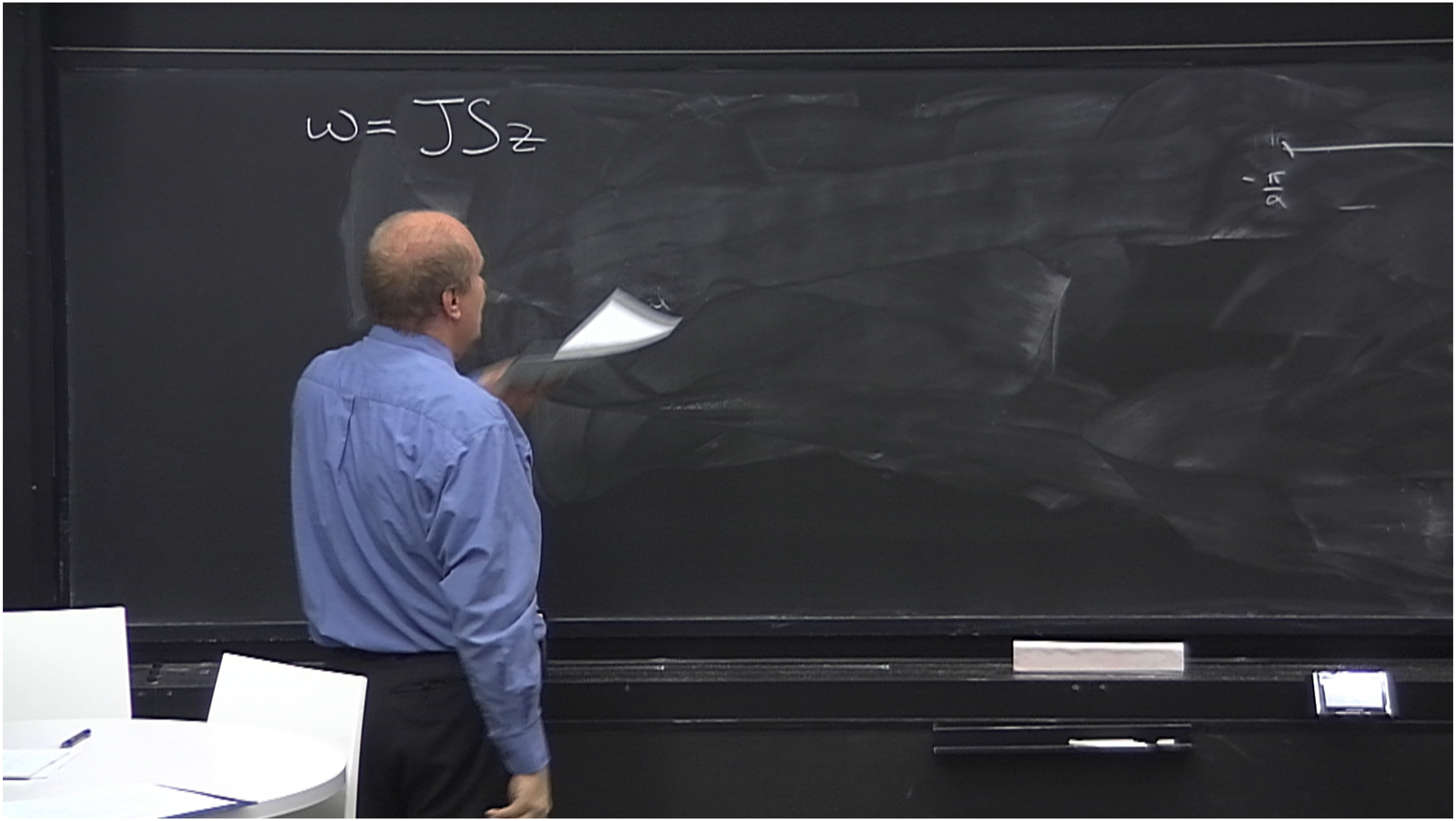
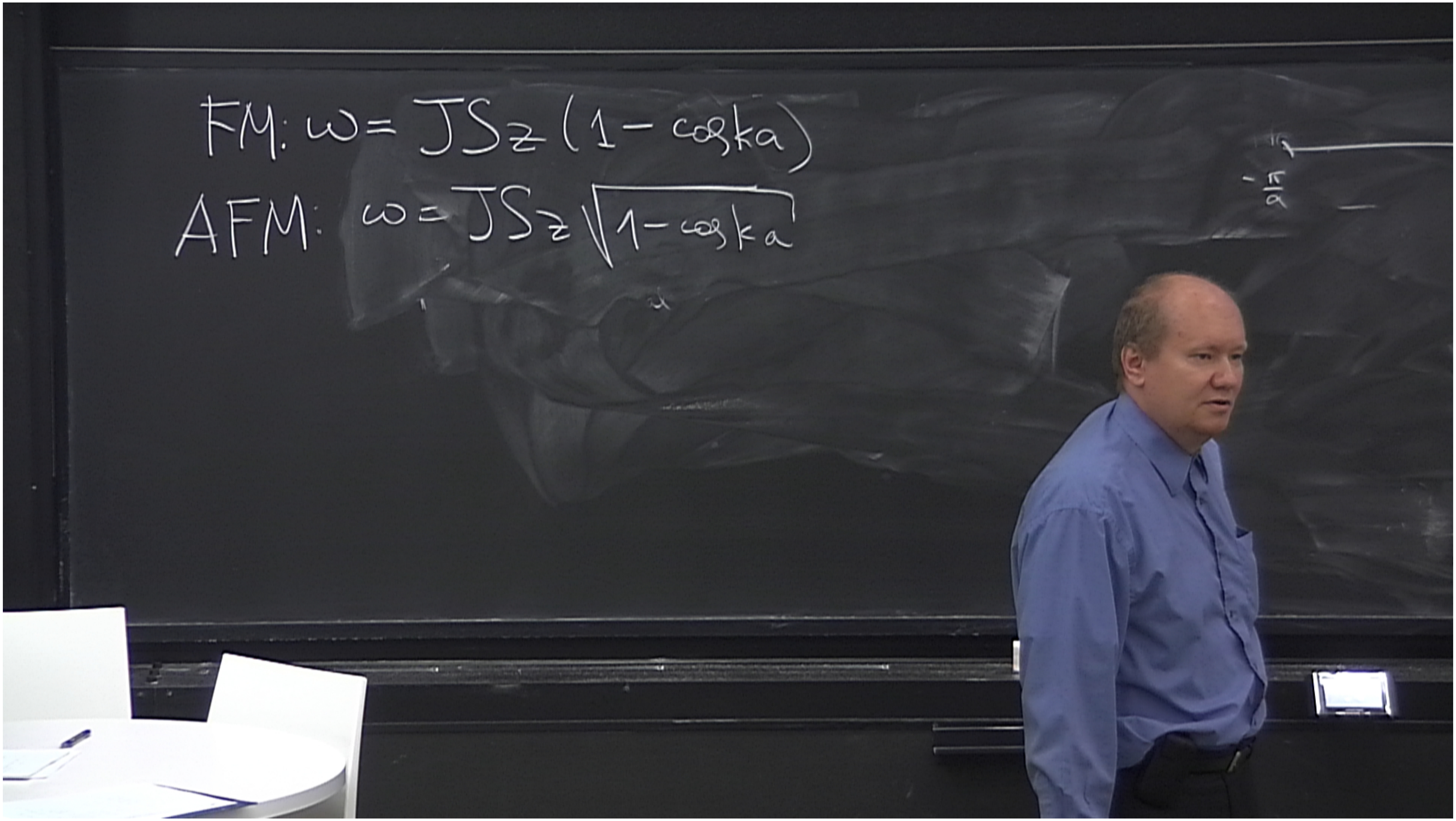
Title: Condensed Matter - Lecture 14

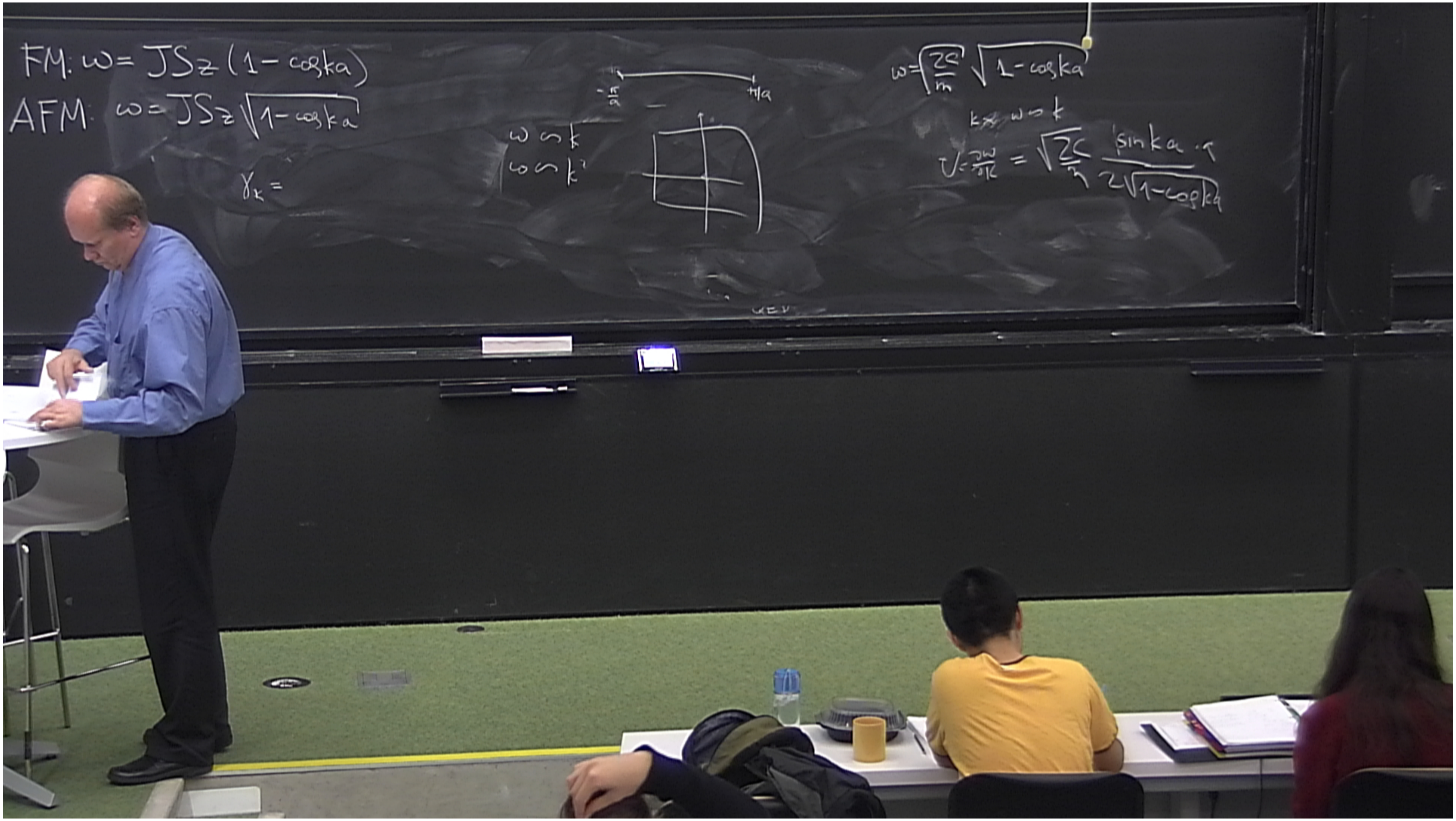
Date: Oct 26, 2012 10:30 AM

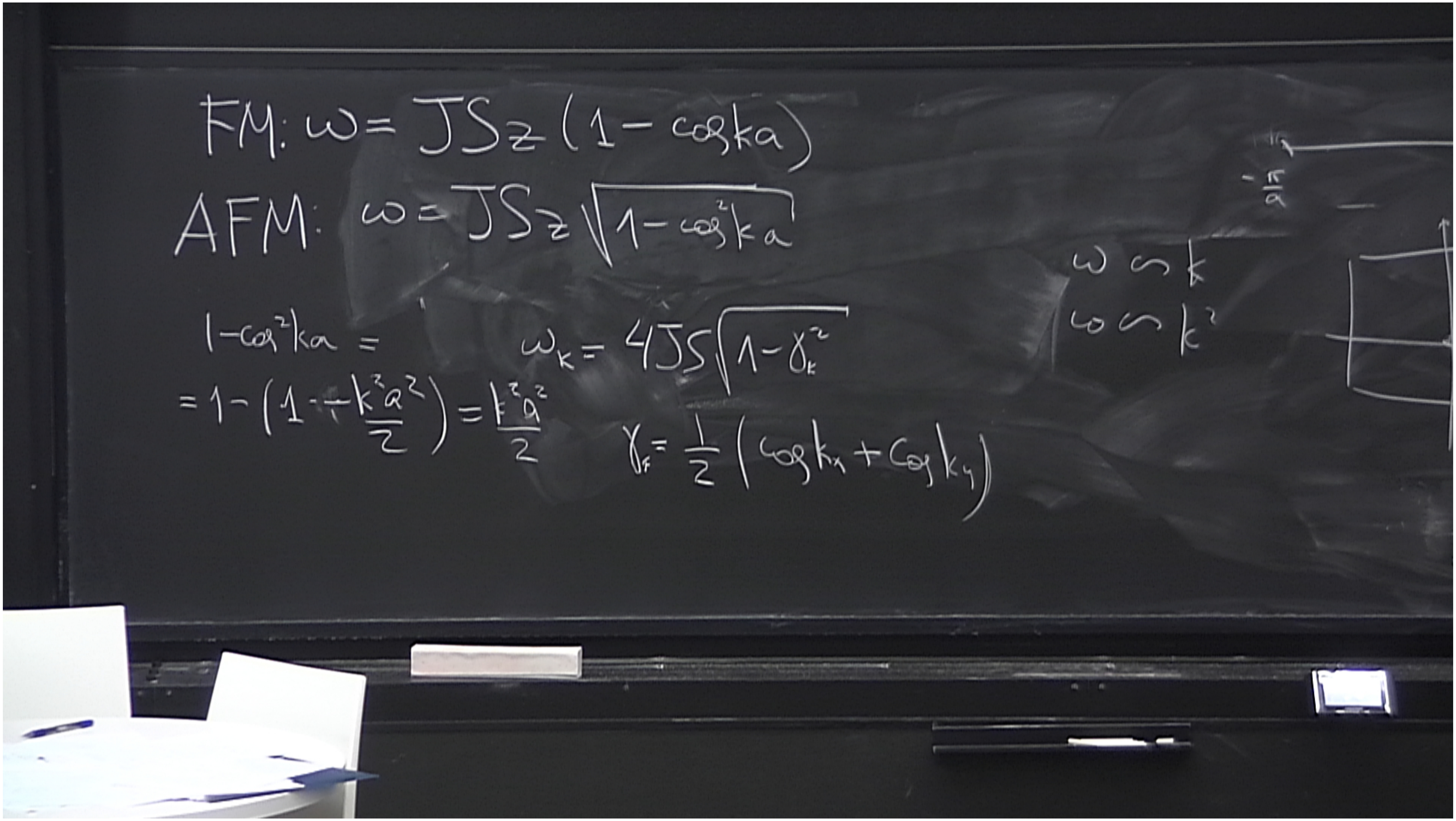
URL: http://pirsa.org/12100039

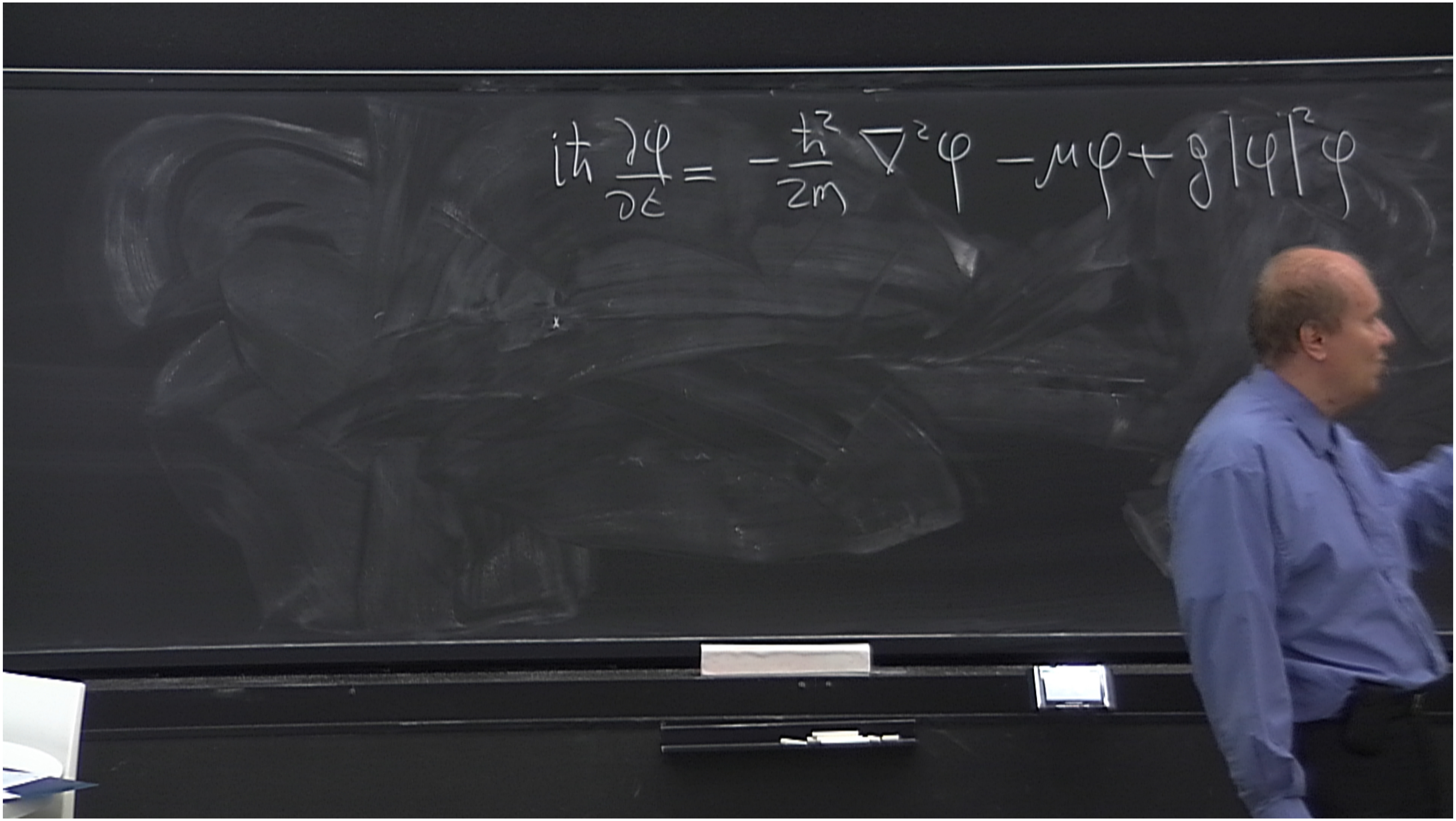
Abstract:

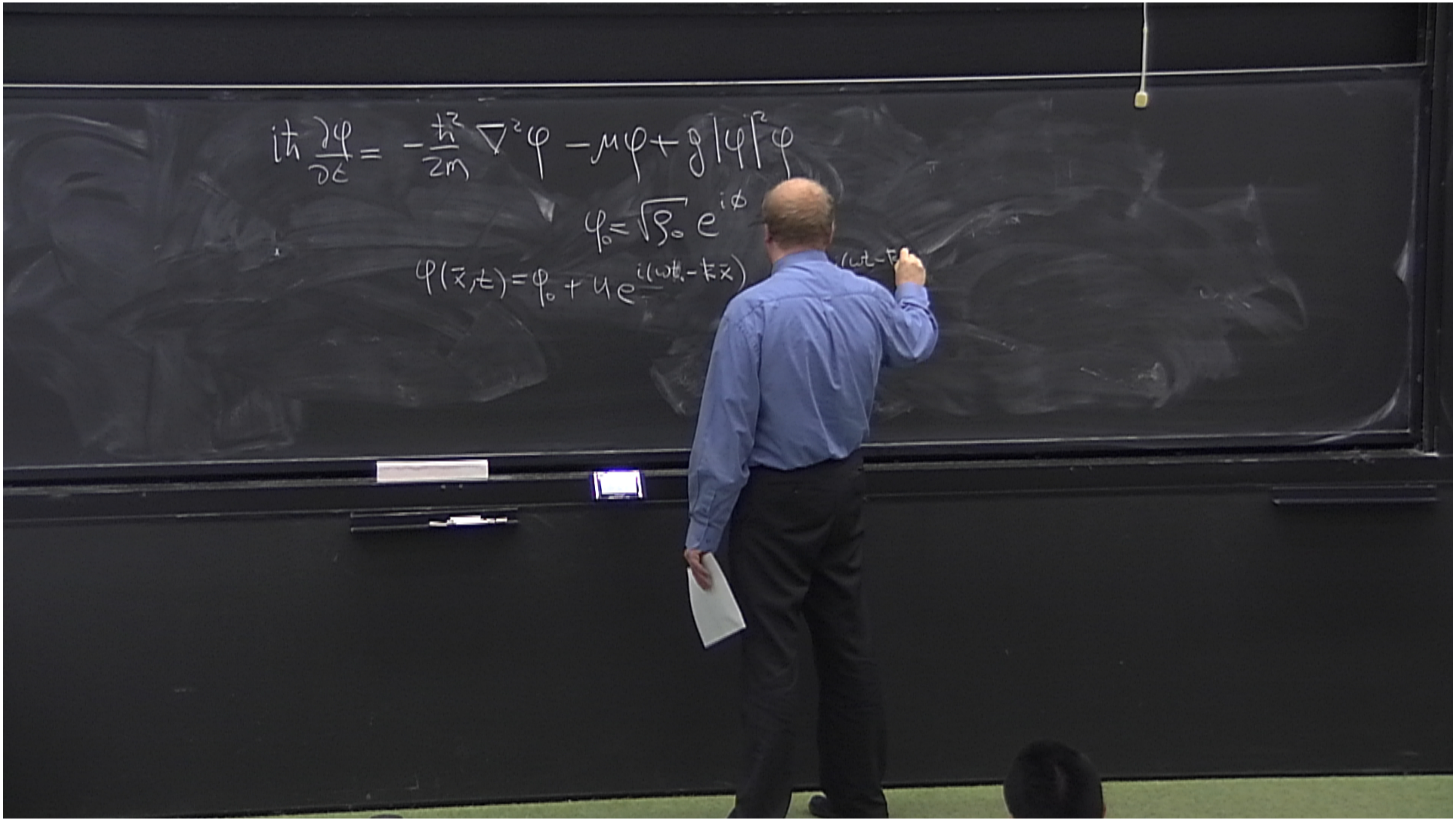

ω = JSz

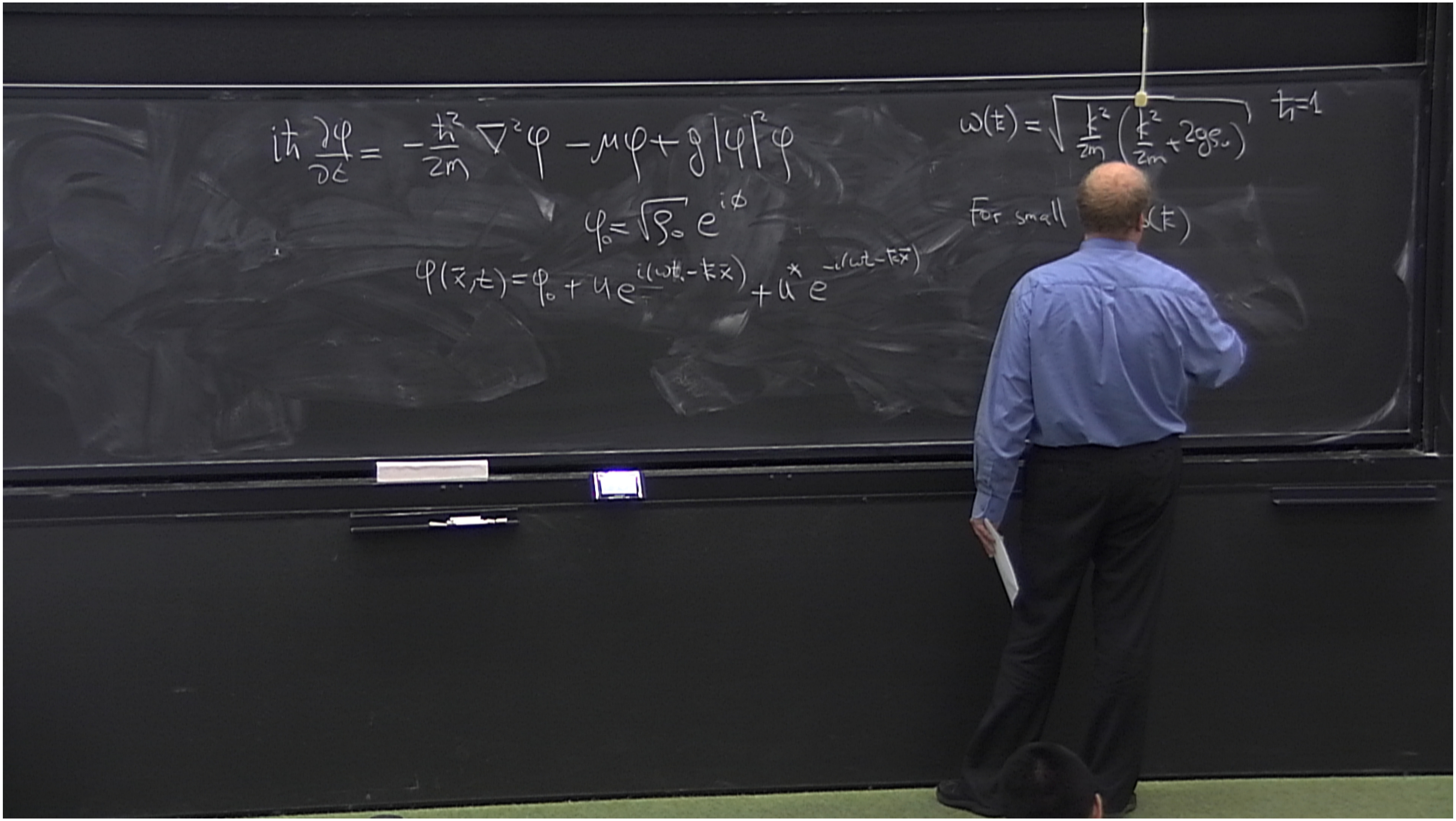

FM: ω = JSz (1 − cos ka)
AFM: ω = JSz √(1 − cos ka)
−π/a


FM. ω= JSz (1 − coska)
AFM ω = JSz √(1 − coska)
ω ∽ k
ω ∽ k²
γk =
−π/a
π/a
ω = √(2C/m) √(1 − coska)
v = ∂ω/∂k = √(2C/m) sinka · a / 2√(1 − coska)


FM: $\omega = JSz(1-\cos ka)$
AFM: $\omega = JSz\sqrt{1-\cos^2 ka}$
$1-\cos^2 ka =$
$= 1-\left(1-\frac{k^2a^2}{2}\right) = \frac{k^2a^2}{2}$
$\omega_k = 4JS\sqrt{1-\gamma_k^2}$
$\gamma_k = \frac{1}{2}(\cos k_x + \cos k_y)$
$\omega \sim k$
$\omega \sim k^2$
$-\frac{\pi}{a}$


$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi - \mu\varphi + g|\varphi|^2\varphi$


$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \varphi - \mu \varphi + g|\varphi|^2 \varphi$
$\varphi_0 = \sqrt{\rho_0}\, e^{i\phi}$
$\varphi(\vec{x},t) = \varphi_0 + u\, e^{i(\omega t - \vec{k}\cdot\vec{x})}$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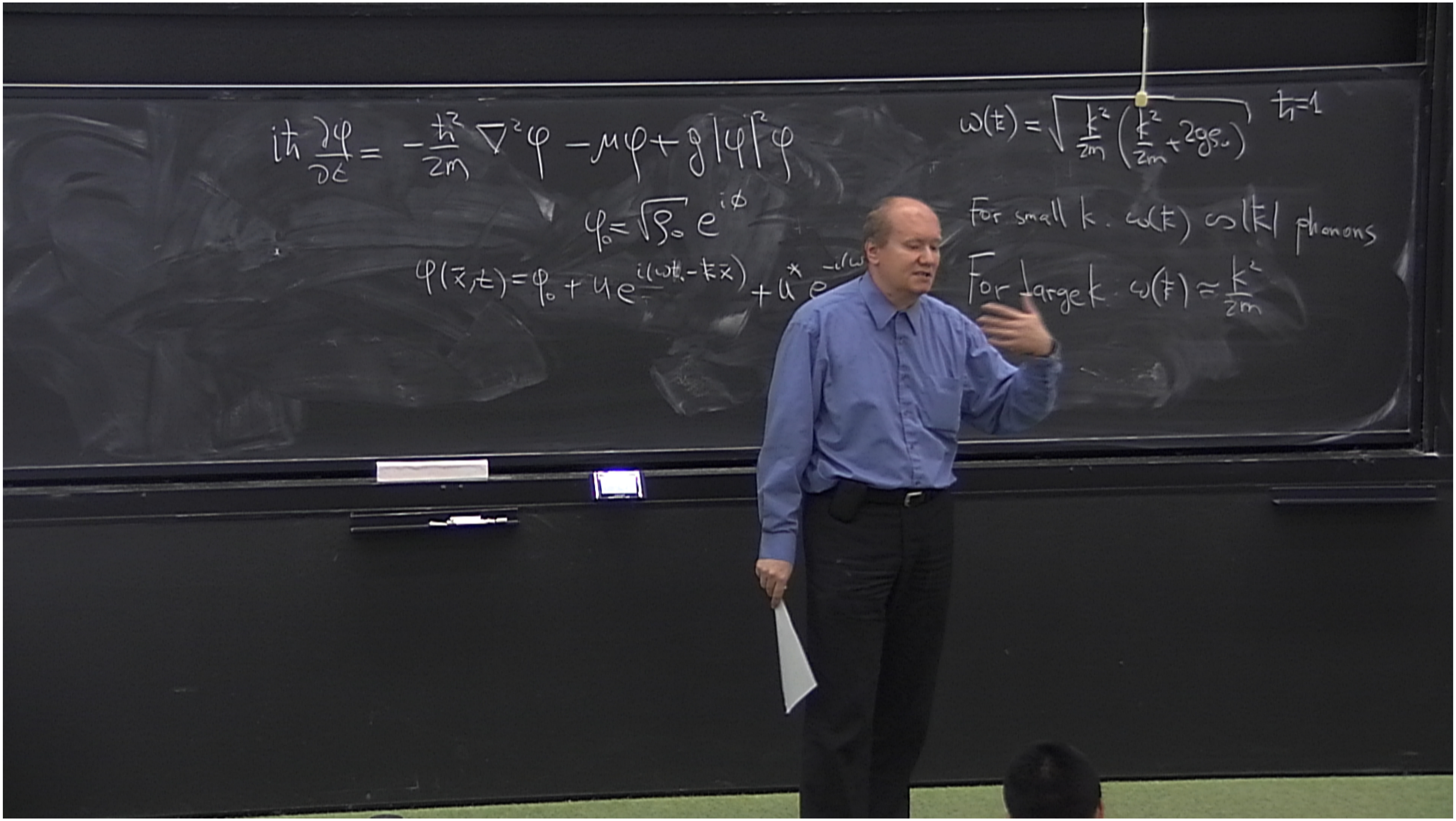
$\varphi(\vec{x},t) = \varphi_0 + u e^{i(\omega t - \vec{k}\cdot\vec{x})} + u^* e^{-i(\omega t - \vec{k}\cdot\vec{x})}$
$\omega(k) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
For small
$\omega(k)$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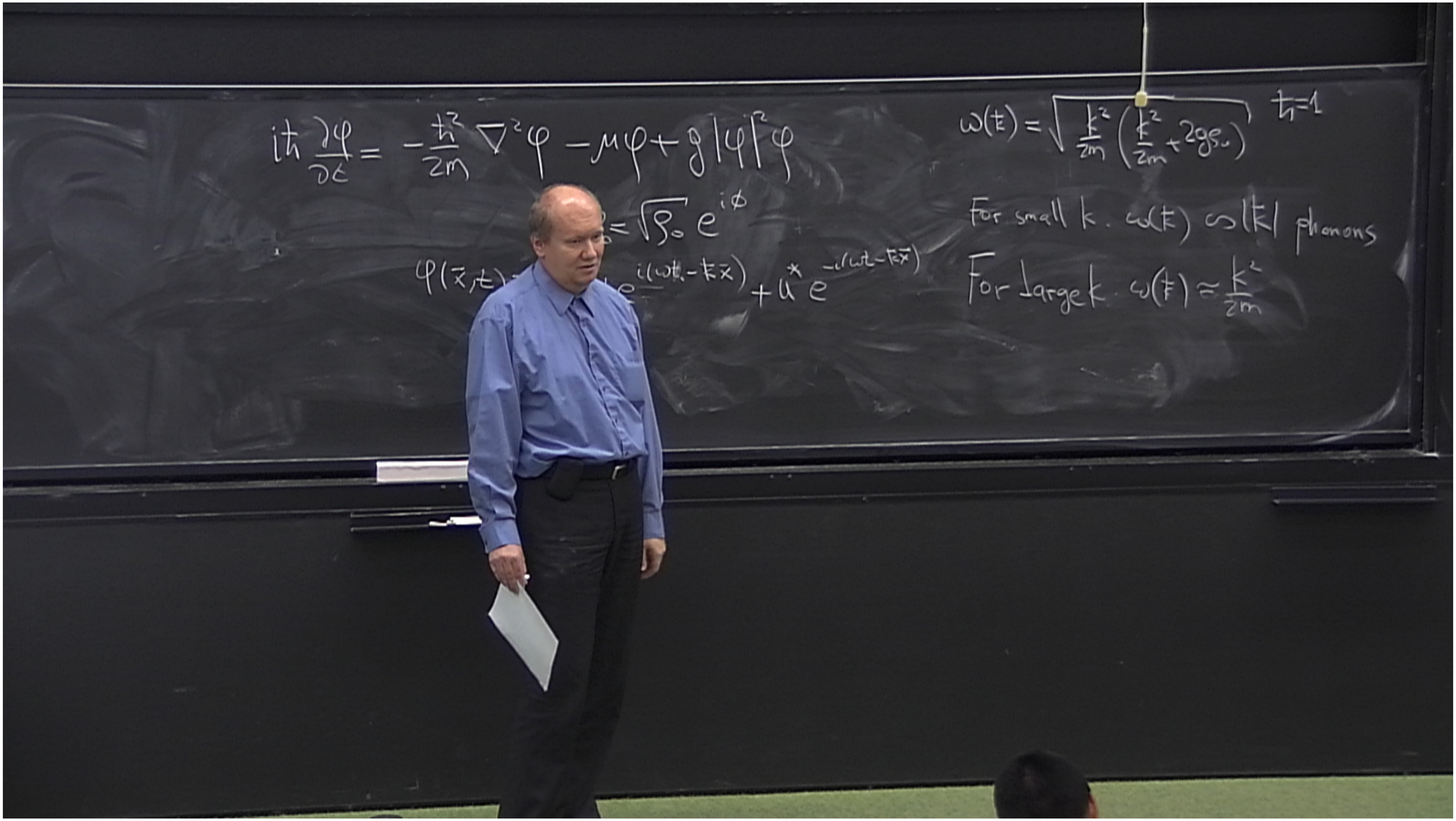
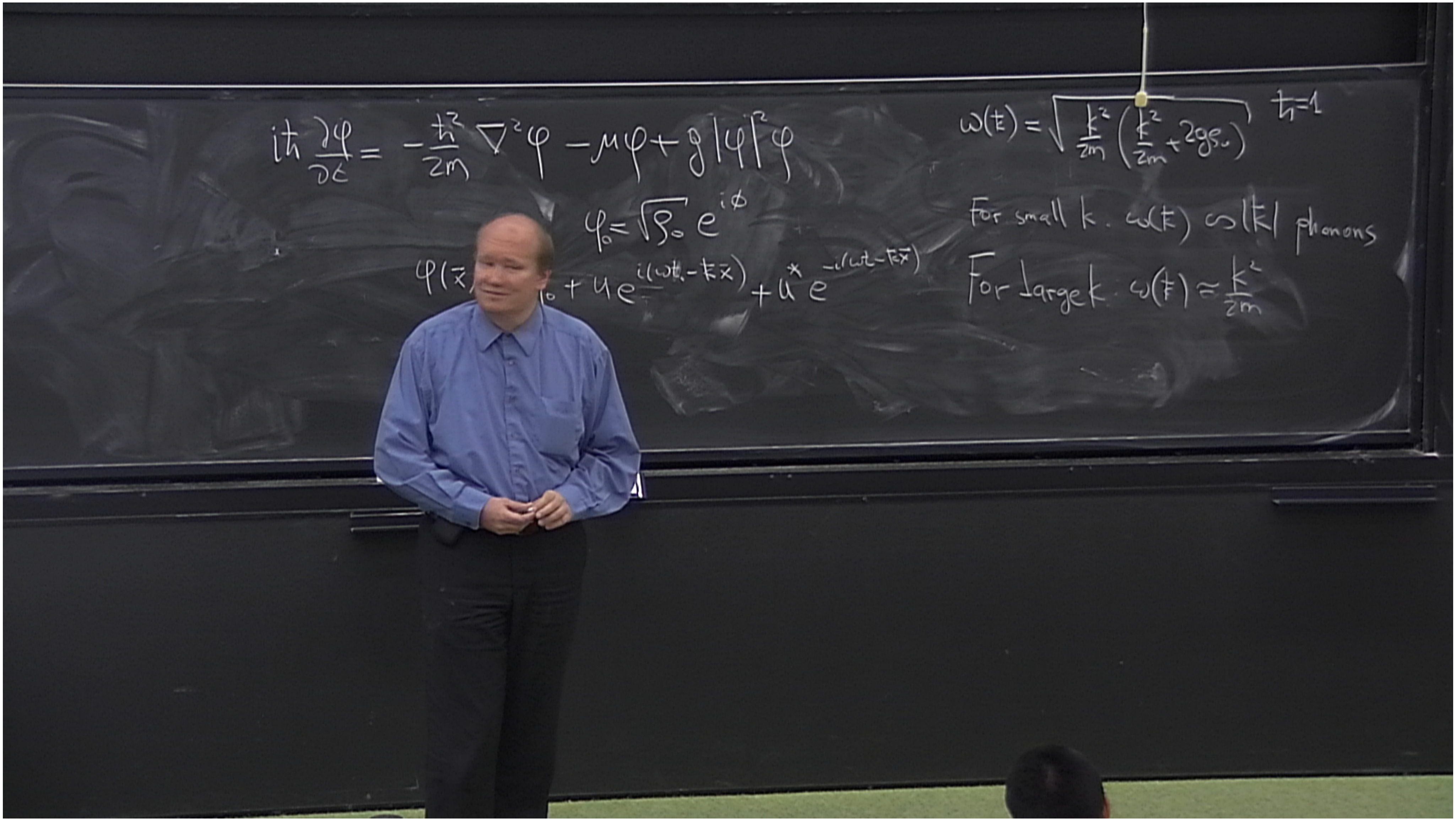
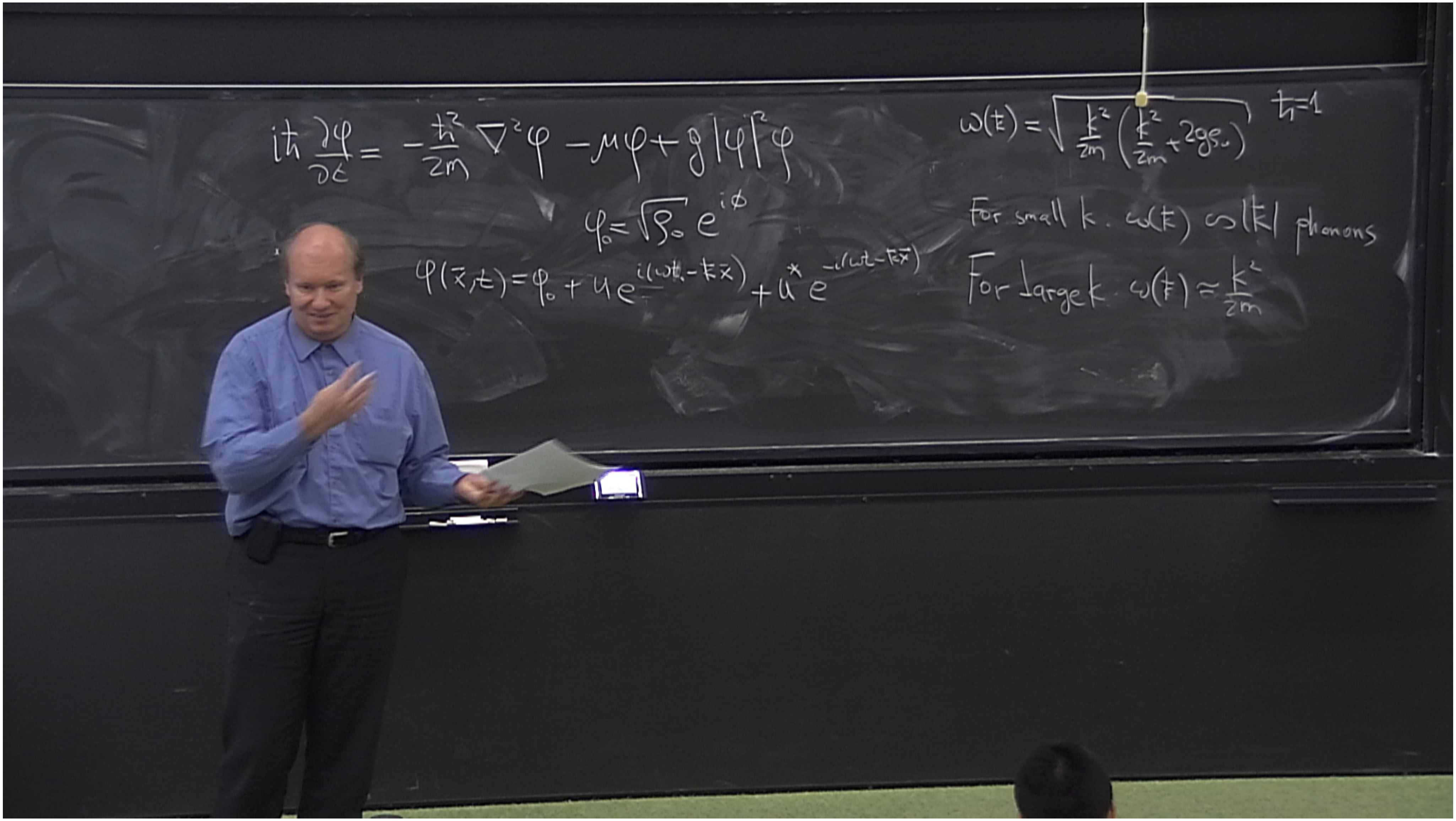
$\omega(k) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
For small k. $\omega(k) \propto |k|$ phonons
For large k $\omega(k) \approx \frac{k^2}{2m}$


$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi - \mu\varphi + g|\varphi|^2\varphi$
$= \sqrt{\rho_0}\, e^{i\phi}$
$\varphi(\vec{x},t)$ $e^{i(\omega t - \vec{k}\vec{x})} + u^* e^{-i(\omega t - \vec{k}\vec{x})}$
$\omega(\vec{k}) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
For small k. $\omega(\vec{k}) \sim |\vec{k}|$ phonons
For large k. $\omega(\vec{k}) \approx \frac{k^2}{2m}$



iħ ∂φ/∂t = − ħ²/2m ∇²φ − μφ + g|φ|²φ
φ₀ = √ρ₀ e^{iϕ}
φ(x) … + u e^{i(ωt − k·x)} + u* e^{−i(ωt − k·x)}
ω(k) = √(k²/2m (k²/2m + 2gρ₀))
ħ=1
For small k. ω(k) ∝ |k| phonons
For large k. ω(k) ≈ k²/2m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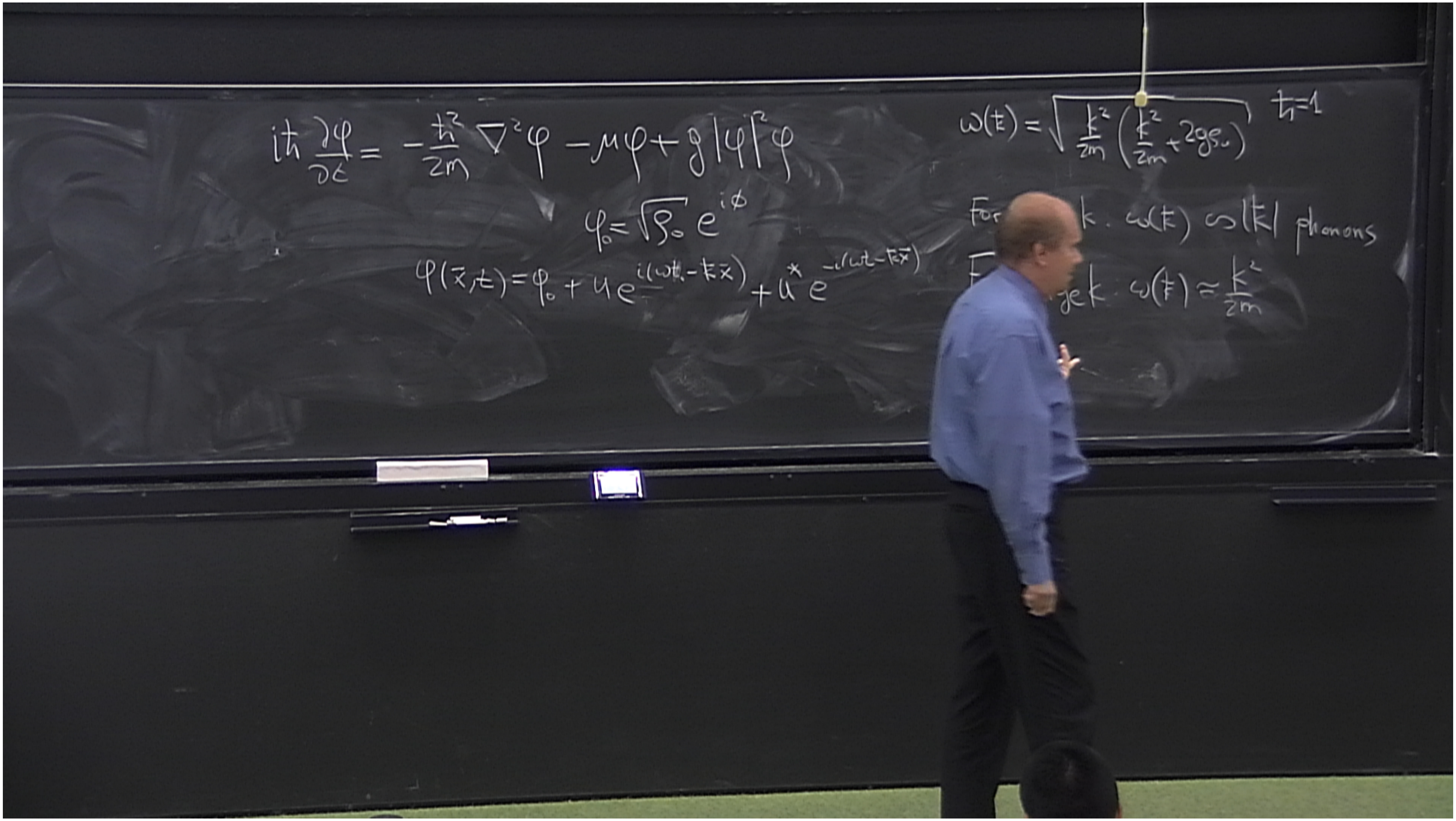
$\omega(\vec{k}) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
For small k. $\omega(\vec{k}) \propto |\vec{k}|$ phonons
For large k. $\omega(\vec{k}) \approx \frac{k^2}{2m}$



$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi - \mu\varphi + g|\varphi|^2\varphi$
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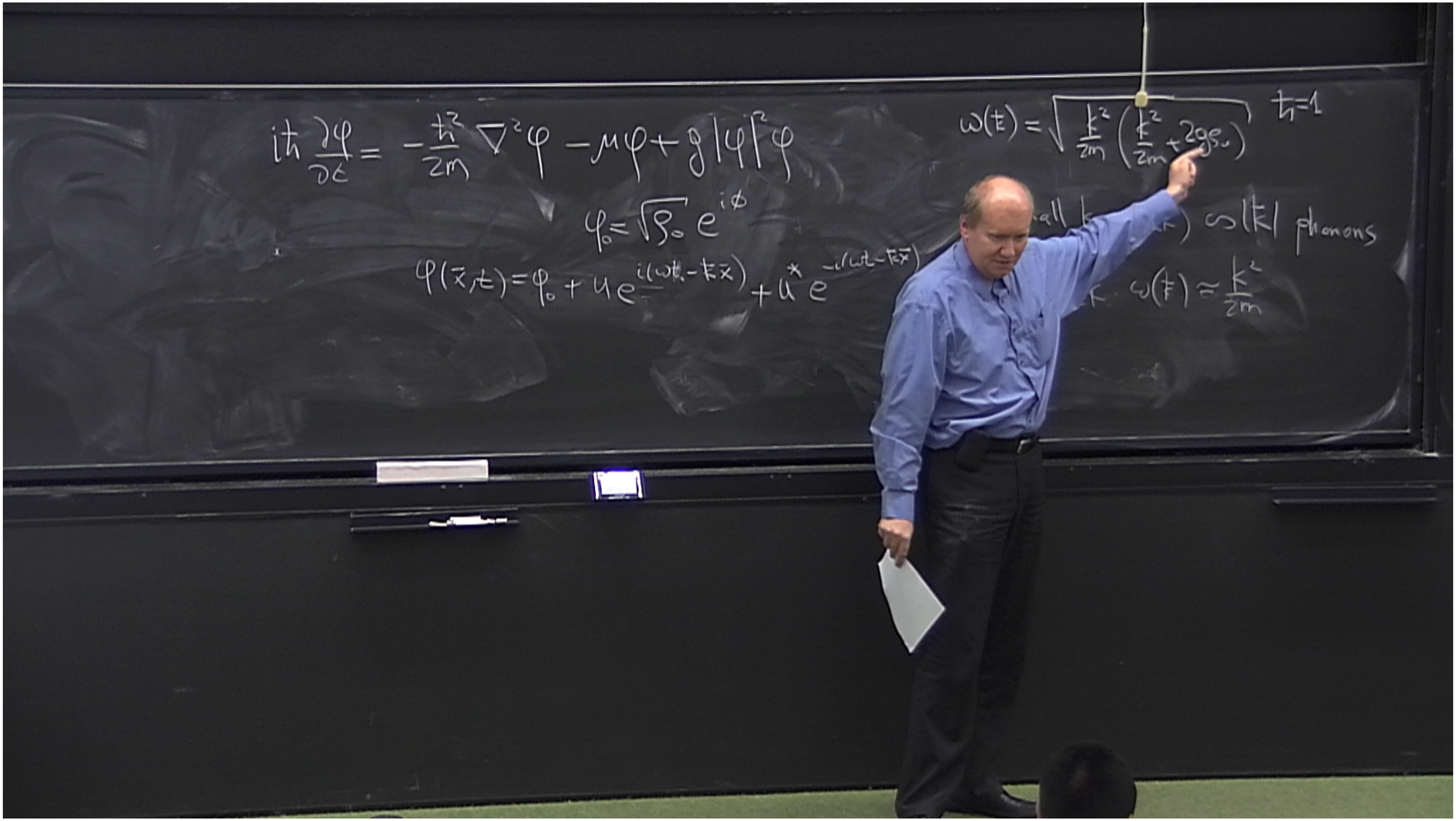
$\omega(\vec{k}) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
For
$k.\ \omega(\vec{k}) \sim |\vec{k}|$ phonons
$k.\ \omega(\vec{k}) \approx \frac{k^2}{2m}$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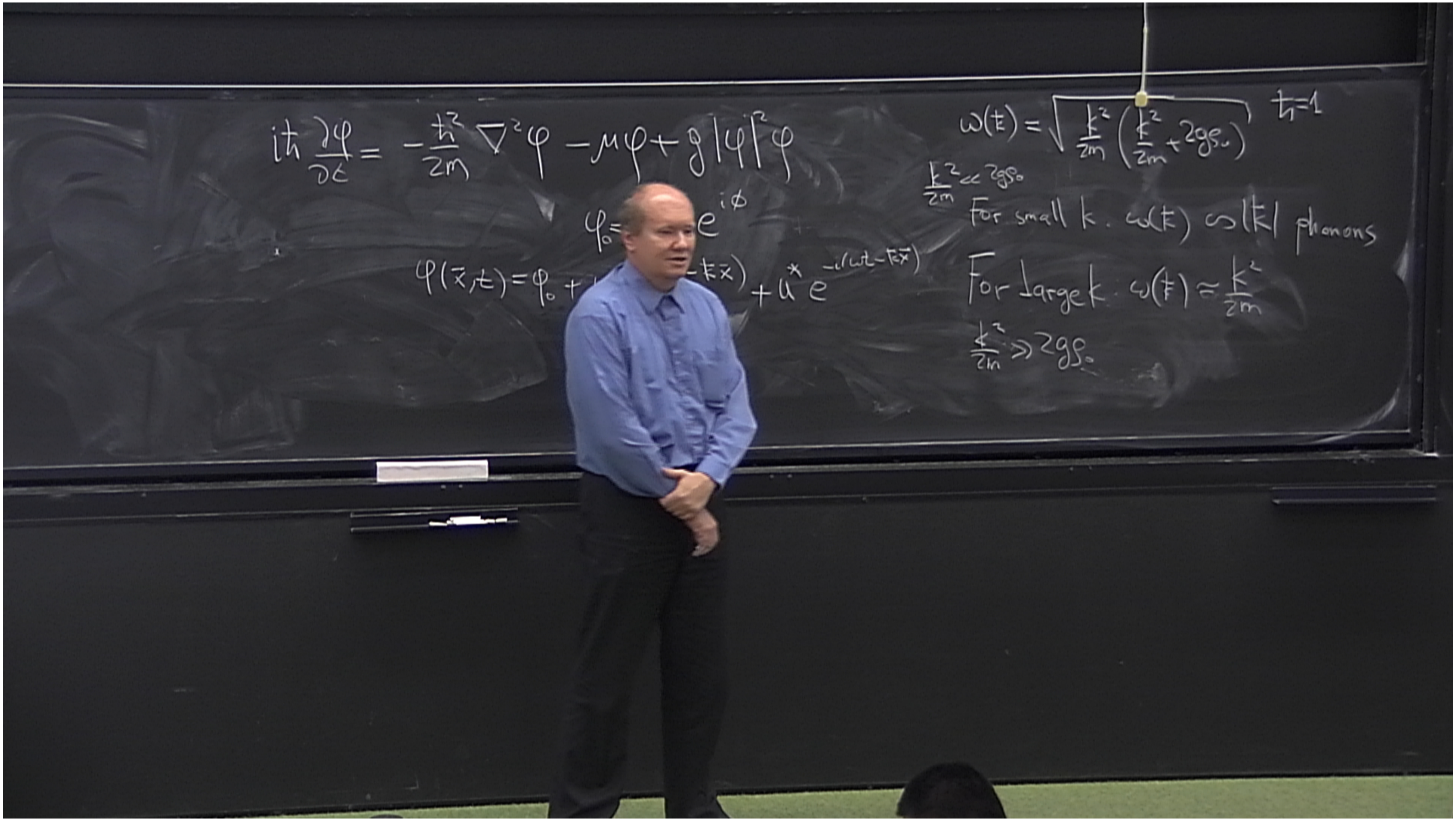
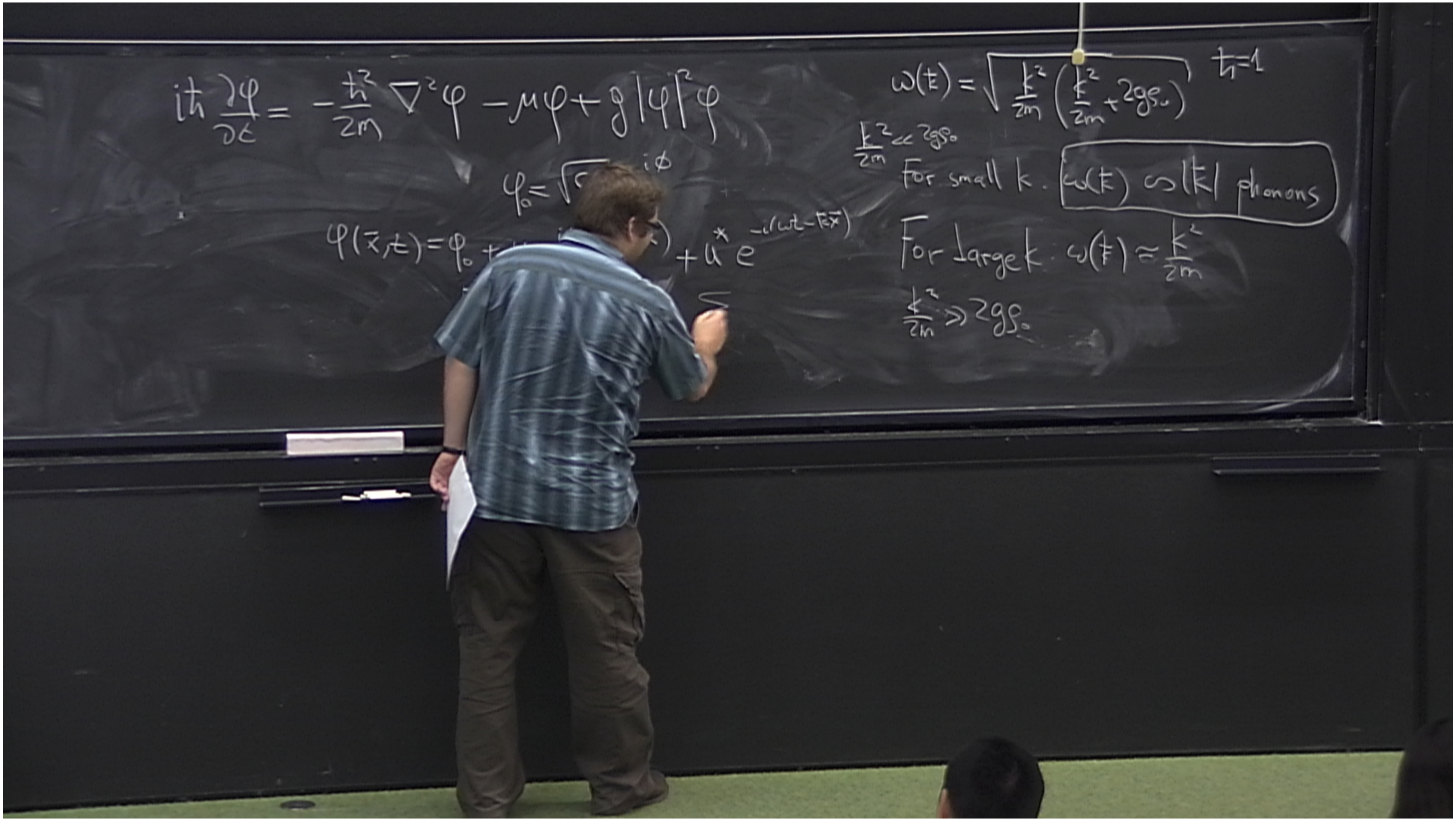
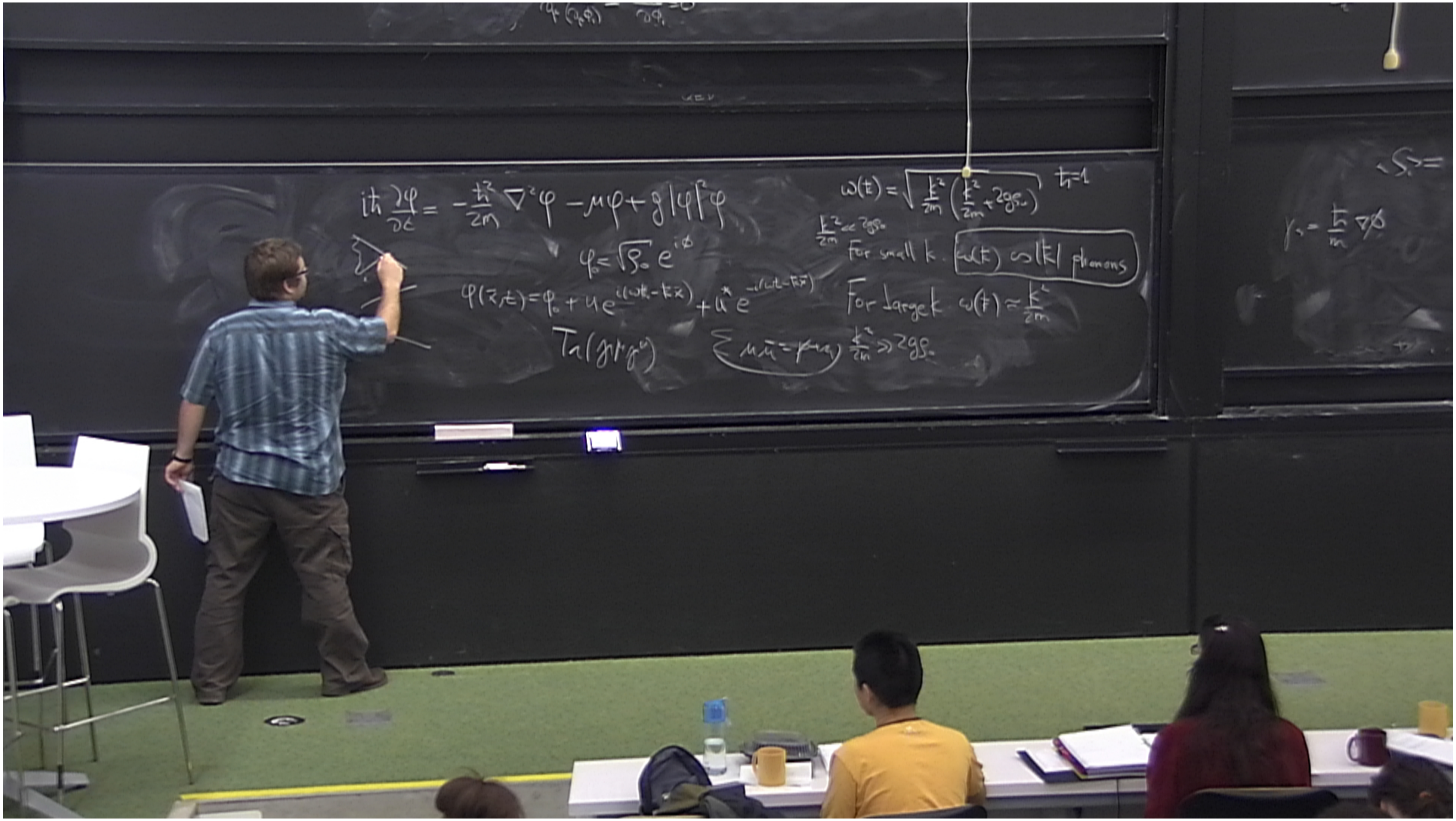
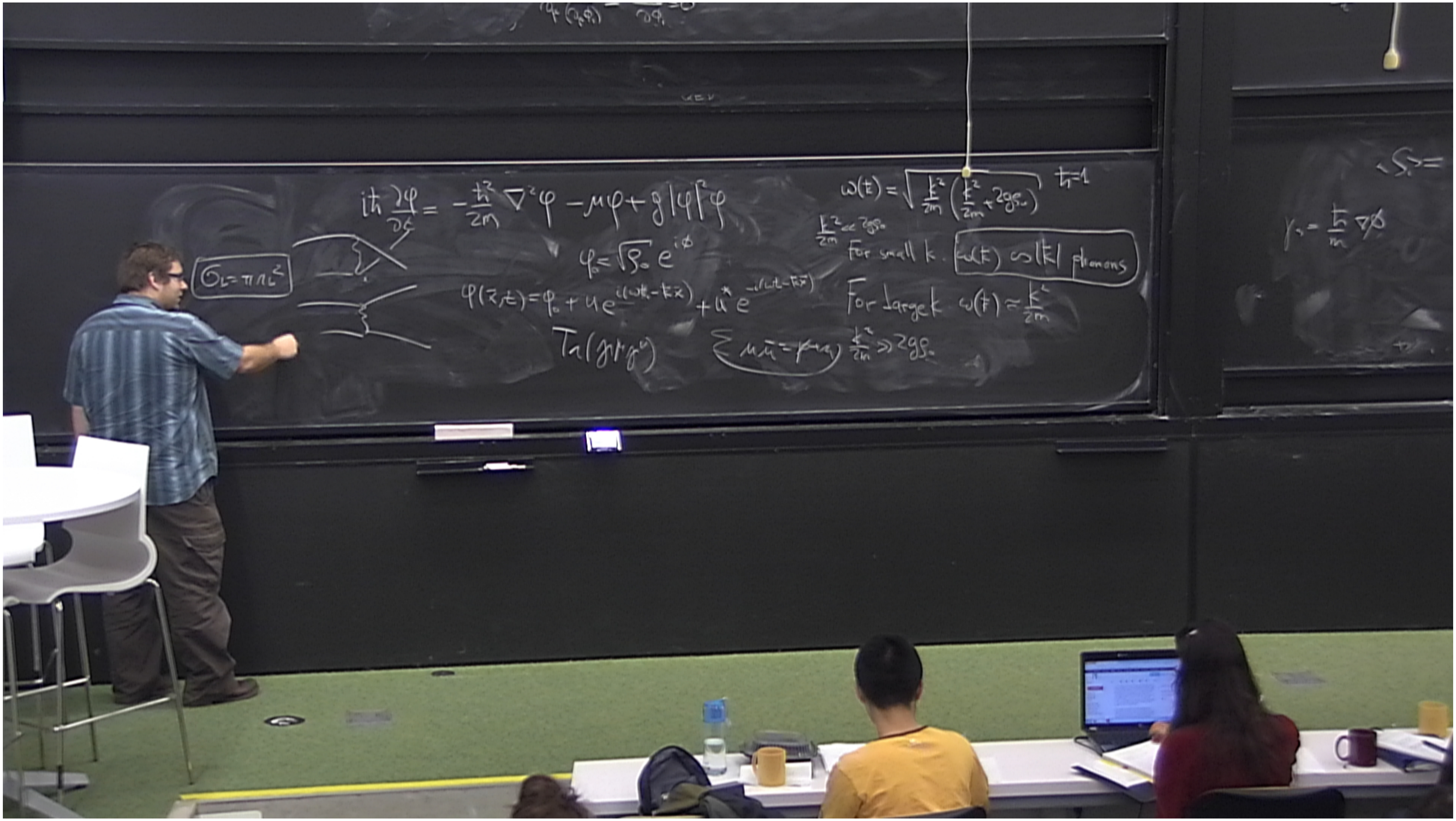
$\omega(\vec{k}) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
$\sim |\vec{k}|$ phonons
$\omega(\vec{k}) \approx \frac{k^2}{2m}$

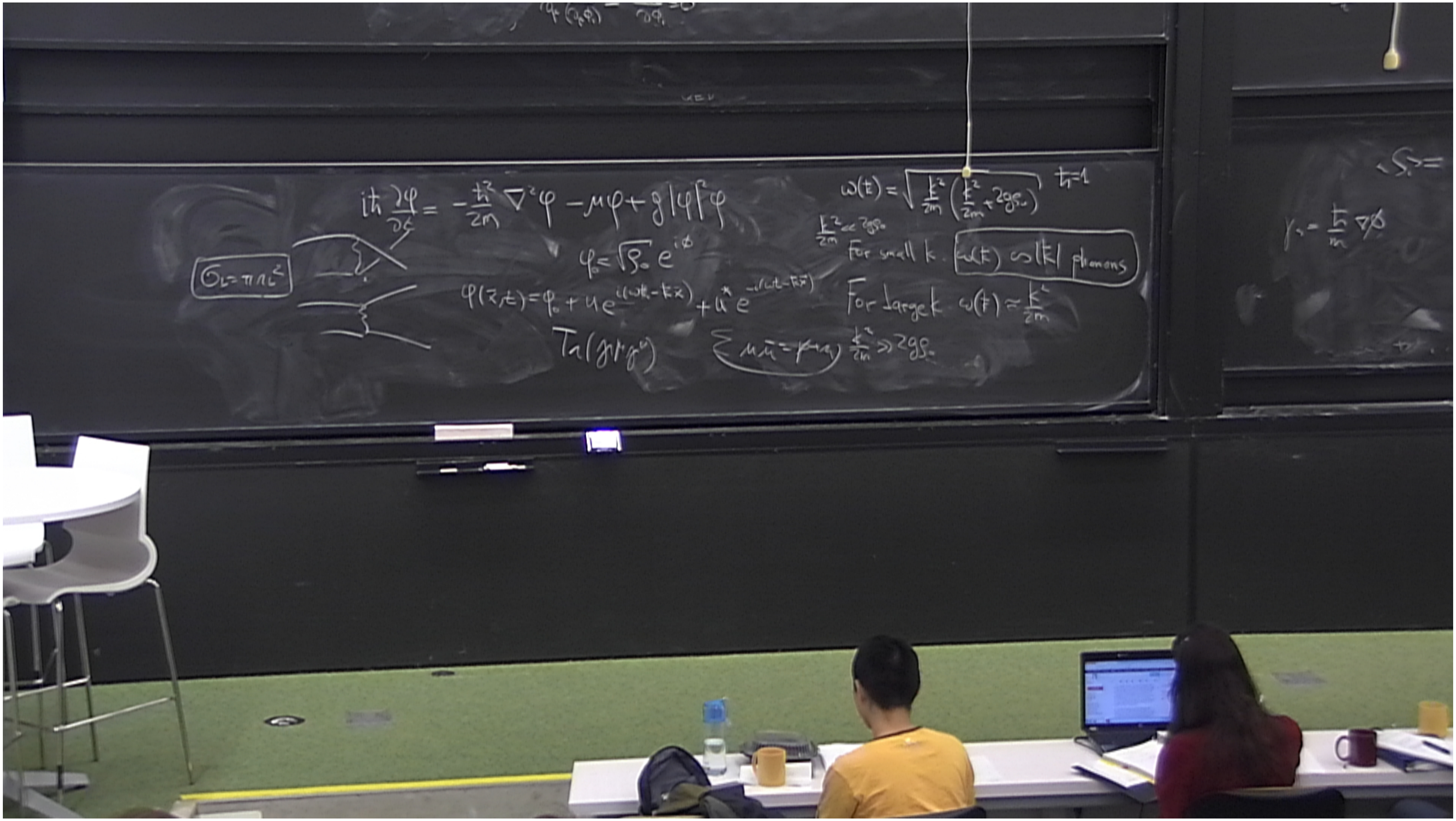

$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi - \mu\varphi + g|\varphi|^2\varphi$
$\omega(k) = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m} + 2g\rho_0\right)}$
$\hbar = 1$
$\frac{k^2}{2m} \ll 2g\rho_0$
For small k. $\omega(k) \propto |k|$ phonons
For large k. $\omega(k) \approx \frac{k^2}{2m}$
$\frac{k^2}{2m} \gg 2g\rho_0$

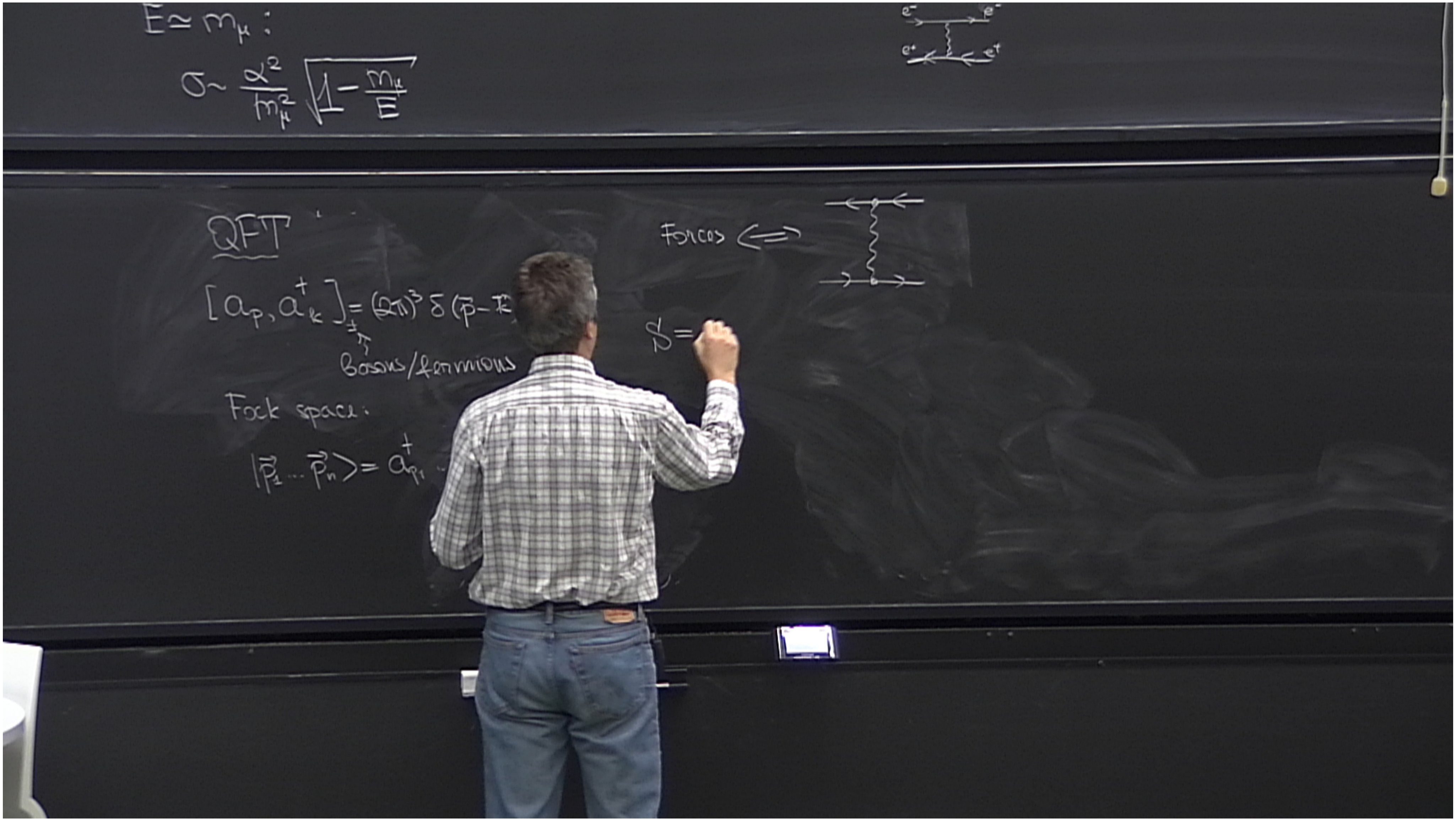


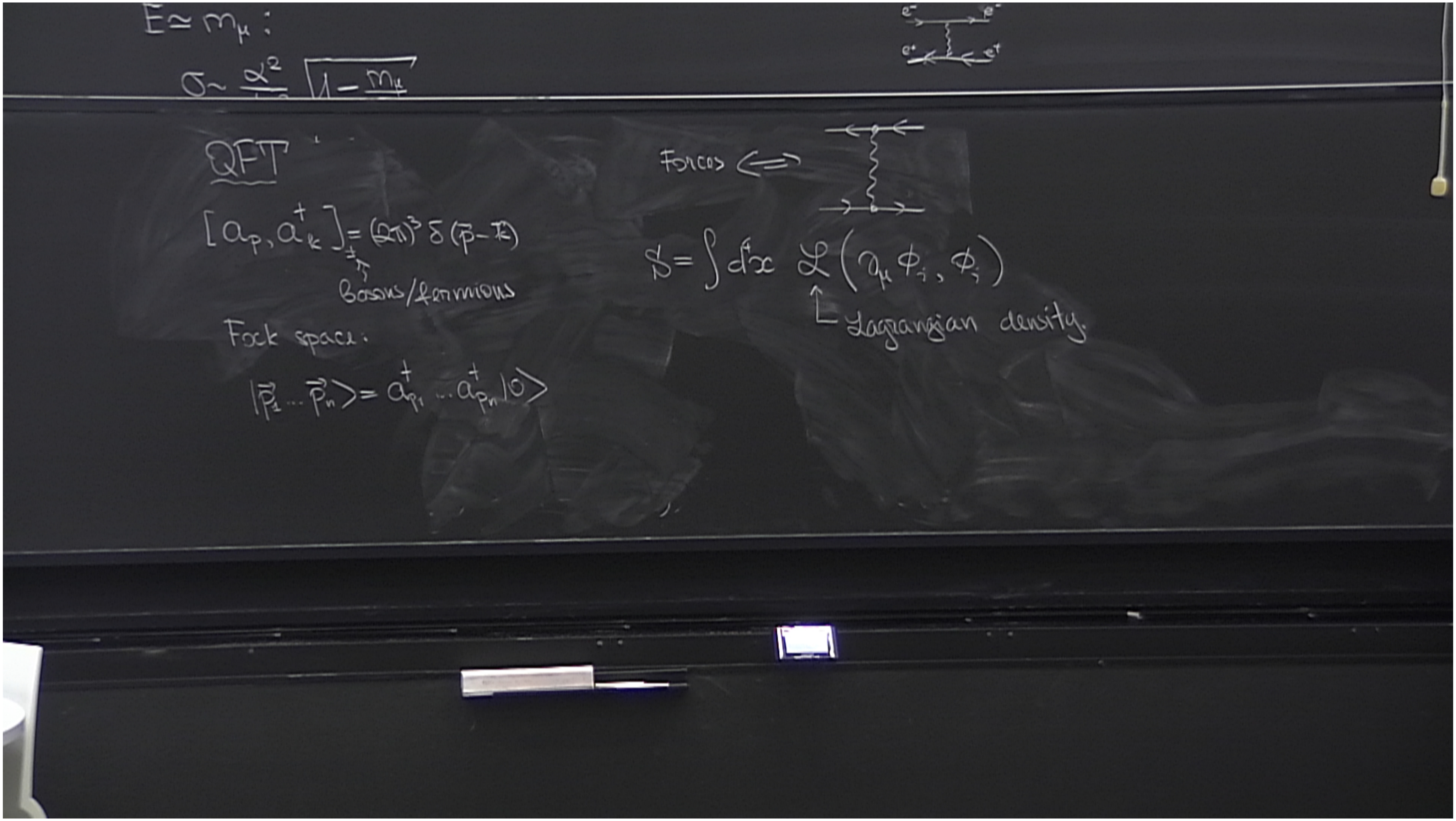
ħ=1
For small k
phonons
For large k

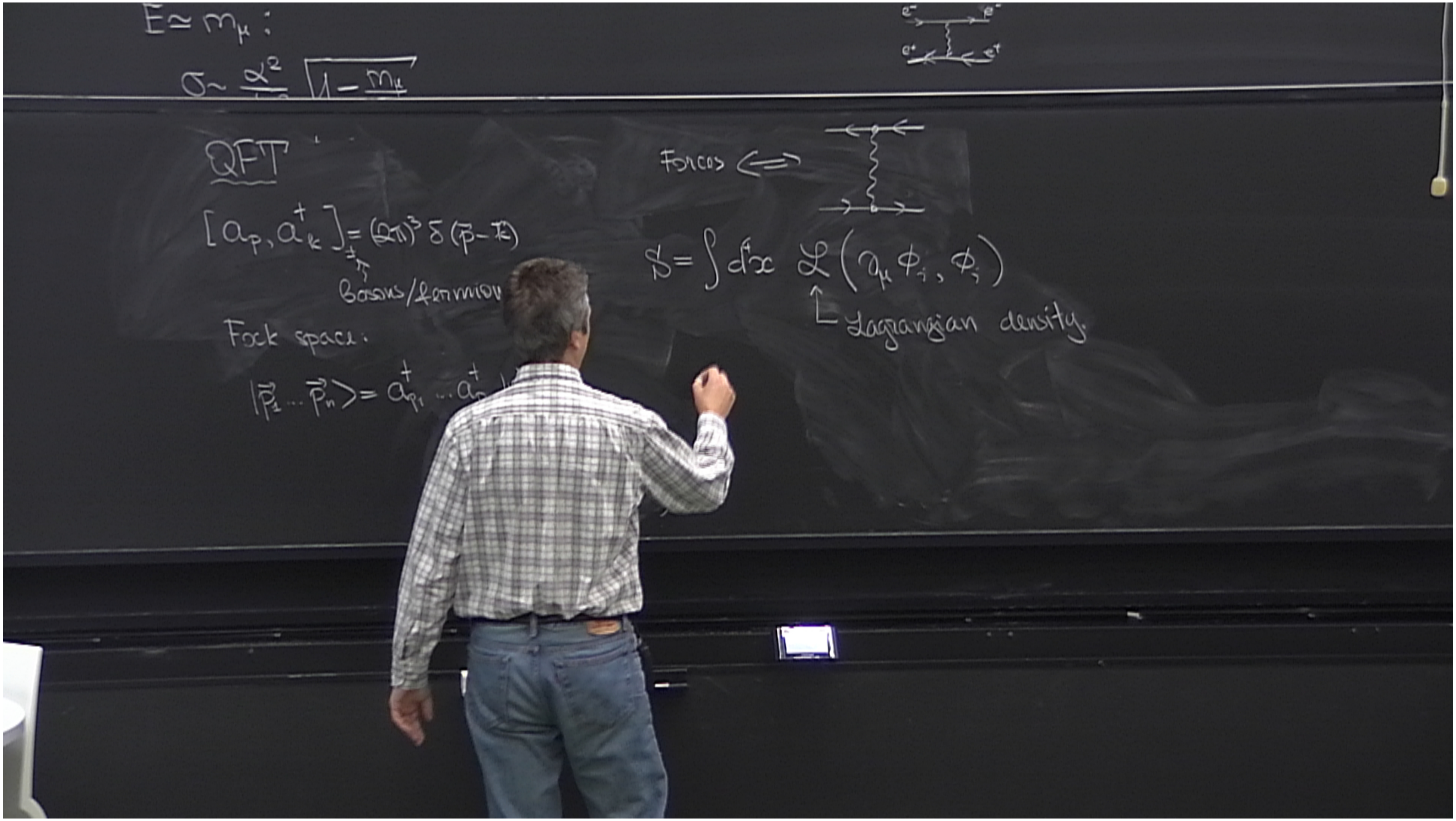

For small k
phonons
For large k

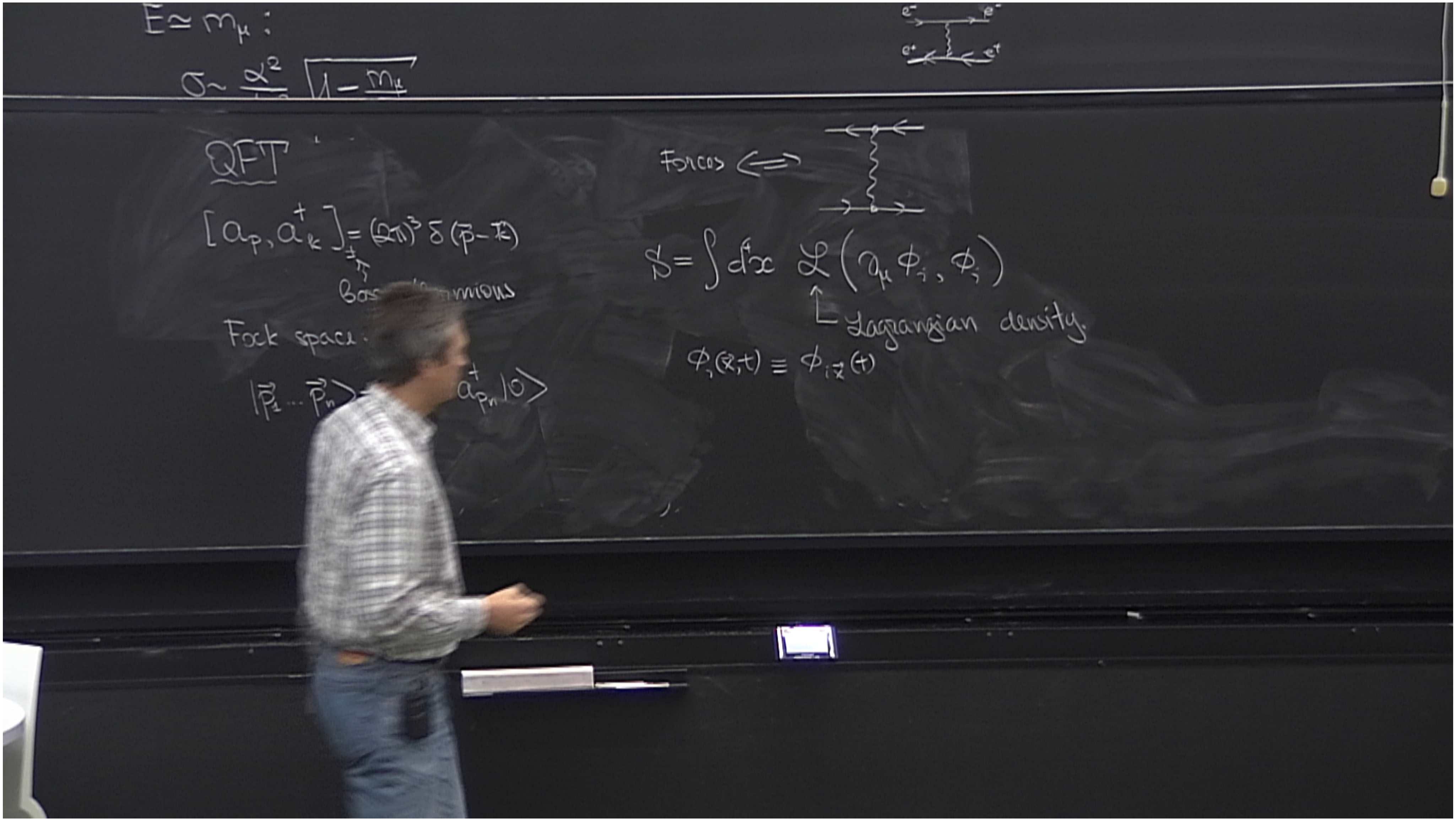

For small k
phonons
For large k
ħ=1


ħ=1
For small k
phonons
For large k

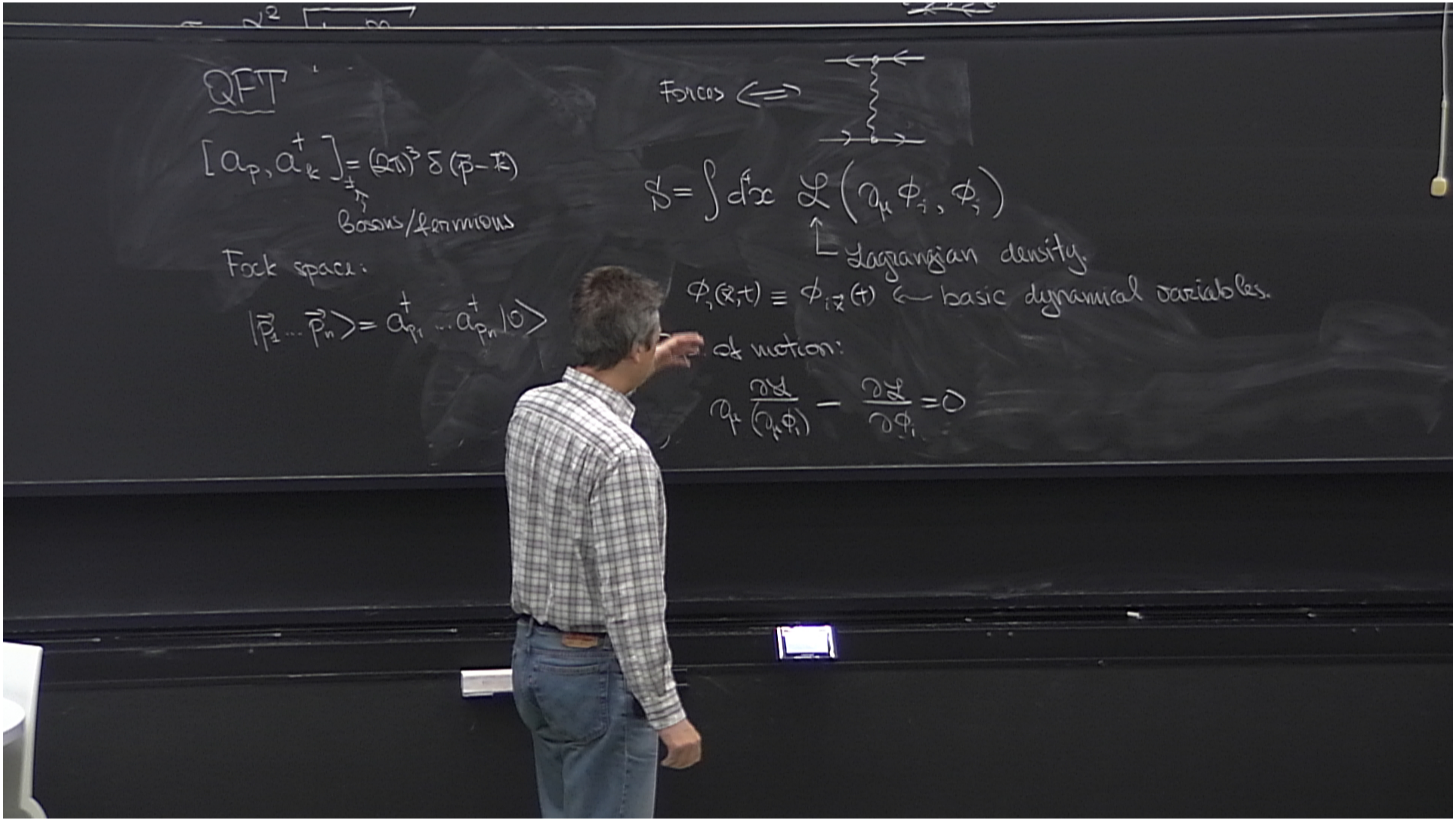

E ≃ mμ :
σ ~ α²/mμ² √(1 − mμ/E)
e⁻ e⁻
e⁺ e⁺
QFT
Forces ⟺
[ap, a†k]∓ = (2π)³ δ(p⃗ − k⃗)
Bosons/fermions
S =
Fock space:
|p⃗1 ... p⃗n> = a†p1

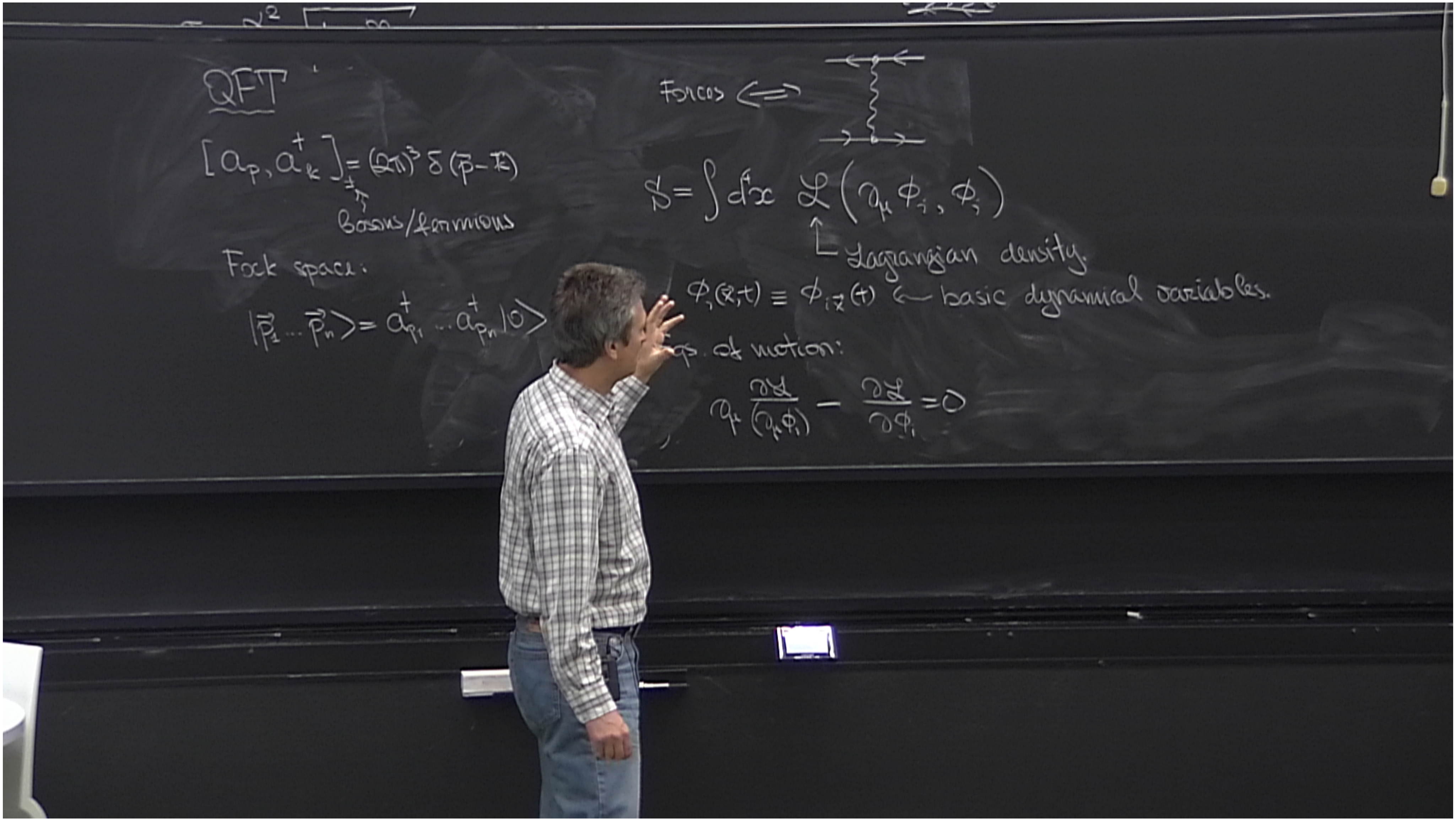

E ≃ m_μ :
QFT
[a_p, a_k^†]_∓ = (2π)^3 δ(p⃗ − k⃗)
Bosons/fermions
Fock space:
|p⃗_1 … p⃗_n> = a_{p_1}^† … a_{p_n}^† |0>
Forces ⟸⟹
S = ∫ d^4x ℒ(∂_μ φ_i, φ_i)
Lagrangian density
e⁻
e⁺

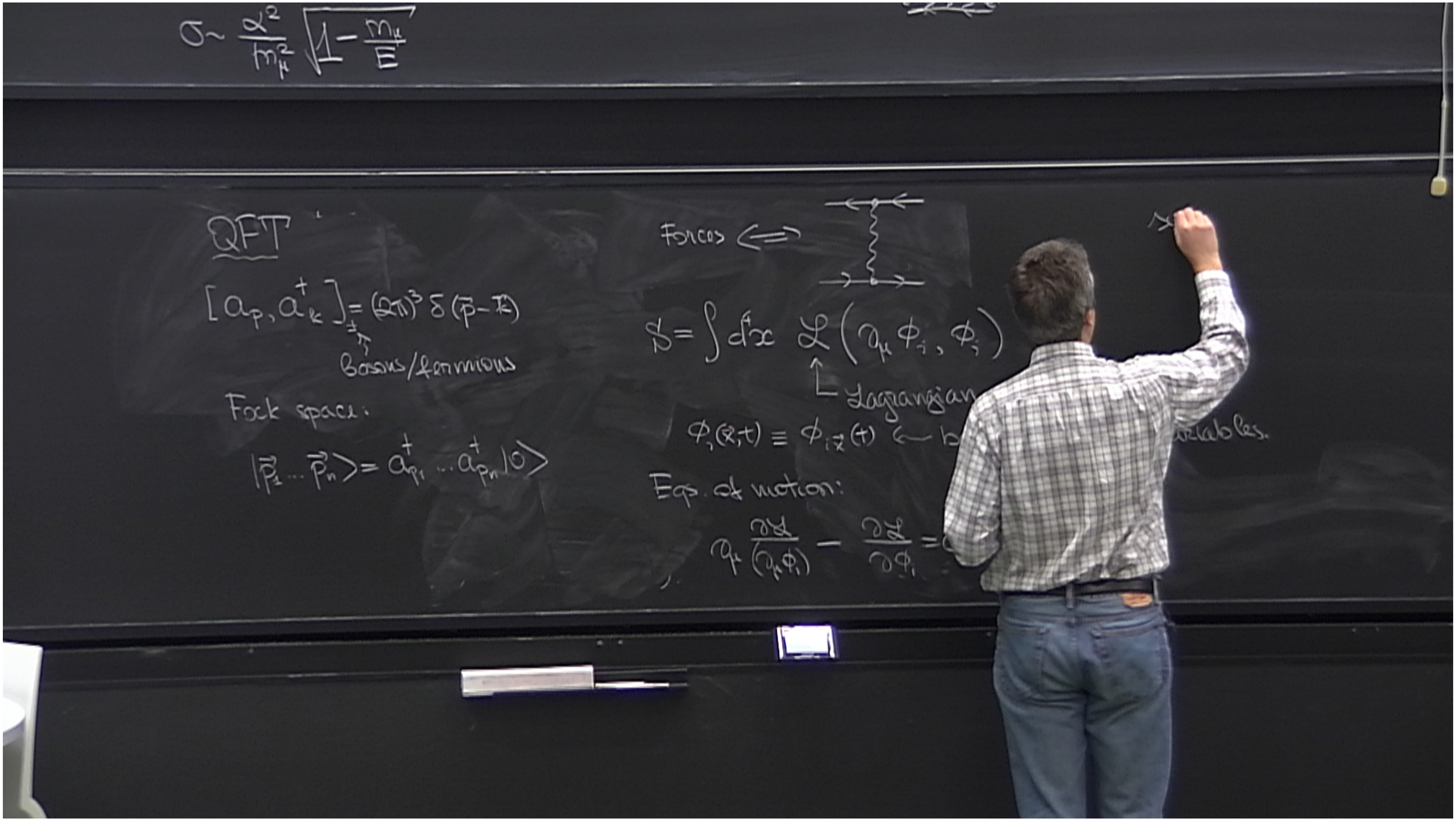

$E \simeq m_\mu$:
QFT
$[a_p, a_k^\dagger]_\mp = (2\pi)^3 \delta(\vec{p} - \vec{k})$
Fock space.
Forces ⟺
$S = \int d^4x \, \mathcal{L}(\partial_\mu \phi_i, \phi_i)$
Lagrangian density.

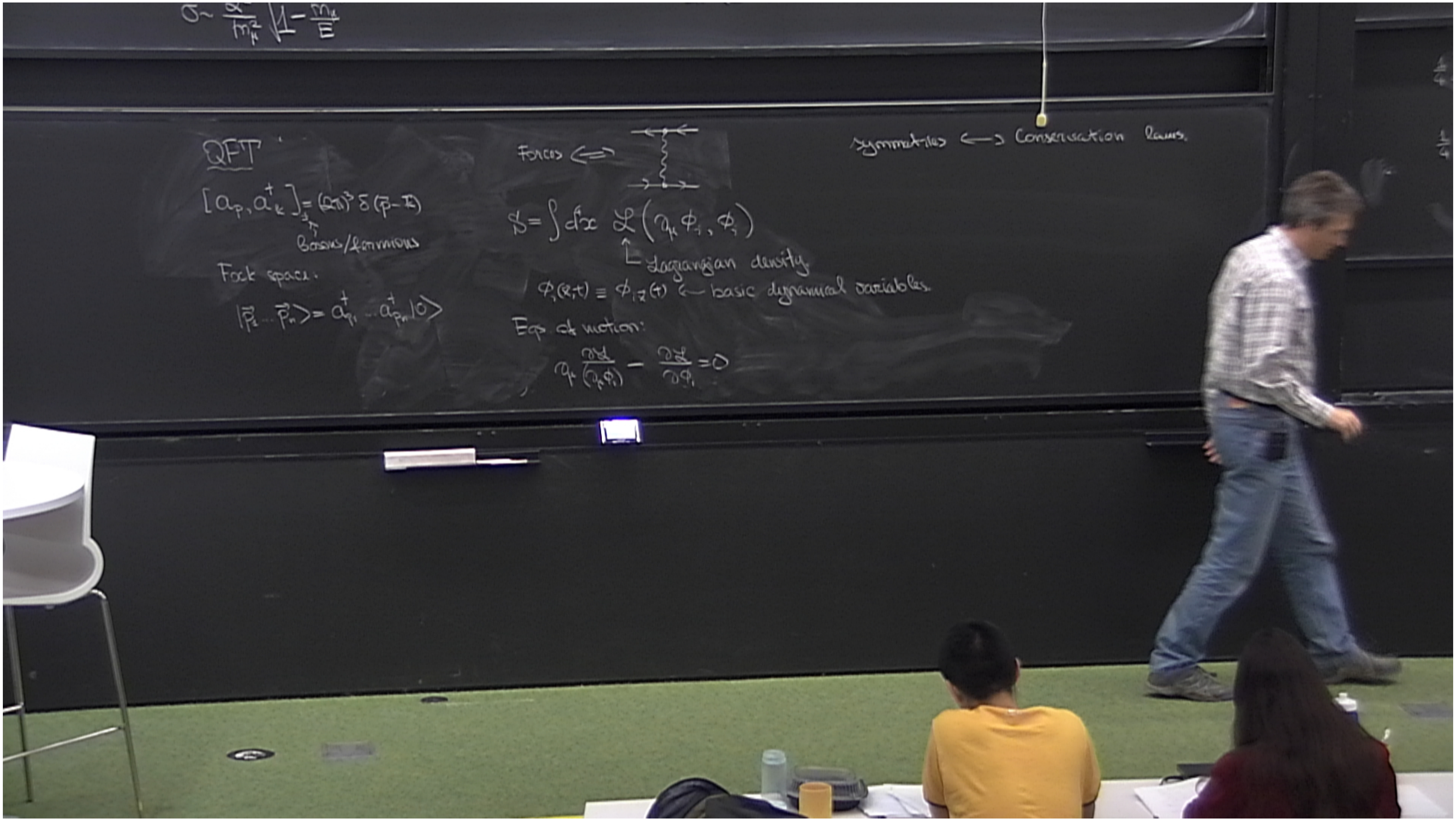

E ≃ mμ :
QFT
Forces ⟸⟹
Fock space:
Lagrangian density.

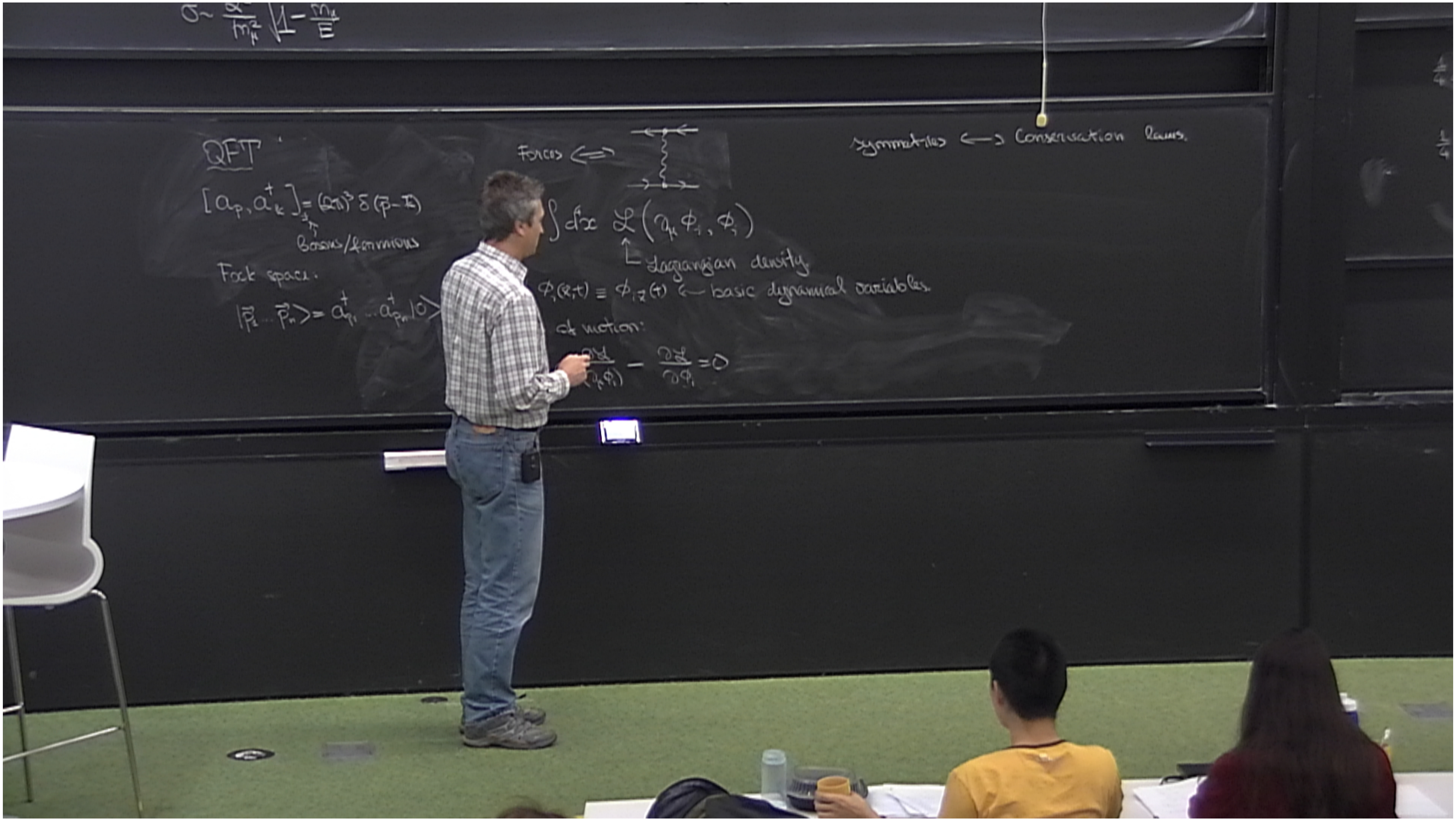

QFT
Forces ⟸⟹
[a_p, a†_k]∓ = (2π)^3 δ(p − k)
Bosons/fermions
Fock space:
|p1 ... pn> = a†p1 ... a†pn |0>
S = ∫ d^4x L(∂μ φi, φi)
Lagrangian density.
φi(x,t) ≡ φi,x(t) ← basic dynamical variables.

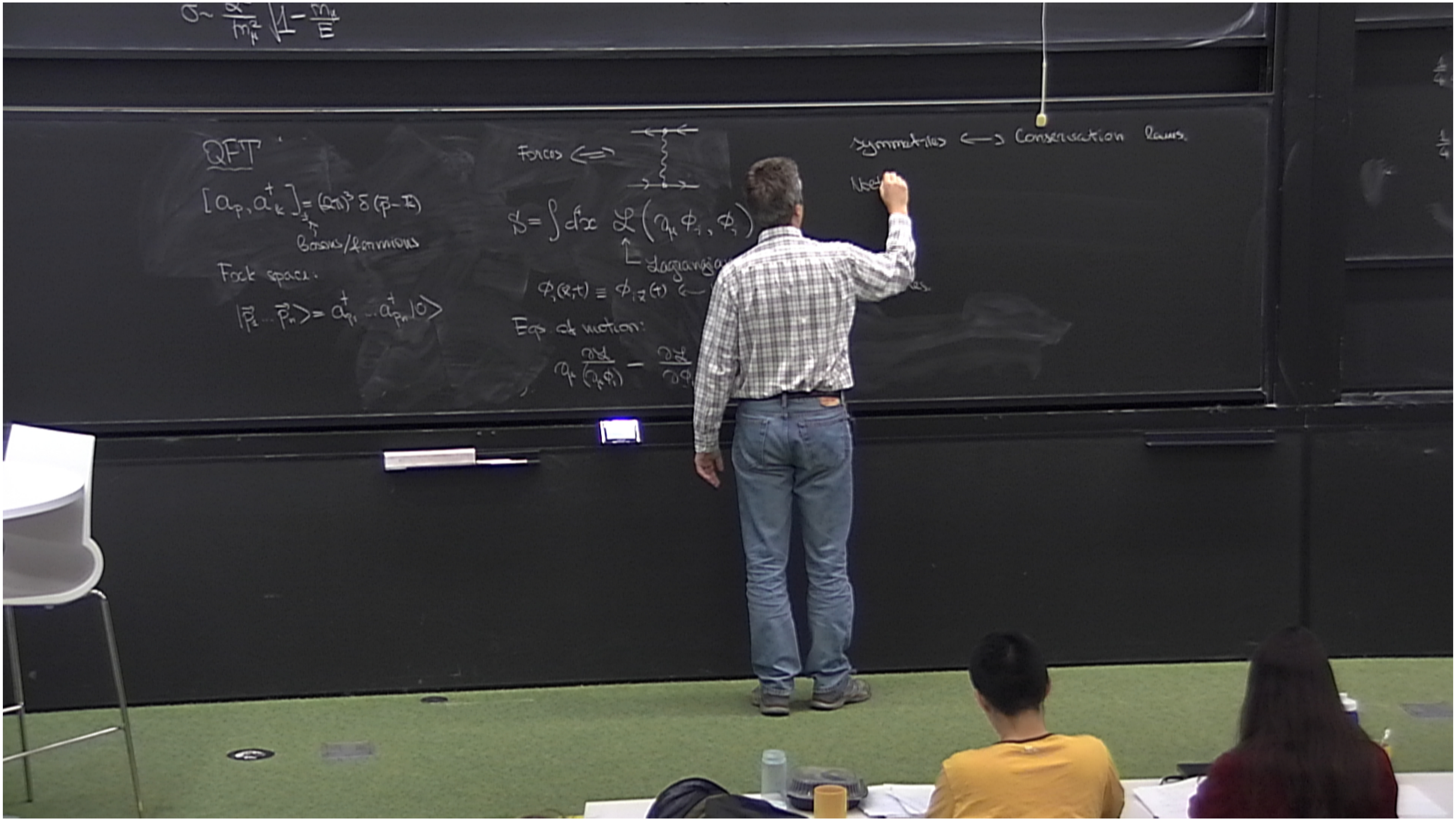

QFT
$[a_p, a_k^\dagger]_\mp = (2\pi)^3 \delta(\vec{p}-\vec{k})$
Bosons/fermions
Fock space:
$|\vec{p}_1 \ldots \vec{p}_n\rangle = a_{p_1}^\dagger \ldots a_{p_n}^\dagger |0\rangle$
Forces ⟺
$S = \int d^4x \, \mathcal{L}(\partial_\mu \phi_i, \phi_i)$
Lagrangian density.
$\phi_i(\vec{x},t) \equiv \phi_{i\vec{x}}(t)$ ⟵ basic dynamical variables.
of motion:
$\partial_\mu \frac{\partial \mathcal{L}}{(\partial_\mu \phi_i)} - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$

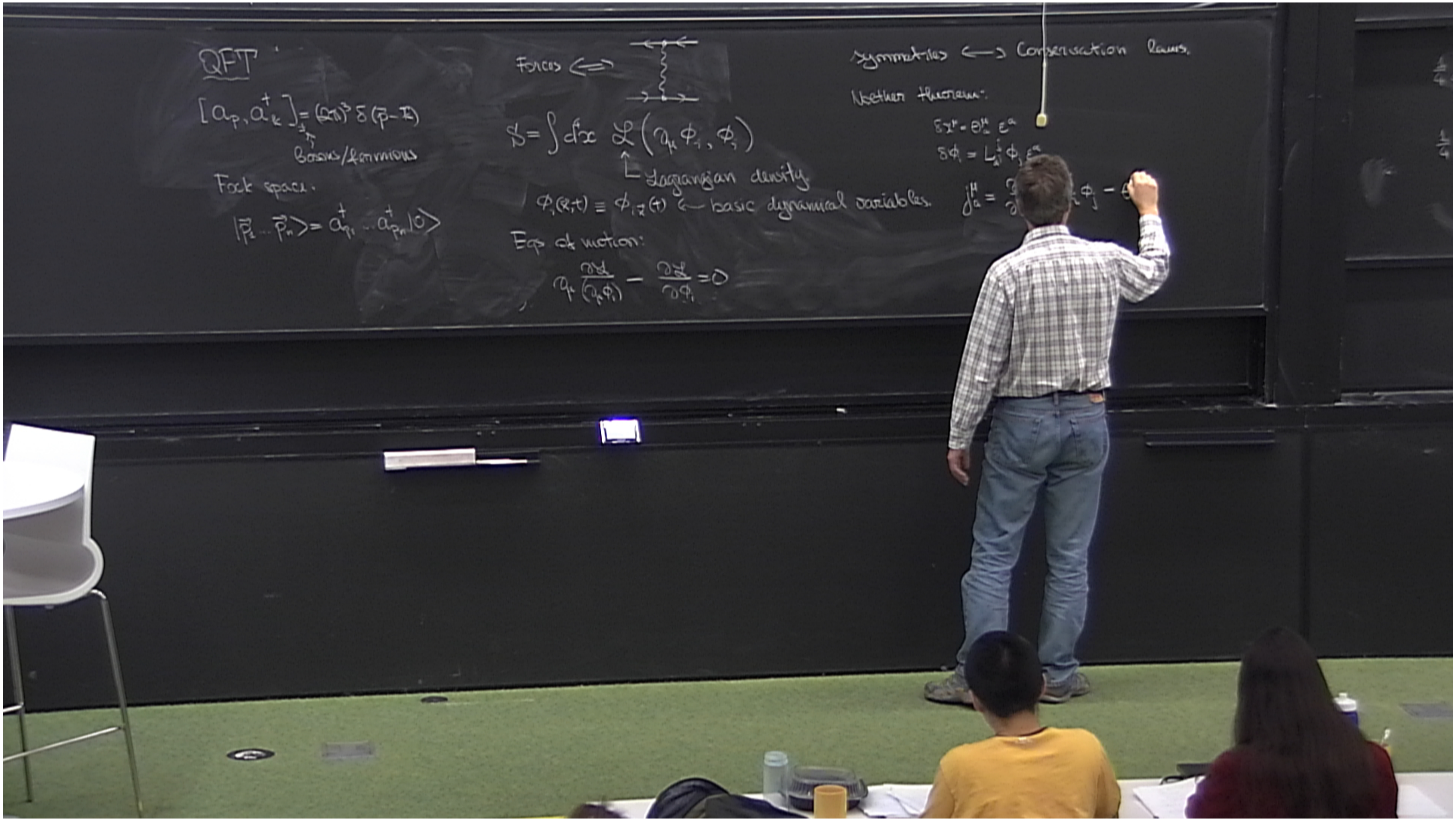

QFT
$[a_p, a^\dagger_k]_\pm = (2\pi)^3 \delta(\vec{p}-\vec{k})$
Bosons/fermions
Fock space:
$|\vec{p}_1 \dots \vec{p}_n\rangle = a^\dagger_{p_1} \dots a^\dagger_{p_n}|0\rangle$
Forces $\Longleftrightarrow$
$S = \int d^4x\, \mathcal{L}(\partial_\mu \phi_i, \phi_i)$
Lagrangian density.
$\phi_i(\vec{x},t) \equiv \phi_{i\vec{x}}(t)$ ← basic dynamical variables.
Eqs. of motion:
$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$

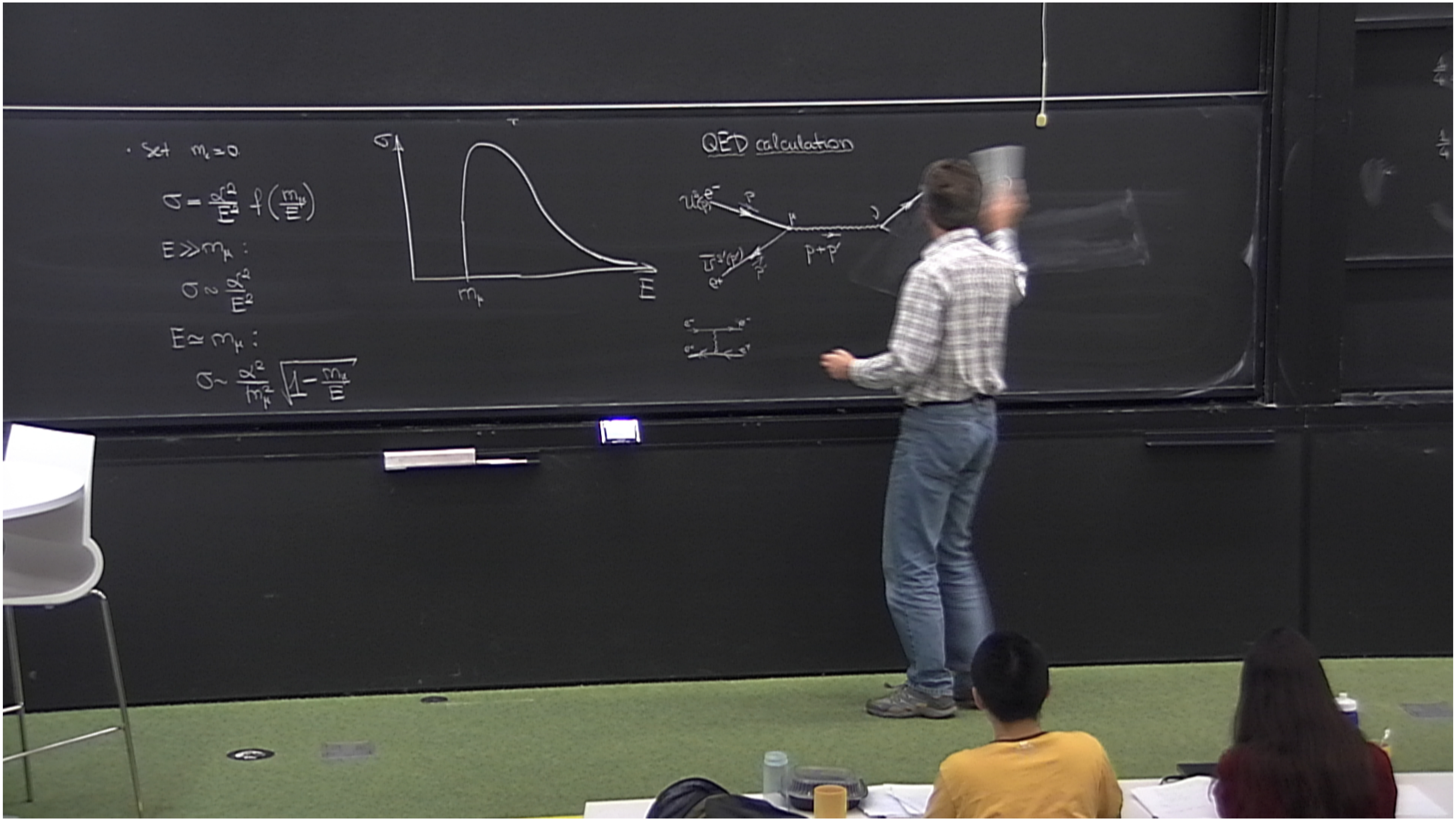

QFT
Forces ⟸
Bosons/fermions
Fock space:
Lagrangian
Eqs. of motion:
$\sigma \sim \frac{\alpha^2}{m_\mu^2}\sqrt{1-\frac{m_\mu}{E}}$
$[a_p, a_k^\dagger]_\mp = (2\pi)^3\delta(\vec{p}-\vec{k})$
$|\vec{p}_1 \dots \vec{p}_n\rangle = a_{p_1}^\dagger \dots a_{p_n}^\dagger|0\rangle$
$S = \int d^4x\, \mathcal{L}(\partial_\mu\phi_i, \phi_i)$
$\phi_i(\vec{x},t) \equiv \phi_{i\vec{x}}(t)$


QFT
Forces ⟺
Symmetries ⟷ Conservation laws.
Bosons/fermions
Fock space.
Lagrangian density
basic dynamical variables
Eqs of motion:

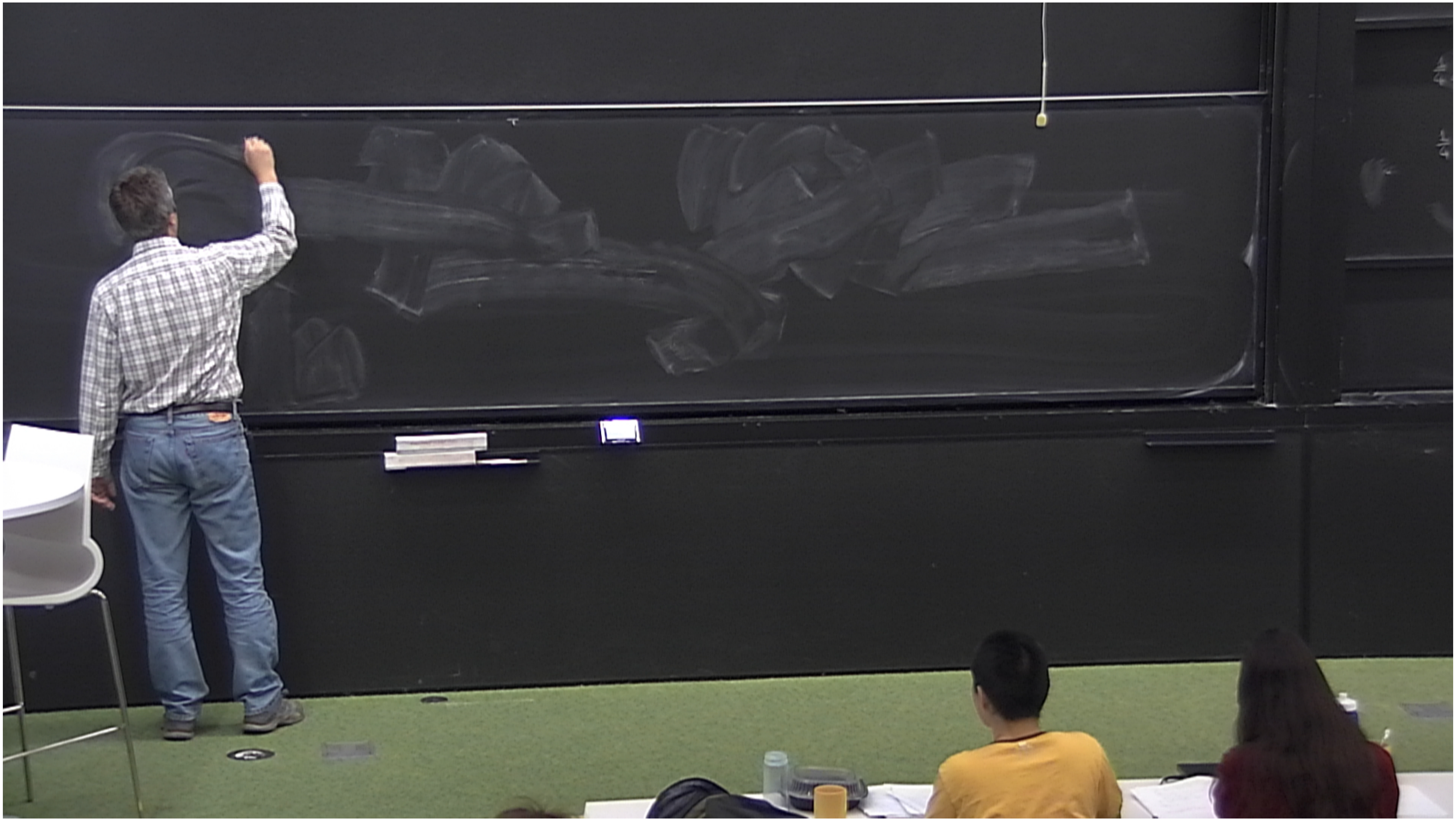

QFT
Forces ⟺
symmetries ⟷ Conservation laws.
Bosons/fermions
Fock space.
Lagrangian density
basic dynamical variables
of motion:


QFT
Bosons/fermions
Fock space.
Forces
Lagrangian
Eqs. of motion:
Symmetries ⟷ Conservation laws.

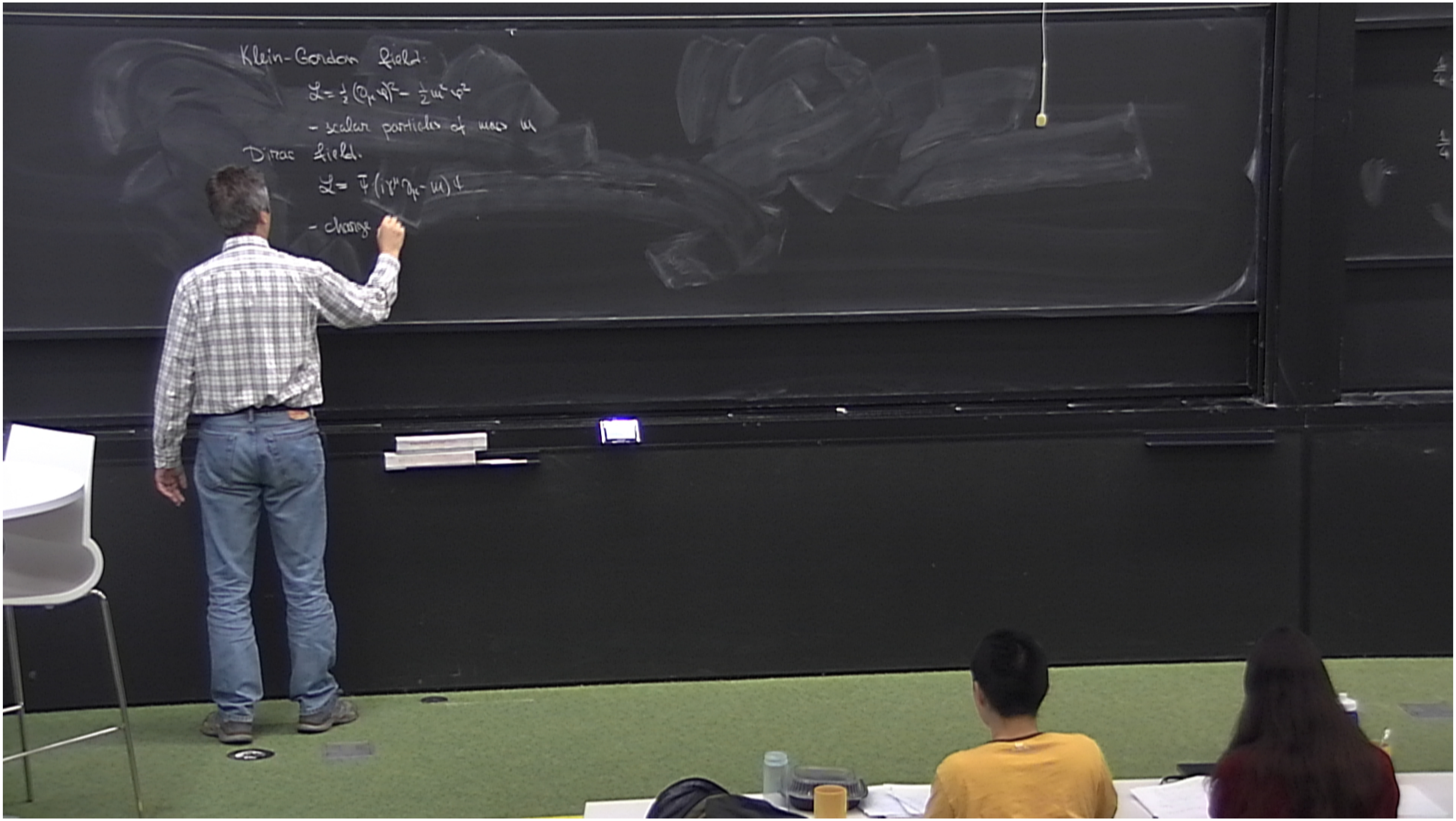

QFT
Forces
Symmetries ⟷ Conservation laws.
Noether theorem.
Bosons/fermions
Fock space.
Lagrangian density
basic dynamical variables.
Eqs of motion:

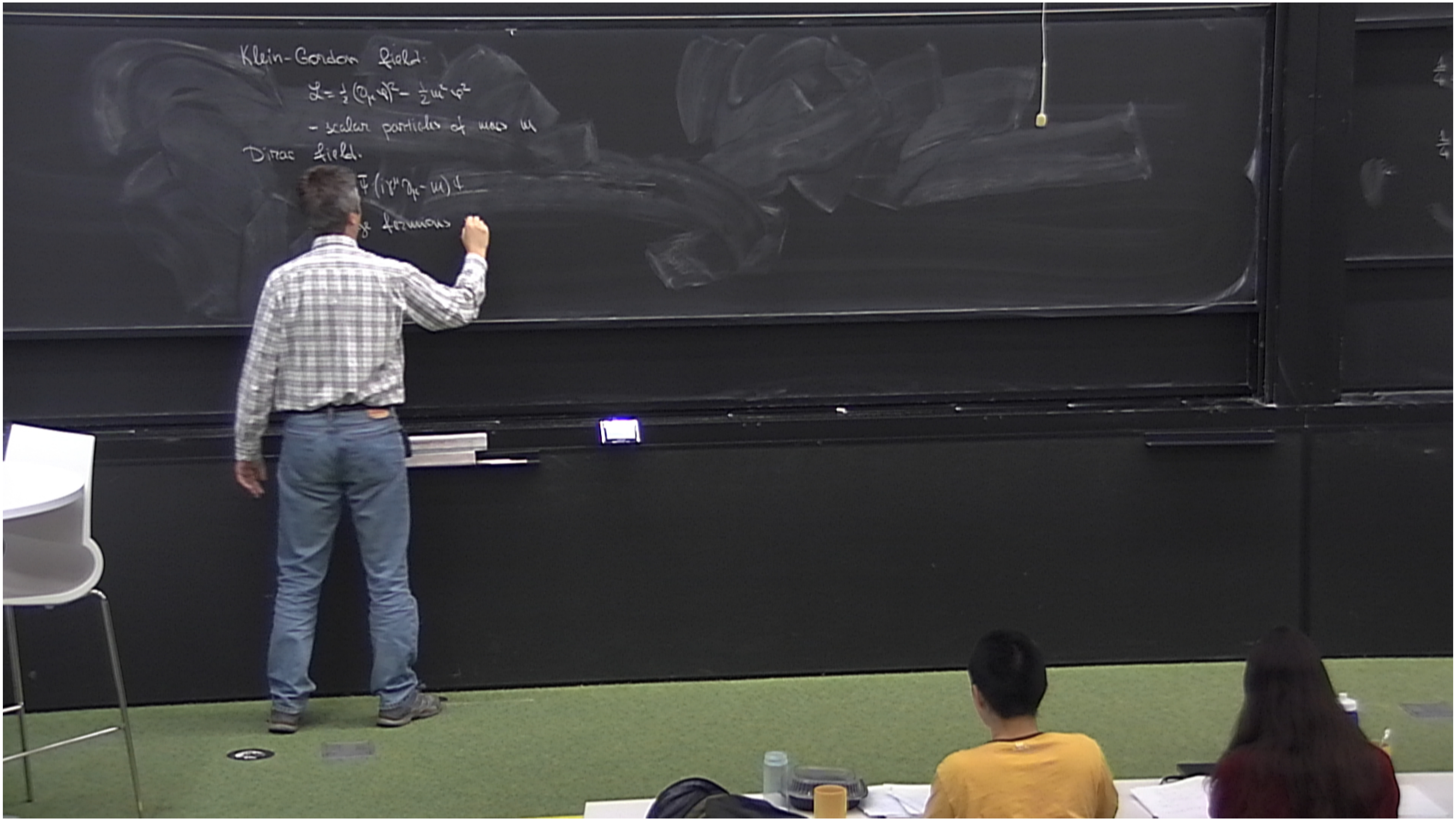


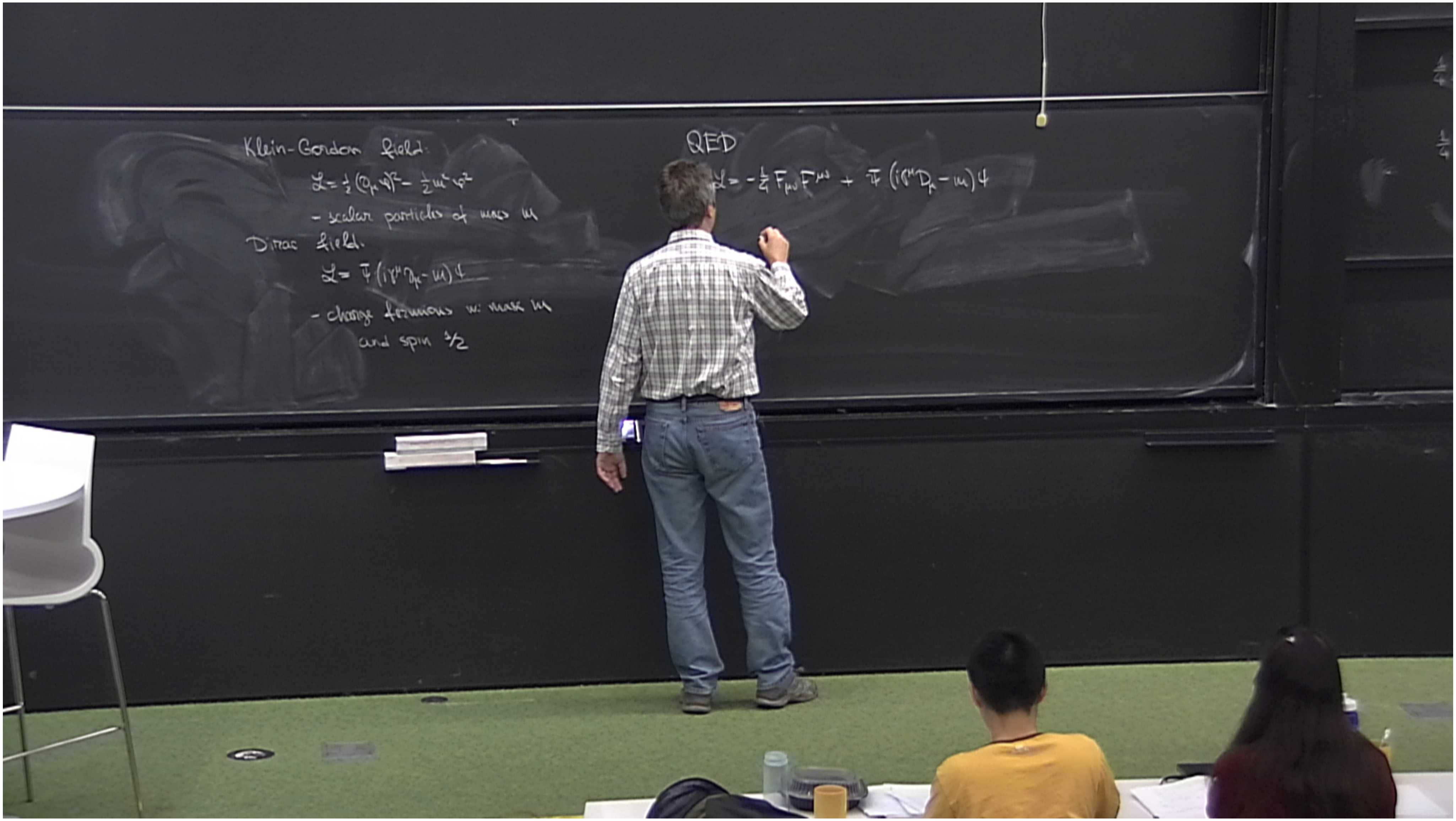
QED calculation

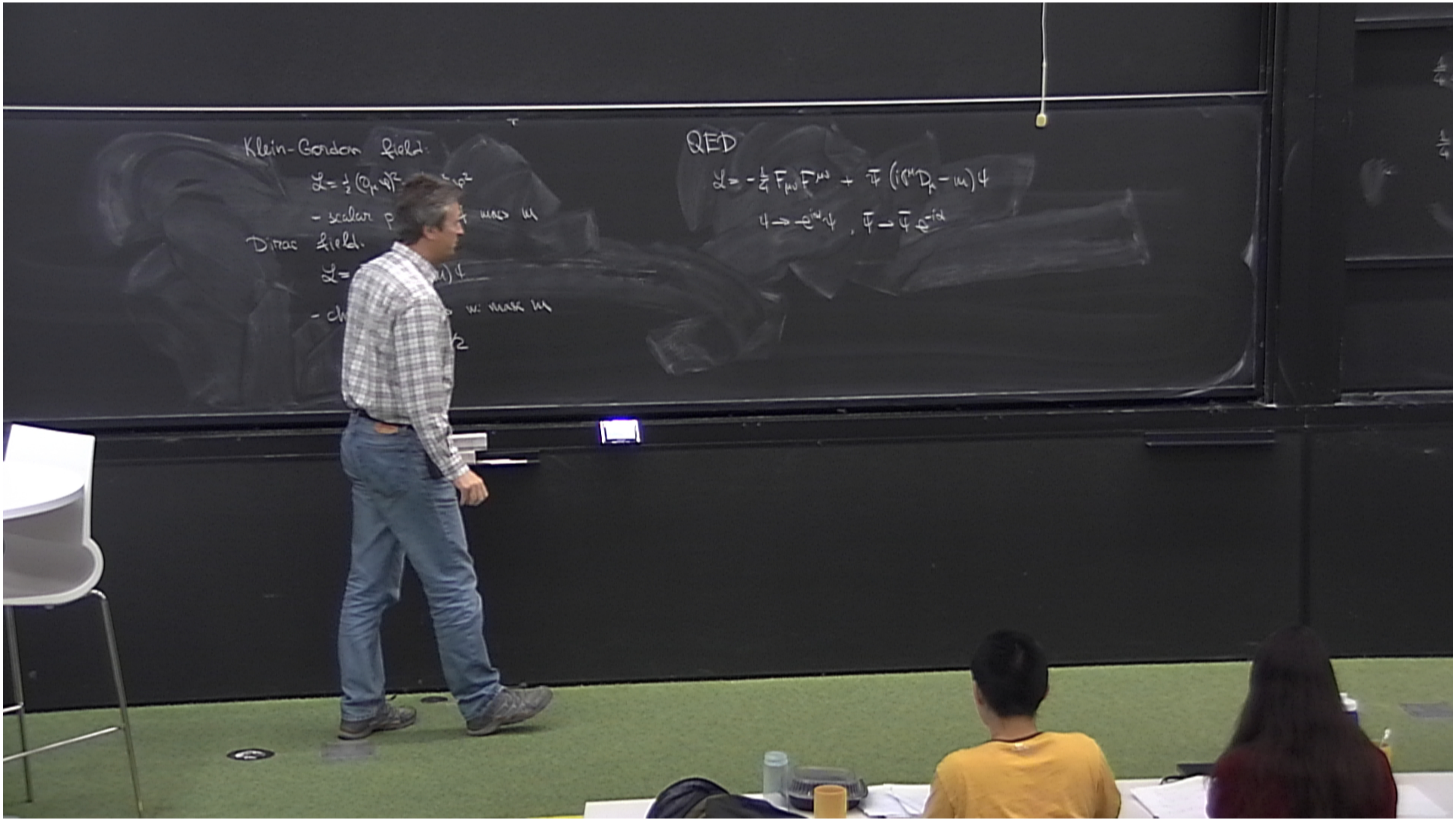


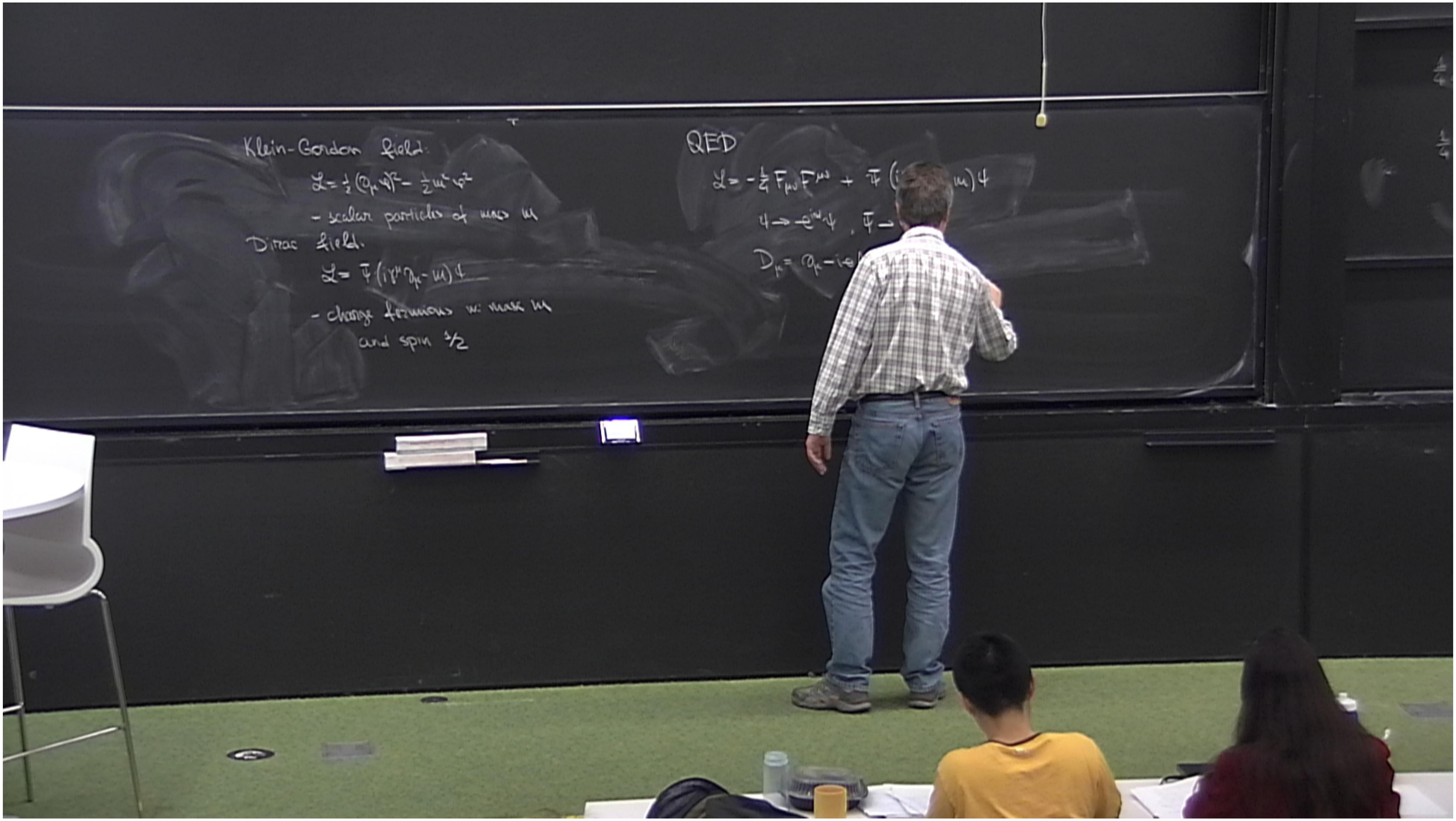

Klein-Gordon field:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2$
- scalar

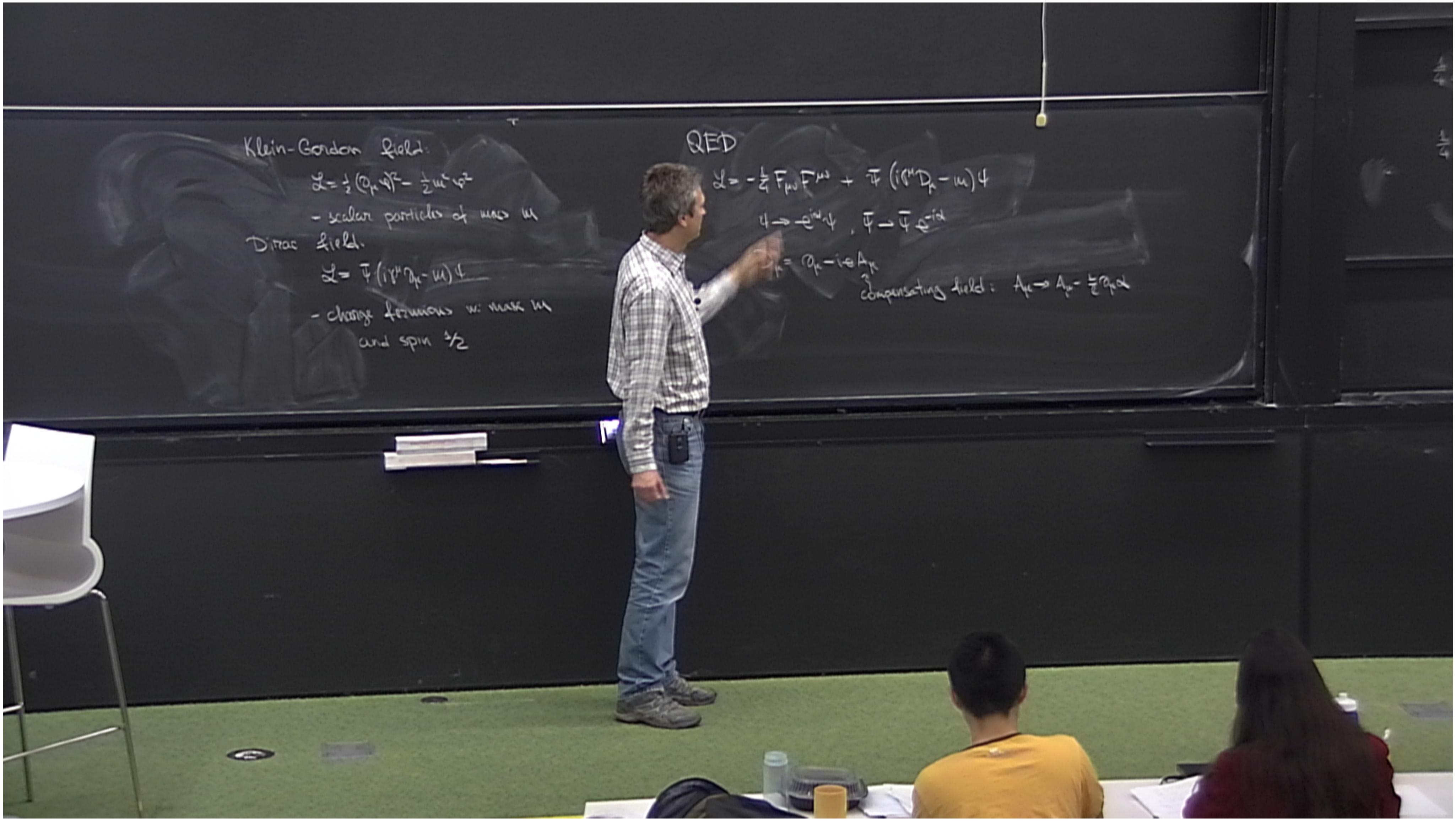

Klein-Gordon field:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2$
- scalar particles of mass m
Dirac field.
$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
- charge

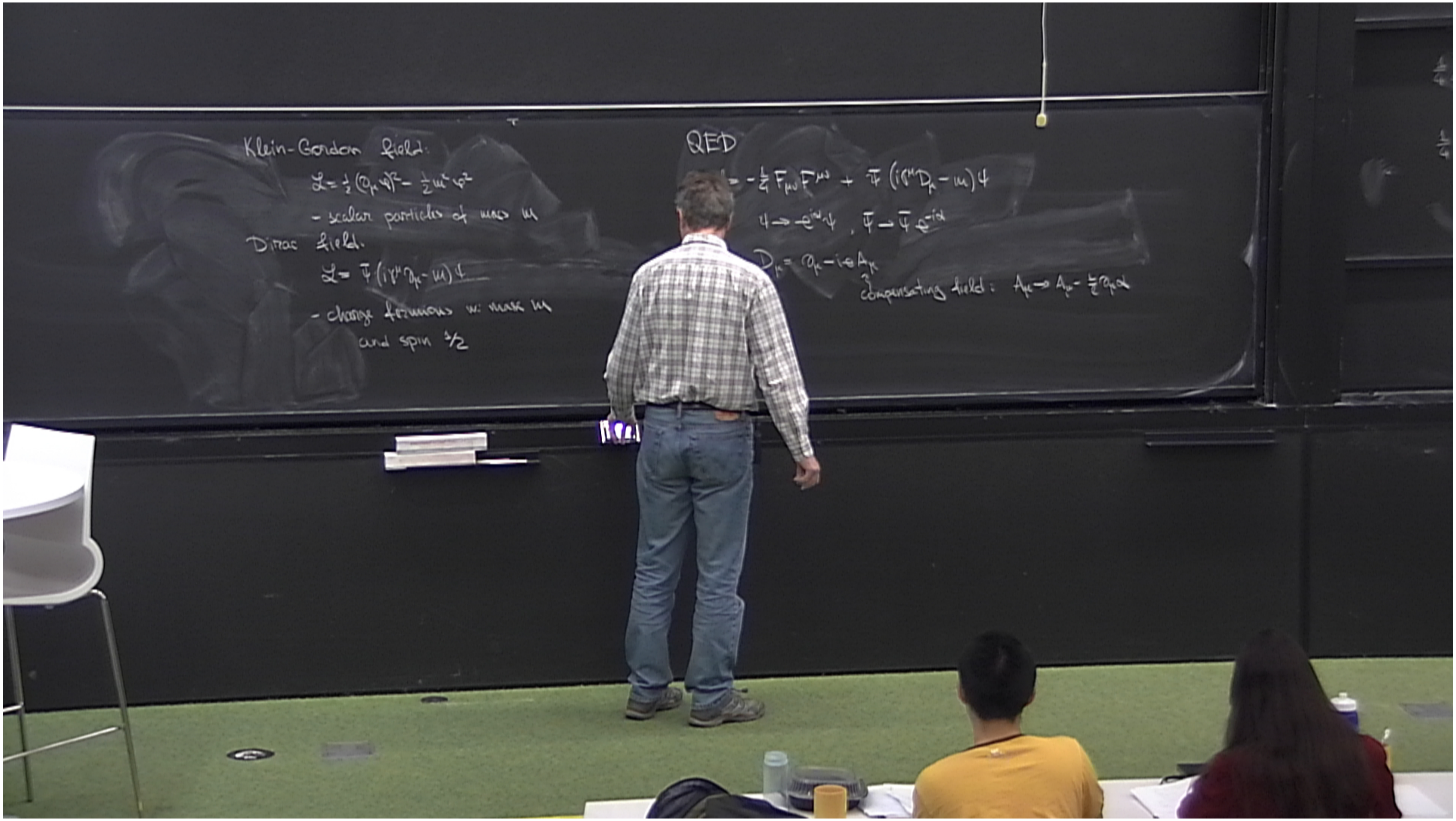

Klein-Gordon field:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2$
- scalar particles of mass m
Dirac field.
$\bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$
fermions


Klein-Gordon field:
L = ½ (∂_μ φ)² − ½ m² φ²
- scalar particles of mass m
Dirac field.
L = ψ̄ (iγ^μ ∂_μ − m) ψ
- charge fermions w/ mass m
and spin 1/2
QED
L = −¼ F_μν F^μν + ψ̄ (iγ^μ D_μ − m) ψ


Klein-Gordon field.
- scalar p
Dirac field.
- ch w: mass m
QED
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$
$\psi \to e^{i\alpha}\psi \,,\; \bar{\psi} \to \bar{\psi} e^{-i\alpha}$


Klein-Gordon field
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2$
- scalar particles of mass m
Dirac field.
$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
- charge fermions w. mass m
and spin 1/2
QED


Klein-Gordon field:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2$
- scalar particles of mass m
Dirac field.
$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
- charge fermions w: mass m
and spin 1/2
QED
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$
$\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to \bar{\psi} e^{-i\alpha}$
$D_\mu = \partial_\mu - ieA_\mu$
compensating field: $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$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and spin 1/2
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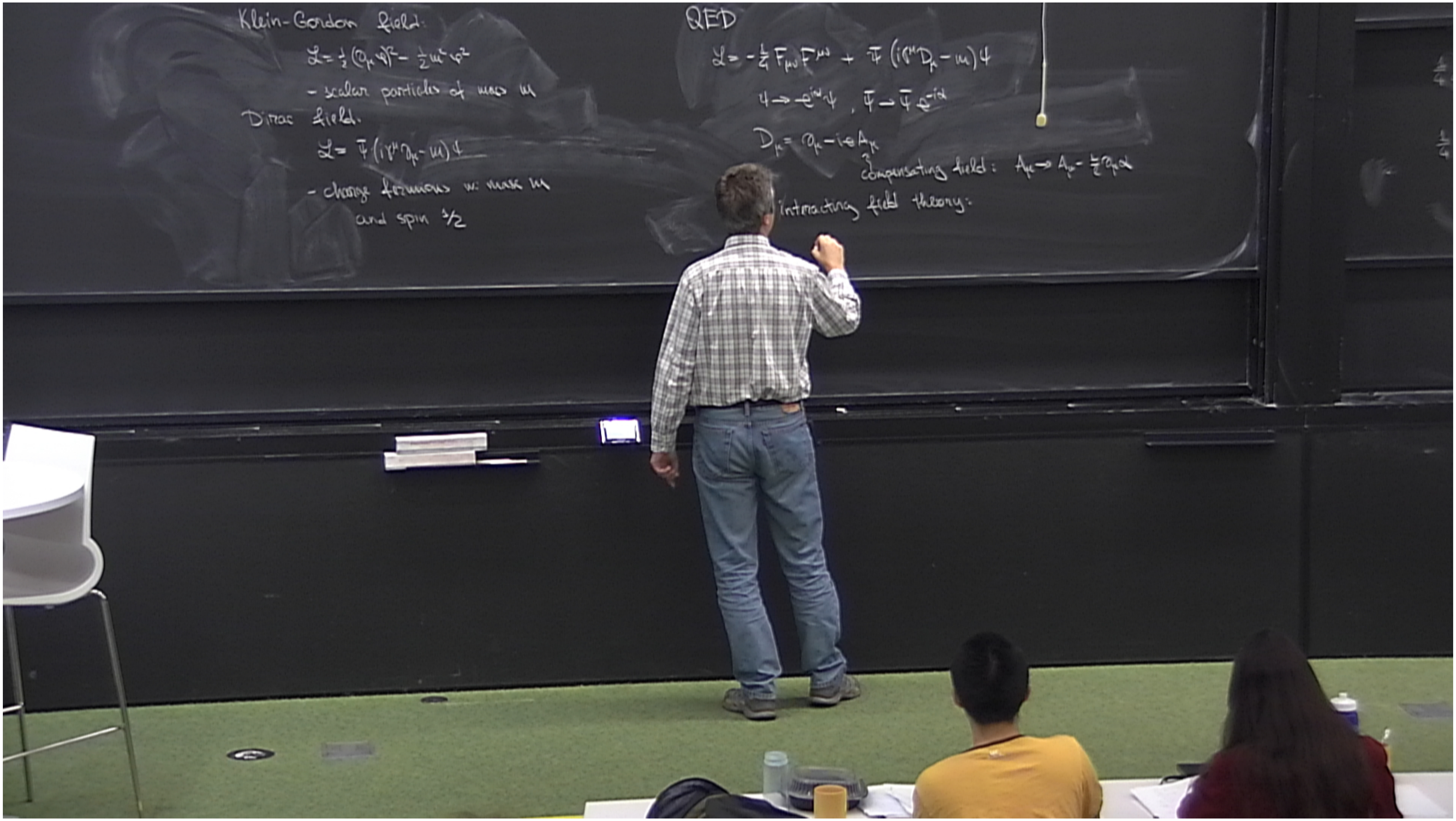
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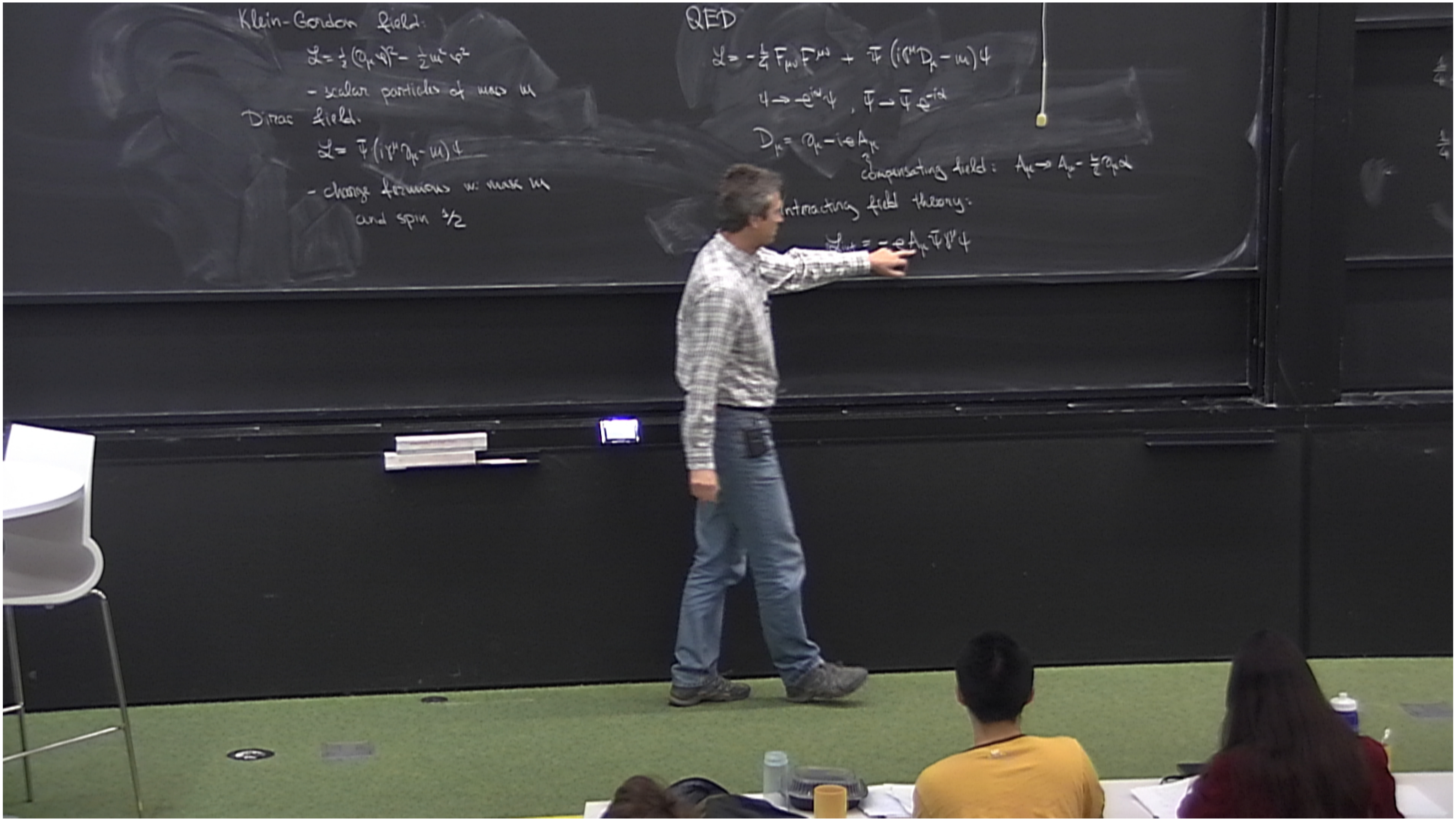
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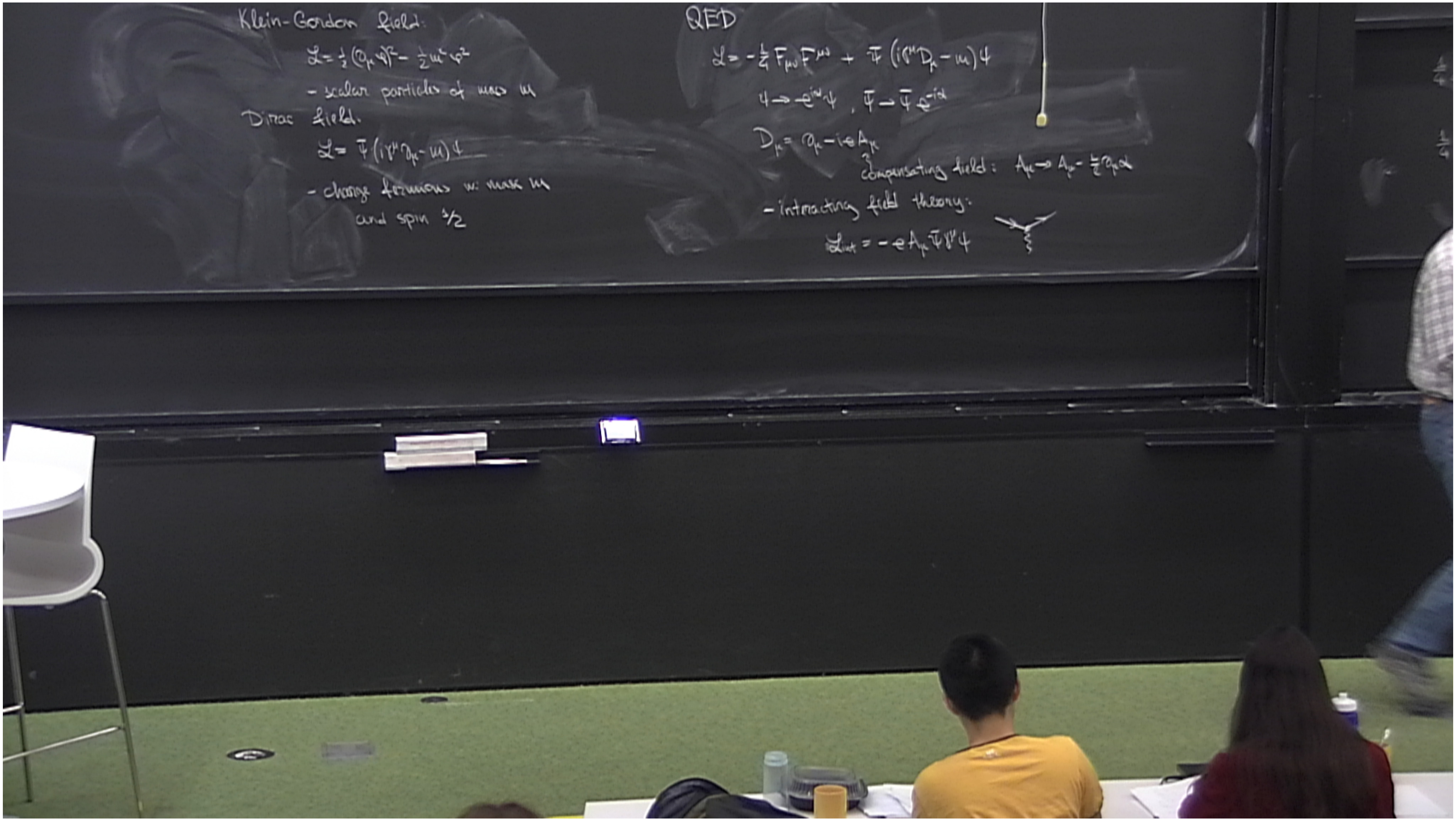
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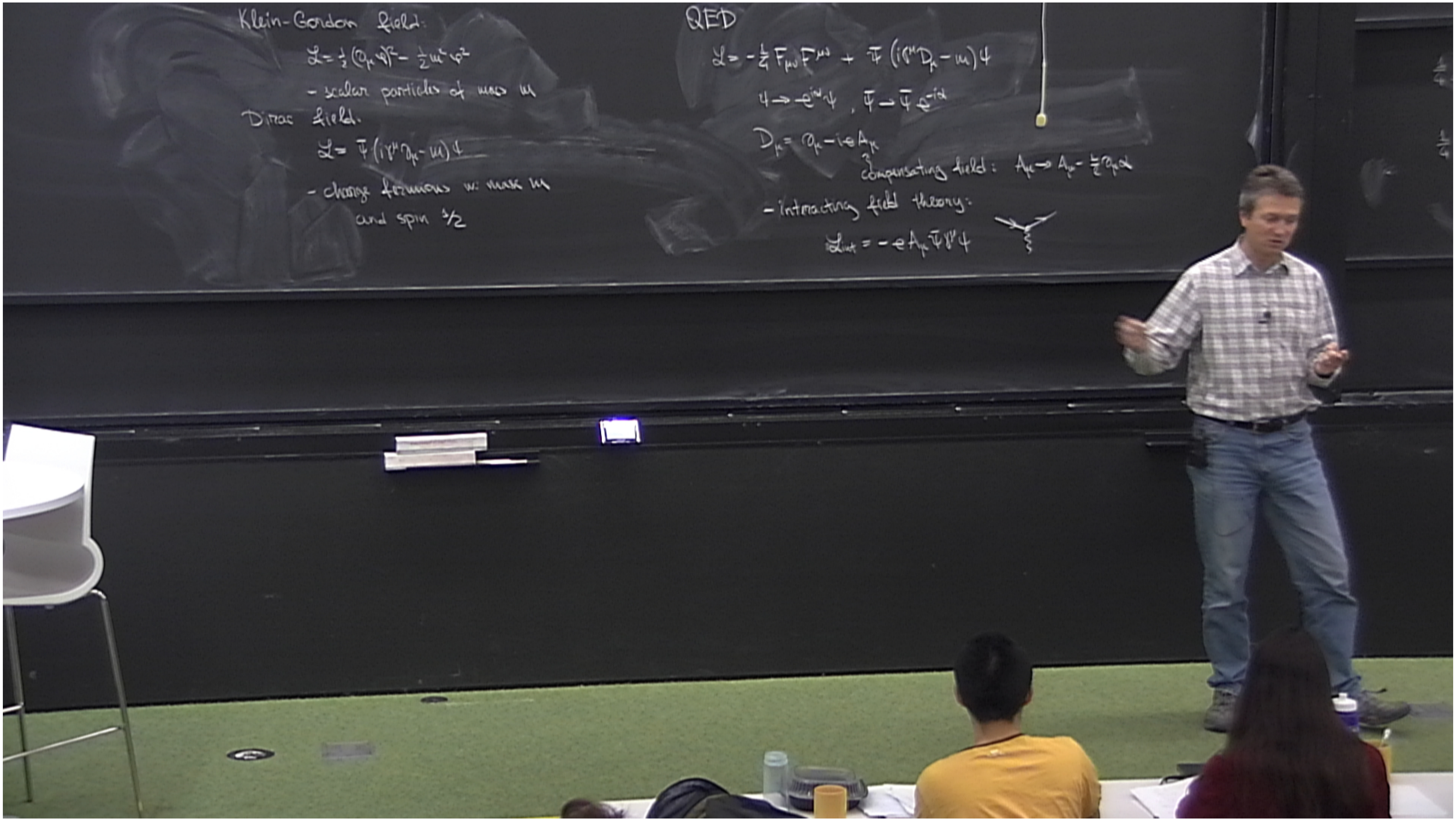
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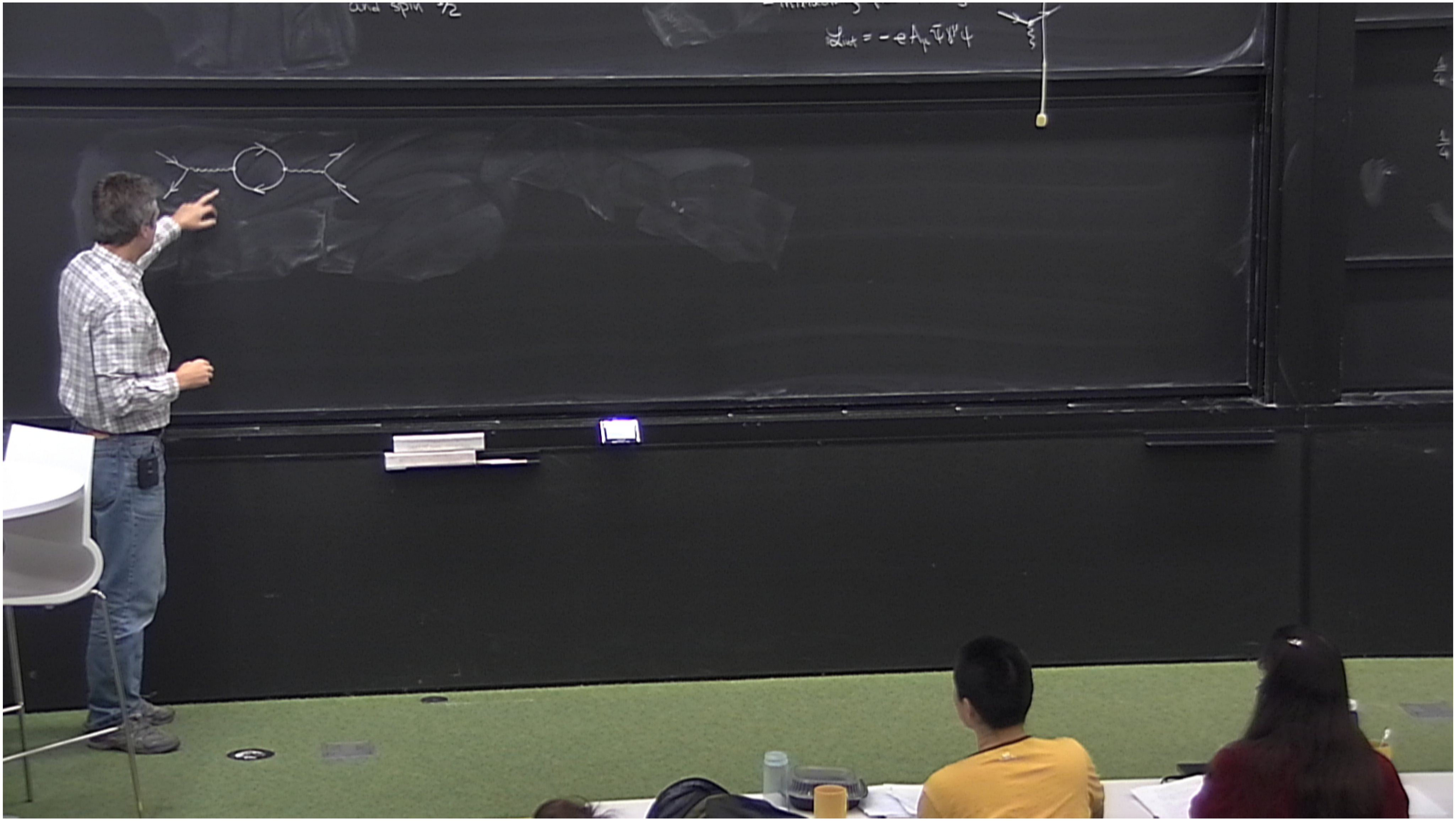
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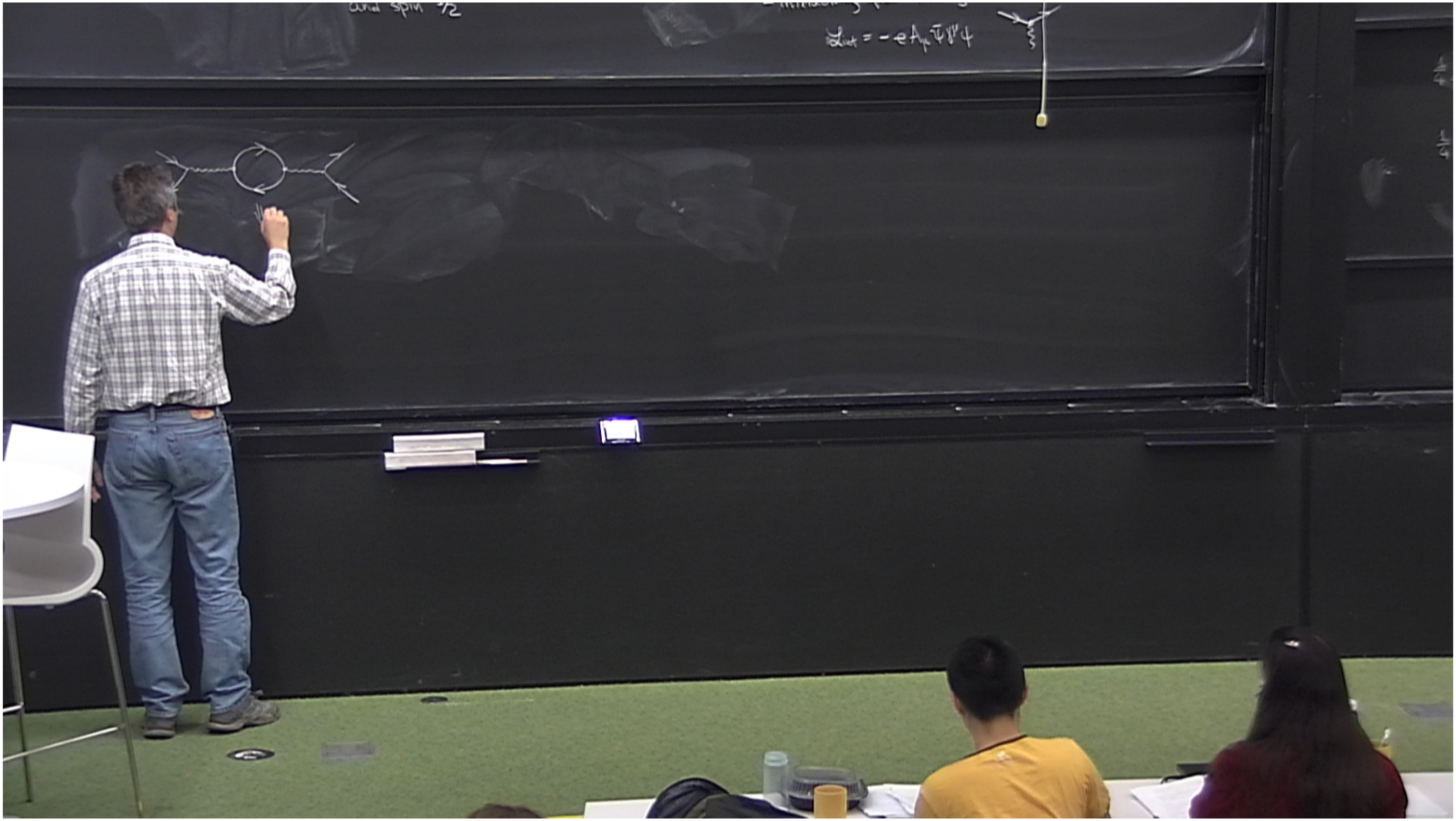
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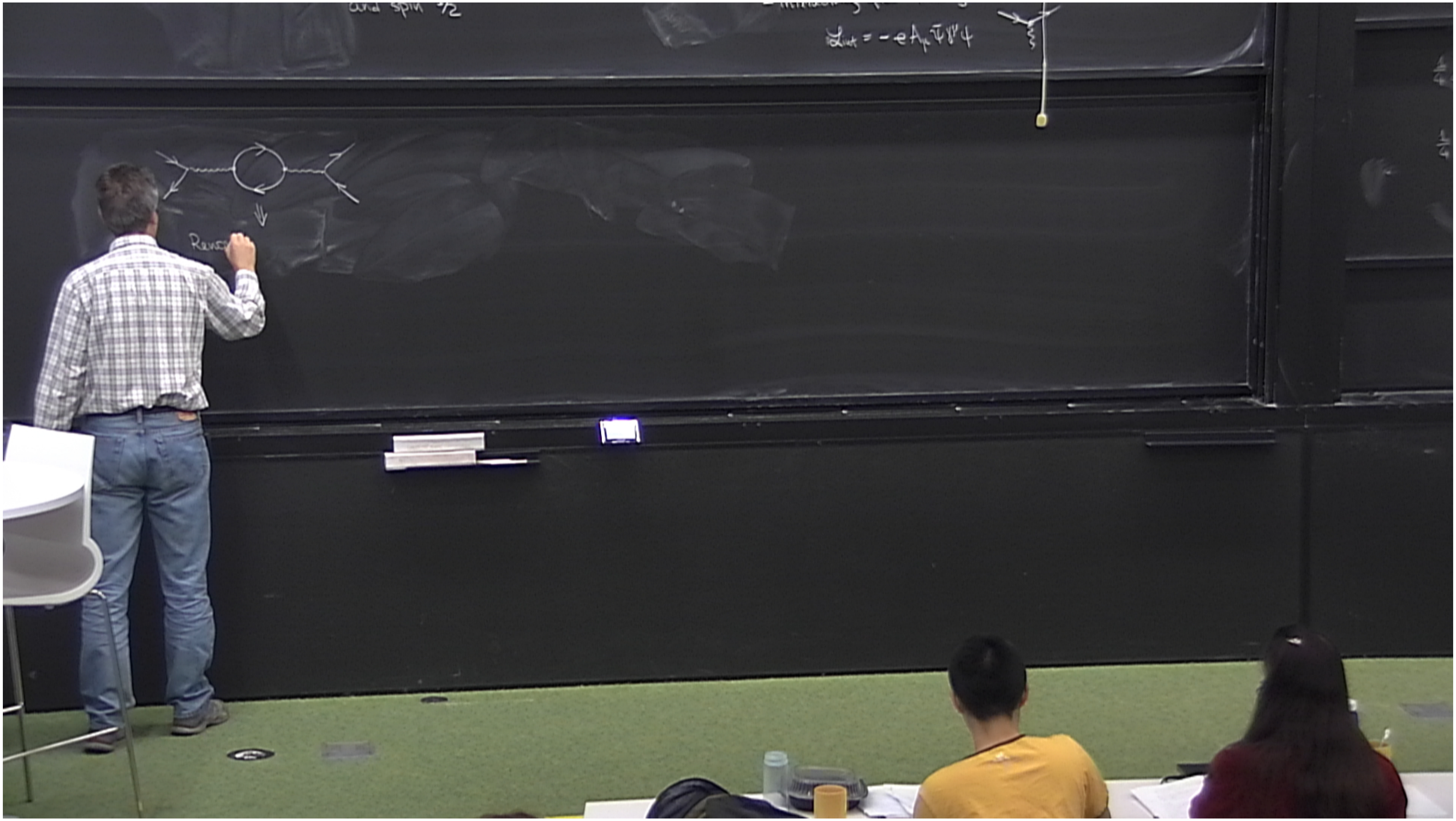


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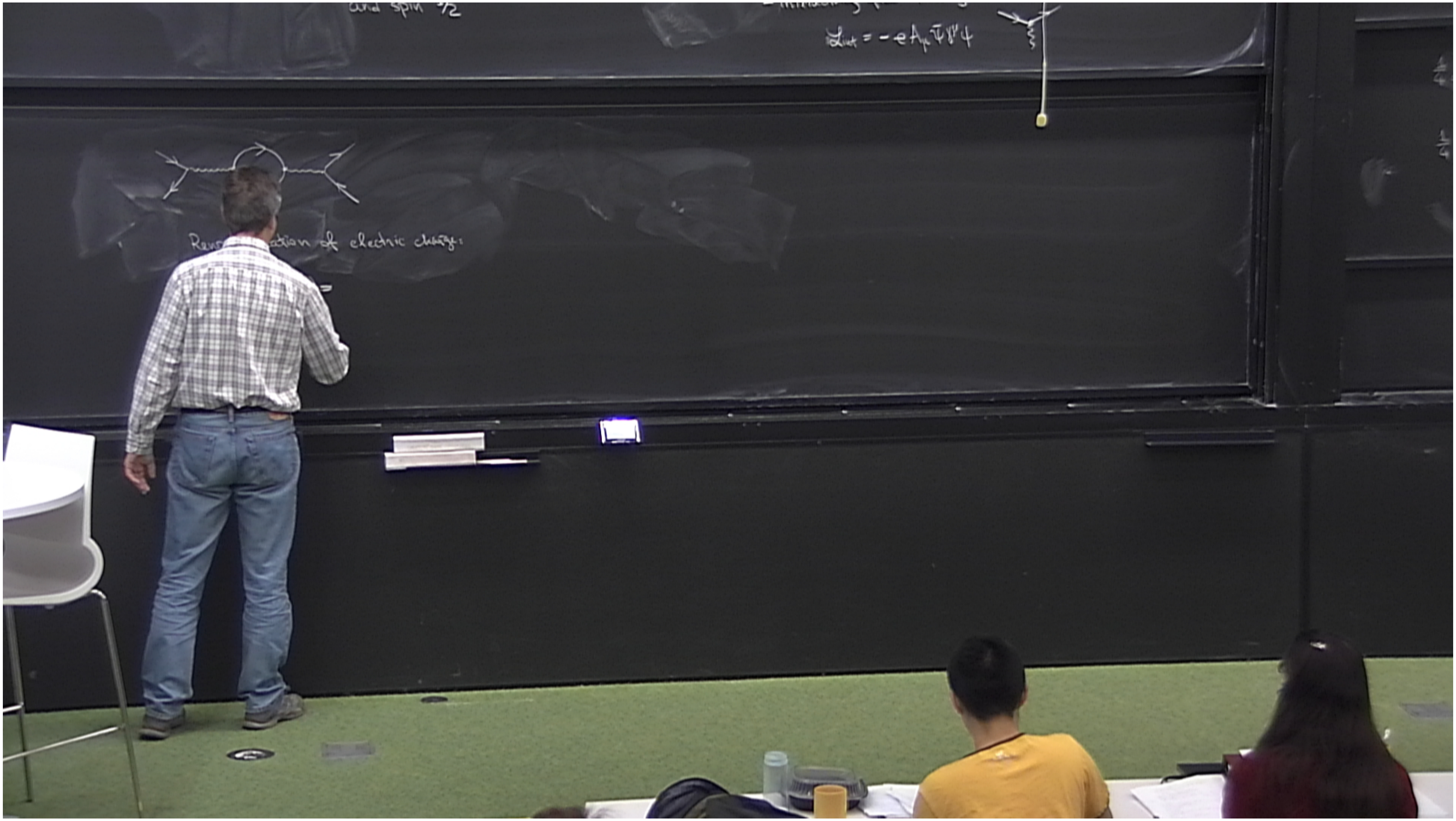
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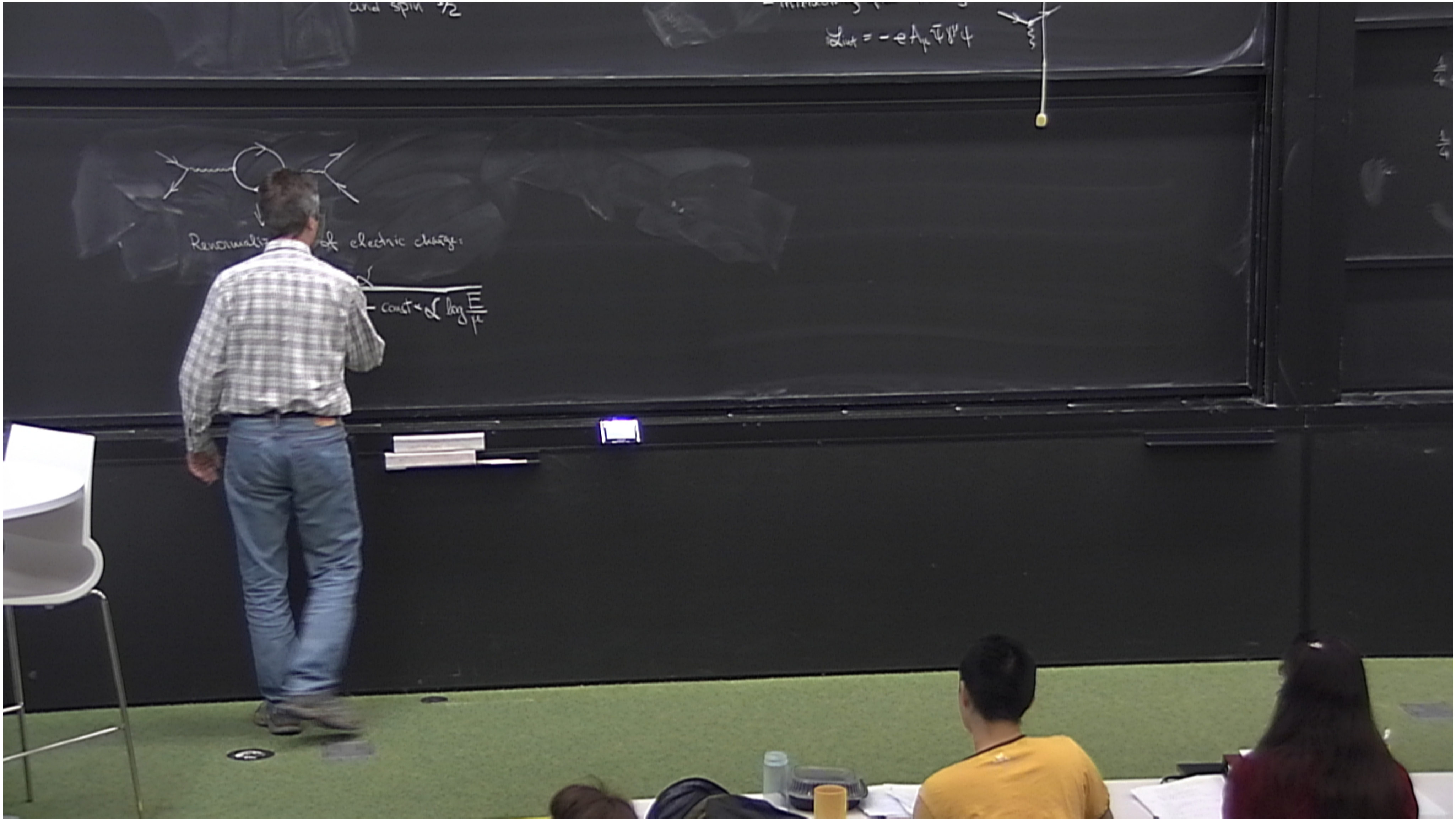
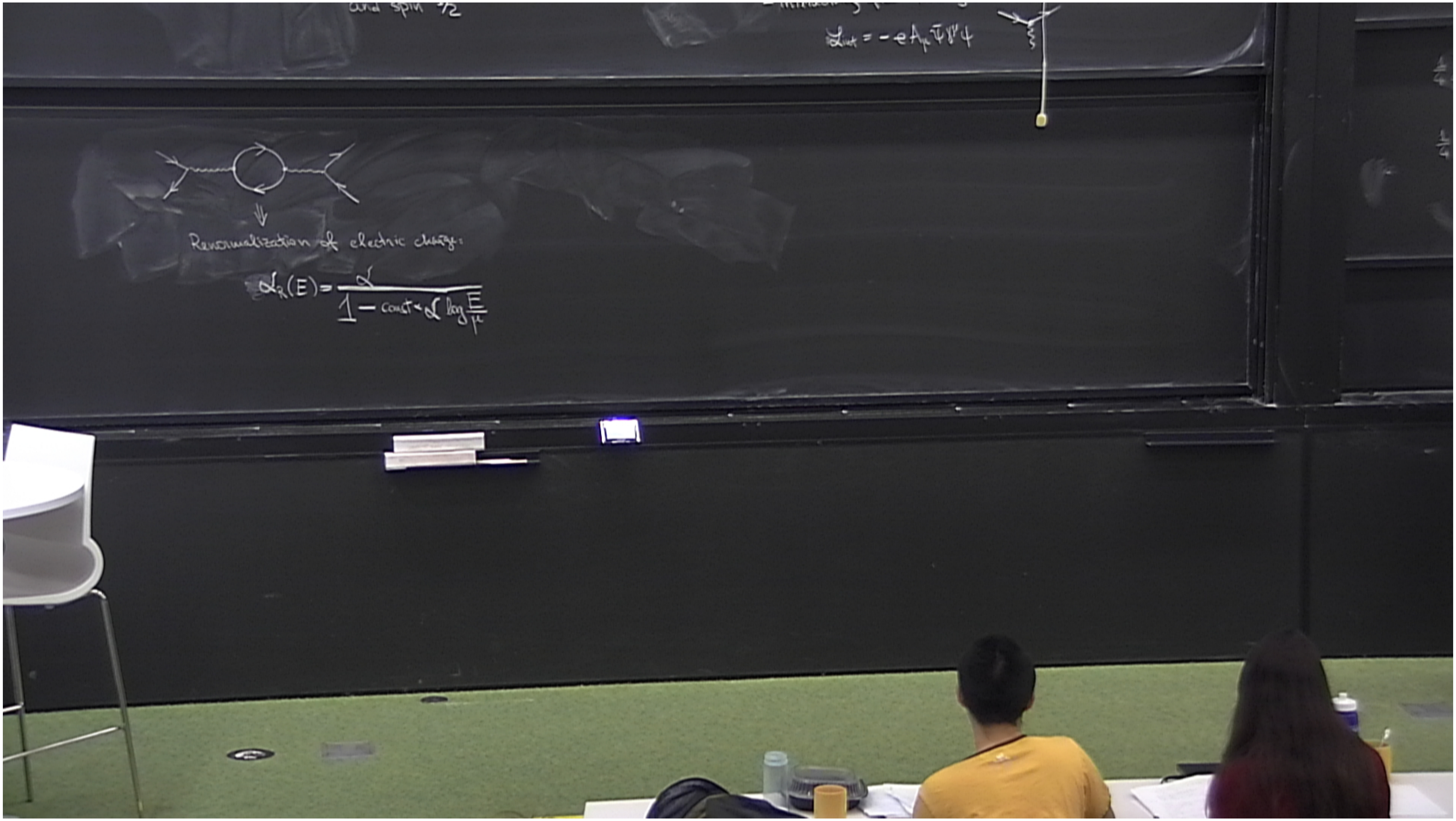

Renormalization of electric charge: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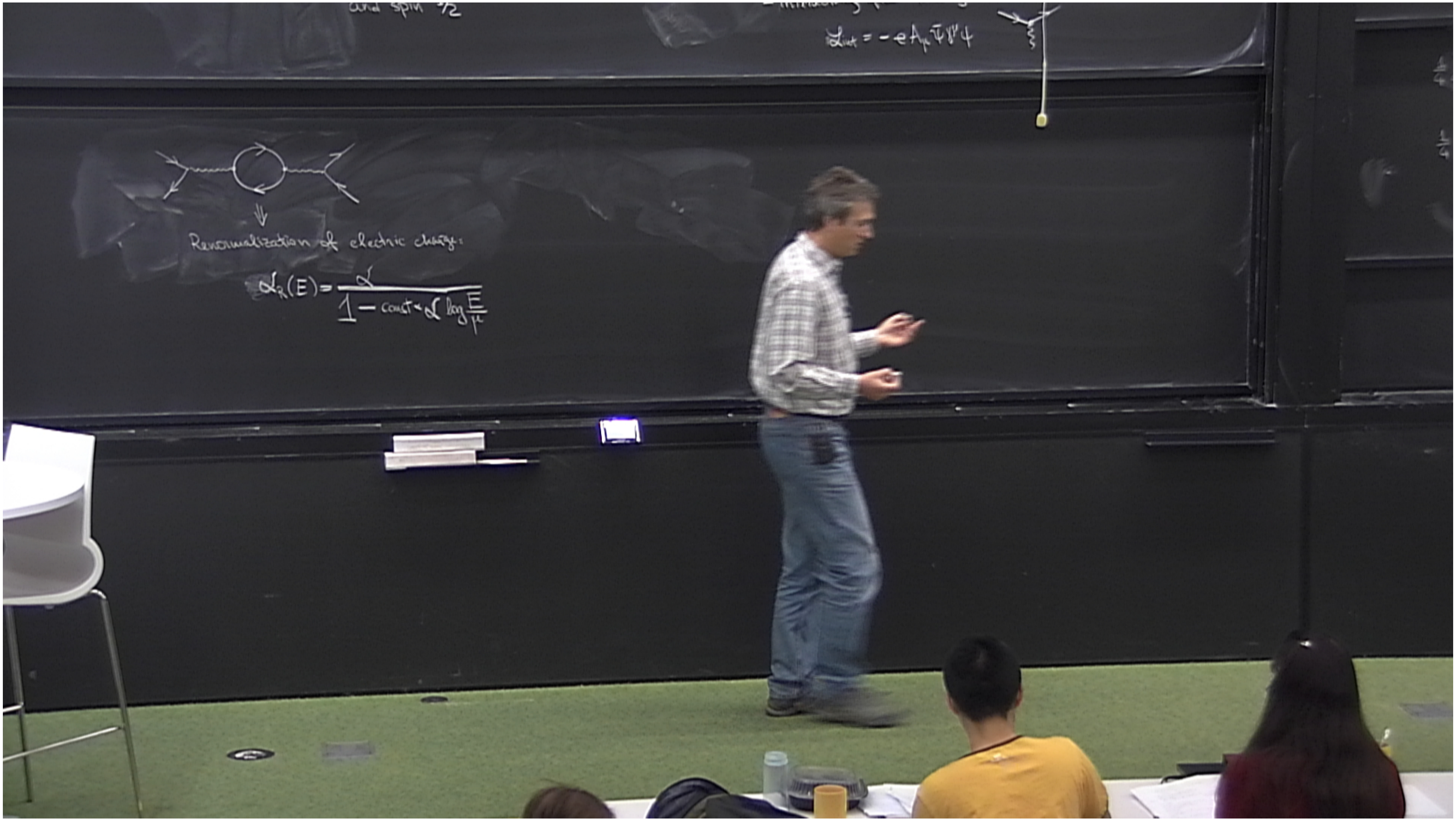
electric charge.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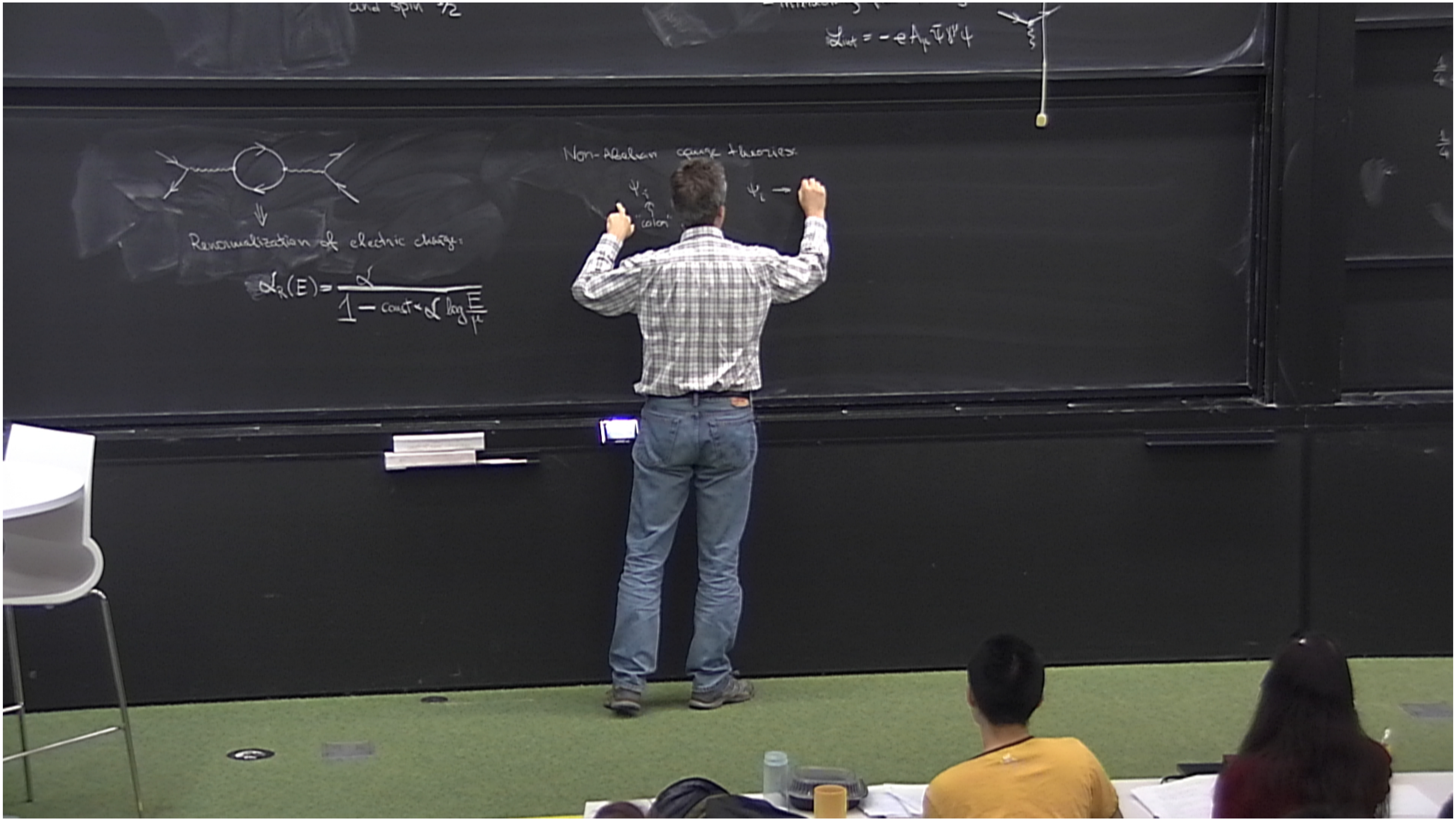
Non-Abelian gauge theories
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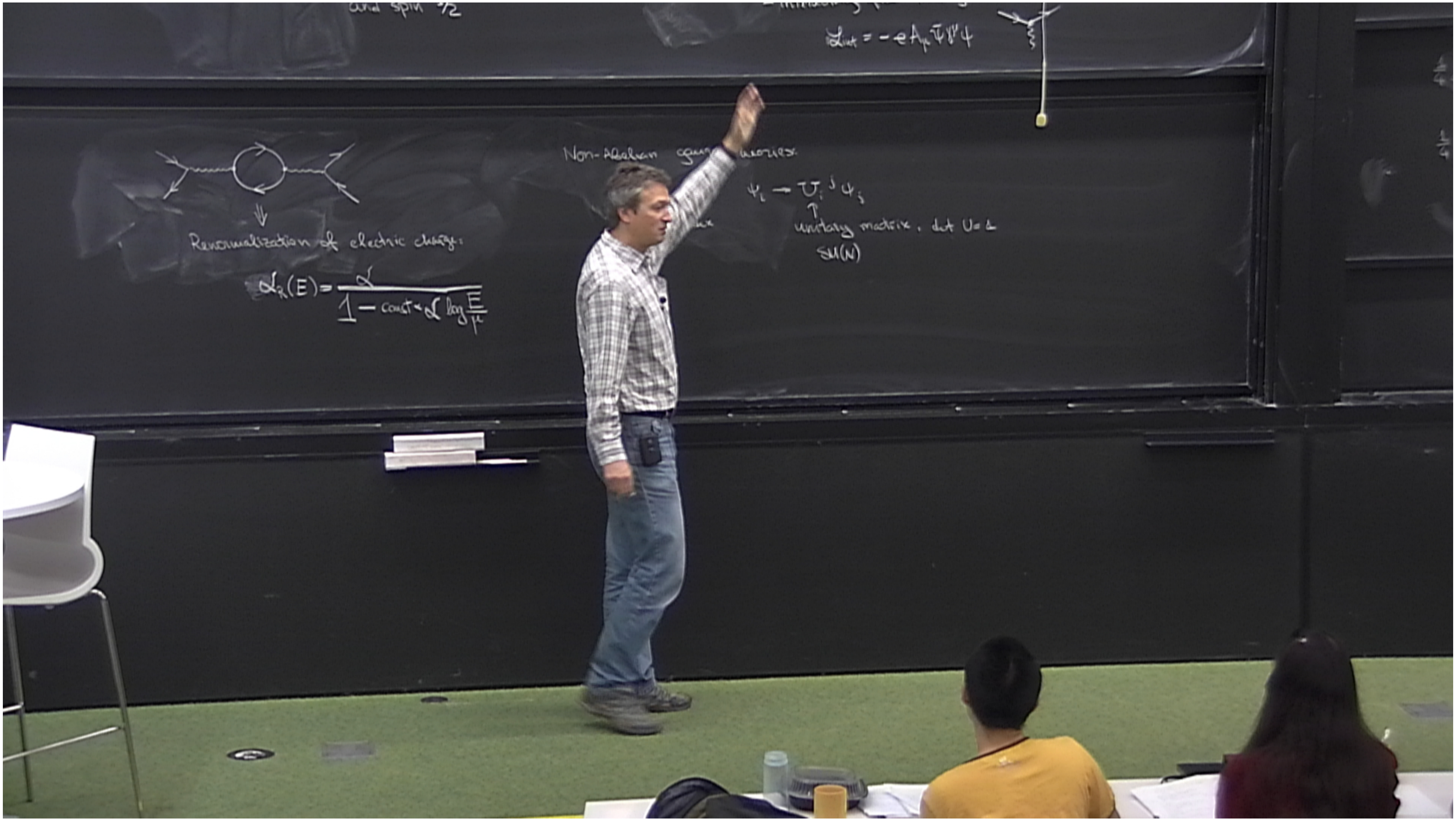
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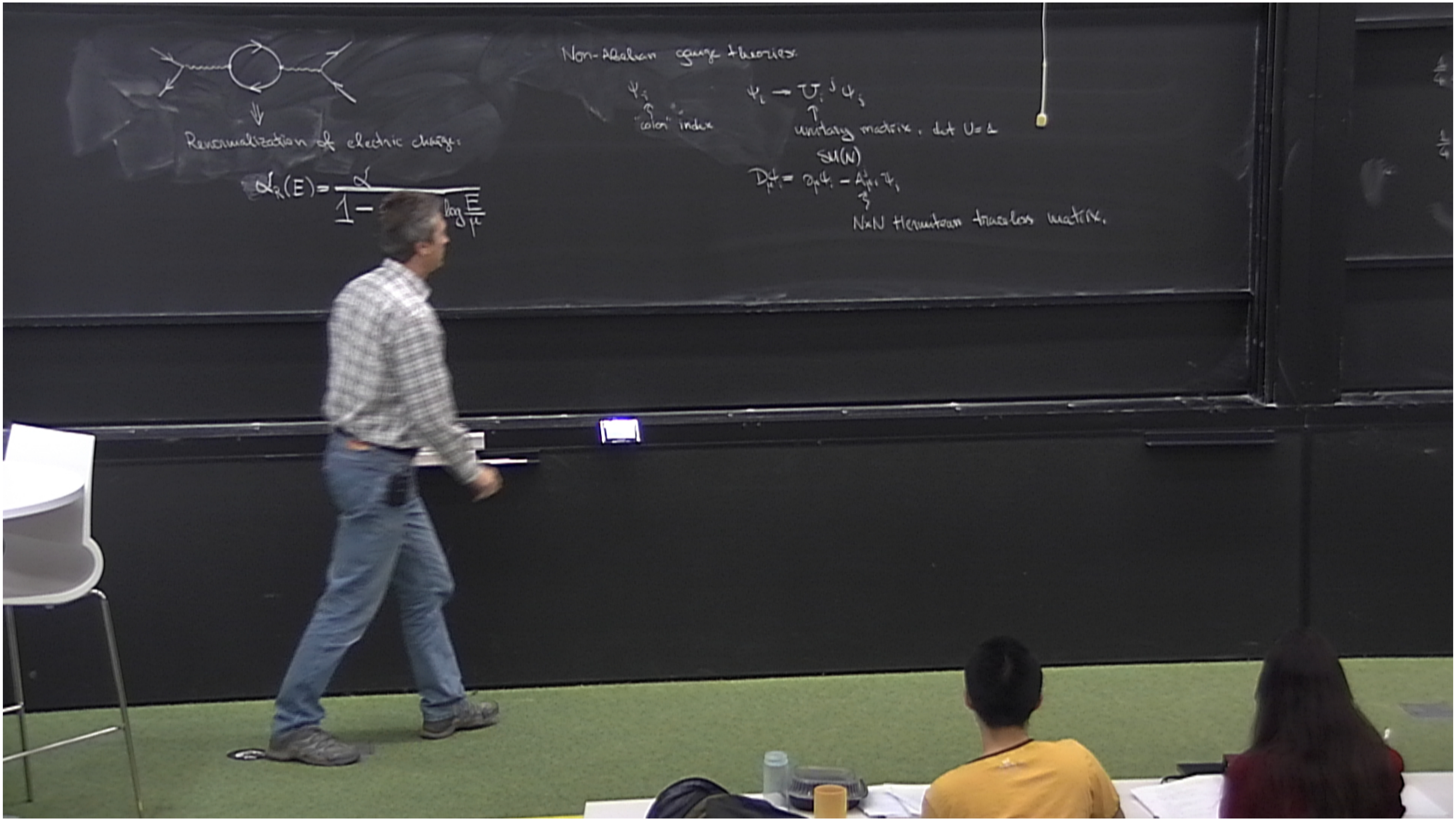
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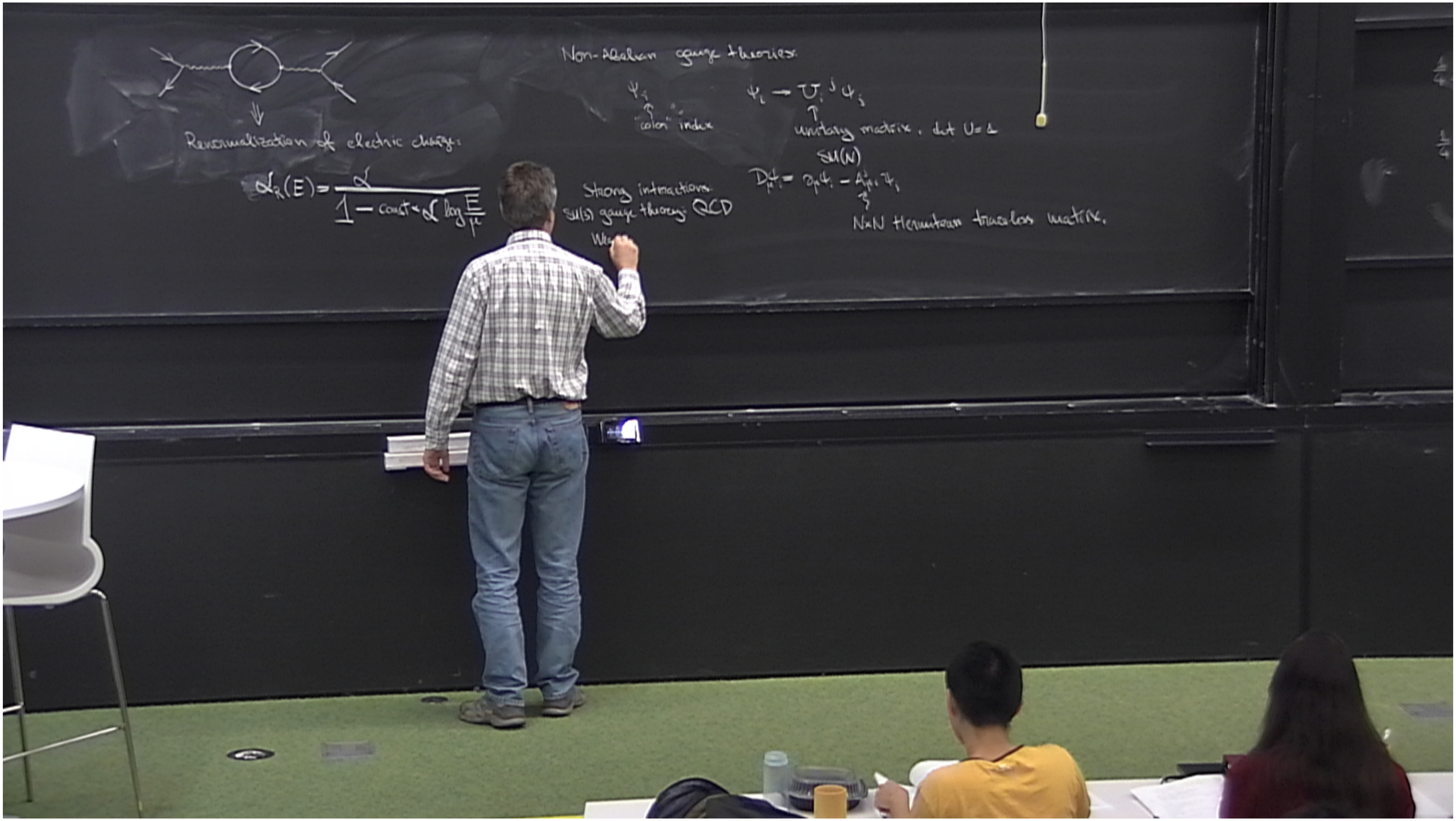
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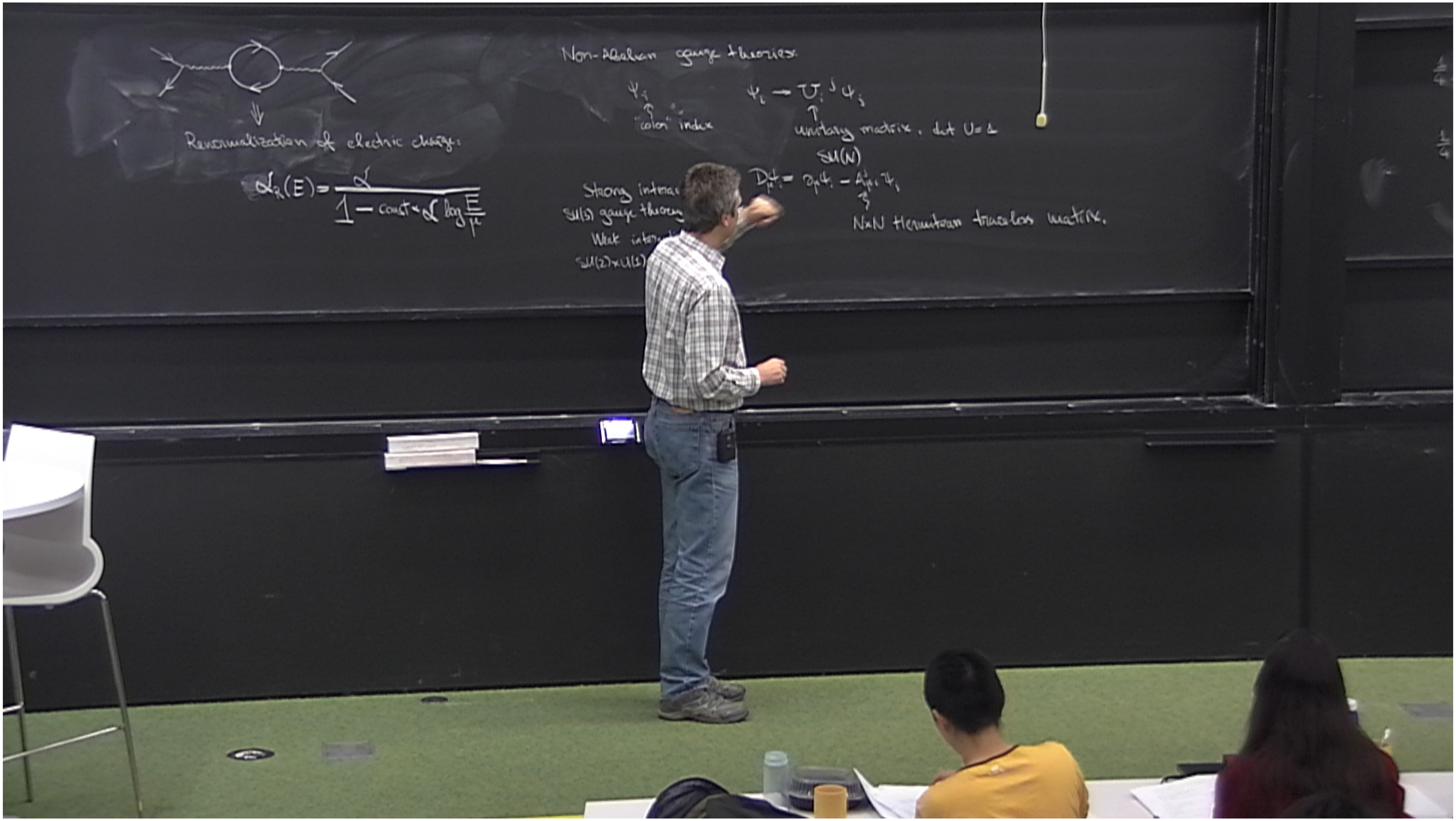
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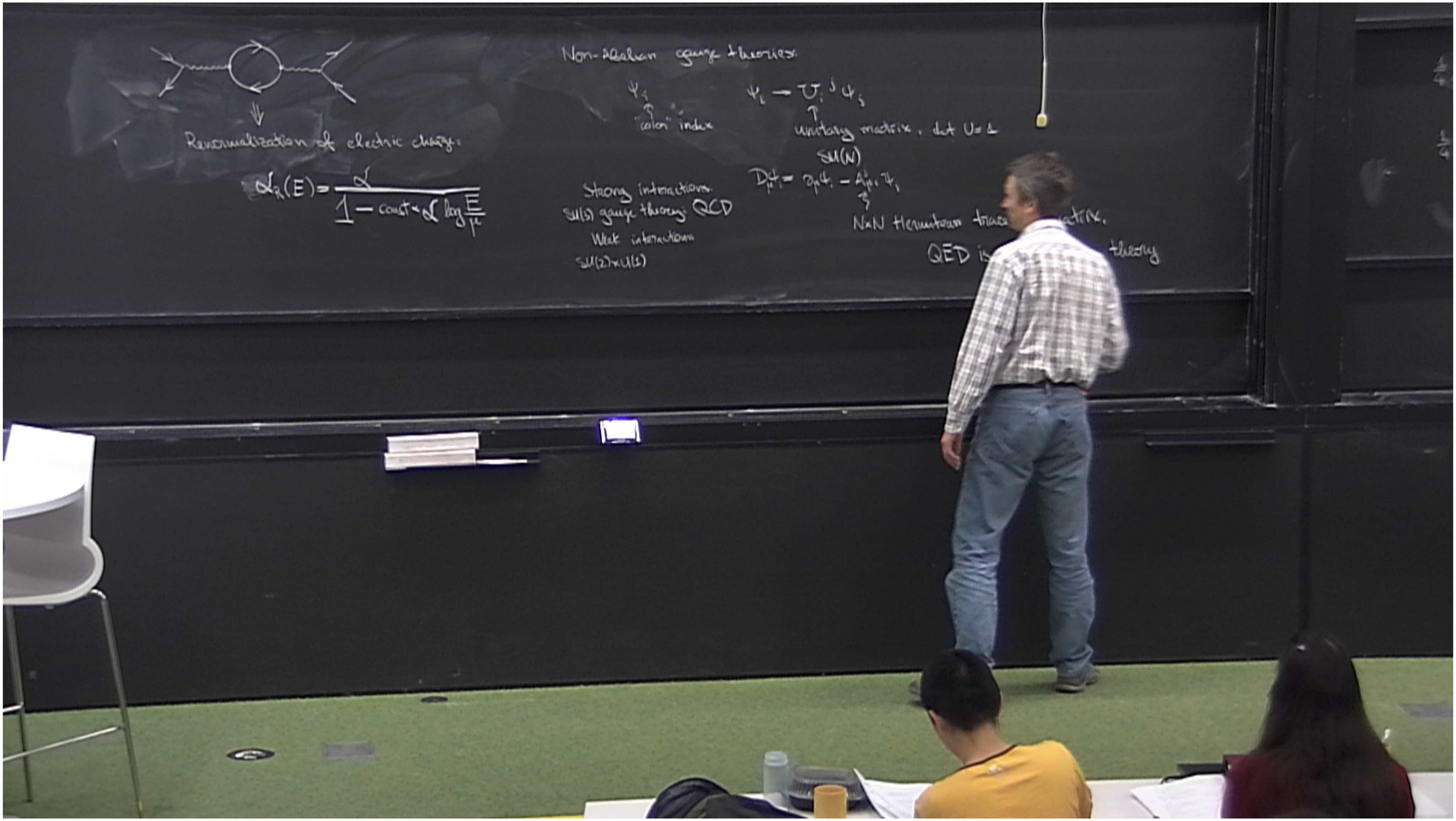
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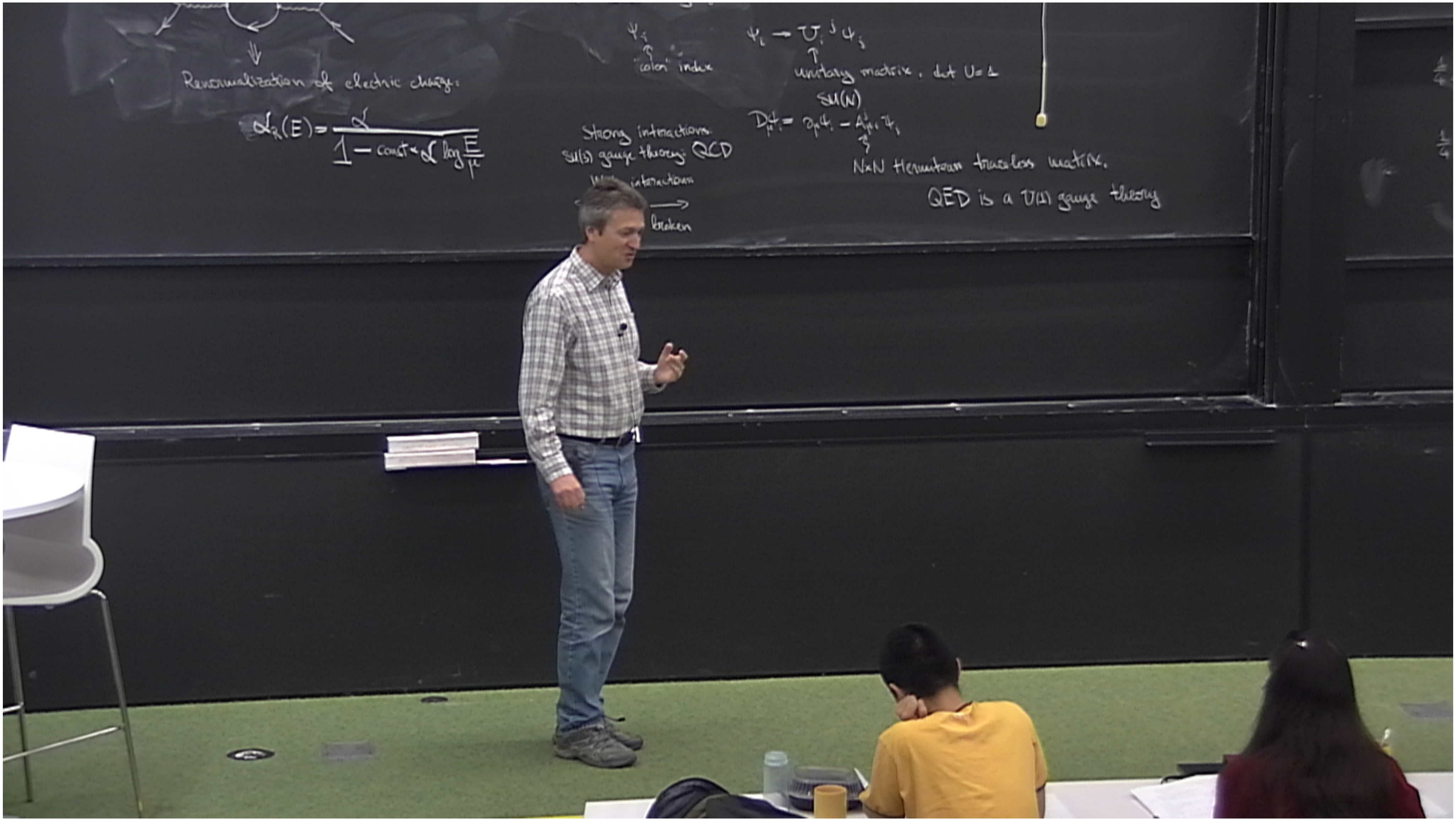
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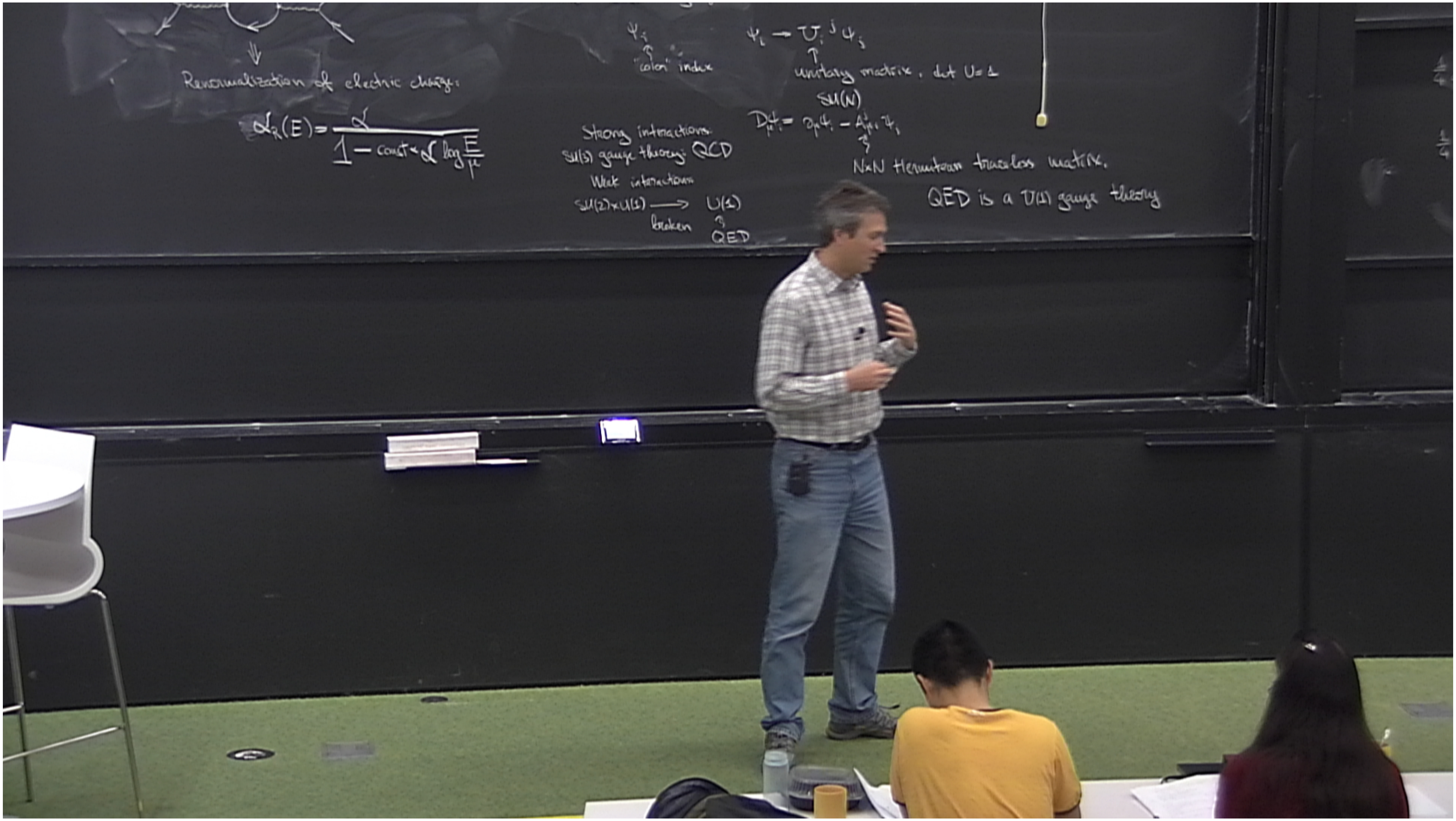
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