

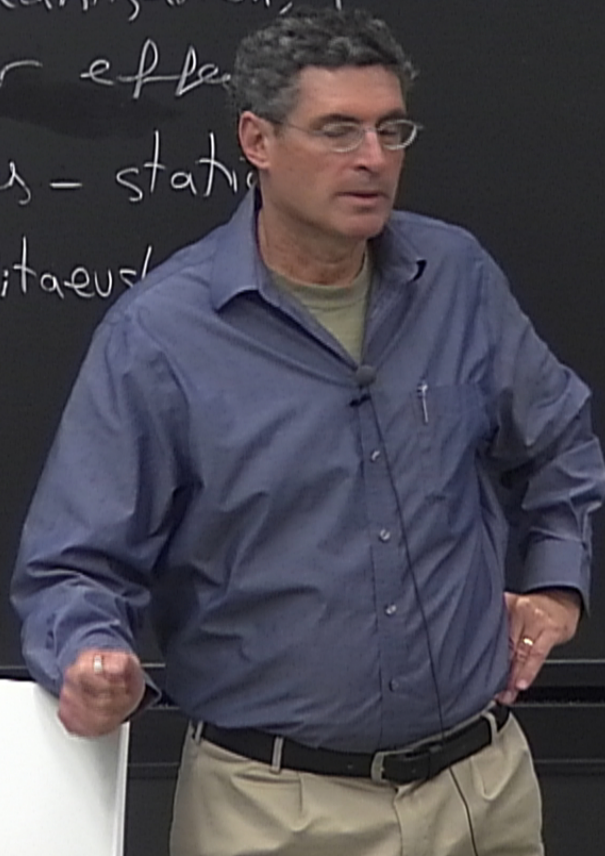
Title: Condensed Matter - Lecture 12

Date: Oct 24, 2012 10:30 AM

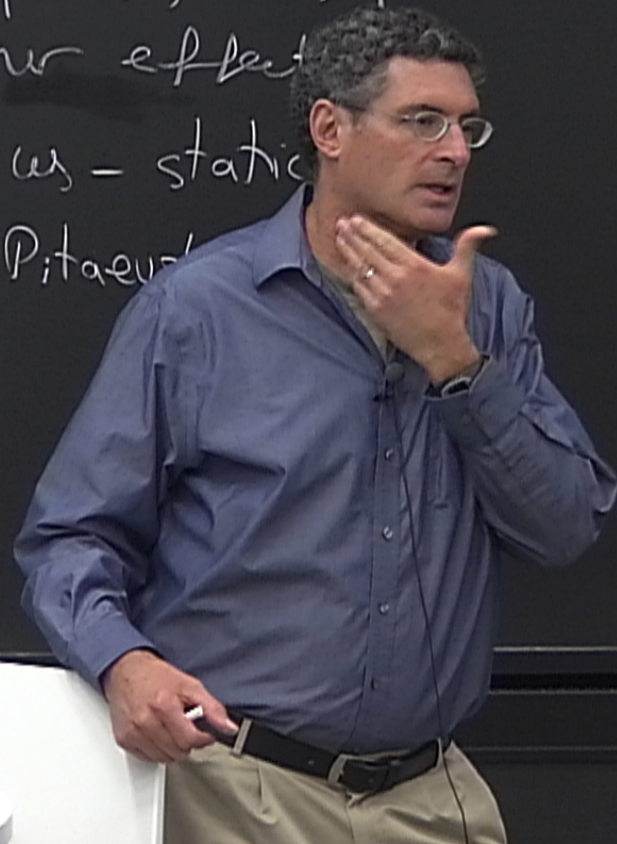
URL: <http://pirsa.org/12100037>

Abstract:

Flux quantization,
Meissner effect
Vortices - static
Gross Pitaevskii

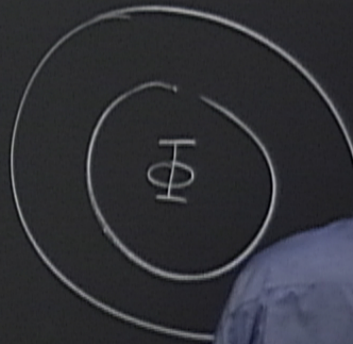


Flux quantization,
Meissner effect
Vortices - static
Gross Pitaevskii



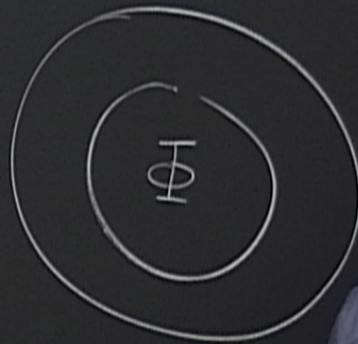
Flux quantization,
Meissner effect
Vortex - statics
Ginzburg-Landau Eqn.

Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaeuslevii Egn.



1

Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaeuskiei Eqn.



$$E = \frac{1}{2m}$$

Flux quantization,
 Meissner effect
 Vortices - statics
 Gross Pitaevskii Eqn



$$E = \frac{1}{2m} \int d^2x \left[\hbar^2 (\nabla - \frac{q}{\hbar c} \vec{A}) \psi \right]^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4$$

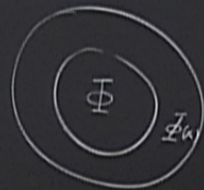
$$= \frac{p_0 \hbar^2}{2m} \int d^2x \left(\nabla \psi - \frac{q}{\hbar c} \vec{A} \psi \right)^2$$

$$\left(\frac{m \cdot \vec{v}^2}{\hbar^2} \right)$$

$$p = \sqrt{p_0} e^{i\phi}$$

$$p_0 = \mu/g$$

Flux quantization,
 Meissner effect
 Vortices - statics
 Gross Pitaevskii Eqn



$$E = \frac{1}{2m} \int d^2x \left[\hbar^2 \left(\nabla - \frac{q}{\hbar c} \vec{A} \right) \psi \right]^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4$$

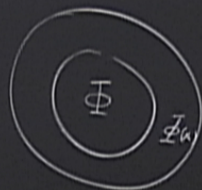
$$= \frac{1}{2} \int d^2x \left[\left(\nabla \phi - \frac{q}{\hbar c} \vec{A} \right)^2 - \mu \rho_0 + \frac{7}{2} \rho_0^2 \right]$$

$\left(\frac{m \cdot \vec{v}}{\hbar} \right)^2$

$$\rho = \sqrt{\rho_0} e^{i\phi}$$

$$\rho_0 = \mu/g$$

Flux quantization,
 Meissner effect
 Vortices - statics
 Gross Pitaevskii Eqn



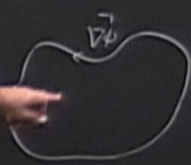
$$E = \frac{1}{2m} \int d^2x \left| (-i\hbar\nabla - \frac{q}{c}\vec{A})\psi \right|^2 - \mu|\psi|^2 + \frac{g}{2}|\psi|^4$$

$$= \frac{\rho_s \hbar^2}{2m} \int d^2x \left(\vec{\nabla}\phi - \frac{q}{\hbar c}\vec{A} \right)^2 - \mu\rho_s + \dots$$

in the ring $\vec{V}=0 \Rightarrow \vec{\nabla}\phi = \frac{q}{\hbar c}\vec{A}$

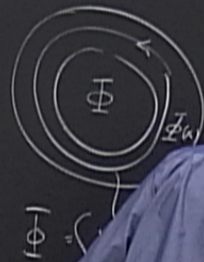
$$\psi = \sqrt{\rho_s} e^{i\phi}$$

$$\rho_s = \mu/g$$



$$\oint d\vec{e} \cdot \vec{A} = \oint d\vec{e} \cdot \vec{\nabla}\phi$$

Flux quantization,
 Meissner effect
 Vortices - statics
 Gross Pitaevskii Eqn



$$E = \frac{1}{2m} \int d^2x \left| (-i\hbar\nabla - \frac{q}{c}\vec{A})\psi \right|^2 - \mu|\psi|^2 + \frac{g}{2}|\psi|^4$$

$$\rho = \sqrt{\rho_0} e^{i\phi}$$

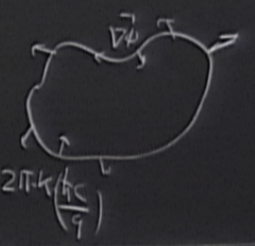
$$\rho_0 = \mu/g$$

$$= \frac{\rho_0 \hbar^2}{2m} \int d^2x \left(\vec{\nabla}\phi - \frac{q}{\hbar c}\vec{A} \right)^2 - \mu\rho_0 + \frac{1}{2}\rho_0^2$$

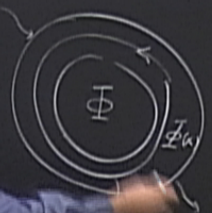
$$\vec{V} = 0 \Rightarrow \vec{\nabla}\phi = \frac{q}{\hbar c}\vec{A}$$

$\left(\frac{m \cdot \vec{V}}{\hbar} \right)$

$$\oint d\vec{e} \cdot \vec{A} = \Phi = \frac{\hbar c}{q} \oint d\vec{e} \cdot \vec{\nabla}\phi = 2\pi\kappa \left(\frac{\hbar c}{q} \right)$$



Flux quantization
 Meissner effect
 Vortices
 Grosse



$$E = \frac{1}{2m} \int d^3x \left| \left(-i\hbar \nabla - \frac{q}{c} \vec{A} \right) \psi \right|^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4$$

$$= \frac{p_0 \hbar^2}{2m} \int d^3x \left(\vec{\nabla} \psi - \frac{q}{\hbar c} \vec{A} \psi \right)^2 - \mu p_0 + \frac{g}{2} p_0^2$$

$$p = \sqrt{p_0} e^{i\phi}$$

$$p_0 = \mu/g$$

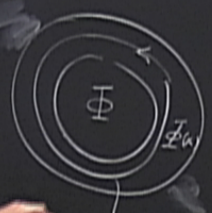
in the ring

$$\oint d\vec{l} \cdot \vec{A} = 2\pi \kappa \frac{\hbar c}{g} = 4\pi \left(\frac{\hbar c}{g} \right) \Phi_0$$

$$\vec{V} = 0 \Rightarrow \vec{\nabla} \psi = \frac{q}{\hbar c} \vec{A} \psi$$

$$\oint d\vec{l} \cdot \vec{A} = \Phi = \frac{\hbar c}{g} \oint d\vec{l} \cdot \vec{\nabla} \phi = 2\pi \kappa \left(\frac{\hbar c}{g} \right) \Phi_0$$

Flux quantization
 Meissner effect
 Vortices -
 Gross Pita



$$\Phi = \int d\vec{r} \cdot \vec{A} = 2\pi r \cdot \frac{\hbar c}{q} = 4\pi \left(\frac{\hbar c}{q}\right) \Phi_0$$

in the ring

$$E = \frac{1}{2m} \int d^3x \left| \left(-\hbar \nabla - \frac{q}{c} \vec{A} \right) \psi \right|^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4$$

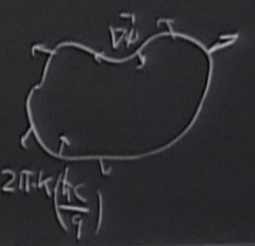
$$= \frac{p_0 \hbar^2}{2m} \int d^3x \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)^2 - \mu p_0 + \frac{7}{2} p_0^2$$

$$\vec{V} = 0 \Rightarrow \vec{\nabla} \phi = \frac{q}{\hbar c} \vec{A} \quad \left(\frac{m \cdot \vec{V}}{\hbar^2} \right)$$

$$\oint d\vec{r} \cdot \vec{A} = \Phi = \frac{\hbar c}{q} \cdot \oint d\vec{r} \cdot \vec{\nabla} \phi = 2\pi r \left(\frac{\hbar c}{q} \right)$$

$$p = \sqrt{p_0} e^{i\phi}$$

$$p_0 = \mu / g$$



Flux quantization,
 Meissner effect
 Vortices - statics
 Gross-Pitaevskii E



$$\oint d\vec{l} \cdot \vec{A} = \frac{2\pi \kappa \hbar c}{g} = 4\pi \kappa \left(\frac{\hbar c}{g}\right) \Phi_0$$

in the ring

$$E = \frac{1}{2m} \int d^2x \left| \left(\hbar \nabla - \frac{q}{c} \vec{A} \right) \psi \right|^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4$$

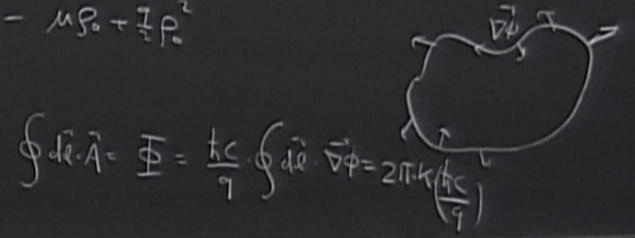
$$= \frac{\rho_0 \hbar^2}{2m} \int d^2x \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)^2 - \mu \rho_0 + \frac{g}{2} \rho_0^2$$

$$\rho = \sqrt{\rho_0} e^{i\phi}$$

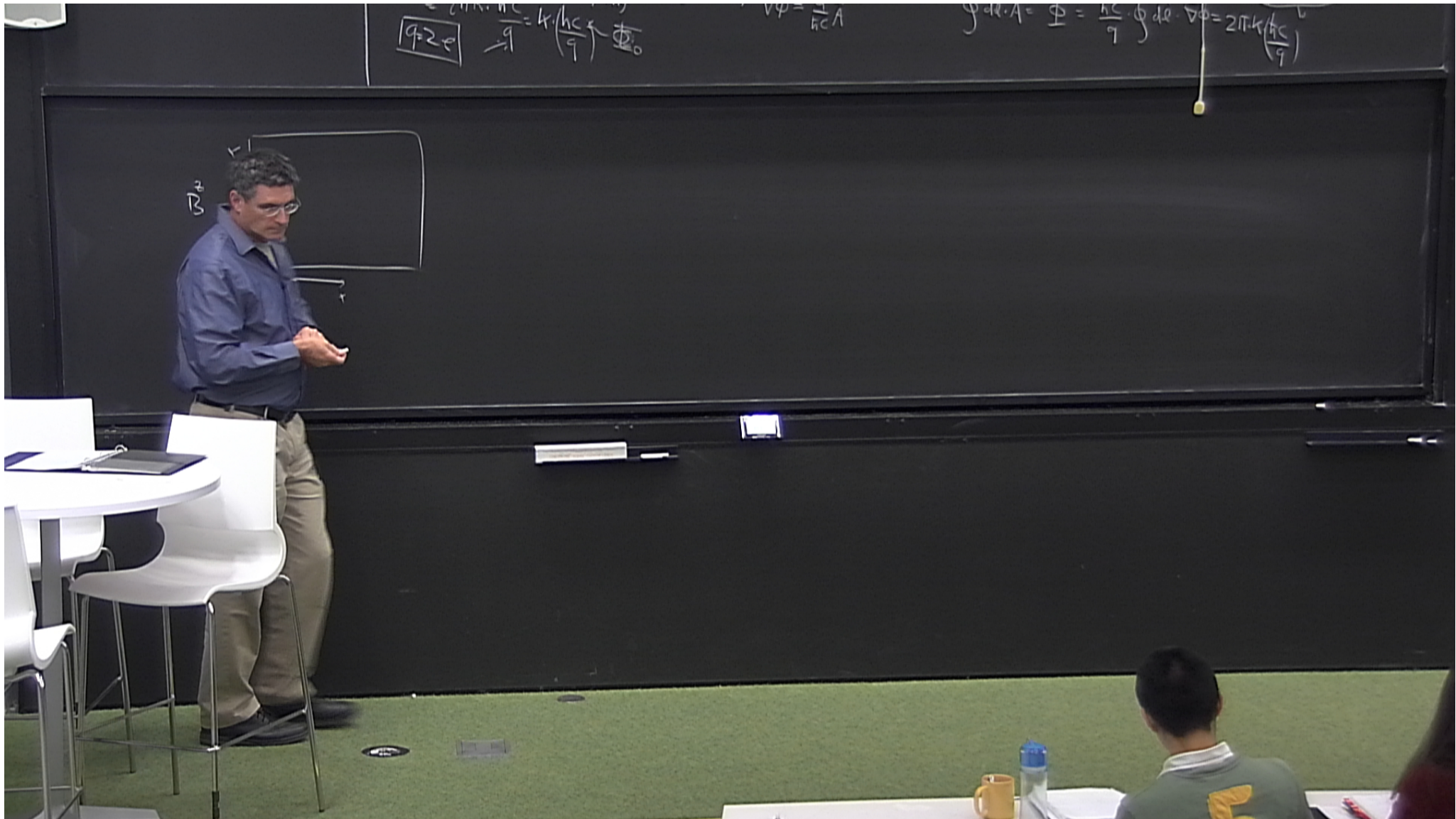
$$\rho_0 = \mu/g$$

$$\vec{V} = 0 \Rightarrow \vec{\nabla} \phi = \frac{q}{\hbar c} \vec{A}$$

$\left(\frac{m \cdot \vec{V}}{\hbar^2} \right)$



$$\oint d\vec{l} \cdot \vec{A} = \Phi = \frac{\hbar c}{g} \oint d\vec{l} \cdot \vec{\nabla} \phi = 2\pi \kappa \left(\frac{\hbar c}{g} \right)$$

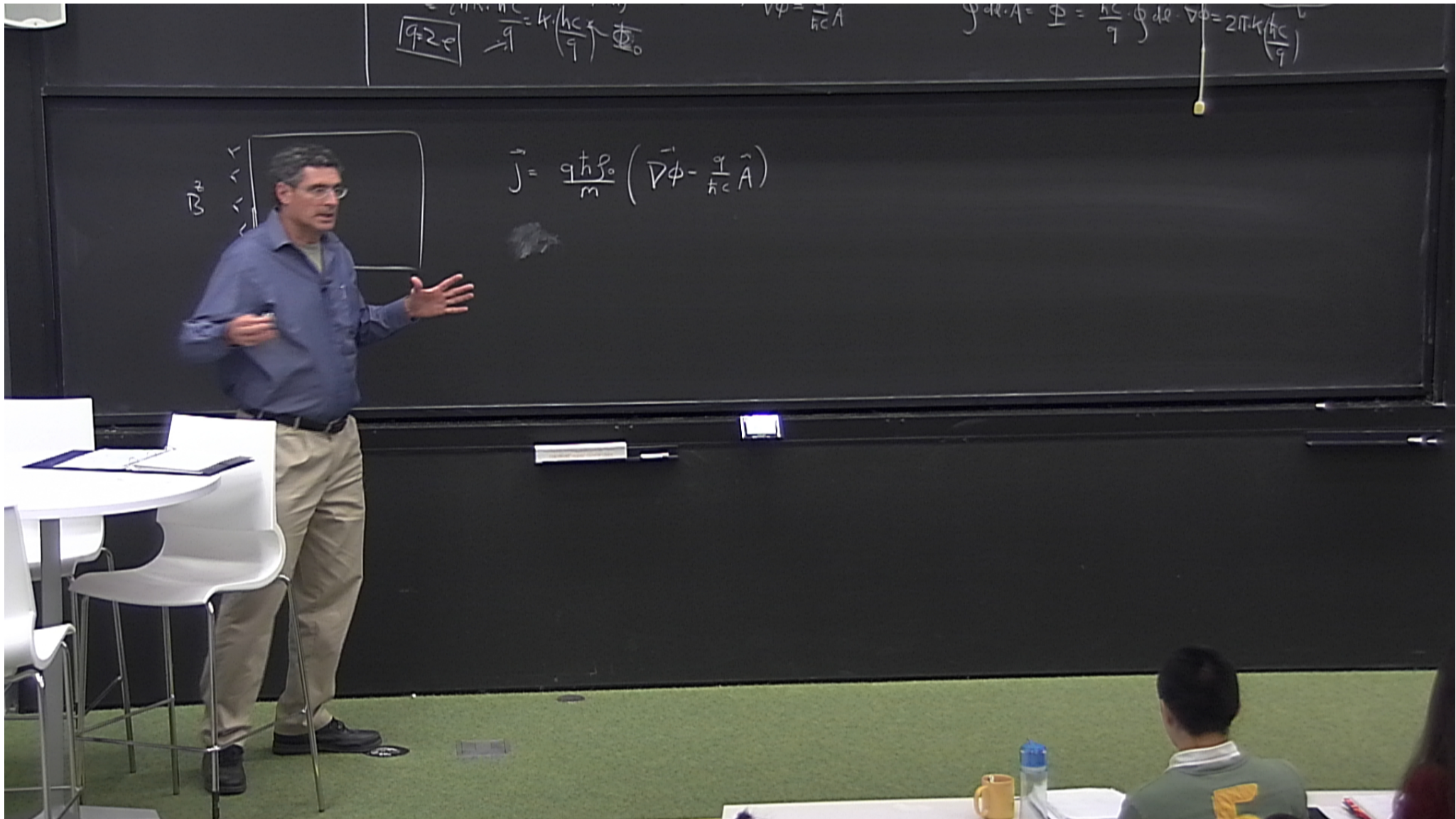


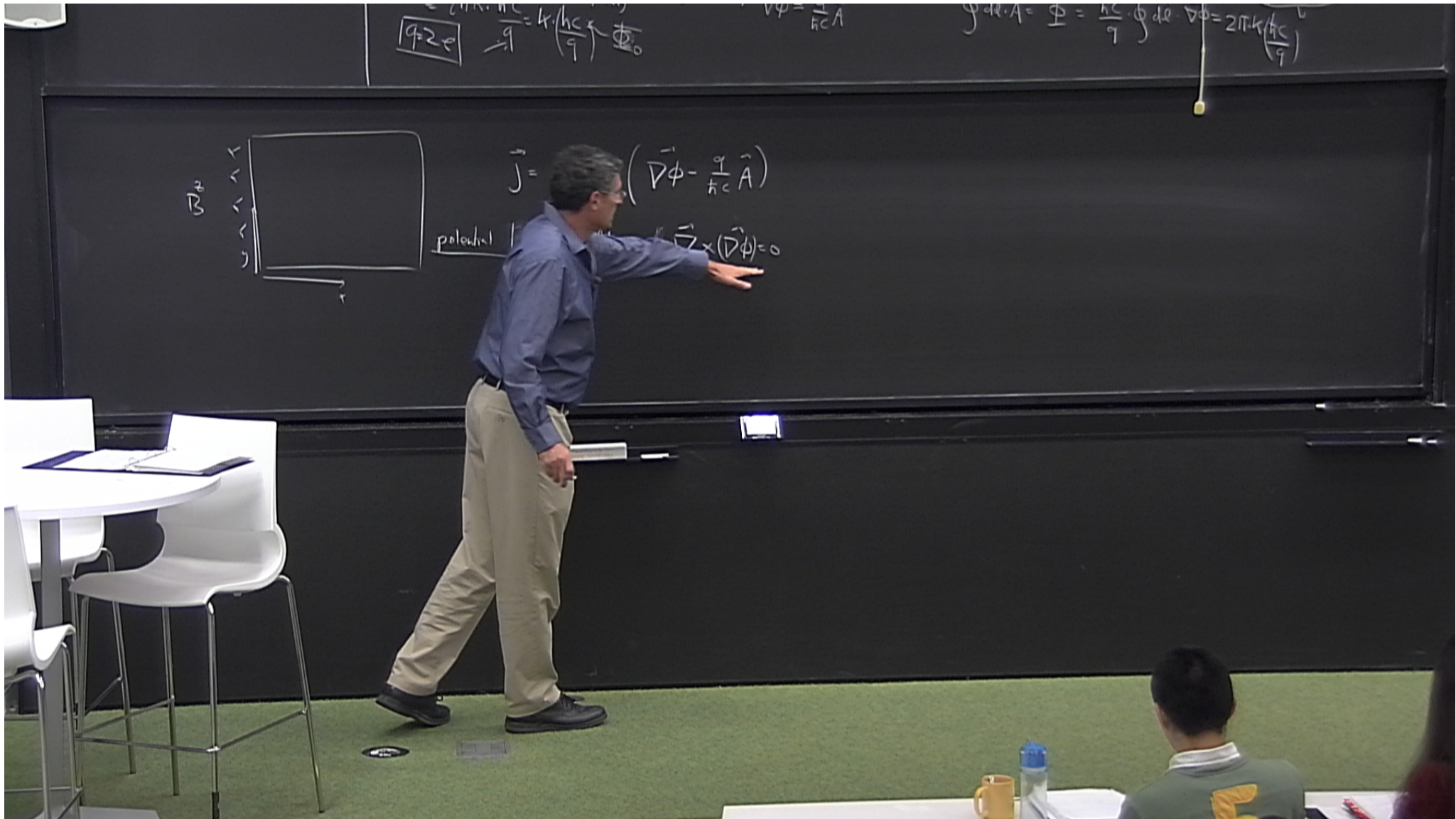
$$\boxed{q=2e} \quad \rightarrow \frac{q}{\epsilon_0} = k \left(\frac{hc}{q} \right) \left(\frac{\Phi_0}{\epsilon_0} \right)$$

$$\nabla \phi = \frac{1}{hc} A$$

$$\oint \mathbf{d}\ell \cdot \mathbf{A} = \Phi = \frac{hc}{q} \oint \mathbf{d}\ell \cdot \nabla \phi = 2\pi k \left(\frac{hc}{q} \right)$$



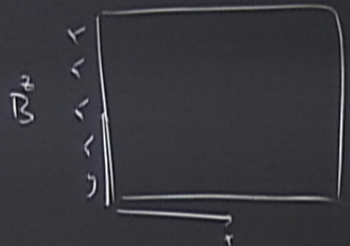




$$\boxed{q_2 e} \rightarrow \frac{1}{r} = k \left(\frac{hc}{q} \right) \Phi_0$$

$$V\phi = \frac{1}{\hbar c} A$$

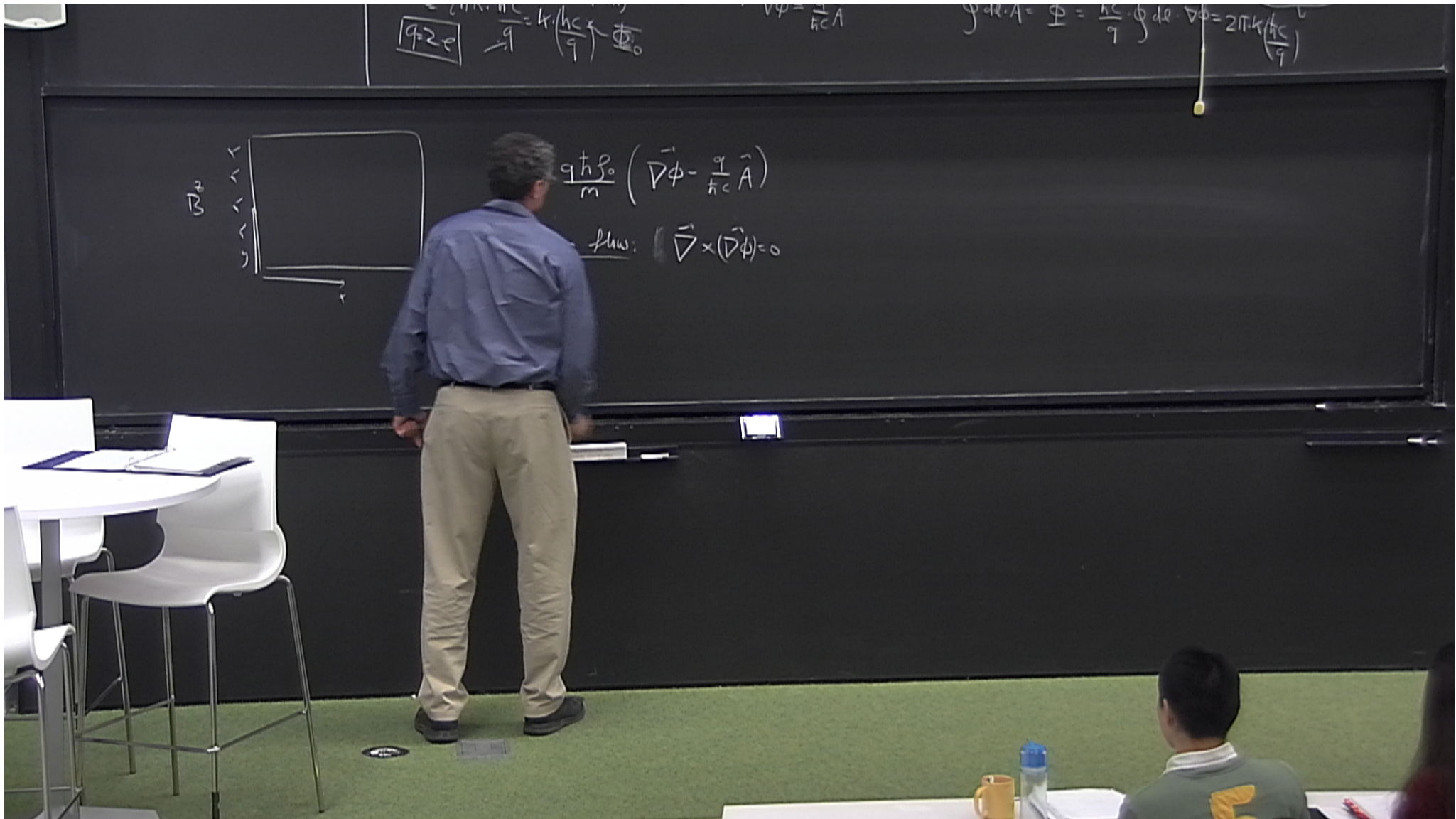
$$\oint \vec{A} \cdot d\vec{l} = \Phi = \frac{\hbar c}{q} \oint \vec{A} \cdot d\vec{l} \quad \nabla \phi = 2\pi k \left(\frac{\hbar c}{q} \right)$$



$$\vec{J} = \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)$$

potential

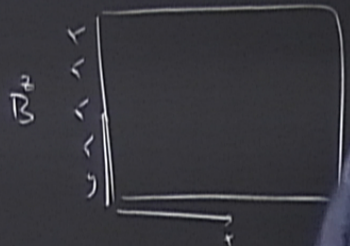
$$\vec{\nabla} \times (\vec{\nabla} \phi) = 0$$



$$\boxed{q_2 e} \quad \frac{1}{q} = k \left(\frac{hc}{q} \right) \Phi_0$$

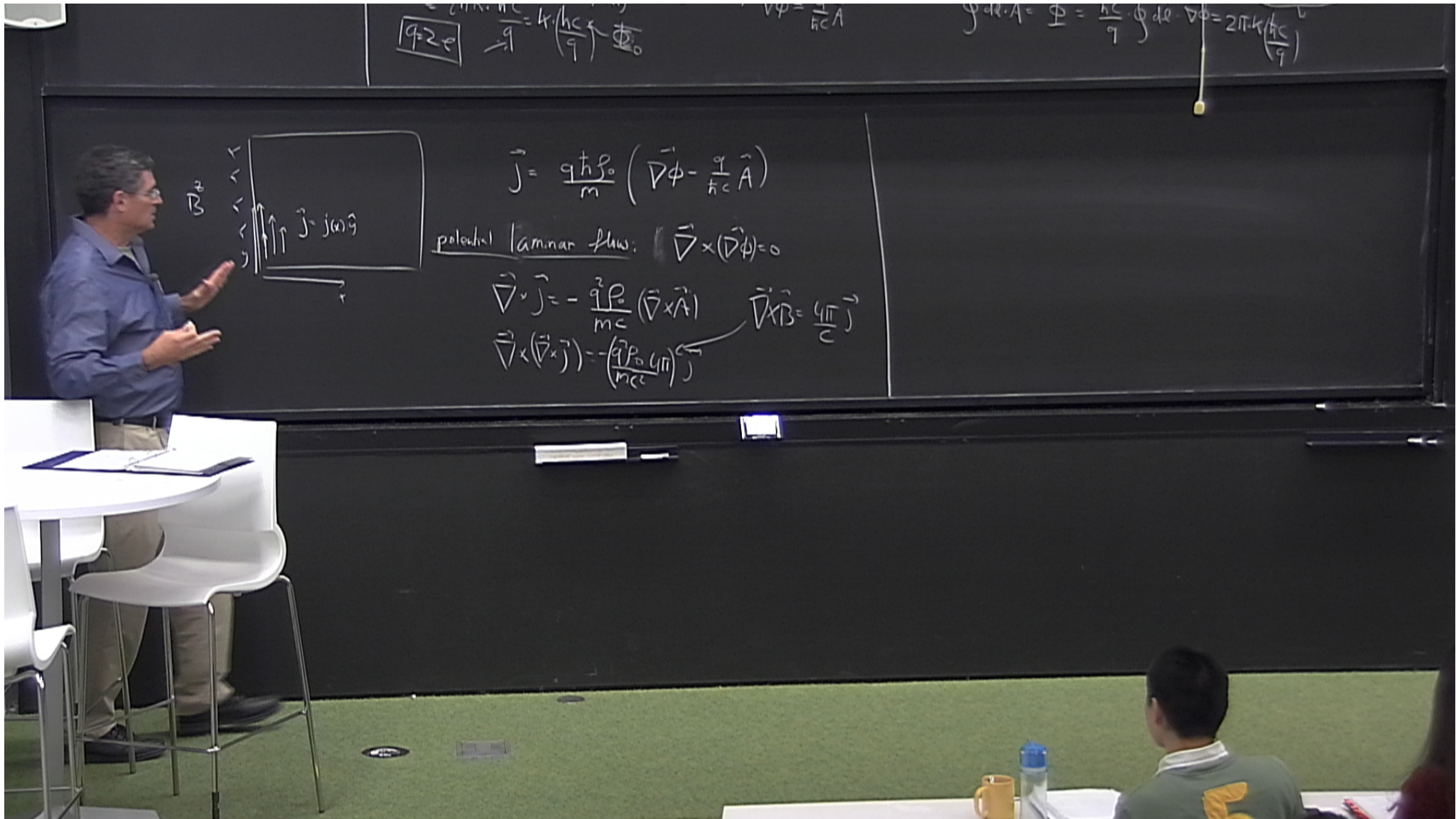
$$\nabla \phi = \frac{1}{hc} \vec{A}$$

$$\oint \vec{A} \cdot d\vec{l} = \Phi = \frac{hc}{q} \oint \vec{A} \cdot d\vec{l} = 2\pi k \left(\frac{hc}{q} \right)$$

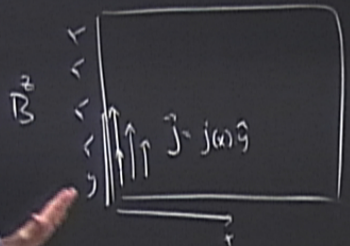


$$\frac{q \hbar \vec{p}_0}{m} \left(\vec{\nabla} \phi - \frac{q}{hc} \vec{A} \right)$$

$$\text{Ans: } \vec{\nabla} \times (\vec{\nabla} \phi) = 0$$



$$\boxed{q_2 e} \quad \frac{1}{q} = k \left(\frac{hc}{q} \right) \left(\frac{\Phi_0}{q} \right) \quad \psi = \frac{1}{hc} A \quad \oint \vec{dl} \cdot \vec{A} = \Phi = \frac{hc}{q} \oint \vec{dl} \cdot \vec{\nabla} \phi = 2\pi k \left(\frac{hc}{q} \right)$$

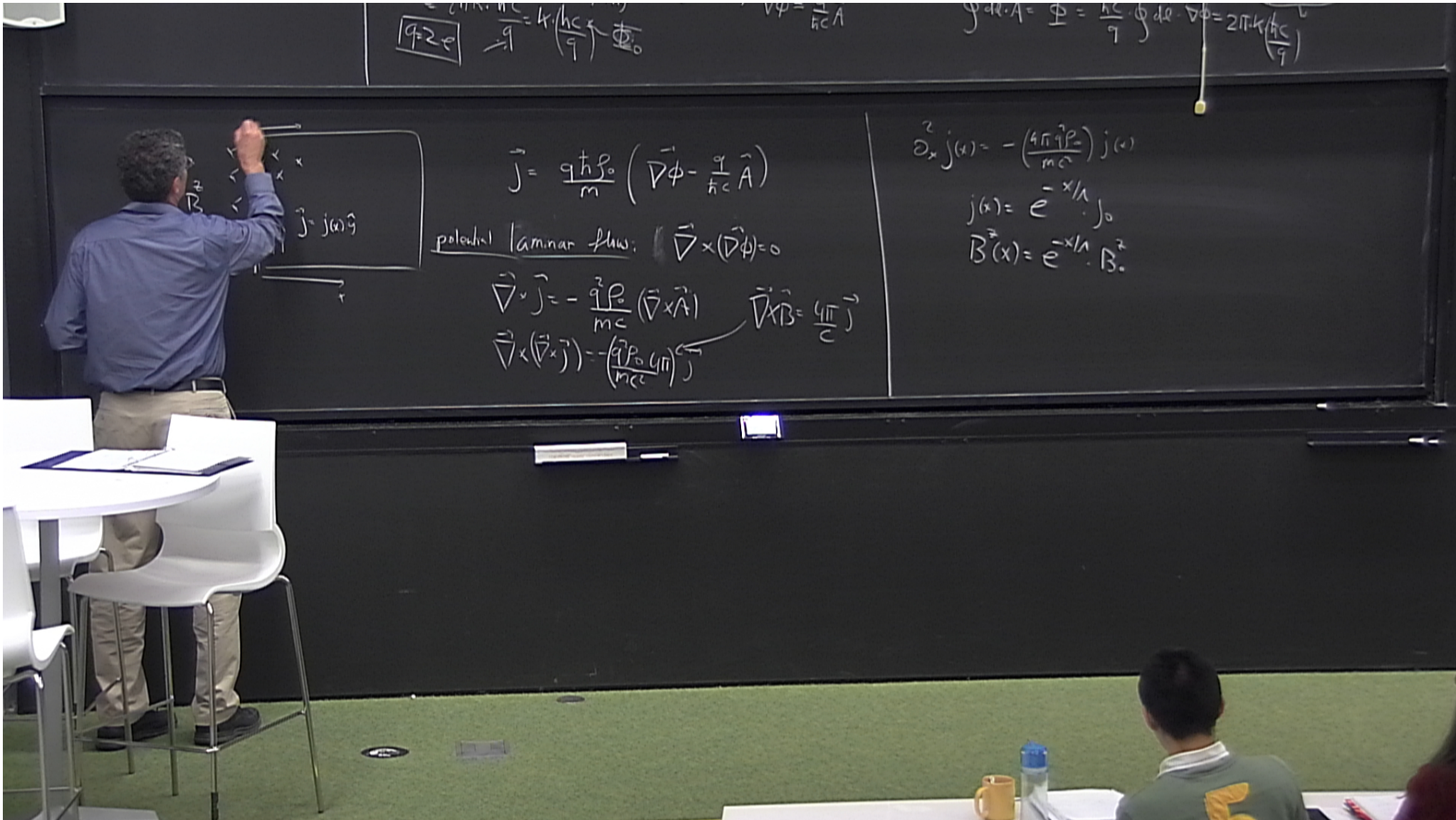


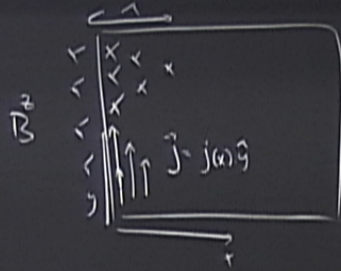
$$\vec{j} = \frac{q \hbar p_0}{m} \left(\vec{\nabla} \phi - \frac{q}{hc} \vec{A} \right)$$

potential laminar flow: $\vec{\nabla} \times (\vec{\nabla} \phi) = 0$

$$\vec{\nabla} \cdot \vec{j} = - \frac{q^2 p_0}{mc} (\vec{\nabla} \times \vec{A}) \quad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{j}) = - \left(\frac{q^2 p_0}{mc} \right) \left(\frac{4\pi}{c} \right) \vec{j}$$





$$\boxed{q_2 e} \rightarrow \frac{1}{q} = k \left(\frac{hc}{q} \right) \Phi_0$$

$$V\phi = \frac{1}{hc} A$$

$$\oint \vec{A} \cdot d\vec{l} = \Phi = \frac{hc}{q} \oint \vec{j} \cdot d\vec{l} \quad \nabla \phi = 2\pi k \left(\frac{hc}{q} \right)$$

$$\vec{j} = \frac{q \hbar^2 \rho_0}{m} \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)$$

potential laminar flow: $\vec{\nabla} \times (\vec{\nabla} \phi) = 0$

$$\vec{\nabla} \times \vec{j} = - \frac{q^2 \rho_0}{mc} (\vec{\nabla} \times \vec{A}) \quad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

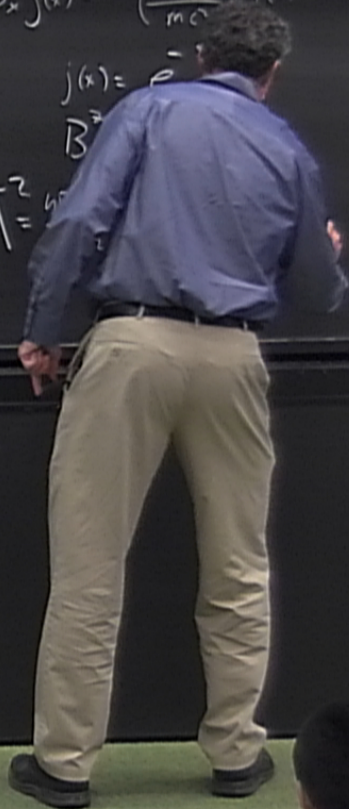
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{j}) = - \left(\frac{q^2 \rho_0}{mc} \right) \left(\frac{4\pi}{c} \vec{j} \right)$$

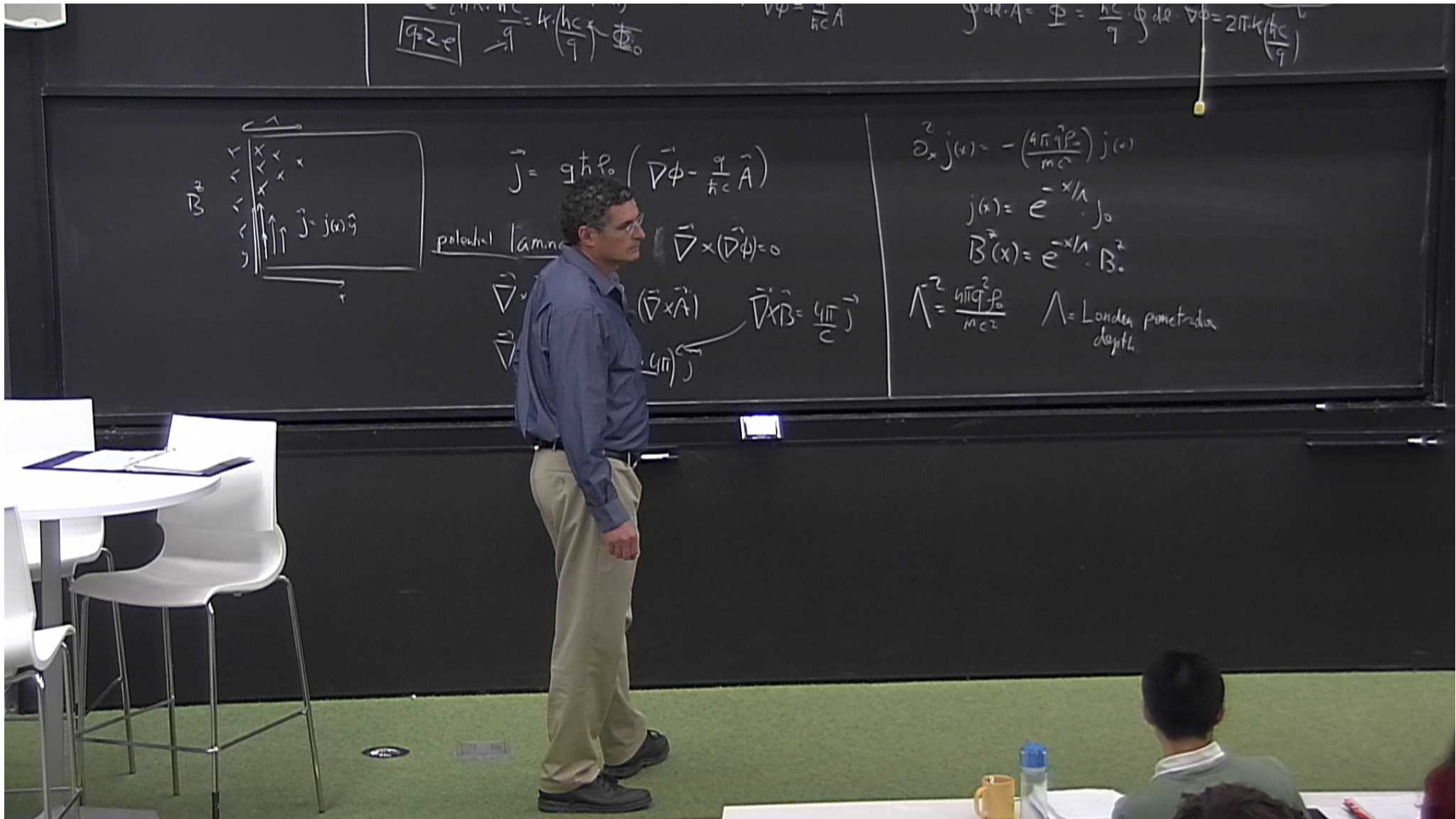
$$\partial_x^2 j(x) = - \left(\frac{4\pi q^2 \rho_0}{mc} \right) j(x)$$

$$j(x) = e^{-\lambda x}$$

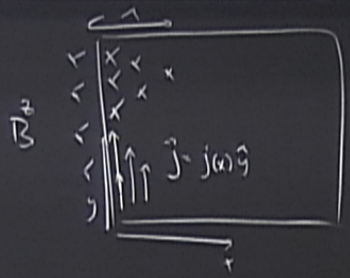
$$\vec{B} =$$

$$\lambda =$$





$$\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{j} = -\frac{1}{\mu_0} \nabla^2 \vec{A} \quad \nabla \cdot \vec{A} = 0 \quad \nabla^2 \vec{A} = -\mu_0 \vec{j}$$



$$\vec{j} = -\frac{1}{\mu_0} \nabla^2 \vec{A}$$

London gauge condition: $\nabla \cdot \vec{A} = 0$

$$\nabla \times (\nabla \times \vec{A}) = -\mu_0 \vec{j}$$

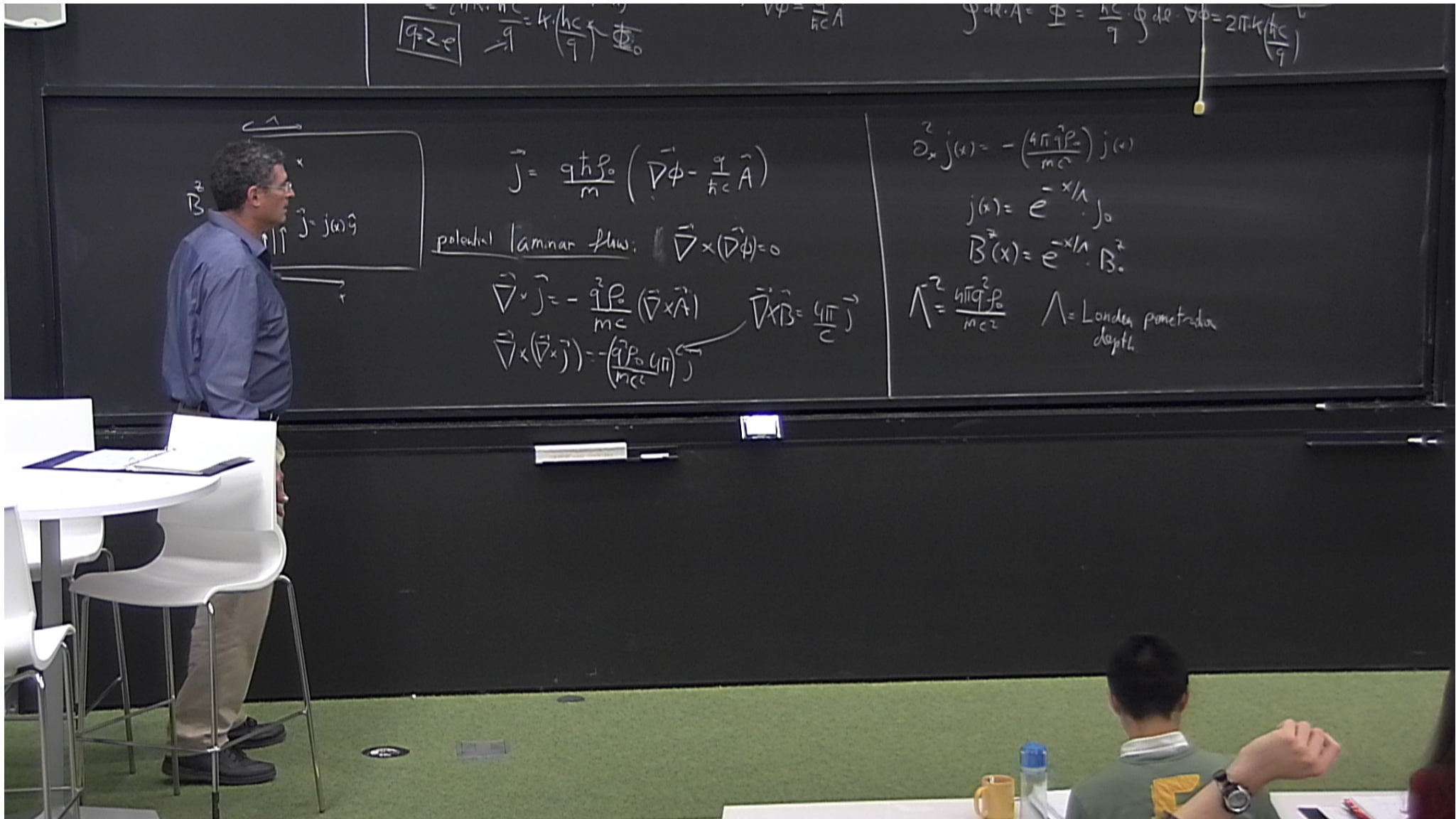
$$\nabla^2 \vec{A} = -\mu_0 \vec{j}$$

$$\nabla^2 j(x) = -\left(\frac{4\pi q^2 p_0}{mc}\right) j(x)$$

$$j(x) = e^{-x/\Lambda} j_0$$

$$\vec{B}(x) = e^{-x/\Lambda} \vec{B}_0$$

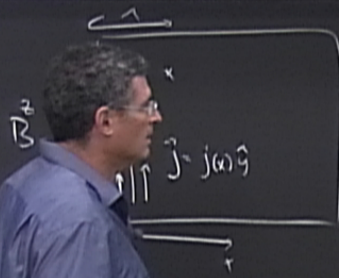
$$\Lambda = \frac{1}{\sqrt{\mu_0 q^2 p_0 / mc^2}} \quad \Lambda = \text{London penetration depth}$$



$$\frac{q_2 e}{m} = \frac{4\pi q_2 e^2}{m c} \frac{\Phi_0}{9}$$

$$V\phi = \frac{1}{\hbar c} A$$

$$\oint \vec{A} \cdot d\vec{l} = \Phi = \frac{\hbar c}{q} \oint d\theta = 2\pi \hbar c \left(\frac{q}{9} \right)$$



$$\vec{j} = \frac{q_2 \hbar \rho_0}{m} \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)$$

potential laminar flow: $\vec{\nabla} \times (\vec{\nabla} \phi) = 0$

$$\vec{\nabla} \times \vec{j} = - \frac{q_2^2 \rho_0}{m c} (\vec{\nabla} \times \vec{A}) \quad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

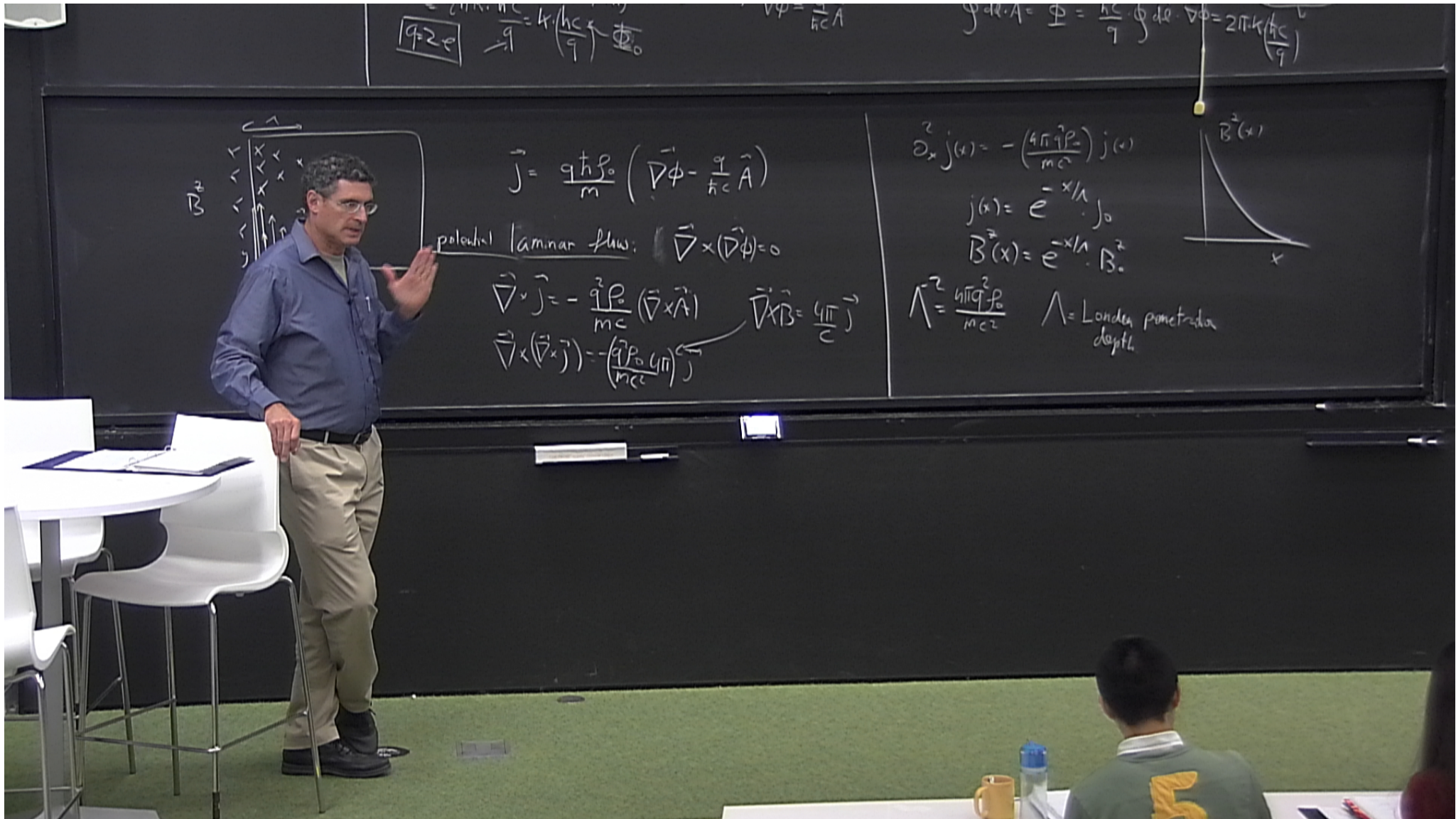
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{j}) = - \left(\frac{q_2^2 \rho_0}{m c} \right) \vec{j}$$

$$\partial_x^2 j(x) = - \left(\frac{4\pi q_2^2 \rho_0}{m c} \right) j(x)$$

$$j(x) = e^{-x/\Lambda} j_0$$

$$\vec{B}(x) = e^{-x/\Lambda} \vec{B}_0$$

$$\Lambda^2 = \frac{4\pi q_2^2 \rho_0}{m c^2} \quad \Lambda = \text{London penetration depth}$$



$$\frac{q_2 e}{m} = \frac{4\pi k (hc/q)}{\Phi_0}$$

$$v\phi = \frac{1}{hc} A$$

$$\oint \vec{A} \cdot d\vec{l} = \Phi = \frac{hc}{q} \oint \vec{A} \cdot d\vec{l} = 2\pi k \left(\frac{hc}{q} \right)$$



$$\vec{j} = \frac{q\hbar p_0}{m} \left(\vec{\nabla}\phi - \frac{q}{hc} \vec{A} \right)$$

poloidal laminar flow: $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$

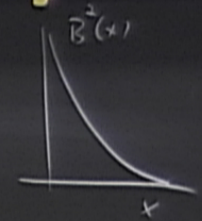
$$\vec{\nabla} \times \vec{j} = -\frac{q^2 p_0}{mc} (\vec{\nabla} \times \vec{A}) \quad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{j}) = -\left(\frac{q^2 p_0}{mc} \right) \vec{j}$$

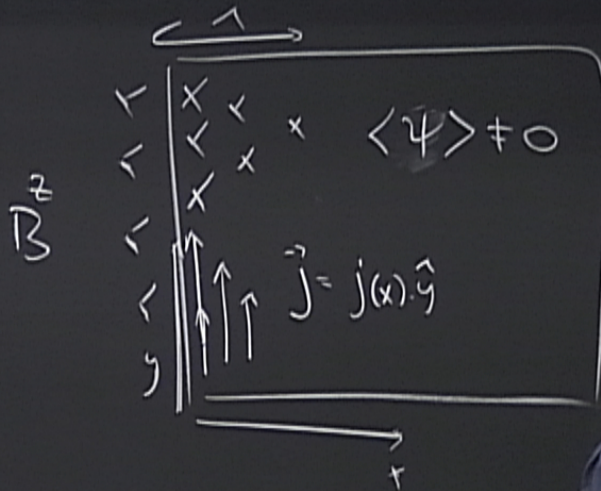
$$\partial_x^2 j(x) = -\left(\frac{4\pi q^2 p_0}{mc} \right) j(x)$$

$$j(x) = e^{-x/\Lambda} j_0$$

$$\vec{B}(x) = e^{-x/\Lambda} \vec{B}^0$$



$$\Lambda^2 = \frac{4\pi q^2 p_0}{mc^2} \quad \Lambda = \text{London penetration depth}$$

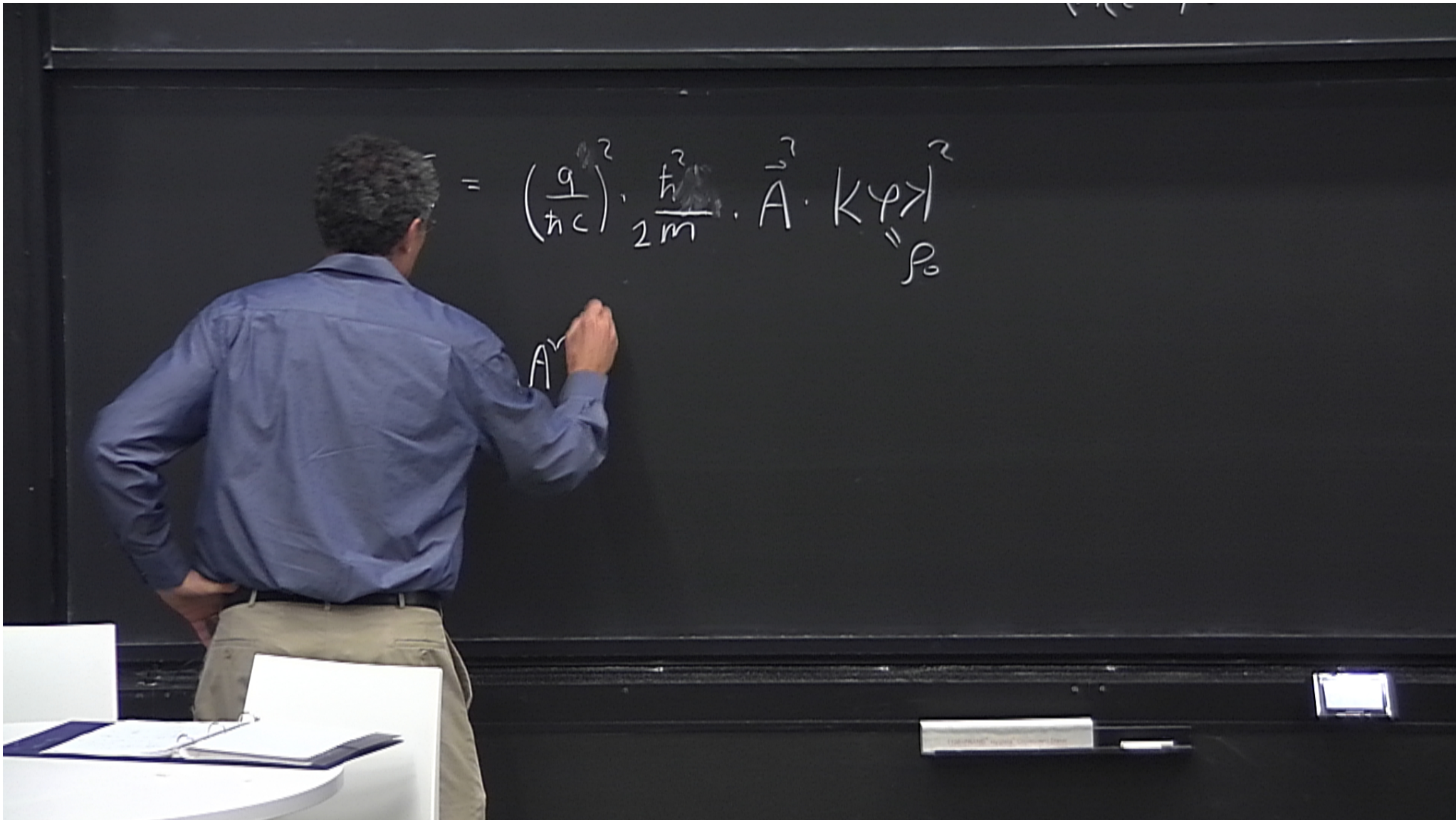


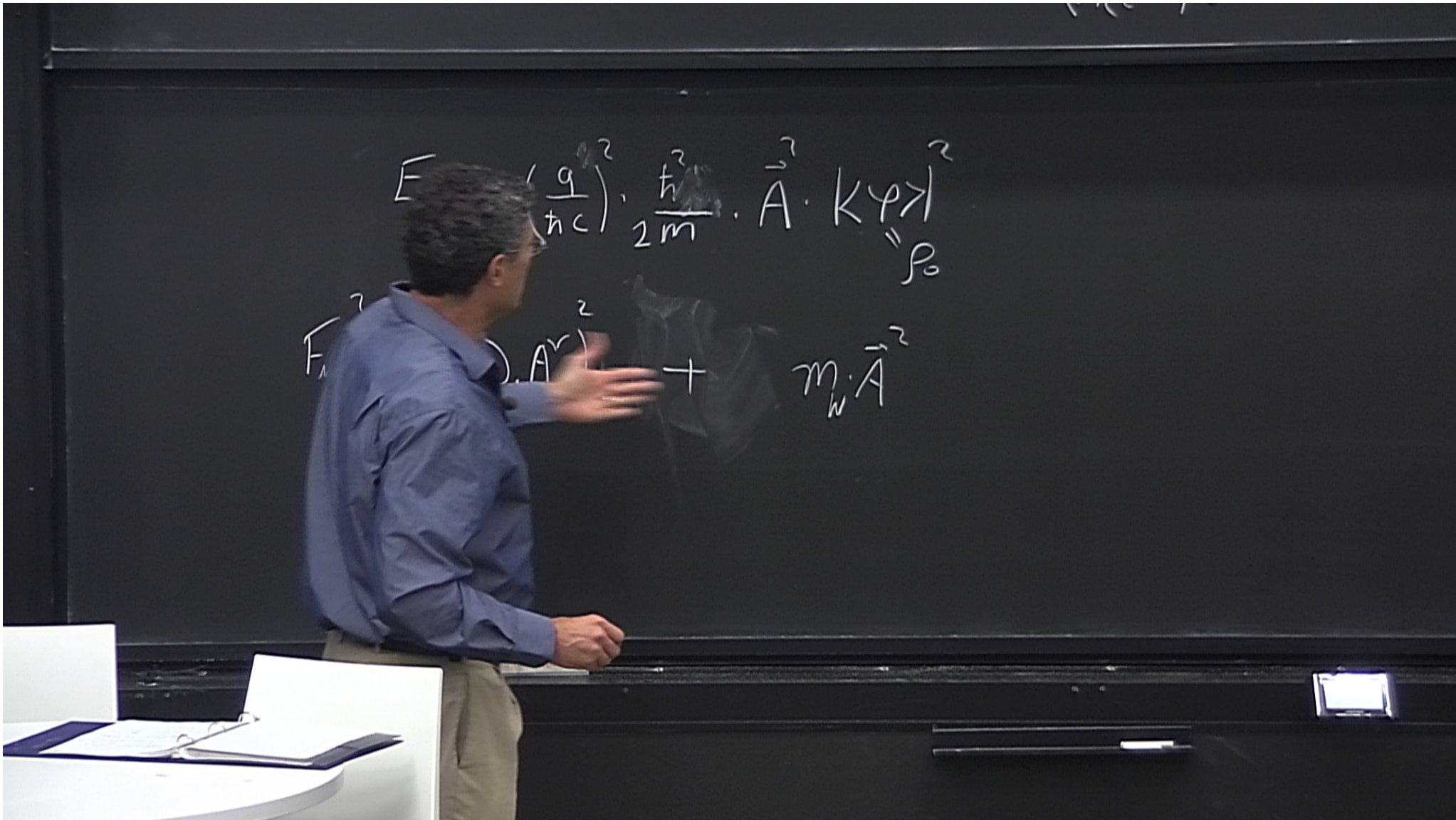
$$\frac{n q_0}{m} \left(\vec{\nabla} \phi - \frac{q}{\hbar c} \vec{A} \right)$$

gauge condition: $\vec{\nabla} \cdot (\vec{\nabla} \phi) = 0$

$$\vec{J} = - \frac{q^2 \rho_0}{m c} (\vec{\nabla} \times \vec{A}) \quad \vec{\nabla} \times \vec{B} =$$

$$\left(\vec{\nabla} \times \vec{J} \right) = - \left(\frac{q^2 \rho_0}{m c} 4\pi \vec{J} \right)$$

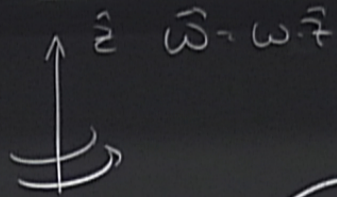




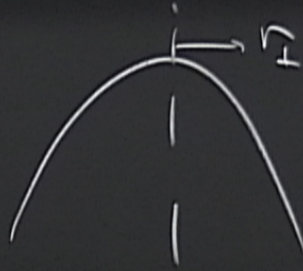
$$E = \left(\frac{q}{\hbar c}\right)^2 \cdot \frac{\hbar^2}{2m} \cdot \vec{A} \cdot \underbrace{K\psi}_{\beta_0}^2$$

$$E_{nr} = \left(\nabla_{nr} \cdot \vec{A}\right)^2 + m \cdot \vec{A}^2$$

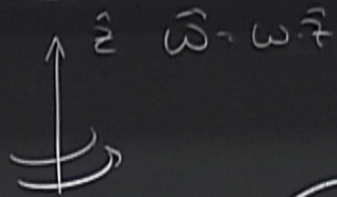
Rotating frame



$\text{Lagrange: } -m\omega^2 r_{\perp}^2$
Cancelled by
potential
centrifugal

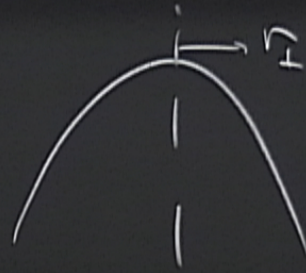


Rotating frame



centrifugal

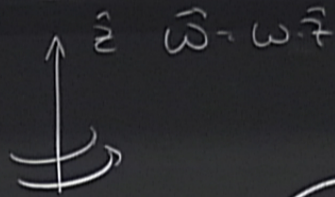
$\frac{v^2}{r}$
derived by
potential



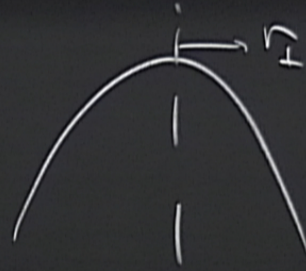
Conical

$$\vec{v} \times \vec{B}$$

Rotating frame



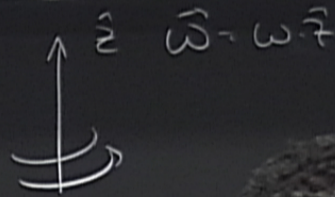
centrifugal: $-m\omega^2 r_{\perp}^2$
cancelled by
potential



Coriolis force:

$$\vec{F} = 2m\vec{\omega} \times \vec{v} = \frac{q}{c} \vec{v} \times \vec{B} \leftarrow \text{like } \vec{v} \times \vec{B}$$
$$\vec{B}^{\text{con}} = -\frac{2mc}{q} \vec{\omega}$$

Rotating frame

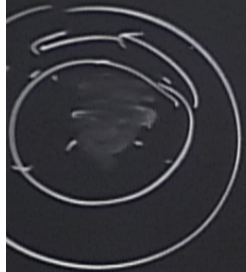


centrifugal: $-m\omega^2 r$
cancelled by
potential

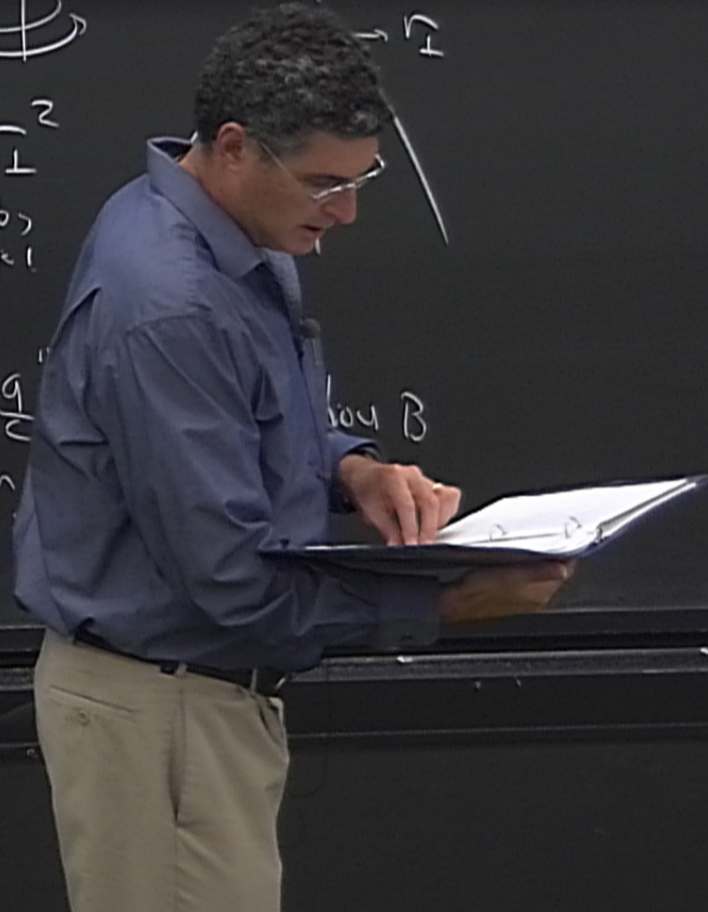
Coriolis force:

$$\vec{F} = 2m\vec{\omega} \times \vec{v} = \text{"g"}$$

\vec{B}^{con} $\text{by } B$

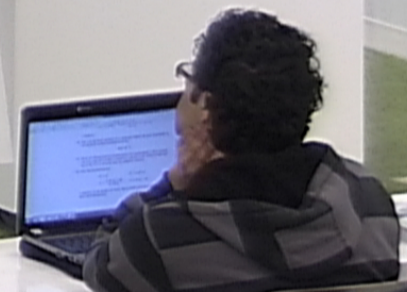
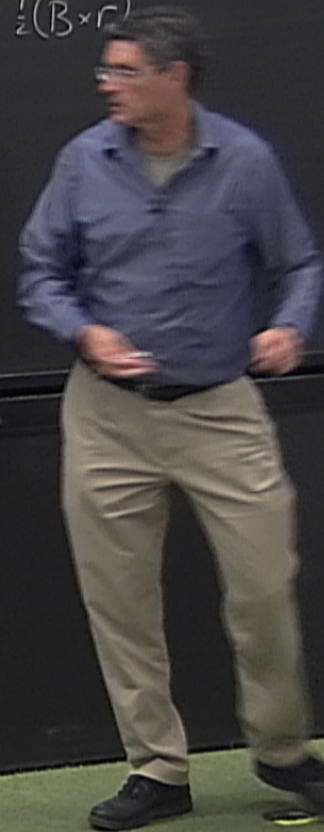


ham



$$E[\psi] = \int d^3x \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mathcal{M} \rho_0 + \frac{1}{2} g \rho_0^2 \quad \vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

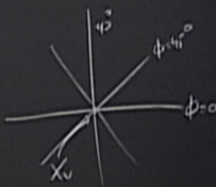
$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$



$$E[\psi] = \int d\mathbf{x} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2 \quad \vec{A} = \frac{1}{2} (\vec{\nabla} \times \vec{r})$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$

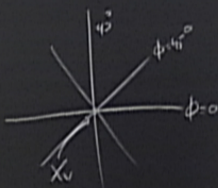
Vortex solutions $\psi(\mathbf{x}) = \sqrt{\rho_0} e^{ik\phi(\mathbf{x})}$



$$E[\psi] = \int d^2x \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \cdot \vec{r} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2 \quad \vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$

Vortex solutions: $\psi_k(x) = \sqrt{\rho_0} e^{ik\phi(x)}$
 $\psi_0(x) = \sqrt{\rho_0}$

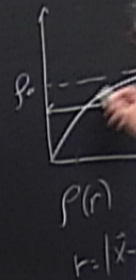
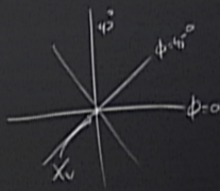


$$E[\psi] = \int d\mathbf{r} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right)^2$$

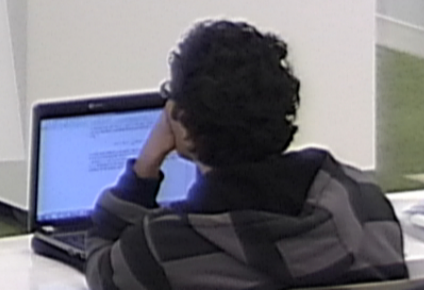
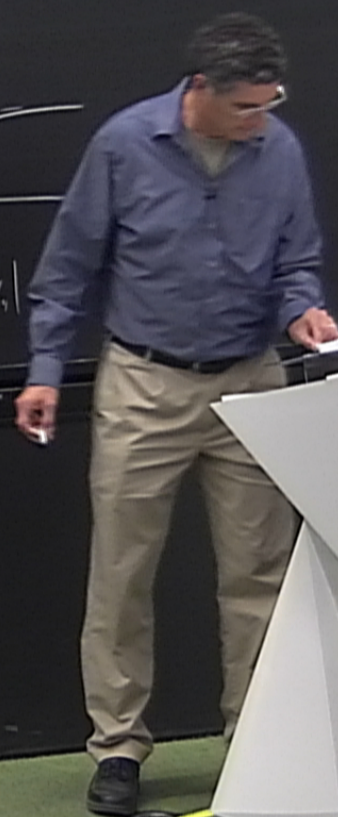
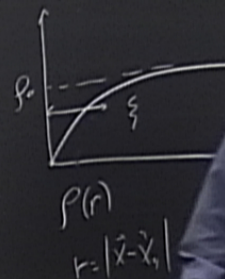
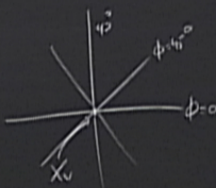
Vortex solutions: $\psi(\mathbf{r}) = \frac{1}{\sqrt{k}} e^{ik\phi(\mathbf{r})}$
 $\psi_0 = \sqrt{\rho_0}$



$$E[\psi] = \int d\mathbf{r} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2 \quad \vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$

Vortex solution: $\psi(\mathbf{r}) = \sqrt{\rho(r)} e^{ik\phi(\mathbf{r})}$
 $\oint_0^{2\pi} \mathbf{x} \cdot \vec{\nabla} \phi = 2\pi k$



$$E[\psi] = \int d\mathbf{r} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2$$

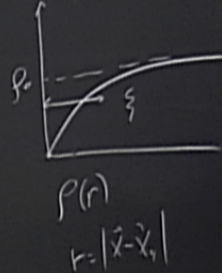
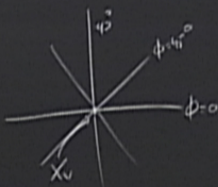
$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$\vec{j} = \rho \vec{v}$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right)^2$$

Vortex solutions: $\psi_k(\mathbf{x}) = \sqrt{\rho(r)} e^{ik\phi(\mathbf{x})}$

$$\oint_{\gamma_0 \times \vec{x}_1} d\mathbf{l} \cdot \vec{\nabla} \phi = 2\pi k$$



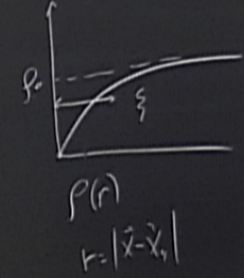
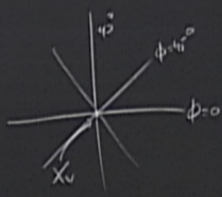
$$E[\psi] = \int d\mathbf{r} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

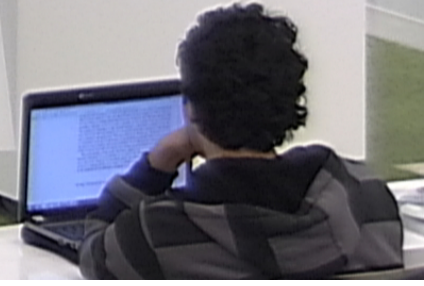
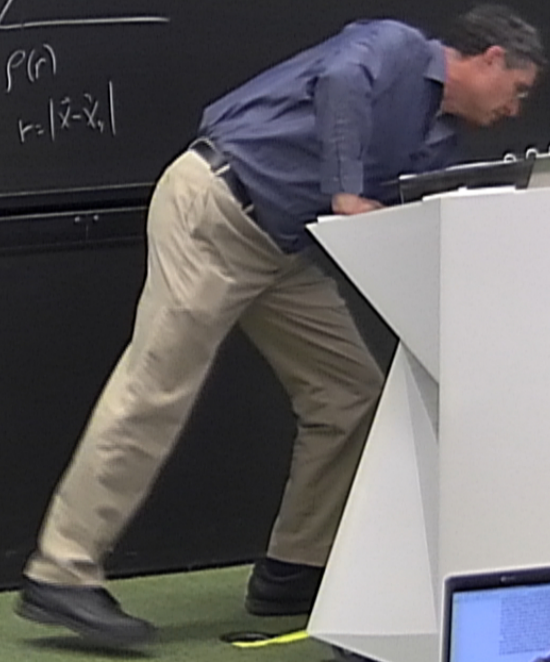
$$\vec{j} = \rho_0 \left(\frac{\hbar}{m} \right)$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m\vec{\omega} \times \vec{r}}{\hbar} \right)^2$$

Vortex solutions: $\psi_k(\mathbf{x}) = \sqrt{\rho_0} e^{ik\phi(\mathbf{x})}$



$$\oint d\mathbf{l} \cdot \vec{\nabla} \phi = 2\pi k$$



$$E[\psi] = \int d\mathbf{x} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

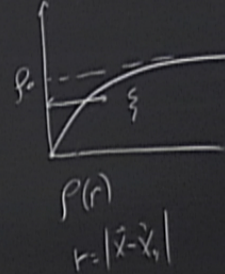
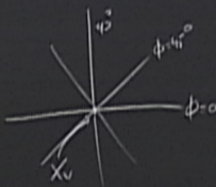
$$\vec{\nabla} \phi = \frac{\hbar}{m} \vec{k}$$

$$\vec{j} = \rho(r) \left(\frac{\hbar}{m} \vec{k} - \vec{\omega} r \right)$$

$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$

Vortex solutions: $\psi_k(\mathbf{x}) = \sqrt{\rho(r)} e^{ik\phi(\mathbf{x})}$

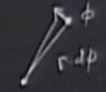
$$\oint_{\gamma_0} d\mathbf{l} \cdot \vec{\nabla} \phi = 2\pi k$$



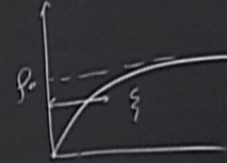
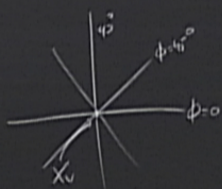
$$E[\psi] = \int d\mathbf{r} \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mu \rho_0 + \frac{1}{2} g \rho_0^2$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$$\vec{\nabla} \phi = \frac{\hbar \mathbf{k}}{r}$$



$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$



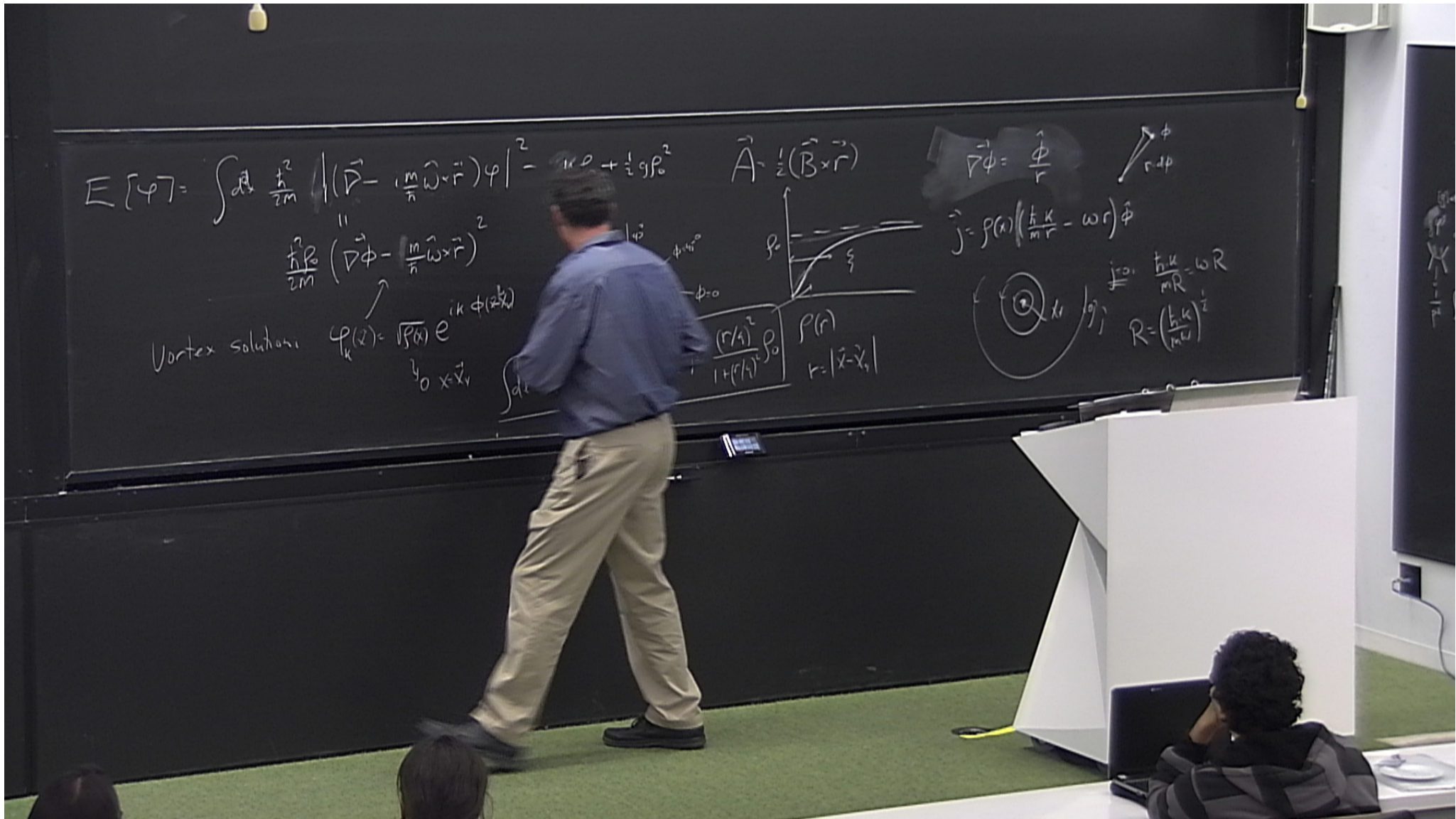
$$\vec{j} = \rho(r) \left(\frac{\hbar \mathbf{k}}{m r} - \vec{\omega} \right) \phi$$



$$R = \frac{\hbar \mathbf{k}}{m R} - \vec{\omega} R$$

Vortex solutions: $\psi_k(\mathbf{r}) = \sqrt{\rho(r)} e^{i k \phi(\mathbf{r})}$

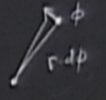
$$\oint d\mathbf{l} \cdot \vec{\nabla} \phi = 2\pi k$$



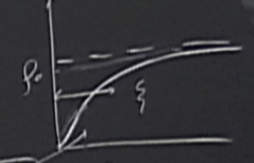
$$E[\psi] = \int d^3x \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right) \psi \right|^2 - \mu \rho + \frac{1}{2} g \rho^2$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$$

$$\vec{\nabla} \phi = \frac{\hbar \vec{k}}{m} \hat{\phi}$$



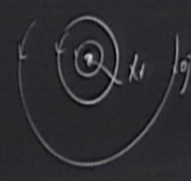
$$\frac{\hbar^2 \rho_0}{2m} \left(\vec{\nabla} \phi - \frac{m}{\hbar} \vec{\omega} \times \vec{r} \right)^2$$



$$\vec{j} = \rho(r) \left(\frac{\hbar \vec{k}}{m} - \vec{\omega} \right) \hat{\phi}$$

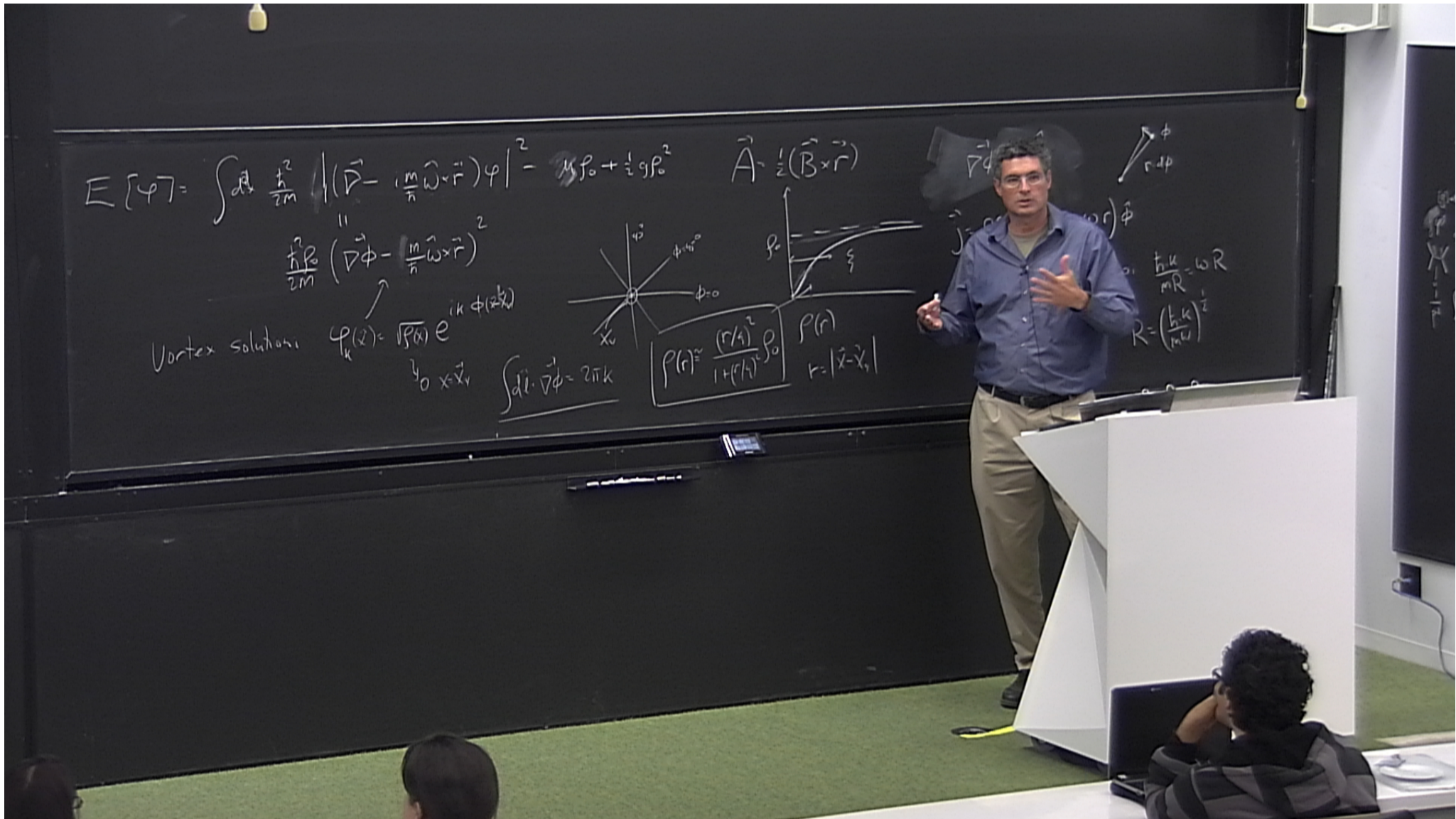
Vortex solution: $\psi(\vec{x}) = \frac{1}{\sqrt{k}} e^{ik\phi(\vec{x})}$
 $\psi_0 \times \vec{x}_\perp$

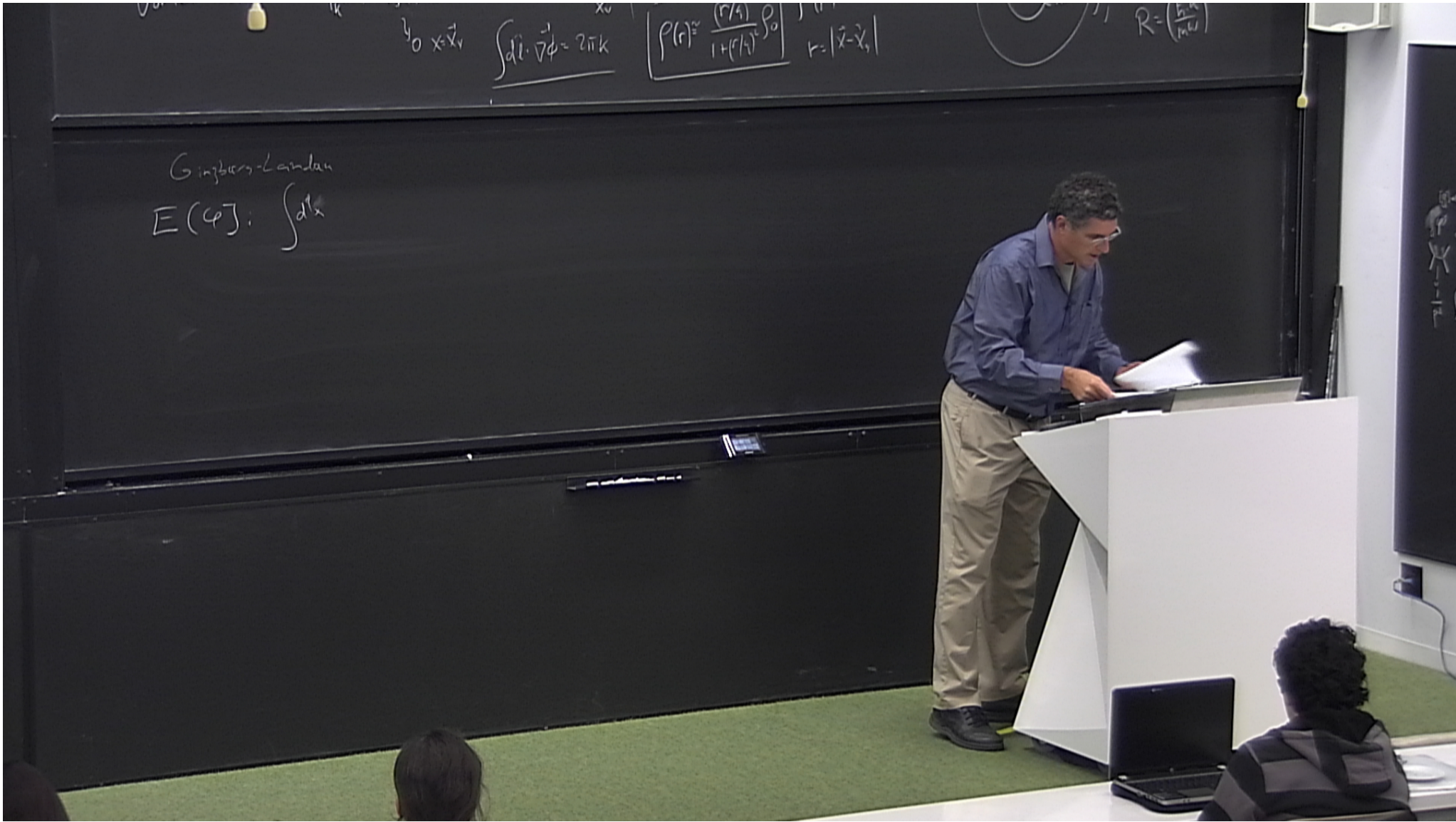
$$\frac{(r/\lambda)^2}{1 + (r/\lambda)^2} \rho_0$$



$$\vec{j} \approx \frac{\hbar k}{mR} - \omega R$$

$$R = \left(\frac{\hbar k}{m\omega} \right)^{1/2}$$





Ginzburg-Landau

$$E(\varphi) = \int d^3x \left[\frac{1}{2} \rho_s (\nabla\varphi - \frac{q}{\hbar c} \vec{A})^2 + \mu |\varphi|^2 + \frac{g}{2} |\varphi|^4 + o(\varphi^6) \right]$$

$$\frac{g}{2} (|\varphi|^2 - M/g)^2 = \frac{M^2}{2g^2}$$

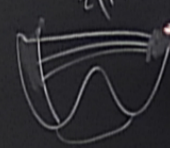
$$\oint d\vec{l} \cdot \vec{\nabla}\phi = 2\pi k$$

$$\rho(r) = \frac{\rho_0}{1 + (r/\lambda)^2}$$

$$r = |\vec{x} - \vec{x}_0|$$

Ginzburg-Landau

$$E(\varphi) = \int d^3x \left[\frac{1}{2} \rho_s \left(\nabla\varphi - \frac{q}{\hbar c} \vec{A} \right)^2 + \underbrace{\mu |\varphi|^2 + \frac{g}{2} |\varphi|^4}_{\frac{g}{2} (|\varphi|^2 - M/g)^2} \right]$$



$$y_0 \times \vec{x}_v \quad \int d\vec{l} \cdot \vec{\nabla} \phi = 2\pi k \quad \rho(r) = \frac{\rho_0}{1 + (r/r_0)^2} \quad r = |\vec{x} - \vec{x}_0| \quad R = \frac{R_0}{\sqrt{1 + (r/R_0)^2}}$$

Ginzburg-Landau

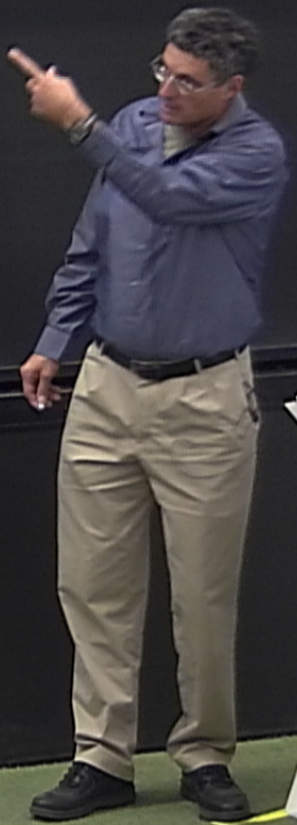
$$E(\varphi) = \int d^3x \left[\frac{1}{2} \rho_s (\nabla \varphi - \frac{q}{\hbar c} \vec{A})^2 + \mu |\varphi|^2 + \frac{g}{2} |\varphi|^4 + O(\varphi)^6 \right]$$

$$\frac{g}{2} (|\varphi|^2 - M/g)$$



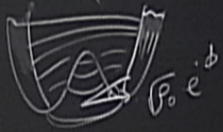
$\int_{\partial V} \vec{x} \cdot \vec{v}$ $\int d\vec{l} \cdot \vec{\nabla} \phi = 2\pi k$ $\rho(r) = \frac{\rho_0}{1+(r/r_0)^2}$ $r = |\vec{x} - \vec{x}_0|$ $R = \left(\frac{2\pi k}{\rho_0}\right)$

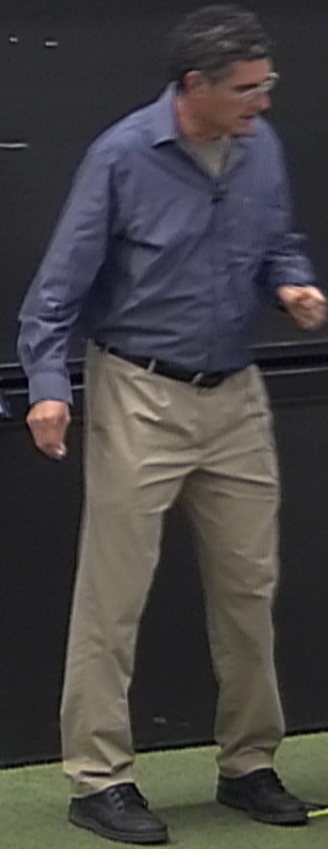
Ginzburg-Landau
 $E(\varphi) = \int d^4x \underbrace{\frac{1}{2} \rho_s (\nabla\phi - \frac{q}{\hbar c} \vec{A})^2}_{\text{stiffness}} - \underbrace{M|\varphi|^2 + \frac{g}{2}|\varphi|^4}_{\frac{g}{2}(|\varphi|^2 - M/g)^2 = M^2/g^2} + O(\varphi)^6$



Ginzburg-Landau

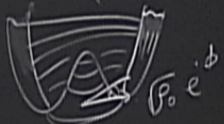
$$E(\varphi) = \int d^3x \left[\frac{1}{2} \rho_s \left(\nabla\varphi - \frac{q}{\hbar c} \vec{A} \right)^2 + \underbrace{\mu |\varphi|^2 + \frac{g}{2} |\varphi|^4}_{\text{stiffness}} + o(\varphi)^6 \right]$$

$$\frac{g}{2} (|\varphi|^2 - M/g)^2 \approx M^2/g^2$$




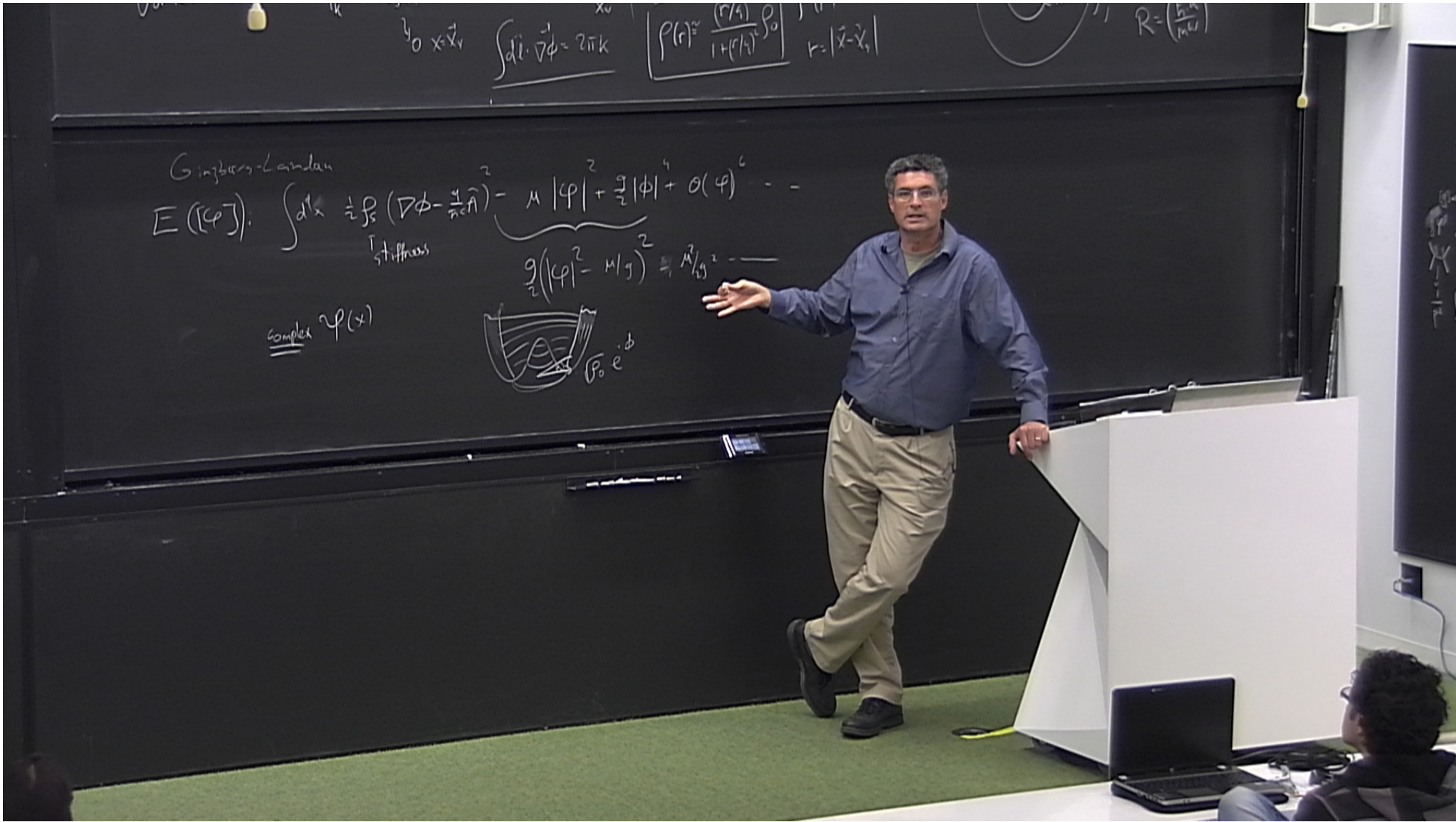
Ginzburg-Landau

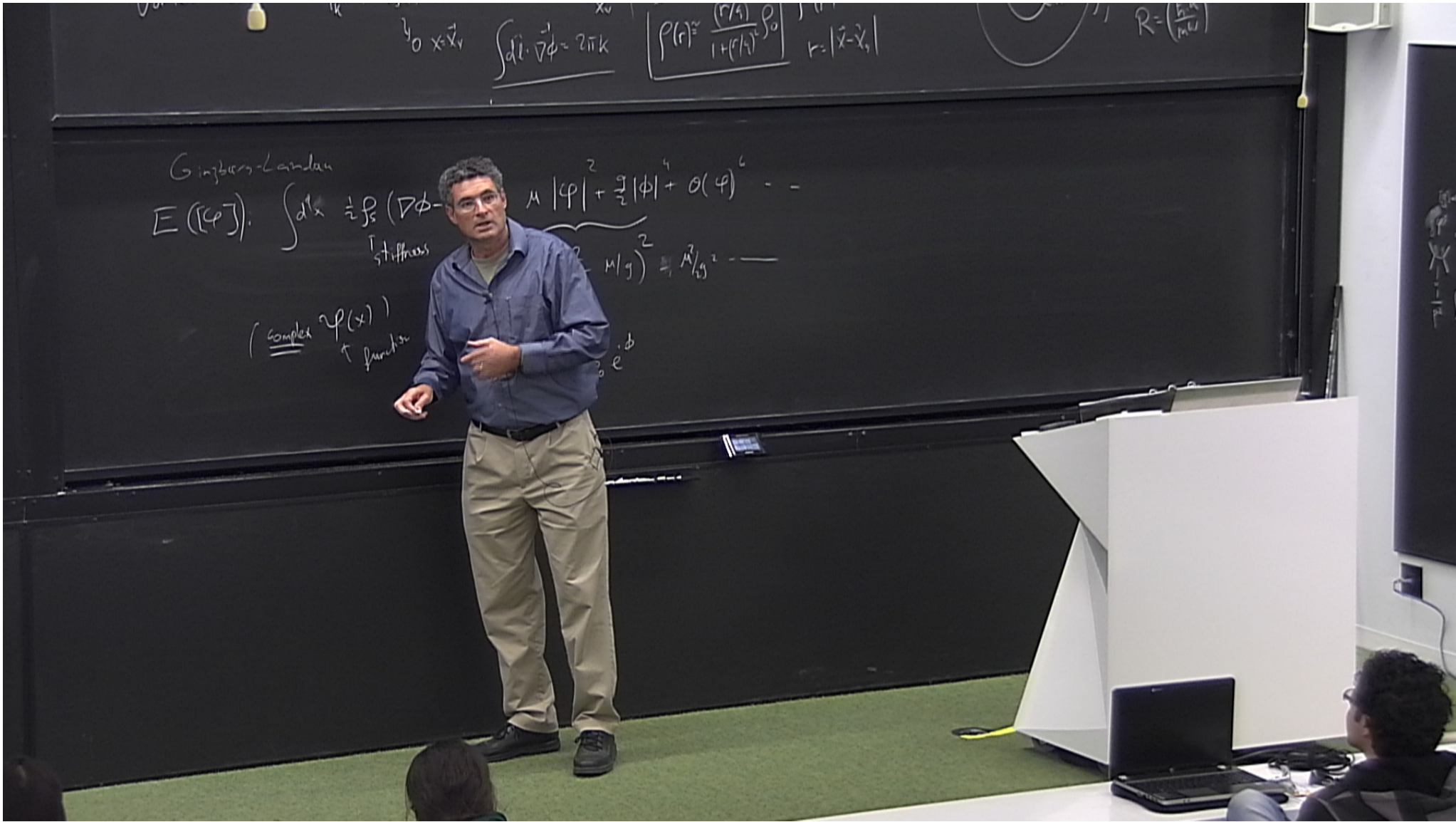
$$E(\varphi) = \int d^3x \left[\frac{1}{2} \rho_s (\nabla\varphi - \frac{q}{\hbar c} \vec{A})^2 + \underbrace{\mu |\varphi|^2 + \frac{g}{2} |\varphi|^4}_{\text{stiffness}} + o(\varphi)^6 \right]$$

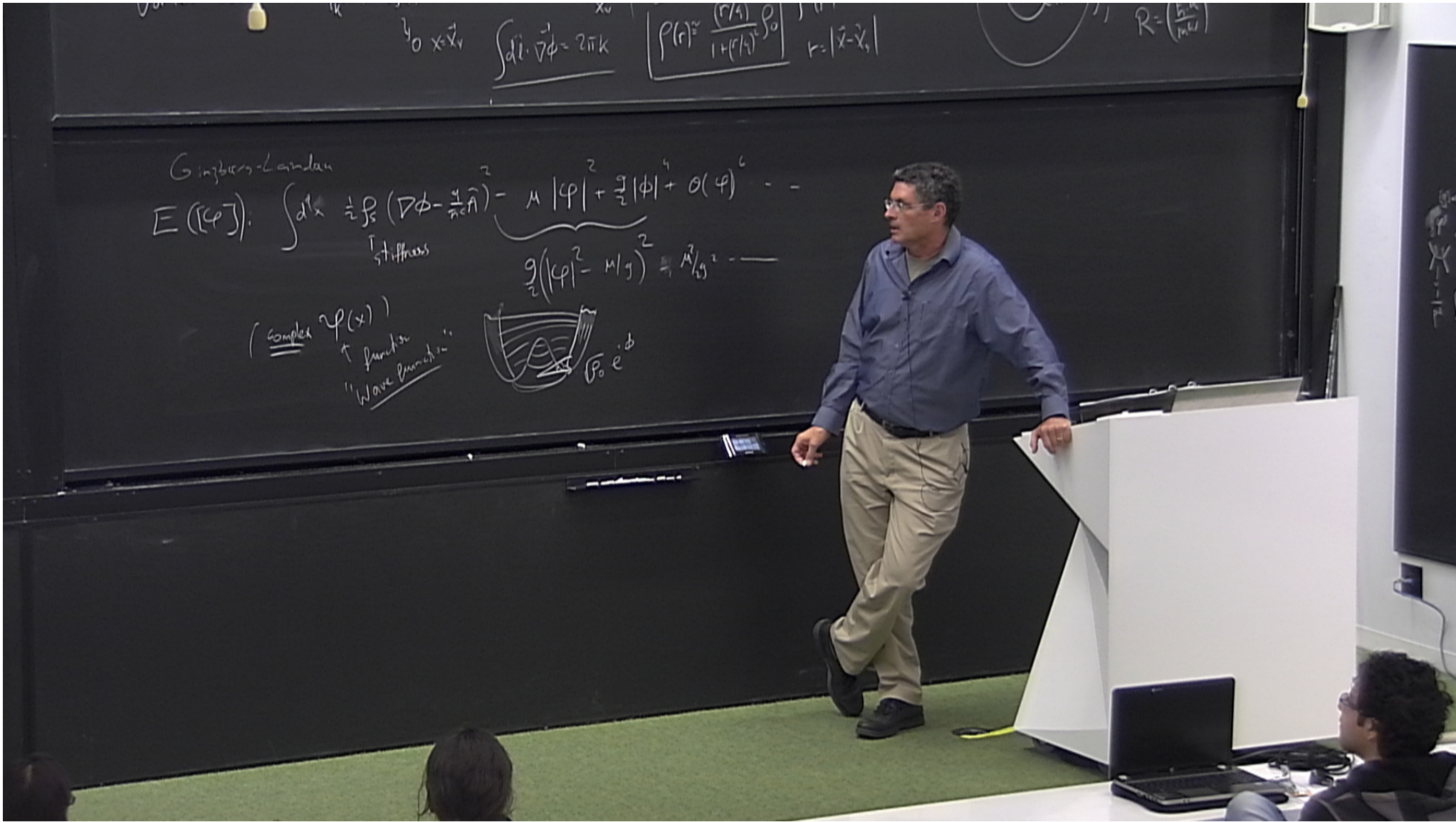
$$\frac{g}{2} (|\varphi|^2 - M/g)^2 \approx M^2/g^2$$


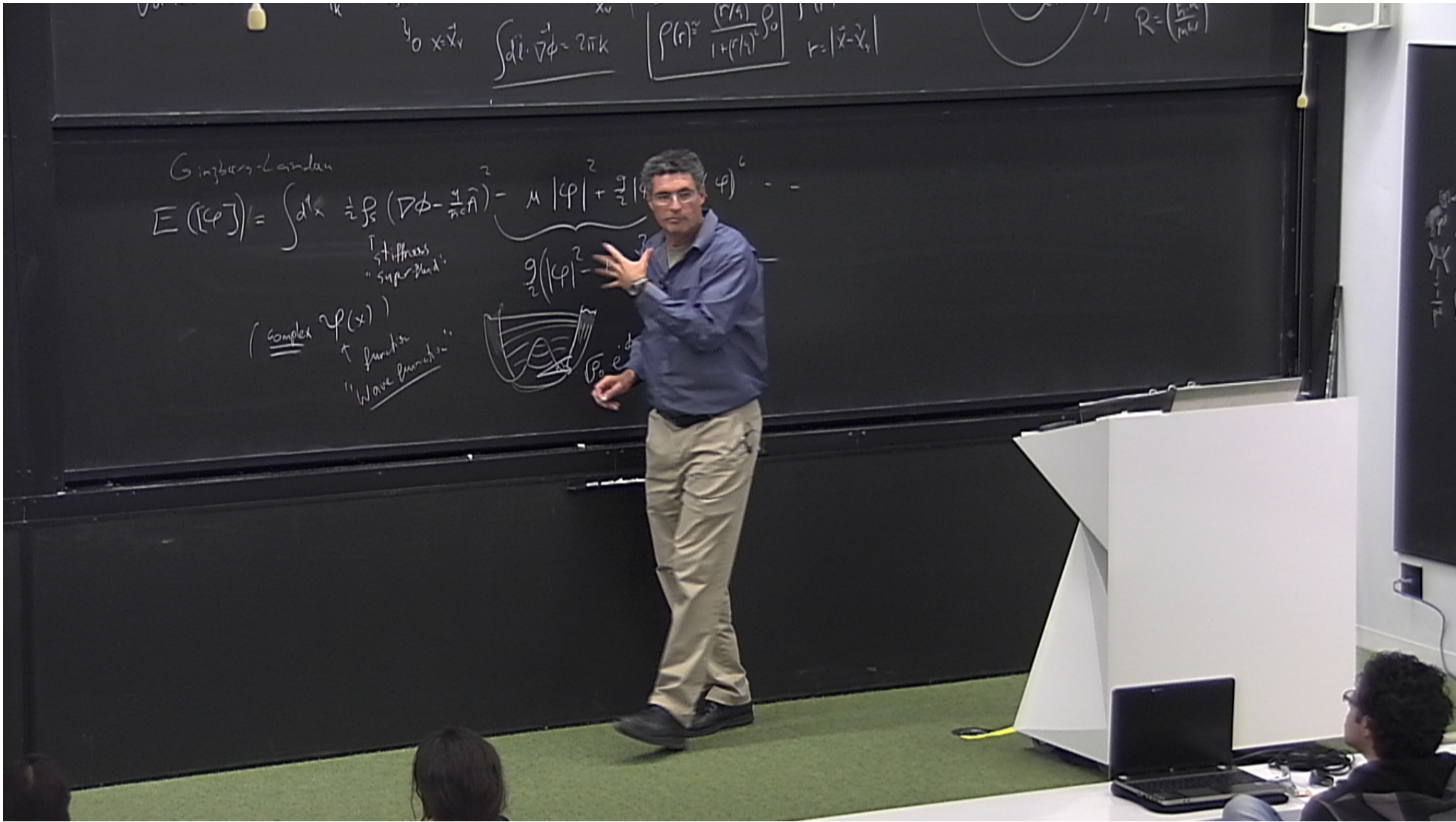
$$\oint_0 \vec{x} \cdot \vec{v} \quad \int d\vec{l} \cdot \vec{\nabla} \phi = 2\pi k$$

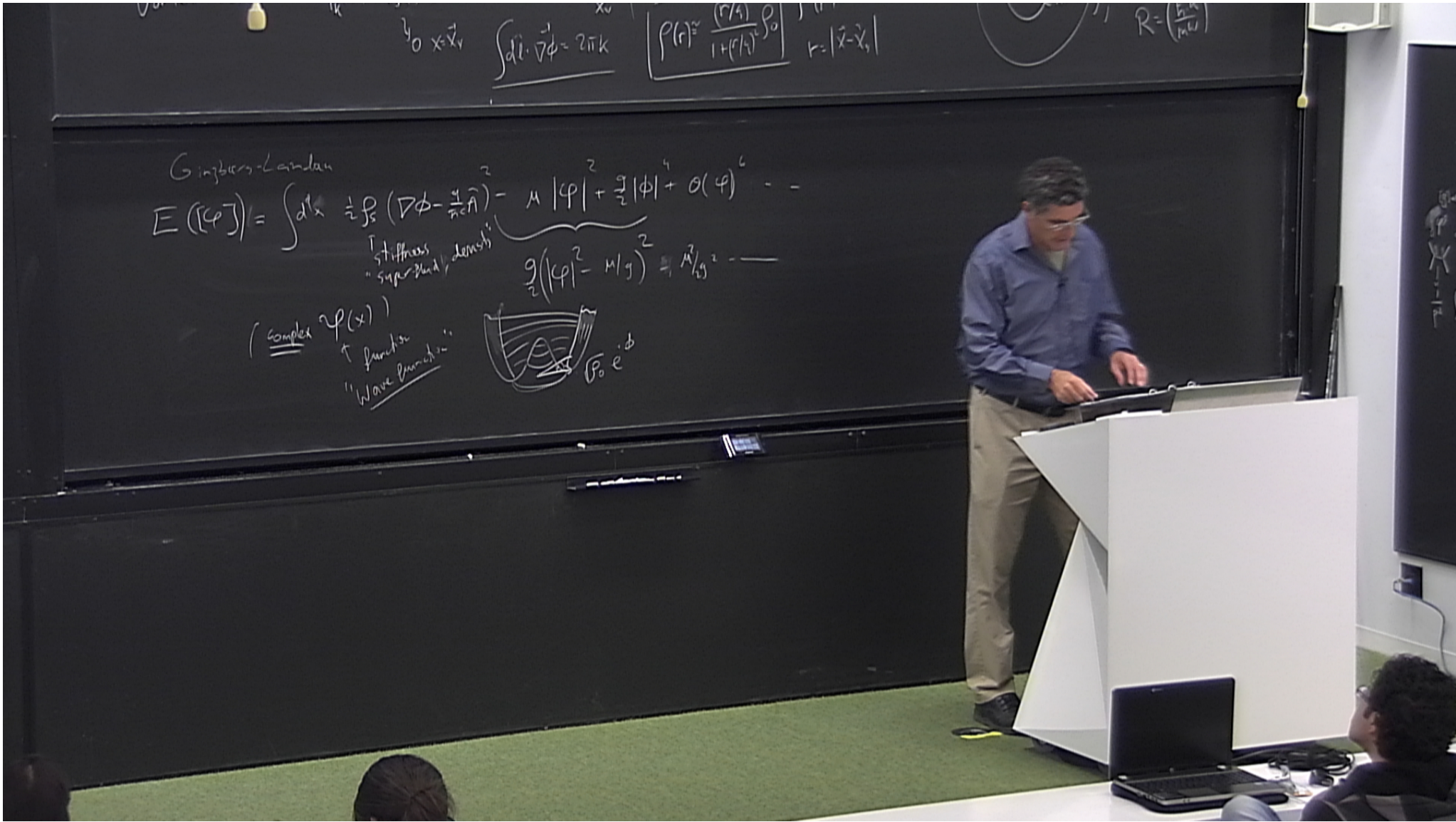
$$\rho(r) = \frac{\rho_0}{1 + (r/\lambda)^2} \quad r = |\vec{x} - \vec{x}_0| \quad R = \left(\frac{2\pi}{\mu} \right)$$

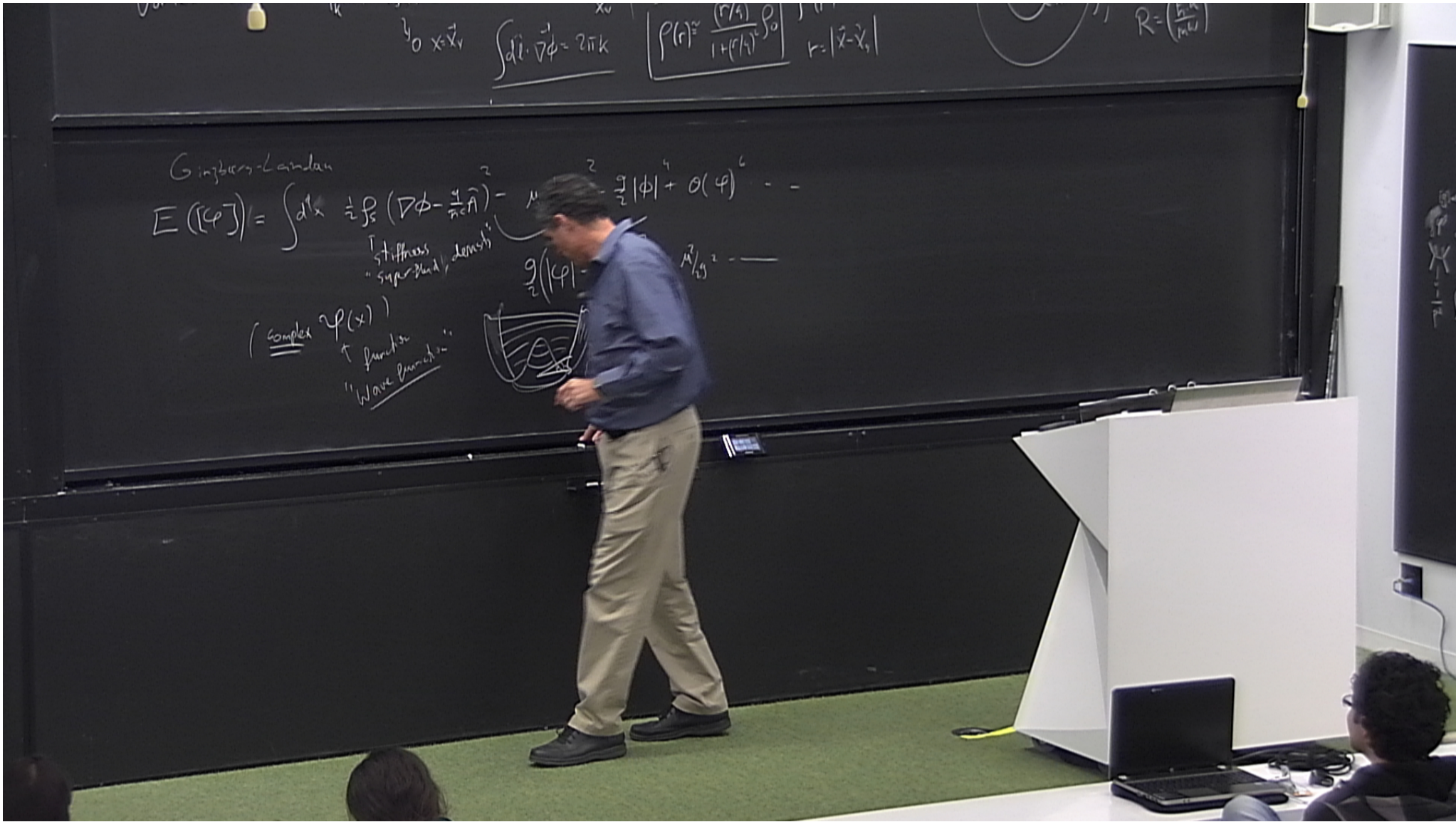


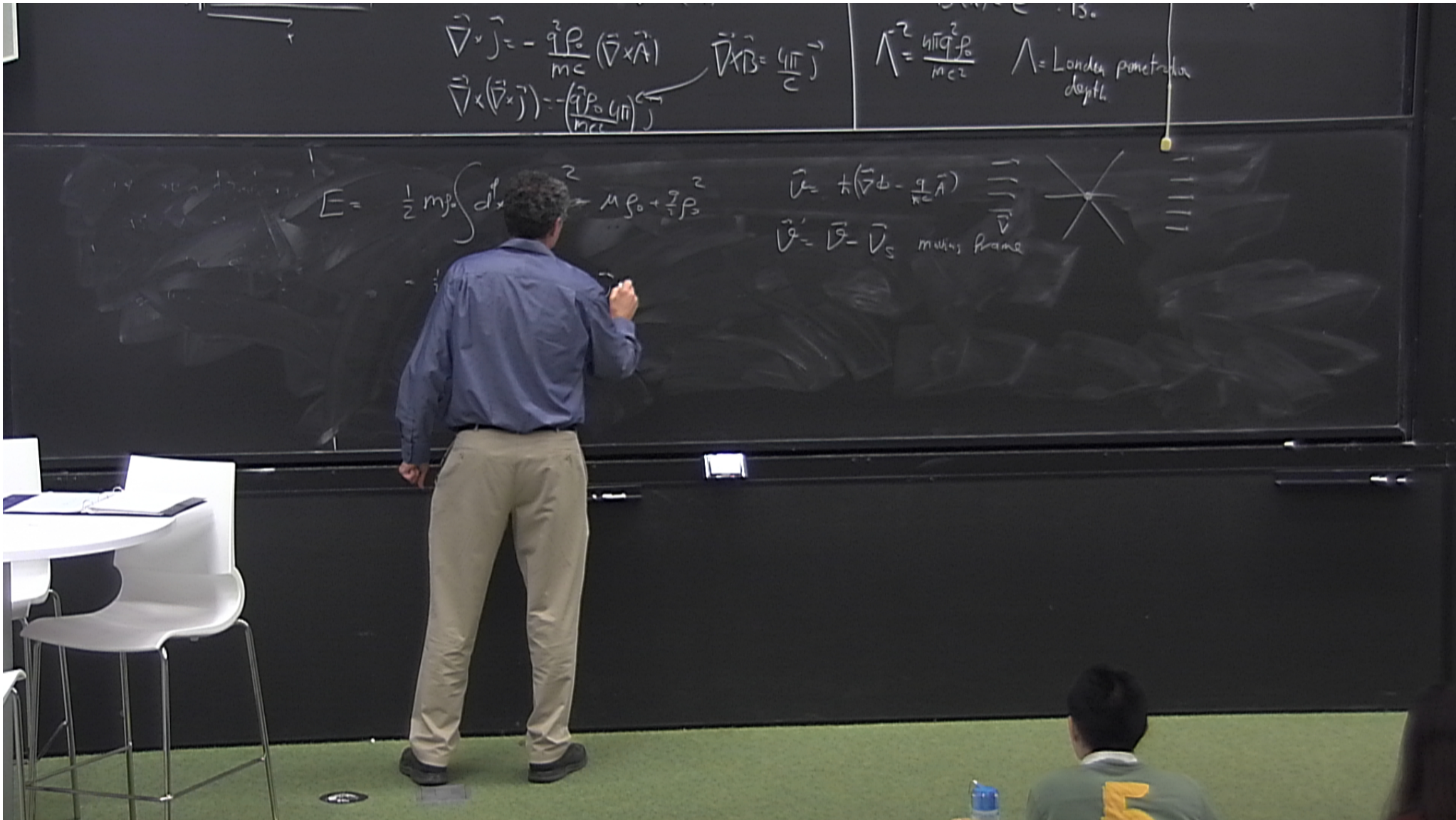






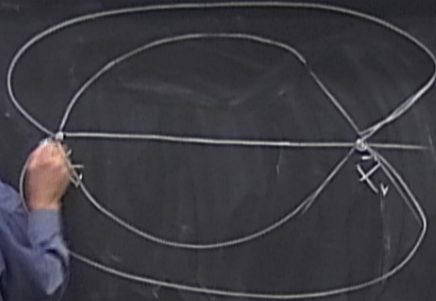






$$L_{\text{stat}} = \int dx m \beta (V \cdot V_s)$$

$$\Delta E = \int dx m \beta (\vec{\nabla} \phi \cdot \vec{V}_s)$$



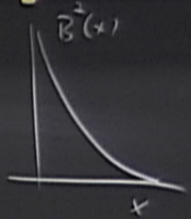
$$-\partial_x^2 j(x) = -\left(\frac{4\pi q^2 p_0}{mc}\right) j(x)$$

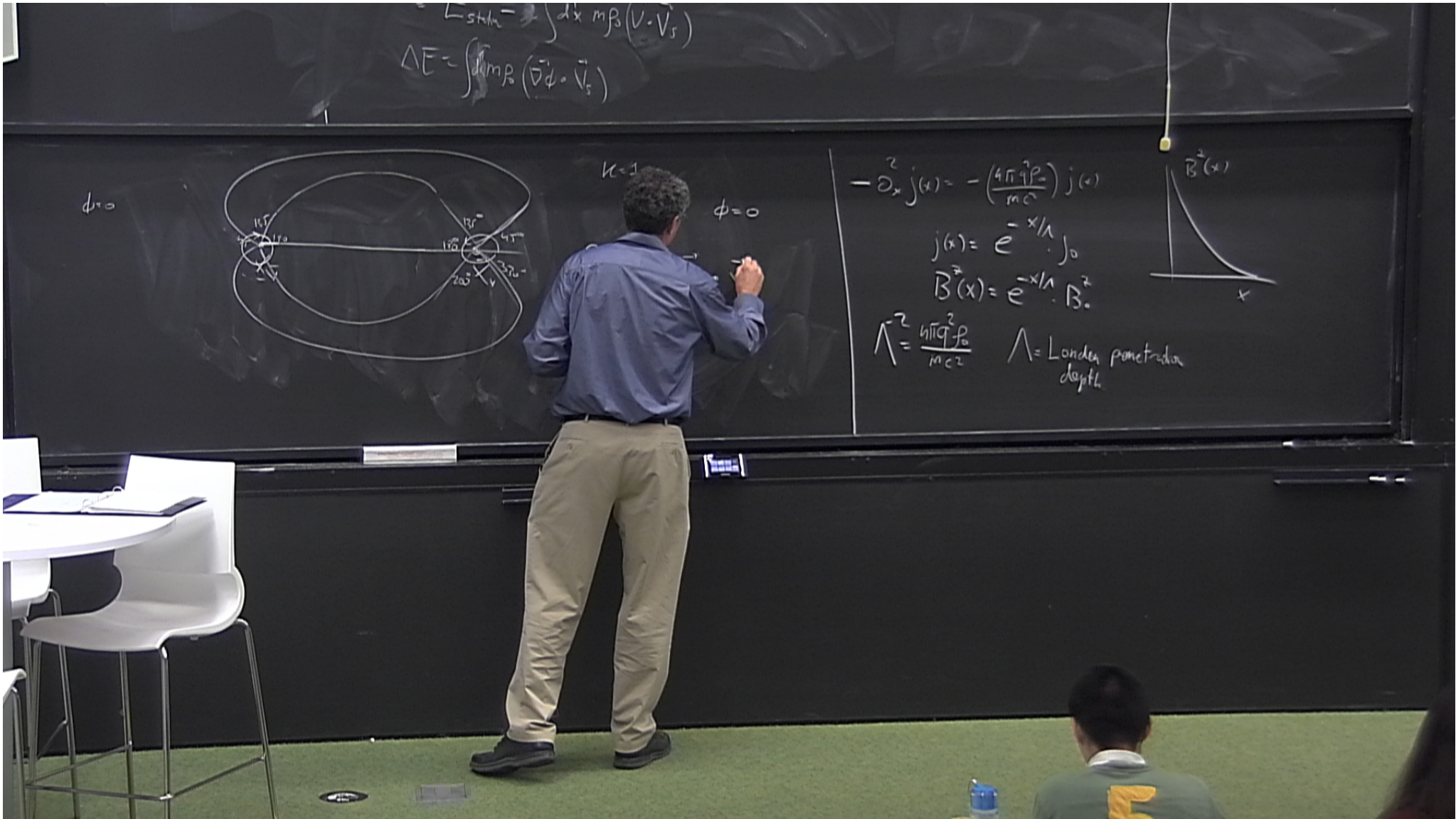
$$j(x) = e^{-x/\lambda} j_0$$

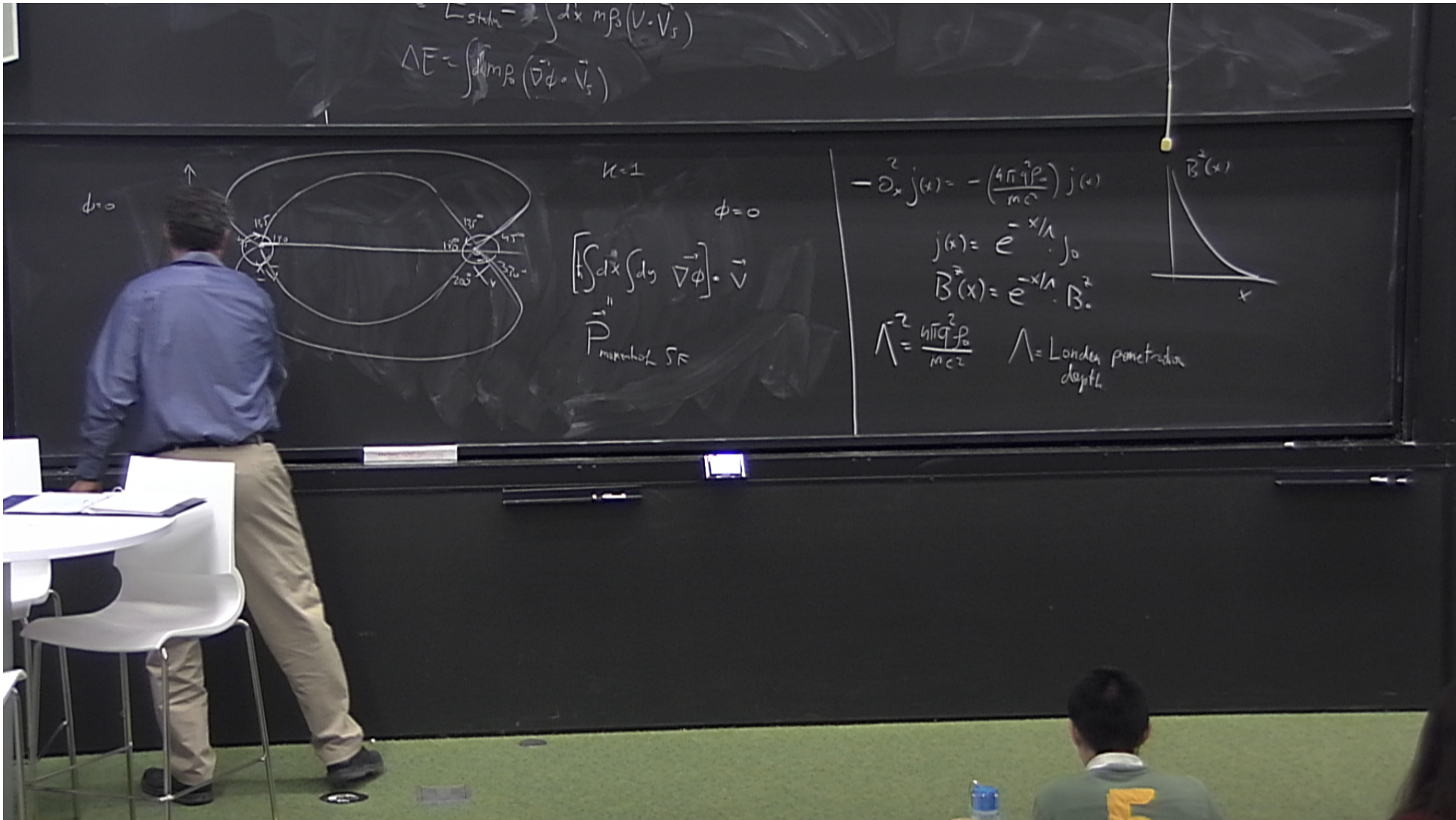
$$\vec{B}(x) = e^{-x/\lambda} \cdot \vec{B}_0$$

$$\lambda^2 = \frac{4\pi q^2 p_0}{mc^2}$$

$\lambda =$ London penetration depth







$$L_{stat} = \int dx m_p (V \cdot V_s)$$

$$\Delta E = \int dx m_p (\nabla \phi \cdot \vec{V}_s)$$



$\kappa=1$

$$\oint dx \int dy \nabla \phi \cdot \vec{V}$$

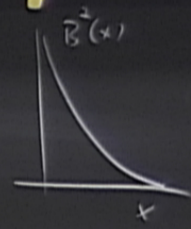
\vec{P}
microscopic SE

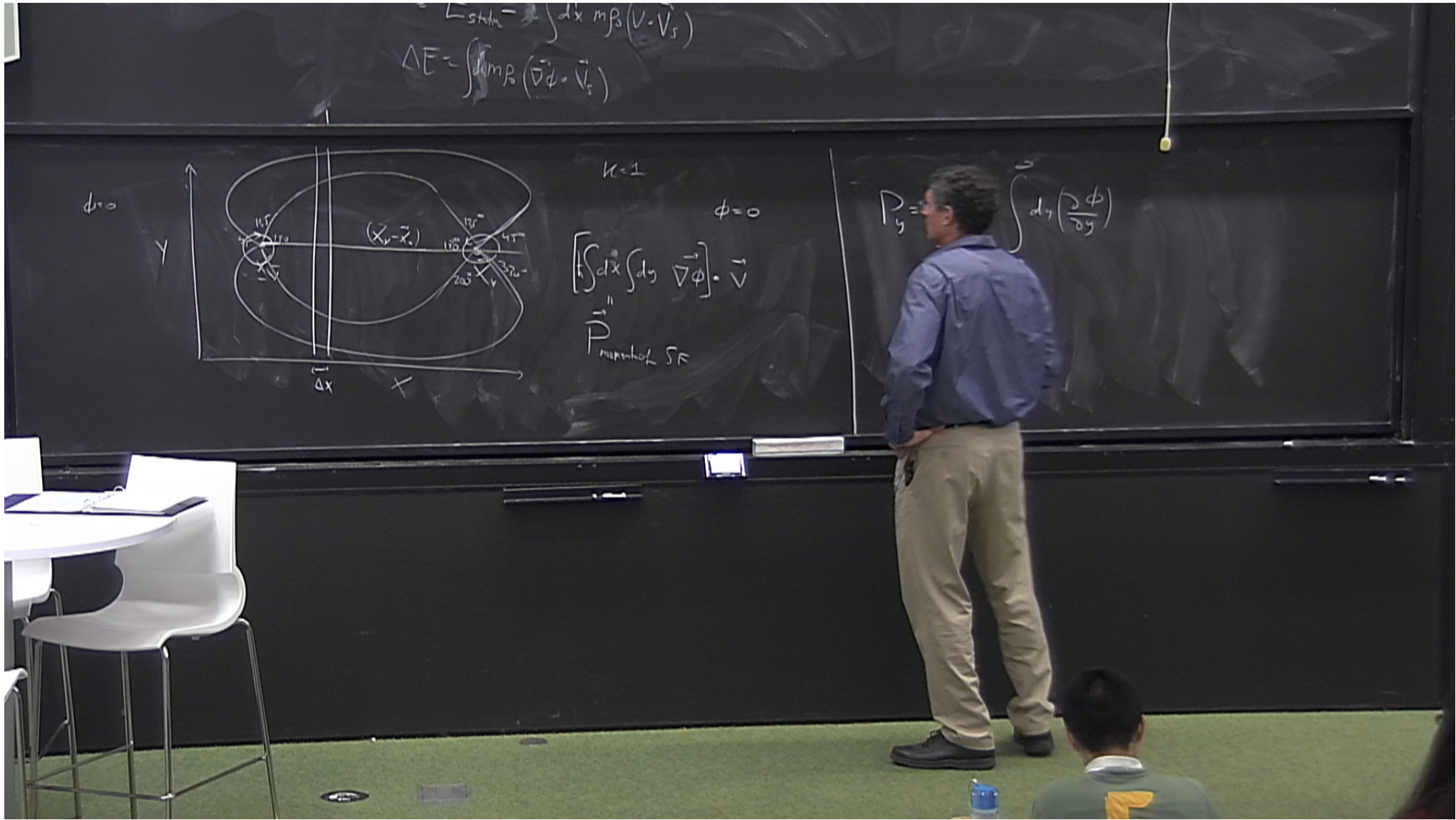
$$-\partial_x^2 j(x) = -\left(\frac{4\pi q^2 p_0}{mc^2}\right) j(x)$$

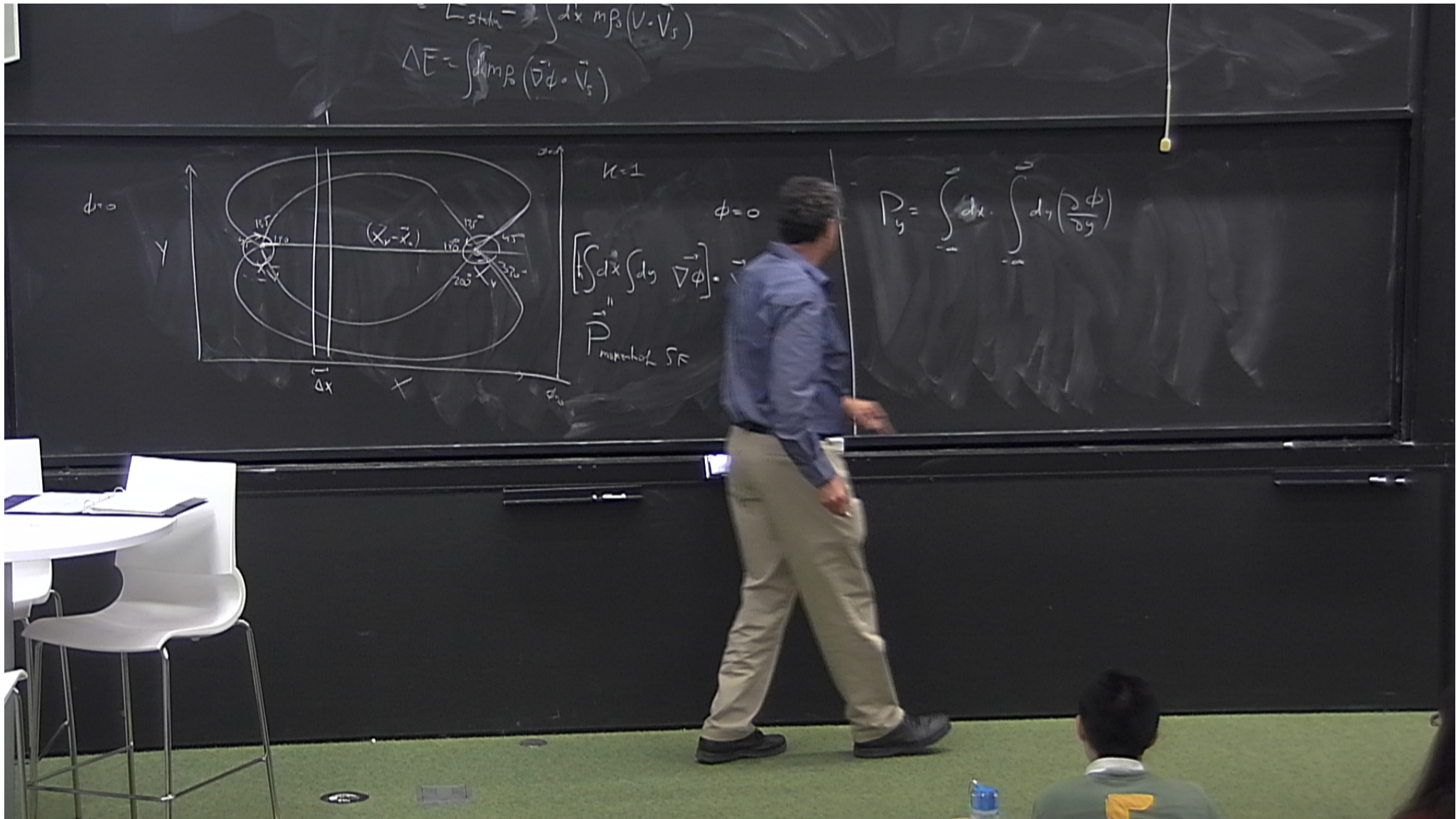
$$j(x) = e^{-x/\lambda} j_0$$

$$B^z(x) = e^{-x/\lambda} B^z_0$$

$$\Lambda^2 = \frac{4\pi q^2 p_0}{mc^2} \quad \Lambda = \text{London penetration depth}$$

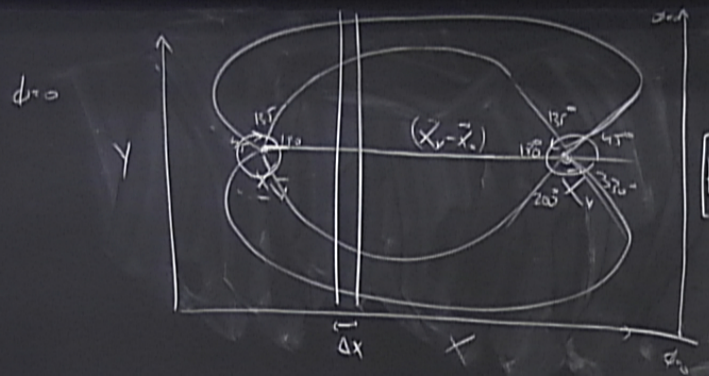






$$L_{stat} = \int dx m\beta (V - V_s)$$

$$\Delta E = \int dx m\beta (\vec{\nabla}\phi \cdot \vec{V}_s)$$



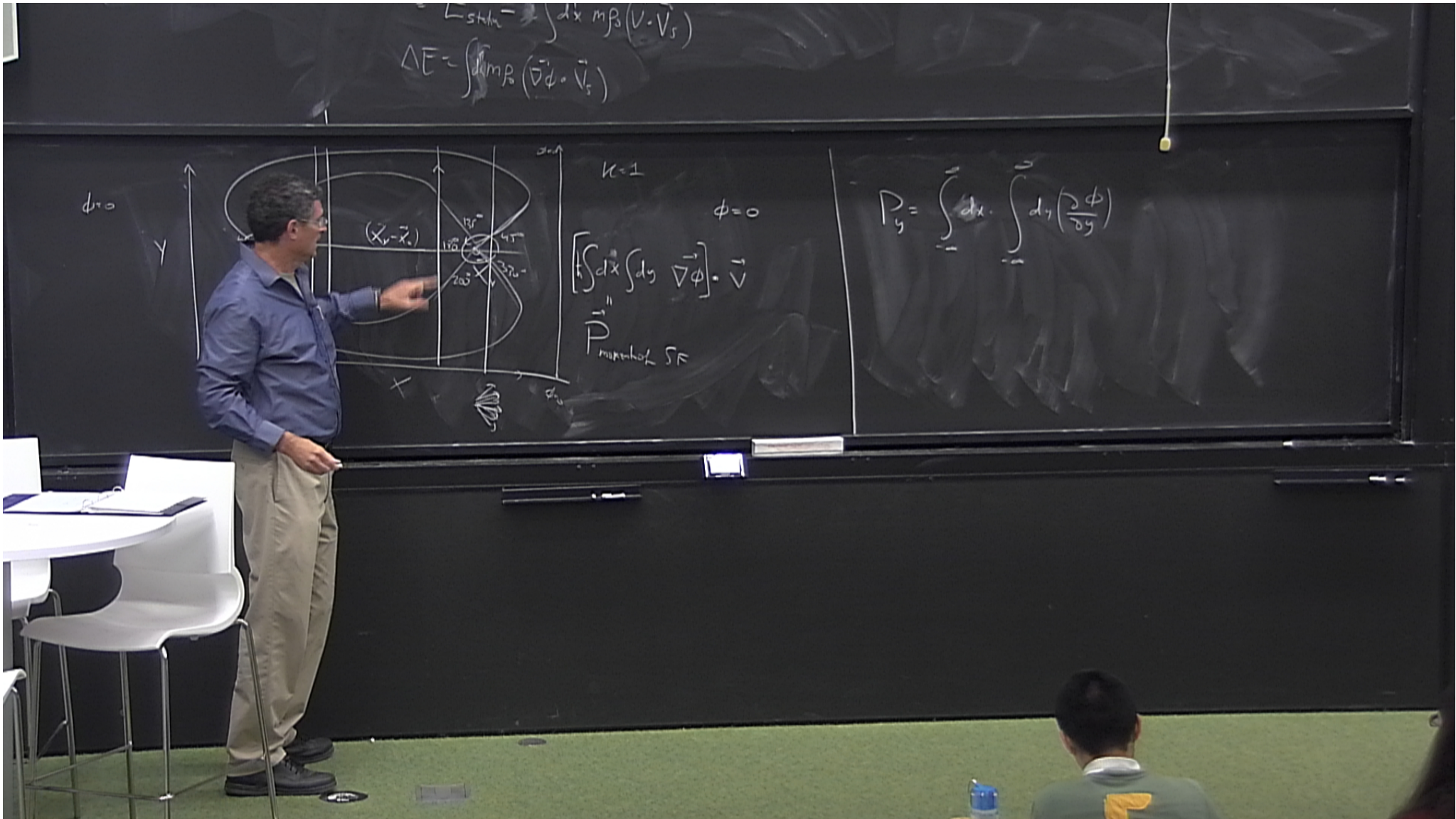
$$k=1$$

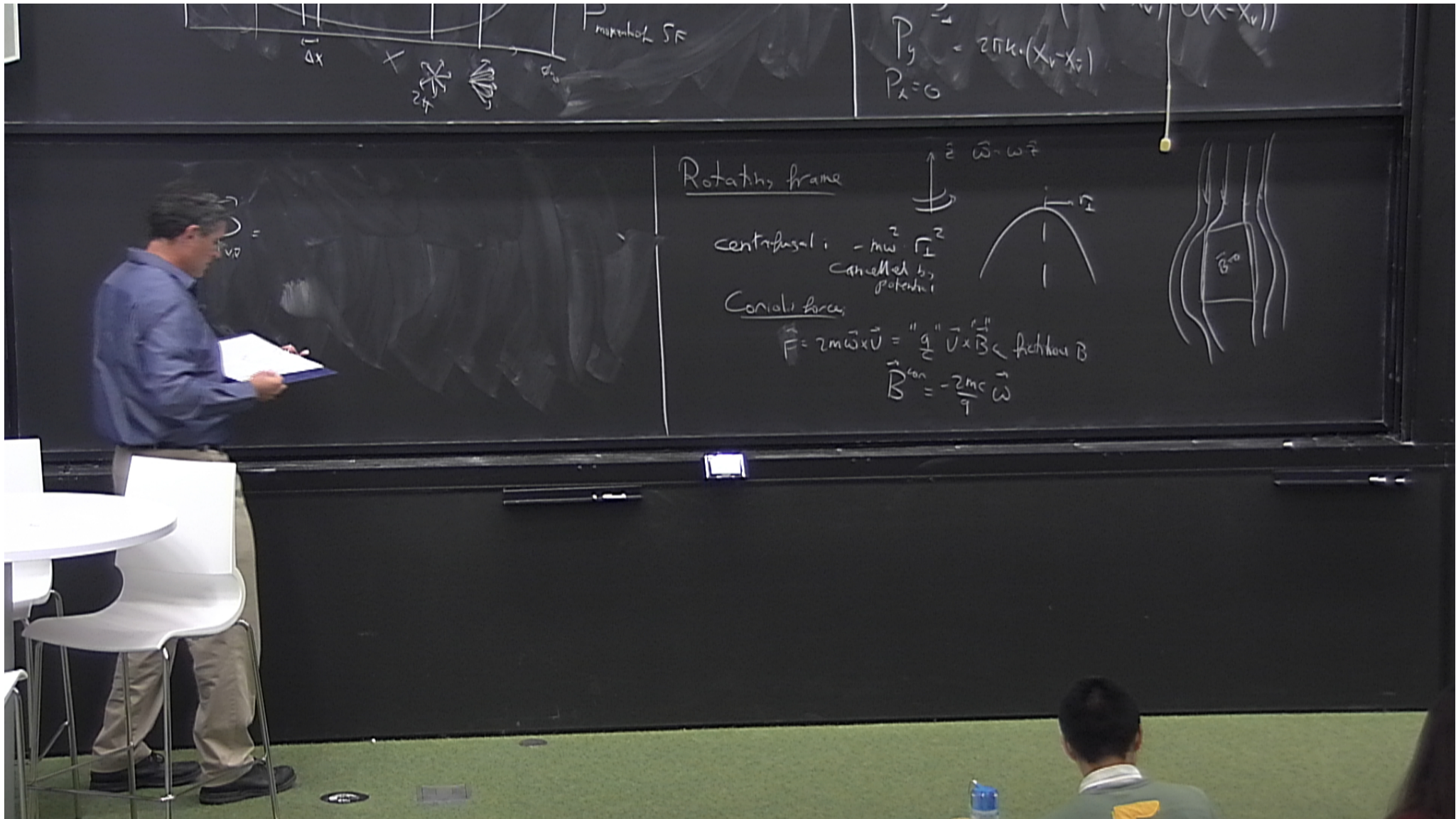
$$\phi=0$$

$$\left[\int dx \int dy \vec{\nabla}\phi \right] \cdot \vec{v}$$

$$\vec{P}_{\text{mikroskop SE}}$$

$$P_y = \int dx \int dy \left(\frac{\partial \phi}{\partial y} \right)$$





$$P_y = 2\pi k \cdot (x_v - x_v)$$

$$P_x = 0$$

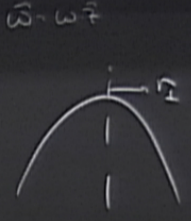
Rotating frame

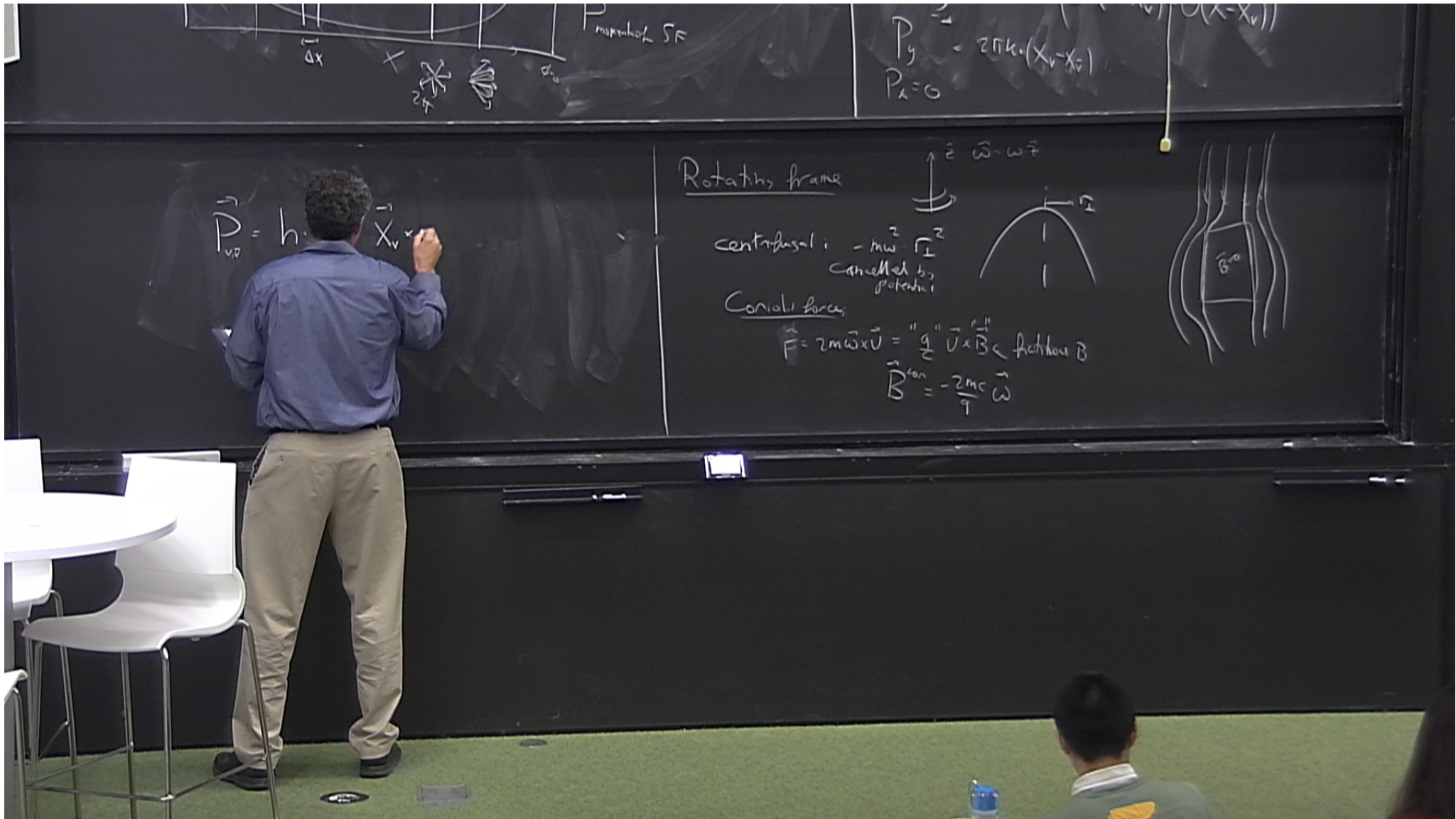
centrifugal: $-m\omega^2 r$
 cancelled by potential

Coriolis force:

$$\vec{F} = 2m\vec{\omega} \times \vec{v} = \frac{q}{c} \vec{v} \times \vec{B} \leftarrow \text{fictitious } \vec{B}$$

$$\vec{B}^{\text{con}} = -\frac{2mc}{q} \vec{\omega}$$





$$\vec{p}_{\nu} = \hbar \vec{k}_{\nu}$$



$$P_y = 2\pi k \cdot (x_{\nu} - x_{\nu'})$$

$$P_x = 0$$

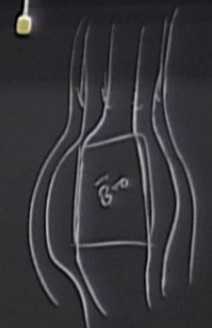
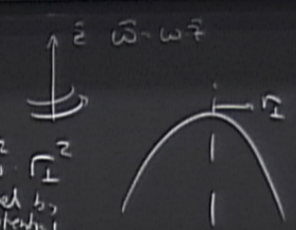
Rotating frame

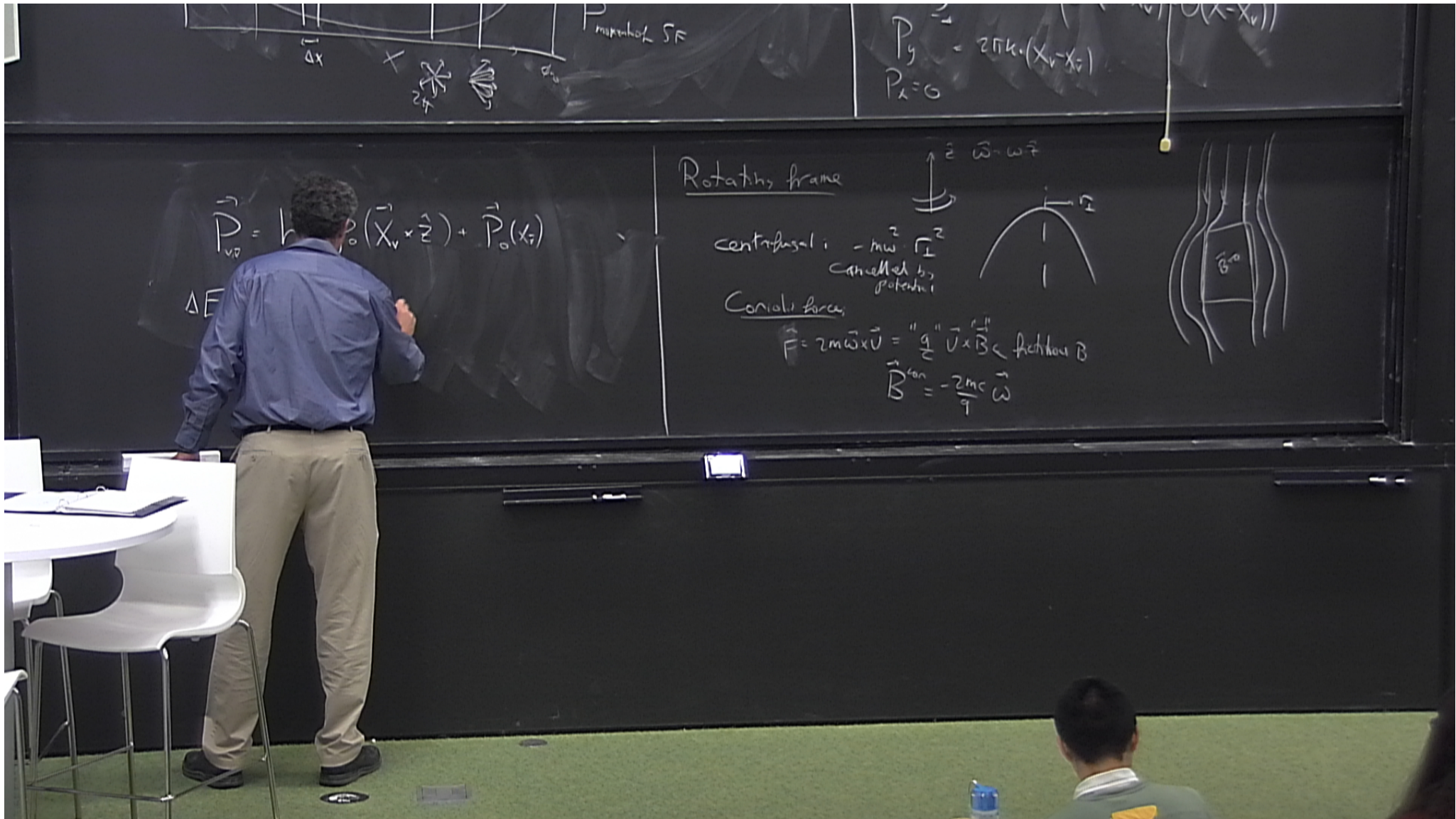
centrifugal: $-m\omega^2 r$
 cancelled by potential

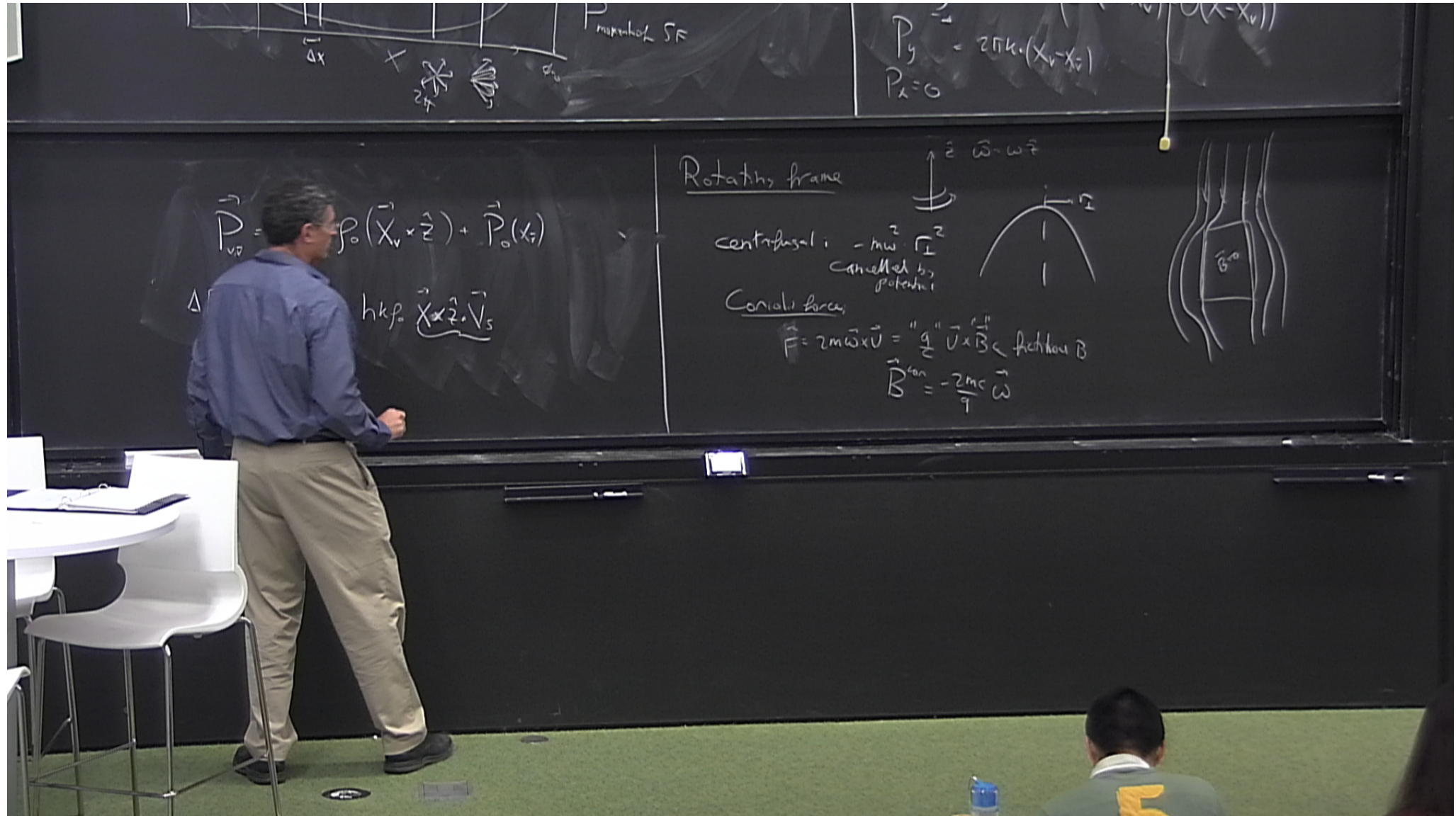
Coriolis force:

$$\vec{F} = 2m\vec{\omega} \times \vec{v} = \frac{q}{c} \vec{v} \times \vec{B} \leftarrow \text{effective } \vec{B}$$

$$\vec{B}^{\text{con}} = -\frac{2mc}{q} \vec{\omega}$$







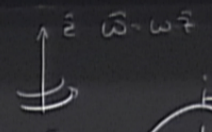
$$P_y = 2\pi k \cdot (x_v - x_i)$$

$$P_x = 0$$

$$\vec{P}_v = p_0(\vec{x}_v \times \vec{z}) + \vec{P}_0(x_i)$$

$$\Delta \vec{p} = \hbar k p_0 \vec{x} \times \vec{z} \cdot \vec{V}_s$$

Rotating frame



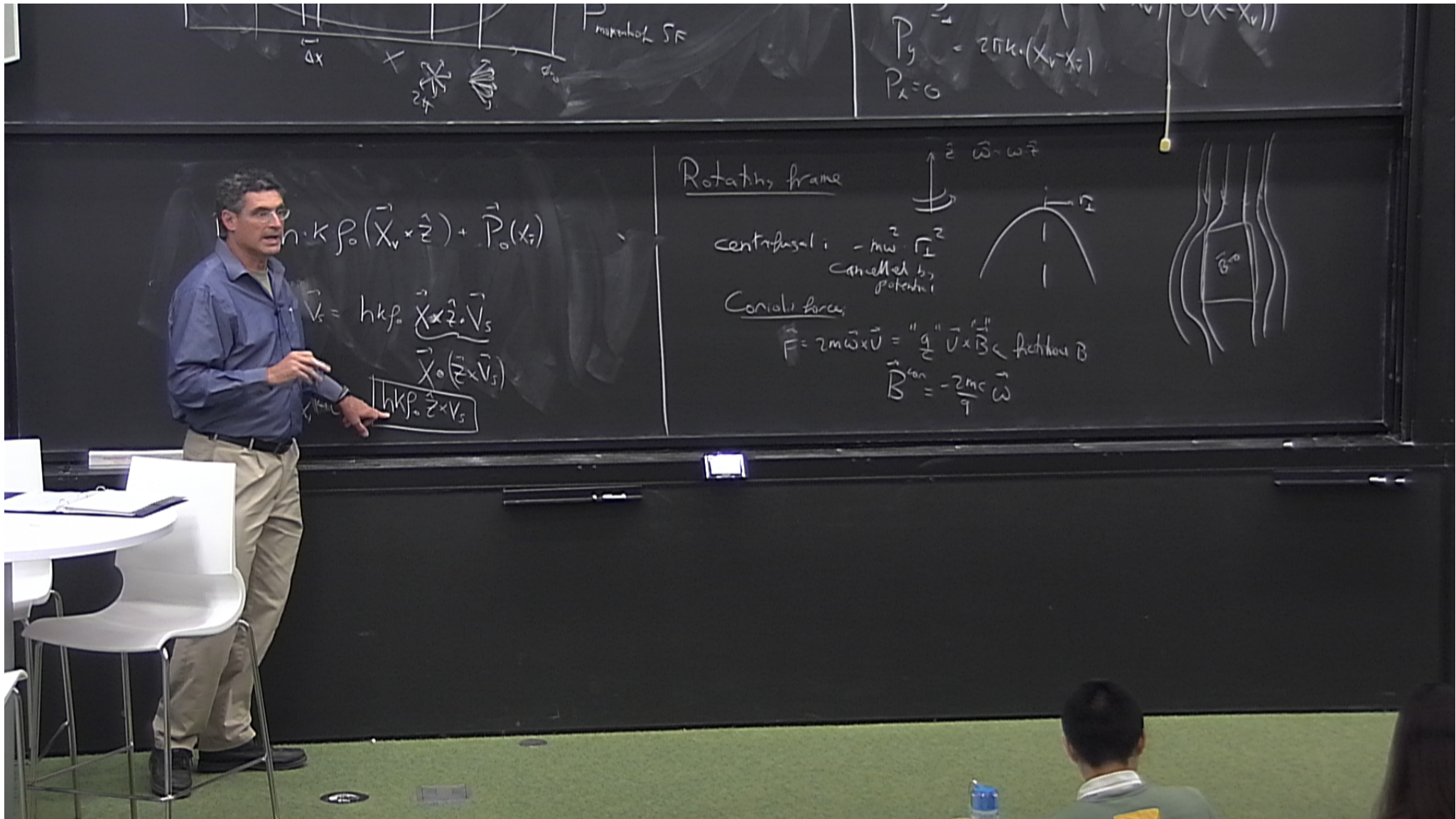
centrifugal: $-m\omega^2 r^2$
 cancelled by potential



Coriolis force:

$$\vec{F} = 2m\vec{\omega} \times \vec{v} = \frac{q}{c} \vec{v} \times \vec{B}_{con} \text{ fictitious } \vec{B}$$

$$\vec{B}_{con} = -\frac{2mc}{q} \vec{\omega}$$



$$P_y = \dots = 2\pi k \cdot (X_v - X_i)$$

$$P_x = 0$$

A professor in a blue shirt and khaki pants stands on the left side of the chalkboard, gesturing towards the equations.

$$h \cdot k p_0 (\vec{X}_v + \vec{z}) + \vec{P}_0(x_i)$$

$$\vec{V}_s = h k p_0 \vec{X} \times \vec{z} \cdot \vec{V}_s$$

$$\vec{X}_0 (\vec{z} \times \vec{V}_s)$$

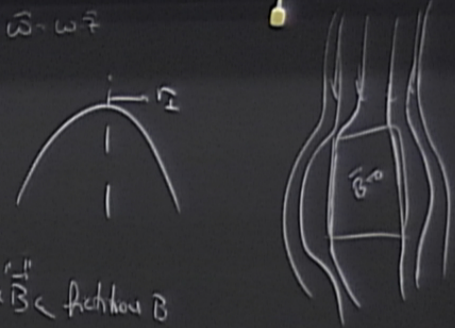
$$h k p_0 \vec{z} \times \vec{V}_s$$

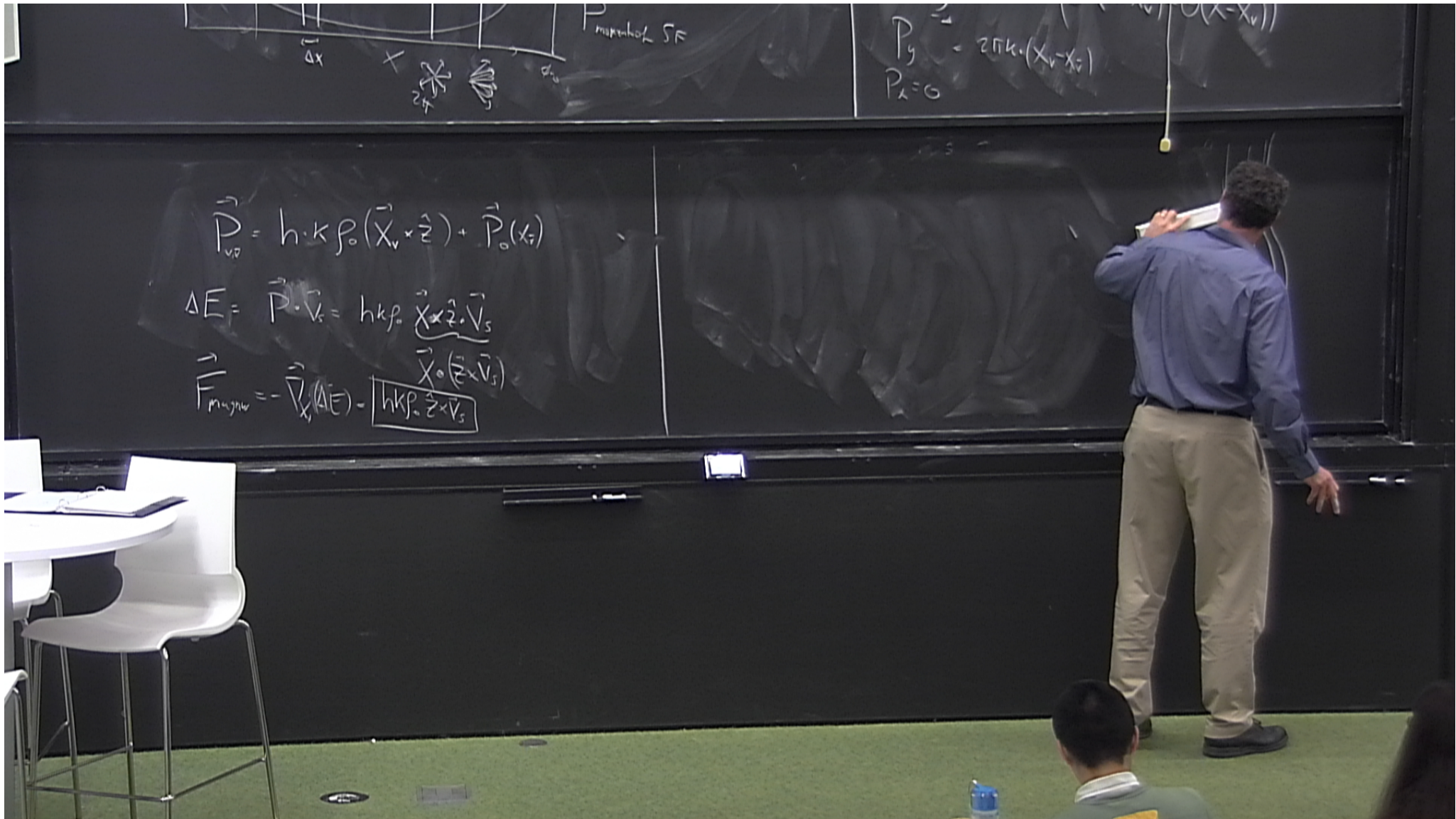
Rotating frame

$\vec{\omega} = \omega \vec{z}$

centrifugal: $-m\omega^2 r^2$
 cancelled by potential

Coriolis force:
 $\vec{F} = 2m\vec{\omega} \times \vec{v} = \frac{q}{c} \vec{v} \times \vec{B}$ fictitious B
 $\vec{B}^{con} = -\frac{2mc}{q} \vec{\omega}$



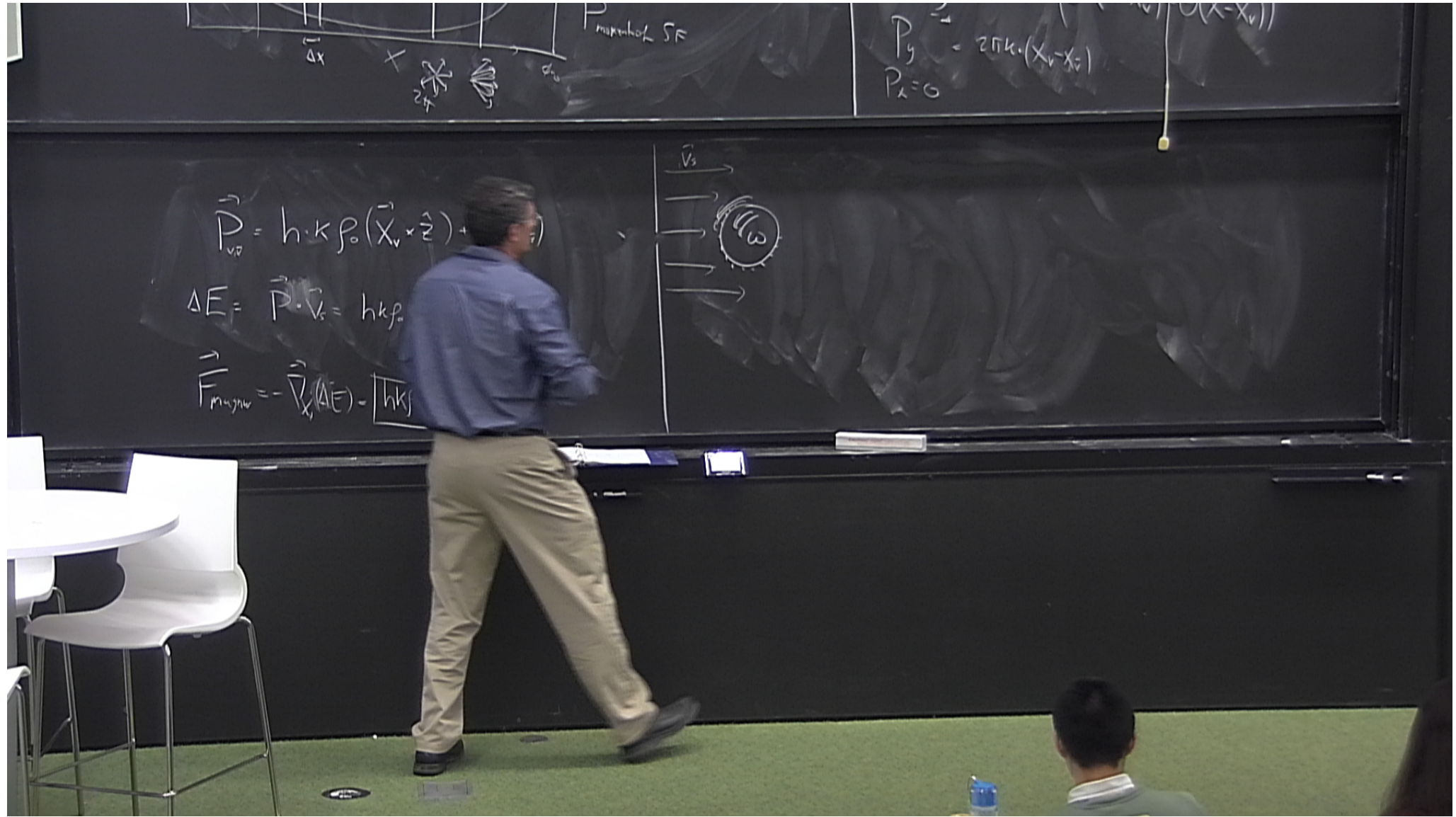


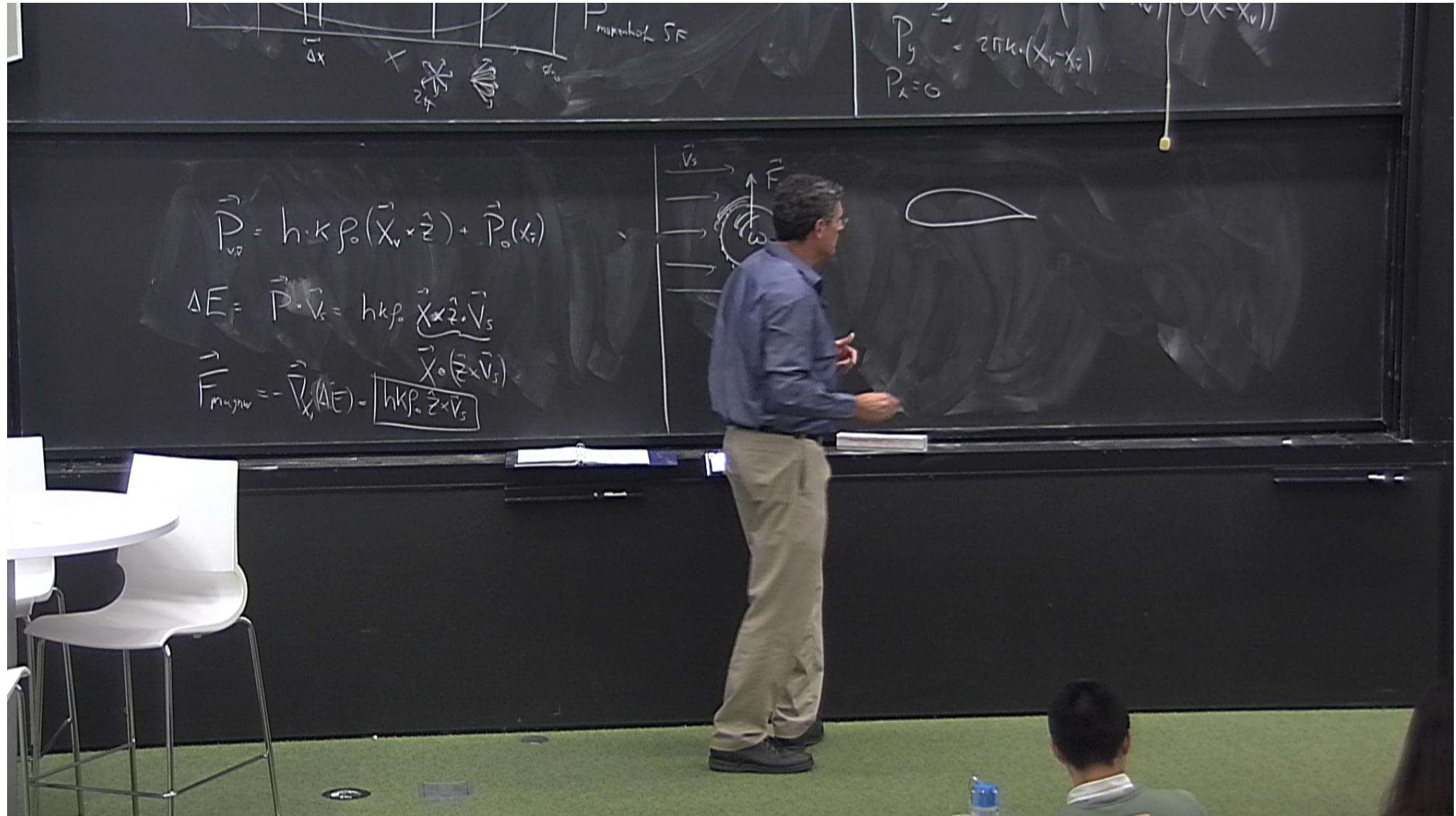
$\vec{p} = h \cdot \vec{k}$
 $P_y = 2\pi k \cdot (x_v - x_i)$
 $P_x = 0$

$$\vec{P}_v = h \cdot k p_0 (\vec{X}_v \times \vec{z}) + \vec{P}_0(x_i)$$

$$\Delta E = \vec{P} \cdot \vec{v}_s = h k p_0 \vec{X} \times \vec{z} \cdot \vec{v}_s$$

$$\vec{F}_{\text{magnet}} = -\vec{\nabla}_X (\Delta E) = \boxed{h k p_0 \vec{z} \times \vec{v}_s}$$

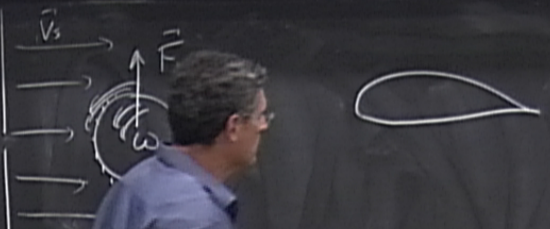




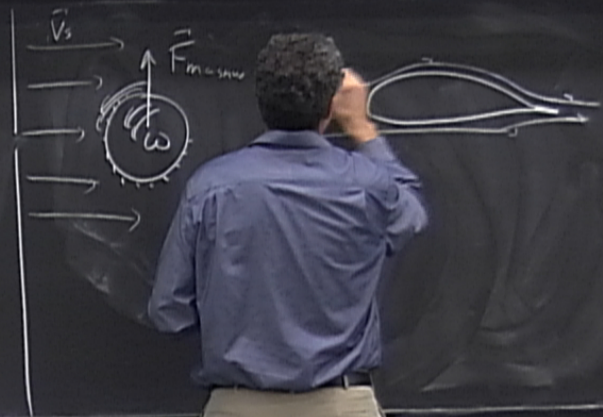
$$\vec{P}_{\nu 0} = \hbar \cdot k \rho_0 (\vec{X}_\nu \times \vec{z}) + \vec{P}_0(x_\nu)$$

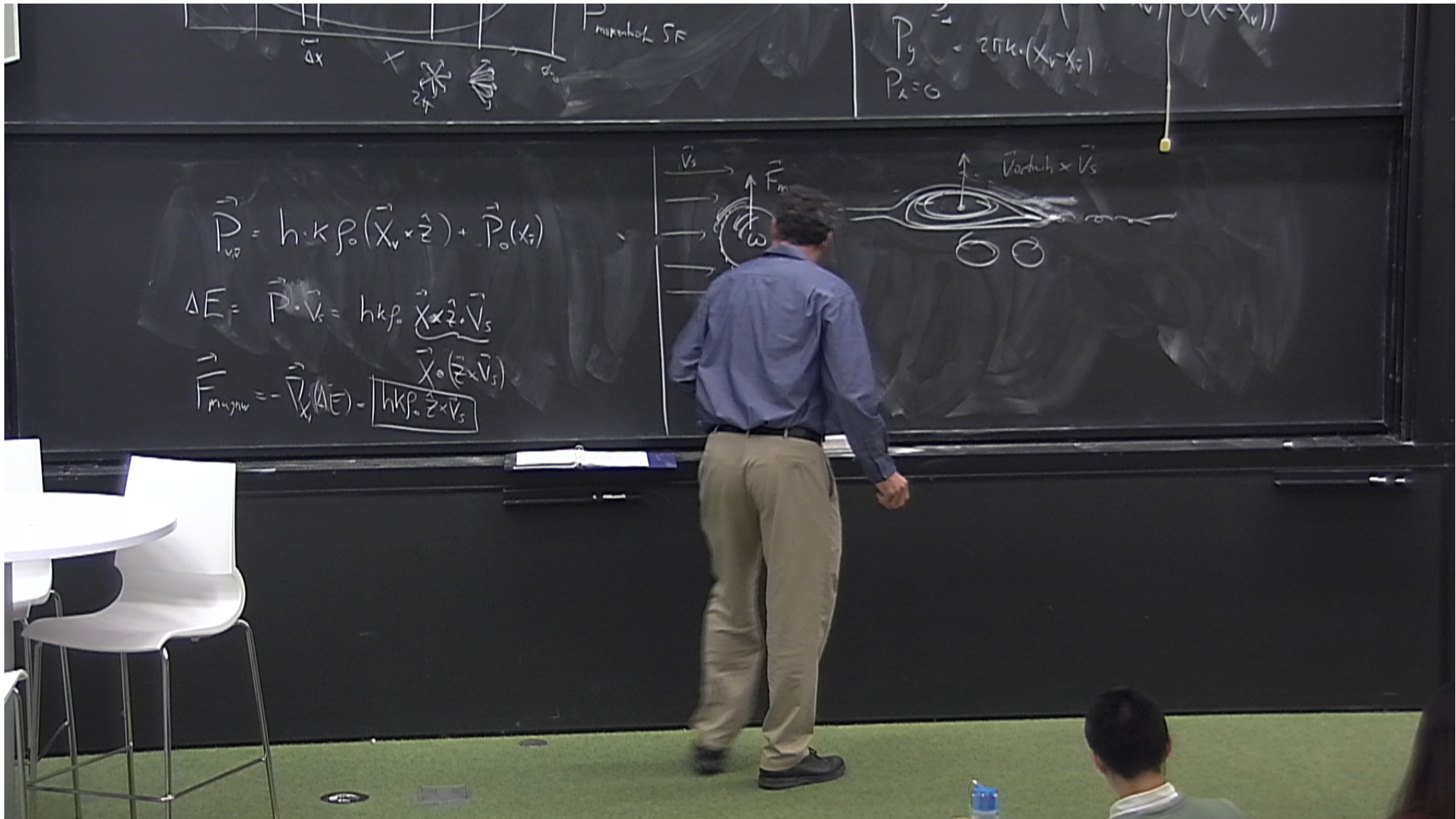
$$\Delta E = \vec{P} \cdot \vec{v}_s = \hbar k \rho_0 \vec{X} \times \vec{z} \cdot \vec{v}_s$$

$$\vec{F}_{\text{magn}} = -\vec{\nabla}_X(\Delta E) = \boxed{\hbar k \rho_0 \vec{z} \times \vec{v}_s}$$



$\vec{P} = h \cdot k \rho_0 (\vec{X}_v \times \vec{z}) + \vec{P}_0(x_i)$
 $\Delta E = \vec{P} \cdot \vec{V}_s = h k \rho_0 \vec{X} \times \vec{z} \cdot \vec{V}_s$
 $\vec{F}_{\text{Magnus}} = -\vec{\nabla}_X (\Delta E) = \boxed{h k \rho_0 \vec{z} \times \vec{V}_s}$



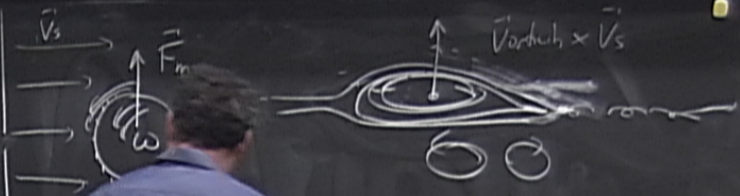


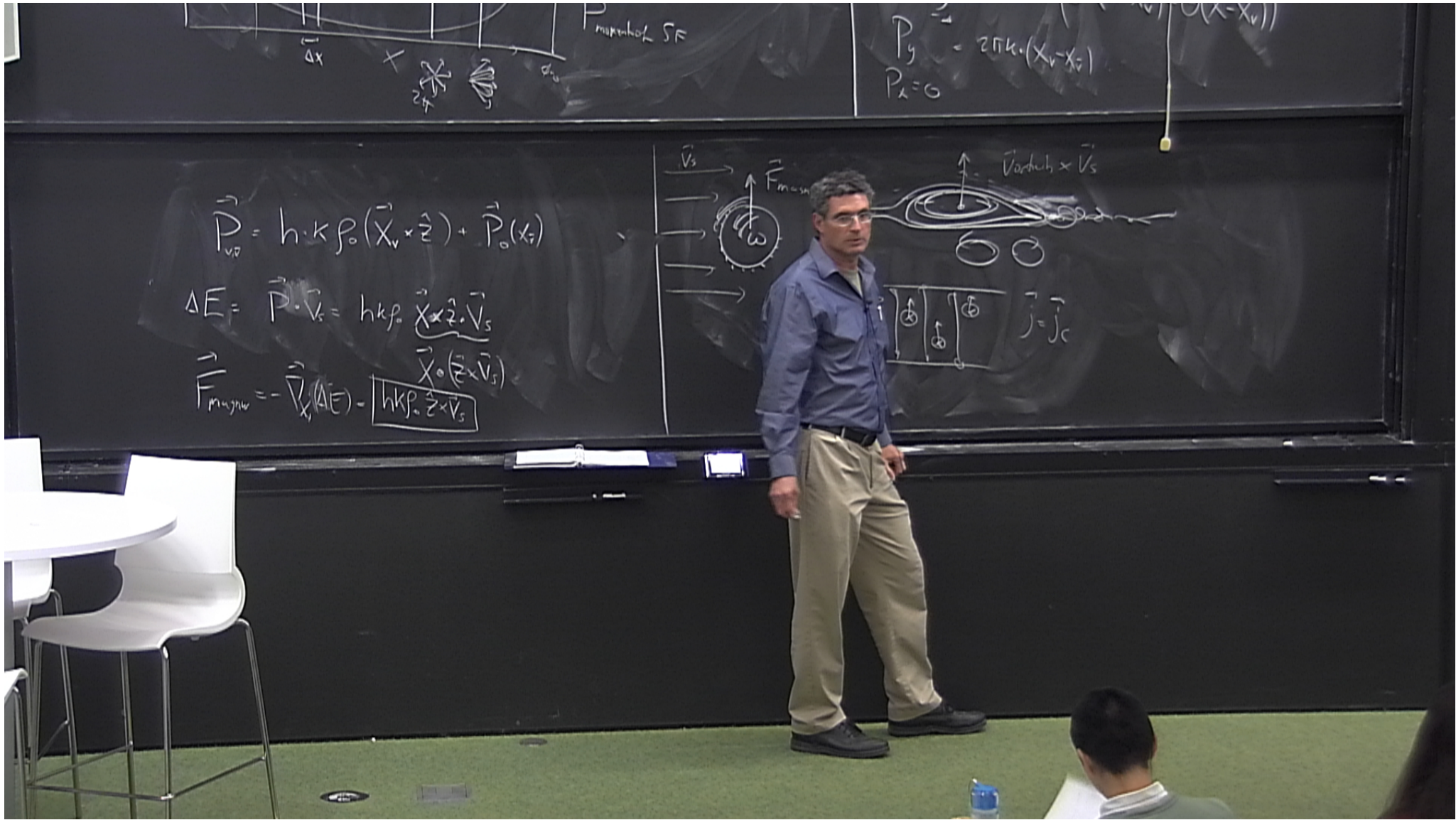
$P_{\text{magnetohol}} \text{ SE}$
 $P_y = 2\pi k \cdot (X_v - X_i)$
 $P_x = 0$

$$\vec{P}_{v0} = \hbar \cdot k \rho_0 (\vec{X}_v \times \vec{z}) + \vec{P}_0(x_i)$$

$$\Delta E = \vec{P} \cdot \vec{v}_s = \hbar k \rho_0 \underbrace{\vec{X} \times \vec{z} \cdot \vec{v}_s}_{\vec{X}_0 (\vec{z} \times \vec{v}_s)}$$

$$\vec{F}_{\text{magnetohol}} = -\vec{\nabla}_X (\Delta E) = \boxed{\hbar k \rho_0 \vec{z} \times \vec{v}_s}$$





$$\vec{P} = \hbar \cdot k \rho_0 (\vec{X}_v \times \vec{z}) + \vec{P}_0(x_i)$$

$$\Delta E = \vec{P} \cdot \vec{V}_s = \hbar k \rho_0 \vec{X} \times \vec{z} \cdot \vec{V}_s$$

$$\vec{F}_{\text{magnet}} = -\vec{\nabla}_X (\Delta E) = \hbar k \rho_0 \vec{z} \times \vec{V}_s$$

