Title: Astrophysical shear-driven turbulence

Date: Oct 17, 2012 02:00 PM

URL: http://pirsa.org/12100043

Abstract: Astronomical hydrodynamics is usually almost ideal in the sense that the Reynolds number (Re) is enormous and any effective viscosity must be due to shocks or turbulence. Astronomical magnetohydrodynamics (MHD) is often also nearly ideal, so that magnetic fields and plasma are well coupled. In particular, dissipation of orbital energy in accretion disks around black holes is readily explained by MHD turbulence. On the other hand, the planet-bearing disks around protostars are magnetically far from ideal because of very low fractional ionization. MHD turbulence is at best marginal in these disks, yet accretion is observed. The Reynolds numbers based on orbital-velocity gradients are enormous, so by analogy with high-Re terrestrial flows, one might expect hydrodynamic (i.e., unmagnetized) turbulence. Direct numerical simulations indicate that such turbulence is somehow suppressed by keplerian rotation, though the mechanism is
unclear and the simulations are limited in Re. Recently, a few groups have studied the question via Taylor-Couette experiments at somewhat higher Re, obtaining conflicting results. Complicating and enriching this debate is the recent discovery that turbulence tends to have a finite lifetime in shear flows that admit a formally linearly stable laminar solution: this includes flow in smooth pipes and probably also unmagnetized keplerian disks. Some suggestions will be offered as to how these open questions might be resolved.

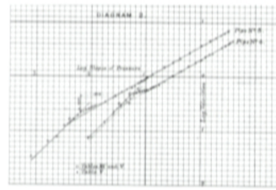
Astrophysical Turbulence

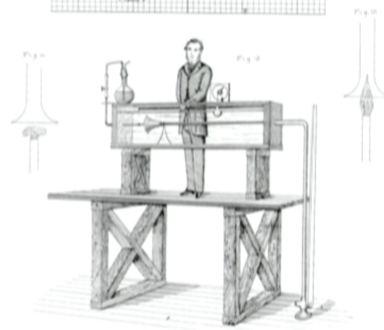
Jeremy Goodman
Princeton University

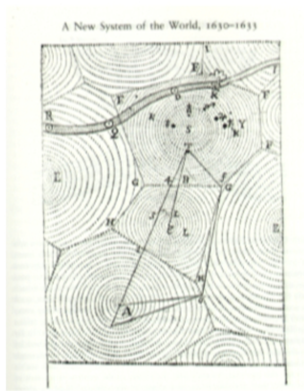
Perimeter Institute
17 October 2012

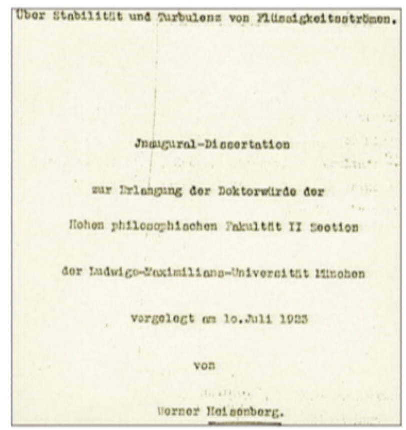
CMSO

Turbulence is an old problem









Über Stabilität und Turbulenz von Flüssigkeitsströmen.

Inaugural-Dissertation

zur Erlangung der Doktorwürde der

Hohen philosophischen Fakultät II Section

der Ludwig-Maximilians-Universität München

vorgelegt am 10.Juli 1923

von

Werner Heisenberg.

Jackson D, Launder B. 2007.
Annu. Rev. Fluid Mech. 39:19–35

Lensfield Cottage, Cambridge, 31 Oct. 1894.

Dear Lord Rayleigh,

I must plead guilty to not having digested Professor Osborne Reynolds's paper, though much time has passed since it was referred to me.

I find it very difficult to make out what the author's notions are. As far as I can conjecture his meaning, I must say I do not think he has made out his point. He is however an able man, and in his former paper did very good work in showing that the conditions of dynamical similarity which follow from the dimensions of the hydrodynamical equations when viscosity is taken into account are not confined to what I may call regular motions, but continue to apply (in relation to mean effects) even when the motion is of that irregular kind which constituted eddies, and which at first sight appears to defy mathematical treatment. The fact that the author has gone to the expense of printing the paper shows that he himself considers it as of much importance. I confess I am not prepared to endorse that opinion myself, but neither can I say that it may not be true.

I do not know whether these remarks will be of any use in assisting the Council to come to a decision.

Yours very truly,

G. G. Stokes

Plan of talk

- Types of astrophysical turbulence
- Accretion disks & shear-driven turbulence
- Magnetorotational instability
- Two major developments in turbulence
  - Transience
  - Structures
- Quasi-keplerian hydrodynamic shear flow

Acknowledgments

Collaborators:

Hantao Ji, *Princeton Plasma Physics Lab*
Xue-Ning Bai, *Princeton*
Michael Burin, *Cal State San Marcos*
Eric Edlund, *PPPL*
Jose Garmilla, *Princeton*
Christophe Gissinger, *ENS, Paris*
Isom Herron, *Rensselaer Polytechnic Inst.*
Akira Kageyama, *Kobe University*
Wei Liu, *M.D. Anderson Cancer Ctr.*
Mark Nornberg, *U Wisconsin Madison*
Austin Roach, *PPPL*
Ethan Schartman, *Nova Photonics & PPPL*
Erik Spence, *PPPL*

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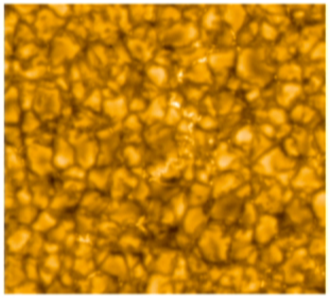
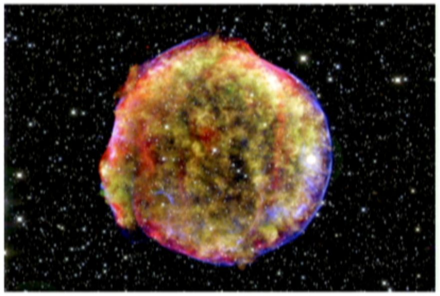
Center for Magnetic Self-Organization

Varieties of turbulence



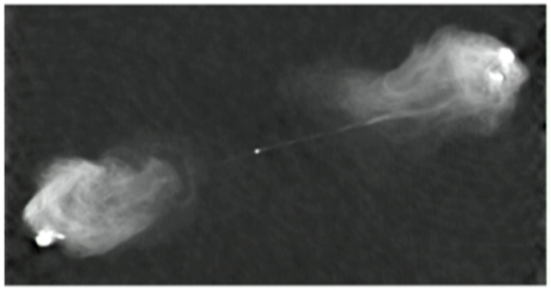
Stellar convection
[Hinode/JAXA]

Molecular clouds
[NASA/HST/STScI]



Shocks
[NASA/CSX/SAO/MPIA]

Jets
[NRAO/VLA/Perley et al. 1984]



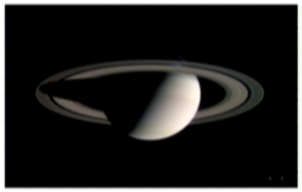
Turbulent processes

- Dissipation
- Mixing & Transport
  - heat, momentum, composition
- Magnetic-field generation
- Particle acceleration

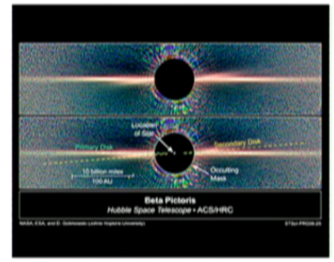
Intractable problems in astrophysics

Process	~~Fudge factor~~ **Parameter**
Convection	$\alpha = l_M / H_P$
Accretion	$\alpha = T_{r\phi} / P$
Dynamo	$\alpha = \bar{E} / \bar{B}$
Tides	$Q = \omega_{\text{tide}} \Delta E_{\text{pot}} / \dot{E}$
Reconnection	$V_{\text{reconn}} / V_{\text{A}}$

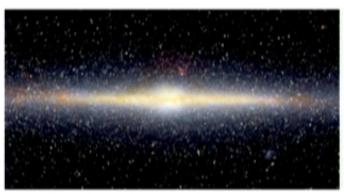
Disks



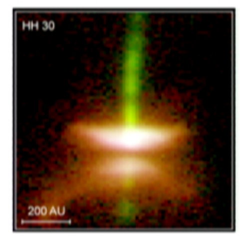
planetary rings
$r \sim 10^4$-10^5 km
rocks, ices



debris disks
$r \sim 10^9$ km ~100 AU)
dust

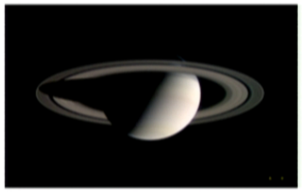


galactic disks
$r \sim 10^{17}$ km ~ 10^4 lt-yr
stars+gas+dust



protostellar disks
$r \sim 10^9$ km ~100 AU)
gas+dust (+planets?)

Disks



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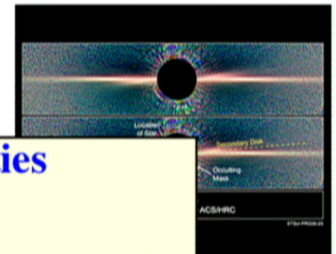
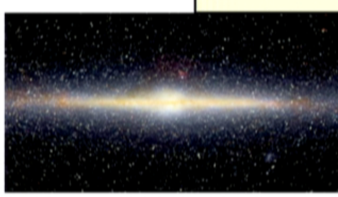


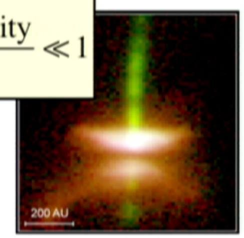
debris disks
$r \sim 10^9$ km ~100 AU)

Common dynamical properties

Rotation balances gravity: $\frac{V_\phi^2}{r} \approx \frac{GM(r)}{r^2}$

Thickness is often small: $\frac{\Delta z}{r} \approx \frac{\text{sound speed or random velocity}}{V_\phi} \ll 1$



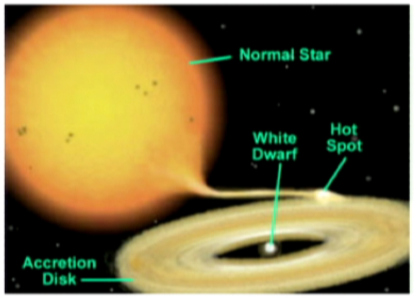


protostellar disks
$r \sim 10^9$ km ~100 AU)
gas+dust (+planets?)

Accretion disks

- A disk, usually gaseous, whose material flows gradually onto the central object
  - $t_{flow} \gg \Omega^{-1} \equiv r/V_\phi$
- Accretion liberates gravitational potential energy
  - available for radiation or outflow
  - $\sim 0.1 c^2 \Delta M$ for neutron-star or black-hole accretors
- Orbital angular momentum must be removed
  - **transport through the disk (``viscosity")**
  - magnetocentrifugal wind or jet
  - tides from a companion

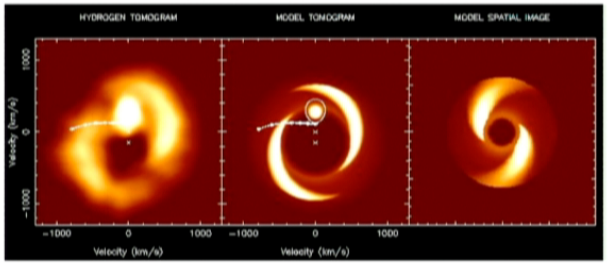
Accretion disks: CVs

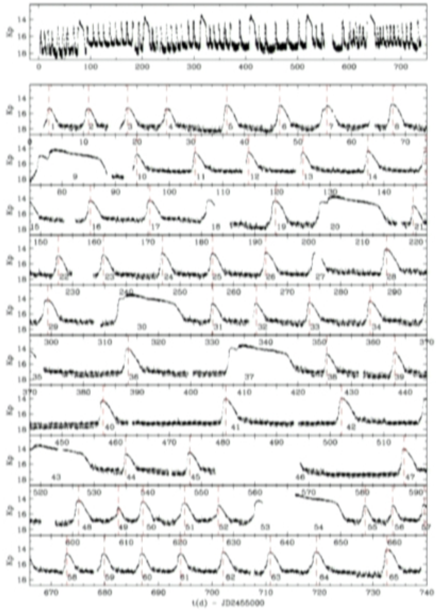


Cataclysmic Variables

r ~ 10^4-10^6 km ~ $R_\oplus$–$R_\odot$

Above: artist's conception (K. Smale)
Below: Doppler tomography (D. Steeghs et al.)





Kepler light curve of V1504 Cygni
(Cannizzo et al. 2011)

X-ray binaries

- Neutron star or black hole accretor

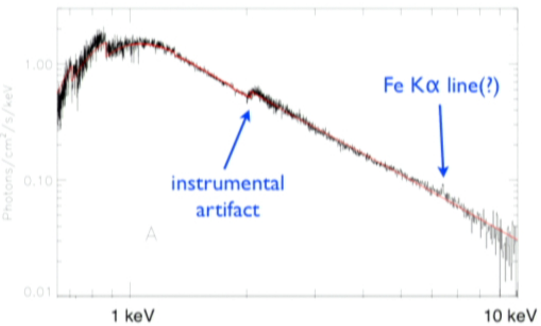
$$M_{ns} \sim 1.4\, M_{\odot};\quad M_{bh} \sim 10\, M_{\odot}$$

- Higher luminosity & harder spectrum than CVs

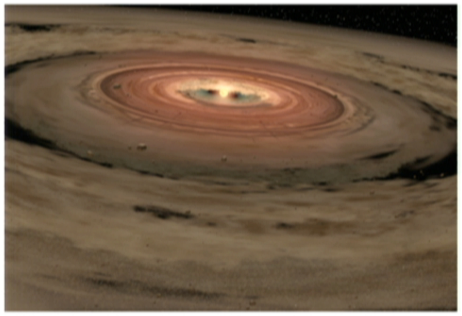
- Deep gravitational potential

$$\frac{GM_*}{r_{min}} \sim 0.1c^2$$

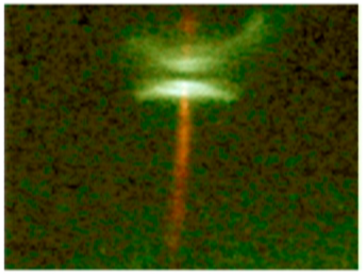
Cygnus X-1: artist's conception & *Chandra* X-ray spectrum (J. Miller et al. 2002)



Protoplanetary disks



Artist's impression



HH-30 (HST/NASA)

Orion proplyd (Bally/HST/NASA)

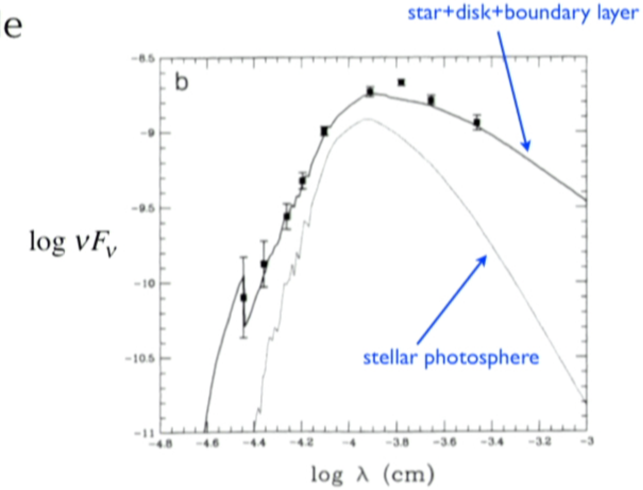
Protostellar disks

- Mostly neutral gas: H_2, He
  - traces of H_2O, CO, ...
  - ~1% dust by mass
  - cool: 10-10^3 K
- M_{disk}~10^{-3}-$10^{-1}M_\odot$
- Accretion ~ $10^{-8}M_\odot$ yr^{-1}

$$L_{\rm disk} \approx \dot{M}\frac{GM_*}{R_*} \sim L_*$$

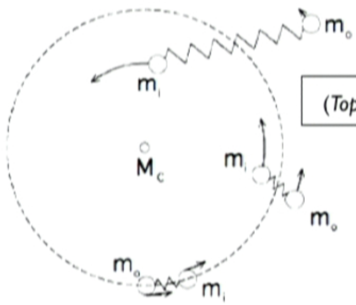
- Nonthermal ionization by stellar X-rays, cosmic rays

$$n(e) \lesssim 10^{-11} n(\mathrm{H}_2)$$



spectral energy distribution of DF Tau (Basri & Bertout 1989)

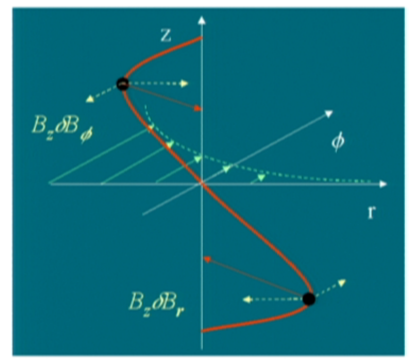
Magnetorotational instability



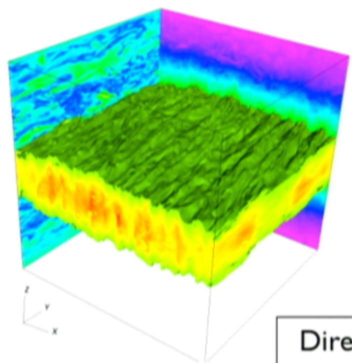
(Top view) The donkey principle

(Side view)
Magnetic tension as the donkey's tether

toward M_C



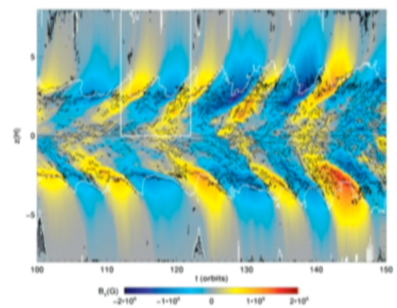
M. Pessah



Direct numerical simulations
(Davis, Stone, & Pessah)

MRI: Open issues

- It is a <u>linear</u> instability only if the magnetic field is given as part of the background state; else the turbulence itself must generate a large-scale field
  - magnetic dynamo problem
  - subcritical instability
- Even linear instability fails when the disk is a poor electrical conductor
  - protostellar disks are problematic
    - nonthermal ionization is required
    - dust tends to soak up free electrons



Shi, Krolik, & Hirose 2009

New paradigm for nonlinear turbulent transition in linearly stable flows

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    - also for cyclonic ($d\ln\Omega/dr > 0$) but not anti-cyclonic ($d\ln\Omega/$
      wall-bounded flow (Rincon, Ogilvie, & Cossu 2007).

Reynolds number

or, in other words, steady direct motion in round tubes is stable or unstable according as

$$\rho \frac{DU_m}{\mu} < 1900 \text{ or } > 2000,$$

---Reynolds (1883, 1895)

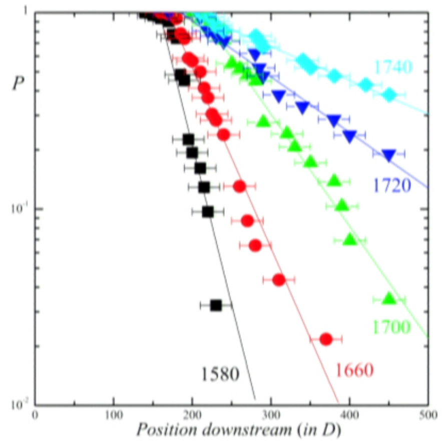
Re=(lengthscale)x(timescale)/(kinematic viscosity)

kinematic viscosity: $\nu=\mu/\rho$ (water: 0.01 cm^2s^{-1})

$Re \ll 1$

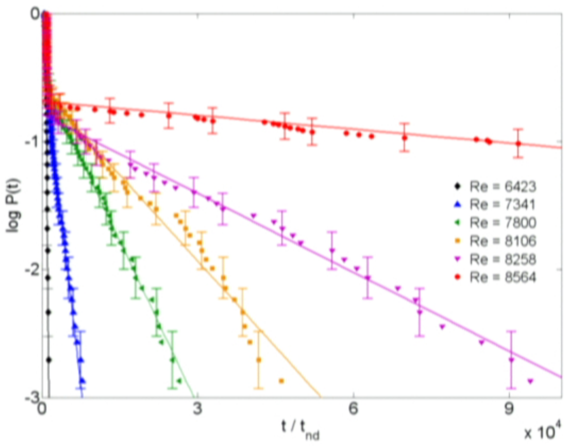
$Re \gg 1$

Transient turbulence



Pipe flow

Peixinho & Mullin (2006)

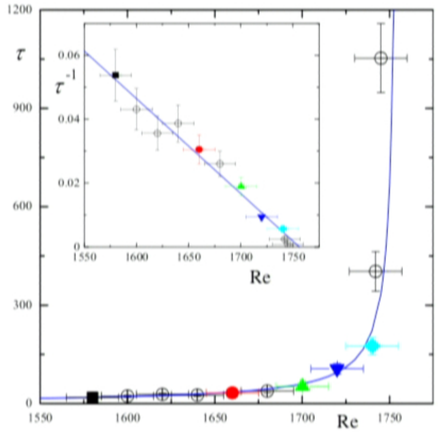


TC flow*

Borerro-Echeverry et al. (2010)

* linearly stable: $\frac{d \ln \Omega}{dr} > 0$ ("cyclonic")

Lifetime vs. Reynolds number



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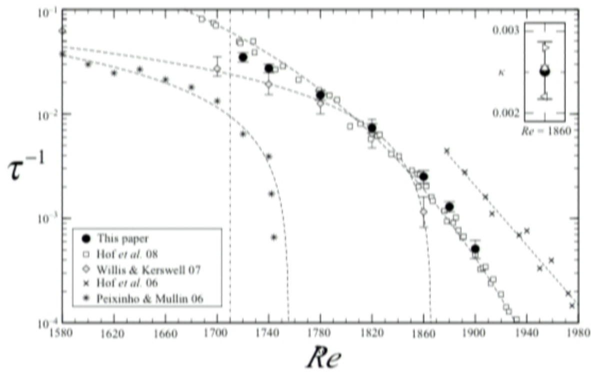

Pipe flow

Avila et al. (2010)

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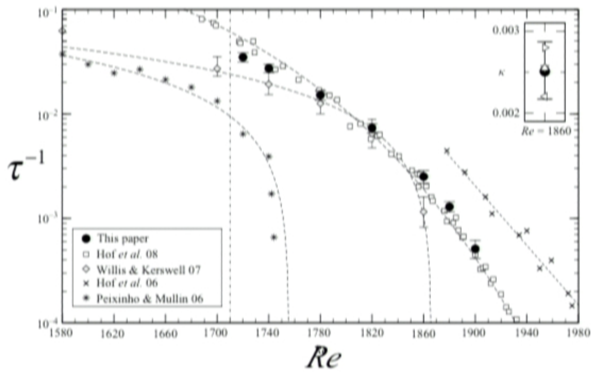

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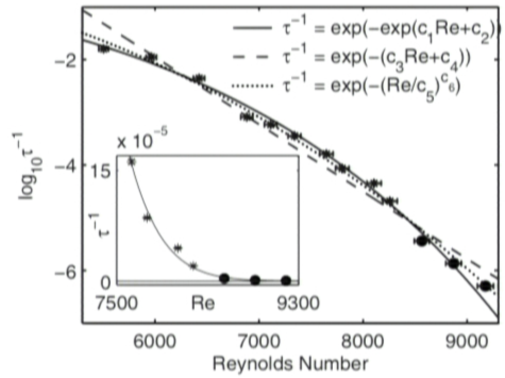
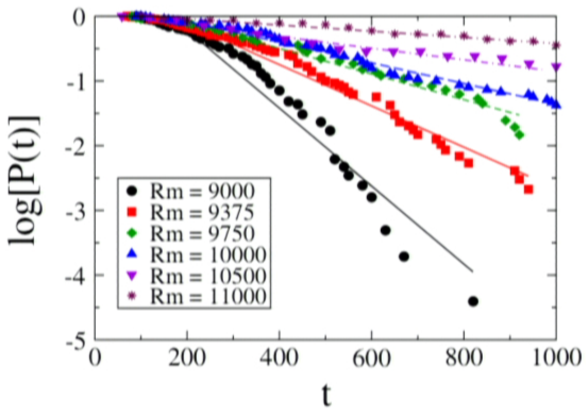
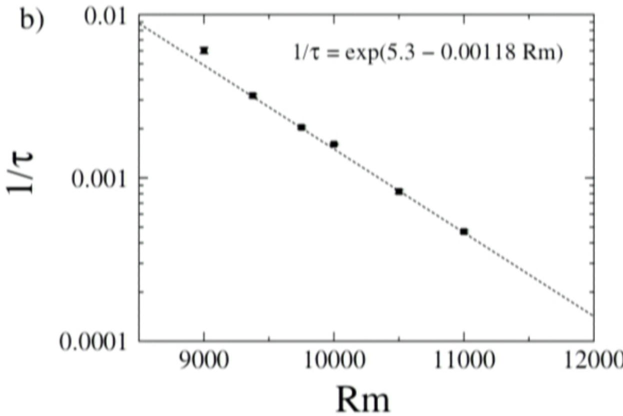


TC flow

Borerro-Echeverry et al. (2010)

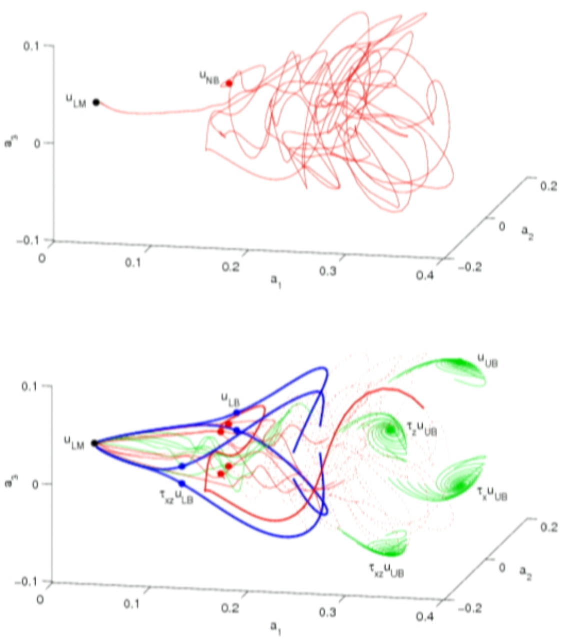
Zero-net-flux MRI





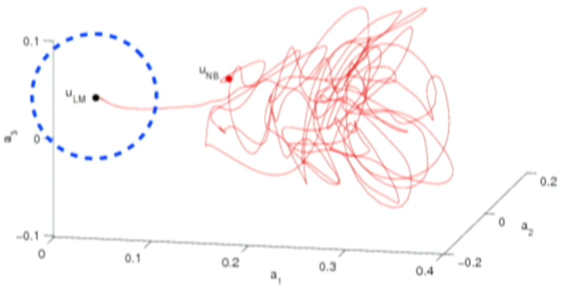
Rempel, Lesur, & Proctor (2010)

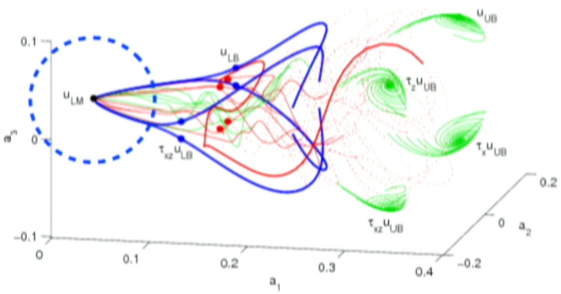
Phase space of plane Couette flow



Gibson, Harclaw, & Ctanovic (2008)

Phase space of plane Couette flow





Gibson, Harclav

Characteristic structures

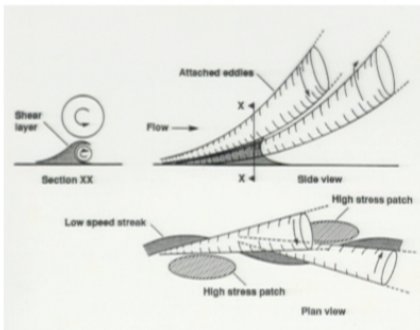
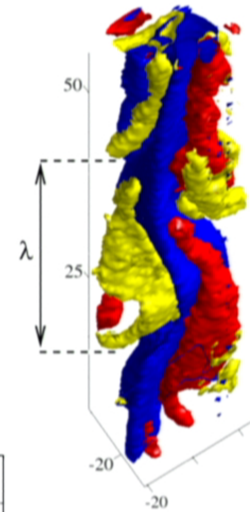
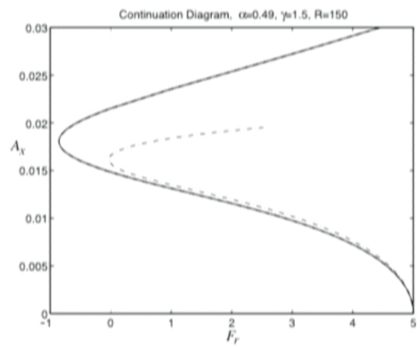


FIG. 1. Sketch of the coherent structure educed from DNS data, from Ref. [3], see also [4].

Liftoff vortices and streaks in plane Couette flow
Stretch (1990),
reproduced by Waleffe (1998)

Experimentally observed
"streaks" in pipe flow at *Re* =3000
B. Hof et al. (2005)





Forced nonlinear solutions in plane Couette flow
Waleffe (1998)

Why exponential scaling? A modest proposal

- Structures have size $l \sim \sqrt{\nu / S}$ (at least near "transition")
- Maximum number in volume V is $N(V) \sim V/l^3 \propto Re^{3/2}$ [or $Re^{9/4}$?]
- Individual structures have mean lifetime $\overline{t_1}$ but may reproduce independently
- A colony of size N dies only if all members die before reproducing

$\Rightarrow$ Mean lifetime of the colony $\overline{t_N} \sim \exp(c\,N)\overline{t_1} \sim \exp(c' Re^{3/2})\overline{t}$.

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EUROPHYSICS LETTERS 1 May 1993
Europhys. Lett., 22 (4), pp. 311-316 (1993)

Survival-Extinction Transition in Bacteria Growth.

M. YA. AZBEL
Raymond and Beverly Sackler Faculty of Exact Sciences, School of Physics and Astronomy, Tel Aviv University - Ramat Aviv, Tel Aviv 69978, Israel

(received 23 November 1992; accepted 12 March 1993)

PACS. 87.20C – General theories of interfaces.
PACS. 64.60C – Order-disorder and statistical mechanics of model systems.
PACS. 64.70D – Solid-solid transitions.

Abstract. – I study an ensemble of bacteria colonies. Finite-size colonies always die out. Their lifetime t_1 is either size independent or exponentially increases with size. In the latter case, their lifetime mean quadratic fluctuation δt is large compared to their representative average lifetime $\bar{t} = \exp[\langle \ln t_1 \rangle]$, $\langle \dots \rangle$ denotes the ensemble average. The extinction-survival phase transition from finite to infinite $\bar{t}$ is accompanied by the phase transition. These results are common for spreading-percolation phenomena.

New paradigm for nonlinear turbulent transition in linearly stable flows

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    - also for cyclonic ($d\ln\Omega/dr > 0$) but not anti-cyclonic ($d\ln\Omega/dr < 0$) wall-bounded flow (Rincon, Ogilvie, & Cossu 2007).
  - dissipative (viscous, resistive) but steady solutions
    - themselves unstable

Differential rotation

Frequency	***Keplerian value***
Rotation: $\Omega(r)$	$\propto r^{-3/2}$
Shear: $r\dfrac{d\Omega}{dr} \equiv S$	$-\dfrac{3}{2}\Omega$
Vorticity: $\dfrac{1}{r}\dfrac{d}{dr}(r^2\Omega) = S + 2\Omega$	$+\dfrac{1}{2}\Omega$
Epicyclic: $\sqrt{\dfrac{1}{r^2}\dfrac{d}{dr}(r^2\Omega)^2} = \sqrt{2\Omega(S+2\Omega)} \equiv \kappa$	Ω

Differential rotation

Frequency		***Keplerian value***
Rotation:	$\Omega(r)$	$\propto r^{-3/2}$
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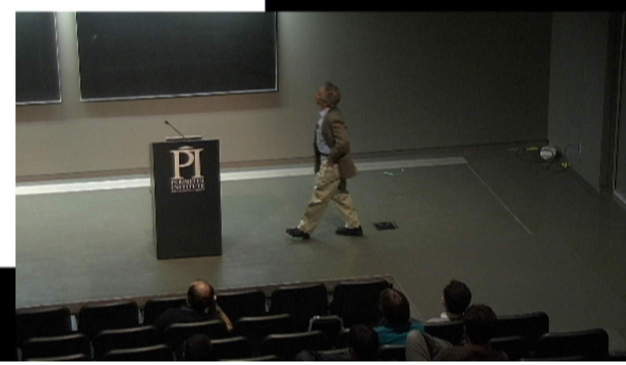
$$\boxed{\kappa^2 < 0 \Rightarrow \text{ linear instability}}$$



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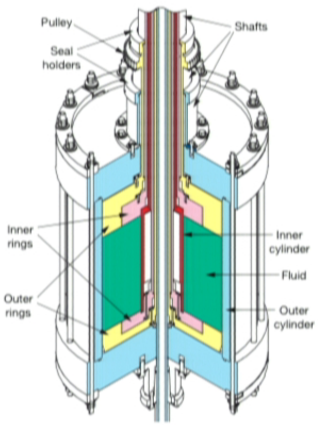
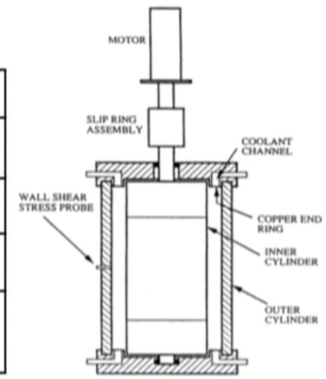
$\boxed{\kappa^2 < 0 \Rightarrow \text{ linear instability}}$ Marginal stability: $\Omega = 0$ or $S = -2\Omega$

It seems to be very difficult to have turbulence at $\kappa^2 > 0$, except near marginal linear stability, <u>especially when $S/\Omega < 0$</u>.

Taylor-Couette experiments



Dimensions	Princeton	Maryland
r_1	7 cm	16 cm
r_2	21 cm	22 cm
h	28 cm	70 (41) cm
Endcaps	independently controlled	fixed to outer

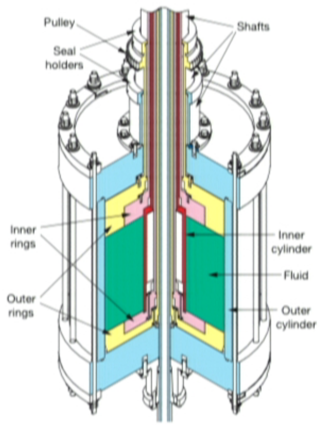


$$\Omega(r) = A + \frac{B}{r^2}$$

"quasi-keplerian": $AB > 0 \quad \Leftrightarrow 0 < \kappa^2 < (2\Omega)^2$

Reynolds number: $Re \equiv \frac{(r_2^2 - r_1^2)(\Omega_1 - \Omega_2)}{\nu}$

Taylor-Couette experiments



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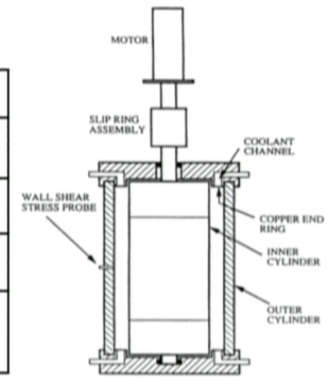

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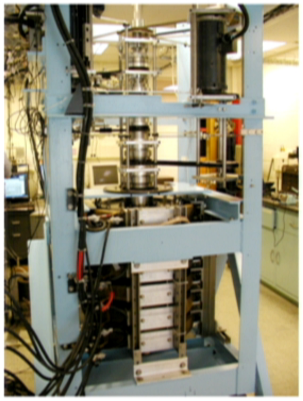
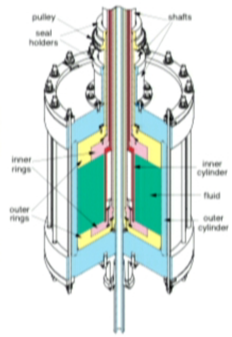
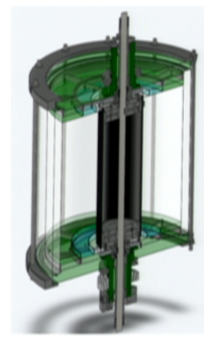
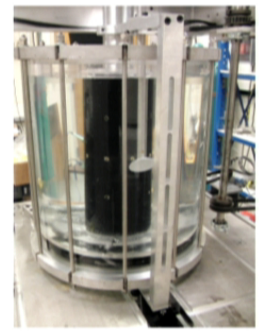
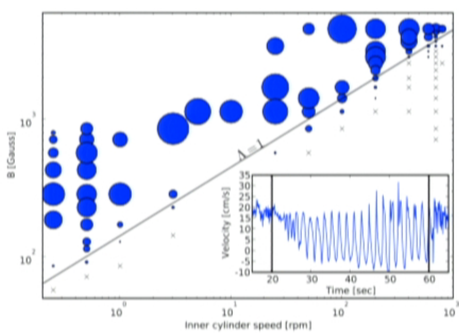
The Princeton MRI & HTX Experiments

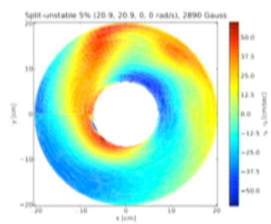


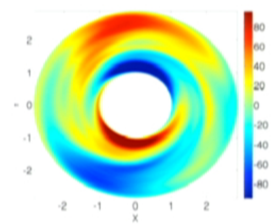




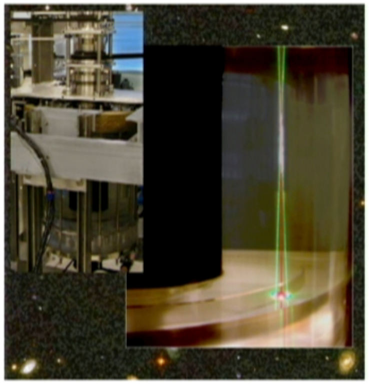


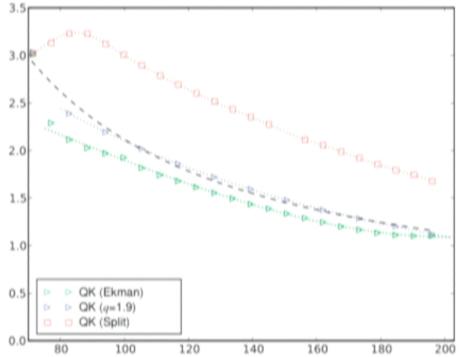


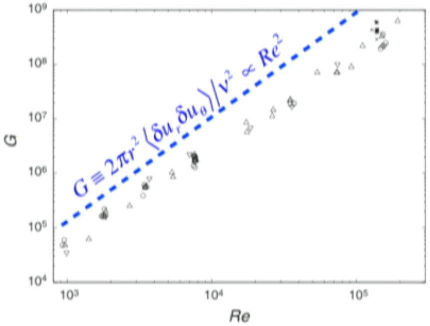




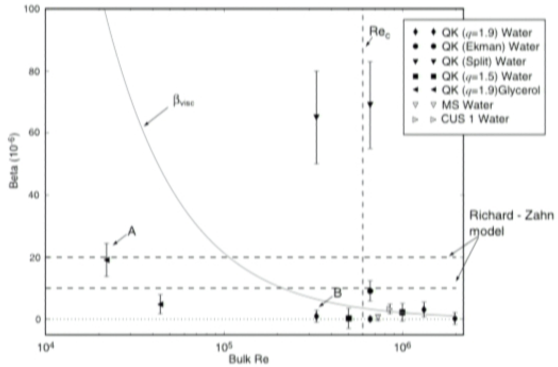
Laser doppler velocimetry





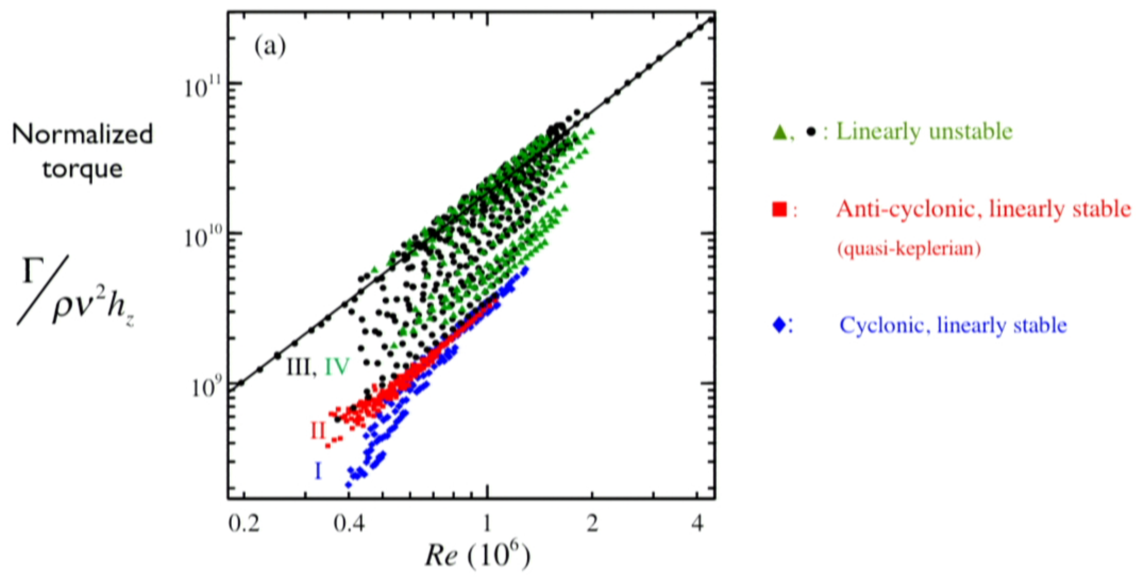


Torque vs. Re *with resting outer cylinder*



Ji et al. 2006; Burin et al. 2010; Schartman et al. 2012

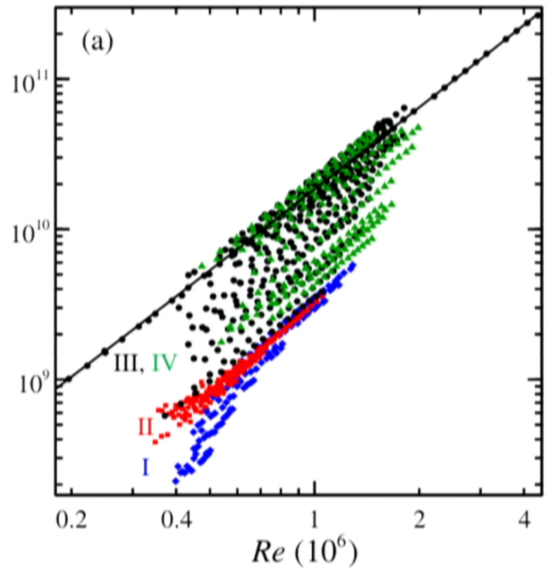
Paoletti & Lathrop (2011)



Paoletti & Lathrop (2011)

Normalized torque

$\Gamma / \rho \nu^2 h_z$

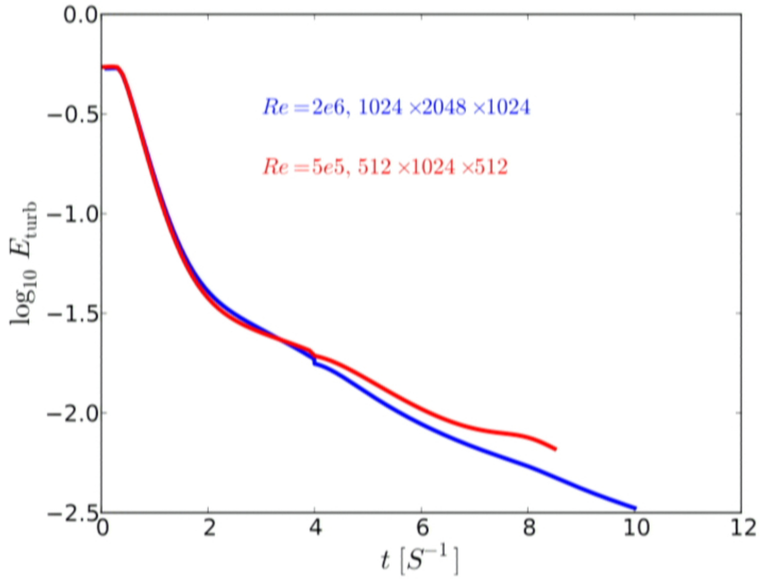


▲, ●: Linearly unstable

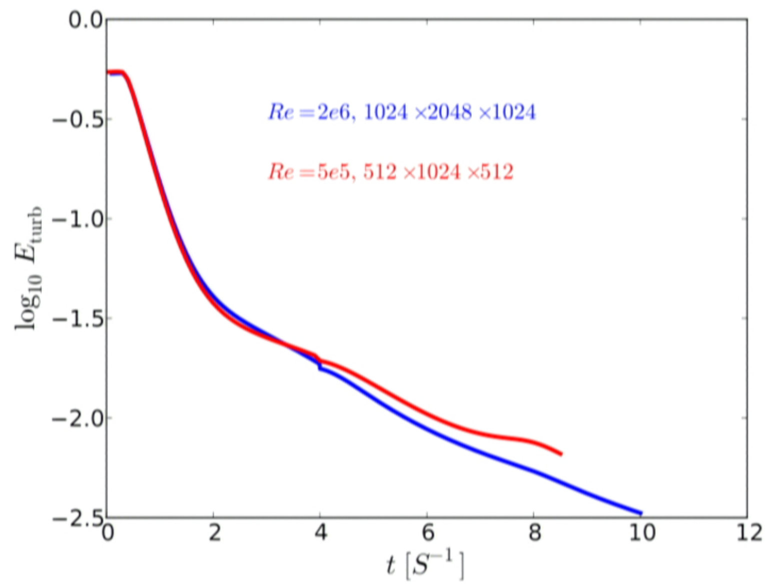
■: Anti-cyclonic, linearly stable (quasi-keplerian)

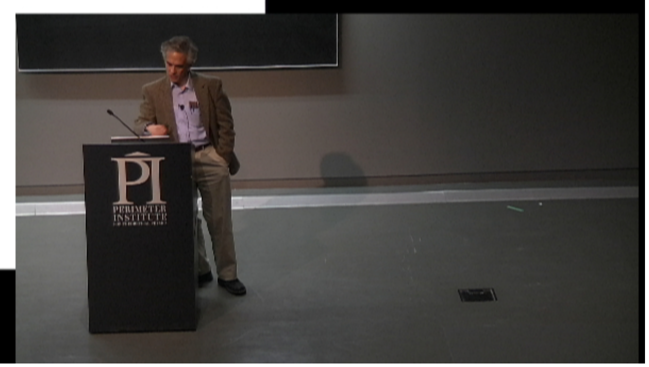
♦: Cyclonic, linearly stable

Quiescence at $Re = 2\text{x}10^6$

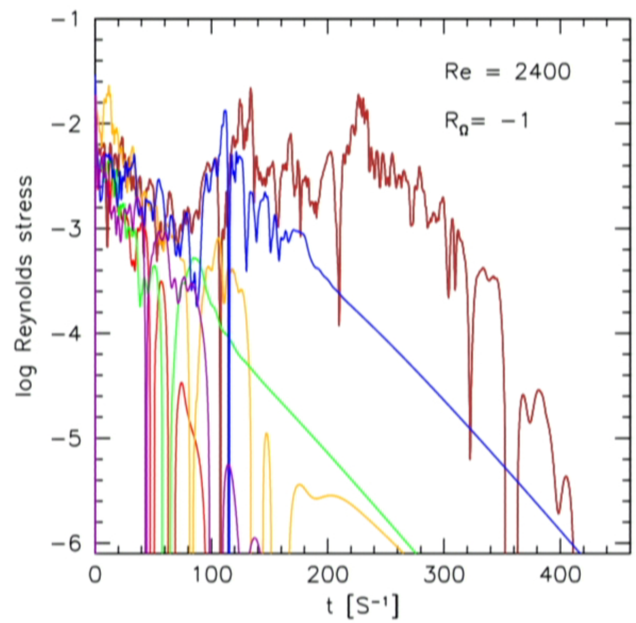


Quiescence at Re = 2x10^6

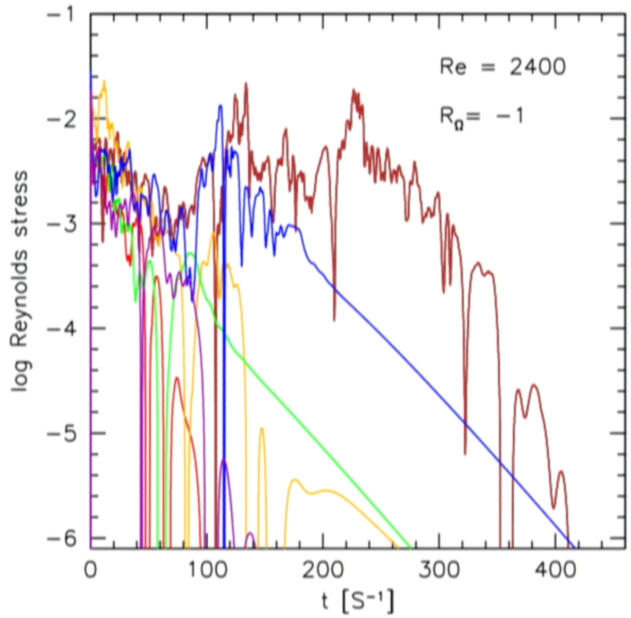




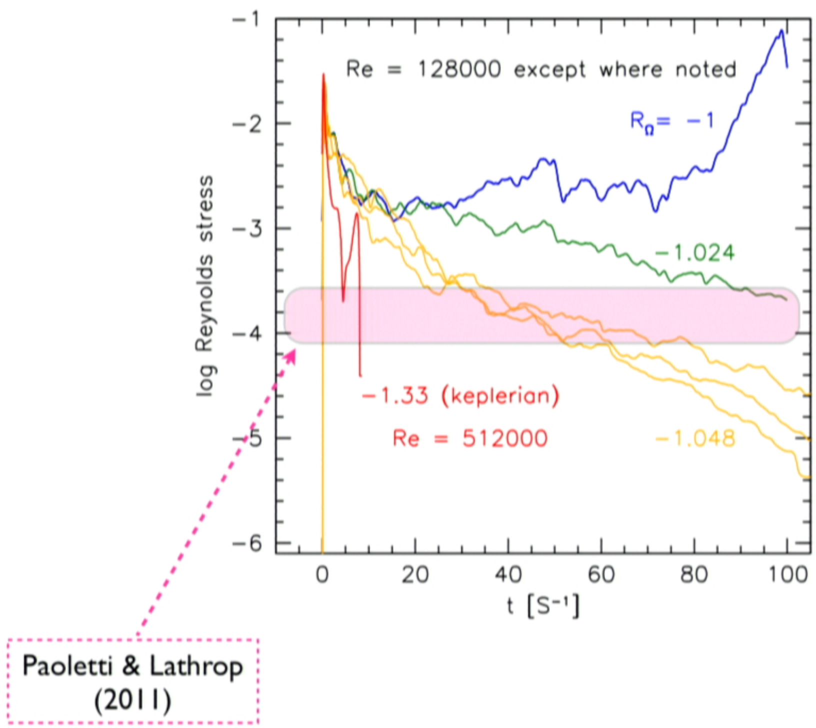
Transience


Re = 2400
R_Ω = −1
log Reynolds stress
t [S⁻¹]
−1
−2
−3
−4
−5
−6
0
100
200
300
400

Transience



Stress vs. Rotation number



Evidence for characteristic structures

- By aligning multiple snapshots of the flow at the position of peak Reynolds stress, then averaging
  - only the lowest wavenumbers are significant & reproducible
- By computing bispectra, averaged over snapshots
  - the bispectral coefficients so obtained appear not to be consistent with multiple instances (and positions) of a single structure

Summary

- Hydro. turbulence in linearly stable flow has
  - finite lifetime, perhaps at all finite Reynolds number
  - characteristic nonlinear structures, at least near turb. onset
- This may also be true of zero-net-flux MRI
- Experiments for quasi-keplerian flow at $Re \sim 10^6$ disagree as to whether hydro turbulence exists
- Shearing-box simulations find no turbulence up to $Re \sim 0.5 \times 10^6$ for $\kappa = \Omega$.
- Preliminary evidence is found for characteri structures in turbulence with $0 \leq \kappa << \Omega$

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