

Title: Condensed Matter - Lecture 6

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URL: <http://pirsa.org/12100029>

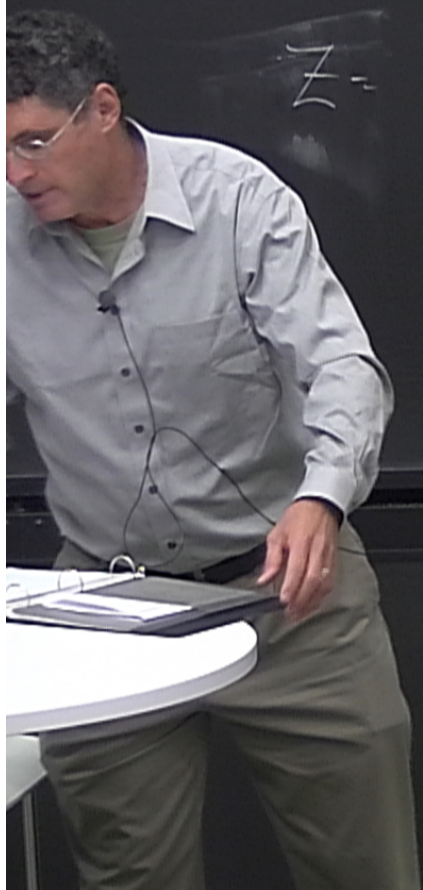
Abstract:

Field theory for the Quantum AFM (non-linear sigma model) $O(3)$

$$Z = \int \mathcal{D}\hat{n}$$

Field theory for the Quantum AFM (non-linear sigma model) $O(3)$

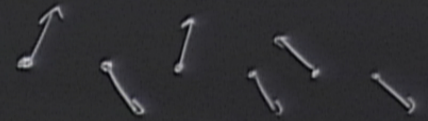
$$Z = \int \mathcal{D}\hat{n}_i e^{iS[\hat{n}_i]} = \int \mathcal{D}\hat{n}_i e^{-\int_0^{\beta} dt H[\hat{n}_i]} \quad H = +J \sum_{ij} \hat{n}_i \cdot \hat{n}_j$$



Field theory for the Quantum AFM (non-linear sigma model) $O(3)$

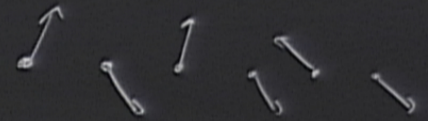
$$Z = \int \mathcal{D}\hat{n}_i e^{iS[\hat{n}_i]} = \int \mathcal{D}\hat{n}_i e^{-\int_0^{\beta} dt H[\hat{n}_i]}$$

$$H = \frac{1}{2} \sum_{\langle ij \rangle} J_{ij} \hat{n}_i \cdot \hat{n}_j$$



Field theory for the Quantum AFM (non-linear sigma model) $O(3)$

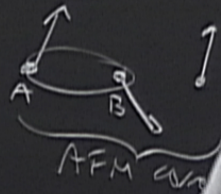
$$Z = \int \mathcal{D}\hat{n}_i e^{iS[\hat{n}_i]} = \int \mathcal{D}\hat{n}_i e^{-\int_0^T dt H[\hat{n}_i]} \quad H = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \cdot \hat{n}_j$$



Quantum AFM (non-linear sigma model) $O(3)$

$$Z[\mathcal{J}] = \int \mathcal{D}[\vec{n}] e^{-\int_0^{\beta} dt H[\vec{n}]}$$

$$H = -\frac{1}{2} \sum_{\langle ij \rangle} J_{ij} \vec{n}_i \cdot \vec{n}_j$$



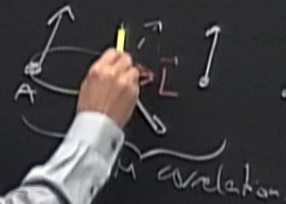
$$\vec{S}_i \quad \vec{L}_i \quad \vec{n}_A - \vec{n}_B =$$



Quantum AFM (non-linear sigma model) $(O(3))$

$$S \approx W[\vec{n}] = \int_0^{\beta} dt' H[\vec{n}(t')]$$

$$H = \frac{1}{2} \sum_{ij} \vec{n}_i \cdot \vec{n}_j$$



$$\left(\frac{\hat{n}_A + \hat{n}_R}{2} = \vec{L} \right) \left(\frac{\hat{n}_A - \hat{n}_R}{2} = \vec{n} \right)$$

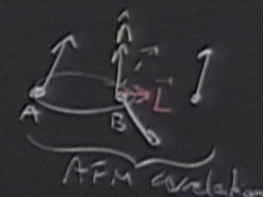


Field theory for the Quantum AFM (non-linear sigma model) $O(3)$

$Z = \int \mathcal{D}\hat{n}, e$

$$S = \int_0^{\beta} dt [H(\hat{n})]$$

$$H = \frac{1}{2} \sum_{\langle ij \rangle} \hat{L}_i \cdot \hat{L}_j$$



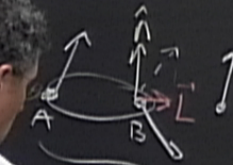
$$\hat{L}_i = \hat{n}_i(x_i) \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2} + \left(\frac{\vec{L}_i}{S}\right)$$

$$|\hat{L}_i|^2 = |\hat{n}_i|^2 \left(1 - \left(\frac{\vec{L}_i}{S}\right)^2\right) + \left(\frac{\vec{L}_i}{S}\right)^2 + 2 \hat{n}_i \cdot \vec{L}_i \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2}$$

Quantum AFM (non-linear sigma model) $(O(3))$

$$S \omega[\vec{n}] = \int_0^{\beta} dt H[\vec{n}]$$

$$H = +\frac{1}{2} \sum_{\langle ij \rangle} J_{ij} \hat{n}_i \cdot \hat{n}_j$$



AFM correlation

$$\left(\frac{\hat{n}_A + \hat{n}_B}{2} = S \cdot \vec{L} \right) \quad \left(\frac{\hat{n}_A - \hat{n}_B}{2} = \vec{n} \right)$$

$$\eta_{i \in A} = +1$$

$$\eta_{i \in B} = -1$$

$$\sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2} + \left(\frac{\vec{L}_i}{S}\right)$$

$$\left(\frac{\vec{L}_i}{S}\right) + \left(\frac{\vec{L}_i}{S}\right)^2 + 2 \eta \cdot \hat{n} \cdot \vec{L} \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2}$$

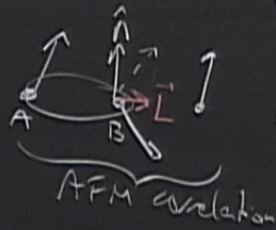
$$|\hat{n}| = 1$$

$$\hat{n} \cdot \vec{L} = 0$$

Quantum AFM (non-linear sigma model) $O(3)$

$\omega[\vec{n}] = \int_0^{\vec{n}} d\vec{n}$

$H = +\frac{1}{2} \sum_{\langle ij \rangle} J_{ij} \hat{n}_i \cdot \hat{n}_j$



$(\frac{\hat{n}_A + \hat{n}_B}{2} = S \cdot \vec{L}) \quad (\frac{\hat{n}_A - \hat{n}_B}{2} = \vec{n})$

$\eta_{i \in A} = +1$
 $\eta_{i \in B} = -1$

$\sqrt{1 - (\frac{\vec{L}}{L})^2}$
 $(\frac{\vec{L}}{L})^2 + (\frac{\vec{n}}{n})^2 = 1$

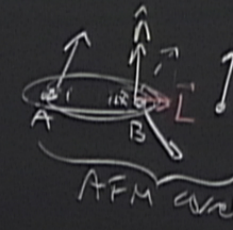
$|\hat{n}| = 1$
 $\hat{n} \cdot \vec{L} = 0$

$\underbrace{d\hat{n}_i d\hat{n}_{i+1}}_4 = d\hat{n}_i \underbrace{d\vec{L}}_{\substack{\uparrow \\ 2}} \cdot \underbrace{\delta(|\hat{n}|-1)}_{\substack{\uparrow \\ 3}} \delta(\hat{n} \cdot \vec{L}) \underbrace{\int_{\substack{S^1 \\ (\hat{n} \cdot \vec{L} / L)}}}_{\substack{\uparrow \\ -1}} \text{ for } \hat{n}_i \text{ important}$

Quantum AFM (non-linear sigma model) $O(3)$

$$S\omega[\vec{n}] = \int_0^{\beta} dt' H[\vec{n}]$$

$$\sum_{\langle ij \rangle} J_{ij} \vec{n}_i \cdot \vec{n}_j$$



$$\left(\frac{\vec{n}_A + \vec{n}_B}{2} = S \cdot \vec{L} \right) \quad \left(\frac{\vec{n}_A - \vec{n}_B}{2} = \vec{n} \right)$$

$$\eta_{i \in A} = +1$$

$$\eta_{i \in B} = -1$$

$$\sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2} + \left(\frac{\vec{L}_i}{S}\right)$$

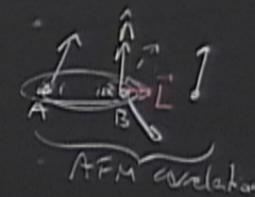
$$\left(\frac{\vec{L}_i}{S}\right) + \left(\frac{\vec{L}_i}{S}\right)^2 + 2\eta \cdot \hat{n}_i \cdot \vec{L}_i \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2}$$

$$\underbrace{d\vec{n}_i \cdot d\vec{n}_{i+a}}_4 = \underbrace{d\hat{n}_i}_{\frac{1}{2}} \cdot \underbrace{d\vec{L}_i}_{\frac{1}{3}} \cdot \underbrace{\delta(|\hat{n}_i|=1)}_{\frac{1}{2}} \underbrace{\delta(\hat{n}_i \cdot \vec{L}_i)}_{\frac{1}{2}} \underbrace{\int_{\vec{n}_i} \delta(\vec{n}_i \cdot \vec{L}_i)}_{\frac{1}{2}} \underbrace{\int_{\vec{n}_i} \delta(\vec{n}_i \cdot \vec{L}_i)}_{\frac{1}{2}} \text{ for } \vec{L}_i \text{ important}$$

theory of the Quantum AFM (non-linear sigma model) $O(3)$ $(\frac{\hat{L}_A - \hat{L}_B}{2} = S \cdot \vec{L})$ $(\frac{\hat{L}_A + \hat{L}_B}{2})$

$Z = \int e^{S[\vec{W}(x)] - \int dt' H[\vec{W}(t)]}$

$H = \frac{1}{2} \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$

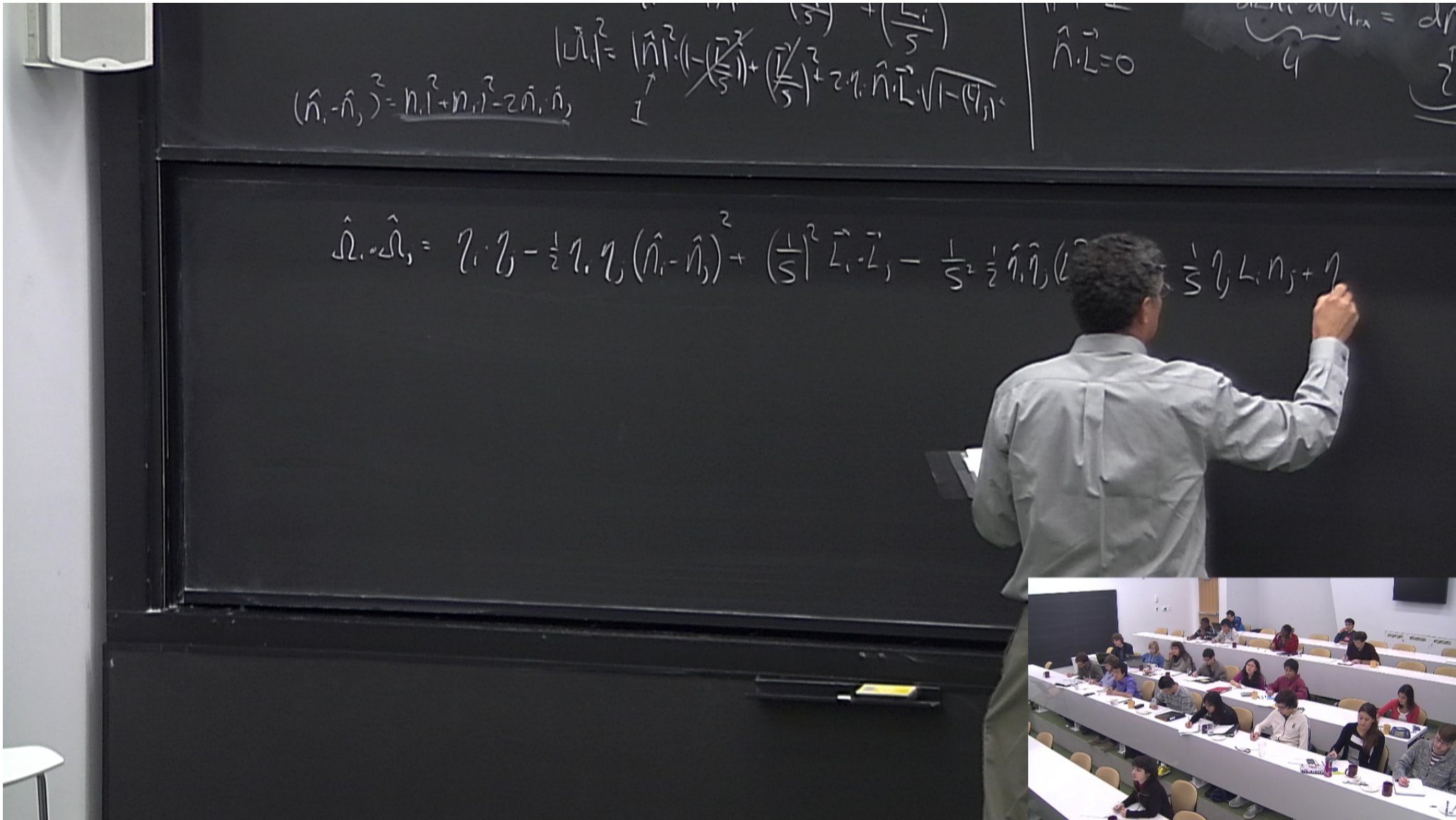


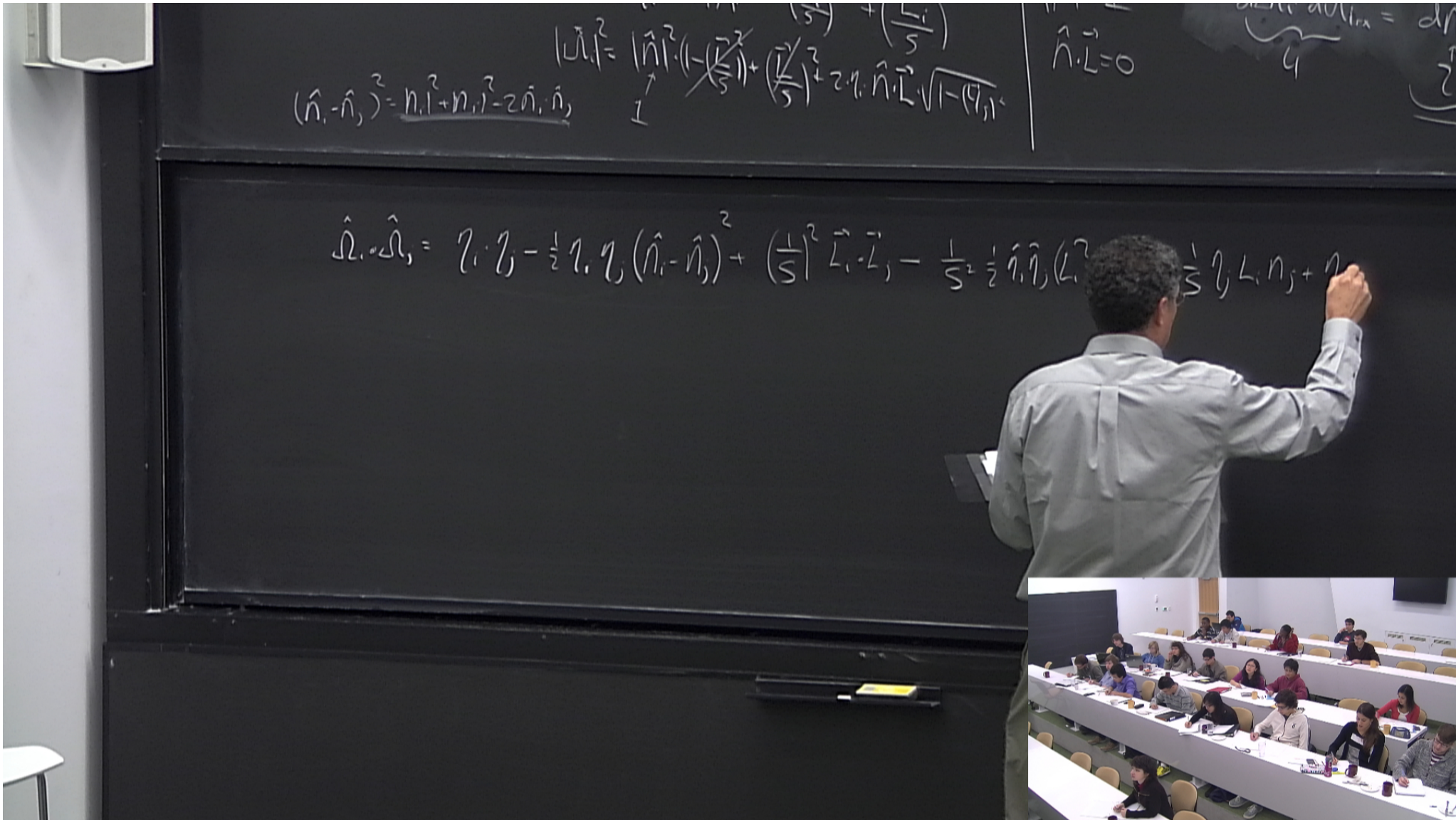
$\eta_{i \in A} = +1$
 $\eta_{i \in B} = -1$

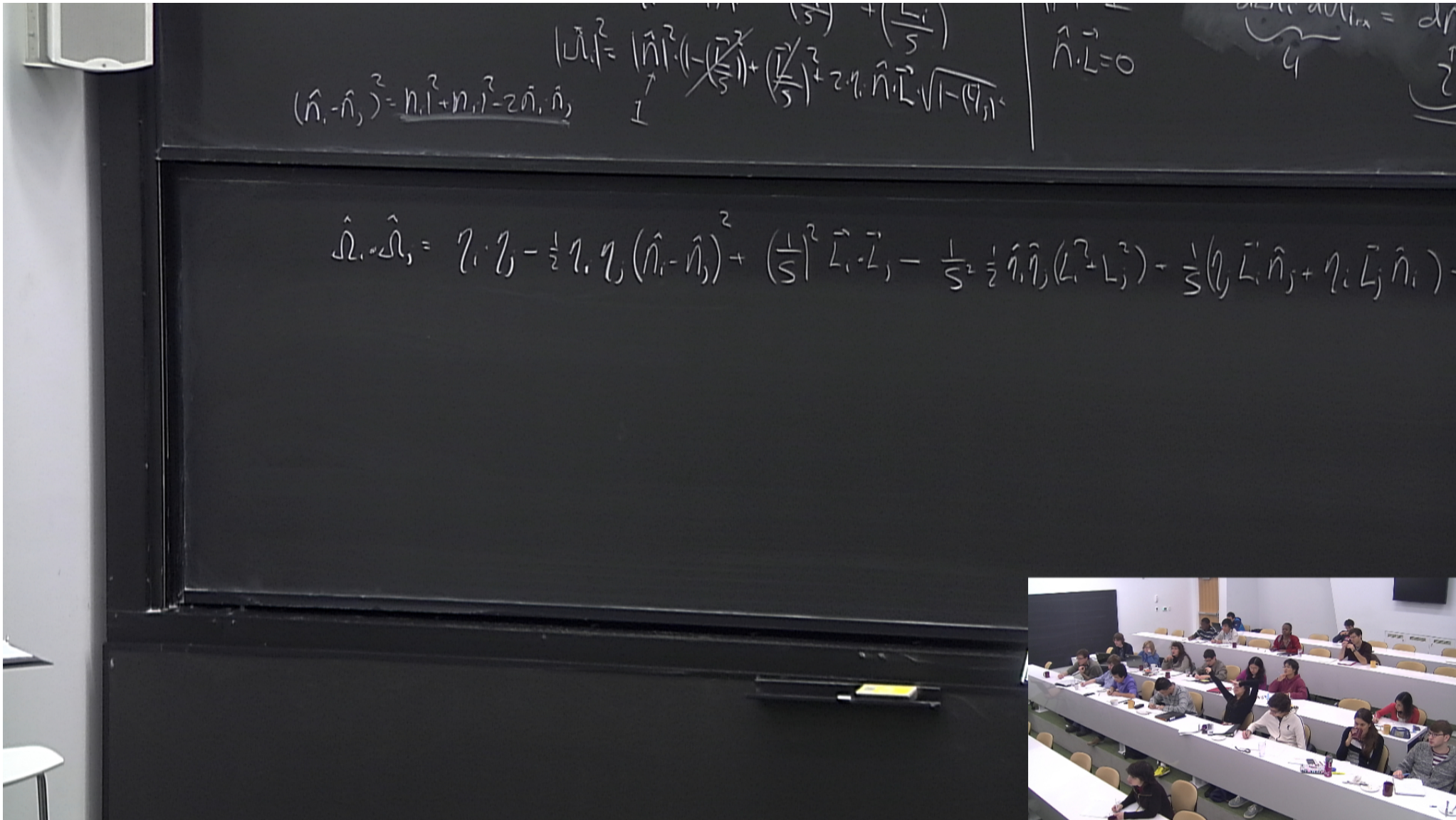
$\vec{n}_i(x_i) \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2} + \left(\frac{\vec{L}_i}{S}\right)$
 $\left(\frac{\vec{L}_i}{S}\right) + \left(\frac{\vec{L}_i}{S}\right)^2 + 2\eta_i \hat{n}_i \cdot \vec{L}_i \sqrt{1 - \left(\frac{\vec{L}_i}{S}\right)^2}$

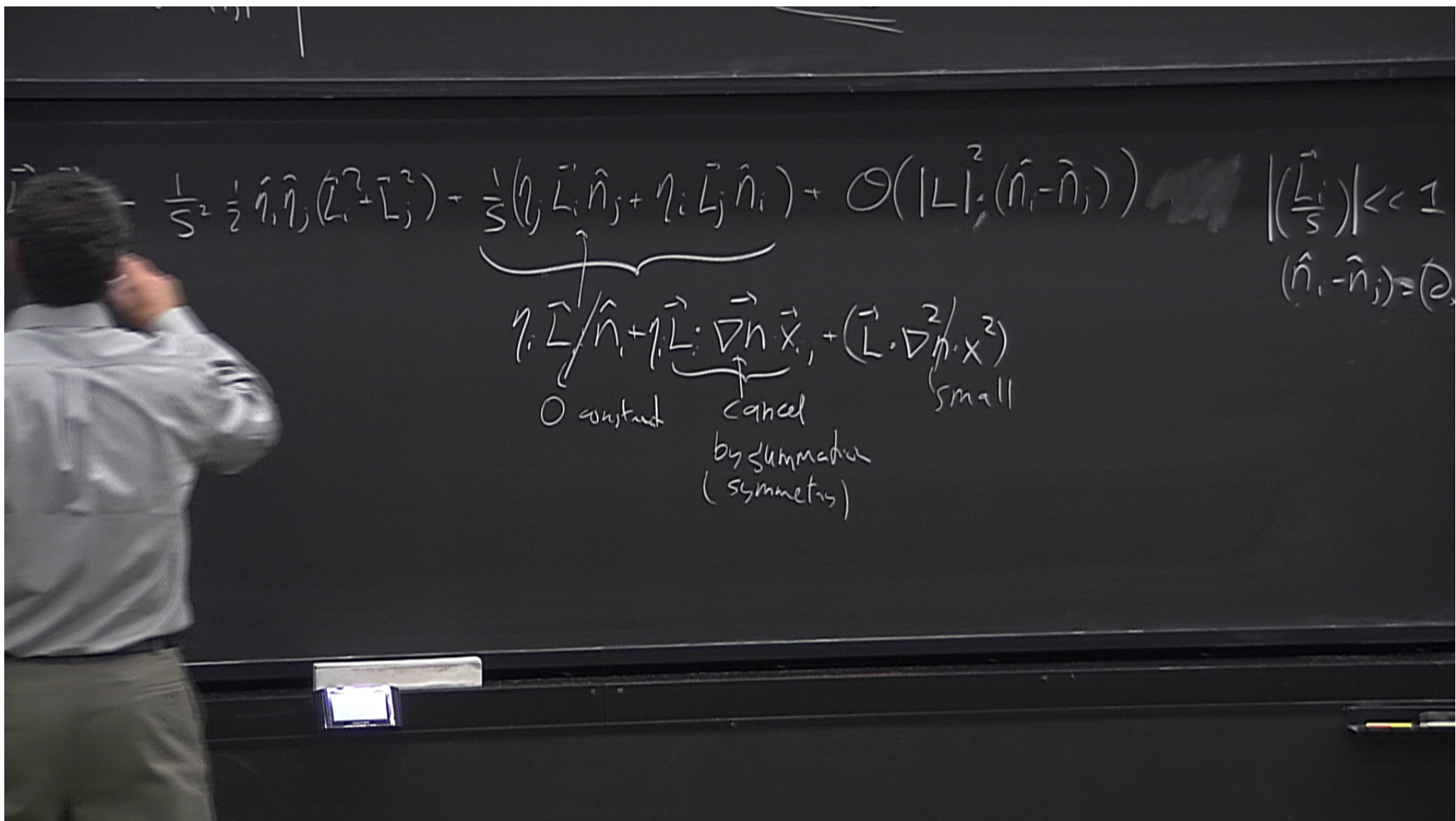
$|\hat{n}| = 1$
 $\hat{n} \cdot \vec{L} = 0$

$\frac{d\hat{n}_i \cdot d\hat{n}_i}{4} = \frac{d\hat{n}_i \cdot d\vec{L}_i \cdot \delta(|\hat{n}_i| - 1) \delta(\hat{n}_i \cdot \vec{L}_i)}{\frac{1}{2} \frac{1}{3}} \frac{1}{|\hat{n}_i|} \frac{1}{S}$









$$\frac{1}{S^2} \frac{1}{2} \hat{n}_i \hat{n}_j (\vec{L}_i^2 + \vec{L}_j^2) + \frac{1}{S} (\eta_j \vec{L}_i \hat{n}_j + \eta_i \vec{L}_j \hat{n}_i) + \mathcal{O}(|\vec{L}_i|^2 (\hat{n}_i - \tilde{n}_i)) \quad \left| \frac{\vec{L}_i}{S} \right| \ll 1$$

$$\underbrace{\eta_j \vec{L}_i / \hat{n}_j + \eta_i \vec{L}_j \cdot \nabla \hat{n}_j \cdot \vec{x}_i}_{\text{cancel by summation (symmetry)}} + \underbrace{(\vec{L}_i \cdot \nabla^2 \eta_i \cdot x^2)}_{\text{small}}$$

$$\hat{n}_i - \tilde{n}_i = \mathcal{O}$$

$$(\hat{n}_i - \hat{n}_j) = n_{i,1} + n_{i,2} - 2n_{i,1} \quad |$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \gamma_j - \frac{1}{2} \gamma_i \gamma_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{S}\right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{S} \frac{1}{2} \gamma_i \gamma_j (\vec{L}_i^2 - \vec{L}_j^2) - \frac{1}{S} \left(\gamma_i \gamma_j \right)$$



$$(\hat{n}_i - \hat{n}_j) = \frac{1}{2}(\hat{n}_i + \hat{n}_j) - \frac{1}{2}(\hat{n}_i - \hat{n}_j)$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \gamma_i \cdot \gamma_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{S}\right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{S^2} \frac{1}{2} \hat{n}_i \hat{n}_j (\vec{L}_i^2 + \vec{L}_j^2) - \frac{1}{S} \left(\gamma_i \cdot \vec{L}_j + \gamma_j \cdot \vec{L}_i \right)$$

$$(\hat{n}_i - \hat{n}_j) = \hat{n}_i - \hat{n}_j + x_{ij}^e + \left(\frac{\partial^2 h}{\partial n^2} \right)^{\text{small}}$$

H =

$$(\hat{n}_i - \hat{n}_j)^2 = n_i^2 + n_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

$$1 \uparrow \quad \quad \quad \left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2 + 2\eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - (\eta/n)^2}$$

$$n \cdot L = 0$$

$$\left(\begin{matrix} 2 \\ 3 \end{matrix} \right)$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \eta_i \eta_j - \frac{1}{2} \eta_i \eta_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{5}\right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{5^2} \frac{1}{2} \eta_i \eta_j (\vec{L}_i^2 + \vec{L}_j^2) + \frac{1}{5} (\eta_i \vec{L}_j \cdot \hat{n}_j + \eta_j \vec{L}_i \cdot \hat{n}_i) + \mathcal{O}(\dots)$$

$$(\hat{n}_i - \hat{n}_j)^2 = \left(\frac{1}{5}\right)^2 \hat{n}_i^2 + \left(\frac{1}{5}\right)^2 \hat{n}_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

Small

$$H = \frac{1}{2} \sum_{ij} J_{ij} \hat{\Omega}_i \cdot \hat{\Omega}_j$$

$$\sum_{ij} J_{ij} \eta_i \eta_j (\partial_{\vec{x}_i} \hat{n}_i) \cdot (\partial_{\vec{x}_j} \hat{n}_j)$$

$\eta_i \vec{L}_j / \hat{n}_j + \eta_j \vec{L}_i \cdot \vec{\nabla} \hat{n}_i \cdot \vec{x}_i + (\vec{L}_i \cdot \vec{L}_j)$
 O constant cancel by summation (symmetric)

$$(\hat{n}_i - \hat{n}_j)^2 = n_i^2 + n_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

$$|\vec{L}_i - \vec{L}_j|^2 = L_i^2 + L_j^2 - 2\eta_i \hat{n}_i \cdot \vec{L}_j \sqrt{1 - \eta_j^2}$$

$$n_i \cdot L = 0$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \eta_i \eta_j - \frac{1}{2} \eta_i \eta_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{5}\right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{5^2} \frac{1}{2} \eta_i \eta_j (L_i^2 + L_j^2) + \frac{1}{5} (\eta_j \vec{L}_i \cdot \hat{n}_j + \eta_i \vec{L}_j \cdot \hat{n}_i) + \mathcal{O}(\dots)$$

$$(\hat{n}_i - \hat{n}_j)^2 = \left(\frac{1}{5}\right)^2 + (\partial_e \hat{n})^2 \text{ Small}$$

$$H = \frac{1}{2} \sum_{ij} J_{ij}$$

$$\approx \sum_{ij} \bar{P}_{ij}^{ee'} (\partial_e \hat{n}_i) \cdot (\partial_{e'} \hat{n}_j)$$

$$\approx \sum_{ij} J_{ij} \eta_i \eta_j (\partial_e \hat{n}_i \cdot x_{ij}^{ee'}) (\partial_{e'} \hat{n}_j \cdot x_{ij}^{e'e'})$$

(constant)

$\eta_i \vec{L}_i / \hat{n}_i + \eta_j \vec{L}_j \cdot \vec{\nabla} \hat{n}_i \cdot \vec{x}_{ij} + (\vec{L}_i \cdot \dots)$
 0 constant cancel by summation (symmetric)

$$(\hat{n}_i - \hat{n}_j)^2 = n_i^2 + n_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

$$1 \uparrow \quad \quad \quad \left(\frac{1}{5} \right) + \left(\frac{1}{5} \right)^2 + 2 \cdot \eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - (\eta/n)^2}$$

$$n \cdot L = 0$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \eta_i \eta_j - \frac{1}{2} \eta_i \eta_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{5} \right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{5^2} \frac{1}{2} \eta_i \eta_j (\vec{L}_i^2 + \vec{L}_j^2) + \frac{1}{5} (\eta_i \vec{L}_i \cdot \hat{n}_j + \eta_j \vec{L}_j \cdot \hat{n}_i) + \mathcal{O}(\dots)$$

$$(\hat{n}_i - \hat{n}_j)^2 = \left(\sum_{e, e'} \hat{n}_i \cdot \vec{x}_{ij}^{e, e'} \right)^2 + \left(\partial_{\vec{n}} \right)^2 \text{small}$$

$$H = \frac{1}{2} \sum_{i,j} J_{ij} \hat{\Omega}_i \cdot \hat{\Omega}_j = \underbrace{\frac{1}{2} \sum_{i,j} \eta_i \eta_j}_{\text{classical energy (constant)}} + \frac{1}{4} \sum_i \sum_{j \neq i} J_{ij} \eta_i \eta_j (\partial_{\vec{n}} \dots)$$

$\eta_i \vec{L}_i / \hat{n}_i + \eta_j \vec{L}_j \cdot \vec{\nabla}_{\vec{n}} \vec{x}_{ij} + (\vec{L}_i \cdot \vec{L}_j)$
 0 constant cancel by summation (symmetry)
 $\sum_{i,j} J_{ij} \eta_i \eta_j x_{ij}^e x_{ij}^{e'} \approx \sum_{i,j} J_{ij}$

$$(\hat{n}_i - \hat{n}_j)^2 = n_i^2 + n_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

$$1 \uparrow \quad \left(\frac{1}{5} \right) + \left(\frac{1}{5} \right)^2 + 2\eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - (\eta/\eta_0)^2}$$

$$n \cdot L = 0$$

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = \eta_i \eta_j - \frac{1}{2} \eta_i \eta_j (\hat{n}_i - \hat{n}_j)^2 + \left(\frac{1}{5} \right)^2 \vec{L}_i \cdot \vec{L}_j - \frac{1}{5^2} \frac{1}{2} \eta_i \eta_j (\vec{L}_i^2 + \vec{L}_j^2) - \frac{1}{5} (\eta_i \vec{L}_j \cdot \hat{n}_j + \eta_j \vec{L}_i \cdot \hat{n}_i) + \mathcal{O}(\dots)$$

$$(\hat{n}_i - \hat{n}_j)^2 = \left(\sum_{e \neq e'} \hat{n}_i \cdot \vec{x}_{ij}^e \right)^2 + \left(\partial_e \hat{n} \right)^2 \text{ Small}$$

$$H = \frac{1}{2} \sum_{ij} J_{ij} \hat{\Omega}_i \cdot \hat{\Omega}_j = \underbrace{\frac{1}{2} \sum_{ij} \eta_i \eta_j}_{\text{Classical energy (constant)}} + \frac{1}{4} \sum_i \sum_{j \neq i} J_{ij} \eta_i \eta_j \left(\partial_e \hat{n}_i \cdot \vec{x}_{ij}^e \right) \left(\partial_{e'} \hat{n}_j \cdot \vec{x}_{ij}^{e'} \right)$$

$$= \bar{P}_S^{ee'} (\partial_e \hat{n}) \cdot (\partial_{e'} \hat{n})$$

$$\eta \cdot \vec{L} / \hat{n}_i + \eta \cdot \vec{L}_j \cdot \vec{\nabla} \hat{n}_j$$

0 constant

$$P_S^{ee'} = \sum_j J_{ij}$$

$$H = \frac{1}{2} \sum_{i,j} J_{ij} \hat{n}_i \hat{n}_j = \underbrace{\frac{1}{2} \sum_{i,j} J_{ij}}_{\text{classical energy (const)}} - \frac{1}{4} \sum_i \sum_{j,l} J_{ij} J_{il} (\partial_e \hat{n}_i \cdot x_{ij}) (\partial_e \hat{n}_i \cdot x_{il})$$

$\approx \rho_s^{cl} (\partial_e \hat{n}) \cdot (\partial_e \hat{n})$
 0 const. cancel by summation (symmetry) small
 $\rho_s^{cl} = \frac{1}{2} \sum_{i,j} J_{ij} J_{il} x_{ij}^l x_{il}^{e'} = \frac{1}{2} \sum_{i,j} J_{ij} J_{il} (x_{ij}^l)^2$

n.n model $J_{i,i} = J \cdot \delta_{(i,j)}$ $H = E_{cc} + \rho_s (\partial_e \hat{n}_i)^2$

$$\bar{\rho}_s = \left(\frac{1}{2} J \cdot 2\right) = J$$

classical
energy (constant)

$$j_{ij} J_{ij} (\partial_e \hat{n}_i \cdot X_{ij}^e) (\partial_{e'} \hat{n}_i \cdot X_{ij}^{e'}) + \{ \vec{L} \}$$

$$P_s^{ex} = \frac{1}{2} \sum_{ij} J_{ij} \gamma_{ij} X_{ij}^e X_{ij}^{e'} = \frac{1}{2} \sum_{ij} J_{ij} \gamma_{ij} (X_{ij}^e)^2 \delta_{e,e'}$$

$$J_{ij} = J \cdot \delta_{(i,j)}$$

$$= (\frac{1}{2} J \cdot 2) = J$$

$$H = E_{ce} + \frac{1}{2} \sum_e P_s \sum_e (\partial_e \hat{n}_i)^2$$

↑
Stiffness

$$\sum_{ij} J_{ij} \vec{L}_i^2 + J_{ij} \gamma_{ij} (\vec{L}_i \cdot \vec{L}_j)$$

$$\vec{L}_i \cdot \vec{L}_j = \chi$$

$$\begin{aligned}
 & \frac{1}{2} \sum_{i,j} J_{ij} \gamma_{ij} X_{ij}^e X_{ij}^{e'} \approx \frac{1}{2} \sum_{i,j} J_{ij} \gamma_{ij} (X_{ij}^e)^2 \delta_{e,e'} \\
 & + \{ \vec{L} \}
 \end{aligned}$$

$$\begin{aligned}
 H &= E_{ce} + \frac{1}{2} \sum_{S+M} \rho_s (\hat{n}_i)^2 + \frac{1}{2} \sum_{i,j} \left(\left(\sum_j J_{ij} \right) \vec{L}_i - J_{ij} \gamma_{ij} \vec{L}_j \right) \\
 & \quad \frac{1}{2} \sum_{i,j} \vec{L}_i \chi_{ij}^{-1} \vec{L}_j \\
 & \quad \Leftarrow \chi_{ij}^{-1} = \left[\left(\sum_j J_{0j} \right) \delta_{ij} - \gamma_{ij} J_{ij} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2} \sum_i J_{ij} \rho_j X_{ij}^e X_{ij}^{e'} \approx \frac{1}{2} \sum_{ij} J_{ij} \rho_j (X_{ij}^e)^2 \delta_{e,e'} \\
 & + \{ \vec{L} \}
 \end{aligned}$$

$$\begin{aligned}
 H = E_{ce} + \frac{1}{2} \sum_e \rho_e \sum_i (\partial_e \hat{n}_i)^2 &+ \frac{1}{2} \sum_i \left(\left(\sum_j J_{ij} \right) \vec{L}_i^2 - J_{ij} \rho_j \vec{L}_i \cdot \vec{L}_j \right) \\
 \text{Stiffness} & \\
 \frac{1}{2} \sum_{ij} \vec{L}_i \chi_{ij}^{-1} L_j &
 \end{aligned}$$

compressibility

$$\chi_{ij}^{-1} = \left[\left(\sum_j J_{0j} \right) \delta_{ij} - \rho_j J_{ij} \right]$$

$$\begin{aligned}
 & \dots + \{ \vec{L} \} \\
 & \rho_s = \frac{1}{2} \sum_{i,j} J_{ij} \gamma_{ij} X_{ij}^e X_{ij}^{e'} \approx \frac{1}{2} \sum_{i,j} J_{ij} \gamma_{ij} (X_{ij}^e)^2 \delta_{e,e'}
 \end{aligned}$$

$$\begin{aligned}
 H = E_{ce} + \frac{1}{2} \sum_i \hat{n}_i^2 + \frac{1}{2} \sum_i \left(\left(\sum_j J_{ij} \right) \vec{L}_i^2 - J_{ij} \gamma_{ij} \vec{L}_i \cdot \vec{L}_j \right) \\
 \frac{1}{2} \sum_i \sum_j \vec{L}_i \cdot \chi_{ij}^{-1} \cdot \vec{L}_j \\
 \left(\begin{array}{c} \text{compressibility} \\ \text{stability} \end{array} \right) \leftarrow \chi_{ij}^{-1} = \left[\left(\sum_j J_{0j} \right) \delta_{ij} - \gamma_{ij} J_{ij} \right]
 \end{aligned}$$

$$H = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \hat{n}_j = \underbrace{\frac{1}{2} \sum_{ij} J_{ij}}_{\text{classical energy (const)}} - \frac{1}{2} \sum_i \sum_{jll} J_{ij} l_i (\partial_e \hat{n}_i \cdot \chi_{ij}^l) (\partial_e \hat{n}_i \cdot \chi_{ij}^{l'}) + \{L\}$$

$\sum P_s^{cl} (\partial_e \hat{n}) (\partial_e \hat{n})$
 0 constant cancel by summation (symmetry) small
 $P_s^{cl} \frac{1}{2} \sum_{ij} J_{ij} l_i \chi_{ij}^l \chi_{ij}^{l'} \sim \frac{1}{2} \sum_{ij} J_{ij} l_i l_j (\chi_{ij}^l)^2$

h.n model $J_{ij} = J \cdot \delta_{i,j}$

$$\bar{P}_s = \left(\frac{1}{2} J \cdot 2\right) = J$$

$$\bar{\chi}^{-1} = 2 \cdot Z \cdot J$$

$$H = E_{ce} + \frac{1}{2} \sum P_s \sum_e (\partial_e \hat{n}_i)^2 + \frac{1}{2} \sum_i \left(\sum_j J_{ij} \right) \bar{L}_i^2 + J_{ij} l_i l_j \bar{L}_i \bar{L}_j$$

Stiffness

$$\frac{1}{2} \sum_i \sum_j \bar{L}_i \bar{\chi}_{ij}^{-1} \bar{L}_j \rightarrow \frac{1}{2} \sum_i \bar{L}_i \bar{\chi}_i^{-1} \bar{L}_i$$

(compressibility) (susceptibility) $\bar{\chi}_{ij}^{-1} = \left[\left(\sum_j J_{oj} \right) \delta_{ij} - (J \cdot l_i l_j) \right]$

h.n model $\bar{\chi}_i^{-1} = \left(\sum_j \bar{\chi}_{ij}^{-1} \right) \cdot Z \cdot J$

$$H = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \hat{n}_j = \frac{1}{2} \sum_{ij} J_{ij} - \frac{1}{4} \sum_i \sum_{jll} J_{ij} J_{ll} (\partial_e \hat{n}_i \cdot X_{ij}^{ll}) (\partial_e \hat{n}_i \cdot X_{ij}^{ll}) + \{L\}$$

$\sum_{ij} J_{ij}$ classical energy (constant)
 $\sum_{ij} J_{ij} J_{ll}$ $\partial_e \hat{n}_i \cdot X_{ij}^{ll}$ $\partial_e \hat{n}_i \cdot X_{ij}^{ll}$
 0 constant cancel by summation (symmetry) small
 $P_S = \frac{1}{2} \sum_{ij} J_{ij} J_{ll} X_{ij}^{ll} X_{ij}^{ll} = \frac{1}{2} \sum_{ij} J_{ij} J_{ll} (X_{ij}^{ll})^2 \delta_{ij}$

n.n model $J_{ij} = J \delta_{ij}$

$$H = E_{ce} + \frac{1}{2} \sum_i P_S \sum_e (\partial_e \hat{n}_i)^2 + \frac{1}{s^2} \sum_i \left(\sum_j J_{ij} \bar{L}_i - J_{ij} J_{ll} \bar{L}_i \bar{L}_j \right)$$

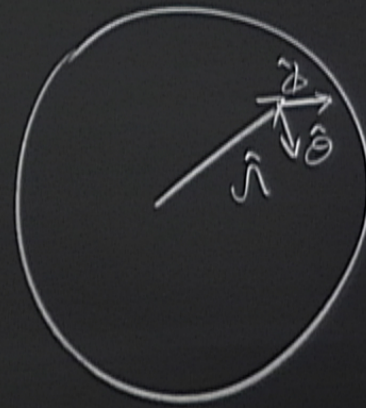
$\bar{P}_S = (\frac{1}{2} J \cdot 2) = J$ $\bar{L}_i = 2 \cdot z \cdot T$
 stiffness $\frac{1}{2} \sum_i \sum_j \bar{L}_i \bar{X}_{ij}^{-1} \bar{L}_j \rightarrow \frac{1}{2} \sum_i \bar{L}_i^2 \bar{X}^{-1}$
 (compressibility (susceptibility)) $\bar{X}_{ij}^{-1} = \left[\left(\sum_j J_{oj} \right) \delta_{ij} - J_{ij} J_{ll} \right] \frac{1}{s^2}$
 n.n model $\bar{X}_i = \left(\sum_j \bar{X}_{ij}^{-1} \right) \bar{z} J \frac{1}{s^2}$

$$H = E_{ce} + \sum_{i \in A} \frac{1}{2} \bar{p}_i$$

$$H = E_{ce} + \sum_{i \in A} \frac{1}{2} \bar{P}_S |\partial_x \hat{n}_i|^2 + \frac{1}{2} \bar{\chi}^{-1 \rightarrow 2} L_i$$

$$i S \omega[\Omega]$$

$$\omega[\hat{n}] = \int dt (1 - \omega \theta) \dot{\phi}$$



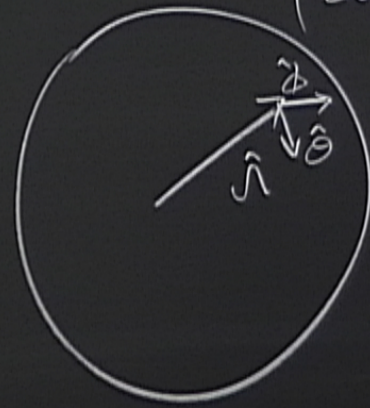
$$H = E_{ce} + \sum_{i \in A} \frac{1}{2} \bar{P}_S |\partial_x \hat{n}_i|^2 + \frac{1}{2} \bar{\chi}^{-1 \rightarrow 2} L_i \quad (\hat{n}, \hat{\theta}, \hat{\phi})$$

$$i S \omega[\hat{n}]$$

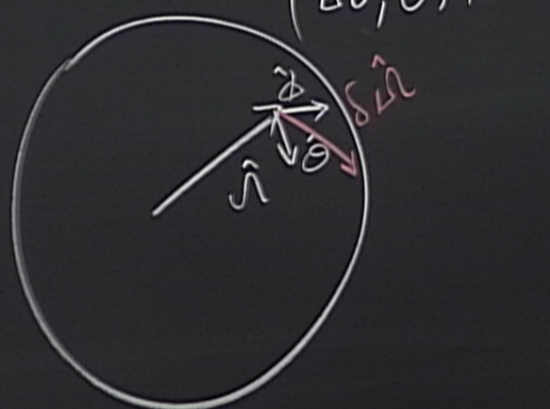
$$\omega[\hat{n}] = \int dt (1 - \omega \theta) \dot{\phi}$$

$$\delta \hat{n} = \sin \theta \delta \phi \hat{\phi} + \delta \theta \hat{\theta}$$

$$\frac{d \hat{n}}{dt} = \sin \theta \dot{\phi} \hat{\phi} + \dot{\theta} \hat{\theta}$$



$$|\partial_{\mathbf{r}} \hat{n}_i|^2 + \frac{1}{2} \chi^{-1 \rightarrow 2} h_i$$



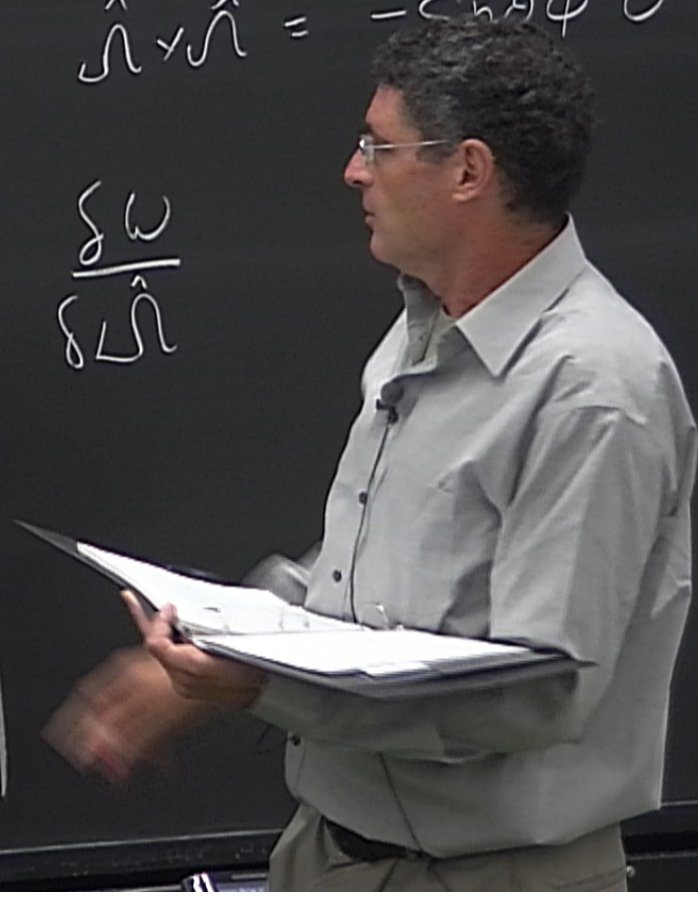
$$- (\omega \theta) \dot{\phi}$$

$$\dot{\phi} + \dot{\theta} \hat{\theta}$$

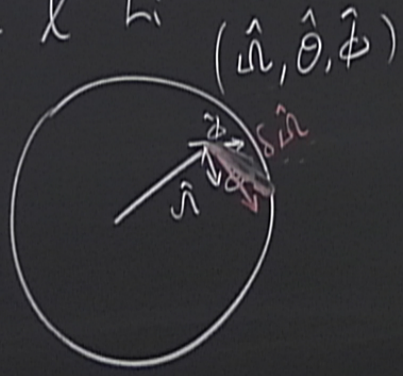
$$\dot{\phi} + \dot{\theta} \hat{\theta}$$

$$\hat{L} \times \dot{\hat{L}} = -c \sin \theta \dot{\phi} \hat{\theta} + \dot{\theta} \hat{\phi}$$

$$\frac{\delta \omega}{\delta L \hat{L}}$$



$$\frac{1}{2} \bar{\chi}^{-1} L_i^2$$



$$(-\omega\theta)\dot{\phi}$$

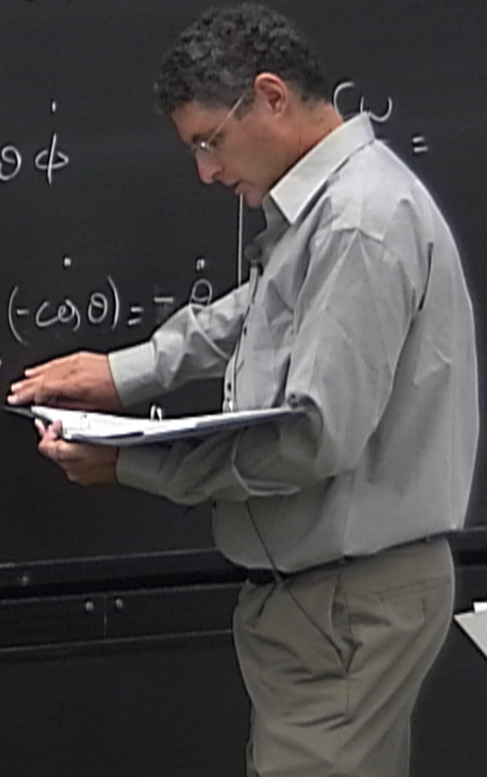
$$\dot{\phi}\hat{\phi} + \dot{\theta}\hat{\theta}$$

$$\dot{\phi}\hat{\phi} + \dot{\theta}\hat{\theta}$$

$$\dot{\hat{n}} \times \hat{n} = -\sin\theta\dot{\phi}\hat{\theta} + \dot{\theta}\hat{\phi}$$

$$\left(\frac{\delta\omega}{\delta L\hat{n}(t)}\right)_\theta = \sin\theta\dot{\phi}$$

$$\left(\frac{\delta\omega}{\delta L\hat{n}}\right)_\phi = \frac{1}{\sin\theta}(-\omega\theta) = -\dot{\theta}$$



$$\begin{aligned}
 (\hat{n}_i - \hat{n}_j) &= \left(\sum_{\alpha \neq \beta} \hat{n}_{\alpha\beta} \cdot \hat{x}_{ij}^\alpha \right) + (\partial \hat{n}) \\
 H &= \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \cdot \hat{n}_j = \underbrace{\frac{1}{2} \sum_{ij} J_{ij}}_{\text{classical energy (constant)}} - \frac{1}{2} \sum_{ij} \sum_{\alpha \beta} J_{ij} \hat{n}_i^\alpha \hat{n}_j^\beta (\partial_\alpha \hat{n}_i \cdot \hat{x}_{ij}^\alpha) (\partial_\beta \hat{n}_j \cdot \hat{x}_{ij}^\beta) + \dots
 \end{aligned}$$

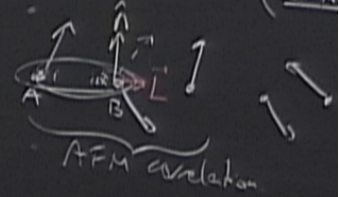
$$\begin{aligned}
 \gamma \frac{\vec{L}}{\hat{n}} + \gamma \vec{L} \cdot \nabla \hat{n} \hat{x} + (\vec{L} \cdot \nabla^2 \hat{n} \cdot \hat{x}^2) \\
 \text{O constant} \quad \text{cancel} \quad \text{small} \\
 \text{by summation} \\
 \text{(symmetry)}
 \end{aligned}$$

$$\rho_s^{\text{eff}} = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \cdot \hat{n}_j = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i^\alpha \hat{n}_j^\alpha = \frac{1}{2} \sum_{ij} J_{ij} (\hat{n}_i^\alpha)^2 \int \dots$$

Field theory for the Quantum AFM (non-linear sigma model) (O(3))

$$\int \mathcal{D}\hat{n} e^{i(S[\hat{n}] - \int dt H[\hat{n}])}$$

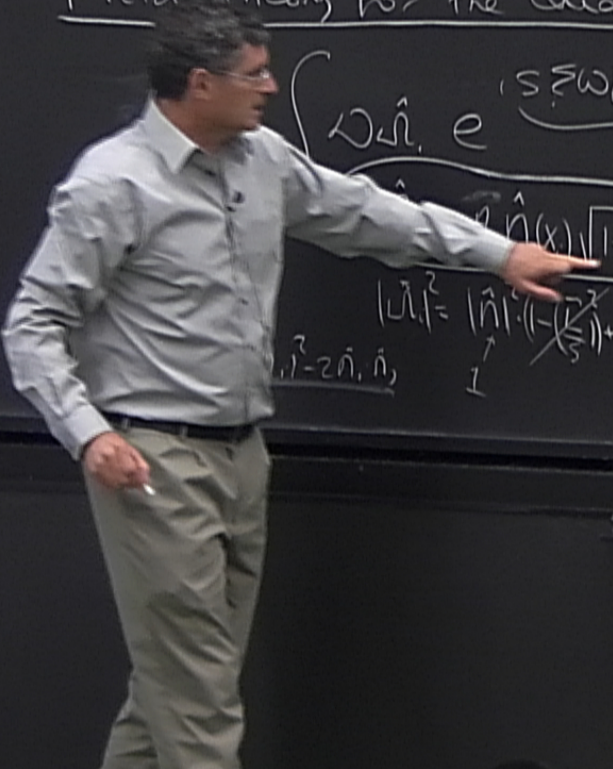
$$H = \frac{1}{2} \sum_{ij} J_{ij} \hat{n}_i \cdot \hat{n}_j$$



$$\begin{aligned}
 |\vec{L}|^2 &= |\hat{n}|^2 \left(1 - \left(\frac{\vec{L}}{s}\right)^2 + \left(\frac{\vec{L}}{s}\right)^2 + 2\gamma \hat{n} \cdot \vec{L} \sqrt{1 - \left(\frac{\vec{L}}{s}\right)^2} \right) \\
 |\vec{L}|^2 &= 2\hat{n} \cdot \vec{L}
 \end{aligned}$$

$$\begin{aligned}
 |\hat{n}| &= 1 \\
 \hat{n} \cdot \vec{L} &= 0
 \end{aligned}$$

$$\underbrace{d\hat{n}_i \cdot d\hat{n}_j}_{\gamma} = d\hat{n}_i \cdot d\vec{L}_j \delta(|\hat{n}_i|=1) \delta(|\hat{n}_j|=1)$$



$$\omega[\hat{n}] = \int dt (1 - \cos\theta) \dot{\phi}$$

$$\delta\hat{n} = \sin\theta \delta\phi \hat{\phi} + \delta\theta \hat{\theta}$$

$$\frac{d\hat{n}}{dt} = \sin\theta \dot{\phi} \hat{\phi} + \dot{\theta} \hat{\theta}$$



$$\left(\frac{\delta\omega}{\delta\hat{n}}\right)_\theta = \frac{1}{\sin\theta} (\dot{\omega}, \theta) = -\dot{\theta}$$

$$\frac{\delta\omega}{\delta\hat{n}_i} = -\eta_i \hat{n} \times \eta_i \hat{n} = -\hat{n} \times \hat{n}$$

$$\sum_i \int dt \omega[\eta_i \hat{n}_i + (\frac{\vec{L}_i}{\hbar})]$$

$$\sum_i \int dt \omega[\eta_i \hat{n}_i] - \sum_i \int dt \hat{n}_i \times \dot{\hat{n}}_i \cdot (\frac{\vec{L}_i}{\hbar})$$

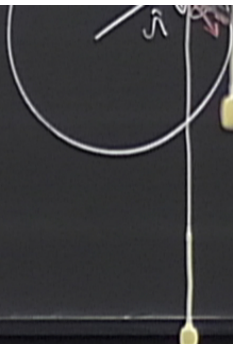
topological Berry phase $= \sum_i \int dt (\hat{n}_i \times \dot{\hat{n}}_i) \cdot \vec{L}_i$

$$Z = \int \mathcal{D}\hat{n}$$

$$\omega[\hat{n}] = \int dt (1 - \cos\theta) \dot{\phi}$$

$$\delta\hat{n} = \sin\theta \delta\phi \hat{\phi} + \delta\theta \hat{\theta}$$

$$\frac{d\hat{n}}{dt} = \sin\theta \dot{\phi} \hat{\phi} + \dot{\theta} \hat{\theta}$$



$$\left(\frac{\delta\omega}{\delta\theta}\right)_\theta = \frac{1}{\sin\theta} (\cos\theta) = -\hat{\theta}$$

$$\frac{\delta\omega}{\delta\hat{n}_i} = -\eta_i \hat{n} \times \eta_i \hat{n} = -\hat{n} \times \hat{n}$$

$$\sum_i \delta \omega[\hat{n}_i] = \underbrace{\sum_i \delta \omega[\hat{n}_i]}_{\text{topological Berry phase}} - \sum_i \hat{n} \times \hat{n} \cdot \left(\frac{\vec{L}_i}{\hbar}\right)$$

$$\sum_i \hat{n} \times \hat{n} \cdot \vec{L} - \frac{1}{\hbar} \chi L^2$$

$\partial \hat{\theta}$
 $\partial \hat{\theta}$

$$\left. \begin{aligned}
 \frac{\delta \omega}{\delta \hat{n}_i} &= -\gamma_i \hat{n}_i \times \dot{\hat{n}}_i = -\dot{\hat{n}}_i \times \hat{n}_i \\
 \text{Simp} &
 \end{aligned} \right\}$$

$$= \underbrace{\sum_i S \omega[\gamma_i \hat{n}_i]}_{\text{topological Berry phase}} - \int dt \sum_i \dot{\hat{n}}_i \times \hat{n}_i \cdot \left(\frac{\vec{L}_i}{S} \right)$$

$$e^{-\int dt \sum_i \dot{\hat{n}}_i \times \hat{n}_i \cdot \vec{L}_i} e^{-\frac{1}{2} \chi \sum_i L_i^2} = \int \mathcal{D}\hat{n} \delta(\hat{n}^2 - 1) e^{-\int dt \sum_i \left(\frac{1}{2} \chi (\dot{\hat{n}}_i \times \hat{n}_i)^2 + \frac{1}{2} \beta_S \dot{\hat{n}}_i \cdot \vec{L}_i \right)}$$

$$- \int dt' \left(\frac{1}{2} \chi (\dot{\hat{n}} \times \hat{n})^2 + \frac{1}{2} \beta_S \dot{\hat{n}} \cdot \vec{L} \right)$$

$$\frac{\delta \omega}{\delta \mathbf{n}} = -\gamma \hat{\mathbf{n}} \times \dot{\mathbf{n}} = -\dot{\mathbf{n}} \times \hat{\mathbf{n}}$$

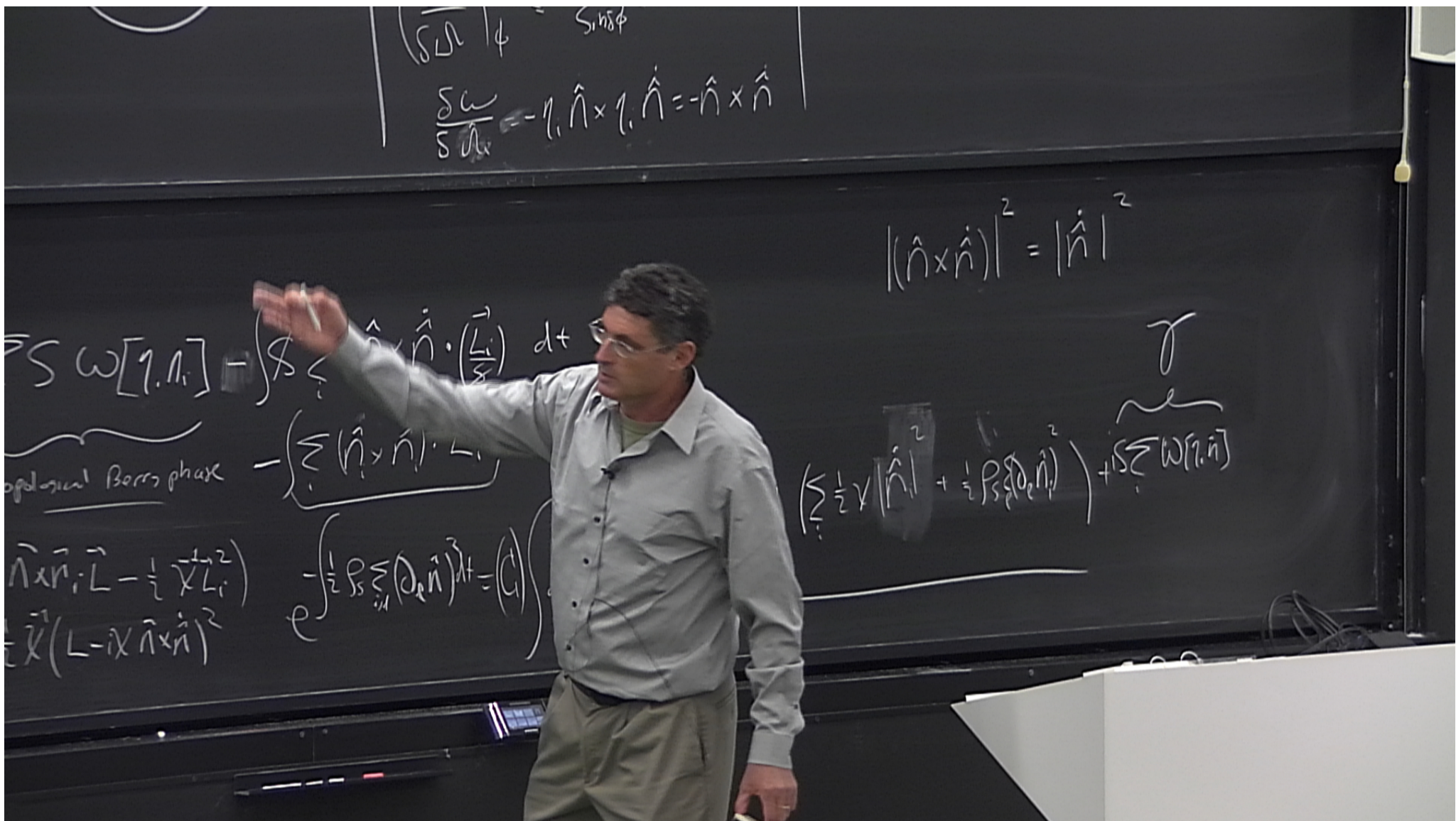
$$|(\hat{\mathbf{n}} \times \dot{\mathbf{n}})|^2 = |\dot{\mathbf{n}}|^2$$

$$S \omega[\gamma, \mathbf{n}] = \int dt \sum_i \hat{\mathbf{n}} \times \dot{\mathbf{n}} \cdot \left(\frac{\vec{L}_i}{\hbar} \right)$$

optimal Berry phase $-\int dt \sum_i (\hat{\mathbf{n}} \times \dot{\mathbf{n}}) \cdot \vec{L}_i$

$$e^{-\int dt \sum_i \frac{1}{\hbar} \mathbf{p}_i \cdot \dot{\mathbf{n}}_i} = \int \mathcal{D}\hat{\mathbf{n}} \delta(|\hat{\mathbf{n}}|=1) e^{-\int dt \left(\frac{1}{2} \gamma |\dot{\mathbf{n}}|^2 + \frac{1}{2} \mathbf{p}_i \cdot \dot{\mathbf{n}}_i \right)}$$





$$\frac{\delta \omega}{\delta \mathbf{a}_i} = -\eta_i \hat{n} \times \dot{\mathbf{n}} = -\dot{\mathbf{n}} \times \hat{n}$$

$$|(\hat{n} \times \dot{\mathbf{n}})|^2 = |\dot{\mathbf{n}}|^2$$

$$\int \delta \omega[\mathbf{a}_i] = \int \delta \left(\sum_i \hat{n}_i \times \dot{\mathbf{n}}_i \cdot \left(\frac{\vec{L}_i}{\hbar} \right) \right) dt$$

topological Berry phase

$$- \left(\sum_i (\hat{n}_i \times \dot{\mathbf{n}}_i) \cdot \vec{L}_i \right)$$

$$\hat{n} \times \dot{\mathbf{n}}; \vec{L} = \frac{1}{2} \chi \vec{L}_i$$

$$\chi (\vec{L} - i\chi \hat{n} \times \dot{\mathbf{n}})^2$$

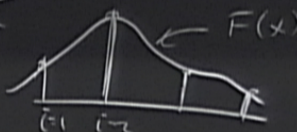
$$e^{-\int \frac{1}{2} P_S \sum_i (\partial_t \hat{n}_i)^2 dt} = (C)$$

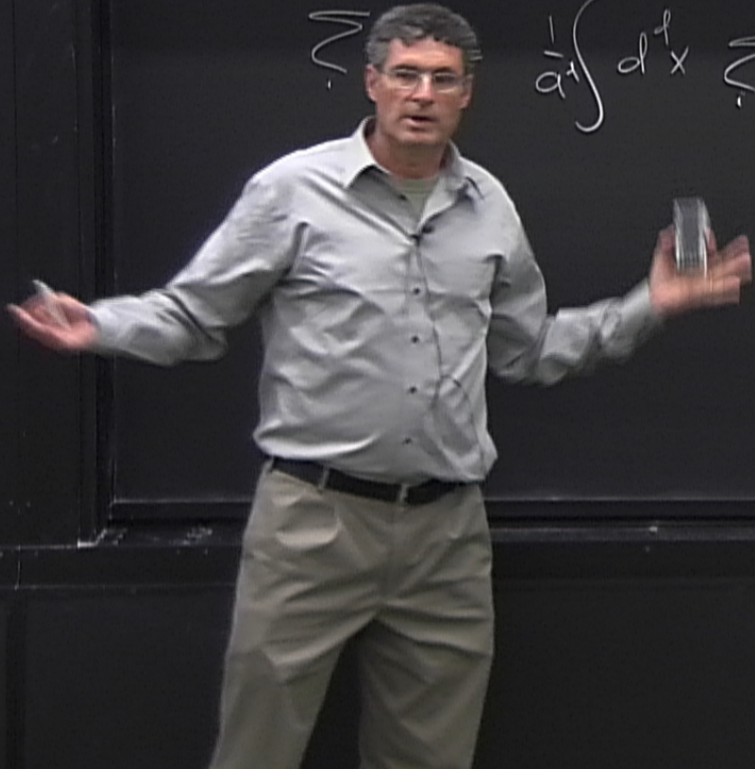
$$\left(\sum_i \frac{1}{2} \chi |\dot{\mathbf{n}}_i|^2 + \frac{1}{2} P_S \sum_i (\partial_t \hat{n}_i)^2 \right) + \underbrace{\int \delta \omega[\mathbf{a}_i]}_{\gamma}$$

topological Berry phase

$$Z = \int \mathcal{D}n \int \mathcal{D}\vec{L} \delta(n^2) e^{\int_0^P dt \left(\sum_i \vec{n}_i \cdot \vec{L}_i - \frac{1}{2} \chi \frac{L_i^2}{L_i} \right)} e^{-\int_0^P dt \left(\sum_i \vec{n}_i \cdot \vec{L}_i \right)}$$

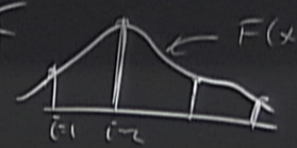
$$= \int \mathcal{D}\hat{n} \delta(\hat{n}^2) e^{-\int_0^P dt \left(\sum_i \frac{1}{2} \chi |\dot{\hat{n}}_i|^2 \right)}$$

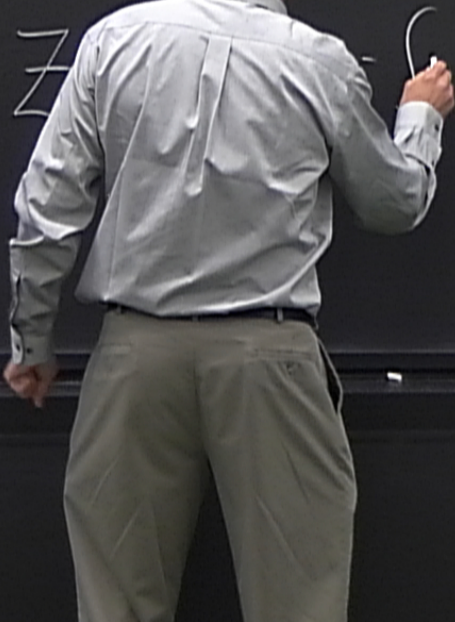
$$\int d^d x \delta(x-x_0) F(x)$$




topological Berry phase

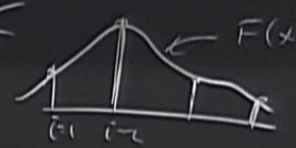
$$Z = \int \mathcal{D}n \int \mathcal{D}\vec{L} \delta(\vec{n}^2) e^{-\int_0^P dt \left(\sum_i \vec{\pi}_i \vec{n}_i \cdot \vec{L}_i - \frac{1}{2} \sum_i \frac{L_i^2}{\hbar^2} \right)} e^{-\int_0^P dt \left(\sum_i \frac{1}{2} v |\dot{\vec{n}}_i|^2 \right)}$$

$$\sum_i F_i = \int dx \sum_j \delta(x-x_j) F(x)$$




topological Berry phase

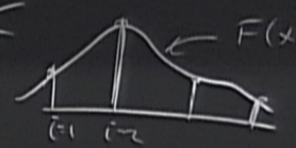
$$Z = \int \mathcal{D}n \int \mathcal{D}\vec{L} \delta(n^2) e^{-\int_0^P dt \left(\sum_i \vec{n}_i \cdot \vec{L}_i - \frac{1}{2} \chi \frac{d^2 z}{dt^2} - \frac{1}{2} \chi (L - i\chi \vec{n} \times \dot{\vec{n}})^2 \right)} e^{-\int_0^P dt \left(\sum_{i,j} P_{ij} (\partial_t \hat{n}_i)^2 \right)} = \mathcal{C} \int \mathcal{D}\hat{n}, \delta(\hat{n}^2) e^{-\int_0^P dt \left(\sum_i \frac{1}{2} v |\dot{\hat{n}}|^2 \right)}$$

$$\sum_i F_i = \frac{1}{a^d} \int d^d x \delta(x-x_i) F(x)$$


$$Z = \int \mathcal{D}\hat{n} \int dt \int d^d x \left(\chi (\partial_t \hat{n})^2 + \sum_{i,j} P_{ij} (\partial_x \hat{n}_{ij})^2 \right); \hat{n}(\vec{x}, t)$$

topological Berry phase

$$Z = \int \mathcal{D}n \int \mathcal{D}\vec{L} \delta(n^2) e^{-\int_0^{\beta} dt \left(\sum_i \vec{n}_i \cdot \vec{L}_i - \frac{1}{2} \sum_{\langle ij \rangle} \vec{L}_i \cdot \vec{L}_j \right)} e^{-\int_0^{\beta} dt \left(\sum_i \frac{1}{2} v |\dot{\vec{n}}_i|^2 \right)}$$

$$\sum_i F_i = \frac{1}{a^d} \int d^d x \sum_j \delta(x-x_j) F(x)$$


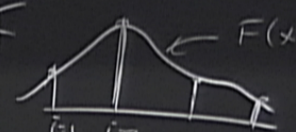
$g \in \Lambda = \text{ultra-uv}$

$$Z = \int \mathcal{D} \left(\int d^d x \left(\lambda (\partial_\mu \hat{n})^2 + \rho \sum_{\sigma=1}^d (\partial_\sigma \hat{n})^2 \right); \hat{n}(\vec{x}, t) \right)$$

$$\Lambda \in \Lambda_{BZ}$$

topological Berry phase

$$Z = \int \mathcal{D}n \int \mathcal{D}\vec{L} \delta(\vec{n} \cdot \vec{L}) e^{-\int_0^P dt \left(\sum_i \vec{n}_i \vec{n}_i \cdot \vec{L}_i - \frac{1}{2} \chi \frac{L_i^2}{L_i} \right)} e^{-\int_0^P dt \left(\sum_i \frac{1}{2} \chi v |\dot{\vec{n}}_i|^2 \right)}$$

$$\sum_i F_i = \frac{1}{a^d} \int d^d x \sum_i \delta(x-x_i) F(x)$$


$$Z = \int \mathcal{D}n_{\vec{q}} e^{-\int_0^{\beta} dt \int d^d x \left(\chi (\partial_t \hat{n})^2 + \rho_s^2 (\partial_x \hat{n})^2 \right)}; \hat{n}(\vec{x}, t)$$

$\left(\begin{matrix} dn_1, dn_2, \dots \\ dn_{q=0}, dn_{q \sim \frac{1}{\Lambda}} \dots \end{matrix} \right) \quad \Lambda \ll \Lambda \in \mathbb{R} \quad \sum \rightarrow \text{typical fluct length scale}$

$$(\hat{n}_i - \hat{n}_j)^2 = n_i^2 + n_j^2 - 2\hat{n}_i \cdot \hat{n}_j$$

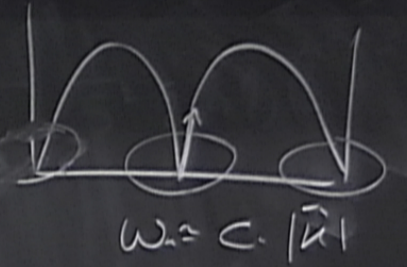
$$|\mathbf{L}| = |\hat{n}| \cdot \left(1 - \left(\frac{v}{c}\right)\right) + \left(\frac{v}{c}\right)^2 + 2\eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$n \cdot L = 0$$

$$c = \sqrt{\rho_s / \chi} = \sqrt{\tau_s a}$$

$$n \cdot n \quad \rho_s = J a^{2-d}$$

$$\chi = \frac{s^2}{4dJ}$$



$$|\mathbf{L}| = |\mathbf{n}_1| \cdot \left(1 - \left(\frac{v}{c}\right)^2\right) + \left(\frac{v}{c}\right)^2 + 2\eta \cdot \hat{\mathbf{n}}_1 \cdot \vec{L} \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

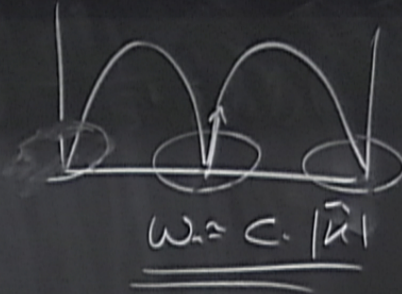
$$\hat{\mathbf{n}}_1^2 = n_{1,x}^2 + n_{1,y}^2 + n_{1,z}^2 - 2\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2$$

$$\mathbf{n} \cdot \mathbf{L} = 0$$

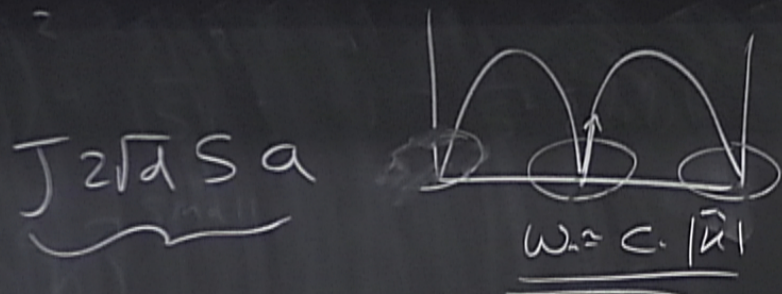
$$c = \sqrt{P_S / \chi} = J_s a$$

$$n \cdot n \quad P_S = J a^{2-d}$$

$$\chi = \frac{S^2}{4dJ}$$



$$\left(\frac{\vec{k}}{5}\right) + \left(\frac{\vec{k}}{5}\right)^2 + 2 \cdot \eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - (\eta \vec{k})^2} \quad | \quad \hat{n} \cdot \vec{L} = 0$$



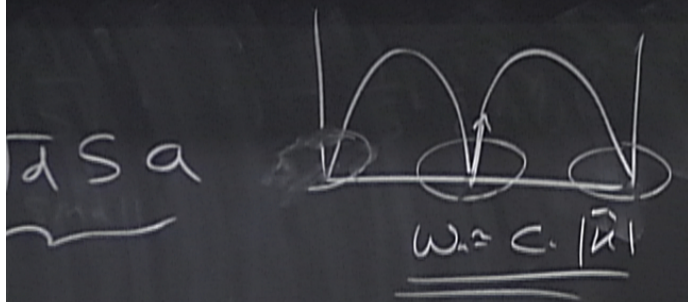
$$X_0 = c \cdot t$$

$$\int dt \int dx^d \rightarrow \frac{1}{c}$$

$$\int d^n x \left[\rho_x (\partial_t \hat{n})^2 + \rho_s (\partial_x \hat{n})^2 \right]$$

$$\int_{n=0}^{d+1} d^n x \left[(\partial_\mu \hat{n})^2 \right]$$

$$\left(\frac{\vec{L}}{5}\right)^2 + 2\eta \cdot \hat{n} \cdot \vec{L} \sqrt{1 - (\eta/\lambda)^2} \quad | \quad n \cdot L = 0$$



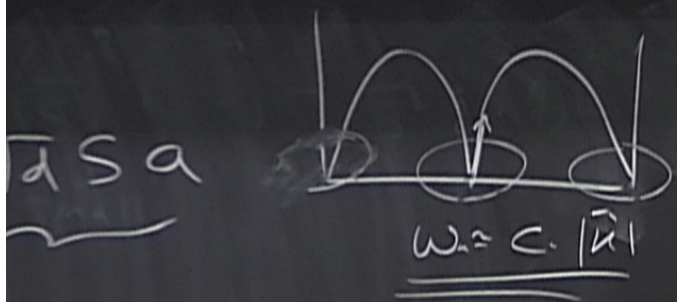
$$X_0 = c \cdot t$$

$$\int dt \int d^d x \rightarrow$$

$$\int dt d^d x \cdot \left[\chi (\partial_t \hat{n})^2 + \rho_s (\partial_x \hat{n})^2 \right]$$

$$= \frac{1}{d^{d+1}} \int d^d x \sum_{n=0}^d (\partial_x \hat{n})^2$$

$$\left(\frac{1}{5}\right)^2 + 2 \cdot \eta \cdot \hat{n} \cdot \vec{L} \cdot \sqrt{1 - (\eta/a)^2} \quad | \quad \hat{n} \cdot \vec{L} = 0$$

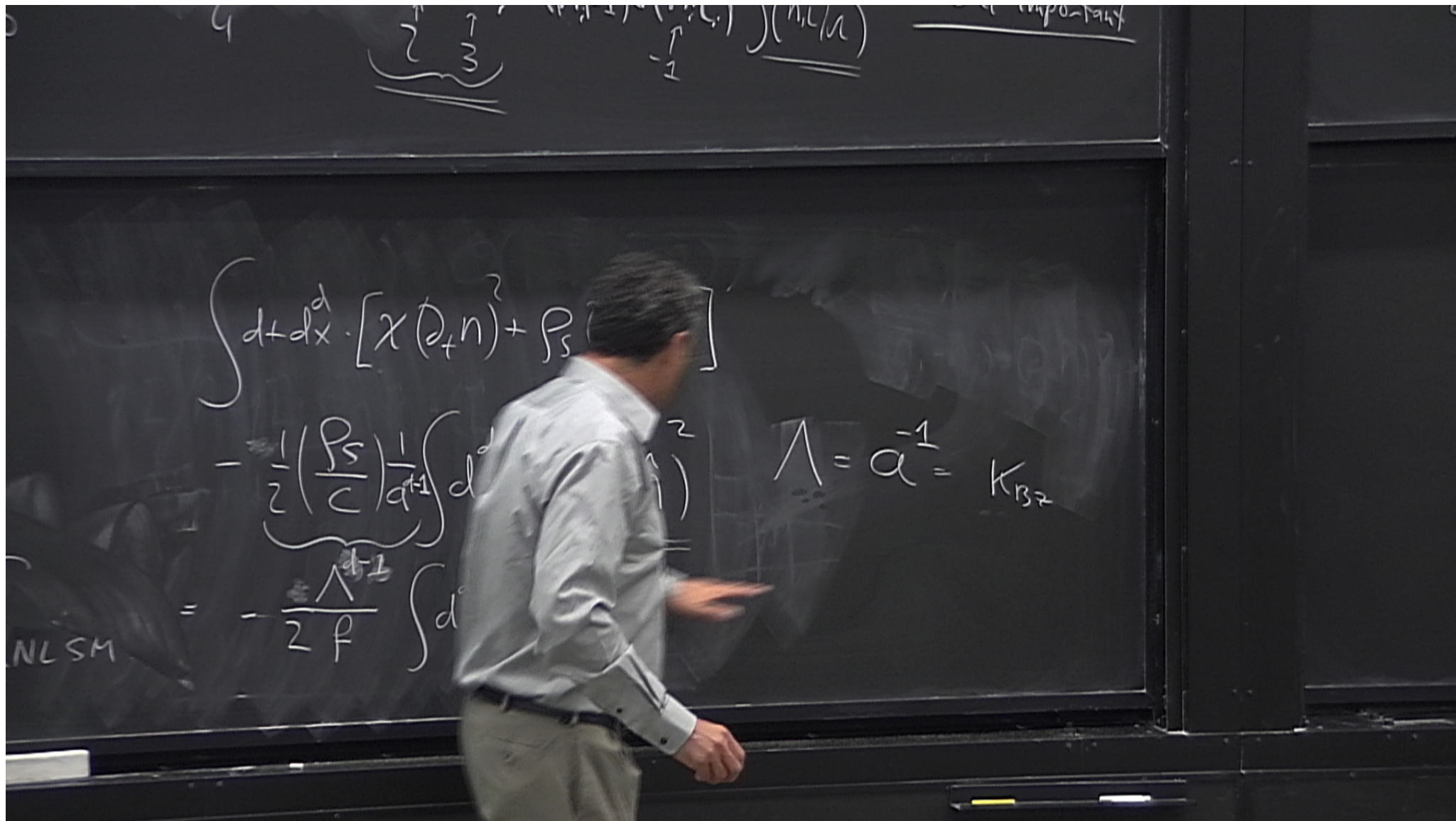


$$X_0 = c \cdot t$$

$$\int d^d x \cdot [\chi(\partial_t n)^2 + P_S(\partial_x n)^2]$$

$$- \frac{1}{2} \left(\frac{P_S}{c} \right) \frac{1}{a^{d+1}} \int d^d x$$

$$\mathcal{L}_{NLSM} = - \frac{\Lambda^{d+1}}{2 f} \int d^{d+1} x$$



4 $\left(\begin{matrix} 2 & 1 \\ 3 & 1 \end{matrix} \right)$ $\left(\begin{matrix} 1 \\ -1 \end{matrix} \right)$ $\left(\frac{1}{a} \right)$ important

$$\int d^d x \cdot [x(D+n)^2 + P_S]$$

$$= -\frac{1}{2} \left(\frac{P_S}{c} \right)^{\frac{1}{d}} \int d^d x (x^2)^{\frac{d}{2}}$$

$$= -\frac{\Lambda^{d-2}}{2f} \int d^d x$$

$$\Lambda = a^{-1} K_{BZ}$$

NLSM

$$X_0 = c \cdot t$$

$$\frac{s^2}{4\pi a^2}$$

$$-\frac{1}{2} \left(\frac{P_s}{c} \right) \frac{1}{a^{d+1}} \int d^d x \sum_{m=0}^{d+1} (\partial_\mu \hat{n})^2$$

$$\Lambda = a^{-1} = k_{Bz}$$

$$\underline{\underline{\mathcal{L}_{NL\text{SM}}} = -\frac{\Lambda^{d+1}}{2f} \int d^{d+1} x (\partial_\mu \hat{n})^2 + \text{S.T.}}$$

$$Z = \int \mathcal{D}\hat{n} e^{-\left(\frac{\Lambda^{d+1}}{2f}\right) \int d^d x \sum_{\mu} (\partial_\mu \hat{n})^2}$$

$$e^{-\frac{H[\hat{n}]}{T}} \quad k_{Bz} T = (f/\Lambda^{d-2})$$

$$\langle \hat{n}(x,t) \hat{n}(0,0) \rangle = \langle n(\vec{x}) n(0) \rangle_{\text{class}} = e^{-|\vec{x}|/\xi}$$

$$\sum_n \psi_n(0) \psi_n(0) e^{-(E_n - E_0)t}$$

Cap!!

