Title: From Effective Strings to the Simplest theory of Quantum Gravity

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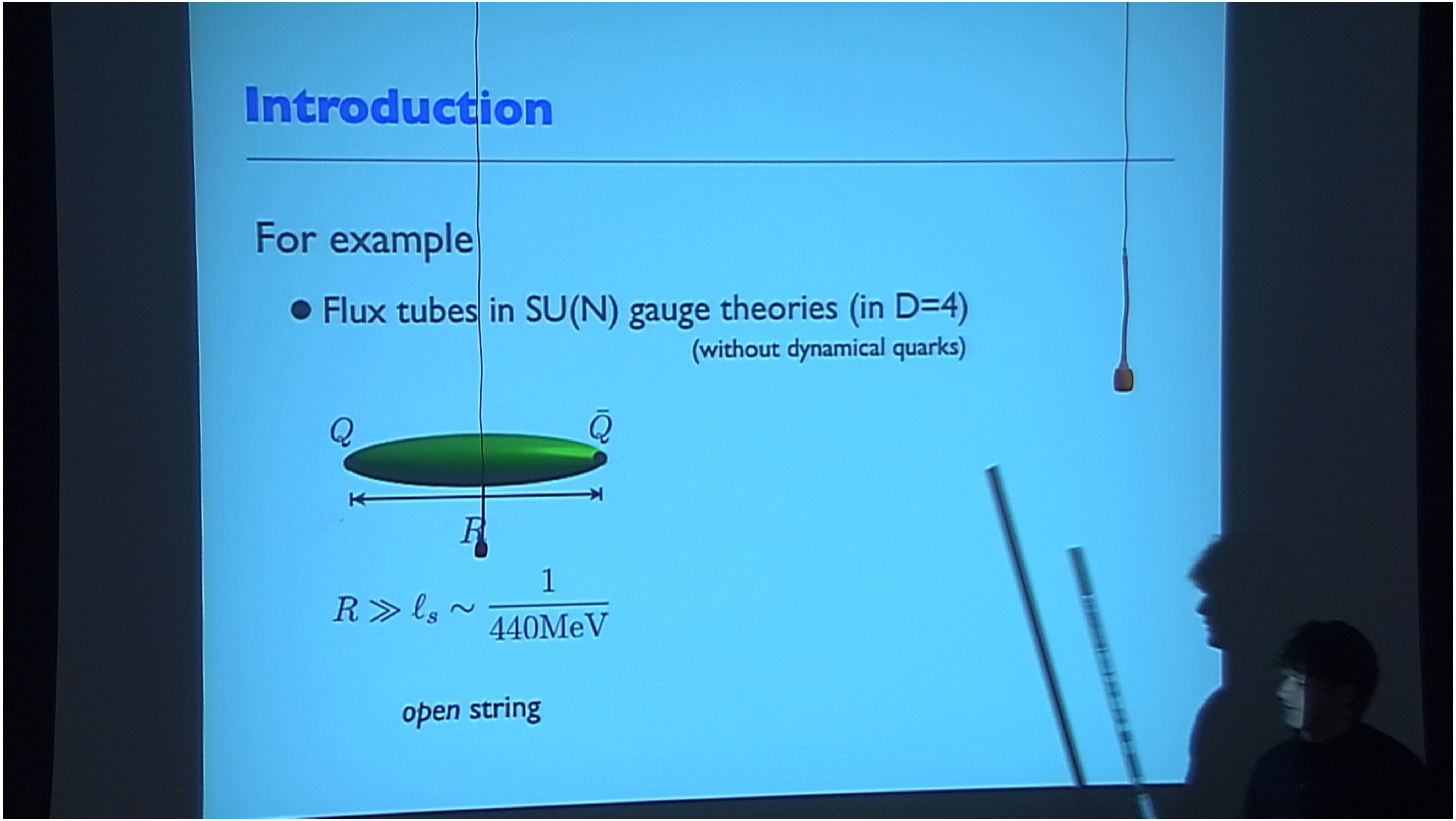
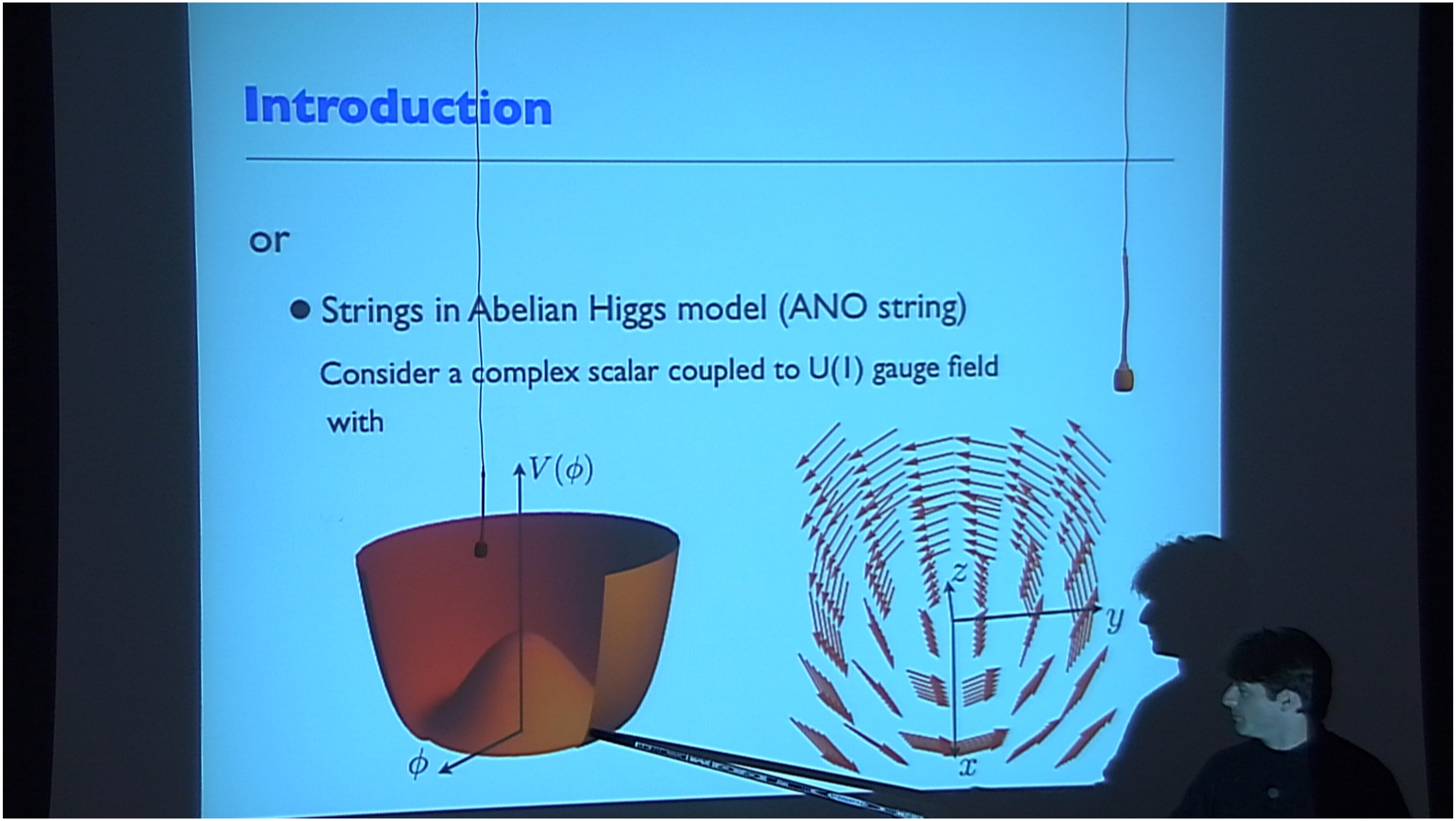
Abstract: String-like objects arise in many quantum field theories. Well known examples include flux tubes in QCD and cosmic strings. To a first approximation, their dynamics is governed by the Nambu-Goto action, but for QCD flux tubes numerical calculations of the energy levels of these objects have become so accurate that a systematic understanding of corrections to this simple description is desirable.

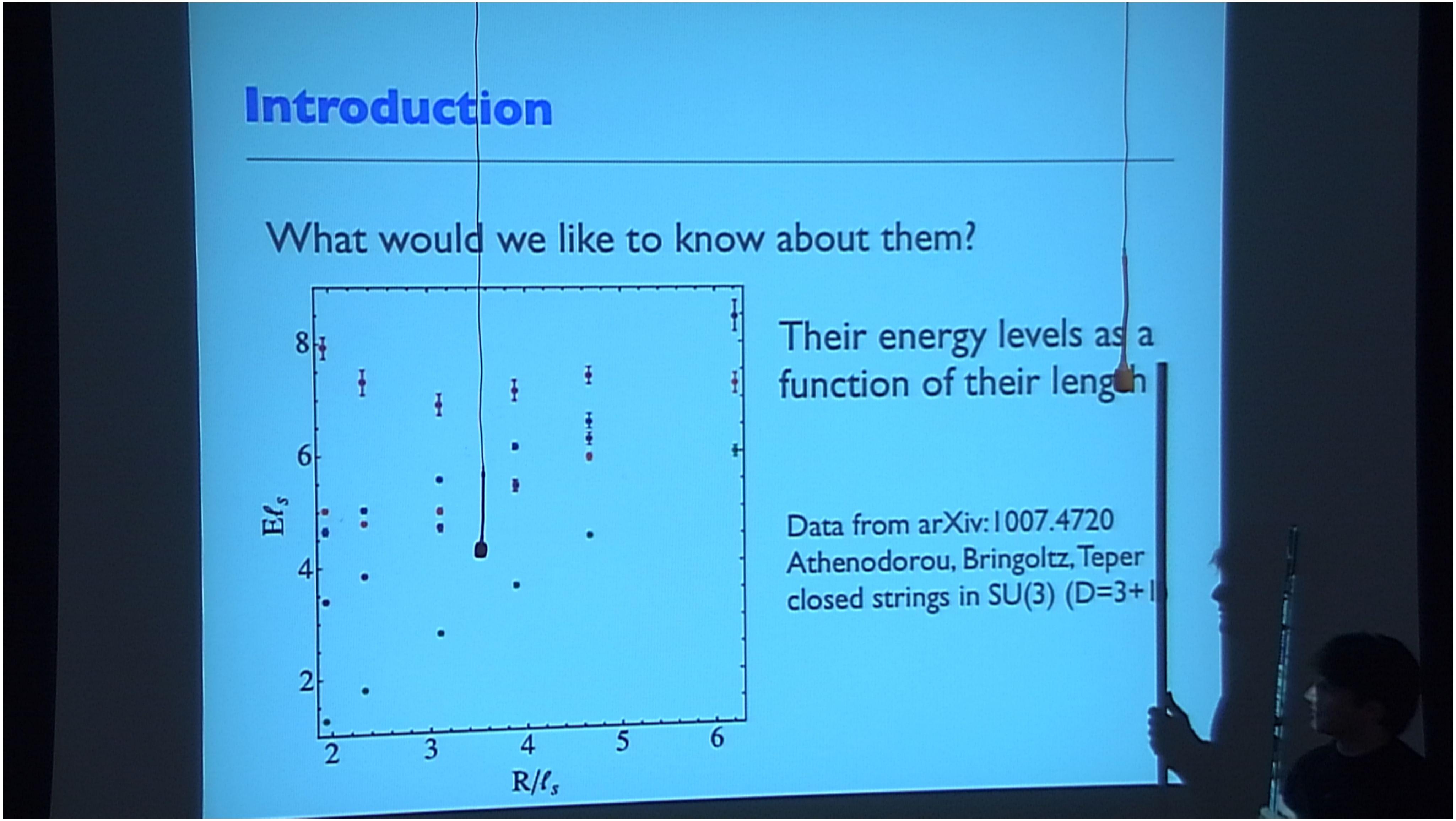
In the first part of my talk, I discuss an effective field theory describing long relativistic strings. The construction parallels that of the chiral Lagrangian in that it is based on the pattern of symmetry breaking. To compare with previous works, I will present the results of the calculation of the S-matrix describing the scattering of excitations on the string worldsheet.

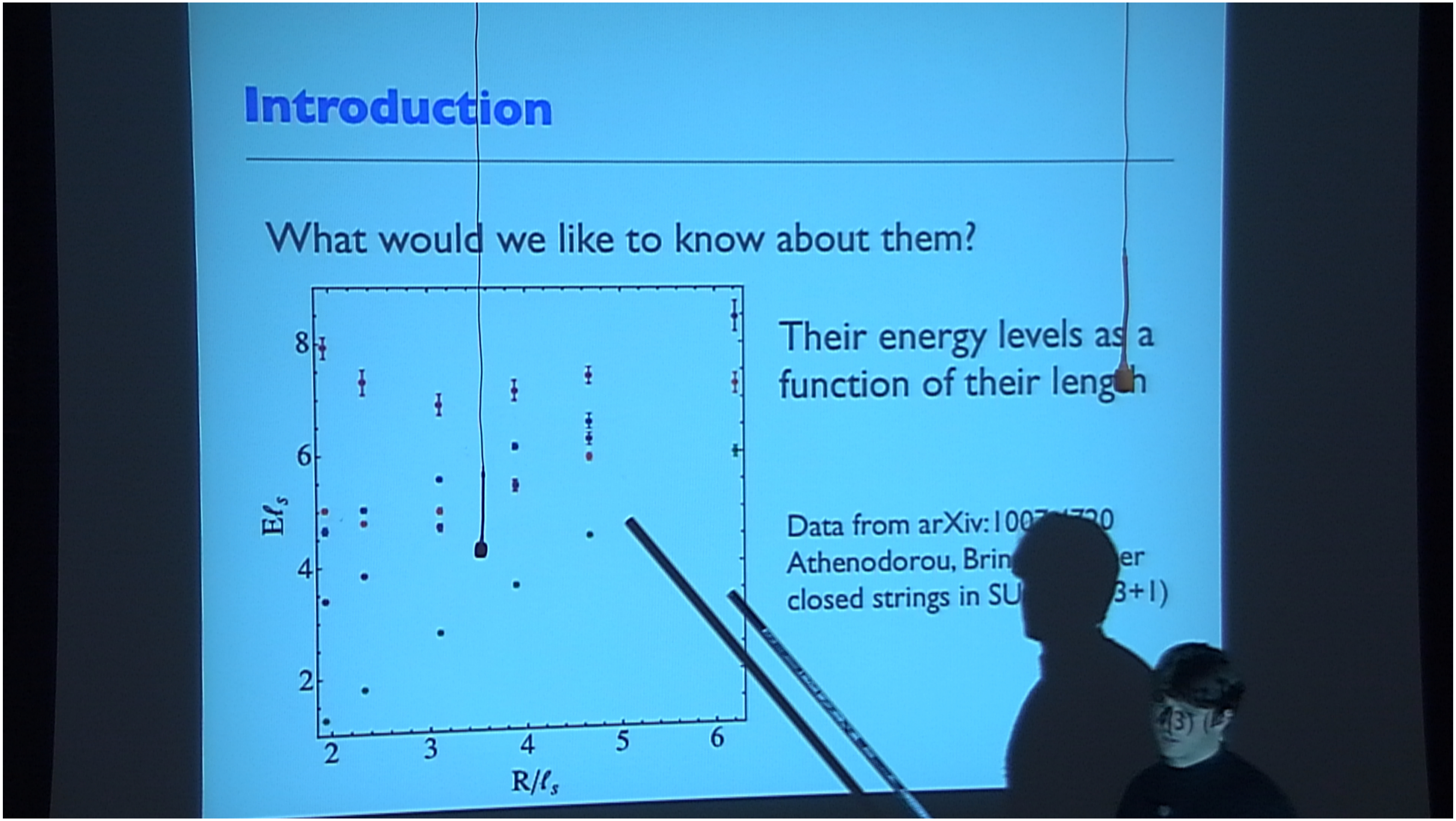
In the second part of my talk, I will discuss critical strings from the same point of view and show that the worldsheet S-matrix in this case is non-trivial but can be calculated exactly. I will show that it encodes the familiar square-root formula for the energy levels of the string, the Hagedorn behavior of strings, and argue that the theory on the string worldsheet behaves like a 1+1 dimensional theory of quantum gravity rather than a field theory.

If time permits, I will return to the task of computing the energy levels of flux-tubes using lessons learned from the second part of my talk.


Introduction
For example
Flux tubes in SU(N) gauge theories (in D=4)
(without dynamical quarks)
Q
$\bar{Q}$
R
$R \gg \ell_s \sim \frac{1}{440\text{MeV}}$
open string


Introduction
or
Strings in Abelian Higgs model (ANO string)
Consider a complex scalar coupled to U(1) gauge field
with
V(φ)
φ
z
y
x


Introduction
What would we like to know about them?
Their energy levels as a
function of their leng h
Data from arXiv:1007.4720
Athenodorou, Bringoltz, Teper
closed strings in SU(3) (D=3+1
8
6
4
2
$E\ell_s$
2
3
4
5
6
R/ℓ_s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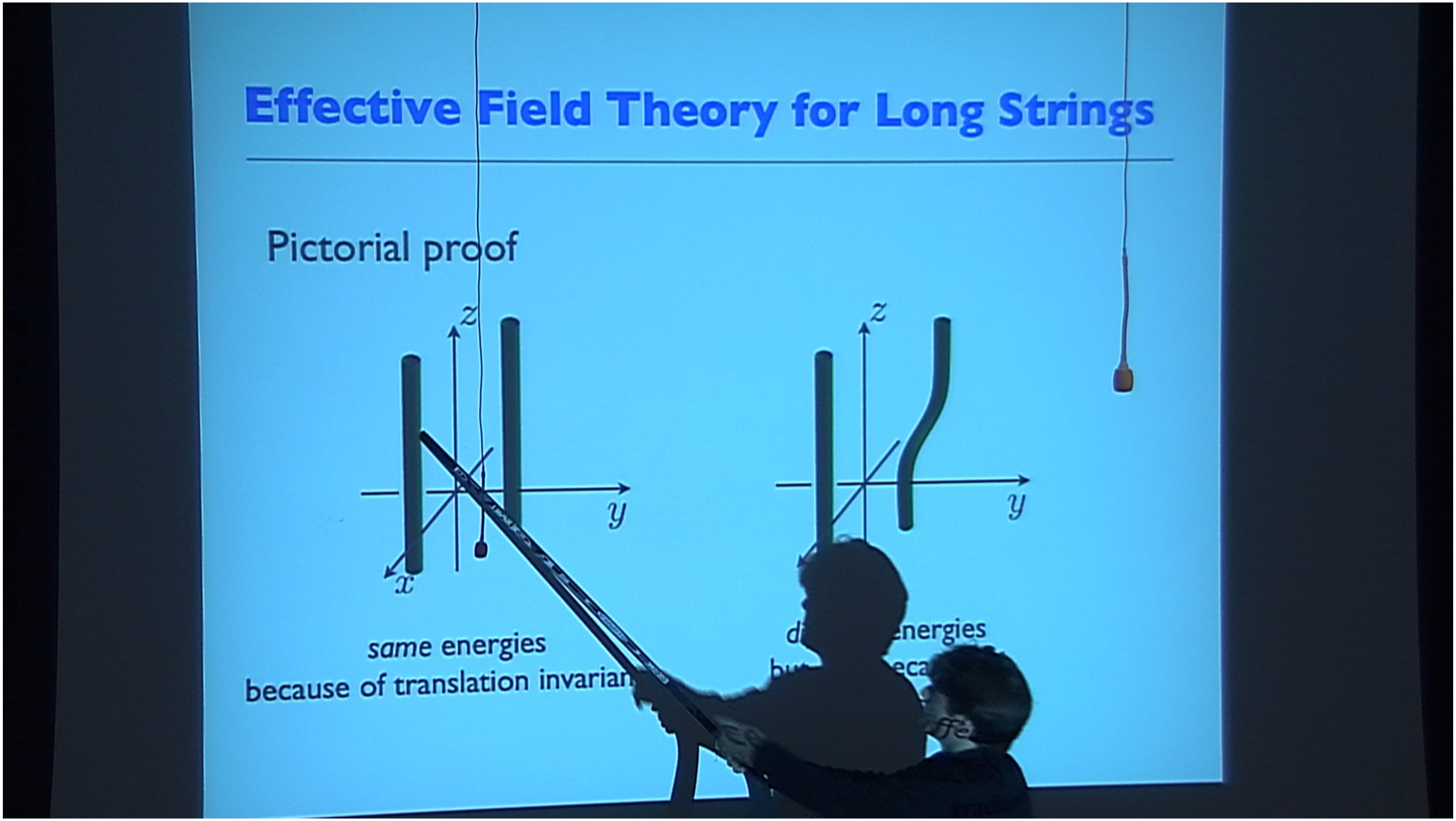
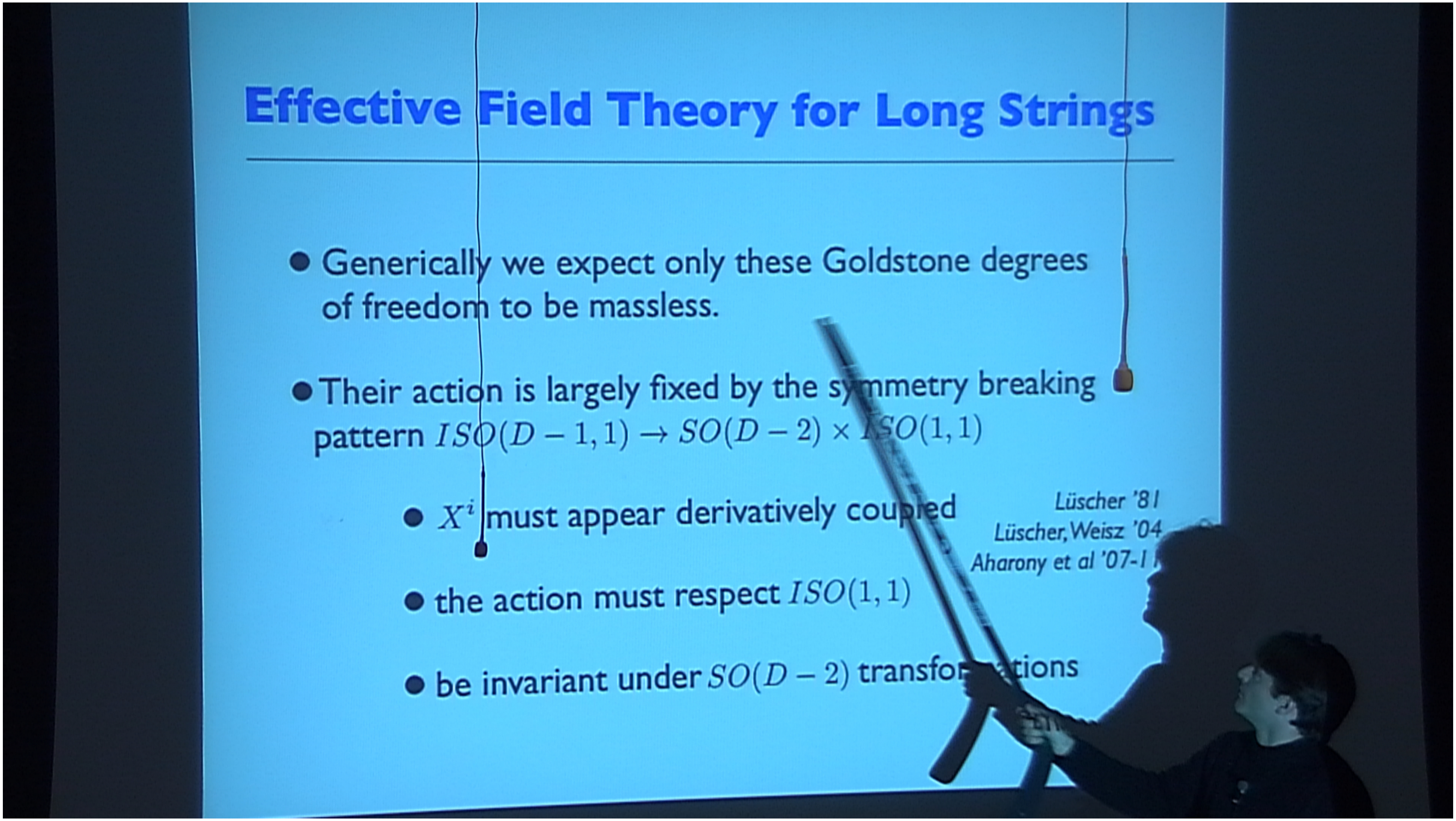
Their energy levels as a function of their length
Data from arXiv:100
Athenodorou, Brin er
closed strings in SU 3+1)
Eℓs
8
6
4
2
2
3
4
5
6
R/ℓs

Effective Field Theory for Long Strings

Why should there be a single effective theory for different kinds of long strings?

- Typical excitations have $E \gtrsim 1/\ell_s$,
- but there are massless excitations with $E \sim 1/R$ because of Goldstone's theorem.
- For long strings $R \gg \ell_s$, the low lying energy levels are thus determined by the dynamics of the Goldstone bosons.


Effective Field Theory for Long Strings
Pictorial proof
z
y
x
z
y
same energies
because of translation
energies


Effective Field Theory for Long Strings
Generically we expect only these Goldstone degrees of freedom to be massless.
Their action is largely fixed by the symmetry breaking pattern $ISO(D-1,1) \to SO(D-2) \times ISO(1,1)$
X^i must appear derivatively coupled
Lüscher '81
Lüscher, Weisz '04
Aharony et al '07-1
the action must respect $ISO(1,1)$
be invariant under $SO(D-2)$ transformations

Effective Field Theory for Long Strings

This implies

$$S = -\frac{1}{\ell_s^2}\int d^2\sigma \sqrt{-\det h_{\alpha\beta}}\left(1 + \frac{\ell_s^2}{\alpha_0}\left(K_{\alpha}^{\mu\,\alpha}\right)^2 + \dots\right)$$

using the convenient notation

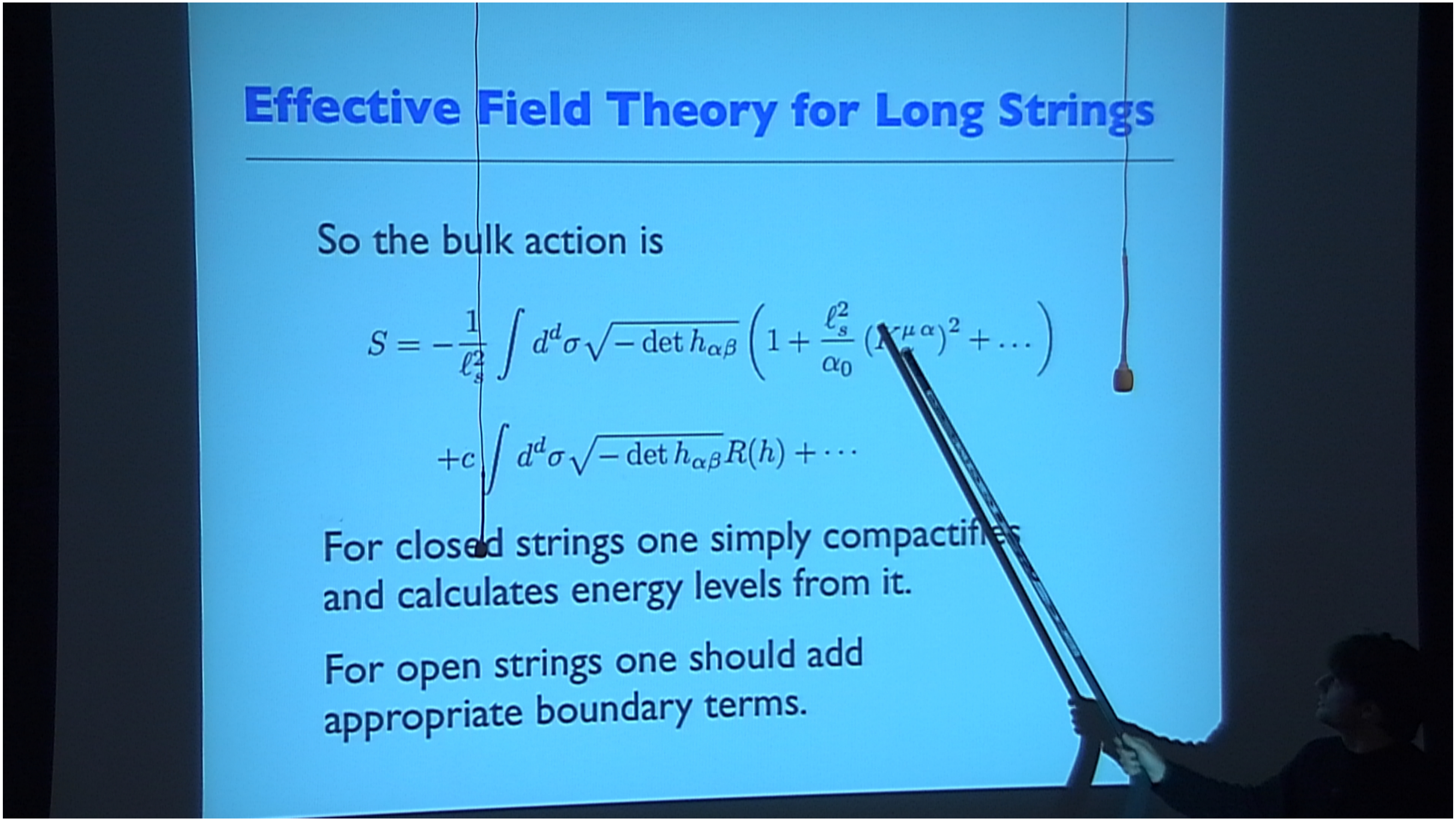
$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \,,\; X^0 = \tau \,,\; X^1 = \sigma.$$

The extrinsic curvature term enters at $\mathcal{O}\left(\frac{1}{R^7}\right)$ making the behavior of long strings rather universal.

Effective Field Theory for Long Strings

So far this was all classical. To preserve the convenient covariant form of the action in the quantum theory, we need a regulator that respects the symmetry.

$$S = -\frac{1}{\ell_s^2}\int d^2\sigma \sqrt{-\det h_{\alpha\beta}}\left(1+\frac{\ell_s^2}{\alpha_0}\left(K_{\alpha}^{\mu\,\alpha}\right)^2+\dots\right)$$


Effective Field Theory for Long Strings
So the bulk action is
$S = -\frac{1}{\ell_s^2}\int d^d\sigma\sqrt{-\det h_{\alpha\beta}}\left(1+\frac{\ell_s^2}{\alpha_0}(K^{\mu\,\alpha}{}_{\alpha})^2+\dots\right)$
$+c\int d^d\sigma\sqrt{-\det h_{\alpha\beta}}R(h)+\cdots$
For closed strings one simply compactifies
and calculates energy levels from it.
For open strings one should add
appropriate boundary terms.

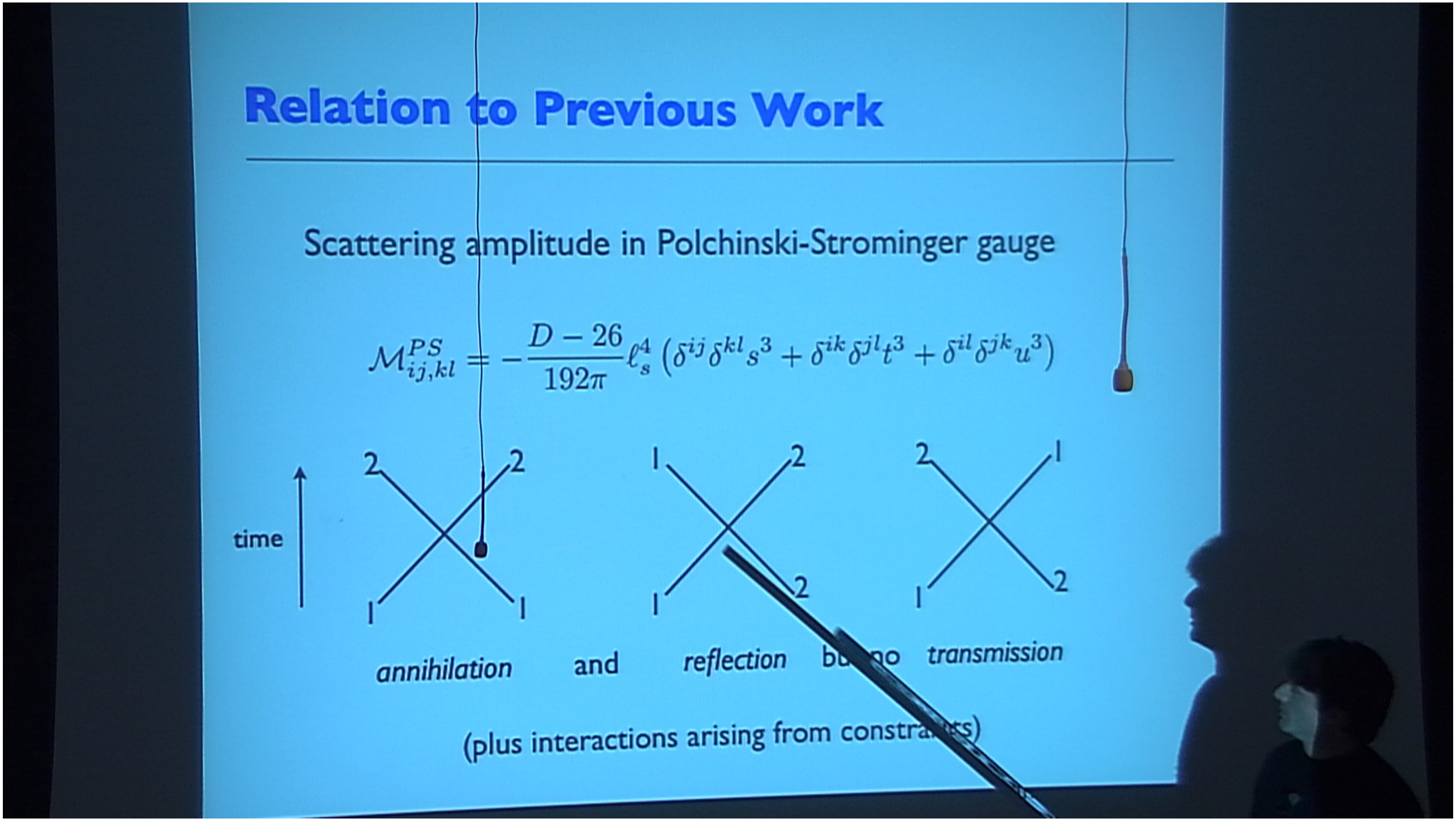
Relation to Previous Work

Polchinski-Strominger (1991)

Only works in D=26.
Away from D=26, one must keep track of the gauge fixing determinant.

One can borrow from the quantization of the Polyakov action with $\phi = \ln \partial_\alpha X^\mu \partial^\alpha X_\mu / 2$ and conjecture

$$S_{PS} = \int d^2\sigma \left(-\frac{1}{2\ell_s^2} \left(\partial_\alpha X^\mu\right)^2 - \frac{26 - D}{24\pi} \frac{\left(\partial_\alpha \partial_\beta X^\mu \partial^\beta X_\mu\right)^2}{\left[\left(\partial_\gamma X^\nu\right)^2\right]^2} + \dots \right)$$


Relation to Previous Work
Scattering amplitude in Polchinski-Strominger gauge
$\mathcal{M}^{PS}_{ij,kl} = -\frac{D-26}{192\pi}\ell_s^4\left(\delta^{ij}\delta^{kl}s^3 + \delta^{ik}\delta^{jl}t^3 + \delta^{il}\delta^{jk}u^3\right)$
time
annihilation and reflection bu no transmission
(plus interactions arising from constra ts)

Relation to Previous Work

Scattering amplitude in Polchinski-Strominger gauge

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annihilation and *reflection* but no *transmission*

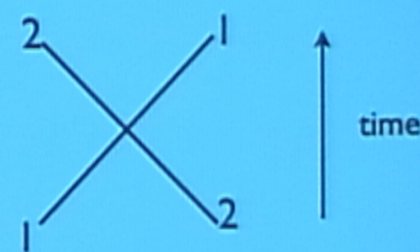
(plus interactions arising from constraints)

Relation to Previous Work

Scattering amplitude in static gauge at tree level

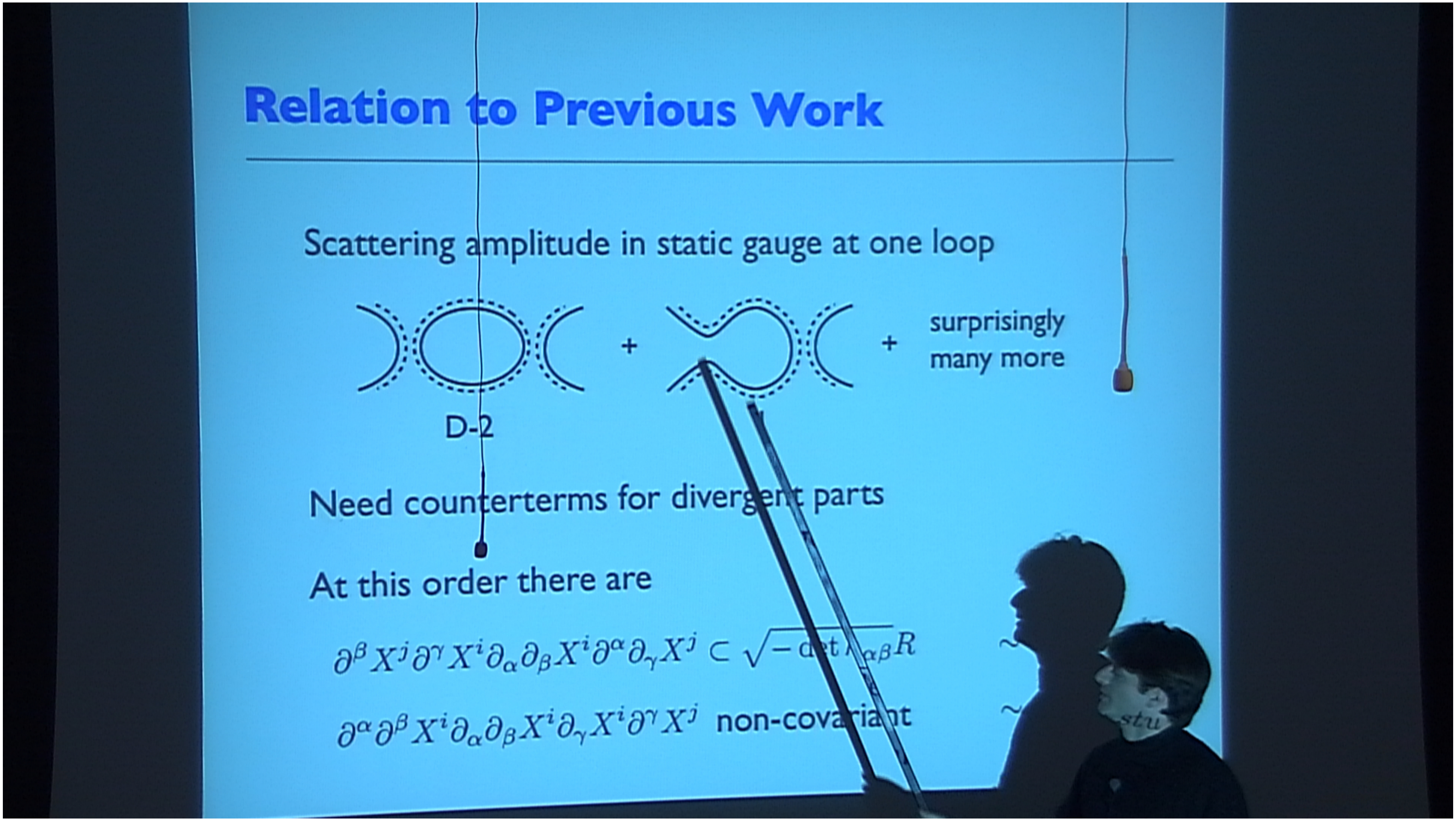
c2 c3

$$\mathcal{M}_{ij,kl} = -\ell_s^2(\delta^{ik}\delta^{jl} su + \delta^{il}\delta^{jk} st)$$



Lorentz invariant theory only has *transmission*

(consistent with interactions arising from constraints in PS gauge)



Relation to Previous Work
Scattering amplitude in static gauge at one loop
+
+
surprisingly many more
D-2
Need counterterms for divergent parts
At this order there are
$\partial^\beta X^j \partial^\gamma X^i \partial_\alpha \partial_\beta X^i \partial^\alpha \partial_\gamma X^j \subset \sqrt{-\det h_{\alpha\beta}}R$ ~
$\partial^\alpha \partial^\beta X^i \partial_\alpha \partial_\beta X^i \partial_\gamma X^i \partial^\gamma X^j$ non-covariant ~ stu

Relation to Previous Work

Divergent part

$$\mathcal{M}_{ij,kl} = -\frac{D-8}{96\pi\bar{\epsilon}}\delta^{ij}\delta^{kl}\ell_s^4 stu + \text{crossings}$$

can be cancelled by inclusion of Einstein-Hilbert term alone, consistent with non-linearly realized Lorentz invariance.

Vanishes for 2-d kinematics since $tu = 0$

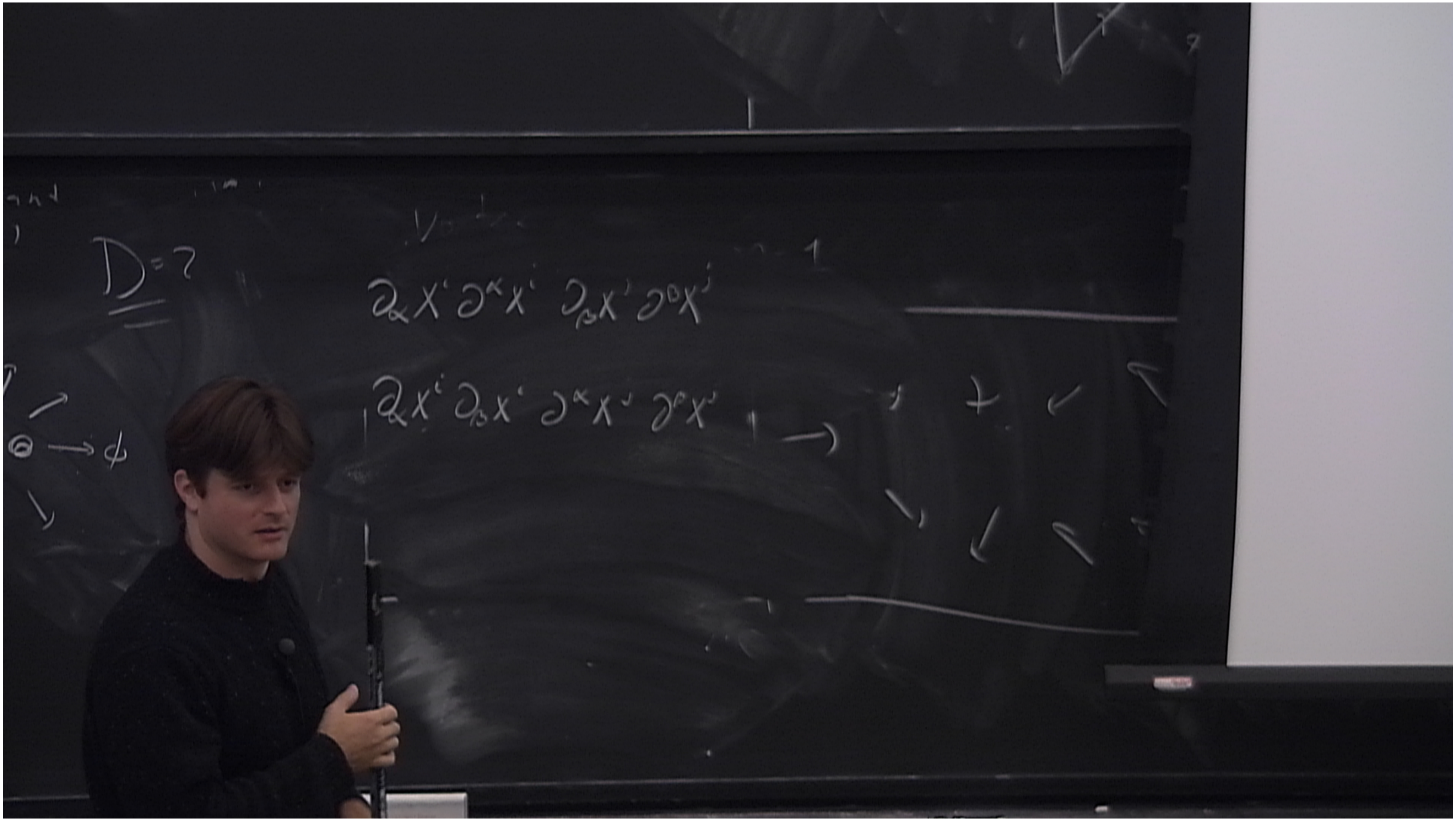
Relation to Previous Work

Conclusion:

No extra terms should be added in static gauge provided the regularization procedure respects the non-linearly realized Lorentz invariance.

If the cutoff does not respect non-linearly realized Lorentz invariance, non-covariant counterterms must be added.

While $R\frac{1}{\partial^2}R$ is local in conformal gauge and appears in the Wilsonian action, in static gauge it is not and appears in the 1-PI effective action.

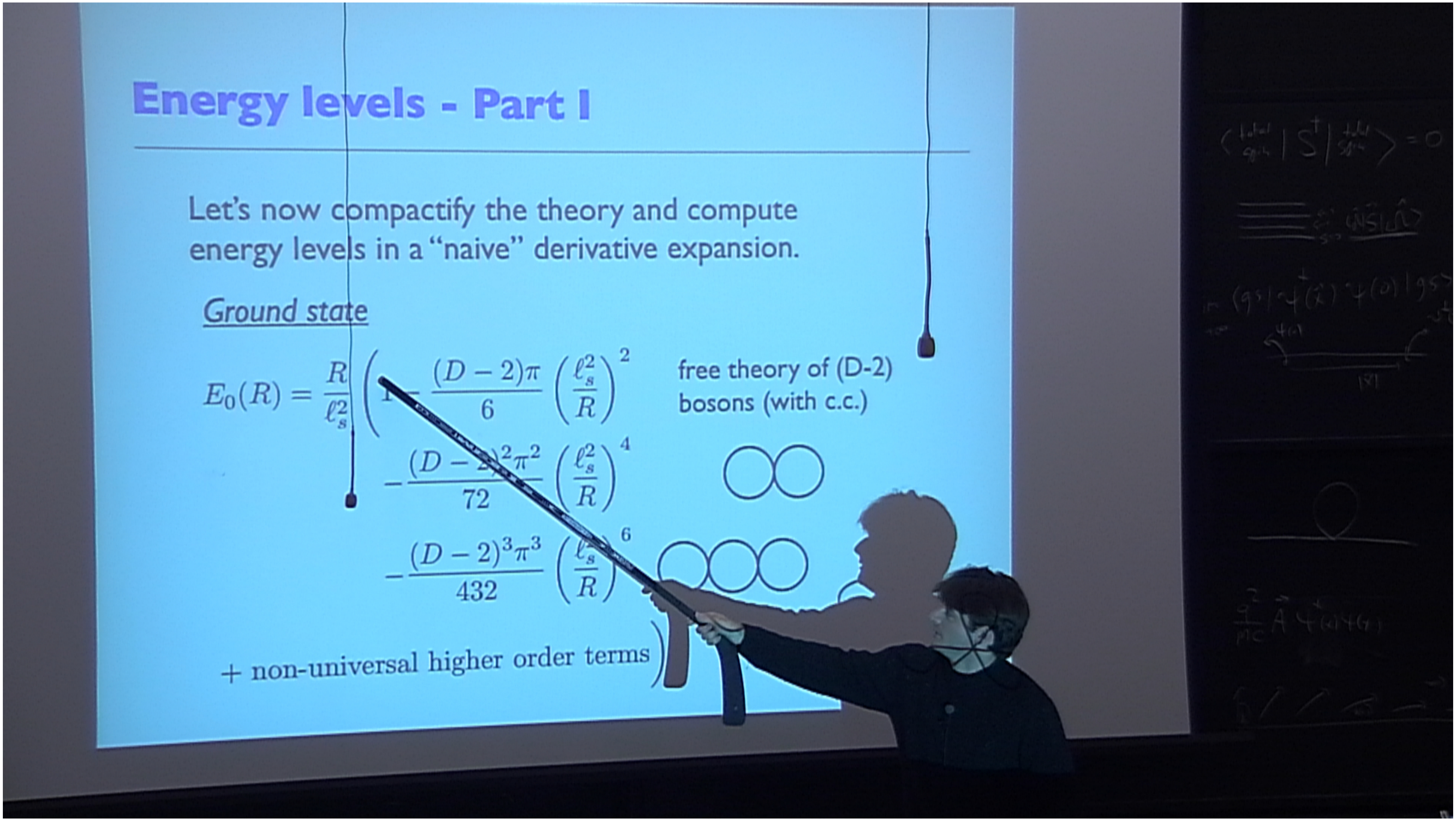

D = ?
∂_α X^i ∂^α X^i ∂_β X^j ∂^β X^j
∂_α X^i ∂_β X^i ∂^α X^j ∂^β X^j

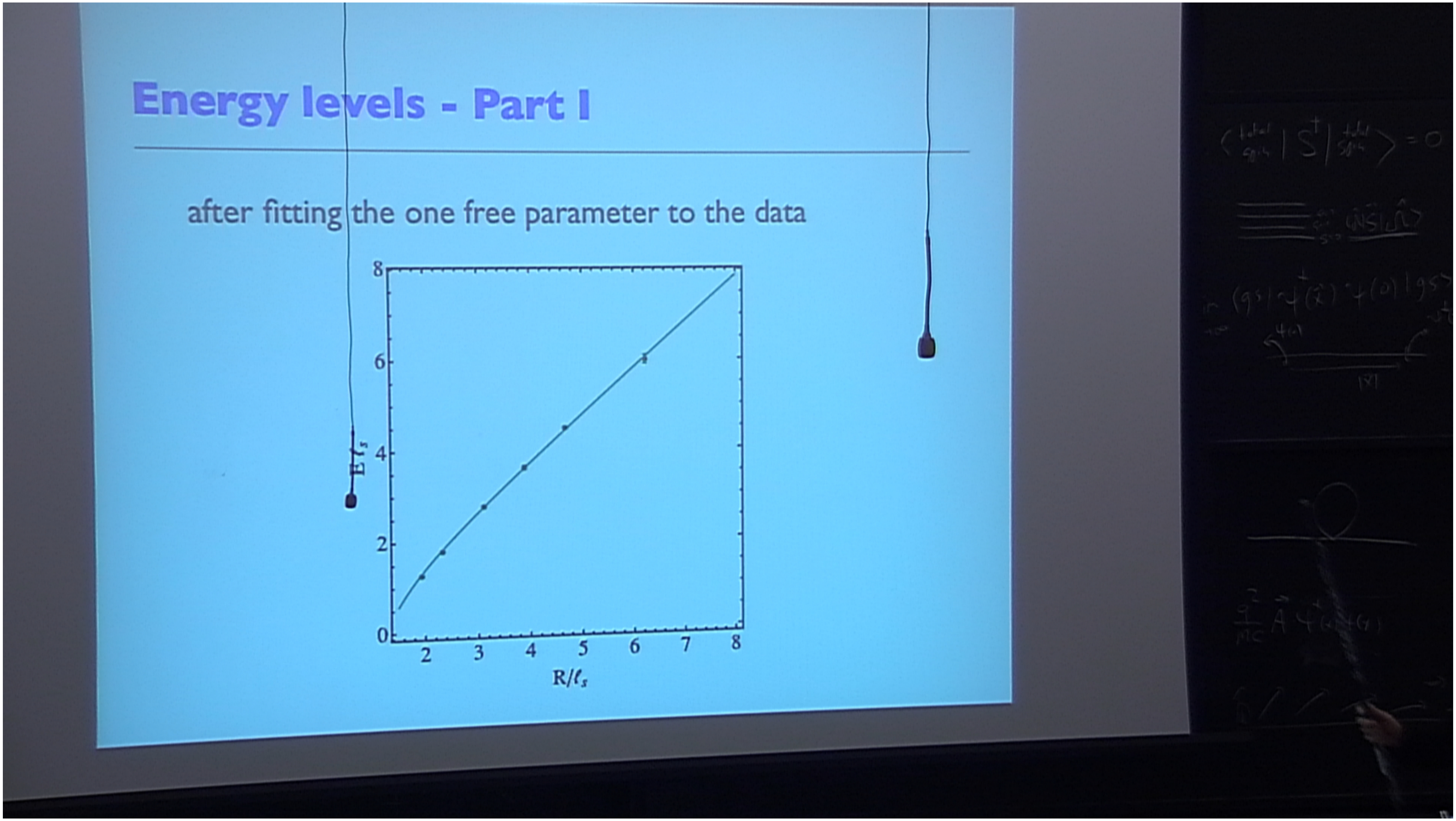
Relation to Previous Work

Conclusion:

$$\mathcal{M}^{PS}_{ij,kl} = -\frac{D-26}{192\pi}\ell_s^4\left(\delta^{ij}\delta^{kl}s^3 + \delta^{ik}\delta^{jl}t^3 + \delta^{il}\delta^{jk}u^3\right)$$

From an effective field theory viewpoint, what is special about the critical string is the absence of annihilations and reflections.


Energy levels - Part I
Let's now compactify the theory and compute energy levels in a "naive" derivative expansion.
Ground state
$E_0(R) = \frac{R}{\ell_s^2}\Big(1 - \frac{(D-2)\pi}{6}\left(\frac{\ell_s^2}{R}\right)^2 - \frac{(D-2)^2\pi^2}{72}\left(\frac{\ell_s^2}{R}\right)^4 - \frac{(D-2)^3\pi^3}{432}\left(\frac{\ell_s^2}{R}\right)^6$
+ non-universal higher order terms$\Big)$
free theory of (D-2) bosons (with c.c.)


Energy levels - Part I
after fitting the one free parameter to the data
E ℓs
R/ℓs
0
2
4
6
8
2
3
4
5
6
7
8

Energy levels - Part I

<u>1 left-mover and 1 right-mover (D=4)</u>

$$E_2(R) = E_0(R) + 2(E_1(R) - E_0(R))$$
$$- \frac{8\pi^2}{R}\left(\frac{\ell_s}{R}\right)^2 + \frac{80\pi^3}{3R}\left(\frac{\ell_s}{R}\right)^4$$
$$\pm \frac{4(D-26)\pi^3}{3R}\left(\frac{\ell_s}{R}\right)^4$$

upper sign for scalar and antisymmetric flavor wave function, lower sign for symmetric tensor.

Energy levels - Part I

1 left-mover and 1 right-mover (D=4)

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finally D-26 $\pm\frac{4(D-26)\pi^3}{3R}\left(\frac{\ell_s}{R}\right)^4$

Aharony et al '11

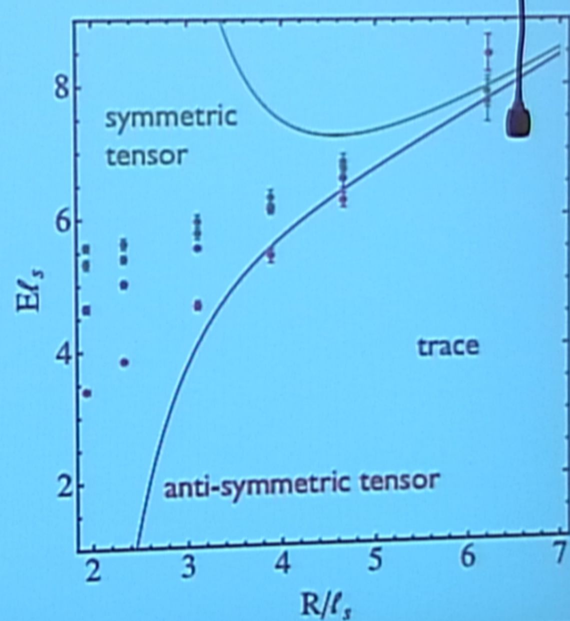
upper sign for scalar and antisymmetric flavor wave function, lower sign for symmetric tensor.

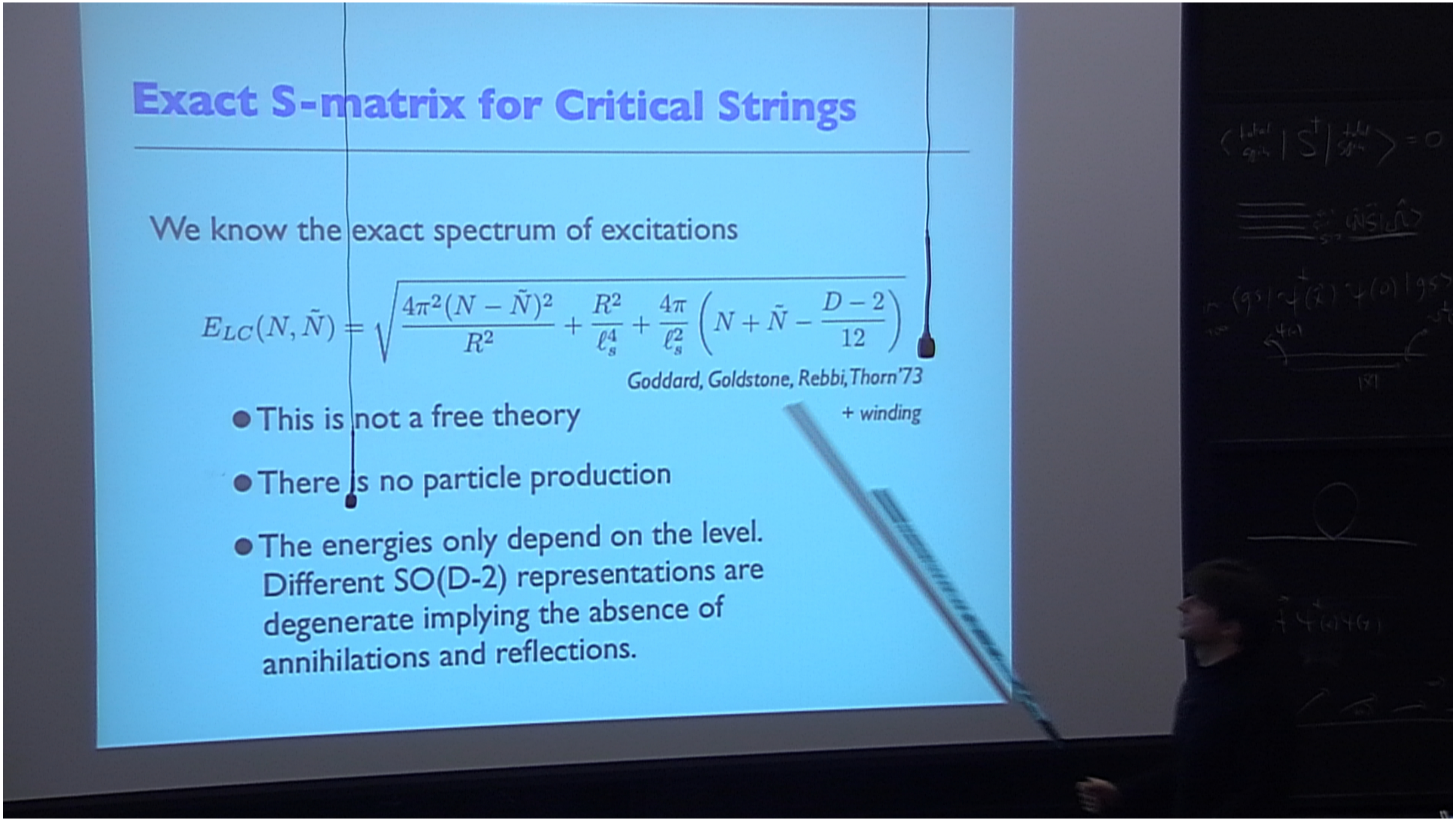
Energy levels - Part I

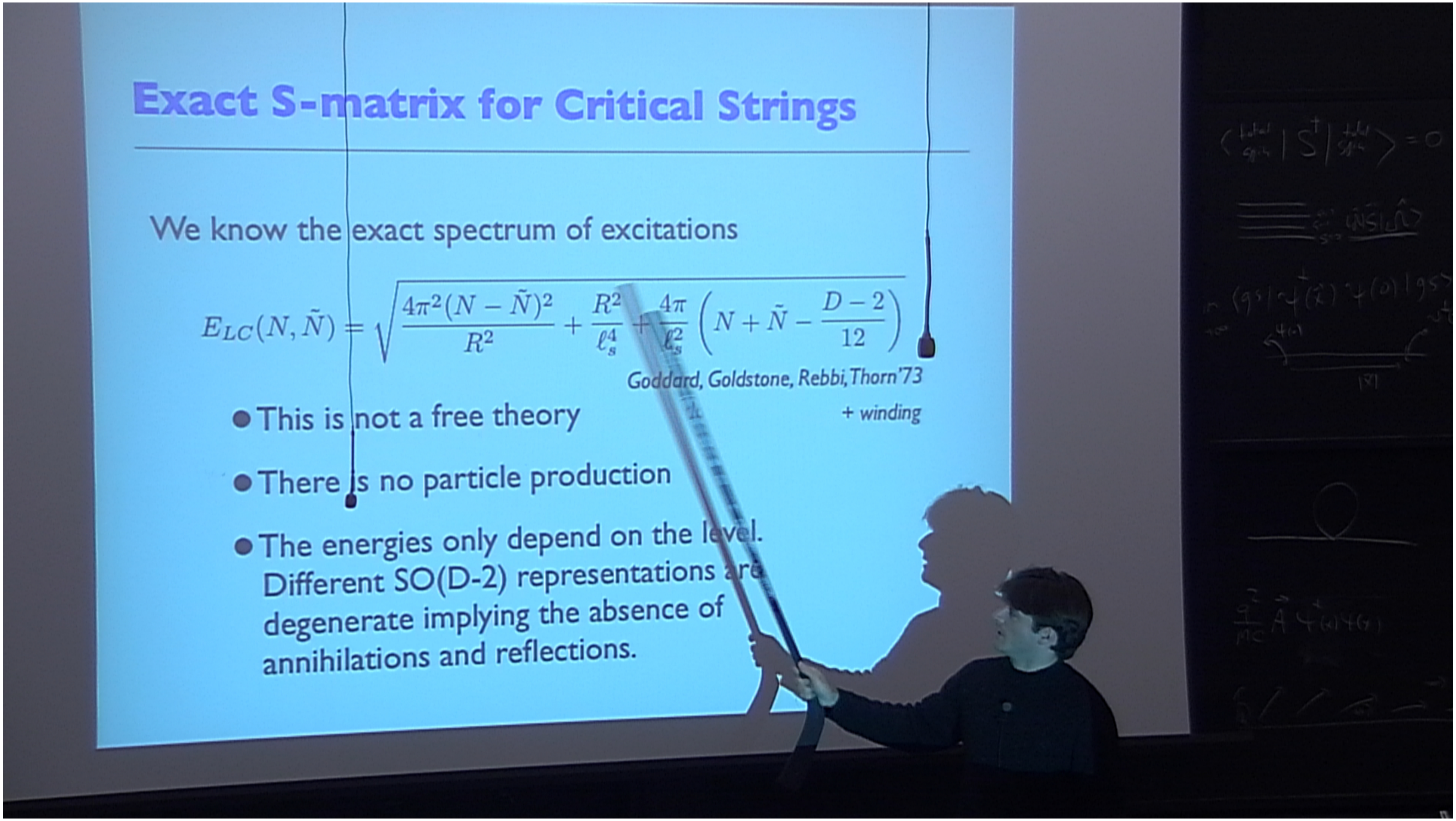
1 left-mover and 1 right-mover (D=4)

Splitting qualitatively correct, but otherwise very disappointing.

The naive derivative expansion only works where we have no data




Exact S-matrix for Critical Strings
We know the exact spectrum of excitations
$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$
Goddard, Goldstone, Rebbi, Thorn'73
+ winding
This is not a free theory
There is no particle production
The energies only depend on the level. Different SO(D-2) representations are degenerate implying the absence of annihilations and reflections.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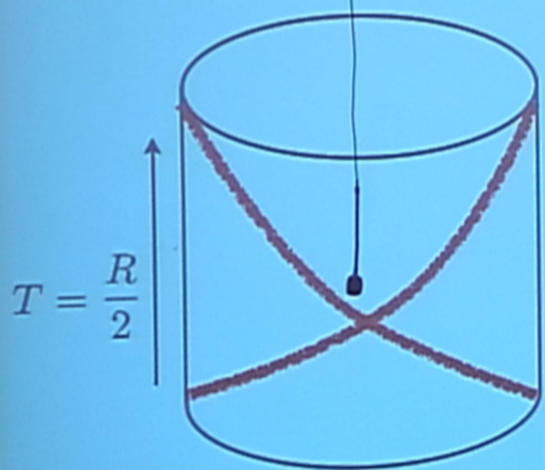
Exact S-matrix for Critical Strings

Consider scattering of a left-moving excitation and a right-moving excitation



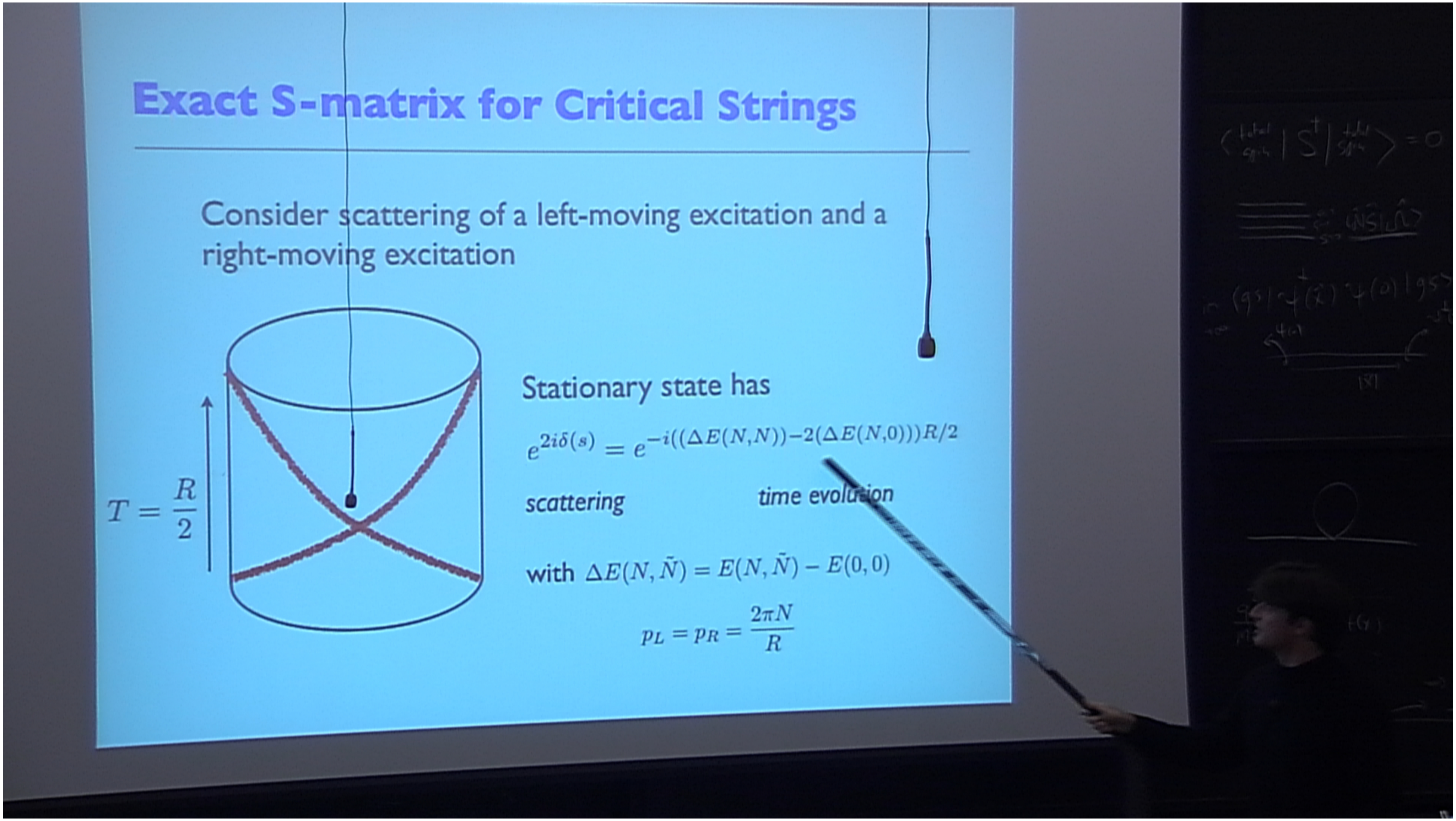
Stationary state has

$$e^{2i\delta(s)} = e^{-i((\Delta E(N,N)) - 2(\Delta E(N,0)))R/2}$$

scattering *time evolution*

with $\Delta E(N, \tilde{N}) = E(N, \tilde{N}) - E(0,0)$

$$p_L = p_R = \frac{2\pi N}{R}$$


Exact S-matrix for Critical Strings
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$T = \frac{R}{2}$
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scattering
time evolution
with $\Delta E(N,\tilde{N}) = E(N,\tilde{N}) - E(0,0)$
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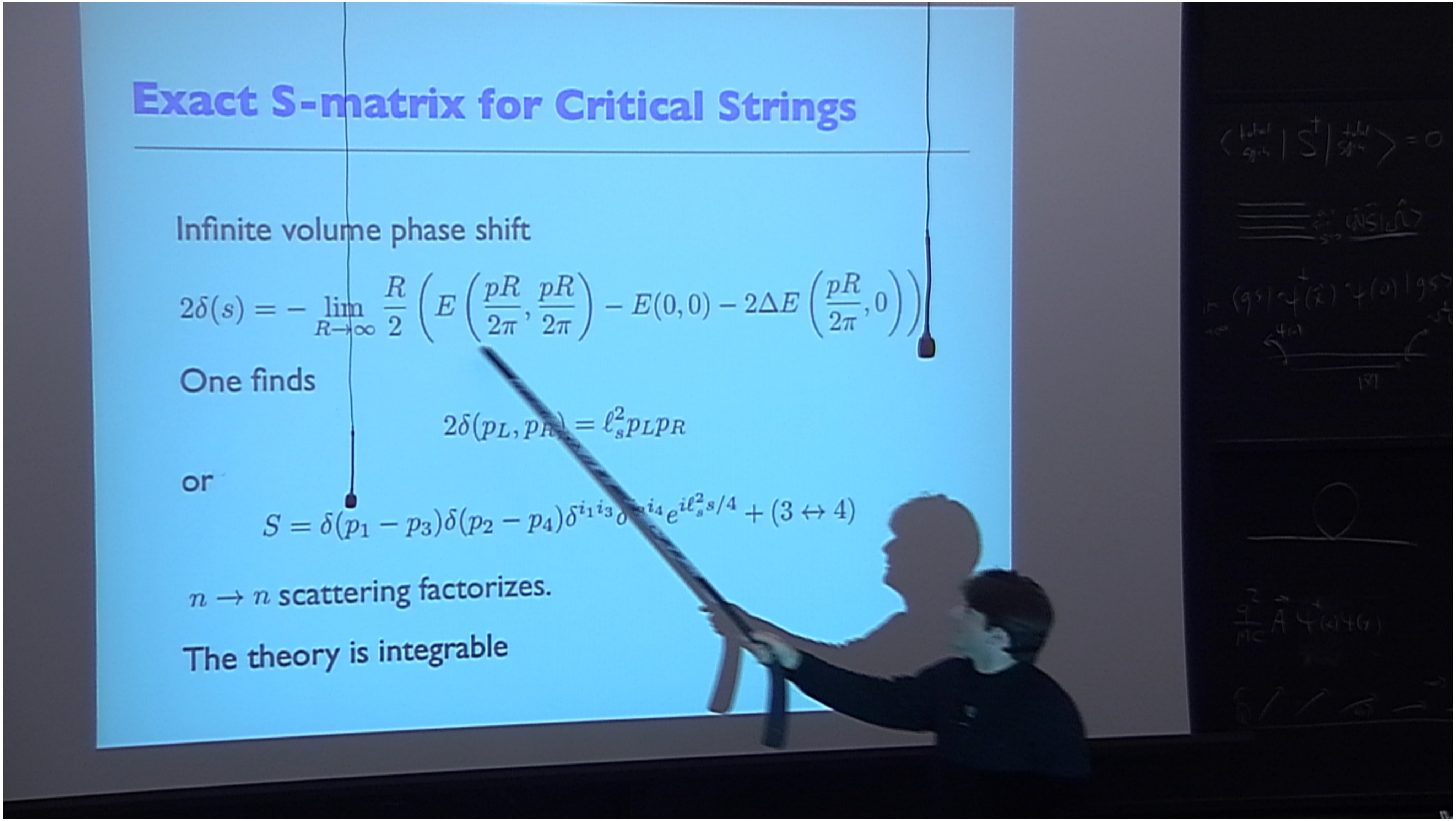
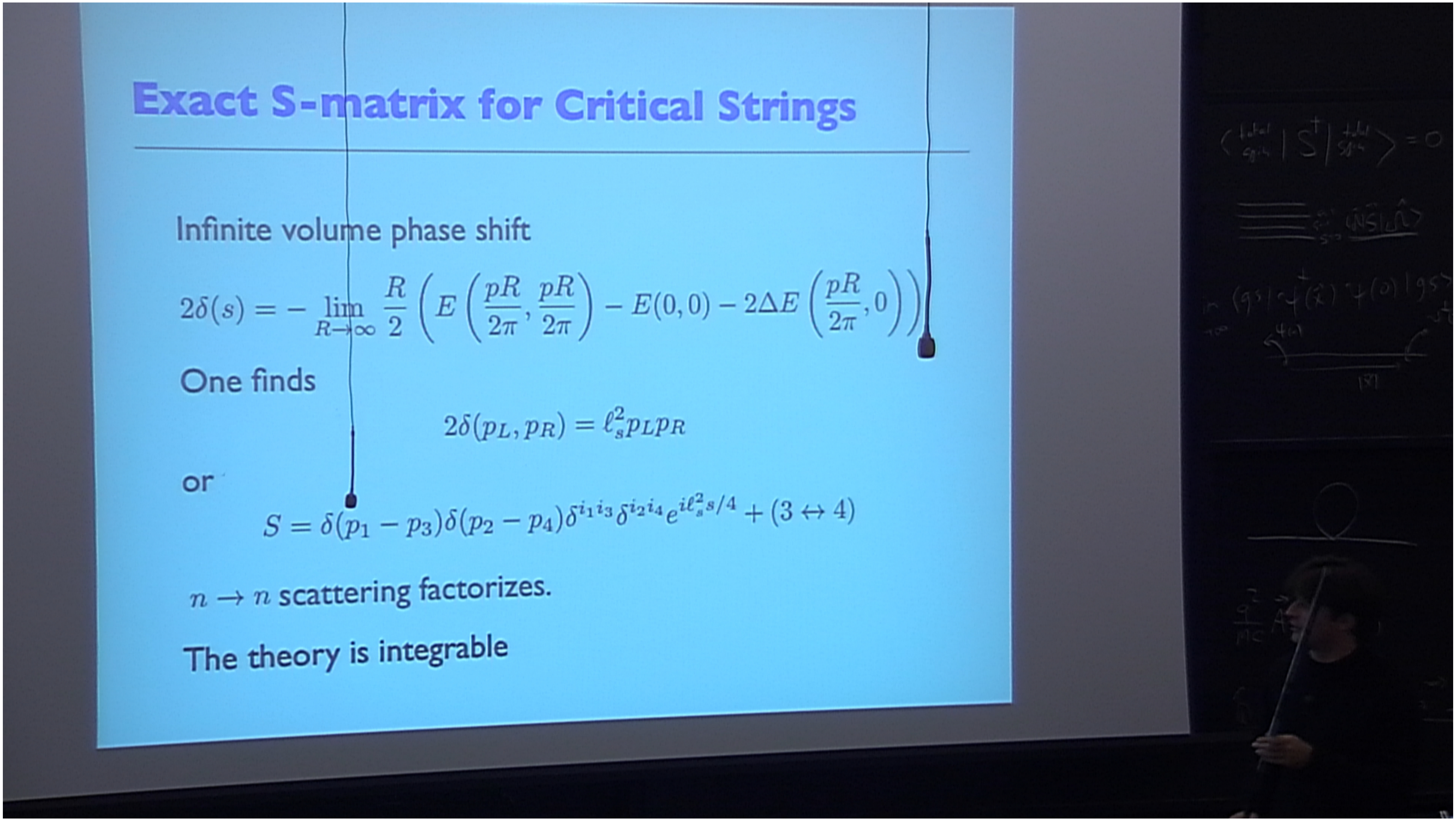

Exact S-matrix for Critical Strings
Infinite volume phase shift
$2\delta(s) = -\lim_{R\to\infty}\frac{R}{2}\left(E\left(\frac{pR}{2\pi},\frac{pR}{2\pi}\right) - E(0,0) - 2\Delta E\left(\frac{pR}{2\pi},0\right)\right)$
One finds
$2\delta(p_L, p_R) = \ell_s^2 p_L p_R$
or
$S = \delta(p_1 - p_3)\delta(p_2 - p_4)\delta^{i_1 i_3}\delta^{i_2 i_4}e^{i\ell_s^2 s/4} + (3 \leftrightarrow 4)$
$n \to n$ scattering factorizes.
The theory is integrable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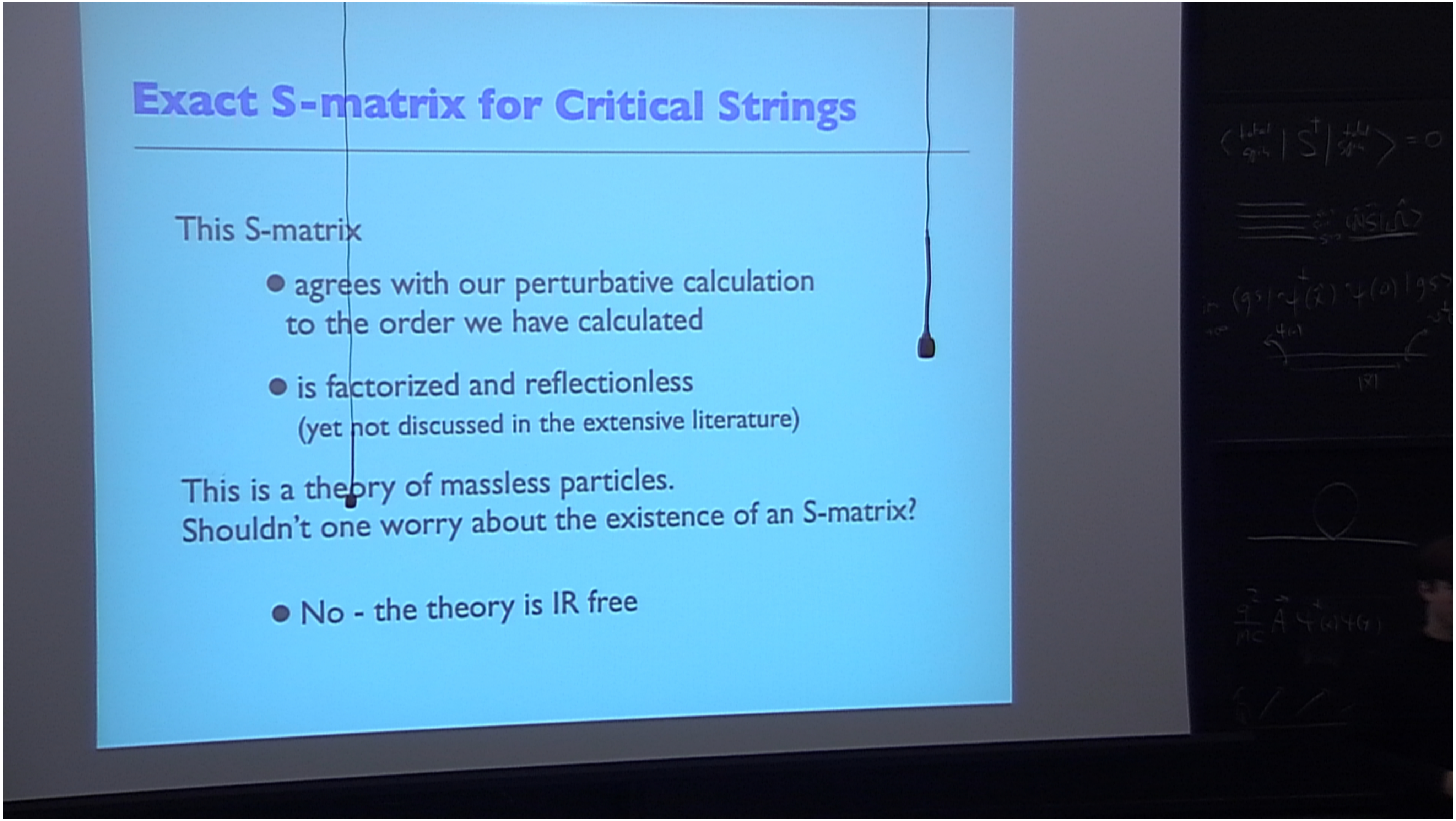
Exact S-matrix for Critical Strings

This S-matrix

- agrees with our perturbative calculation to the order we have calculated
- is factorized and reflectionless (yet not discussed in the extensive literature)

This is a theory of massless particles.
Shouldn't one worry about the existence of an S-matrix?

- No - the theory is IR free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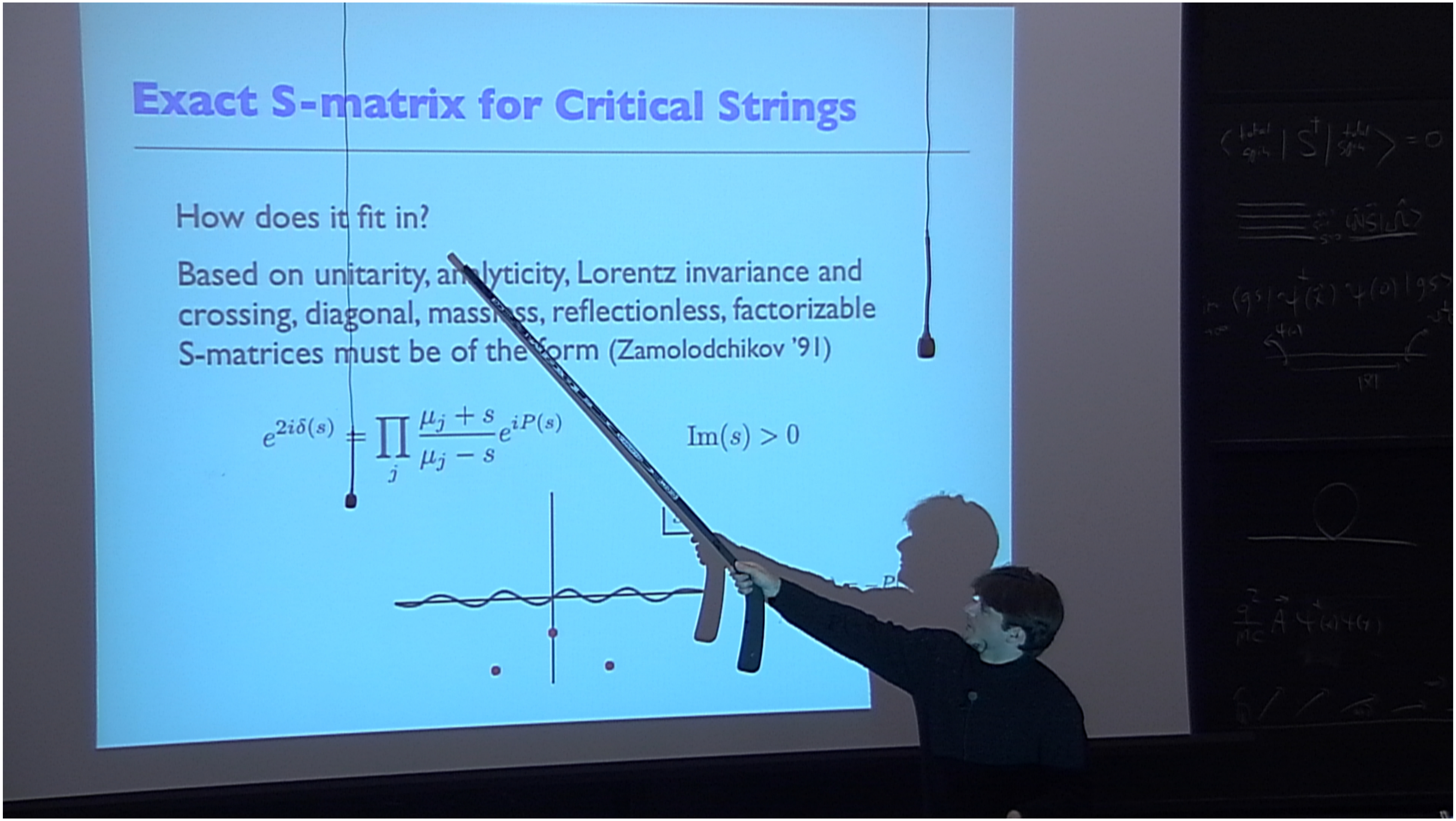
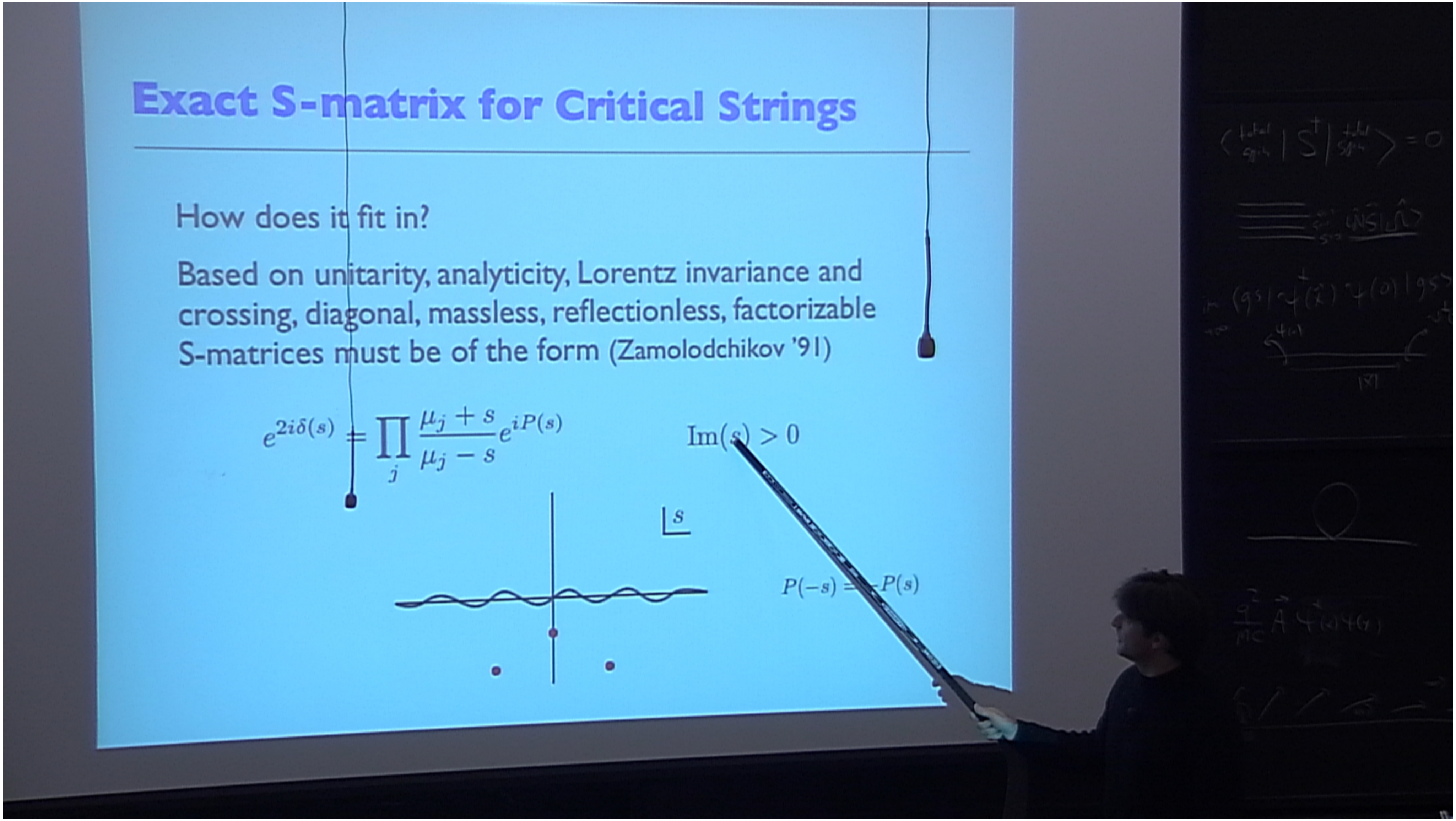

Exact S-matrix for Critical Strings
How does it fit in?
Based on unitarity, analyticity, Lorentz invariance and crossing, diagonal, massless, reflectionless, factorizable S-matrices must be of the form (Zamolodchikov '91)
$e^{2i\delta(s)} = \prod_j \frac{\mu_j + s}{\mu_j - s} e^{iP(s)}$
$\mathrm{Im}(s) > 0$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$\mathrm{Im}(s) > 0$
s
$P(-s) = -P(s)$

Exact S-matrix for Critical Strings

Locality suggests $P(s) = 0$

The simplest S-matrix is then

$$e^{2i\delta_{\text{Gold}}(s)} = \frac{iM^2 - s}{iM^2 + s}$$

Corresponding to 2d Volkov-Akulov Goldstino theory

$$S = \int d^2\sigma \left(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} - \frac{4}{M^2}\psi\partial\psi\bar{\psi}\bar{\partial}\bar{\psi} + \cdots \right)$$

Describes RG flow from tri-critical to critical Ising model

(seemingly only valid up to M, but UV completes itself)

Exact S-matrix for Critical Strings

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Describes RG flow from tri-critical to critical Ising model

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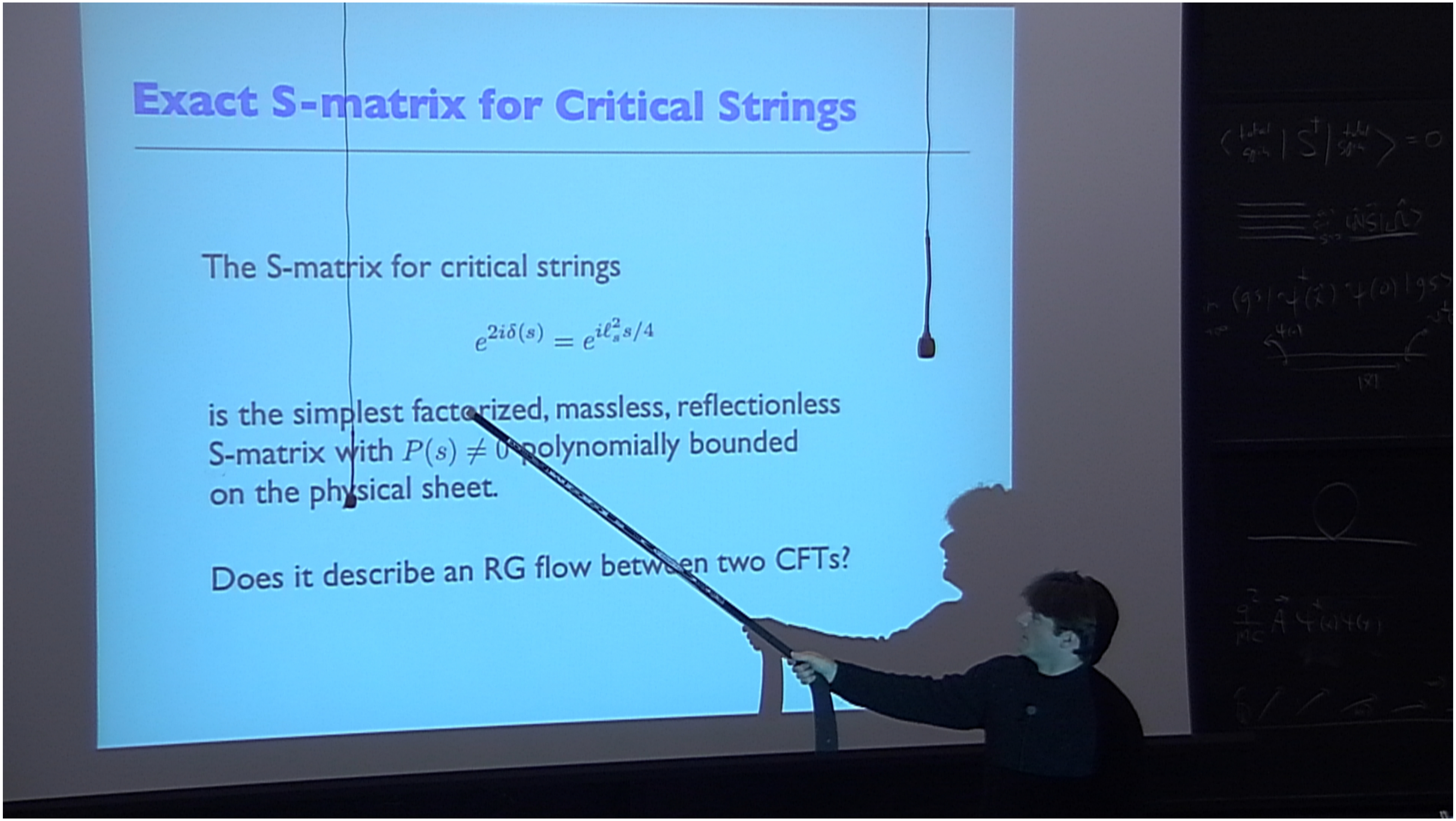
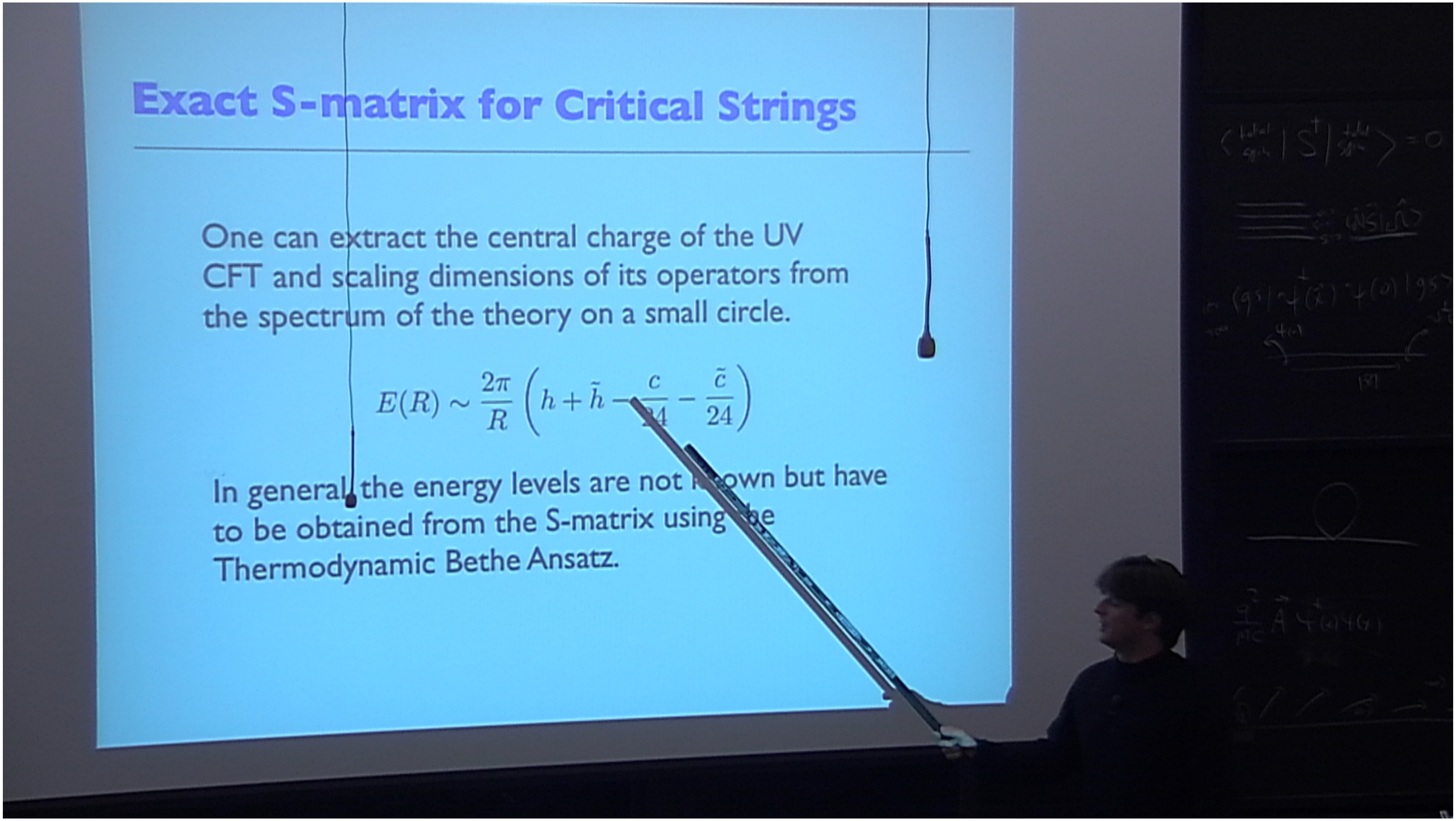

Exact S-matrix for Critical Strings
The S-matrix for critical strings
$e^{2i\delta(s)} = e^{i\ell_s^2 s/4}$
is the simplest factorized, massless, reflectionless
S-matrix with $P(s) \neq 0$ polynomially bounded
on the physical sheet.
Does it describe an RG flow between two CFTs?


Exact S-matrix for Critical Strings
One can extract the central charge of the UV CFT and scaling dimensions of its operators from the spectrum of the theory on a small circle.
$E(R) \sim \frac{2\pi}{R}\left(h + \tilde{h} - \frac{c}{24} - \frac{\tilde{c}}{24}\right)$
In general, the energy levels are not own but have
to be obtained from the S-matrix using e
Thermodynamic Bethe Ansatz.

Exact S-matrix for Critical Strings

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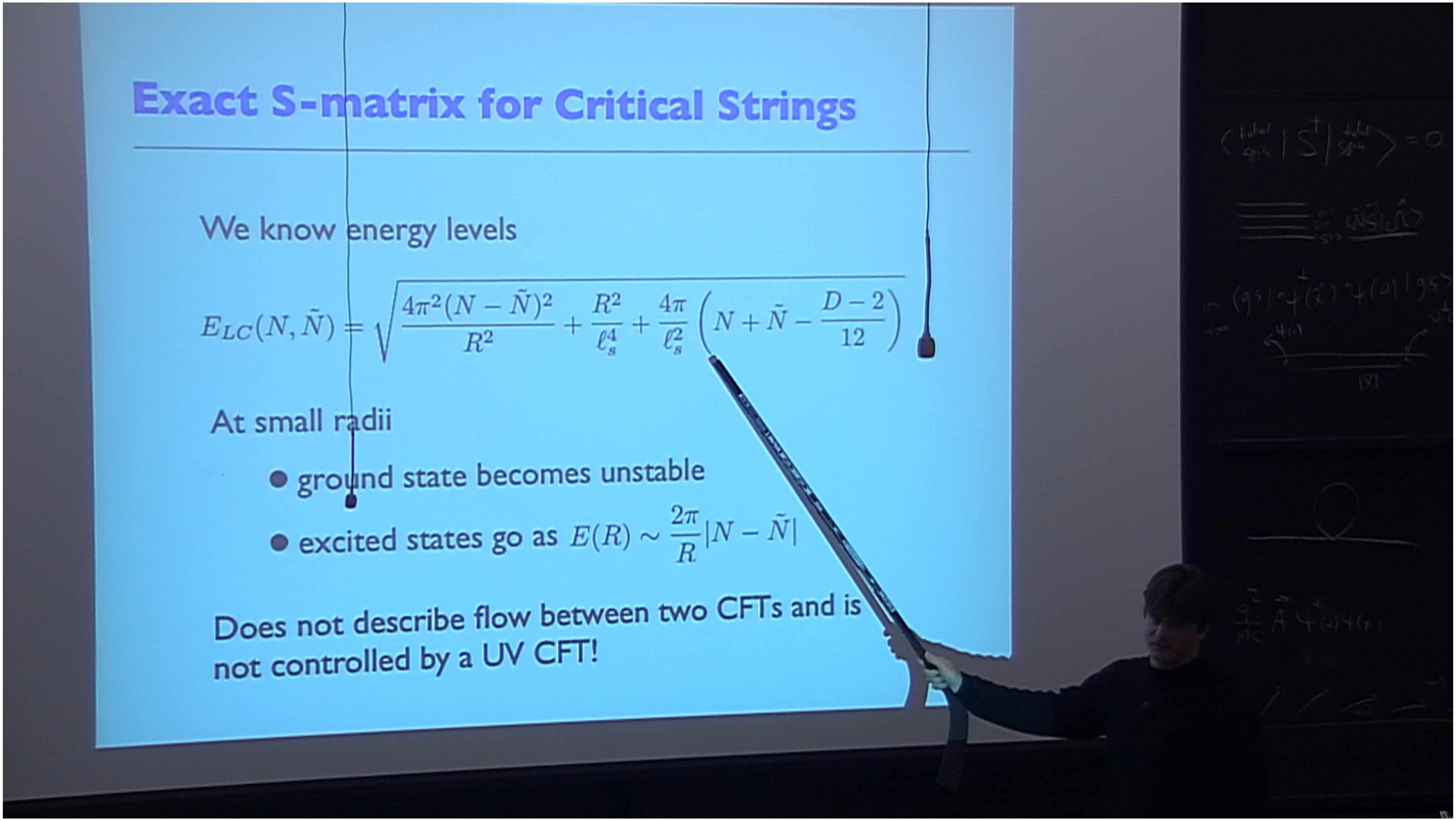
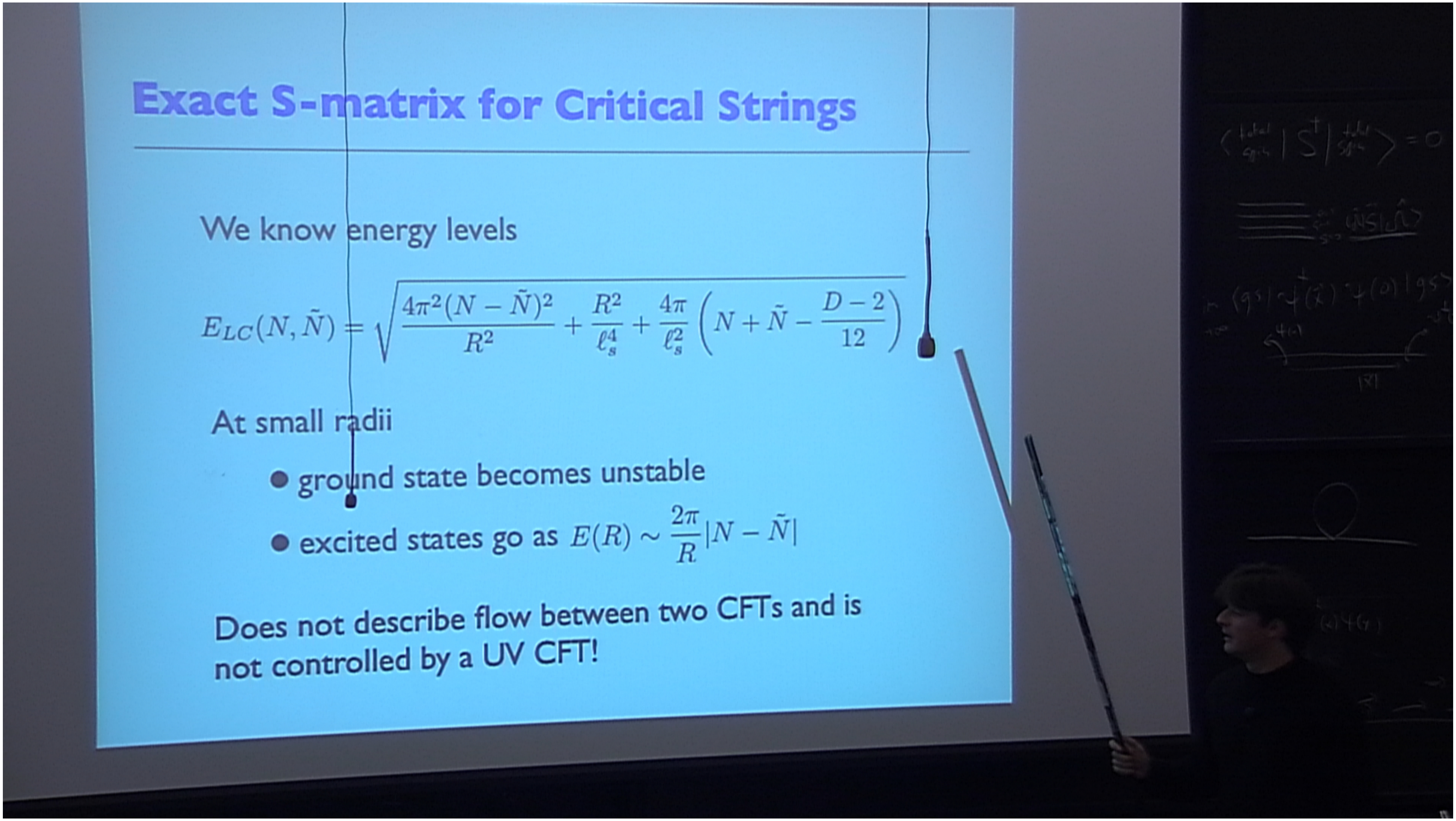
$$E(R) \sim \frac{2\pi}{R}\left(h + \tilde{h} - \frac{c}{24} - \frac{\tilde{c}}{24}\right)$$

In general, the energy levels are not known but have to be obtained from the S-matrix using the Thermodynamic Bethe Ansatz.

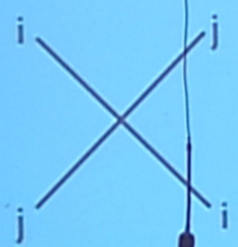

Exact S-matrix for Critical Strings
We know energy levels
$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$
At small radii
ground state becomes unstable
excited states go as $E(R) \sim \frac{2\pi}{R}|N-\tilde{N}|$
Does not describe flow between two CFTs and is not controlled by a UV CFT!


Exact S-matrix for Critical Strings
We know energy levels
$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$
At small radii
ground state becomes unstable
excited states go as $E(R) \sim \frac{2\pi}{R}|N-\tilde{N}|$
Does not describe flow between two CFTs and is not controlled by a UV CFT!

Energy levels from TBA

(Asymptotic) Bethe Ansatz (cont'd)

Consider $\Psi(x_1, x_2) = \langle 0 | X^i(x_1) X^j(x_2) | p_L^{(i)}, p_R^{(j)} \rangle$



for large R (ignoring wrapping interactions)

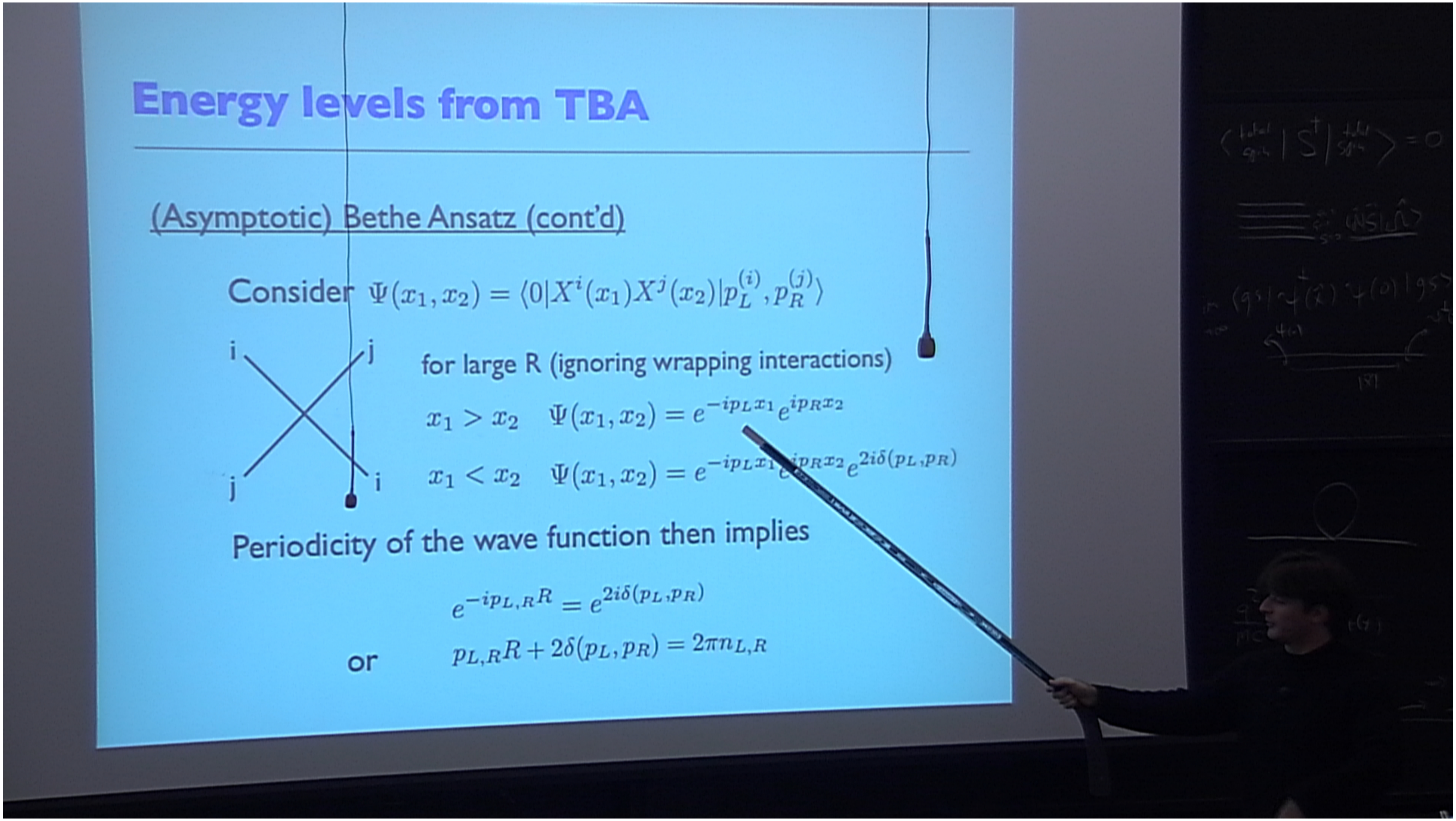
$x_1 > x_2 \quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2}$

$x_1 < x_2 \quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2} e^{2i\delta(p_L, p_R)}$

Periodicity of the wave function then implies

$$e^{-ip_{L,R} R} = e^{2i\delta(p_L, p_R)}$$

or $$p_{L,R} R + 2\delta(p_L, p_R) = 2\pi n_{L,R}$$


Energy levels from TBA
(Asymptotic) Bethe Ansatz (cont'd)
Consider $\Psi(x_1, x_2) = \langle 0 | X^i(x_1) X^j(x_2) | p_L^{(i)}, p_R^{(j)} \rangle$
i
j
j
i
for large R (ignoring wrapping interactions)
$x_1 > x_2 \quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2}$
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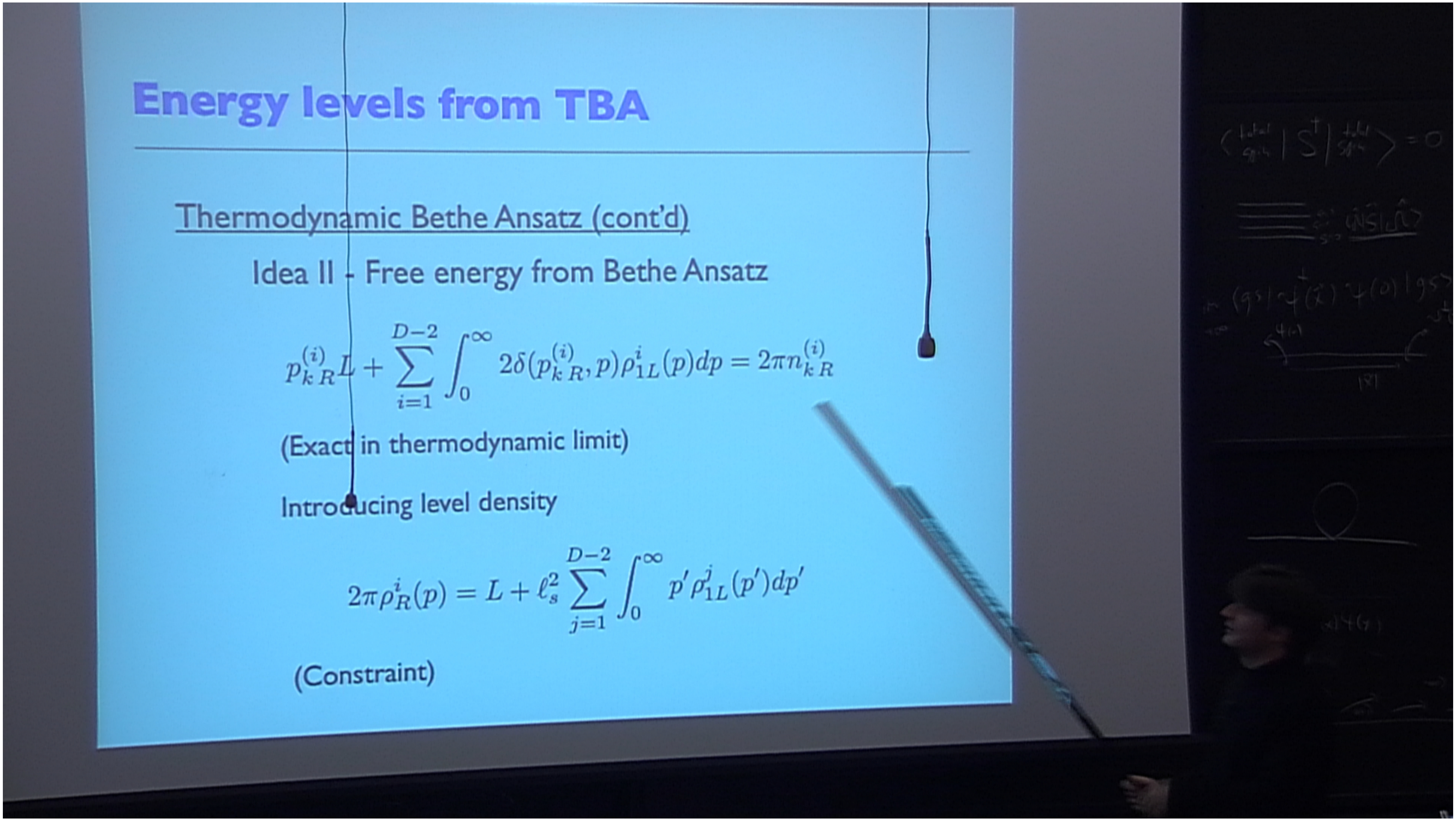
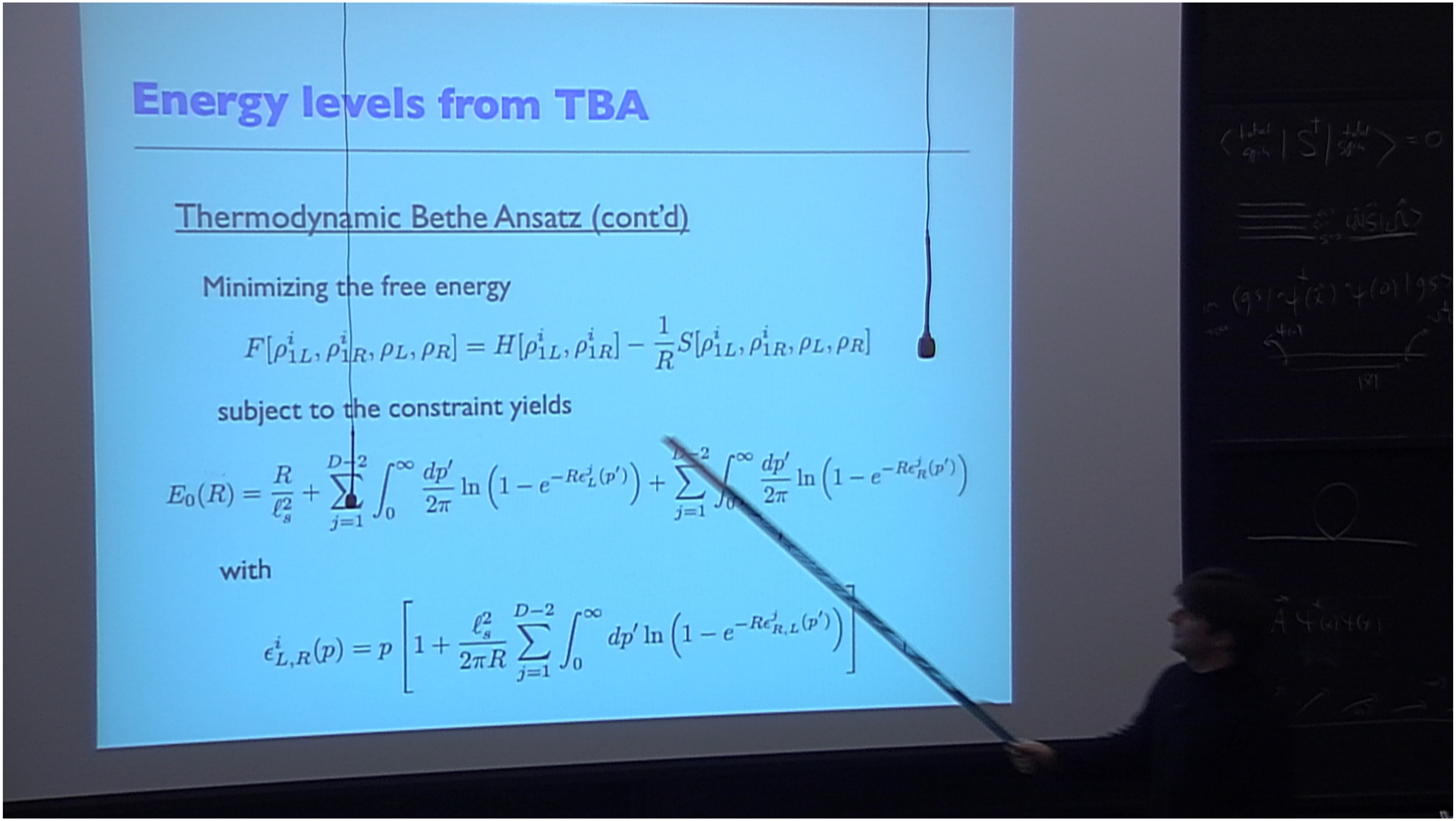


Energy levels from TBA
Thermodynamic Bethe Ansatz (cont'd)
Idea II - Free energy from Bethe Ansatz
$p_{k\,R}^{(i)}L + \sum_{i=1}^{D-2}\int_0^\infty 2\delta(p_{k\,R}^{(i)},p)\rho_{1L}^i(p)dp = 2\pi n_{k\,R}^{(i)}$
(Exact in thermodynamic limit)
Introducing level density
$2\pi\rho_R^i(p) = L + \ell_s^2\sum_{j=1}^{D-2}\int_0^\infty p'\rho_{1L}^j(p')dp'$
(Constraint)


Energy levels from TBA
Thermodynamic Bethe Ansatz (cont'd)
Minimizing the free energy
$F[\rho^i_{1L}, \rho^i_{1R}, \rho_L, \rho_R] = H[\rho^i_{1L}, \rho^i_{1R}] - \frac{1}{R} S[\rho^i_{1L}, \rho^i_{1R}, \rho_L, \rho_R]$
subject to the constraint yields
$E_0(R) = \frac{R}{\ell_s^2} + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_L(p')}\right) + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right)$
with
$\epsilon^i_{L,R}(p) = p\left[1 + \frac{\ell_s^2}{2\pi R} \sum_{j=1}^{D-2} \int_0^\infty dp' \ln\left(1 - e^{-R\epsilon^j_{R,L}(p')}\right)\right]$

Energy levels from TBA

Thermodynamic Bethe Ansatz (cont'd)

For excited states

Bethe Ansatz

$$p^{(i)}_{k\,L} R + \sum_{j,m} 2\delta(p^{(i)}_{k\,L}, p^{(j)}_{m\,R}) + \sum_{j=1}^{D-2}\int_0^\infty \frac{dp'}{2\pi}\frac{d\,2\delta(p^{(i)}_{k\,L},p')}{dp'}\ln\left(1-e^{-R\epsilon^j_R(p')}\right) = 2\pi n^{(i)}_{k\,L}$$

$$\epsilon^i_L(p) = p + \frac{1}{R}\sum_{j,k} 2\delta(p,\hat{p}^{(j)}_{k\,R}) + \frac{1}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp' \frac{d\,2\delta(p,p')}{dp'}\ln\left(1-e^{-R\epsilon^j_R(p')}\right)$$

$$E(R) = R + \sum_{j,k} p^{(j)}_{k\,L} + \sum_{j=1}^{D-2}\int_0^\infty \frac{dp'}{2\pi}\ln\left(1-e^{-R\epsilon^j_L(p')}\right)$$

+right-movers

Energy levels from TBA

Thermodynamic Bethe Ansatz (cont'd)

For strings integral equations are still algebraic

$$\epsilon^i_L(p) = c_L p \qquad \text{and} \qquad \epsilon^i_R(p) = c_R p$$

$$c_{L,R} = 1 + \frac{2\pi \ell_s^2 N_{R,L}}{c_{R,L} R^2} - \frac{\pi \ell_s^2}{c_{R,L} R^2} \frac{D-2}{12}$$

And one finds

$$E(N_L, N_R) = \sqrt{\frac{4\pi^2 (N_L - N_R)^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N_L + N_R - \frac{D-2}{12}\right)}$$

Energy levels from TBA

Thermodynamic Bethe Ansatz (cont'd)

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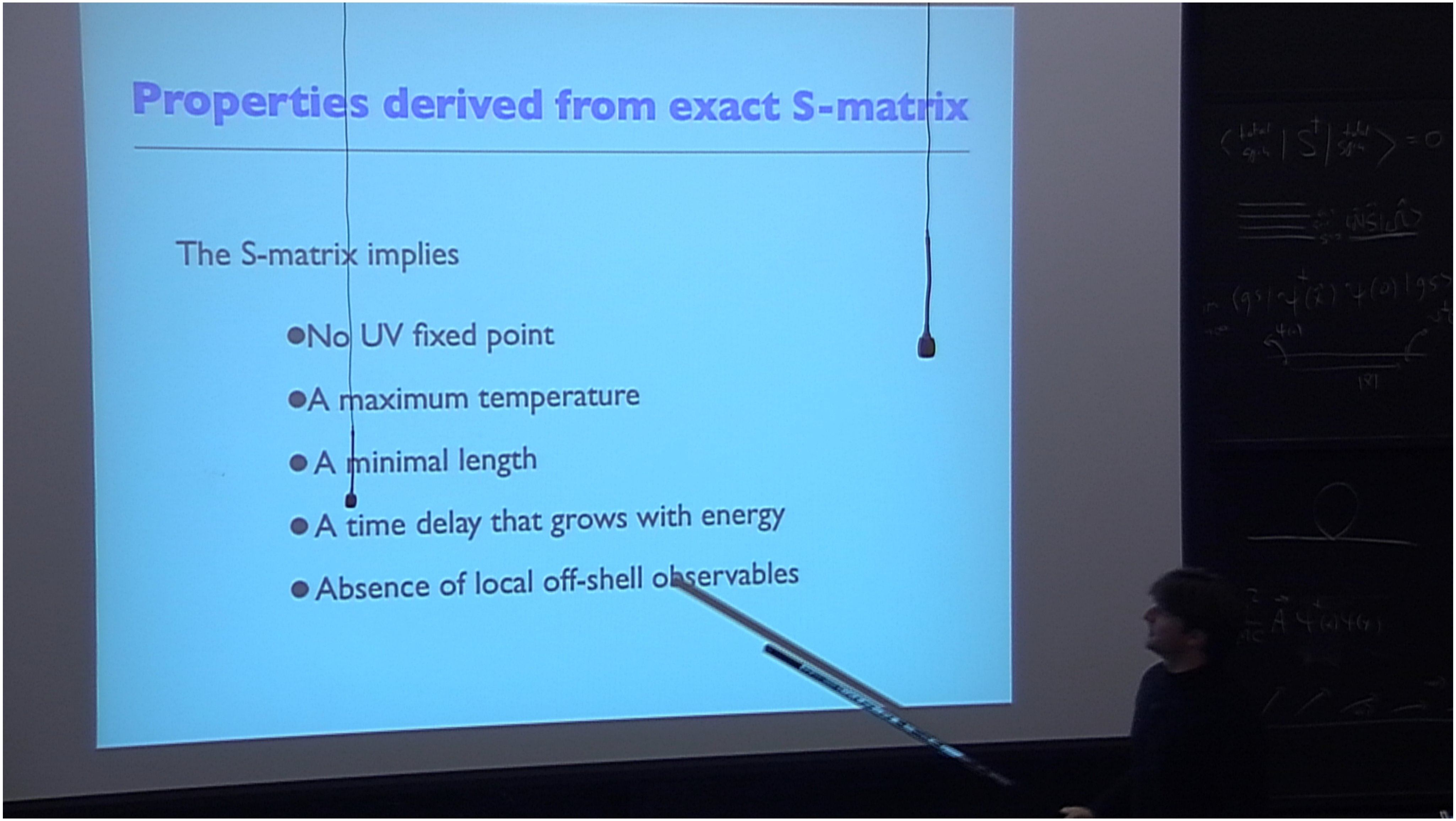

Properties derived from exact S-matrix
The S-matrix implies
No UV fixed point
A maximum temperature
A minimal length
A time delay that grows with energy
Absence of local off-shell observables

Properties derived from exact S-matrix

Maximum temperature

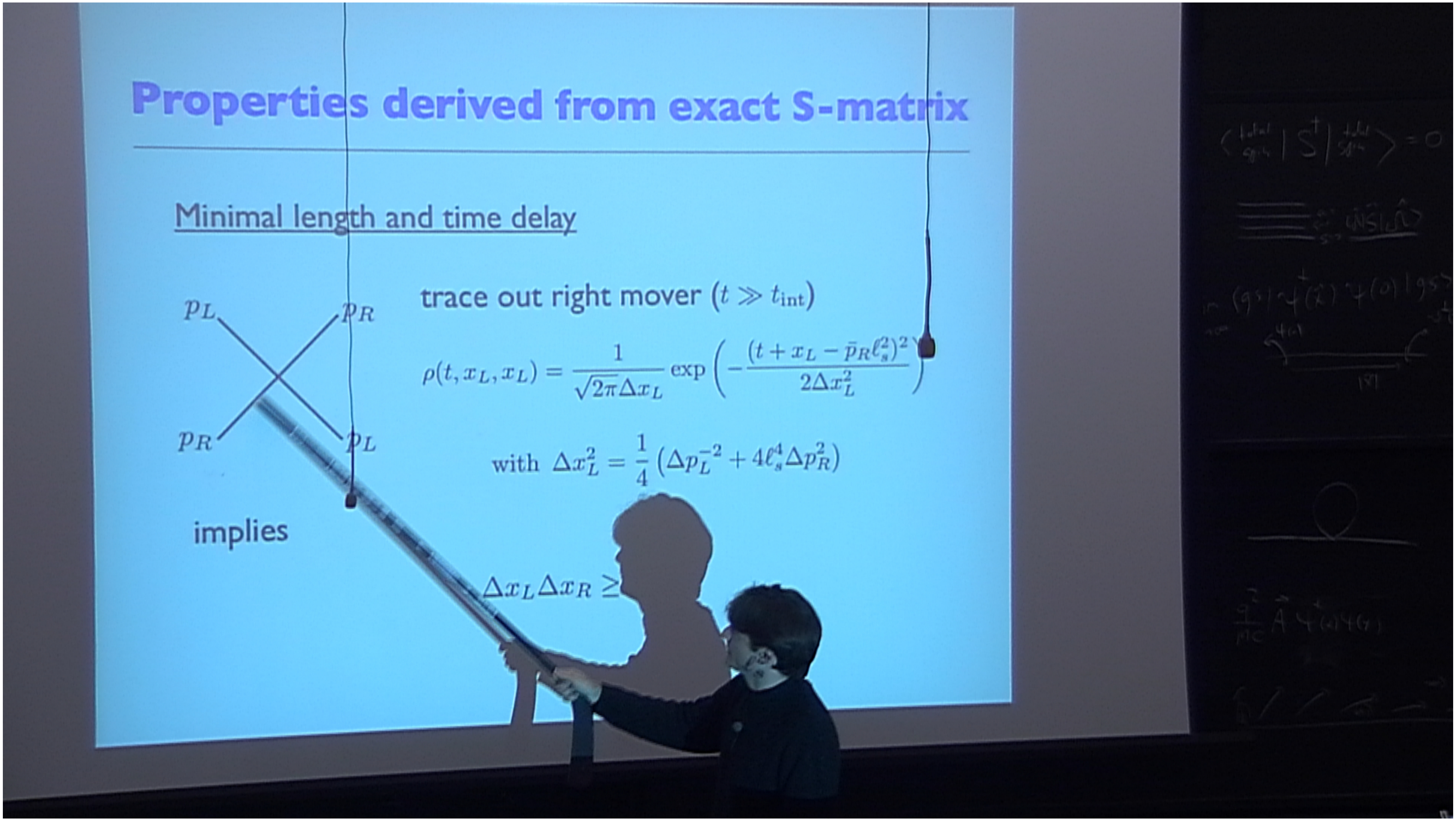
$$F(T) = \frac{L}{\ell_s^2}\sqrt{1-\frac{T^2}{T_H^2}} \qquad T_H = \frac{1}{\ell_s}\sqrt{\frac{3}{\pi(D-2)}}$$

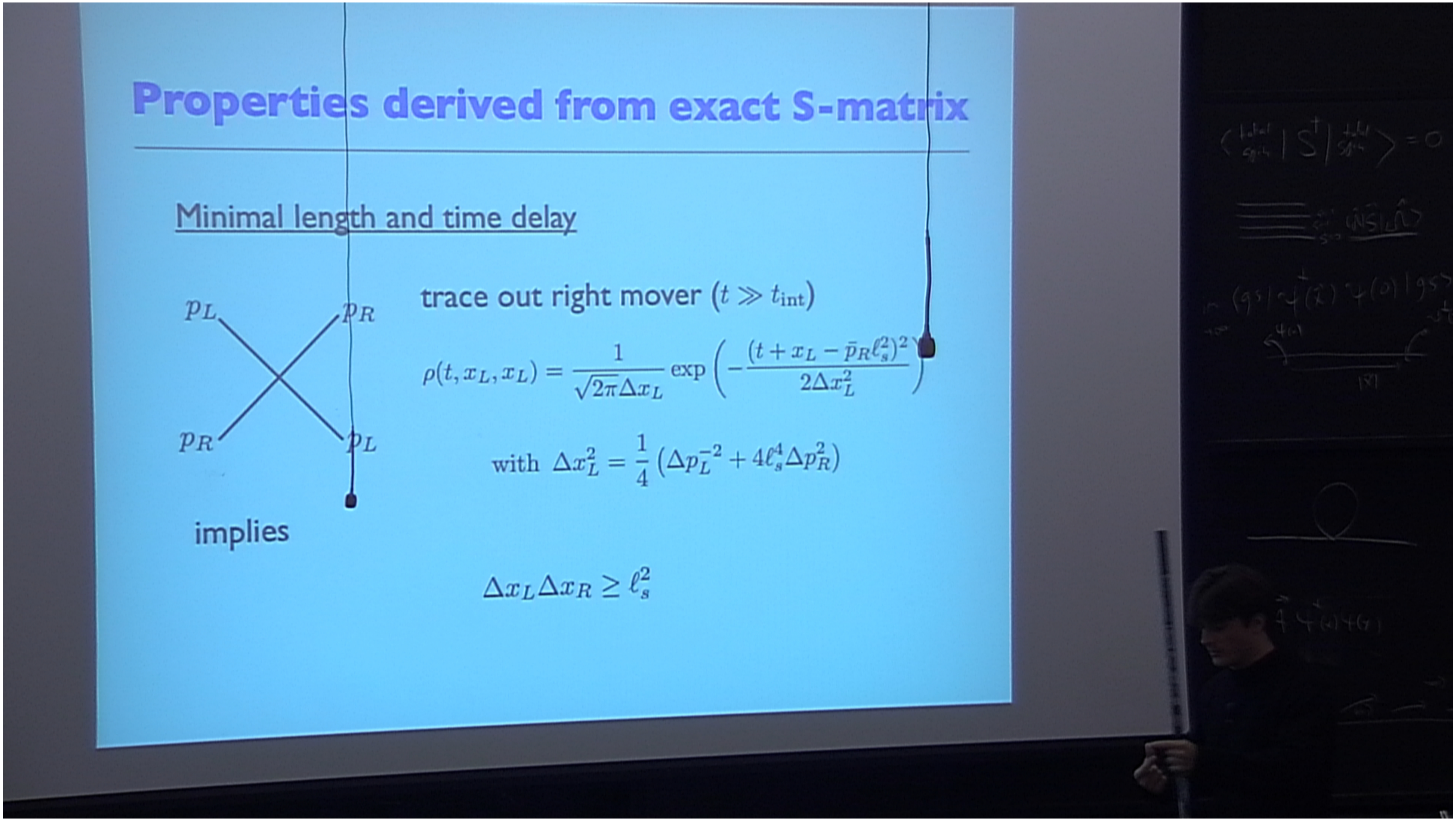
Heat capacity and its integral diverges

$$C_v = -T\frac{\partial^2 F}{\partial T^2} = \frac{T T_H}{\ell_s^2 (T_H^2 - T^2)^{3/2}} \sim (T_H - T)^{-3/2}$$

Need infinite amount of energy (per unit volume) to reach Hagedorn temperature.

Speed of sound vanishes at Hagedorn temperature


Properties derived from exact S-matrix
Minimal length and time delay
p_L
p_R
p_R
p_L
trace out right mover ($t \gg t_{\rm int}$)
$\rho(t, x_L, x_L) = \frac{1}{\sqrt{2\pi}\Delta x_L} \exp\left(-\frac{(t + x_L - \bar{p}_R \ell_s^2)^2}{2\Delta x_L^2}\right)$
with $\Delta x_L^2 = \frac{1}{4}\left(\Delta p_L^{-2} + 4\ell_s^4 \Delta p_R^2\right)$
implies
$\Delta x_L \Delta x_R \geq$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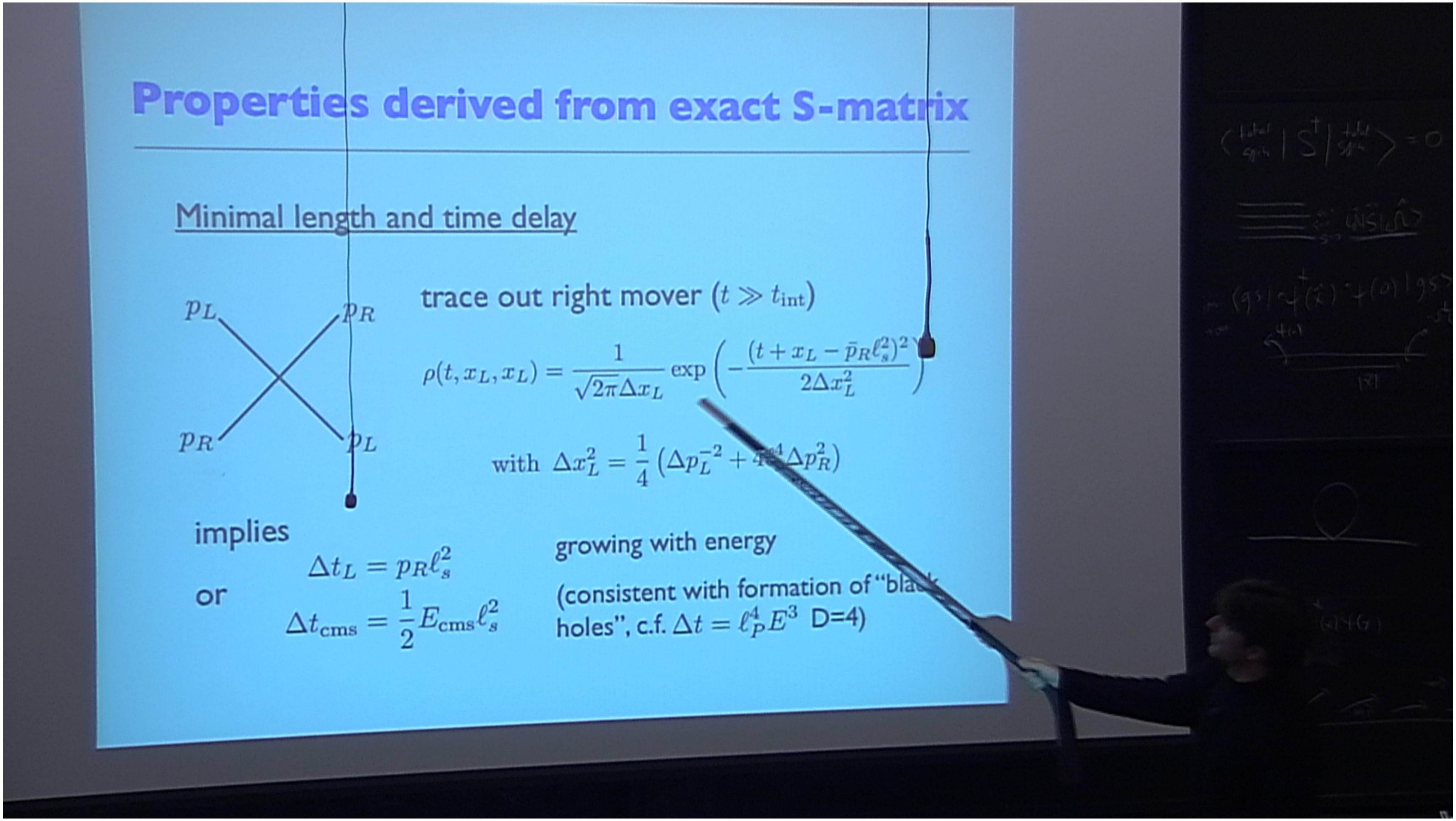
implies
$\Delta x_L \Delta x_R \geq \ell_s^2$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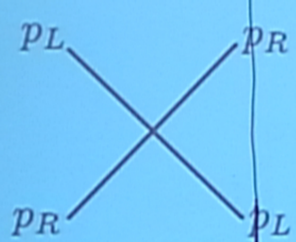
with $\Delta x_L^2 = \frac{1}{4}\left(\Delta p_L^{-2} + 4\ell_s^4 \Delta p_R^2\right)$
implies
$\Delta t_L = p_R \ell_s^2$
growing with energy
or
$\Delta t_{\rm cms} = \frac{1}{2} E_{\rm cms} \ell_s^2$
(consistent with formation of "bla
holes", c.f. $\Delta t = \ell_P^4 E^3$ D=4)

Properties derived from exact S-matrix

Minimal length and time delay



trace out right mover ($t \gg t_{\rm int}$)

$$\rho(t, x_L, x_L) = \frac{1}{\sqrt{2\pi}\Delta x_L} \exp\left(-\frac{(t + x_L - \bar{p}_R \ell_s^2)^2}{2\Delta x_L^2}\right)$$

$$\text{with } \Delta x_L^2 = \frac{1}{4}\left(\Delta p_L^{-2} + 4\ell_s^4 \Delta p_R^2\right)$$

implies

$$\Delta t_L = p_R \ell_s^2$$

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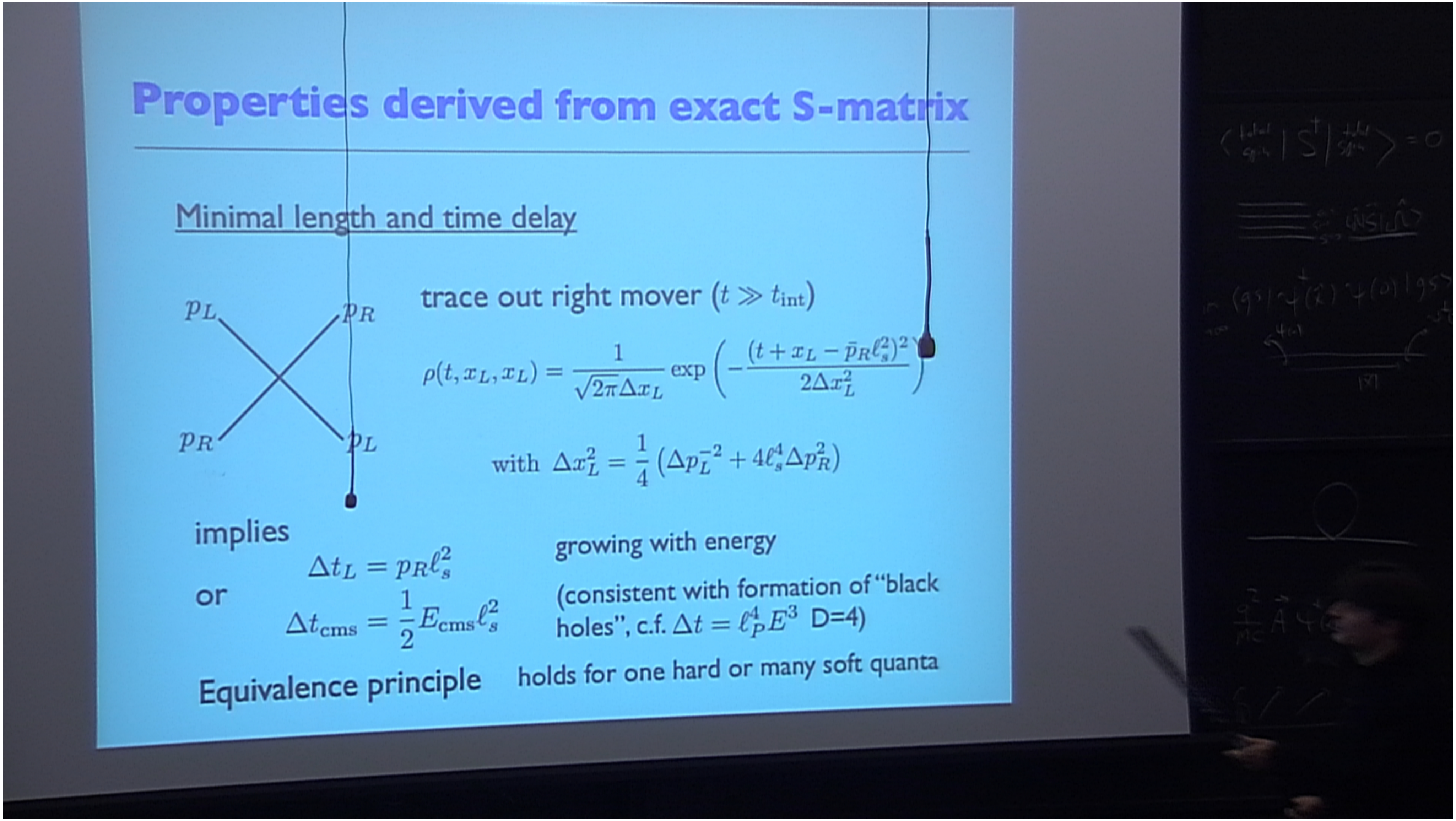
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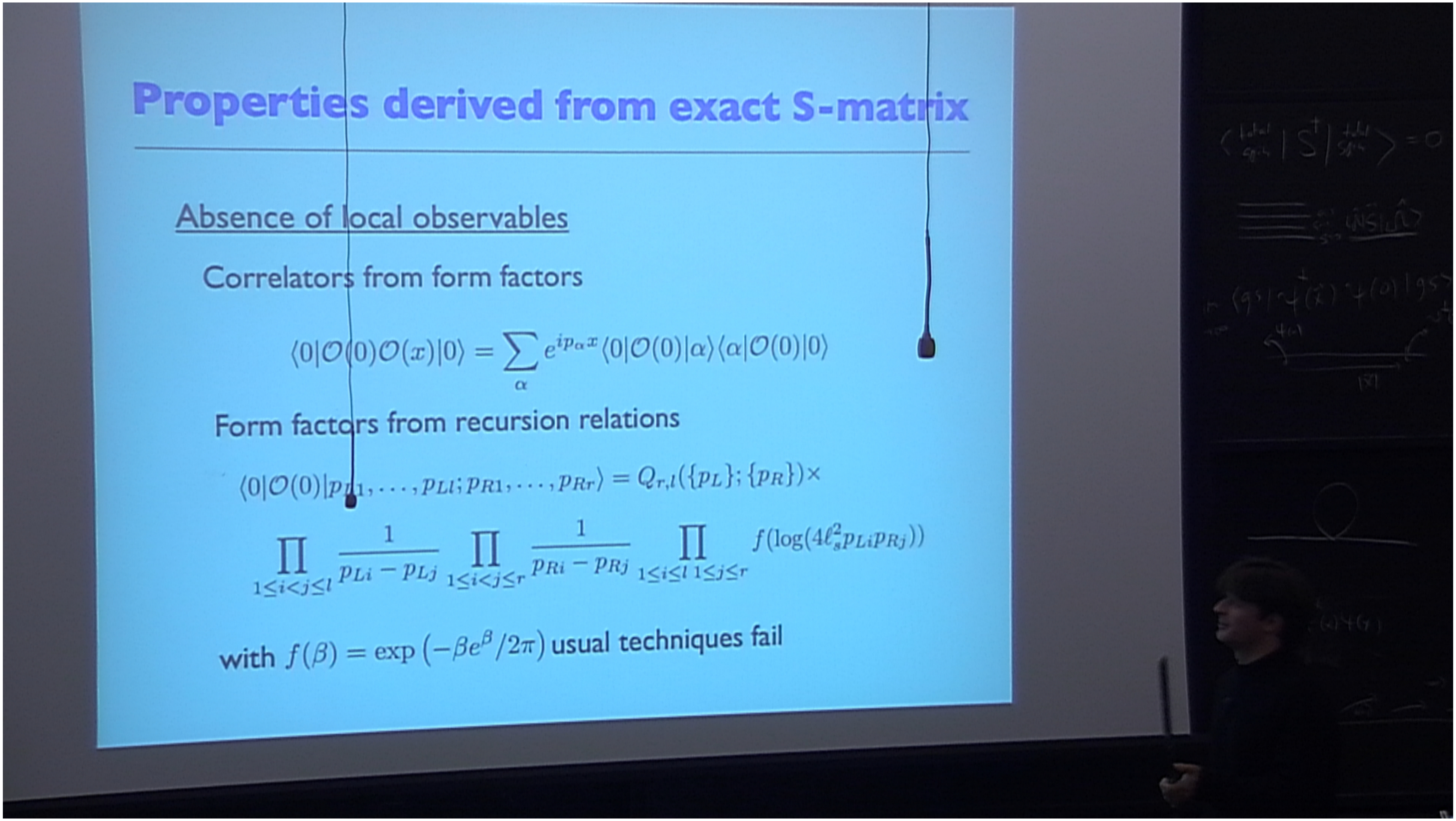
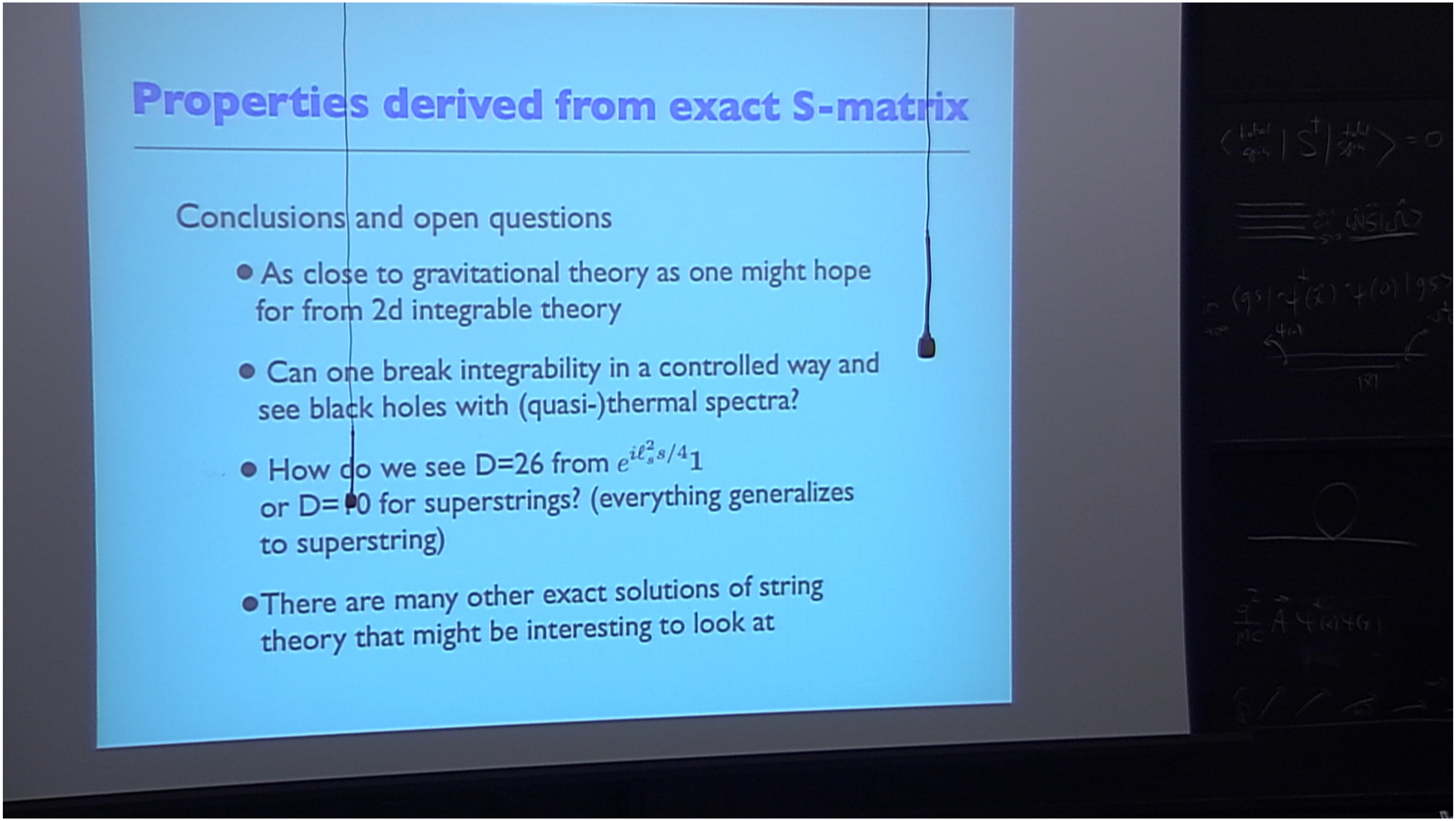
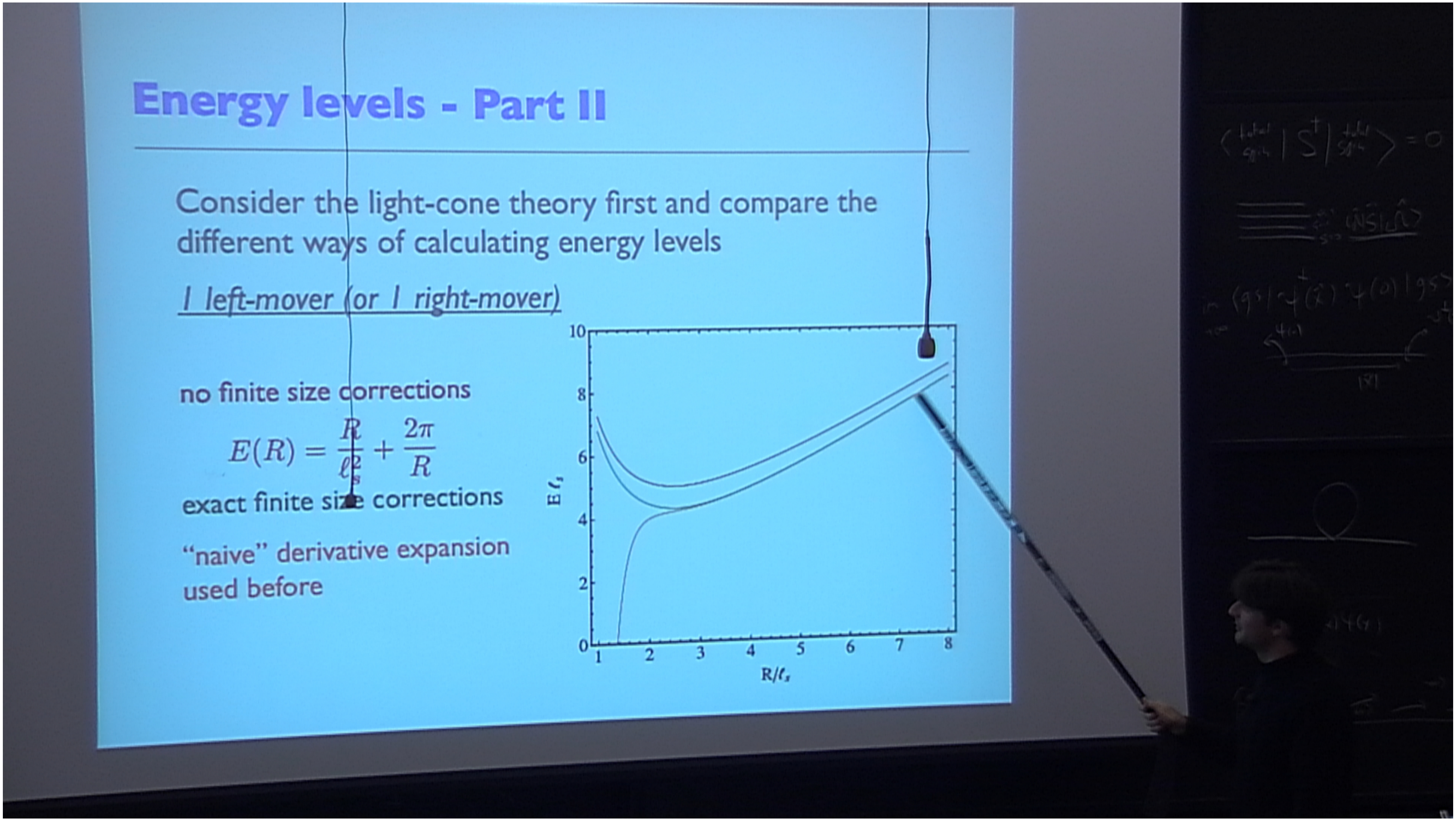
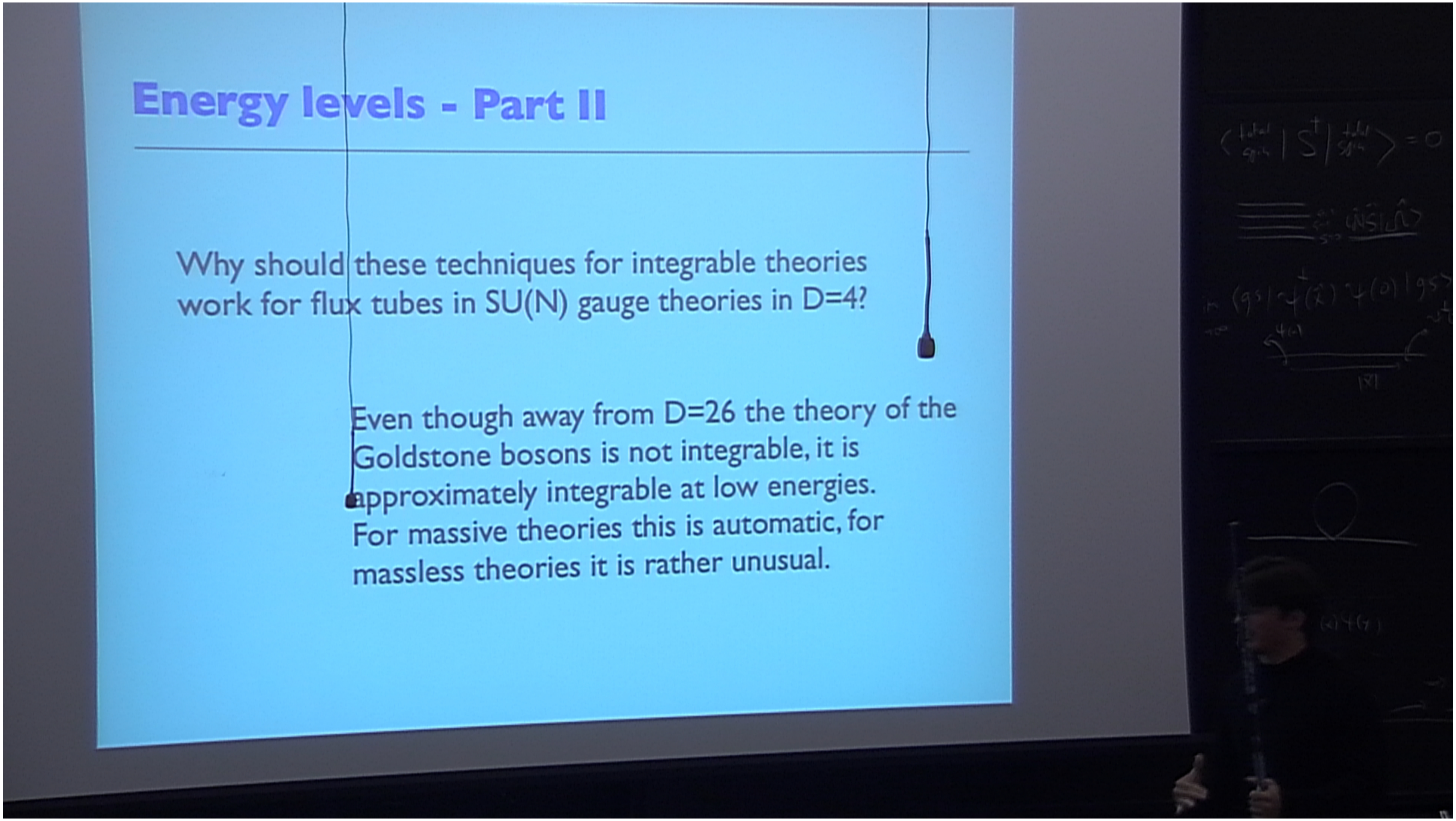
(consistent with formation of "black holes", c.f. $\Delta t = \ell_P^4 E^3$ D=4)
Equivalence principle
holds for one hard or many soft quanta


Properties derived from exact S-matrix
Absence of local observables
Correlators from form factors
$\langle 0|\mathcal{O}(0)\mathcal{O}(x)|0\rangle = \sum_\alpha e^{ip_\alpha x}\langle 0|\mathcal{O}(0)|\alpha\rangle\langle\alpha|\mathcal{O}(0)|0\rangle$
Form factors from recursion relations
$\langle 0|\mathcal{O}(0)|p_{L1},\ldots,p_{Ll};p_{R1},\ldots,p_{Rr}\rangle = Q_{r,l}(\{p_L\};\{p_R\})\times$
$\prod_{1\leq i<j\leq l}\frac{1}{p_{Li}-p_{Lj}}\prod_{1\leq i<j\leq r}\frac{1}{p_{Ri}-p_{Rj}}\prod_{1\leq i\leq l}\prod_{1\leq j\leq r} f(\log(4\ell_s^2 p_{Li}p_{Rj}))$
with $f(\beta)=\exp\left(-\beta e^{\beta}/2\pi\right)$ usual techniques fail


Properties derived from exact S-matrix
Conclusions and open questions
As close to gravitational theory as one might hope for from 2d integrable theory
Can one break integrability in a controlled way and see black holes with (quasi-)thermal spectra?
How do we see D=26 from $e^{i\ell_s^2 s/4}1$ or D=10 for superstrings? (everything generalizes to superstring)
There are many other exact solutions of string theory that might be interesting to look at


Energy levels - Part II
Consider the light-cone theory first and compare the different ways of calculating energy levels
1 left-mover (or 1 right-mover)
no finite size corrections
$E(R) = \frac{R}{\ell_s^2} + \frac{2\pi}{R}$
exact finite size corrections
"naive" derivative expansion used before
$E\ell_s$
R/ℓ_s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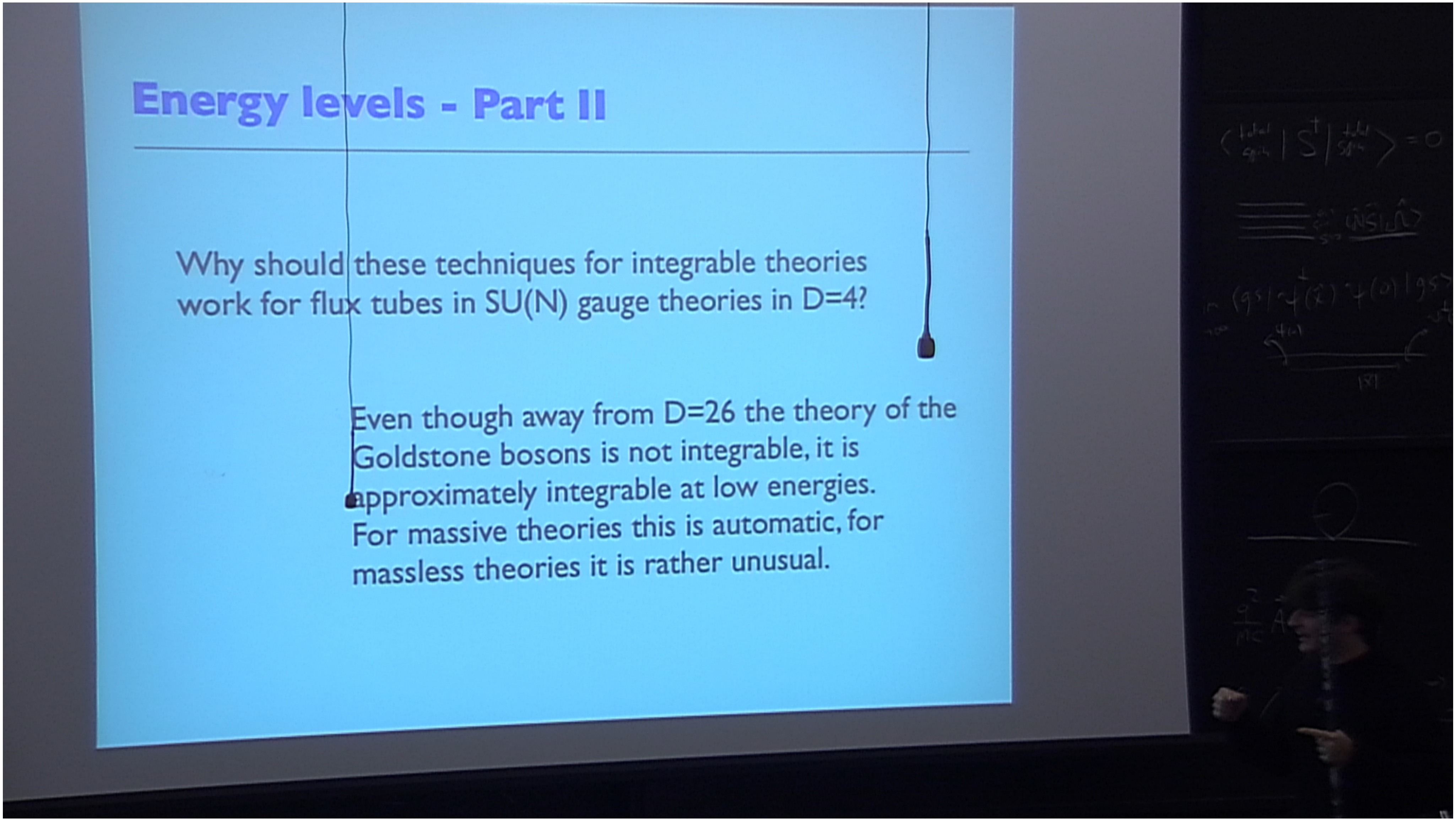
Why should these techniques for integrable theories work for flux tubes in SU(N) gauge theories in D=4?
Even though away from D=26 the theory of the Goldstone bosons is not integrable, it is approximately integrable at low energies.
For massive theories this is automatic, for massless theories it is rather unusual.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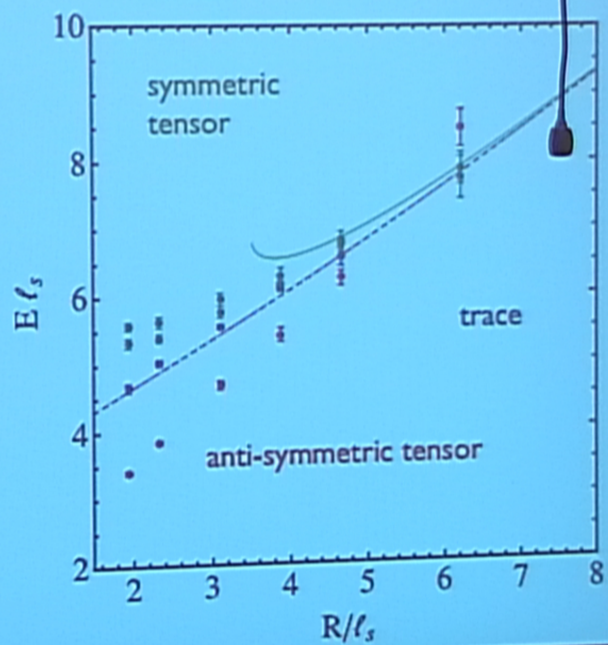
Energy levels - Part II

1 left-mover and 1 right-mover (D=4) reconsidered

phase shift from 1-loop computation $\mathcal{O}(p^4)$ with TBA finite size corrections

$p\ell_s < 1.8$



Energy levels - Part II

1 left-mover and 1 right-mover (D=4) reconsidered

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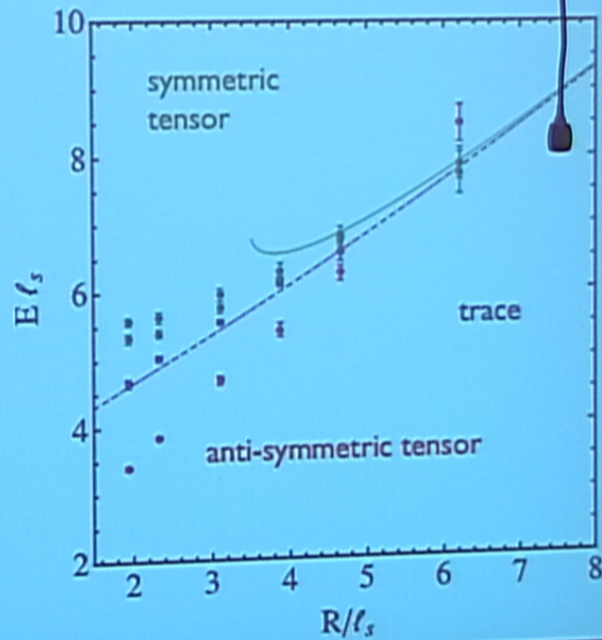
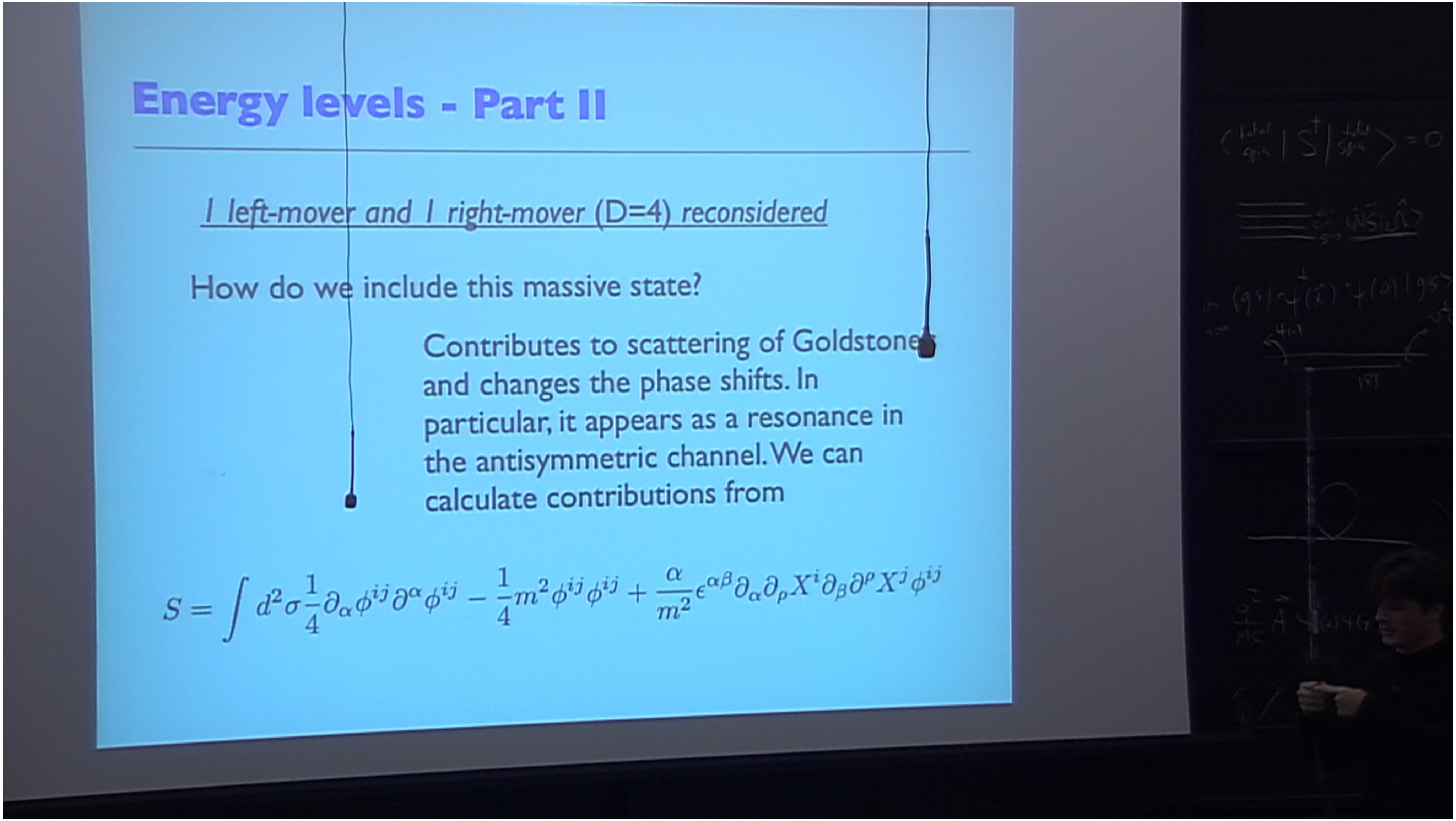
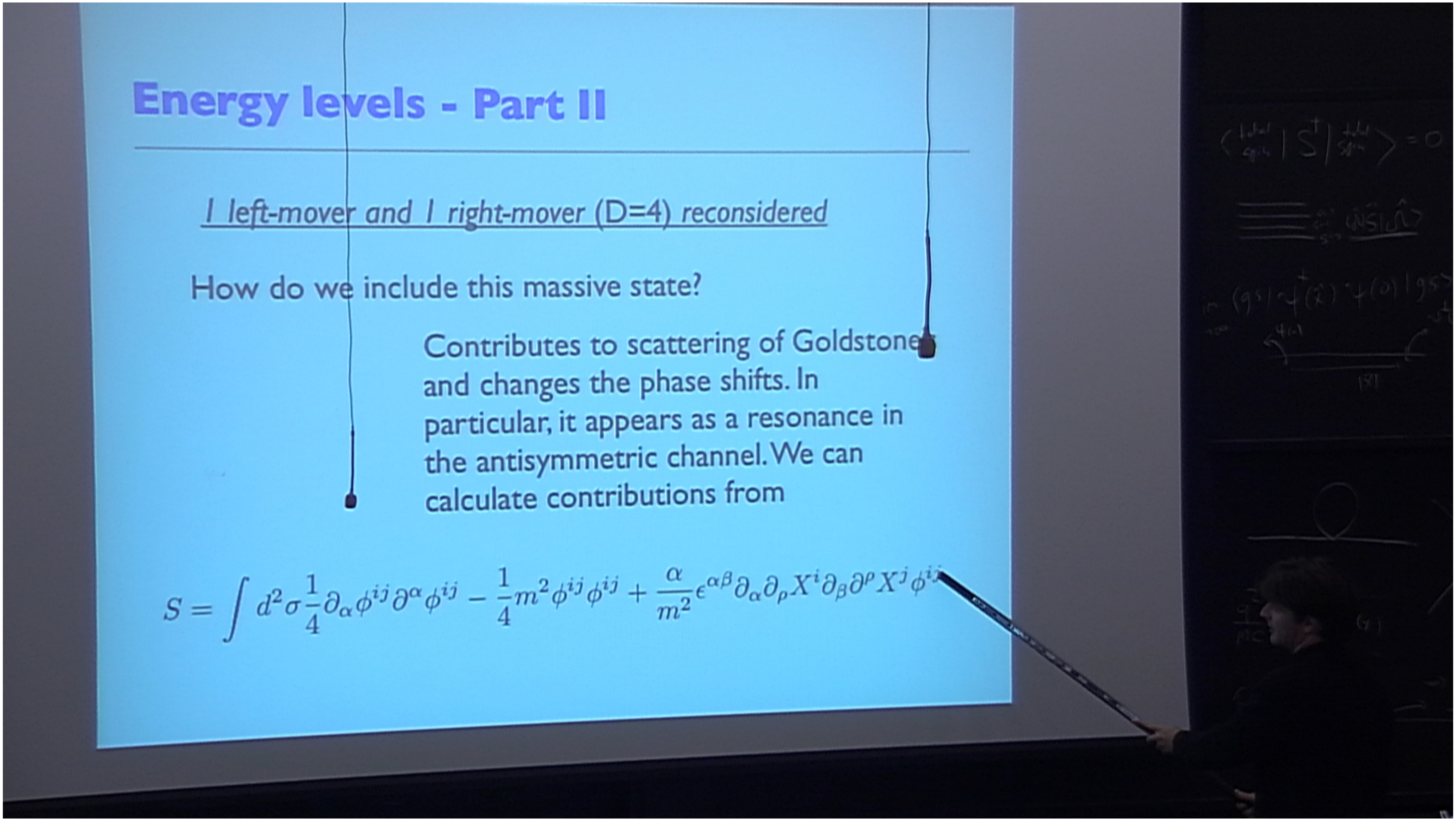
$$p\ell_s < 1.8$$

even though these points are at small radii, they are under theoretical control!




Energy levels - Part II
1 left-mover and 1 right-mover (D=4) reconsidered
How do we include this massive state?
Contributes to scattering of Goldstones and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from
$S = \int d^2\sigma \frac{1}{4}\partial_\alpha \phi^{ij} \partial^\alpha \phi^{ij} - \frac{1}{4} m^2 \phi^{ij}\phi^{ij} + \frac{\alpha}{m^2} \epsilon^{\alpha\beta} \partial_\alpha \partial_\rho X^i \partial_\beta \partial^\rho X^j \phi^{ij}$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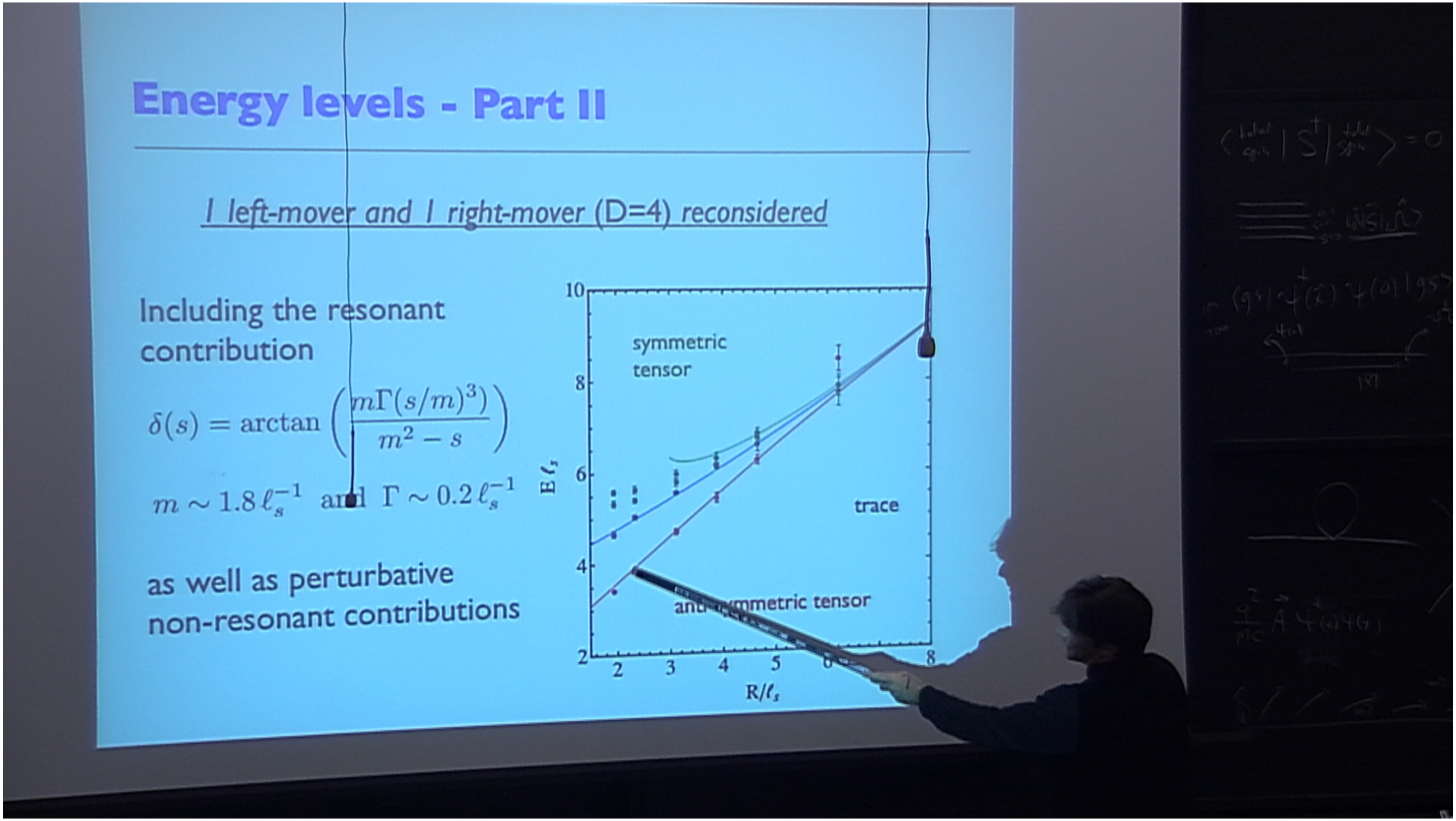

Energy levels - Part II
1 left-mover and 1 right-mover (D=4) reconsidered
Including the resonant contribution
$\delta(s) = \arctan\left(\frac{m\Gamma(s/m)^3)}{m^2 - s}\right)$
$m \sim 1.8\,\ell_s^{-1}$ and $\Gamma \sim 0.2\,\ell_s^{-1}$
as well as perturbative non-resonant contributions
symmetric tensor
trace
tensor
$E\,\ell_s$
R/ℓ_s

Energy levels - Part II

1 left-mover and 1 right-mover (D=4) reconsidered

Including the resonant contribution

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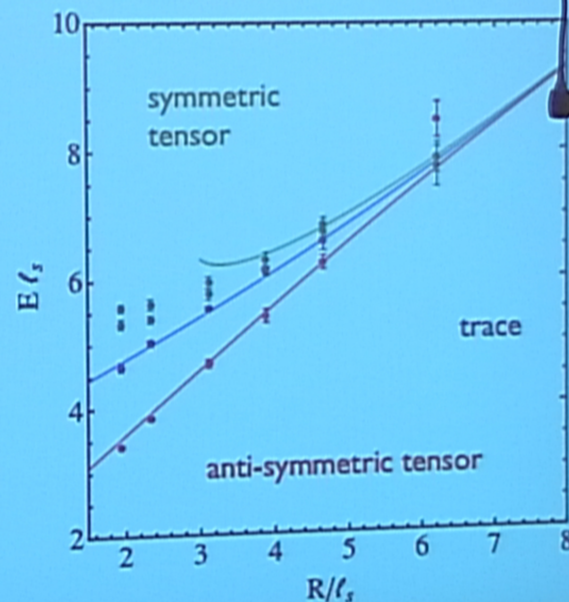

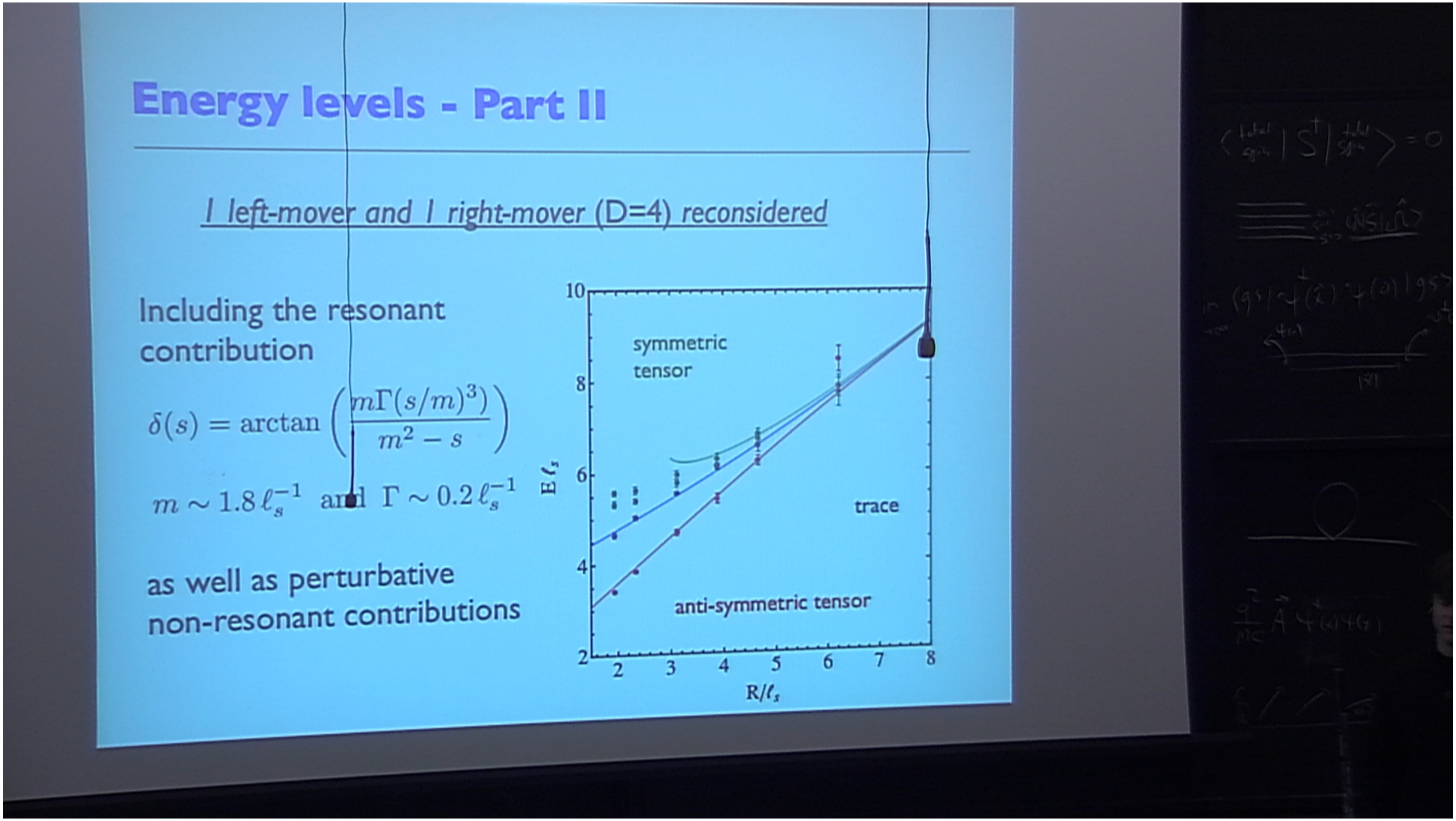
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