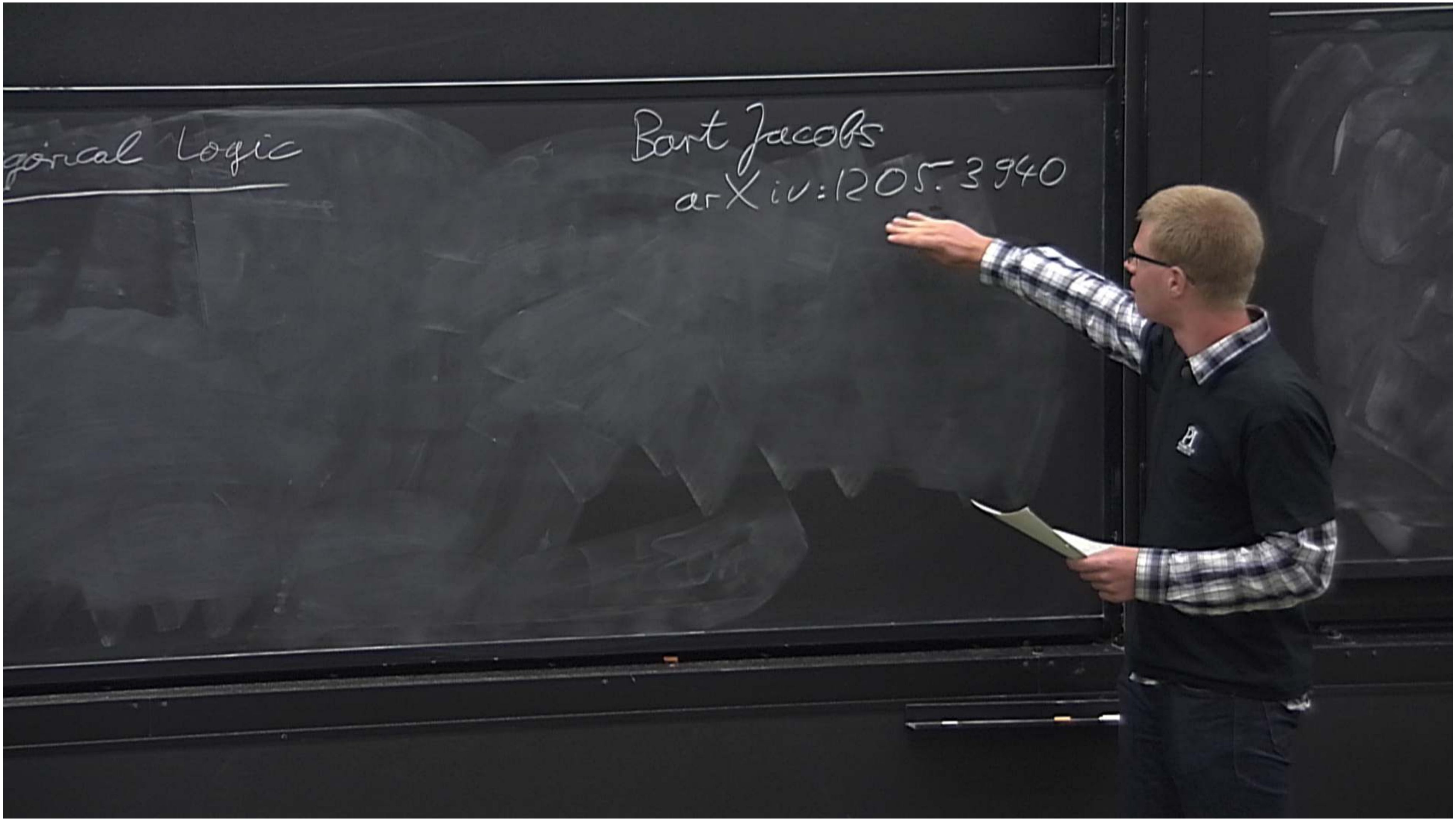
Title: New Directions in Categorical Logic for Classical, Probabilistic and Quantum Physics

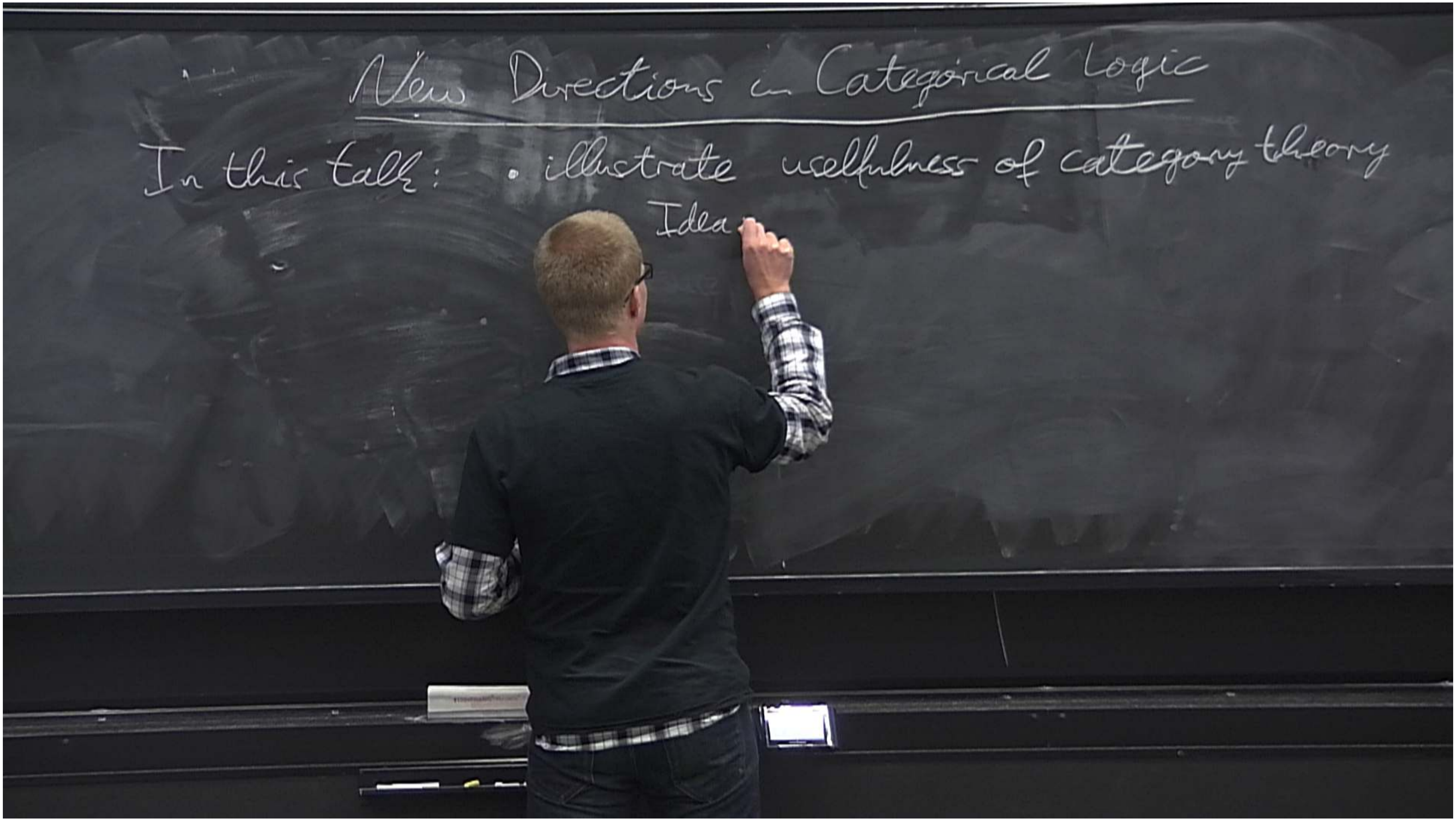
Date: Oct 09, 2012 03:30 PM

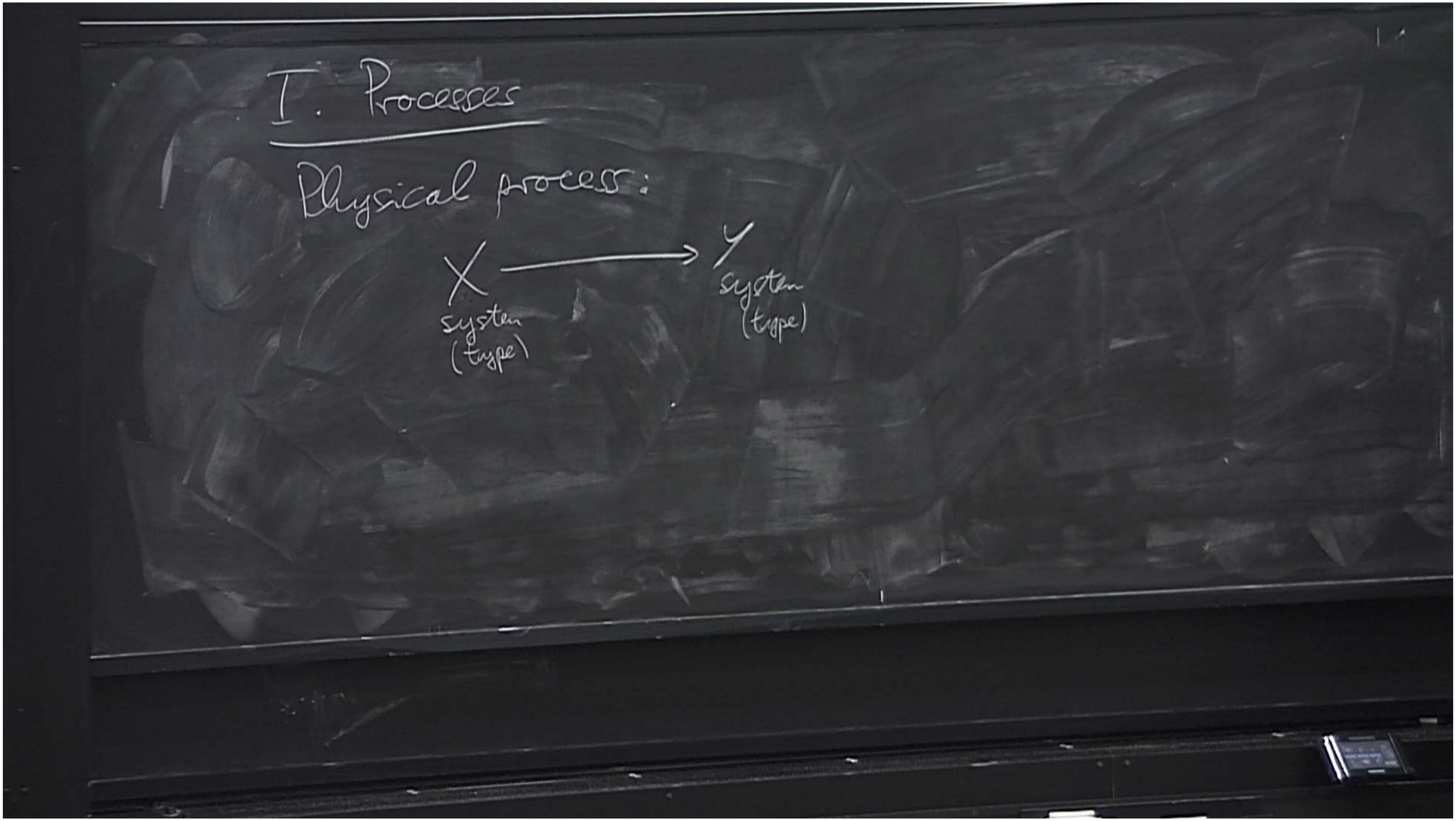
URL: http://pirsa.org/12100055

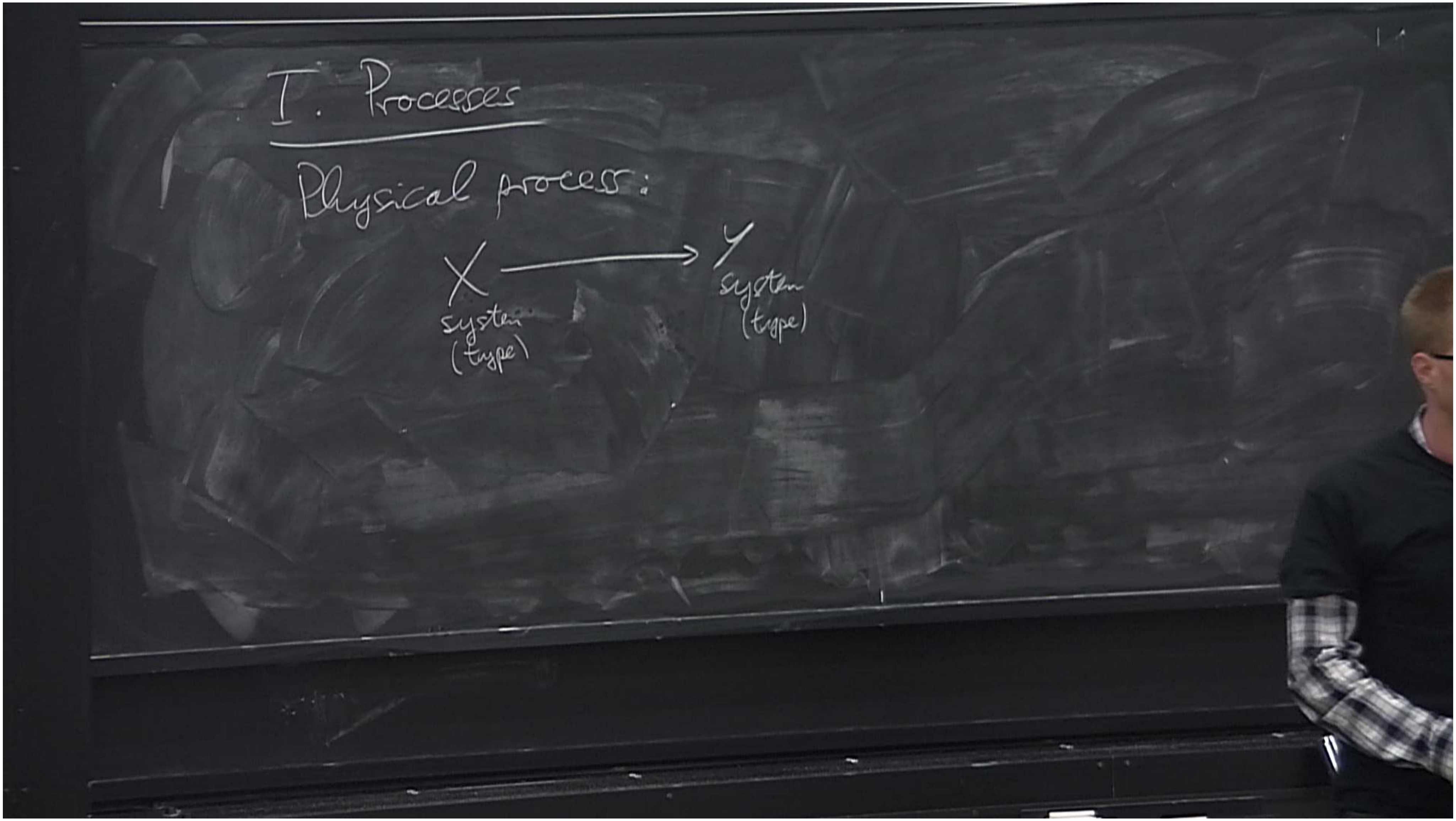
Abstract: I will
give an idea of what category theory is and how it can be successfully applied in mathematics and the mathematical sciences by means of example. The example is a notion from mathematical logic formalizing the intuitive concept of "property". The new category-theoretical
definition of this notion can physically be interpreted as a measurement. Unraveling this definition in particular categories can be regarded as defining the concept of "property" in different context, e.g. in classical, probabilistic
and quantum physics, and in each case this recovers familiar things.
Disclaimer:
Up to some differences in the technical details, all of this is work of Bart Jacobs. (See arXiv:1205.3940)<strongr>

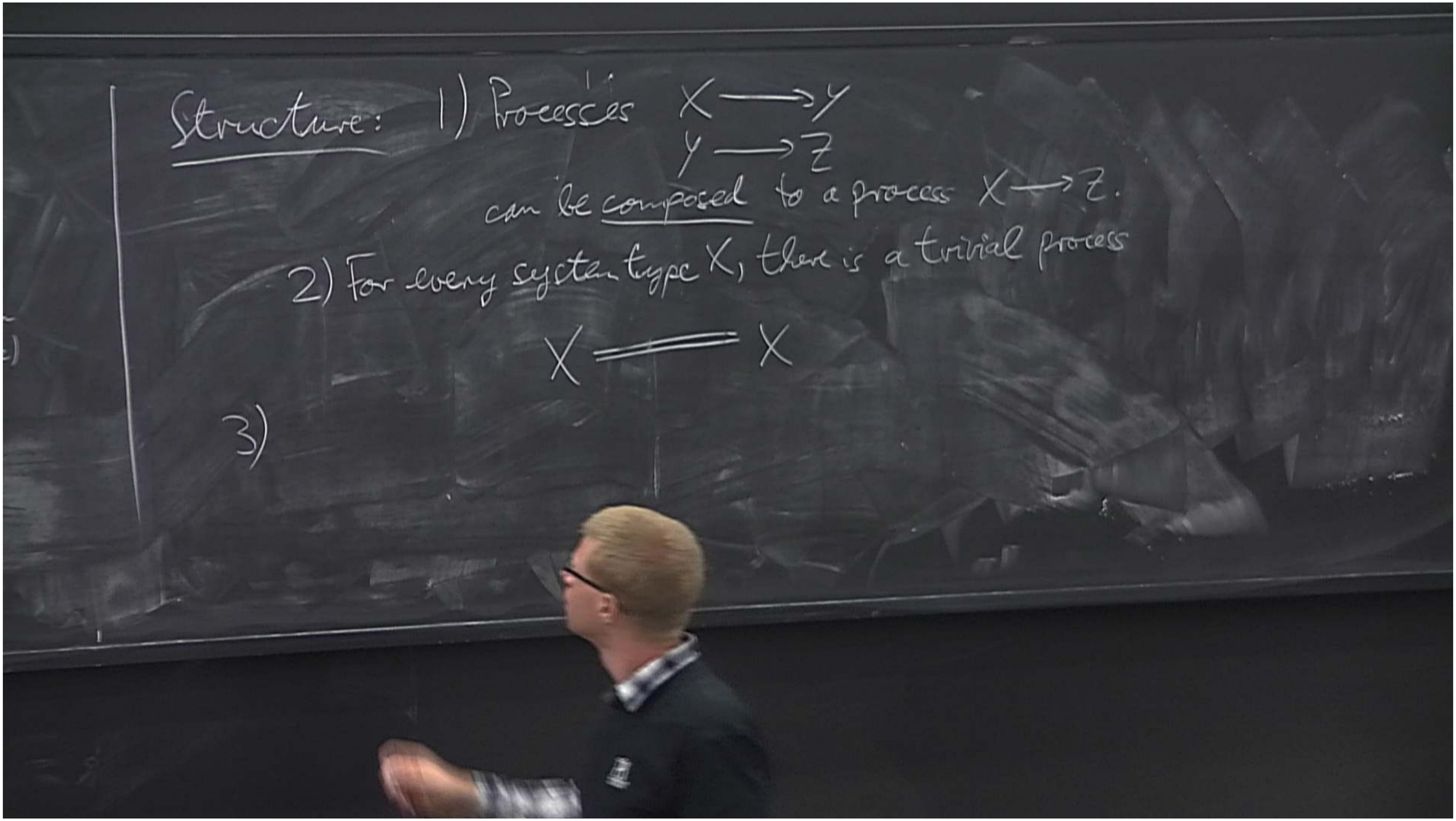
</strongr>

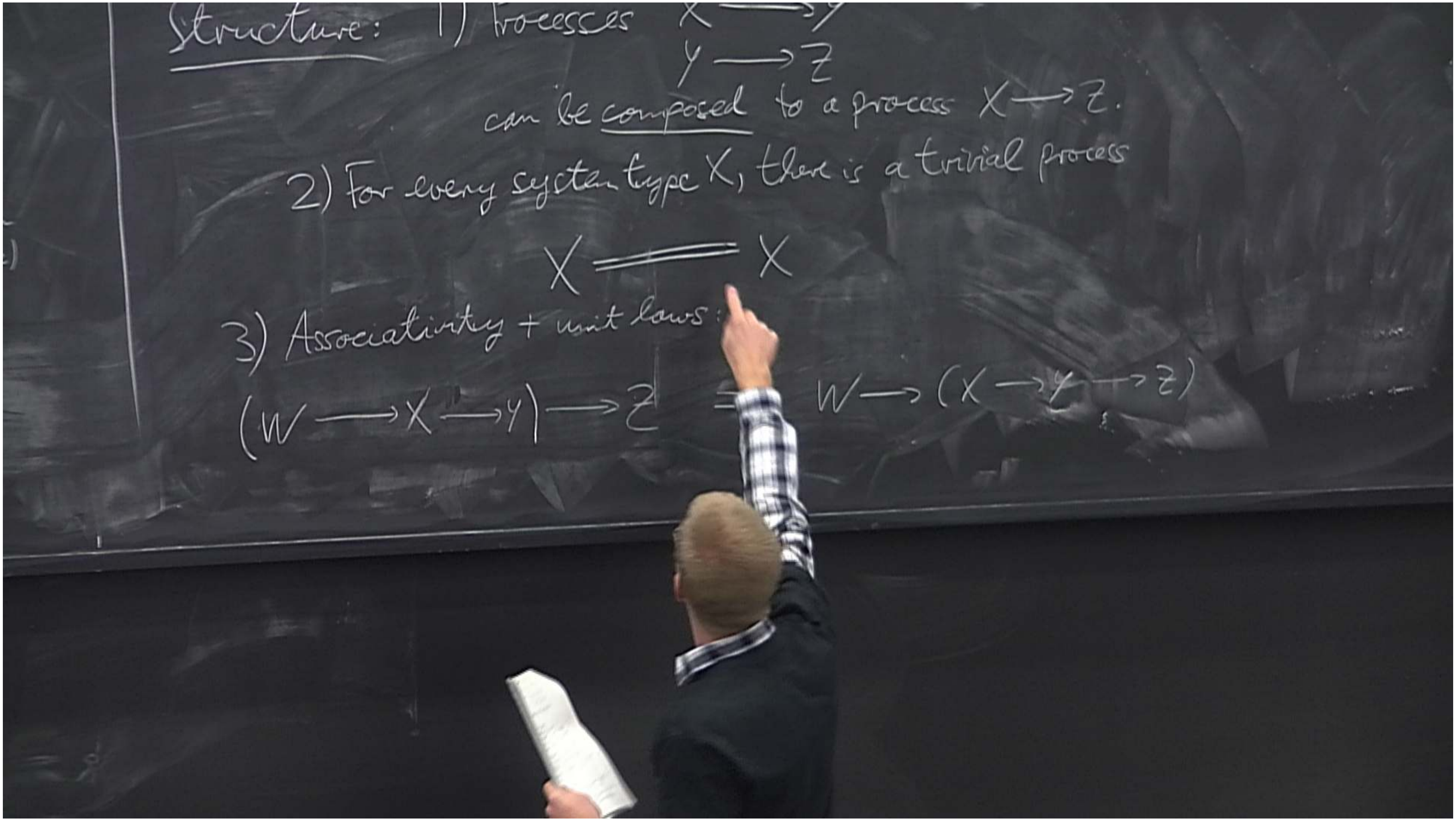

gorical Logic
Bart Jacobs
arXiv:1205.3940

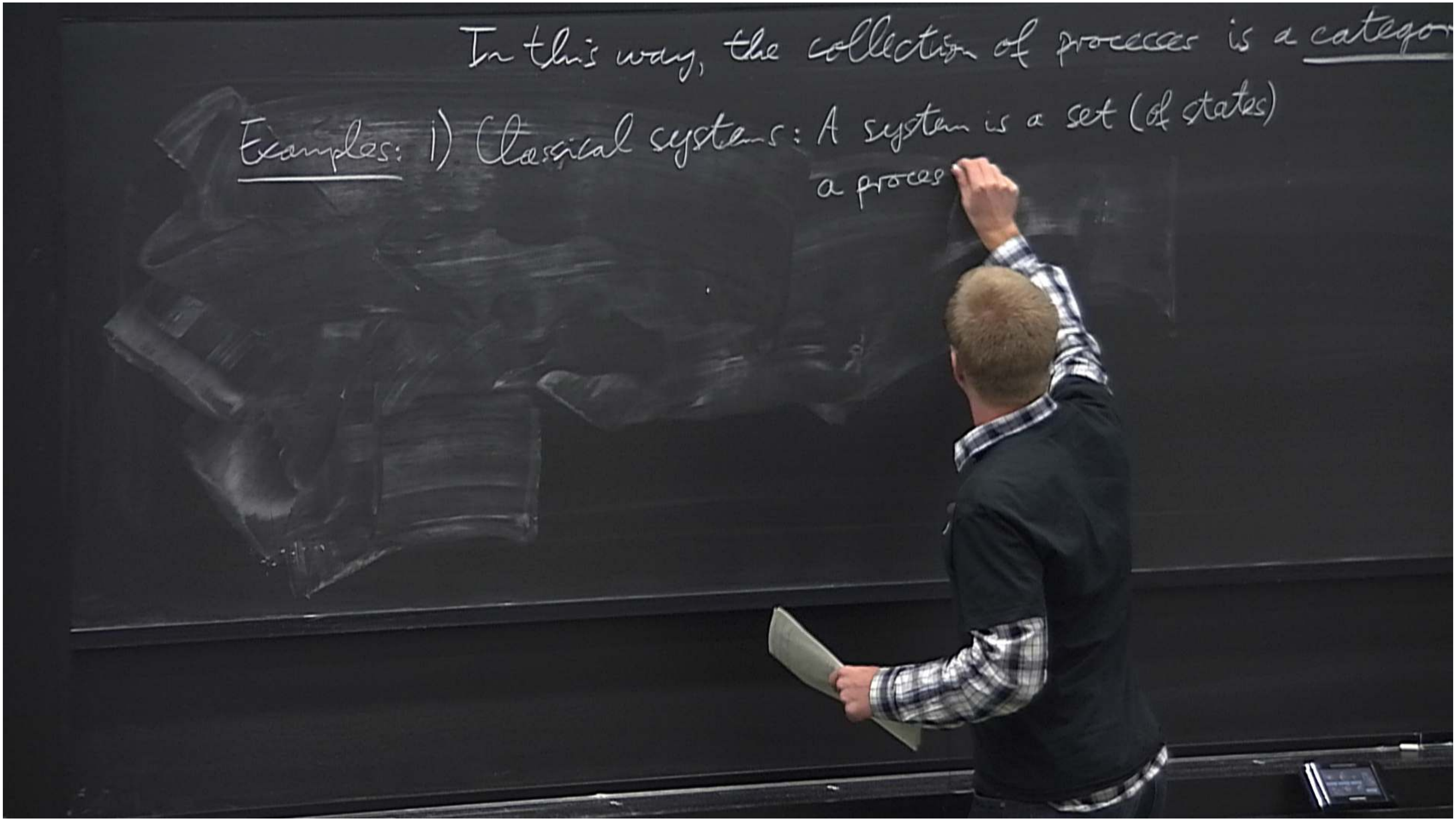

New Directions in Categorical Logic
In this talk: • illustrate usefulness of category theory
Idea

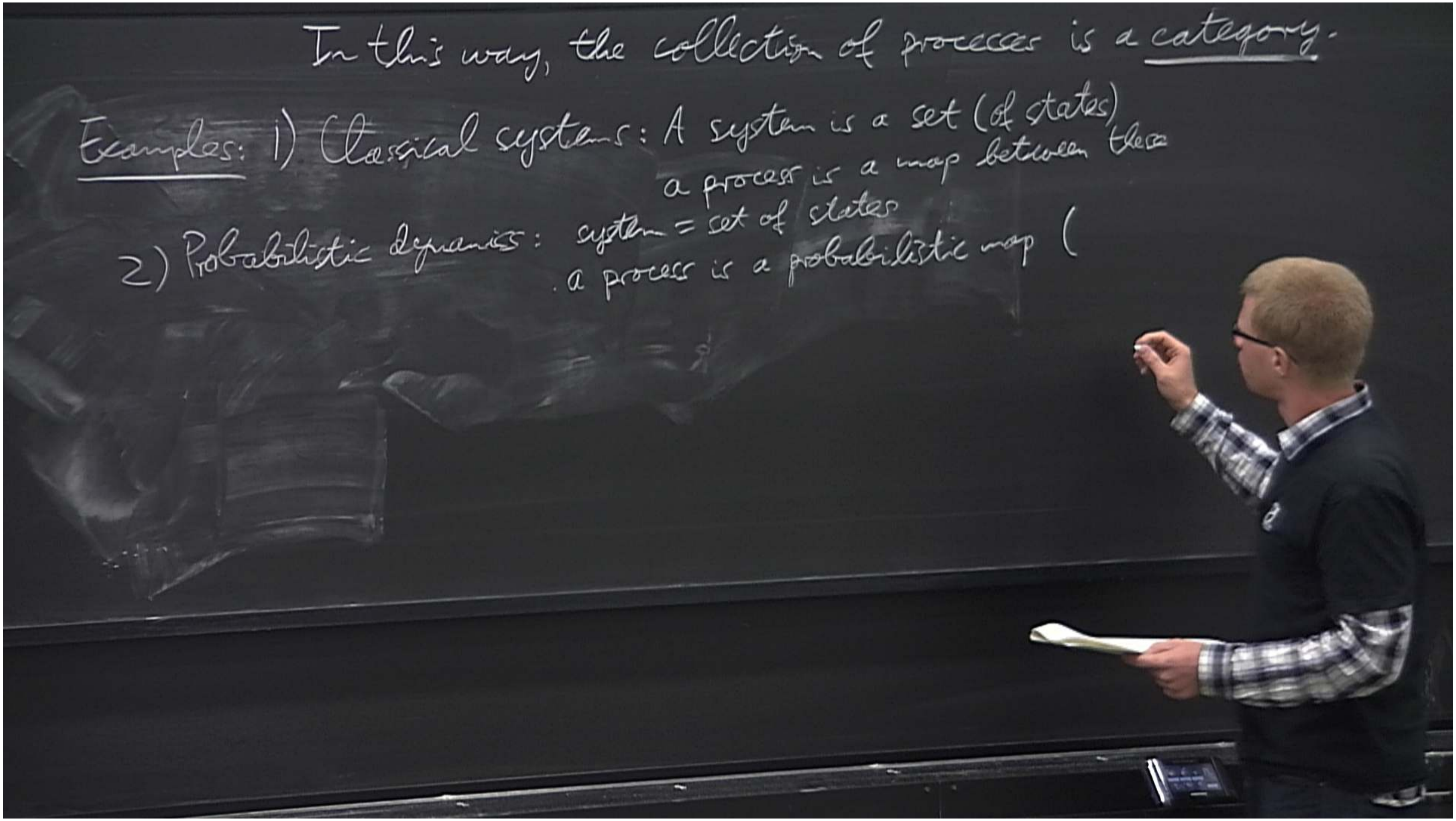

I. Processes
Physical process:
X → Y
system
(type)
system
(type)


I. Processes
Physical process:
X
Y
system
(type)
system
(type)


Structure: 1) Processes X ⟶ Y
Y ⟶ Z
can be composed to a process X ⟶ Z.
2) For every system type X, there is a trivial process
X ═══ X
3)


Structure: 1) Processes X ⟶ Y
Y ⟶ Z
can be composed to a process X ⟶ Z.
2) For every system type X, there is a trivial process
X ═══ X
3) Associativity + unit laws:
(W ⟶ X ⟶ Y) ⟶ Z = W ⟶ (X ⟶ Y ⟶ Z)

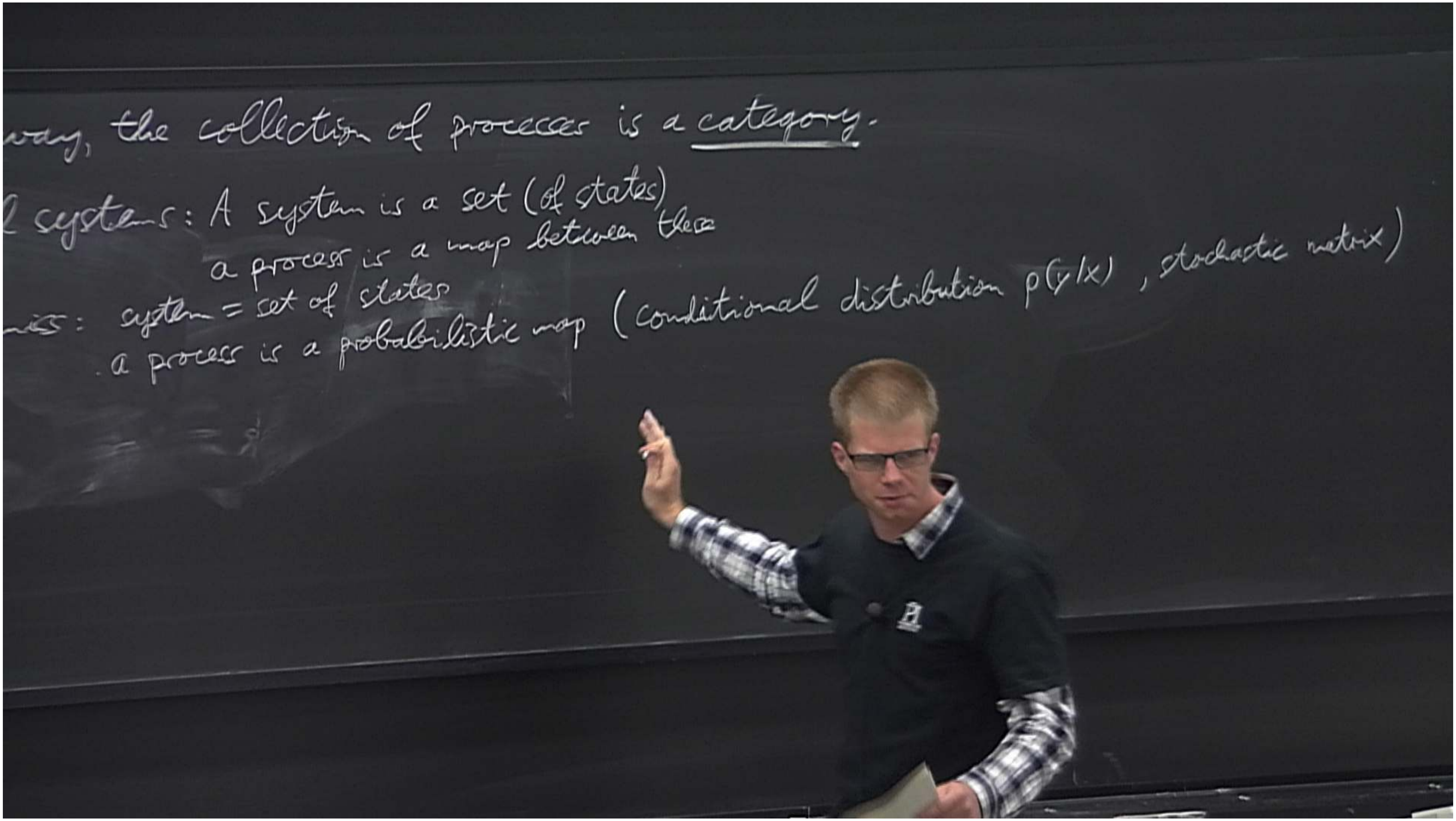

In this way, the collection of processes is a categor
Examples: 1) Classical systems: A system is a set (of states)
a process

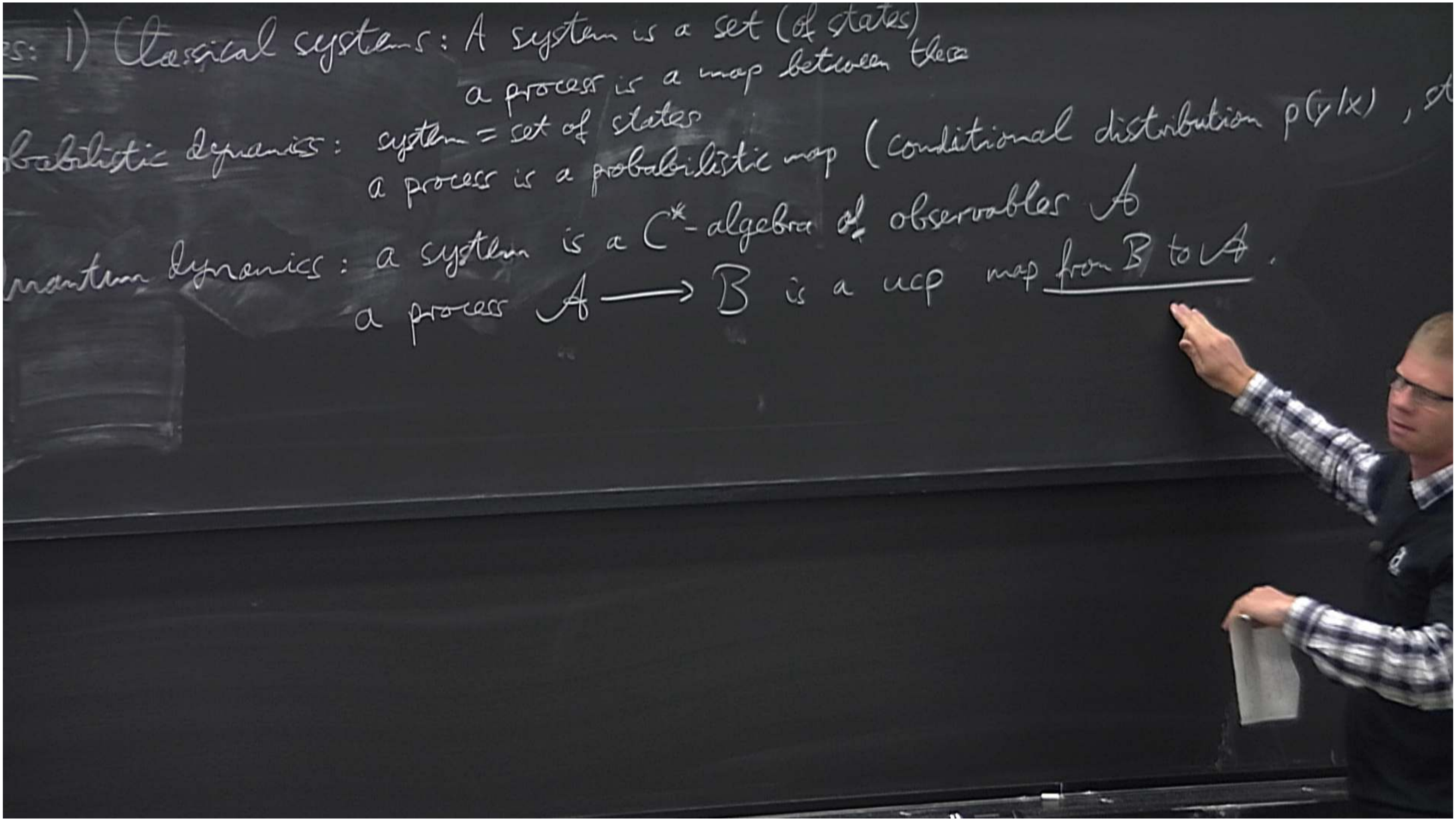

In this way, the collection of processes is a category.
Examples: 1) Classical systems: A system is a set (of states)
a process is a map between these
2) Probabilistic dynamics: system = set of states
a process is a probabilistic map (

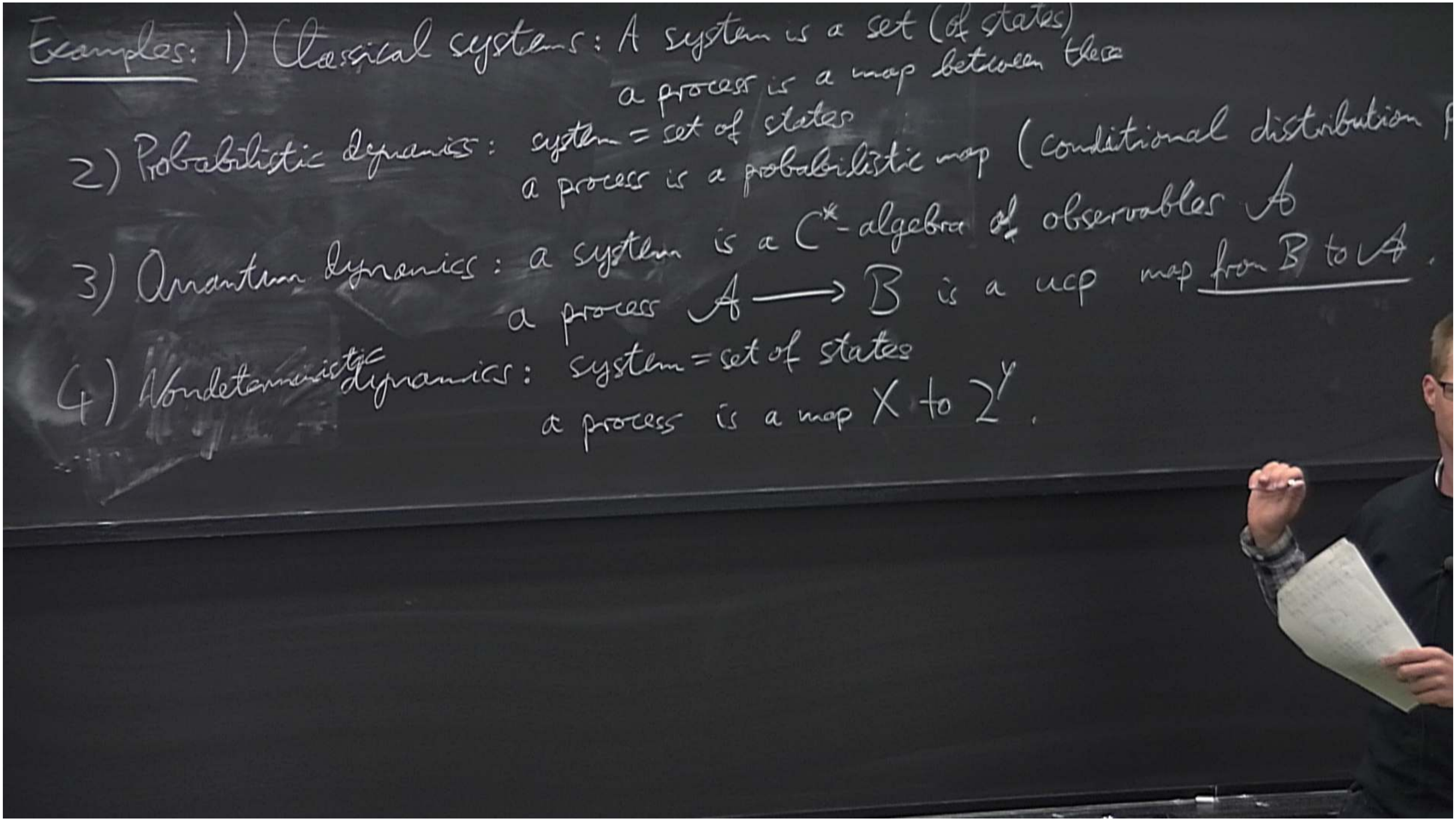
In this way, the collection of processes is a category.

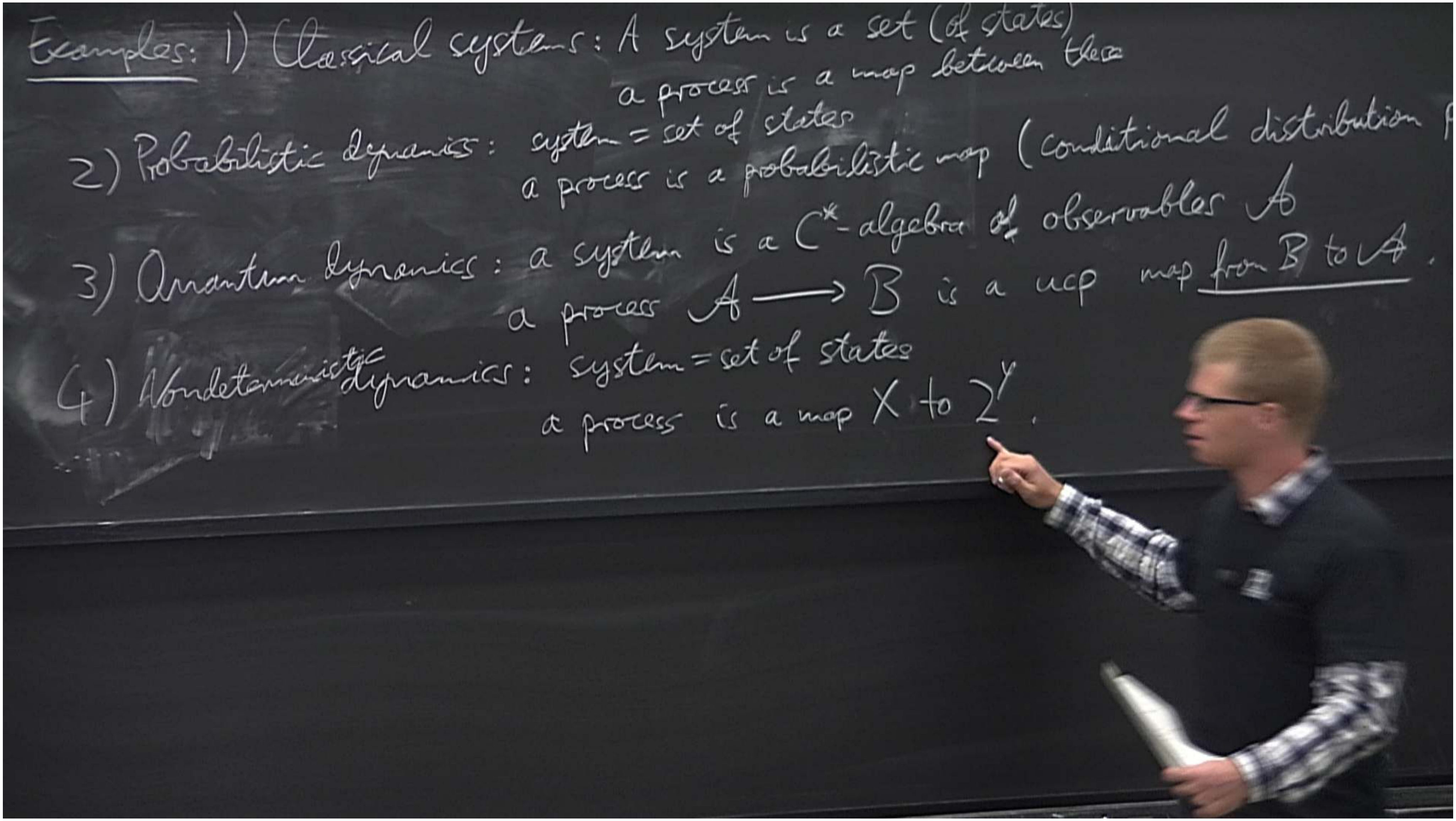
Classical systems: A system is a set (of states)
a process is a map between these

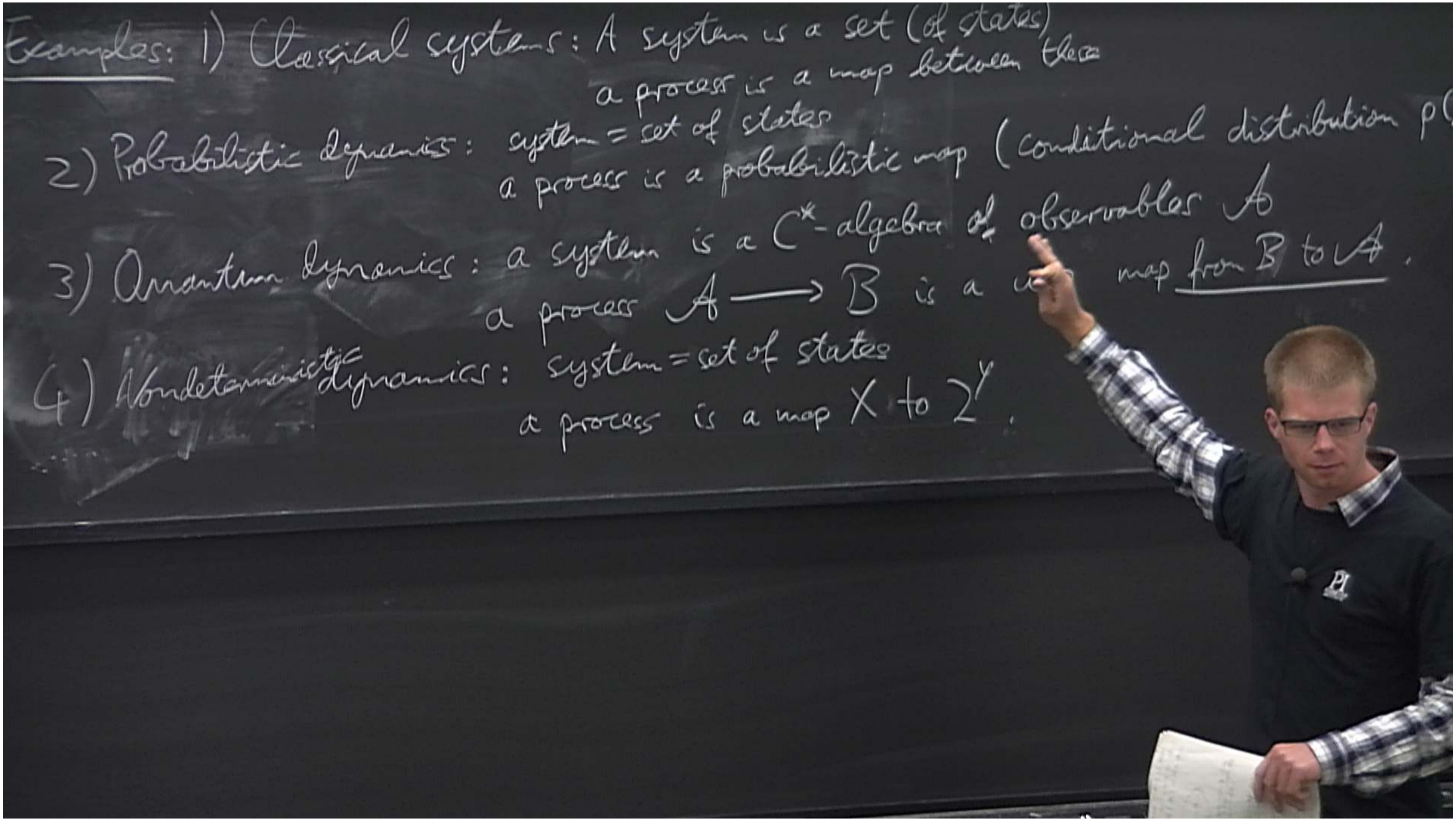
listic dynamics: system = set of states
a process is a probabilistic map (conditional distribution $p(y|x)$


way, the collection of processes is a category.
systems: A system is a set (of states)
a process is a map between these
system = set of states
a process is a probabilistic map (conditional distribution p(y/x), stochastic matrix)

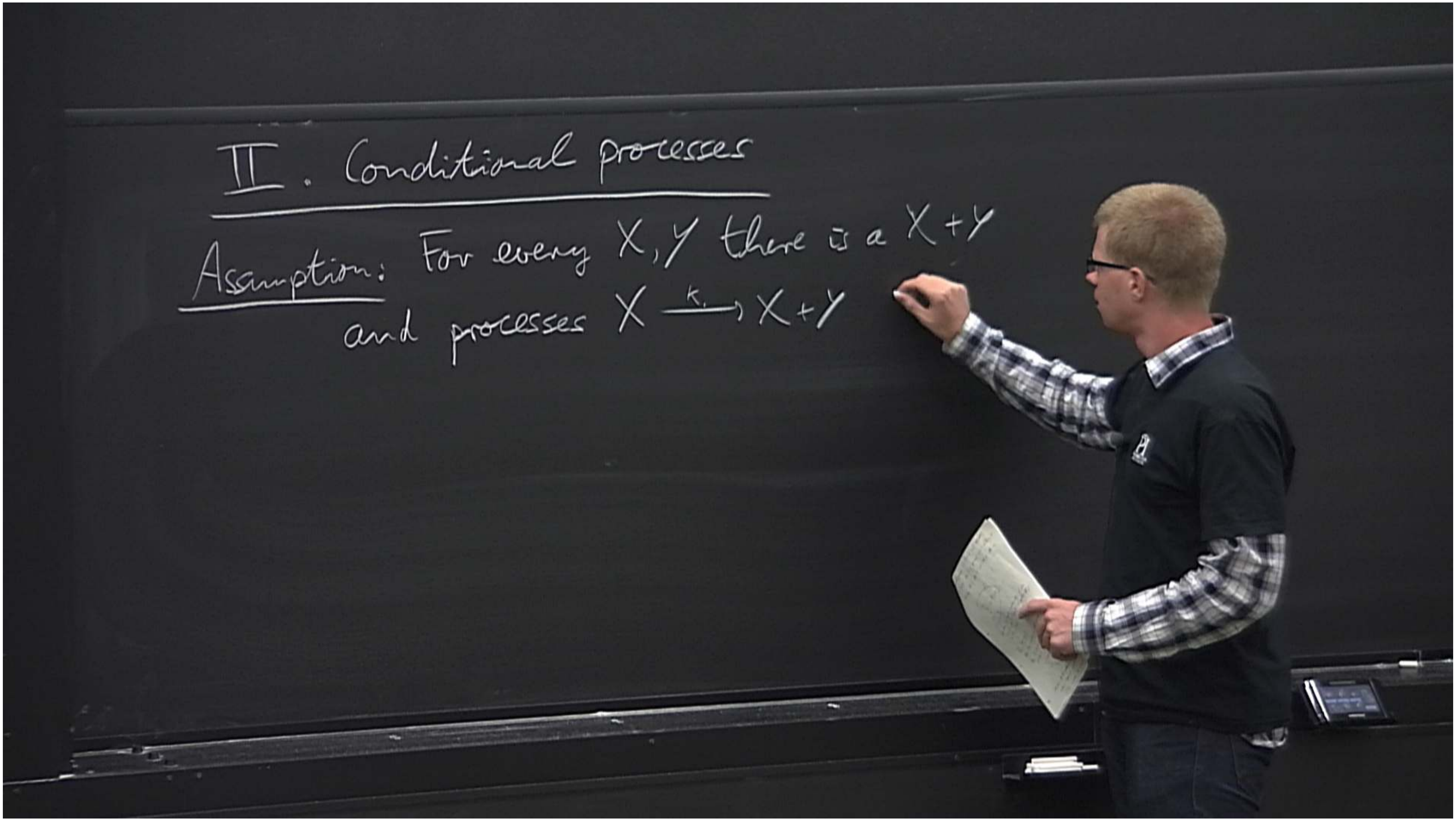

es: 1) Classical systems: A system is a set (of states)
a process is a map between these
obabilistic dynamics: system = set of states
a process is a probabilistic map (conditional distribution p(y|x), st
uantum dynamics: a system is a C*-algebra of observables A
a process A ⟶ B is a ucp map from B to A.

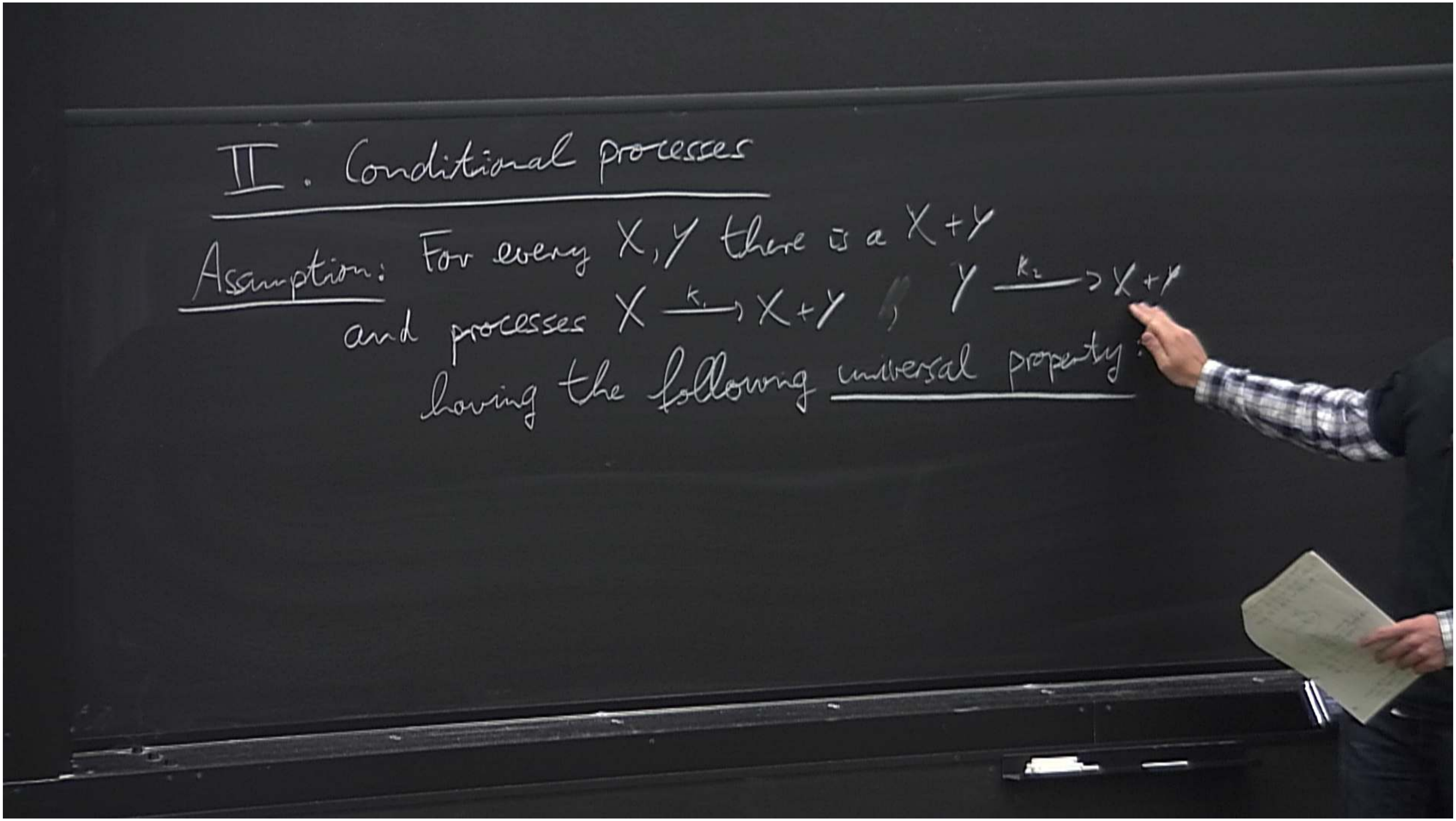

Examples: 1) Classical systems: A system is a set (of states)
a process is a map between these
2) Probabilistic dynamics: system = set of states
a process is a probabilistic map (conditional distribution
3) Quantum dynamics: a system is a C*-algebra of observables A
a process A ⟶ B is a ucp map from B to A.
4) Nondeterministic dynamics: system = set of states
a process is a map X to 2^Y.


Examples: 1) Classical systems: A system is a set (of states)
a process is a map between these
2) Probabilistic dynamics: system = set of states
a process is a probabilistic map (conditional distribution
3) Quantum dynamics: a system is a C*-algebra of observables A
a process A ⟶ B is a ucp map from B to A.
4) Nondeterministic dynamics: system = set of states
a process is a map X to 2^Y.



Examples: 1) Classical systems: A system is a set (of states)
a process is a map between these
2) Probabilistic dynamics: system = set of states
a process is a probabilistic map (conditional distribution p(
3) Quantum dynamics: a system is a C*-algebra of observables A
a process A ⟶ B is a map from B to A.
4) Nondeterministic dynamics: system = set of states
a process is a map X to 2^Y.


II. Conditional processes
Assumption: For every X, Y there is a X + Y
and processes X —k₁→ X + Y


II. Conditional processes
Assumption: For every X, Y there is a X+Y
and processes X $\xrightarrow{k_1}$ X+Y , Y $\xrightarrow{k_2}$ X+Y
having the following universal property:

Y

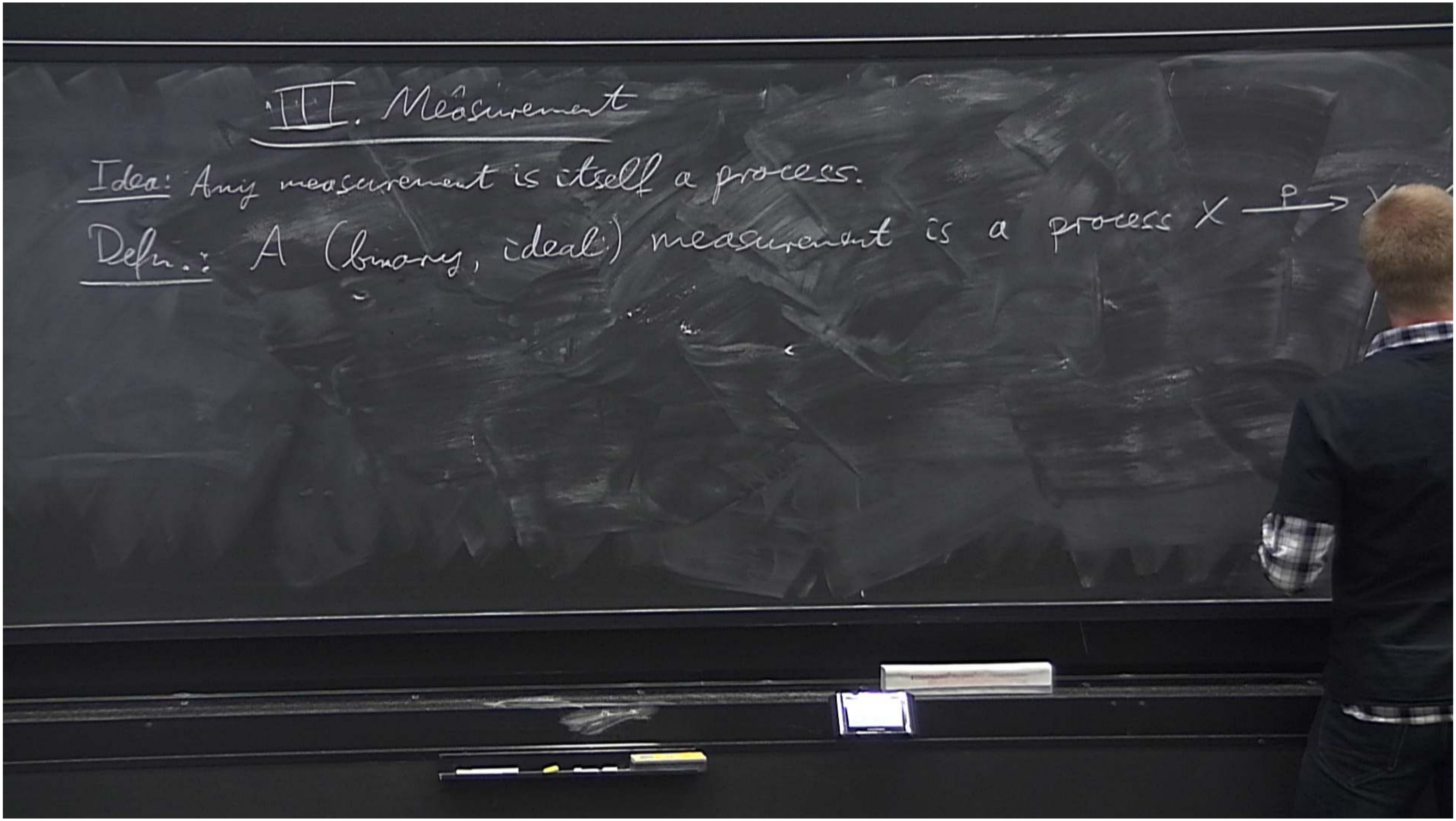
$Y \xrightarrow{\kappa_2} X+Y$

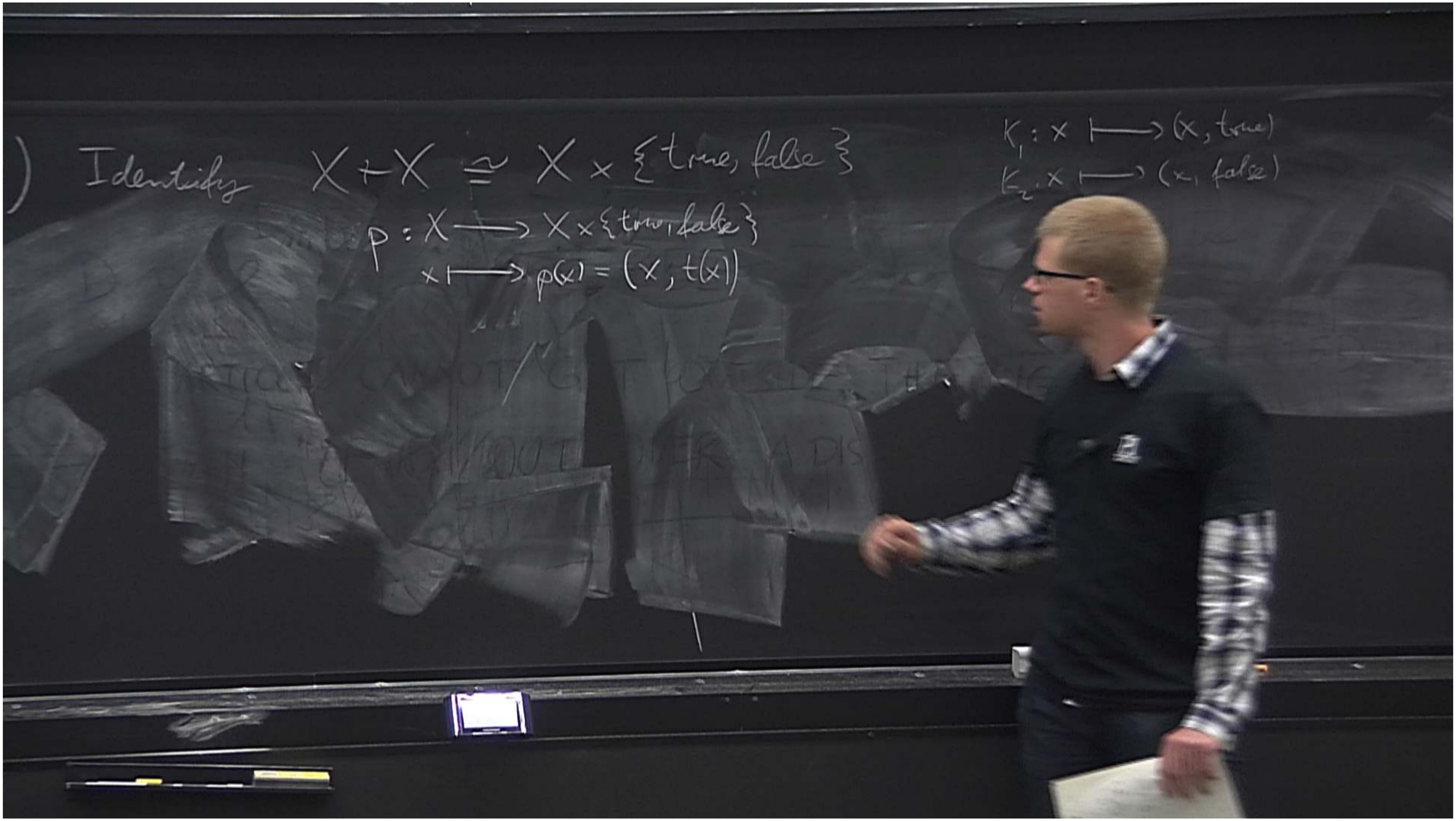
l property:

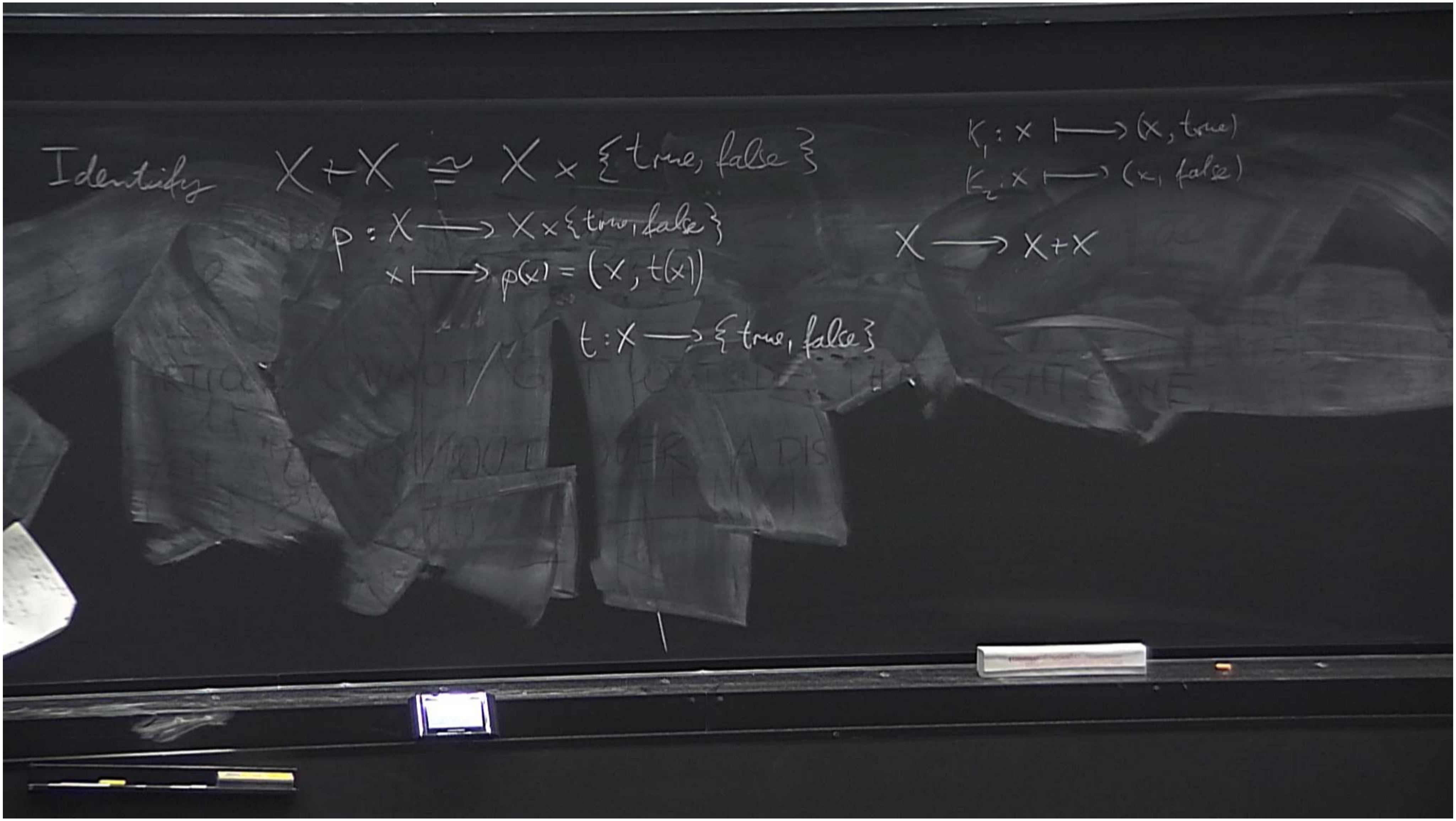
for all processes $X \xrightarrow{f} Z$ and $Y \xrightarrow{g} Z$

there is a unique process $X+Y \xrightarrow{f|g} Z$ behaving like f after κ_1 and like g after κ_2:

$$\begin{tikzcd}
X \arrow[dr, "\kappa_1"] \arrow[drr, bend left, "f"] & & \\
& X+Y \arrow[r, "f|g"] & Z \\
Y \arrow[ur, "\kappa_2"] \arrow[urr, bend right, "g"'] & &
\end{tikzcd}$$


III. Measurement
Idea: Any measurement is itself a process.
Defn.: A (binary, ideal) measurement is a process X


Identify X+X ≅ X × {true, false}
p : X ⟶ X × {true, false}
x ⟼ p(x) = (x, t(x))
K1 : x ⟼ (x, true)
K2 : x ⟼ (x, false)


Identify X+X ≅ X × {true, false}
p : X ⟶ X × {true, false}
x ⟼ p(x) = (x, t(x))
t : X ⟶ {true, false}
K1 : x ⟼ (x, true)
K2 : x ⟼ (x, false)
X ⟶ X+X