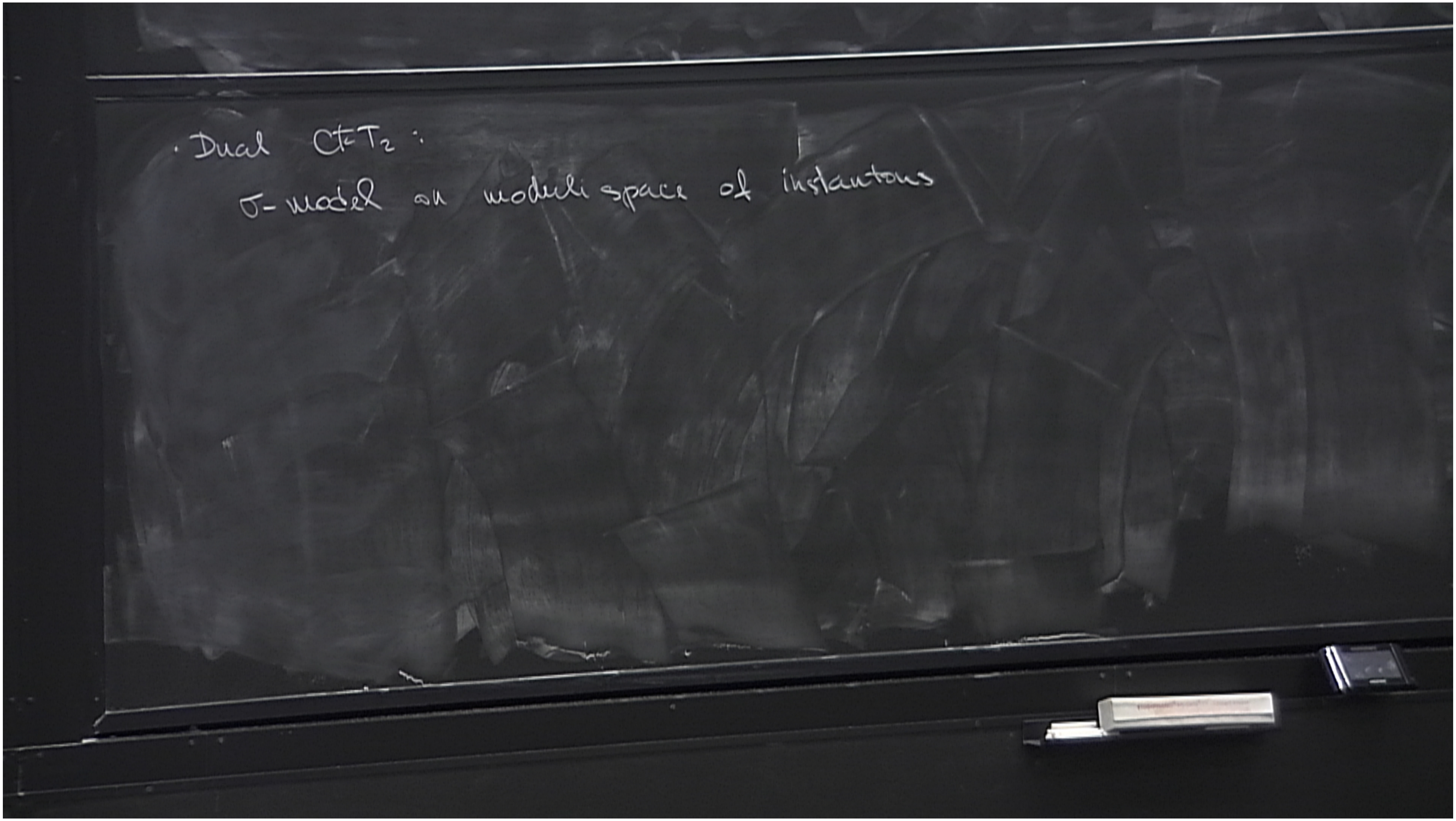

and integrability
Brane construction
ϕ
D5
D5
D5
D1
λ
$AdS_3 \times S^3 \times S^3 \times S^1$

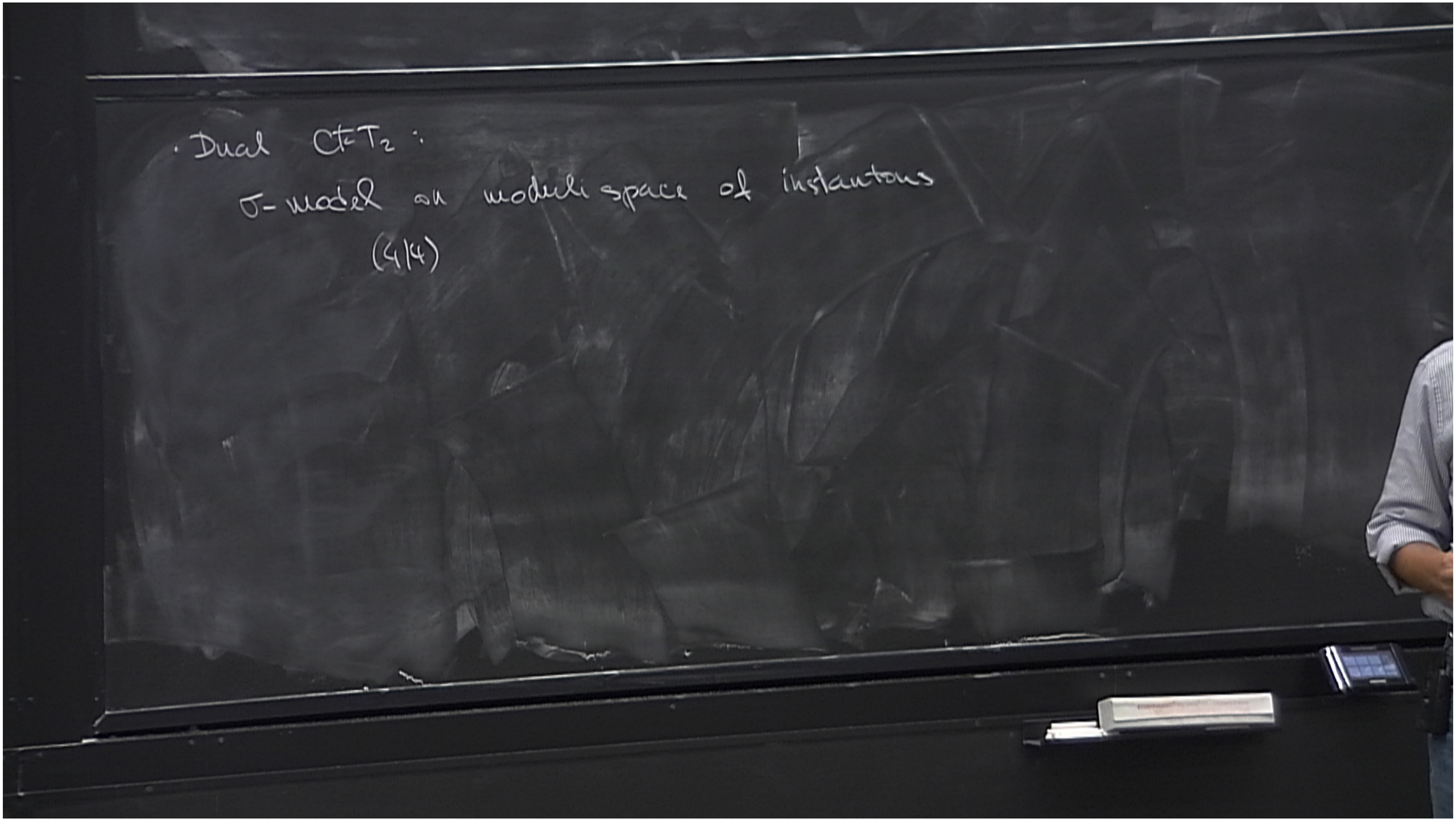

and integrability
Brane construction
ϕ
D5
D5
D1
λ
R_+
R_-
$\frac{1}{\lambda^2} = \frac{1}{R_+^2} + \frac{1}{R_-^2}$
$\times S^3 \times S^3 \times S^1$
$AdS_3 \times S^3 \times T^4$


and integrability
D5
D1
AdS3 × S3 × T4
λ
R+
R−
1/l² = 1/R+² + 1/R−²
R+ = l/sinφ
R− = l/cosφ

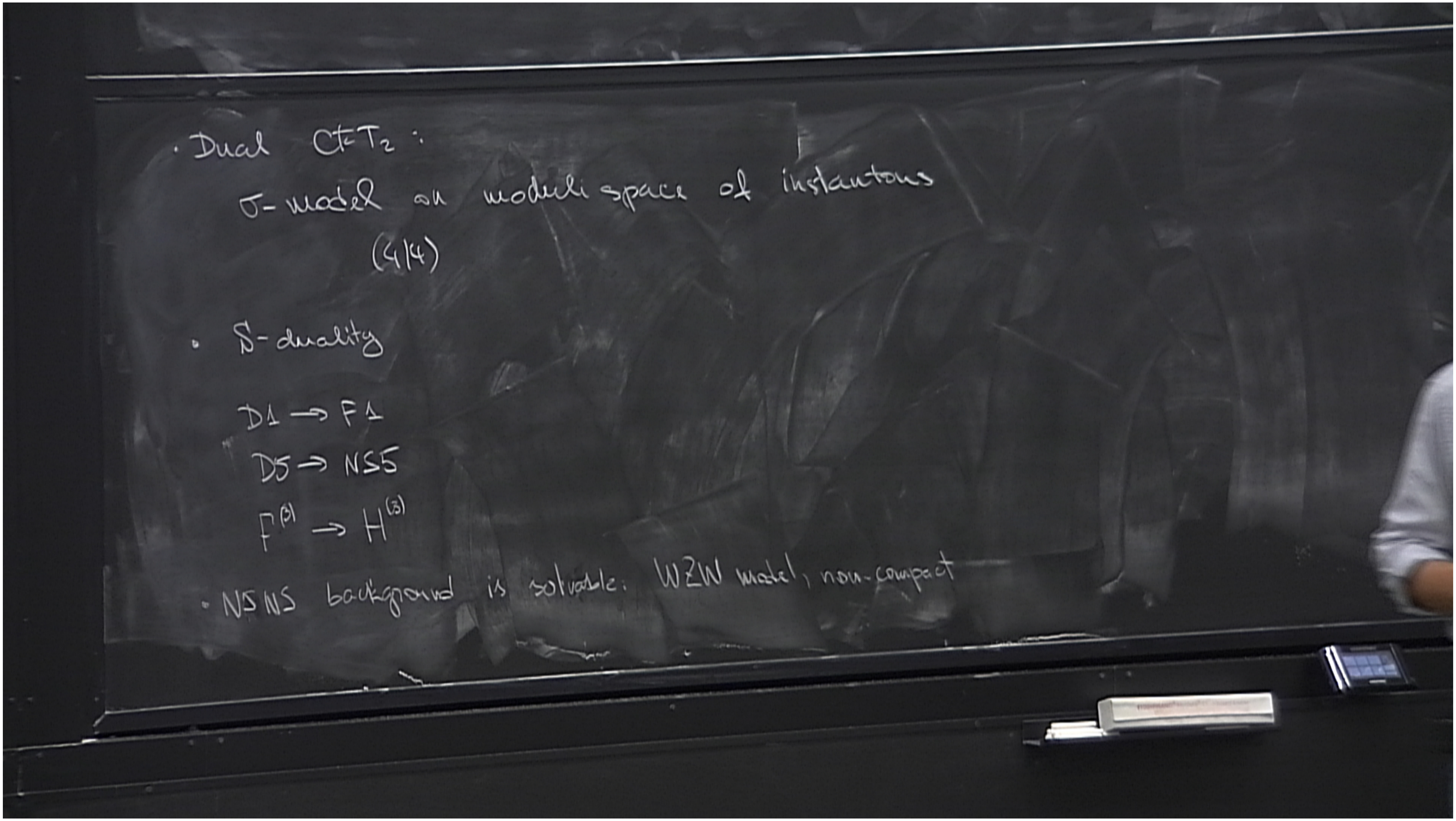

ability
D5
D1
ℓ
R_+
R_-
RR : $F^{(3)}$
$\frac{1}{\ell^2} = \frac{1}{R_+^2} + \frac{1}{R_-^2}$
$R_+ = \frac{\ell}{\sin\phi}$
$R_- = \frac{\ell}{\cos\phi}$

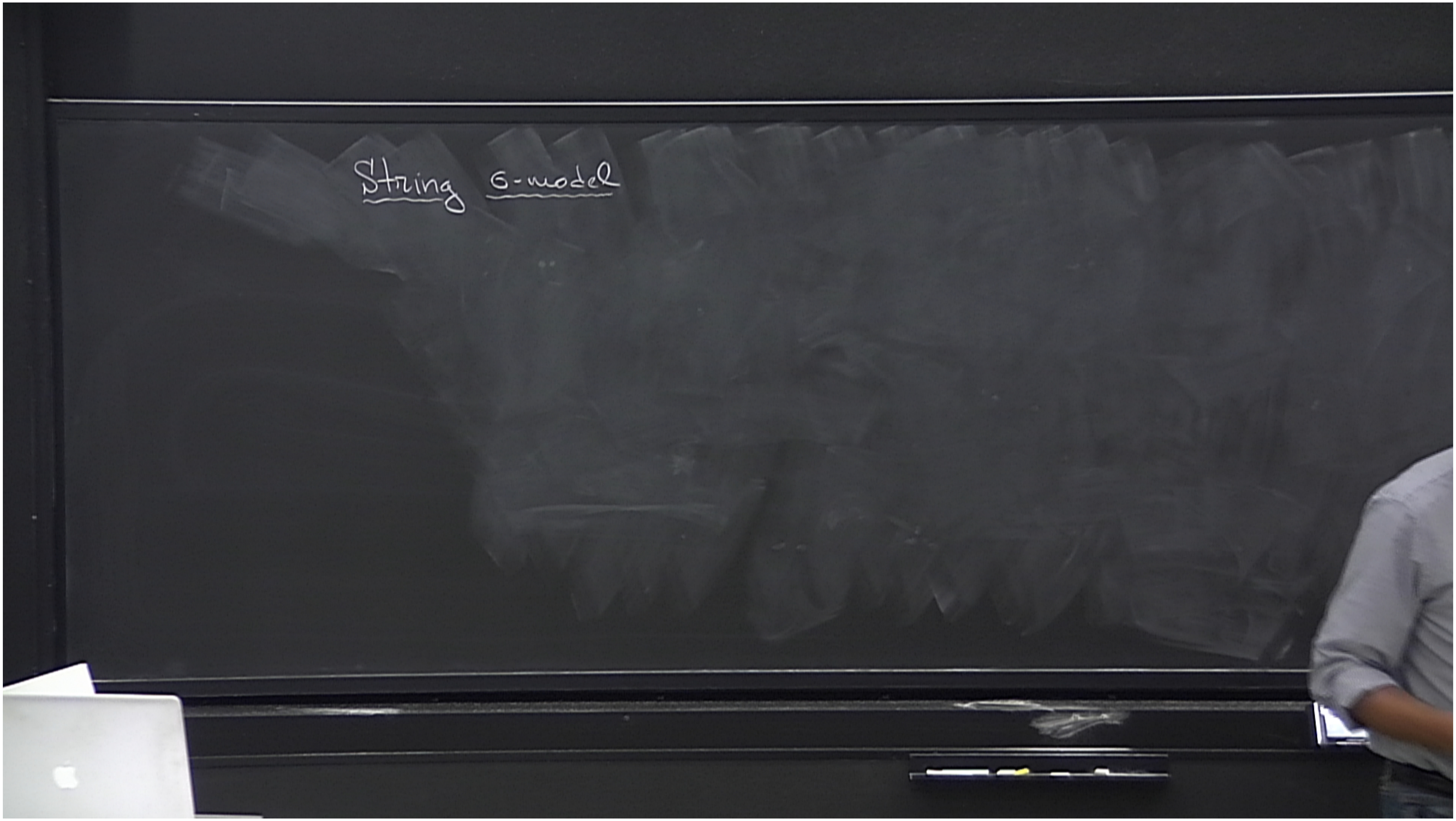

λ
R +
R _
RR: F(3)
1/l² = 1/R²₊ + 1/R²₋
R₊ = l / sinφ
R₋ = l / cosφ
D5
D1
S₃ × S³ × T⁴

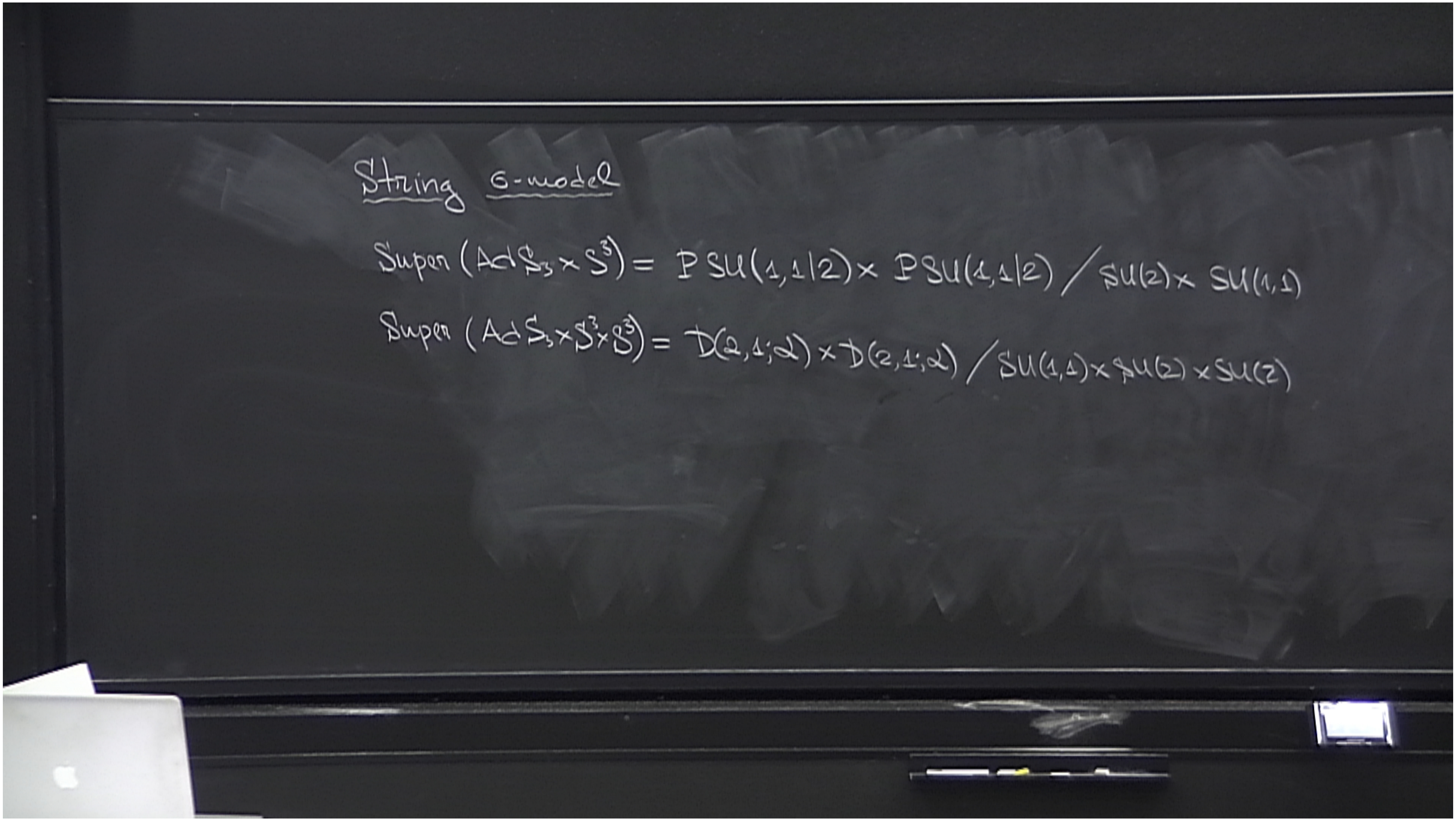

· Dual CFT_2 :
σ- model on moduli space of instantons

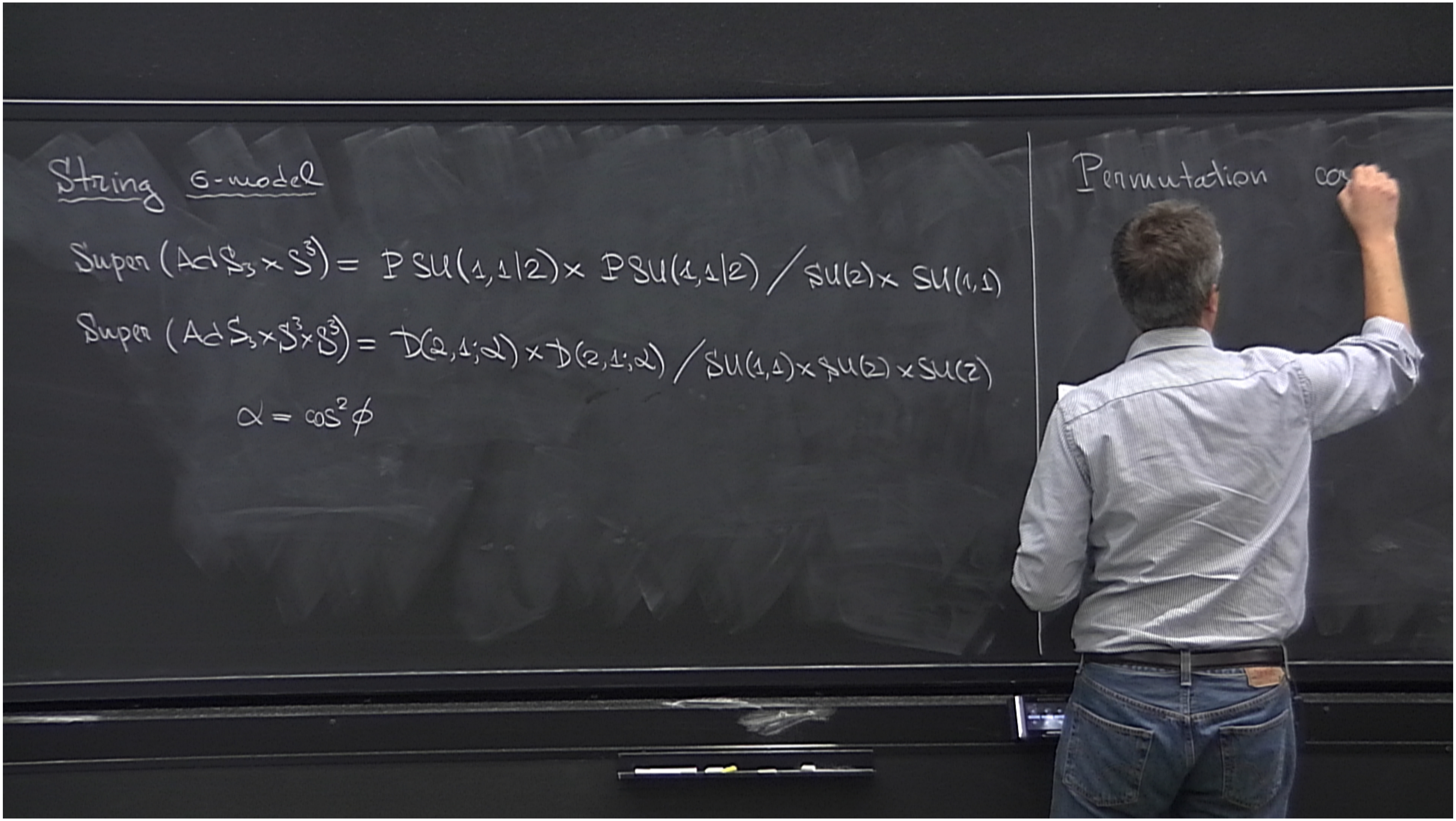


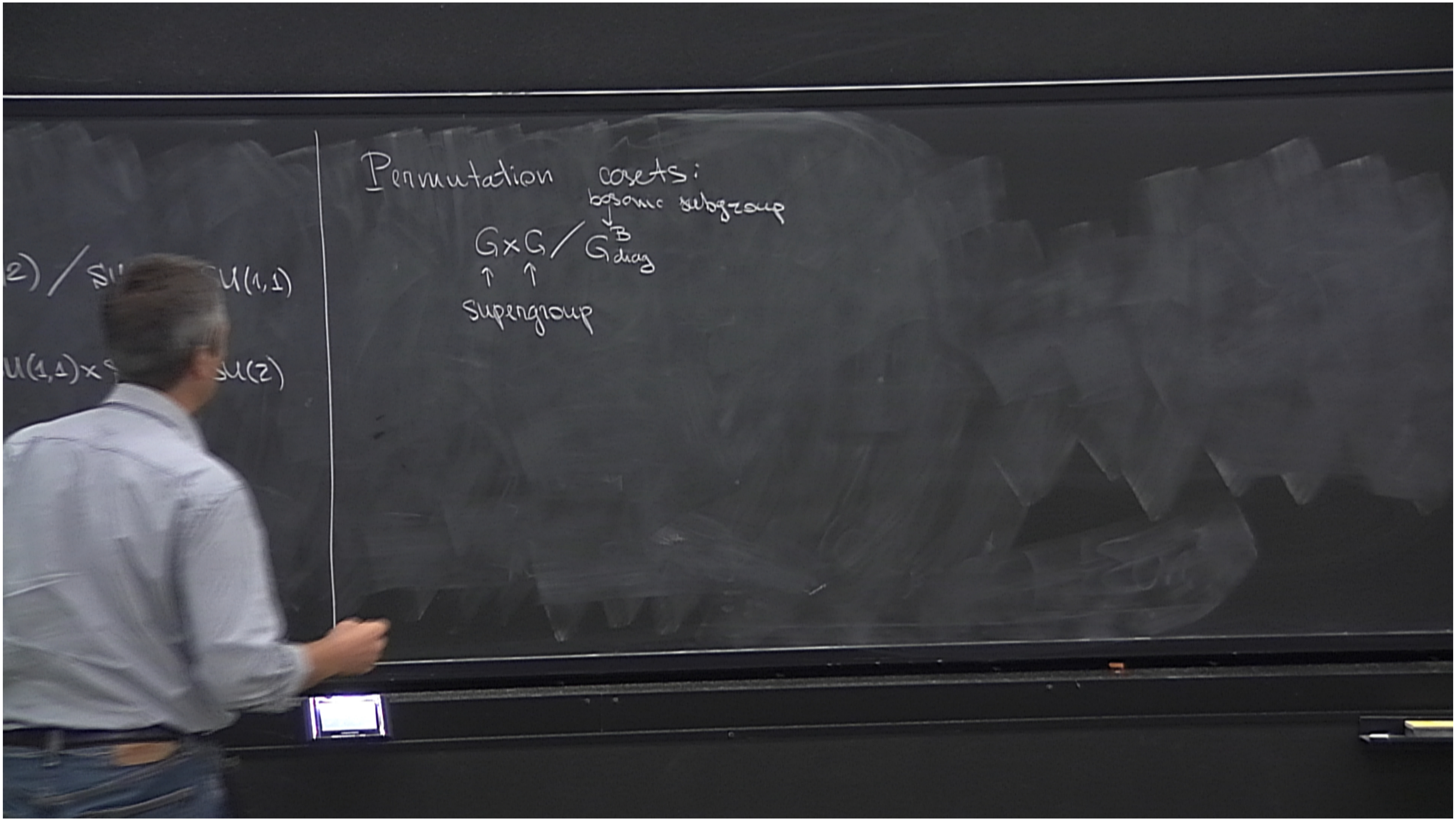
· Dual CFT2 :
σ-model on moduli space of instantons
(4|4)

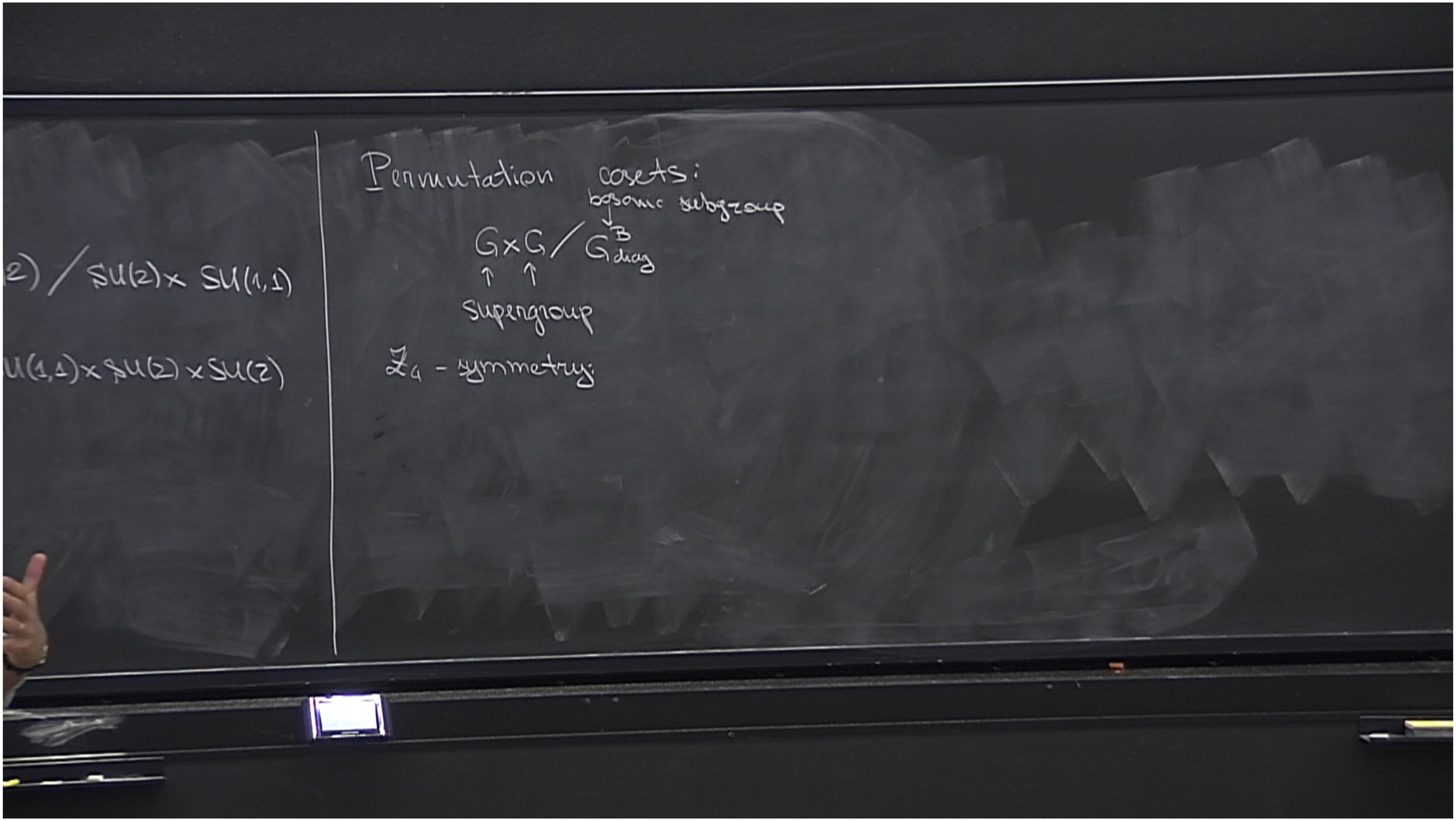

· Dual CFT2 :
σ- model on moduli space of instantons
(4|4)
· S-duality
D1 → F1
D5 → NS5
F(3) → H(3)
· NSNS background is solvable: WZW model, non-compact

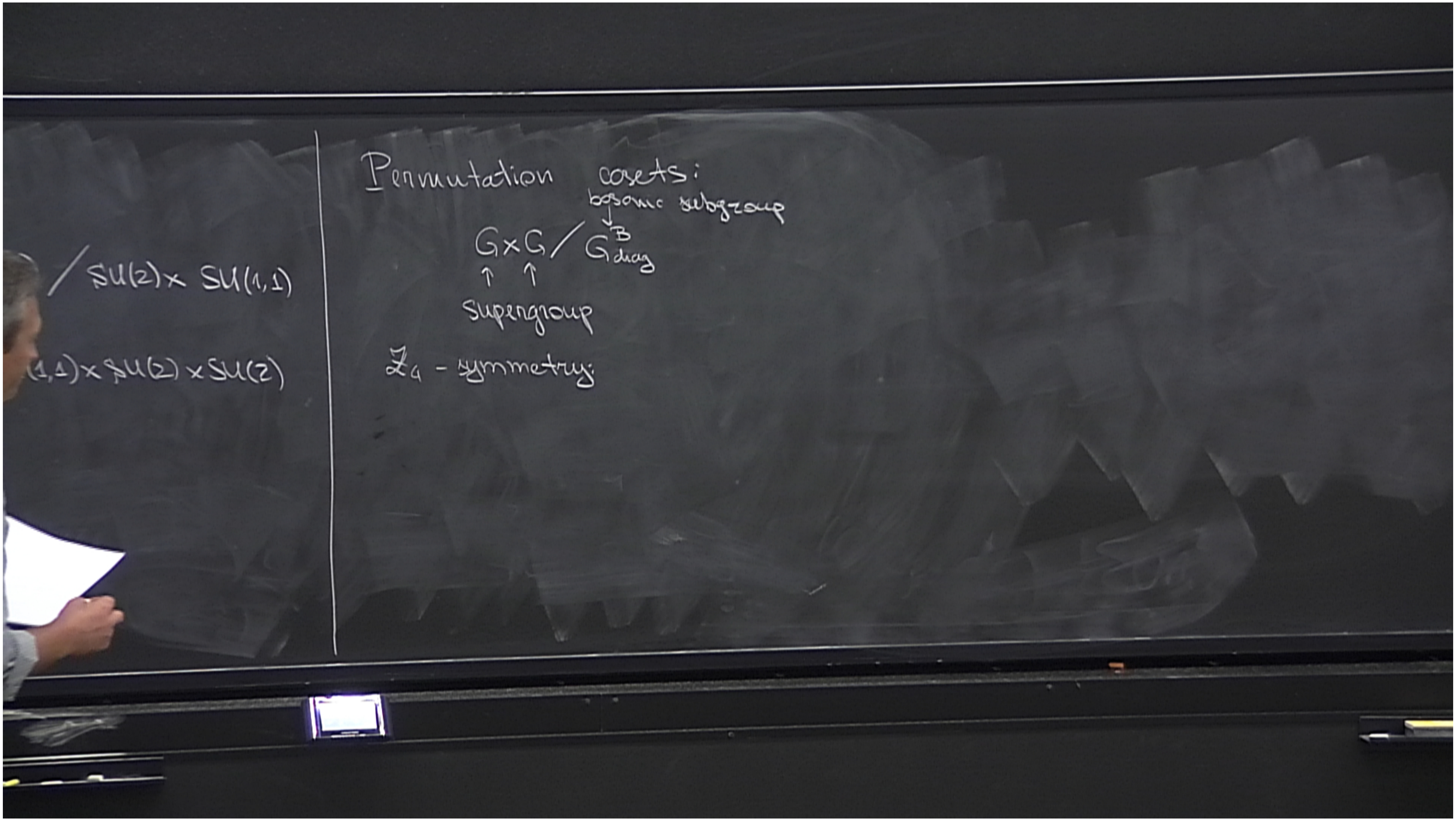

String σ-model


String σ-model
$\mathrm{Super}(AdS_3 \times S^3) = PSU(1,1|2) \times PSU(1,1|2) \,/\, SU(2) \times SU(1,1)$
$\mathrm{Super}(AdS_3 \times S^3 \times S^3) = D(2,1;\alpha) \times D(2,1;\alpha) \,/\, SU(1,1) \times SU(2) \times SU(2)$

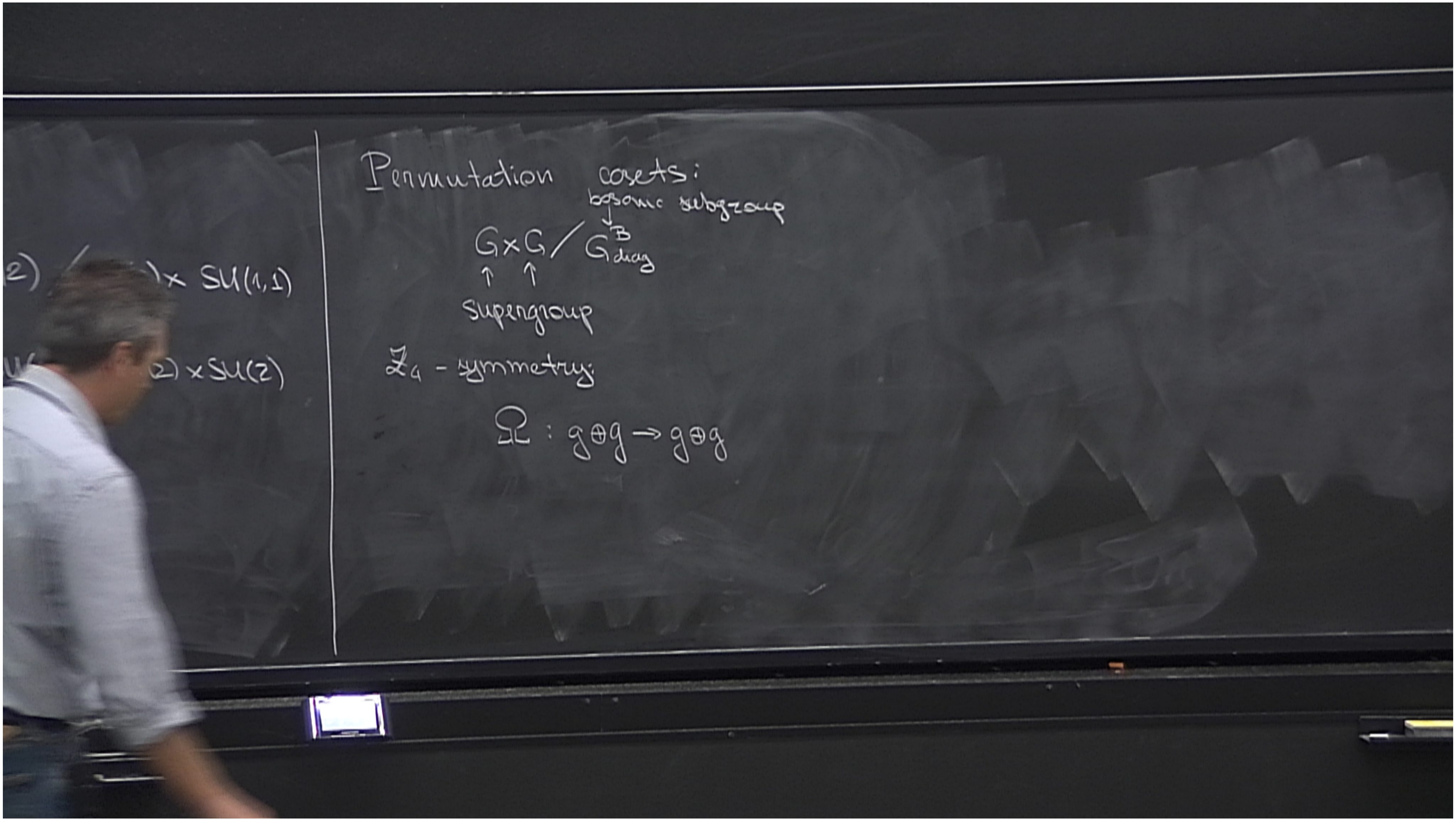

String σ-model
Super (AdS3 × S3) = PSU(1,1|2) × PSU(1,1|2) / SU(2) × SU(1,1)
Super (AdS3 × S3 × S3) = D(2,1;α) × D(2,1;α) / SU(1,1) × SU(2) × SU(2)
α = cos²φ
Permutation


Permutation cosets:
bosonic subgroup
$G \times G / G^{B}_{diag}$
supergroup


2) / SU(2)× SU(1,1)
U(1,1)×SU(2)×SU(2)
Permutation cosets:
bosonic subgroup
G×G / G^B_diag
supergroup
Z_4 - symmetry:



/ SU(2)× SU(1,1)
(1,1)× SU(2) × SU(2)
Permutation cosets:
bosonic subgroup
$G \times G / G^{B}_{diag}$
supergroup
$\mathbb{Z}_4$ - symmetry:


Permutation cosets:
bosonic subgroup
G×G / G^B_diag
supergroup
Z_4 - symmetry:
Ω : g⊕g → g⊕g
× SU(1,1)
2) × SU(2)

SU(1,1)

× SU(2)

Permutation cosets:

bosonic subgroup

$$G\times G / G^{B}_{diag}$$

supergroup

$\mathbb{Z}_4$ - symmetry:

$$\Omega : g\oplus g \to g\oplus g$$

$$\Omega = \begin{pmatrix} 0 & id \\ (-1)^F & 0 \end{pmatrix}$$ semigraded permutation

$$\Omega^4 = id$$

$\ldots SU(1,1)$

$\times SU(2)$

Permutation cosets:

$$\underset{\text{supergroup}}{G \times G} \Big/ G^{B}_{\text{diag}}$$

B = bosonic subgroup

$\mathbb{Z}_4$ - symmetry:

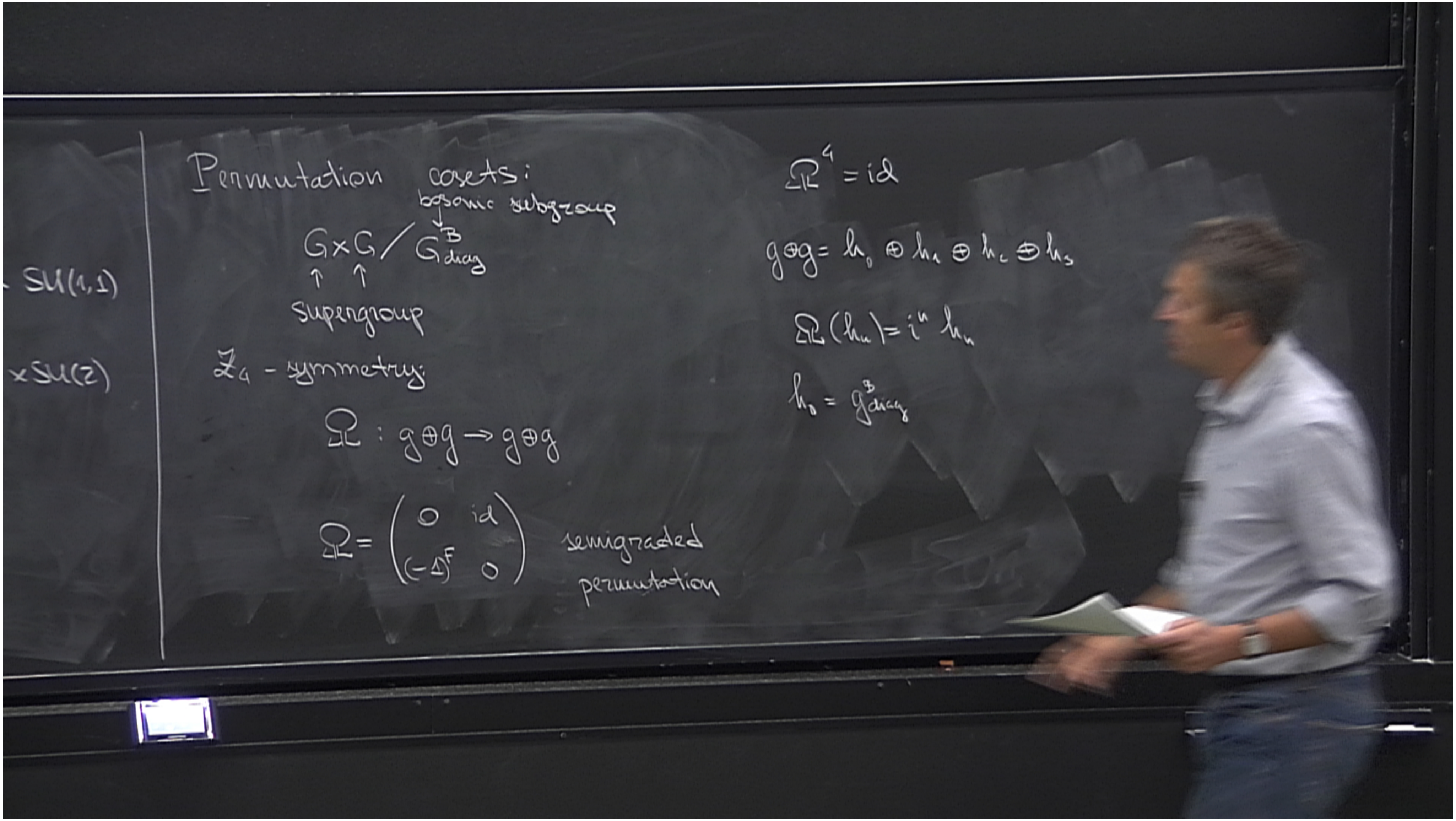
$$\Omega : \mathfrak{g} \oplus \mathfrak{g} \to \mathfrak{g} \oplus \mathfrak{g}$$

$$\Omega = \begin{pmatrix} 0 & \mathrm{id} \\ (-1)^F & 0 \end{pmatrix}$$ semigraded permutation

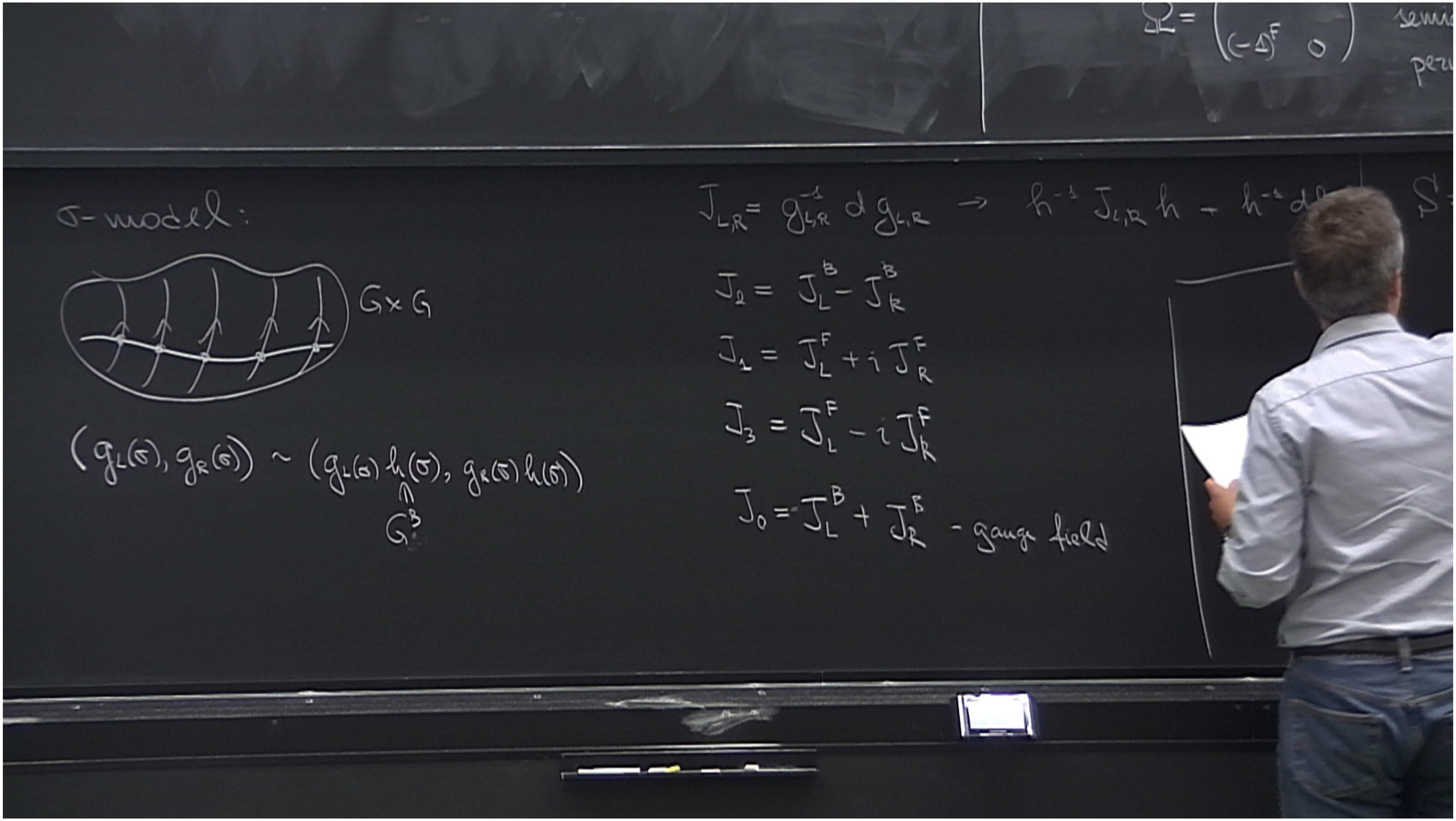
$$\Omega^4 = \mathrm{id}$$

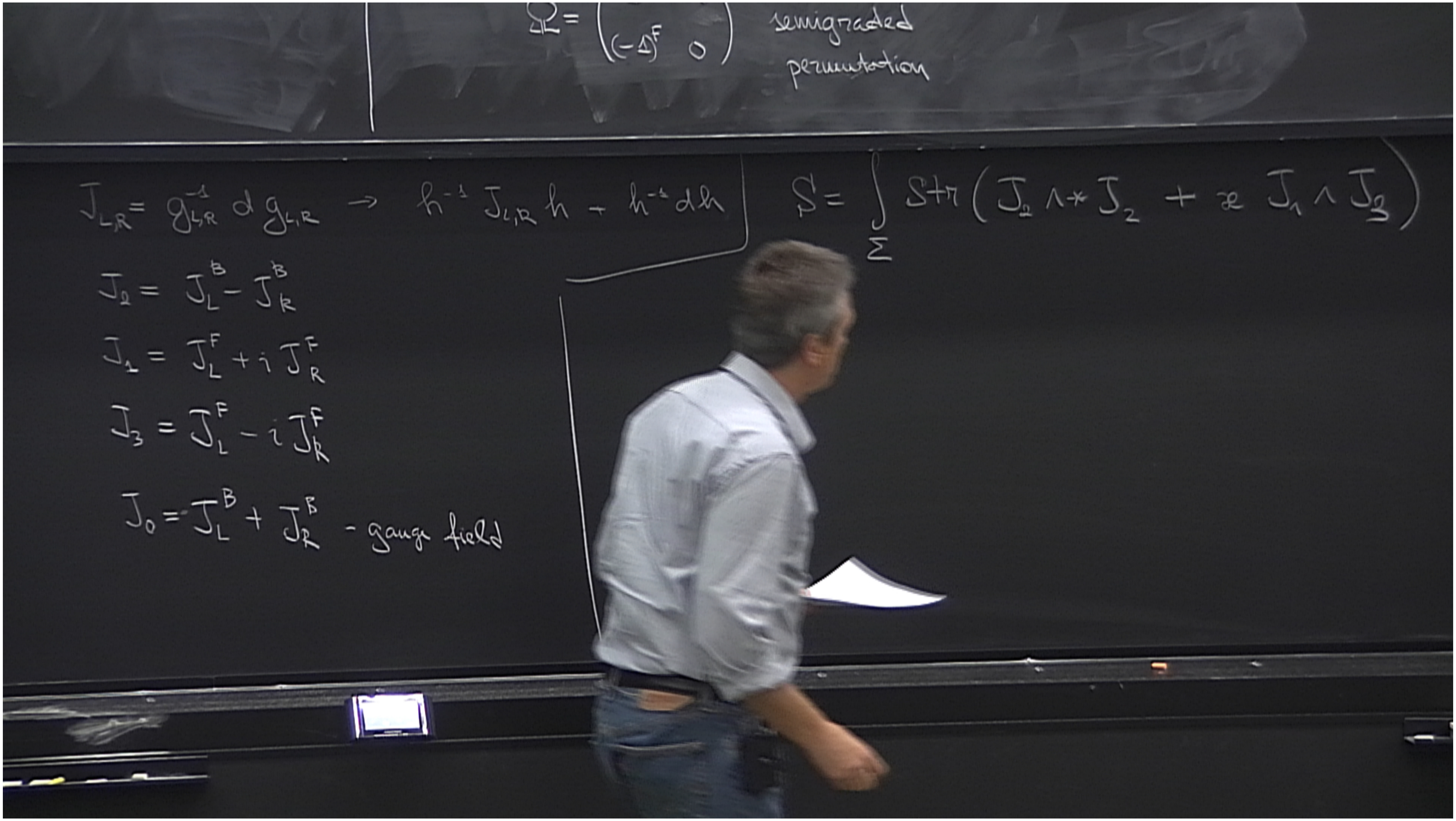
$$\mathfrak{g} \oplus \mathfrak{g} = \mathfrak{h}_0 \oplus \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

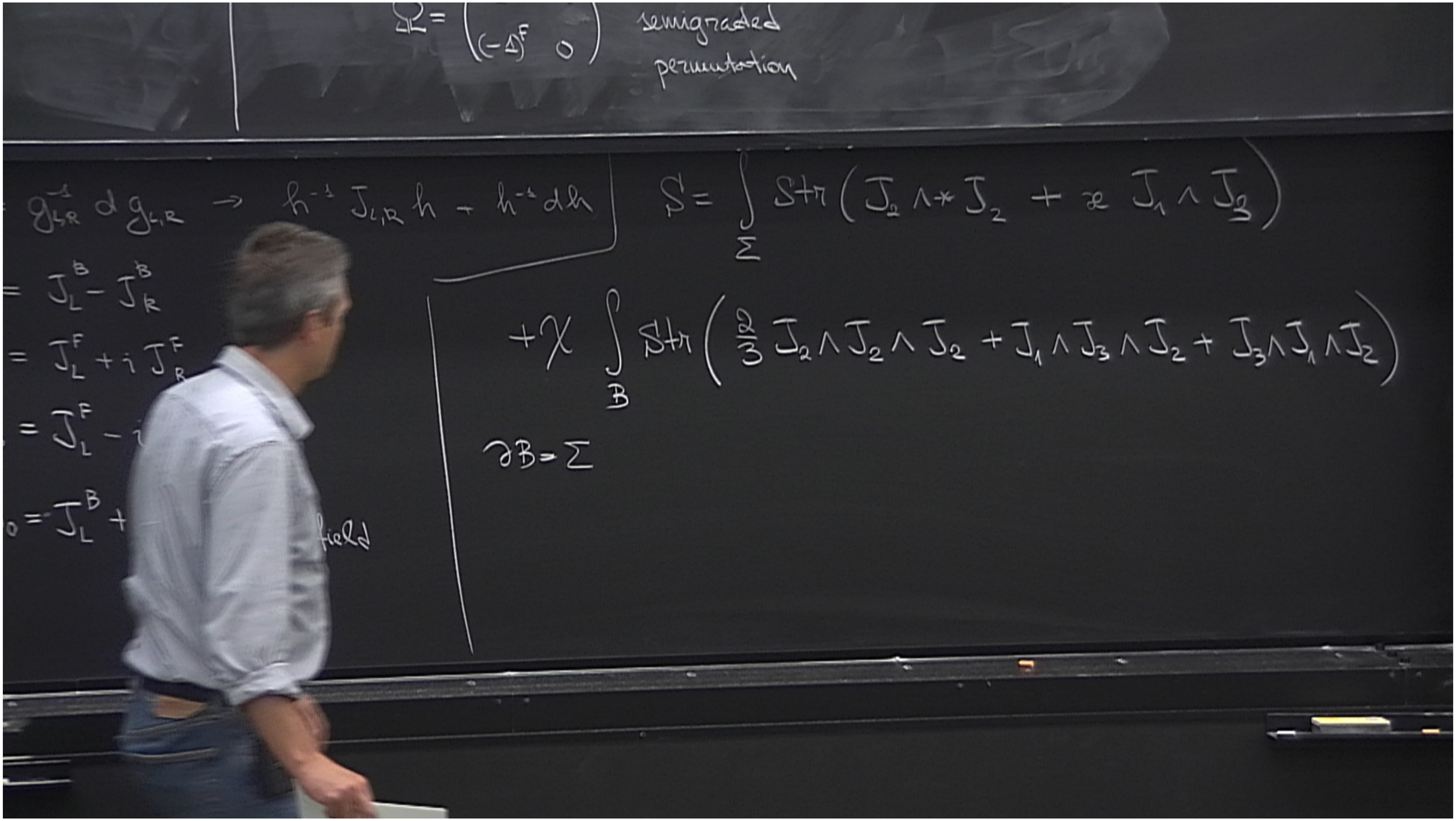
$$\Omega(\mathfrak{h}_n) = i^n \mathfrak{h}_n$$



$SU(1,1)$
$\times SU(2)$
Permutation cosets:
bosonic subgroup
$G \times G / G^{B}_{diag}$
supergroup
$\mathbb{Z}_4$ - symmetry:
$\Omega : g \oplus g \to g \oplus g$
$\Omega = \begin{pmatrix} 0 & id \\ (-1)^F & 0 \end{pmatrix}$
semigraded permutation
$\Omega^4 = id$
$g \oplus g = h_0 \oplus h_1 \oplus h_2 \oplus h_3$
$\Omega(h_n) = i^n h_n$
$h_0 = g^{B}_{diag}$


σ-model:
G×G
(g_L(σ), g_R(σ)) ~ (g_L(σ) h(σ), g_R(σ) h(σ))
G^B
J_{L,R} = g_{L,R}^{-1} d g_{L,R} → h^{-1} J_{L,R} h + h^{-1} d
J_2 = J_L^B − J_R^B
J_1 = J_L^F + i J_R^F
J_3 = J_L^F − i J_R^F
J_0 = −J_L^B + J_R^B - gauge field


$\Omega = \begin{pmatrix} & \\ (-1)^F & 0 \end{pmatrix}$ semigraded permutation
$J_{L,R} = g^{-1}_{L,R}\, d g_{L,R} \rightarrow h^{-1} J_{L,R} h + h^{-1} dh$
$S = \int_\Sigma \mathrm{Str}\left(J_2 \wedge * J_2 + x\, J_1 \wedge J_3 \right)$
$J_2 = J_L^B - J_R^B$
$J_1 = J_L^F + i J_R^F$
$J_3 = J_L^F - i J_R^F$
$J_0 = -J_L^B + J_R^B$ - gauge field


$\Omega = \begin{pmatrix} & \\ (-1)^F & 0 \end{pmatrix}$ semigraded permutation
$g_{L,R}^{-1} d g_{L,R} \rightarrow h^{-1} J_{L,R} h - h^{-1} dh$
$S = \int_\Sigma \mathrm{Str}\left(J_2 \wedge * J_2 + x\, J_1 \wedge J_3 \right)$
$= J_L^B - J_R^B$
$= J_L^F + i J_R^F$
$= J_L^F - i$
$= -J_L^B +$
field
$+ \chi \int_B \mathrm{Str}\left(\frac{2}{3} J_2 \wedge J_2 \wedge J_2 + J_1 \wedge J_3 \wedge J_2 + J_3 \wedge J_1 \wedge J_2 \right)$
$\partial B = \Sigma$

$$\Omega = \begin{pmatrix} & \\ (-1)^F & 0 \end{pmatrix}$$ semigraded permutation

$$g_{L,R}^{-1}\, d g_{L,R} \to h^{-1} J_{L,R}\, h + h^{-1} dh$$

$$= J_L^B - J_R^B$$

$$= J_L^F + i J_R^F$$

$$= J_L^F - i J_R^F$$

$$= -J_L^B + J_R^B$$ - gauge field

$$S = \int_\Sigma \mathrm{Str}\left(J_2 \wedge * J_2 + \varkappa\, J_1 \wedge J_3 \right)$$

$$+ \chi \int_B \mathrm{Str}\left(\frac{2}{3} J_2 \wedge J_2 \wedge J_2 + J_1 \wedge J_3 \wedge J_2 + J_3 \wedge J_1 \wedge J_2 \right)$$

$$\partial B = \Sigma$$

- integrability
- $\varkappa$-symmetry
- $\beta = 0$

$$\left.\begin{array}{l}\text{integrability}\\ \varkappa\text{-symmetry}\\ \beta=0\end{array}\right\}\quad \varkappa^2 = 1-\chi^2$$

$\chi = 0,\ \varkappa = 1$: pure RR

$\chi = 1,\ \varkappa = 0$: pure NS-NS (WZW point)

$$p = 0 \iff \varkappa = 0,\ \chi = 1$$

or

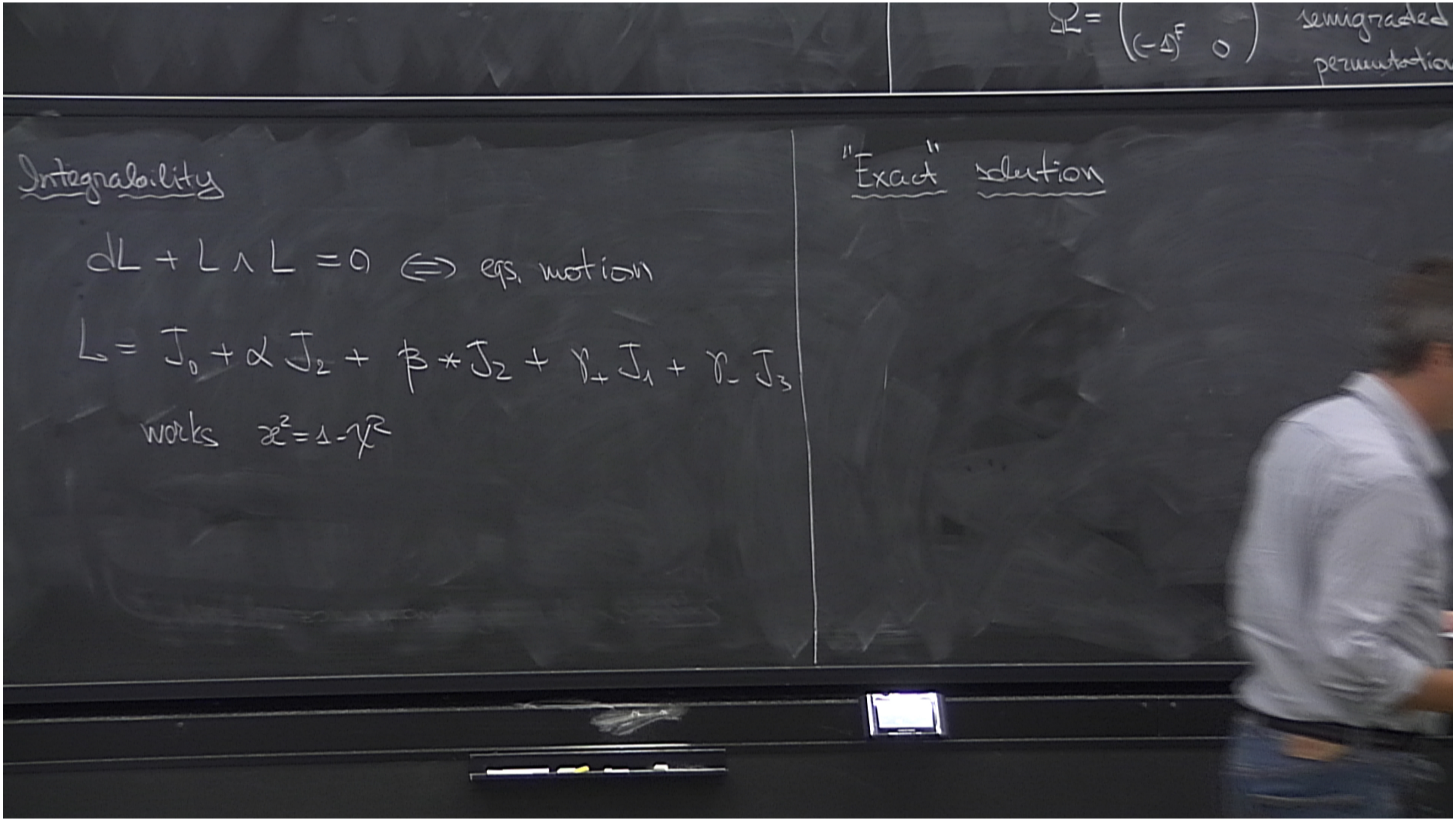
$$\mathrm{Str}_{ad_1} = 0 \iff G = PSU(n|n) \text{ or } OSp(2n+2|2n)$$

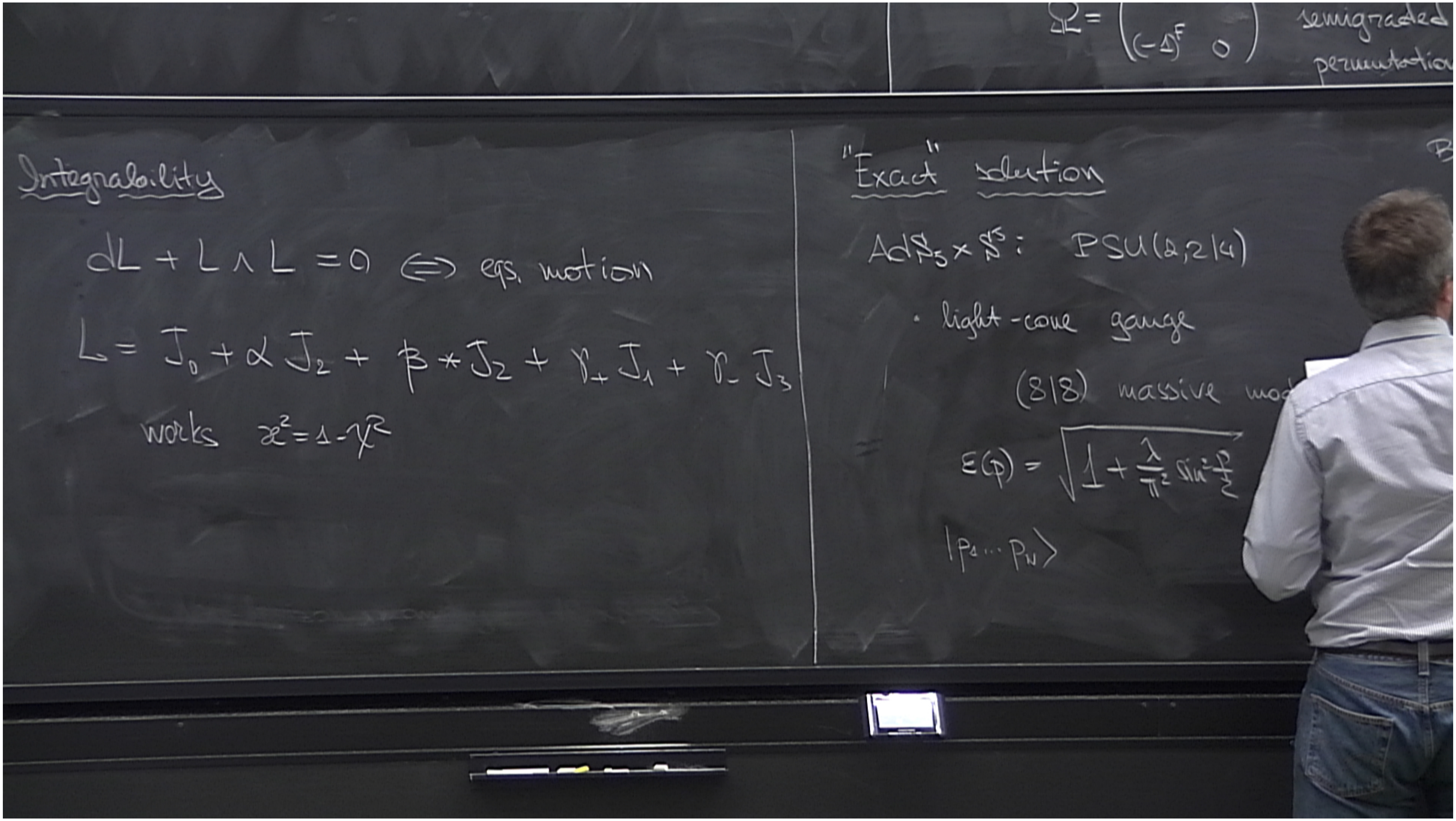
Z-symmetry

$$\delta X_{13} = \epsilon^{\pm}_{13} Q^{\pm} \qquad S \text{ is inv } \iff \varkappa^2 = 1 - \chi^2$$

$$K_{\pm} = \frac{1 \mp *}{2} \bar{J}_2$$

$$[K_{\pm}, Q^{\pm}] = 0$$


$\Omega = \begin{pmatrix} & \\ (-1)^F & 0 \end{pmatrix}$ semigraded permutation
Integrability
$dL + L \wedge L = 0 \iff$ eqs. motion
$L = J_0 + \alpha J_2 + \beta * J_2 + \gamma_+ J_1 + \gamma_- J_3$
works $\varkappa^2 = 1 - \chi^2$
"Exact" solution


Ω = ((−1)^F 0)
semigraded
permutation
Integrability
dL + L ∧ L = 0 ⟺ eqs. motion
L = J_0 + α J_2 + β * J_2 + γ_+ J_1 + γ_- J_3
works x² = 1 - γ²
"Exact" solution
AdS_5 × S^5 : PSU(2,2|4)
· light-cone gauge
(8|8) massive mod
E(p) = √(1 + λ/π² sin² p/2)
|p_1 ... p_N⟩

$\Omega = \begin{pmatrix} & \\ (-1)^F & 0 \end{pmatrix}$ semigraded permutation

"Exact" solution

$AdS_5 \times S^5$: $PSU(2,2|4)$

- light-cone gauge

(8|8) massive modes

$$\varepsilon(p) = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$$

$|p_1 \dots p_N\rangle$

Bethe equations:

$$e^{ip_j L} = \prod_{k \neq j} S(p_k, p_j)$$

$\otimes$ — $\bigcirc$ — $\otimes$ — $\overset{1}{\bigcirc}$ — $\otimes$ — $\bigcirc$ — $\otimes$

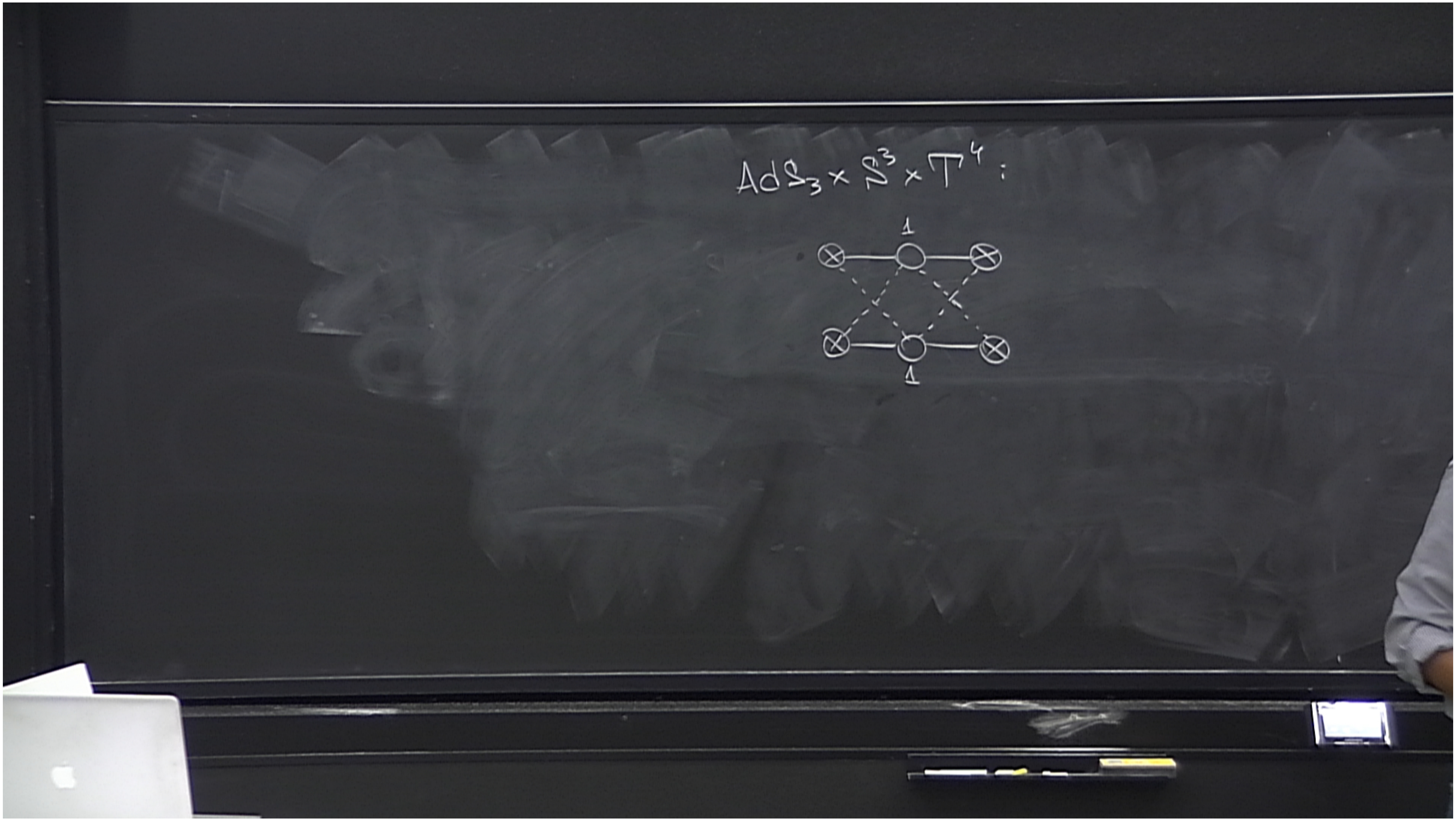
$u_{j\alpha}$

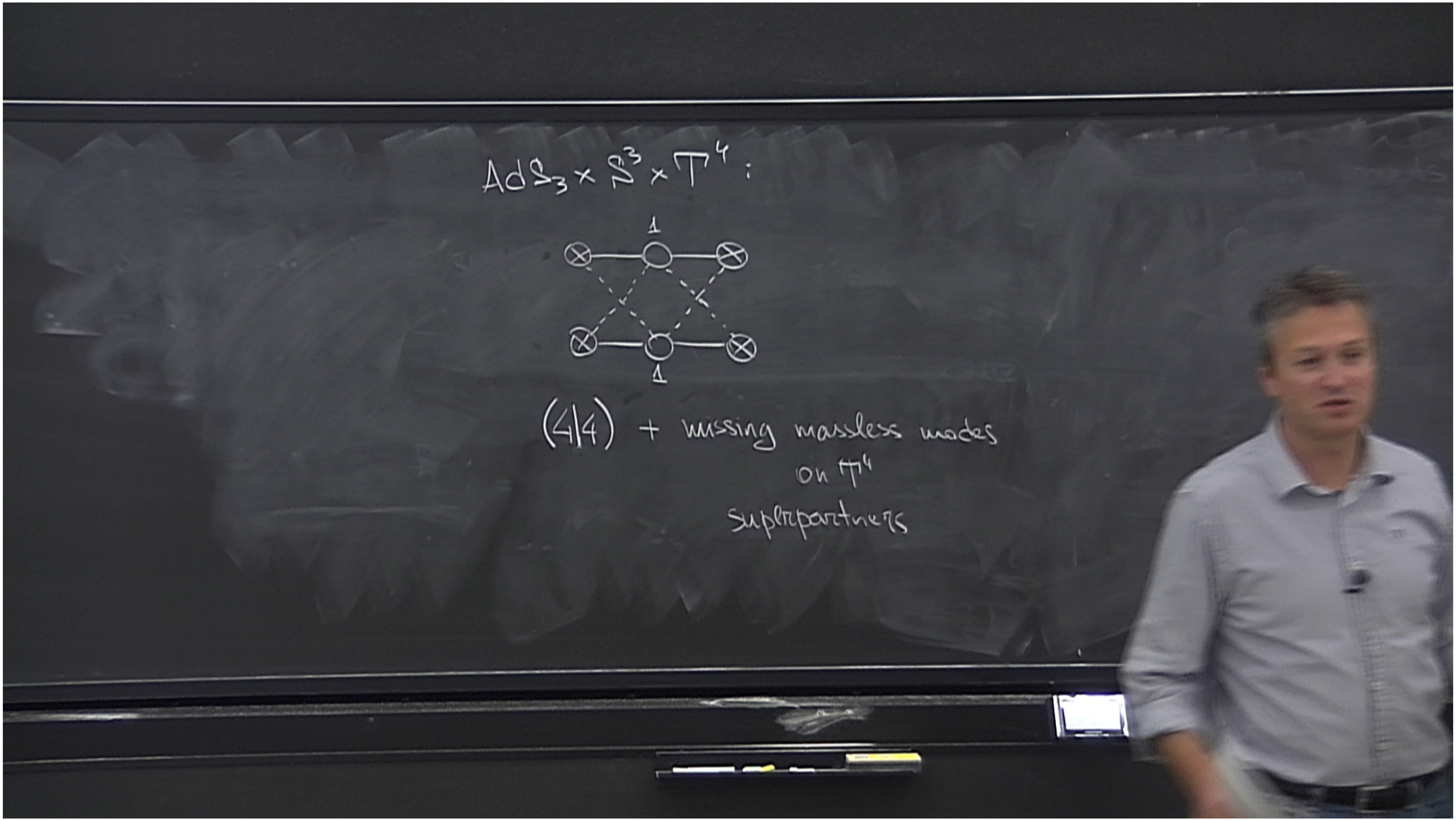
$$x^\pm + \frac{1}{x^\pm} = u_4 \pm \frac{2\pi i}{\sqrt{\lambda}}$$

$$\frac{x^+}{x^-} = e^{ip}$$

$$\varepsilon = i(x^- - x^+)$$

$$\left(\frac{x_j^+}{x_j^-}\right)^L = \prod\left(\dots\right)$$


$AdS_3 \times S^3 \times T^4$:
1
1


$AdS_3 \times S^3 \times T^4$:
1
1
(4|4) + missing massless modes
on T^4
superpartners