Title: Quantum Field Theory - Lecture 4A

Date: Oct 12, 2012 09:00 AM

URL: http://pirsa.org/12100007

Abstract:

$$T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - g^{\mu\nu} \mathcal{L}$$

$$\partial_\mu T^{\mu\nu} = 0$$

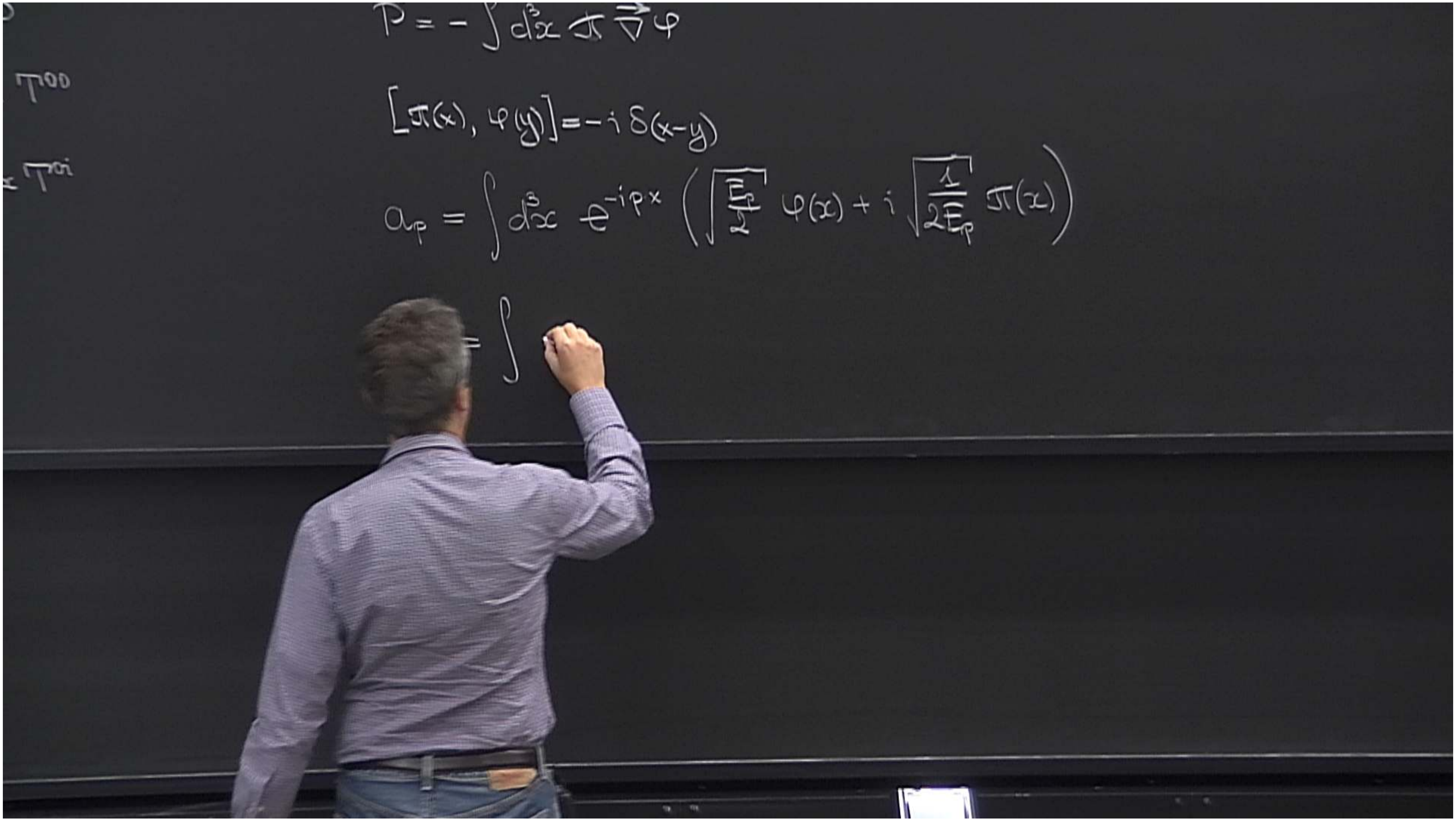
$$E = \int d^3x \, T^{00}$$

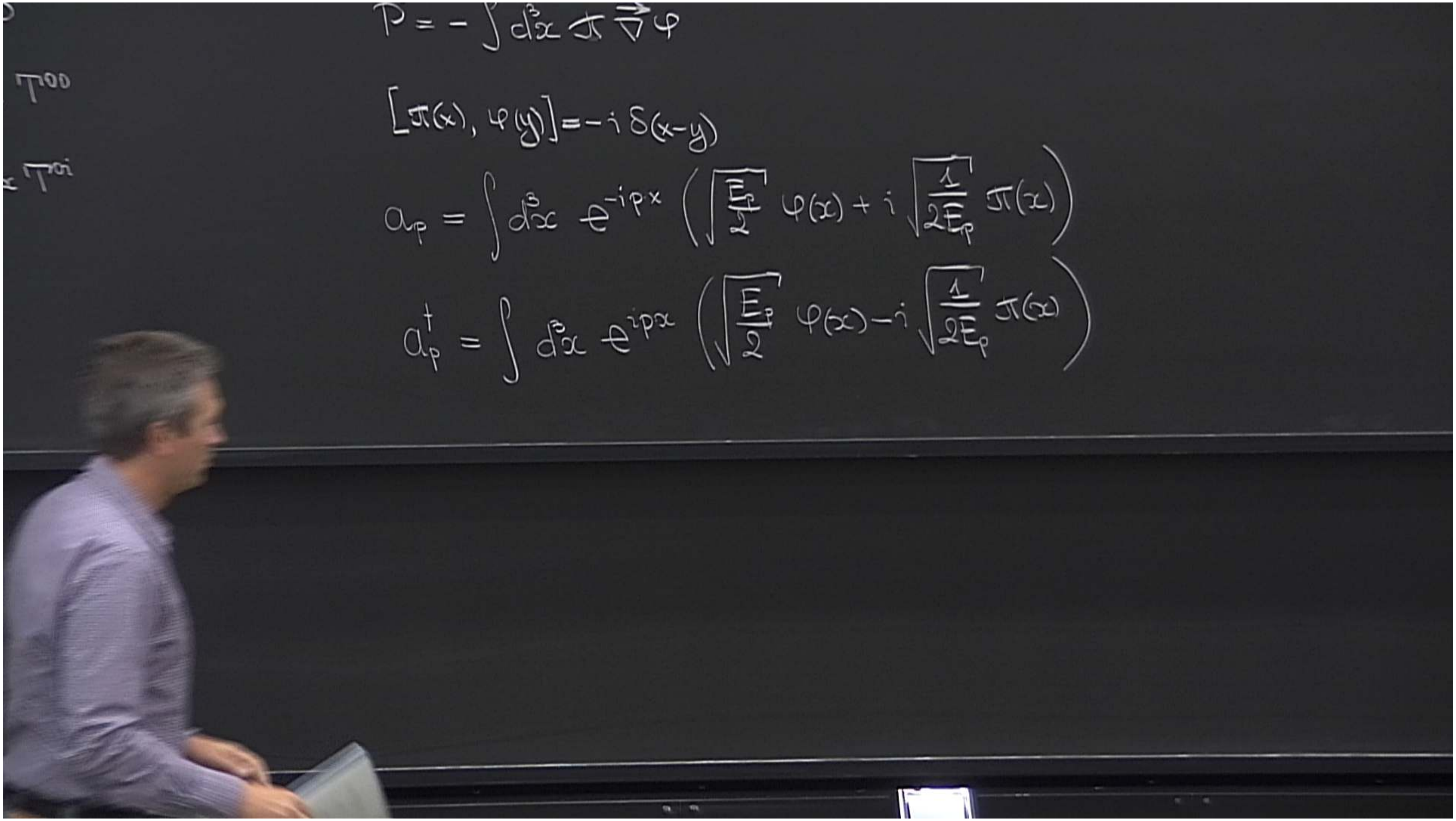
$$P^i = \int d^3x \, T^{0i}$$

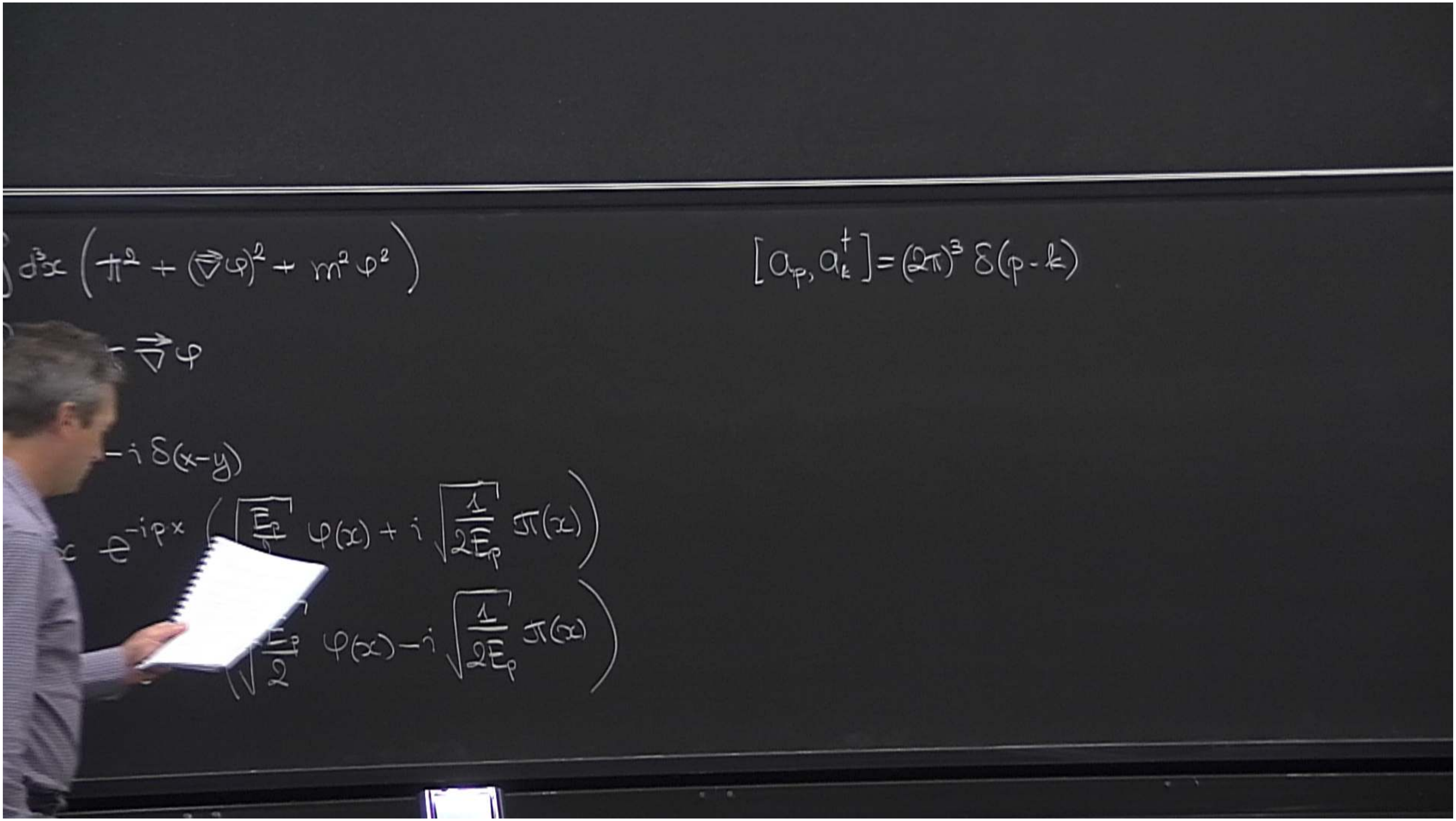
$$H = \frac{1}{2} \int d^3x \left(\pi^2 + (\vec{\nabla}\varphi)^2 + m^2 \varphi^2 \right)$$

$$\vec{P} = - \int d^3x \, \pi \vec{\nabla} \varphi$$

$$[\pi(x), \varphi(y)] = -i \, \delta(x-y)$$


T^{00}
T^{0i}
$P = -\int d^3x\, \pi \vec{\nabla} \varphi$
$[\pi(x), \varphi(y)] = -i\,\delta(x-y)$
$a_p = \int d^3x\, e^{-ipx} \left(\sqrt{\frac{E_p}{2}}\, \varphi(x) + i \sqrt{\frac{1}{2E_p}}\, \pi(x) \right)$
$= \int$


T^{00}
$x\, T^{0i}$
$\vec{P} = -\int d^3x\, \pi\, \vec{\nabla}\varphi$
$[\pi(x), \varphi(y)] = -i\,\delta(x-y)$
$a_p = \int d^3x\, e^{-ip\cdot x}\left(\sqrt{\frac{E_p}{2}}\,\varphi(x) + i\sqrt{\frac{1}{2E_p}}\,\pi(x)\right)$
$a_p^\dagger = \int d^3x\, e^{ipx}\left(\sqrt{\frac{E_p}{2}}\,\varphi(x) - i\sqrt{\frac{1}{2E_p}}\,\pi(x)\right)$


$\int d^3x \left(\pi^2 + (\vec{\nabla}\varphi)^2 + m^2\varphi^2\right)$
$[a_p, a_k^\dagger] = (2\pi)^3\,\delta(p-k)$
$\vec{\nabla}\varphi$
$-i\,\delta(x-y)$
$e^{-ipx}\left(\sqrt{E_p}\,\varphi(x) + i\sqrt{\frac{1}{2E_p}}\,\pi(x)\right)$
$\varphi(x) - i\sqrt{\frac{1}{2E_p}}\,\pi(x)\Big)$

$$\int d^3x \left(\pi^2 + (\vec{\nabla}\varphi)^2 + m^2\varphi^2\right)$$

$$\int d^3x \, \pi \vec{\nabla}\varphi$$

$$\varphi(y)] = -i\,\delta(x-y)$$

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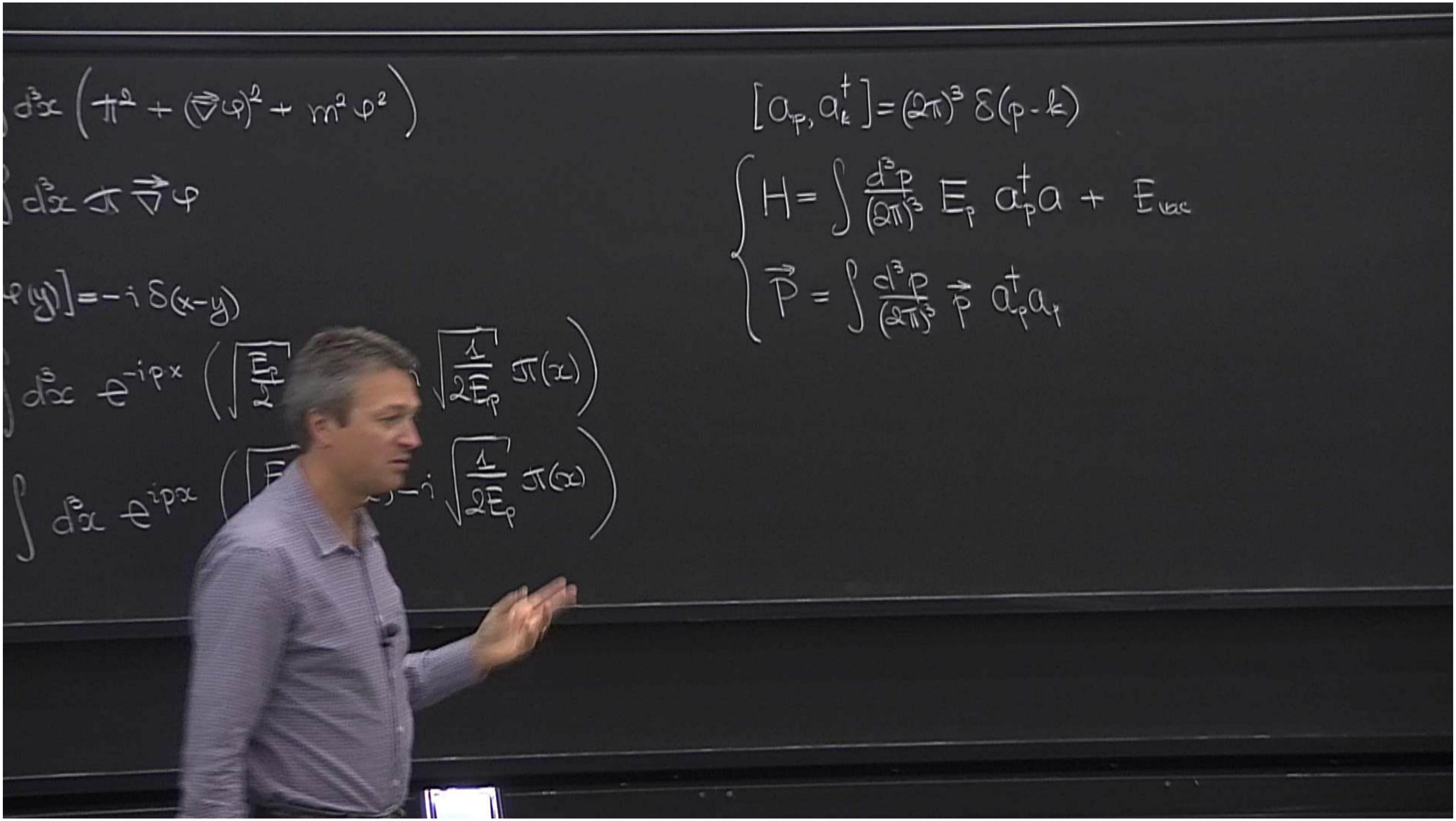
$$[a_p, a_k^\dagger] = (2\pi)^3\,\delta(p-k)$$


$\int d^3x \left(\pi^2 + (\vec{\nabla}\varphi)^2 + m^2\varphi^2\right)$
$\int d^3x \, \pi \vec{\nabla}\varphi$
$\varphi(y)] = -i\,\delta(x-y)$
$[a_p, a_k^\dagger] = (2\pi)^3\,\delta(p-k)$
$H = \int \frac{d^3p}{(2\pi)^3} E_p \, a_p^\dagger a + E_{vac}$
$\vec{P} = \int \frac{d^3p}{(2\pi)^3} \vec{p} \, a_p^\dagger a_p$

$$\int d^3x \left(\pi^2 + (\vec{\nabla}\varphi)^2 + m^2\varphi^2\right)$$

$$\int d^3x\, \pi \vec{\nabla}\varphi$$

$$\varphi(y)] = -i\,\delta(x-y)$$

$$\int d^3x\, e^{-ip\cdot x}\left(\sqrt{\frac{E_p}{2}}\,\varphi(x) + i\sqrt{\frac{1}{2E_p}}\,\pi(x)\right)$$

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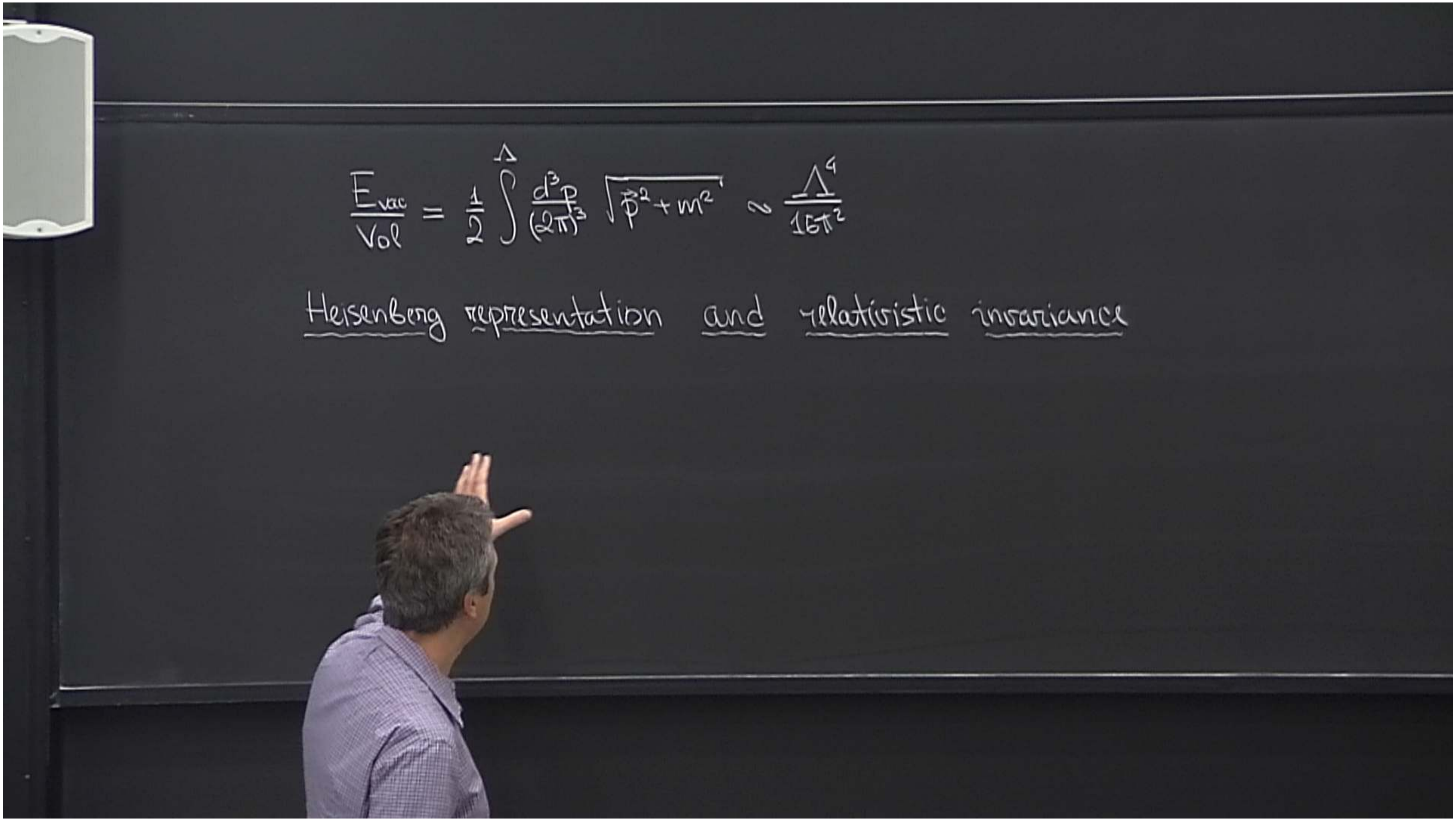
$$[a_p, a_k^\dagger] = (2\pi)^3\,\delta(p-k)$$

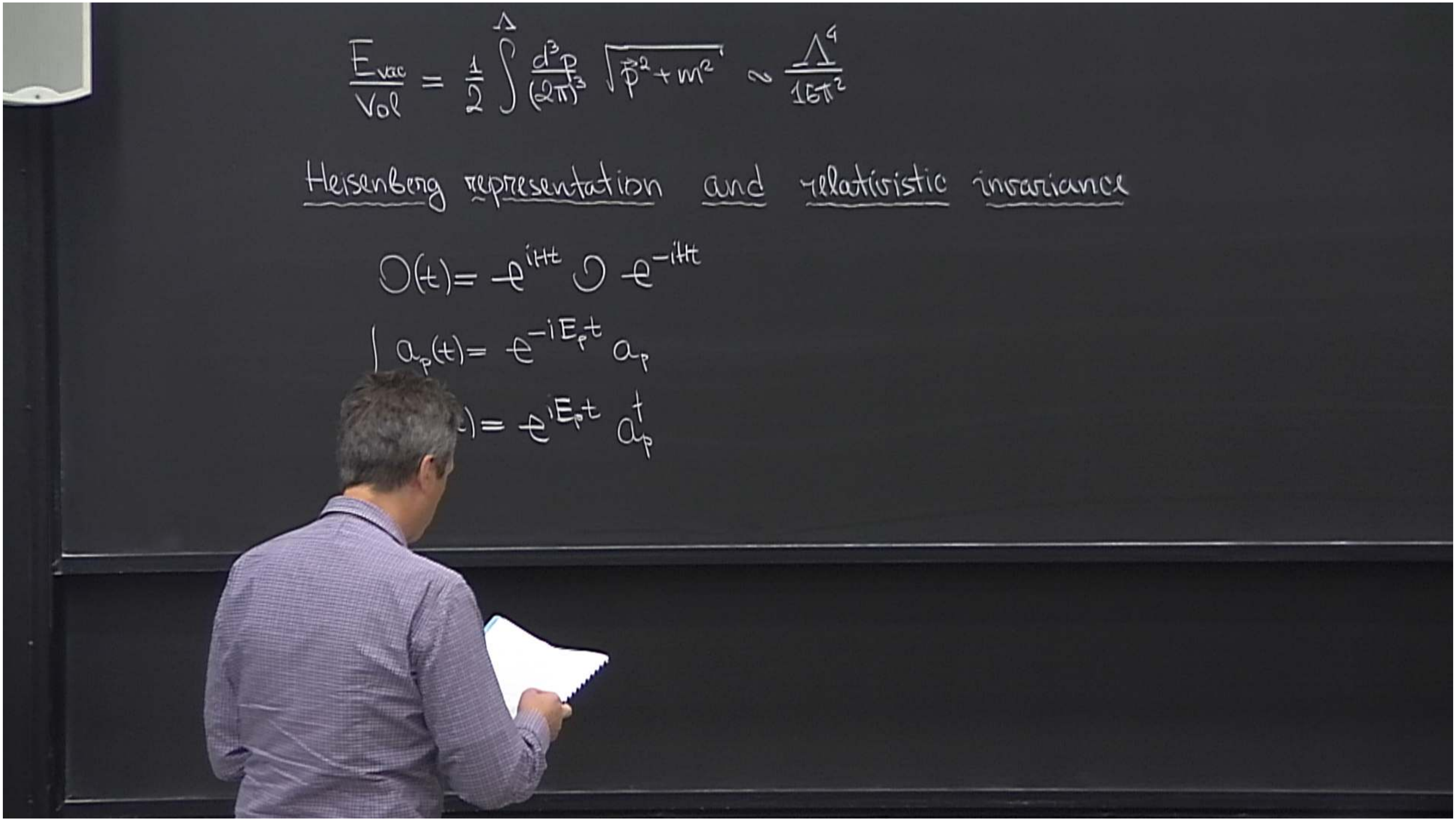
$$\begin{cases} H = \int \frac{d^3p}{(2\pi)^3} E_p\, a_p^\dagger a + E_{vac} \\ \vec{P} = \int \frac{d^3p}{(2\pi)^3}\, \vec{p}\, a_p^\dagger a_p \end{cases}$$

$$E_p = \sqrt{\vec{p}^2 + m^2}$$

$$\frac{E_{vac}}{Vol} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^{\,2}+m^2}$$

$$\frac{E_{vac}}{Vol} = \frac{1}{2}\int^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^{\,2} + m^2} \sim \frac{\Lambda^4}{16\pi^2}$$


$\frac{E_{vac}}{Vol} = \frac{1}{2} \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^2 + m^2} \sim \frac{\Lambda^4}{16\pi^2}$
Heisenberg representation and relativistic invariance

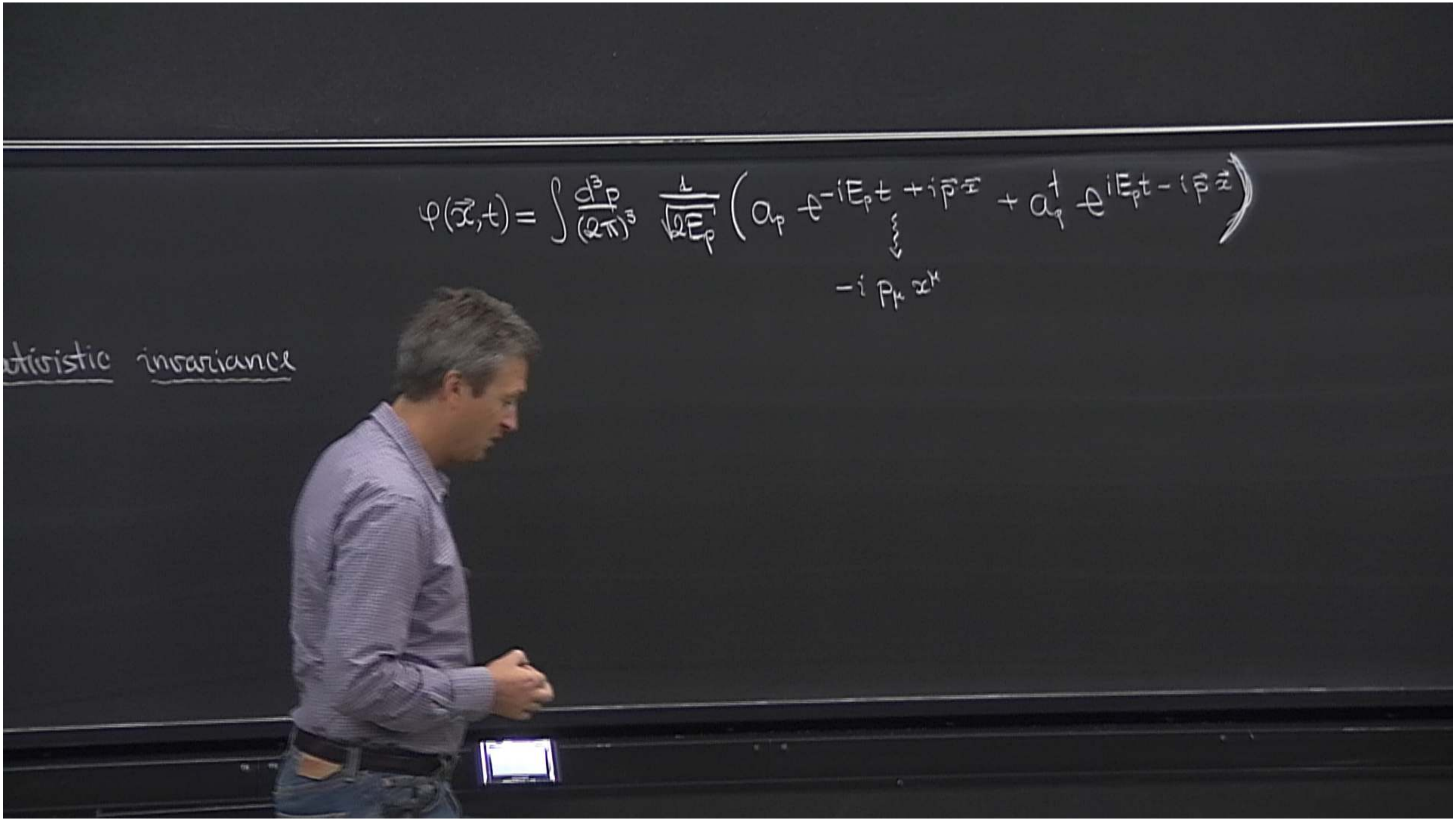

$\frac{E_{vac}}{Vol} = \frac{1}{2}\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+m^2} \sim \frac{\Lambda^4}{16\pi^2}$
Heisenberg representation and relativistic invariance
$O(t) = e^{iHt}\, O\, e^{-iHt}$
$a_p(t) = e^{-iE_pt}\, a_p$
$) = e^{iE_pt}\, a_p^\dagger$

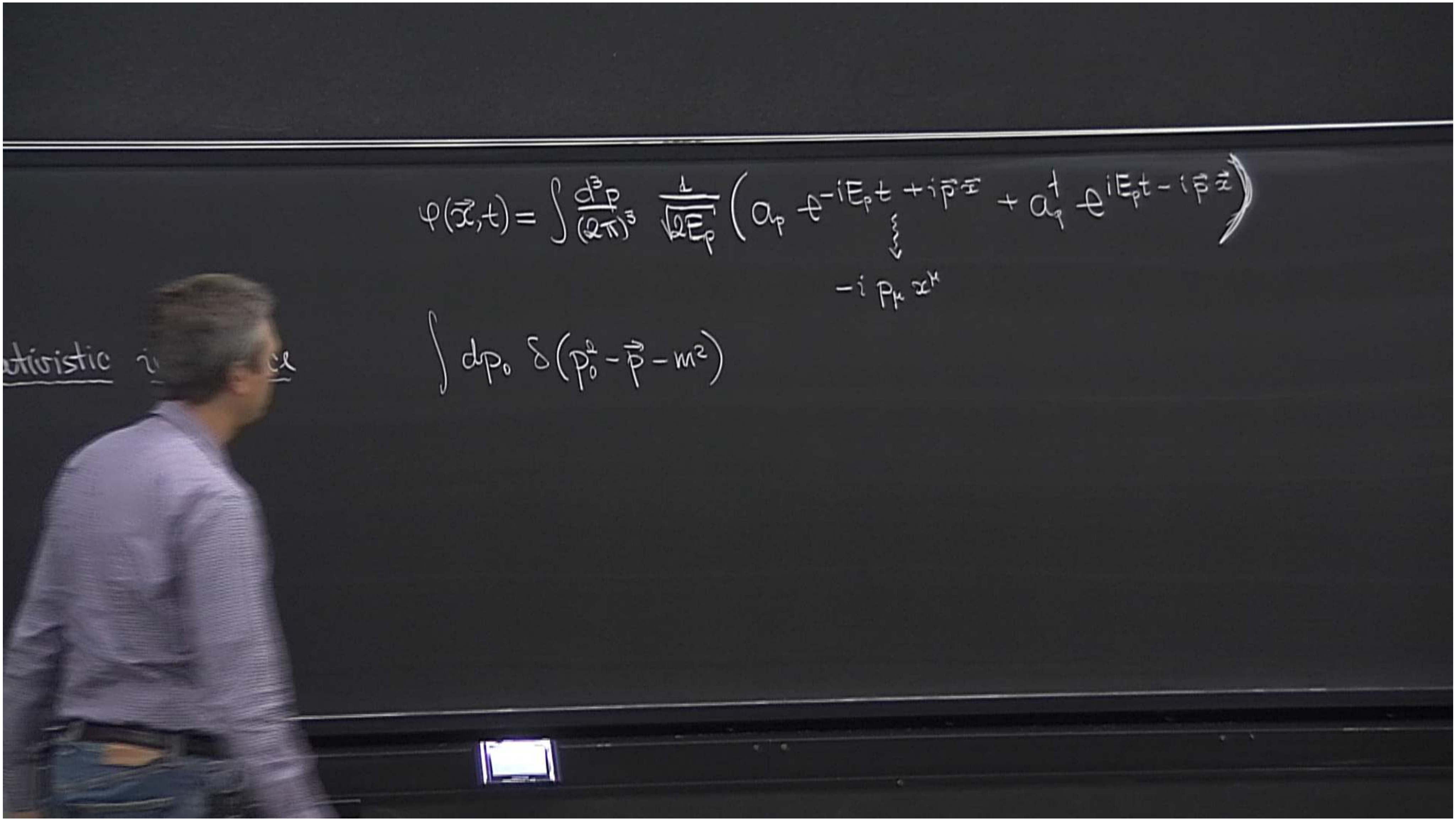
$$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p \, e^{-iE_p t + i\vec{p}\vec{x}} + a_p^\dagger \, e^{iE_p t - i\vec{p}\vec{x}} \right)$$

<u>...tivistic</u> <u>invariance</u>

$$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \Big(a_p\, e^{\underbrace{-iE_p t + i\vec{p}\vec{x}}_{-i\, p_\mu x^\mu}} + a_p^\dagger\, e^{iE_p t - i\vec{p}\vec{x}} \Big)$$

<u>ativistic</u> <u>invariance</u>


$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p e^{-iE_p t + i\vec{p}\vec{x}} + a_p^\dagger e^{iE_p t - i\vec{p}\vec{x}} \right)$
$-i\, p_\mu x^\mu$
ativistic invariance


$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p\, e^{-iE_p t + i\vec{p}\cdot\vec{x}} + a_p^\dagger\, e^{iE_p t - i\vec{p}\cdot\vec{x}} \right)$
$-i\, p_\mu x^\mu$
$\int dp_0\, \delta(p_0^2 - \vec{p} - m^2)$

$$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p e^{-iE_p t + i\vec{p}\vec{x}} + a_p^\dagger e^{iE_p t - i\vec{p}\vec{x}} \right)$$

$$-i\, p_\mu x^\mu$$

...tivistic invariance

$$\int dp_0\, \delta(p_0^2 - \vec{p}^2 - m^2)\, \Theta(p_0)\, f(p_0) = \frac{1}{2E_p} f(E_p)$$

$$\Theta(p_0) = \begin{cases} 1, & p_0 > 0 \\ 0, & p_0 < 0 \end{cases}$$

$$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p\, e^{-iE_p t + i\vec{p}\cdot\vec{x}} + a_p^\dagger\, e^{iE_p t - i\vec{p}\cdot\vec{x}} \right)$$

$$-i\, p_\mu x^\mu$$

tivistic invariance

$$\int dp_0\, \delta\left(p_0^2 - \vec{p}^{\,2} - m^2\right) \Theta(p_0)\, f(p_0) = \frac{1}{2E_p} f(E_p)$$

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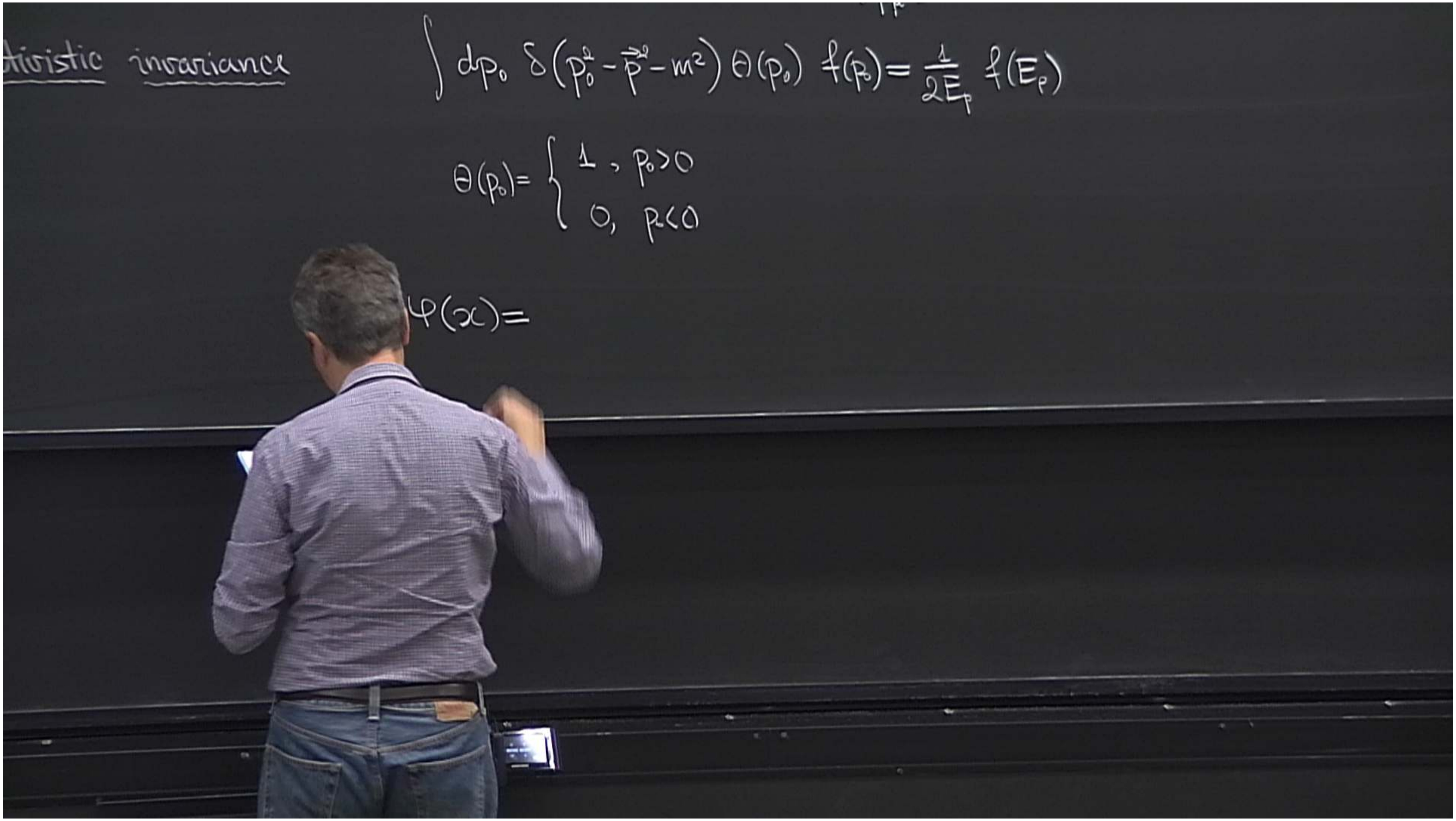
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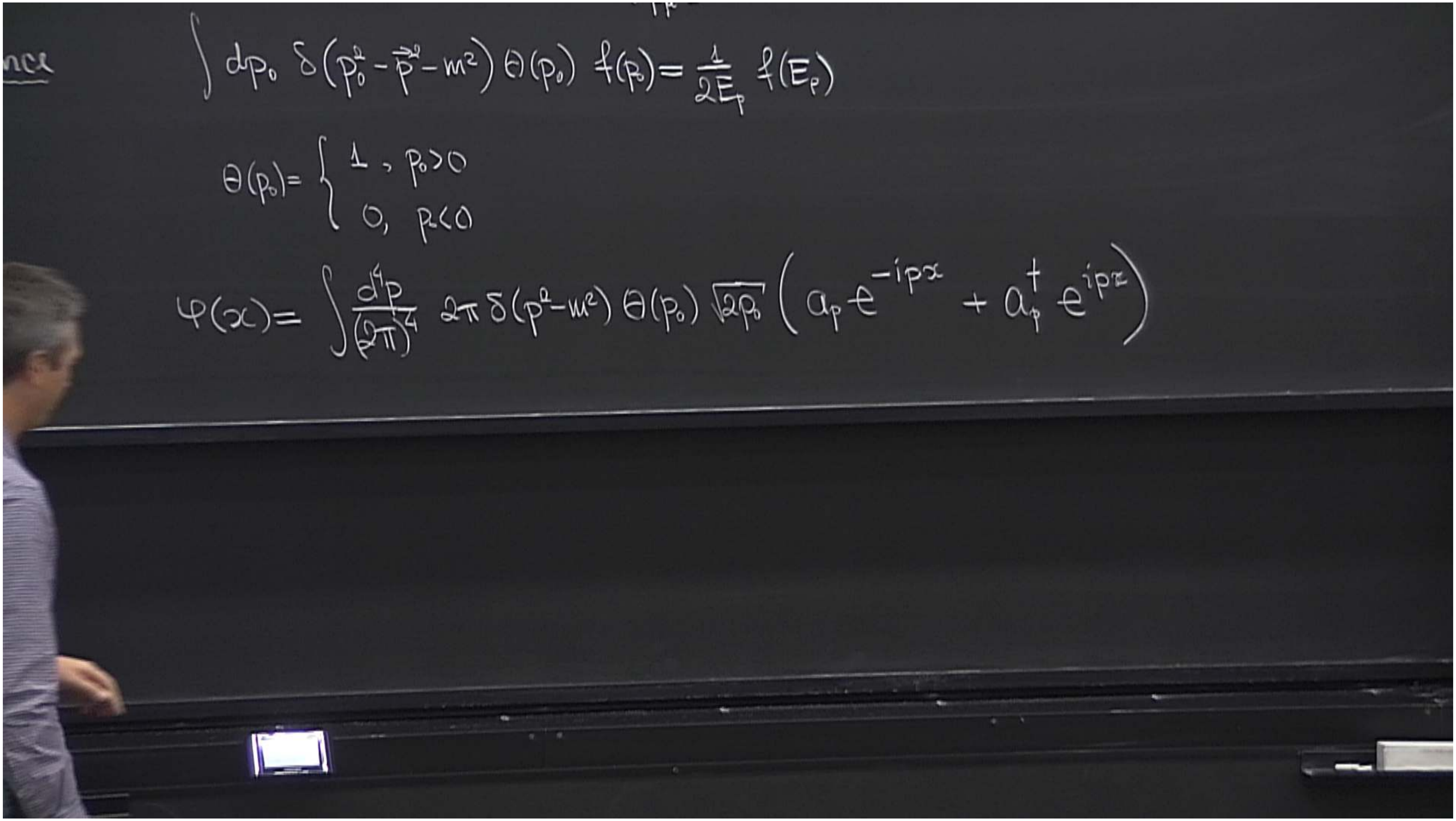
$\varphi(x) =$

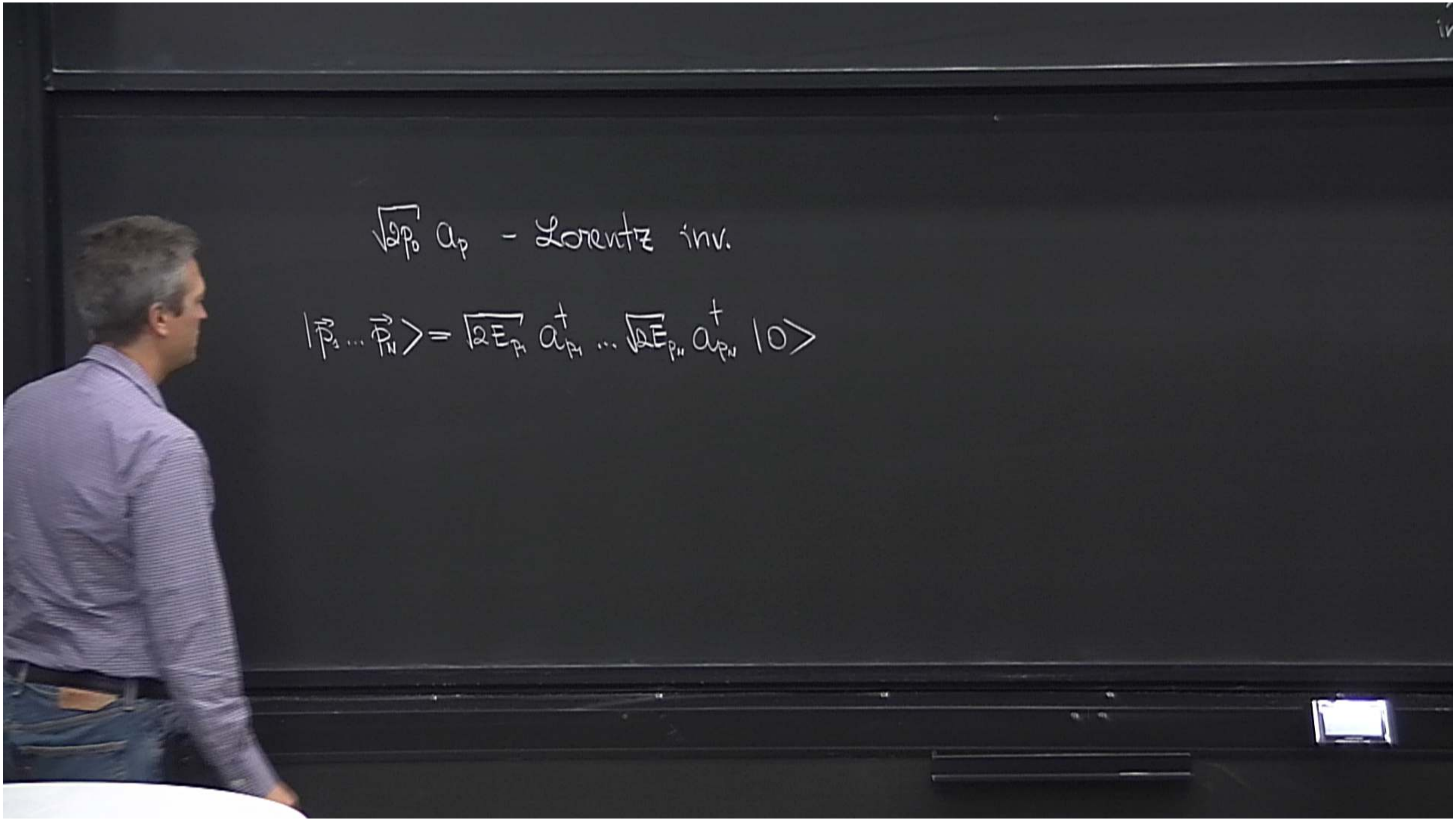

nce
$\int dp_0\, \delta(p_0^2 - \vec{p}^2 - m^2)\, \Theta(p_0)\, f(p_0) = \frac{1}{2E_p} f(E_p)$
$\Theta(p_0) = \begin{cases} 1, & p_0 > 0 \\ 0, & p_0 < 0 \end{cases}$
$\varphi(x) = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\, \delta(p^2 - m^2)\, \Theta(p_0)\, \sqrt{2p_0} \left(a_p e^{-ipx} + a_p^\dagger e^{ipx} \right)$

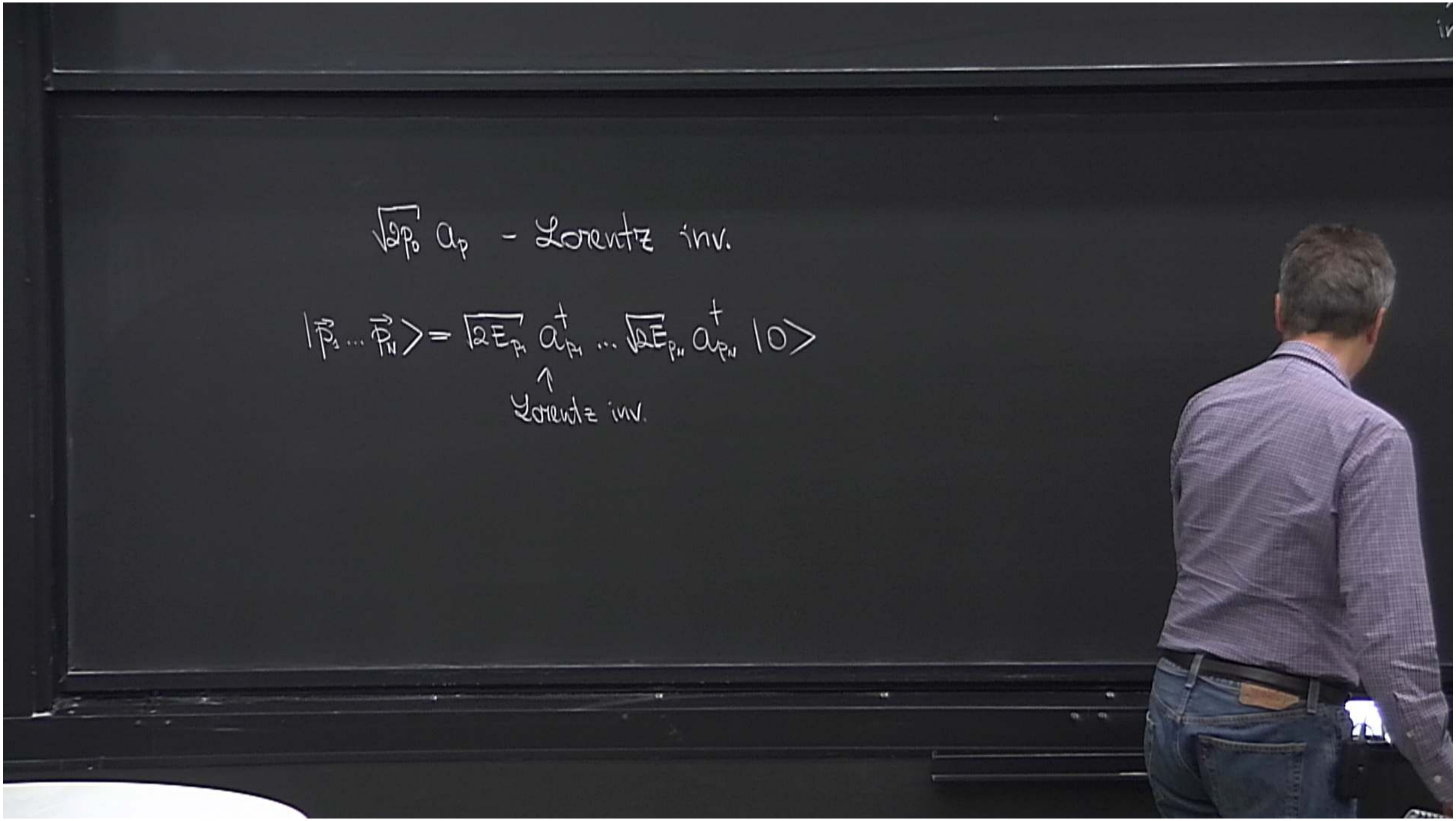
$$\varphi(x) = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\, \delta(p^2 - m^2)\, \theta(p_0)\, \sqrt{2p_0}\, \left(a_p e^{-ipx} + a_p^\dagger e^{ipx} \right)$$

inv.

$$x^\mu = (t, \vec{x})$$

$\sqrt{2p_0}\, a_p$ - Lorentz inv.


$\sqrt{2p_0}\, a_p$ - Lorentz inv.
$|\vec{p}_1 \ldots \vec{p}_N\rangle = \sqrt{2E_{p_1}}\, a^\dagger_{p_1} \ldots \sqrt{2E_{p_N}}\, a^\dagger_{p_N} |0\rangle$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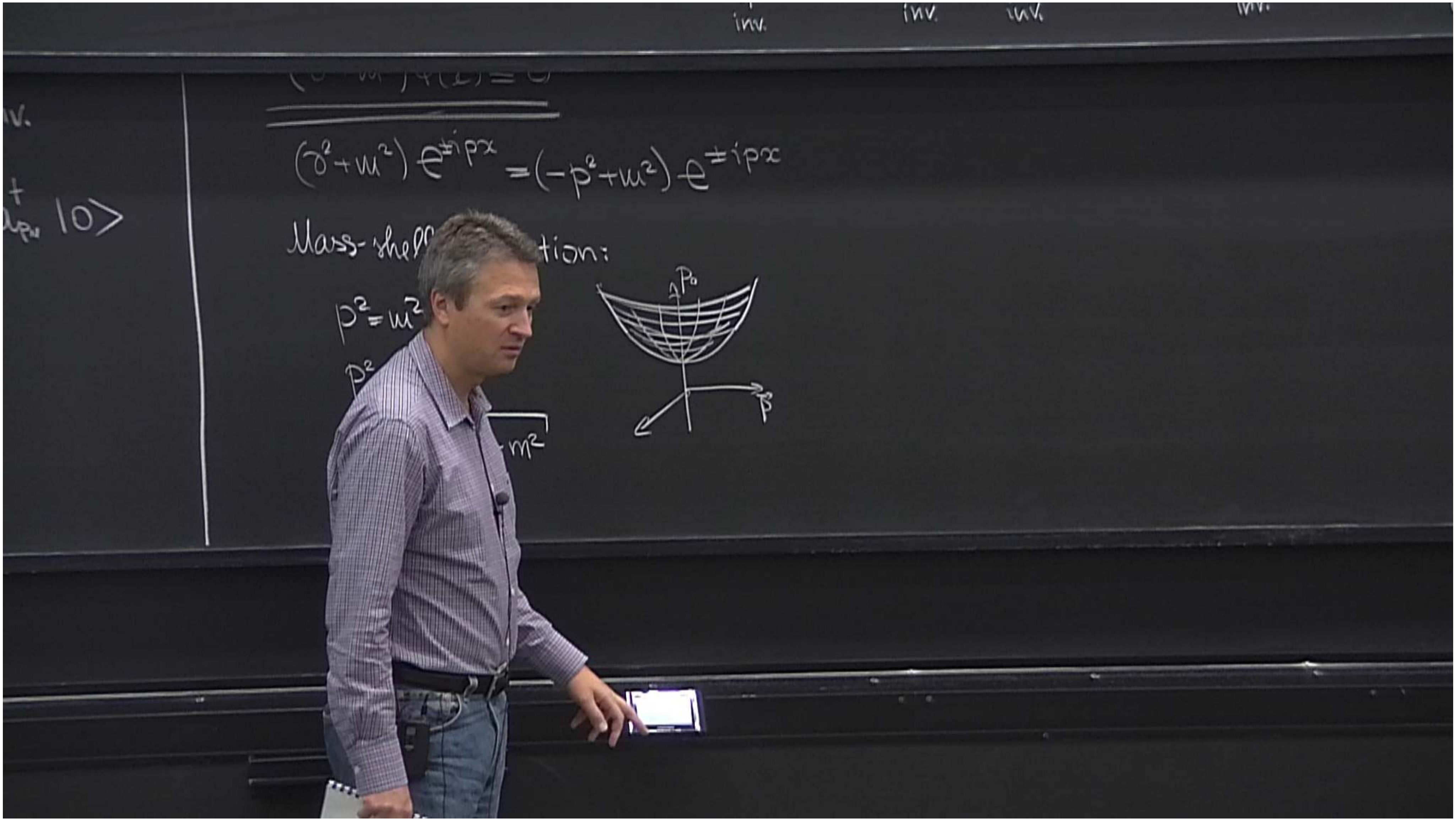

inv.
inv.
inv.
$(\partial^2+m^2)e^{\pm ipx}=(-p^2+m^2)e^{\pm ipx}$
Mass-shell
tion:
$p^2=m^2$
p_0
$\vec{p}$

$$(*)\quad \varphi(x) = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\, \delta(p^2-m^2)\, \Theta(p_0)\, \sqrt{2p_0}\, \left(a_p e^{-ipx} + a_p^\dagger e^{ipx} \right)$$

inv. inv. inv. inv. inv.

$x^\mu = (t, \vec{x})$

$\varphi(x) = 0$

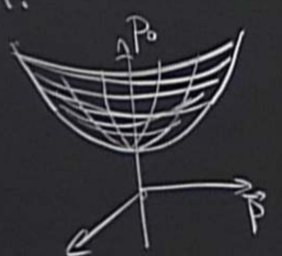
$e^{\pm ipx} = (-p^2+m^2)\, e^{\pm ipx}$

ll condition:

2

$^2 = m^2$

$= \pm\sqrt{\vec{p}^{\,2}+m^2}$



(*) is the general solution of KG equation

amplitudes of partial waves

replaced by ladder operators

$$(*)\quad \varphi(x) = \int \underset{\text{inv.}}{\frac{d^4p}{(2\pi)^4}}\, 2\pi\, \underset{\text{inv.}}{\delta(p^2-m^2)}\, \underset{\text{inv.}}{\Theta(p_0)}\, \sqrt{2p_0}\left(a_p \underset{\text{inv.}}{e^{-ipx}} + a_p^\dagger \underset{\text{inv.}}{e^{ipx}} \right)$$

$x^\mu = (t, \vec{x})$

$\varphi(x) = 0$

$e^{\pm ipx} = (-p^2+m^2)\, e^{\pm ipx}$

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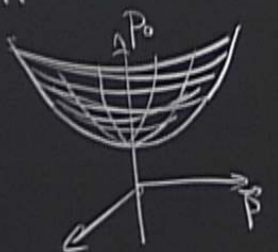

2

$^2 = m^2$

$= \pm\sqrt{\vec{p}^2 + m^2}$

(*) is the general solution of KG equation

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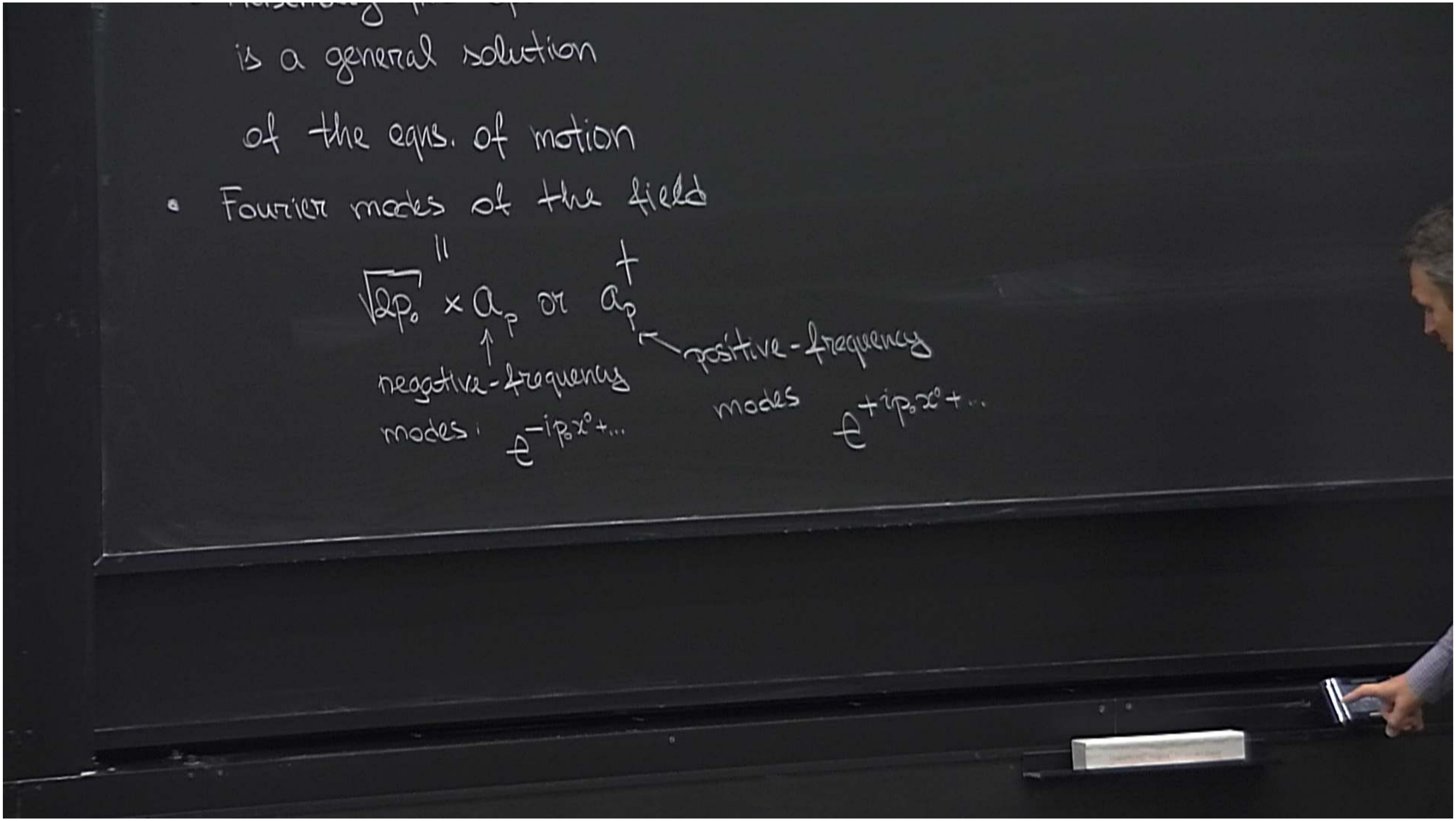
is a general solution
of the eqns. of motion
• Fourier modes of the field
$\sqrt{2p_0} \times a_p$ or $a_p^\dagger$
negative-frequency modes: $e^{-ip_0x^0+\dots}$
positive-frequency modes $e^{+ip_0x^0+\dots}$

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- Fourier modes of the field

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negative-frequency modes: $e^{-ip_0x^0+\dots}$

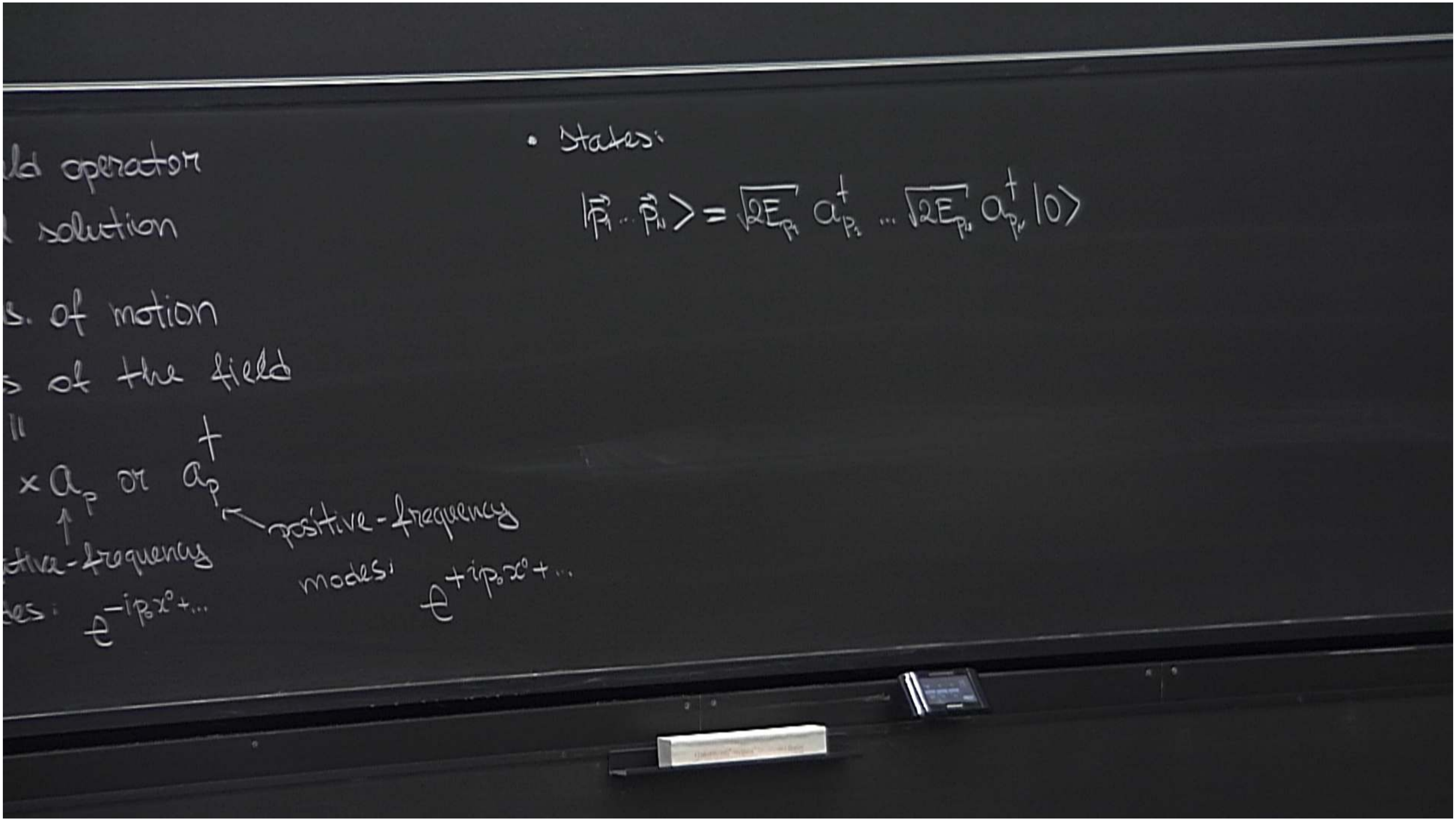
positive-frequency modes $e^{+ip_0x^0+\dots}$

- Heisenberg field operator is a general solution of the eqns. of motion
- Fourier modes of the field

$$\sqrt{2p_0} \times a_p \text{ or } a_p^\dagger$$

a_p: negative-frequency modes: $e^{-ip_0 x^0 + \dots}$

$a_p^\dagger$: positive-frequency modes $e^{+ip_0 x^0 + \dots}$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