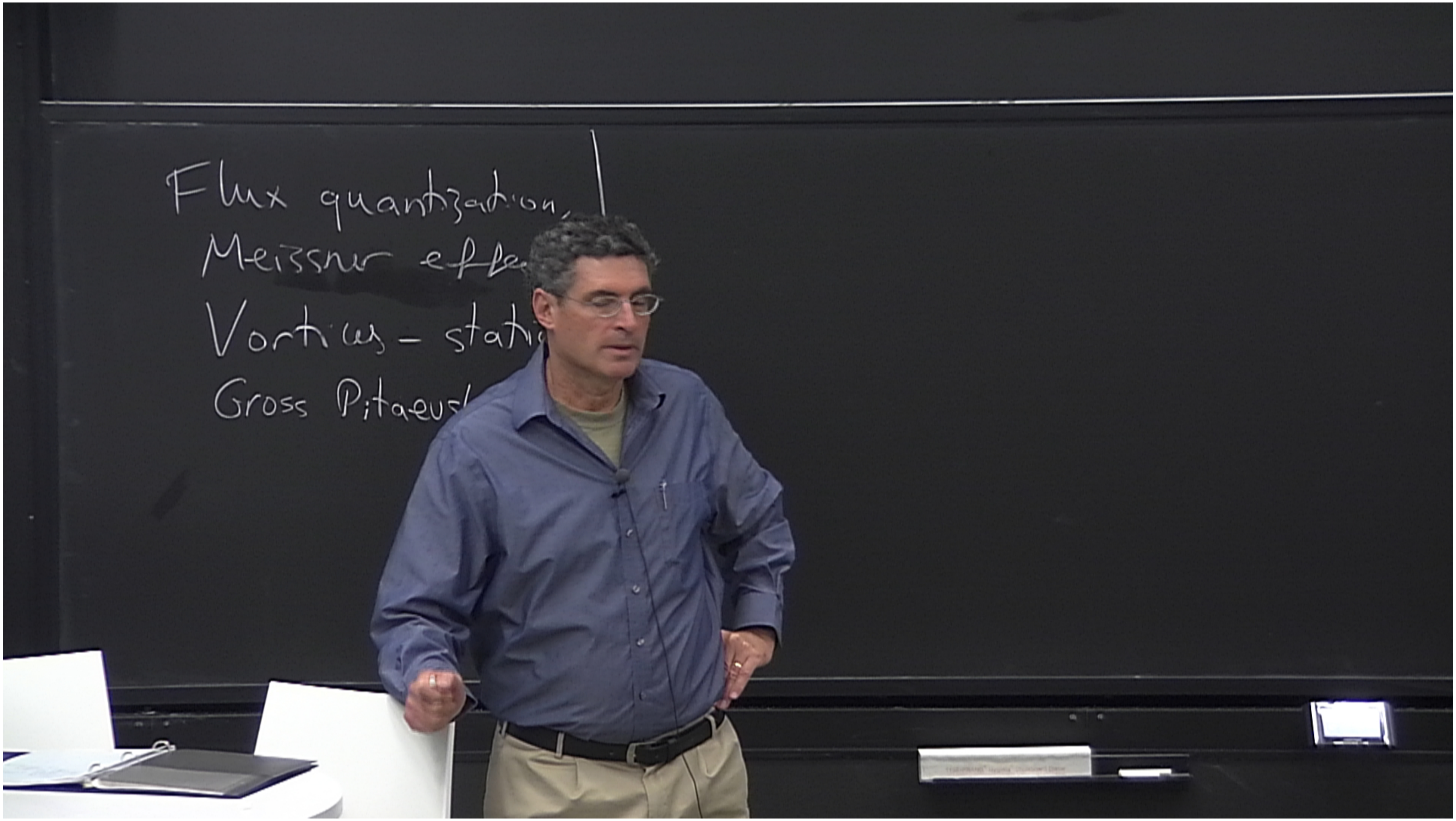
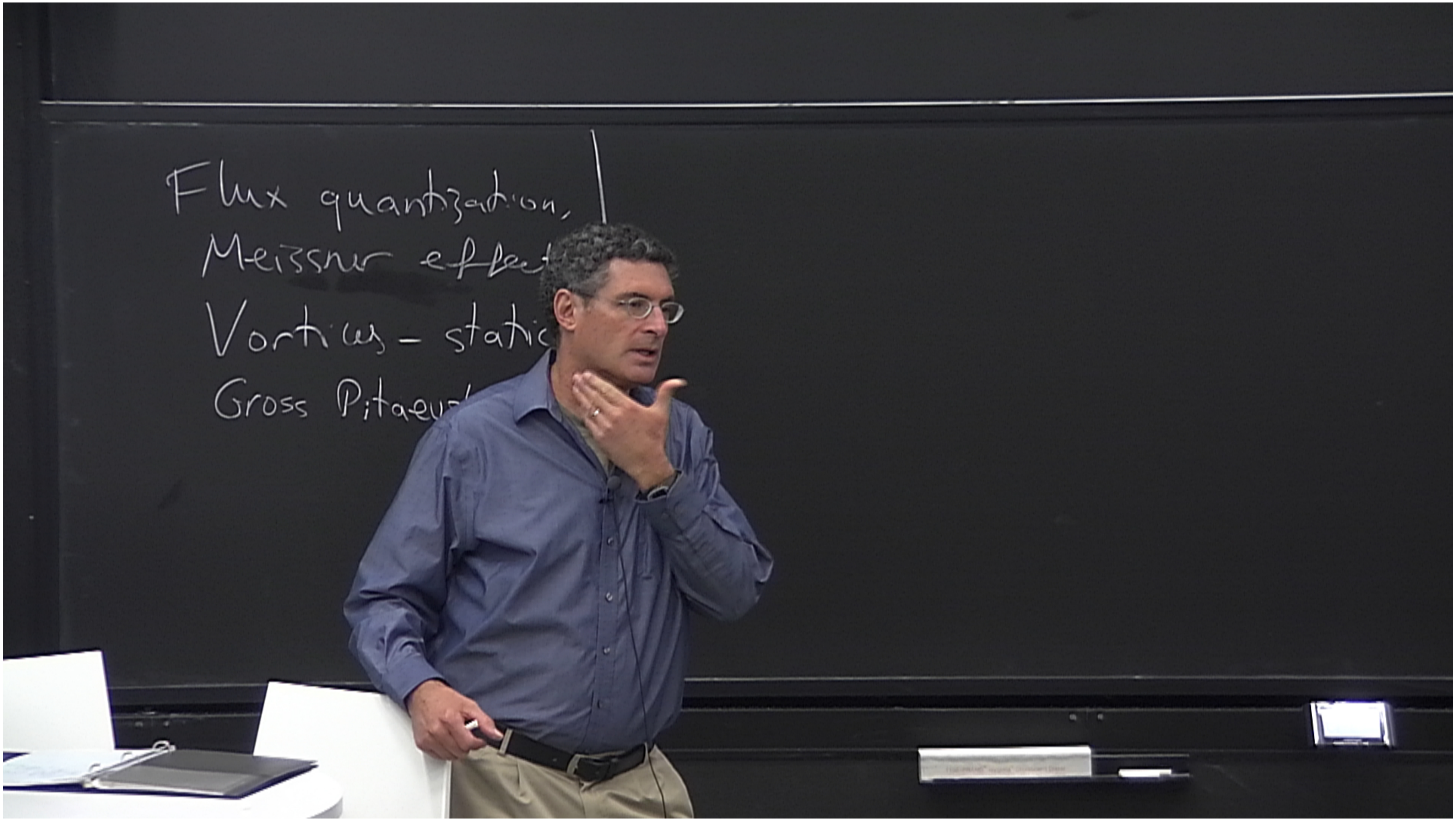
Title: Condensed Matter - Lecture 12

Date: Oct 24, 2012 10:30 AM

URL: http://pirsa.org/12100037

Abstract:


Flux quantization,
Meissner
Vortices –
Gross


Flux quantization,
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Vortices –
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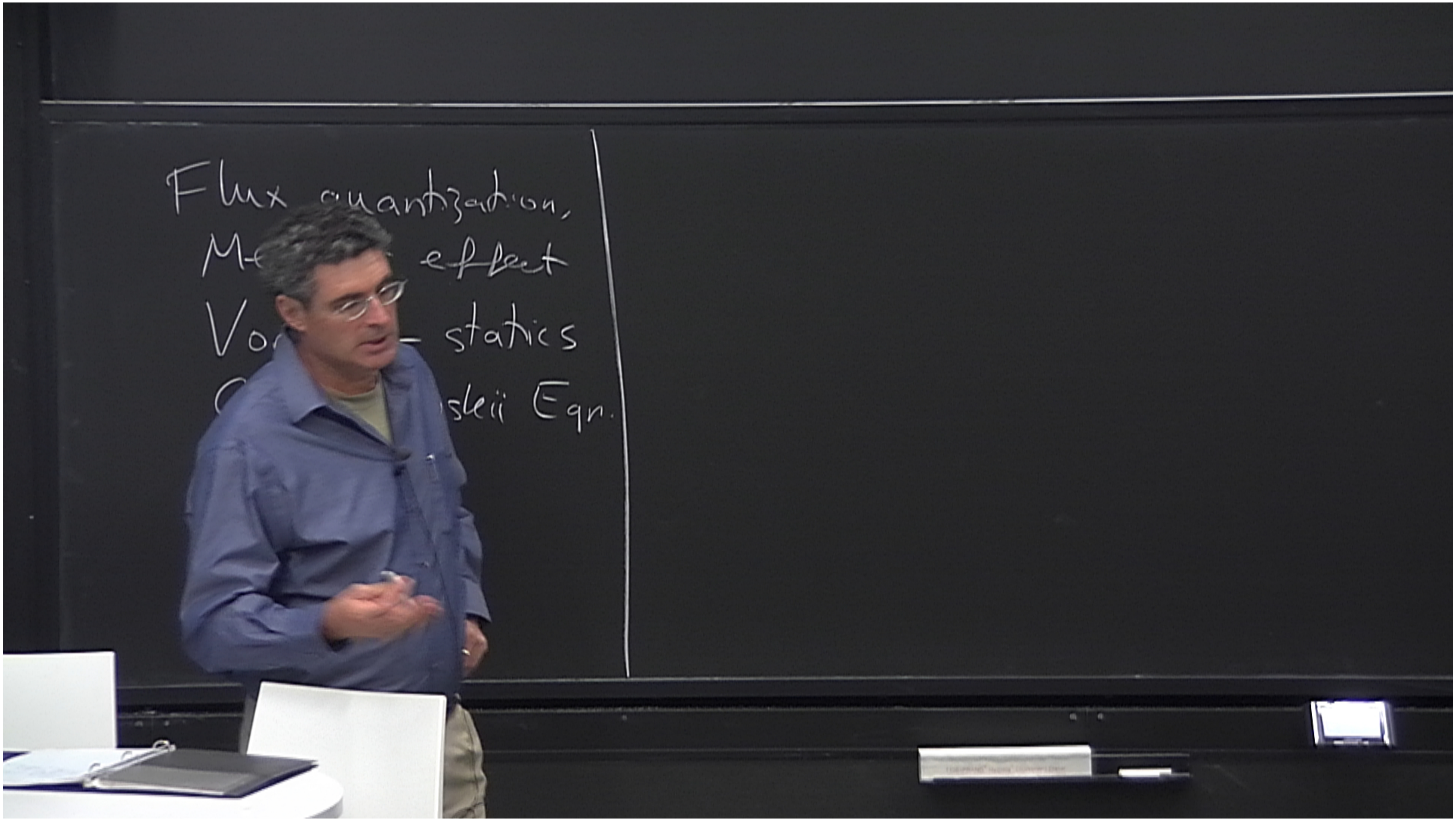
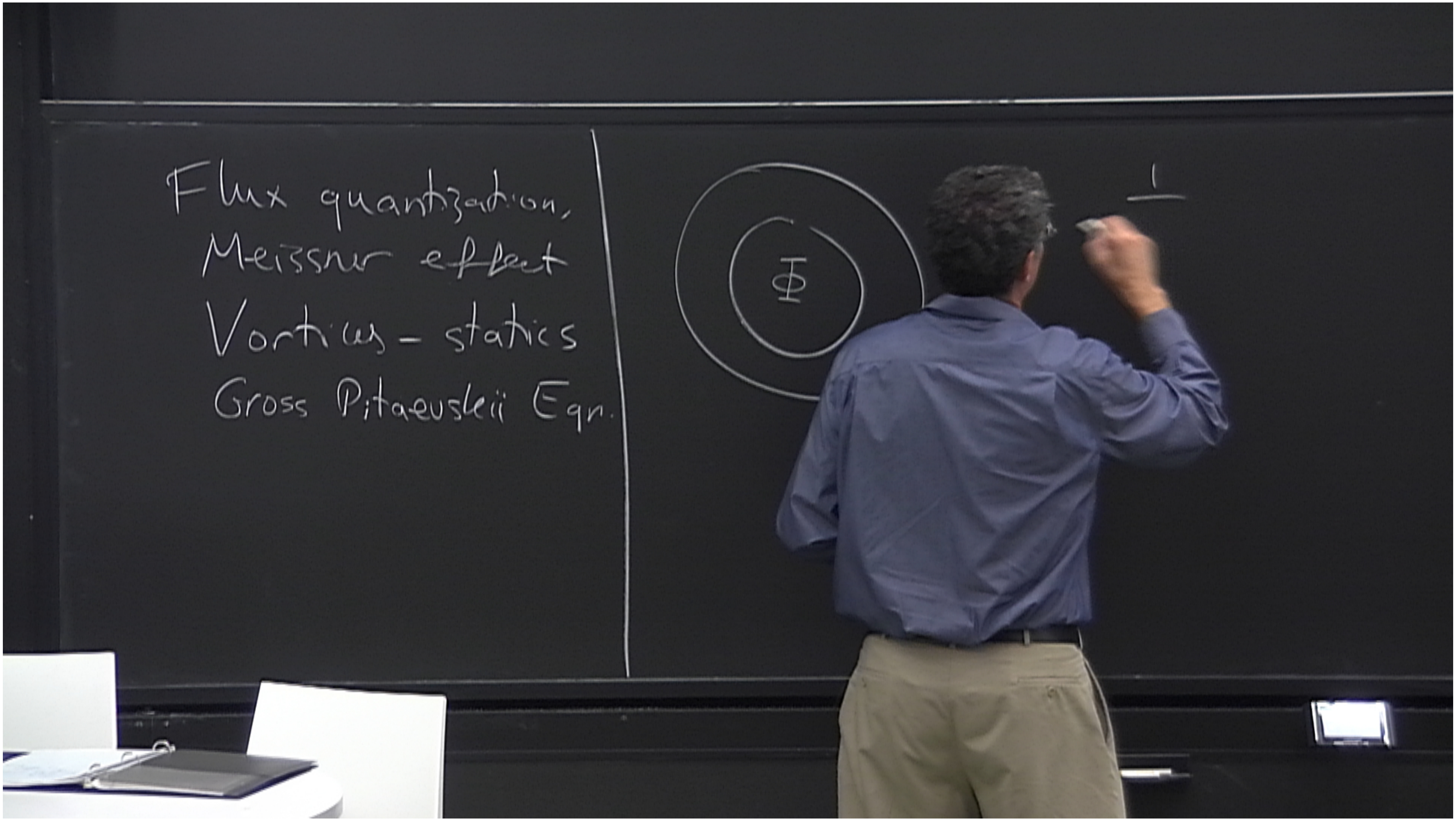
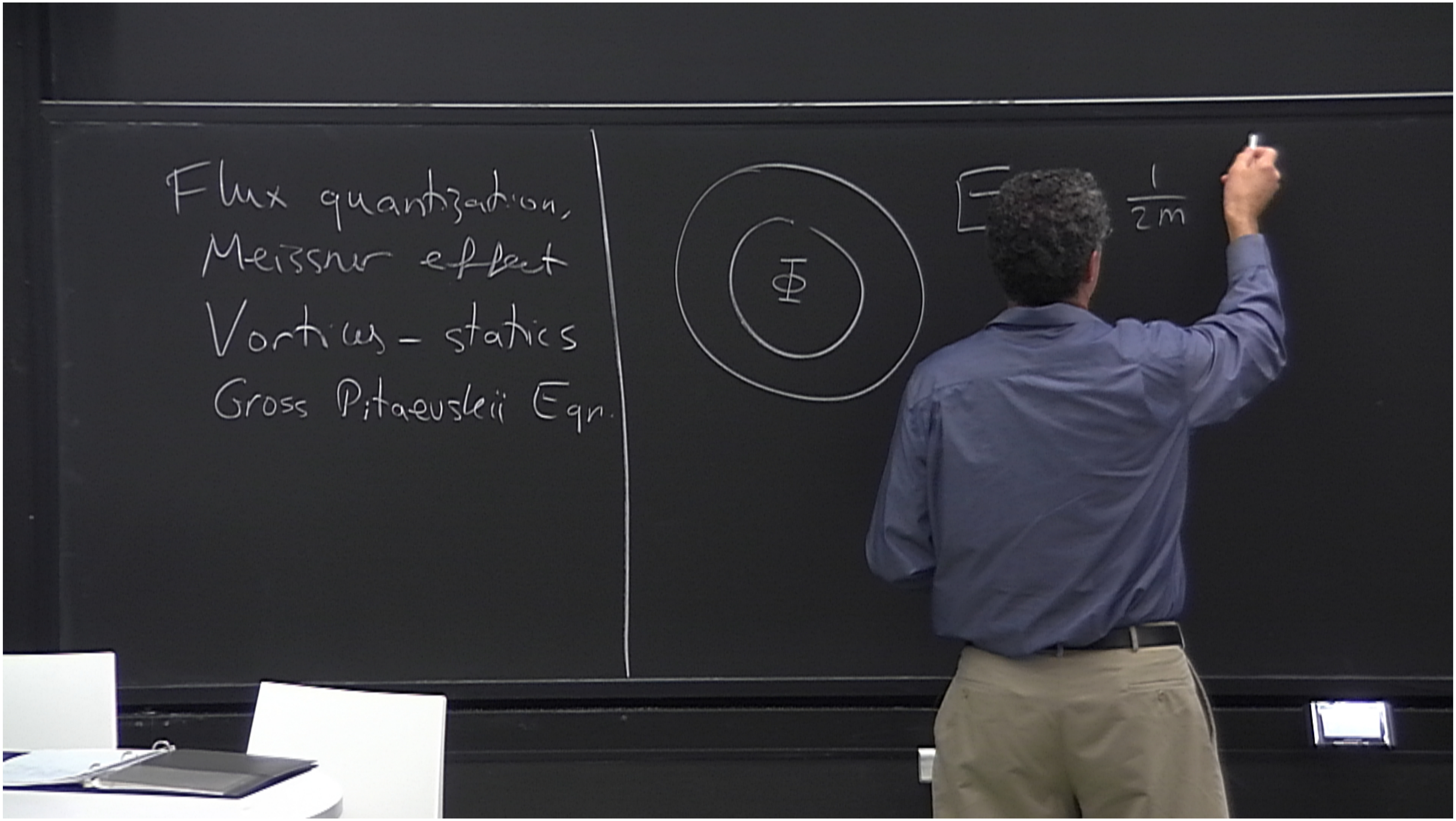
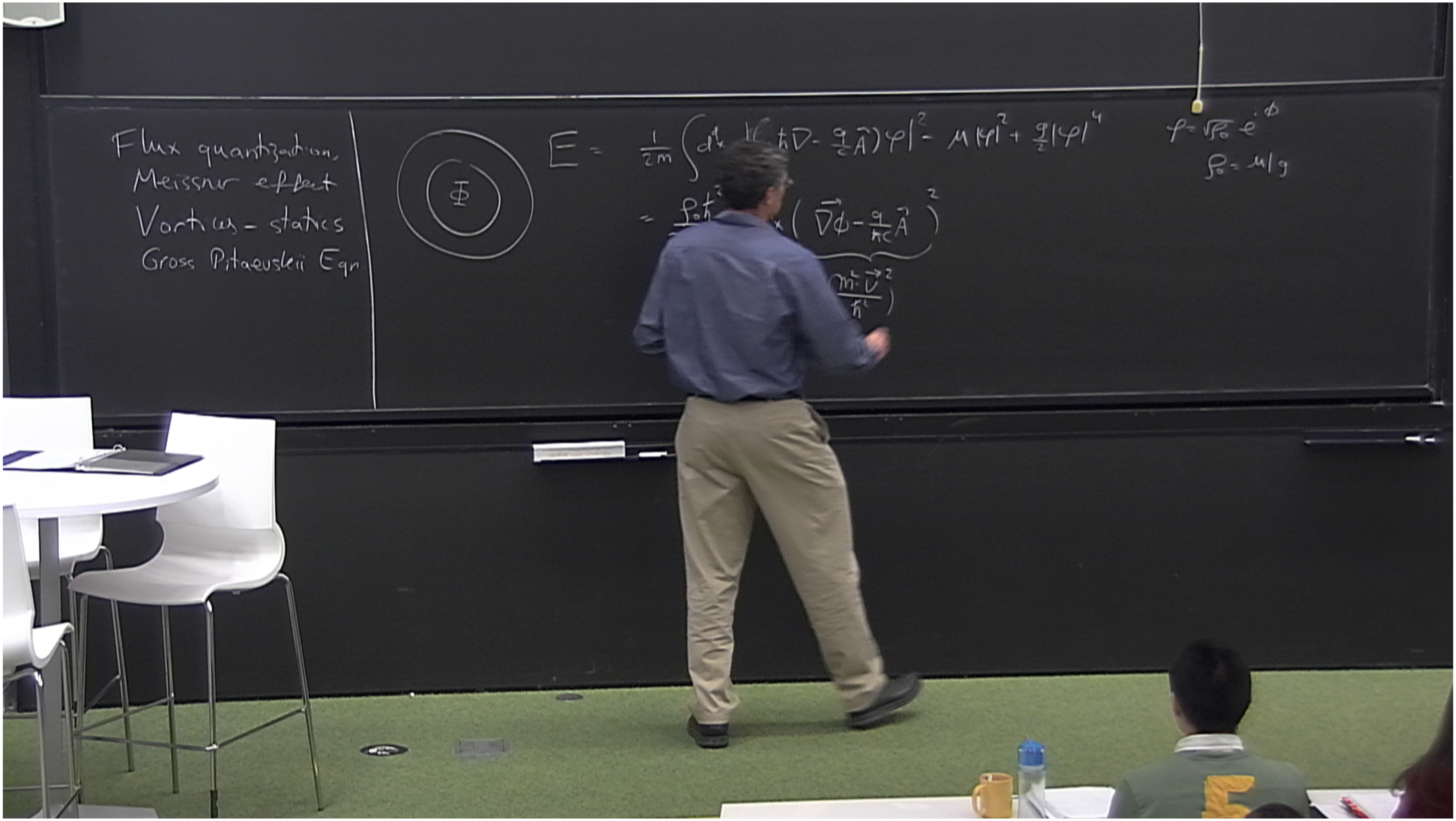
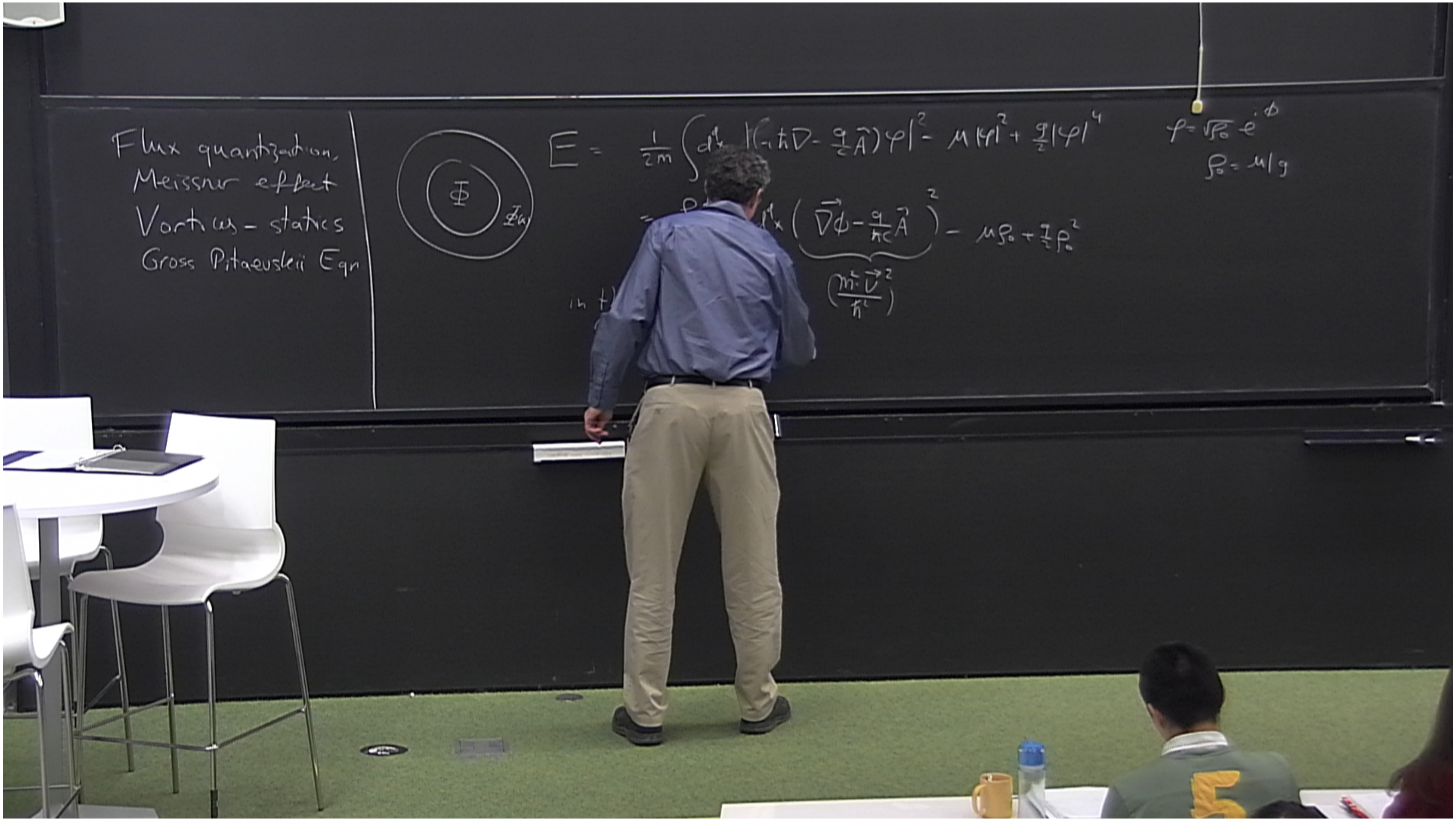

Flux quantization,
effect
statics
Eqn.


Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii Eqn.
Φ


Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii Eqn.
Φ
1/2m



Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii Eqn


Flux quantization,
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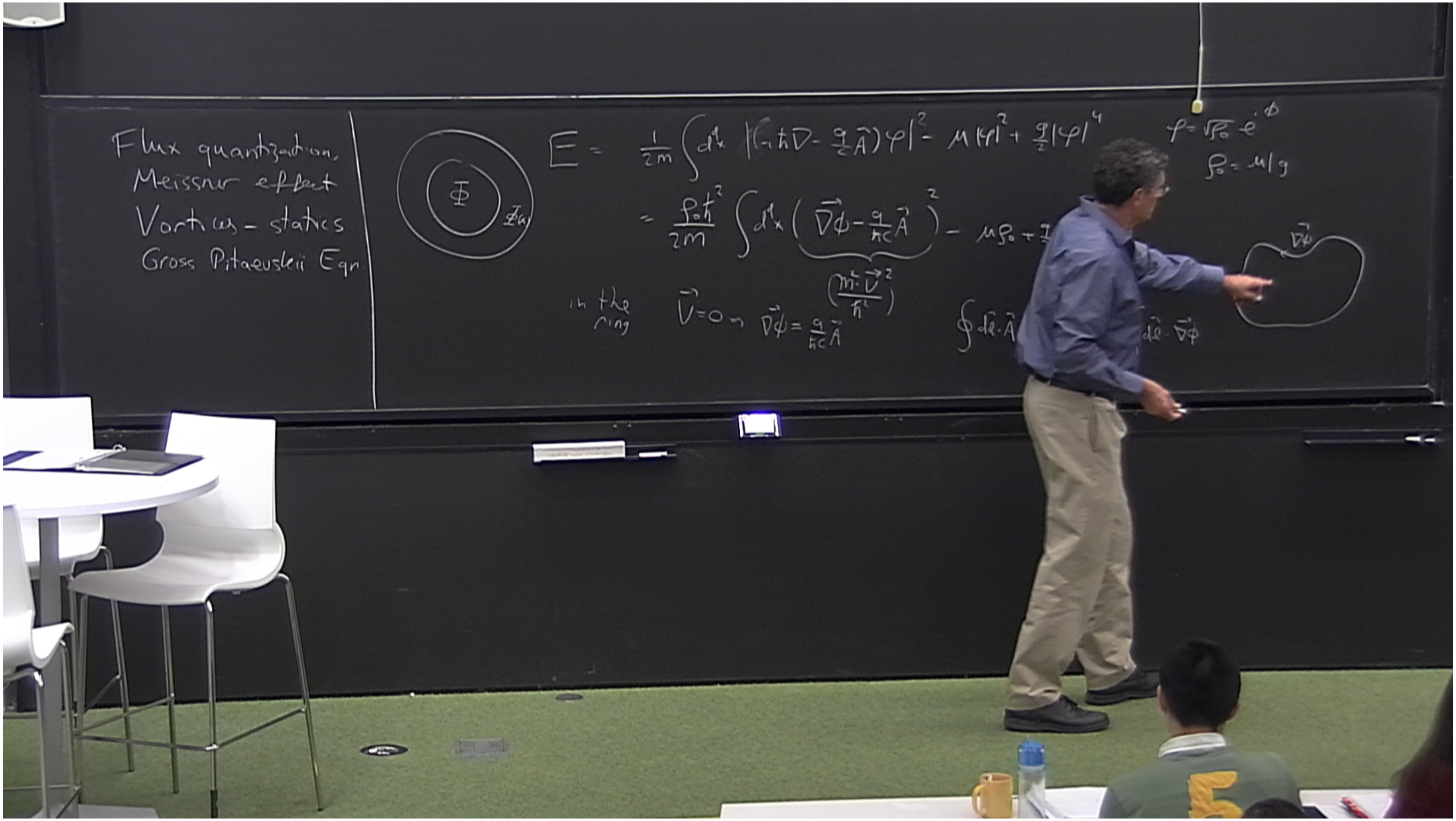
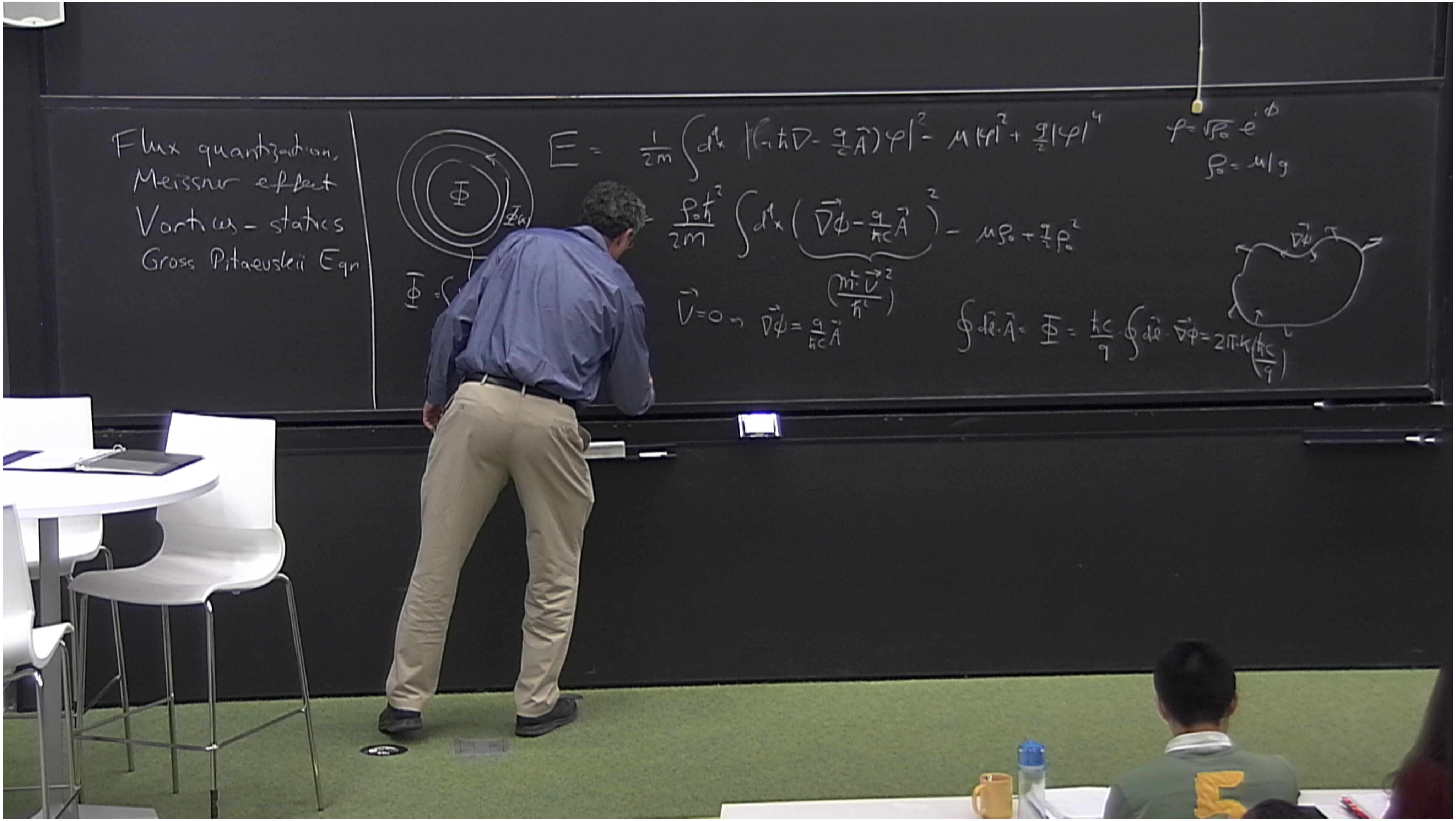
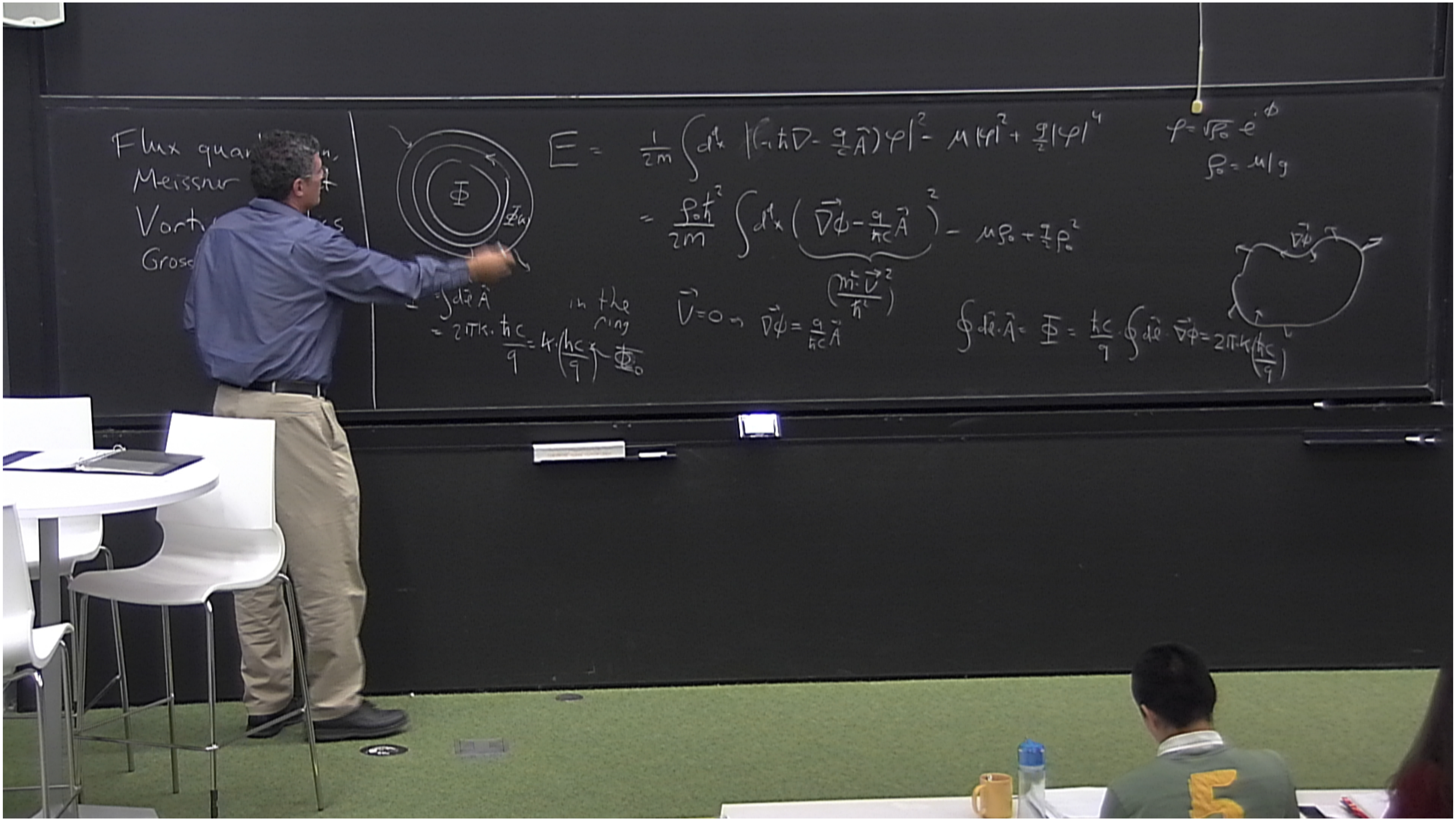
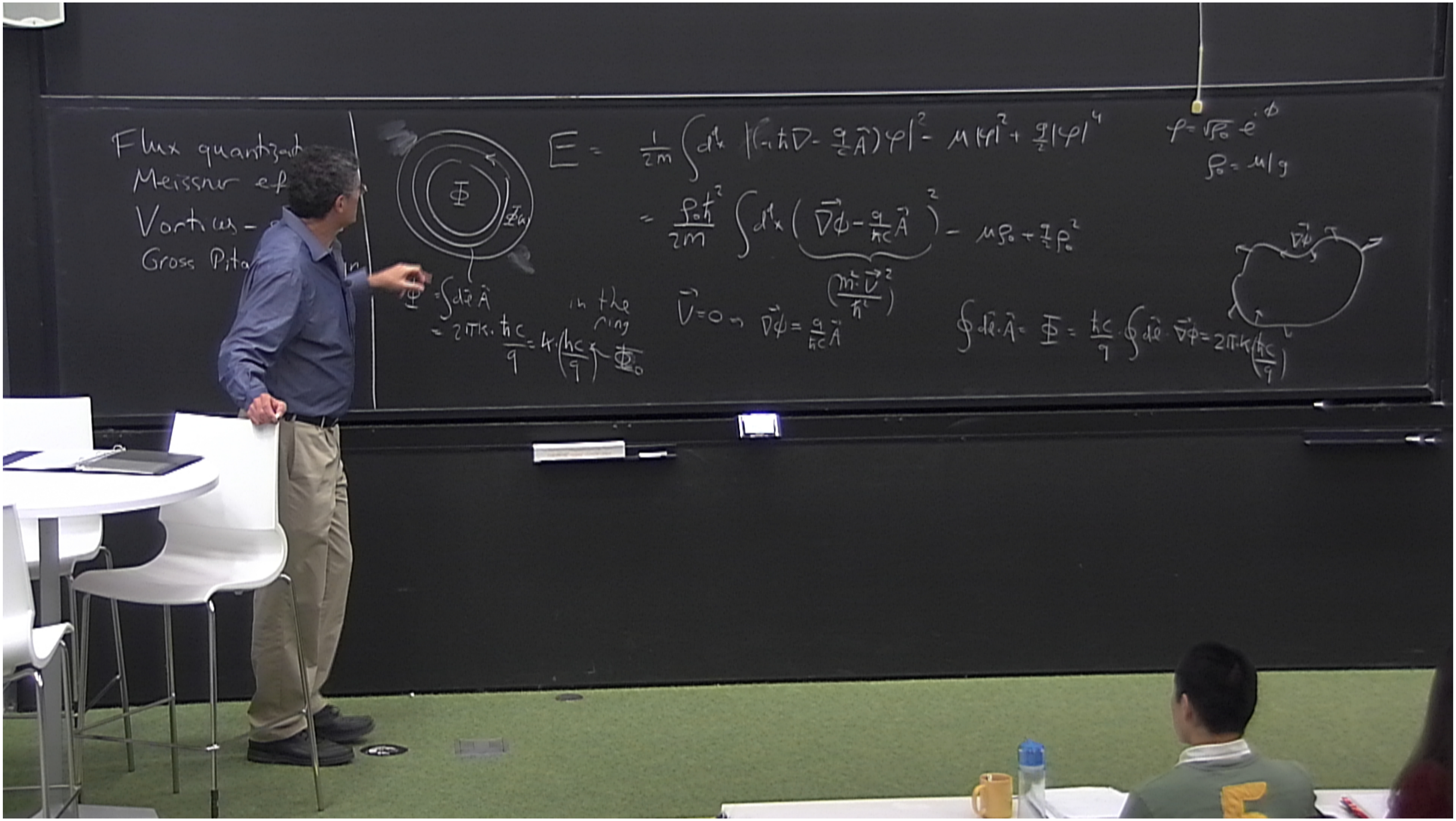
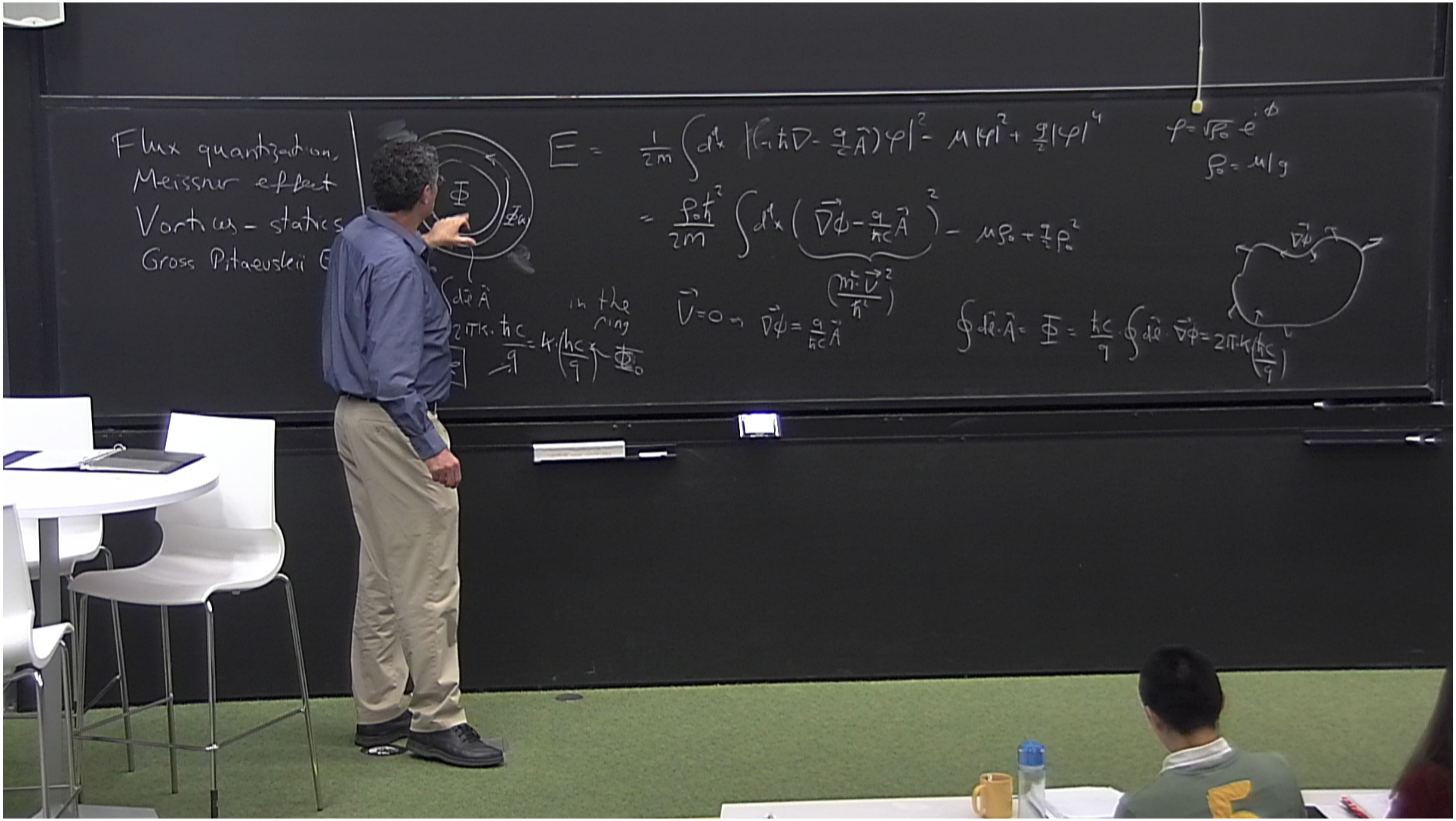
Vortices - statics
Gross Pitaevskii Eqn

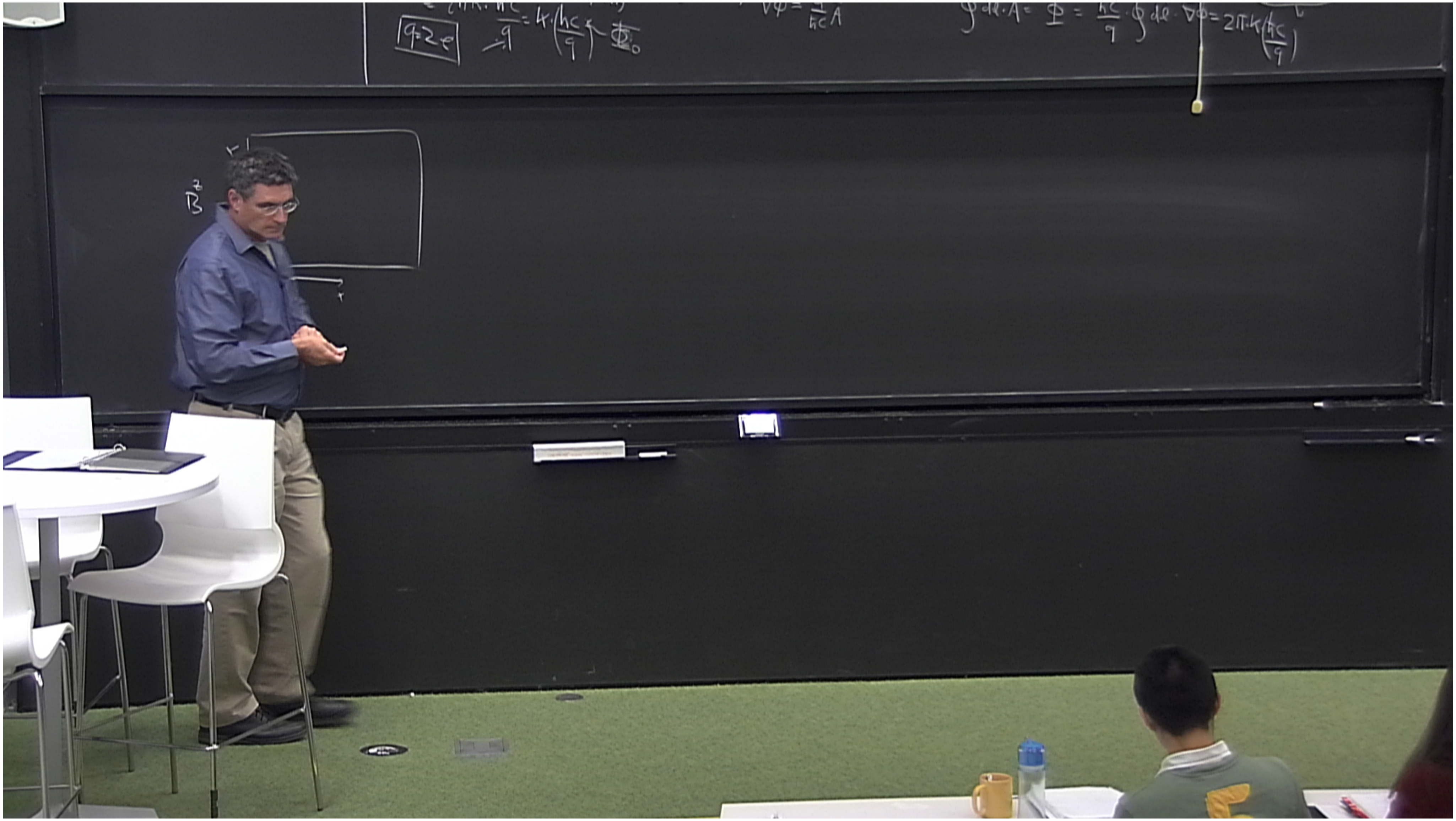

Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii Eqn
in the ring

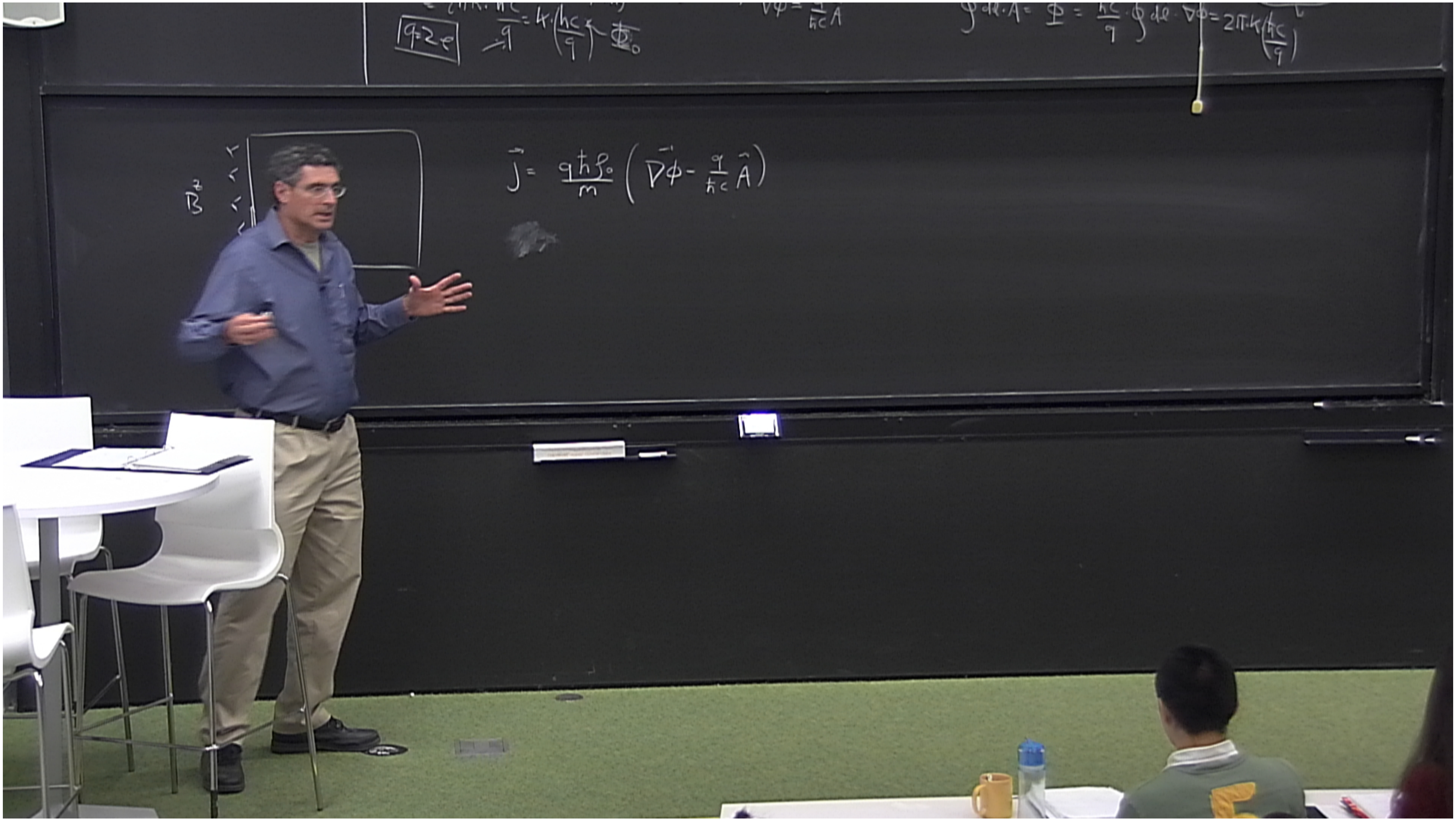

Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii Eqn

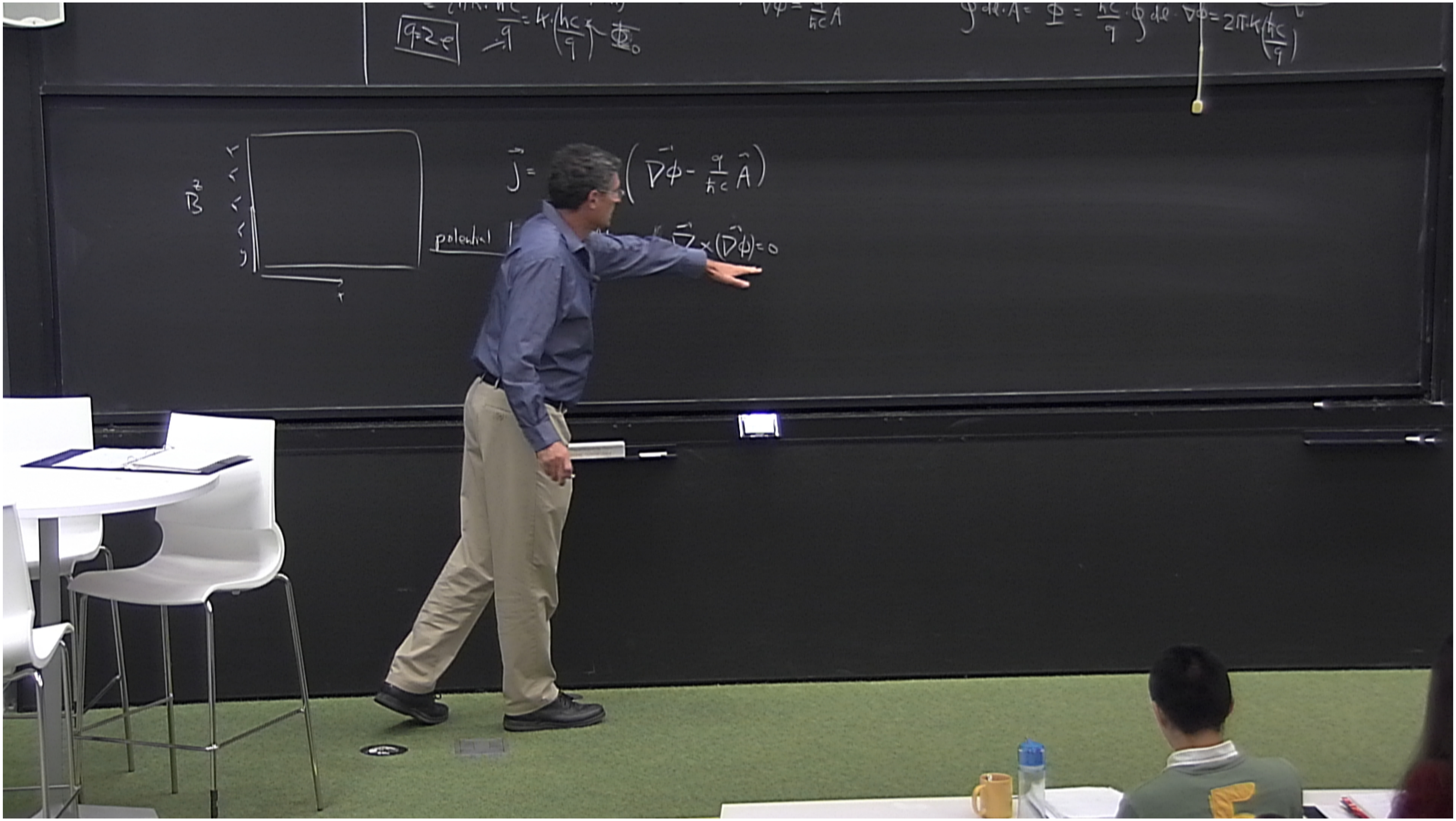

Meissner
in the
ring

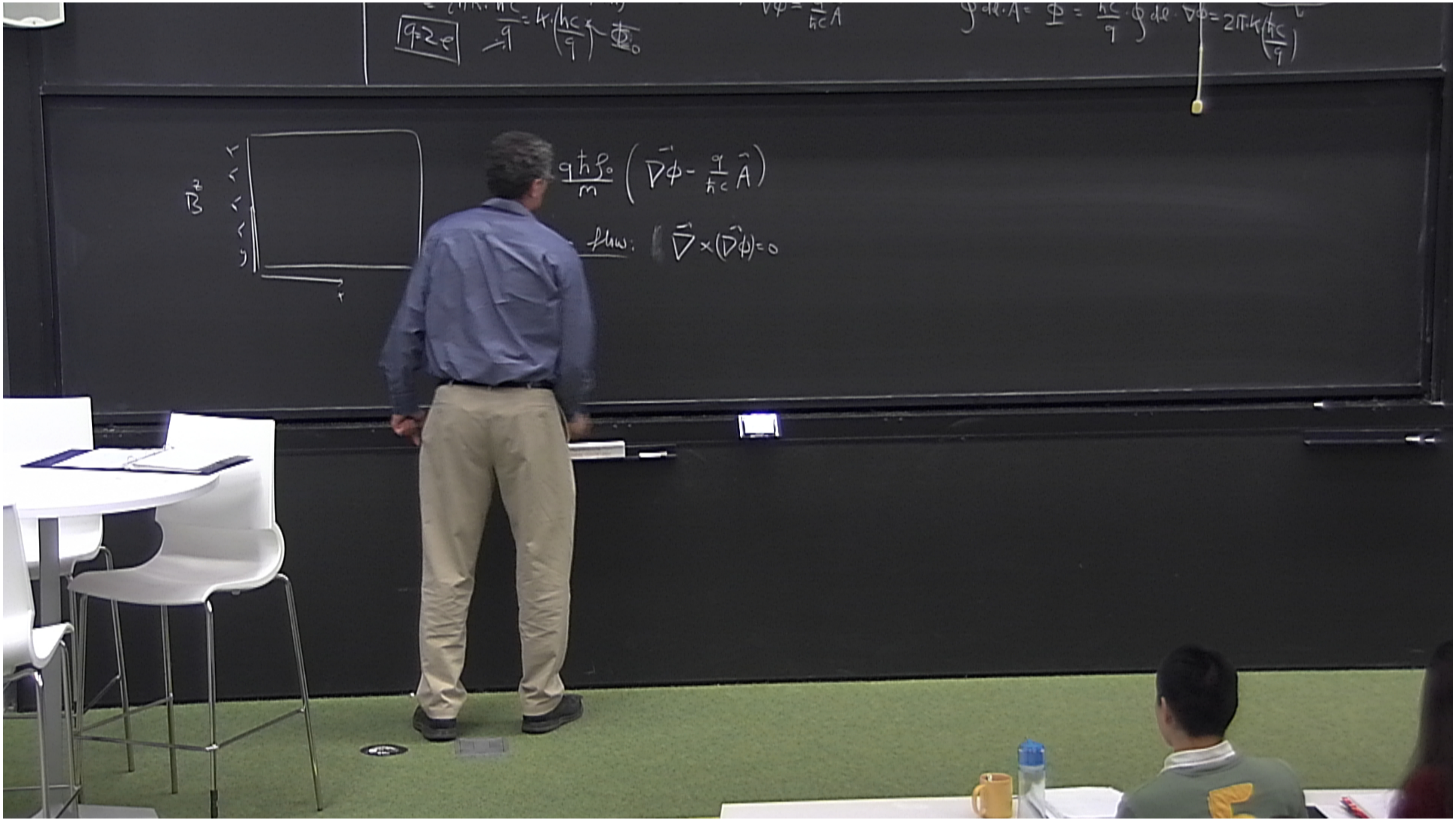


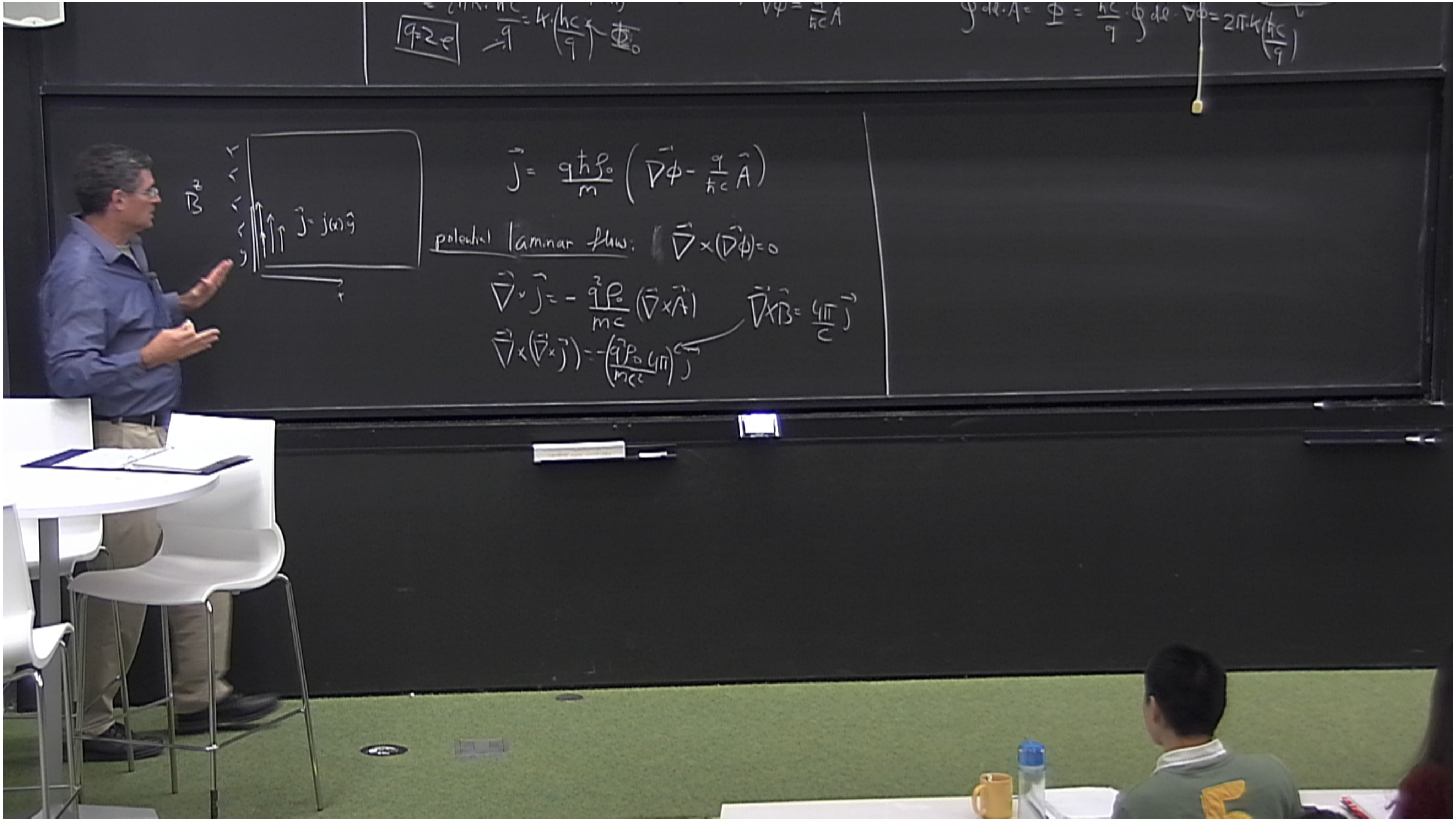

Flux quantization,
Meissner effect
Vortices - statics
Gross Pitaevskii

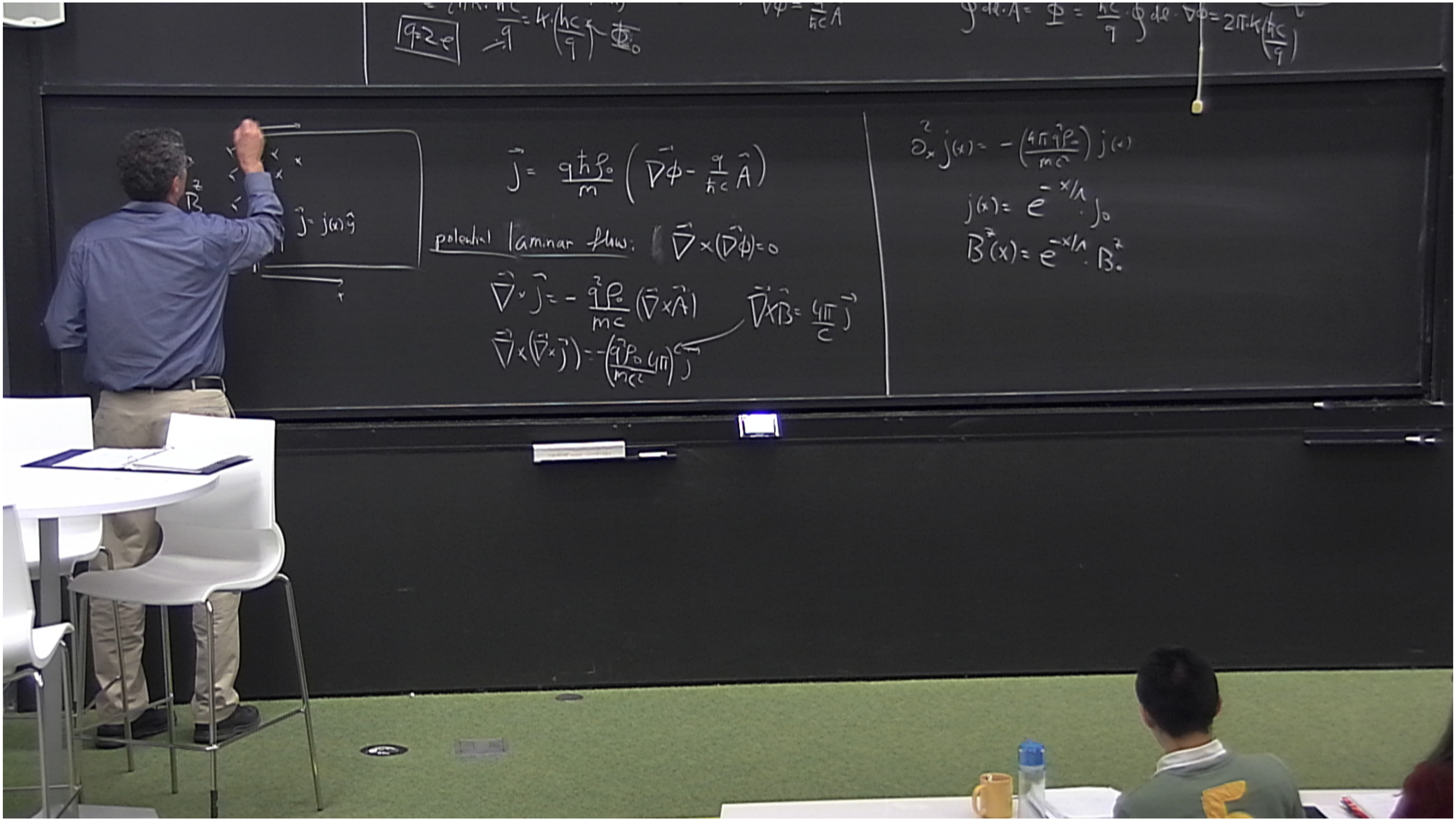


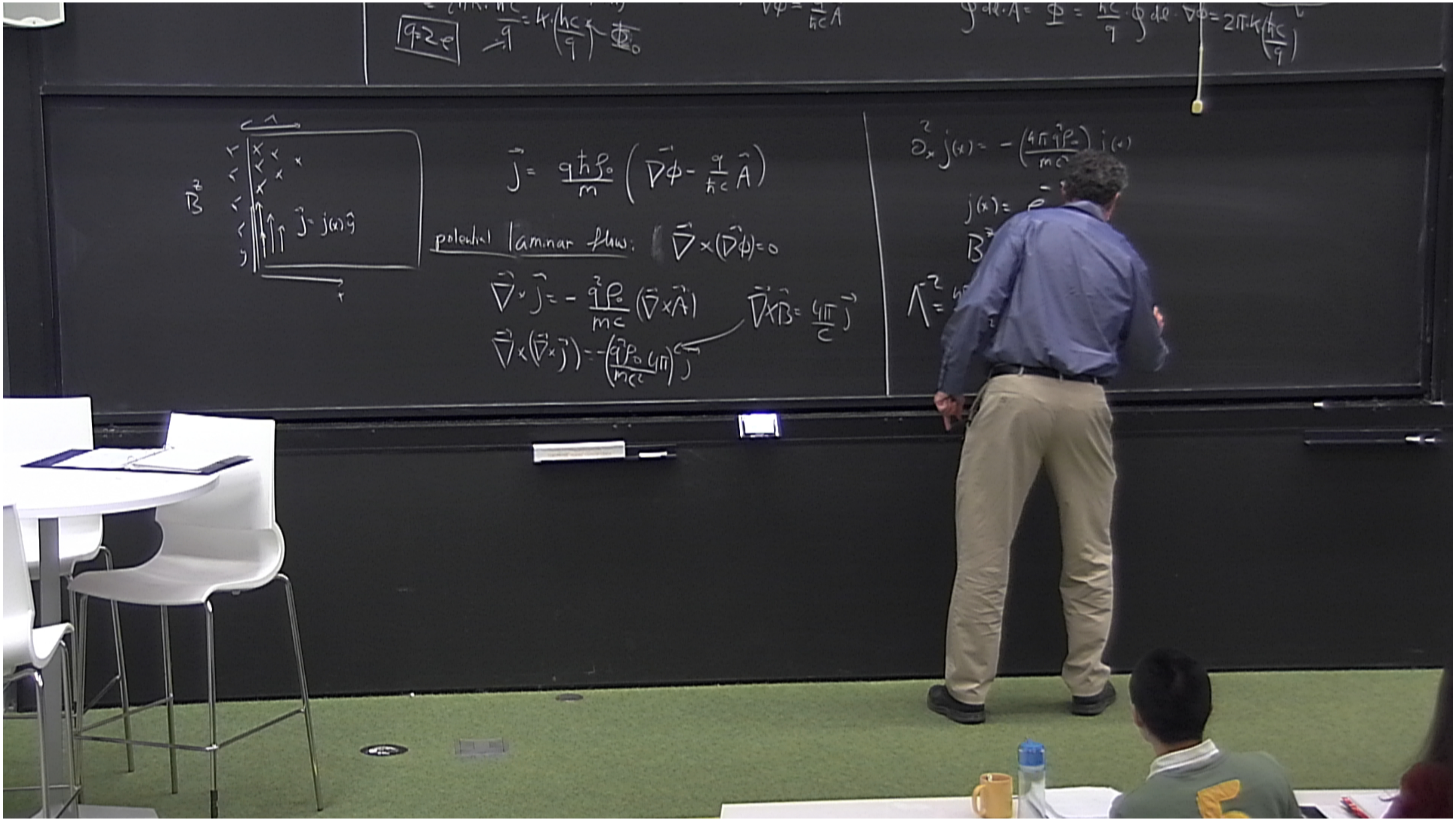


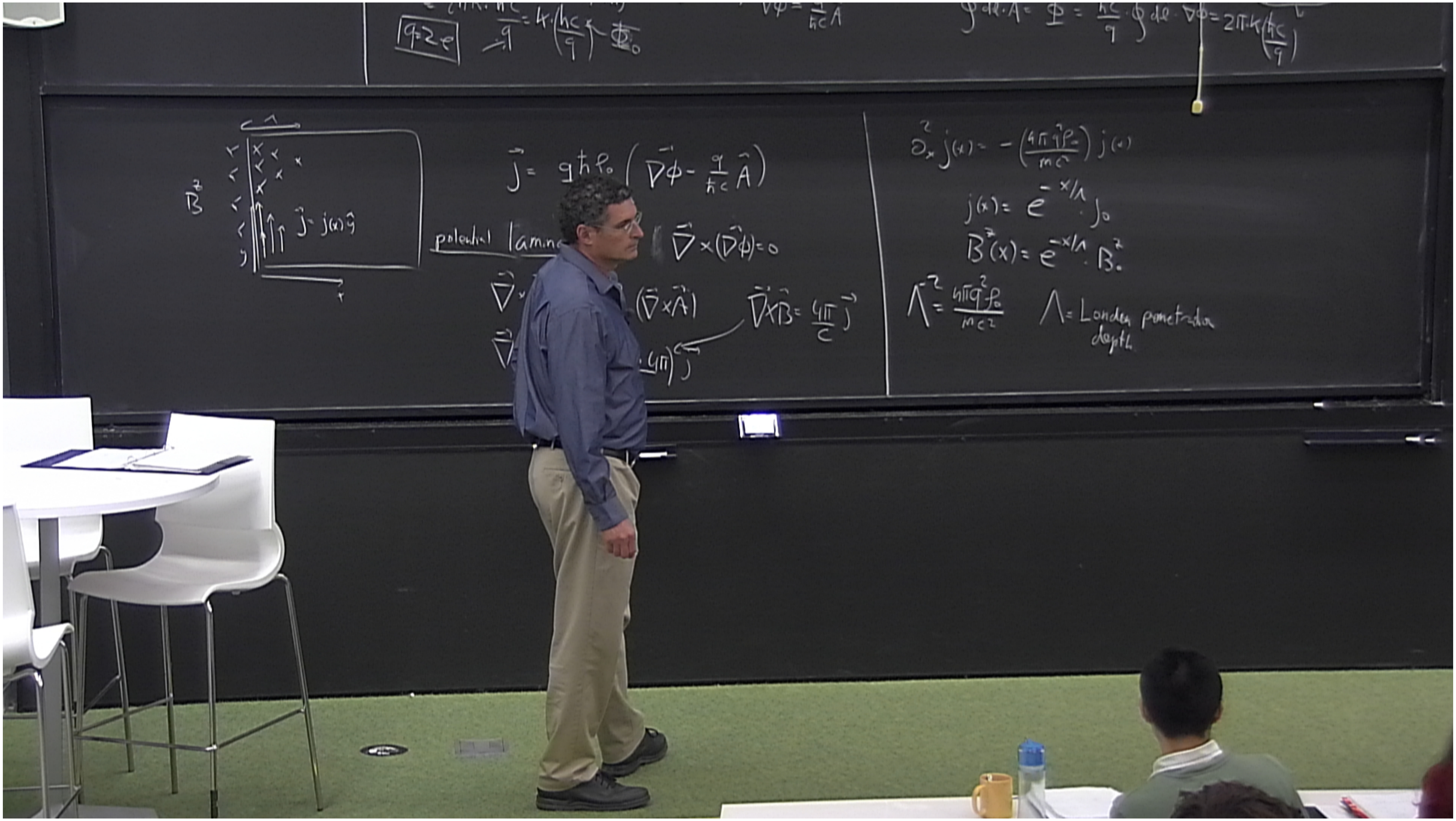


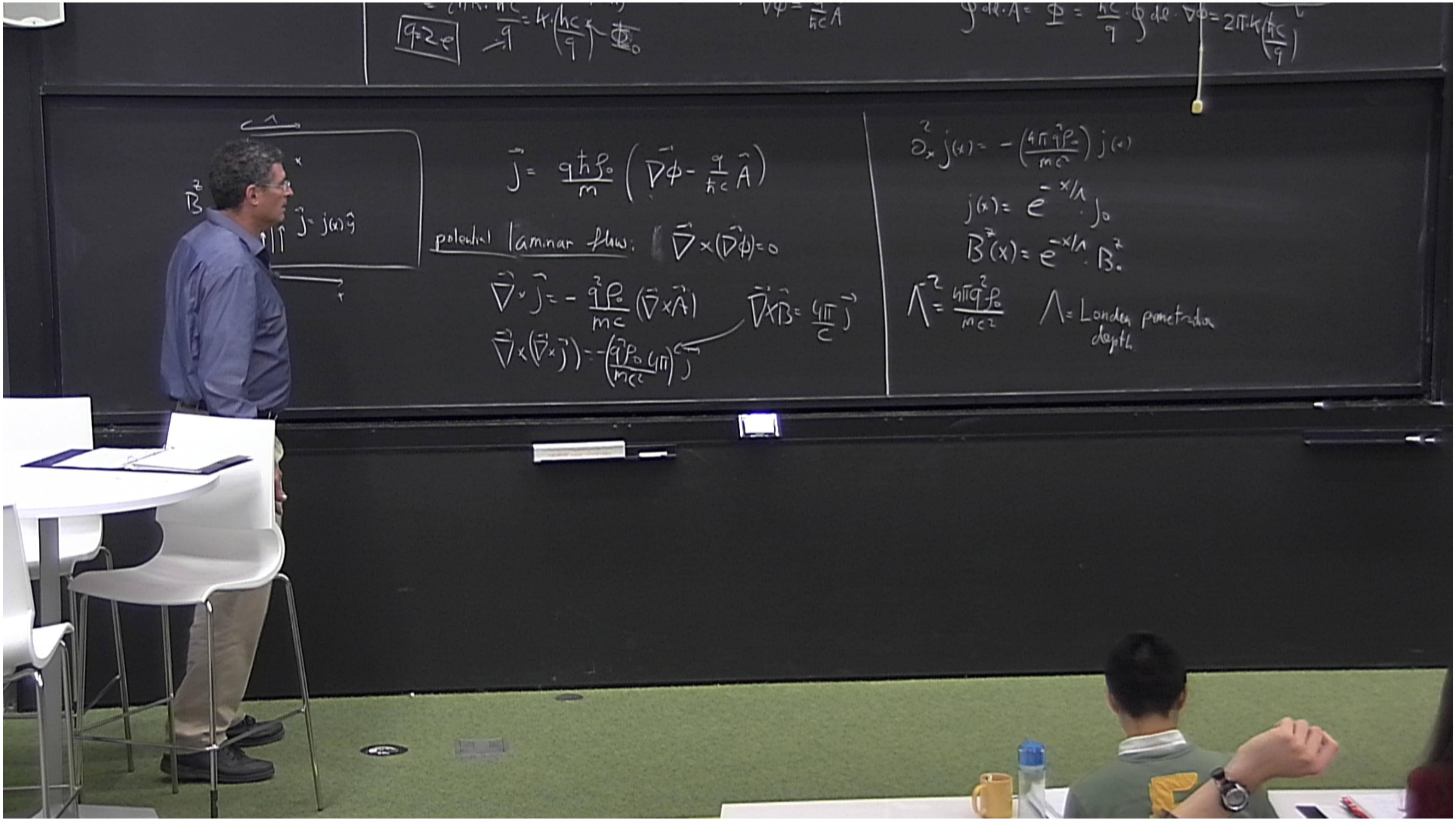

∇ × (∇φ) = 0
q ħ ρ₀ / m (∇φ − q/ħc A)

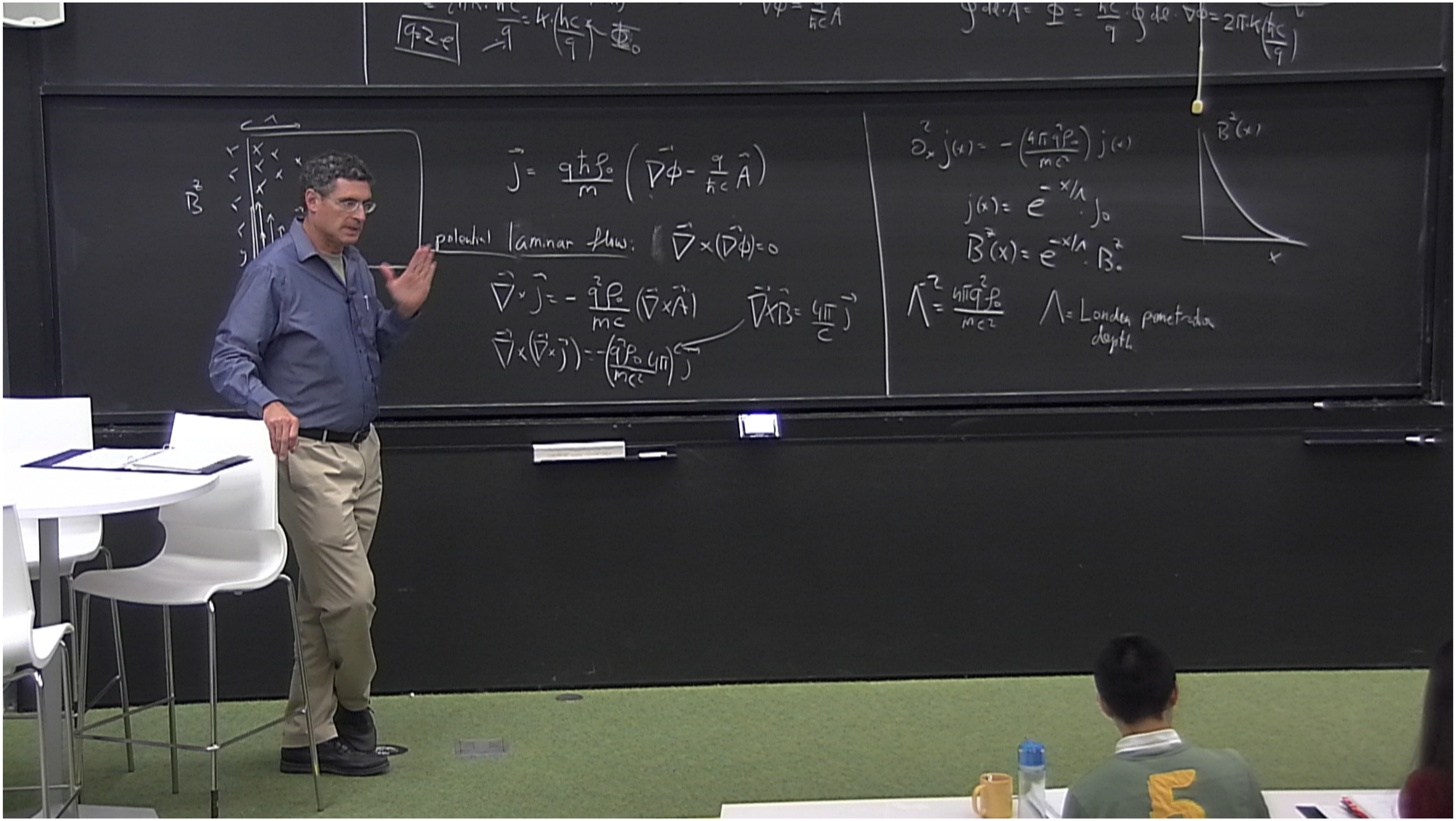


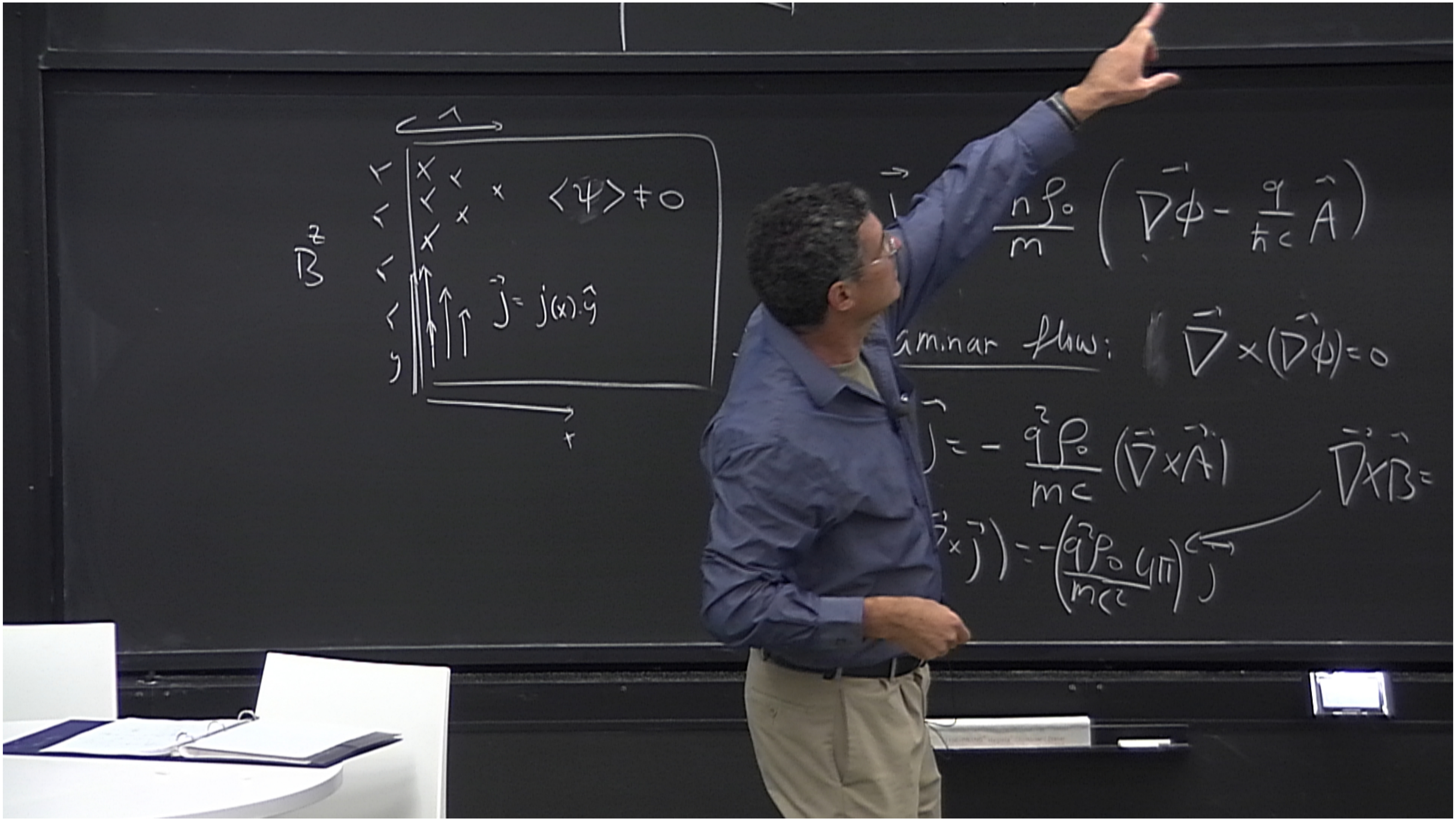

potential laminar flow:

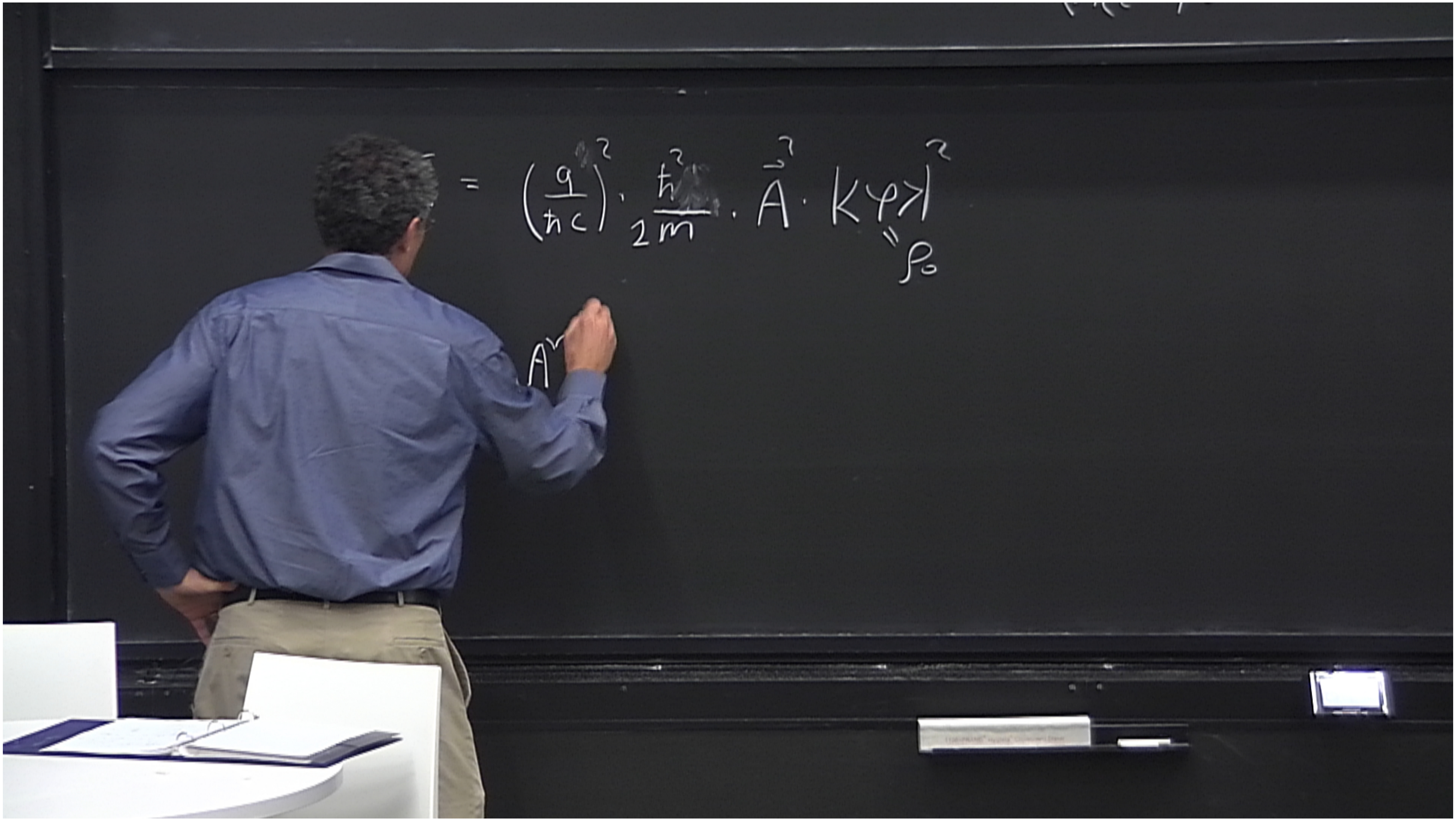

potential laminar flow:

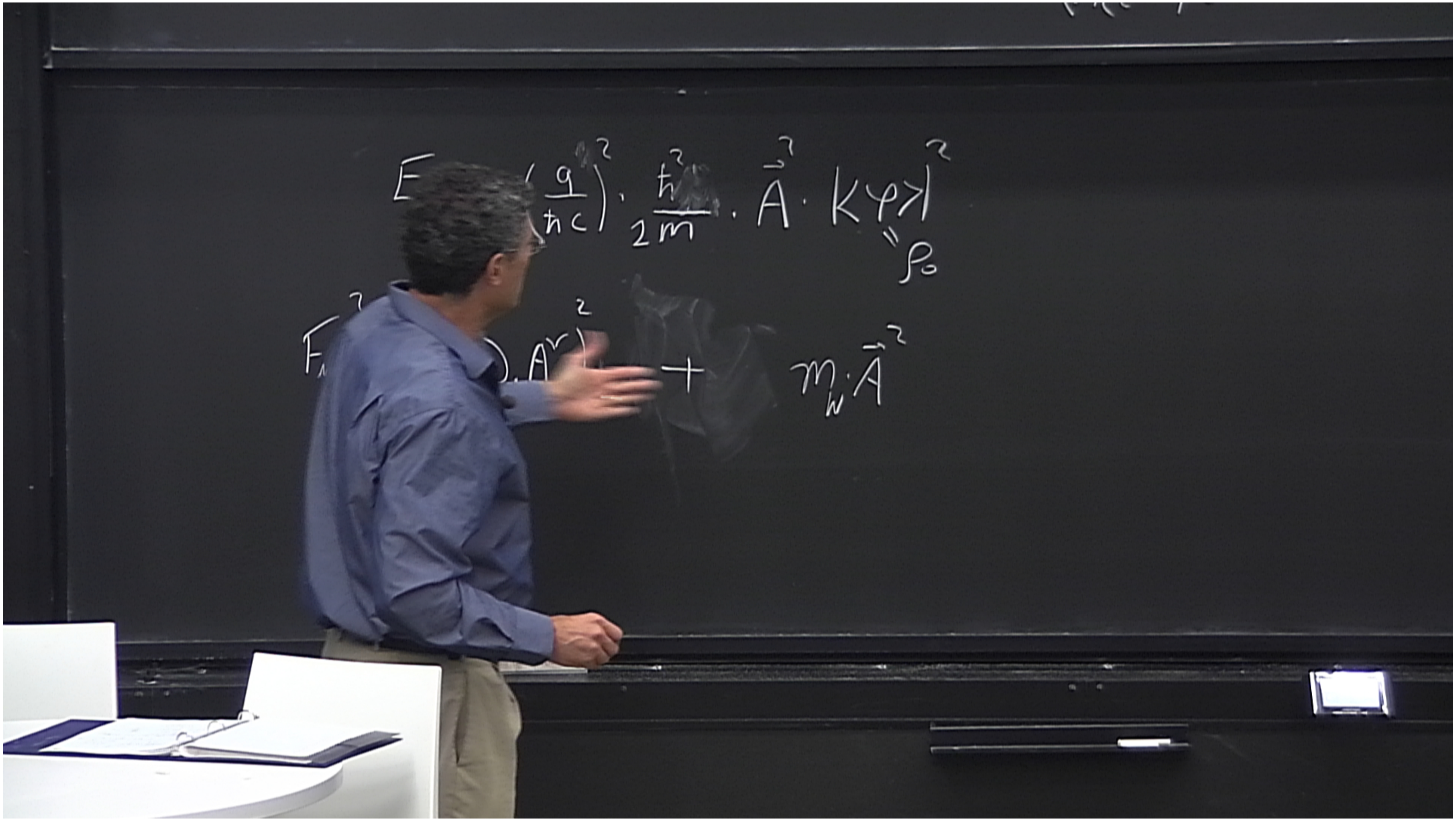


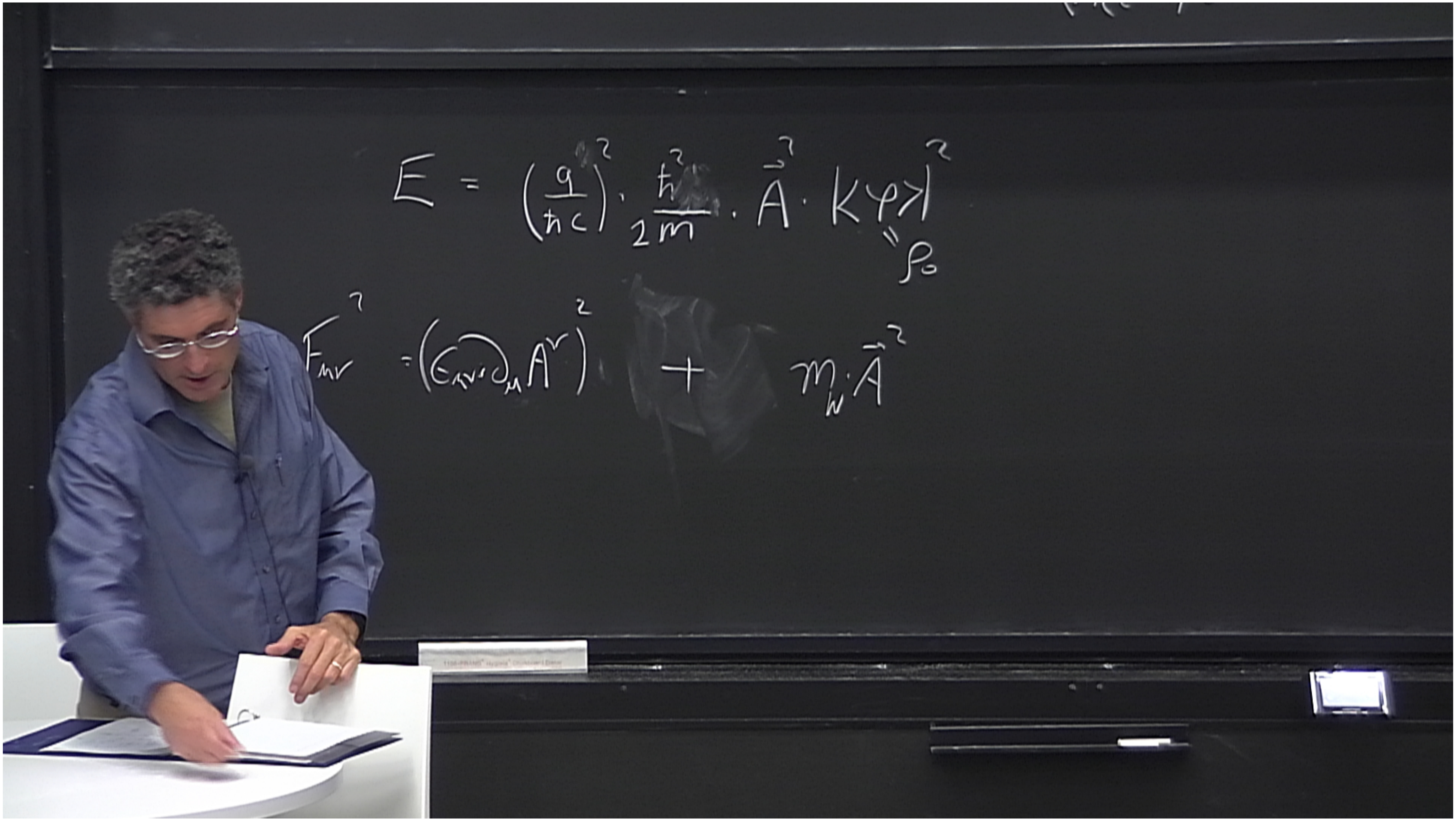

j = (q ħ ρ₀ / m) (∇φ − (q / ħc) A)
potential laminar flow: ∇ × (∇φ) = 0
∇ × j = − (q² ρ₀ / mc) (∇ × A)
∇ × B = (4π / c) j
∇ × (∇ × j) = − (q² ρ₀ 4π / mc²) j
∂ₓ² j(x) = − (4π q² ρ₀ / mc²) j(x)
j(x) = e^(−x/Λ) · j₀
B(x) = e^(−x/Λ) · B₀
Λ⁻² = 4π q² ρ₀ / mc²
Λ = London penetration depth
j = j(x) ŷ

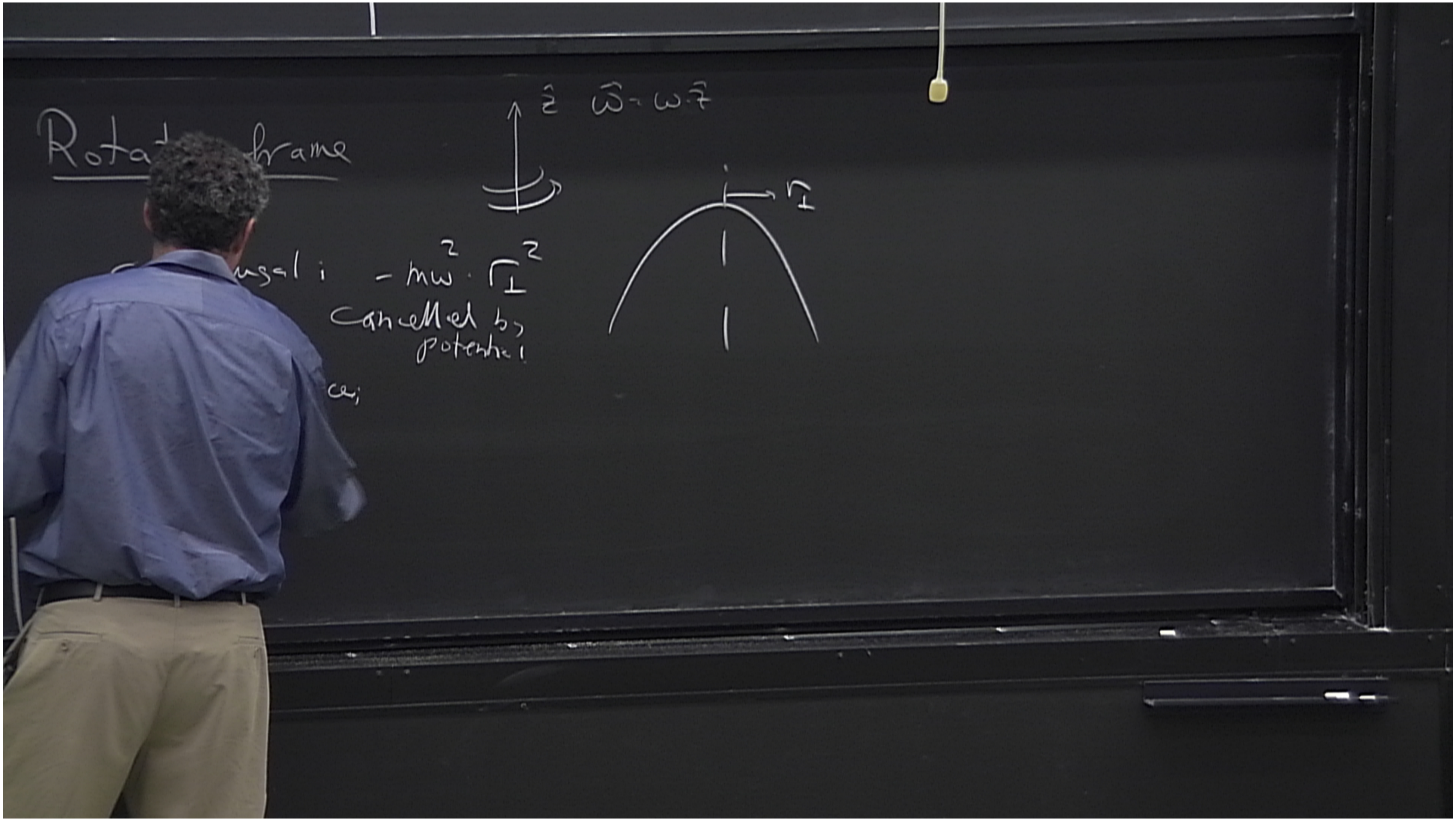

potential laminar flow:
Λ = London penetration depth

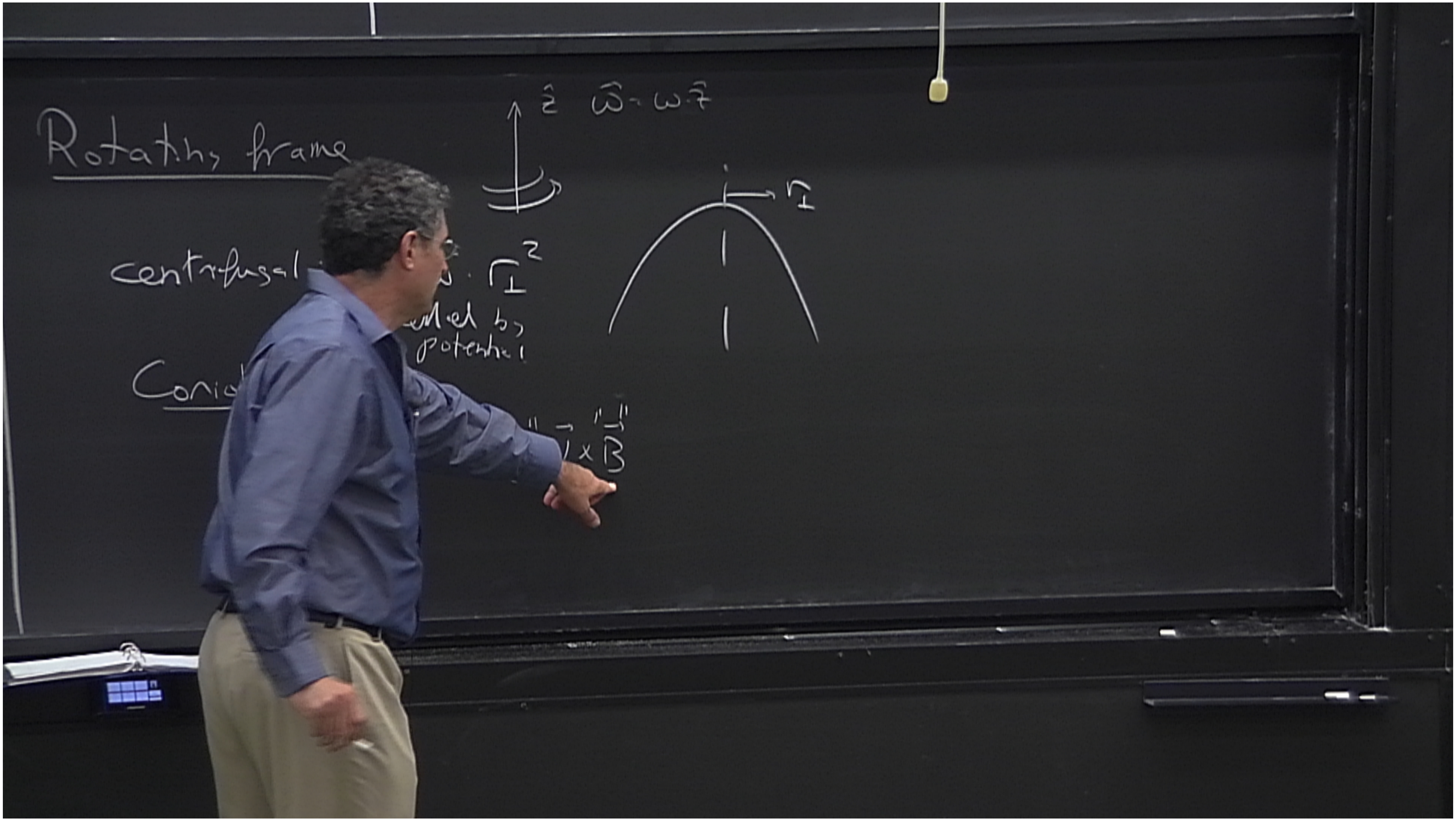


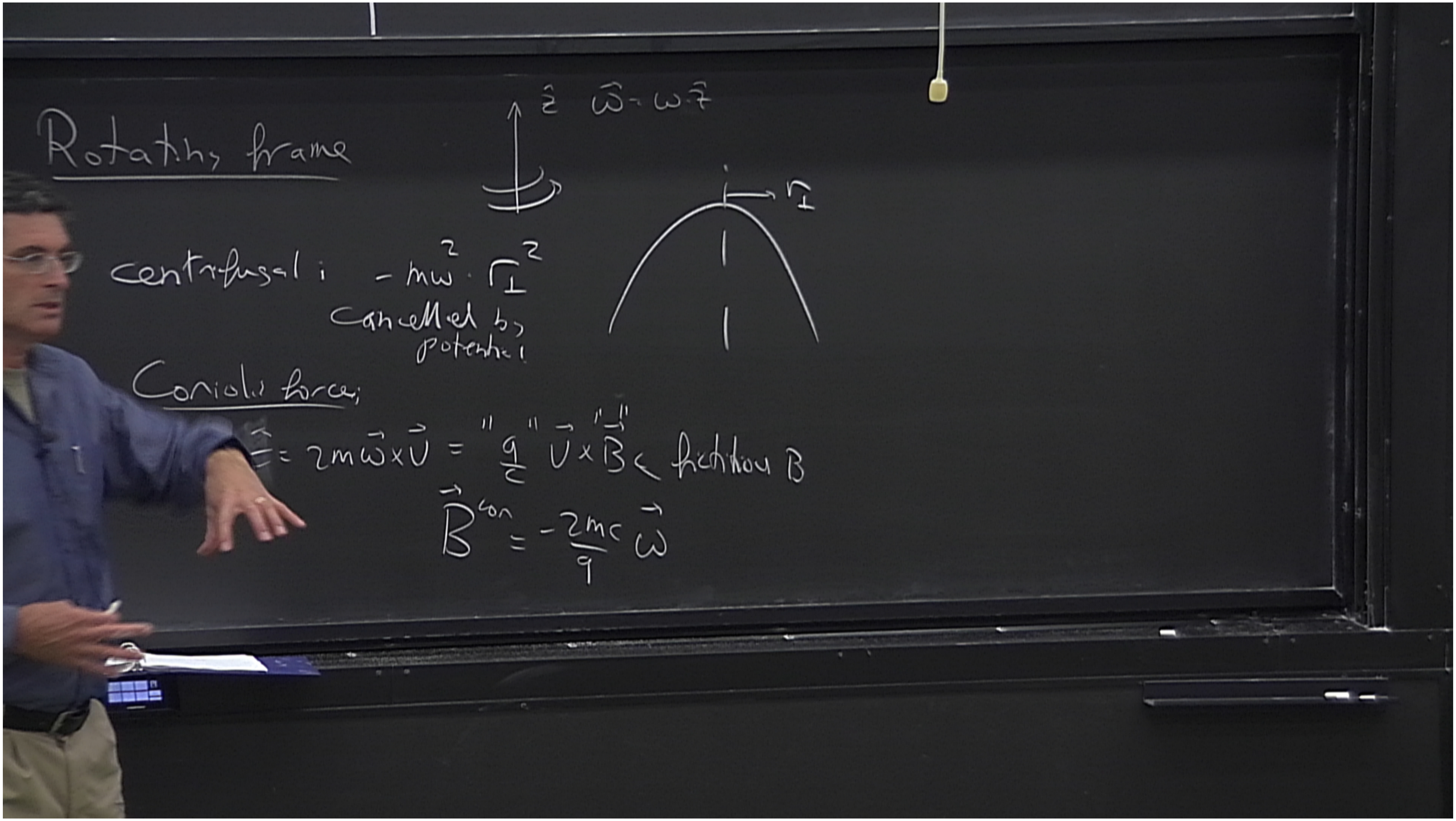


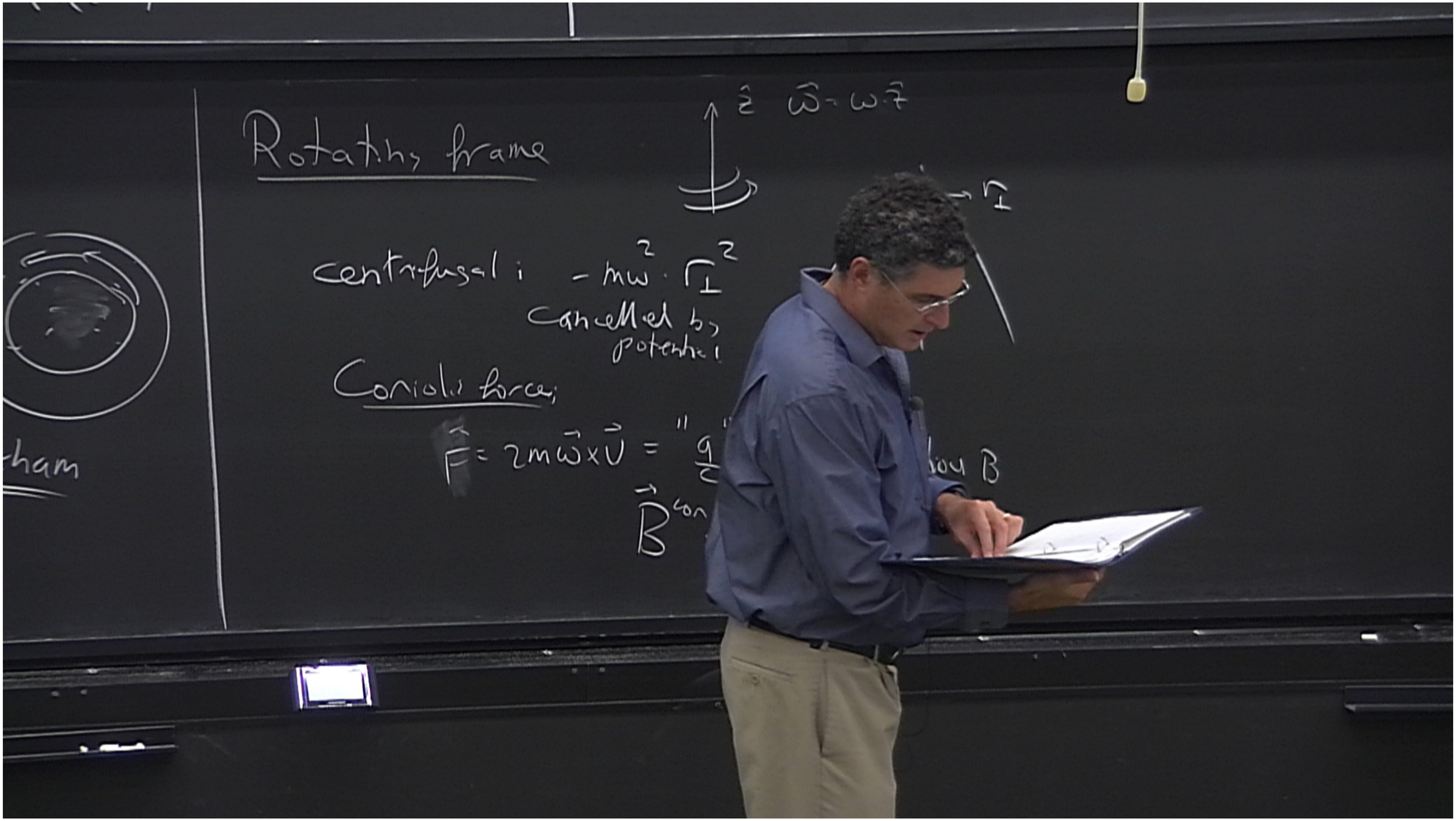



E = (q/ħc)^2 · ħ^2/2m · A^2 · |⟨φ⟩|^2
ρ0
F_μν^2 = (ε_μνλ ∂_μ A^ν)^2 + m_W · A^2

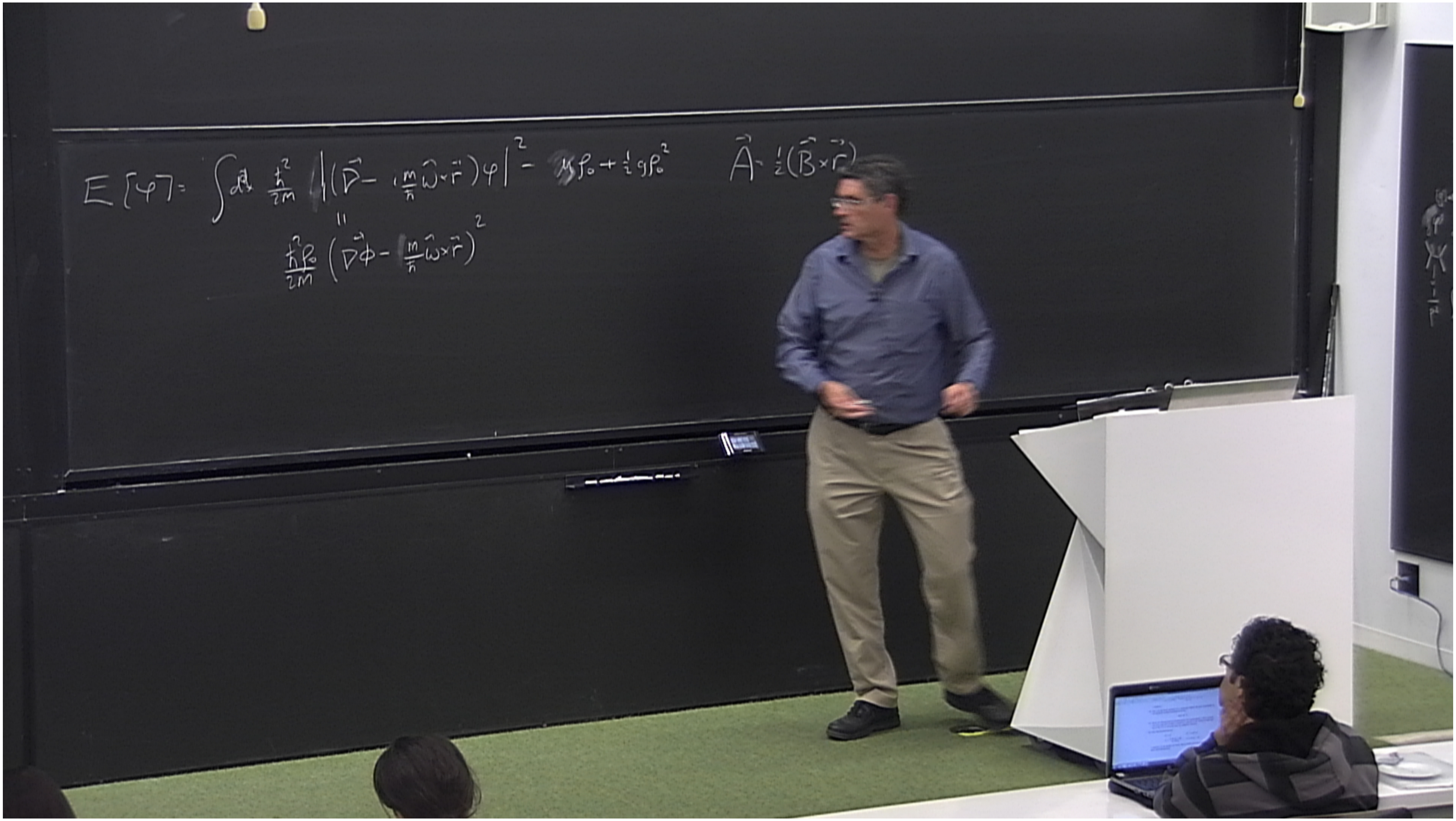

frame
ẑ ω̂ = ω·ẑ
r⊥
− mω² · r⊥²
cancelled by
potential

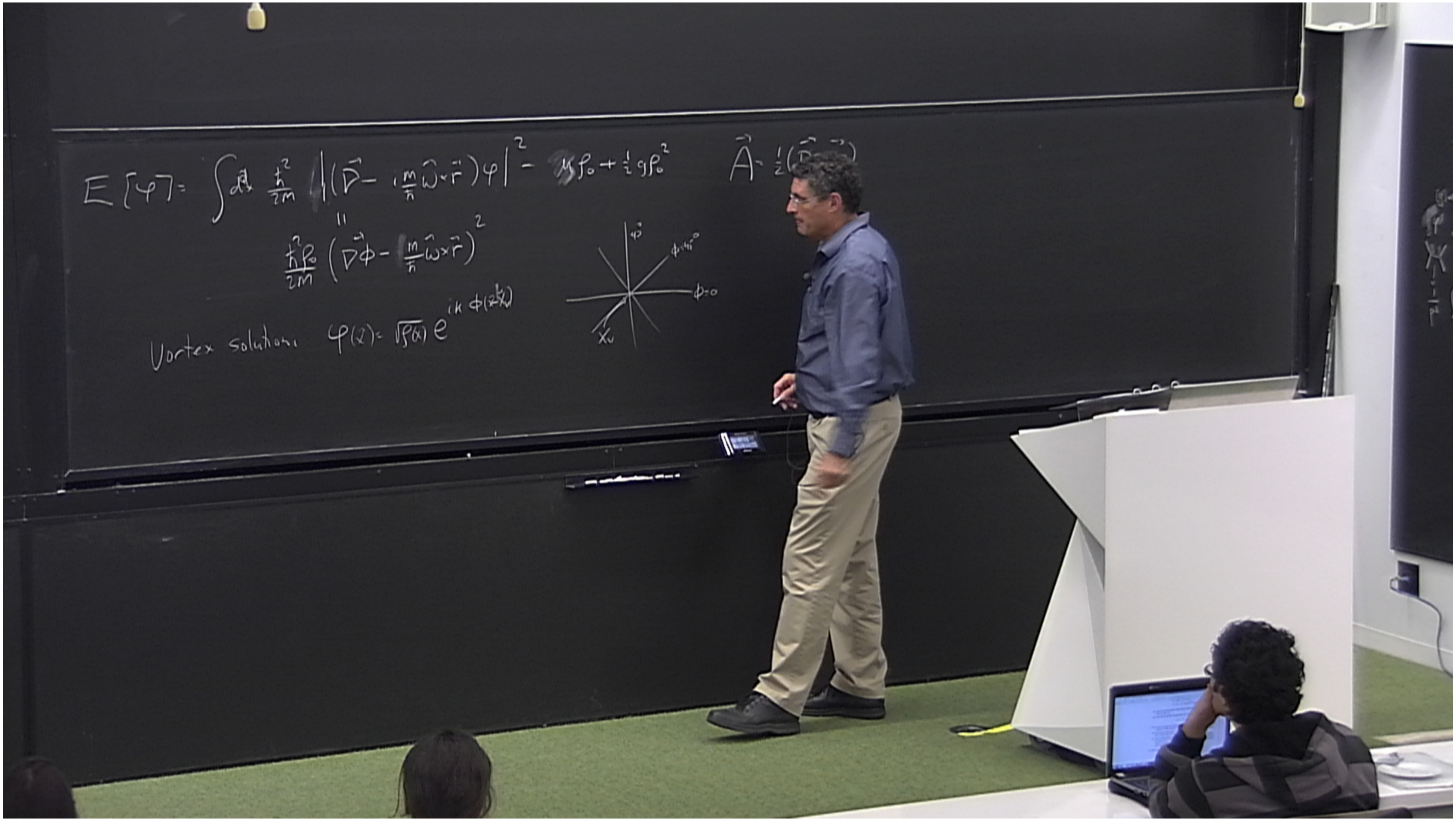

Rotating frame

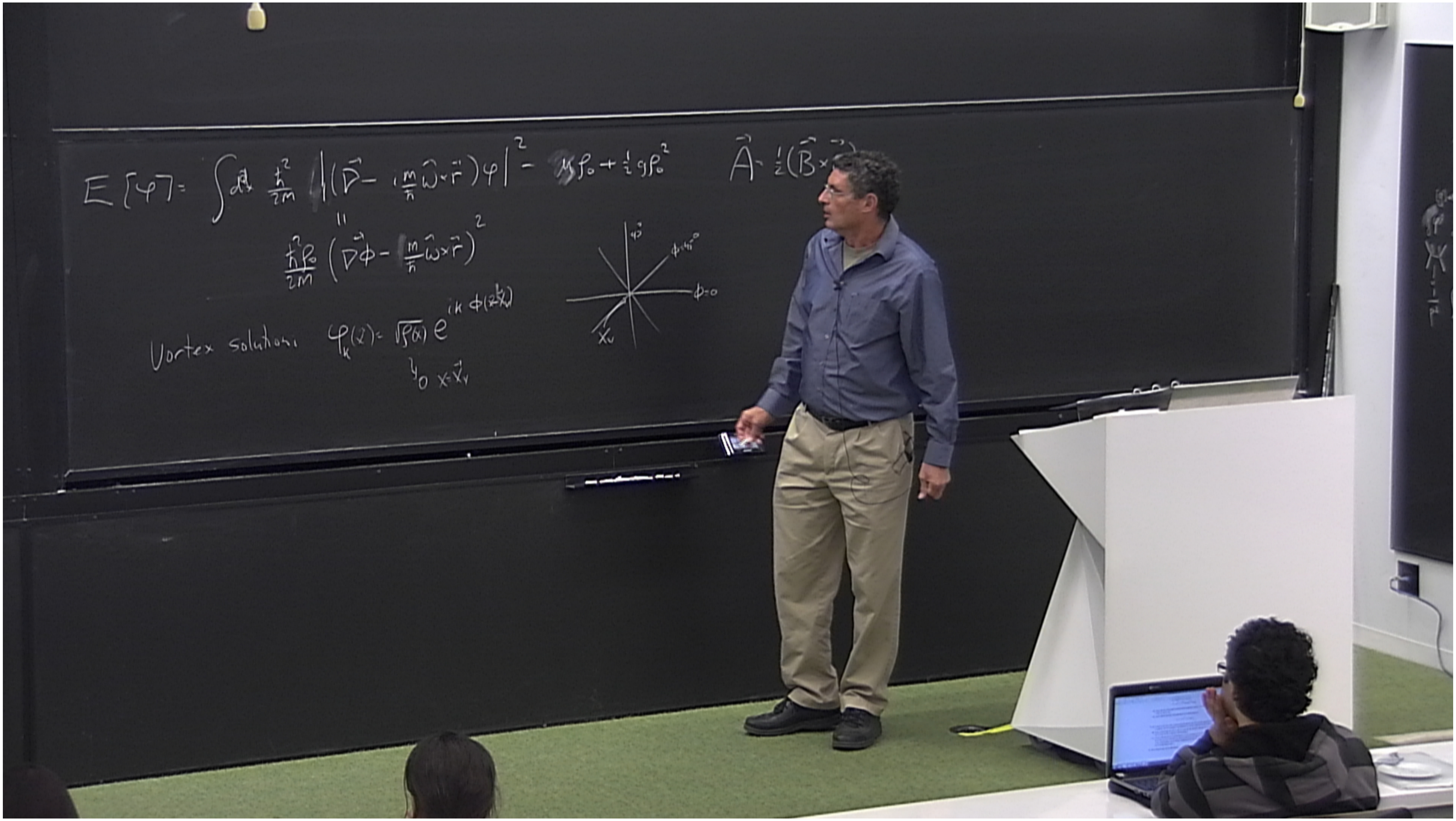

Rotating frame
ẑ ω⃗ = ω ẑ
r⊥
centrifugal: −mω² · r⊥²
cancelled by potential
Coriolis force:
F⃗ = 2m ω⃗ × v⃗ = "q/c" v⃗ × "B⃗" ← fictitious B
B⃗^cor = −2mc/q ω⃗

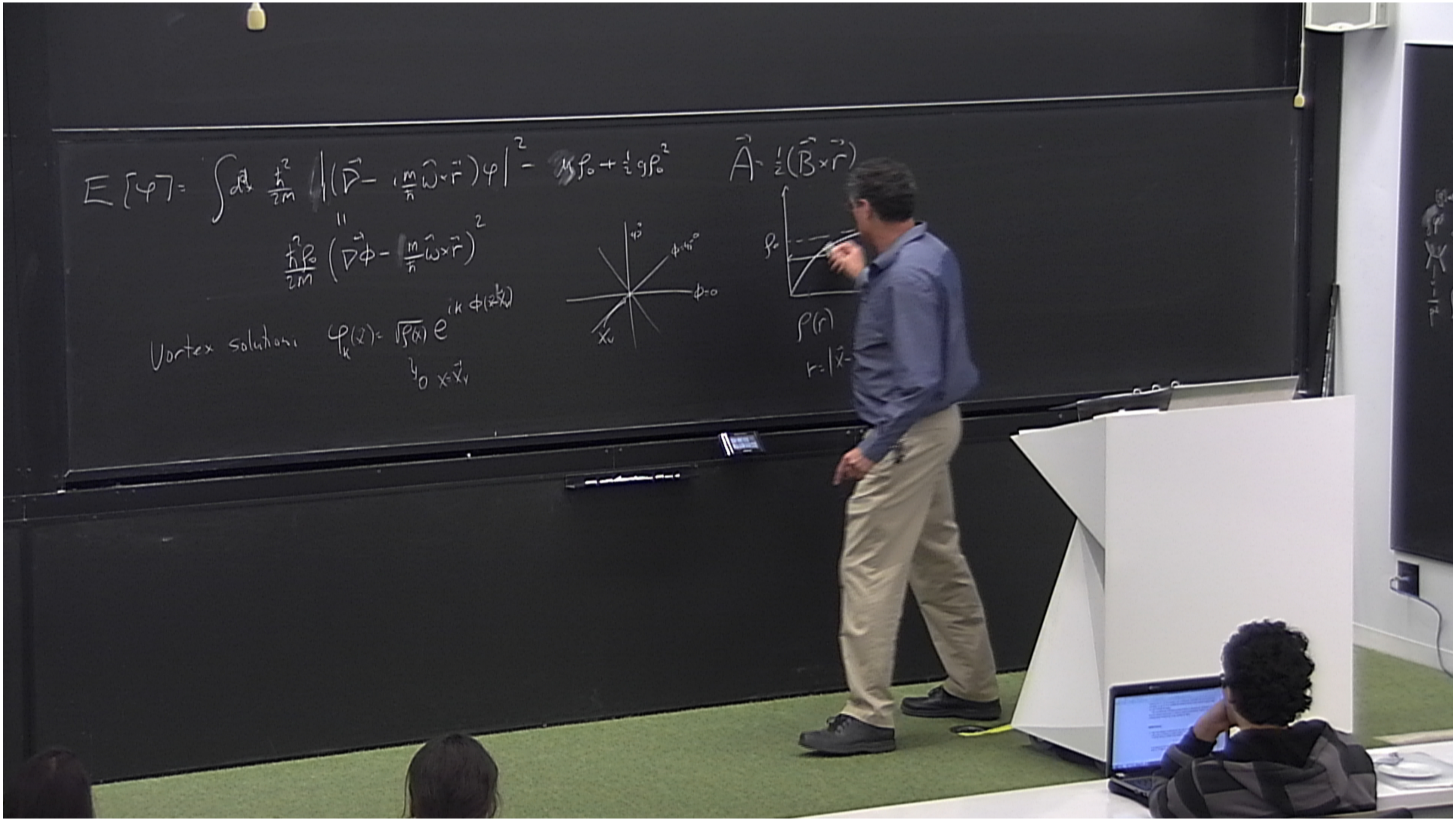


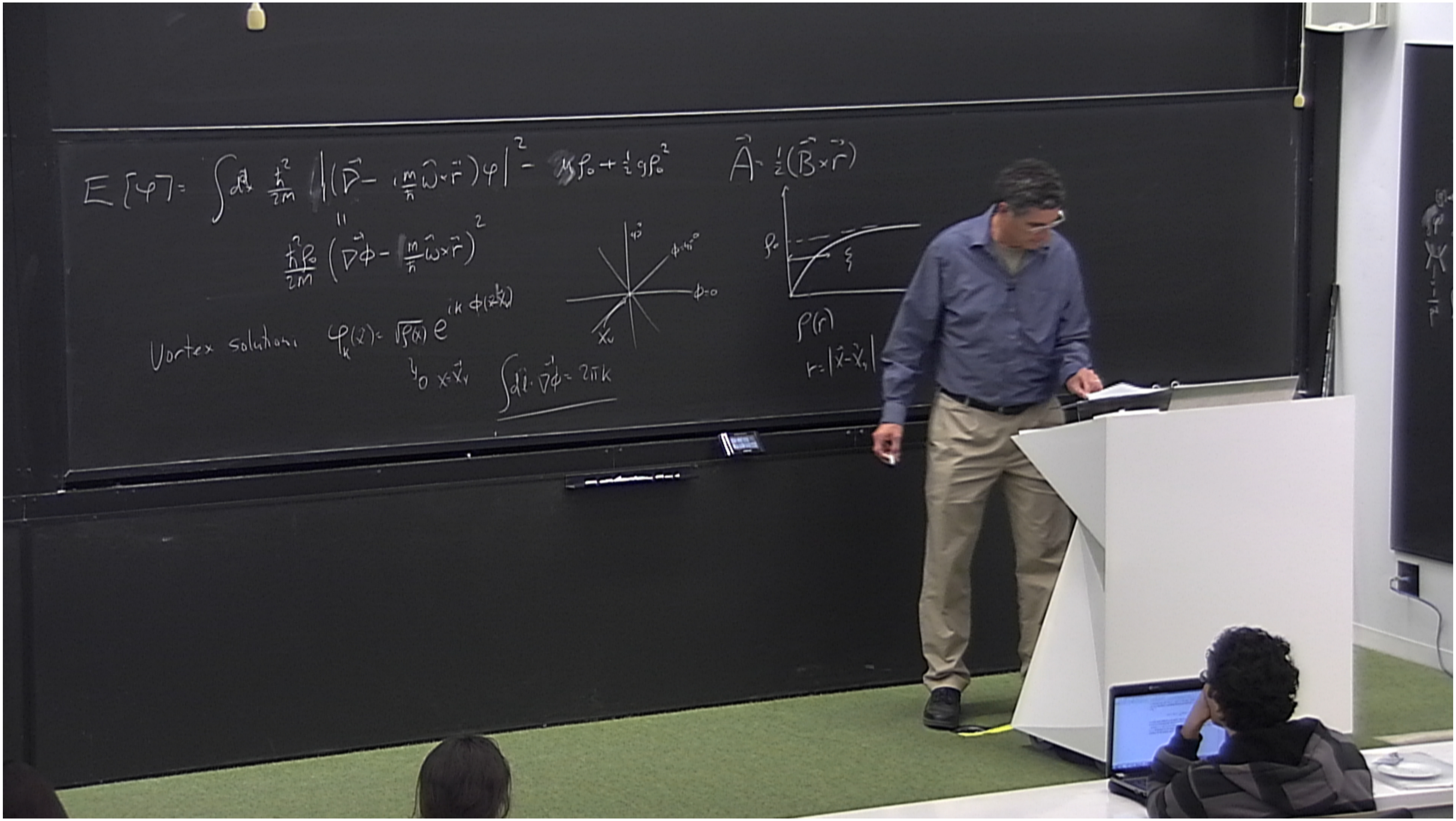
Rotating frame
centrifugal:
cancelled by
potential
Coriolis force;

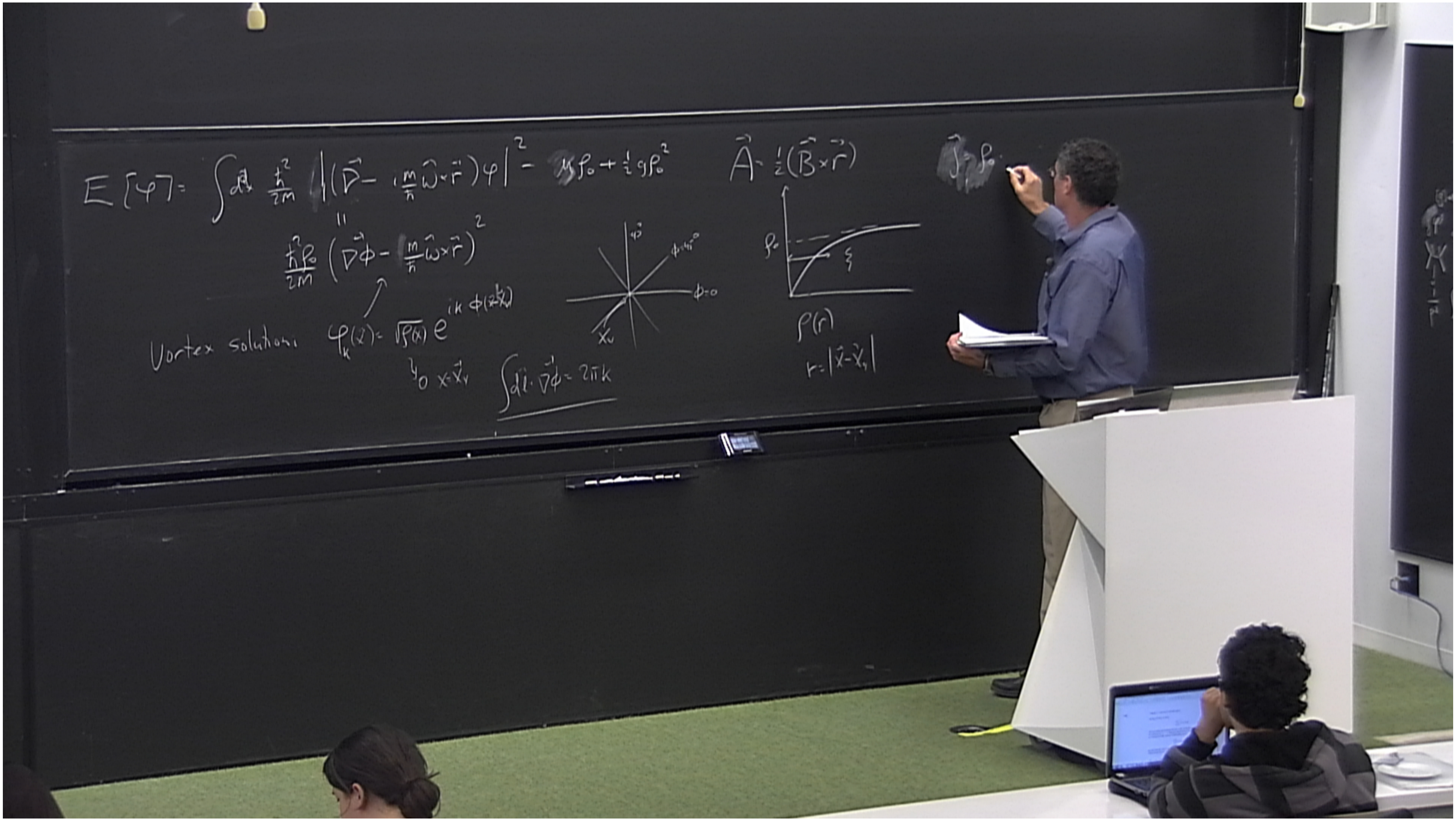

$E[\varphi] = \int d^3x \, \frac{\hbar^2}{2m} \left| \left(\vec{\nabla} - i\frac{m}{\hbar}\hat{\omega}\times\vec{r} \right)\varphi \right|^2 - \mu\rho_0 + \frac{1}{2} g \rho_0^2$
$\vec{A} = \frac{1}{2}(\vec{B}\times\vec{r})$
$\frac{\hbar^2\rho_0}{2m}\left(\vec{\nabla}\phi - \frac{m}{\hbar}\hat{\omega}\times\vec{r}\right)^2$

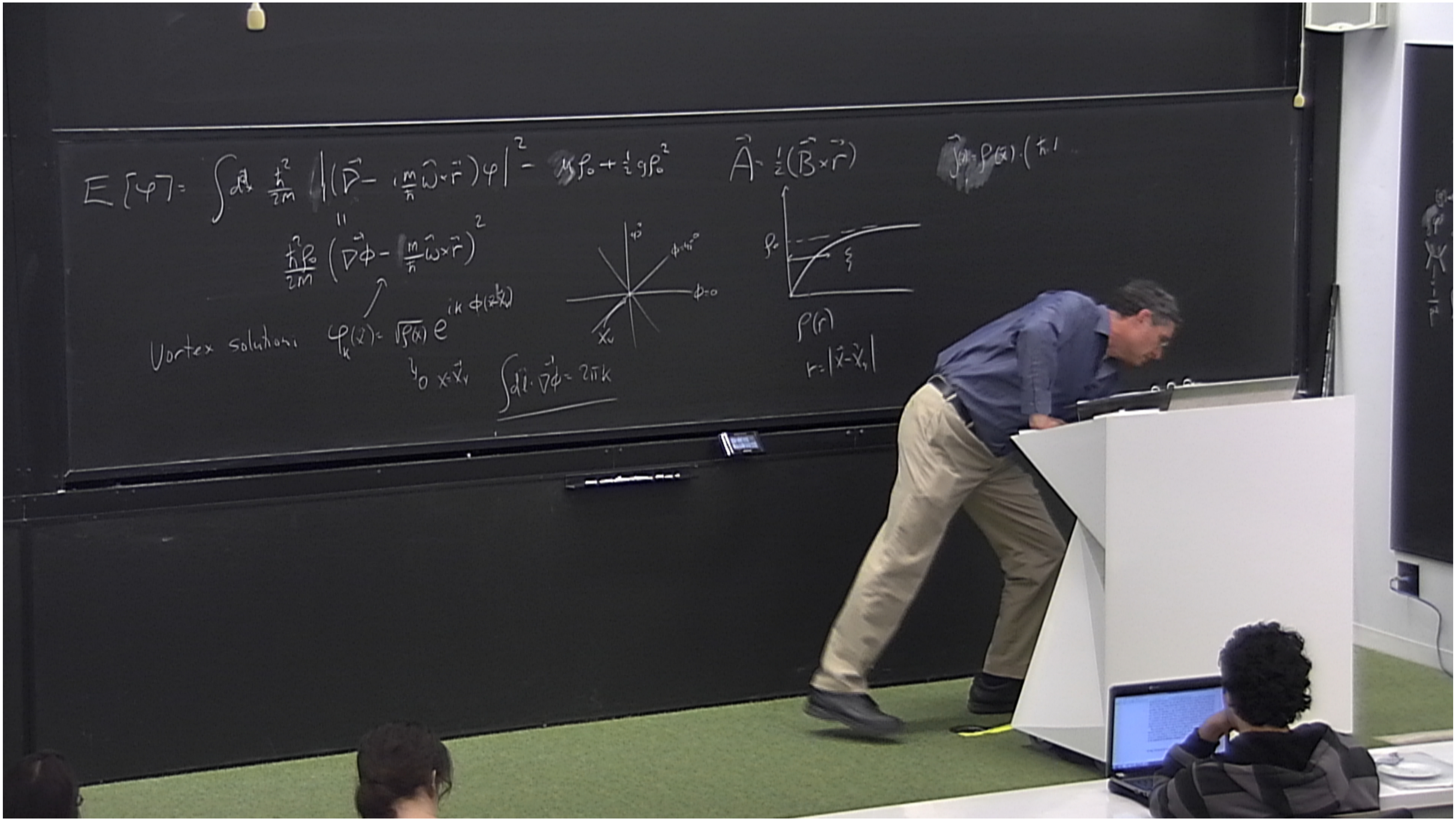

Vortex solution

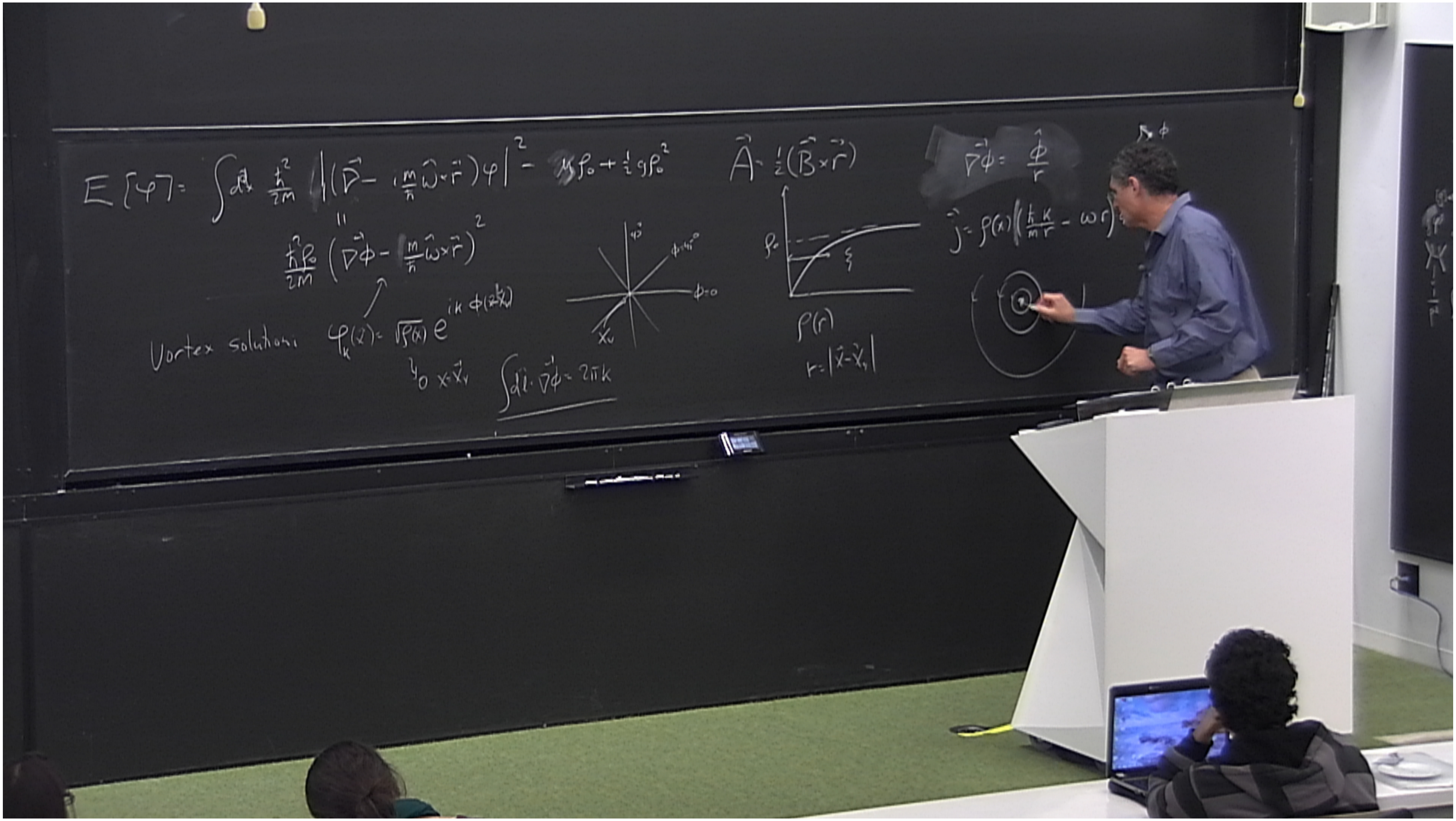

Vortex solution

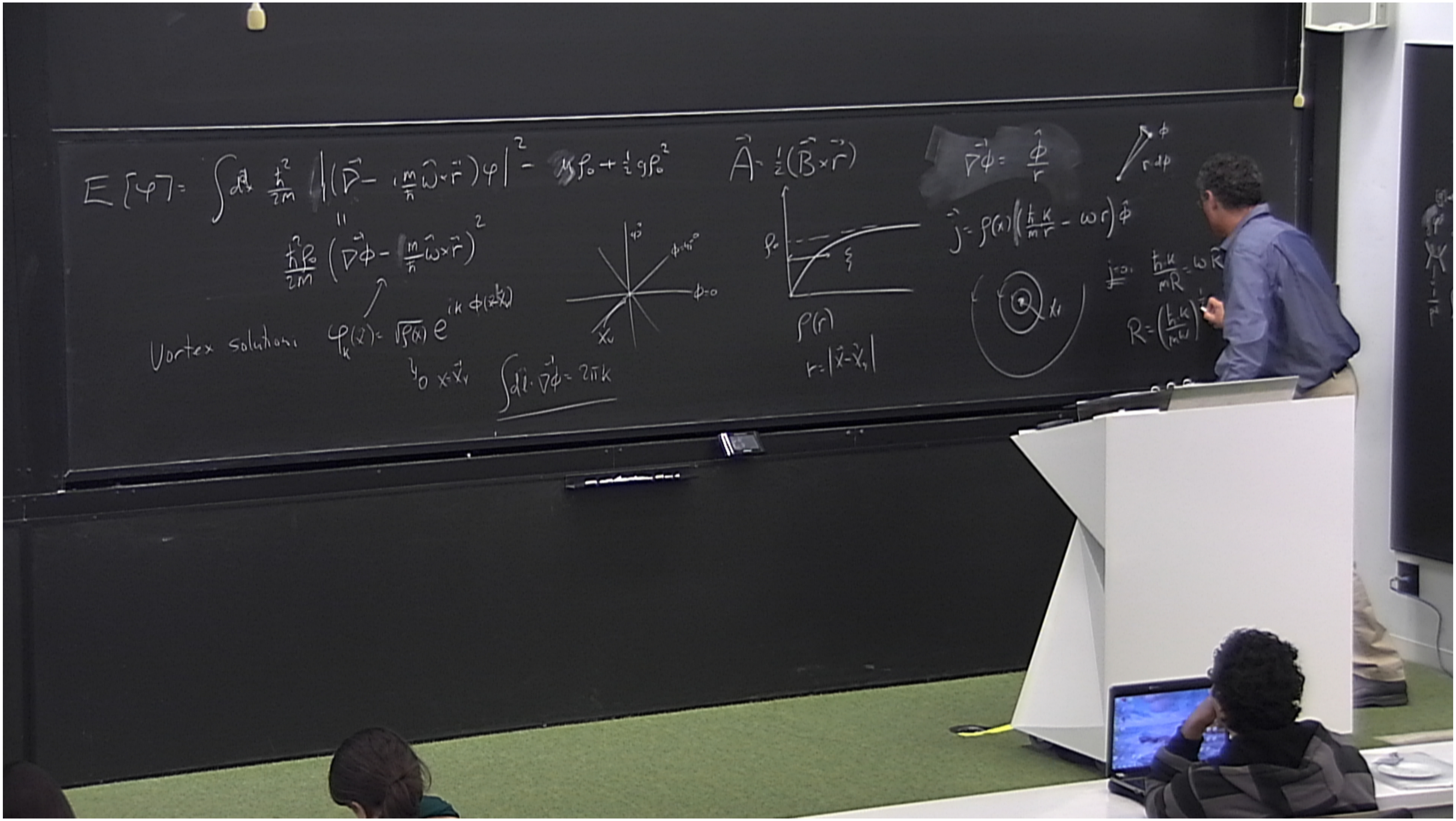


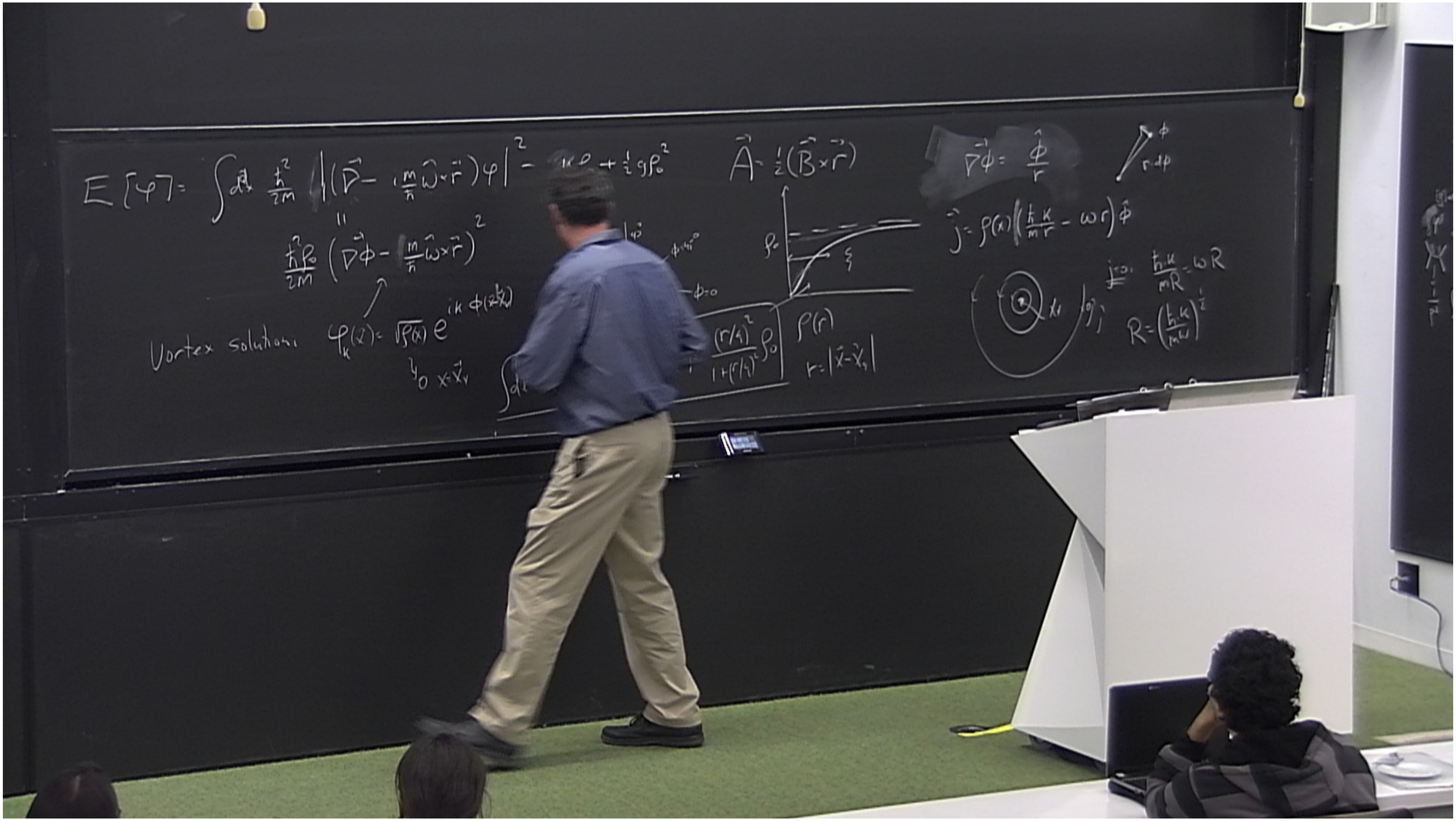

Vortex solution

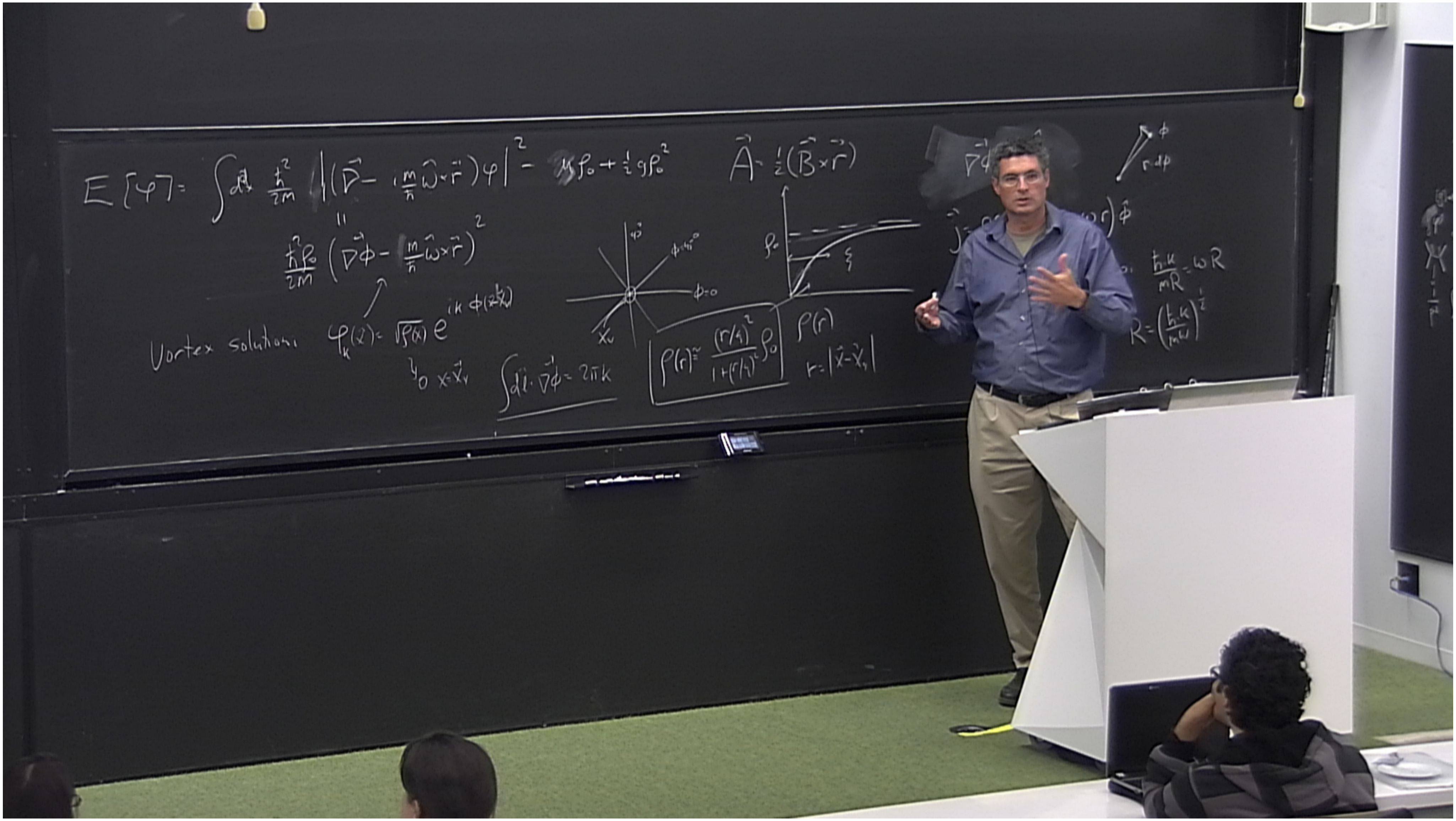

Vortex solutions
$\int d\vec{l}\cdot\vec{\nabla}\phi = 2\pi k$
$\rho(r)$
$r = |\vec{x}-\vec{x}_v|$

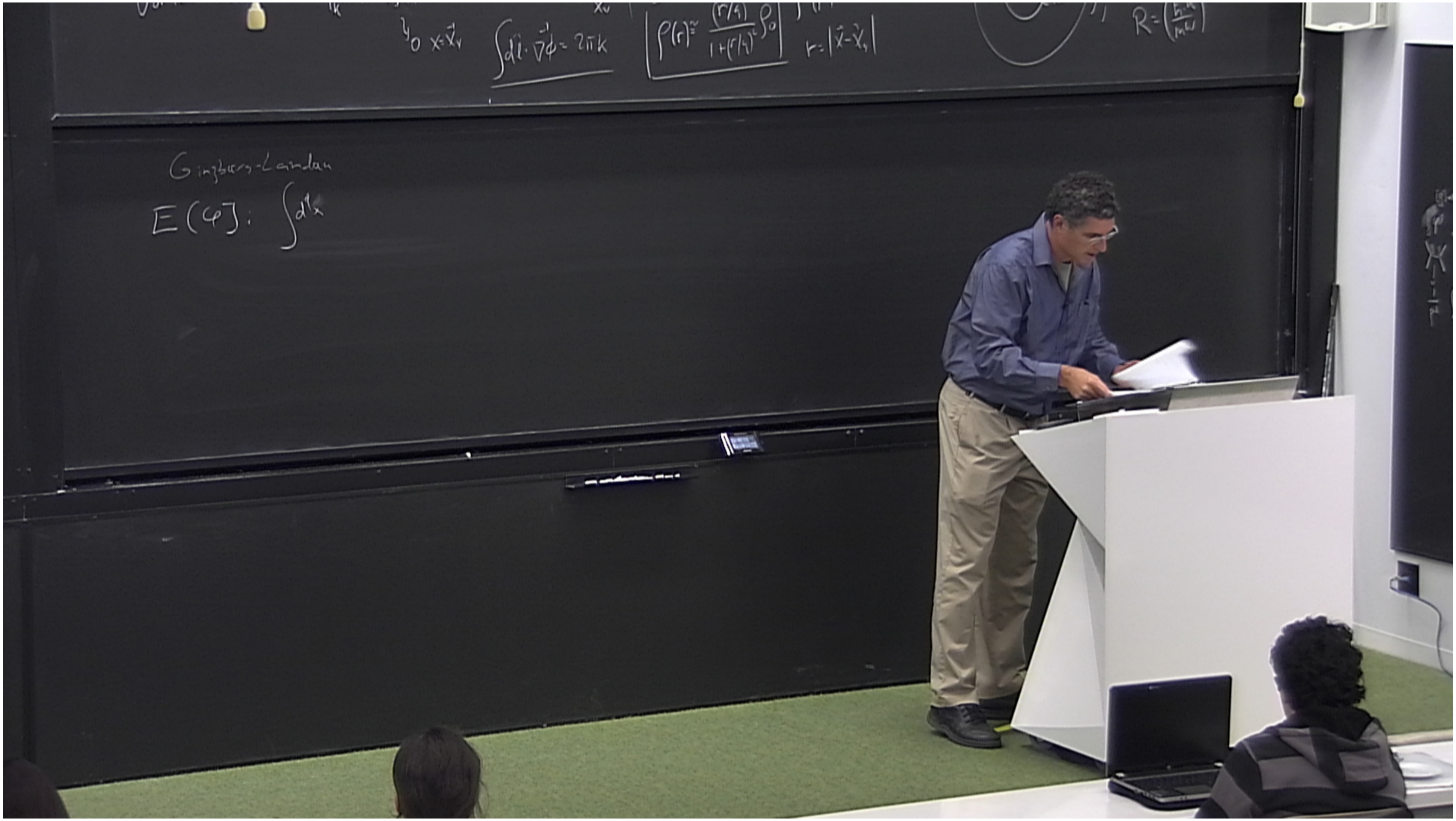

Vortex solution:
∮dl · ∇φ = 2πk
ρ(r)
r = |x − x_v|

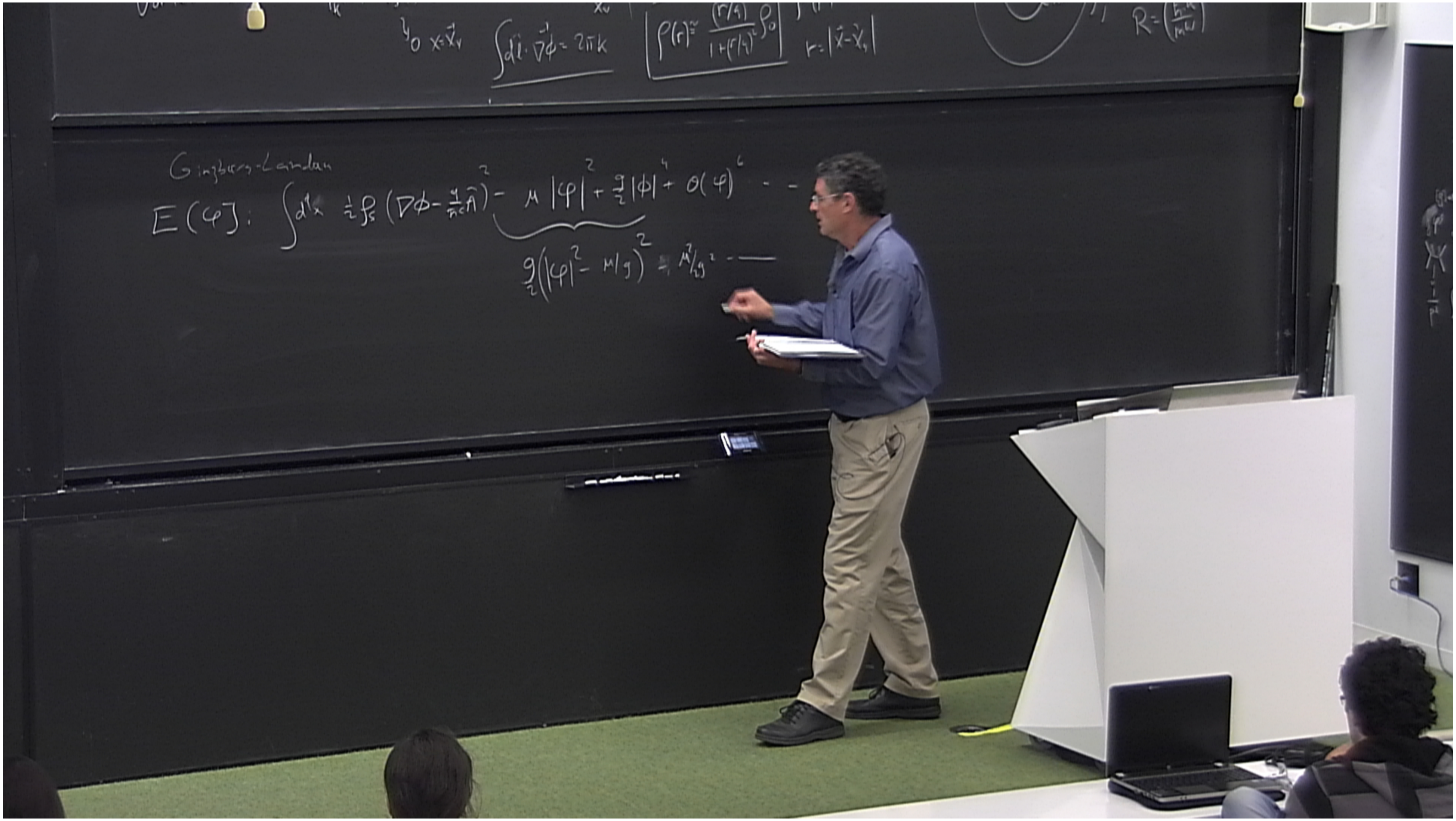

Vortex solution

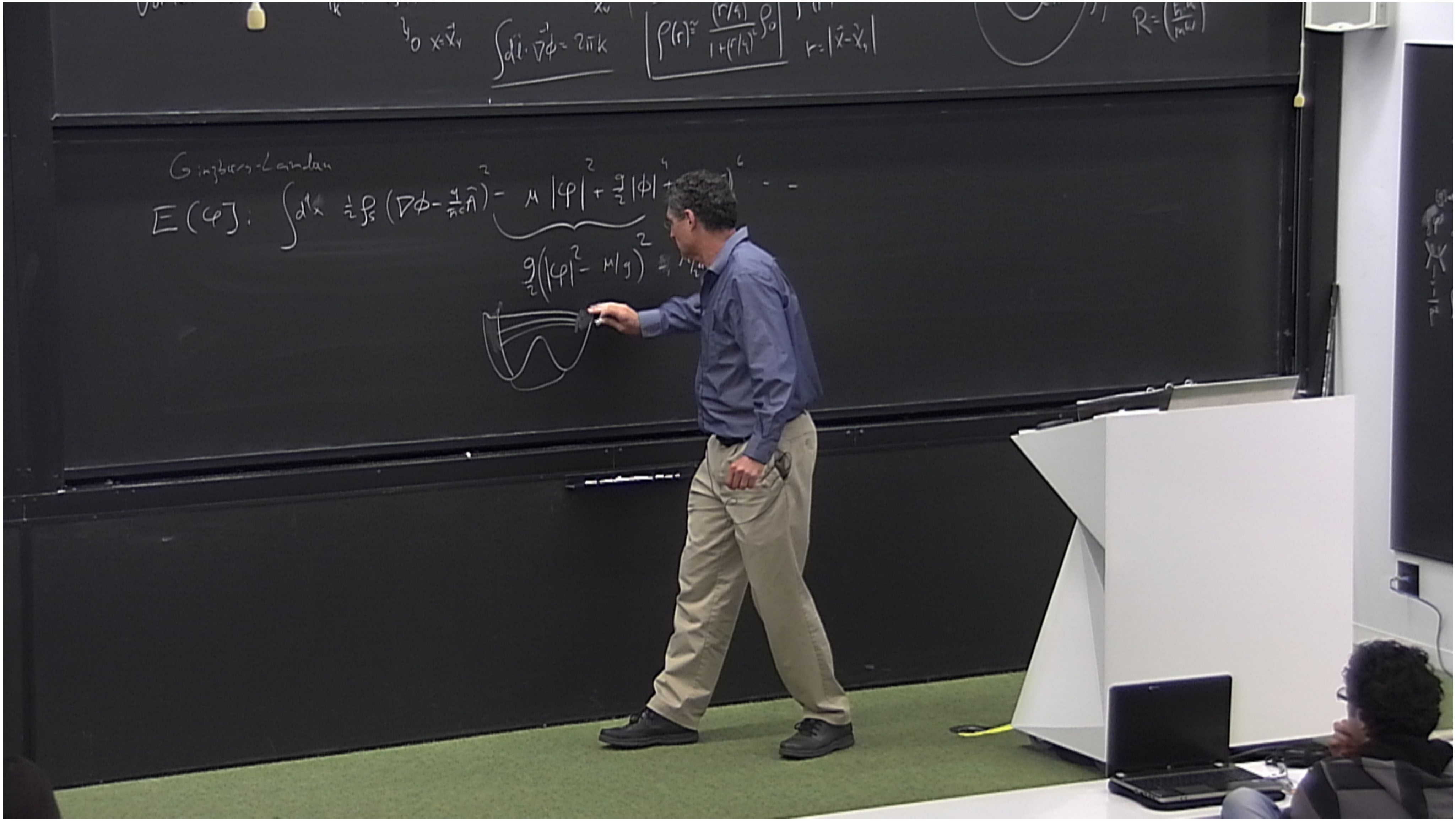


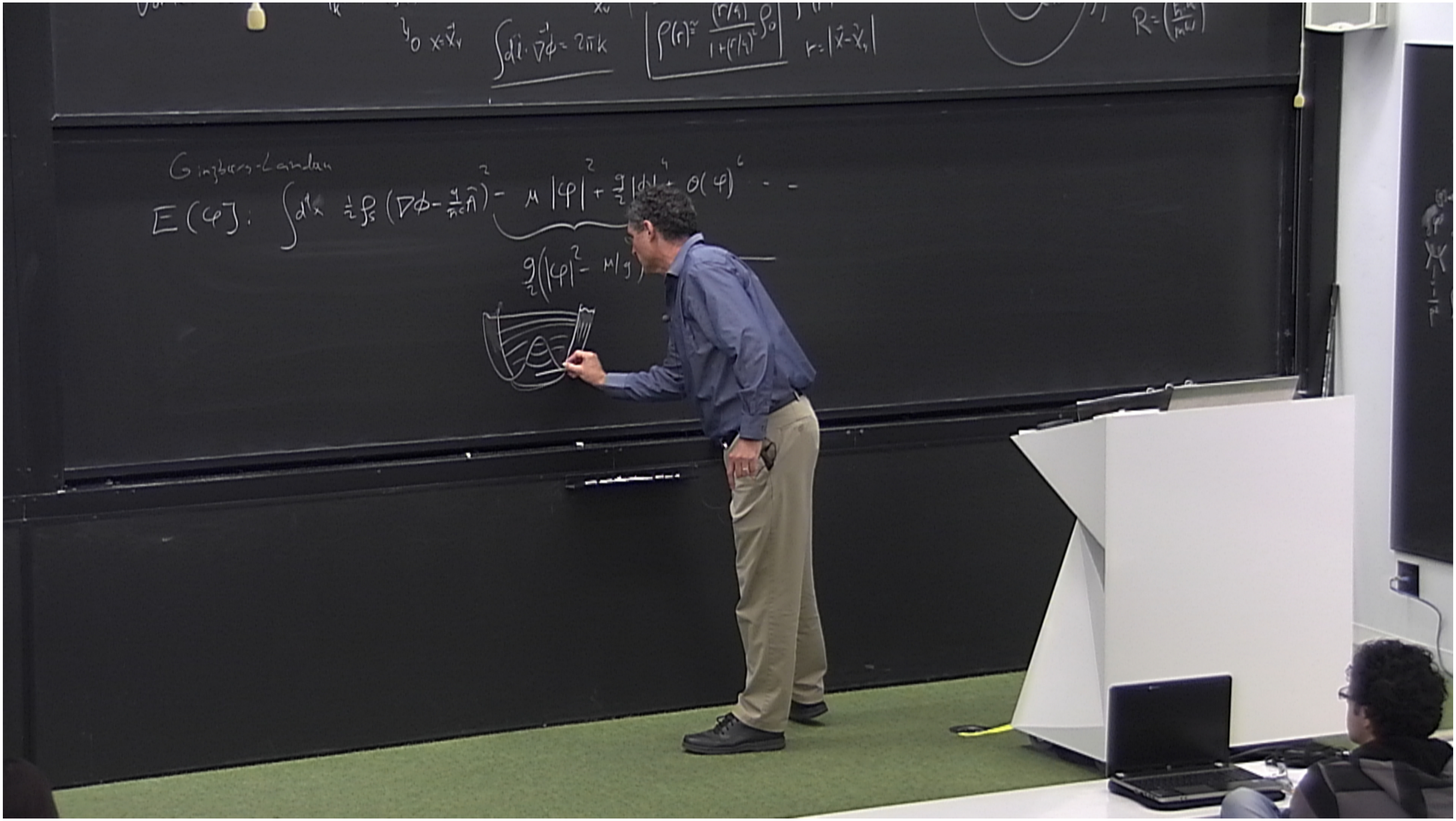


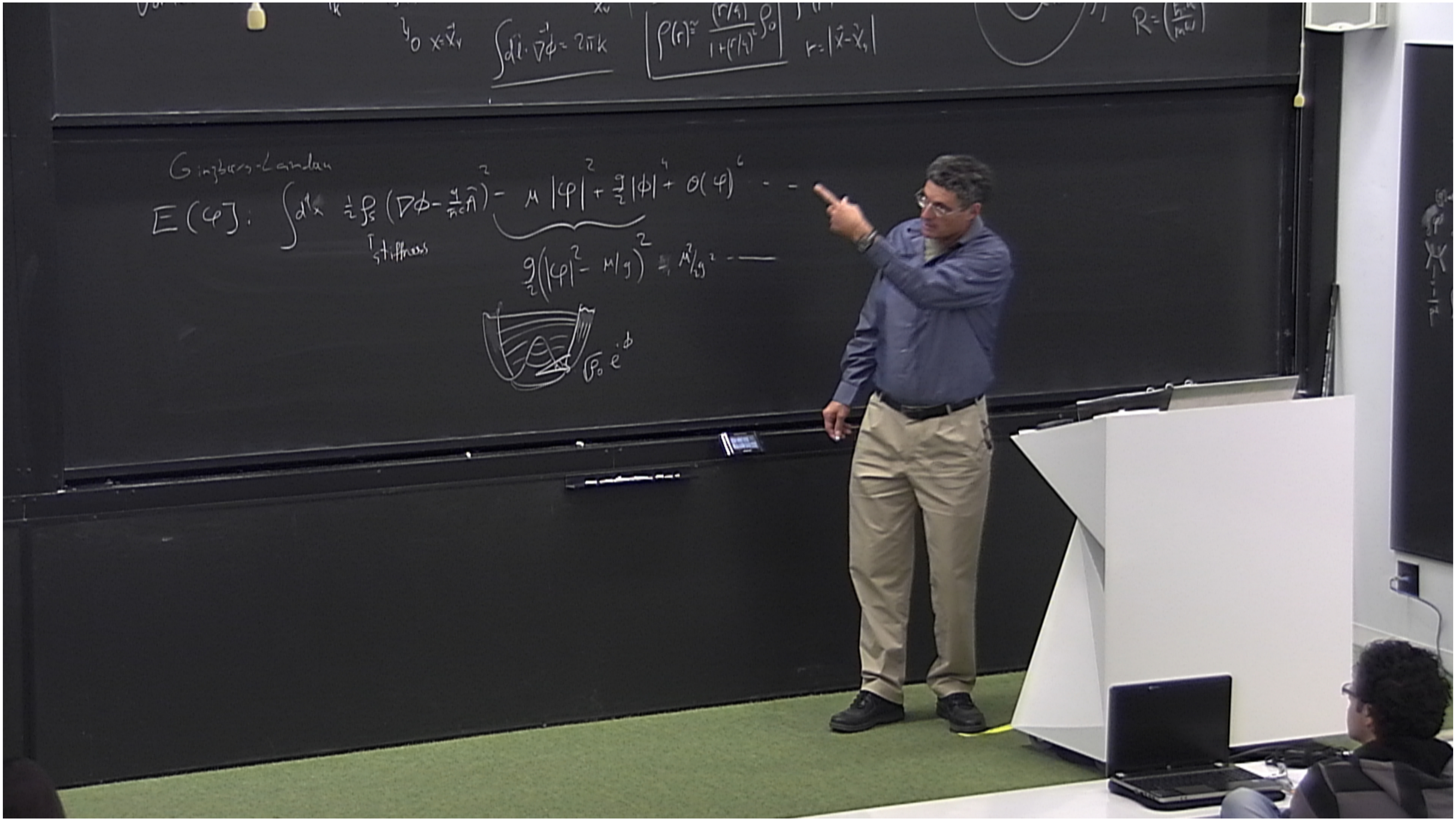

Vortex solution

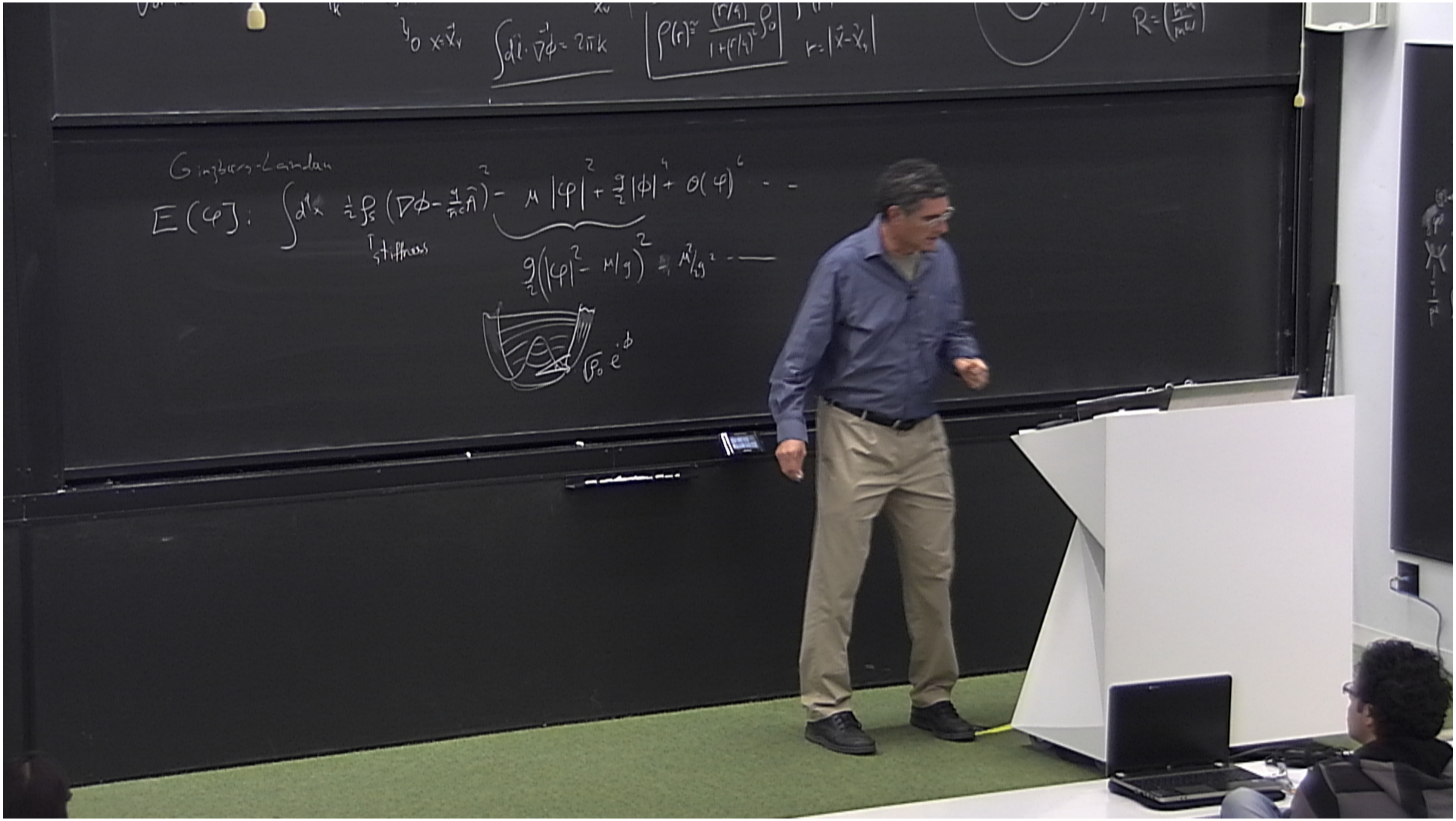

Ginzburg-Landau
E[φ]: ∫d^dx

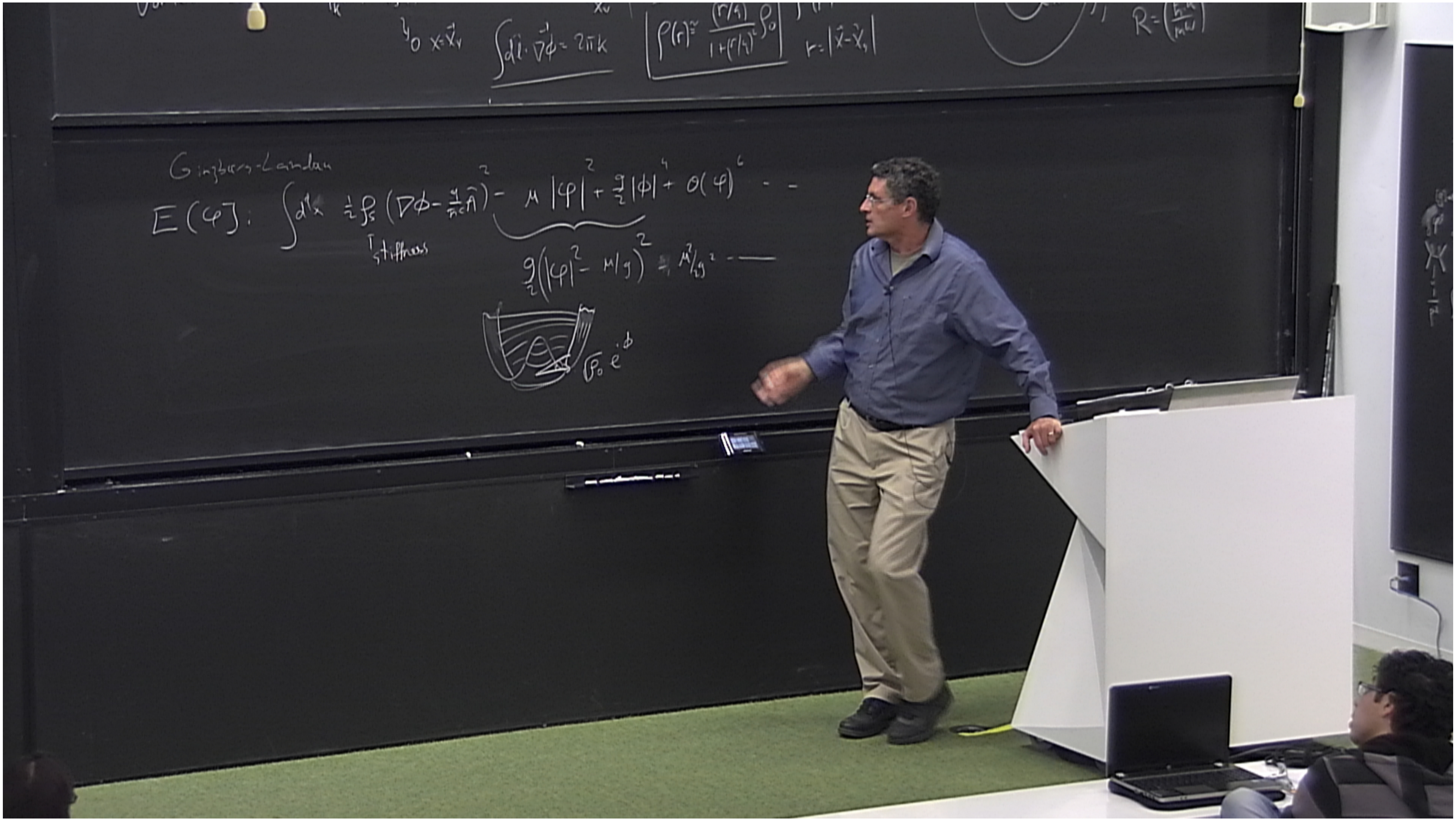

Ginzburg-Landau

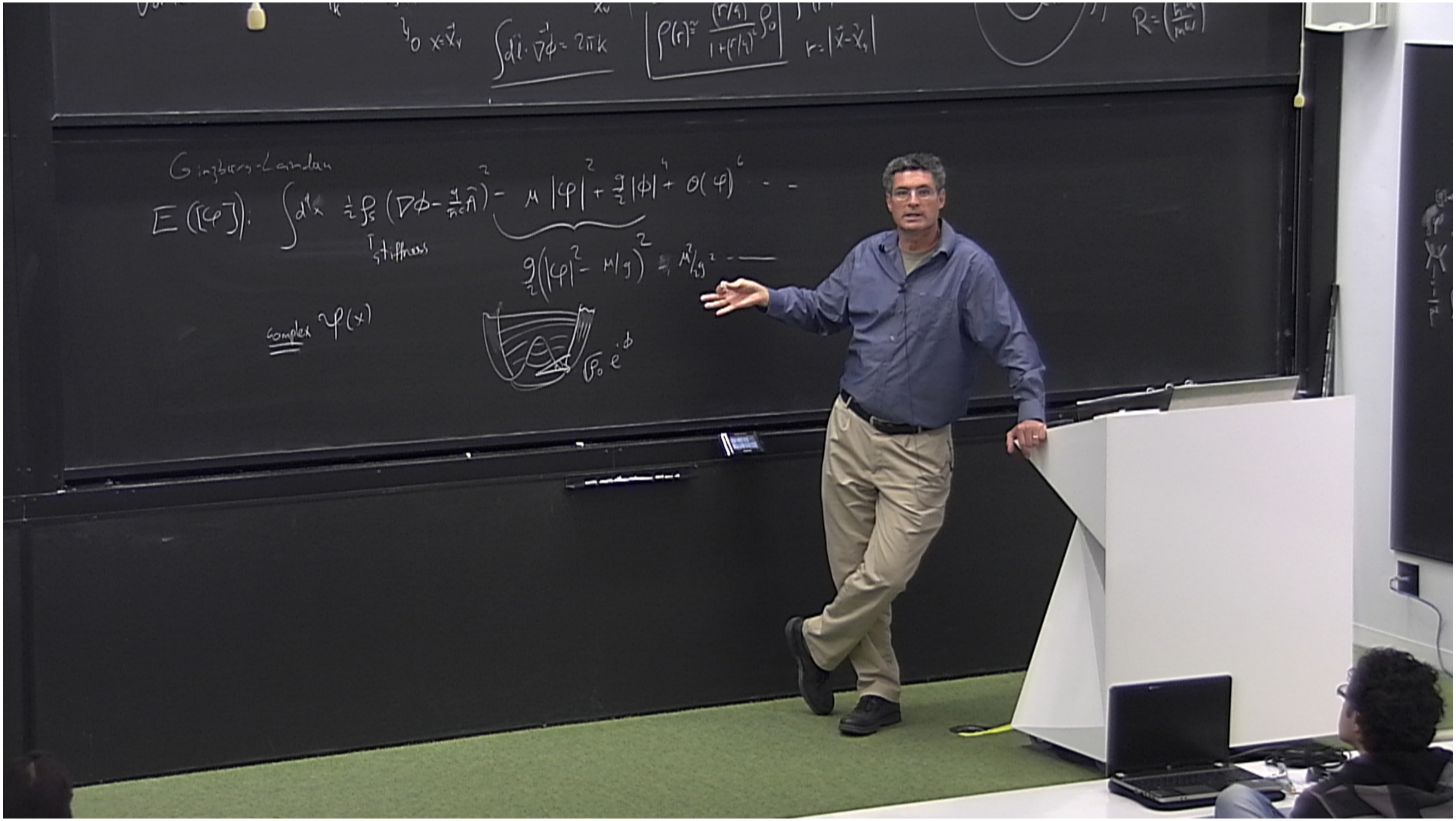

Ginzburg-Landau

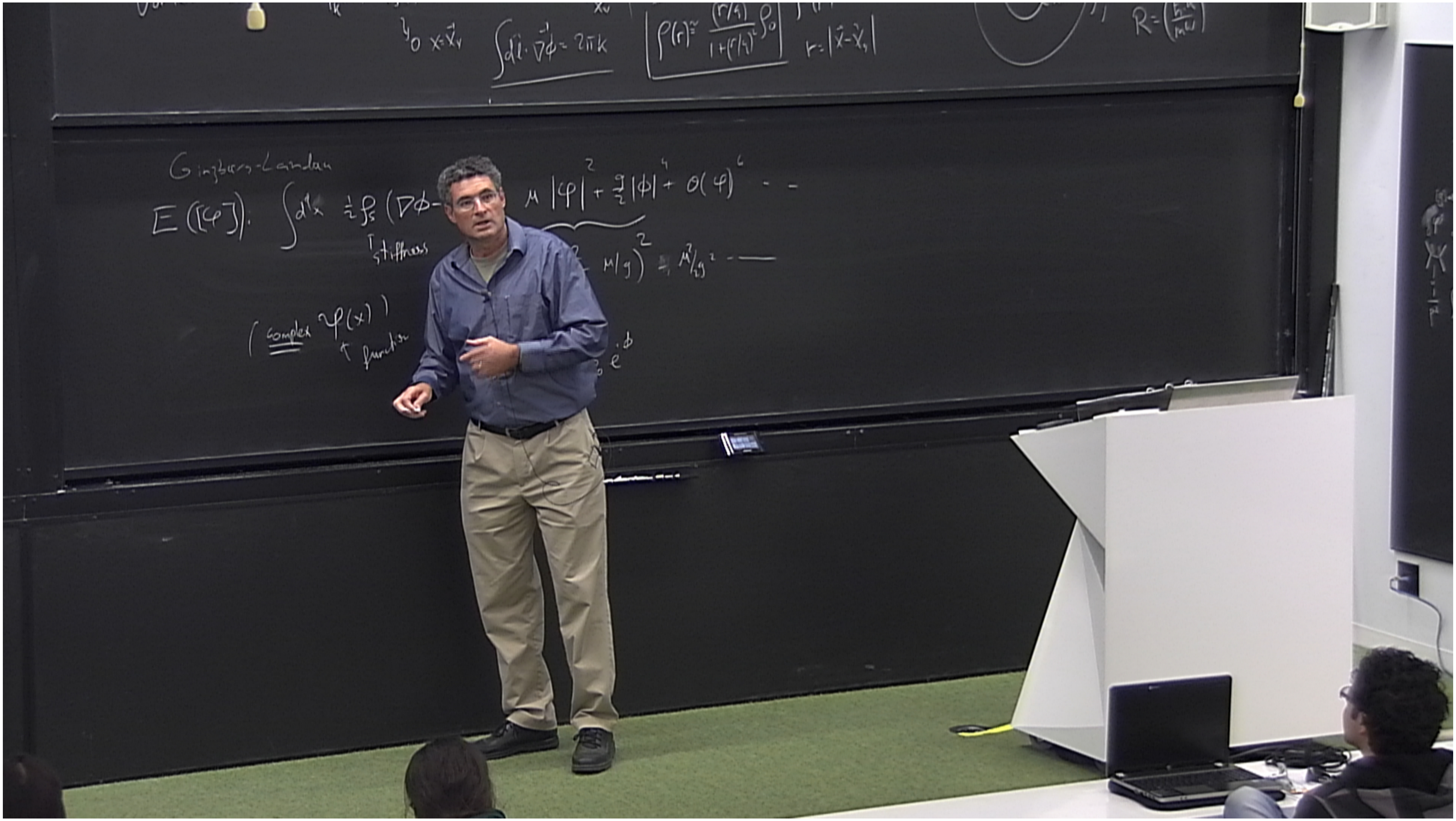

Ginzburg-Landau

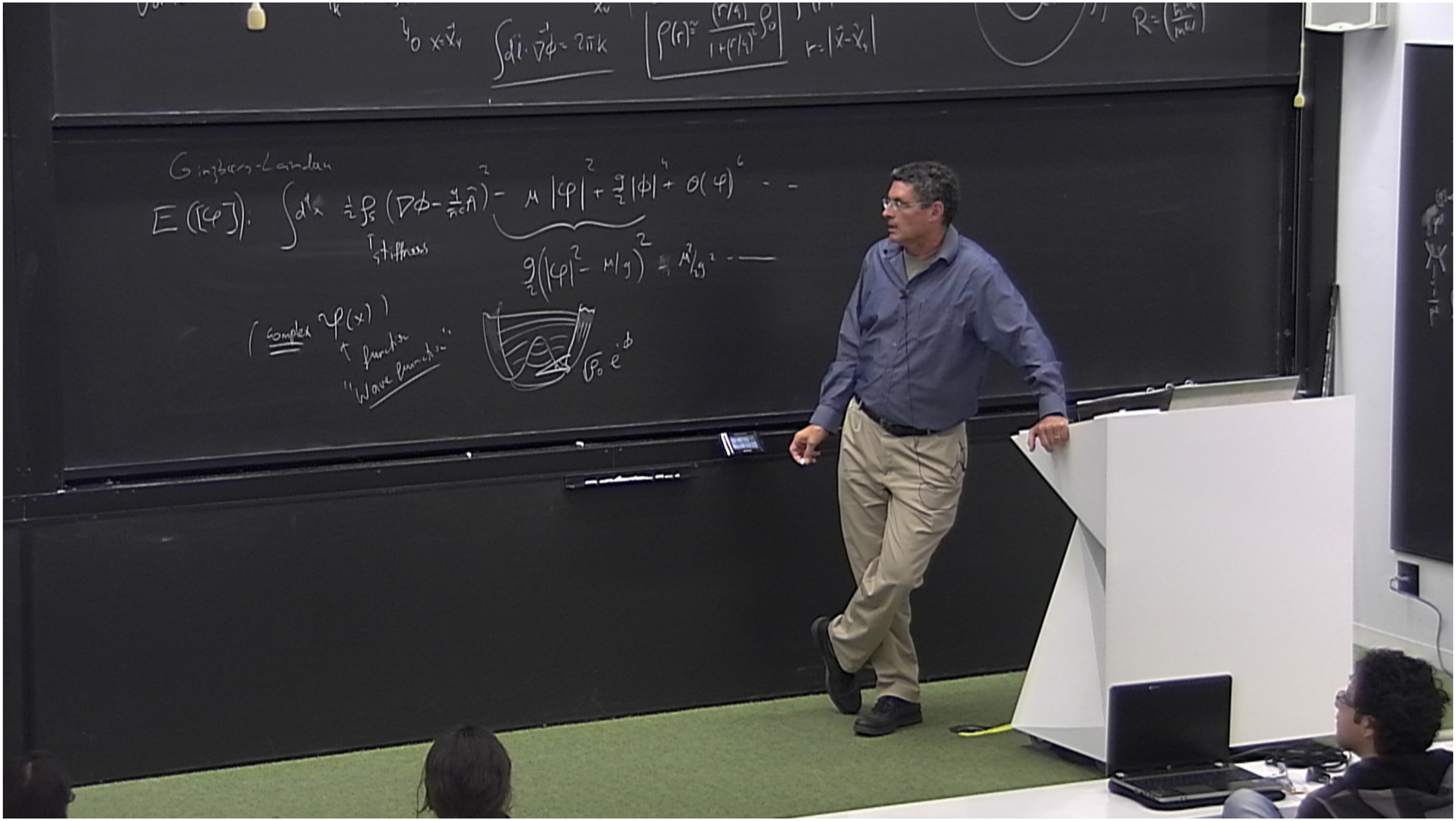


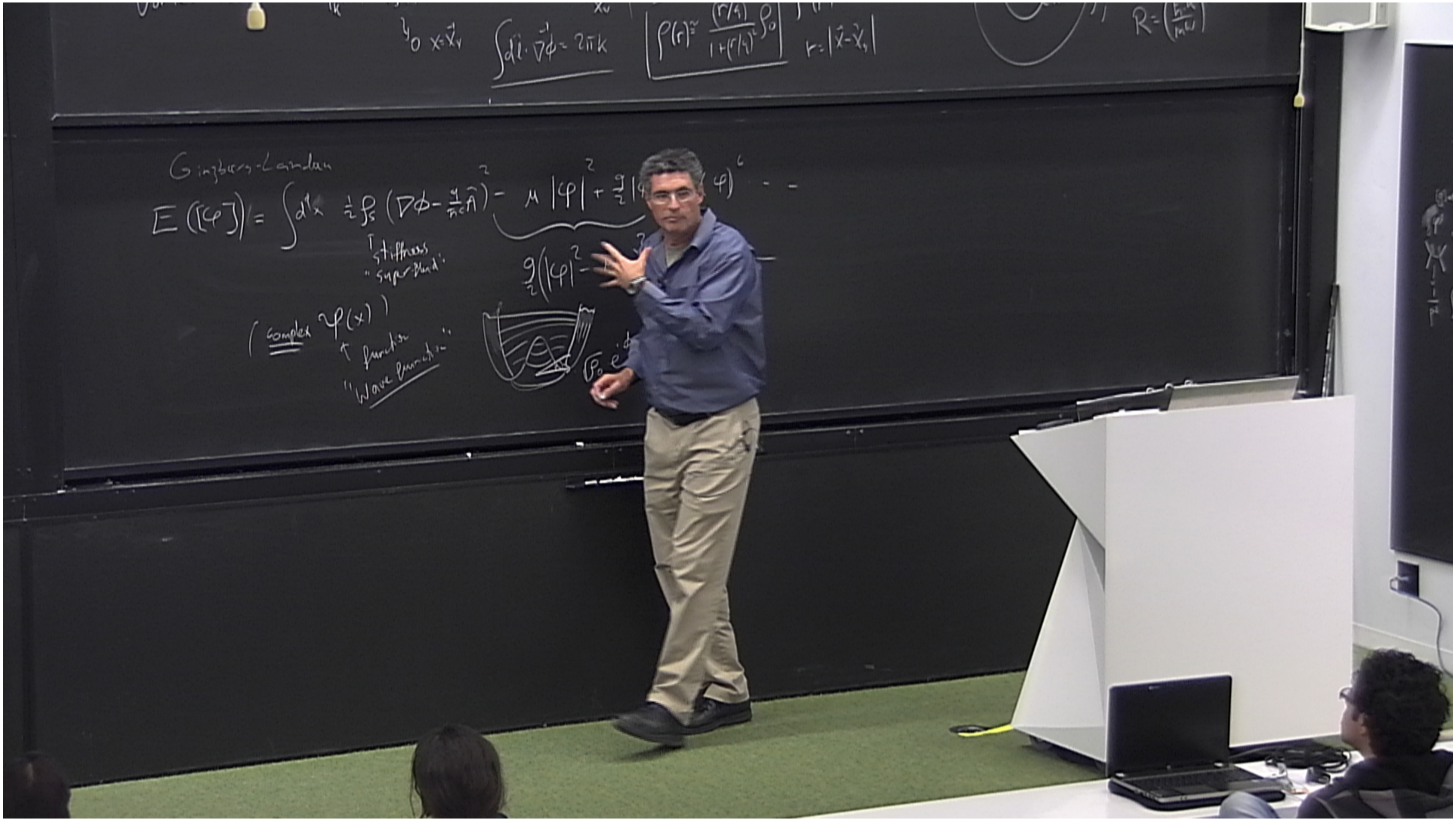

Ginzburg-Landau
stiffness

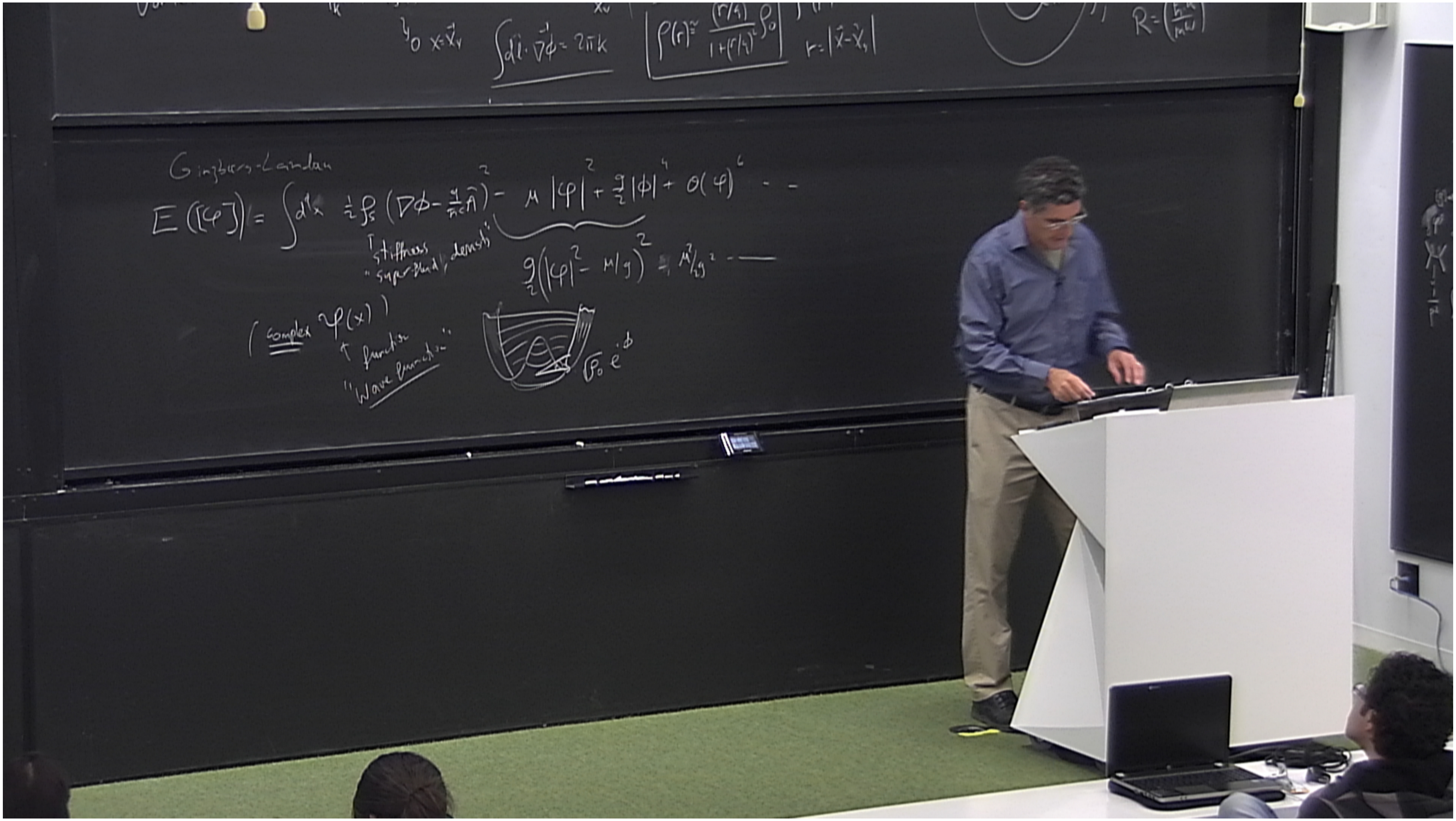

Ginzburg-Landau
stiffness

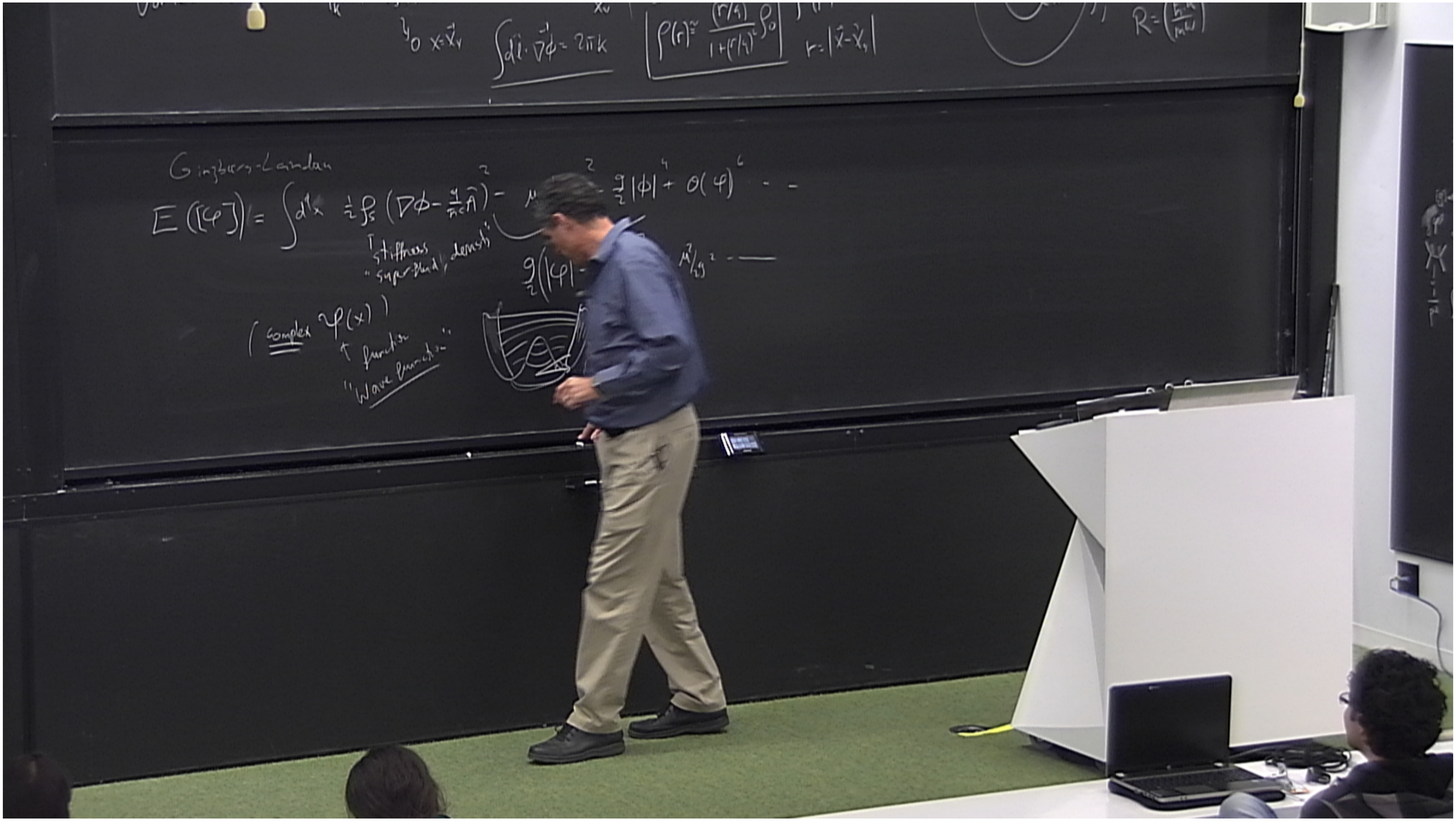

Ginzburg-Landau
complex φ(x)
stiffness

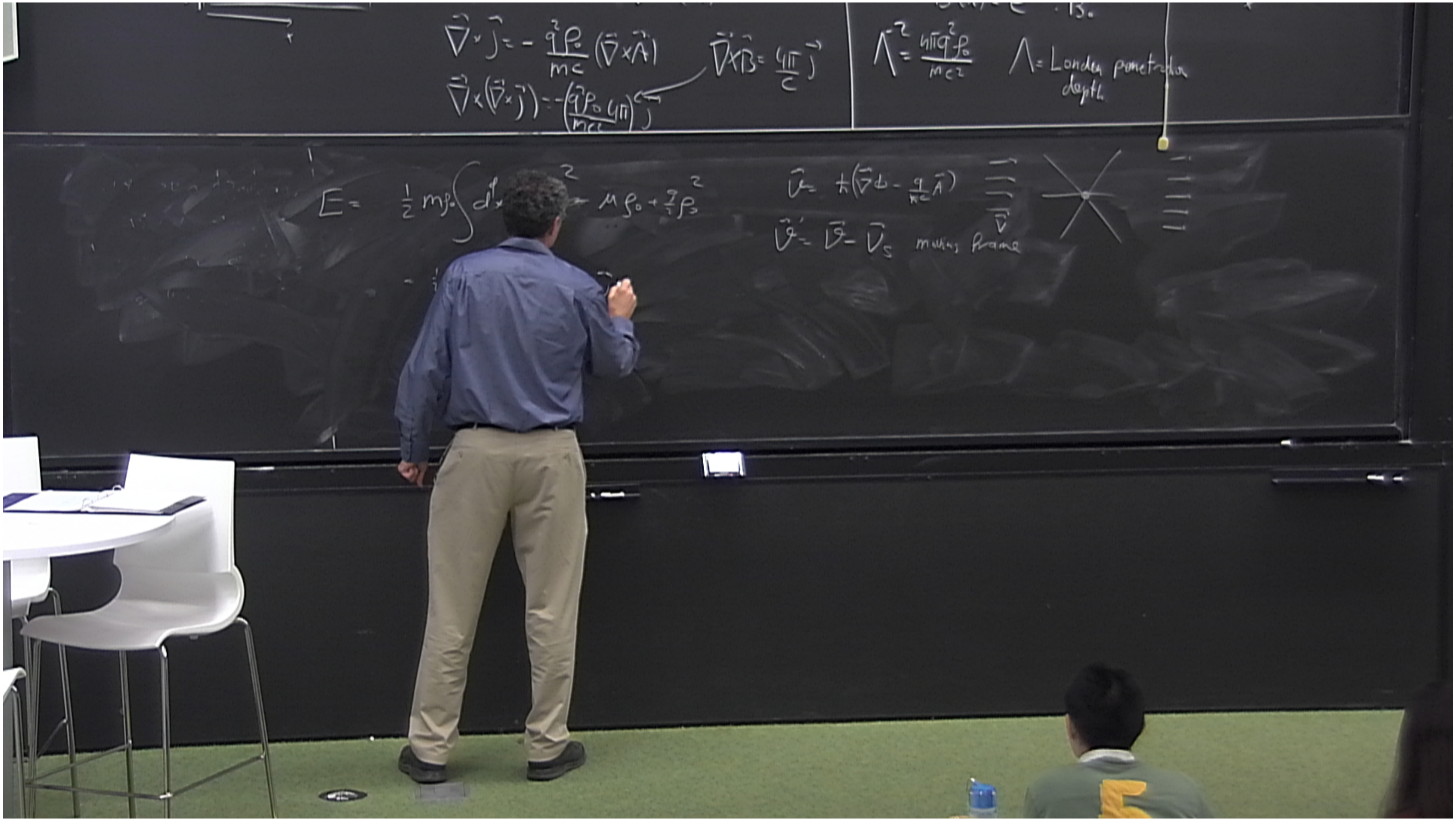

Ginzburg-Landau
stiffness
complex
function

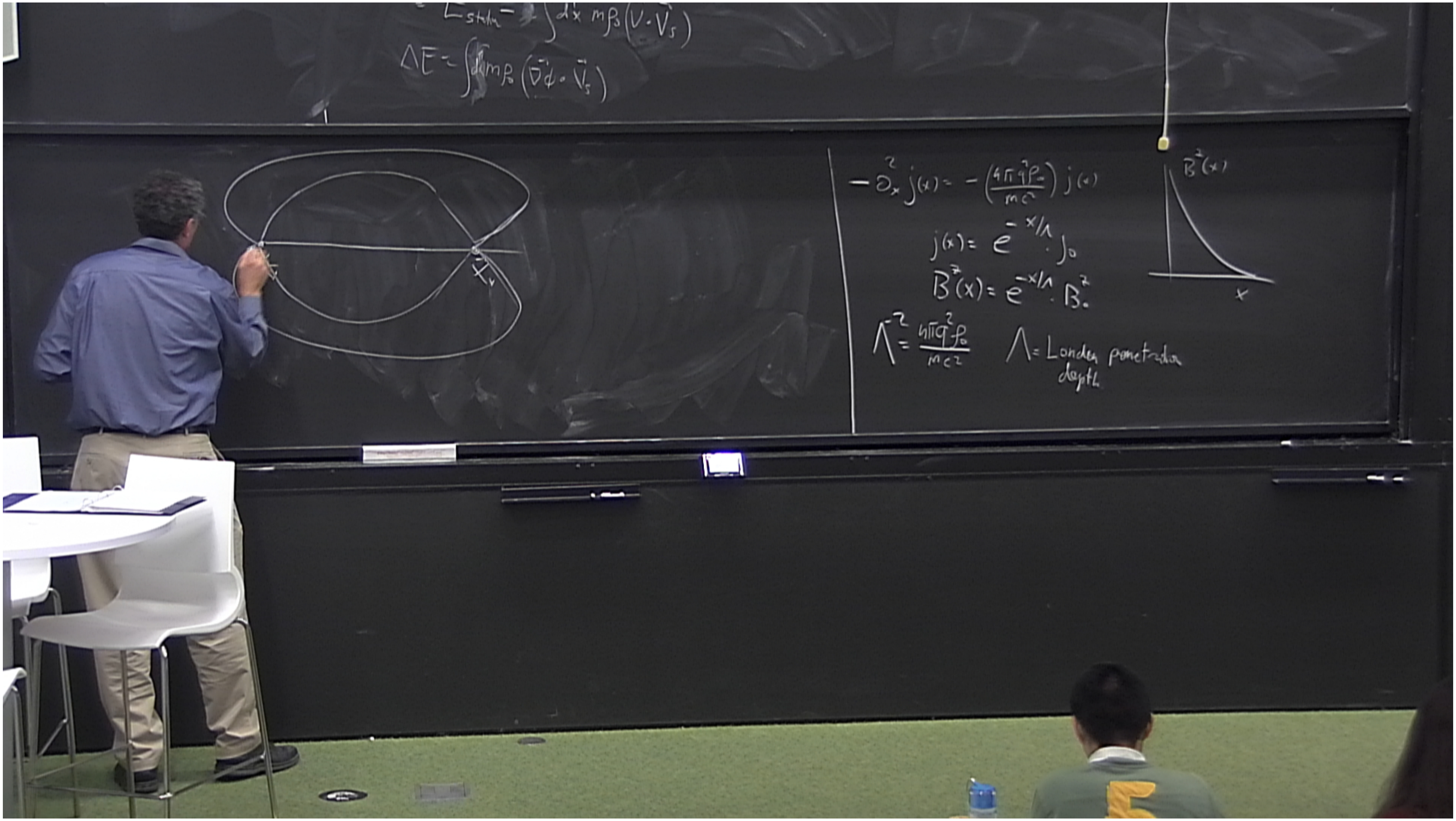


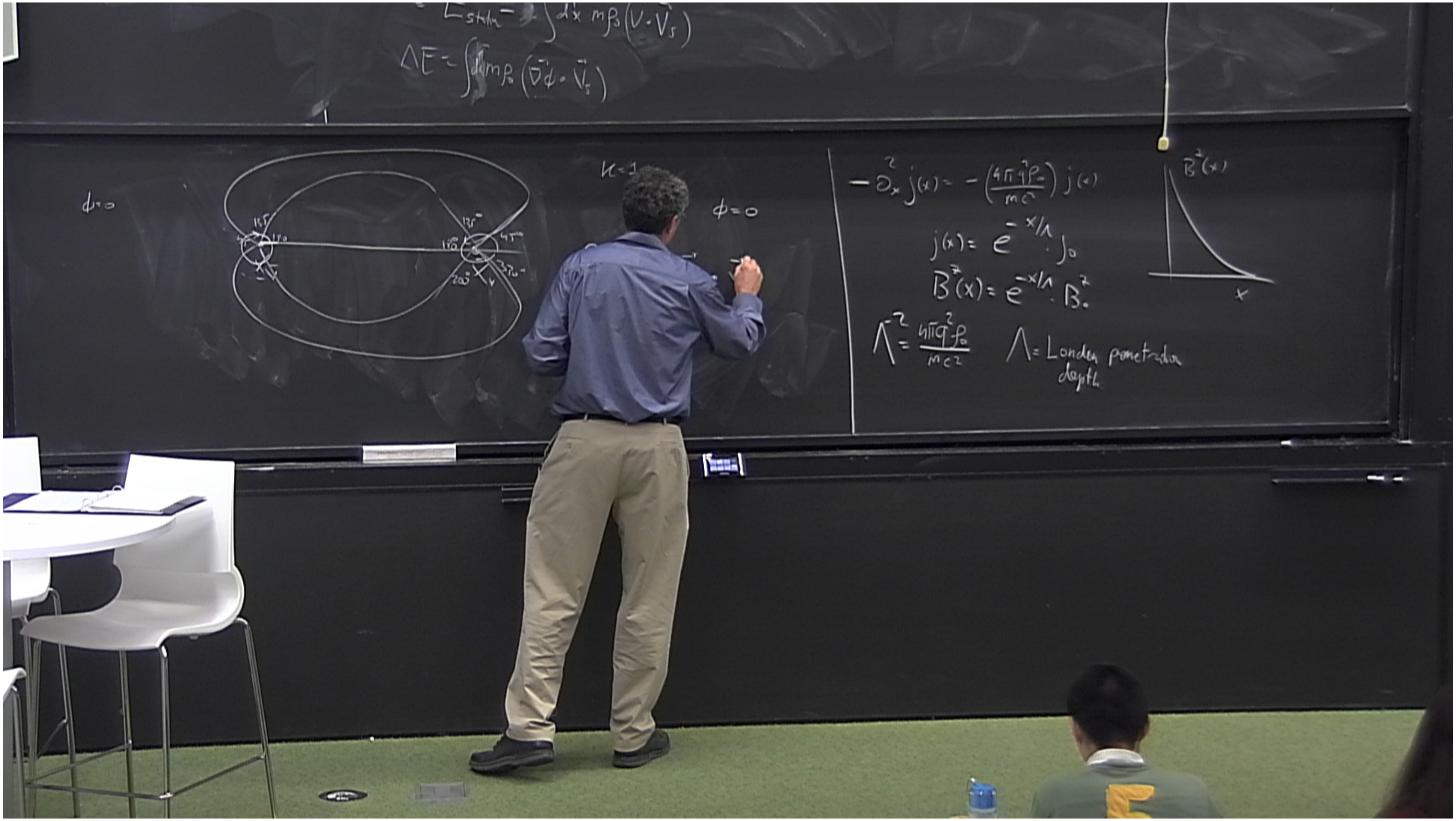

Ginzburg-Landau
stiffness
"super fluid"
complex
function
"Wave function"


Ginzburg-Landau
stiffness
"super-fluid density"
complex
function
"Wave function"

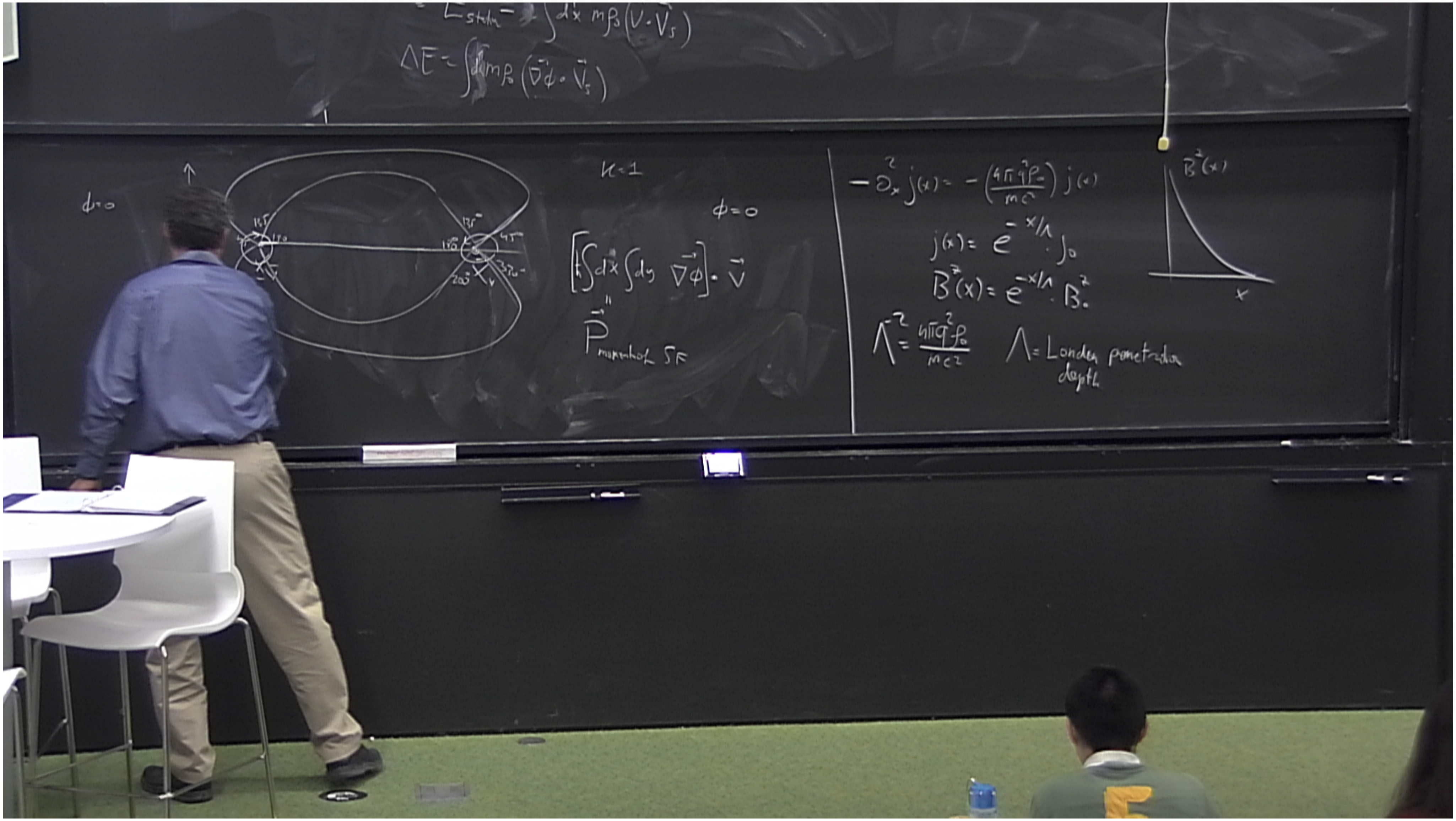

Ginzburg-Landau
stiffness
complex

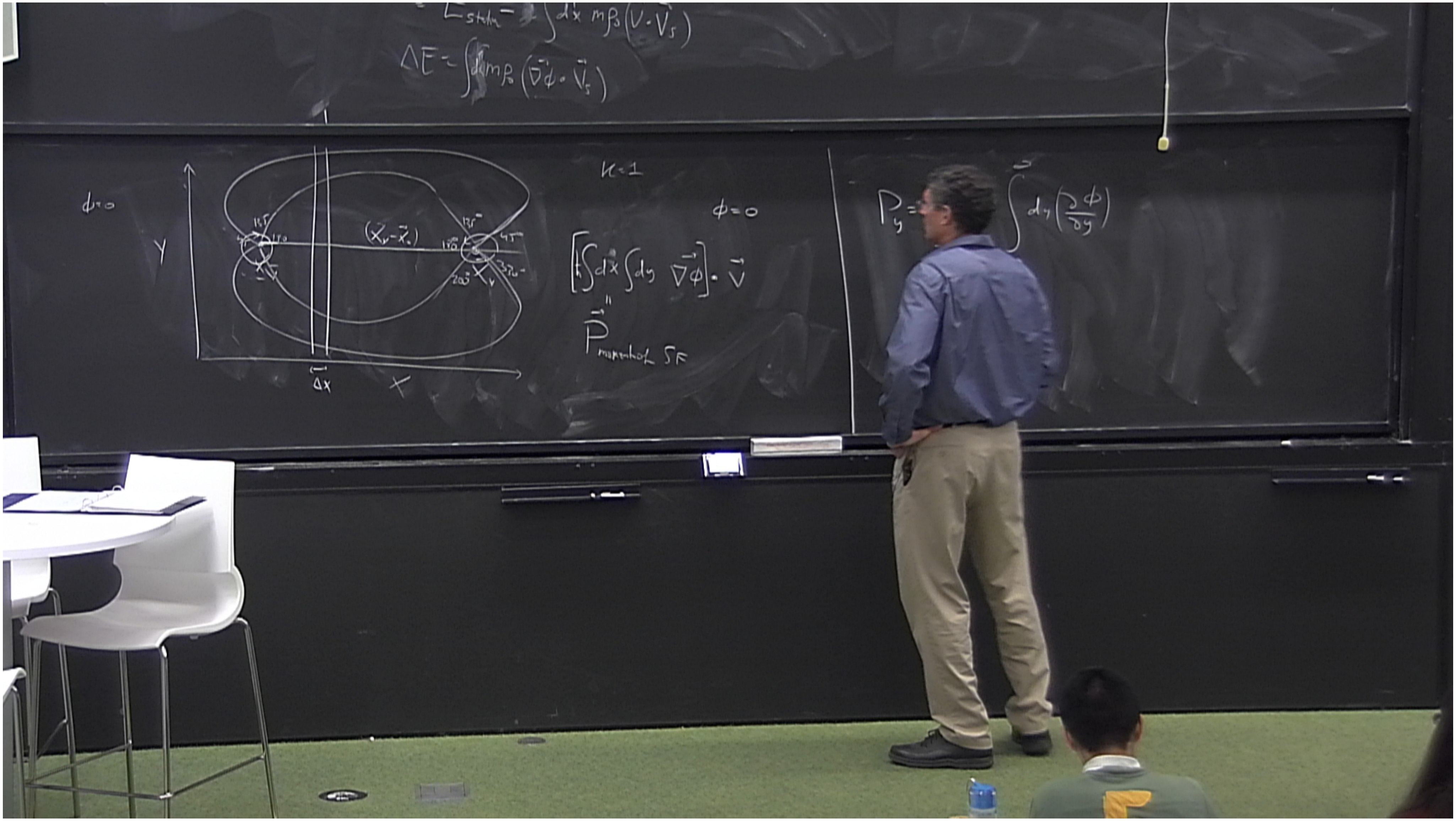

Λ = London penetration depth
moving frame

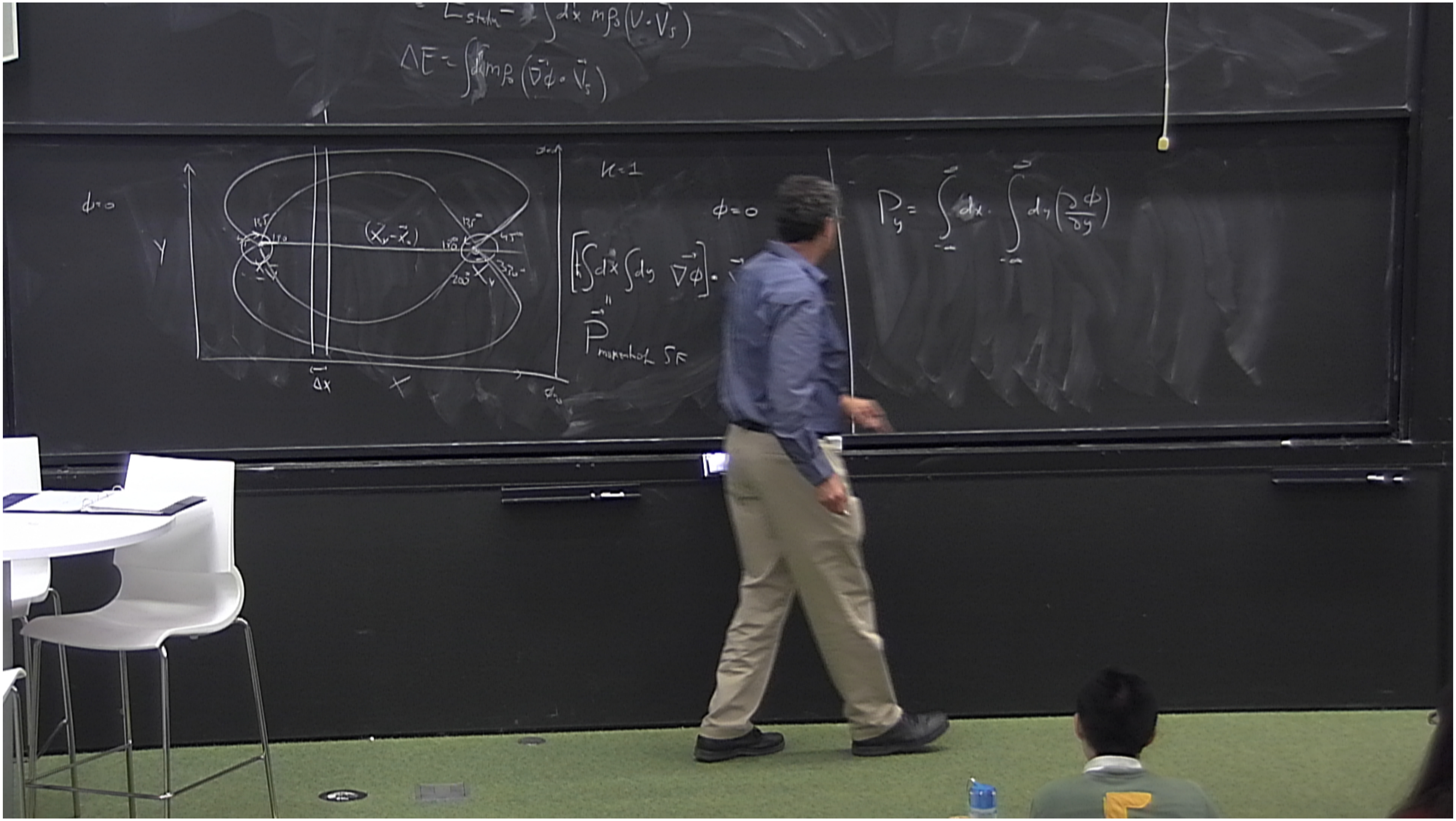

j(x) = e^{-x/Λ} · j_0
B^2(x) = e^{-x/Λ} · B_0^2
Λ^{-2} = 4πq^2ρ_0 / mc^2
Λ = London penetration depth

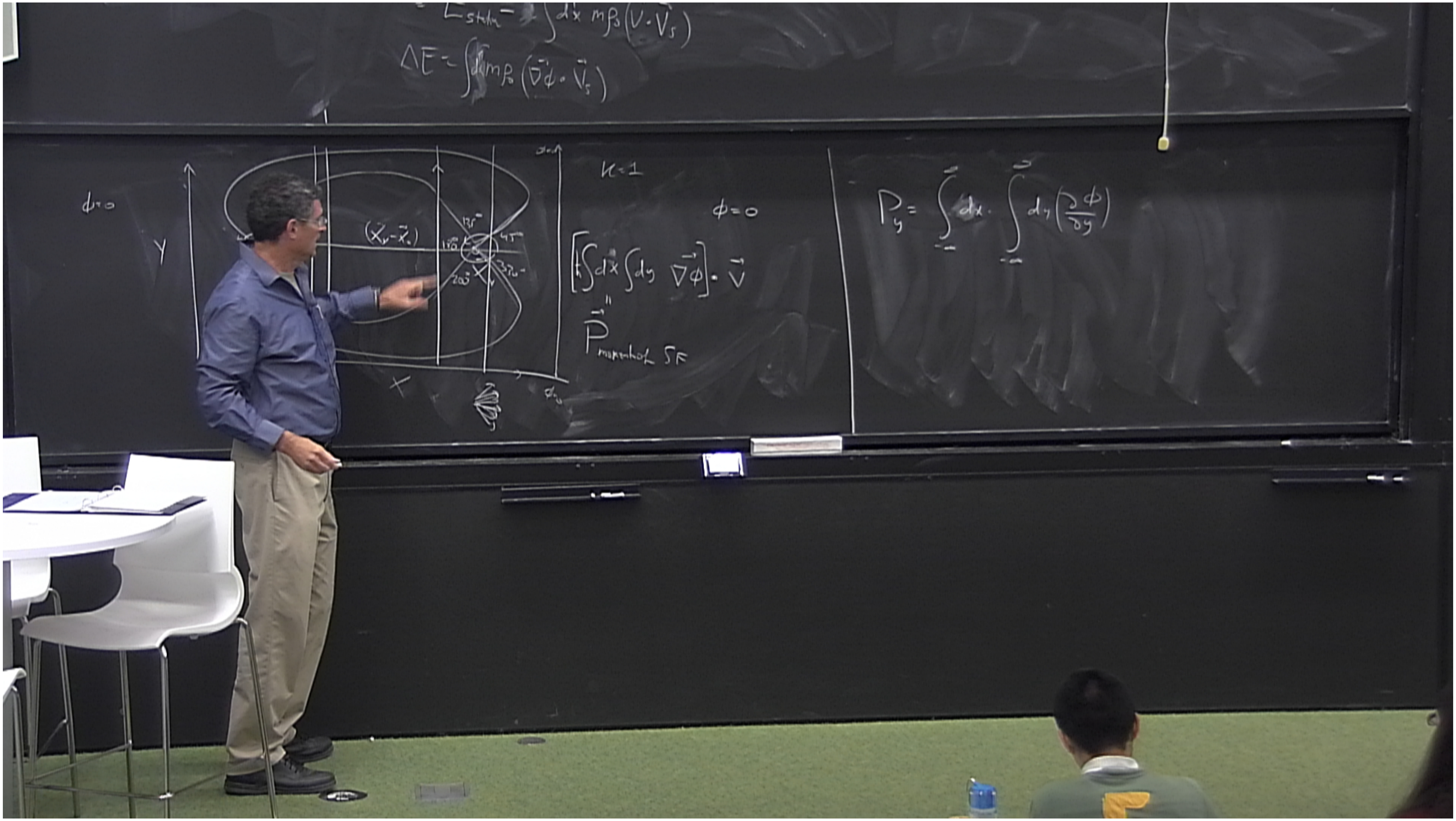


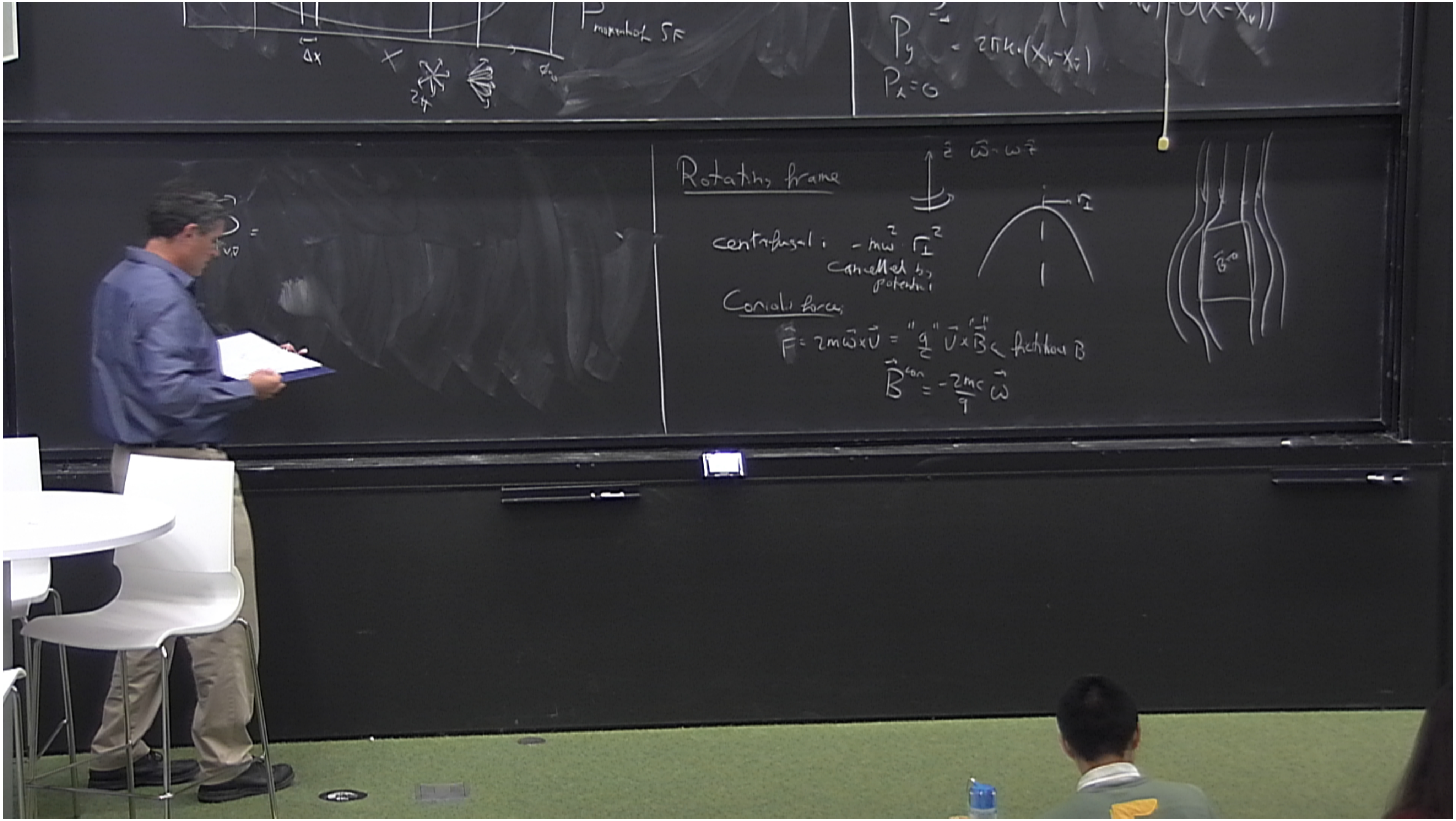
ΔE = ∫ d x m ρ₀ (∇φ · V_s)
φ = 0
−∂ₓ² j(x) = −(4π q² ρ₀ / mc²) j(x)
j(x) = e^{−x/Λ} · j₀
B²(x) = e^{−x/Λ} · B₀²
Λ⁻² = 4π q² ρ₀ / mc²
Λ = London penetration depth
B²(x)
x


κ=1
φ=0
Λ= London penetration depth

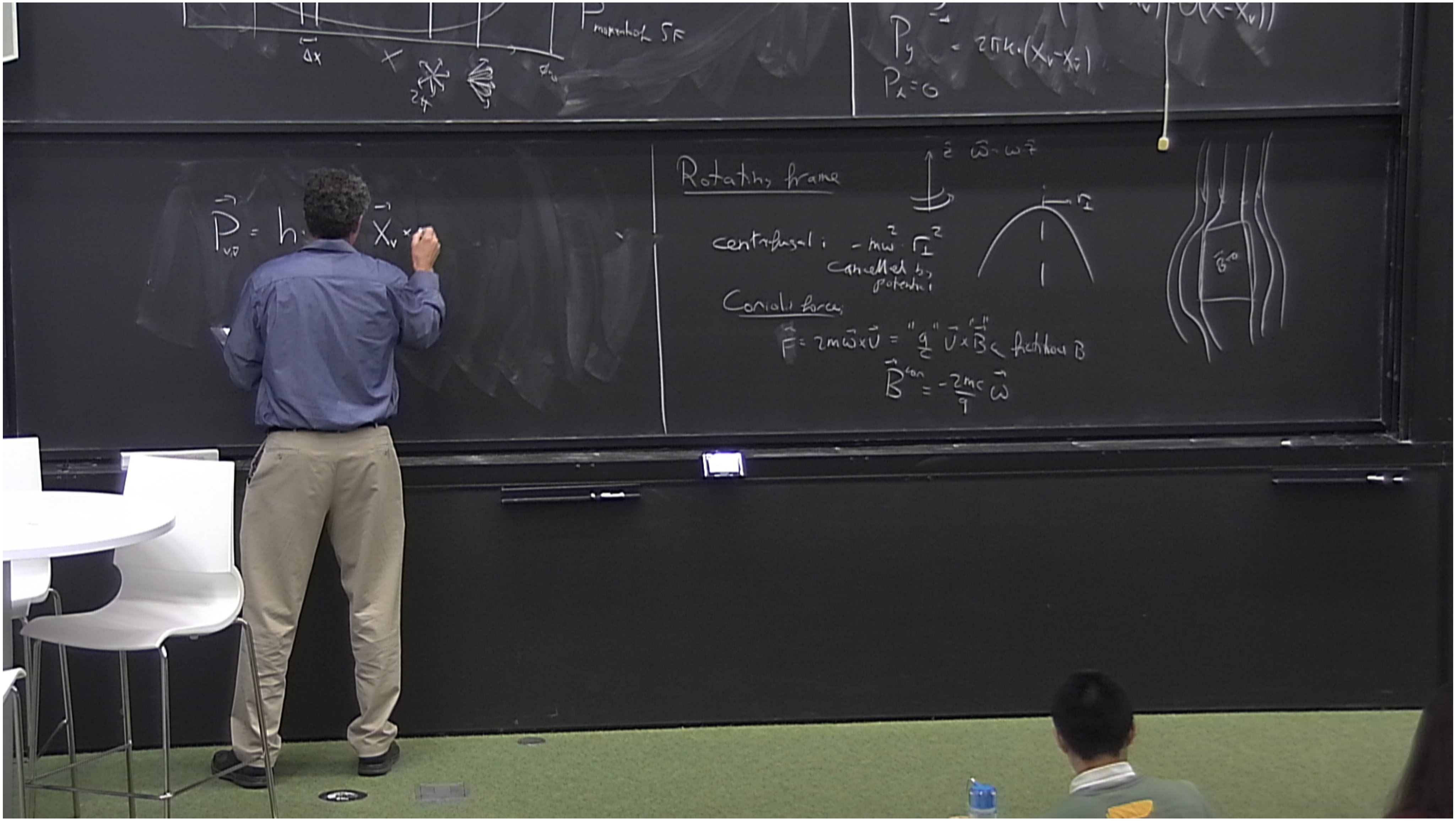


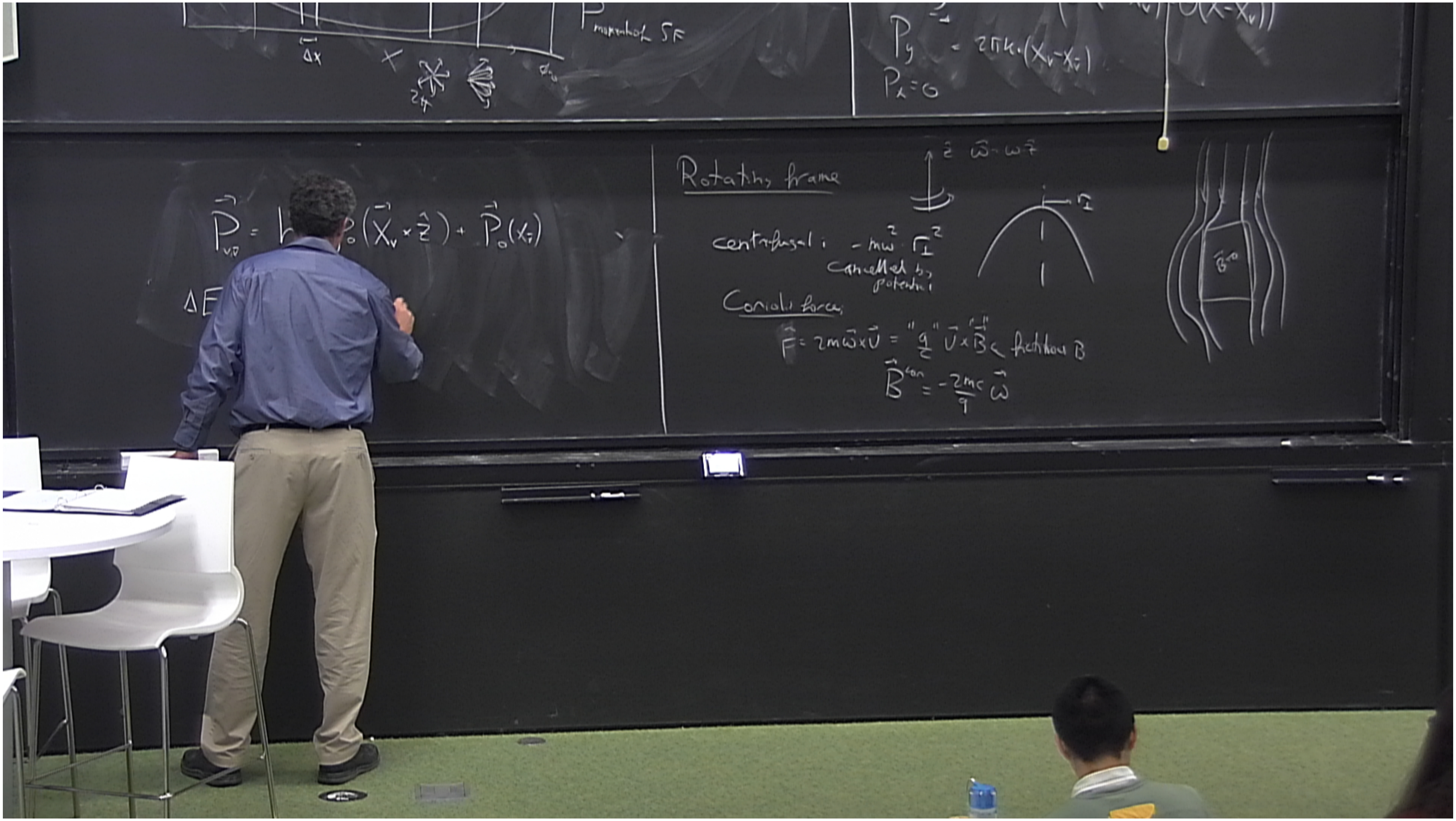

K=1
φ=0
Y
X

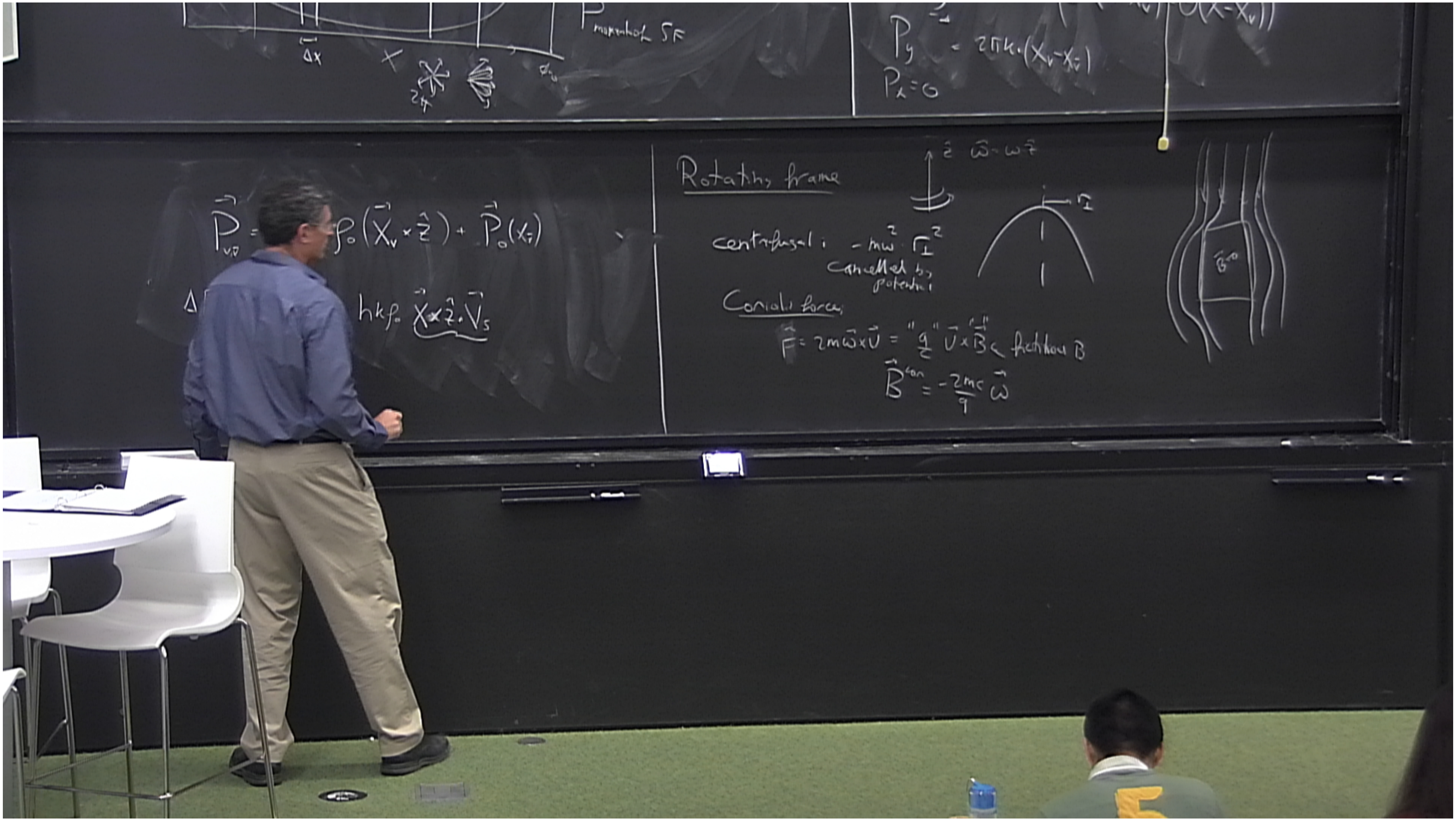


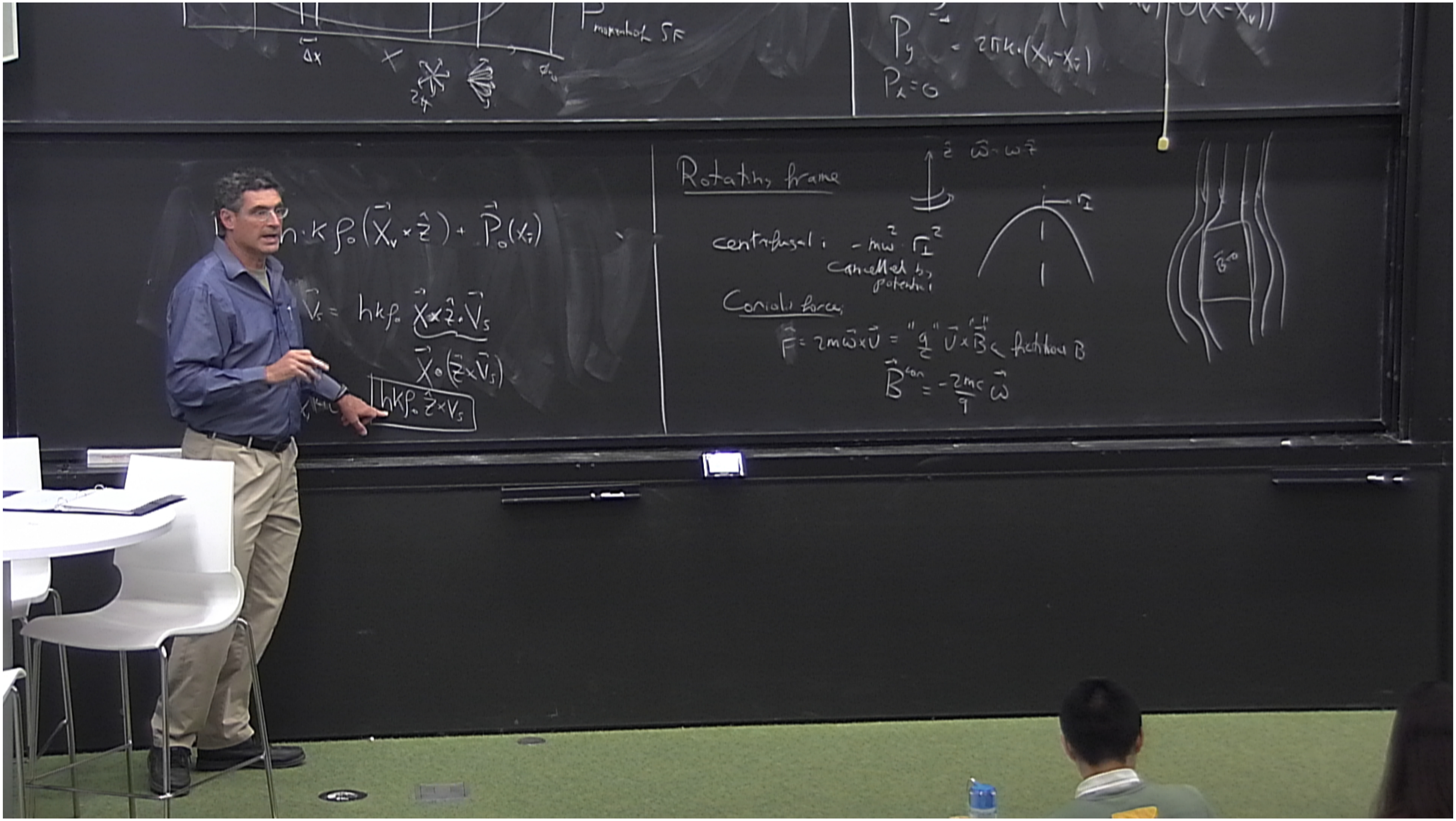


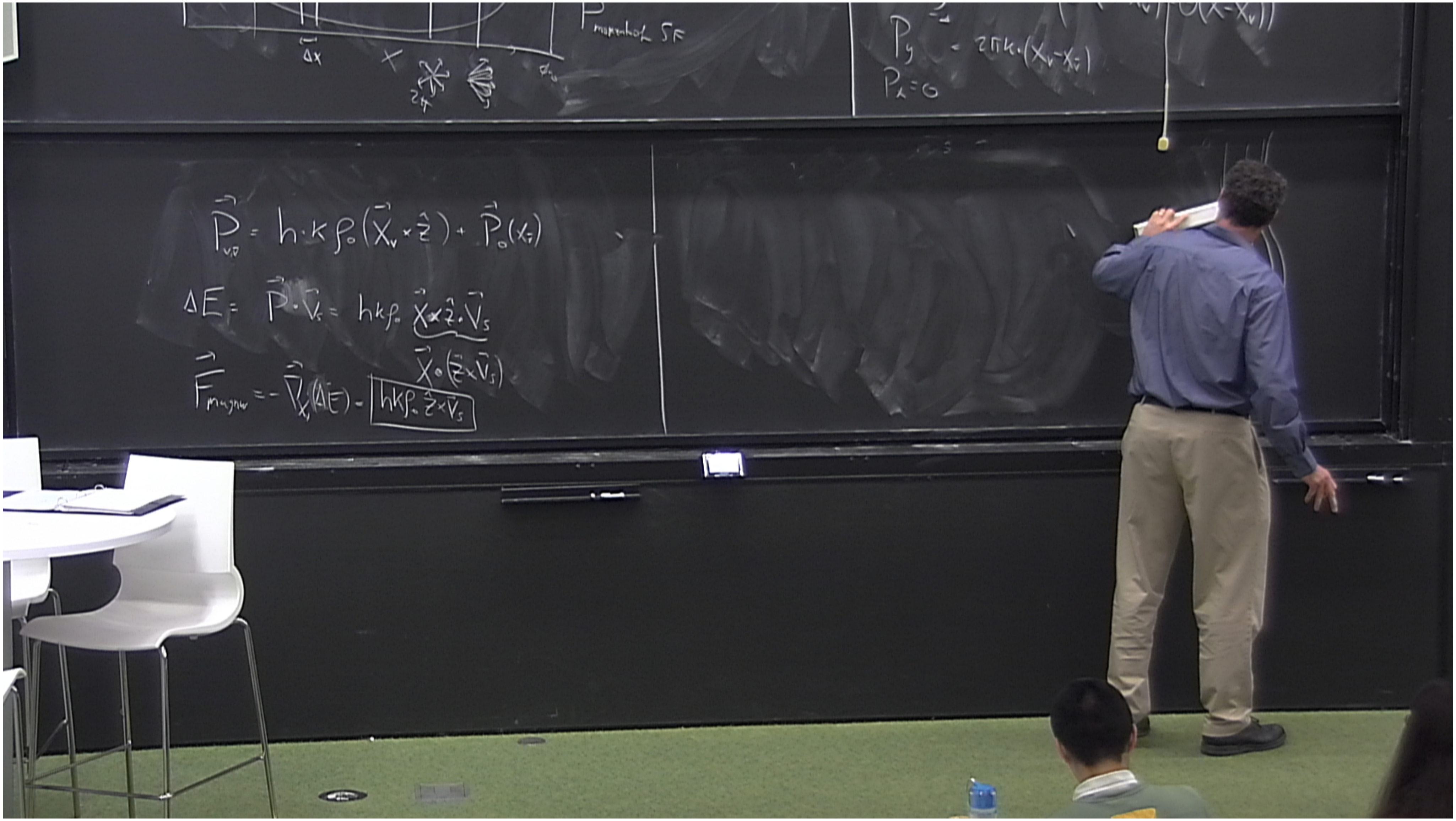
Rotating frame
centrifugal:
cancelled by potential
Coriolis force:
fictitious B

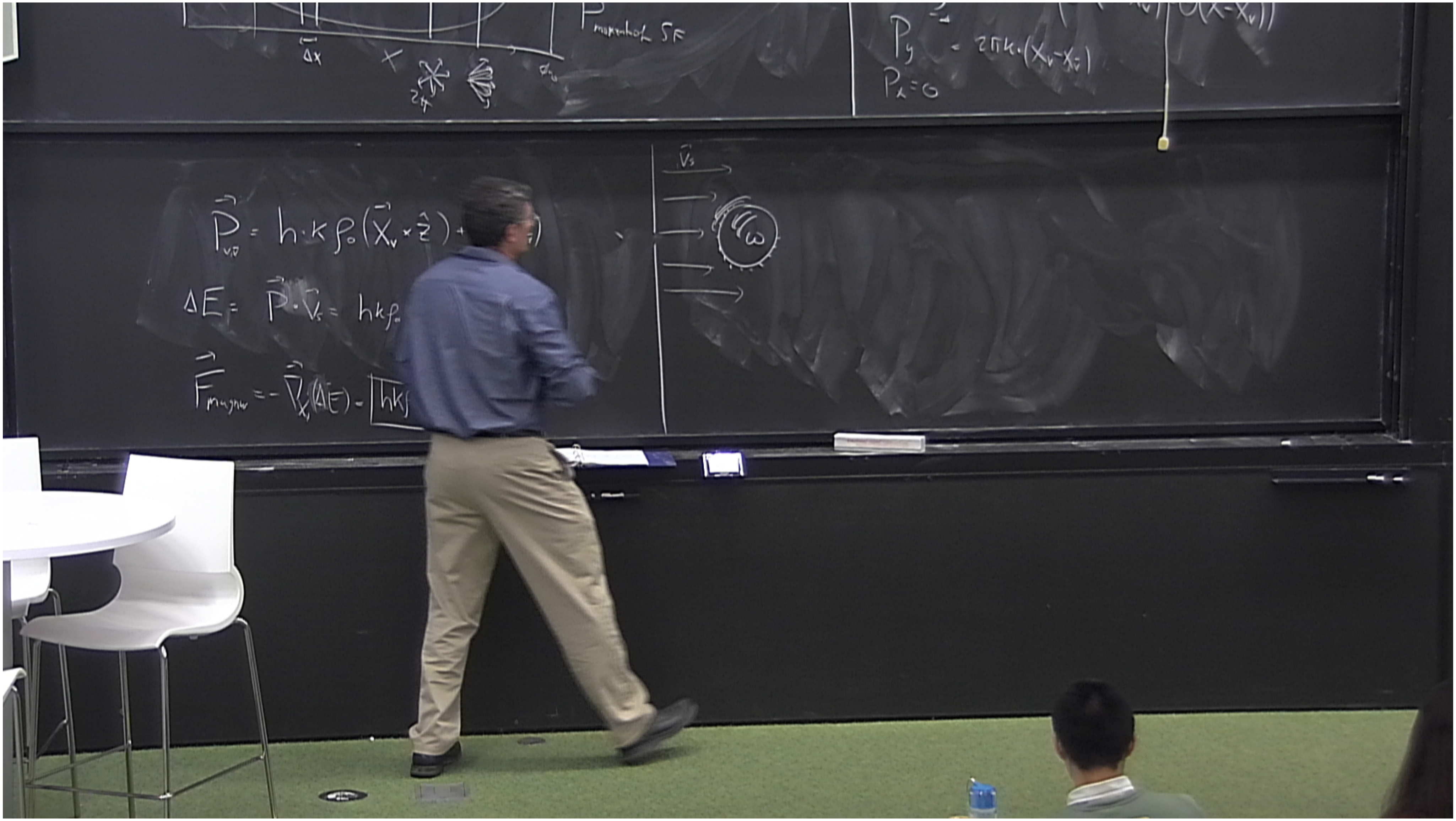

Rotating frame
centrifugal: $-m\omega^2 \cdot r_\perp^2$
cancelled by potential
Coriolis force:
$\vec{F} = 2m\vec{\omega}\times\vec{v} = \text{"}\frac{q}{c}\text{"}\,\vec{v}\times\vec{B}$ ← fictitious B
$\vec{B}^{cor} = -\frac{2mc}{q}\vec{\omega}$
$P_y = 2\pi k \cdot (X_v - X_{\bar{v}})$
$P_x = 0$

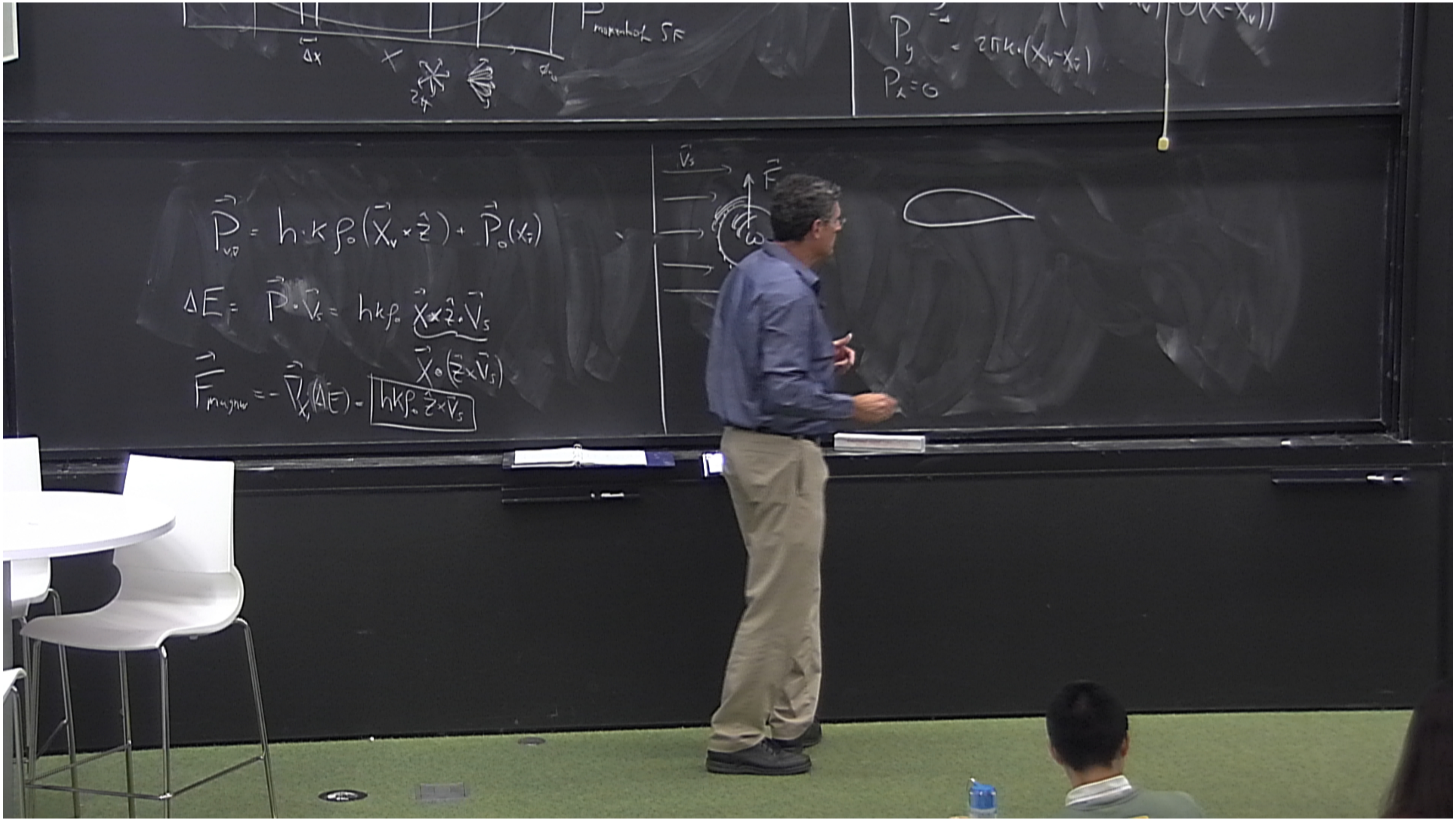

Rotating frame
centrifugal
cancelled by potential
Coriolis force
fictitious B

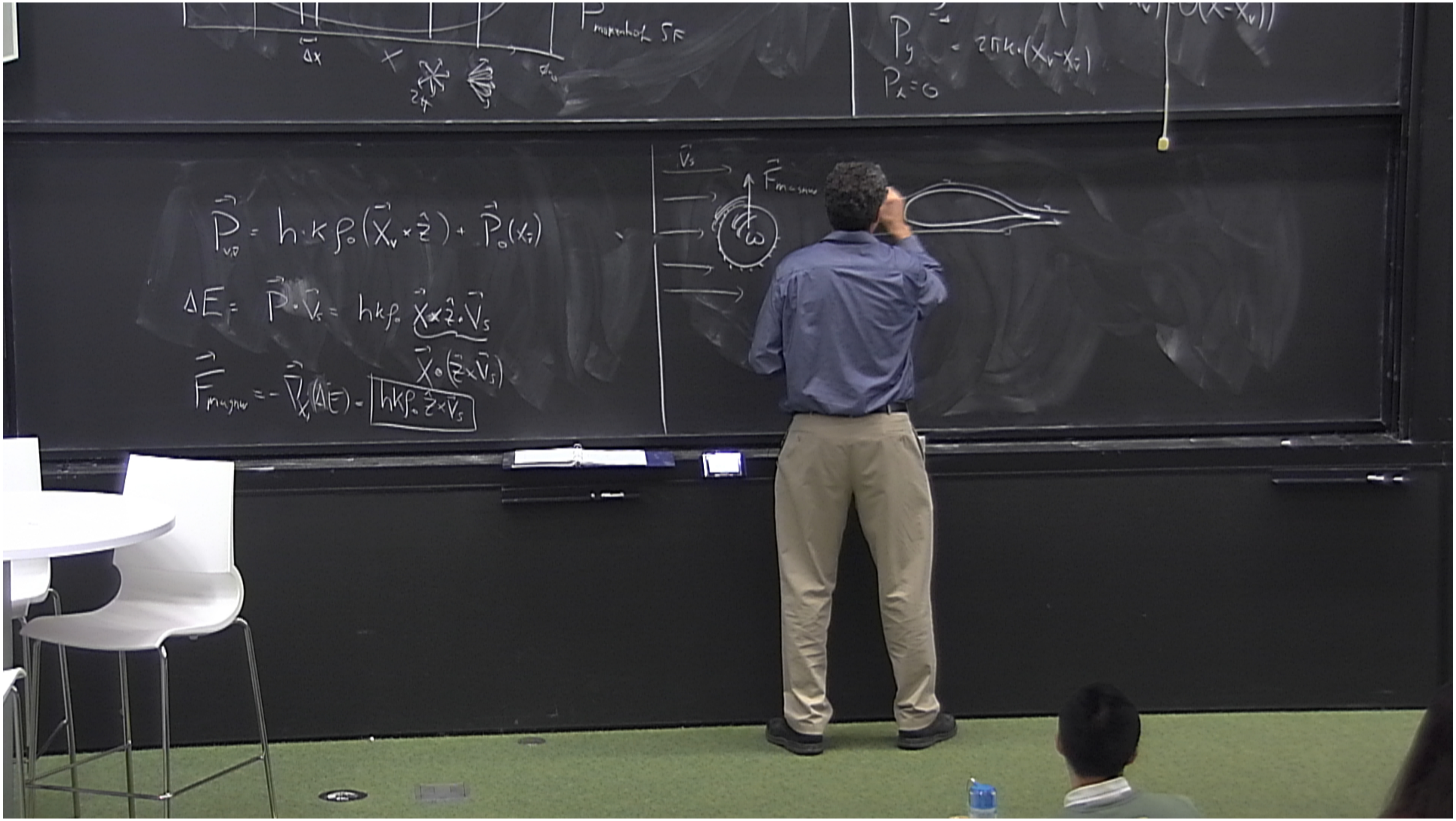

Rotating frame
centrifugal:
cancelled by potential
Coriolis force
fictitious B

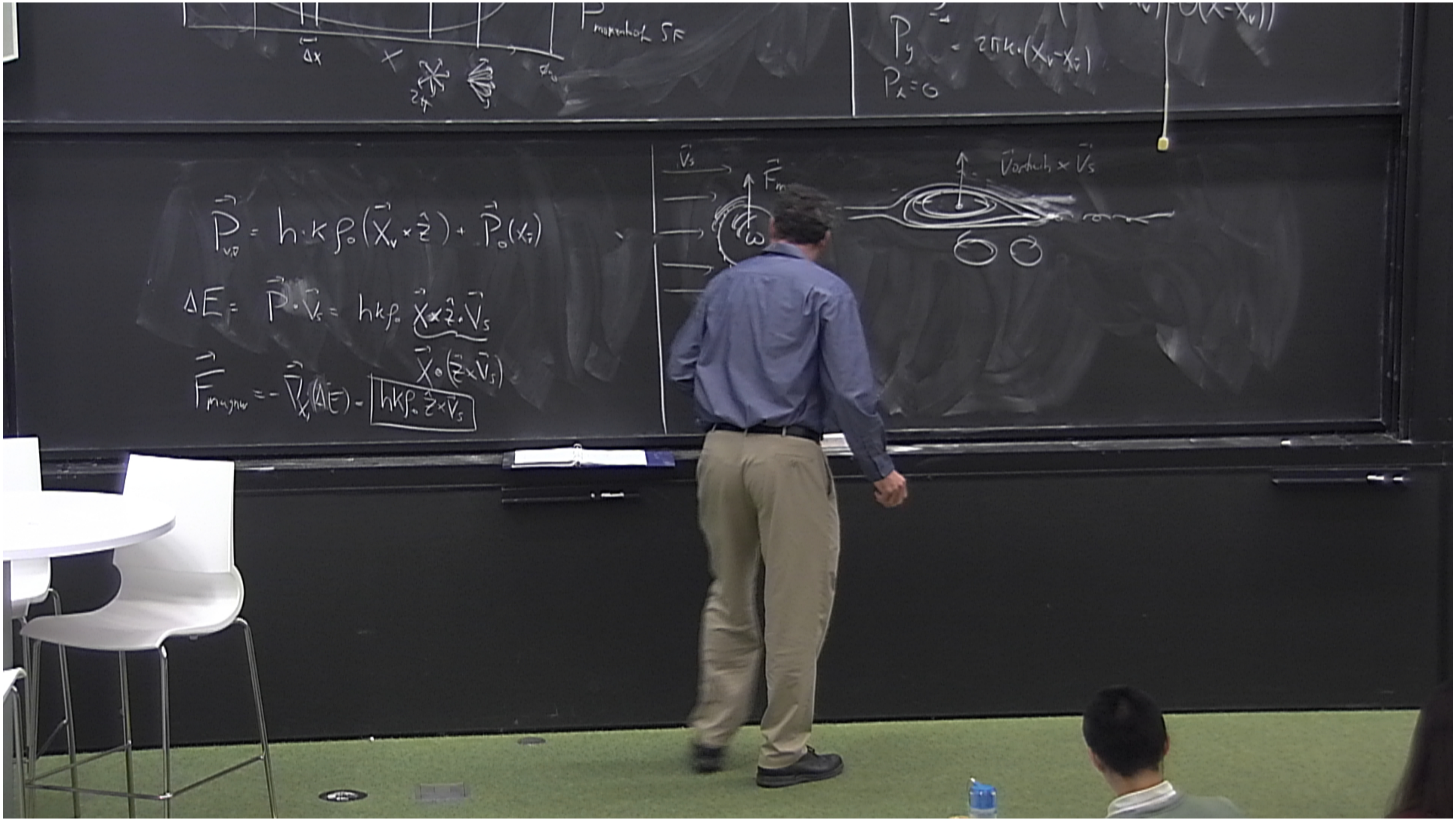


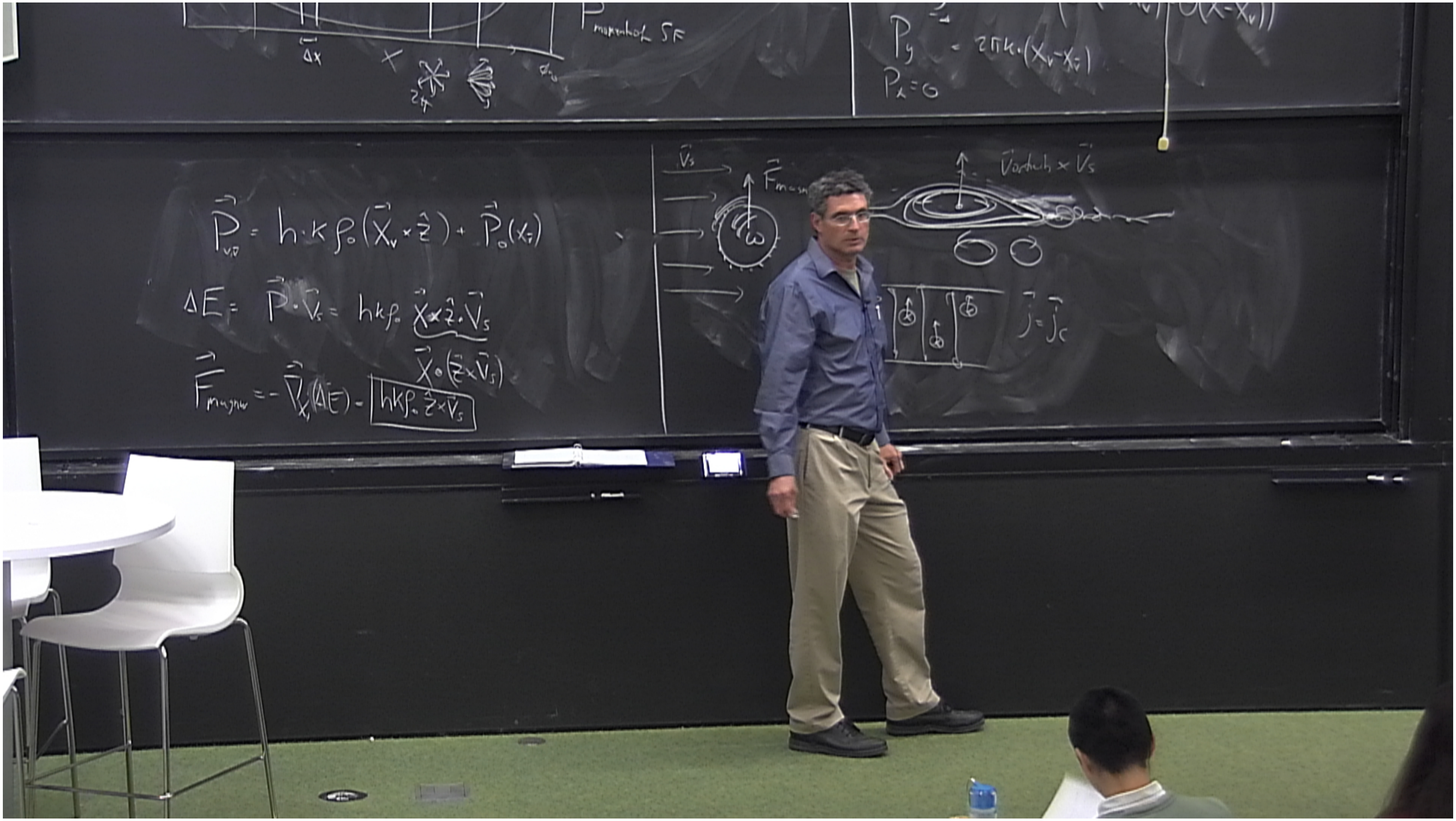

$P_{momentum}$ SF
$\vec{\Delta x}$
$P_y = 2\pi k \cdot (x_v - x_i)$
$P_x = 0$
$\vec{P}_{v\tau} = h \cdot k \rho_0 (\vec{X}_v \times \hat{z}) + \vec{P}_0(x_i)$
$\Delta E = \vec{P} \cdot \vec{V}_s = hk\rho_0 \vec{X} \times \hat{z} \cdot \vec{V}_s$
$\vec{X} \cdot (\hat{z} \times \vec{V}_s)$
$\vec{F}_{magnus} = -\vec{\nabla}_{x_v}(\Delta E) = hk\rho_0 \hat{z} \times \vec{V}_s$






$\vec{P}_{v\bar{v}} = h \cdot \kappa \rho_0 (\vec{X}_v \times \hat{z}) + \vec{P}_0(x_{\bar{v}})$
$\Delta E = \vec{P} \cdot \vec{V}_s = h\kappa\rho_0 \vec{X} \times \hat{z} \cdot \vec{V}_s$
$\vec{X} \cdot (\hat{z} \times \vec{V}_s)$
$\vec{F}_{magnus} = -\vec{\nabla}_{X_v}(\Delta E) = h\kappa\rho_0 \hat{z} \times \vec{V}_s$
$\vec{V}_s$
$\vec{F}_{magnus}$
ω
$P_y = 2\pi\kappa \cdot (X_v - X_{\bar{v}})$
$P_x = 0$


P_y = 2πk·(x_v − x_i)
P_x = 0
P_v,i = h·kρ_0(X_v × ẑ) + P_0(x_i)
ΔE = P·V_s = hkρ_0 X×ẑ·V_s
X·(ẑ×V_s)
F_magnus = −∇_X(ΔE) = hkρ_0 ẑ×V_s
V_s
F_m
ω
Vortex × V_s


P_y = 2πk·(X_v − X_i)
P_x = 0
P_v = h·kρ_0(X_v × ẑ) + P_0(X_i)
ΔE = P·V_s = hkρ_0 X×ẑ·V_s
X·(ẑ×V_s)
F_magnus = −∇_X(ΔE) = hkρ_0 ẑ×V_s
V_s
F_magnus
ω
V_vortex × V_s
j = j_c