Title: Synthetic non-Abelian anyons in fractional Chern insulators and beyond

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Abstract: An exciting new prospect in condensed matter physics is the possibility of realizing fractional quantum Hall states in simple lattice models without a large external magnetic field, which are called fractional Chern insulators. A fundamental question is whether qualitatively new states can be realized on the lattice as compared with ordinary fractional quantum Hall states. Here we propose new symmetry-enriched topological states, topological nematic states, which are a dramatic consequence of the interplay between the lattice translational symmetry and topological properties of these fractional Chern insulators. The topological nematic states are realized in a partially filled flat band with a Chern number N, which can be mapped to an N-layer quantum Hall system on a regular lattice. However, in the topological nematic states the lattice dislocations become non-Abelian defects which create "worm holes" connecting the effective layers, and effectively change the topology of the space. Such topology-changing defects, which we name as "genons", can also be defined in other physical systems. We develop methods to compute the projective non-abelian braiding statistics of the genons, and we find the braiding is given by adiabatic modular transformations, or Dehn twists, of the topological state on the effective genus g surface. We find situations where the

> genons have quantum dimension 2 and can be used for universal topological quantum computing (TQC), while the host topological state is by itself non-universal for TQC.

Synthetic non-Abelian anyons in fractional Chern insulators and beyond

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Stanford University

Perimeter Institute, Oct. 26th, 2012

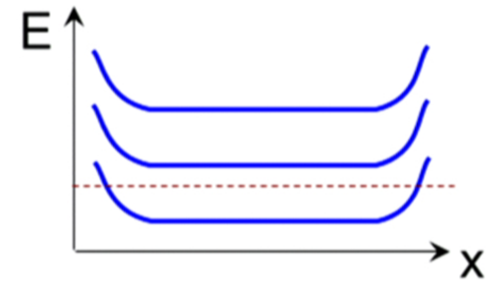
Maissam Barkeshli & XLQ, PRX, **2**, 031013 (2012)
Maissam Barkeshli, Chaoming Jian, XLQ, arxiv:1208.4834 (2012)

Outline

- A new class of topological states of matter: Fractional Chern insulators (FCI)
- 1D Wannier state description of FQAH states
- FCI with higher Chern number and the topological nematic states
- Lattice dislocations in topological nematic states as non-Abelian “genons” (generators of genus).
- Non-Abelian statistics of the genons.
- A topological field theory description of topological nematic states
- Relation to other twist defects and topological quantum computation

Integer and fractional quantum Hall states

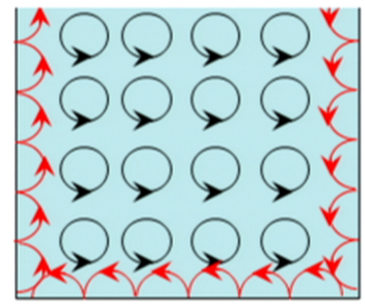
- Quantum Hall effect occurs in 2d electron system with strong perpendicular magnetic field



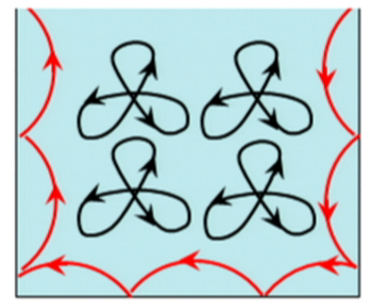
- Integer quantum Hall (IQH) state: Filling integer number of Landau levels. Momentum space Chern number (Thouless et al 1982)

$$n = \frac{1}{2\pi}\int d^2k\left(\partial_x a_y - \partial_y a_x\right)$$

$$a_i(\boldsymbol{k}) = -i\langle \boldsymbol{k}|\partial_i|\boldsymbol{k}\rangle$$



- Fractional quantum Hall state (FQH): Partially filled Landau level. Strongly correlated state with fractionalized excitations



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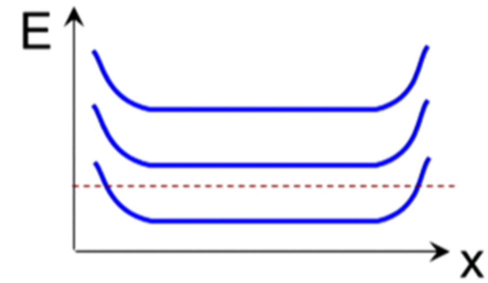

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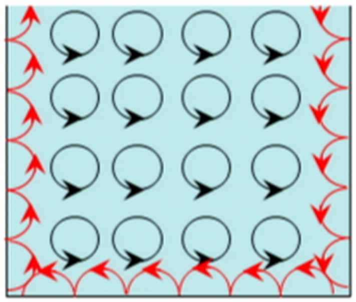

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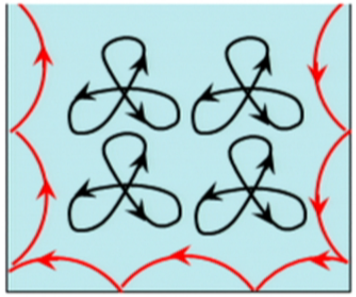
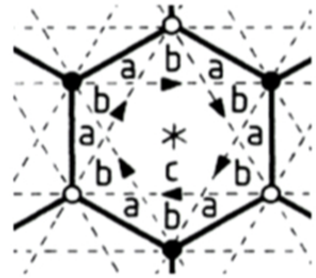
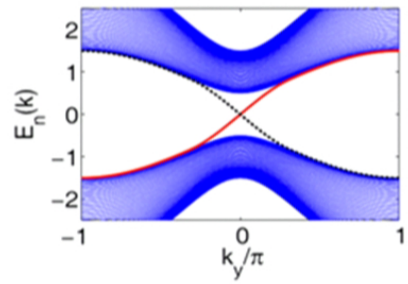
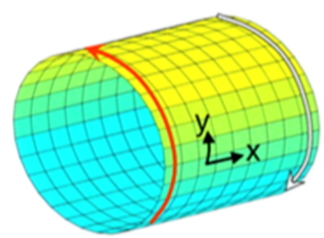


Chern insulators

- IQH effect can be realized in a lattice model without orbital magnetic field (Haldane 1988)
- A quantized anomalous Hall effect (QAH)
- General lattice Hamiltonian with translation symmetry $H = \sum_k c_k^+ h(k) c_k$
- There are n bands $|n, \boldsymbol{k}\rangle$
  Chern number is defined for each band
- Example: two-band models $H = \sum_a d_a(\boldsymbol{k})\sigma^a$ (Haldane 1988, Qi Wu Zhang 2005)
- Mateiral proposals: Hg(Mn)Te/CdTe (Liu et al PRL '08), Cr or Fe doped Bi2Se3 film (Yu et al '10)

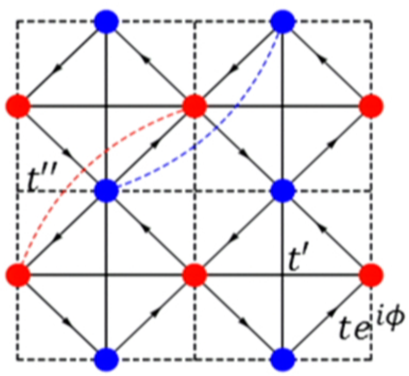


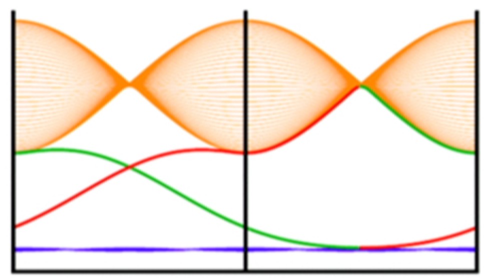




Fractional quantum anomalous Hall effect (FQAH) and Fractional Chern insulators (FCI)

- Can FQH state also be realized in a lattice system?
- Evidences have been observed (Sun *et al*, Neupert, *et al*, Tang *et al*, PRL 2011) (Sheng *et al* Nat. Comm. 2011, Regnault&Bernevig PRX 2011, Wu, Bernevig&Regnault PRB 2012)
- Supported by analytic results, e.g. Parameswaran, Roy, Sondhi, PRB 2012)
- Example: Checkerboard model





Nearly flat band for $t = 1, t' = \frac{1}{2+\sqrt{2}}, t'' = \frac{1}{2+2\sqrt{2}}, \phi = \frac{\pi}{4}$
(Sun *et al 2011*)

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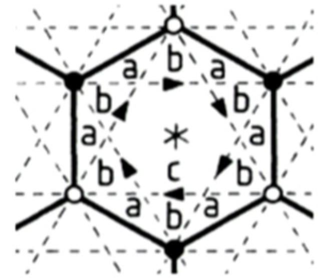

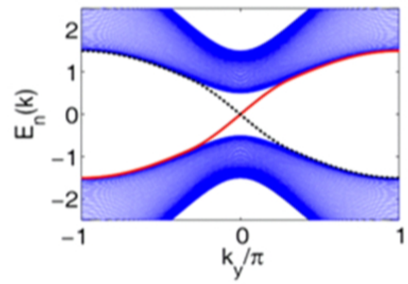

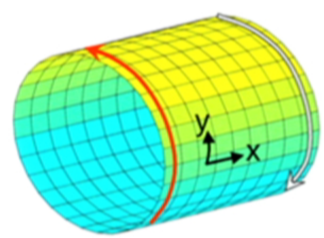

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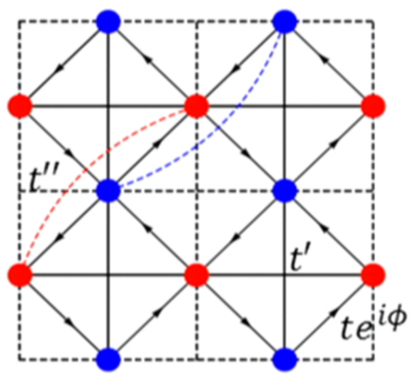

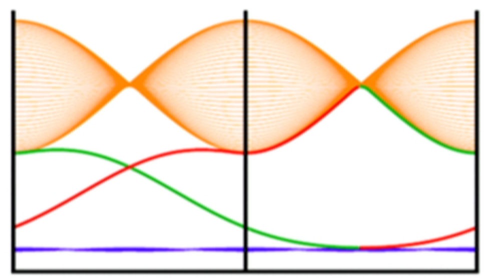

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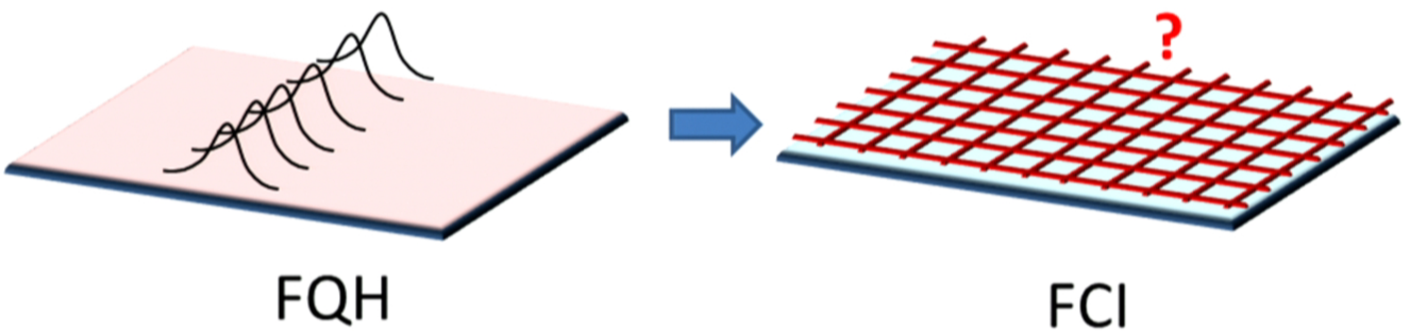
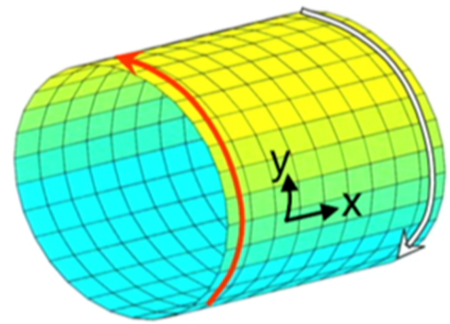
Wave-function description of FCI

- FQH states can be described by many-body wavefunctions such as the Laughlin wavefunction (Laughlin 1983)
- $\Psi_{\frac{1}{m}}(\{z_i\}) = \prod_{i<j}\left(z_i - z_j\right)^m \exp(-\sum_i |z_i|^2/2l_B^2)$
- What are the many-body wavefunctions describing FCI states?
- What FQH states can be realized on the lattice?
- Idea: Finding the single-particle basis corresponding to the Landau level wavefunctions in the ordinary QH states.



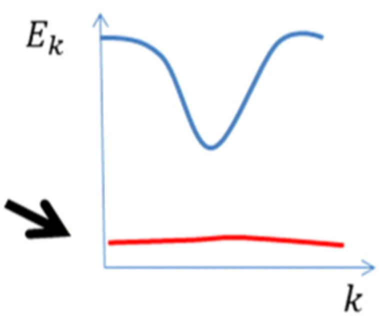
Wave-function description of FCI: 1D Wannier functions

- The proper basis can be found by using 1D Wannier functions
- Consider FCI on a cylinder
- The states for each fixed k_y forms a 1D chain.
- 1D Wannier functions:
  a local basis for the 1D system.
  Fourier transform of Bloch states
- $|W_{nk_y}\rangle = \frac{1}{\sqrt{L_x}} \sum_{k_x} e^{ik_x n} e^{i\varphi(\boldsymbol{k})} |k_x, k_y\rangle$



k_y eigenstates

$h_{nm}(k_y)$



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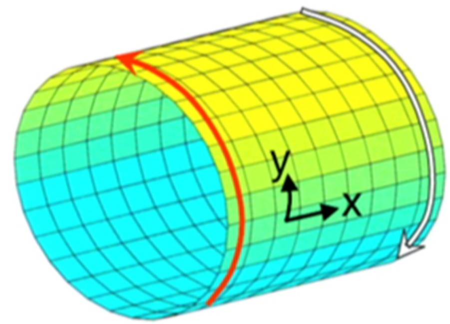

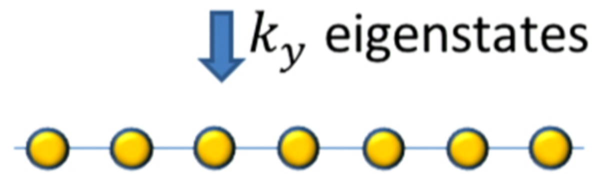

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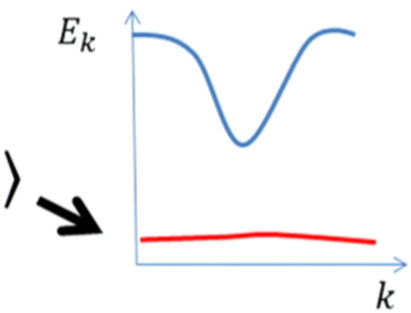

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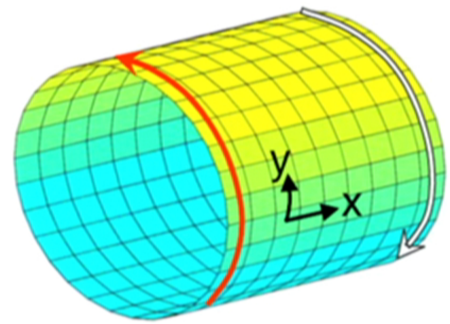

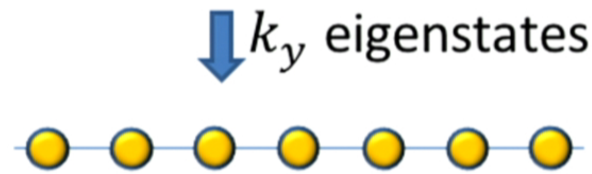

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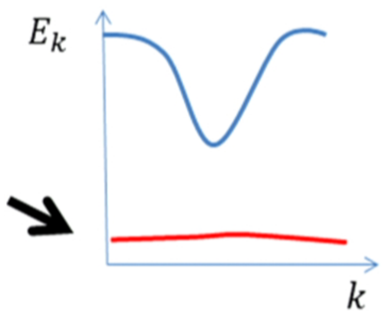
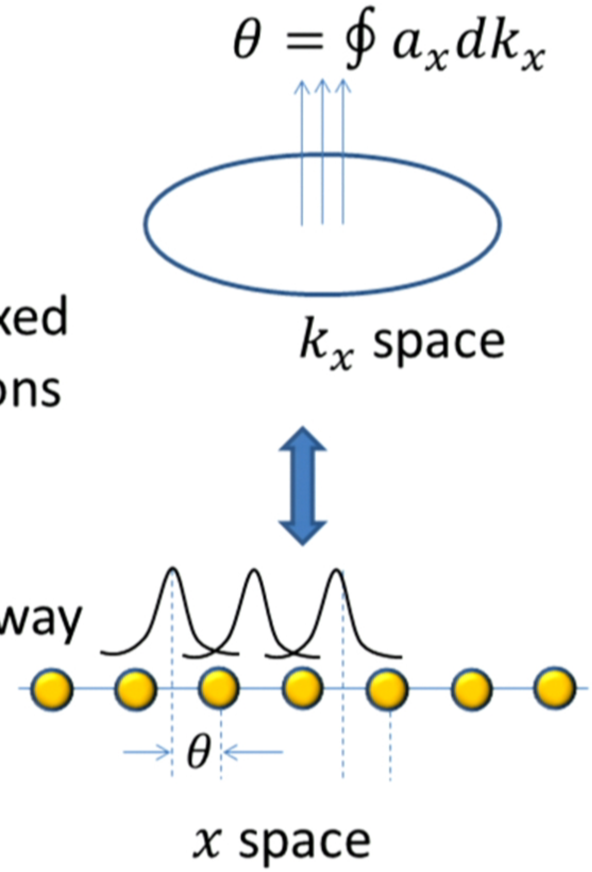


1D Wannier functions

- $|W_{nk_y}\rangle = \frac{1}{\sqrt{L_x}}\sum_{k_x} e^{ik_x n} e^{i\varphi(\boldsymbol{k})}|k_x, k_y\rangle$
- The ambiguity of $\varphi(\boldsymbol{k})$ can be fixed by requiring the Wannier functions to be maximally localized
- The center-of-mass position of Wannier function is in general away from the lattice site position:
- $\langle W_{nk_y}|x|W_{nk_y}\rangle = n - \frac{\theta(k_y)}{2\pi}$
- The shift $P = -\theta/2\pi$ is the charge polarization



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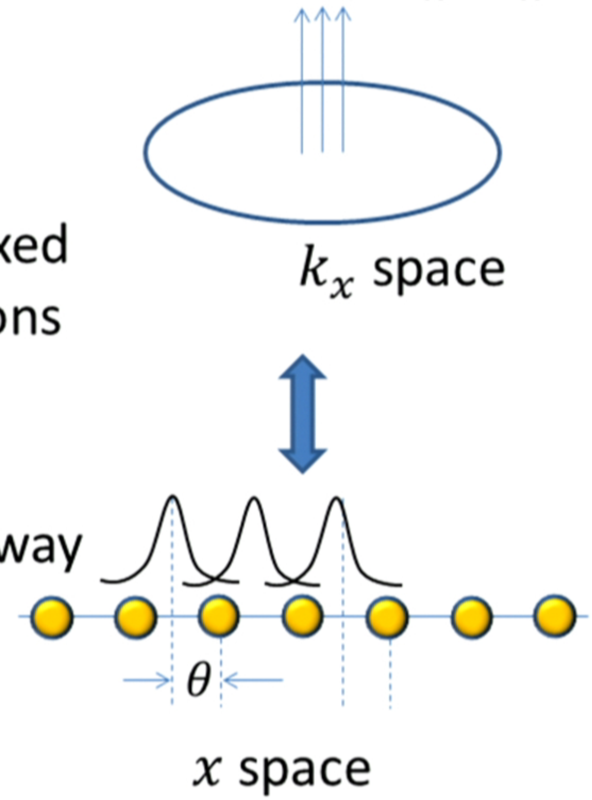
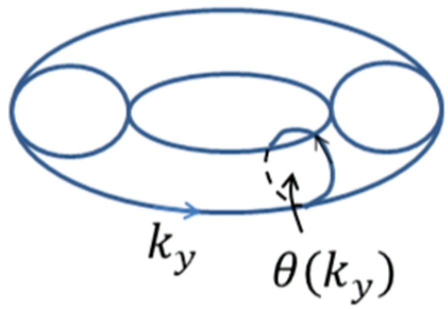
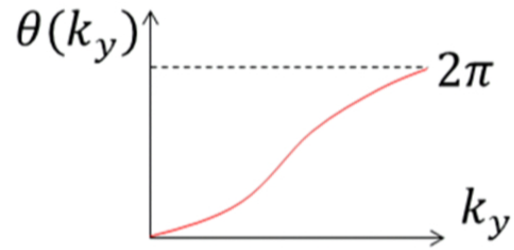


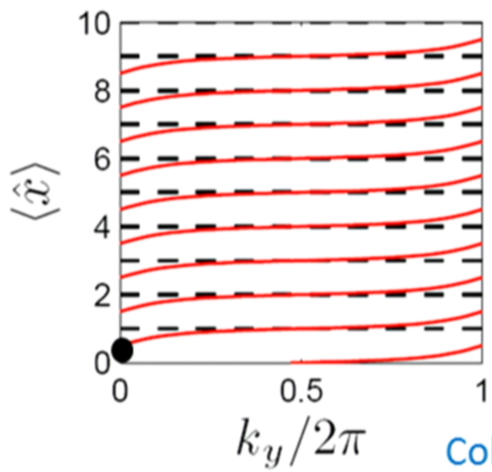
1D Wannier functions and the Chern number

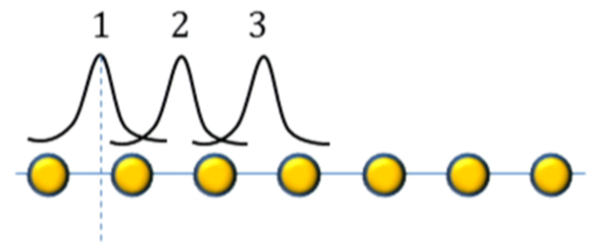
- Chern number on the Brillouin zone torus is the winding number of the flux $\theta(k_y)$



$$\frac{\sigma_H}{h/e^2} = C_1 = -\frac{1}{2\pi}\int_0^{2\pi} \partial_{k_y}\theta(k_y)\, dk_y$$







Coh & Vanderbilt, 2009 PRL

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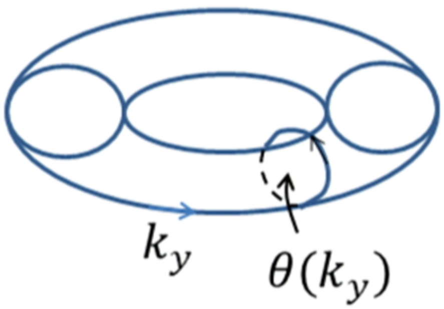

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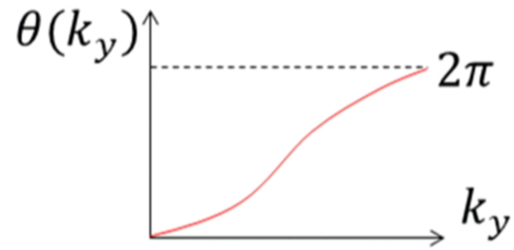

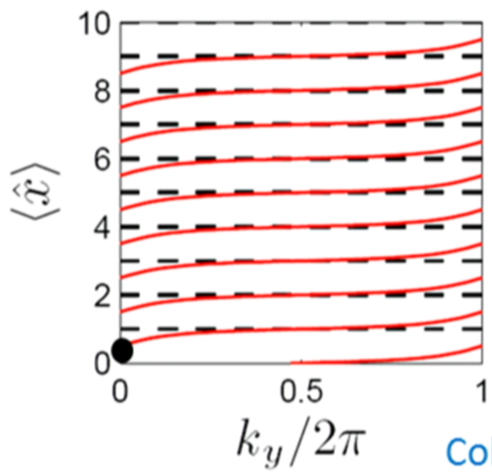

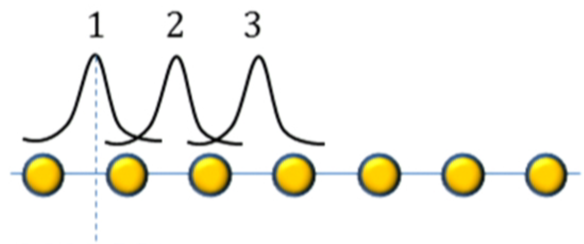

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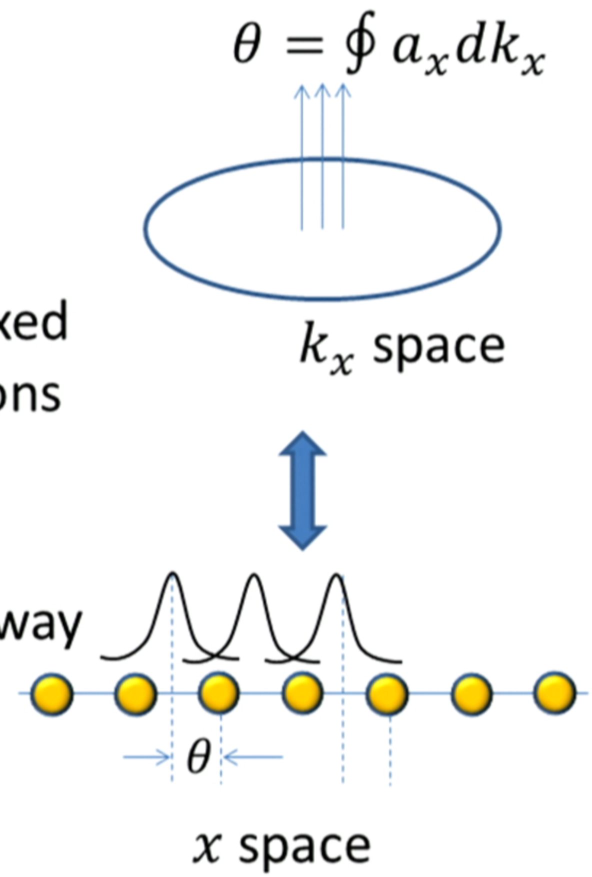

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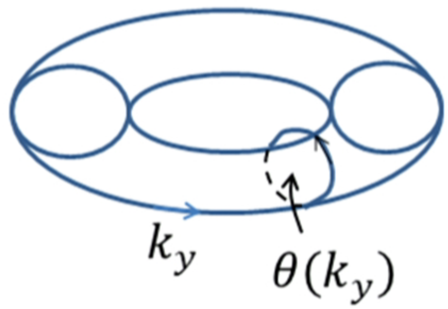

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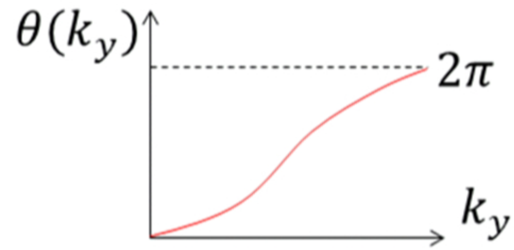
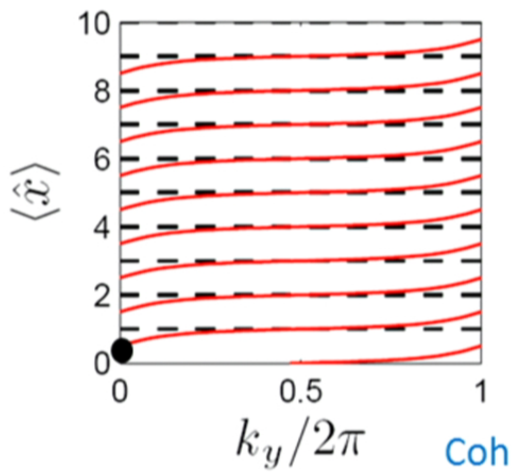
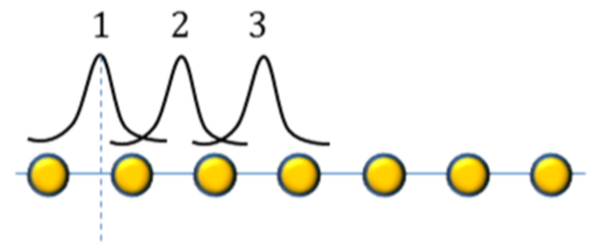




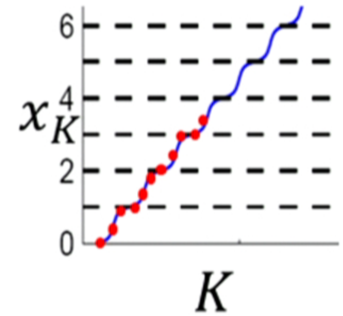


Coh & Vanderbilt, 2009 PRL

Using 1D Wannier functions to describe FCI

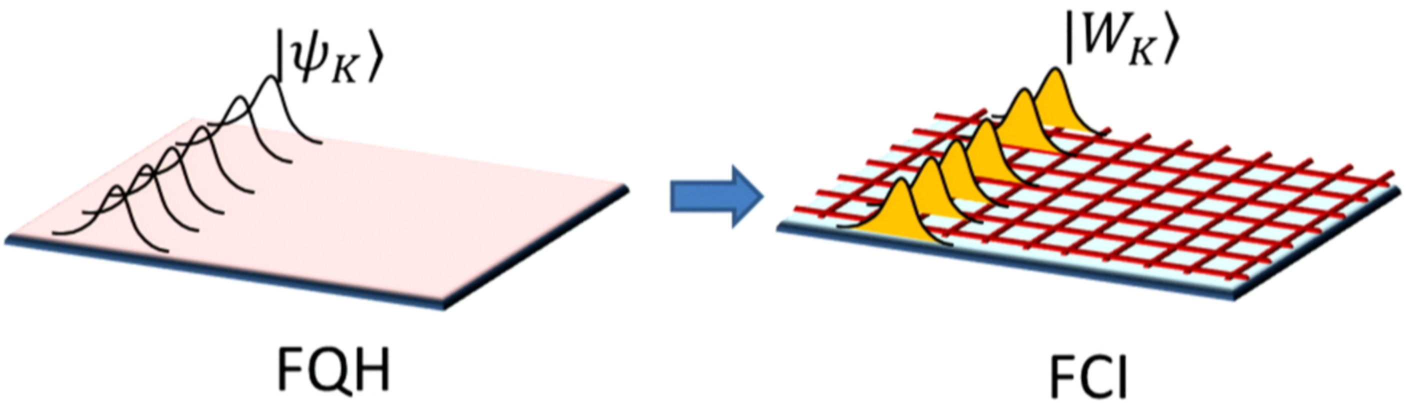
- After the redefinition, Wannier functions $|W_K\rangle$ are analog of Landau level wavefunctions

$$\psi_K(x,y) = e^{iky}\, e^{-\left(x-Kl_B^2\right)^2/2l_B^2}$$



$n_K = 0,1$

- Using this mapping of basis, every FQH state $|\Psi\rangle = \sum_{\{n_K\}} \Phi(\{n_K\}) \prod_{n_K=1} |\psi_K\rangle$ is mapped to a lattice FCI state $|\Psi\rangle = \sum_{\{n_K\}} \Phi(\{n_K\}) \prod_{n_K=1} |W_K\rangle$



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- Fortunately, the occupation number wavefunction $\Phi(\{n_K\})$ is known for many FQH states
- For the 1/3 Laughlin state (Rezayi&Haldane 1994 PRB)

$$|\Psi_{1/3}\rangle = \bullet \circ \circ \bullet \circ \circ \bullet \circ \circ \bullet \circ \circ \bullet \circ \circ$$

$$+ \bullet \circ \bullet \circ \circ \circ \circ \bullet \circ \bullet \circ \circ \bullet \circ \circ \quad + \ldots\ldots$$

- A generic construction by Jack polynomials (Bernevig&Haldane 2008 PRL)
- Numerical works confirmed the validity of Wannier representation in 1/2 (83%, Scaffidi&Moller 1207.3539) and 1/3 (99% after gauge optimization, Wu et al 1206.5773)
- Exact lattice Hamiltonians can be constructed by mapping the pseudopotential Hamiltonians (Haldane 1983) to the lattice system. (Lee, Thomale, XLQ, 1207.5587)

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- A generic construction by Jack polynomials (Bernevig&Haldane 2008 PRL)
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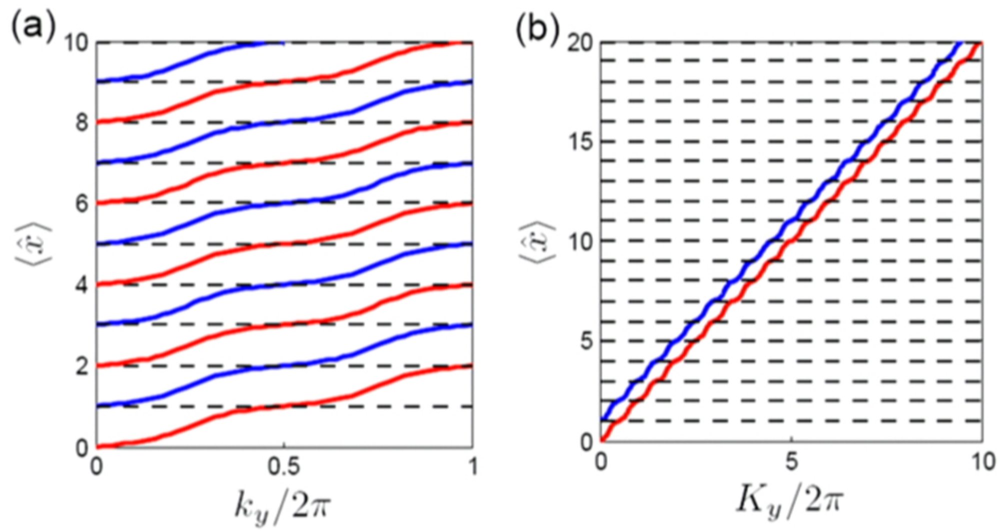
- Exact lattice Hamiltonians can be constructed by mapping the pseudopotential Hamiltonians (Haldane 1983) to the lattice system. (Lee, Thomale, XLQ, 1207.5587)

New states in FCI with higher Chern number

- Are there new topological states in the FQAH system that are absent in the ordinary FQH?
- The Wannier approach can be generalized to bands with Chern number >1. (For an example of Chern number 2 model, see Wang&Ran PRB'11)
- Higher winding number of the Wannier state position



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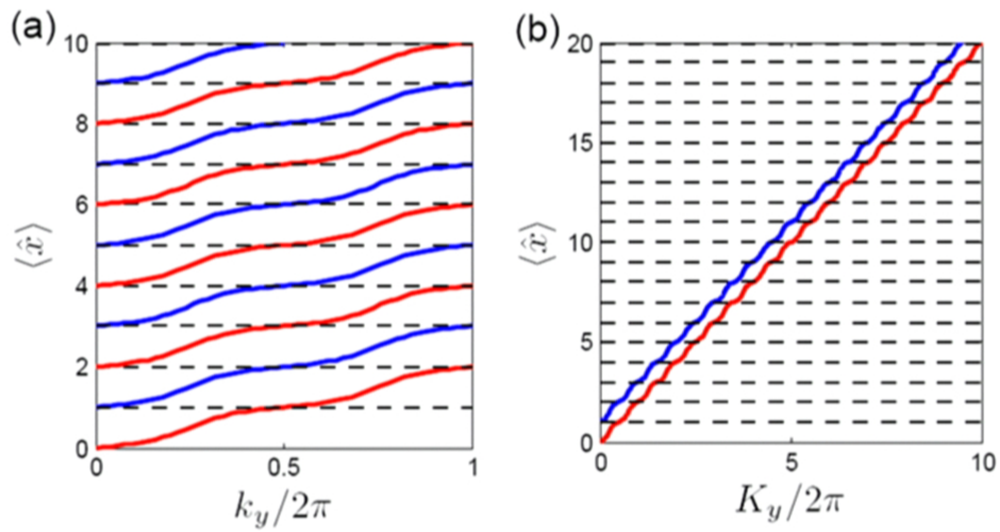
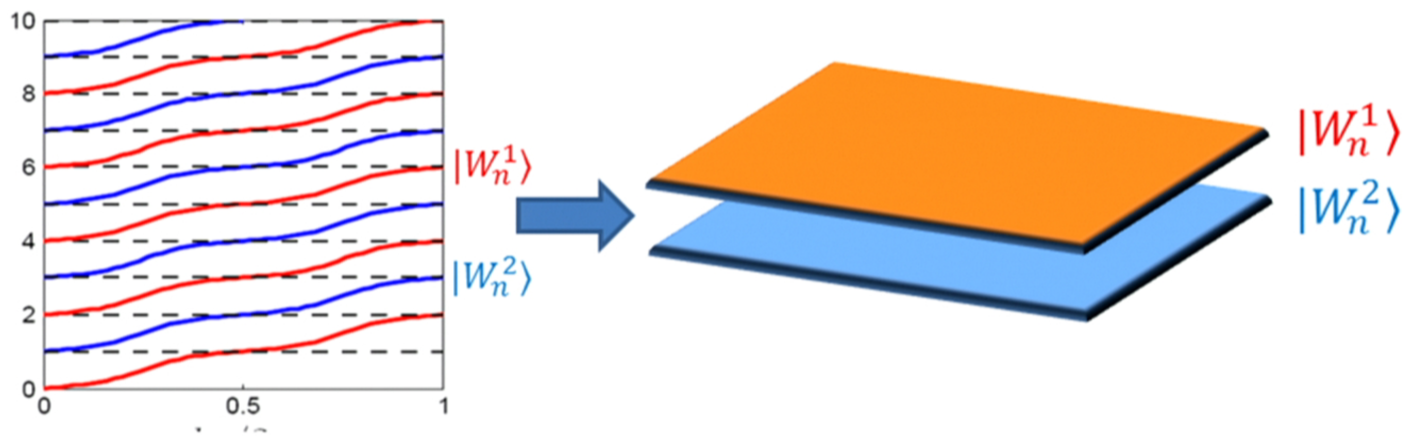


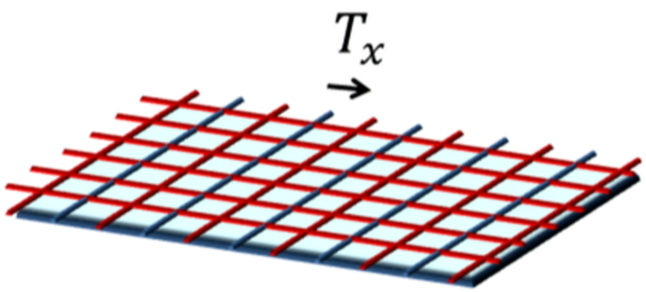
Realizing multi-layer FQH states in one band

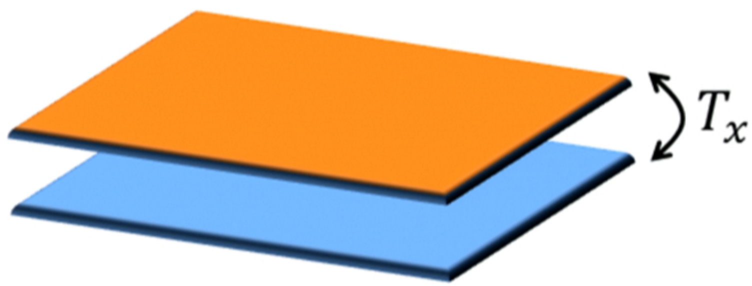
- For Chern number $C_1 = 2$, the Wannier states form two groups $|W_n^1\rangle, |W_n^2\rangle$, with each group equivalent to a Landau level
- ➔Double-layer FQH states can be realized in a single band



Nontrivial representation of lattice translation symmetry

- Lattice translations T_x, T_y acts differently on this basis
- $T_x|W_n^1\rangle = |W_n^2\rangle,\ T_x|W_n^1\rangle = |W_n^1\rangle$
- $T_y|W_n^{1,2}\rangle = e^{in2\pi/L_y}|W_n^{1,2}\rangle$





Topological nematic states

- Consider the Halperin (mnl) states (Halperin ’83)
- $\Phi(z_i, w_j) =$
  $\prod_{i<j}(z_i - z_j)^m (w_i - w_j)^n \prod_{i,j}(z_i - w_j)^l$
- Lattice translation T_x exchanges the two “layers”.
- For $m = n$ the state is translation invariant. However, the 4-fold lattice rotation symmetry (for a square lattice) is broken.
- We name such a state as a topological nematic state
- Lattice dislocations in a topological nematic state carry nontrivial topological degeneracy

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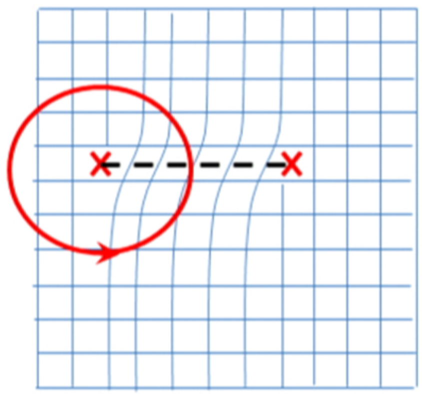
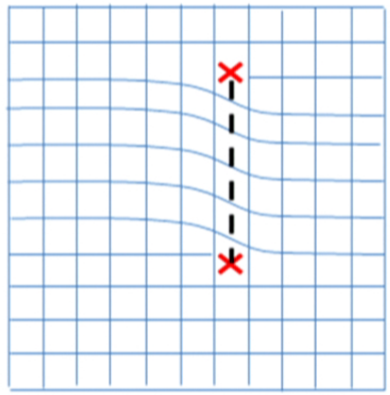
Dislocations in topological nematic states

- Dislocations are described by the Burgers vector $\vec{b} = (b_x, b_y)$



x-dislocation $\vec{b} = \hat{x}$



y-dislocation $\vec{b} = \hat{y}$

- Across the "branch-cut" of the x-dislocation, the two layers are exchanged!

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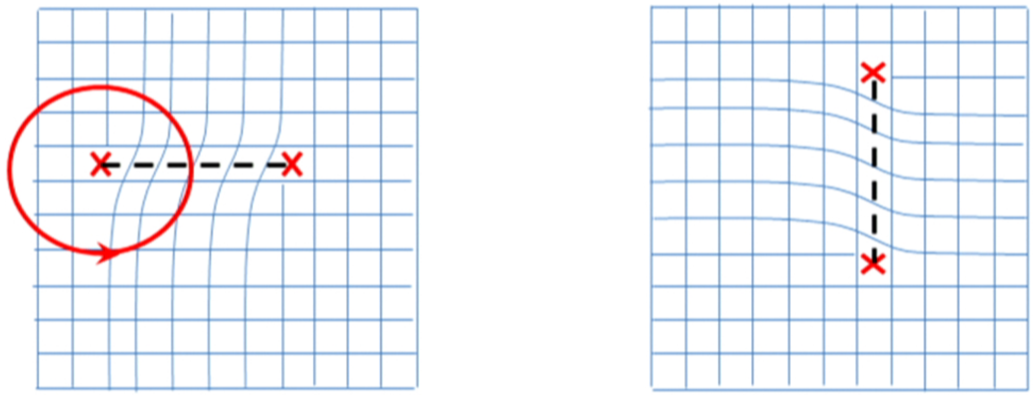
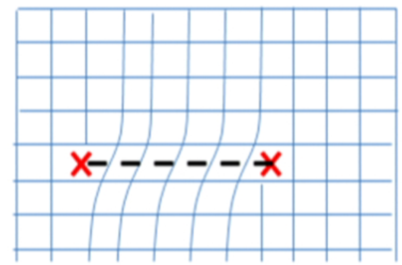


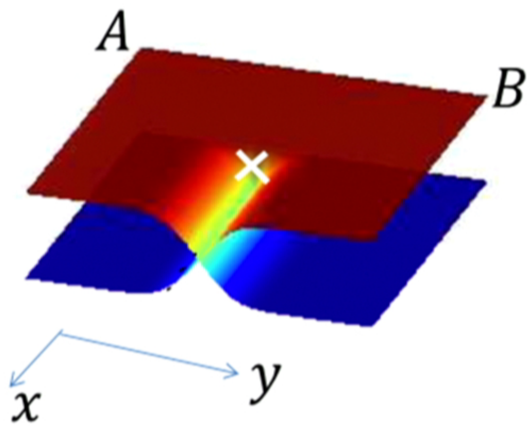
x-dislocation $\vec{b} = \hat{x}$ y-dislocation $\vec{b} = \hat{y}$

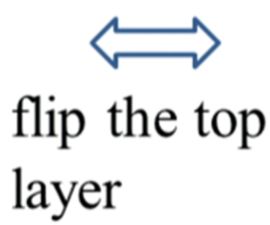
- Across the "branch-cut" of the x-dislocation, the two layers are exchanged!

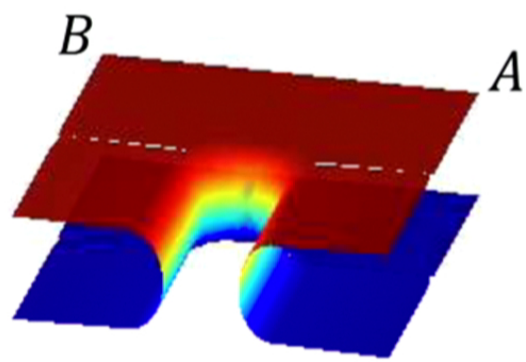
Dislocations in topological nematic states

- A pairs of x-dislocations is equivalent to a "worm-hole"
- Related to the $U(1) \times U(1) \rtimes Z_2$ Chern-Simons theory of Barkeshli-Wen '10, but with global Z_2 symmetry
- Dislocation has similar effect in some Z_N gauge theories (Bombin '10, You&Wen '12)



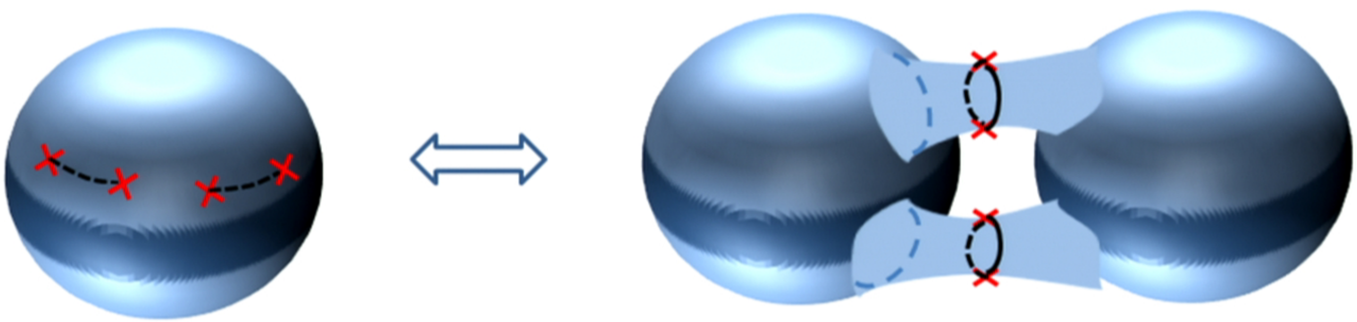






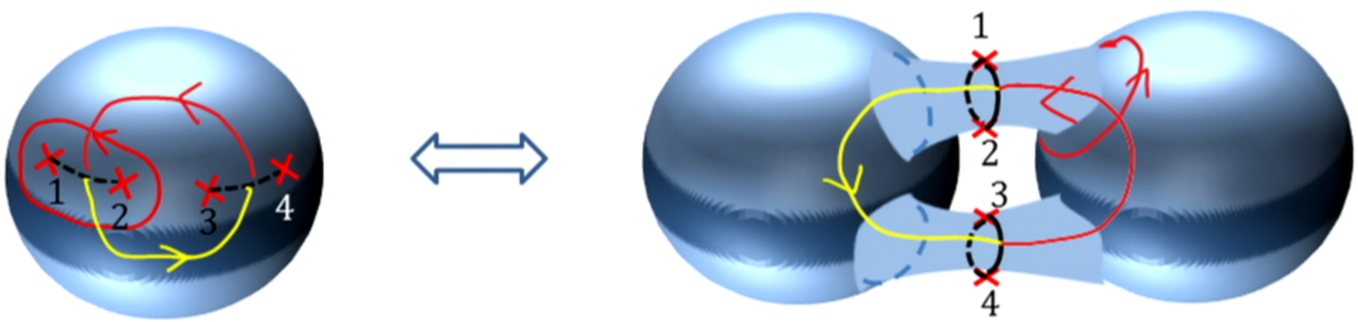
Dislocations in topological nematic states

- Consider a simple case of $(mm0)$ state, which is a direct product of two Laughlin states
- $|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle$
- Consider n pairs of dislocations on sphere
- Ground state degeneracy $N = m^{n-1}$
- Degeneracy $d = \sqrt{m}$ per dislocation
- Dislocations become "genus generators"---genons



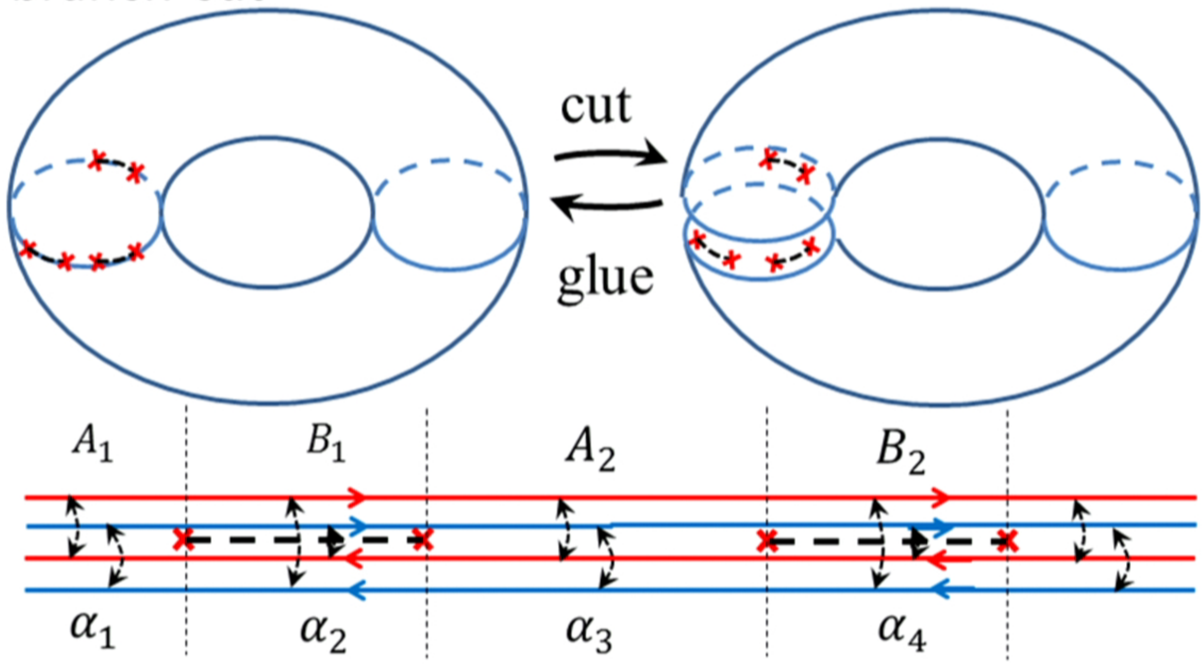
Properties of genons: braiding statistics

- Braiding of two defects corresponds to the Dehn twist along the loop surounding them
- Example: (220) state. 2 ground states for 4 defects. The braiding matrices are
- $U_{12} = e^{i\theta}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}, U_{23} = e^{i\phi}\,\frac{1}{2}\begin{pmatrix}1+i & 1-i\\ 1-i & 1+i\end{pmatrix}$
- Abelian phases are undetermined.
- The non-Abelian statistics is identical to Ising anyon!

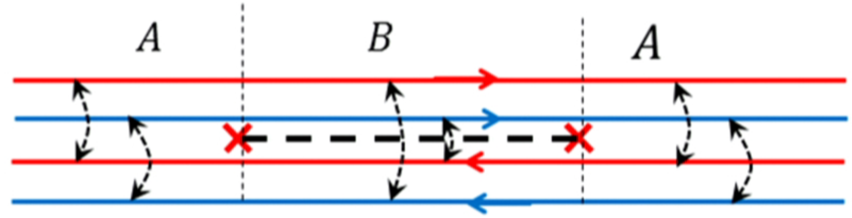


More general genons: edge state understanding

- Consider more general Halperin (mml) states
- Consider the torus as a cylinder glued along the edge
- Inter-edge tunneling exchanges the two layers across the branch-cut



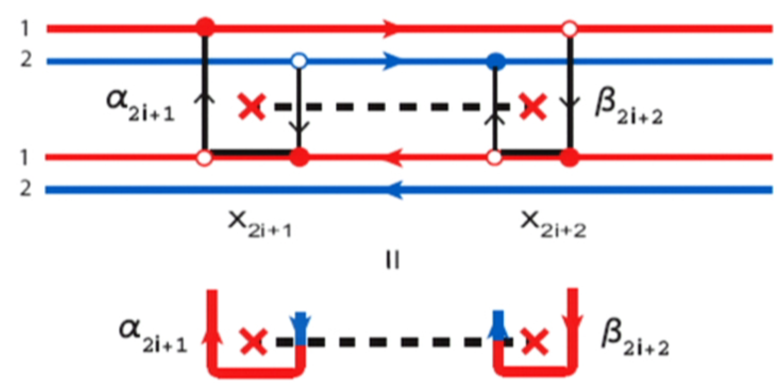
Edge state picture of dislocation-induced degeneracy



- The edge states are described by the chiral Luttinger liquid theory (Wen 1990) with the boson fields $\phi_{1L}, \phi_{2L}, \phi_{1R}, \phi_{2R}$.
- Electron operator
  $c^1_{L,R} = e^{i(m\phi^1_{L,R} + l\phi^2_{L,R})}, c^1_{L,R} = e^{i(l\phi^1_{L,R} + m\phi^2_{L,R})}$,
- Inter-edge electron tunneling determines the number of degenerate minima $(m^2 - l^2)^n (m+l)^n$
- There are $2n - 1$ independent charge on the $2n$ dislocations, each takes $m + l$ values.
- ➔Topological degeneracy
  $N = \frac{(m^2 - l^2)^n (m+l)^n}{(m+l)^{2n-1}} = (m^2 - l^2)(m - l)^{n-1}$,
- quantum dimension $d = \sqrt{|m - l|}$

Edge state picture of genon braiding

- The braiding of genons can also be understood in the edge state picture
- Zero mode operators can be defined as open path quasi-particle tunneling e.g. $\alpha_{2i-1} = e^{i\phi_1(x_{Ai})}e^{-i\,\tilde{\phi}_1(x_{Bi})}$
- $[\alpha_i, H] = [\beta_i, H] = 0$, generalization of Majorana modes
- Physical operators are quadratic combinations of α_i, β_i.
- The braiding matrix is determined by its action to the zero modes



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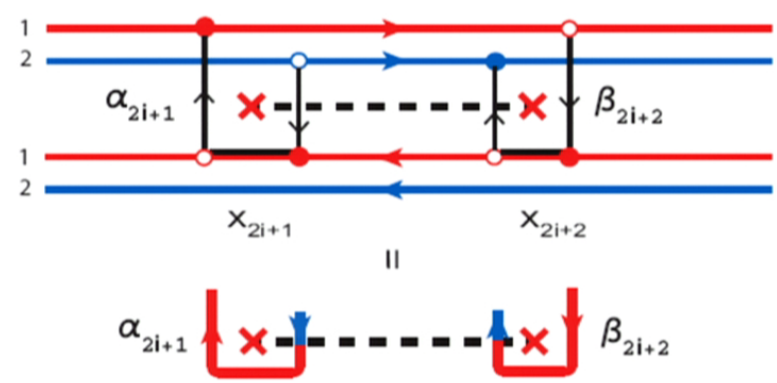
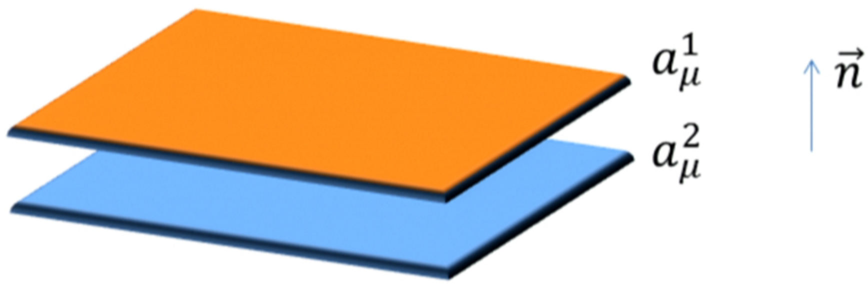


A topological field theory description

- Without dislocations, the effective theory is an Abelian $U(1) \times U(1)$ Chern-Simons theory (Wen&Zee'92)
  $\mathcal{L} = \frac{1}{4\pi} a_\mu^I K_{IJ} \partial_\nu a_\tau^J$
- Around a dislocation, a_μ^1 and a_μ^2 are exchanged
- To describe this effect we introduce a $U(2)$ gauge field A_μ and a Higgs field $H = \sigma \cdot \vec{n} e^{i\theta}$ which breaks $U(2) \rightarrow U(1) \times U(1)$. $\vec{n}$ should change sign when a reference point is moving around a dislocation.



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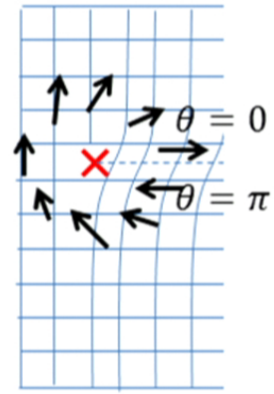
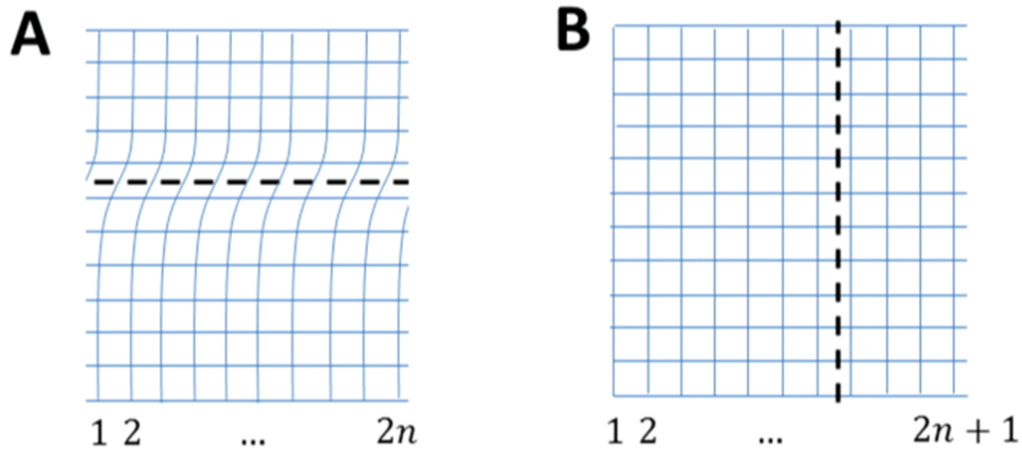


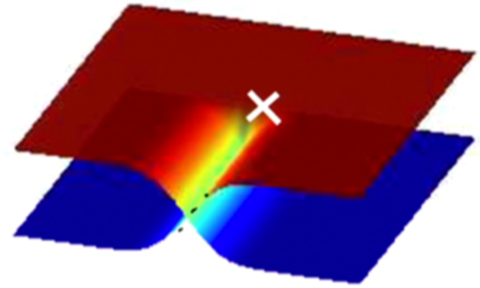
- We propose the Lagrangian
- $$\mathcal{L} = \frac{m-l}{4\pi}\epsilon^{\mu\nu\tau} tr\left[A_\mu \partial_\nu A_\tau + \frac{2}{3} A_\mu A_\nu A_\tau\right] + \frac{l}{4\pi}\epsilon^{\mu\nu\tau} tr[A_\mu]\partial_\nu tr[A_\tau] + J tr[D_\mu H^\dagger D_\mu H]$$
- $\theta = \vec{u} \cdot \vec{K}/2$ with $\vec{u}$ the lattice displacement field, and $\vec{K} = 2\pi(n_x, n_y)$ the type of the topological nematic state.
- Around a dislocation $\vec{u}$ changes by the Burgers vector $\vec{b}$. Thus if $\vec{b} \cdot \vec{K}/2\pi$ is odd, the dislocation corresponds to a half vortex of θ which must be associated with a half vortex of $\vec{n}$.
- ➔The two $U(1)$ Chern-Simons fields are exchanged around the dislocation

Numerical probe of topological nematic states

- For a (1,0) topological nematic state on a torus with hopping shifted like in A, or odd number of sites in x direction, the ground state degeneracy is reduced from $|m^2 - l^2|$ to $|m + l|$.
- This can be used as a numerical probe to topological nematic states in exact diagonalization.



More general discussion on genons and other twist defects



- Genons can be defined in any bilayer topological state $G \times G$ where G is an Abelian or non-Abelian state.
- In general the quantum dimension of genon is $d = D_G = \sqrt{\sum_i d_i^2}$ with D_G the total quantum dimension of the single-layer theory
- If we take G to be Ising TQFT, the genon braiding realize Dehn twists in Ising TQFT, which leads to universal topological quantum computation if measurement is allowed (Bravyi&Kitaev '00, Freedman et al '06)

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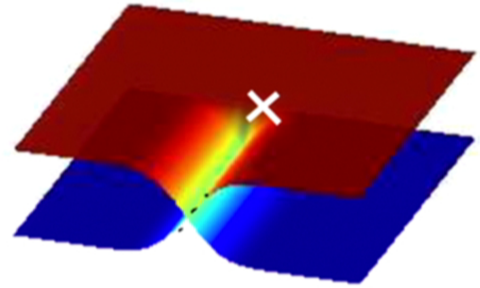

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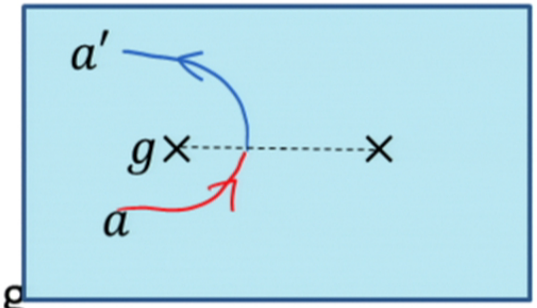


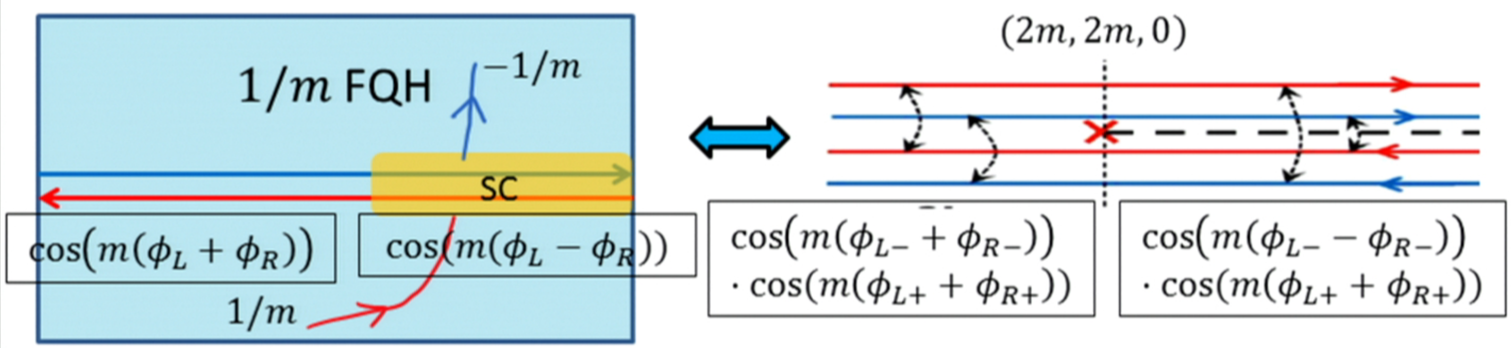
- For any topological state, if there is a mapping of the topological particles $a \to a' = g(a)$ preserving the braiding and fusion rules, a twist defect can be defined.
- Many different twist defects can be mapped to genons

Topological states	Symmetries	Transformation of quasiparticles
Z_N states	layer permutation Z_2	$(a,b) \to (b,a)$
	particle-hole symmetry Z_2	$(a,b) \to (N-a, N-b)$
N-layer FQH states	layer permutation S_N	$(a_1, a_2, .., a_N) \to (a_{P_1}, a_{P_2}, ..., a_{P_N})$
$1/k$-Laughlin FQH state	particle-hole symmetry Z_2	$a \to (k-a)$.

Toric code and generalization (Bombin '10, You&Wen '12)
topological nematic states, or real multi-layer FQH (Barkeshli&Qi, '12)
FM/SC domain wall (Linder et al, Clarke et al, Cheng, Vaezi, '12)

Relation between genons and other twist defects

- Genons are related to another kind of extrinsic defect (Linder et al, Clarke et al, Cheng, Vaezi, 2012) at the domain wall between FM and SC regions of the FQSH edge states, or SC and ordinary inter-edge tunneling in FQH edge
- The edge theory for $\frac{1}{m}$ FQH is equivalent to the anti-symmetric sector $\phi_- = \phi_1 - \phi_2$ of the $(2m, 2m, 0)$ topological nematic states, if $\phi_+ = \phi_1 + \phi_2$ is gapped.



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