Title: Understanding black hole entropy through the renormalization group

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Abstract: It is known that the entanglement entropy of quantum fields on the black hole background contributes to the Bekenstein-Hawking entropy,and that its divergences can be absorbed into the renormalization of gravitational couplings. By introducing a Wilsonian cutoff scale and the concepts of the renormalization group, we can expand this observation into a broader framework for understanding black hole entropy. At a given RG scale, two contributions to the black hole entropy can be identified: the "gravitational" contribution coming from the running effective gravitational action, and the entanglement entropy of the quantum degrees of freedom below the cutoff scale. At different RG scales the balance is different, though the total black hole entropy is invariant. I will describe this picture for free fields, considering both minimal and non-mininal coupling, and discuss the extension to interacting fields and the difficulties it raises.

Understanding black hole entropy through the renormalization group

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Based on work in collaboration with Ted Jacobson

Perimeter Institute -- October 18th 2012

Outline

1. Introduction: Entanglement entropy and black hole entropy
2. Renormalization group and black hole entropy: free fields
3. Interacting fields
4. Conclusions

Approaches to black hole entropy

1973: Bekenstein postulates that black holes have an entropy proportional to the event horizon area

1974: Hawking verifies that black holes exhibit thermodynamical properties when quantum field theory in curved space is taken into account.

→ Bekenstein-Hawking entropy: $S_{BH} = \frac{A}{4G}$ Temperature: $\beta^{-1} = \frac{\kappa}{2\pi}$

Attempts to understand it: →
- within a fundamental theory of quantum gravity: string theory, LQG.
- within an effective description.

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Entanglement entropy

Quantum (pure) state $|\psi\rangle$ on a region $A \cup B$, can be written in a basis of products of states for each region. The reduced density matrix ρ_A formed by tracing over the B states has nonzero entropy:

$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A$$

It can be proven that $S_A = S_B$.

The entanglement entropy has ultraviolet divergences. With a UV short-distance cutoff ε, the leading order scales as the area of the *d-2* surface between A and B:

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On a black hole background, the restriction of the global Hartle-Hawking vacuum to the exterior is a thermal state at the Hawking temperature

$\longrightarrow$ $\rho_A = \mathrm{e}^{-\beta H_A}$ $\longrightarrow$ entanglement entropy equals thermal entropy of the fields in the exterior

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Canonical partition function

Consider a partition function including both matter and metric fields. The matter fields integrate out to give the effective action for gravity, and gravity is treated "classically" with a saddle point evaluation:

$$Z(\beta) = \int \mathcal{D}g \int \mathcal{D}\varphi \; \mathrm{e}^{-(S_b[g]+S[g,\varphi])} = \int \mathcal{D}g \; \mathrm{e}^{-\Gamma_0[g]} \approx \mathrm{e}^{-\Gamma_0[\bar{g}]}$$

$\bar{g} = \bar{g}(\beta)$ solution of the effective EOMs.

$\Gamma_0 = S_b + W$

Then by thermodynamics: $$S_{BH} = -\left(\beta\frac{d}{d\beta} - 1\right) \ln Z(\beta) = \left(\beta\frac{d}{d\beta} - 1\right) \Gamma_0[\bar{g}(\beta)]$$

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$$\frac{d\Gamma_0[\bar{g}]}{d\beta} = \frac{\partial\Gamma_0[g]}{\partial\beta}\Big|_{g=\bar{g}} + \left(\frac{\partial g}{\partial\beta}\frac{\partial\Gamma_0[g]}{\partial g}\right)\Big|_{g=\bar{g}}$$

and the second term vanishes at the on-shell metric.

If Γ_0 can be approximated as the Einstein-Hilbert action with $\Lambda=0$ and Gibbons-Hawking boundary term, then $\bar{g}$ is Euclidean Schwarzschild, and $S_{BH}=A/4G_0$ comes only from the boundary term.

So **formally** the total black hole entropy equals the $S_b + S_{ent}$, and the divergences in the latter (matching those in the effective action) are renormalized with G.

Can we make the renormalization property explicit in a calculation without dealing with divergences?

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2. RG and BHE: free fields

RG conceptualization of BH entropy

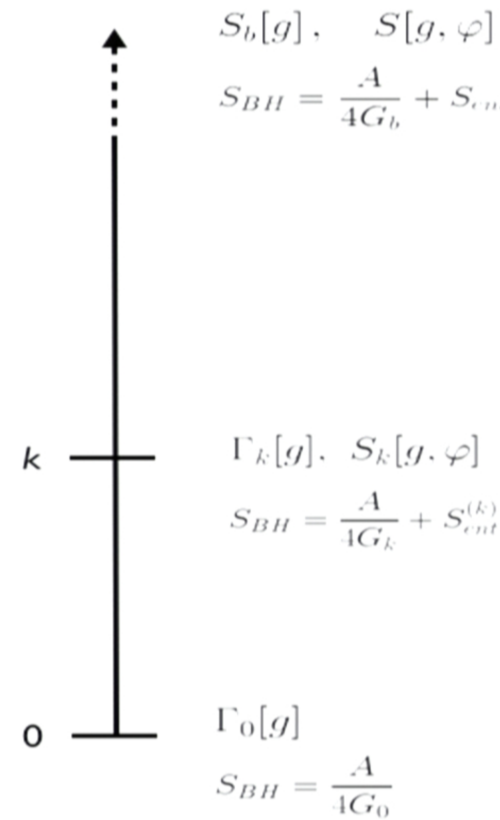
Instead of the integrating out all the matter degrees of freedom at once, integrate out only the modes above a Wilsonian scale k.

The contributions from $p > k$ add up with $S_b[g]$ into a flowing effective gravitational action $\Gamma_k[g]$, which reduces to Γ_0 as $k \to 0$.

$\Gamma_k[g]$ can be computed by integrating a flow equation "upwards" from Γ_0. No explicit handling of UV divergences

The modes with $p < k$ are still "quantum" and contribute entanglement/thermal entropy.

As one moves the RG scale k, the total entropy remains constant while the balancing between the different contributions changes.



2. RG and BHE: free fields

Minimally coupled scalar

$$e^{-\Gamma_0[g]} = e^{-S_b[g]} \int \mathcal{D}\varphi \, e^{-\frac{1}{2}\int \sqrt{g}\, \varphi(-\nabla_g^2)\varphi}$$

$$e^{-\Gamma_k[g]} = e^{-S_b[g]} \int \mathcal{D}\varphi_> \, e^{-\frac{1}{2}\int \sqrt{g}\, \varphi_>\left(-\nabla_g^2 + \mathcal{R}_k(-\nabla_g^2)\right)\varphi_>}$$

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Γ_k satisfies the Wetterich flow equation: $$\partial_t\Gamma_k[g] = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_t\mathcal{R}_k(-\nabla_g^2)}{-\nabla_g^2+\mathcal{R}_k(-\nabla_g^2)}\right]$$

In this case it is integrated trivially: $$\Gamma_k[g]-\Gamma_0[g] = \frac{1}{2}\mathrm{Tr}\ln\left[\frac{-\nabla^2}{-\nabla^2+\mathcal{R}_k(-\nabla^2)}\right]$$

The trace has an inbuilt UV cutoff at scale k. It can be computed with a heat kernel expansion.
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The entropy is computed evaluating at the Euclidean black hole metric and applying (β d$_\beta$ - 1):

$$S_{BH} = \frac{A}{4G_0} = \frac{A}{4G_k} + \frac{A\,k^2}{48\pi}$$

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Non-minimal coupling

$$S = S_b[g] + \frac{1}{2}\int \sqrt{g}\,\varphi(-\nabla^2 + \xi\, R)\varphi$$

The framework is applied in the same way:

$$\Gamma_k[g] - \Gamma_0[g] = \frac{1}{2}\mathrm{Tr}\ln\left[\frac{\Delta_g}{\Delta_g + \mathcal{R}_k(\Delta_g)}\right], \qquad \Delta_g = -\nabla_g^2 + \xi\, R$$

Assuming Dirichlet boundary conditions, the result is different for the running of G in the bulk term and the boundary term. (See also Becker and Reuter 2012).

The running of boundary G is not affected by the ξ term, so the entropy comes out equal to the one for minimal coupling.

We can use an alternative (Robin) boundary condition: $\left.(\nabla_n\varphi - \xi K\,\varphi)\right|_\partial = 0$

Then boundary G and bulk G run in the same way:

$$S_{BH} = \frac{A}{4G_0} = \frac{A}{4G_k} + \frac{A}{4}\cdot\frac{k^2}{2\pi}\left(\frac{1}{6} - \xi\right)$$

Interacting fields

For interacting fields, we need to keep track of how the matter action changes with RG scale.
We define the Wilsonian effective action S_k by:

$$\mathrm{e}^{-S_k[g,\phi]} = \int \mathcal{D}\varphi\, \mathrm{e}^{-\frac{1}{2}\int \varphi\left(-\nabla_g^2+\mathcal{R}_k(-\nabla_g^2)\right)\varphi - S_b[g,\phi+\varphi]}$$

S_b = gravitational bare action, **plus** non-kinetic terms of matter bare action.

$$S_k[g,\phi=0] = \Gamma_k[g]$$

Gravitational effective action.

$$S_k[g,\phi] - \Gamma_k[g] \;=\; \tilde{S}_k[g,\phi]$$

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Their flow is found solving a curved-space version of the Polchinski flow equation:

$$\dot{S}_k[g,\phi] = \frac{1}{2}\left\{ \frac{\delta S_k}{\delta\phi} \cdot \frac{\dot{\mathcal{R}}_k}{(-\nabla^2 + \mathcal{R}_k)^2} \cdot \frac{\delta S_k}{\delta\phi} - \mathrm{Tr}\left[\frac{\dot{\mathcal{R}}_k}{(-\nabla^2 + \mathcal{R}_k)^2} \cdot \frac{\delta^2 S_k}{\delta\phi\delta\phi}\right] + \mathrm{Tr}\left[\frac{\dot{\mathcal{R}}_k}{-\nabla^2 + \mathcal{R}_k}\right]\right\}$$

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Then the black hole entropy is partitioned as:

$$S_{BH} = (\beta\partial_\beta - 1)\Gamma_k[\bar{g}] - (\beta\partial_\beta - 1)\ln\left[N_k[\bar{g}]\int \mathcal{D}\phi\, e^{-\frac{1}{2}\int \phi P_k^{-1}\phi - \tilde{S}_k[\bar{g},\phi]}\right]$$

Momentum space entanglement?

Balasubramanian, McDermott and Van Raamsdonk (2011) point out that in an interacting quantum field theory, there is mutual entanglement between modes above and below a certain scale.

A simple order of magnitude estimate shows that this entanglement entropy scales as the volume of spacetime and hence dominates over the spatial horizon entanglement entropy.

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Answer:

It is misleading to describe the entropy computed from a partition function over the lower modes using S_k as "the entropy of the lower modes".

The partition function with a Wilsonian action is equivalent to the full theory, it does not involve tracing over the upper modes and losing information. We are just identifying an "effective gravitational" and an "effective quantum matter" term in this description.

4. Conclusions

Summary

- Black hole entropy, in a semiclassical approximation, comes from the full effective action for gravity evaluated on shell. Formally, $S = S_b + S_{ent}$, but this needs regularizing divergences.

- With a sliding RG scale k, the entropy comes partly from a flowing gravitational effective action, partly from the "lower" path integral using the Wilsonian effective action for the quantum theory.

- For free fields, there is a clean interpretation for the latter contribution as the entropy of the modes below k (just entanglement entropy, or also Wald entropy for nonminimally coupled fields).
  - (However, it is problematic to trace back this into the Lorentzian theory!)

- For interacting fields, we can still track both contributions with the Polchinski equation, but we cannot in general make this interpretation due to the momentum entanglement of the "upper" and the "lower" subsystems.