

Title: Quantum Field Theory - Lecture 12A

Date: Oct 24, 2012 09:00 AM

URL: <http://pirsa.org/12100019>

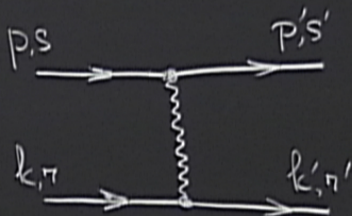
Abstract:

Scattering of electrons

$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, \tau & & p', s' & k', \tau' \end{array}$$

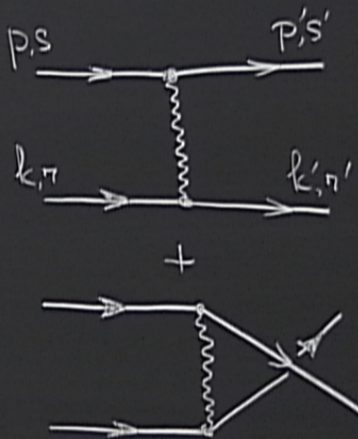
Scattering of electrons

$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, \tau & & p', s' & k', \tau' \end{array}$$



Scattering of electrons

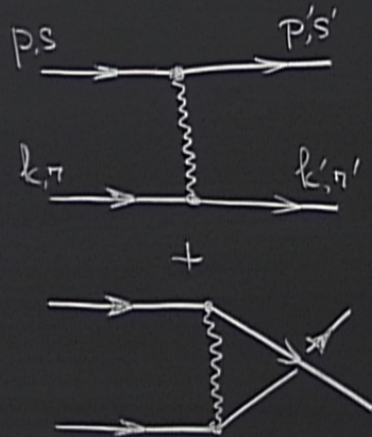
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Scattering of electrons

$$i\mathcal{M} =$$

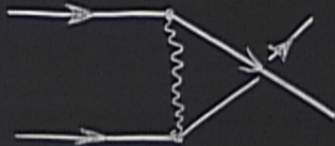
$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, \tau & & p', s' & k', \tau' \end{array}$$



Scattering of electrons

$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, \tau & & p', s' & k', \tau' \end{array}$$

$$\begin{array}{c} u^s(p) \xrightarrow{p, s} \text{---} \xrightarrow{-ie\gamma^\mu} \text{---} \xrightarrow{p', s'} \bar{u}^{s'}(p') \\ \text{---} \xrightarrow{k, \tau} u^\tau(k) \xrightarrow{-ie\gamma_\mu} \text{---} \xrightarrow{k', \tau'} \bar{u}^{\tau'}(k') \\ \quad \quad \quad \uparrow \\ \quad \quad \quad + \end{array}$$



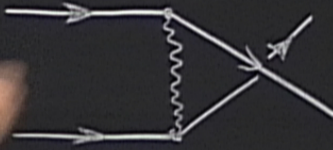
$$i\mathcal{M} = (-ie)^2 \bar{u}^{s'}(p') \gamma^\mu u^s(p) \bar{u}^{\tau'}(k') \gamma_\mu u^\tau(k)$$

Scattering of electrons

$$i\mathcal{M} = (-ie)^2 \bar{u}^{s'}(p') \gamma^\mu u^s(p) \bar{u}^{r'}(k') \gamma^\nu u^r(k)$$

$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, r & & p', s' & k', r' \end{array}$$

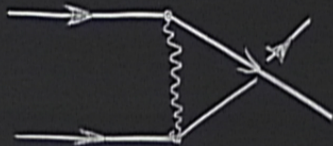
$$\begin{array}{c} u^s(p) \xrightarrow{p, s} \xrightarrow{-ie\gamma^\mu} p', s' \xrightarrow{\bar{u}^{s'}(p')} \\ \downarrow \text{wavy line} \\ u^r(k) \xrightarrow{k, r} \xrightarrow{-ie\gamma^\nu} k', r' \xrightarrow{\bar{u}^{r'}(k')} \end{array}$$



Scattering of electrons

$$\begin{array}{ccc} e^- & e^- & \longrightarrow & e^- & e^- \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ p, s & k, \tau & & p', s' & k', \tau' \end{array}$$

$$\begin{array}{c} \text{Top line: } u^s(p) \xrightarrow{p, s} \text{---} \xrightarrow{-ie\gamma^\mu} \text{---} \xrightarrow{p', s'} \bar{u}^{s'}(p') \\ \text{Bottom line: } u^\tau(k) \xrightarrow{k, \tau} \text{---} \xrightarrow{-ie\gamma^\nu} \text{---} \xrightarrow{k', \tau'} \bar{u}^{\tau'}(k') \\ \text{Wavy line: } p' - p = k - k' \end{array}$$



$$i\mathcal{M} = (-ie)^2 \bar{u}^{s'}(p') \gamma^\mu u^s(p) \bar{u}^{\tau'}(k') \gamma^\nu u^\tau(k)$$

trans

$$i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{u}^{r'}(k') \gamma^\nu u^r(k) \frac{-ig_{\mu\nu}}{(p'-p)^2}$$

$\bar{u}^s(p')$

$\bar{u}^{r'}(k')$

$$i\mathcal{M} = (-ie)^2 \bar{u}^{s'}(p') \gamma^\mu u^s(p) \bar{u}^{r'}(k') \gamma^\nu u^r(k) \frac{-ig_{\mu\nu}}{(p'-p)^2} - (p's \leftrightarrow k'r')$$

Non-relativistic approximation:

$$\bar{u} \gamma \gamma \gamma u$$

$$u^s \simeq \sqrt{2m} \begin{pmatrix} \xi^s \\ \bar{\xi}^s \end{pmatrix}$$

$$\bar{u}^{s'} \gamma^0 u^s = u^{s't} u^s = 2m \xi^{s't} \xi^s = 2m \delta^{ss'}$$

$$\bar{u}^{s'} \gamma^i u = u^{s't} \gamma^i u^s = m \begin{pmatrix} \xi^{s't} & \bar{\xi}^{s't} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}}_{\begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}} \begin{pmatrix} \xi^s \\ \bar{\xi}^s \end{pmatrix} = 0$$

$$(p' - p)^0 = (E(p') - E(p))^0 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

$$\sqrt{m^2 + \vec{p}'^2} - \sqrt{m^2 + \vec{p}^2} \simeq \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

$$\mathcal{M} =$$

$$(p' - p)^0 = (E(p') - E(p))^2 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

$$\sqrt{m^2 + \vec{p}'^2} - \sqrt{m^2 + \vec{p}^2} \simeq \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

$$\mathcal{M} \simeq -(2m)^2 \frac{p^2}{(\vec{p}' - \vec{p})^2} (\gamma^{54} \gamma^{45} - \gamma^{54} \gamma^{45})$$

$$(p' - p)^0 = (E(p') - E(p))^2 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

$$\sqrt{m^2 + \vec{p}'^2} - \sqrt{m^2 + \vec{p}^2} \simeq \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

$$\mathcal{M} \simeq -(2m)^2 \frac{e^2}{(\vec{p}' - \vec{p})^2} (\gamma^{\mu_1} \gamma^{\mu_2} - \gamma^{\mu_2} \gamma^{\mu_1})$$

$$U \approx -(2m)^2 \frac{e^2}{(\vec{p}' - \vec{p})^2} (\gamma^{5s} \gamma^{4t} - \gamma^{4t} \gamma^{5s})$$

Comparison to Born formula in QM:

$$U_{\text{Born}} = - \int d^3x e^{-i(\vec{p} - \vec{p}') \cdot \vec{x}} V(\vec{x})$$

$$(p' - p)^2 = (E(p') - E(p))^2 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

$$\sqrt{m^2 + \vec{p}'^2} - \sqrt{m^2 + \vec{p}^2} \simeq \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

$$\mathcal{M} \simeq -(2m)^2 \frac{e^2}{(\vec{p}' - \vec{p})^2} (\gamma^{5s} \gamma^{7t} - \gamma^{5t} \gamma^{7s})$$

Comparison to Born formula in QM:

$$\mathcal{M}_{\text{Born}} = - \int d^3x e^{-i(\vec{p} - \vec{p}') \cdot \vec{x}} V(\vec{x})$$

$$(p' - p)^2 = (E(p') - E(p))^2 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

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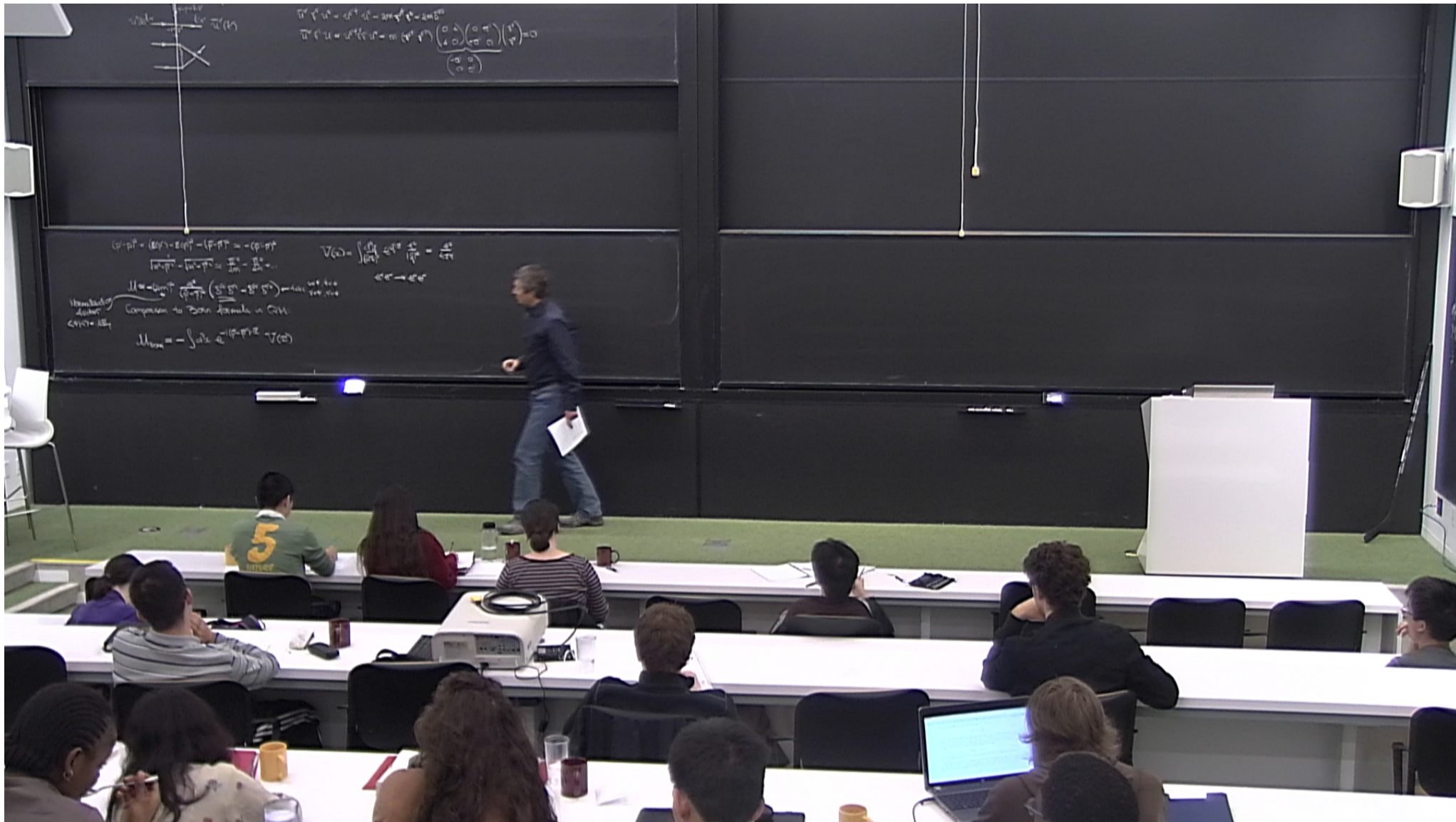
$$\mathcal{M} \simeq -(2m)^2 \frac{e^2}{(\vec{p}' - \vec{p})^2} (\gamma^{s's} \gamma^{t't} - \gamma^{s't} \gamma^{t's})$$

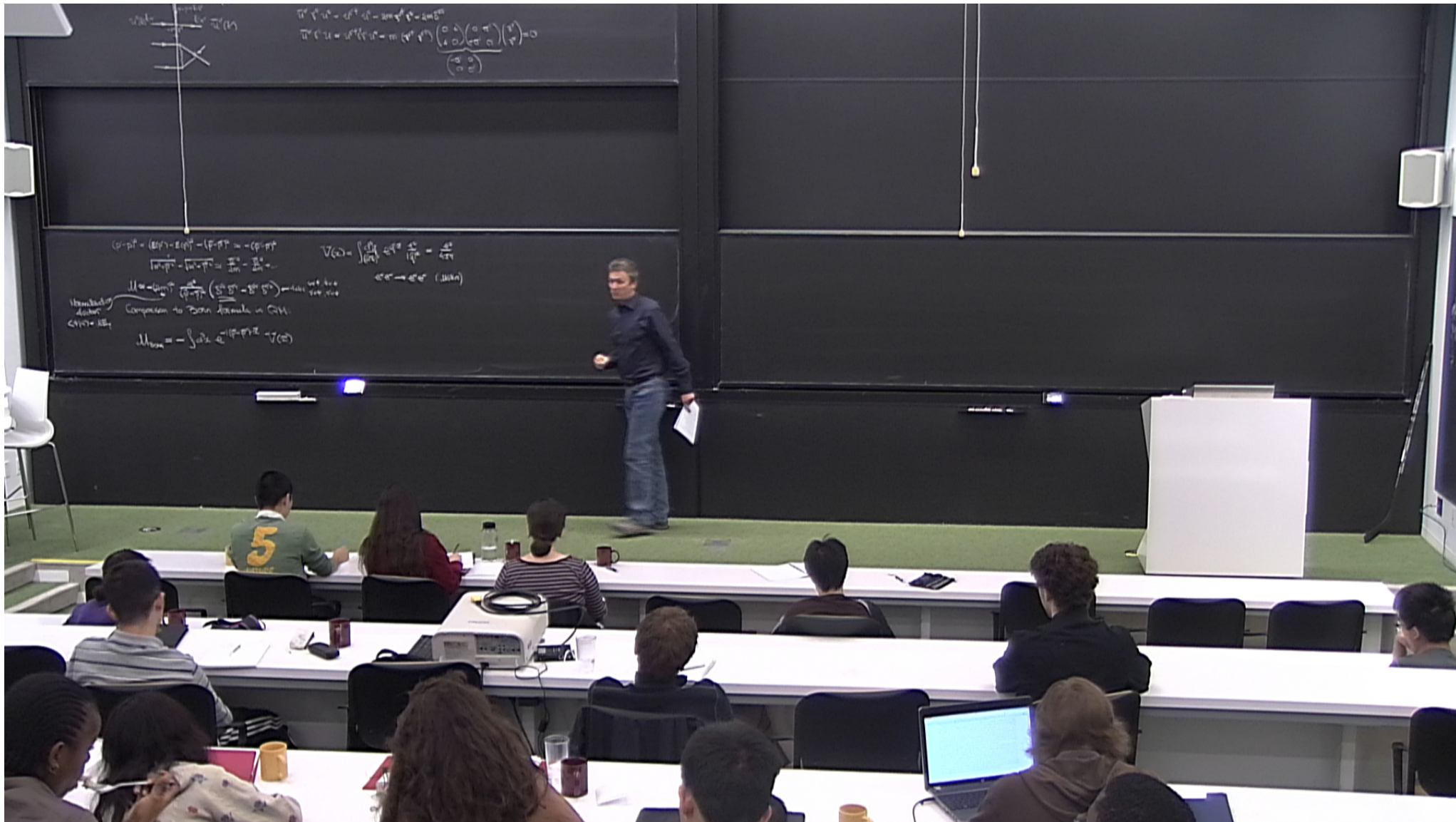
Normalization
factor

$$\langle \psi | \psi \rangle = 2E_p$$

Comparison to Born formula in QM:

$$\mathcal{M}_{\text{Born}} = - \int d^3x e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}} V(\vec{x})$$





$$p)^2 - (\vec{p}' - \vec{p})^2 \simeq -(\vec{p}' - \vec{p})^2$$

$$\sqrt{m^2 + \vec{p}^2} \simeq \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

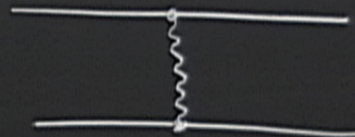
$$\frac{e^2}{(\vec{p}' - \vec{p})^2} (\underline{\underline{\delta^{s's'} \delta^{\eta'\eta'}}}) \leftarrow \text{take } \begin{matrix} s = \uparrow, \eta = \downarrow \\ s' = \uparrow, \eta' = \downarrow \end{matrix}$$

Born form (QM:

$$\int d^3x e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}} V(\vec{x})$$

$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$\underline{e^+ e^- \rightarrow e^+ e^-} \quad (\text{Möller})$$



$$m^2 - (\vec{p} - \vec{p}')^2 \approx -(\vec{p}' - \vec{p})^2$$

$$|\vec{p}' - \vec{p}|^2 \approx \frac{\vec{p}'^2}{2m} - \frac{\vec{p}^2}{2m} + \dots$$

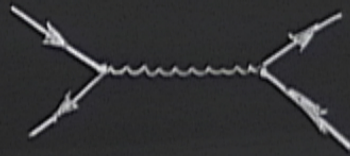
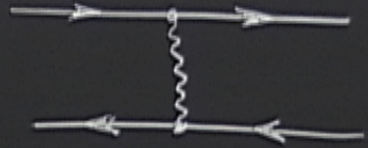
$$\frac{e^2}{(\vec{p}' - \vec{p})^2} (\underline{\underline{\delta^{ss'} \delta^{\eta\eta'}}} - \delta^{s\eta'} \delta^{\eta s'}) \leftarrow \text{take } \begin{matrix} s = \uparrow, \eta = \downarrow \\ s' = \uparrow, \eta' = \downarrow \end{matrix}$$

Born formula in QM:

$$\int d^3x e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}} V(\vec{x})$$

$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{iq \cdot x} \frac{e}{|\vec{q}|^2} = \frac{e}{4\pi r}$$

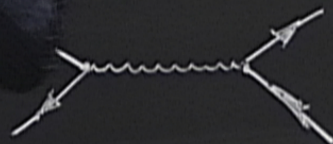
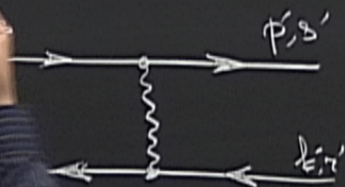
$$\underline{e^+ e^- \rightarrow e^+ e^-} \quad (\text{Möller})$$



$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$(1) i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \frac{1}{i}$$

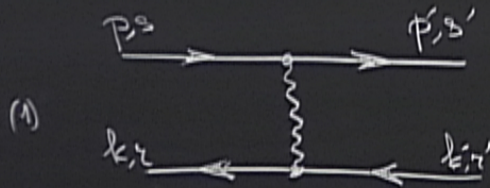
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$$(1): i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma_\mu v^r(k') \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

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$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

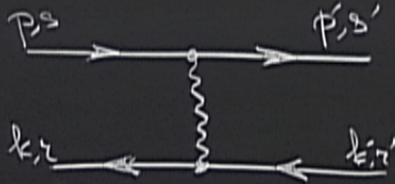
$$(1): i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma_\mu v^r(k) \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

$$\underline{e^+e^- \rightarrow e^+e^-} \quad (\text{Möller})$$

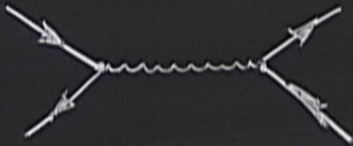
$$r = \frac{1}{r}$$

$$r' = \frac{1}{r'}$$

(1)



(2)



$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$(1): i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma_\mu v^r(k') \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

$e^+e^- \rightarrow$ (Möller)



(2)



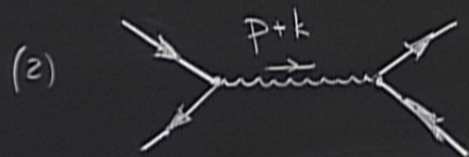
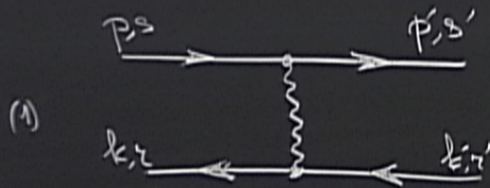
$$V(x) = \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$\underline{e^+ e^- \rightarrow e^+ e^-} \quad (\text{Möller})$$

$$(1): i\mathcal{M} = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma^\nu v^r(k) \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

$$(2): i\mathcal{M} = (-ie)^2 \bar{v}^r(k) \gamma^\mu u^s(p) \bar{u}^s(p') \gamma^\nu v^r(k') \frac{-ig_{\mu\nu}}{(p+k)^2}$$

$$(p+k)^2 = (E(p) + E(k))^2 - (\vec{p} + \vec{k})^2 \simeq 4m^2$$



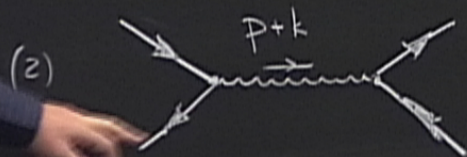
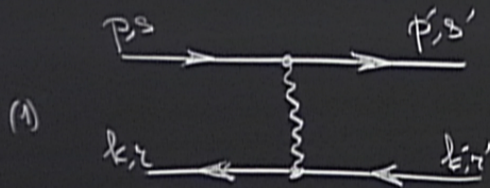
$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$\underline{e^+e^- \rightarrow e^+e^-} \quad (\text{Möller})$$

$$(1): i\mathcal{M}_s = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma^\nu v^r(k) \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

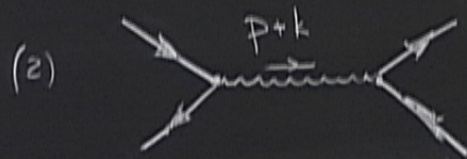
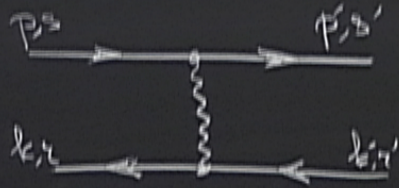
$$(2): i\mathcal{M}_t = (-ie)^2 \bar{v}^r(k) \gamma^\mu u^s(p) \bar{u}^s(p') \gamma^\nu v^r(k') \frac{-ig_{\mu\nu}}{(p+k)^2}$$

$$(p+k)^2 = (E(\vec{p}) + E(\vec{k}))^2 - (\vec{p} + \vec{k})^2 \simeq 4m^2$$



$$V(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi r}$$

$$\underline{e^+e^- \rightarrow e^+e^-} \quad (\text{Möller})$$



$$(1): i\mathcal{M}_s = (-ie)^2 \bar{u}^s(p') \gamma^\mu u^s(p) \bar{v}^r(k) \gamma^\nu v^r(k) \frac{-ig_{\mu\nu}}{(p-p')^2} (-1)$$

$$(2): i\mathcal{M}_a = (-ie)^2 \bar{v}^r(k) \gamma^\mu u^s(p) \bar{u}^s(p') \gamma^\nu v^r(k') \frac{-ig_{\mu\nu}}{(p+k')^2}$$

$$(p+k')^2 = (E(p)+E(k'))^2 - (\vec{p}+\vec{k}')^2 \simeq 4m^2$$

